Essays in Microeconomics Theory

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Abstract

In the first essay, we model information transmission in a rumor-spread approach, wherein the goal of the Rumor Maker (RM) is to start a rumor that can spread to the largest possible number of Decision Makers (DM). In the model, an RM designs and commits to a rumor-generating mechanism. After the state is realized, a rumor of the state is generated and sent to the first DM. Each DM needs to choose a binary action by playing a guessing-the-state game, and also decides whether to spread the rumor or block it. The DMs move sequentially in a prefixed order. A DM transmits a rumor to the next DM only when she expects the following DM's gain from the rumor to be larger than the cost of reading the rumor; otherwise this rumor will be blocked. In addition to the rumor, DMs have a common signal and private signals about the state. We show that when DMs' private signals dominate common signal in terms of accuracy, always telling the truth is one of the best-spread rumors. However, when DMs' common signal dominates, the unique best-spread rumor tells the truth with a certain probability. We also show that as the cost of reading a rumor and the accuracy of DM's signals increase, the best spread rumor converges to the truth.

In the second essay, we study an endogenous timing learning model over a star network, in which there is 1 central player connected with n periphery players. Players in each period face two options: make an irreversible investment or wait for another period. Players

receive a binary private signal on the profitability of investment at the beginning of the game, and also observe neighbors' actions in past periods. We show that there exists a threshold of network size ($\overline{N}$) : when the size of the network is small ($n \leqslant \overline{N}$), in equilibrium, periphery players use a pure strategy which fully reveals their private signals to the central player; when the network is large ($n > \overline{N}$), periphery players use mix strategy and only partially reveal their signals to central player. The central player waits in the first period and then makes the final decision on whether to invest at all in the second period (based on the number of first-period investment of periphery players). Among the equilibriums, the central player works as a crowdsourcing platform, collecting information from some peripheral players and deliver it to the rest players. We also show that Asymptotic Learning does not occur in star network: the probability of central player making the right action does not converge to 1 as the size of the network increases to infinity, which indicates a failure of information collecting in star structure.

In the third essay, we develop a search and matching model in housing market with restricted purchase policy, where an agent cannot own more than one house. I show that in equilibrium, the housing price is lower than its value in free market. By comparing restricted purchase policy with restricted price policy in a search and matching model, I show that to reach a certain amount of price decrease, restricted purchase policy generates a larger welfare cost and a lower temporary excess return for prospective buyers. I also show that in stationary equilibrium, the excess returns in both restricted purchase and restricted price market equal to zero, which indicates the source of excess return is not stable restricted policies, but the policy shocks.

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Chapter 1. Introduction

There are three essays in this dissertation, studying information transmission in network and search game. The first essay models rumors spread through social network, where a strategic rumor maker designs a rumor to be spread through social network and people in the network who receive the rumor decide whether transmit the rumor to friends or block the rumor. We find the conditions for the best spread rumor to converge to the truth, which draws a clear policy implication on how to reduce the social cost from fake news rumor. In the second essay, we add a star network structure on endogenous timing learning model and find an interesting pattern in star network equilibriums, where the central player works as a crowdsourcing platform, collecting information from some peripheral players and deliver it to the rest players. The third essay uses a search model to explain why restricted purchase policy can decrease housing prices and we show the policy advantage of restricted purchase is generating a lower temporary excess return for prospective buyers and attracting less real estate speculators.

Chapter 2. Is Truth the Best Spread Rumor?

2.1 Introduction

Rumors spread through social networks in many forms, such as word of mouth, emails, Facebook articles, and Twitter messages. Some of the rumors are true, while some are not true and generate a large social cost. There are two types of rumors and each has a different purpose: rumors generated to persuade people; and rumors generated to be spread among people.

The target of the first type is to change the beliefs of decision makers and persuade them to make the choice favored by the Rumor Maker (RM), such as political rumors that persuade voters to support or disfavor a politician and advertisement rumors that persuade consumers to purchase or not purchase a product. This type of information transmission is well studied by economic research, such as in the Cheap Talk model (Crawford and Sobel, 1982) and the Bayesian Persuasion model (Kamenica and Gentzkow, 2011).

The second type of rumor is designed to be spread widely. Most are generated by "We Media", such as subscriptions to YouTube or WeChat, personal websites, Blogs, and Facebook. The RM's goal is to devise a rumor that will spread to many people. RMs do not care which choice the receivers make. They benefit only when receivers transmit the rumor to other people. An example is the rumor-filled article spread among friends through social media (Facebook, WeChat). As the rumor spreads, it increases the RM's popularity and generates profits from advertisements or tips.

To the best of our knowledge, this paper is the first economics paper to study the second type of rumor. We are interested in two questions: How does an RM decide how to devise a rumor and what conditions make truth become the best-spread rumor.

Consider the example of rumors about weather. If you hear a rumor that tomorrow there will not, as usual, be rain (assume it's a dry area), you are highly unlikely to transmit this rumor to your friends because it will not change your friend's action (e.g., whether to bring an umbrella); it will only cost your friend the time and attention needed to hear the rumor. If you hear a rumor that tomorrow it will rain, it is more likely that you will transmit the rumor to friends in order to remind them to bring an umbrella.

Thus, an RM who knows the weather well may want to devise more rumors about rain, rather than a lack of rain, even if the latter is the truth. However, if an RM keeps telling it will rain, people will stop believing him and block the rumor. The best strategy for an RM is to find a balance between the boring truth and surprising news. This strategy allows a rumor to spread more easily and be sufficiently credible.

In this paper, we use an information transmission model to study the general problem of the spread of rumors. In particular, the state of the world is binary and privately observed by the RM. There are two possible rumors corresponding to the two possible states. The RM's goal is to design a rumor-generating mechanism (which rumor to send for each state) that can make the rumor spread to as many DMs as possible. Each DM needs to choose a binary action by playing a guessing-the-state game, and the DMs move sequentially in a prefixed order. Reached by a rumor, each DM also decides whether to transmit the rumor to the next DM or block it. A DM transmits the rumor only when she believes, for the next DM, the gain from the rumor is larger than the cost of reading the rumor (the time cost and the attention cost). DMs know the accuracy of the rumor, and this can be regarded as the RM's reputation. In addition to the rumor, DMs receive private signals and a common signal about the state. The common signal represents the global news, while private signals represent people's individual information.

We derive two main results for the basic model. First, when DM's private signals are more accurate than the common signal, there is a continuum of optimal rumors and always telling the truth is one of them. This is because without the rumor each DM will act according to his private signal. But since a DM does not observe the private signal of the next DM, he will pass on the rumor only if it is more accurate than the private signals to help the next DM's decision making. Second, when the common signal is more accurate than the private signals, the optimal rumor is unique, which has the accuracy level such that when the common signal and the private signal are in the same direction and the rumor has the opposite direction a DM is indifferent between two actions. In particular, truth telling is not optimal. This is because in this case without the rumor a DM's action will follow the common signal. But since the common signal is observed by all DMs, a DM will pass on the rumor only if the rumor and the common signal point to different directions and the rumor is more accurate. In order to maximize the probability that the rumor and the common signal have different signals, the rumor should be as less accurate as possible. These two opposite forces determine the level of accuracy of the optimal rumor.

We also show that as the cost of reading a rumor and the accuracy of a DM's signals (common signal and private signals) increases, the best-spread rumor converges to the truth.

We devise three extensions on the basis of the benchmark model. (1) We study competition among RMs and find that when there are 2 RMs, there is a unique equilibrium in which the strategy of both RMS is to always tell the truth. This indicates that competition in a rumor game will lead to truth. (2) We add a survey cost for the RM to learn the state and assume that the higher the accuracy of the rumor, the higher the survey cost. We find that the RM may send a less informative rumor when there is a large survey cost. (3) We study the case in which a DM updates his belief even if the rumor is blocked. A new equilibrium is found in which DMs only transmit one direction of the rumors and block the other direction. For example, the DMs only transmit the rumor points to state 1 (when the rumor is informative enough) and always block the rumor points to state 0. Here the RM's strategy is similar to Bayesian Persuasion: the RM persuades DMs to change their pre-established belief and choice.

The first economic research to show an interest in rumors was Banerjee (1993), which modeled rumors as the spread of a DMs' herding behavior. Thereafter, researchers modelled rumors as messages or fake news transmitted from information senders to receivers.

Currently, there are two main approaches to rumor research. In the first, rumor generation is exogenous: biased senders or social bots send misinformation to affect the beliefs of receivers (Acemoglu et al, 2010, Azzimontia and Fernandes 2018). These analysts study how information structure and the social network structure affect the degree of misinformation and polarization.

Among these papers, the work most closely related to ours is that of Bloch et al (2018), in which a biased sender spreads fake news rumors through social networks, and other players decide whether to transmit or block them. The main difference between their paper and ours is that in their model the rumor is exogenously given, whereas in ours the rumor is designed to be best spread by the RM. Thus, by endogenizing the rumor-generating process, our model is able to identify the best rumor-generating mechanisms for the RM.

The second approach to rumor transmission is to model how the strategic rumor sender persuades receivers. Ambrus et al (2013) introduces the hierarchy cheap talk model and Ivanov (2010) studies strategic mediator. In these papers, both the sender and intermediators are biased and try to persuade the DM to make the choice favored by sender and intermediators. The main difference between these papers and our paper is that we model the RM and all intermediators as unbiased agents. The RM does not care what the DM chooses. The RM’s only goal is to spread the rumor, while the DM’s goal is to benefit his or her friends by transmitting or blocking the rumor.

The remainder of the paper is organized as follows. The main model is presented in Section 2. We characterize the equilibrium outcomes in Section 3, while section 4 provides three extensions of the model. Section 5 concludes.

2.2 Model

In the game, there are 1 Rumor Maker (RM) and n Decision Makers (DM). DMs are ordered and noted as DM1, DM2 … DMn.

The RM's action is to set and commit to a rumor-generating mechanism. The rumor is about the state θ and there are two underlying states with equal probability, $\theta \in \{0,1\}$. All players (1 RM and n DMs) have this common prior of the state.

After the state is realized, RM observes the state, produces a rumor based on the rumor generating mechanism, and sends it to DM1. Then, depending on DMs' reactions, the rumor may or may not spread among DMs.

