Bounded Rationality and Mechanism Design

Dissertation

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Abstract

Mechanism Design Theory, introduced by 2007 Nobel laureates Hurwicz, Maskin, and Myerson, has guided economic institutions worldwide to achieve desirable goals in allocating scarce resources. However, most of the literature on Mechanism Design Theory that guides its application, in reality, assumes that people are fully rational; this omission of people's bounded rationality raises doubt over the reliability of the theory's empirical implications. To bridge this gap between theory and reality, we introduce new formalizations to characterize new types of boundedly rational behavior that is missing in existing models but supported by experimental evidence.

NLK, the first formalization we propose, is a new solution concept in Game Theory that connects two existing ones, Nash Equilibrium (NE) and Level-K model. Of these two, NE, introduced by 1994 Nobel Laureates John Nash has revolutionized the economics of Industrial Organization and has influenced many other branches such as the theories of monetary policy and international trade. However, there is mounting and robust evidence from laboratory experiments of substantial discrepancy between the predictions of NE and the behavior of players. Among all the alternative models that retain the individual rationality of optimization, but relax correct beliefs, Level-K model is probably the most prominent. Absent in NE, Level-K model explicitly allows players to consider their opponent as less sophisticated than themselves. But Level-K does not allow players to use

an important element of strategic thinking, namely, “put yourself in the others’ shoes” and believe the opponent can think in the same way they do. Bridging NE and Level-K, NLK allows a player in a game to believe that her opponent may be either less- or as sophisticated as they—a view supported by various studies in Psychology. We compare the performance of NLK to that of NE and some versions of Level-K by applying it to data from three experimental papers published in top economics journals and to data from a field study. These studies allow us to test NLK on: (1). A static game of complete information, (2). A static game of incomplete information, (3). A dynamic game of perfect information, and (4). On field data.

NLK provides additional insights to those of NE and Level-K. Moreover, a simple version of it explains the experimental data better in many cases. As a new solution concept, NLK shares a similar foundation to NE but is also applicable to games with players of different cognitive or reasoning abilities. As an analytical tool, NLK exists and gives a sharp prediction in general, and therefore it can be applied to empirical analysis in a broad range of settings.

In the second formalization, we first propose two alternative axiomatic approaches, formalizing a distinct anomaly in hypothetical reasoning that agents fail to reason state-by-state. Our theory expands the foundation of Decision Theory and ties together a broad range of evidence documented in multiple disciplines that decision makers often choose a dominated strategy. Secondly, we extend our concept to Game Theory and Mechanism Design, where we identify a rich class of mechanisms that successfully achieve desirable goals even with boundedly rational agents and agents who mistrust the market makers.

Thirdly, we test and verify our theory and its implications, by two laboratory experiments with a cross-over design that enables pooled data, within-subject, and cross-subject comparisons. Finally, we address how our approach contributes to accomplishing two goals simultaneously in modeling bounded rationality: providing a unified framework that subsumes existing ones as limiting cases and stimulating transdisciplinary conversations connecting the concepts of heuristics and emotions in Psychology, the utilization of eye-tracking technology in Neuroscience, and considerations of the moral foundation underlying a mechanism design in Ethics. The general insights of our work can be transferred to practical impacts on applications of Mechanism Design. Among these applications are the U.S. Federal Communications Commission auctions that raise more than 10 billion dollars yearly in government revenue; College Admissions that affect more than 10 million students every year around the world; and a Kidney Exchange Program with more than 1 million people waiting for kidney transplants. By formalizing bounded rationality into economic theory, our study honors the elegance of classic economic theory; at the same time, by modeling human behavior even more closely, it directs us to a new way of improving human welfare.

In the history of economic thought lies a dilemma for future economists: should we adopt simple models with unrealistic assumptions, or should we describe human behavior closely but give up elegant abstractions? In the projects above, we endeavor to create a middle way that synthesizes the merits in both directions and leave unanswered questions for future researchers.

Dedication

The Master said, “Virtue is not left to stand alone. He who practices it will have neighbors.”

—Analects, 4:25

I dedicate my dissertation to all the fools who yet hold on to their dream, bravely strive against adversity, and seek after coordination between theory and reality. As your sentiments resonate with mine, my joy expands.

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I am grateful for Prof. Ariel Rubinstein's inspiration on formal modeling of bounded rationality, in general, and its application to mechanism design, in particular. Special thanks go to Prof. Dan Levin for his tireless mentoring and steadfast support during my entire Ph.D. journey. Chapter 1 of this dissertation is developed from his original idea and based on our joint work. I am indebted to Prof. Shengwu Li for his initial and insightful idea of *Obvious Strategy-proof* Mechanisms. Chapter 2 and 3 are closely related to and inspired by his work. I especially thank my dissertation adviser Prof. James Peck and Prof. Paul J. Healy on my committee for their careful, patient, and instructive guide to the completion of this work. I likewise want to show gratitude to Profs. Hal Arkes, Yaron Azrieli, Pierpaolo Battigali, John Kagel, Philippe Jehiel, Ian Krajbich, Ran Shorrer and Marciano Siniscalchi for their invaluable comments on the contents of this dissertation and much more. Although, all errors are my own and should not tarnish the reputations of these esteemed scholars. I have benefited from the comments of participants in the Texas Experimental Economic Symposium 2015, the Midwest Economic Theory and Trade Conference 2015, the D-TEA workshop 2015, the Summer School of the Econometric Society 2015 and 2016, ASSA meeting 2017, the 28th Stony Brook International Conference on Game Theory 2017, and the 70th European Meeting of the Econometric Society 2017. This work is supported in part by the JMCB Research Grant for Graduate

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Fields of Study

Major Field: Economics

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Chapter 1. Bridging Level K to Nash Equilibrium

1. Introduction

There is mounting and robust evidence from laboratory experiments of substantial discrepancies between the prediction of Nash Equilibrium (NE) and the behavior of agents.[1] Among all the alternative models that retain perfectly maximization behavior is the prominent Level-K model.[2] First proposed by Stahl and Wilson (1994, 1995) and Nagel (1995), it introduces a non-equilibrium, structural model of strategic thinking, which admits possible cognitive limitations of players that are not allowed in NE.[3] This model has a hierarchy of levels of sophistication that are constructed iteratively starting with an exogenous, non-strategic and least sophisticated $level_0$ player. Higher levels are then constructed by assuming that a $level_k$ player best responds to $level_{k-1}$ opponents, $k = 1,2, \ldots$. Absent in NE, the Level-K model explicitly allows players to consider their opponents as less sophisticated than themselves. However, it does not allow players to use an important element of strategic thinking, namely, "put yourself in the other's shoes."

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[1] There is much experimental evidence that predictions of both (Bayesian) NE in static games and Subgame Perfect Nash Equilibrium (SPNE) in dynamic games fail miserably. For instance, see McKelvey and Palfrey (1992) and Kagel and Levin (2002).

[2] Another strand of literature like Quantal Response Equilibrium (McKelvey and Palfrey 1995), on the other hand, relax the assumption of perfect best response.

[3] There are many variations and extensions of the Level-K model and we refer the reader to Crawford, Costa-Gomes and Iriberri (2013) and the references therein.

Our paper introduces a new solution concept, NLK that bridges the gap between NE and the Level-K model. Our model has two possible interpretations:

1. **A population game**: In this interpretation, an NLK player behaves as if she faces a population composed of naïve players and NLK players. In equilibrium, an NLK player best responds to her belief that with a probability λ, her opponent is a naïve player, and that with a probability of $(1-\lambda)$, her opponent is another NLK player (like herself). Note that λ is the subjective belief formed by the player and it does not have to coincide with the objective proportions of naïve players in the population, denoted by ρ. Thus, with $\lambda \neq \rho$, NLK is not a "full-equilibrium,"[4] allowing an NLK player to hold inconsistent beliefs regarding the proportion of naïve players in the population, which is supported by a psychology literature: the "False Consensus Effect," first introduced by Ross, Greene, and House (1977), claims that people overestimate the proportion of people like themselves ($\lambda < \rho$).[5] More recent works, both in psychology and experimental economics, have re-evaluated the "False Consensus Effect" with some works providing evidence in support of such effect (Krueger and Clement 1994, Jimenez-Gomez 2016), while other works point at evidence to an opposite effect ($\lambda > \rho$) (Dawes 1990, Sherman, Presson, and Chassin 1984) or the absence of a biased belief (Engelmann and

[4] Stahl and Wilson (1995) include a rational expectation type together with different types of level_k and Nash players in analyzing experimental data of a 3×3 symmetric game. Their results reject the existence of rational expectation type.
[5] There is a rich psychology literature supporting the finding of FCE or the "self-anchoring" argument. Mullen et al. (1985) reported 115 studies that show FCE. For more detailed empirical and theoretical discussion, refer to Marks and Miller (1987) and all the listed references therein.

Strobel 2000). Apparently, one may insist on consistency by requiring that in a "full-equilibrium" $\lambda = \rho$.

2. **A hierarchy of heterogeneous players**: A construction of such hierarchy can be accomplished in two ways.

   a. **As an analog of the Level-K model**: A player is an NLK player of type m, denote by NLK_m, when his naïve opponent is exogenously given as a $level_{m-1}$ player of the Level-K model. Thus, an NLK_m player coincides with a $level_m$ player when $\lambda = 1$ and NLK equilibrium reduces to NE when $\lambda = 0$.

   b. **As an analog of the Poisson Cognitive Hierarchy (P-CH) model** (Camerer, Ho, and Chong 2004): Denote by $f(m)$ the probability function of a Poisson distribution $f(m) = \frac{e^{-\tau}\tau^m}{m!}$. Then by a similar treatment of truncated probability distributions as in the P-CH model, an NLK_m believes that he faces a naïve player of level $h, h = 0,1, \dots, m-1$ with probability $g(h) = \frac{f(h)}{\sum_{l=0}^{m} f(l)}$, and another NLK_m player with probability $(1 - \sum_{h=0}^{m-1} g(h)) = \frac{f(m)}{\sum_{l=0}^{m} f(l)}$.

In this work we only use m=1, resulting in NLK that has just one parameter, λ. We show that this simplest version of NLK already outperforms Level-K in many cases, although in some of them Level-K uses more than one parameter.

To illustrate the NLK equilibrium, consider a simple example of the chicken game introduced by Rapoport and Chammah (1966). It is a two-player symmetric game, where each player chooses either "Dove" or "Hawk," and the player's payoffs depend on her own action and that of the opponents as follows:

	Dove	Hawk
Dove	30,30	20,70
Hawk	70,20	0,0

Table 1.1. The dove and hawk game.

A random $level_0$ chooses either Dove or Hawk with equal probability. A $level_1$ best responds to the $level_0$ player by choosing Hawk. Then a $level_2$ player optimally chooses Dove. A $level_3$ player chooses Hawk again. A $level_4$ player reverts to Dove, and so on. There are two pure NE strategies: (Hawk, Dove) and (Dove, Hawk) and a third mixed-strategy where Dove and Hawk are played with the probability of $\frac{1}{3}$ and $\frac{2}{3}$ respectively. Now, consider an NLK_1 player who faces a naive random $level_0$ player with the probability of λ and another NLK_1 player with the probability of $(1-\lambda)$. For $1 \geq \lambda \geq \frac{2}{3}$, only one pure strategy NLK equilibrium exists, where each player chooses Hawk. For $\frac{2}{3} \geq \lambda \geq 0$, there exist two pure strategy NLK equilibria: (Hawk, Dove) and (Dove, Hawk) and a mixed-strategy where Dove and Hawk are played with the probabilities of $\frac{2-3\lambda}{6(1-\lambda)}$ and $\frac{4-3\lambda}{6(1-\lambda)}$, respectively.

As a new solution concept, NLK shares a similar foundation to NE but is also applicable to games with players of different cognitive or reasoning abilities. For example, in the experiment of Alaoui and Penta (2015), when students from math and science departments interact with opponents from the field of humanities, it is reasonable to expect that they adopt a larger subjective λ than when they play with fellow math and science students. We also adapt our basic definition of NLK to Bayesian games and dynamic games, as extensions of Bayesian Nash Equilibrium (BNE) and Subgame Perfect Nash Equilibrium (SPNE).

As an analytical tool, NLK can be applied to empirical analysis in a broad range of settings. We are fortunate to be able to compare the performance of NLK to that of NE and some versions of Level-K by applying it to data from three experimental papers published in top economic journals and to data from a field study. These studies allow us to test the NLK on a static game of complete information and another with incomplete information, a dynamic game of perfect information, and on field data. In the experiments we analyzed, NLK provides several insightful implications. First, in the static Guessing Game by Arad and Rubinstein (2012), a simple version of NLK with one parameter, $\lambda \in (0,1)$, that is chosen optimally, fits data better than both NE and Level-K models with an optimal distribution among three types of players, i.e., two parameters. After allowing for an error structure that is sensitive to payoffs, NLK still performs better than Level-K models with the same number of one parameter but not when we allow Level-K to choose freely more parameters. The results suggest that in some cases, NLK can also serve as an analytical tool. Second, in application to the data from an experiment of the Centipede Game by

Palacios-Huerta and Volij (2009), NLK's predictions, adapted to dynamic games, are different and more precise than those of SPNE and Level-K models, with only few exceptions when they coincide or when Level-K adopts more parameters. It is also reassuring that the optimal $\lambda < 1$ is the largest when players are all students, the smallest when only chess players are involved, and in the middle when a chess player plays with a student. Thus, the optimal λ for NLK seems to track well and capture the shift in subjective beliefs that can be expected in the different mixes of subjects' populations. The better performance of NLK than Level-K in the Centipede Game is reconfirmed using the data from Levitt, List and Sadoff (2011). Moreover, NLK can capture belief updating in every round of a game that a dynamic Level-K cannot. Finally, we compare predictions of NLK to those of Level-K for the data from the Common Value Auction experiment by Avery and Kagel (1997). For inexperienced bidders, NLK's performance coincides with that of Level-K; but for experienced bidders, NLK with $\lambda \in (0,1)$ provides the most accurate prediction. Moreover, since the estimated λ is larger for the data of experienced bidders than that of inexperienced bidders, NLK may also be used to track dynamic learning from experience, for example, learning in repeated games and convergence to a "full-equilibrium" $(\lambda = \rho = 0)$. Finally, in a recent experimental work on a rank-order tournament with an outside option-a dynamic game with imperfect information, Brünner (2018) finds that a mixture of Level-K and PBNLK – the dynamic version of NLK – that predicts both the population of types in the tournament, as well as the mean variance of efforts remarkably well. In fact, that paper show that PBNLK predicts the experimental data better than a level-K model without updating of beliefs,[6] which highlights the importance of the belief updating criterion that PBNLK adds onto Level-K.

Like other models that use "relaxed beliefs," NLK has its limitations. For instance, NLK cannot explain deviations from theoretical predictions in games with a dominant strategy solution, such as overbidding one's value in Second Price Sealed Bid auctions with private values, first reported by Kagel, Harstad, and Levin (1987).

We follow with a brief discussion of related literature. In Section 2, we present our basic solution concepts as used in different types of games (static or dynamic, with complete or incomplete information). In Section 3, 4 and 5, we provide the NLK solutions and compare them to those of NE and Level-K models for a static Guessing Game, a dynamic Centipede Game, and a Common Value Auction. In Section 6, we discuss and explore possible extensions to games with more than two players, allowing for heterogeneous beliefs and distributions of lower-level players. We conclude in Section 7.

1.1 Related Literature

Level-K and its related extension, Cognitive Hierarchy models by Camerer, Ho and Chong (2004) are applied to many laboratory experiments and Field data.[7] The survey by Crawford, Costa-Gomes, and Iriberri (2013) documents many successes of Level-K and its

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[6] Brünner (2018) shows that Nash Equilibrium performs even worse than the Level-K without belief updating.

[7] Applications to static games with complete information include the p-beauty contest (Bosch-Domenech, Montalvo, Nagel, and Satorra, 2012), the two persons' guessing game (Costa-Gomes and Crawford 2006) and the 11-20 money request game (Arad and Rubinstein 2012). For dynamic games, Kawagoe and Takizawa (2012) show that Level-K out predicts the Agent Quantal Response Equilibrium (AQRE) in a centipede game with an increasing pie. Ho and Su (2013) apply dynamic Level-K to both four- and six-stage centipede games and find it fits the data well. In Bayesian Games like auctions, particularly first-price and common-value auctions, Crawford and Iriberri (2007) claim that Level-K performs better than Cursed Equilibrium (CE) for inexperienced bidders in most cases (and it does better than NE, which fails badly.)

extensions over other solution concepts, including NE. However, as we saw in the Chicken Game above, Level-K is less useful than NLK in certain scenarios; and as we will see in other examples later, NLK outperforms Level-K.

Theoretically, Level-K has been extended in two ways. Strzalecki (2012) allows beliefs to vary arbitrarily for players at a certain level. Specifically, a $level_k$ player can believe the opponent to be $level_j$; $j < k$; by any arbitrary subjective distribution. However, here as well, beliefs are restricted to lower levels. Alaoui and Penta (2015) use another approach and show how cognitive bounds, beliefs about opponents, and beliefs about opponents' beliefs vary according to incentives by a cost-benefit analysis. In their model, if agents believe that their opponents behave at lower levels than their own cognitive bound, they would behave at one level higher than these opponents; but if they believe that the strategies of their opponents are reaching or exceeding their own cognitive bound, they would act at their own cognitive bound. So, although the researchers considered a situation where the opponents have the same or even a higher cognitive level than the agents themselves, they treated it as if the opponents were nevertheless one level below the agents. Thus, as far as we are aware, no extension of the Level-K model allows the player to believe she faces the same type as herself.

Another strand of literature also relaxes the restriction of beliefs in NE, while maintaining the equilibrium concepts for players' strategies. Eyster and Rabin (2005) proposed Cursed Equilibrium (CE), extending BNE to rationalize behavior (data) from experiments where BNE fails. In particular, this rationalization occurs in Common-Value Auctions, where Kagel and Levin (2002) found systematic overbidding and losses, a phenomenon called the

Winner's Curse. The CE also fits experimental data from voting and signaling models better than BNE. In its extreme situation, entitled as "fully CE," people correctly predict other players' distribution of actions, but ignore the correlation between actions and the specific players' types who chose those actions. In their general model, χ-Cursed Equilibrium, beliefs are a weighted average of beliefs in fully cursed opponents (with weight χ) and Bayesian Nash opponents (with weight (1-χ)). The CE characterizes heterogeneous behaviors by different cursed levels (with $\chi = 1$ being fully cursed, and $\chi = 0$ being BNE). However, CE reduces to NE when there is complete information. Hence it cannot be applied to explain deviations from NE in both static and dynamic games with complete information. Conceptually, the CE models bounded rational agents who only partially take into account how other players' actions depend on their type. In contrast, our solution concept allows our NLK player to consider that it might be possible to meet both a naive player and "another self" of the NLK player.

