INTERPOLATION OF DEFLECTIONS OF THE VERTICAL USING THE GRADIENTS OF GRAVITY AND COMPARISON WITH THE GRAVIMETRIC METHOD

A Thesis

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by

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I
INTRODUCTION

A. General

The fundamental problem of geodesy is to find the space coordinates of any point (P) on the physical surface of the earth. In order to develop a solution to this problem, two auxiliary surfaces have been established. (1) A reference surface which is as regular as possible and at the same time sufficiently close to the actual earth surface. This is a mathematical surface called the reference ellipsoid. It is assumed that the reference ellipsoid is a substitute for the spheroid which is an equipotential surface in the normal gravity field. (2) An intermediate surface called the geoid which is the equipotential surface of the actual gravity field coinciding with the mean surface of the oceans [8]. Using the gravity anomalies (Δg), the deviations (N, ξ and η) of the geod from the reference ellipsoid (spheroid) can be computed. Then a return to the physical surface is made to get the space coordinates of the point.

In order to find the differences between the reference ellipsoid and the geoid the formulas developed by Stokes and Vening-Meinesz are employed in what is called the gravimetric method. The gravity anomalies covering the entire earth's surface are the primary requirements of this method. Known for over a century, this method has not been applied until recently due to the scarcity of gravity material. Only during the past 20 years has enough material become available to compute the deviations of the geoid. Although there has been an improvement in the gravity coverage of the water areas of the world, the primary increase has been on land. This is due to the development of lightweight, easy to handle gravity meters. Gaps still remain, particularly in the ocean areas, which limit the application of this method. If, however, the distribution of gravity anomalies is known rather accurately in the neighborhood of a computation point and in broad lines over the rest of the world the geoidal properties (N, ξ and η) can be completed satisfactorily [3].

For this reason the gravity meter has become the dominate gravity instrument now used for geodetic purposes. However, difficulties often arise in gathering gravity data in sufficient density in the neighborhood of the observation point to use the gravimetric method. Gravity data is not available say out to 6° x 6° at locations near bodies of water. Inaccessible land areas such as jungles, mountains, etc., also place gaps in the gravity data in the neighborhood of a computation point. Some other method could be used in these areas to provide the required geoidal properties.

One method which could easily be adapted to these areas involves the use of an instrument known as the torsion balance. Prior to World War I, R. Eötvös developed the torsion balance primarily for geophysical prospecting. It was used successfully for this purpose for many years. It has now been replaced by the gravity meter. Although the torsion balance gives important information concerning the shape and structure of the earth, it has not as yet been used for geodetic purposes [7]. An important advantage of the torsion balance is that observations need only be taken at the computation point. Therefore, it is not dependent on gravity material in large surrounding areas obtained with the gravity meter. Along with astro-geodetic observations the geoid properties (N, ξ, η) can be determined with the torsion balance.

The torsion balance can provide detailed information about the geoid or geops which may be of increasing interest to the geodesist in the future. Also data obtained with the gravity meter can be complimented by torsion balance data in the search for new geodetic knowledge.

It is likely that an interest in the torsion balance will develop in the United States. Interest in this country in gravimetric geodesy has increased considerably since the establishment of the World Wide Gravity Project in 1950 at The Ohio State University.

B. Purpose

Specifically this paper explores the problem of interpolating deflections of the vertical between known points by applying the gradients of gravity. Eötvös derived formulas for accomplishing this. However, very little field work has been done except by Eötvös and his contemporaries to verify his theories. This is an initial study to test the methods which he evolved and determine if they could be of value in the light of today's advanced technology. If some success emerges from this study then follow-on projects of a more extensive and specific nature should be attempted. It should be emphasized that this is an initial effort in what could be an expanding field, and therefore the results cannot be viewed as conclusive.

C. Origin of Data

Data was obtained from a region in Southern Ohio along a first order triangulation net extending from station BARR, 7 miles NE of Circleville to TEMPLETON, 4 miles west of Xenia for a total distance of 62 miles (see map, page 6).

Astro-Geodetic Deflections

BARR - A first order longitude observed by the US Coast and Geodetic Survey in 1929.

TEMPLETON - A first order latitude observed by Saul Cushman and Ronald Adler in 1962.

Gravimetric Deflections of Eight Triangulation Stations

TEMPLETON, FOUST, TAYLOR, LANUM, HOYT, OWERS, MEINFELER, BARR

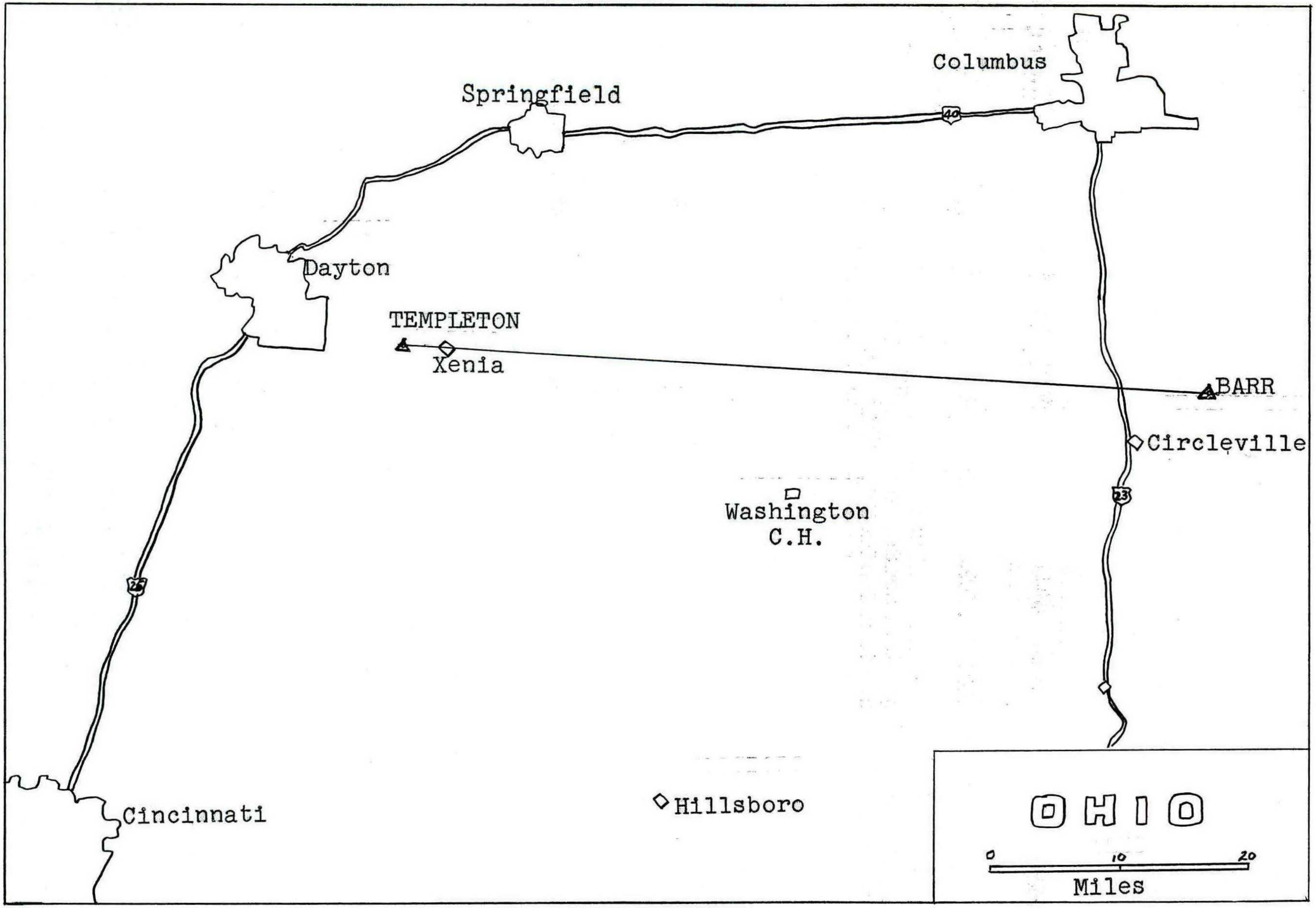
Inner Ring - Gravity meter observations,

Rice's Rings - Free Air Anomalies in Ohio, Institute of Geodesy, Photogrammetry, and Cartography of The Ohio State University, 1956.

Outer Area $6.5^{\circ} \times 6^{\circ}$ - Data from a thesis of L. Martucci [6].

Gradients of Gravity

Torsion Balance Observations for 21 stations along the north line of the triangulation net from BARR to TEMPLETON obtained from a thesis by Holway [4].


Columbus
Springfield
40
Dayton
TEMPLETON
Xenia
BARR
Circleville
23
Washington
C.H.
25
Cincinnati
Hillsboro
OHIO
0
10
20
Miles

II

GRAVIMETRICALLY COMPUTED DEFLECTIONS OF THE VERTICAL

A. General Considerations

The eight triangulation stations on the survey line were computed for vertical deflections using gravimetric methods based on Stokes theorm. In 1849 Stokes concluded that knowing the total mass and rotational speed of a rotating body the potential and its first derivatives on any equipotential surface could be determined independent of the mass distribution, providing no masses lie outside the equipotential surface in question. In order to apply Stokes theorm to deflection computations the gravity must be observed over the entire earth's surface. In Stokes' day it seemed quite unlikely that his theorm would be put to any practical use since there was available very limited gravity data, and the computations would have been extremely laborious. Today gravity data is quite plentiful, though not complete, and high speed computers make the computations entirely feasible. The scarcity of gravity data in certain areas, particularly the oceans, results in obtainable accuracies by this method in the order of $\pm 1''$.

The deflections were computed for all stations using a common area of $6.5^{\circ} \times 6^{\circ}$. Since the maximum distance between the two outside triangulation stations was 54 minutes of arc, the assumption was made that the effect of the earth beyond 6.5^{o} x 6^{o} would be the same for each station in between.

B. Inner Circle

The inner circle was taken as the area within a radius of six km of the station. Data for the inner circle area was obtained by making field observations with the Worden Gravity Meter at approximately 16 points per station. The observed values around each station were made independent of the other stations, and were not tied into a station of known gravity since only gravity differences were required.

Sites for gravity readings were selected which were accessible and easily pinpointed on the map. Benchmarks and road intersections were theprimary site locations. Elevations were interpolated from USGS maps (1:62500). Kaula found this method as accurate as barometrically obtained elevations [5].

A simple free air reduction was used, and since the terrain is quite level a topographic correction was unnecessary (see sample reduction Table I).

The observed gravity at the first point of each station was arbitrarily selected as 980.000 gals.

TABLE I

SAMPLE COMPUTATIONS OF $\Delta\xi$ AND $\Delta\eta$ EFFECTS AROUND A TRIANGULATION STATION (MEINFELER)

Sta. No.	Mean Obs.	h in Feet	Read X K Gals	Diff. From 1	γ_o	δg_ξ Gals.	Relative Δg	
676	530.0	676	.0450	0	980.1571	.0636	-93.5	Δg_S = -0.5
Fox	496.2	718	.0421	-.0029	980.1601	.0675	-95.5	Δg_N = -8.7
1	505.1	710	.0428	-.0021	.1615	.0668	-96.8	Δg_W = -6.9
2	529.9	691	.0449	-.0000	.1608	.0650	-95.8	Δg_E = +2.9
697	540.4	697	.0458	+.009	.1592	.0656	-92.7	Δg_{SE} = +2.8
ISL.	564.0	690	.0478	+.0029	.1591	.0649	-91.3	Δg_{NE} = +0.7
3	578.4	661	.0491	+.0041	.1607	.0622	-94.4	Δg_{SW} = -4.3
657	570.5	659	.0484	+.0034	.1582	.0620	-92.8	Δg_{NW} = -8.8
4	574.0	680	.0487	+.0037	.1573	.0640	-89.6	
S.Town	576.4	665	.0487	+.0039	.1564	.0625	-90.0	
80	558.0	758	.0473	+.0023	.1585	.0713	-84.9	
81	535.0	680	.0454	+.0004	.1639	.0640	-99.5	
83	525.3	791	.0446	-.0004	.1612	.0744	-87.2	

$(\Delta g_S - \Delta g_N) = -.5 + 8.7 \qquad (\Delta g_{SE} - \Delta g_{NE} + \Delta g_{SW} - g_{NW}) = +2.8 - 0.7 - 4.3 + 8.8$

$= 0.02625(+8.2) + 0.01856\ (+6.6) = \underline{\underline{+.''338}}$

$(\Delta g_W - \Delta g_E) = -6.9 - 2.9 \qquad (\Delta g_{SW} - \Delta g_{SE} + \Delta g_{NW} - \Delta g_{NE}) = -4.3 - 2.8 - 8.8 - 0.7$

$= 0.02625(-9.8) + 0.01856\ (-16.6) = \underline{\underline{-.''565}}$

Then the following gravity anomaly formula was used:

$$\Delta g = g_o - (\gamma_o - \delta g_f)$$

g_o = observed gravity at the point

γ_o = normal gravity at the points

δg_f = free air reduction

Δg = free air anomaly at the point

Then to obtain the relative gravity anomalies the following formulas were used:

$$\Delta g_{R1} = 980.000 - (\gamma_1 - \delta g_{f1})$$

$$\Delta g_{R2} = 980.000 + (g_2 - g_1) - (\gamma_2 - \delta g_{f2})$$

$$\Delta g_{Rn} = 980.000 + (g_n - g_1) - (\gamma_n - \delta g_{fn})$$

where Δg_R is the relative gravity at each point based on a constant gravity of 980.000 gals at point 1. g is the gravity meter reading times the constant K. γ is the normal gravity value computed for one point around each station. Since γ is a function of latitude only it was linearly interpolated for the other points around the station. δg_f = 0.09406 h where h is in feet. All values in Table I are in gals except Δg_R which is in mgals. From the Δg_R values a relative gravity anomaly map was drawn for each station with a contour interval of one mgal.

(see Figures 1-4).

Initially it was assumed that all of the stations displayed a constant gravity gradient within the 6 km radius circle. This would imply straight parallel gravity anomaly contours at uniform spacing. Although from the Figures (1-4) it can be seen this is not everywhere the case, the deflection effects were nevertheless first computed for alleight triangulation stations using the Rice Three Gradient method which assumes linearity.