The RM's payoff is the number of DMs the rumor reaches (not including the DM1). The rumor generating mechanism that maximizes rumor spread is the "best rumor". A rumor-generating mechanism is known by all DMs, and it can be described as follows. If the state $\theta = 1$, the RM sends a "High" signal with probability p and a "Low" signal with probability $1 - p$. If the state $\theta = 0$, the RM sends a "High" signal with probability q and a "Low" signal with probability $1 - q$. WLOG, we set a "High" signal in favor of state 1 and a "Low" signal in favor of state 0. If $p = 1$ and $q = 0$, then RM always tells the truth. A rumor-generating mechanism can be summarized as (p, q).

After the state is realized, each DM receives two binary signals of the state: a common signal $s \in \{0,1\}$, which is same among all DMs, and a private signal $s_i \in \{0,1\}$ for DMi. (The RM does not know the realized signals.) The accuracy of the common signal is α and the accuracy of the private signal is β:

$$pr(\theta = 0|\mathrm{s} = 0) = pr(\theta = 1|s = 1) = \alpha \tag{2.1}$$

$$pr(\theta = 0|\mathrm{s}_i = 0) = pr(\theta = 1|\mathrm{s}_i = 1) = \beta \tag{2.2}$$

A DM plays a guessing state game: there are two candidate actions $a \in \{0,1\}$: if the action matches the state, the payoff is 1; otherwise, payoff is 0. In addition to the guessing state game, a DM, once reached by a rumor, needs to decide whether to "transmit" the rumor to the next DM or “block” the rumor. When making the decision, the DM considers whether transmitting the rumor will benefit the next DM. The gain for the next DM is the information offered by the rumor. There is a fixed cost c to read the rumor, which can be regarded as the time and attention spent reading the rumor. Because reading the rumor requires time and attention, we assume that this fixed cost is unavoidable for the DM who is reached by a rumor.

The DM1 is guaranteed to receive the rumor; whether the following DMs receive the rumor depends on the actions of other DMs. The DMi receives the rumor only when DM1...DMi-1 all transmit the rumor.

We also assume that if the rumor is blocked by the previous DM, the following DM will not be aware of the rumor game and make her decision only on the basis of her own signals. DMs will not update their belief on the grounds that the rumor is blocked. (We discuss this assumption in section 4.3, which examines the case in which DMs update their belief when they observe that the rumor is blocked.)

The timing of the game is as follows:

(1) The RM sets the rumor-generating Mechanism (p,q) and commits to it.

(2) The state is realized. The rumor is generated and sent to DM1. All DMs receive the common Signal and private signals.

(3) DM1 plays the guessing state game. DM1 decides whether to transmit the rumor to DM2.

(4) DM2 plays the guessing state game. If reached by the rumor, DM2 decides whether to transmit it to DM3…

2.3 Equilibriums

To find the equilibrium strategy for the RM, we begin with the case: $n = 2$ and show that the best strategy of the RM remains the same when n is larger than 2. When $n = 2$, RM's strategy is simplified: he or she sets a rumor-generating mechanism (p, q) to maximize the probability that DM1 transmits the rumor to DM2.

Proposition 2.1 When the private signals of DMs dominate the common signal ($\beta > \alpha$), always telling the truth creates one of the best rumors.

The details of the proof are provided in the Appendix. Here we sketch the basic argument. We use backward induction to find the equilibrium. If the rumor is blocked by DM1, DM2 will follow his private signal because $\beta > \alpha$. Thus, when the rumor is blocked, DM1's expectation about the payoff to DM2 is always β, which is the probability that DM2's private signal is correct. If $c > 1 - \beta$, then all rumors will be blocked because the rumor is never informative enough to overcome the cost. If $c < 1 - \beta$, then we can always find a strategy of RM that can guarantee that all rumors will be transmitted. As long as the rumor is sufficiently informative, it will be always transmitted. One example is always telling the truth ($p = 1, q = 0$). In equilibrium, DM1 always transmits the rumor and DM2 always follows the rumor, regardless of each DM's own signals.

When the cost of the rumor increases or when the accuracy of private and common signals increases, the area of the best rumor shrinks and converges to the truth.

We can also see that if $n > 2$, the best rumor remains the same. This is because following the equilibrium strategy, when DM1 transmits the rumor to DM2, DM2 will also transmit the rumor to following DMs since the rumor is sufficiently informative. Thus, the best rumor remains the same when we have more DMs.

RM's payoff is always $n-1$, while DM's payoff depends on which rumor the RM chooses. Among the equilibriums, the highest expected payoff for DM2 is $1-c$, which occurs when the RM always tells the truth ($p=1, q=0$). The lowest expected payoff for DM2 is $\frac{1}{\frac{1-\alpha}{\alpha}\frac{1-\beta}{\beta}\frac{1-\beta-c}{\beta+c}+1}-c$, when the rumor is ($p=\frac{1}{\frac{1-\alpha}{\alpha}\frac{1-\beta}{\beta}\frac{1-\beta-c}{\beta+c}+1}, q=\frac{1}{\frac{\alpha}{1-\alpha}\frac{\beta}{1-\beta}\frac{\beta+c}{1-\beta-c}+1}$).

If there is no RM, the expected payoff for DMs is β because every DM will follow her or his private signals. Comparing this to the rumor game, we find that the best rumor strictly increases the DM's payoff because the lowest expected payoff for DM2 is larger than β.

Proposition 2.2 When the common signal dominates private signals ($\alpha > \beta$), the best rumor is to set $p^* = 1 - q^* = \frac{1}{1+\frac{1-c}{1+c}\frac{1-\alpha}{\alpha}\frac{1-\beta}{\beta}}$.

The proof can be found in the Appendix. The main intuition is that if the rumor is blocked by DM1, DM2 will follow the common signal because $\alpha > \beta$. Thus, if a rumor is the same as the common signal, it will be blocked by DM1 because it will not change DM2's action and will result only in a reading cost.

Only two possible cases for a rumor being transmitted are possible: a "Low" rumor when the common signal is "High"; and a "High" rumor when the common signal is "Low".

There are two candidate types of strategy for RM: (1) Sending an informative rumor which will be transmitted even when it differs from both common signal and private signal of DM1; (2) Sending a less informative rumor which will only be transmitted when the rumor and private signal have same direction, which is different from common signal. We show that type 1 strategy is always better than type 2 strategy for RM (details of proof are provided in Appendix). The intuition is that because all rumor and signals are informative (the accuracy is larger than 0.5), the probability for common signal and private signal have different directions is small.

In equilibrium, RM's strategy is to set

$$p^* = 1 - q^* = \frac{1}{1 + \frac{1-c}{1+c}\frac{1-\alpha}{\alpha}\frac{1-\beta}{\beta}} \tag{2.3}$$

We can see from the equilibrium that the equilibrium rumor is a symmetry binary signal whose accuracy equals p^*. When the rumor and the common signal are the same, the rumor is blocked; when the rumor differs from both the common and the private signals, DM1 is indifferent between transmitting and blocking the rumor. If the rumor is less accurate, it will not be transmitted when differs from both the common and the private signals ; if it is more accurate, there is less probability that it will be different from the common signal, which makes RM worse off.

We can also see that if $n > 2$, the best rumor remains the same. This is because following the equilibrium strategy, when DM1 transmits the rumor to DM2, DM2 will also transmit the rumor to following DMs since the rumor is sufficiently informative and differs from the common signal. Thus, the best rumor remains the same when we have more DMs.

The probability that the best rumor will be transmitted is $\alpha(1-p^*) + (1-\alpha)p^*$, and so the expected payoff for RM is

$$\mathrm{EU}_{RM}{}^* = (n-1)[\alpha(1-p^*) + (1-\alpha)p^*] \tag{2.4}$$

The RM's expected payoff decreases when the accuracy of signals (α and β) or the cost of the rumor increases. This is because higher accuracy signals or a higher cost make RM's best rumor more accurate, lowering the chance that it will differ from the common signal.

The expected payoff for DMs (DM2…DMn) is

$$\mathrm{EU}_{DM}{}^{*} = \alpha p^{*} + (1-\alpha)p^{*}(1-c) - \alpha(1-p^{*})c \qquad (2.5)$$

The first term on the RHS (5) is the payoff when the common signal and the rumor both match the realized state. Here the rumor will be blocked, and the DM will choose the correct action, in which case the payoff is 1. The second term is the payoff when the common signal is different from the realized state and the rumor is same as the state. Here the rumor will be read, and the DM will choose the correct action, in which case the payoff is $1-\mathrm{c}$. The third term is the payoff when the common signal matches the state but the rumor does not. Here the rumor will be read, and the DM will choose the wrong action, in which case the payoff is $-c$. When neither the common signal nor the rumor matches the state, the rumor will be blocked and the DM will choose the wrong action, in which case the payoff is 0.

We can see that as the accuracy of signals increases, the DMs' expected payoff also increases. If there is no RM, the expected payoff for DMs is α because every DM will follow the common signal. We can find that the best rumor strictly increases the DMs payoff.

2.4 Extensions

Based on the benchmark model in Section 3, we devise two extensions of the model.

2.4.1 Competition among RMs

In section 3, we study the model with only 1 RM. In this section, we discuss how the competition among RMs affects the best rumor. We are especially interested in whether competition among RMs necessarily leads to high informative rumors. For example, where two independent RMs both choose to spread a low informative rumor, will their rumors confirm each other, and will they be transmitted among DMs together?

In the new setting, there are 2 RMs and they set the rumor-generating mechanism simultaneously and independently. After the state is realized, two rumors are sent to DM1. Each DM needs to choose whether to (1) transmit both rumors, (2) transmit one rumor and block the other, or (3) block both rumors. To read each rumor, DMs need to pay a fixed cost c (when they read the 2 rumors, they need to pay 2c).

From the benchmark model we know the best strategies for the RM among all cases are always symmetric between states ($p^* = 1 - q^*$). Thus, the best strategy for a single RM is to set an optimal accuracy level of the rumor. In order to keep focusing our study on the accuracy of rumor when there are more RMs, we restrict the RMs' strategy to be symmetry between states by limiting $q \equiv 1 - p$.

A rumor-generating mechanism of RMi can be summarized as the accuracy of rumor p_i:

$$pr(\theta = 0|\mathrm{r}_i = 0) = pr(\theta = 1|\mathrm{r}_i = 1) = p_i \tag{2.6}$$

When the DM is indifferent between transmitting Rumor 1 and Rumor 2, the tie breaking rule is to randomly chooses a rumor to transmit with equal probability. The rest of the model is the same as what is described in section 3.

Proposition 2.3 When there are 2 RMs, there is a unique equilibrium wherein the strategies of both RMs is to always tell the truth.