For application to dynamic games with perfect information and recall Analogy-Based Expectation Equilibrium (ABEE), a solution concept proposed by Jehiel (2005), is the most closely related to ours.[8] In ABEE, agents first group the set of opponents' decision nodes into a partition, namely, an analogy class. Then, they form expectations about each opponent's average behavior at every element of the analogy class rather than, more precisely, at each decision node. Though conceptually, ABEE is similar to CE, when applied to a different type of games, ABEE also suggests that people might not fully

[8] Jehiel and Koessler (2008) extends his analogy-based concept to Bayesian games.

consider how others' choices depend on their information,[9] and such deficiency in reasoning is common knowledge among all players. Differently, our model allows NLK players to consider heterogeneity in their opponents' inference process. In our version of NLK equilibrium as adapted to dynamic games, beliefs for different types of opponents are anchored at the beginning of the game and are updated at each stage, using Bayes' Rule. Analytically, ABEE coincides with SPNE for the finest analogy partition. And like our NLK, ABEE also rationalizes, in the Centipede Game, "passing" to the last few stages for a large range of partitions, in contrast to the implication of backward induction. However, ABEE does not provide a specific way to choose an analogy class, while our model offers a way of parametric estimation to specify beliefs in equilibrium.[10]

NLK is not the first equilibrium solution concept to introduce an exogenous type; Kreps, Milgrom, Roberts and Wilson's (KMRW) models have already used an exogenous type (Kreps and Wilson 1982, Milgrom and Roberts 1982, Kreps etc. 1982). However, NLK and KMRW's models are different drastically in facets of *motivation* and *generality*.

Regarding motivation, KMRW's works are motivated by Selten's (1978) Chain-Store Paradox (CSP) and by vast experimental evidence of cooperation in finitely repeated Prisoner's Dilemma (PD) games. *Deterrence strategy* in CSP,[11] and *cooperation* in PD

[9] More specifically, information means the history upon reaching a decision node at which the choice is made.

[10] As an extension of the Level-K model to dynamic games, Ho and Su (2013) apply their model to the experiment data of the Centipede Game. However, theirs is intended for learning across repetitions while ours explains strategic behavior better, even for novel games. Moreover, unlike their model, our solution concept does not restrict the strategy set, while allowing us to capture Bayesian updating for beliefs across stages within one round.

[11] *Deterrence strategy*, where the monopoly fights an early entrant, although it is not the best response in the stage game, was offered by Selten (1978), as a sensible, though not an equilibrium, strategy to deter later entrant.

games contradict the solution of backward induction, whereby unraveling to the one-shot stage game solution. KMRW's objective is to resolve the paradox of using deterrence strategy in the CSP game and to rationalize cooperation in the finitely repeated PD game. To do so, they transform these games from complete, to incomplete, information games by introducing a tiny probability of exogenous type and showing that it is sufficient to "choke off" the otherwise unavoidable logic of unraveling. The emphasis on tiny probability is a critical novelty, as otherwise deterrence strategy in the CSP game or cooperation in the PD game may be rationalized even in a one-shot game. In NLK, the probability λ of such exogenous type is typically quite large, similarly to, but less extreme than, that in Level-K model. Thus, whereas the motivation of the KMRW's models is to "defend" the standard NE, NLK, is a behavioral model of bounded rationality.

Regarding generality, NLK introduces one nonstrategic exogenous type to be applied to all, or at least to a large class of different games. In contrast, KMRW admit that their "defense" of the standard NE, requires a particular exogenous type for each case.[12] For instance, in the CSP case, Kreps and Wilson, use a "strong" monopoly, who is hard-wired to fight; In the finitely repeated PD game, KMRW use two nonstrategic types for two cases respectively: the one who play Tit-for-Tat for the one-sided incomplete information game, and the one who prefers the stage payoffs from joint-cooperation to the payoff of defection when the other player cooperates for the two-sided incomplete information game.[13]

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[12] KMRW explicitly acknowledge that such particular, and different, exogenous type may be needed for different cases.

[13] In addition, NLK can require that λ matches the probability of the exogenous type in the population making the model an equilibrium model with rational expectations.

As all the aforementioned solution concepts, NLK maintains the best response to beliefs but relaxes, relative to NE, assumptions of players' beliefs about other players. In dynamic games, Aumann (1992), like several other writers afterward, has shown that a failure of backward induction does not imply a failure of individual rationality. For example, in the Centipede Game, backward induction implies that the first mover stops the game at the first decision node, which is rarely found in the data from experiments. However, some relaxations of the "common knowledge of rationality."[14] may rationalize several rounds of continuation, although all of the players are rational.[15]

2. The solution concept

In this section, we formally define the NLK equilibrium in different types of games and prove its existence. We focus on the simple case of only two players with symmetric beliefs. We discuss several extensions in Section 5.

[14] One has to be careful about the terminology according to the epistemic condition of NE. Aumann and Brandenburger (1995) prove that in a two persons' game, mutual knowledge of preferences and payoffs, rationality, and beliefs for the other players' strategies are sufficient for NE. In other words, common knowledge of rationality is not necessary for NE in two persons' game. Moreover, Battigalli and Bonanno (1999) argue that there is a contradiction between results of backward induction and a common belief in sequential rationality at later stages. Thus, in this paper by the "common knowledge of rationality," we mean in general, the extra assumptions needed for NE/BNE/SPNE besides individual rationality.

[15] Aumann (1992) shows that continuation of the game beyond the first node for several rounds could occur even with "mutual knowledge" of high degrees. Considering the fact that some sequentially rational behaviors off the equilibrium path are only reachable by the violation of sequential rationality, Reny (1992) defines a weaker version of sequential rationality in light of forward induction. Ben-Porath (1997) proves that cooperation in the Centipede Game is consistent with Common Certainty of Rationality, a weaker concept than the Common Knowledge of Rationality. Asheim and Dufwenberg (2003) introduce the concept of "Fully Permissible Sets" to the extensive form game, where players reason deductively by trying to figure out one another's moves. They show that deductive reasoning does not necessarily imply backward induction.

2.1 The Basic Case

Consider a normal form game with two players, $G = (S_i, u_i)_{i=1,2}$, where (S_i, u_i) are the strategy set and the utility function of player i, respectively. The strategy of a naïve player i is given exogenously by $\sigma_i^o \in \Delta(S_i), i = 1,2$. In our strategic environment, an NLK player believes that her opponent is either a naïve player with probability λ or another NLK player with probability $(1 - \lambda), \lambda \in [0, 1]$. In an NLK equilibrium, an NLK player chooses an optimal strategy by best responding to her belief. A formal definition of the NLK equilibrium is as follows:

Definition 1.1. *A mixed-strategy profile* $(\sigma_i^*)_{i=1,2}$, *is a* λ*-NLK equilibrium if for each* $i =$ 1,2, *and each* $s_i' \in S_i$,

$$\lambda u_i(\sigma_i^*, \sigma_{-i}^0) + (1 - \lambda) u_i(\sigma_i^*, \sigma_{-i}^*) \geq \lambda u_i(s_i', \sigma_{-i}^0) + (1 - \lambda) u_i(s_i', \sigma_{-i}^*).$$

2.2. Bayesian Games

Consider a *Bayesian Game* of incomplete information $B = (S_i, u_i, \Theta_i, p)_{i=1,2}$, where Θ_i denotes the set of player i's types and where p is the joint *density function* of the probability distribution over $\Theta_1 \times \Theta_2$. Similar to the relationship between NE and BNE, a BNLK equilibrium is the NLK equilibrium of the "extended game" in which each player i's space of pure strategies is ${S_i}^{\Theta_i}$, which denotes the set of mappings from Θ_i to S_i. Again, let $\sigma_i^0 \in \Delta(S_i), i = 1,2$, denote the strategy of a naïve player i which is independent of his type. Then a formal definition of BNLK with respect to subjective symmetric belief λ is as follows:

Definition 1.2. *A profile of strategies* $\{s_i^*(\cdot)\}_{i=1,2}$, *is a* λ*-BNLK equilibrium*, *if for each* $i =$ 1,2, *and each* $\theta_i \in \Theta_i$,

$$s_i^*(\theta_i) \in \arg max_{s_i \in S_i} \int p(\theta_{-i}|\theta_i)[\lambda u_i(s_i, \sigma_i^0; \theta_i, \theta_{-i}) + (1 - \lambda) u_i(s_i, s_{-i}^*(\theta_{-i}); \theta_i, \theta_{-i})] d\theta_{-i}.$$

2.3. Dynamic Games

Consider a dynamic game with perfect information and perfect recall played by two players[16] $P = (u_i, \Upsilon)_{i=1,2}$, where Υ denotes a game tree. A node in the game tree Υ is denoted by h^t, and the set of nodes is denoted by H. The set of nodes at which player i must move is denoted by H_i. An NLK player holds a prior belief that the opponent is either a naïve player with probability λ or another NLK player with probability $(1 - \lambda), \lambda \in [0, 1]$. At every decision node with history h^t, as more information is revealed, beliefs are updated. We denote the updated belief that the opponent is a naïve player as $p_i(h^t)$. In equilibrium, an NLK player chooses an optimal strategy according to her belief at every decision node. In other words, here the choice is sequentially rational as defined below:

Definition 1.3. (sequential rationality). *A strategy profile* $\{\sigma_i^*\}_{i=1,2}$ *is sequentially rational with respect to the profile of beliefs* $\{p_i(h_i^t)\}_{h_i^t \in H_i}, i = 1,2$ *if for* $i = 1,2$, *all strategies* σ_i', *and all nodes* $h_i^t \in H_i$:

(1.1) $$p_i(h_i^t) u_i(\sigma_i^*, \sigma_{-i}^0 | h_i^t) + (1 - p_i(h_i^t)) u_i(\sigma_i^*, \sigma_{-i}^* | h_i^t) \geq p_i(h_i^t) u_i(\sigma_i', \sigma_{-i}^0 | h_i^t) + (1 - p_i(h_i^t)) u_i(\sigma_i', \sigma_{-i}^* | h_i^t)$$

[16] That is, at any decision node, all previous moves are assumed to be known to every player.

We also require that the beliefs of an NLK player are consistent. That is, to start with a subjective prior distribution and then get updated by *Bayes' Rule* at each succeeding decision node. To present formally the consistency restriction, let $p(h^t|\sigma_i, \sigma_{-i})$ denote the probability that decision node h_t is reached according to the strategy profile, (σ_i, σ_{-i}).

Definition 1.4. (consistency). *A profile of beliefs* $\{p_i^*(h_i^t)\}_{h_i^t \in H_i}, i = 1{,}2$ *is consistent with the subjective prior* λ *and the strategy profile* $\{\sigma_i\}_{i=1,2}$ *if and only if for i=1,2, and all nodes* $h_i^t \in H_i$*:*

(1.2) $$p_i^*(h_i^t) = \frac{\lambda p(h_i^t|\sigma_i,\sigma_{-i}^0)}{\lambda p(h_i^t|\sigma_i,\sigma_{-i}^0) + (1-\lambda) p(h_i^t|\sigma_i,\sigma_{-i})},$$

where $p(h_i^t \big| \sigma_i, \sigma_{-i}^0) > 0$ or $p(h_i^t|\sigma_i, \sigma_{-i}) > 0$.[17]

Although the game itself has perfect information, the belief structure in our strategic environment makes our solution concept more like an analogy of a *Perfect Bayesian Equilibrium* (PBE), So we denote it as PBNLK, formally treated below:

Definition 1.5. An assessment $\left(\sigma_i^*, \{p_i^*(h_i^t)\}_{h_i^t \in H_i}\right)_{i=1,2}$ is a λ*-PBNLK* equilibrium if

1. The strategy profile $\{\sigma_i^*\}_{i=1,2}$ is sequentially rational with respect to the profile of beliefs $\{p_i^*(h_i^t)\}_{h_i^t \in H_i}, i = 1{,}2$ and
2. The profile of beliefs $\{p_i^*(h_i^t)\}_{h_i^t \in H_i}, i = 1{,}2$ is consistent with the subjective prior λ and the strategy profile $\{\sigma_i^*\}_{i=1,2}$.

[17] It should be noted that Definition 1.4 places no restrictions on player i's expectations about those decision nodes that are not possibly reached according to σ, regardless of facing a naïve player or another NLK player. A stronger notion of consistency could be defined in the spirit of a trembling hand or a sequential equilibrium (Kreps and Wilson, 1982a). Such stronger restriction and its impact on prediction are discussed in Section 5.

2.4. Existence

Proposition 1.1. for any $\lambda \in [0, 1]$:

a) In every finite strategic-form game, there exists an NLK equilibrium.

b) In every finite Bayesian game, there exists a BNLK equilibrium.

c) In every finite extensive form game, there exists a PBNLK equilibrium.

Proof. The existence of an NLK equilibrium is guaranteed by a standard fixed-point theorem (Kakutani 1941), similar to the proof of the existence of a NE (Glicksberg 1952). For (b), a similar argument follows from Harsanyi (1973). For (c), consider an alternative dynamic game of incomplete information $\mathrm{P} = (\mathrm{u_i}, \Phi_\mathrm{i}, \Upsilon)_{\mathrm{i}=1,2}$, where Φ_i denotes the possible types for agent i, which can be either a naïve player or an NLK player. Let Σ^0be the strategy set of the naïve player and restrict it to be $\{\sigma^0\}$. Then according to Kreps and Wilson (1982a), for every finite extensive form game, there exists at least one sequential equilibrium $(\sigma_i^*, p_i^*)_{i=1,2}$ should satisfy equation 1.1 and 1.2 for sequential rationality and consistency. In other words, PBNLK exists.

Remark 1.1. In the special case when $\lambda = 0$, NLK/BNLK/PBNLK coincides with NE/BNE/SPNE. In another special case, if the strategy of our naïve player is exogenously given as that of a level$_{k-1}$ player, where $k \in \aleph^+$ and $\lambda = 1$, then the strategy for an NLK player coincides with that of a level$_k$ player in all three types of games considered above.

3. The Arad-Rubinstein Money Request Game.

In the basic version of the Money Request Game by Arad and Rubinstein (2012), there are two risk-neutral players, and each can request and receive an integer amount of money from \$11 to \$20. In addition, a player receives an extra \$20 if she asks for exactly one integer less than the other player.

NLK (λ)	15	16	17	18	19	20
NLK (%) ($0 \leq \lambda \leq \frac{1}{2}$)	$\frac{5(5-10\lambda)}{1-\lambda}$	$\frac{5(5-2\lambda)}{1-\lambda}$	$\frac{5(4-2\lambda)}{1-\lambda}$	$\frac{5(3-2\lambda)}{1-\lambda}$	$\frac{5(2-2\lambda)}{1-\lambda}$	$\frac{5(1-2\lambda)}{1-\lambda}$
NLK (%) ($\frac{1}{2} \leq \lambda \leq \frac{14}{20}$)	0	$\frac{5(14-20\lambda)}{1-\lambda}$	$\frac{15}{1-\lambda}$	$\frac{10}{1-\lambda}$	$\frac{5}{1-\lambda}$	0
NLK (%) ($\frac{14}{20} \leq \lambda \leq \frac{17}{20}$)	0	0	$\frac{5(17-20\lambda)}{1-\lambda}$	$\frac{10}{1-\lambda}$	$\frac{5}{1-\lambda}$	0
NLK (%) ($\frac{17}{20} \leq \lambda \leq \frac{19}{20}$)	0	0	0	$\frac{5(19-20\lambda)}{1-\lambda}$	$\frac{5}{1-\lambda}$	0
NLK (%) ($\frac{19}{20} \leq \lambda \leq 1$)	0	0	0	0	100	0

Table 1.2. NLK equilibrium strategy for different subjective beliefs.

Consider the Level-K model with a level_0 payer who randomizes uniformly within the strategy set: $\{\$11, \$12, \ldots, \$20\}$. A level_1 player that requests \$20 earns \$20. Alternatively, if she asks for \$19, she would earn \$19 for sure and \$20 bonus with a probability of 1/10, for a total expected payoff of \$21.[18] Thus, level_1 picks \$19, level_2 picks \$18,…, level_9 picks

[18] To ask for any amount of money less than \$19 lead to s strictly lower payoff.

\$11. But then $level_{10}$ picks \$20, $level_{11}$ picks \$19, and so on. It is difficult to infer from players' actions their sophistication level: A player who requests \$19 can be a $level_1$ player or a highly sophisticated $level_{11}$ player. Table 1.2 shows the unique mixed strategy λ-NLK equilibrium for each $\lambda \in [0,19/20]$ and the unique pure strategy for $\lambda \in [19/20,1]$.[19]

Action	11	12	13	14	15	16	17	18	19	20	MSE
$Level_1$ (%)	0	0	0	0	0	0	0	0	100	0	980.2
$Level_2$ (%)	0	0	0	0	0	0	0	100	0	0	620.2
$Level_3$ (%)	0	0	0	0	0	0	100	0	0	0	580.2
$level_k, k = 1,2,3,$ $optimal\ distribution$	0	0	0	0	0	0	40.7	38.7	20.6	0	35.93
NE (%)	0	0	0	0	25	25	20	15	10	5	137.2
NLK (%)[20] ($\lambda = 0.6485$)	0	0	0	0	0	12.1	43.9	29.4	14.6	0	28.39
Data (%)	4	0	3	6	1	6	32	30	12	6	

Table 1.3. 11-20 Game: comparison of different solution concepts by MSE.

Table 1.3 compares the performance of the Level-K model, NE, and NLK[21] using the Mean Squared Error (MSE). NLK with the best the best $\lambda(= 0.6585)$ fits the adta better NE and

[19] See Appendix A.1 for detail of the argument.
[20] We choose the value that minimizes the Mean Squared Errors (MSE), that is, the nonlinear least squares estimate of λ.
[21] Our naïve player is defined in the same way as the random $level_0$ player.

any type of the Level-K model. Moreover, with only one additional parameter compared to two in the often-used Level-K model with an optimal distribution of $level_1$, $level_2$ and $level_3$, it reduces MSE by 23.45% (from MSE=35.93 to 28.39).

Finally, we further test the robustness of our results by an alternative statistic method. Our econometric specification follows the mixture-of-types models of Stahl and Wilson (1994, 1995).[22] Both $level_k$ and our NLK types are assumed to make logistic errors[23] as described below. The decision rule suggests that the choice probabilities of type t players are positively but imperfectly related to expected payoffs according to the beliefs specific for type t. Formally, denote the expected payoff for player i of type t, given strategy s by $\pi_i^t(s)$. Then the probability of observing s by such players is specified as follows:

$$p_i^t(s) = \frac{\exp(\eta\pi_i^t(s))}{\sum_{s'\in s_i}\exp(\eta\pi_i^t(s'))},$$

where S_i is the strategy set for player i and η is the parameter of precision. Specifically, η determines the sensitivity of choice probabilities to payoff differences.[24] The *Likelihood* of observing sample $\{s_i\}_{i=1}^N$, given type t is

$$L^t(\{s_i\}|\eta) = \prod_{i=1}^{N} p_i^t(s_i).$$

[22] The same econometric specification was also adopted by Costa-Gomes, Crawford and Broseta (2001), Camerer, Ho and Chong (2004), Costa-Gomes and Crawford (2006) and Crawford and Iriberri (2007). The error model is developed from *Quantal Response Equilibrium* (See, e.g., Goeree, Holt and Palfrey 2008), and discussed in Goeree and Holt (2001).

