THE IMPRIMITIVE SUBSTITUTION GROUPS OF DEGREE EIGHTEEN WHOSE SIX SYSTEMS OF IMPRIMITIVITY ARE PERMUTED ACCORDING TO THE PRIMITIVE GROUPS OF DEGREE SIX.

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by

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The symbols $a_1, a_2, a_3; b_1, b_2, b_3; c_1, c_2, c_3; d_1, d_2, d_3; e_1, e_2, e_3; f_1, f_2, f_3$ will be used to represent the elements of the six systems of imprimitivity of the groups to be determined, and H_h will represent a head of order h.

The following intransitive groups of degree eighteen having six systems of intransitivity may be used as heads of the imprimitive groups to be constructed.

		Order.
$H_{2^6\cdot3^6} =$	$\{(a_1a_2a_3)\text{all}(b_1b_2b_3)\text{all}(c_1c_2c_3)\text{all}(d_1d_2d_3)\text{all}(e_1e_2e_3)\text{all}(f_1f_2f_3)\text{all}\}$	46,656
$H_{2^5\cdot3^6} =$	$\{(a_1a_2a_3)\text{all}(b_1b_2b_3)\text{all}(c_1c_2c_3)\text{all}(d_1d_2d_3)\text{all}(e_1e_2e_3)\text{all}(f_1f_2f_3)\text{all}\}$ positive	23,328
$H_{2\cdot3^6} =$	$\{(a_1a_2a_3)\text{all}(b_1b_2b_3)\text{all}(c_1c_2c_3)\text{all}(d_1d_2d_3)\text{all}(e_1e_2e_3)\text{all}(f_1f_2f_3)\text{all}\}_{3-3-3-3-3-3}$	1,458
$H_{3^6} =$	$\{(a_1a_2a_3), (b_1b_2b_3), (c_1c_2c_3), (d_1d_2d_3), (e_1e_2e_3), (f_1f_2f_3)\}$	729
$H_{2\cdot3^5} =$	$\{(a_1a_2a_3\cdot b_1b_3b_2), (b_1b_2b_3\cdot c_1c_3c_2), (c_1c_2c_3\cdot d_1d_3d_2), (d_1d_2d_3\cdot e_1e_3e_2), (e_1e_2e_3\cdot f_1f_3f_2), (a_1a_2\cdot b_1b_2\cdot c_1c_2\cdot d_1d_2\cdot e_1e_2\cdot f_1f_2)\}$ ..	486
$H_{3^5} =$	$\{(a_1a_2a_3\cdot b_1b_3b_2), (b_1b_2b_3\cdot c_1c_3c_2), (c_1c_2c_3\cdot d_1d_3d_2), (d_1d_2d_3\cdot e_1e_3e_2), (e_1e_2e_3\cdot f_1f_3f_2)\}$	243
$H_{2\cdot3} =$	$\{(a_1a_2a_3)\text{all}(b_1b_2b_3)\text{all}(c_1c_2c_3)\text{all}(d_1d_2d_3)\text{all}(e_1e_2e_3)\text{all}(f_1f_2f_3)\text{all}\}_{1-1-1-1-1-1}$	6
$H_3 =$	$\{(a_1a_2a_3), (b_1b_2b_3), (c_1c_2c_3), (d_1d_2d_3), (e_1e_2e_3), (f_1f_2f_3)\}_{1-1-1-1-1-1}$	3.

The identity can not occur as a head for these groups.

In determining the imprimitive groups containing the above heads the following notation will be used:-

$S_1 = a_1 a_2 a_3$, $S_2 = b_1 b_2 b_3$, $S_3 = c_1 c_2 c_3$, $S_4 = d_1 d_2 d_3$, $S_5 = e_1 e_2 e_3$, $S_6 = f_1 f_2 f_3$.
$s_1 = a_1 a_2$, $s_2 = b_1 b_2$, $s_3 = c_1 c_2$, $s_4 = d_1 d_2$, $s_5 = e_1 e_2$, $s_6 = f_1 f_2$.
1,2,3,4,5,6 stand for the systems a_1, a_2, a_3; b_1, b_2, b_3; c_1, c_2, c_3; d_1, d_2, d_3; e_1, e_2, e_3; f_1, f_2, f_3, respectively.