The following formulas were initially used to obtain the vertical deflection effects of the inner circle of 6 km radius for all eight triangulation stations.

$$\Delta \xi''_{o-r_o} = 0.105 r_o \left(\frac{\delta \Delta g}{\delta y} \right)$$

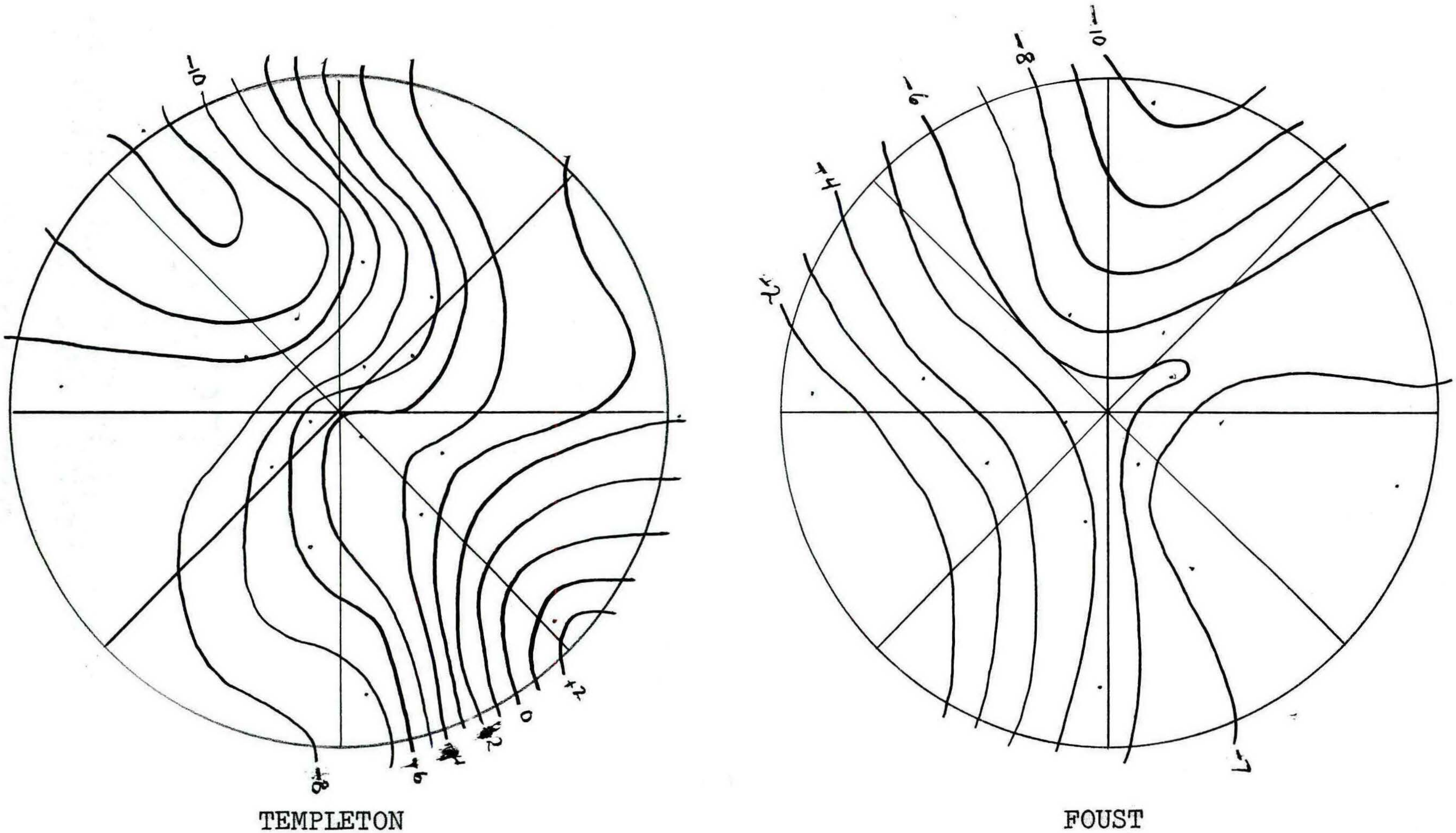
$$\Delta \eta''_{o-r_o} = 0.105 r_o \left(\frac{\delta \Delta g}{\delta x} \right) \qquad (1)$$

Using the Rice Three Gradient method, and developing the horizontal gravity gradients from the point values, equation (2) can be derived from (1).

$$\Delta \xi''_{o-r_o} = 0.02625\ (\Delta g_S - \Delta g_N) + 0.01856\ (\Delta g_{SE} - \Delta g_{NE} + \Delta g_{SW} - \Delta g_{NW}) \qquad (2)$$

$$\Delta \eta''_{o-r_o} = 0.02625\ (\Delta g_W - \Delta g_E) + 0.01856 (\Delta g_{SW} - \Delta g_{SE} + \Delta g_{NW} - \Delta g_{NE})$$

Figure 1.



Scale
0 1 2 3 4 5 km

Figure 2.

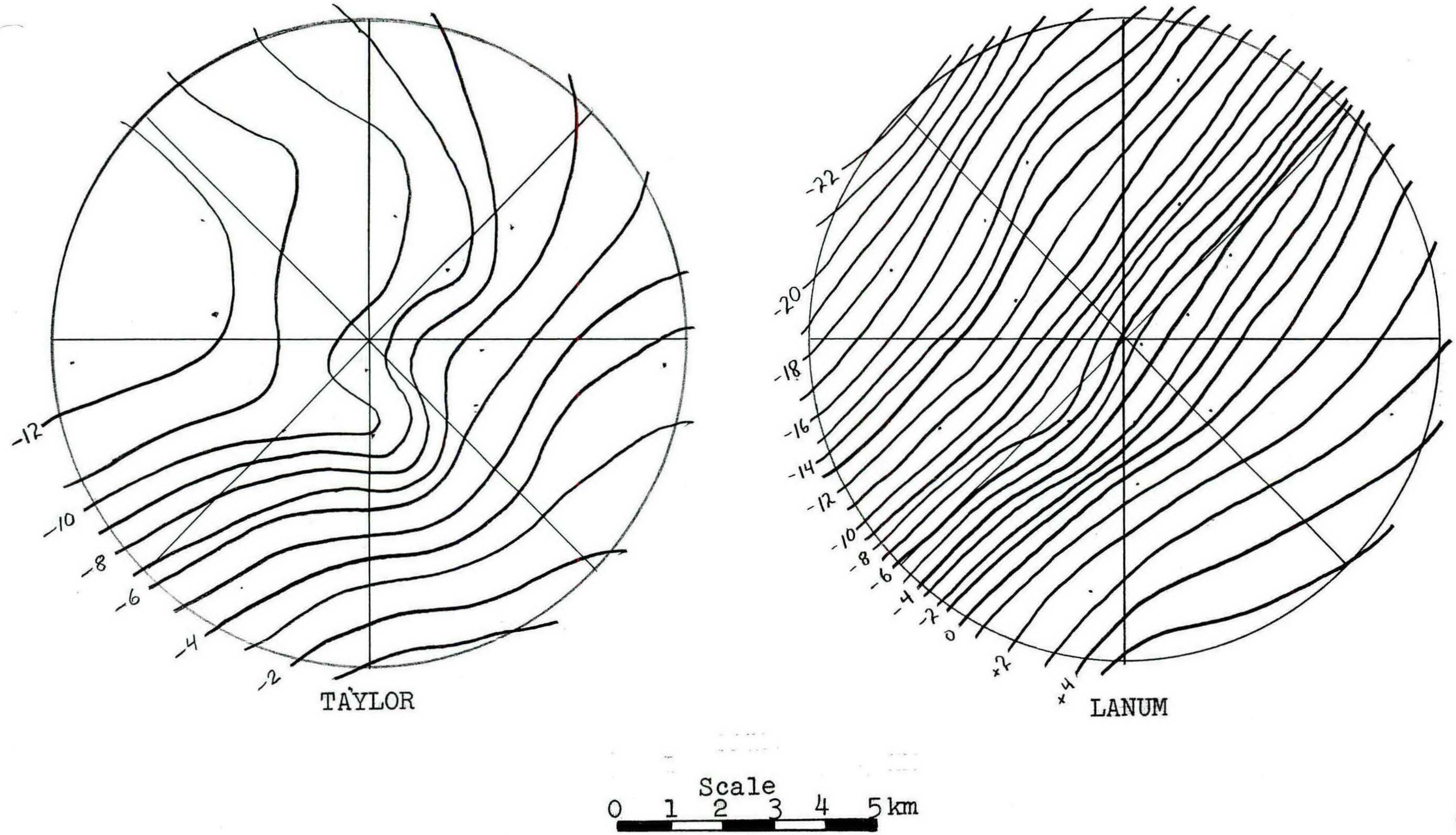

-12
-10
-8
-6
-4
-2
TAYLOR
-22
-20
-18
-16
-14
-12
-10
-8
-6
-4
-2
0
+2
+4
LANUM
Scale
0 1 2 3 4 5 km

Figure 3.

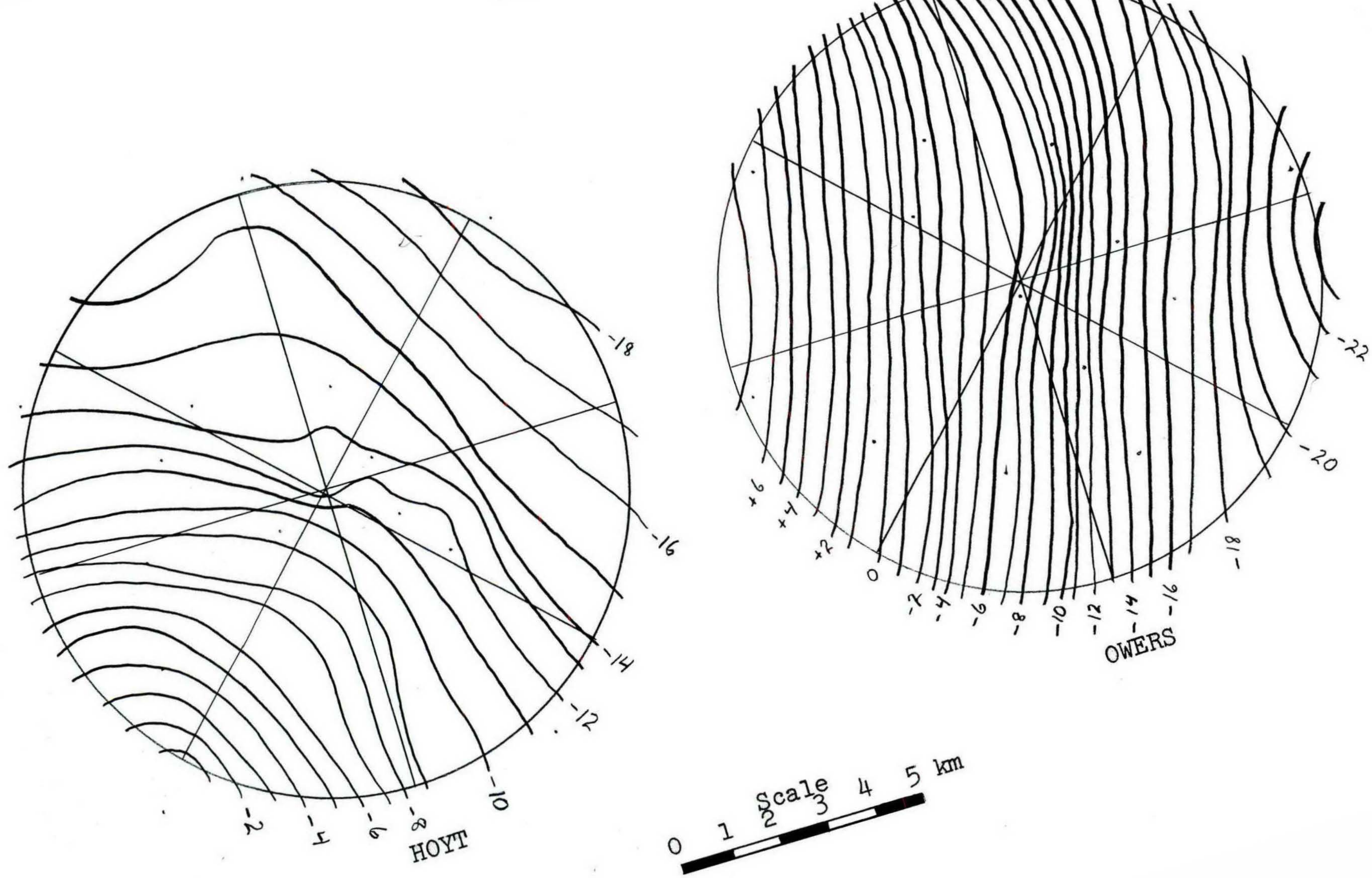
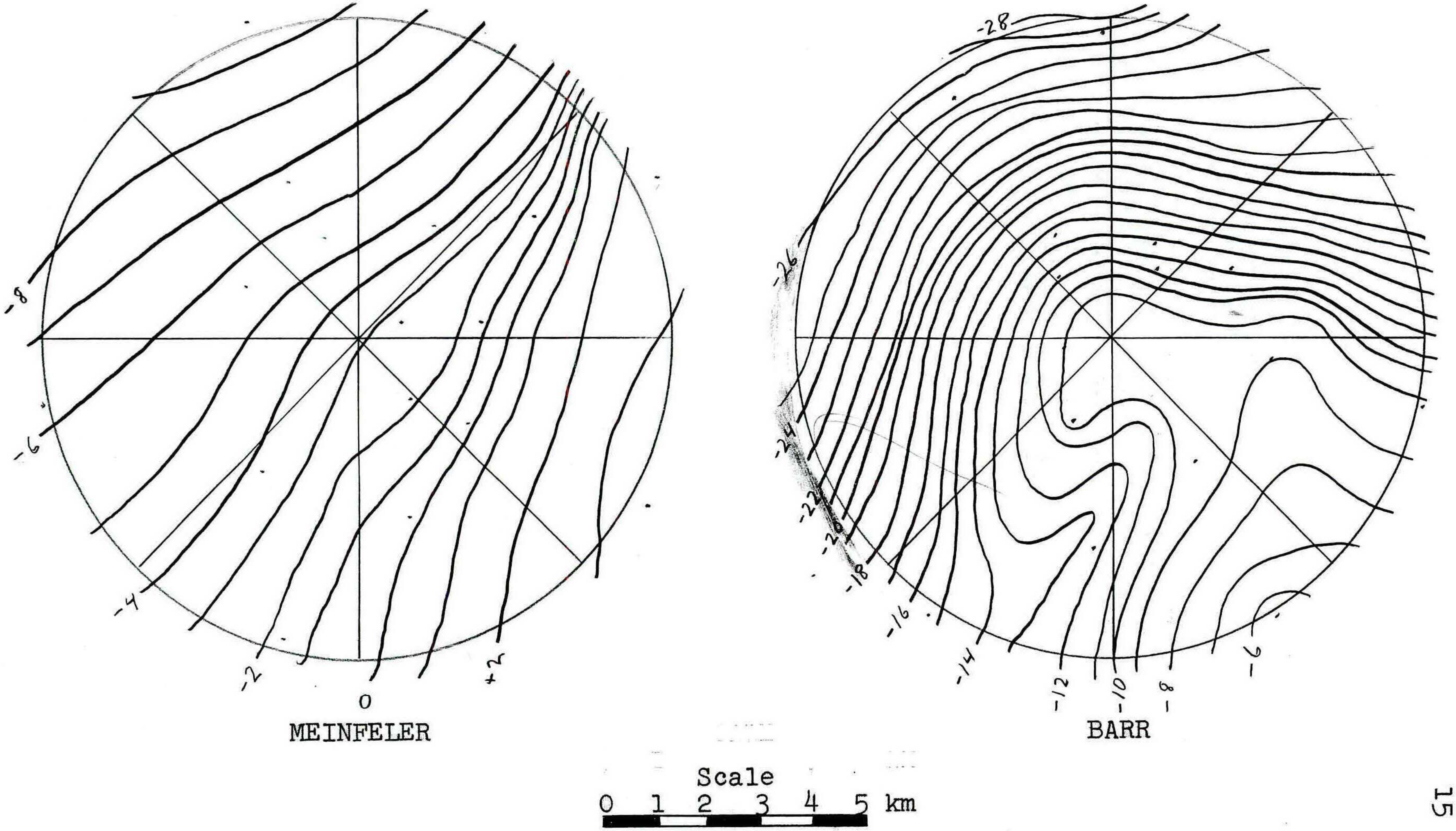

HOYT
-2
-4
-6
-8
-10
-12
-14
-16
-18
OWERS
+6
+4
+2
0
-2
-4
-6
-8
-10
-12
-14
-16
-18
-20
-22
Scale
0 1 2 3 4 5 km

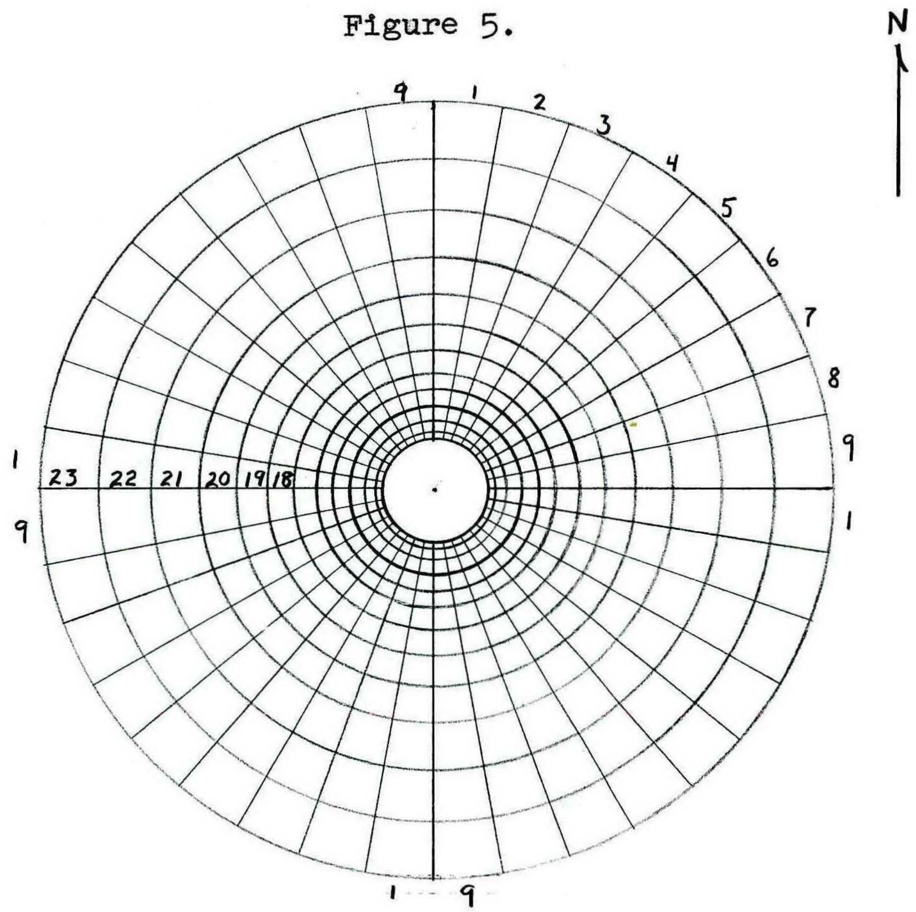
Figure 4.