[23] Random $level_0$ directly specifies a uniform distribution of decisions, and so has no precision parameter. Or it is equivalent by specifying the precision parameter to be 0 for a random $level_0$ player.

[24] As η goes to ∞, the probability of the optimal decision converges to 1. In other words, the choice is error-free and fully characterized by the model under consideration. At the other extreme, as η goes to 0, the choice probability converges to a uniformly random choice as that of the random $level_0$ players.

Let α_t denote the proportion of type t in the population, with $\sum_t \alpha_t = 1$. The *Likelihood* of observing the sample unconditional on type is $\prod_{i=1}^{N} \sum_t \alpha_t p_i^t(s_i)$.

Action	Log-Likelihood (LL)	The precision parameter (η)
Level_1	-233.97	0.296 (0.039)
Level_2	-226.245	0.066 (0.009)
Level_3	-221.215	0.075 (0.010)
$level_k, k = 1,2$ $optimal\ distribution$	-218.093	0.252 (0.052)
$level_k, k = 1,2,3,$ $optimal\ distribution$	-197.770	0.207 (0.051)
NE	-230.035	0.231 (0.046)
NLK ($\lambda = 0.85$)	-210.046	0.359 (0.025)

Table 1.4. Comparison of different solution concepts by Maximum Log-Likelihood

The result is reported in Table 1.4. With the error structure, the single type Level-K model with the best $k^* = 3$ has both a smaller Log-likelihood and a precision parameter, *LL*=-*221.275*, $\eta = 0.075$ than those of NLK with the best $\lambda^* = 0.85, LL = -210.046, \eta =$

0.359. Moreover, NLK also performs better than the Level-K model with the optimal distribution of $level_1$ and $level_2$,[25] *LL=-218.093*, $\eta = 0.252$. However, if we allow the Level-K to have one more parameter than NLK, the result is different. NLK still have a higher precision but a lower Log-likelihood than the Level-K with the optimal distribution of $level_1$, $level_2$ and $level_3$,[26] *LL=-197.770*, $\eta = 0.207$.

4. The Centipede Game

Introduced by Rosenthal (1981), the Centipede Game serves as an example where deviations from *Backward Induction* (or SPNE) seem reasonable.[27] Following the bulk of literature, we study a version of the Centipede Game, where the total payout doubles when the game continues to the next stage, which subsumes the game in the experiment of both Palacios-Huerta and Volij (2009) and Levitt, List, and Sadoff (2011), as a special case (when there are six decision nodes).

There are two players, A and B, with an initial pot worth \$5. At Node 1, Player A moves and chooses either to stop the game (T) by taking 80% of the pot and leaving 20% of it to Player B or pass the game (P) to player B and doubling the pot. If Player A chooses P, then at Node 2, Player B faces a similar decision but with a pot now worth \$10. Unless one of the players chooses T earlier, the game ends after $S = 2N$ stages, with Player B either choosing T, taking 80% of the pot and leaving the other 20% to Player A, or choosing P

[25] It is estimated to be 85% $level_1$ and 15% $level_2$ types.
[26] It is estimated to be 46% $level_1$, 24.45% $level_2$ and 28.98% $level_3$ types.
[27] For additional literature, see McKelvey and Palfrey (1992), Fey, McKelvey, and Palfrey (1996), Nagel and Tang (1998), Borstein, Kugler, and Ziegelmeyer (2004), and Rapoport, Stein, Parco, and Nicholas (2003). These papers show that even in high-stakes situations, involving altruism or group decisions, *Backward Induction* is still inadequate to explain players' behavior.

and doubling the pot, with the result that 20% of the pot goes to Player B and 80% of it goes to Player A.

The payoffs for Players A and B are ($\$2^{2k}, \$2^{sk-2}$) if the game ends at an odd decision node, $2k-1$, and ($\$2^{2k-1}, \$2^{sk+1}$) if the game ends at an even decision node, $2k$, $k = 1, 2, \ldots, N-1$. By backward induction, the unique SPNE strategy profile is for Player A to play T at node 1; and off equilibrium, the active player always chooses T at each node.

Following the dynamic Level-K model by Ho and Su (2013), it is equally likely that a $level_0$ player will choose T or P at each decision node, and strategies of higher levels are generated from iterative best responses to a player of one level below. First, a $level_1$ Player B would choose T at the last node.[28] Denote the whole pie at each decision node as x. For a $level_1$ Player A, paying T at Node $(2N-1)$ generates $\frac{4x}{5}$, while playing P generates $\frac{9x}{5}$, so a $level_1$ Player A would choose P at the decision Node $(2N-1)$.

Role	Threshold stage s	The level of players
Player A	$2(N-h)+1$	$k = 2h \; or \; 2h+1$ $(1 \leq h \leq N-1)$
Player B	$2(N-h)+2$	$k = 2h-1 \; or \; 2h$ $(1 \leq h \leq N)$

*h is an auxiliary parameter for indicating the same threshold stage of two adjacent levels.

Table 1.5. Threshold stage for different levels of players.

[28] To end the game at Node $2N$, Player B gets payoff $\$2^{2N+1}$, while he only ends up with $\$2^{2N}$ if he chooses P instead

Table 1.5 summarizes the solution for the Level-K model for a game of length $S = 2N$. For a certain level of players (indicated in the second column), there exists a corresponding threshold stage (indicated in the first column). A level$_k$ player chooses P before the threshold stage s^*, but T at stage s^* and afterward. For example, in a six-stage game ($N = 3$), the threshold stage for a level$_3$ ($k = 3, h = 1$) Player A is $2(3 - 1) + 1 = 5$. So a level$_3$ Player A chooses P before Node 5 and T at Node 5.

In addition, a Player A, at level $k = 2N$ or higher, and a Player B at $k = (2N - 1)$ or higher, ought to choose T at each decision node. The Level-K solution requires relatively high levels[29] to rationalize terminating the game at earlier stages, especially for longer games, since the strategies of different level players are independent of the length of the game. For example, no matter how long the game is, a level$_1$ Player A ought to keep passing to the last decision node, and no matter what the observed history is, a level$_k$ player never updates his belief. [30]

Consider a simple version of our PBNLK with only symmetric beliefs, $0 < \lambda < 1$. First, at the last stage, T is the best response for Player B regardless of his subjective belief about his opponents' type. Now, assume that Player B first chooses T at Stage $2n$ and Player A

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[29] Table 1.5 also entails that to increase the level by just 1 would not necessarily predict earlier termination. Two adjacent levels of players might behave the same way.

[30] One may argue that the more general Cognitive Hierarchy (CH) solution concept would produce qualitatively different predictions. However, since beliefs put more weight on lower levels according to a Poisson distribution in CH and lower levels continue passing to later stages, an even higher level of players than in the Level-K model would be required to rationalize early termination.

plans to choose T at Stage $(2n+1)$. Then, at stage $(2n-1)$, Player A's posterior belief of the opponent being level$_0$ is

(1.3) $$p_A^\lambda(2n-1) = \frac{\lambda\left(\frac{1}{2}\right)^{n-1}}{\lambda\left(\frac{1}{2}\right)^{n-1}+(1-\lambda)} = \left[\frac{\left(\frac{1}{2}\right)^{n-1}}{\left(\frac{1}{2}\right)^{n-1}+\frac{(1-\lambda)}{\lambda}}\right] \in (0,\lambda).$$

If Player B plays T at Stage $2n$, then at Stage $(2n-1)$, Player A gets $\frac{4x}{5}$ by playing T, while by playing P now and then T at $(2n+1)$ yields the expected payoff:

$$\left[\frac{p_A^\lambda(2n-1)}{2}+1-p_A^\lambda(2n-1)\right]\frac{2x}{5}+p_A^\lambda(2n-1)\frac{1}{2}\times\frac{4}{5}\times 4x = \frac{2x}{5}+p_A^\lambda(2n-1)\frac{7x}{5}$$

Thus, Player A plays P whenever $\frac{2}{7} < p_A^\lambda(2n-1) \leq 1$ and plays T otherwise. Moreover, since $p_A^\lambda(2N-1)$ decreases in N for a given λ, then, in a longer game, and NLK Player A (with a certain λ) is more likely to play T at stage $(2N-1)$. This result is a key difference between NLK and the Level-K model where a level$_1$ Player A always passes at stage $(2N-1)$ no matter how long the game is. Since $p_A^\lambda(2n-1)(\leq\lambda)$ is strictly decreasing in n and $p_A^\lambda(2n-1)_{n\to\infty}=0$, then for $\lambda \leq \frac{2}{7}$, Player A would always play T, given that Player B plays T at the next sage. For $\lambda > \frac{2}{7}$, by continuity, there is a critical value n_A, such that $p_A^\lambda(2n-1) > \frac{2}{7}$ for $n < n_A$, and $p_A^\lambda(2n-1) \leq \frac{2}{7}$ for $n \geq n_A$.

Similarly, assume that Player A first chooses T at Stage $(2n+1)$, $(n \leq N-1)$ and Player B plans to choose T at stage $(2n+2)$. Then at Stage $2n$, Player B's posterior belief that the opponent is level$_0$ is

$$p_B^\lambda(2n) = \frac{\lambda\left(\frac{1}{2}\right)^{n-1}}{\lambda\left(\frac{1}{2}\right)^{n-1}+(1-\lambda)} = p_A^\lambda(2n+1).$$

This implies that the threshold stage for Player B, s_B^* is one stage earlier than that of Player A, s_A^*, i.e., $s_B^* = (2n_B) = (2n_A - 1) - 1 = s_A^* - 1$.

We use these arguments to construct our PBNLK equilibrium. For $\lambda = 0$, the game ends at the first stage (the same result as in SPNE).[31] For $\lambda > 0$, there are two possibilities. In a short game with a relatively larger λ satisfying $p_A^\lambda(2N-1) > \frac{2}{7}$, Player A plays P to the end, and Player B first plays T at the last stage (the same result as when both players are level$_1$). In a longer game with $p_A^\lambda(2N-1) \leq \frac{2}{7}$, the game would end earlier. For similar arguments as in Kreps, Milgrom, Roberts, and Wilson (1982) paper,[32] PBNLK must be in mixed strategies for this range of λ. The reason is that in a presumed pure strategy PBNLK, and NLK player (who ought to play T earlier than the other player) would rather deviate in the first node she ought to play T and lay P instead. Doing so would mislead the other player to believe that he is facing a level$_0$ player (as only a level$_0$ player would have played P in the last node) and thus the other NLK player would play P.[33]

We apply our model to experimental by Palacios-Huerta and Volij (2009) and Levitt, List, and Sadoff (2011) on the *Centipede Game*, where *N=3* as in Figure 1.1.

[31] The off-equilibrium path will not be reached by an NLK player (A or B) whether her opponent is another NLK player or a naive player. So, it is not restricted by Definition 1.4 of consistency. We assume that an NLK player believes the other NLK player would always play T off the equilibrium path.

[32] Inserting a "crazy" type even with a slight probability can rationalize long cooperation in the finitely repeated prisoners' dilemma games.

[33] For example, consider the case when the threshold stage of Player B is 4 and it follows that of Player A is 5. Now at Node 5, which is reachable for Player A when facing a level$_0$ player, since Player B first choose T at 4 not 6, the belief $P_A^\lambda(5)$ represented by Equation 1.3 no longer satisfies our consistency requirement. Upon reaching Node 5, by *Bayes' rule*, an NLK Player A confirms that her opponent is a level$_0$ player for sure, so she would pass instead. Thus, at decision Node 4, an NLK Player B has an incentive to pass with a positive probability to mimic the level$_0$ player which motivates an NLK Player A to pass with a positive probability at decision Node 5, as well.

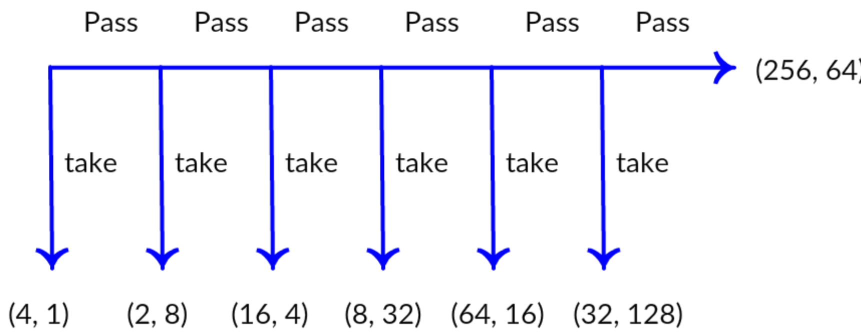

Figure 1.1. the centipede game

The prediction of our PBNLK with all $\lambda \in \{0.05n\}_{n=0,1,2,\dots,20}$ and the Level-K model with all $k \in \aleph^{+}$ as well as data from the above-mentioned two papers are summarized in Table 1.6. When $\lambda = 0$, PBNLK coincides with SPNE and $level_k$, $k \geq 6$, and when $\lambda \in [0.615, 1]$, PBNLK coincides with $level_1$. For all other $\lambda \in (0, 0.615)$, PBNLK generates different predictions. We first compare predictions to data from Palacios-Huerta and Volij's laboratory experiment with four treatments. Unlike other experiments of the *Centipede Game*, in their work, the composition of two opponents varies[34] across treatments, and it is common knowledge among all players. This allows us to explore how belief represented by λ and the results change as the nature of the subject pool changes. Next, we compare predictions to data from Levitt, List and Sadoff's field experiments of chess players to further evaluate the predictions of NLK, since the data are quite different from the former experiment data.

[34] See Table 1.6 for detail. The two opponents are chess players or students.

Data or Prediction	Node 1	Node 2	Node 3	Node 4	Node 5	Node 6
NLK $(\lambda = 0)$ or $level_k$ $(k \geq 6)$	1*	1	1	1	1	1
NLK $(\lambda = 0.05)$	0	0.704	0.867	0.899	0.892	1
NLK $(\lambda = 0.1)$	0	0.375	0.877	0.889	0.938	1
NLK $(\lambda = 0.15)$	0	0.007	0.889	0.889	0.999	1
NLK $(\lambda = 0.2)$	0	0	0	0.844	0.879	1
NLK $(\lambda = 0.25)$	0	0	0	0.792	0.887	1
NLK $(\lambda = 0.3)$	0	0	0	0.732	0.895	1
NLK $(\lambda = 0.35)$	0	0	0	0.663	0.905	1
NLK $(\lambda = 0.4)$	0	0	0	0.583	0.916	1
NLK $(\lambda = 0.45)$	0	0	0	0.489	0.930	1
NLK $(\lambda = 0.5)$	0	0	0	0.375	0.946	1
NLK $(\lambda = 0.55)$	0	0	0	0.236	0.966	1
NLK $(\lambda = 0.6)$	0	0	0	0.0625	0.991	1

Continued

Table 1.6. Centipede game-prediction and data

Table 1.6 Continued

$NLK\ \ or\ level_1$ $(0.615 < \lambda \leq 1)$	0	0	0	0	0	1
$level_2$	0	0	0	0	1	1
$level_3$	0	0	0	1	1	1
$level_4$	0	0	1	1	1	1
$level_5$	0	1	1	1	1	1
Data** (S vs S)	0.030 *** (200)	0.17 (194)	0.42 (161)	0.65 (93)	0.82 (33)	0.83 (6)
Data (S vs C)	0.30 (200)	0.52 (140)	0.61 (67)	0.69 (26)	1.00 (8)	-
Data (C vs S)	0.375 (200)	0.44 (125)	0.56 (70)	0.61 (31)	1.00 (12)	-
Data (C vs C)	0.725 (200)	0.64 (55)	0.90 (20)	1.00 (2)	-	-
Data****(Field)	0.039 (102)	0.102 (98)	0.193 (88)	0.352 (71)	0.587 (46)	0.632 (19)

Note: * presents predicted probabilities of playing T at each node by the model. Columns correspond to the probability that a player is predicted to play T upon reaching that node. Odd nodes refer to Player A's choices, while even nodes refer to Player B's choices.
** The data is from Palacios-Huerta and Volij (2009). S represents students and C represents chess players. S vs C represents the situation when Player A is a student and Player B is a chess player. The other way around, C vs S is when Player A is a chess player and Player B is a student.
*** shows the distribution of implied stop probabilities for players in the Centipede Game. The number of opportunities observed is displayed in the parentheses below.
**** The data is from the field Centipede Game of chess players by Levitt, List, and Sadoff (2011).

Referring to Ho and Su (2013), we define a measure, $D(H, M, G_S)$ to quantify the deviation of data, denoted by H, from the prediction of a model, denoted by M, in *Centipede Game* G_S with S decision nodes as follows:

$$D(H, M, G_S) = \sum_{s=1}^{S} w_s^H d_s(p_s^H, p_s^M) \text{ , } w_s^H = \frac{n_s^H}{\sum_{k=1}^{S} w_k^H} \text{ , } d_s(p_s^H, p_s^M) = |p_s^H - p_s^M|,$$

Where n_s^H is the number of observations at each stage, given by data H, $d_s(p_s^H, p_s^M)$ is the distance of stopping probabilities at stage s between data H and the prediction of model M measured by their absolute difference $|p_s^H - p_s^M|$.

Table 1.7 presents the result of $D(H, M, G_S)$ for our PBNLK and the Level-K model with 5 different data sets above. In the lab experiments when opponents are students (Column 2), PBNLK with $\lambda = 0.35$ gives the most precise prediction $(D = 0.1625)$, which is better than the best prediction of the single type Level-K model $(k = 3, D = 0.2127)$; in the treatment when chess players and students play with each other (Column 3 and 4), PBNLK with $\lambda = 0.1$ fits the data the best $(D^{S\ vs\ C} = 0.2361, D^{C\ vs\ S} = 0.2619\ )$, which is more accurate than the Level-K model with an optimal $k = 4$ $(D^{S\ vs\ C} = 0.3787, D^{C\ vs\ S} = 0.3947\ )$; when the opponents are chess players, the best fit goes to the case when PBNLK $(\lambda = 0)$, SPNE and the level$_k$ type, $(k \geq 6)$ coincide $(D = 0.2773)$. For the field data, PBNLK with $\lambda = 0.5$ gives the most precise prediction $(D = 0.1323)$, which is more accurate than the best prediction of the Level-K model with an optimal $k = 2$ $(D = 0.1933)$. Moreover, in all 5 datasets, the optimal PBNLK performs better than the Level-K with an optimal distribution of level$_1$ and level$_2$ types. When we allow the Level-K model to have one more parameter, the optimal PBNLK still performs better the Level-K with an optimal distribution of $level_1$, $level_2$ and $level_3$ in three datasets (Column 2, 3, 4) except for the lab experiments when opponents are students and the field data.