The details of proof are provided in the Appendix. We sketch the basic argument here. It is easily seen that when both RMs tell the truth $p_1 = p_2 = 1$, no RM has an incentive to deviate. To demonstrate that this is the only equilibrium, we prove that, given the accuracy of RM1 $p_1 < 1$, the best strategy for RM2 is $p_2{}^* > p_1$. Thus, the only equilibrium is $p_1 = p_2 = 1$.

There is no "colluding equilibrium" wherein both DMs send low informative rumors in order to target those rumors being transmitted together. This is so because the RMs have an incentive to deviate to a more accurate rumor, which guarantees that it will be transmitted even if two rumors are different.

2.4.2 DMs update their belief even if the rumor is blocked

In this section we change the assumption that DMs are not aware of the rumor when it is blocked. Now we assume that the DMs know the game even if the rumor is blocked. DMs always use the Bayesian rule to update their beliefs. When a rumor is blocked, DMs update their belief accordingly and their actions may differ from their own signals.

We still begin with $\mathrm{n} = 2$ and then show that the equilibrium strategy of the RM is same when $\mathrm{n} > 2$. We can see that all equilibriums in section 3 still hold in the new setting. The DM's strategy is to follow his or her own signal when the rumor is blocked. Consequently, in section 3, the RM and the other DM have no incentive to deviate from their strategies.

However, other equilibriums exist. One candidate equilibrium is DM1 only transmits one type of rumor, for example, only transmits "Low" rumor when it is informative enough and always blocks "High" rumor. DM2 chooses state 1 if the rumor is blocked; otherwise she chooses state 0. Given the strategy of DM1, the RM will set a rumor (p, q) to maximize the probability of sending a "Low" rumor under the condition that the rumor is informative enough to be transmitted.

Given the strategy of DMs, the RM can have two candidate strategies: (1) Send an informative rumor whose target is the transmission of a "Low" rumor (even the common signal and private signal are both "High"); and (2) Send a less informative rumor whose target is the transmission of a "Low" rumor only when at least one of the signals is also low. We show that the first strategy is always better than the second strategy (proof can be found in Appendix). Thus, RM's strategy is to set rumor (p, q)

$$\text{Max}_{p,q}\ \frac{1}{2}(1-p+1-q) \tag{2.7}$$

s.t

$$\frac{(1-q)(1-\alpha)(1-\beta)}{(1-q)(1-\alpha)(1-\beta)+(1-p)\alpha\beta} - \mathrm{c} \geq 1 - \frac{(1-q)(1-\alpha)(1-\beta)}{(1-q)(1-\alpha)(1-\beta)+(1-p)\alpha\beta} \tag{2.8}$$

(2.7) is the probability of sending a "Low" rumor. (2.8) is the condition that the DM1 with "High" common signal and "High" private signal will transmit a "Low" rumor.

From (2.7) (2.8) we have

$$p^{**} = 1 - \frac{1-c}{1+c}\frac{1-\alpha}{\alpha}\frac{1-\beta}{\beta}, \qquad q^{**} = 0 \tag{2.9}$$

$$EU_{RM}{}^{**} = \frac{1}{2}\left(1 + \frac{1-c}{1+c}\frac{1-\alpha}{\alpha}\frac{1-\beta}{\beta}\right) \tag{2.10}$$

$$EU_{DM2}{}^{**} = \frac{1}{2} + \frac{1}{2}(1-c)(1 - \frac{1-\alpha}{\alpha}\frac{1-\beta}{\beta}) \tag{2.11}$$

In such an equilibrium, the RM maximizes the probability of sending a "Low" rumor when that "Low" rumor is informative enough to be transmitted. DM1 always blocks a "High" rumor and transmits a "Low" rumor. DM2 chooses state 1 when the rumor is blocked and chooses state 0 when the rumor is transmitted (after paying the fixed cost). We find that in the Bayesian Persuasion Model, the equilibrium is very similar: the RM sends a rumor to persuade DM1 to believe in state 0.

As the accuracy of the DM's signals increases, the payoff for the RM decreases and the payoff for the DM increases. This is because the DM's signals make it harder for the RM to persuade and the RM has to offer more informative rumors, which benefits the DM and hurts the RM.

2.5 Conclusion

We build an information transmission model to explain rumor-spreading behavior. The goal of the Rumor Maker (RM) is to design a rumor about the state that can spread to as many DMs as possible. Each DM plays a guessing state game, and once reached by a rumor, the DM also decides whether to transmit the rumor to the next DM or block it. A DM transmits the rumor only when she believes the gain from rumor is larger than the cost of reading the rumor.

We show that when a DMs' private signal dominates the common signal, always telling the truth is one of the best rumors. When the DM's common signal dominates the private signal, the unique best-spread rumor tells the truth with a certain probability. We also show that as the cost of reading a rumor or the accuracy of the DM's signals increases, the best-spread rumor converges to the truth.

We devise two extensions based on the benchmark model. (1) We study the competition among RMs and find that when there are 2 RMs, there is a unique equilibrium wherein the strategy of both RMs is to always tell the truth. This indicates that competition in the rumor game will lead to the truth. (2) We study the case wherein DMs update their belief even if the rumor is blocked. A new equilibrium is found where DMs only transmit one direction of the rumor and block the other direction. Here the RM's strategy is similar to Bayesian Persuasion: the RM persuades DMs to believe in the state described by the rumor.

In this paper we assume the RM and DMs are ordered and located in a "Line-Network". This model also applies to any "Tree-Network" in which RM the is the Root. In a Tree-Network, the direction of rumor spread is certain (it moves from root to leaves), and so the RM and the DMs employ the same strategy used in the "Line-Network". In types of social network that contain a ring structure, the equilibrium strategy is different and deserves further research because the direction of rumor spread, which is not predetermined, depends on the network's structure.

Chapter 3. Social Learning and Strategic Delay in the Star Network

3.1 Introduction

In many cases people make decisions on the basis of observation of friends' actions. Several network learning models describe this type of issue, including Gale et al. (2003), Golub et al. (2010), and Acemoglu et al. (2011). However, among the existing network learning models, one question remains unsolved: how to model the order of players' actions endogenously—in other words, how to explain the direction of learning.

Acemoglu et al. (2011) assume the order is exogenous. Players locate in a directed network and make decisions one by one in a given order. This model suits settings like purchasing a new smartphone, where the order of actions is predetermined by the expiration date of an existing service.

In Gale et al. (2003) and Golub et al. (2010), players make simultaneous repeated actions period by period on the basis of neighbors' previous actions. This type of model suits a setting like forming an opinion about politics, where actions are repeated and reversible.

Yet the power of existing models to describe more common situations, such as those in which there is no predetermined order of players and action is irreversible, is limited. For example, firms need to decide whether to make an investment but are not sure whether the state is good for investment. One option is for the firm is wait and observe the actions of neighbor firms, which reveal neighbors' information. In this setting, exist models cannot work because there is no exogenous order of player, as noted by Acemoglu et al. (2011), and the action is not reversible, as noted by Gale et al. (2003) and Golub et al. (2010).

Our paper tries to explain the direction of learning in a social network by introducing an endogenous timing learning model first developed by Chamley and Gale (1994). In Chamley and Gale (1994), players decide whether and when to make an irreversible action on the basis of the previous action of every player. In our model, a player cannot see everyone's action, but can only observe her neighbors' action in network.

Our model has the following features. Players (firms or potential investors) receive a binary signal on the state of the economy at the beginning of the game. (We call a player who receives the favorable signal a type 1 player and a player who receives the unfavorable signal a type 0 player.) Players choose between investing and waiting for another period. Waiting and observing the investment decisions of neighbor firms could be used to improve inference about the economic state.

We study one type of network architecture, the star network, in which the central player can observe the past actions of all peripheral players, whereas peripheral players can only observe the actions of the central player. The star network structure reflects the reality cases of the crowdsource platform, where in equilibrium the central player plays the role of collecting information from some peripheral players and delivering information to the rest. We show that when the number of periphery players is small, in equilibrium, all type 1 periphery players invest in the first round and central player observe these actions and make the "invest now or never" decision in the second round, and the rest players follow the central player's decision. When the number of periphery players is large (larger than a threshold), the previous equilibrium cannot hold because the action of the central player is so informative that type 1 periphery players prefer waiting to invest. The equilibrium lies in the pattern in which only some of the type 1 periphery players invest in the first round and the rest wait to observe the central player.

We find that asymptotic learning does not occur in both two endogenous timing models. As the number of periphery players increases, the probability that the central player will make the right action does not converge to 1. This is because in equilibrium, periphery players only reveal a certain level of information to the central player, and they do so to keep the balance between waiting for the central player and investing in the first round.

The rest of the paper is organized as follows. In section 2, we present the model, while in section 3 we show the equilibrium strategy. Conclusions are offered in section 4.

3.2 Model

In a star network there is 1 central player and n peripheral players. The central player is connected to all other players and each peripheral player is only connected to the central player. Links are undirected, and players who are connected to each other are called neighbors.

The timeline of the game is as follows: first, each player i (the central player and peripheral players) privately observes a binary signal s_i that is correlated with the binary economy state. We call a player who has received the high signal, $s_i = 1$, a type 1 player and a player who has received the low signal, $s_i = 0$, a type 0 player.

In each round, starting with round 0, each player observes the history of the behavior of her neighbors, and players not yet invested simultaneously decide whether to invest or wait. More formally, for t = 0, 1, ..., denote the action of player i in round t as $e_i^t \in \{0, 1\}$, where the action, 0, represents no change in status (either not yet invested or invested in a previous round) and the action, 1, represents investing in that round. We assume that once a firm invests it remains invested.

Players decide whether and when to make an irreversible investment with an uncertain return, which depends on the binary state $\theta \in \{0,1\}$. We also assume that both values of the underlying state are equally likely and players prior are 1/2. When $\theta = 1$, the return of investment is 1, and when $\theta = 0$, the return is -1. All players are risk-neutral and impatient. Impatience is measured by the discount factor δ. If the player does not invest, the payoff is 0.