$\{12345,\ 12\cdot46\} \equiv P_{60}$, $\{12345,\ 1426\} \equiv P_{120}$, $\{12345,\ 126\} \equiv P_{360}$, and $\{12345,\ 16\} \equiv P_{720}$ stand for the primitive groups of degree six and of orders 60, 120, 360, and 720 respectively; and the systems of imprimitivity are permuted according to these groups.

p_i stands for any substitution in P_h.

t_i stands for a substitution which permutes in the simplest way according to p_i; for example;-

$t_{12345} = a_1 b_1 c_1 d_1 e_1 \cdot a_2 b_2 c_2 d_2 e_2 \cdot a_3 b_3 c_3 d_3 e_3$

$t_{12\cdot46} = a_1 b_1 \cdot a_2 b_2 \cdot a_3 b_3 \cdot d_1 f_1 \cdot d_2 f_2 \cdot d_3 f_3$.

The usual methods for determining imprimitive groups will be employed, and the construction of the groups containing only a certain number of the heads will be given. Then will follow a list of the groups containing all the heads.

The Imprimitive Groups which contain $H_{2^6 \cdot 3^6}$.

The largest group which transforms $H_{2^6 \cdot 3^6}$ into itself without interchanging the systems of imprimitivity is $H_{2^6 \cdot 3^6}$ and there are exactly $2^6 \cdot 3^6$ substitutions which transform $H_{2^6 \cdot 3^6}$ into itself and which interchange the systems of imprimitivity according to each substitution of P_h. It follows that there are just four groups containing this head, namely:-

$\{H_{2^6 \cdot 3^6},\ t_{12345},\ t_{12 \cdot 46}\}$	...... order	.....$2^6 \cdot 3^6 \cdot 60$
$\{H_{2^6 \cdot 3^6},\ t_{12345},\ t_{1426}\}$	...... "	.... $2^6 \cdot 3^6 \cdot 120$
$\{H_{2^6 \cdot 3^6},\ t_{12345},\ t_{126}\}$	...... "	.... $2^6 \cdot 3^6 \cdot 360$
$\{H_{2^6 \cdot 3^6},\ t_{12345},\ t_{16}\}$	...... "	.... $2^6 \cdot 3^6 \cdot 720$.

The Imprimitive Groups which contain $H_{2^5 \cdot 3^6}$.

The largest group which transforms $H_{2^5 \cdot 3^6}$ into itself without interchanging its systems of imprimitivity is $\{H_{2^5 \cdot 3^6}, s_1\}$. There are then exactly two sets of $2^5 \cdot 3^6$ substitutions which transform $H_{2^5 \cdot 3^6}$ into itself and which interchange the systems according to p_i, and these sets are $H_{2^5 \cdot 3^6} t_i$ and $H_{2^5 \cdot 3^6} s_1 t_i$.

As i equals 12345 for each P_h, this substitution can now be treated conclusiviely for $H_{2^5 \cdot 3^6}$. Of the two sets, $H_{2^5 \cdot 3^6} t_{12345}$ and $H_{2^5 \cdot 3^6}\ s_1 t_{12345}$, only the former can be used, since $H_{2^5 \cdot 3^6} s_1 t_{12345}$ consists of negative substitutions, and since their fifth powers are not in $H_{2^5 \cdot 3^6}$.