Models	Data (S vs S)	Data (S vs C)	Data (C vs S)	Data (C vs C)	Data (Field)
NLK $(\lambda = 0)$ or $level_k$ $(k \geq 6)$	0.7102	0.5474	0.5431	$0.2773^{\#*}$	0.7760
NLK $(\lambda = 0.05)$	0.3016	0.2478	0.3191	0.5393	0.4296
NLK $(\lambda = 0.1)$	0.2132	$0.2361^{\#}$	$0.2619^{\#}$	0.5785	0.3589
NLK $(\lambda = 0.15)$	0.2071	0.3536	0.3672	0.6500	0.3269
NLK $(\lambda = 0.2)$	0.1857	0.4051	0.4062	0.7166	0.2036
NLK $(\lambda = 0.25)$	0.1791	0.4019	0.4023	0.7170	0.1946
NLK $(\lambda = 0.3)$	0.1714	0.3982	0.3978	0.7175	0.1866
NLK $(\lambda = 0.35)$	$0.1625^{\#}$	0.3971	0.3927	0.7181	0.1761
NLK $(\lambda = 0.4)$	0.1703	0.4016	0.3905	0.7185	0.1638
NLK $(\lambda = 0.45)$	0.1837	0.4069	0.3968	0.7192	0.1497

Continued

Table 1.7. Centipede game-prediction for different models

Table 1.7 Continued

NLK $(\lambda = 0.5)$	0.1999	0.4134	0.4044	0.7200	$0.1323^{\#}$
NLK $(\lambda = 0.55)$	0.2197	0.4212	0.4137	0.7210	0.150
NLK $(\lambda = 0.6)$	0.2444	0.4310	0.4253	0.7222	0.1818
$NLK\ \ or\ level_1$ $(0.615 < \lambda \leq 1)$	0.2840	0.4526	0.4569	0.7227	0.2121
$level_2$	0.2533	0.4345	0.4295	0.7227	0.1933*
$level_3$	0.2127*	0.4121	0.4139	0.7155	0.2428
$level_4$	0.2502	0.3787*	0.3947*	0.6578	0.3703
$level_5$	0.4366	0.3660	0.4290	0.6022	0.5540
$level_k, k = 1{,}2$ $optimal\ distribution$	0.2446	0.4345	0.4295	0.7227	0.1484
$level_k, k = 1{,}2{,}3,$ $optimal\ distribution$	0.1567	0.3946	0.3872	0.7155	0.0895

Note: * and # indicate the best prediction of a single type Level-K and NLK, respectively.

Our solution concept provides an alternative explanation for scenarios where neither the original Level-K model nor backward induction applies. Note that we constrained NLK by using only symmetric beliefs. However, it is reasonable for each group to have a different subjective λ in cases where students interact with chess players, wherein we conjecture that our solution concept would perform even better when allowing for heterogeneous beliefs, but accounting for additional parameters.

5. Common Value Auction

Avery and Kagel (1997, AK afterward) conducted a laboratory experiment using a Common-Value, Second Price Auction, *the Wallet Game*. In their design, there are two bidders, $i = 1,2$, each privately observes a signal X_i that is drawn i.i.d from a *uniform distribution* on $[1, 4]$. The common value is the sum of the two private signals, that is, $v_i(x_1, x_2) = v(x_1, x_2) = x_1 + x_2$. Let $v(x, y) = x + y$, and r(x) = x + E$[X_2] = x + 2.5$. $v(x, x) = 2x$ is the unique symmetric BNE.[35] In fact, with just two bidders, the strategy $b(x) = v(x, x) = 2x$, is an *ex-post* equilibrium—independent of signals' distribution and risk attitude and with no regret. AK defines *Naïve bidding* by $r(x) = x + 2.5$, representing a naïve bidder who assumes that whenever she wins, the other bidder's signal is at its expected value (2.5). It turns out that $r(x)$ is also the level$_1$ player's strategy in Crawford and Iriberri (2007, CI afterward), the best response to a level$_0$ player who bids uniformly randomly on $[1, 4]$. We denote by $b^{\lambda}(\cdot)$ the strategy in a λ-BNLK equilibrium and solve the symmetric linear strategy. (The detail is provided in Appendix A.2.)

The data produced by AK is evaluated using the CE by Eyster and Rabin (2005) and the Level-K by Crawford and Iriberri (2007). Eyster and Rabin show that for any cursed level, $0 < \chi \leq 1$, their CE predicts better than BNE (i.e. CE with $\chi = 0$) and that for a given χ,

[35] Refer to Milgrom and Weber (1982).

CE fits better for experienced, rather than for inexperienced subjects, with respect to the Mean Squared Error (MSE). For data on only inexperienced bidders, CI use the Level-K with a logistic error structure and a subject-specific precision. They compare their model using the best mixture of 5 types, including random $level_1$ and $level_2$,[36] truthful $level_1$ and $level_2$,[37] and BNE players and show that it outperforms CE (with the best mixture of types, such that $\chi \in \{0.1, 0.2, \ldots, 0.9, 1.0\}$), using both likelihood and the Bayesian Information Criterion (BIC).[38]

Table 1.8 compares the prediction of λ-BNLK (with all $\lambda \in \{0.05\text{n}\}_{n=0,1,2,\ldots,20}$) and the Level-K model. As shown, the optimal bidding of a $level_2$ player already reduces to a boundary solution (the objective function becomes a linear function), where all bidders with a value lower than 2.5 bid 3.5 and the others (with a value higher than 2.5) bid 6.5. For a $level_3$ player, when her signal is smaller than 2.5, she bids any number below 3.5, (expecting to lose), while bidding any number above 6.5 when her signal is larger than 2.5 (expecting to win). The predictions are ambiguous for higher levels. In contrast, there always exists a symmetric linear strategy for our λ-BNLK players.

[36] Random $level_1$ and $level_2$ are generated iteratively by best responding to a random $level_0$, as considered in this paper.
[37] Truthful $level_1$ and $level_2$ are generated iteratively by best responding to a ruthful $level_0$ who always bids her signal: $b(x) = x$.
[38] BIC penalize models with more parameters to adjust the likelihood.

Models	$b(x)$	MSE (inexperienced)	MSE (experienced)
NLK $(\lambda = 0)$ or NE	2x	2.897	1.171
NLK $(\lambda = 0.05)$	1.951x+0.122	2.823	1.124
NLK $(\lambda = 0.1)$	1.904x+0.239	2.756	1.082
NLK $(\lambda = 0.15)$	1.859x+0.352	2.693	1.042
NLK $(\lambda = 0.2)$	1.815x+0.462	2.634	1.010
NLK $(\lambda = 0.25)$	1.772x+0.570	2.579	0.978
NLK $(\lambda = 0.3)$	1.730x+0.676	2.531	0.953
NLK $(\lambda = 0.35)$	1.688x+0.781	2.484	0.927
NLK $(\lambda = 0.4)$	1.646x+0.886	2.440	0.906
NLK $(\lambda = 0.45)$	1.604x+0.990	2.396	0.889
NLK $(\lambda = 0.5)$	1.562x+1.096	2.356	0.872
NLK $(\lambda = 0.55)$	1.519x+1.203	2.320	0.859
NLK $(\lambda = 0.6)$	1.475x+1.313	2.286	0.848

Continued

Table 1.8. Model comparison for the wallet game.

Table 1.8 Continued

NLK ($\lambda = 0.65$)	1.430x+1.426	2.250	0.840
NLK ($\lambda = 0.70$)	1.383x+1.543	2.220	0.835
NLK ($\lambda = 0.75$)	1.333x++1.667	2.190	0.834*
NLK ($\lambda = 0.80$)	1.281x+1.798	2.164	0.835
NLK ($\lambda = 0.85$)	1.224x+1.940	2.137	0.843
NLK ($\lambda = 0.90$)	1.161x+2.098	2.117	0.857
NLK ($\lambda = 0.95$)	1.088x+2.280	2.097	0.882
NLK ($\lambda = 1$) or level_1	x+2.5	$2.085^{\#*}$	$0.922^{\#*}$
level_2	$\begin{cases} 3.5\ if\ x < 2.5 \\ 6.5\ if\ x > 2.5 \end{cases}$	2.955	1.381
Level_3	$\begin{cases} < 3.5\ if\ x < 2 \\ > 6.5\ if\ x > 2 \end{cases}$	-	
Data (inexperienced)	$\underset{(0.079)}{0.997}x+\underset{(0.203)}{2.950}$	1.899	
Data (experienced)	$\underset{(0.053)}{1.313}x+\underset{(0.150)}{2.023}$	-	0.745

Table 1.8, Figure 1.2 and Figure 1.3 show that for inexperienced bidders (results from the first 18 periods), the most accurate prediction of BNLK is with $\lambda = 1$, and it coincides with

level$_1$ (MSE[39]=2.085). For experienced bidders (results from periods 19-42). BNLK with $\lambda = 0.75$ fits the data the best ($MSE = 0.834$) which is better than the most precise prediction of the Level-K ($k = 1, MSE = 0.922$).

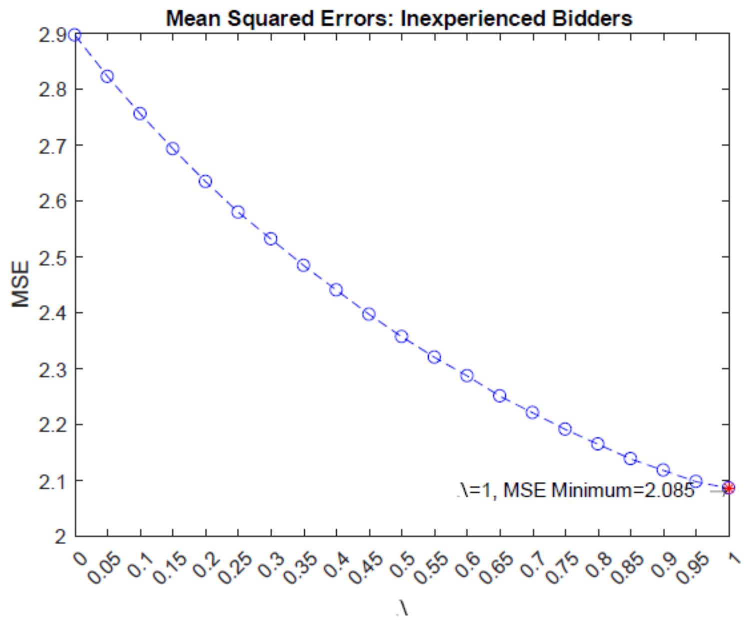

Figure 1.2. MSE of BNLK with different λ: inexperienced bidders.

[39] We choose the value of λ that minimizes the Mean Squared Errors (MSE), that is, the nonlinear least squares estimate of λ.

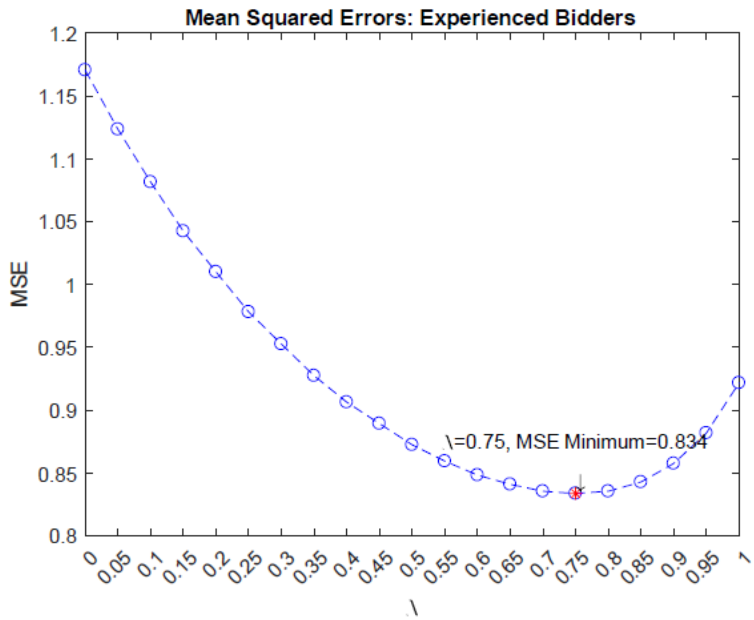

Figure 1.3. MSE of BNLK with different λ: experienced bidders.

6. Discussion and Possible Extension

A possible extension of our model is to consider distributions of different naïve players in the spirit of *Cognitive Hierarchy* by Camerer, Ho, and Chong (2004). For example, now let λ be the prior belief that the opponents are naïve players. Let $G(h|Naive), h \leq k,$ be the probability that the opponent is type h, conditional on being a naïve player. Then the probability that the opponent is type h is $\lambda\ G(h|Naive)$, and $(1-\lambda)$ for being another NLK player.

Allowing heterogeneous beliefs is natural and may help predictions in situations where players, who are known to belong to groups with different sophistication levels, interact with each other. For example, in the *Centipede Game* above, in the treatment where students play with expert chess players, it is reasonable to expect them to have different subjective beliefs (λ). Thus, we could apply NLK with different types of players (with different subjective λ) to analyze such scenarios. Furthermore, an NLK player might rationally expect her NLK opponents to hold different subjective beliefs from her. Let λ_i be player i's belief regarding the probability that the opponent is a naïve player. Then, we could re-define the NLK equilibrium as follows

Definition 1.6. *A mixed-strategy profile* $\{\sigma_i^*\}_{i=1,2}$, *is a* λ*-NLK equilibrium if for each* $i = 1,2$, *and each* $s_i' \in S_i$,

$$\lambda_i u_i(\sigma_i^*, \sigma_{-i}^0) + (1 - \lambda_i) u_i(\sigma_i^*, \sigma_{-i}^*) \geq \lambda_i u_i(s_i', \sigma_{-i}^0) + (1 - \lambda_i) u_i(s_i', \sigma_{-i}^*),$$

Where $\lambda = (\lambda_i)_{i=1,2}$.

Our solution concept can easily be extended to games with more than two players. For example, with N players in the game, the probability of n players, $n = 0,1, \dots, (N-1)$ being a naïve player can be described by the *Binominal* distribution,

$$p^\lambda(n) = C_{N-1}^n \lambda^n (1-\lambda)^{N-1-n},$$

where C_{N-1}^n is the binomial coefficient.

Comparing across the applications to experimental data, it appears that a good fitting λ depends on the games and players. It is beyond the scope of this paper to identify criteria when we should expect a large or a small λ.[40] We leave this topic for future research.

[40] In general, our intuition and limited evidence in the current paper suggest that we should expect a larger λ for less experienced players or a simpler game.

7. Conclusion

This paper proposes a new solution concept, NLK that connects NE and the Level-K. It allows a player to believe that her opponent may be either less- or as sophisticated as she is, a view with support in psychology. NLK is well-defined in both static and dynamic games, making it easy to apply to the data from four published papers on static, dynamic, and auction games. In all four cases, NLK provides better predictions than those of NE and the Level-K, except for few cases when they coincide or when we allow the Level-K to choose freely more parameters.

Chapter 2. Partition Obvious Preference or Mistrust in Mechanism Design-Theory

1. Introduction

In theory, a dominated strategy is inferior regardless of what other players contemplate or do. However, substantial evidence in field, lab and thought experiments in multiple disciplines, shows that decision makers often eccentrically choose a dominated strategy. The long list of literature includes: (1) the deviations from truthful-reporting in *Strategy-proof* Mechanisms such as School Choice and College Admission (Chen and Sönmez 2006; Fack et al. 2015; Gross et al. 2015, Artemov et al. 2017; Chen and Pereyra 2017; Shorrer and Sóvágó 2018), Matching Program (Hassidim et al. 2017, 2018; Rees-Jones 2018), Public Goods (Attiyeh 2000), and Auctions (Kagel et al. 1987; Kagel and Levin 1993; Harstad 2000; Garratt et al. 2011);[41] (2) Behavioral and field experiments in various games and decision problems in economics (Dawes 1980; Dawes and Thaler 1988; Charness

[41] Being a worse response under any subjective beliefs implies that the choice of it cannot be explained by models that relax beliefs in equilibrium, such as *Analogy-Based Expectation Equilibrium* (Jehiel 2005), the *Level-K* (Stahl and Wilson 1994, 1995; Nagel 1995; Crawford and Iriberri 2007) and *Cursed Equilibrium* (Eyster and Ragin 2005). For the same reason, *Anticipated Regret* (Filiz-Ozbay 2007; Engelbrecht-Wiggans 2007), proposed to explain overbidding in First Price auctions cannot account for the insincere bidding in private-value Second Price auctions, where bidding one's value is a dominant strategy.

and Levin 2009; Esponda and Vespa 2014; Bhargava et al 2015); (3) *Disjunction Effect* in Psychology (Shafir and Tversky 1992) ; (4) Newcomb's Paradox in Philosophy and Mathematics (Robert Nozick 1969). Moreover, experimental evidence shows that the choices of dominated strategies in mechanisms such as single and multi-unit Vickrey auctions abate significantly in their counterparts (Kagel, Harstad, and Levin 1987, Kagel and Levin 2009).

Aiming at bridging the ironic gap between theory and evidence, in this chapter, we focus on answering the following two research questions: (1) How to provide a unified framework that ties together above empirical evidence?[42] (2) What're its implications for (experiment) mechanism design?

We start with generalizing the current decision theory for three reasons. First, the choice of dominated strategies is not only found in games but also decision problems, implying violations of fundamental axioms in decision theory[43]. Second, since dominated strategy is non-optimal regardless of what others contemplate or do, to figure out its inferiority in games reducing to an individual optimization problem. Finally, as stated and supported by empirical evidence in Esponda and Vespa (2017), decision problems share a common ground with strategic choices in

[43] The decision maker who satisfies the axioms of Subjective Expected Utility Theory (Savage 1954; Anscombe and Autumn 1963; Fishburn 1970), or even weaker axioms proposed in Ambiguity models (Schmeidler and Gilboa 1989; Gilboa and Marinacci 2011), ought not to choose a dominated strategy, due to the monotonicity axiom.

terms of hypothetical thinking, i.e. to perform state-by-state reasoning, which is critical for the success in spotting a dominated strategy.

We propose two alternative axiomatic approaches. In both approaches, decision makers partitioning the state space into events can reason event-by-event, but not state-by-state within each event. The coarser the partition is, the more bounded rational the decision maker is. Using a metaphor, “can’t reason state-by-state” is like Heraclitus’ famous quote “no man can jump into the same river twice;” while “can reason event-by-event” is like that “the man can be sure if he is in or out of the water.”