-8
-6
-4
-2
0
+2
MEINFELER
-28
-26
-24
-22
-20
-18
-16
-14
-12
-10
-8
-6
BARR
Scale
0 1 2 3 4 5 km

Where all the $\triangle g$ values are the $\triangle g_R$ anomaly differences in mgal. Anomaly differences were read on the circular ring and when substituted into the formula the desired gravity gradient values are achieved [11].

From inspection of the Figures (1-4), it can be seen that three of the stations (TEMPLETON, FOUST, and BARR) do not appear very linear. Therefore at these stations the Rice's Rings method (explained in Chapter II C) was used between radii 0.8 and 6.0 km. This was done by constructing a template of Rice's Rings (Zones 12-23, 0.8 to 6.0 km) and reading the mean gravity anomalies from the Figures 1 and 4, respectively, for the three stations. For the template used see Figure 5. Then the deflection effects for this area were computed by the Rice's Ring Method.(For sample calculations see Table II). The effect of the circle of radius 0.8 km was computed using the Rice Three Gradient method in the same way the inner circle of 6 km was previously computed for all eight triangulation stations. (Sample calculation, bottom of Table II). For these three stations the effects using the Rice Three Gradient Method out to a radius of 0.8 km., plus the Rice's Rings out to a radius of 6 km would, of course, be more accurate than only using the Rice three Gradient method out to 6 km. as was initially done. The maximum difference between

Figure 5.



Template
Rice's Rings - Zones 12-23

TABLE II

SAMPLE COMPUTATION OF RICE'S RINGS FROM 0.78 km. to 6.0 km.
(Zones 12-23) Using the Inner Circle Gravity Anomaly Map

TEMPLETON - Compartments Starting from North

ZONE	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
12	-7	-7	-7	-7	-6	-6	-6	-5	-5	-5	-5	-5	-5	-5	-5	-5	-5	-5
13	-7	-7	-7	-7	-6	-6	-6	-5	-5	-5	-5	-5	-5	-5	-5	-5	-5	-5
14	-8	-7	-7	-7	-6	-6	-6	-5	-5	-5	-5	-5	-5	-5	-5	-5	-5	-5
15	-8	-8	-7	-7	-6	-6	-6	-5	-5	-5	-4	-4	-4	-4	-4	-5	-5	-5
16	-8	-8	-7	-7	-6	-6	-6	-5	-5	-4	-4	-4	-4	-4	-4	-4	-5	-5
17	-8	-8	-7	-7	-6	-6	-5	-5	-5	-4	-4	-4	-4	-4	-4	-4	-5	-5
18	-9	-8	-7	-7	-6	-6	-5	-5	-4	-4	-4	-3	-3	-3	-4	-4	-5	-5
19	-9	-8	-7	-6	-6	-5	-5	-4	-4	-4	-3	-3	-3	-3	-3	-4	-5	-5
20	-9	-7	-7	-6	-5	-4	-4	-4	-4	-3	-2	-2	-2	-2	-3	-4	-5	-6
21	-7	-6	-5	-5	-4	-4	-4	-4	-3	-3	-2	-1	-1	-1	-2	-4	-5	-6
22	-6	-5	-4	-4	-4	-3	-3	-3	-3	-2	-1	-1	-0	-0	-1	-3	-6	-7
23	-5	-4	-4	-3	-3	-3	-3	-3	-3	-2	-1	-0	-1	-2	-0	-2	-6	-7
Sector Σ	-91	-93	-76	-73	-64	-61	-59	-53	-51	-46	-40	-37	-35	-34	-40	-49	-62	-66
Azimuth	5°	15°	25°	35°	45°	55°	65°	75°	65°	85°	75°	65°	55°	45°	35°	25°	15°	5°
Cos $A_z \times \Sigma$	-91	-80	-69	-60	-45	-35	-25	-14	-4	+4	+10	+16	+20	+24	+33	+44	+60	+66
sin $A_z \times \Sigma$	-8	-21	-32	-42	-45	-50	-53	-51	-51	-51	-39	-33	-29	-24	-23	-21	-16	-6

Computation of Inner Circle (Radius 0.8 km.)

$\Delta g_S = -5$ $\Delta g_{SE} = -5$
$\Delta g_N = -7$ $\Delta g_{NE} = -6$
$\Delta g_E = -5$ $\Delta g_{NW} = -7$
$\Delta g_W = -6$ $\Delta g_{SW} = -6$

$\Delta \xi = 0.02625(+2) + 0.01856(+2) = +0.''090$
$\Delta \eta = 0.02625(-1) + 0.01856(-2) = +0.''063$

TABLE II (continued)

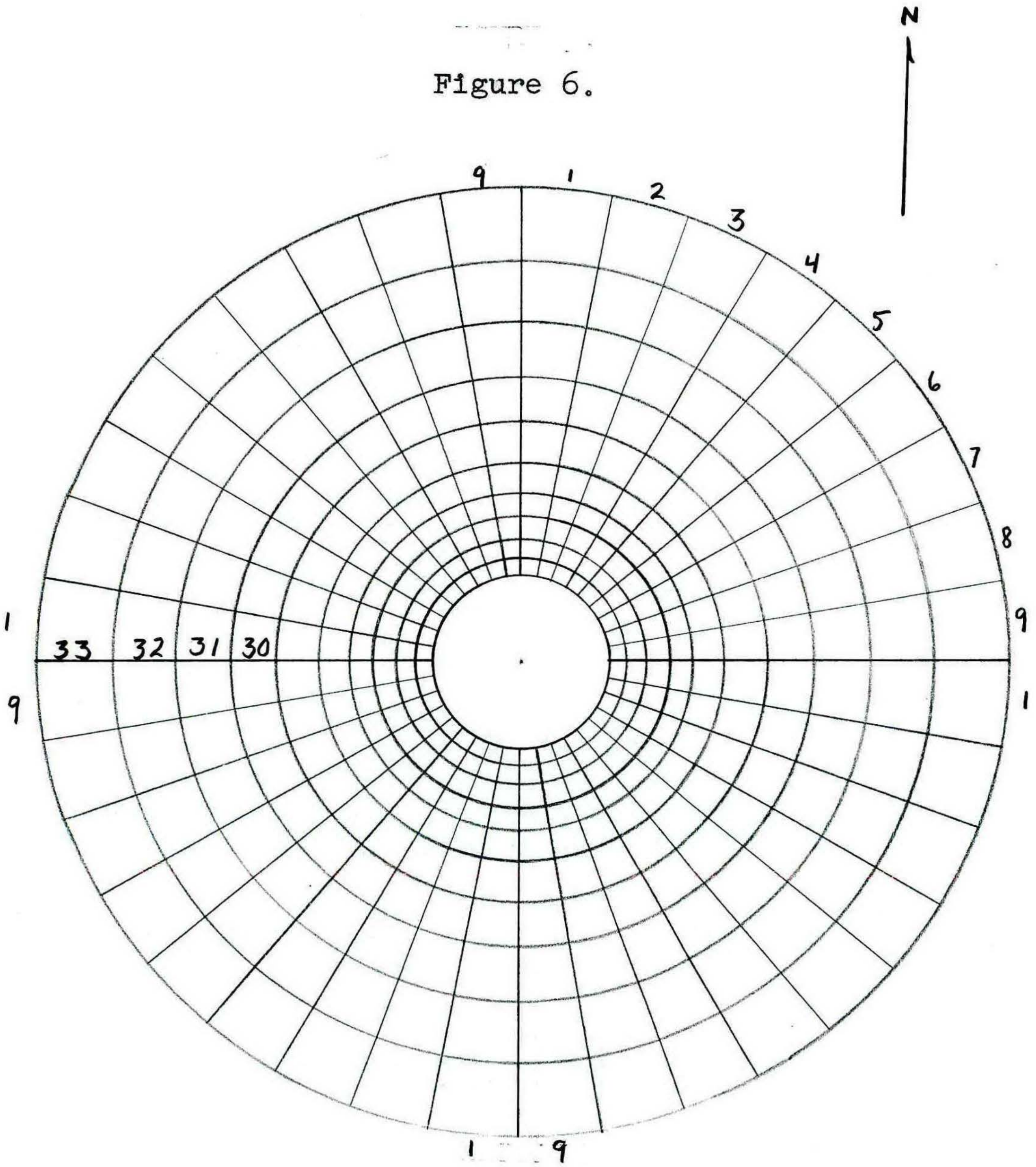
ZONE	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	TOTAL
12	- 5	- 5	- 5	- 5	- 6	- 6	- 6	- 6	- 7	- 7	- 7	- 7	- 7	- 8	- 8	- 8	- 8	- 7	
13	- 5	- 5	- 5	- 6	- 6	- 6	- 6	- 7	- 7	- 7	- 8	- 8	- 8	- 8	- 8	- 8	- 8	- 8	
14	- 5	- 5	- 5	- 6	- 6	- 6	- 7	- 7	- 7	- 8	- 8	- 8	- 8	- 8	- 8	- 8	- 8	- 8	
15	- 5	- 5	- 6	- 6	- 6	- 7	- 7	- 7	- 8	- 8	- 8	- 8	- 9	- 9	- 9	- 9	- 9	- 8	
16	- 5	- 5	- 6	- 6	- 6	- 7	- 7	- 8	- 8	- 8	- 8	- 9	- 9	- 9	- 9	- 9	- 9	- 9	
17	- 5	- 6	- 6	- 6	- 7	- 7	- 8	- 8	- 8	- 8	- 9	- 9	- 9	-10	-10	-10	-10	- 9	
18	- 5	- 6	- 6	- 7	- 7	- 7	- 8	- 8	- 8	- 9	- 9	- 9	- 9	-10	-10	-10	-10	-10	
19	- 6	- 6	- 6	- 7	- 7	- 8	- 8	- 8	- 8	- 9	- 9	- 9	-10	-10	-10	-10	-10	-10	
20	- 6	- 6	- 7	- 7	- 8	- 8	- 8	- 8	- 9	- 9	- 9	- 9	-10	-11	-11	-11	-10	-10	
21	- 7	- 7	- 7	- 8	- 8	- 8	- 8	- 8	- 9	- 9	- 9	- 9	-10	-11	-11	-11	-10	- 9	
22	- 7	- 7	- 8	- 8	- 8	- 8	- 8	- 8	- 9	- 9	- 9	- 9	-10	-11	-11	-11	- 9	- 8	
23	- 8	- 8	- 8	- 8	- 8	- 8	- 8	- 9	- 9	- 9	- 9	- 9	-10	-11	-11	-10	- 9	- 7	
Sector Σ	-69	-71	-75	-80	-83	-86	-89	-93	-97	-100	-102	-103	-109	-116	-116	-115	-110	-103	
Azimuth	5°	15°	25°	35°	45°	55°	65°	75°	85°	85°	75°	65°	55°	45°	35°	25°	15°	5°	
Cos Az × Σ	+69	+69	+68	+66	+59	+49	+38	+24	+ 8	- 9	-26	-43	-62	-82	-95	-103	-106	-102	+324″
sin Az × Σ	+ 6	+18	+32	+46	+59	+70	+81	+89	+97	+99	+98	+93	+89	+82	+66	+49	+28	+ 9	-516″

the two methods was $0''.345$ seconds of arc. This was for the $\Delta\xi$ effect at TEMPLETON. In the final summation of gravimetric deflection effects (Table X), the inner circle effects for TEMPLETON, FOUST and BARR includes an inner circle of 0.8 km. radius computed by the Rice three Gradient method, and then Rice's Rings out to 6 km. For the other five stations the inner circle effects are computed from the Rice three Gradient method for a 6 km. inner circle radius.

C. Rice's Rings

Computation of a 25' x 25' "window" excluding the inner circle was done using Rice's Ring method [9]. From the vertical deflection component formulas developed by Vening Meinesz in 1928 Rice developed a template system to compute the numerical integration required. The template is constructed on a sheet of tracing paper with a uniform angular aperture of 10°. Rice computed from Sollins' table the values of zone radii so that each compartment has a deflection effect of $0''.001$ for a mean anomaly of 1 mgal. For the 25' x 25' "window" it was necessary to use zones 24 through 34 (6.08 km. to 39.67 km.) (see Figure 3). The totals for each compartment were added up and multiplied by the cosine of the azimuth for the effect on the meridian component and by the sine of the azimuth for the effect on the prime vertical component.

Figure 6.



Template
Rice's Rings - Zones 24-33

Appropriate signs were applied with the azimuth beginning from south, then final summation was made to find deflection effects (see Table III).

D. Outer Area ($6.5^{\circ} \times 6^{\circ}$)

Values of deflection effects for this area were obtained from a computer program devised by Martucci [6]. The program computes the deflection component contribution for the $6.5^{\circ} \times 6^{\circ}$ area, except for the 25' x 25' "window" located around the station.

Due to the fact that no values were available for the area of Lake Erie it was assumed that this area has a zero contribution. The "window" is oriented using the 5' x 5' "square" containing the station as the center 5' x 5' "square" in the 25' x 25' "window." Input of the geographical coordinates of the station into the program yields deflection component effects. For details of the program see the thesis of Martucci [6].

E. Summary

The sign convention was fixed as follows: ξ is positive with a deflection away from north and η is positive with a deflection toward the east. Special care was taken to insure that the signs for all three computational areas were consistent.

TABLE III

SAMPLE COMPUTATIONS OF RICE'S RINGS (Zones 24-34) for WINDOW (25' - 25') NOT INCLUDING INNER CIRCLE

COMPARTMENT A-1 BARR OUTER WINDOW (25' x 25')

ZONE	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
24	-25	-23	-22	-20	-19	-18	-17	-16	-15	-14	-13	-12	-13	-14	-15	-16	-18	-18
25	-25	-23	-22	-20	-18	-18	-17	-16	-15	-13	-11	-11	-12	-14	-15	-17	-18	-19
26	-25	-23	-21	-19	-18	-17	-17	-16	-14	-13	-11	-10	-11	-14	-15	-17	-19	-20
27	-25	-22	-20	-19	-17	-17	-17	-16	-14	-13	-12	-10	-11	-14	-16	-18	-19	-21
28	-25	-22	-19	-18	-18	-18	-18	-17	-16	-14	-14	-13	-14	-15	-18	-19	-21	-22
29	-25	-21	-18	-17	-19	-20	-20	-19	-19	-18	-18	-17	-16	-17	-20	-21	-22	-23
30	-25	-21	-19	-17	-21	-22	-22	-22	-22	-23	-22	-19	-17	-20	-21	-22	-23	-24
31	-18	-18	-20	-17	-17	-19	-16	- 9	-	-	-11	-16	-16	-20	-22	-24	-24	-25
32	-	-	- 4	-15	-15	- 6	-	-	-	-	-	-	- 7	-21	-20	-13	- 3	-
33	-	-	-	-	- 4	-	-	-	-	-	-	-	-	- 8	- 5	-	-	-
34	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Sector Σ	-193	-173	-165	-162	-166	-155	-144	-131	-115	-108	-112	-108	-117	-157	-167	-167	-167	-172
Sector A_Z	5°	15°	25°	35°	45°	55°	65°	75°	85°	85°	75°	65°	55°	45°	35°	25°	15°	5°
$(\cos A_Z) \times (\Sigma)$	+192	+167	+149	+132	+117	+ 89	+61	+34	+10	- 9	-29	-46	-67	-111	-137	-151	-161	-171
$(\sin A_Z) \times (\Sigma)$	+17	+45	+70	+93	+117	+127	+131	+127	+115	_108	+108	+98	+96	+111	+71	+43	+15	-16

TABLE III (continued)

ZONE	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
24	-20	-21	-22	-22	-23	-23	-24	-25	-25	-25	-26	-27	-27	-28	-27	-27	-26	-26
25	-21	-21	-23	-23	-24	-25	-25	-25	-25	-27	-28	-29	-29	-29	-29	-28	-28	-27
26	-21	-22	-23	-24	-25	-25	-25	-25	-28	-29	-30	-30	-31	-31	-31	-30	-29	-27
27	-22	-23	-24	-25	-26	-26	-26	-26	-29	-30	-31	-32	-33	-33	-33	-31	-30	-28
28	-23	-24	-26	-26	-27	-26	-25	-25	-27	-29	-32	-35	-35	-35	-35	-33	-31	-29
29	-24	-26	-26	-27	-26	-25	-22	-11	- 9	-10	-14	-33	-36	-37	-36	-35	-32	-29
30	-25	-26	-27	-26	-25	-11	-	-	-	-	-	-	-13	-36	-38	-36	-33	-30
31	-26	-28	-28	-25	- 9	-	-	-	-	-	-	-	-	-10	-36	-36	-25	-18
32	-	- 5	-15	-11	-	-	-	-	-	-	-	-	-	-	-10	- 6	-	-
33	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
34	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Sector Σ	-182	-196	-214	-208	-185	-161	-147	-137	-145	-150	-161	-186	-204	-239	-275	-262	-254	-214
Sector A_z	5°	15°	25°	35°	45°	55°	65°	75°	85°	85°	75°	65°	55°	45°	35°	25°	15°	5°
$(\cos A_z) \times (\Sigma)$	-181	-189	-194	-171	-131	-92	-62	-36	-12	+13	+41	+79	+117	+169	+235	+237	+226	+213
$(\sin A_z) \times (\Sigma)$	-16	-31	-90	-119	-131	-132	-133	-132	-142	-149	-156	-169	-167	-169	-158	-111	-61	-19

Discussion of Errors.