When P_h equals P_{60}, more than half the operators in P_h are positive and as $H_{2^5 \cdot 3^6}$ contains only positive substitutions, it follows that all the substitutions in any imprimitive group to be constructed must be positive. Therefore the set $H_{2^5 \cdot 3^6}\, s_1 t_{12\cdot 46}$ consisting of negative substitutions, is eliminated. t_{12345} and $t_{12\cdot 46}$ generate a group of order sixty which is simply isomorphic to P_{60}. It follows that t_{12345} and $t_{12\cdot 46}$ with $H_{2^5 \cdot 3^6}$ generate an imprimitive group containing the given head, namely:-

$\{H_{2^5 \cdot 3^6},\ t_{12345},\ t_{12\cdot 46}\}$ ...... order.... $2^5 \cdot 3^6 \cdot 60$

When P_h equals P_{120}, since P_{60} is an invariant subgroup of P_{120}, it follows that the substitutions which permute according to p_i, i being 1426, must transfotm $\{H_{2^5 \cdot 3^6}, t_{12345}, t_{12\cdot 46}\}$ into itself, in addition to transforming $H_{2^5 \cdot 3^6}$ into itself and having their fourth powers in $H_{2^5 \cdot 3^6}$. Therefore these equations-

$$(t_{1426})^{-1}\ H_{2^5 \cdot 3^6}\ t_{1426} = H_{2^5 \cdot 3^6}$$
$$(t_{1426})^{-1}\ t_{12345}\ t_{1426} = t_{25463}$$
$$(t_{1426})^{-1}\ t_{12\cdot 46}\ t_{1426} = t_{12\cdot 46}$$
$$(s_1 t_{1426})^{-1}\ H_{2^5 \cdot 3^6}\ s_1 t_{1426} = H_{2^5 \cdot 3^6}$$
$$(s_1 t_{1426})^{-1}\ t_{12345}\ s_1 t_{1426} = s_4 s_5 t_{25463}$$
$$(s_1 t_{1426})^{-1}\ t_{12\cdot 46}\ s_1 t_{1426} = s_4 s_6 t_{12\cdot 46}$$

show the existence of the two groups:-

$\{H_{2^5 \cdot 3^6},\ t_{12345},\ t_{1426}\}$ ......order....$2^5 \cdot 3^6 \cdot 120$

$\{H_{2^5 \cdot 3^6},\ t_{12345},\ s_1 t_{1426}\}$ ...... " ...$2^5 \cdot 3^6 \cdot 120$.

When P_h equals P_{360}, and i equals 126, $H_{2^5 \cdot 3^6}\, s_1 t_{126}$ consists of negative substitutions and their third powers are not in $H_{2^5 \cdot 3^6}$. Therefore only the set $H_{2^5 \cdot 3^6}\, t_{126}$ can be used, and there exists the group:-

$\{H_{2^5\cdot 3^6}\ ,\ t_{12345}\ ,\ t_{126}\}$ .....order .... $2^5\cdot 3^6\cdot 360$.

When P_h equals P_{720}, since P_{360} is an invariant subgroup of P_{720}, it follows that the substitutions which permute according to p_i, i being 16, must transform $\{H_{2^5\cdot 3^6},\ t_{12345}, t_{126}\}$ into itself, in addition to transforming $H_{2^5\cdot 3^6}$ into itself and having their squares in $H_{2^5\cdot 3^6}$. As both t_{16}, and $s_1 t_{16}$ fulfill these conditions the following two groups are listed:-

$\{H_{2^5\cdot 3^6}, t_{12345}, t_{16}\}$ .....order....$2^5\cdot 3^6\cdot 720$

$\{H_{2^5\cdot 3^6}, t_{12345}, s_1 t_{16}\}$ ..... " ....$2^5\cdot 3^6\cdot 720$.

The Imprimitive Groups which contain $H_{2\cdot 3^6}$.

The largest group which transforms $H_{2\cdot 3^6}$ into itself without interchanging its systems of imprimitivity is $\{H_{2\cdot 3^6}, s_1 s_2 s_3 s_4 s_5\}$. There are then thirty-two and just thirty-two sets of $2\cdot 3^6$ substitutions which transform $H_{2\cdot 3^6}$ into itself and interchange the systems according to p_i. These sets are $\{H_{2\cdot 3^6},\ s_1 s_2 s_3 s_4 s_5\} t_i$.