Our two approaches differ in primitives. The first approach starts with the state space and the set of outcomes, and act is defined as a mapping from the state space to the set of outcomes. We then propose Partition Obvious Preference (POP) that generalizes Subjective Expected Utility Theorem (Savage 1954, Anscombe and Autumn 1963, Fishburn 1970), and embeds it as an extreme case when the partition is the finest. Specifically, we envision that decision makers partition the state space into events, where for each event, they value each act as the weighted average of the most and the least preferred outcomes, then form subjective expected probabilities over the partition, and choose the action that gives the highest subjective expected utility. Our POP contributes to the broad literature on limited human cognition and its impact on economic decisions. It develops and illustrates a distinct approach by focusing on a deficiency in contingent reasoning under uncertainty, other than incorrect probability judgment (e.g., Kahneman and Tversky 2000)[44] and previously reported violations of Expected Utility Theory.[45] Moreover, although the general idea that a decision maker has a coarse vision of the state-space appears in research in psychology (see e.g., Tversky and Koehler 1994), ambiguity, and non-additive probabilities (Schmeidler 1989, Epstein et al. 2007, Ghirardato 2001, Mukerji 1997, Ahn and Ergin 2010, Burkovskaya 2018), formation of subjective state-space (Kreps 1992, Dekel et al. 2001), and growing awareness (Karni and Marie-Louise 2013), these papers mainly focus on the formation of subjective probabilities over the state-space, while ours first proposes a weakening of the monotonicity axiom based on such a coarser understanding of the state-space.

The first approach goes in line with the standard approach in decision theory but seemly imposes unnecessary controls on its practical use. For instance, in a field or lab experiment, there is usually a set of available choice for consideration. The decision maker does not have to consider all abstract acts when he makes choice. Thus, we also propose an alternative approach that instead starts with the set of

[44] The subjects' inability to make correct probability judgment in a risk environment includes overestimation of small probabilities, failure of *Bayesian updating* and *representative bias*, *conjunction fallacy*, etc.

[45] To accommodate experimental evidences of the violation of Expected Utility Theory, especially the violation of the *independent axiom* (Allais 1953), several theories were introduced as alternatives. Prominent in those are Rank-dependent utility models (Quiggin 1982; Yaari 1987; Hong et al. 1987; Green and Jullien 1988), Betweenness Conforming theories (Chew and MacCrimmon 1979; Fishburn 1983; Dekel 1986; Gul 1991), Prospect Theory (Kahneman and Tversky 1979) and Regret Theories (Bell 1982; Loomes 1982).

available actions and the set of outcomes, and, in each event, the new conceivable state space is defined as a mapping from the set of available actions to the set of outcomes. We then show that to meet the *monotonicity* axiom in the new problem with conceivable state space is equivalent to meet a weaker axiom, *partition obvious monotonicity* in the original problem. We illustrate by some simple examples how each approach characterizes the choice of dominated strategies found in decision problems.

Based on the two axiomatic approaches, we then characterize the choice of dominated strategies in dynamic games. Notably, in an insightful paper, Li (2017) proposed *Obviously Strategy-Proof* (OSP) Mechanism which has an equilibrium in obviously dominant strategy (ODS). Li (2017) also design several experiments that document the decrease in choice of dominated strategies in mechanisms with ODS than in mechanisms with just DS. But does there exist some intermediate concept that fully characterizes levels of bounded rationality from being able to spot ODS to being able to spot DS? To answer this question, we define Partition-ODS which coincides with ODS, at one polar case, when the partition is the coarsest, and at the other polar case, when the partition is the finest, it coincides with DS. We prove two propositions. First, we show that a strategy is partition obviously dominant if and only if all POP prefers it to any deviating strategy at any reachable information set. According to our characterization, the set of POP enlarges when the partition gets coarser and when decision makers who are more bounded rational were taken into consideration. So, it’s beneficial to seek a mechanism with DS that is also obviously dominant respect to a coarser partition, because it helps bounded rational players who reason in a coarser partition and thus could decrease the eccentrical choice of dominated strategies.

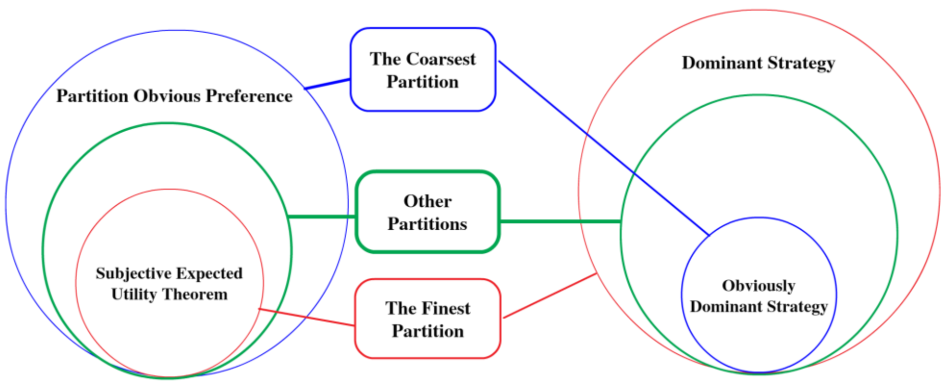

Figure 2.1. POP and Partition-ODS

Besides the failure to identify DS, the choice of dominated strategies might also be due to mistrust in the market maker. Gross et al. (2015) and Hassidim et al. (2017) document the evidence that respondents doubt the veracity of the *Strategy-proof* Mechanism in Israeli Psychology Master’s Match and school choice in Denver and New Orleans. Can Partition-ODS help mitigate the problem of mistrust? We first define partition identical game as a game that is indistinguishable from the original game under auditing measurable respect to the partition. Based on the second axiomatic approach, we prove that a strategy is partition obviously dominant if and only if it is dominant in all partition identical games. That is, in a mechanism with Partition-ODS, if each player could verify which event of the partition is realized, the player has no incentive to choose an alternative dominated strategy even if he doubts that the market marker might use another identical game to implement the result. Thus, in a mechanism with Partition-ODS under relevant auditing, to trust the market maker is strategically rational.[46] Moreover, when the partition is coarser, less information is needed to be verified, which reduces the auditing cost. Particularly, when the partition is the coarsest, no auditing is needed, and the mechanism is OSP. However, in general, in a *Strategy-proof* Mechanism with no auditing, it might not be strategically rational to trust the market maker and report truthfully.

A literature has characterized several impossibility results for OSP mechanism in general.[47] Our theory thus provides a second-best choice when an implementation in ODS is not feasible. For instance, an OSP implementation for a multi-unit Vickrey (1961) auction does not exist. However, Kagel and Levin (2009) document

[46] Although it might not be epistemically rational (Baker 1987).

[47] An implementation in ODS rarely exists. Li (2017) proves that no top trading cycle rule with more than three agents can be implemented by an OSP mechanism. Ashlagi and Gonczarowski (2017) show that for general preferences, no mechanism that implements a stable matching is OSP. Pycia (2018) finds that *Random Priority* is the unique mechanism that is OS*P*, *ex-post Pareto efficient*, and *symmetric*. Bade and Gonczarowski (2017) characterize a similar limitation in applications of OSP mechanisms in dictatorship mechanisms, house matching, and multi-unit auctions.

a significantly higher rate of sincere bidding in an Ausubel (2004) Auction, which is strategically equivalent to the multi-unit Vickrey auction when we restrict the strategy set to cut-off strategies. Notably, when considering the partition by "clinching prices," coined by Ausubel, sincere bidding is partition-ODS in the Ausubel auction but not in the Vickrey auction. Thus, by our theory, Ausubel auction help bounded rational agents who reason in a coarser partition and agents who mistrust the market maker converse to sincere bidding.

As a by-product, we show that an implementation using a partition-ODS of an extensive game is equivalent to an implementation through an iterative exclusion of obviously dominated misreport in a direct mechanism with a specified guide. Thus, we also join the literature that explains why a dynamic mechanism might perform better than its strategically equivalent static mechanism.[48]

Empirically, do games and decision problems with a choice that partition obviously dominate others significantly diminish the choice of dominated choice? We test and verify our theoretical implication by a simple laboratory experiment with both cross-subject and within-subject comparison by a cross-over design.

In Section 2, we present our two axiomatic approaches. In Section 3, we extend our results to games, define *partition obviously dominant strategy* and address its application to mechanism design. In Section 4, we propose one possible extension

[48] In the same spirits, but in a discussion of a stronger solution concept in game theory, Glazer and Rubinstein (1996) show that an extensive game can be viewed as a guide for a solving normal form game.

of our concept to games without a dominant strategy, where we define *Partition Obvious Equilibrium*.

2. Two Axiomatic Approaches

Denote by $\mathcal{X}$ a set of deterministic outcomes and by $\mathcal{Z}$ a set of distributions over $\mathcal{X}$ with finite supports, i.e., $\mathcal{Z}$ is a collection of random outcomes. Let Ω denote the finite state space of all states, $\omega \in \Omega$; and let $\mathcal{A}$ denote the set of all available actions, $a \in \mathcal{A}$. Denote by $o: \mathcal{A} \times \Omega \to \mathcal{Z}$ the outcome function; and by $\mathcal{O}(a)$ the set of outcomes induced by action a.

Definition 2.1. (partition) $\Sigma = (\mathcal{B}_k)_{k=1}^n$ is a finite partition of Ω if $\cup_{k=1}^n \mathcal{B}_k = \Omega$ and $\mathcal{B}_i \cap B_j = \emptyset$, when $i \neq j$.[49]

2.1. Partition Obvious Preference

Let $\mathcal{F}$ denote the set of all acts, $f: \Omega \to \mathcal{Z}$; and by $\mathcal{F}^c$ the set of constant acts in $\mathcal{F}$. The first two axioms are standard in decision theory.

AXIOM 1. (weak order) $\succsim$ is complete and transitive.

AXIOM 2. (non-degeneracy) there are $f, g \in \mathcal{F}$, such that $f \succ g$.

Denote by $\mathcal{O}^{\mathcal{B}}(f)$ the set of all possible outcomes induced by act f given event $\mathcal{B}$. The next axiom generalizes the standard monotonicity axiom.

[49] For simplicity, we only consider the case where each $\mathcal{B}_k$ is a non-null event.

AXIOM 3. (partition monotonicity) for any $f, g \in \mathcal{F}$, if for each $\mathcal{B} \in \Sigma$, we have, for all $p \in \mathcal{O}^{\mathcal{B}}(f)$, $q \in \mathcal{O}^{\mathcal{B}}(g)$, $p \succsim q$, then $f \succsim g$;In addition, if for a non-null event $\mathcal{B}' \in \Sigma$, we have for all $p \in \mathcal{O}^{\mathcal{B}'}(f)$, $q \in \mathcal{O}^{\mathcal{B}'}(g)$, $p \succ q$, then $f \succ g$.

The *partition monotonicity* requires the decision maker to compare outcomes of two acts event-by-event, but not state-by-state within each event of the partition, as in the standard *monotonicity axiom*.

The following definition of mixed acts is standard in the literature.

Definition 2.2. (mixed acts) for any $f, h \in \mathcal{F}$, $\lambda \in [0,1]$, and $\omega \in \Omega$, $[\lambda f + (1-\lambda)h](\omega) \equiv \lambda f(\omega) + (1-\lambda)h(\omega)$.

Definition 2.3. (partition measurable act) $\mathcal{F}^c(\Sigma)$ is the set of acts that is constant in each event $\mathcal{B}$ of the partition Σ. (The set of acts that is measurable with respect to the partition.)

The implication of Definitions 2.2 and 2.3: to understand the concept of mixed acts, it's enough to reason by Partition Σ for the Σ-measurable act, but the reasoning in a finer partition is required for other acts.

The next two axioms generalize the continuity and independence axioms by imposing them on mixed acts of only partition measurable acts.

AXIOM 4. (partition continuity) for any act $g \in \mathcal{F}$ and any two acts $f, h \in \mathcal{F}^c(\Sigma)$ such that $f \succ g \succ h$, there are $\lambda, \beta \in (0,1)$ such that $\lambda f + (1-\lambda)h \succ g \succ \beta f + (1-\beta)h$.

AXIOM 5. (partition independence) for any three acts $f, g, h \in \mathcal{F}^c(\Sigma)$ and any $\lambda \in (0,1]$, $f \succ g$ implies that $\lambda f + (1-\lambda)h \succ \lambda g + (1-\lambda)h$.

Hence, partition continuity and partition independence are analogous to the continuity and independence axioms without request the decision maker to reason in an even finer partition.

Definition 2.4. (partition indifferent act) $\mathcal{F}^e(\Sigma) = \{f \in \mathcal{F} | p \sim q \ for\ any\ \mathcal{B} \in \Sigma\ and\ any\ p, q \in \mathcal{O}^{\mathcal{B}}(f)\}$ is the set of acts that generate indifferent outcomes in each event $\mathcal{B}$ of the Partition Σ.

Theorem 1. (partition obvious preference) $\Sigma = (\mathcal{B}_k)_{k=1}^n$ is a partition given exogenously. Let $\succsim$ be a binary relation defined on $\mathcal{F}$. The following conditions are equivalent:

(i) $\succsim$ satisfies Axiom 1-5. (We call such preferences $\Sigma -$Obvious Preference)

(ii) there exist a non-constant affine function $u: \mathcal{Z} \to \mathbb{R}$, a probability function $P: \Sigma \to [0,1]$ and a function $\alpha: \mathcal{F} \to [0,1]$ such that $\succsim$ is represented by the preference functional $V: \mathcal{F} \to \mathbb{R}$ given by

$$V(f) = \sum_{k=1}^n V(f|\mathcal{B}_k) P(\mathcal{B}_k), \tag{2.1}$$

where

$$V(f|\mathcal{B}) = \alpha(f) \max_{p \in \mathcal{O}^{\mathcal{B}}(f)} u(p) + [1 - \alpha(f)] \min_{q \in \mathcal{O}^{\mathcal{B}}(f)} u(q). \tag{2.2}$$

That is, for all $f, g \in \mathcal{F}$, $f \succsim g$ if and only if $V(f) \succsim V(g)$.

Furthermore:

(a) The function u in (ii) is unique up to positive affine transformation;

(b)The probability function P is unique;

(c) α is unique on $\mathcal{F}$ with the exclusion of $\mathcal{F}^e(\Sigma)$.[50]

Proof. See Appendix B.1.

Example 1. Consider the matrix game with incomplete information in Figure 1. There are three possible states of nature: A, B, and C. The computer randomly draws one, which is unknown to both players, each with a probability of 1/3. Each player chooses between R and L and the payoff table is shown in three matrices. Each decision maker is randomly matched with a player who is drawn from the pool of subjects who played the same game in pairs. The payoff of the decision maker thus depends on his choice, the states of nature, and the strategy chosen by his opponent in the past. The state space in this decision problem is a cross product of states of nature {A, B, C} and choices of the other player {L, R}. Given each case (A, B, or C) and each strategy of the opponent (R or L), choosing R always generates a higher payoff. Thus, any subjective utility maximizer would not be willing to pay for the non-instrumental[51] information notifying the states of nature, at a positive price.

[50] For any f in $\mathcal{F}^e(\Sigma)$, all $\alpha(f) \in [0,1]$ end up with the same $V(f)$.

[51] We call a piece of information "instrumental" in the case where this information can alter the optimal decision.

		State A		State B		State C	
		The Other		The Other		The Other	
		L	R	L	R	L	R
You	L	20	8	22	6	18	10
	R	25	13	27	11	23	15

Figure 2.2. The Matrix Game with Incomplete Information

Now consider the partition by states of nature and POP with $\alpha_L = \frac{2}{3}$ assigned to choice L and $\alpha_R = \frac{1}{6}$ assigned to choice R.[52] By our POP Presentation (1), the utilities of choosing L in States A, B, and C are as follows: $V(L|A) = 20 \times \frac{2}{3} + 8 \times \frac{1}{3} = 16; V(L|B) = 22 \times \frac{2}{3} + 6 \times \frac{1}{3} = 16\frac{2}{3}; V(L|C) = 18 \times \frac{2}{3} + 10 \times \frac{1}{3} = 15\frac{1}{3}$. The expected utility of choosing L is thus $V(L) = 16 \times \frac{1}{3} + 18 \times \frac{50}{3} + 16 \times \frac{46}{3} = 16$. The utilities of choosing R in States A, B and C are $V(R|A) = 25 \times \frac{1}{6} + 13 \times \frac{5}{6} = 15$; $V(R|B) = 27 \times \frac{1}{6} + 11 \times \frac{5}{6} = 13\frac{2}{3}; V(R|C) = 23 \times \frac{1}{6} + 15 \times \frac{5}{6} = 16\frac{1}{3}$. The expected utility of choosing R is thus $V(R) = 15 \times \frac{1}{3} + \frac{41}{3} \times \frac{1}{3} + \frac{49}{3} \times \frac{1}{3} = 15$. Since $V(L|C) < V(R|C)$ but $V(L) > V(R)$, the information

[52] A similar argument follows for any $(\alpha_L - \alpha_R) \in (\frac{5}{16}, \frac{5}{8})$ that for POP, the information of states of nature can vary optimal decisions. See Appendix B.5 for details.

regarding which case the decision maker is in can alter the optimal choice and thus it is instrumental for the bounded rational player we characterize. This example alerts us to the fact that non-instrumental information, in theory, might nevertheless be instrumental for bounded rational players, e.g., POP with a coarser partition.[53]

Event	B_1		B_2	
State	ω_1	ω_2	ω_3	ω_4
U	**21**	13	11	**16**
D	25	**15**	**12**	20

Event	B_1'		B_2'	
State	ω_1	ω_4	ω_3	ω_2
U	**21**	16	11	**13**
D	25	**20**	**12**	25

Figure 2.3. Uniqueness is not guaranteed.

The partition in our theorem is given exogenously. A natural question is: does there always exist a unique finest partition that can rationalize some POP? However, uniqueness is not guaranteed at least when the state space is finite. Consider the decision problem presented in two ways in Figure 2.3. There are four possible states, ω_1, ω_2, ω_3, ω_4; and two available actions, U and D, that the decision maker can take; any payoffs are given in the corresponding matrices. The choice of U can be rationalized by $\Sigma = \{\mathcal{B}_1, \mathcal{B}_2\}$, where $\mathcal{B}_1 = \{\omega_1, \omega_2\}$, $\mathcal{B}_2 = \{\omega_3, \omega_4\}$; and $\Sigma' =$

$\{\mathcal{B}_1', \mathcal{B}_2'\}$, where $\mathcal{B}_1' = \{\omega_1, \omega_4\}$, $\mathcal{B}_2' = \{\omega_2, \omega_3\}$. However, it cannot be rationalized by their joint, the coarsest common refinement: $\Sigma \vee \Sigma' = \{\omega_1, \omega_2, \omega_3, \omega_4\}$.