At some stations a drift check was made and the drift found was very small, so drift corrections were neglected.

Systematic errors could occur in estimating mean anomalies but since they would be of opposite signs they should for the most part cancel out.

Some small errors may occur in the gravimetric computations. However, the purpose of collecting the gravimetric deflection values is to show a trend along the survey line. This trend can then be compared to the torsion balance deflection trend. Therefore, small errors in the gravimetric method will not compromise the purpose for which they have been computed.

III

INTERPOLATION OF DEFLECTIONS OF THE VERTICAL USING THE CURVATURE GRADIENTS OF GRAVITY

A. Development of Formulas [7]

If deflections of the vertical are to be interpolated between any two points, it is necessary to assume that the deflection changes between intermediary points is linear. Therefore, the distance between intermediary points that can be safely used varies with the mass distribution close to the physical surface of the earth. To obtain such a dense net of deflections using astronomic means would be extremely laborious. As explained in Part I A, obtaining the deflections using the gravimetric method is simply not possible in some areas of the world due to a lack of available gravity data.

However, by using the curvature gradients of gravity obtained with a torsion balance, the deflection can be obtained to the desired density. The only requirement is that both deflection components ξ, η are known at one station, and at least one of the components is known at another station. So, along with the torsion balance observations, a minimum of three astronomic determinations are necessary. The following change is used to simplify notation for the gradients of gravity.

$$\frac{d^2W}{dxdy} = W_{xx} \qquad \frac{d^2W}{dxdy} = W_{xy} \quad \text{etc.}$$

The torsion balance observations produce the curvature gradients $W_{\Delta} = W_{yy} - W_{xx}$, and W_{xy}. Using these quantities the deflection differences between points can be computed using Formulas derived by Eötvös in 1906.

A start is made with an initial point (0) which should be a point where both the astronomic and geodetic positions are known so that the astro-geodetic deflections are available. The initial point is used as the origin of the (x, y, z) coordinate system with the x axis oriented positive toward the north (tangent to the local meridian) and the z axis coincides with the local vertical, positive down. The y axis is perpendicular to the x and z axis and is positive toward the east. This means that the relative orientation of the coordinate system is different at each station. For simplicity only one coordinate system is used for the survey area. Taking the initial point (0) as the reference station, the coordinate system based on this point is used over the whole area. Therefore, at any point in the survey area the z axis is parallel to the vertical at point (o) and the x axis is parallel to the tangent of the meridian at point (0).

In Figure 7 the plane of the paper can be taken as the xz plane of the above defined coordinate system. In this new coordinate system with the origin at any arbitrary point A_1 the z axis is parallel to the vertical of the initial point (O). Therefore, it will not coincide with the direction of the gravity vector at the arbitrary point A_1. Line A_1G is the projection of the gravity vector

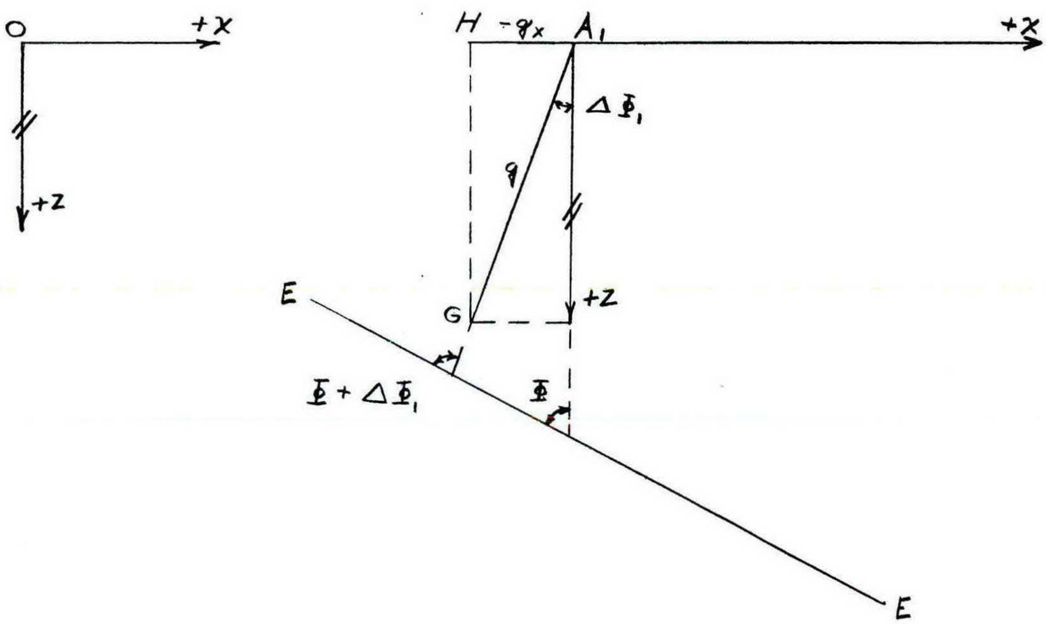


Figure 7.

at point A_1 on the xz plane. Since the dominate gravity change is a function of latitude the change along the x axis is much greater than the change along the y axis. Therefore, the difference between the vector A_1G and the true gravity vector at point A_1 is small and can be neglected. A_1H is the g_x component of g and line EE represents the equatorial plane intersecting the xz plane. It can be seen from the figure that the astro-latitude of the initial point (O) is Φ and of point A_1 is $\Phi + \Delta\Phi_1$.

For point A_1 $\Phi_1 = \Phi + \Delta\Phi_1$ then

$$-g_x = g \sin \Delta\Phi_1, \quad \text{but since } \Delta\Phi_1 \tag{3}$$

is a small angle

$$\Delta\Phi_1 = -\frac{g_x}{g} \tag{4}$$

and using a similar reasoning along the prime vertical:

$$\Delta L_1 \cos \Phi_1 = -\frac{g_y}{g}$$

where ΔL_1 is the astro-longitude between the points O and A_1.

The geop of the observation point is defined as the equipotential surface of the gravity passing through the center point of the torsion balance, i.e., where the horizontal beam is attached to the torsion fiber. The perpendicular to the equipotential surface at that point is, of course, the geop normal. The above equations give the North and East components of the angle between the geop normals at O and A_1. This development can be used to determine the respective North and East components of the angle between the geop normals at O and another arbitrary point A_2, $\Delta\Phi_2$, and ΔL_2.

Subtracting the equations for points A_1 and A_2 gives the expressions for the quantities $(\Delta\Phi_2 - \Delta\Phi_1)$ and $(\Delta L_2 - \Delta L_1)$ which are the North and East components of the angle between the geop normals at A_2 and A_1.

$$\Delta\Phi_2 - \Delta\Phi_1 = -\frac{1}{g_m}(g_{x2} - g_{x1}) = -\frac{1}{g_m}(W_{x2} - W_{x1})$$

(6)

$$(\Delta L_2 - \Delta L_1)\cos\phi_m = \frac{-1}{g_m}(g_{y2} - g_{y1}) = \frac{-1}{g_m}(W_{y2} - W_{y1})$$

Where g_m, and ϕ_m are the mean gravity and the mean astro-latitude between points A_1, and A_2. $(\Delta\phi_2 - \Delta\phi_1)$ and $(\Delta L_2 - \Delta L_1)$ are the astro-latitude and longitude differences between points A_2 and A_1. The quantities g_{x1}, g_{x2}, g_{y1}, g_{y2}, are the g_x and g_y gravity components at A_1 and A_2. These can be substituted by the first partial derivative of the potential function of the gravitational force (W) since according to Newton's Second Law of Motion:

$$G = mg \ (\text{gr cm sec}^{-2}) \text{ units} \tag{7}$$

where G is the gravitational force, and g is the gravitational acceleration. Let the mass m = 1 gr and by definition

$$\frac{dW}{dx} = G_x \; ; \quad \frac{dW}{dy} = G_y \; ; \qquad \frac{dW}{dz} = G_z \tag{8}$$

therefore G = g and

$$\frac{dW}{dx} = g_x \; ; \quad \frac{dW}{dy} = g_y \; ; \qquad \frac{dW}{dz} = g_z$$

The spheroidal normals are perpendicular to the spheroid, but can also be assumed to refer to the reference ellipsoid, and the spherop of the observation point. The reference to the spheroid implies the normal gravity field. Using the same analysis as previously the equations for the North and East components of the angle between the spheriodal normals at A_1 and at A_2 can be developed.

$$\Delta\phi_2 - \Delta\phi_1 = \frac{1}{\gamma m} (U_{x2} - U_{x1})$$
$$(\Delta\lambda_2 - \Delta\lambda_1) \cos \phi_m = - \frac{1}{\gamma m} (U_{y2} - U_{y1}) \tag{10}$$

Here the quantities ($\Delta\phi_2 - \Delta\phi_1$) and ($\Delta\lambda_2 - \Delta\lambda_1$) are the geodetic latitude and longitude differences between points A_1 and A_2. Then U_{x1}, U_{x2}, U_{y1}, U_{y2} are the normal gravity components at A_1 and A_2, and γ_m is the mean normal gravity. Now ξ, the north-south component of the deflection of the vertical is equal to the astro-geodetic latitude difference:

$$\xi = \phi' - \phi \tag{11}$$

and the east west component involves the astro-geodetic longitude differences in the expression

$$\eta = (\lambda' - \lambda) \cos \phi \tag{12}$$

therefore, the differences between the astronomic and geodetic latitude and longitude differences in the above equations (6) and (10) are the differences of the deflection components between the points A_1 and A_2.

Assuming that in the relatively small survey area:

$$\gamma_m = g_m \quad \text{and} \quad \phi_m = \Phi_m$$

and then subtracting the previous equations gives:

$$[(\Delta\Phi_2 - \Delta\phi_2) - (\Delta\Phi_1 - \Delta\phi_1)]g_m = -(W_{x2} - W_{x1}) + (U_{x2} - U_{x1}) \tag{13}$$

$$[(\Delta L_2 - \Delta\lambda_2)\cos\phi_m - (\Delta L_1 - \Delta\lambda_1)\cos\phi\, g_m =$$

$$= -(W_{y2} - W_{y1}) + (U_{y2} - U_{y1})$$

or

$$(\xi_2 - \xi_1)g_m = -(W_{x2} - W_{x1}) + (U_{x2} - U_{x1})$$

$$(\eta_2 - \eta_1)g_m = -(W_{y2} - W_{y1}) + (U_{y2} - U_{y1}) \tag{14}$$

Defining W - U as the potential anomaly ΔW and similarly for the other partial derivatives.

Substituting

$$\Delta\xi_{21} = \xi_2 - \xi_1$$

$$\Delta\eta_{21} = \eta_2 - \eta_1 \tag{15}$$

gives

$$g_m \Delta \xi_{21} = -\Delta W_{x2} + \Delta W_{x1}$$
$$g_m \Delta \eta_{21} = -\Delta W_{y2} + \Delta W_{y1} \qquad (16)$$

Calling upon the assumption that the points A_1 and A_2 are close enough together so that the variations of the gradients between them is linear, a new coordinate system (n, s, t) is introduced.

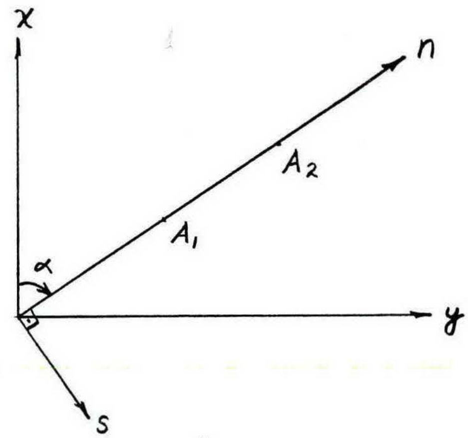


Figure 8.

The t axis coincides with the z axis of the (x, y, z) coordinate system and the n axis goes through points A_1 and A_2. The s axis is then perpendicular to n and t. (see Figure 8). The coordinates of points A_1 and A_2 in the new coordinate system are ($s=o$, $n=n_1$) and ($s=o$, $n=n_2$) respectively. One of the second partial derivatives of the potential in the new coordinate system is $\frac{d^2W}{dnds}$. This is also called a gradient of gravity and it expresses the change of the gravity component in a certain direction for a unit distance.

These relationships exist between the gradients

$$\frac{d^2W}{dnds} = \frac{d^2W}{dsdn} = \frac{dg_s}{dn} = \frac{dg_n}{ds}$$

In order to find the first partial derivatives of the potential the gradient of gravity is integrated.

$$T_{12} = \int_{n_1}^{n_2} \frac{d^2W}{dnds} \quad dn = \left(\frac{dW}{ds}\right)_2 - \left(\frac{dW}{ds}\right)_1 = W_{s2} - W_{s1} \tag{17}$$

the change in $\frac{d^2W}{dnds}$ is assumed to be linear so approximate integration can be used.

The Trapezoidal Rule gives:

$$T_{12} = \int_{n_1}^{n_2} \frac{d^2W}{dnds} \quad dn = 1/2 \left[\left(\frac{d^2W}{dnds}\right)_1 + \left(\frac{d^2W}{dnds}\right)_2\right] (n_2-n_1) \tag{18}$$

$$T_{12} = 1/2(W_{ns1} + W_{ns2}) \; (n_2-n_1)$$

The transformation equations between the two coordinate systems are:

$$\begin{aligned} s &= y \cos\alpha - x \sin\alpha \\ n &= y \sin\alpha + x \cos\alpha \\ t &= z \end{aligned} \tag{19}$$

So the transformation equation of the first partial derivative is:

$$W_s = -W_x \sin\alpha + W_y \cos\alpha \qquad (20)$$

Substituting (20) into (17) we get:

$$T_{12} = \int_{n_1}^{n_2} \frac{d^2W}{dnds} \, dn = W_{s2} - W_{s1} = \qquad (21)$$

$$= +(W_{y2} - W_{y1}) \cos\alpha_{12} - (W_{x2} - W_{x1}) \sin\alpha_{12}$$

α_{12} is the azimuth of the direction $A_1 A_2$. In the same manner an expression for the normal gravity field can be developed.

$$T^N_{12} = +(U_{x2}-U_{x1}) \cos\alpha_{12} - (U_{x2}-U_{x1}) \sin\alpha_{12} \qquad (22)$$

the term T^N_{12} is the same as T_{12} but in the normal gravity field. Now as before (22) is subtracted from (21) and using the notation $\Delta T_{12} = T_{12} - T^N_{12}$

$$\Delta T_{12} = -(-\Delta W_{y2} + \Delta W_{y1}) \cos\alpha_{12} + (-\Delta W_{x2} + \Delta W_{x1}) \sin\alpha_{12} \qquad (23)$$

substitute (16) into (23)

$$\Delta T_{12} = g_m \Delta\xi_{21} \sin\alpha_{12} - g_m \Delta\eta_{21} \cos\alpha_{12} \qquad (24)$$

Now ΔT_{12} can be computed using:

$$W_{ns} - U_{ns} = \Delta W_{ns}$$

and equation (18) becomes

$$\Delta T_{12} = 1/2 \left[\Delta W_{ns1} + \Delta W_{ns2} \right] (n_2 - n_1) \tag{25}$$

The torsion balance observation gives the quantities

$$W_{yy} - W_{xx} = W_{\Delta} \text{ and } W_{xy}$$

after subtracting the effect of the normal gravity field the observed gradient anomalies are obtained:

$$\Delta W_{\Delta} \quad \text{and} \quad \Delta W_{xy}$$

These values must be transformed to the (n, s, t) co-ordinate system.