When i equals 12345, since the fifth power of each substitution of $\{s_1 s_2 s_3 s_4 s_5\} t_{12345}$ must be in the head, the sixteen sets that contain odd substitutions can not be used. The remaining sixteen even sets are all conjugate as shown by the equation:-

$$(s_1^{\alpha_1} s_2^{\alpha_2} s_3^{\alpha_3} s_4^{\alpha_4} s_5^{\alpha_5})^{-1}\, t_{12345}\, s_1^{\alpha_1} s_2^{\alpha_2} s_3^{\alpha_3} s_4^{\alpha_4} s_5^{\alpha_5} = s_1^{\alpha_1-\alpha_2} s_2^{\alpha_2-\alpha_3} s_3^{\alpha_3-\alpha_4} s_4^{\alpha_4-\alpha_5} s_5^{\alpha_5-\alpha_1} t_{12345}$$

(where $\alpha_i = 0$ or 1.) It is evident that the exponents may be so chosen that all of the sets will be included.

When P_h equals P_{60}, i equals 12345 and 12·46. Of the thirty-two sets $\{s_1 s_2 s_3 s_4 s_5\}\, t_{12\cdot 46}$ the negative sets are excluded as proven for P_{60} and $H_{2^5\cdot 3^6}$, because $H_{2\cdot 3^6}$ also contains only positive substitutions. Of the sixteen even sets just four have their squares in the head;namely:-

$$t_{12\cdot 46},\quad s_1 s_2 t_{12\cdot 46},\quad s_3 s_5 t_{12\cdot 46},\quad s_1 s_2 s_3 s_5 t_{12\cdot 46}.$$

Now, $(s_1 s_2 s_3 s_4 s_5)^{-1}\ H_{2\cdot 3^6}\ s_1 s_2 s_3 s_4 s_5 = H_{2\cdot 3^6}$

$$(s_1 s_2 s_3 s_4 s_5)^{-1}\ t_{12345} s_1 s_2 s_3 s_4 s_5 = t_{12345}$$

$$(s_1 s_2 s_3 s_4 s_5)^{-1}\ t_{12\cdot 46} s_1 s_2 s_3 s_4 s_5 = s_4 s_6 t_{12\cdot 46} = s_1 s_2 s_3 s_5 t_{12\cdot 46}$$

$$(s_1 s_2 s_3 s_4 s_5)^{-1} s_1 s_2 t_{12\cdot 46} s_1 s_2 s_3 s_4 s_5 = s_1 s_2 s_4 s_6 t_{12\cdot 46}$$

$$s_3 s_5 t_{12\cdot 46}.$$

Hence there remains $t_{12\cdot 46}$ and $s_1 s_2 t_{12\cdot 46}$.

That t_{12345} and $s_1 s_2 t_{12\cdot 46}$ generate a group of order greater than sixty follows from the fact that the product $s_1 s_2 t_{12\cdot 46} t_{12345}$ equals $s_1 s_2 t_{13465}$ which is a substitution of order ten, and therefore could not occur in a group simply isomorphic to P_{60}; But it may be verified by the usual methods for group construction that t_{12345} and $s_3 s_4 s_5 s_6 t_{12\cdot 46}$ do generate a group of order sixty which is simply isomorphic to P_{60}; so also do t_{12345} and $t_{12\cdot 46}$, and there are the two groups, namely:-

$\{H_{2\cdot 3^6}, t_{12345}, t_{12\cdot 46}\}$ .....order.... $2\cdot 3^6\cdot 60$

$\{H_{2\cdot 3^6}, t_{12345}, s_3 s_4 s_5 s_6 t_{12\cdot 46}\}$ .....order.... $2\cdot 3^6\cdot 60$.