2.2. Partition Conceivable State-space

State of the world, said by Arrow (1971), is "a description of the world so complete that, if true and known, the consequences of every action would be known." However, a true state that might be able to be known by experimentalists or mechanism designers in a controlled environment is usually less likely to be observable for a decision maker.[54] When the true state is not known or fully understood, a decision maker needs to construct an association between actions and outcomes based on which a choice could be made (Osborne and Rubinstein, 1998). Following previous literature[55], we name such an association, a conceivable state.

For instance, in the Greek mythology (Homer, 1997), when Helen was abducted in Troy, Odysseus was called upon by Menelaus to honor his oaths as a suitor of Helen and help him retrieve her. Abstract the decision problem: Odysseus has two available actions, A= "pretend to be mad," B = "fight." If he pretended to be mad, he predicts he might be either scorned or forgiven, $\mathcal{O}(A) = \{scorned, forgiven\}$; if he chooses to fight, he foresees he might either be praised or blamed $\mathcal{O}(B) =$

[54] For instance, Ecclesiastes 11:1 says "send your bread upon the surface of the water, for after many days you may find it." For decision makers who "send out their bread", although they could verify whether they get anything in return after years, they could hardly know what alternative payoff they might get if they instead "keep their bread" in the same state of the world.

[55] Similar construction of conceivable states could be found in Schmeidler and Wakker (1987), Karni and Schmeidler (1991), Gilboa et al. (2009), Karni and Vierø (2013, 2015) and Karni (2017).

$\{praised, blamed\}$. Ergo, he could conceive 4 ($= C_2^1 \times C_2^1$) non-null states,$(\omega_k)_{k=1}^4$ as shown in Table 2.1.

	ω_1	ω_2	ω_3	ω_4
A=pretend to be mad	scorned	scorned	forgiven	forgiven
B=fight	praised	blamed	praised	blamed

Table 2.1. When Odysseus was called upon

However, even though a decision maker might not be able to fully grasp the state space, he might be able to understand a partition of it; within which, each event could be known or observed. Hence, he only needs to form conceivable state-space within each event. Formally, given decision problem $\mathcal{D} = (\Omega, \mathcal{A}, \mathcal{Z}, o)$, and a partition Σ of Ω, we define Σ -*Conceivable State-space* $\mathcal{C}^{\Sigma} = \cup_{\mathcal{B} \in \Sigma} \mathcal{C}^{\mathcal{B}}$, where $\mathcal{C}^{\mathcal{B}} = \{c: \mathcal{A} \to \mathcal{Z} \big| c(a) \in \mathcal{O}^{\mathcal{B}}(a)\}$. And the new outcome function $o^{\Sigma}(a, c) = c(a), a \in \mathcal{A}, c \in \mathcal{C}^{\Sigma}$. Denote by $\mathcal{D}^{\Sigma} = (\mathcal{C}^{\Sigma}, \mathcal{A}, \mathcal{Z}, o^{\Sigma})$ the Σ -*Conceivable problem* of $\mathcal{D}$.

Definition 2.5. In a decision problem, $\mathcal{D} = (\Omega, \mathcal{A}, \mathcal{Z}, o)$, for any $a, b \in \mathcal{A}$, a dominates b if for all $\omega \in \Omega$, $o(a, \omega) \gtrsim o(b, \omega)$.

Definition 2.6. In a decision problem, $\mathcal{D} = (\Omega, \mathcal{A}, \mathcal{Z}, o)$ with a partition Σ of Ω, for any $a, b \in \mathcal{A}$, a $\Sigma-$obviously dominates b if for all $\mathcal{B} \in \Sigma$, we have, for all $p \in o(a, \mathcal{B})$, $q \in o(b, \mathcal{B})$, $p \gtrsim q$.

Remark: If *a* dominates *b*, we say, *b* is dominated by *a*. To figure out dominance, *monotonicity axiom* is required; to figure out $\Sigma-$obviously dominance, a weaker axiom, Σ *−obviously monotonicity* is enough.

Lemma 1. Given a decision problem $\mathcal{D}$ and $\mathcal{D}^{\Sigma}$, the Σ -*conceivable problem* of $\mathcal{D}$, for any $a, b \in \mathcal{A}$, a $\Sigma-$obviously dominates b in $\mathcal{D}$ if and only if a dominates b in $\mathcal{D}^{\Sigma}$.

Proof. If a $\Sigma-$obviously dominates b in $\mathcal{D}$, then in all Σ -*conceivable states*, the outcome of a is weekly preferred to b, thus a dominates b in $\mathcal{D}^{\Sigma}$; if a dominates b in $\mathcal{D}^{\Sigma}$, then in all events of Σ, any randomly selected outcome of a is weekly preferred to that of *b*. Hence, a $\Sigma-$obviously dominates b in $\mathcal{D}$.

Therefore, by Lemma 1, to implement dominance in decision makers' $\Sigma-$conceivable problem, it is both necessary and sufficient, for the designer, to implement a stronger concept, $\Sigma-$obviously dominance in the designed problem.

Field experiments, such as Hungarian college admission (Shorrer and Sóvágó 2018) and admissions for graduate studies in psychology in Israel (Hassidim et al. 2018), have documented a common "obvious mistake" (of choosing dominated strategies): the applicants either forge financial supports or list it as less preferred. We abstract the scenario to a decision problem in Example 2. We show that the dominated choice might not be dominated in applicants' partition conceivable problem. And the choice of partitions matters.

Example 2. For each applicant, the uncertainty comes from his ranking-H or L, and whether the financial support is feasible or not-F or NF. Each applicant is asked, "Do you need financial aids?" The answer can be Yes or No. We assume that each applicant prefers admission with financial aids (A+F) to admission without financial aids (A) to not being admitted (NA). The outcome function is represented in Table 2.2. The applicant will get admitted whenever his ranking is H but he will only get financial support if, in addition, his answer is Yes and the financial aid is feasible. "No" is a dominated choice.

	H, F	L, F	H, NF	L, NF
Yes	A+F	NA	A	NA
No	A	NA	A	NA

Table 2.2. Do you need financial aids?

Now, consider the partition Σ_1 by the feasibility of financial aids. "No" is not $\Sigma_1 - obvious\ dominant$. See the $\Sigma_1 - conceivable\ problem$ in Table 2.3 where "No" is not dominated. An applicant who has a stronger belief that with a high probability he is in ω_4 and ω_8 is more likely to choose "No." But if we consider the partition Σ_2 by the ranking of the applicants. "No" is $\Sigma_2 -$ *obvious dominant*. The $\Sigma_2 -$*conceivable problem* is the same as the original problem where "No" is dominated.

	F				NF			
	ω_1	ω_2	ω_3	$\boldsymbol{\omega_4}$	ω_5	ω_6	ω_7	$\boldsymbol{\omega_8}$
Yes	A+F	NA	A+F	**NA**	A	NA	A	**NA**
No	A	NA	NA	**A**	A	NA	NA	**A**

Table 2.3. Partition by the feasibility of financial aids

3. Dynamic Games and Mechanism Design

We extend our concept to dynamic games. We introduce the decision environment in dynamic games in Subsection 3.1. We propose *partition*-ODS and relate it to POP in Subsection 3.2. In Subsection 3.3, we provide an alternative characterization of *partition*-ODS as a solution of mistrust in market makers. In Subsection 3.4, we show that a choice rule can be implemented by a mechanism in partition-ODS if and only if it can be implemented by a direct mechanism of exclusion in a solution concept we coin iteratively deletion of obviously-dominated misreport.

3.1. The Decision Problem in a Dynamic Game

We consider the extensive game form[56] Γ with imperfect information and perfect recall as defined in Osborne and Rubinstein (1994). Denote the set of strategy profiles by $\mathcal{S} = \mathcal{S}_i \times \mathcal{S}_{-i}$ and the set of nature's moves by $\Omega_N = \{\omega_n\}$. The domain of Player i's uncertainty consists of moves of nature Ω_N and the strategy of other players $\mathcal{S}_{-i}$. So, let $\Omega_i = \mathcal{S}_{-i} \times \Omega_N$ denote the subjective state space of player i.[57] At each terminal history, $h = (s_i, s_{-i}, \omega_n)$, Player i is assigned a deterministic or random outcome in a set $\mathcal{Z}$ defined in Section 2. Again, denote by $o: \mathcal{S}_i \times \Omega_i \to \mathcal{Z}$ the outcome function. Thus, Player i's preference over her own strategy set is characterized by a preference relation $\succsim$ on the set of acts $f: \Omega_i \to \mathcal{Z}$. The utility of Player i at each terminal history, $u_i(s_i, s_{-i}, \omega_n)$, is thus determined by the utility function of lotteries in Theorem 1, $u_i: \mathcal{Z} \to \mathbb{R}$. A dynamic game is then a tuple $\mathrm{G} = \{\Gamma, \mathcal{Z}, (u_i)_{i \in N}\}$.

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[56] An extensive game form is a tuple $\Gamma = \{\mathrm{N}, \Upsilon\}$, where N is the set of players and Υ is the game tree.

[57] *Bayesian Models* in decision theory under uncertainty Savage (1954) and *Solution Concepts* in game theory (Nash 1950) originated independently. Aumann (1987) synthesizes the two viewpoints by *Correlated Equilibrium*. In his set-up, the state of the world in games is a specification of which strategy is chosen by each player. Esponda (2013) further defines the state space as the product of the strategy sets and the set of fundamentals to include both strategic and structural uncertainty. He further develops the *Rationalizable Conjectural Equilibrium* by adding certain restrictions to each player's beliefs over states of the world in equilibrium. Siniscalchi (2016a, 2016b, 2016c) adopts a similar definition of the subjective state space in dynamic games in three of his recent works about structural rationality.

For any information set I ∈ $\mathcal{I}$, denote by $\mathcal{S}(I)$ the set of strategy profiles that reach I.[58] The projections of $\mathcal{S}(I)$ on $\mathcal{S}_i$ and $\mathcal{S}_{-i}$ are denoted by $\mathcal{S}_i(I)$ and $\mathcal{S}_{-i}(I)$; perfect recall implies that $\mathcal{S}(I) = \mathcal{S}_i(I) \times \mathcal{S}_{-i}(I)$, and the set of available strategies for Player *i* at information set I is $\mathcal{S}_i(I)$. Player *i* who chooses an action $a \in \mathcal{A}(I)$ at information set I, must restrict herself to a smaller set of strategies denoted by $\mathcal{S}_i(I)[a] = \{s_i \in \mathcal{S}_i(I) | s_i(I) = a\}$. We denote by $\mathcal{S}_i(I)[a)]^c = \mathcal{S}_i(I) \backslash \mathcal{S}_i(I)[a]$ the set of strategies from which the player is deviating by choosing a.

Upon reaching an information set I ∈ $\mathcal{I}_i$, Player *i* must rule out moves of nature and strategies of other players that do not allow reaching information set I. We denote the conditioning event at information set I by [I], at which I is reachable.[59] Finally, denote by $\mathcal{I}(s_i) = [I \in \mathcal{I} | s_i \in \mathcal{S}_i(I)]$, the set of information sets that is reachable by strategy s_i.

3.2. Partition-ODS and POP

Definition 2.7. (conditional partition-system) A conditional partition-system Σ^G for Player i in a dynamic Game G is a collection of partitions, $\{\Sigma(I)\}_{I \in \mathcal{I}_i \cup \emptyset}$, such that

(i) $\Sigma(\emptyset) = \Sigma = \{\mathcal{B}_k\}_{k=1}^{n}$ is a partition of Ω_i,

[58] Formally, $\mathcal{S}(I) = \{s \in \mathcal{S} | \text{there exists } h \in I \; and \; \omega_n \in \Omega_N \text{ such that h is a subhistory } of \; (s, \omega_n)\}$

[59] Formally, [I]= $\{ (s_{-i}, \omega_n) \in \Omega_i | \text{there exists } h \in I, s_i \in \mathcal{S}_i \text{ such that } h \text{ is a sub-history } of \; (s_i, s_{-i}, \omega_n)\}$.

(ii) for any $I \in \mathcal{I}_i$, $\Sigma(I) = \{[I] \cap \mathcal{B}_k\}_{k=1}^{n}$, from here on we denote $[I] \cap \mathcal{B}_k$ by $\mathcal{B}_k(I), k = 1, \dots, n$.

Definition 2.8. (partition-ODS) In a dynamic Game G, a strategy s_i^* is a Σ-obvious dominant strategy for Player i, if for any information set, $I \in \mathcal{I}(s_i^*)$, any non-empty event, $\mathcal{B}(I) \in \Sigma(I)$ and any deviating strategy, $s_i' \in \mathcal{S}_i(I)[s_i^*(I)]^c$:

$$\text{(2.3)} \qquad \inf_{(s_{-i},\omega_n)\in\mathcal{B}(I)} u_i(s_i^*, s_{-i}, \omega_n) \geq \sup_{(s_{-i},\omega_n)\in\mathcal{B}(I)} u_i(s_i', s_{-i}, \omega_n).$$

Remark: When the partition is the coarsest, $\Sigma = \{\Omega_i\}$, Σ-ODS coincides with Li's (2017) ODS.

Definition 2.9. (*dominant strategy*)[60] In a dynamic Game G, s_i^* is a dominant strategy for Player *i* if for any $s_i' \in S_i$, any state $(s_{-i}, \omega_n) \in \Omega_i$:

$$\text{(2.4)} \qquad u_i(s_i^*, s_{-i}, \omega_n) \geq u_i(s_i', s_{-i}, \omega_n).$$

Lemma 2. When the Partition Σ is the finest, Definitions 2.8 and 2.9 are equivalent.

PROOF: See Appendix B.2.

[60] Li (2017, Def. 4) defines DS in a slightly different way. Note that, in our paper, nature's moves Ω_N include both the *chance moves* and *type randomizations* in Li's. Li defines a strategy as *weekly dominant* if its expected payoff with respect to *chance moves* is not smaller than that of any alternative strategy for any realized type. Our notion of *dominance* is stronger than Li's, since our *dominant strategy* needs to be *ex-post* optimal, not only *expected*, given any realization of chance moves. However, Li's Theorem 1 still holds even if he instead used our notion.

Proposition 2.1. In a dynamic Game G, a strategy s_i^* is an Σ-ODS for Player i if and only if for any Σ-Obvious Preference $\gtrsim$, it satisfies the following:

(2.5) $s_i^* \in C(\gtrsim, \mathcal{S}_i) = \{s_i \in \mathcal{S}_i | s_i \gtrsim s_i' \textit{ for any } s_i' \in \mathcal{S}_i(I)[s_i^*(I)]^c \textit{ at any } [I], I \in \mathcal{I}_i\}$.

PROOF: See Appendix B.3.

So, a strategy is Σ-ODS if and only if any Σ-Obvious Preference prefers it to any deviating strategy at any reachable information set. Hence, mechanisms with a strategy that is *partition-ODS* in a coarser partition work for a larger set of preferences.

3.3. Partition-ODS and Mistrust in Market Makers

In a dynamic Game G, A player first observes the set of strategy profiles $\mathcal{S}$ and the moves of nature Ω_N. Then, when Player *i* commits to strategy $s_i \in \mathcal{S}_i$, he observes each reachable information set $I \in \mathcal{I}(s_i)$, at which the set of strategy he hasn't given up $\mathcal{S}_i(I)$, the set of available choices $\mathcal{A}(I)$ and his own choice $s_i\ (I)$, and finally an outcome z $\in \mathcal{Z}$ assigned at the terminal node. We define two games as *i-identical* if there exists a bijection between the domain of uncertainty $\mathcal{S} \times \Omega_N$ and $\mathcal{S}' \times \Omega_N{}'$ for Player *i* in the two games such that any strategy of Player *i* generates the same set of observation as its image.

Definition 2.10. (identical games) Game G' is identical to Game G for Player i if there exists a bijection $\lambda_{G,G'}$ from $\mathcal{S} \times \Omega_N$ to $\mathcal{S}' \times \Omega_N{}'$, where $(s', \omega') = \lambda_{G,G'}(s, \omega)$, $\forall (s, \omega) \in \mathcal{S} \times \Omega_N$, such that for any $s_i \in \mathcal{S}_i$:

(i) for any $I \in \mathcal{I}(s_i)$, there exists a $I' \in \mathcal{I}(s_i{}')$ such that $\mathcal{A}(I) = \mathcal{A}'(I')$, $s_i\,(I) = s_i'(I')$, $(s_{-i}, \omega) \in [I]$ iff $(s_{-i}{}', \omega') \in [I']$, and $s_i \in \mathcal{S}_i(I)$ iff $s_i{}' \in \mathcal{S}_i{}'(I')$.

(ii) $\mathrm{o}(s_i, s_{-i}, \omega) = \mathrm{o}(s_i', s_{-i}{}', \omega')$

Definition 2.11. (partial games) For *Player i* in dynamic Game G with partition $\Sigma_i = \{\mathcal{B}_k\}_{k=1}^{n}$. We call $G^{\Sigma_i} = \{G^k\}_{k=1}^{n}$ the set of partial games of G, where G^k is G restricted to $\mathcal{B}_k$.

Definition 2.12. (Σ_i-identical games) Game G' is Σ_i-identical to Game G for Player i, where $\Sigma = \{\mathcal{B}_k\}_{k=1}^{n}$, if there exists a partition $\Sigma_i' = \{\mathcal{B}_k{}'\}_{k=1}^{n}$ of $\Omega_i{}'$ such that G'^k is identical to G^k, $\forall \mathrm{k} = 1, \ldots, \mathrm{n}$.

Proposition 2.2. s_i^* is Σ_i-obviously dominant in Game G if and only if for every G' that is Σ_i-identical to Game G, the image of s_i^* is dominant in G'.

Proof. The "if" direction proceeds as follows. Assume by contradiction that s_i^* is not Σ_i-obviously dominant in Game G, then there exists an information set $I \in \mathcal{I}(s_i^*)$, an event $\mathcal{B}(I) \in \Sigma(I)$ and a deviating strategy, $s_i' \in \mathcal{S}_i(I)[s_i^*(I)]^c$, such that Equation (2.3) doesn't hold. Formally, we denote the violation as $u_i(s_i^*, s_{-i}^{inf}, \omega_n^{inf}) < u_i(s_i', s_{-i}^{sup}, \omega_n^{sup})$. And we denote by z the outcome assigned to $(s_i', s_{-i}^{sup}, \omega_n^{sup})$ and by z′ the outcome assigned to $(s_i', s_{-i}^{inf}, \omega_n^{inf})$. We construct a new game G' by switching z and z'. i.e., Let $z' = o(s_i', s_{-i}^{sup}, \omega_n^{sup})$ and $z = o(s_i', s_{-i}^{inf}, \omega_n^{inf})$. G' is $\Sigma_i - identical$ to G,[61] but s_i' is preferred to s_i^* in state $(s_{-i}^{inf}, \omega_n^{inf})$ so that s_i^* is not dominant in G'. Contradiction. For the "only if" direction, it's crucial to see that at each reachable information set, the partition conceivable problem for the decision of choosing s_i^* or another deviating strategy s_i' in G is the same as the partition conceivable problem for the choice between their images in G'. Ergo, by Lemma 1, the image of s_i^* dominates the image of s_i'. Finally, apply the same argument to all deviating strategies and all information sets, by Lemma 2, the image of s_i^* dominates the image of s_i'.