The payoff function is

$$\mathrm{u}_i(e_i, \theta) = \begin{cases} \delta^t & if\ e_i^t = 1, \theta = 1 \\ -\delta^t & if\ e_i^t = 1, \theta = 0 \\ 0 & if\ e_i^t = 0, \forall\ t \end{cases} \tag{3.1}$$

The private signal is binary and independent with accuracy $\alpha \in \left(\frac{1}{2}, 1\right)$:

$$pr(\theta = 0|\mathrm{s}_i = 0) = pr(\theta = 1|\mathrm{s}_i = 1) = \alpha \tag{3.2}$$

We further require that δ is large enough: $\delta > \sqrt{\frac{2\alpha-1}{\alpha}}$. This condition gives the room for type 1 periphery players to choose waiting. (If $\delta < \sqrt{\frac{2\alpha-1}{\alpha}}$, type 1 periphery players always prefer investing in round 0 regardless of the network structure.)

3.3 Equilibrium

A strategy for player i is a mapping from network position, signal realization and observed histories of neighbors' actions into a decision of whether to make an irreversible invest or wait for another period. A central player's observed history h_c^{t-1} can be described as $h_c^{t-1} = (x^0, x^1, \dots, x^{t-1})$, where $x^t = \sum_n e_i^t$ denote the number of periphery players who invest in round t (observed by central player). A periphery player's observed history h_p^{t-1} can be described as $h_p^{t-1} = \{\, e_c^0,\ e_c^1, \dots, e_c^{t-1}\}$. The definition of a pure/mix strategy perfect Bayesian equilibrium follows the common definition.

Proposition 3.1 Given the size of network n, there exists a threshold $\bar{\delta_1}(n)$: When $\delta \leq \bar{\delta_1}$, there is a unique pure strategy perfect Bayesian equilibrium in which type 1 players (central and periphery) invest in round 0; the type 0 central player waits in round 0 and in round 1 uses a cutoff strategy: invest only if the number of investments in round 0 is larger than $\frac{n+1}{2}$; otherwise wait for all rounds. The Type 0 periphery player waits in round 0 and then follows the central player's action.

Proof. To guarantee that players have no incentive to deviate, we need the condition that the expected payoff for type 1 central players to invest in round 0 is no less than the expected payoff to wait in round 0 (and invest in round 1 if it is profitable). Thus, we have the condition

$$2\alpha - 1 \geq \delta \sum_{\frac{n-1}{2}}^{n} q(x)[2\alpha'(x) - 1] \tag{3.3}$$

The RHS is the best candidate deviation for the type 1 central player: wait in round 0 and in round 1 invest only if the number of investments in round 0 is larger than $\frac{n-1}{2}$; otherwise wait for all rounds.

$\alpha'(x)$ is the updated belief if the type 1 central player sees x out of n peripheral players invest in round 0

$$\alpha'(x) = \frac{\alpha^{x+1}(1-\alpha)^{n-x}}{\alpha^{x+1}(1-\alpha)^{n-x} + (1-\alpha)^{x+1}\alpha^{n-x}} \tag{3.4}$$

$q(x)$ is the type-1 central player's expected probability of observing x players invest in round 0

$$q(x) = C_n^x[\alpha^{x+1}(1-\alpha)^{n-x} + (1-\alpha)^{x+1}\alpha^{n-x}] \tag{3.5}$$

From (3.3) -(3.5) we get the condition for such pure strategy

$$2\alpha - 1 \geq \delta \sum_{x=\frac{n-1}{2}}^{n} C_n^x[\alpha^{x+1}(1-\alpha)^{n-x} - (1-\alpha)^{x+1}\alpha^{n-x}] \tag{3.6}$$

$\bar{\delta}_1$ is the value of discount factor makes (3.6) hold with equality. As long as $\delta \leq \bar{\delta}_1$, condition (3.3) holds, and the type 1 central player has no incentive to deviate. It is easy to see that under condition (3.3), type 0 players and type 1 periphery players do not deviate. Thus, we have a pure strategy equilibrium in which both the type 1 central player and type 1 periphery players invest in round 0. ■

Proposition 3.1 shows that when players are not patient enough, type 1 players give up the information contained in the network and choose to react on their own signal. If the central player becomes more patient, type 1 central players will have an incentive to deviate and wait for periphery players.

In equilibrium, the information flow direction is from the type 1 player to the type 0 player. Type 1 players invest in round 0 and reveal their signals to neighbors.

Condition (3.3) also defines the threshold $\bar{N}_1$ of the size of network n given a fix discount factor δ, where $\bar{N}_1$ is the largest integer that satisfies condition (3.3). Given the discount factor, such a pure strategy equilibrium can exist only if the size of network is small. If the network is large, type 1 central players will have an incentive to deviate and wait for periphery players information revealed in round 0.

Proposition 3.2 Given the size of network n, there exists a discount factor $\bar{\delta}_2$(n); when $\bar{\delta}_1 \leq \delta \leq \bar{\delta}_2$, there is a pure strategy perfect Bayesian equilibrium in which type 1 periphery players invest in round 0; type 0 periphery players wait in round 0 and then follow the central player's action. The central player (type 1 or 0) waits in round 0. In round 1, the Type 1 central player uses cutoff strategies: invest only if the number of investments in round 0 is larger than $\frac{n-1}{2}$; otherwise wait for all rounds. The Type 0 central player also uses cutoff strategies with a threshold $\frac{n+1}{2}$.

Proof. First, to guarantee that the type 1 central player has no incentive to deviate from waiting to investing in round 0, we need the condition

$$\delta \geq \bar{\delta}_1 \tag{3.7}$$

Second, to guarantee that type 1 periphery players have no incentive to deviate from investing to waiting in round 0, we need the condition

$$2\alpha - 1 \geq \ \delta^2 \, \tilde{q}[2\tilde{\alpha} - 1] \tag{3.8}$$

LHS of (3.8) is the type 1 periphery player's expected payoff for investing in round 0. RHS is the type 1 peripheral player's expected payoff for waiting in round 0.

$\tilde{q}$ is the expected possibility that the central player invests in round 1.

$$\tilde{q} = \sum_{x=\left\lfloor\frac{n}{2}\right\rfloor+1}^{n} C_n^x[\alpha^{x+1}(1-\alpha)^{n-x} + (1-\alpha)^{x+1}\alpha^{n-x}] \tag{3.9}$$

To understand (3.9), we know that the probability that the central player will invest in round 1 when a type 1 periphery player deviates can be calculated as the probability that among the other n players (1 central player and $\mathrm{n}-1$ periphery players), more than $\mathrm{n}/2$ players get a high signal.

$\tilde{\alpha}$ is the updated belief if type 1 peripheral players observe the central player investing in round 1.

$$\tilde{\alpha} = \frac{\sum_{\bar{x}}^{n} C_n^x \alpha^{x+1}(1-\alpha)^{n-x}}{\sum_{\bar{x}}^{n} C_n^x \{\alpha^{x+1}(1-\alpha)^{n-x} + (1-\alpha)^{x+1}\alpha^{n-x}\}} \tag{3.10}$$

Combining (3.8)-(3.10), we get the condition

$$2\alpha - 1 \geq \delta^2 \sum_{x=\left\lfloor \frac{n}{2} \right\rfloor + 1}^{n} C_n^x \{\alpha^{x+1}(1-\alpha)^{n-x} - (1-\alpha)^{x+1}\alpha^{n-x}\} \tag{3.11}$$

$\bar{\delta}_2$ is the value of δ that makes (3.11) holds with equality. Thus, when $\bar{\delta}_1 \leq \delta \leq \bar{\delta}_2$, there exists such pure strategy perfect Bayesian equilibrium ■

Proposition 3.2 shows that when $\bar{\delta}_1 \leq \delta \leq \bar{\delta}_2$, type 1 periphery players invest in round 0, while the type 1 central player waits in round 0. The difference in strategy between the central player and periphery players is due to the difference in the network position.

There exists such a pure strategy equilibrium only if the players' patience lies within a certain range. If players become less patient, type 1 central players will tend to deviate and invest in round 0; if players are more patient, type 1 periphery players will tend to deviate and wait for the central player.

Another candidate learning pattern occurs when the type 1 central player invests in round 0, while all periphery players wait for the central player to reveal her signal. We show that such an equilibrium cannot exist because type 1 periphery players strictly prefer investing in round 0 to waiting for the central player's action, as can be seen from (3.12)

$$2\alpha - 1 \geq \delta^2 [\alpha^2 + (1-\alpha)^2][2\frac{\alpha^2}{[\alpha^2 + (1-\alpha)^2]} - 1] \tag{3.12}$$

The LHS of (3.12) is the expected utility for a type 1 periphery player who invests in round 0; the RHS is the expected utility for a type 1 periphery player who waits in round 0 and follows the central player's action. We see that (3.12) always holds as long as $\delta < 1$, which indicates that there is no equilibrium in which type 1 periphery players wait for the central player.

Condition (3.11) also defines the threshold $\bar{N}_2$ of the size of network n given a fix discount factor δ, where $\bar{N}_2$ is the largest integer that satisfies condition (3.11). In another interpretation of Proposition 3.2, there exists such a pure strategy equilibrium only if the network is medium size. In a medium-sized network, the information contained in the network is large enough to keep the type 1 central player waiting, yet it is not large enough to cause type 1 periphery players to wait. If the size of the network becomes smaller than $\bar{N}_1$, even the central player will not wait, in which case the equilibrium is the same as in Proposition 3.1; if the size of the network becomes larger than $\bar{N}_2$, type 1 periphery players will have an incentive to deviate and wait for the central player because the information contained in the network is sufficiently large.

From (3.11) we can see that threshold $\bar{N}_2(\delta)$ decreases with δ. This is because when players become more patient, periphery players have more of an incentive to deviate and wait for the central player, which leads the threshold $\bar{N}_2$ decreases.

Proposition 3.3 Given the size of network n, when $\delta > \bar{\delta}_2(n)$, there exists a symmetry mixed strategy perfect Bayesian equilibrium: Type 1 periphery players invest in round 0 with probability $p(\delta)$,wait with probability $1 - p(\delta)$, and then follow the player's action.

Type 0 periphery players wait in round 0 and then follow the central player's action. The central player waits in round 0 and in round 1 uses the cutoff strategy to make the final decision on whether invest or not.