Using the transformation equations

$$x = n \cos\alpha - s \sin\alpha$$

$$y = n \sin\alpha + s \cos\alpha$$

$$z = t$$

the partial derivatives are obtained

$$\frac{dx}{dn} = \cos\alpha \qquad \frac{dx}{ds} = -\sin\alpha$$

$$\frac{dy}{dn} = \sin\alpha \qquad \frac{dy}{ds} = \cos\alpha$$

$$\frac{dz}{dn} = 0 \qquad \frac{dz}{ds} = 0$$

Now the first partial derivatives of the potential function W are calculated by using the Chain Rule.

$$\frac{dW}{dn} = \frac{dW}{dx}\frac{dx}{dn} + \frac{dW}{dy}\frac{dy}{dn} + \frac{dW}{dz}\frac{dz}{dn} = \frac{dW}{dx}\cos\alpha + \frac{dW}{dy}\sin\alpha \tag{28}$$

$$\frac{dW}{ds} = \frac{dW}{dx}\frac{dx}{ds} + \frac{dW}{dy}\frac{dy}{ds} + \frac{dW}{dz}\frac{dz}{ds} = -\frac{dW}{dx}\sin\alpha + \frac{dW}{dy}\cos\alpha$$

Again using the Chain Rule the second partial derivative is found:

$$\frac{d^2W}{dnds} = \frac{d}{dx}\left(\frac{dW}{dn}\right)\frac{dx}{ds} + \frac{d}{dy}\left(\frac{dW}{dn}\right)\frac{dy}{ds} + \frac{d}{dz}\left(\frac{dW}{dn}\right)\frac{dz}{ds} = \tag{29}$$

$$= \left(\frac{d^2W}{dy^2} - \frac{d^2W}{dx^2}\right)\sin\alpha\cos\alpha + \frac{d^2W}{dxdy}\left(\cos^2\alpha - \sin^2\alpha\right)$$

using the Trigonometric identities

$$\sin 2\alpha = 2\sin\alpha\cos\alpha$$

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$$

gives:

$$\Delta W_{ns} = 1/2\,\Delta W_{\Delta}\sin 2\alpha + \Delta W_{xy}\cos 2\alpha \tag{30}$$

Now moving on to a third point A_3 a triangle A_1, A_2, A_3 is formed and equations corresponding to equation (24) can be written.

For side A_1 A_3

$$\Delta T_{13} = g_m \Delta\xi_{31}\sin\alpha_{13} - g_m \Delta\eta_{31}\cos\alpha_{13} \tag{31}$$

For side A_2 A_3

$$\Delta T_{23} = g_m \Delta\xi_{32}\sin\alpha_{23} - g_m \Delta\eta_{32}\cos\alpha_{23} \tag{31}$$

Summing the deflection components in a given triangle should give 0.

This gives us two more equations

$$\Delta\xi_{21} + \Delta\xi_{31} + \Delta\xi_{32} = 0$$
$$\Delta\eta_{21} + \Delta\eta_{31} + \Delta\eta_{32} = 0 \tag{32}$$

With equations (24) (31) (32) we have five equations and six unknowns. To solve this, five of the unknowns must be expressed as a function of the sixth. The six unknowns are

$(\Delta\xi_{21}, \Delta\xi_{31}, \Delta\xi_{32}, \Delta\eta_{21}, \Delta\eta_{31}, \Delta\eta_{32})$

Taking the difference between the North components at triangle side $\Delta\xi_{21}$ as an unknown then

$$\Delta\xi_{21} = \frac{-u}{g_m} \tag{33}$$

then when the equations (24), (31) and (32) are solved for the unknowns, this gives:

$$\Delta\eta_{21} = -\frac{\Delta T_{12} + u\sin\alpha_{12}}{g_m \cos\alpha_{12}} \tag{34}$$

$$\Delta\xi_{32} = \tag{35}$$

$$= \frac{\Delta T_{23}\cos\alpha_{31} + (\Delta T_{31} + g_m\Delta\xi_{21}\sin\alpha_{31} - g_m\Delta\eta_{21}\cos\alpha_{31})\cos\alpha_{23}}{g_m \sin(\alpha_{31} - \alpha_{23})}$$

$$\Delta\eta_{32} = \tag{36}$$

$$= \frac{\Delta T_{23}\sin\alpha_{31} + (\Delta T_{31} + g_m\Delta\xi_{21}\sin\alpha_{31} - g_m\Delta\eta_{21}\cos\alpha_{31})\sin\alpha_{23}}{g_m \sin(\alpha_{31} - \alpha_{23})}$$

The normal gravity value for the station can be used for g_m.

The formulas (35) (36) are then applied repeatedly to get values for subsequent lines. For example, the next line would be 3-1 so we would solve for $\Delta\xi_{31}$ and $\Delta\eta_{31}$

using appropriate subscripts in the formulas. Then the formulas would be used for the next triangle A_2, A_3, A_4, and so on until reaching the end of the chain where another astro-geodetic deflection is known (see Figure 9).

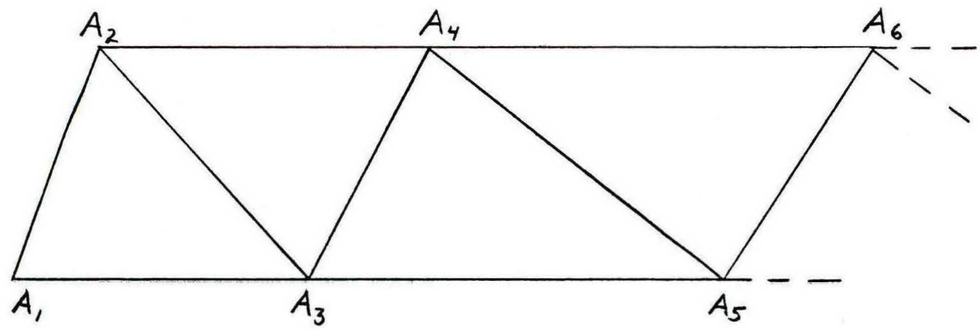


Figure 9.

At this point there are formulas for the deflection differences on each line which are unknown. Also, each formula contains the unknown u. Now u can be solved provided three values of ξ and η are known at two points in the chain, preferably at the initial point and the end.

The deflection components at the two end stations can be determined through astronomic observations or by gravimetric means. If astronomic observations are used the values are relative since the values depend on the size, shape and orientation of the reference ellipsoid used. If the deflection components are computed gravimetrically the values are considered absolute. In this case the triangle corner values will also be absolute.

B. Practical Computations

The torsion balance observations give values which must be corrected for local terrain (100 meter radius of observation point), the cartographic effect and for normal gravity. The values entered in Table IV then are actually corrected values. The constants for the torsion fibers in the instrument were not known exactly [4]. If the assumed values were incorrect the ξ and η values would have a proportional change.

The first formula used is (30)

$$\Delta W_{ns} = 1/2\, \Delta W_{\Delta} \sin 2\alpha + \Delta W_{xy} \cos 2\alpha$$

There is one ΔW_{ns} at each point for each line that terminates at the point (see Figure 10).

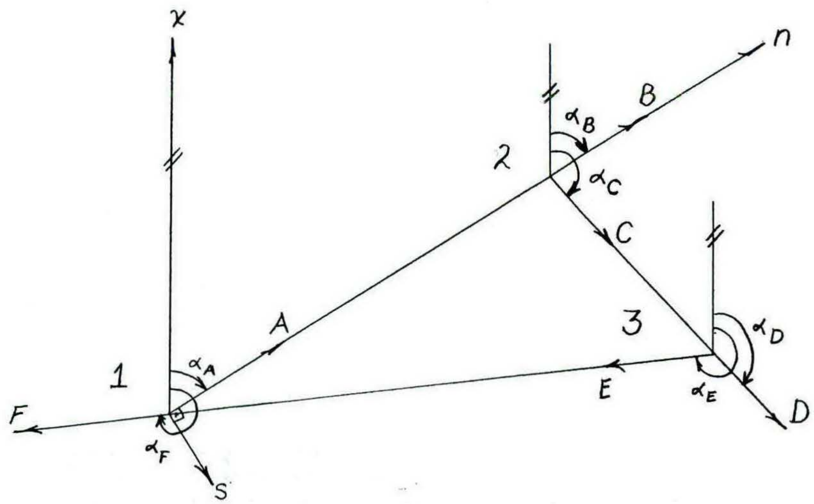


Figure 10.

α is the azimuth from north (measured from the local meridian) of the initial point (1). It was measured from a USGS map (1:62500) to the nearest tenth of a degree. For example for the line 1 - 2 the calculations require ΔW_{ns1} and ΔW_{ns2}. These are vector quantities with the magnitude represented by the gradients of gravity and the direction fixed by the azimuth taken from north.

For example (Figure 10)

Line 1-2 would require

$$\Delta W_{ns1} = 1/2\, \Delta W_{\Delta 1} \sin 2\alpha_A + \Delta W_{xy1} \cos 2\alpha_A$$

and $$\Delta W_{ns2} = 1/2\, \Delta W_{\Delta 2} \sin 2\alpha_B + \Delta W_{xy2} \cos 2\alpha_B$$

Of course, $\alpha_A = \alpha_B$, $\alpha_C = \alpha_D$, etc.

A is the vector of ΔW_{ns1} for line 1-2 and F is the vector of ΔW_{ns1} for line 3-1. Values of ΔW_{ns} are computed at each point for each line terminating at that point. The next calculation involves equation (25).

$$\Delta T_{12} = 1/2\, (\Delta W_{ns1} + \Delta W_{ns2})\, (n_2 - n_1)$$

Since this formula is derived from the Trapozoidal Rule, ΔT_{12} represents the area of a trapozoid (see Figure 11). The n values were taken from the USGS map (1: 62500) with an accuracy of $\pm$ 25 meters. The shortest

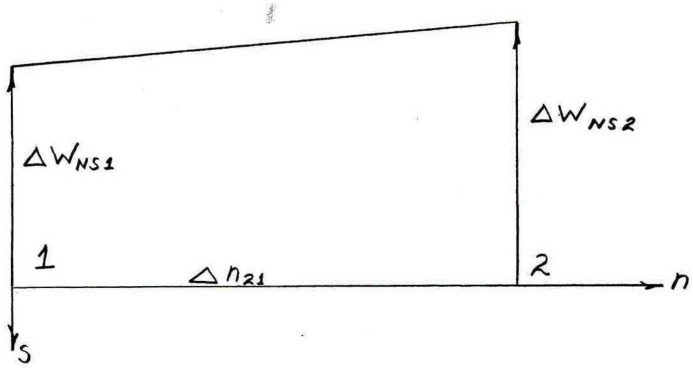


Figure 11.

line distance was 218,750 cm which is accurate to four places. ΔT is tabulated for each line in Table III.

ΔW_{Δ} and ΔW_{xy} are in Eötvös units where 1E = .0001 gals/km or 10^{-9} cm sec^{-2}/cm. ΔT is then in 10^{-9}cm sec^{-2} units, but for convenience they are tabulated in cm sec^{-2} 10^{-3}.

Solution of formulas (33) (34) is tabulated in Tables V and VI. Since the survey area was close to a straight line the triangle chain method described in Part III A was modified. The line was computed as a traverse so the triangles were not closed. Then the formulas take the following form (Table V, column 8):

$$\Delta \xi_{21} = -u$$

$$\Delta \xi_{32} = a_{32} + b_{32}u$$

$$\Delta \xi_{43} = a_{43} + b_{43}u$$

$$\Delta \xi_{54} = a_{54} + b_{54}u$$

- - -

- - -

$$\frac{\Delta\xi_{n-(n-1)} = a_{n,n-1} + b_{n,n-1}u}{}$$

$$\sum_{21}^{n,n-1} \Delta\xi = \sum_{21}^{n,n-1} a + u \sum_{21}^{n,n-1} b \qquad (37)$$

Where it is assumed the astro-geodetic latitude is known at points 1 and n: then it is known that

$$\sum_{21}^{n,n-1} \Delta\xi = \xi_1 - \xi_n$$

The only unknown in equation (37) is u which can easily be solved. a and b are constants which automatically result from the formulas. If instead the astro-geodetic longitudes at the two points were known then the $\Delta\eta$ equations (Table VI, column 12) would have to be summed in order to find u. After u is determined, it can be applied to each formula and then the $\Delta\xi$, and $\Delta\eta$ can be solved for each line.

Obviously,

$$\Delta\xi_{ik} = \xi_i - \xi_k$$

$$\Delta\eta_{ik} = \eta_i - \eta_k$$

Referring back to Tables V and VI the units of the constant term in each column are cm sec^{-2} 10^{-3} and units of terms involving u are cm sec^{-2}. In these computations the ΔW_{ns} for station 22 was an unknown.

Since the observations for station 22 were not available, this procedure was necessary. The final summation equations for ξ and η were solved as two equations with two unknowns.

The final equations for computing the u and $\Delta W_{ns_{22}}$ are

$\Delta \xi = \Delta \xi_{T-B} = +1''.094 - 2''.387 = +3''.481$

$\Delta \eta = \Delta \eta_{T-B} = +2''.173 - (-1''.020) ^{-} +3''.193$

Let $g_m = \gamma_m = 980.101$ cm sec^{-2}

$$\frac{g_m}{\rho''} = .004751955$$

$$\Delta \xi'' \text{ x } \underline{g_m} = +0.16541555 \text{ cm sec}^{-2}$$

$$\Delta \eta'' \text{ x } \underline{g_m} = +.015172992 \text{ cm sec}^{-2}$$

$$+ .2375215 + .0014097 \Delta W_{ns22} - 35.095u = +.016541555$$

$$-5.544882 + .0073066 \Delta W_{ns22} + 739.760 u = +.015172992$$

$$\text{1st } \Delta W_{ns22} = \frac{+35.0959u - .220979945}{.0014097} = +24,896u -156,756714$$

Substitute in 2nd equation

$$-5.544882 + 181.9051u - 1.1453586 + 739.760u = +.015172992$$

$$921.6651u = 6.7054136$$

$$u = \underline{\underline{.00727532}} \text{ cm sec}^{-2}$$

ΔW_{ns22}	181.1264
ΔW_{ns22}	-156,7657
	+24,3697

$$\frac{\rho''}{g_m} = \underline{210.4397} \text{ cm}^{-1} \text{ sec}^{-2}$$

Normally u is found from a single equation using $\Delta\xi_{1-n}$ or $\Delta\eta_{1-n}$, whichever is known. If both are known a check can be made but is not necessary. The g_m value is simply left out of the calculations until the final solution for u is made, and the $\Delta\xi$, $\Delta\eta$ values are computed. The International Gravity Formula was used to find a γ_m for an average latitude of the line. The value obtained was

$$g_m \cong \gamma_m = 980.161 \text{ cm sec}^{-2}$$

Table VII gives the final values where column 1 under ξ and η are in cm sec^{-2} 10^{-3} radians. Then column 2 under ξ, and η are in seconds of arc.

IV

COMPARISON OF GRAVIMETRIC AND TORSION BALANCE RESULTS

The adopted values of the deflections for the end points of the line are as follows:

TEMPLETON: $\xi'' = +1''.094$ Astro-Geodetic

$\eta'' = +2''.173$

BARR: $\xi'' = +2''.387$

$\eta'' = -1''.020$ Astro-Geodetic

η for TEMPLETON and ξ for BARR are based on the gravimetric values out to $6.5^\circ \times 6^\circ$ plus the effect of the rest of the earth. The effect over the rest of the earth is computed from the known astro-geodetic deflections at the two stations minus the gravimetric effect out to $6.5^\circ \times 6^\circ$ (see Table VIII).

The adopted values are used for the values at TEMPLETON in Table VIII. Then the deflection differences between the points based on the gradients of gravity are used to compute the deflections at the remaining points. The values obtained at BARR should be the same as the adopted values there. These deflection values are then plotted on the graphs Figures 12, 13.

The final values for the three areas around each triangulation station are summed up in Table X. These values added to the effect over the rest of the earth as computed in Table IX, give the total gravimetric deflection values. These are plotted in Figures 12, 13. Also the profile of the free air anomaly map over the line is plotted as a comparison. Results show a close comparison of the η values. The magnitude of the values obtained with the gradients of gravity are too high. This is probably caused by the use of incorrect instrument constants. The ξ values do not have too much correlation; however, this also could be a problem in instrument constants. At the end of the appendix is a discussion of a method for recomputing the constants and a graph using the new constants.

Figure 12.