In plain words, Proposition 2 says that in a mechanism with Partition-ODS, if each player could verify which event of the partition is realized, the player has no incentive to choose an alternative dominated strategy even if he doubts that the market marker might use another identical game to implement the result. Thus, in a mechanism with Partition-ODS under auditing respect to the partition, to trust the market maker is strategically rational. Moreover, when the partition is coarser, less information is needed to be verified, which reduces the auditing cost.

Kagel and Levin (2009) document a significantly higher rate of sincere bidding in an Ausubel (2004) Auction with dropout information than in a multi-unit Vickrey

[61] Note that here we have $\lambda_{G,G'}(s_i', s_{-i}^{sup}, \omega_n^{sup}) = (s_i', s_{-i}^{inf}, \omega_n^{inf})$ and $\lambda_{G,G'}(s_i', s_{-i}^{inf}, \omega_n^{inf}) = (s_i', s_{-i}^{sup}, \omega_n^{sup})$.

(1961) Auction. When we restrict the strategy set to cut-off strategies,[62] the Ausubel Auction is strategically equivalent with the Vickrey Auction; both are *strategy-proof*. The Ausubel auction is not OSP; thus, its superior performance cannot be explained by Li (2017). However, there exists a partition, by the "clinching prices", coined by Ausubel, so that sincere bidding is partition-ODS in the Ausubel auction, but not in the Vickrey Auction. Thus, by our theory, the Ausubel auction helps bounded rational agent who reason in coarser partitions. Moreover, since "the clinching prices" are reported to the bidders in the Ausubel Auction with dropout information, the bidders who has incentive to bid insincerely in the Vickrey Auction due to mistrust, doubting that the market maker might instead implement another partition-identical game, will not have such an incentive in the Ausubel Auction. We provide a detailed argument in Appendix B.4.

Elmes and Reny (1994) prove that if two finite extensive form games with perfect recall share the same normal form, then we can get one game from another by three kinds of transformations in finite steps. It raises the questions: why in general, among two games that share the same normal form, one has a certain Partition-ODS but the other does not; which transformation breaks the nice property of *partition*

[62] If we consider a larger set of strategies, where one bidder's active units of demand can depend on not only the clock price, but also on other bidders' active units of demand, then in the Ausubel auction with dropout information, sincere bidding is not even a DS (see Ausubel 2004). However, we argue that to consider such strategies, it also requires higher cognitive ability because bidders need to consider more contingencies.

dominance. We show by the following example that one of their transformation, called "ADD," is critical for answering this question.

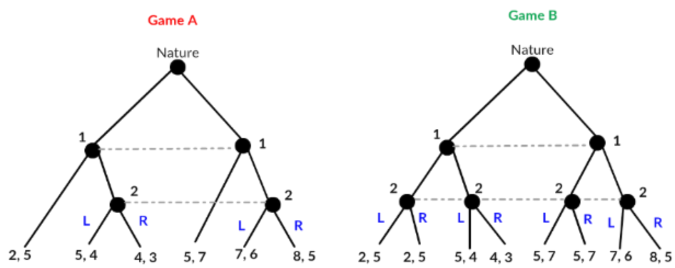

Figure 2.4. Add Transformation.

Example 3. In Figure 2.4, we get extensive form Game B from Game A by Elmes and Reny's "ADD" transformation.[63] Clearly, the two games share the same normal form. Partitioning by moves of nature, L is a Partition-ODS for Player 2 in Game A but not in Game B.

[63] See Page 12 of Elmes and Reny's paper for the definition of "ADD" transformation.

3.4. A Direct Mechanism of Exclusion

Definition 2.14. (*choice rule*) A choice rule is a function f which specifies, for each type profile $\theta \in \Theta$, and for each feasible set $\mathcal{X}_A \subseteq \mathcal{X}, A \in \mathcal{A}$, an allocation $f(\theta, \mathcal{X}_A) \in \mathcal{X}_A$.[64]

A mechanism is an extensive game forms Γ^m with an allocation in $\mathcal{X}_A$. Each player $i \in N$ who participates in a mechanism Γ^m with choice rule $f(\theta, \mathcal{X}_A)$, $\mathcal{X}_A \subseteq \mathcal{X}, A \in \mathcal{A}$ is playing a dynamic game $G^m = \{\Gamma^m, \mathcal{Z}, (u_i)_{i \in N}\}$:

(1) A type profile of agents $\theta \in \Theta$ is generated by a probability measure, denoted by δ_θ of support Θ; and each agent privately observes θ_i.

(2) By a probability measure δ_A, A feasible set of allocations is generated $\mathcal{X}_A \subseteq \mathcal{X}$.The Planner privately observes $\mathcal{X}_A$, proceeds as Γ^m, and assigns each terminal history with an outcome z $\in \mathcal{X}_A$ using an outcome function o_A.

Definition 2.15. (*partition obviously strategy-proof*). A choice rule is $\Sigma - \mathrm{OSP}$, if there exists a mechanism Γ^m and a strategy profile $(s_i)_{i \in N}$ in G^m that implements the choice rule, moreover, for any Player *i*, s_i is $\Sigma_i - \mathrm{ODS}$.

We now define a direct mechanism of exclusion as follows:

The planner privately observes $\mathcal{X}_A$. Denote $M_0 = \Theta$, t = 0.

---

[64] In the mechanism design literature, $\mathcal{Z}_A$ is usually treated as fixed. For instance, Dasgupta, Hammond and Maskin (1979) states that "since under our assumptions the feasible $\mathcal{Z}_A$ is known to the planner in advance, we need only ensure that for that fixed $\mathcal{Z}_A$, there exists a game form implementing f." However, in our setting where $\mathcal{Z}_A$ is private information to the market maker and there is mistrust in the market marker, we need to inquire whether a mechanism designed *ex-ante*, when $\mathcal{Z}_A$ is uncertain, can implement each social choice function *ex-poste*, when $\mathcal{Z}_A$ is known.

1. The market maker chooses one player $i \in N$ and sends the standing type-profile $M_t = (\Theta_i^t, \Theta_{-i}^t)$.

2. i observes $M_t = (\Theta_i^t, \Theta_{-i}^t)$, and reports the set of types R_i^t he intends to exclude.

3. The planner observes R_i^t, and update $M_{t+1} = (\Theta_i^t \backslash \mathrm{R}_i^t, \Theta_{-i}^t)$.

4. The planner either selects an allocation according to a choice rule z= $f(\theta, \mathcal{X}_A) \in \mathcal{X}_A$ or choose to send another query.

(a) If the planner selects an outcome, the game ends.

(b) If the planner chooses to send another query, update t=t+1, go to step 1.

Definition 2.16. In a direct mechanism of exclusion, we call a solution concept iterative deletion of $\Sigma - obviously\ dominated$ misreports, where each player iteratively reports the set of types R_i^t that are $\Sigma_i - obviously\ dominated$ by their true type θ_i.

Proposition 2.3. *(revelation principle of exclusion*) If a social choice rule is $\Sigma -$ OSP, then it can be implemented by a direct mechanism of exclusion by iterative deletion of $\Sigma - obviously\ dominated$ misreports.

Proof. It's crucial to identify an equivalent relation between a direct mechanism of exclusion and an $\Sigma - \mathrm{OSP}$ mechanism as show in Table 3.4. Then the result follows directly from Eq (2.3) and the definition of $\Sigma_i - obvious\ dominace$.

$\Sigma - \text{OSP}$ mechanism[65]	Direct Mechanism of Exclusion
$[I_i]$	$\Theta^t_{-i} \times \mathcal{A}$
$\mathcal{S}_i$	Θ^t_i
$\mathcal{S}_i(I)[a]^c$	R^t_i

Table 3.4. Equivalence

4. Partition Obvious Equilibrium

Applying the concept of Nash Equilibrium to POP, we propose *Partition Obvious Equilibrium* (POE). Following the notation in Aumann (1976), we denote by $\varepsilon(\omega)$ the event in Partition Σ that contains state ω.[66]

Definition 2.17. (POE) In a dynamic Game G, a strategy profile s^* is a Σ-Obvious Equilibrium if for any Player i, at any information set $I \in \mathcal{I}(s_i^*)$, for any deviating strategy $s_i' \in \mathcal{S}_i(I)[s_i^*(I)]^c$ and any move of nature $\omega_n \in \Omega_N$:

$$(2.6) \qquad u_i(s_i^*, s_{-i}^*, \omega_n) \geq \sup_{(s_{-i},\omega_n)\in\varepsilon(s_{-i}^*,\omega_n)(I)} u_i(s_i', s_{-i}, \omega_n).$$

Since any equilibrium payoff can be viewed as a constant act, a strategy profile is an Σ-Obvious Equilibrium if and only if any Σ_i-Obvious Preference prefers any realized equilibrium payoff to any deviating strategy at any reachable information

[65] We exclude the branches of the game tree that won't be followed by any strategy-profile in equilibrium.

[66] That is, if the state of the world is ω, then the player is informed of the element ε(ω) of Σ that contains ω.

set conditioning on the event that contains the realized state.[67] When the partition is the finest $\Sigma_i = \Omega_i$ for any Player i , Σ-Obvious Equilibrium coincides with *Ex-post Equilibrium*, and when the partition is the coarsest $\Sigma_i = \{\Omega_i\}$ for any Player i, it coincides with *Obvious Nash Equilibrium* (Zhang and Levin 2017). In one "intermediate" case, when partitioning by strategies of opponents $\Sigma_i = \{s_{-i} \times \Omega_N | s_{-i} \in S_{-i}\}$[68] for any Player i , Σ-Obvious Equilibrium coincides with *Obvious Ex-post Equilibrium* (Li 2017). Following from similar arguments as in the proof of Proposition 2.2, any Σ-Obvious Equilibrium is Ex-post Equilibrium for games that are Σ-indistinguishable to the original game.

When multiple *Ex-post Equilibria* exist, Σ-Obvious Equilibrium can serve as a criterion for refinement, as in the following stag-hunt game with imperfect information. When nature moves Left (Right), the payoff is shown in the matrix on the Left (Right). There are two *Ex-post Nash Equilibria*, (Stag, Stag) and (Hare, Hare). There is no *Obvious Nash Equilibrium* or *Obvious Ex-post Equilibrium*. However, there exists a unique Σ-Obvious Equilibrium, (Stag, Stag), when partitioning by moves of nature, $\Sigma_i = \{S_{-i} \times \omega_n | \omega_n \in \Omega_N\}$ for any Player i.

[67] The proof is similar to that of Proposition 2.1.
[68] In the setting of Li (2017), type randomizations are the only type of nature's moves.

	Stag	Hare
Stag	10,10	0,8
Hare	8,0	8,8

	Stag	Hare
Stag	6,6	0,4
Hare	4,0	4,4

Figure 2.5. The stag-hare game with incomplete information

In scenarios where a DS mechanism does not exist, POE can be used to identify sets of mechanisms that are more robust than mechanisms with just *Ex-post Equilibria*. For example, in some interdependent value settings, a generalization of a Vickrey auction achieves efficiency with *Ex-post Equilibrium* (Crémer and Mclean 1988; Dasgupta and Maskin 2000; Bergemenn and Morris 2008). In such settings, there might exist a dynamic auction that achieves efficiency with our stronger solution concept, POE.[69][70]

[69] In the scenario of single-object auctions, Li (2017) proposes *Obvious Auctions* that achieve efficiency with his *Obvious Ex-post Equilibrium.*

[70] In single-unit, common-value auctions, Levin and Reiss (2017) shows that the dynamic English auction generates less Winner's Curse, a non-equilibrium behavior than a static auction with a non-regret payment rule. Notably, the dynamic English auction has the sincere bidding as an Obvious Ex-post Equilibrium, while the static auction has sincere bidding only as an ex-post equilibrium.

Chapter 3. Partition Obvious Preference or Mistrust in Mechanism Design-Experiments

1. Introduction

In this chapter, we present design and result of experiments that test the theoretical implication addressed in Chapter 2.

Empirically, do games and decision problems with a choice that partition obviously dominate others significantly diminish the choice of dominated choice? We test and verify our theoretical implication by a simple laboratory experiment with both cross-subject and within-subject comparison by a cross-over design in Section 2 of this chapter.

Information plays an important role in economic activities. However, by the standard economic view, any information is valuable only if it improves decision-making. This implies no willingness to pay for non-instrumental information.[71] Experimental data, however, documents the opposite phenomena.[72] In Section 3, we design an experiment to

[71] We call a piece of information "instrumental" in the case where this information can alter the optimal decision.

[72] In social learning, several experimental works documented a significant proportion of subjects who purchase signals that are informative (Kübler and Weizsäcker, 2004; Celen et al., 2005; Goeree and Yariv, 2007). Some studies in medicine demonstrate that physicians tend to order excess diagnostic tests that give no new information (Allman et al., 1985; Myers and Eisenber, 1985; Kassirer, 1989). In second price auctions where bidding one's own value is a dominant strategy, Cooper and Fang (2008) corroborated that more than 72% of subjects purchased a signal about their opponents' value at a positive price at least once. In the psychology literature, Tversky and Shafire (1992) and Shafir, Simonson, and Tversky (1993) documented instances in which decision-makers pursued costly information even when that information had no impact on their decision. In one case, students who planned to purchase a vacation to Hawaii regardless of passing an exam or not, chose to postpone their decision and pay a $5 non-refundable fee in order to retain the right to buy the vacation package the next day—after they had confirmed their exam results.

test whether the demand for such non-instrumental information diminishes in situation where the best choice is also obviously dominant. A finding that whether subjects' demand for non-instrumental information relates to whether the *dominant strategy* is obvious (or not), would lead to a better understanding of the phenomena which do not align with the prediction of standard economic theory. Accounting for cognitive cost of reasoning alert us to the fact that, non-instrumental information in theory might nevertheless be instrumental for less than fully rational players. Moreover, by incorporating a questionnaire that test students' cognitive abilities, we can further explore whether subjects' behavior differences are related to their cognitive levels.

2. The empirical effectivity of partition obviously dominance

2.1. Games and Decision Problems Used in the Experiments

Random Serial Dictatorship. Each group has four agents. There are two Cases $\mathcal{A} = \{\mathrm{L}, \mathrm{R}\}$, each selected with a probability of ½. The total budget is $T_L = 10$ in Case L and $T_R = 22.5$ in Case R. In both cases, values of four prizes A, B, C, D are drawn uniformly at random without replacement, from the set:

$$\{P_1, P_2, P_3, P_4\} = \{0.1T_A, 0.2T_A, 0.3T_A, 0.4T_A\},$$

each being a fixed proportion of the total prize $\mathrm{T_A}$, A = L or R. See Figure 3.1 for one realization of four prize values in each case.

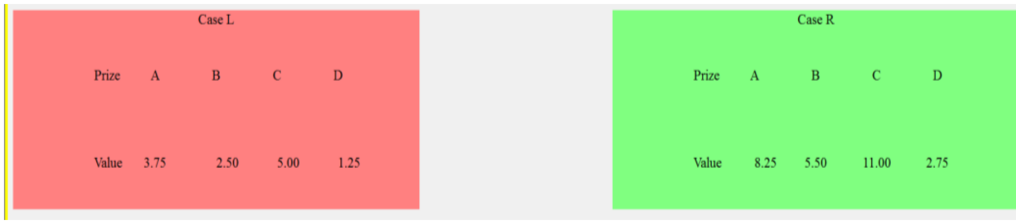

Figure 3.1. One realization of four prize values in each case

The market maker discriminates agents by the ranking of their priority score. The choice rule $f(\theta, \mathcal{X}_A) \in \mathcal{X}_A, A = L, R$ follows random serial dictatorship in Case L and R respectively, where $\mathcal{X}_A = \{0.1T_A, 0.2T_A, 0.3T_A, 0.4T_A\}, A = L, R$.

At the start of each game, the subjects observe the value of four prizes in both Case L and R. They are also assigned and informed of a priority score, which is drawn uniformly from integers 1 to 10. There are two games, S and D.

In Game S, each player is asked simultaneously to submit a list that ranks her preferences over the four prizes A, B, C, D. The players are then processed sequentially, from the highest to the lowest priority score. Ties in priority score are broken randomly. Each player is assigned the highest-ranked prize on her list among the prizes that have not yet been assigned to players with higher priorities who selected earlier.

In Game D, the players take turns to select a prize in order of their priority score, from the highest to the lowest. When a player takes her turn, she is shown the prizes that have not yet been taken and is asked to pick one of them.

In both Games S and D, truthful reporting – ranking/picking prizes with respect to the monetary value $\{P_1 > P_2 > P_3 > P_4\}$ in either Case – is DS but not ODS in either game.[73] Note that, truthful reporting is *partition obviously dominant* (by Cases L and R) in Game D, but not in Game S.[74] Therefore, if subjects are capable of reasoning

The Decision Task. Consider the following individual choice problem. There are six prizes worth 0, 2, 4, 6, 8 and 10. They are randomly, without replacement, inserted inside six Boxes (one in each box): A, B, C, D, E and F, and cannot be seen from the outside. See Figure 3.2.

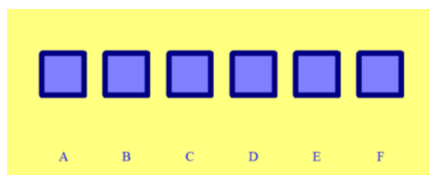

Figure 3.2. Six boxes

[73] For example, See Figure 3.1, in Game D, the agent who has the highest ranking of priority can infer as follows: if I am not that greedy and pick A, the most I can get is 8.25, which is larger than 5.00, the least I might earn if I Pick C instead. Thus picking P_1, (=A in Figure 3.1) is not an ODS.

[74] In Game D, contingent on Case L or R, once you pick an item with higher value, you assure obtaining a higher monetary value for sure; In Game S, even contingent on Case L or R, the least you can get by truthful reporting is the lowest value P_4, and the most you can get by reporting P_2 on the top is $P_2 > P_4$. Thus, truthful reporting is *partition dominant* in Game D but not in Games S.

Another six prizes worth: 0, 2, 4, 6, 8 and 10, are randomly, without replacement chosen to be written on six Stickers (one on each sticker): A, B, C, D, E and F. See Figure 3.3 for one realization.

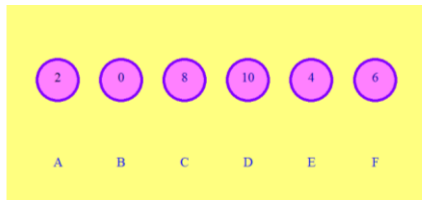

Figure 3.3. Six stickers

A subject is randomly assigned, with a probability of ½, to Case L or Case R, but they will not be informed whether they are in Case L or Case R. There are two decision tasks, S and D. The subjects' payoff depends on the case they are in, and on their choices.