Proof. Because type-1 peripheral players use a mixed strategy, we need an indifference condition between waiting in round 0 and investing in round 0.

$$2\alpha - 1 = \delta^2 \tilde{q}_p \left[2\tilde{\alpha}_p - 1\right] \tag{3.13}$$

where $\tilde{q}_p$ is the type-1 peripheral player's expected probability that the central player will invest in round 1

$$\begin{aligned}\tilde{q}_p = \alpha[\alpha \textstyle\sum_{\bar{x}}^{n-1} f(x) + (1-\alpha)\sum_{\tilde{x}}^{n-1} f(x)] + \\ (1-\alpha)[(1-\alpha)\textstyle\sum_{\bar{x}}^{n-1} g(x) + \alpha \sum_{\tilde{x}}^{n-1} g(x)]\end{aligned} \tag{3.14}$$

In (3.14), $f(x)$is the expected probability that the central player observes x investment when a type-1 peripheral player waits and the state $\theta = 1$.

$$f(x) = C_{n-1}^{x}(\alpha p)^x(1-\alpha p)^{n-x-1} \tag{3.15}$$

$g(x)$is the expected probability that the central player observes x investment when a type-1 peripheral player waits and the state $\theta = 0$.

$$g(x) = C_{n-1}^{x}[(1-\alpha)p]^x[1-(1-\alpha)p]^{n-x-1} \tag{3.16}$$

$\tilde{\alpha}_p$ is the updated belief if type-1 peripheral observes the central player investing in round 1

$$\tilde{\alpha}_p = \frac{\alpha[\alpha \sum_{\bar{x}}^{n-1} f(x) + (1-\alpha)\sum_{\tilde{x}}^{n-1} f(x)]}{\tilde{q}_p} \tag{3.17}$$

$\bar{x}$ is the threshold for the type-1 central player's cutoff strategy and $\tilde{x}$ is the threshold for type-0 central player's cutoff strategy.

$\bar{x}$ is the smallest integer that satisfies condition (3.18)

$$\frac{\alpha(\alpha p)^x(1-\alpha p)^{n-x}}{\alpha(\alpha p)^x(1-\alpha p)^{n-x}+(1-\alpha)[(1-\alpha)p]^x[1-(1-\alpha)p]^{n-x}} \geq \frac{1}{2} \tag{3.18}$$

The LHS of (3.18) is the posterior of a type 1 central player who observes x out of n periphery players investing in round 0. Only when the posterior is larger than 1/2 is it profitable for the central player to invest in round 1. From (3.18), we can obtain the expression of $\bar{x}$

$$\bar{x} = 1 + \left\lfloor n \frac{1}{1+\frac{ln\alpha - \ln(1-\alpha)}{\ln[1-(1-\alpha)p]-\ln(1-\alpha p)}} + \frac{1}{\frac{\ln[1-(1-\alpha)p]-\ln(1-\alpha p)}{\ln(1-\alpha)-ln\alpha}-1} \right\rfloor \tag{3.19}$$

$\tilde{x}$ is the threshold for the type-0 central player. To make investment profitable, we need condition (3.20) and $\tilde{x}$ is the smallest integer that satisfies condition (3.20)

:

$$\frac{(1-\alpha)(\alpha p)^x(1-\alpha p)^{n-x}}{(1-\alpha)(\alpha p)^x(1-\alpha p)^{n-x}+\alpha[(1-\alpha)p]^x[1-(1-\alpha)p]^{n-x}} \geq \frac{1}{2} \tag{3.20}$$

The LHS of (3.20) is the posterior of a type 0 central player who observes x out of n periphery players invest in round 0. Only when the posterior is larger than ½ is it profitable for the central player to invest in round 1. From (3.20), we can get the expression of $\tilde{x}$

$$\tilde{x} = 1 + \left\lfloor n \frac{1}{1+\frac{ln\alpha - \ln(1-\alpha)}{\ln[1-(1-\alpha)p]-\ln(1-\alpha p)}} - \frac{1}{\frac{\ln[1-(1-\alpha)p]-\ln(1-\alpha p)}{\ln(1-\alpha)-ln\alpha}-1} \right\rfloor \tag{3.21}$$

Combining (3.13) -(3.21), the indifference condition can be written as:

$$2\alpha - 1 = \delta^2\alpha\left[\alpha\sum_{\bar{x}}^{n-1} f(x) + (1-\alpha)\sum_{\tilde{x}}^{n-1} f(x)\right] -$$
$$\delta^2(1-\alpha)\left[(1-\alpha)\sum_{\bar{x}}^{n-1} g(x) + \alpha\sum_{\tilde{x}}^{n-1} g(x)\right] \tag{3.22}$$

The first term on the RHS of (3.22) $\alpha[\alpha\sum_{\bar{x}}^{n} f(x) + (1-\alpha)\sum_{\tilde{x}}^{n} f(x)]$ is the probability that the central player will invest when a type-1 peripheral player waits and the state $\theta = 1$, where $\alpha\sum_{\bar{x}}^{n} f(x)$ is the probability that the central player is type 1 and invests; $(1-\alpha)\sum_{\tilde{x}}^{n} f(x)$ is the probability that the central player is type 0 and invests. Similarly, the second term on the RHS of (3.22), $(1-\alpha)[(1-\alpha)\sum_{\bar{x}}^{n-1} g(x) + \alpha\sum_{\tilde{x}}^{n-1} g(x)]$ is the probability that the central player will invest when a type-1 peripheral player waits and the state $\theta = 0$.

Equation (3.22) determines the unique value of $p(\delta)$ and we have such mixed strategy equilibrium ■

The condition for such a mixed strategy equilibrium to exist is the presence of players who are sufficiently patient. Furthermore, as δ increases, $p(\delta)$ decreases. This is so because when players are more patient, they have more of an incentive to wait.

In another interpretation of Proposition 3.3, given the discount factor, there exists such a mixed strategy equilibrium only if the network is large enough. When $n > \bar{N}_2$, periphery players will not fully reveal their signal to the central player. Furthermore, as the number of periphery players increases, each one will reveal less information in order to keep the total information received by central player at the same level.

Proposition 3.4 Asymptotic Learning does not occur in this model.

We use the common definition of Asymptotic Learning (Acemoglu et al. 2011) in this model: Asymptotic Learning occurs if the central player's action convergences (in probability) to the right action as the size of social network increases to infinity.

Proof. In equilibrium the central player makes the final decision in round 1, so the probability that the central player will take the right action is

$$\frac{1}{2}\mathrm{pr}(e_c^1 = 1|\theta = 1) + \frac{1}{2}[1 - \mathrm{pr}(e_c^1 = 1|\theta = 0)] \tag{3.23}$$

Define $\mathrm{pr}(e_c^1 = 1|\theta = 1) \equiv Q; \mathrm{pr}(e_c^1 = 1|\theta = 0) \equiv R$. The expected utility for a type 1 periphery waiting and following the central player's action can be written as

$$\delta^2[\alpha Q + (1-\alpha)R]\left[2\frac{\alpha Q}{\alpha Q + (1-\alpha)R} - 1\right] \tag{3.24}$$

Simplifying this expression and applying the indifference condition, we have

$$\delta^2[\alpha Q - (1-\alpha)R] = 2\alpha - 1 \tag{3.25}$$

Since $\alpha > 1/2$, from (3.25) we have

$$\alpha Q - \alpha R < \frac{2\alpha - 1}{\delta^2} \tag{3.26}$$

Combining (3.23) (3.26), and recall $\delta > \sqrt{\frac{2\alpha-1}{\alpha}}$, we have

$$\frac{1}{2}\mathrm{Q} + \frac{1}{2}[1 - \mathrm{R}] < \frac{1}{2} + \frac{1}{2}\frac{2\alpha - 1}{\alpha\delta^2} < 1 \tag{3.27}$$

Equation (3.27) means that in such an equilibrium we can find an upper bound for the probability that the central player takes right the action. As the social network becomes large, the probability that the central player will take the right action is always lower than $\frac{1}{2} + \frac{1}{2}\frac{2\alpha-1}{\alpha\delta^2}$ and Asymptotic Learning does not occur. ■

As the size of the network increases, periphery players lower their probability of investing in round 0 and keep the probability that the central player will take the right action at a certain level. This is the only way to ensure that periphery players have no incentive to deviate. If in the periphery players' mix strategy the probability of investing in round 0 is too high, the central player will collect more information and her action will be so informative that periphery players will have an incentive to wait; if the probability of investing in round 0 is too low, the central player's action will be less informative, and type 1 periphery players will have an incentive to invest in round 0.Thus, the central player will take the right action at a certain level and Asymptotic Learning will not occur.

3.4 Conclusion

In this paper, we introduce an endogenous timing learning model to explain the order of actions and the direction of learning in a star network. We find that in equilibrium, the information flow follows an "in-and-out" pattern: all or some of type 1 periphery players invest in the first round, revealing their signals to the central player, and in the second round, the central player makes the final decision, which is followed in the third round by the rest of the periphery players.

The size of the network plays an essential role in the pattern of equilibrium. When the number of periphery players is small, the network contains little information. Type 1 periphery players prefer investing in round 0 to waiting for the central player's action, which contains information from the whole network. When the number of periphery players is large enough, periphery players will use the mixed strategy, which only partially reveals their signal, thus, keeping the information collected by the central player at a certain amount.

We reach the result that asymptotic learning does not occur in this game. This is because once the probability that the central player will take the right action becomes too high, periphery players will deviate from waiting for the central player. Thus, when the periphery players are large in number, they will only reveal part of their signal through the mixed strategy or they will keep some of the periphery players waiting, regardless of the signal. This result indicates that the probability that the central player will take the right action will increase with the number of periphery players until the number reaches a threshold. Above the threshold, adding periphery players will not increase the central player's utility or the periphery players' utility.

The focus of this paper is limited to the star network, which is one structure of the network. Many other structures of networks are worth studying, such as the circle network, the line network, the double-star network, and the two-sided network. One direction of future study would be to solve the endogenous timing learning model in the network structures that this paper examines.

Additionally, instead of the network being fixed, the network structure can change over time. Another direction for future study would be to solve the endogenous timing learning model in a random network structure. Imagine the case of purchasing a new brand of car of an uncertain quality. The buyer can go to a street or parking lot to count the number of cars of this new brand. This type of observation is random and requires that a network is randomly created in every period.

This model overcomes the difficulty of establishing the equilibrium strategy analytically. Due to the complexity of Bayesian updating, this common difficulty is shared with all perfect Bayesian learning models of social networks. A common approach to solving this problem is to use non-Bayesian updating to simplify the calculation. The challenge is to find a non-Bayesian updating rule that makes the problem tractable without adding too many strong assumptions.

Chapter 4. A Search Model in Housing Market with Restricted Purchase

4.1 Introduction

To decelerate the rapid price growth in Chinese housing market, especially in secondhand housing market, Chinese government has used two temporary policies, restricted price and restricted purchase. Restricted price policy is setting the price ceiling of houses for sale. Restricted purchase policy is a relatively new policy tool, which limits the maximum number of houses a citizen can own at a time.