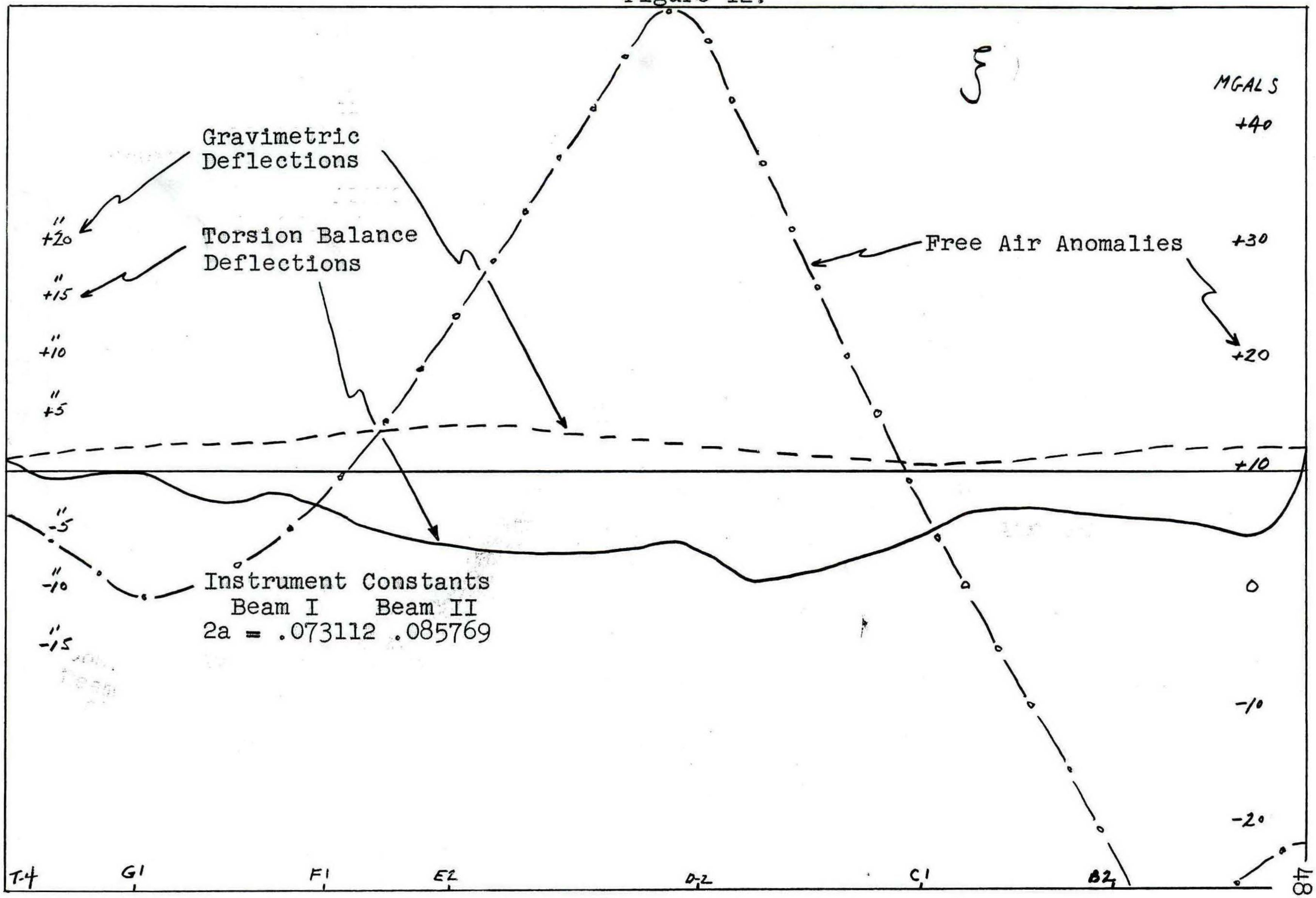
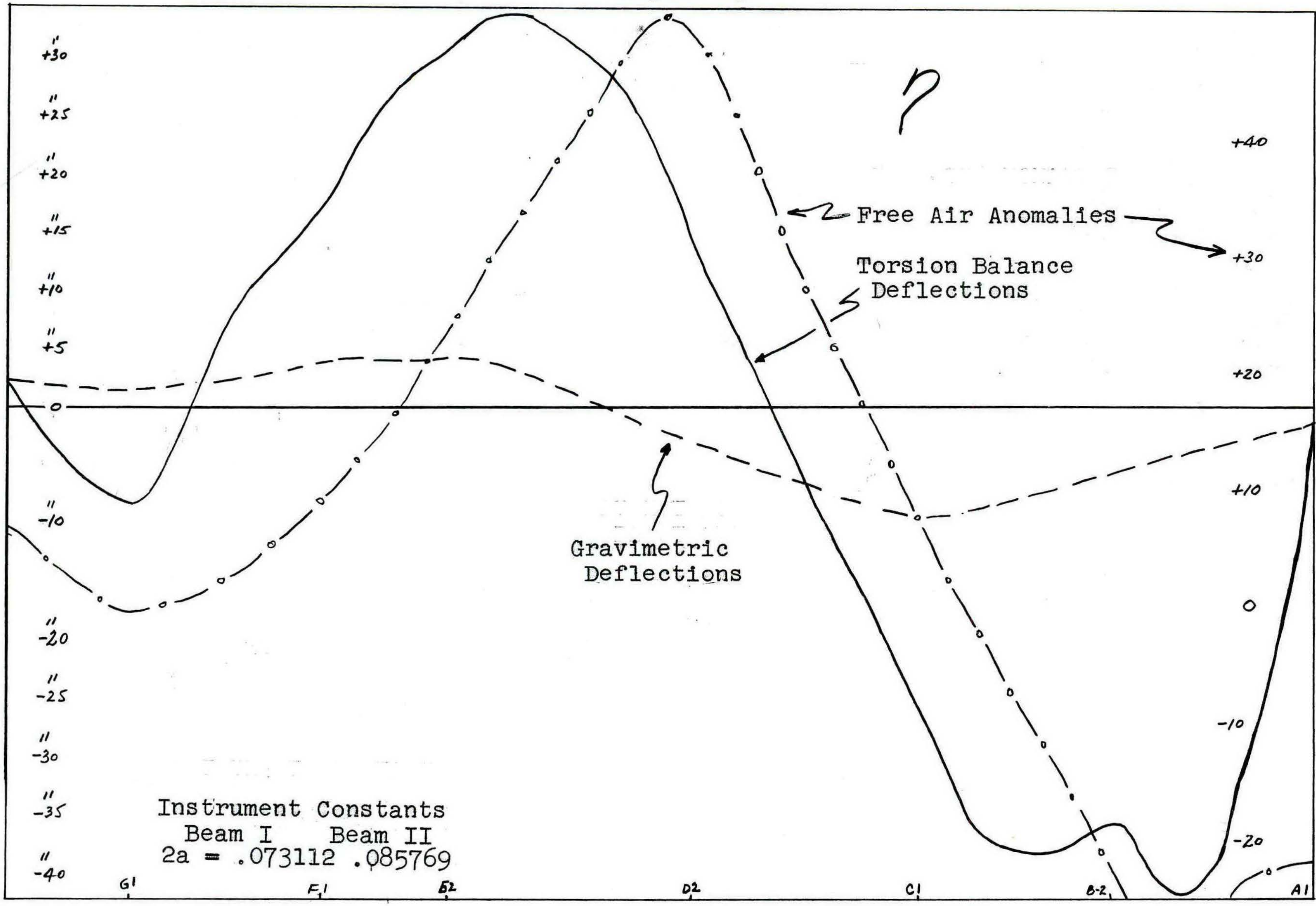

ξ
MGALS
+40
+30
+20
+10
0
-10
-20
Gravimetric Deflections
Torsion Balance Deflections
Free Air Anomalies
+20"
+15"
+10"
+5"
-5"
-10"
-15"
Instrument Constants
Beam I Beam II
2a = .073112 .085769
T-4
G1
F1
E2
D-2
C1
B2

Figure 13.



+30
+25
+20
+15
+10
+5
0
-10
-20
-25
-30
-35
-40
+40
+30
+20
+10
0
-10
-20
Free Air Anomalies
Torsion Balance Deflections
Gravimetric Deflections
Instrument Constants
Beam I Beam II
2a = .073112 .085769
G1
F1
E2
D2
C1
B-2
A1

V
ERROR ANALYSIS

The accuracy of the gradients of gravity could be effected by several factors. Since the surrounding area of the project line is fairly flat, the topographic correction was neglected. There are some hilly areas near station BARR; however, the error in assuming flat topography here would be very small. At some stations the local terrain correction was taken only to 30 meters instead of 100 meters. This could produce some error which may average out over the survey line. Errors in terrain correction are, therefore, also neglected.

The effect of possible errors in instrument constants, reading of photographic plates, and measuring quantities from the map can be found analytically since there is a functional relation between them. The following formulas were developed to observe the effects of errors in these quantities:

Development of Formulas

Assume all variables are independent

a = instrument constant

D, K, t are physical quantities of the instrument

$$a = \frac{DK}{t} \qquad da = \frac{D}{t}\,dK - \frac{Dkdt}{t^2}$$

$$\frac{da}{dK} = \frac{D}{t} \qquad \frac{da}{dt} = -\frac{DK}{t^2}$$

$$m_a = \sqrt{\left(m_K \frac{da}{dK}\right)^2 + \left(m_t \frac{da}{dt}\right)^2} = \sqrt{\left(m_K \frac{D}{t}\right)^2 + \left(m_t \frac{DK}{t}\right)^2}$$

$$W_\triangle = + \frac{.76082}{2a}(d_4 - d_3) - \frac{0.47024}{2a}(d_5 - d_2)$$

$$\frac{dW_\triangle}{da} = -\frac{0.76082}{(2a)}(d_4 - d_3) + \frac{0.47024}{(2a)^2}(d_5 - d_2)$$

Since dd is very small assume $dd_2 = dd_3 = dd_4 = dd_5$

$$dW_\triangle = + \frac{.76082}{2a}(dd_4 - dd_3) - \frac{0.47024}{2a}(dd_5 - dd_2) = 0$$

$$2W_{xy} = -\frac{0.55276}{2a}(d_3 + d_4) - \frac{1.44724}{2a}(d_2 + d_5)$$

$$\frac{dW_{xy}}{da} = + \frac{0.55276}{2(2a)^2}(d_3 + d_4) + \frac{1.44724}{2(2a)^2}(d_2 + d_5)$$

$$\frac{dW_{xy}}{dd} = -\frac{0.55276}{2a} - \frac{1.44724}{2a}$$

$$mW_\triangle = \sqrt{\left(m_a \frac{dW_\triangle}{da}\right)^2 + \left(m_d \frac{dW}{dd}\right)^2}$$

$$mW_\triangle = \sqrt{\left\{m_a \left[-\frac{0.76082}{(2a)^2}(d_4 - d_3) + \frac{0.47024}{(2a)^2}(d_5 - d_2)\right]\right\}^2}$$

$$mW_{xy} = \sqrt{\left(m_a \frac{dW_{xy}}{da}\right)^2 + \left(m_d \frac{dW_{xy}}{dd}\right)^2}$$

$$mW_{xy} = \sqrt{\left[m_a \left(+\frac{.55276}{2(2a)^2}(d_3 + d_4) + \frac{1.44724}{2(2a)^2}(d_2 + d_5)\right)\right]^2 + \left[m_d \left(-\frac{0.55276}{2a} - \frac{1.44724}{2a}\right)\right]^2}$$

$$\Delta W_{ns} = 1/2\ W_\Delta \sin 2\alpha + \Delta W_{xy} \cos 2\alpha$$

$$\frac{d\Delta W_{ns}}{d\alpha} = W_\Delta \cos 2\alpha - 2\Delta W_{xy} \sin 2\alpha$$

$$\frac{d\Delta W_{ns}}{dW_\Delta} = 1/2 \sin 2\alpha \qquad \frac{d\Delta W_{ns}}{dW_{xy}} = \cos 2\alpha$$

$$m_{\Delta Wns} = \sqrt{\left(m_\alpha \frac{d\Delta W_{ns}}{d\alpha}\right)^2 + \left(m_{W_\Delta} \frac{d\Delta W_{ns}}{dW_\Delta}\right)^2 + \left(mW_{xy} \frac{d\Delta W_{ns}}{dW_{xy}}\right)^2}$$

$$m\Delta W_{ns} = \sqrt{\left[m_\alpha \left(W_\Delta \cos 2\alpha - 2W_{xy} \sin 2\alpha\right)\right]^2 + \left[m_{W_\Delta} 1/2 \sin 2\alpha\right]^2 + \left[m_{W_{xy}} \cos 2\alpha\right]^2}$$

$$T = 1/2(\Delta W_{ns} + \Delta W_{ns})\Delta n = \Delta W_{ns}\ \Delta n$$

$$\frac{dT}{d\Delta W_{ns}} = \Delta n \qquad \frac{dT}{d_{\Delta n}} = \Delta W_{ns}$$

$$m_T = \sqrt{\left[m_{\Delta Wns} \frac{dT}{d\Delta W_{ns}}\right]^2 + \left[m_{\Delta n} \frac{dT}{d\Delta n}\right]^2}$$

$$m_T = \sqrt{\left[m_{\Delta Wns}\, \Delta n\right]^2 + \left[m_{\Delta n}\, \Delta W_{ns}\right]^2}$$

$$\Delta \xi = -\frac{2T\cos\alpha}{g_m \sin\alpha} - \Delta\xi \cos\alpha + \frac{\Delta \eta \cos^2\alpha}{\sin\alpha}$$

$$\frac{d\Delta\xi}{dT} = -\frac{2 \cot\alpha}{g_m}$$

$$\frac{d\Delta\xi}{d\alpha} = +\frac{2T}{g_m}\csc^2\alpha + \Delta\xi\sin\alpha - \Delta\eta\frac{\cos\alpha}{\sin^2\alpha} - \Delta\eta\cos\alpha$$

$$m_{\Delta\xi} = \sqrt{\left[m_T\frac{d\Delta\xi}{dT}\right]^2 + \left[m_\alpha\frac{d\Delta\xi}{d\alpha}\right]^2}$$

$$m_{\Delta\xi} = \sqrt{\left[m_T\left(\frac{-2\cot\alpha}{g_m}\right)\right]^2 + \left[m_\alpha\left(+\frac{2T}{g_m}\csc^2\alpha + \Delta\xi\sin\alpha - \Delta\eta\frac{\cos\alpha}{\sin^2} - \Delta\eta\cos\alpha\right)\right]^2}$$

$$\Delta\eta = -\frac{2T}{g_m} - \Delta\xi\sin\alpha + \Delta\eta\cos\alpha$$

$$\frac{d\Delta\eta}{dT} = -\frac{2}{g_m} \qquad \frac{d\Delta\eta}{d\alpha} = -\Delta\xi\cos\alpha - \Delta\eta\sin\alpha$$

$$m_{\Delta\eta} = \sqrt{\left[m_T\left(-\frac{2}{g_m}\right)\right]^2 + \left[m_\alpha\left(-\Delta\xi\cos\alpha - \Delta\eta\sin\alpha\right)\right]^2}$$

Using the underlined formulas with appropriate values the standard error of the quantities a, W , W_{xy}, ΔW_{ns}, T, $\Delta\xi$, and $\Delta\eta$ can be determined. This analysis should be done for each triangle in the net separately.