These two groups of order 87,480 are distinct abstract groups for each contains just one invariant subgroup of half its order which would correspond in any simple isomorphism established between the two groups. Besides the invariant operators of these subgroups would also correspond. But the subgroup $\{H_{3^6}, t_{12345}, t_{12\cdot 46}\}$ has three invariant operators, while just the identity is invariant in $\{H_{3^6}, t_{12345}, s_3 s_4 s_5 s_6 t_{12\cdot 46}\}$; therefore the two groups are different.

When P_h equals P_{120}, it has been proved that any substitution corresponding to p_i must transform $\{H_{2\cdot3^6}, t_{12345}, t_{12\cdot46}\}$ and $\{H_{2\cdot3^6}, t_{12345}, s_3 s_4 s_5 s_6 t_{1246}\}$ into themselves as well as transform $H_{2\cdot3^6}$ into itself. Of the thirty-two sets $\{s_1 s_2 s_3 s_4 s_5\} t_{1426}$ only t_{1426} fulfills these requirements, therefore the following groups exist, namely:-

$\{H_{2\cdot3^6}, t_{12345}, t_{1426}\}$ .....order .....$2\cdot3^6\cdot120$

$\{H_{2\cdot3^6}, t_{12345}, s_3 s_4 s_5 s_6 t_{12\cdot46}, t_{1426}\}$.... " .....$2\cdot3^6\cdot120$

When P_h equals P_{360}, i equals 126 and 12345. Of the thirty-two sets $\{s_1 s_2 s_3 s_4 s_5\} t_{126}$ only the following four have their cubes in the head:-

$$t_{126},\ s_1 s_2 t_{128},\ s_1 s_6 t_{126},\ s_2 s_6 t_{126}.$$

Now,
$$(s_6)^{-1}\ H_{2\cdot3^6}\ s_6 = H_{2\cdot3^6}$$
$$(s_6)^{-1}\ t_{12345}\ s_6 = t_{12345}$$
$$(s_6)^{-1}\ t_{126}\ s_6 = s_2 s_6 t_{126}$$
$$(s_6)^{-1}\ s_1 s_6 t_{126}\ s_6 = s_1 s_2 t_{126}.$$

Hence there remains t_{126} and $s_1 s_2 t_{126}$.

The following group may be listed:-

$\{H_{2\cdot3^6}, t_{12345}, t_{126}\}$ ...... order .... $2\cdot3^6\cdot360$.

But t_{12345} and $s_1 s_2 t_{126}$ do not generate a group of order360 Since in a simple isomorphism between P_{360} and the proposed group the orders of corresponding substitutions should be equal which is not the case in the product $s_1 s_2 t_{126}\cdot t_{12345}$ which is a substitution of order eight, and the product $t_{126}\cdot t_{12345}$ which is the corresponding suhstitution in P_{360}.

When P_h equals P_{720}, and i equals 16, the thirty-two sets $\{s_1 s_2 s_3 s_4 s_5, H_{2\cdot3^6}\} t_{16}$ transform the head into itself, but only sixteen of these have their squares in the head. Only the one set, $H_{2\cdot 3^6}t_{16}$, also transforms $\{H_{2\cdot 3^6}, t_{12345}, t_{126}\}$ into itself and therefore there is just the following group:-

$\{H_{2\cdot 3^6}, t_{12345}, t_{16}\}$ ....order........$2\cdot 3^6 \cdot 720$

The Imprimitive Groups which contain $H_{2\cdot 3^5}$.