From 2010, restricted purchase policy has been carried out in 35 main cities and then spread to other cities in China. The policy sets the maximum amount of houses a person can own at a time (if a person with houses equal to or more than this amount, he can't buy another house). The maximum amount is usually 1 or 2, varies from cities and categories of people. In 2014, 18 cities cancelled or slacken this policy when housing price is regarded to be low, and in 2016 most of them re-established such policy rule when housing price is high. As a temporary policy to control the housing price, restricted purchase has an advantage that it is easier to carry out, comparing to restricted price policy. It avoids the black-market problem generated by price ceiling, which is hard to supervise in secondhand housing market. According to Li and Xu 2016, restricted purchase is more effective than restricted price and restricted bank loan policy on decelerating the housing price.

Besides enforceability and effectiveness, a good price-control policy should not generate a high excess return in short run (policy-shock) or long run (equilibrium), since excess return will attract investors and bring a negative effect on price control. Wheaton and Nechayev 2008 claims that investors and second home buyers who regard real asset as a safe investment are main driven forces of the housing price inflation.

At the best of our knowledge, this is the first attempt to describe a restricted purchase constraint based on search and matching model in housing market. This paper follows illiquid asset search model developed by Krainer and LeRoy 2002 and take it as the benchmark model, which is free from any restricted policy.

Infinite lived risk neutral agents consume two goods: housing services and a background good. Background good equals the negative of net expenditure on housing market. On the date when agent purchasing a house, the consumption of background good is the negative of the price. On the date selling a house, the consumption of background good is the positive of the price, and on other dates, the consumption is zero.

Housing service is from a matched house owned by the agent. The amount of housing service each period is called "fit". If an agent is not matched with a house, housing service is zero.

In the model, we only consider the housing market and do not involves the rental market. Match value is only from self-owned house and we regard it can not be substituted by renting a house. If an agent is not matched with a house currently, we regard him receives 0 housing service even though he may rent a house. This setting is reasonable because in many cases, there is a huge difference between owning a house and renting a house. For example, in China, owning a house brings the benefit of Hukou (which represent the right of resident), such as the admission of school and university, or the right to own a car. Also, we ignore the situation that household may rent out an unmatched and unsold house.

An agent with a match will not search for new houses. An existing match continues to next period with probability π.

In the benchmark model there is no restricted policy, any agents without a match have the right to search for houses, no matter she owns a house or not. In restricted purchase model, only the agents who own no house have the right to search.

Agents with the right to search are the prospective buyers, who visit exactly one house for sale per period, observe the fit and price offered by seller, make the decision that whether to buy the house.

Unmatched agent with a house (which is not matched to her) is the prospective sellers, who set a take-it-or-leave-it price for the house. Each house for sale is visited by exactly one prospective buyer.

This paper models the restricted purchase by adding the constraint that an agent can only hold no more than one house at a time, which indicates that an unmatched agent needs to sell her previous house before search for a new one.

We add this constraint in order to analyze the effect of restricted purchase policy, while this model can also be used to study the effect of budget constraint. Under the market where the housing price is high, down payment is high and loan interest is high, a house can be unaffordable for an unmatched agent if she does not sell the old house. This situation results in actions as “Selling for Buying”, “Trade -up” (Ortalo and Rady 2006). The result of our model can also help explain the effect of the constraint budget on such actions, as well as the effect on housing price, vacancy, welfare and asset returns.

In section 2, we formally describe the model. The main difference between restricted purchase model and benchmark model is the relationship between buying decision and selling decision for an agent. In the benchmark model, these two decisions are independent, and we can decouple them into two separate problems. While in the restricted purchase model, if an agent fails to continue the match, then she needs to sell the house before search for a new one, which indicates she needs to take the buying action into consideration when she makes the selling decision. In section 3, we show the equilibrium result, analyze the policy effect on housing price, vacancy ratio, social welfare and asset returns. We also compare these effects with restricted price policy. In section 4 we draw the conclusion that as a price-control policy tool, comparing to restricted price policy, restricted purchase policy generates a larger welfare lost but a lower temporary excess return for prospective buyers.

4.2 Model

Infinite lived risk neutral agents consume two goods in each period: housing services and a background good. Background good equals the negative of net expenditure on housing market. On the date when agent purchasing a house, the consumption of background good is the negative of the price. On the date selling a house, the consumption of background good is the positive of the price, and on other dates, the consumption is zero.

Housing service is from a matched house owned by the agent. The amount of housing service each period is called “fit”. If an agent is not matched with a house, housing service is zero.

Under restricted purchase policy, in the model only the agents who own no house have the right to search. Agents with the right to search are the prospective buyers, who visit exactly one house for sale per period, observe the fit and price offered by seller, make the decision that whether to buy the house.

Unmatched agents with a house (which is not matched to her) are the prospective sellers, who set a take-it-or-leave-it price for the house. Each house for sale is visited by exactly one prospective buyer.

There are 3 possible states for an agent in housing market with restricted purchase policy: matched, own a house but not matched, own no house and not matched. Figure 1 shows the timeline for agents in three states.

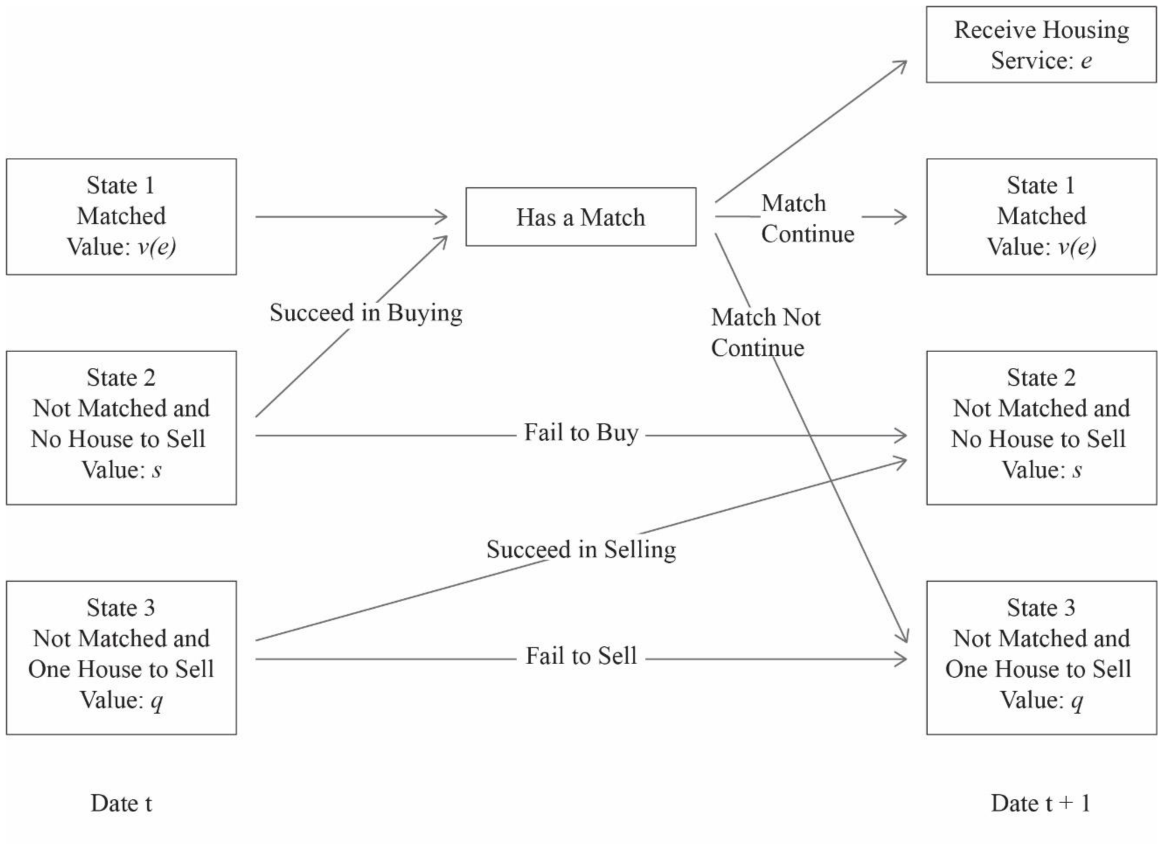

Figure 1: Timeline for Agents

A matched agent receives housing service each period (starts from next period) until the match fails. The value of a matched house with fit e is

$$v(e) = \beta e + \beta \pi v(e) + \beta(1-\pi)q^* \tag{4.1}$$

Where q* is the value of a house for sale in equilibrium.

An unmatched agent without a house is the prospective buyer in this market. The buyer visits one house each period and observe the fit, which is a random variable distributed uniformly on [0,1], IID. The buyer compares the fit with the price posted by the seller and decide whether to buy the house or not.

The buyer's strategy is setting a reservation fit for each price, er(p), including the equilibrium reservation fit e* and equilibrium price p*. Follow Krainer and LeRoy 2002, the strategy of reservation fit and price are expressed as deviations from their respective equilibrium values

$$e_r - e^* = \delta(p - p^*) \tag{4.2}$$

The prospective buyer owns an asset that consists of the right to search for a house. Define the value of this right as s. Then s is given by

$$s = \mu\left(v\left(\frac{e_r + 1}{2}\right) - p^*\right) + \beta(1 - \mu)s^* \tag{4.3}$$

μ is the probability of sale

$$\mu = 1 - e_r \tag{4.4}$$

The FOC for a maximum of s with respect to er gives

$$p^* + \beta s^* = \frac{\beta e_r + \beta(1-\pi)q^*}{1-\beta\pi} \tag{4.5}$$

An unmatched agent with one house for sale is the prospective seller in this market. The seller's strategy is setting a take-it-or-leave-it price to maximize the value of q. Since a seller can't observe the fit, she sets the same price to all buyers. The value of a house for sale is q

$$q = \mu(p + \beta s^*) + (1 - \mu)\beta q^* \tag{4.6}$$

The FOC for a maximum of q with respect to p is

$$(1 - e) + (\beta^{-1} - \pi)(\beta q^* - p^* - \beta s^*) = 0 \tag{4.7}$$

To find a stationary symmetric Nash equilibrium, from (4.1) to (4.7), we get 5 equations as follow:

$$q^* = \mu^*(p^* + \beta s^*) + (1 - \mu^*)\beta q^* \tag{4.8}$$

$$s^* = \mu^* \left(v\left(\frac{e^*+1}{2}\right) - p^* \right) + \beta(1-\mu^*)s^* \tag{4.9}$$

$$p^* + \beta s^* = \frac{\beta e^* + \beta(1-\pi)q^*}{1-\beta\pi} \tag{4.10}$$

$$(1-e^*) + (\beta^{-1} - \pi)(\beta q^* - p^* - \beta s^*) = 0 \tag{4.11}$$

$$\mu^* = 1 - e^* \tag{4.12}$$

4.3 Result

Setting π=0.9, β=0.95, we solve the model in stationary equilibrium. Table1 shows the equilibrium result under restricted purchase policy. We compare it with the equilibrium in market with free search and market with restricted price policy.