APPENDIX

TABLE IV

COMPUTATION OF ΔT VALUES (EQUATION 25) AND TRIGONOMETRIC VALUES

Line	Sta	α°	$2\alpha^\circ$	sin2α	$1/2\Delta W_\Delta$	cos2α	ΔW_{xy}	ΔW_{ns}	Distance Δn	ΔT cm sec^{-2} x 10^{-3}	cos α	sin α
	1				+2.78		+2.15	-.67765				
1-2		76.7	153.4	+44776		-.89415			586,250	-1.1860	+.23005	+.97318
	2				+3.54		+5.54	-3.3685				
	2				+3.54		+5.54	-6.5602				
2-3		104.4	208.8	-.48175		-.8731			650,000	-4.7118	-.24869	+.96858
	3				+0.56		+8.75	-7.9375				
	3				+0.56		+8.75	-8.7642				
3-1		271.0	542	-.03490		-.99939			1,206,250	-6.6403	+.01745	-.99985
	1				+2.78		+2.15	-2.2457				
	3				+0.56		+8.75	-8.3695				
3-4		100.5	201.0	-.35837		-.93358			430,625	-2.2067	-.18224	+.98325
	4				-0.07		+2.04	-1.8794				
	4				-0.07		+2.04	-1.7995				
4-2		283.1	566.2	-.44151		-.89726			1,075,000	-4.4791	+.22665	-.97398
	2				+3.54		+5.54	-6.5338				
	4				-0.07		+2.04	-2.0301				
4-5		92.0	184.0	-.06976		-.99756			298,750	-0.08746	-.03490	+.99939
	5				+1.17		-1.53	+1.4446				
	5				+1.17		-1.53	+1.2533				
5-3		276.0	552.0	-.20791		-.97815			796,875	-2.9572	+.10452	-.99452
	3				+0.56		+8.75	-8.6752				
	5				+1.17		-1.53	+1.8167				
5-6		81.0	162.0	+.30902		-.95106			218,750	+0.1894	+.15643	+.98769
	6				+1.14		+0.46	-.0852				
	6				+1.14		+0.46	-.3794				
6-4		268.0	536.0	+.06976		-.99756			525,000	-0.6351	-.03490	-.99939
	4				-0.07		+2.04	-2.0399				

TABLE IV (continued)

Line	Sta.	α	2α	sin2α	1/2ΔW$_{\Delta}$	cos2α	ΔW$_{xy}$	ΔW$_{ns}$	Distance Δn	ΔT cm sec^{-2} x10^{-3}	cos α	sin α
	6				+1.14		+0.46	-.7117				
6-7		96.7	193.4	-.23175		-.97278			481,250	-1.6387	-.11667	+.99317
	7				+0.08		+6.25	-6.0984				
	7				+0.08		+6.25	-6.2472				
7-5		271.3	542.6	-.04536		-.99897			700,000	-1.6701	+.02269	-.99974
	5				+1.17		-1.53	+1.4754				
	7				+0.08		+6.25	-5.9879				
7-8		98.7	197.4	-.29904		-.95424			475,000	-2.8772	-.15126	+.98849
	8				+0.32		+6.32	-6.1265				
	8				+0.32		+6.32	-6.1209				
8-6		278.8	557.6	-.30237		-.95319			945,000	-3.2622	+.15299	-.98823
	6				+1.14		+0.46	-0.7832				
	8				+0.32		+6.32	-5.9273				
8-9		101.7	203.4	-.39715		-.91775			469,375	-1.7626	-.20279	+.97922
	9				-2.60		+2.85	-1.5830				
	9				-2.60		+2.85	-1.7289				
9-7		280.5	561.0	-.35837		-.93358			931,875	-3.5376	+.18224	-.98325
	7				+0.08		+6.25	-5.8635				
	9				-2.60		+2.85	-2.4082				
9-10		94.5	189.0	-.15643		-.98769			297,500	-0.2404	-.07846	+.99692
	10				+1.63		-1.06	+0.7920				
	10				+1.63		-1.06	+0.5371				
10-8		278.5	557.0	-.29237		-.95630			771,875	-2.1614	+.14781	-.98902
	8				+0.32		+6.32	-6.1374				
	10				+1.63		-1.06	+1.3070				
10-11		85.4	170.8	+.15988		-.98714			581,250	-0.7587	-.08020	+.99678
	11				+0.07		+3.98	-3.9176				

TABLE IV (continued)

Line	Sta	α	2α	sin2α	$1/2\Delta W_\Delta$	cos2α	ΔW_{xy}	ΔW_{ns}	Distance Δn	ΔT cm sec^{-2} x10^{-3}	cosα	sinα
	11				+0.07		+3.98	-3.9800				
11-9		270.0	540.0	0		-1.0000			871,250	-2.9753	-	-1
	9				-2.60		+2.85	-2.8500				
	11				+0.07		+3.98	-3.8787				
11-12		97.0	194.0	-.24192		-.97030			408,125	-0.7928	-.12187	+.99255
	12				-6.39		+1.60	-0.0066				
	12				-6.39		+1.60	-1.4434				
12-10		270.7	541.4	-.02443		-.99970			987,500	-0.2091	+.01222	-.99993
	10				+1.63		-1.06	+1.0199				
	12				-6.39		+1.60	-1.3760				
12-13		91.0	182.0	-.03490		-.99939			550,000	-0.8305	-.01745	+.99985
	13				-1.00		+1.68	-1.6441				
	13				-1.00		+1.68	-1.5663				
13-11		273.0	546.0	-.10453		-.99452			953,750	-2.6380	+.05234	-.99863
	11				+0.07		+3.98	-3.9655				
	13				-1.00		+1.68	-1.9546				
13-14		74.0	148.0	+.52992		-.84805			371,875	-1.1233	+.27564	+.96126
	14				-6.08		+1.02	-4.0869				
	14				-6.08		+1.02	-2.2217				
14-12		264.2	528.4	+.20108		-.97958			911,250	-2.3118	-.10106	-.99488
	12				-6.39		+1.60	-2.8522				
	14				-6.08		+1.02	+0.4812				
14-15		97.0	194.0	-.24192		-.97030			775,000	-0.0174	-.12187	+.99255
	15				-3.12		+1.32	-0.5260				
	15				-3.12		+1.32	-1.4388				
15-13		268.9	537.8	+.03839		-.99926			1,131,250	-1.7851	-.01920	-.99982
	13				-1.00		+1.68	-1.7171				

TABLE IV (continued)

Line	Sta	α	2α	$\sin 2\alpha$	$1/2 \Delta W_{\Delta}$	$\cos 2\alpha$	ΔW_{xy}	ΔW_{ns}	Distance Δn	ΔT cm sec^{-2} $x10^{-3}$	$\cos\alpha$	$\sin\alpha$
	15				-3.12		+1.32	-2.0356				
15-16		83.0	166.0	+.24192		-.97030			517,500	-0.7187	_.12187	_.99255
	16				-0.26		+0.70	-0.7421				
	16				-0.26		+0.70	-0.6854				
16-14		271.5	543.0	-.05234		-.99863			1,286,250	-0.8912	+.02618	-.99966
	14				-6.08		+1.02	-0.7004				
	16				-0.26		+0.70	-0.5057				
16-17		103.5	207.0	-.45399		-.89101			420,000	-0.5452	-.23345	+.97237
	17				+1.15		+1.76	-2.0903				
	17				+1.15		+1.76	-1.8604				
17-15		272.7	545.4	-.09411		-.99556			921,875	-1.3279	+.04711	-.99889
	15				-3.12		+1.32	-1.0205				
	17				+1.15		+1.76	-2.0193				
17-18		98.5	197.0	-.29237		-.95630			650,000	-0.7246	-.14781	+.98902
	18				-6.64		+2.25	-0.2103				
	18				-6.64		+2.25	+0.5721				
18-16		281.7	563.4	-.39715		-.91775			1,050,000	+0.0173	+.20279	-.97922
	16				-0.26		+0.70	-0.5392				
	18				-6.64		+2.25	-0.1369				
18-19		98.8	197.6	-.30237		-.95319			387,500	+0.1931	-.15299	+.98823
	19				+3.66		-2.35	+1.1333				
	19				+3.66		-2.35	+1.0156				
19-17		279.6	559.2	-.32887		-.94438			1,018,750	-0.5220	+.16677	-.98600
	17				+1.15		+1.76	-2.0403				
	19				+3.66		-2.35	+2.4638				
19-20		89.1	178.2	+.03141		-.99951			425,000	-1.5203	+.01571	+.99988
	20				+4.05		+9.75	-9.6180				

TABLE IV (continued)

Line	Sta	α	2α	$\sin 2\alpha$	$1/2\Delta W_{\Delta}$	$\cos 2\alpha$	ΔW_{xy}	ΔW_{ns}	Distance Δn	ΔT cm sec^{-2} x 10^{-3}	$\cos\alpha$	$\sin\alpha$
	20				+4.05		+9.75	-10.2549				
20-18		274.4	548.8	-.15299		-.98823			805,000	-4.6137	+.07672	-.99705
	18				-6.64		+2.25	- 1.2077				
	20				+4.05		+9.75	- 9.3083				
20-21		87.2	174.4	+.09758		-.99523			533,750	-8.4205	+.04885	+.99881
	21				-11.53		+21.22	-22.2439				
	21				-11.53		+21.22	-22.5718				
21-19		269.1	538.2	+.03141		-.99951			950,000	-9.0763	-.01571	-.99988
	19				+3.66		-2.35	+2.4638				
	21				-11.53		+21.22	-23.6296		-8.0119		
21-22		81.7	163.4	+.28569		-.95832			678,125	+.3391	+.14436	+.98953
	22				-		-					
	22				-		-			-5.1656		
22-20		263.7	527.4	+.21814		-.97592			1,196,875	+.59844	-.10973	-.99396
	20				+4.05		+9.75	- 8.6318				

TABLE V

COMPUTATION OF $\Delta\xi$ (USING EQUATIONS 33,35)

		1	2		3		4	
		$+\Delta T_{i+1,i-1}$	$+\Delta\xi_{i,i-1} \, SIN\alpha_{i+1,i-1}$		$-\Delta\eta_{i,i-1} \, COS\,\alpha_{i+1,i-1}$		Σ Col. (1+2+3)	
LINE	i	$\times 10^{-3}$	$\times 10^{-3} + \mu$		$\times 10^{-3} + \mu$		$\times 10^{-3} + \mu$	
$\Delta\xi_{21}$	1	-	-	-	-	-	-	-
32	2	-6.6403	-	+ .99985u	- .08974	+ .07382u	- 6.7300	+ 1.0737u
43	3	-4.4791	+ 6.6886	- 1.1221u	- 1.7676	+ 1.0171 u	+ 0.4419	- .1050u
54	4	-2.9572	+ 12.7297	- 0.4188u	- 5.9537	+ .2378u	+ 3.8188	- .1740u
65	5	-0.6351	+ 2.0401	- 0.0873u	+ 1.9529	- 0.0870u	+ 3.3579	- .1743u
76	6	-1.6701	- 4.2567	+ .2239u	- 0.5822	+ .0321u	- 6.5090	+ .2560u
87	7	-3.2622	+ 7.5837	- .3140u	- 7.8456	+ .4134u	- 3.5241	+ .0994u
98	8	-3.5376	- 52.1404	+ 8.4278u	+ 66.669	-10.2367u	+ 10.991	- 1.8089u
109	9	-2.1614	-120.44	+17.3244u	+ 88.203	-12.5031u	- 34.398	+ 4.8213u
11-10	10	-2.9753	- 37.9917	+ 5.4229u	-	-	- 40.9670	+ 5.4229u
12-11	11	-0.2091	+ 40.9641	- 5.4225u	+ 6.1064	- .8236u	+ 46.8614	- 6.2461u
13-12	12	-2.6380	- 52.0629	+ 6.9275u	+ 22.5640	- 2.9572u	- 32.1369	+ 3.9703u
14-13	13	-2.3118	- 14.7464	+ 1.9755u	- 90.643	+11.4951u	-107.701	+13.471 u
15-14	14	-1.7851	+166.98	-20.964u	- 11.1042	+ 1.4040u	+154.091	-19.560u
16-15	15	-0.8912	-133.23	+16.912u	+ 28.421	- 3.6072u	-105.700	+13.305u
17-16	16	-1.3279	+ 87.184	-10.958u	+ 33.207	- 4.2090u	+119.063	-15.167u
18-17	17	+0.0173	-145.39	+18.503u	+125.880	-15.961u	- 19.49	+ 2.5420u
19-18	18	-0.5220	- 48.300	+ 6.6364u	+ 55.479	- 7.5112u	+ 6.6570	- .8748u
20-19	19	-4.6137	+ 70.444	- 9.5562u	- 35.108	+ 4.7510u	+ 30.722	- 4.8052u
21-20	20	-9.0763	- 3.9618	+ .8172u	+ 5.4823	- .8172u	- 7.5558	-
22-21	21	-5.1656 + .5984 $\Delta W_{NS_{22}}$	+ 7.0980	-	- 2.8879	-	- .9555	+ .5984 $\Delta W_{NS_{22}}$

TABLE V (continued)

		5	6	7	8	
LINE	i	$\Sigma \times \cos \alpha_{i,i+1} \times 10^{-3} + \mu$	$\Delta T_{i,i+1} \times 10^{-3} \times \cos \alpha_{i+1,i-1}$	NUMERATOR [SUM 5+6] $\times 10^{-3} + \mu$	-NUMERATOR/SIN$(\alpha_{i+1,i-1} - \alpha_{i,i+1}) \times 10^{-3} + \mu$	LINE
$\Delta\xi_{21}$	1	- -	-	- -	-u -	21
32	2	+ 1.6737 - .2670u	-.08222	+ 1.5915 - .2670u	- 6.8673 + 1.1521u	32
43	3	- .0805 + .0191u	-.5001	- .5806 + .0191u	- 12.7998 + .4211u	43
54	4	- .1333 + .0061u	-.0091	- .1424 + .0061u	- 2.0413 + .0874u	54
65	5	+ .5253 - .0273u	-.0066	+ .5189 - .0273u	+ 4.2578 - .2240u	65
76	6	+ .7594 - .0299u	-.0372	+ .7222 - .0299u	- 7.6740 + .3177u	76
87	7	+ .5330 - .0150u	-.4402	+ .0928 - .0150u	+ 53.0286 - 8.5714u	87
98	8	- 2.2289 + .3668u	-.3212	- 2.5501 + .3668u	+121.78 -17.5167u	98
109	9	+ 2.6989 - .3783u	-.0355	+ 2.6634 - .3783u	+ 37.9917 - 5.4229u	109
11-10	10	- 3.2856 + .4349u	-	- 3.2856 + .4349u	- 40.9670 + 5.4229u	11-10
12-11	11	- 5.7110 + .7612u	-.0097	- 5.7207 + .7612u	+ 52.1343 - 6.9370u	12-11
13-12	12	+ .5608 - .0693u	-.0435	+ .5173 - .0693u	+ 14.8223 - 1.9857u	13-12
14-13	13	-29.687 +3.7131u	+.1135	-29.574 +3.7131	-167.01 +20.968u	14-13
15-14	14	-18.779 +2.3838u	+.0003	-18.779 +2.3838u	+133.28 -16.918u	15-14
16-15	15	-12.882 +1.6215u	-.0188	-12.901 +1.6215u	- 87.281 +10.970u	16-15
17-16	16	-27.795 +3.5407u	-.0257	-27.821 +3.5407u	+148.474 -18.896u	17-16
18-17	17	+ 2.8813 - .3757u	-.1469	+ 2.7344 - .3757u	+ 48.986 - 6.7306u	18-17
19-18	18	- 1.0185 + .1338u	+.0322	- .9863 + .1338u	- 70.652 + 9.5845u	19-18
20-19	19	+ 0.4826 -0.07549u	-.1166	+ .3660 - .07549u	+ 3.9623 - .8173u	20-19
21-20	20	- 0.3691 -	+.1323	- .2368 -	- 7.1411 -	21-20
22-21	21	- .1379 + .0864 $\Delta W_{NS_{22}}$	+.8791 -.03721 $\Delta W_{NS_{22}}$	+ .7412 + .0492 $\Delta W_{NS_{22}}$	+ 21.258 + 1.4097 $\Delta W_{NS_{22}}$	22-21

$\Sigma =$ +237.5215-35.0959u+ +1.4097 $\Delta W_{ns_{22}}$

TABLE VI

COMPUTATION OF $\Delta\eta$ (USING EQUATIONS 34, 36)