In the $2^5\cdot 3$ sets of substitutions which interchange the systems of imprimitivity according to p_i only t_i, $S_1 t_i$, and $S_1^2 t_i$ transform $H_{2\cdot 3^5}$ into itself. When i=12345, 12·46, 1426, and 16 respectively $S_1 t_i$ and $S_1^2 t_i$ have not their required powers in $H_{2\cdot 3^5}$, and therefore can not be used to form groups. Hence the following groups for t_i may be listed:-

$\{H_{2\cdot 3^5}, t_{12345}, t_{12\cdot 46}\}$ ......order .....$2\cdot 3^5 \cdot 60$

$\{H_{2\cdot 3^5}, t_{12345}, t_{1426}\}$ .... " ...$2\cdot 3^5 \cdot 120$

$\{H_{2\cdot 3^5}, t_{12345}, t_{16}\}$ .... " ...$2\cdot 3^5 \cdot 720$

When P_h equals P_{360}, the three substitutions t_{126}, $S_1 t_{126}$, and $S_1^2 t_{126}$ transform $H_{2\cdot 3^5}$ into itself and also have their third powers in $H_{2\cdot 3^5}$. As t_{12345} and $S_1 t_{126}$ are substitutions in any group possible of construction their product $S_1 t_{1345\cdot 26}$ must be in the group. But this is impossible as the fourth power of $S_1 t_{1345\cdot 26}$ is not in $H_{2\cdot 3^5}$. As the same reasoning can be applied for $S_1^2 t_{126}$, only the following group for t_{126} is listed:-

$\{H_{2\cdot 3^5}, t_{12345}, t_{126}\}$ ....order ....$2\cdot 3^5 \cdot 360$

The Imprimitive Groups which contain H_3.

When P_h equals P_{360}, and i equals 126, there are $2^6 \cdot 3^5$ sets of substitutions, namely:- $\{s_1 s_2 s_3 s_4 s_5 s_6, S_1 S_2 S_3 S_4 S_6\} t_{126}$, which transform H_3 into itself, and interchange the systems of imprimitivity according to p_i. But, as the substitutions of the set $\{s_1 s_2 s_3 s_4 s_5 s_6\} t_{126}$ have not their cubes in H_3, only the sets $\{S_1 S_2 S_3 S_4 S_6\} t_{126}$ need to be considered. Since t_{12345} and the substitutions $\{S_1 S_2 S_3 S_4 S_6\} t_{126}$ are contained in any group possible of construction, their products $\{S_1 S_2 S_3 S_4 S_6\} t_{126}$ must be substitutions the fourth powers of which are in H_3. After eliminating all substitutions which do not fulfill this requirement, the products $\{S_1 S_2 S_3 S_4 S_6\} t_{126} \cdot t_{12 \cdot 46}$ and $\{S_1 S_2 S_3 S_4 S_6\} t_{126} \cdot t_{16324}$ can be used in the same manner. Finally, this leaves only the one set, t_{126}, which together with t_{12345} and H_3 forms the following group:-

$\{H_3, t_{12345}, t_{126}\}$ ....order........ $3 \cdot 360$.