The model of free search market is from Krainer and LeRoy 2002. The setting is same with the restricted purchase model, except that all the unmatched agents (not only the unmatched agents with no house) have the right to search for new house.

The model of market with restricted price policy is same with free search market except that the seller can't post price which is different from the government restricted price.

The details of those two models can be found in Appendix.

	Free Market	Restricted Purchase	Restricted Price
e*	0.76	0.72	0.69
p*	8.85	6.59	6.59
E(v*)	12.49	11.59	13.48
s*	3.83	5.26	6.20
q*	7.65	10.52	5.92
Expected Time to Sell	3.14	2.53	2.23
Vacancy Rate	0.31	0.28	0.26
Optimal Reservation	0.63	0.53	0.63
Social Welfare W*	0.61	0.49	0.62
Return on Asset	1.05	1.05	1.05
Temporary Return	1.05	1.38	1.70

Table 1: Equilibrium Results in Free Search Market, Restricted Purchase Market and Restricted Price Market

Price

Comparing to the free search market, the restricted purchase market results in a ¼ decrease in equilibrium housing price. This result indicates that as a policy tool, restricted purchase has a significant effect on controlling the housing price.

From (4.8) (4.10) we have

$$p^* = \frac{(1-e^*)(1-\beta e^*)}{(\beta^{-1}-\pi)(1-\beta)} - \beta s^* \quad (4.13)$$

Compare to the equilibrium price in free search market, (we use p*, e*, s* to indicates the equilibrium value in restricted purchase market, and p#, e# to indicates the equilibrium value in free search market)

$$p^\# = \frac{(1-e^\#)(1-\beta e^\#)}{(\beta^{-1}-\pi)(1-\beta)} \quad (4.14)$$

There are two reasons to explain why the equilibrium price is lower than the price in a free search market. First, because of the restricted purchase policy, agents need to sell the unmatched house before searching for a new one. In other words, sellers get the right to search besides the house price in a trade, that's the second term on the RHS of (4.13), which acts as a compensation for sellers and decreases the equilibrium price. Second, a restricted purchase market results in a lower reservation fit, which indicates a more effective supply of houses, which decreases the equilibrium price, and that's the difference between the first term on the RHS of (4.13) and the RHS of (4.14). This can be seen from a lower vacancy ratio (the proportion of houses for sale) in Table1.

Welfare

Under free search market, the equilibrium is not social optimal. We can see from Table1 that in free search market, the equilibrium reservation fit is higher than the optimal reservation fit.

In restricted price market, by setting the price ceiling, the policy decreases the reservation fit. As shown in Tabel1, if we set the restricted price as the equilibrium price in restricted purchase market, which indicates same level of price control, the reservation fit in restricted price market is closer to the optimal reservation fit. A restricted price policy will increase the social welfare under current value of parameters.

To calculate the welfare lost in restricted purchase market, we find the equilibrium social welfare in each period W, (since the housing service for unmatched agent is zero) we have

$$W = \Pr(matched) * Avg.housing.service \tag{4.15}$$

where average housing service is (e*+1)/2.

To see the proportion of matched agents, we need the transition matrix among the three states: matched, not matched and one house to sell, not matched and no house to sell

$$T = \begin{bmatrix} \pi & 1-\pi & 0 \\ 0 & e^* & 1-e^* \\ \pi(1-e^*) & 0 & 1-\pi+\pi e^* \end{bmatrix} \tag{4.16}$$

From (4.16) we can find the proportion of matched agents:

$$Matched\ Rate = \frac{1-e^*}{1-e^*+\pi^{-1}-\pi} \tag{4.17}$$

Thus, we can calculate the welfare in restricted purchase market. Under current value of parameters, the restricted purchase model results in a welfare lost. The reason is in free market, agents begin to search for new house as soon as the previous match fails, while in restricted purchase market, agents need to wait until the previous house is sold. This waiting time decrease the matched rate and thus decrease the social welfare.

Asset Return

We know from Krainer and LeRoy 2002 that free search market is a fair game. There is no excess return in equilibrium. But does a restricted policy generate excess return in long run and short run? If a price-control policy generates large excess return which will attracts investors, it may end up with an inflation, rather than a decrease in price. So, a qualified price-control policy should not generate large excess return.

In a restricted purchase market, agents have three type of asset: a house for sale, a matched house and the right to search. We examine the return of those assets in equilibrium.

The equilibrium return on a house for sale is

$$r^* = \begin{cases} \frac{p^*+\beta s^*}{\beta q^*}, with\ probability\ \mu^* \\ 1,\ \ with\ probability\ 1-\mu^* \end{cases} \tag{4.18}$$

The equilibrium return on a matched house is

$$r^* = \begin{cases} \frac{e}{v(e)} + 1, with\ probability\ \pi \\ \frac{e+q^*}{v(e)},\ \ with\ probability\ 1-\pi \end{cases} \tag{4.19}$$

The equilibrium return on search option is

$$r^* = \begin{cases} \frac{\frac{e^*+1}{2}+\pi v\left(\frac{e^*+1}{2}\right)+(1-\pi)\left(q^*-\frac{p^*}{\beta}\right)}{s^*}, with\ probability\ \mu^* \\ 1,\ \ with\ probability\ 1-\mu^* \end{cases} \tag{4.20}$$

From (4.8) - (4.12) we can get that all the returns equal to 1/β, which indicates in equilibrium there is no excess return in any type of asset in a restricted purchase market. We can use the same method to check the restricted price market and get the same result that there is no excess return in a restricted price market in equilibrium. These results indicate in equilibrium, or in other words, under a stable restricted policy in the long run, there is no excess return. Thus, the constraint alone doesn’t generate excess return.

We also interested in the temporary return for prospective buyers (return on search option). If a price-control policy generates a large temporary excess return in the short run, it may attract market maker and investors, which may increase the price instead of decreasing it.

We assume the price adjusts quick enough to policy and calculate the short run return for prospective buyers facing a policy shock of restricted purchase and restricted price. (We use x* to indicates the equilibrium value for variable x in restricted purchase market, use x# to indicates the equilibrium value for variable x in free market, use x^ to indicates the equilibrium value for variable x in restricted price market.)

Return for prospective buyers in restricted purchase policy shock is

$$r = \begin{cases} \dfrac{\frac{e^*+1}{2}+\pi v\left(\frac{e^*+1}{2}\right)+(1-\pi)q^*-\frac{p^*}{\beta}}{s^\#}, with\ probability\ \mu^* \\ \dfrac{s^*}{s^\#},\ \ with\ probability\ 1-\mu^* \end{cases} \tag{4.21}$$

Return for prospective buyers in restricted price policy shock is

$$r = \begin{cases} \dfrac{\frac{e^{\wedge}+1}{2}+\pi v\left(\frac{e^{\wedge}+1}{2}\right)+(1-\pi)(q^{\wedge}+s^{\wedge})-\frac{p^{\wedge}}{\beta}}{s^\#}, with\ probability\ \mu^{\wedge} \\ s^{\wedge}/s^\#,\ \ with\ probability\ 1-\mu^{\wedge} \end{cases} \tag{4.22}$$

From the equilibrium result in Table1, we calculate the return for prospective buyers in restricted purchase policy shock is 1.38 and the return for prospective buyers in restricted price policy shock is 1.70. The reason why the excess return for restricted purchase policy shock is lower (comparing to restricted price) under the same level of price control is that the excess return of prospective buyers caused by a lower housing price is partly cancelled out by the low future asset value due to the purchase restriction.

This result indicates that a large social cost and a low temporary excess return are two sides of a coin. A large social cost means a low expected (across states) value in the future, which reduce the excess asset return caused by the price decrease from policy shock.

4.4 Conclusion

To summarize, as a policy tool to control housing price, restricted purchase policy has following effects on housing market. First, it decreases the equilibrium price significantly by increasing the motivation for sellers to trade and increasing the effective house supply. The restricted purchase policy makes sellers gain more than the price of the house from a trade. Sellers can also get the right to search from a trade and this increases the motivation for sellers and decreases the equilibrium price. On the other hand, restricted purchase decreases the reservation fit of buyers, which increases the effective supply of houses and decreases the housing price.

Second, restricted purchase policy costs larger social welfare than a restricted price policy with a same level of price control. This is because the policy increases the unmatched periods for an agent. An unmatched agent needs to sell her house before searching for a new match. This results in a reduction in matched agents and a large welfare cost.

Third, restricted purchase policy doesn’t generate excess return in equilibrium and only bring a relatively small excess return for prospective buyers in short run comparing to restricted price policy. This is because the excess return caused by decrease in price is partly cancelled out by the low expected future value due to the policy.

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Appendix A. Omitted Proofs

A.1 Proof of Proposition 2.1

Proposition 2.1 When DMs' private signals dominate common signal ($\beta > \alpha$), always telling the truth is among the best rumor.

Proof. We use backward induction to find the equilibrium strategy. If the rumor is blocked by DM1, DM2 will follow his private signal, since $\beta > \alpha$. Thus, when the rumor is blocked, DM1's expectation on DM2's payoff is always β, which is the probability that DM2's private signal is correct. If $c > 1 - \beta$, all rumors will be blocked, since rumor is never informative enough to overcome the cost. If $c < 1 - \beta$, we can always find rumors maker's strategy that can guarantee all rumor to be transmitted. One example is always telling the truth ($p = 1, q = 0$), while the best rumor is not unique, as long as (p, q) satisfies following conditions

$$\frac{(1-\alpha)(1-\beta)(1-q)}{(1-\alpha)(1-\beta)(1-q)+\alpha\beta(1-p)} - c \geq \beta \qquad (A.1)$$

$$\frac{(1-\alpha)(1-\beta)p}{(1-\alpha)(1-\beta)p+\alpha\beta q} - c \geq \beta \qquad (A.2)$$

(A.1) is the condition for a DM1 with "High" private signal and "High" common signal to transmit a "Low" rumor; (A.2) is the condition for a DM1 with "Low" private signal and "Low" common signal to transmit a "High" rumor.