		9	10	11	12
$\Delta\eta_{21}$	i 1	$\Delta T_{12} = -1.1860$	$+\mu \sin\alpha_{12} = +.97318\mu$	$(\Delta T_{12} + \mu \sin\alpha_{12})/\cos\alpha_{12} = -5.1428 + 4.2303\mu$	$+5.1428 - 4.2303\mu$
LINE	i	COL 4_i $\sin\alpha_{i,i+1}$ $\times 10^{-3} + \mu$	$\Delta T_{i,i+1} \times 10^{-3} \times \sin\alpha_{i+1,i-1} + \mu$	NUMERATOR $\Sigma(9+10)$ $\times 10^{-3} + \mu$	$-$NUMERATOR$/\sin(\alpha_{i+1,i-1} - \alpha_{i,i+1}) \times 10^{-3} + \mu$
$\Delta\eta_{32}$	2	− 6.5185 + 1.0400u	+4.7111	− 1.8074 + 1.0400u	+ 7.7989 − 4.4876u
43	3	+ 0.4345 − .1032u	+2.1493	+ 2.5838 − .1032u	+ 56.9621 − 2.2751u
50	4	+ 3.8165 − .1739u	+0.0870	+ 3.9035 − .1739u	+ 55.956 − 2.4928u
65	5	+ 3.3166 − .1722u	−0.1893	+ 3.1273 − .1722u	+ 25.661 − 1.4130u
76	6	− 6.4645 + .2543u	+1.6383	− 4.8262 + .2543u	+ 51.282 − 2.7022u
87	7	− 3.4835 + .0983u	+2.8433	− 0.6402 + .0983u	−365.83 + 56.1715u
98	8	+ 10.7626 − 1.7713u	+1.7330	+ 12.4956 − 1.7713u	−596.73 + 84.589u
109	9	− 34.2921 + 4.8064u	+0.2378	− 34.054 + 4.8064u	−488.16 + 68.8991u
11-10	10	− 40.8351 + 5.4054u	+0.7587	− 40.0764 + 5.4054u	−499.706 + 67.399u
12-11	11	+ 46.5123 − 6.1996u	+0.7927	+ 47.3050 − 6.1996u	−431.104 + 56.499u
13-12	12	− 32.1321 + 3.9697u	+0.8294	− 31.3027 + 3.9697u	−896.926 +113.745u
14-13	13	−103.53 +12.949u	+1.1175	−102.413 +12.949u	−578.343 + 73.125u
15-14	14	+152.94 −19.414u	+0.0174	+152.96 −19.414u	−1,085.59 +137.786u
16-15	15	−104.91 +13.206u	+0.7185	−104.19 +13.206u	−704.89 + 89.344u
17-16	16	+115.77 −14.748u	+0.5446	+116.375 −14.748u	−620.74 + 78.706u
18-17	17	− 19.279 + 2.5141u	+0.7095	− 18.570 + 2.5141u	−322.67 + 45.039u
19-18	18	+ 6.5786 − .8645u	−0.1904	+ 6.3882 − .8645u	+457.607 − 61.927u
20-19	19	+ 30.718 − 4.8046u	+1.5158	+ 32.234 − 4.8046u	+348.97 − 52.015u
21-20	20	− 7.5468 −	+8.4195	+ .8727 −	+ 26.318 −
22-21	21	− .9455 −	+7.9635	+ 7.0180 −	+20.109 −
		+ .5921 $\Delta W_{ns_{22}}$	− .3371 $\Delta W_{ns_{22}}$	+ .2550 $\Delta W_{ns_{22}}$	+ 7.3066 $\Delta W_{ns_{22}}$

$$\Sigma = -5544.8822 + 739.760u + 7.3066\,\Delta W_{ns_{22}}$$

TABLE VII

FINAL COMPUTATION OF $\Delta\xi$ AND $\Delta\eta$ USING COMPUTED VALUES FOR u, and g_m

Line	1 $g_m \Delta\xi 10^{-3}$	2 $\Delta\xi''$	1 $g_m \Delta\eta 10^{-3}$	2 $\Delta\eta''$
2-1	- 7.275	-1.53	-25.634	- 5.39
3-2	+ 1.515	+0.31	-24.850	- 5.23
4-3	- 9.736	-2.05	+40.410	+ 8.50
5-4	- 1.405	-0.30	+37.820	+ 7.96
6-5	+ 2.628	+0.55	+15.381	+ 3.24
7-6	- 5.363	-1.13	+31.623	+ 6.65
8-7	- 9.331	-1.96	+42.835	+ 9.01
9-8	- 5.660	-1.19	+18.682	+ 3.93
10-9	- 1.461	-0.30	+13.103	+ 2.76
11-10	- 1.514	-0.31	- 9.357	- 1.97
12-11	+ 1.665	+0.35	-20.056	- 4.22
13-12	+ 0.376	+0.08	-69.395	-14.60
14-13	-14.461	-3.04	-46.335	- 9.75
15-14	+10.196	+2.15	-83.153	-17.50
16-15	- 7.471	+1.57	-54.884	-11.55
17-16	+11.000	+2.32	-48.129	-10.13
18-17	- 0.019	0.00	- 5.000	- 1.05
19-18	- 0.992	-0.19	+ 7.068	+ 1.49
20-19	- 1.984	-0.42	-29.456	- 6.20
21-20	- 7.141	-1.50	+26.318	+ 5.54
22-21	+36.38	+7.89	+167.00	+35.32

Note: u = 0.00727532 cm sec^{-2}

g_m = 980.101 cm sec^{-2}

TABLE VIII

FINAL COMPUTED VALUES FOR ξ AND η USING THE DIFFERENCES FOUND IN TABLE VII

Station	ξ''	η''
1 TEMPLETON	+ 1.094*	+ 2.173
2	- 0.44	- 3.22
3 FOUST	- 0.13	- 8.45
4	- 2.18	+ 0.05
5	- 2.48	+ 8.01
6	- 1.93	+11.25
7 TAYLOR	- 3.06	+17.90
8	- 5.02	+26.91
9	- 6.21	+30.84
10	- 6.51	+33.60
11	- 6.82	+31.63
12	- 6.47	+27.41
13	- 6.39	+12.81
14	- 9.43	+ 3.06
15	- 7.28	-14.44
16 OWERS	- 5.71	-25.99
17	- 3.39	-36.12
18	- 3.39	-37.17
19	- 3.58	-35.68
20	- 4.00	-41.88
21	- 5.50	-36.34
22 BARR	+ 2.387	- 1.020*

* Astro-Geodetic Deflections

Note: Torsion balance observations were not available for triangulation stations LANUM, HOYT, and MEINFELER.

TABLE IX

TOTAL GRAVIMETRIC DEFLECTIONS TO INCLUDE COMPUTED EFFECT OF ENTIRE EARTH

TEMPLETON

$\xi''_{AG} = +1.094$

$\Delta\xi''(6^o \times 6^p) = -0.346$

Effect of Δg over earth outside $(6.5^o \times 6^o)$ $\xi''_{AG} - \Delta\xi''(6.5^o \times 6^o) = +1''.440$

Station	$\Delta\xi''(6.5^o \times 6^o)$	Total ξ''
TEMPLETON T-4	- 0.346	+ 1.094
FOUST G-1	+ 0.726	+ 2.166
TAYLOR F-1	+ 1.689	+ 3.129
LANUM E-1	+ 2.612	+ 4.052
HOYT D-1	+ 0.736	+ 2.176
OWERS C-1	- 0.696	+ 0.744
MEINFELER B-1	+ 0.677	+ 2.117
BARR A-1	+ 0.947	+ 2.387

BARR

$\eta''_{AG} = -1''.020$ $\Delta\eta''(6.5^o \times 6^o) = -0''.949$

Effect of Δg on η over earth outside $(6.5^o \times 6^o)$ $= \eta''_{AG} - \Delta\eta''(6.5^o \times 6^o) = -0''.071$

Station	$\Delta\eta''(6.5^o \times 6^o)$	Total η''
TEMPLETON T-4	+ 2.244	+ 2.173
FOUST G-1	+ 1.769	+ 1.698
TAYLOR F-1	+ 3.979	+ 3.908
LANUM E-1	+ 4.246	+ 4.175
HOYT D-1	- 3.953	- 4.024
OWERS C-1	- 9.147	- 9.218
MEINFELER B-1	- 4.573	- 4.644
BARR A-1	- 0.949	- 1.020

TABLE X

SUM OF GRAVIMETRIC DEFLECTIONS INCLUDING
INNER CIRCLE, RICE'S RINGS AND (6.5° x 6°)

Station	ξ'' Inner Ring	ξ'' Rice's Rings (25'x25')	ξ'' 6.5°Lat 6°Long.	ξ'' Total	η'' Inner Ring	η'' Rice's Rings (25'x25')	η'' 6.5°Lat 6°Long.	η'' Total
TEMPLETON (T-4)	+.434	- .027	-.753	- .346* +1.094*	+ .579	- .122	+1.787	+2.244
FOUST (G-1)	+.238	+ .597	-.109	+ .726	- .389	+ .004	+2.154	+1.769
TAYLOR (F-1)	+.397	- .822	+.470	+1.689	+ .399	+1.483	+2.097	+3.979
LANUM (E-1)	+.971	+1.781	-.140	+2.612	+1.081	+2.481	+ .684	+4.246
HOYT (D-1)	+.429	+ .545	-.238	+ .736	- .550	-1.738	-1.665	-3.953
OWERS (C-1)	-.247	- .397	-.052	- .696	-1.718	-3.994	-3.435	-9.147
MEINFELER (B-1)	+.338	+ .339	-.093	+ .677	+ .565	-1.609	-3.529	-4.573
BARR (A-1)	+.635	+ .322	-.010	+0.947	+..454	+ .520	-1.923	-0.947 -1.020*

*Astro-Geodetic Deflections

Possible error in instrument constant: The following method was used to find a better constant. The gradients of gravity in the east-west direction are compared using a torsion balance observation and the inner circle gravity anomaly map. The triangulation station TAYLOR was selected for comparison because of its fairly linear pattern of gravity anomaly change. The W_{xy} for TAYLOR with the terrain correction is +14.6E. Using Rice's three gradient method weighting the center difference 1 and the 45° point difference as 1/2 the following value is obtained W_{yz}=5.8E Therefore 5.8 ÷ 14.6 = 0.4. Using this analysis the value of the instrument constant should be reduced by a factor of .4 then the values of the deflection would also be reduced by .4. The results are shown in Figures 18, 19.

Figure 14.

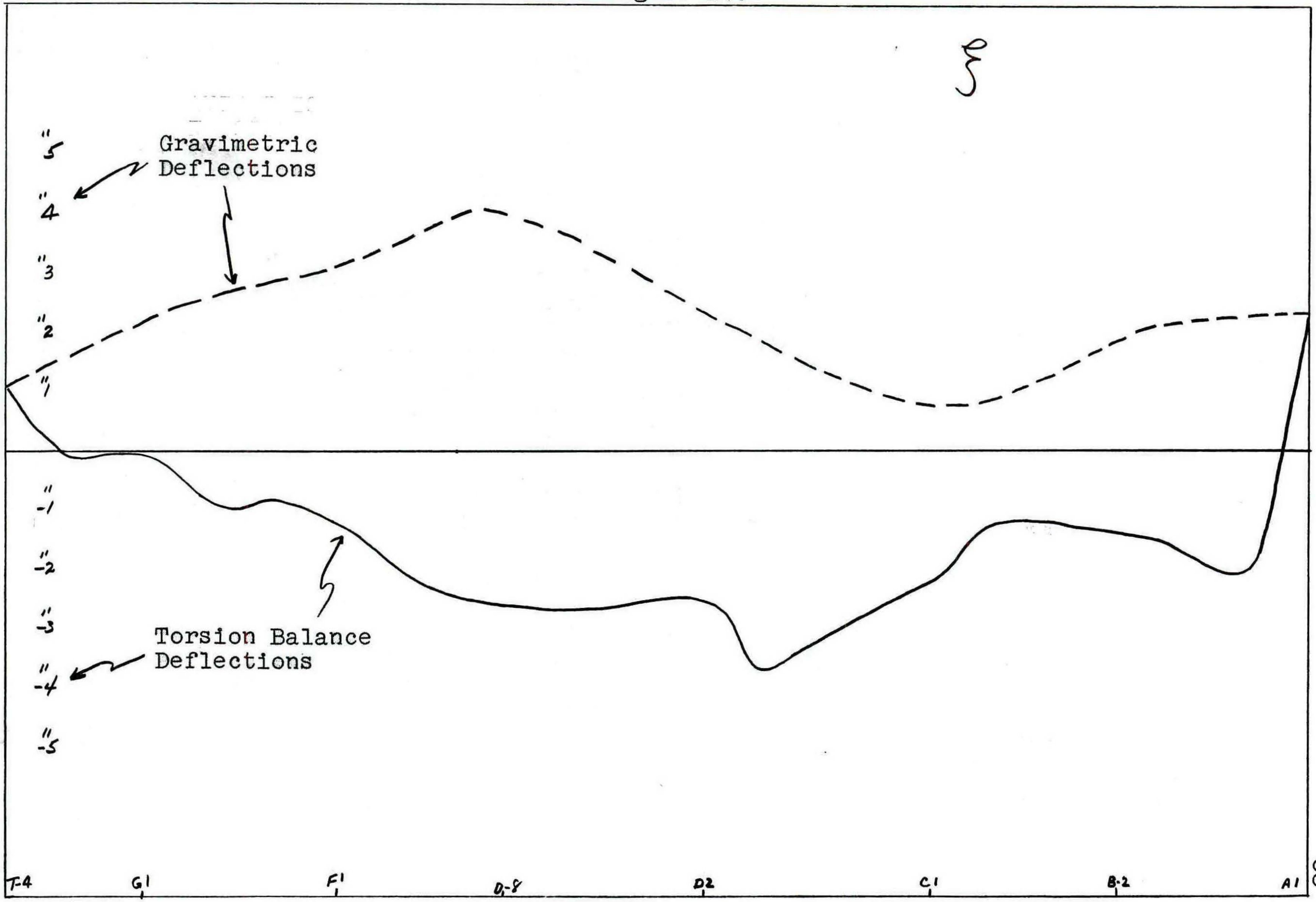
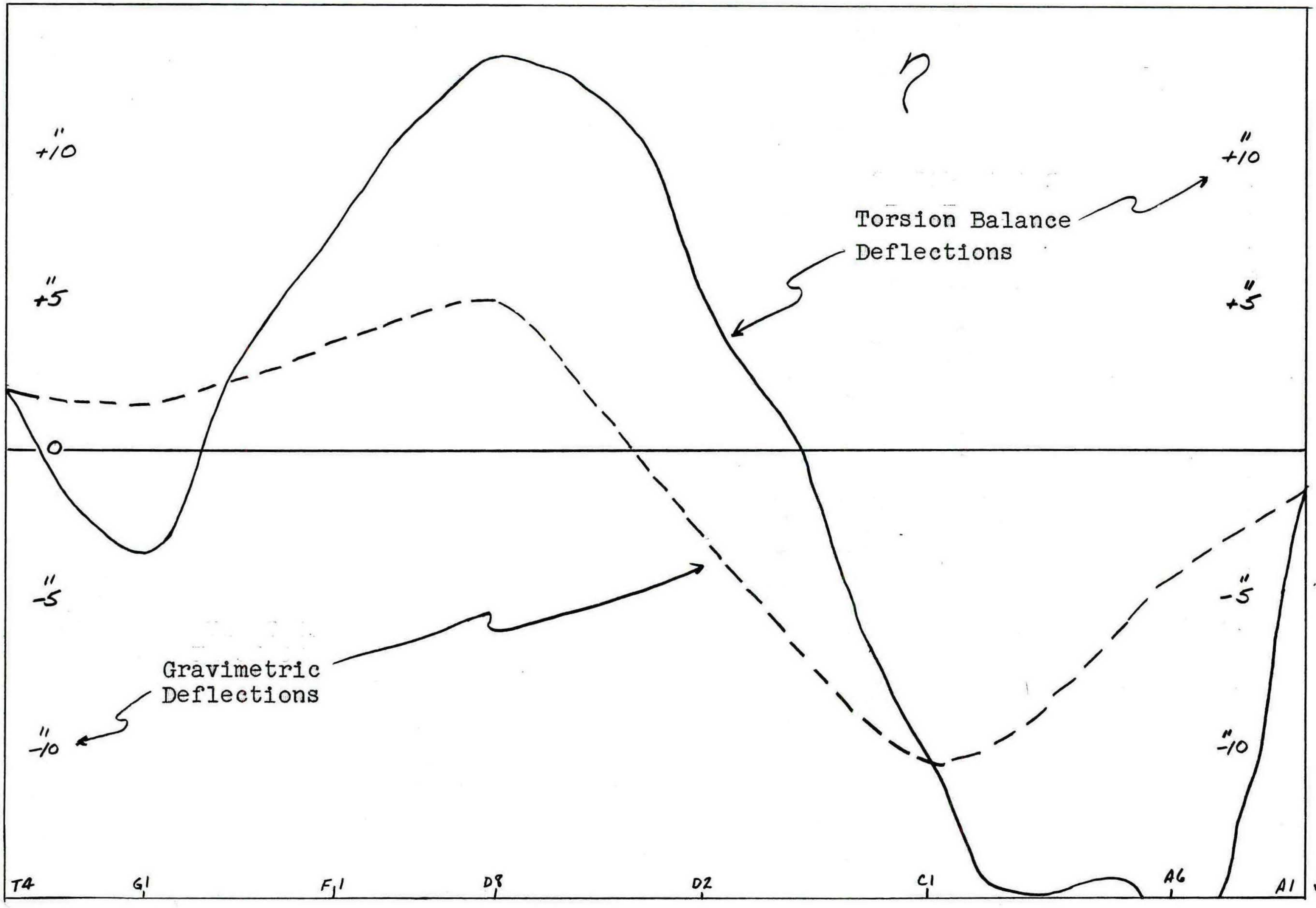

Gravimetric Deflections
Torsion Balance Deflections
"5
"4
"3
"2
"1
-"1
-"2
-"3
-"4
-"5
T-4
G1
F1
D-8
D2
C1
B-2
A1

Figure 15



+10"
+5"
0
-5"
-10"
+10"
+5"
-5"
-10"
Torsion Balance
Deflections
Gravimetric
Deflections
T4
G1
F1
D8
D2
C1
A6
A1

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