		Order.
1.	$\{H_{2^6\cdot 3^6}, t_{12345}, t_{16}\}$	33,592,320.
2.	$\{H_{2^6\cdot 3^6}, t_{12345}, t_{126}\}$	16,796,160
3.	$\{H_{2^5\cdot 3^6}, t_{12345}, t_{16}\}$	16,796,160
4.	$\{H_{2^5\cdot 3^6}, t_{12345}, s_1 t_{16}\}$	16,796,160
5.	$\{H_{2^5\cdot 3^6}, t_{12345}, t_{126}\}$	8,398,080
6.	$\{H_{2^6\cdot 3^6}, t_{12345}, t_{1426}\}$	5,598,720
7.	$\{H_{2^6\cdot 3^6}, t_{12345}, t_{12\cdot 46}\}$	2,799,360
8.	$\{H_{2^5\cdot 3^6}, t_{12345}, t_{1426}\}$	2,799,360
9.	$\{H_{2^5\cdot 3^6}, t_{12345}, s_1 t_{1426}\}$	2,799,360
10.	$\{H_{2^5\cdot 3^6}, t_{12345}, t_{12\cdot 46}\}$	1,399,680
11.	$\{H_{2\cdot 3^6}, t_{12345}, t_{16}\}$	1,049,760
12.	$\{H_{3^6}, t_{12345}, t_{16}\}$	524,880
13.	$\{H_{3^6}, t_{12345}, s_1 s_2 s_3 s_4 s_5 s_6 t_{16}\}$	524,880
14.	$\{H_{2\cdot 3^6}, t_{12345}, t_{126}\}$	524,880
15.	$\{H_{2\cdot 3^5}, t_{12345}, t_{16}\}$	349,920
16.	$\{H_{3^6}, t_{12345}, t_{126}\}$	262,440
17.	$\{H_{3^5}, t_{12345}, t_{16}\}$	174,960
18.	$\{H_{3^5}, t_{12345}, s_1 s_2 s_3 s_4 s_5 s_6 t_{16}\}$	174,960
19.	$\{H_{2\cdot 3^5}, t_{12345}, t_{126}\}$	174,960
20.	$\{H_{2\cdot 3^6}, t_{12345}, t_{1426}\}$	174,960
21.	$\{H_{2\cdot 3^6}, t_{12345}, s_3 s_4 s_5 s_6 t_{12\cdot 46}, t_{1426}\}$	174,960
22.	$\{H_{3^5}, t_{12345}, t_{126}\}$	87,480
23.	$\{H_{3^6}, t_{12345}, t_{1426}\}$	87,480
24.	$\{H_{3^6}, t_{12345}, s_3 s_5 t_{12\cdot 46}, t_{1426}\}$	87,480
25.	$\{H_{3^6}, t_{12345}, s_1 s_2 s_3 s_4 s_5 s_6 t_{1426}\}$	87,480
26.	$\{H_{3^6}, t_{12345}, s_3 s_5 t_{12\cdot 46}, s_1 s_2 s_3 s_4 s_5 s_6 t_{1426}\}$	87,480

27.	$\{H_{2\cdot 3^6}, t_{12345}, t_{12\cdot 46}\}$	87,480
28.	$\{H_{2\cdot 3^6}, t_{12345}, s_3 s_4 s_5 s_6 t_{12\cdot 46}\}$	87,480
29.	$\{H_{2\cdot 3^5}, t_{12345}, t_{1426}\}$	58,320
30.	$\{H_{3^6}, t_{12345}, t_{12\cdot 46}\}$	43,740
31.	$\{H_{3^6}, t_{12345}, s_3 s_5 t_{12\cdot 46}\}$	43,740
32.	$\{H_{3^5}, t_{12345}, t_{1426}\}$	29,160
33.	$\{H_{3^5}, t_{12345}, s_1 s_2 s_3 s_4 s_5 s_6 t_{1426}\}$	29,160
34.	$\{H_{2\cdot 3^5}, t_{12345}, t_{12\cdot 46}\}$	29,160
35.	$\{H_{3^5}, t_{12345}, t_{12\cdot 46}\}$	14,580
36.	$\{H_6, t_{12345}, t_{16}\}$	4,320
37.	$\{H_3, t_{12345}, t_{16}\}$	2,160
38.	$\{H_3, t_{12345}, s_1 s_2 s_3 s_4 s_5 s_6 t_{16}\}$	2,160
39.	$\{H_{2\cdot 3}, t_{12345}, t_{126}\}$	2,160
40.	$\{H_3, t_{12345}, t_{126}\}$	1,080
41.	$\{H_{2\cdot 3}, t_{12345}, t_{1426}\}$	720
42.	$\{H_3, t_{12345}, t_{1426}\}$	360
43.	$\{H_3, t_{12345}, s_1 s_2 s_3 s_4 s_5 s_6 t_{1426}\}$	360
44.	$\{H_{2\cdot 3}, t_{12345}, t_{12\cdot 46}\}$	360
45.	$\{H_3, t_{12345}, t_{12\cdot 46}\}$	180