Simplify (A.1) (A.2), we have

$$1-\mathrm{p}\leq\frac{1-\alpha}{\alpha}\frac{1-\beta}{\beta}\frac{1-\beta-c}{\beta+c}(1-q) \tag{A.3}$$

$$\mathrm{q}\leq\frac{1-\alpha}{\alpha}\frac{1-\beta}{\beta}\frac{1-\beta-c}{\beta+c}p \tag{A.4}$$

Under the conditions (A.3) (A.4), DM1 always transmits the rumor and the DM2 always follows the rumor, regardless of their own signals.

From the equilibrium we know that all the rumor can be transmitted as long as it is informative enough. When the cost of rumor increases or when the accuracy of private and common signals increases, the area of best rumor shrinks and converges to the truth.

When $n>2$, once DM1 transmits the rumor to DM2, DM2 will also transmits the rumor to following DMs, since the rumor is informative enough. Thus, the best rumor is same when we have more DMs. ■

A.2 Proof of Proposition 2.2

Proposition 2.2 When the common signal dominates private signals (α > β), the best rumor is to set $p^*=1-q^*=\frac{1}{1+\frac{1-c}{1+c}\frac{1-\alpha}{\alpha}\frac{1-\beta}{\beta}}$.

Proof. We use backward induction to find the equilibrium strategy. If the rumor is blocked by DM1, DM2 will follow the common signal since α > β. Thus, if a rumor is same with the common signal, it will be blocked by DM1, since it won't change DM2's action and only result in a reading cost.

The only two possible cases for rumor being transmitted are: a "Low" rumor when common signal is "High" and a "High" rumor when common signal is "Low".

There are two types of strategy for RM: (1) Targeting on sending an informative rumor which will be transmitted even when common signal and private signal have same direction which is different from the rumor; (2) Targeting on sending a less informative rumor which will only be transmitted when rumor and private signal have same direction, which is different from common signal. We show that type 1 strategy is always better than type 2 strategy for RM.

Type 1 RM's strategy can be described as

$$\mathrm{Max}_{p,q} 1/2[(1-\alpha)p + \alpha q + \alpha(1-p) + (1-\alpha)(1-q)] \quad \text{(A.5)}$$

s.t.

$$\frac{p(1-\alpha)(1-\beta)}{p(1-\alpha)(1-\beta)+q\alpha\beta} - c \geq 1 - \frac{p(1-\alpha)(1-\beta)}{p(1-\alpha)(1-\beta)+q\alpha\beta} \quad \text{(A.6)}$$

$$\frac{(1-q)(1-\alpha)(1-\beta)}{(1-q)(1-\alpha)(1-\beta)+(1-p)\alpha\beta} - c \geq 1 - \frac{(1-q)(1-\alpha)(1-\beta)}{(1-q)(1-\alpha)(1-\beta)+(1-p)\alpha\beta} \quad \text{(A.7)}$$

(A.5) is the probability that rumor is different from common signal. (A.6) is the condition that DM1 with "Low" common signal and "Low" private signal transmits a "High" rumor. (A.7) is the condition that DM1 with "High" common signal and "High" private signal transmits a "Low" rumor.

From (A.5) -(A.7) We have

$$p^* = 1 - q^* = \frac{1}{1 + \frac{1-c}{1+c}\frac{1-\alpha}{\alpha}\frac{1-\beta}{\beta}} \quad \text{(A.8)}$$

$$\mathrm{EU}_{RM}{}^* = \alpha(1-p^*) + (1-\alpha)p^* \quad \text{(A.9)}$$

(A.9) is the expected payoff for RM in Type 1 strategy.

Type 2 strategy can be described as

$$\mathrm{Max}_{p,q} 1/2[\mathrm{p}\beta(1-\alpha)+(1-\mathrm{p})(1-\beta)\alpha+\mathrm{q}(1-\beta)\alpha+(1-\mathrm{q})\beta(1-\alpha)] \quad (\mathrm{A}.10)$$

s.t.

$$\frac{p(1-\alpha)\beta}{p(1-\alpha)\beta+q\alpha(1-\beta)}-c \geq 1-\frac{p(1-\alpha)\beta}{p(1-\alpha)\beta+q\alpha(1-\beta)} \quad (\mathrm{A}.11)$$

$$\frac{(1-q)(1-\alpha)\beta}{(1-q)(1-\alpha)\beta+(1-p)\alpha(1-\beta)}-c \geq 1-\frac{(1-q)(1-\alpha)\beta}{(1-q)(1-\alpha)\beta+(1-p)\alpha(1-\beta)} \quad (\mathrm{A}.12)$$

(A.10) is the probability that rumor is same with private signal and different from common signal. (A.11) is the condition that an DM1 with low common signal and high private signal transmits a “High” rumor. (A.12) is the condition that an DM1 with high common signal and low private signal transmits a “Low” rumor.

From (A.10) -(A.12) we have

$$p^{**}=1-q^{**}=\frac{1}{1+\frac{1-c}{1+c}\frac{1-\alpha}{\alpha}\frac{\beta}{1-\beta}} \quad (\mathrm{A}.13)$$

$$\mathrm{EU}_{RM}{}^{**}=\alpha(1-\beta)(1-p^{**})+(1-\alpha)\beta p^{**} \quad (\mathrm{A}.14)$$

(A.14) is the expected payoff for RM in Type 2 strategy. To show that type 1 strategy is always better than type 2 strategy for RM, we need to show for all (α, β) that

$$\mathrm{EU}_{RM}{}^{*}-\mathrm{EU}_{RM}{}^{**}>0 \quad (\mathrm{A}.15)$$

We find as β increases, $\mathrm{EU}_{RM}{}^{*}$ decreases and $\mathrm{EU}_{RM}{}^{**}$ increases. We show that when β reaches highest value $\beta=\alpha, \mathrm{EU}_{RM}{}^{*}-\mathrm{EU}_{RM}{}^{**}>0$ still holds.

When $\beta=\alpha$,

$$\mathrm{EU}_{RM}{}^{*} - \mathrm{EU}_{RM}{}^{**} = \alpha^2 + \frac{1-2\alpha}{1+\frac{1-c}{1+c}\left(\frac{1-\alpha}{\alpha}\right)^2} = \frac{\frac{2}{1+c}(\alpha-1)^2}{1+\frac{1-c}{1+c}\left(\frac{1-\alpha}{\alpha}\right)^2} > 0 \qquad (\mathrm{A}.16)$$

Thus, we have the conclusion that Type 1 strategy is always better than Type 2 strategy for a RM.

From the equilibrium, we can see that $p^* = 1 - q^*$, which indicates equilibrium rumor is a symmetry binary signal with accuracy equals p^*. When the rumor signal and common signal are same, rumor is blocked; when the rumor signal and common signal are different, DM1 is indifferent between transmitting and blocking the rumor. If the rumor is less accurate, it will not be transmitted; if the rumor is more accurate, it has less probability to be different from the common signal.

We can also see that, if $n > 2$, once DM1 transmit the rumor to DM2, DM2 will also transmit the rumor to following DMs, since the rumor is informative enough and different from the common signal. Thus, the best rumor is same when we have more DMs. ■

A.3 Proof of Proposition 2.3

Proposition 2.3 When there are 2 RMs, there is a unique equilibrium where both RMs strategies are always telling the truth.

Proof. It's easy to see that when both RMs tell the truth $p_1 = p_2 = 1$, no RM has incentive to deviate. To show this is the only equilibrium, we prove that given the accuracy of RM1 $p_1 < 1$, the best strategy for RM2 is ${p_2}^* > p_1$. Thus, the only equilibrium is $p_1 = p_2 = 1$.

When DMs' private signal dominates common signal ($\beta > \alpha$), given the accuracy of RM1 p_1, under the assumption that $c < 1 - \beta$, the best strategy for RM2 is to send a high informative rumor which can always be transmitted, even when it's different from all other rumor and signals. One example is RM2 always tells the truth, ${p_2}^* = 1$.Thus, the best respond for RM2 is ${p_2}^* > p_1$.

When DMs' common signal dominates private signal ($\alpha > \beta$), given the accuracy of RM1 p_1, the best strategy for RM2 is to target on be transmitted when it's different from all other rumor and signals. When 2 rumors are same (if rumor 1 is informative enough), DM1 randomly chooses a rumor to transmit. Thus, ${p_2}^* > p_1$, when $\alpha > \beta$. This result is similar with proposition 2.2.

There is no collude equilibrium where both DMs send low informative rumors, targeting being transmitted together. This is because they have incentive to deviate to a higher accuracy rumor, which guarantee being transmitted even if two rumors are different. ■

Appendix B. Benchmark Model and Restricted Price Model in Chapter 4

B.1 Free Search Model

The Free Search model is from Krainer and LeRoy 2002. The setting is almost same with the restricted purchase model, except that all the unmatched agents (not only the unmatched agents with no house for sale) have the right to search and purchase for new house.

In equilibrium, we have the following five equations:

$$q^* = \mu^* p^* + (1-\mu^*)\beta q^* \tag{B.1}$$

$$s^* = \mu^* \left(v\left(\frac{e^*+1}{2}\right) - p^* \right) + \beta(1-\mu^*)s^* \tag{B.2}$$

$$p^* + \beta s^* = \frac{\beta e^* + \beta(1-\pi)q^*}{1-\beta\pi} \tag{B.3}$$

$$(1-e^*) + (\beta^{-1} - \pi)(\beta q^* - p^*) = 0 \tag{B.4}$$

$$\mu^* = 1 - e^* \tag{B.5}$$

(B.1) is the equilibrium value of a house for sale. (B.2) is the equilibrium value of the right for search. (B.3) is the FOC for a prospective buyer to maximize s with respect to reservation fit e. (B.4) is the FOC for a prospective seller to maximize q with respect to p. From these five equations, we can find the equilibrium shown in Table 1.

B.2 Restricted Price Model

The model setting is almost same with benchmark model except that the seller can't post price which is different from the restricted price.

In equilibrium, the seller set the price as restricted price, so instead of (B.4), we have

$$p^* = p_{restr} \tag{B.6}$$

From (B.1), (B.2), (B.3), (B.5), (B.6), we can get the equilibrium in restricted price market shown in Table 1.