In Decision S, the subject is asked to first choose one of the stickers, and then pick one box with an unknown value. Then the subject sees the monetary value in the box she picks. In Decision D, the subject is asked to first pick one box. And then is shown the monetary value inside the box. Then she is asked to choose a sticker.

In both decision tasks, S and D, if the case is L, the subject is assigned the lower monetary value between the one in the box and the one on the sticker; if the case is R, the subject is assigned the higher monetary value between these two. Picking a sticker with a monetary value other than 10, is *dominated*,[75] but not *obviously dominated* by picking 10.[76] Consider the partition by Case L and Case R. Picking a sticker with a prize of 10 is a partition-ODS in Decision D but not in Decision S.[77] Thus, our theory predicts more choices of stickers with a prize of 10 in Decision D than in Decision S.

2.2. Treatments

	Game Tasks		Decision Tasks	
	1st Game	2nd Game	1st Decision	2nd Decision
Treatment 1	Game S	Game D	Decision S	Decision D
Treatment 2	Game D	Game S	Decision D	Decision S

Table 3.1. The crossover design

We adopt a crossover design (Piantadosi 2005) as shown in Table 3.1 above. The treatments are across subjects. Each treatment consists of 4 tasks, Game S, Game D,

[75] They are dominated by picking the prize of 10 and the same box.

[76] For example, the highest prize the subject can get by picking a sticker with 8 on it, is 10. It is higher than the lowest prize the subject can get by picking a sticker with 10, which is 0.

[77] In Decision D, after the subject see the value inside the box ($x \leq 10$): in Case L, the lowest prize by picking a sticker with 10 on it is x; the highest prize by picking a sticker with any other value is at most x. In Case R, the lowest prize by picking a sticker with 10 on it is 10; the highest prize by picking a sticker with any other value is at most 10. Thus, picking a sticker with 10 on it is *obviously dominant* in both cases. The argument does not follow in Decision S. For example, in Case L, the lowest prize by picking a sticker with 10 is 0, lower than the highest prize of picking 8 (that is 8). Thus, subjects might pick 8 when they infer as follows: if I am not that greedy, God will help me get the best outcome. Similar magical thinking has been addressed in Arad's (2014) choice experiment.

Decision S and Decision D, and each task will repeatedly be played for 10 rounds.[78] In Treatment 1, the subjects first play 10 rounds of Game S followed by 10 rounds of Game D, and then Decision S for the first 10 rounds, followed by 10 rounds of Decision D. In Treatment 2, the order is reversed. The instructions for each task are given immediately before that task. There is no information feedback until the end of the experiment. At the start of each game of the experiment, the subjects are randomly assigned into groups of four. These groups persist throughout the experiment. Consequently, each group's play can be regarded as a single independent observation in the statistical analysis. Our design allows us to compare each subject's behavior in Game (Decision) S with those in Game (Decision) D, controlling for sequential order effects. Moreover, using the data for only the 1st game and decision tasks in both treatments, we are also able to compare across subjects how players behave differently in Game (Decision) S and Game (Decision) D.

2.3. Administrative Detail

The subjects were paid $5 for participating in the experiment, in addition to their profits or losses from every round of the experiment. On average, they received a total of $16.19, including the participation payment.

We conducted the experiment in January 2017 at the Ohio State University Experimental Economics Laboratory, using z-Tree (Fischbacher 2007). We recruited subjects from the student population using the ORSEE online recruiting system. We administrated 7 sessions,

[78]In each task, at the end of Round 10, we randomly select a round and add to the subjects' earnings the payment they receive in that round. Azrieli, Chambers and Healy (2018) prove that such random problem section mechanism is the only incentive compatible mechanism assuming monotonicity.

where each session involved 3-5 groups. Each session lasted about 60 minutes. The data was collected from a total of 108 subjects in 27 groups of 4, with 13 groups in Treatment 1 and 14 groups in Treatment 2. 48% of subjects are female and 52% are male. Experiment Instructions are in Appendix C.

2.4. Result

To compare the subjects' behavior in Game S to their behavior in Game D, we report the proportions, in the pooled data, of the games that do not end in the DS outcome. In Game S, 36.58% of the games did not end in the DS outcome (non-DS), wherein Game D this percentage is just 3.33%. Tables 3.2, 3.3 and 3.4 show the empirical frequency of non-DS outcomes by Games and by 5-round blocks, in the pooled data, the within-subject, and the cross-subject comparison. Deviations from the DS outcome happen almost 10 times more frequently in Game S than in Game D, and these differences are highly significant in both early and late rounds of the pooled data, the within-subject, and the cross-subject comparison. In Game S, 31.85% of the submitted erroneous rank-order lists, and in Game D, this percentage is just 1.11%.

	Game S	Game D	p-value
Rounds 1-5	36.11%	3.70%	<0.001
Rounds 6-10	37.04%	2.96%	<0.001
p-value	0.987	0.135	

Notes: For each group, for each 5-round block, we record the error rate. When comparing Game S to Game D, we compute p-values using a Wilcoxon rank-sum test. When comparing early to late rounds of the same game, we compute p-values using the Wilcoxon matched-pairs signed-rank test.

Table 3.2. Proportions of non-DS outcomes (pool data)

	Treatment 1				Treatment 2		
	Game S	Game D	p-value		Game D	Game S	p-value
Rounds 1-5	31.92%	1.54%	<0.001	Rounds 1-5	5.71%	40.00%	<0.001
Rounds 6-10	36.92%	1.54%	<0.001	Rounds 6-10	4.29%	37.14%	<0.001
p-value	0.650	1.000		p-value	0.080	0.824	

Notes: We compute p-values using Wilcoxon matched-pairs signed-ranks test.

Table 3.3. Proportion of non-DS outcomes (within-subject Comparison)

	Game S (Treatment 1)	Game D (Treatment 2)	p-value
Rounds 1-5	31.92%	5.71%	<0.001
Rounds 6-10	36.92%	4.29%	<0.001
p-value	0.650	0.080	

Notes: We use the data for only the 1st game in both treatments, Game S in Treatment 1 and Game D in Treatment 2, for Cross-Subject Comparison. When comparing Game S to Game D, we compute p-values using a Wilcoxon rank-sum test. When comparing early to late rounds of the same game, we compute p-values using the Wilcoxon matched-pairs signed-rank test.

Table 3.4. Proportions of non-DS outcomes (cross-subject Comparison)

To compare subject behavior in Decision S and Decision D, we display the proportion of dominated choice. In Decision S of the pooled data, 23.80% of choices are dominated strategies. In Decision D of the pooled data, 0.83% of choices are dominated strategies. Tables 3.5, 3.6 and 3.7 report the empirical frequency of dominated choice, by Decision Tasks, and by 5-round blocks, in the pooled data, the within-subject, and the cross-subject comparison. Dominated choice happens more frequently in Decision S than in Decision D, and these differences are highly significant in both early and late rounds of the pooled data, the within-subject, and the cross-subject comparison.

	Decision S	Decision D	p-value
Rounds 1-5	25.74%	1.11%	<0.001
Rounds 6-10	21.85%	0.56%	<0.001
p-value	0.014	0.473	

Notes: For each group, for each 5-round block, we record the error rate. When comparing Decision S to Decision D, we compute p-values using a Wilcoxon rank-sum test. When comparing early to late rounds of the same game, we compute p-values using Wilcoxon matched-pairs signed-rank test.

Table 3.5. Proportions of dominated choice (pooled data)

Treatment 1			
	Decision S	Decision D	p-value
Rounds 1-5	23.85%	1.54%	<0.001
Rounds 6-10	20.00%	0.38%	<0.001
p-value	0.143	0.312	

Treatment 2			
	Decision D	Decision S	p-value
Rounds 1-5	0.714%	27.5%	<0.001
Rounds 6-10	0.714%	23.57%	<0.001
p-value	1.000	0.047	

Notes: We compute p-values using Wilcoxon matched-pairs signed-rank test.

Table 3.6. Proportions of dominated choice (within-subject comparison)

	Decision S (Treatment 1)	Decision D (Treatment 2)	p-value
Rounds 1-5	23.85%	0.714%	<0.001
Rounds 6-10	20.00%	0.714%	<0.001
p-value	0.143	1.000	

Notes: We use the data for only the 1st decision task in both treatments, Decision S in Treatment 1 and Decision D in Treatment 2, for cross-subject Comparison. When comparing Decision S to Decision D, we compute p-values using a Wilcoxon rank-sum test. When comparing early to late rounds of the same game, we compute p-values using the Wilcoxon matched-pairs signed-rank test.

Table 3.7. Proportions of dominated choice (cross-subject comparison)

In summary, subjects play the dominated strategies at much lower rates in mechanisms with partition-ODS, as compared to dominant strategy mechanisms that should implement the same allocation rule. Moreover, subjects choose dominated strategy at a much lower rate in decision tasks when the optimal choice is also *partition obviously dominant* than when it is not. In Appendix D, we display alternative statistical analyses that yield similar results. We also found a significant negative correlation between the priority score and the deviation from dominant strategies in Game S.[79] See Appendix D for details.[80]

3. Pay for non-instrumental information

3.1. Main treatments

We design pairs of two one-shot games with 2 players and incomplete information as in Figure 3.4. There are three possible states of nature: A, B, and C. The computer randomly draws one, which is unknown to both players. Each player chooses between R and L. We provide the payoff table in three matrices. (a, b) denotes that player 1 gets a and player b gets b.[81] The game on the top of Figure 3.3 is the same as that in Example 1 of chapter 2. R is a dominant strategy in both games, but it is obviously dominant only in the game on the bottom. Following the argument in Example 1 of chapter 2, the information regarding

[79] Hassidim, Romm, and Shorrer (2018) also find a negative correlation between the priority score and the deviation from the dominant strategy in Serial Random Dictatorship by revisiting data from one of the treatments in Li's (2017) experiment.

[80] As a side result, we found that women are more likely to choose dominated strategies in both Game and Decision S. But we are aware that the gender difference we found might be due to other correlated factors, which is beyond the scope of the current paper.

[81] All payoffs are in Experimental Currency Unit (ECU) and will be paid with exchange rate: 2ECU=1 U.S. dollar.

the state of nature is in can alter the optimal choice for some POP preference and thus it is instrumental in the game on the top (Denote by Game-DS); however, regarding Proposition 2.1, such information cannot alter the optimal choice for all POP preference and hence it is not instrumental in the game on the bottom (Denote by Game-ODS).

		State A		**State B**		**State C**	
		The Other		The Other		The Other	
		L	R	L	R	L	R
You	L	20	8	22	6	18	10
	R	25	13	27	11	23	15

		State A		**State B**		**State C**	
		The Other		The Other		The Other	
		L	R	L	R	L	R
You	L	15	13	16	12	13	15
	R	20	18	21	17	18	20

Figure 3.4 Two symmetric games: DS or ODS

In each of the one-shot games, after the state is randomly selected, as in Figure 3.5, we will ask each player whether they are willing to pay some amount of ECU to know the selected state A, B, or C.[82] They need to provide answers for all listed prices. After choices are

[82] The payoff table is showing on the same screen.

made, we will randomly draw a number from the listed prices to determine which price to be selected and implemented.

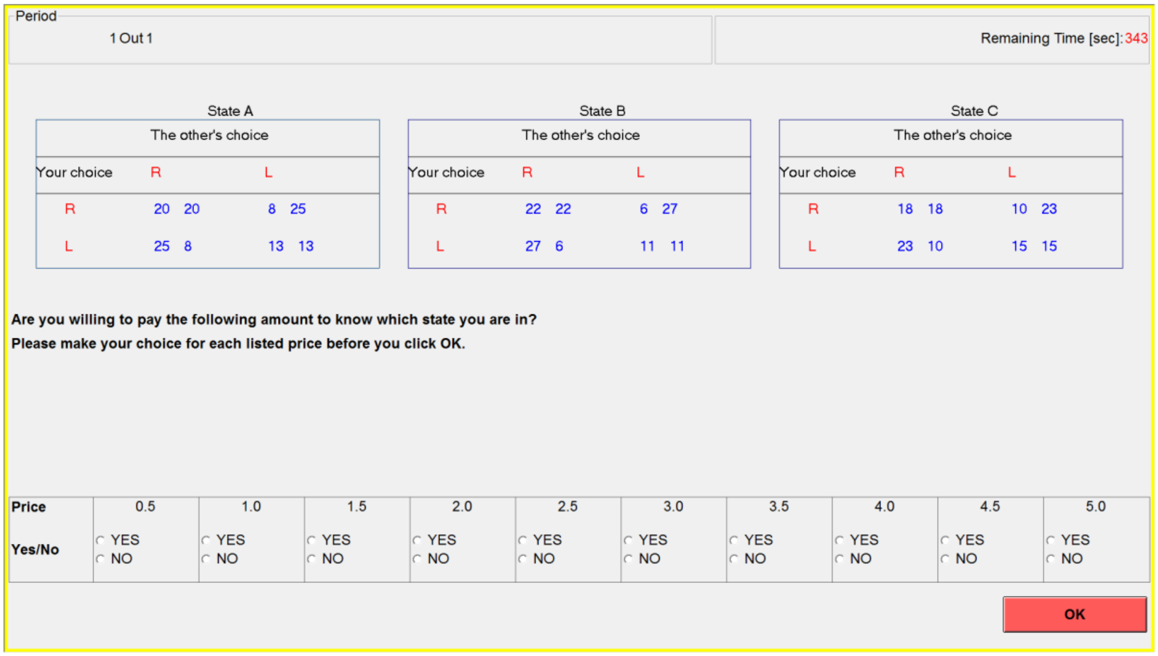

Figure 3.5. zTree screenshot: willingness to pay

If the subject indicated that he/she is willing to pay for the selected price, as in Figure 3.6, he/she will be shown the selected states and then asked to make their choice, R or L. Otherwise, as in Figure 3.7, they need to make their choice immediately without any more information.

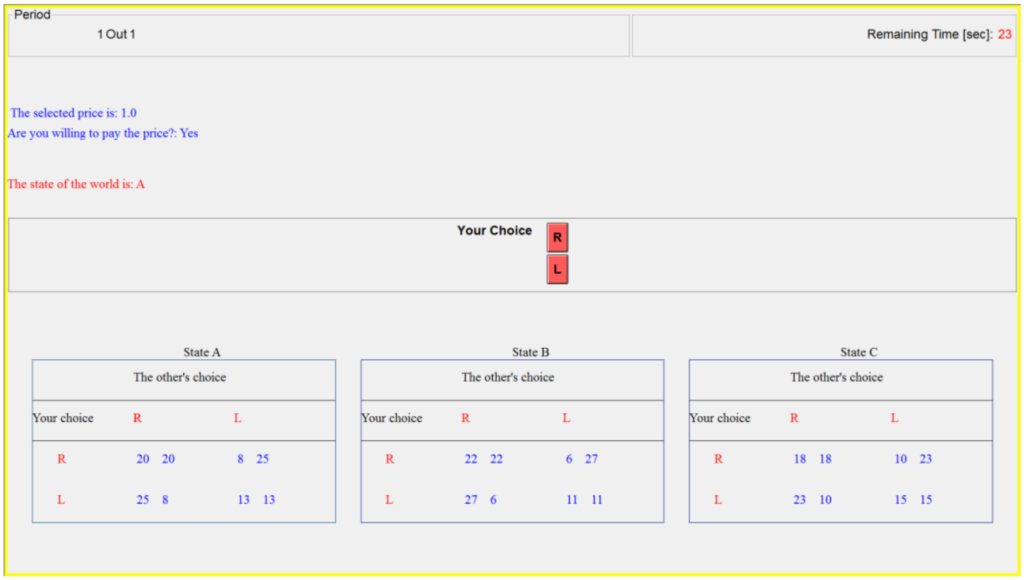

Figure 3.6. zTree screenshot: selected state shown

Figure 3.7. zTree screenshot: selected price not shown

3.2. Control for other explanations

Our design also controls for other explanations for paying for non-instrumental information proposed in existing studies. First, because R is *ex-post* optimal with probability 1, with or without the information about states of nature, our design rules out the possibility of paying for confidence, when subjects have an intrinsic preference for the likelihood that this decision is *ex-post* optimal (Eliaz and Schotter 2007, 2010).

Second, the possible different behavioral results in the pair of games can't be explained by models of best responding with errors including *Quantal Response Equilibrium*, (McKelvey and Palfrey 1995) and mixture-of-types models (See, e.g. Stahl and Wilson 1994; Crawford and Iriberri 2007). Denote the expected payoff of strategy R and L by π_R and π_L; denote the random shock by ε and precision parameter by μ. Strategy R is selected if the disturbed expected utility of R ($\pi_R + \frac{\varepsilon}{\mu}$) is larger than that of L ($\pi_L + \frac{\varepsilon}{\mu}$). So, the probability of choosing R is $F(\mu(\pi_R - \pi_L))$, where $F(\cdot)$ is the distribution function of the difference in the shocks. Note that in all the four games, given any state, the payoff of R is 5 ECU higher than that of L. So $\pi_R - \pi_L = 5$ regardless of subjects' belief about the states of nature. So, the probability of choosing R is constant, $F(5\mu)$) for both games.

Third, to control for pay for curiosity, a desire to know *ex-post* the situation one was in (Eliaz and Schotter 2007) or the intrinsic preference for information (Grant et al. 1998, 2000), we inform subjects at the beginning of the experiment that information regarding states of nature will be released to them at the end of the experiment in all treatments.

Four, to rule out the effect of social preference, we include sessions where each player is randomly matched with a past player in the subject pool.

Five, the possible different behavioral results in the pair of games can't be explained by paying for early resolution of uncertainty (Kreps and Porteus 1978), (Grant, Kaji, and Polak 1998, 2002). Because if subjects (do not) have a preference for early resolution of uncertainty, then they should (not) be willing to pay for the information in both games.

3.3. The voting games

The following voting game is from the experiment of Esponda and Vespa (2014). There is a jar with 10 balls, some of which are Green and some of which are Pink. Each subject plays with two computers that are programmed to behave in a specific manner**:**

(1) If the selected ball is Green: vote Green.

(2) If the selected ball is Pink: vote Green with probability p and pink with 1-p, $0<p<1$. The exact value of p is unknown. If the selected ball matched the vote of the majority, the subject's payoff is 2; otherwise, it is zero.

In Phase 1, subjects vote without any information about actual votes of the computers; in Phase 2, each subject plays the same game but is informed that one computer voted Green and the other voted Pink.

Voting Pink is *dominant strategy* in both phases.[83] However, it is *obviously dominant strategy* only in phase 2.

3.4. The treatments

We adopt a crossover design (Piantadosi 2005) as shown in Table 2. The treatments are across subjects. Each treatment consists of 2 tasks, the normal form games, and the voting games. The instructions for each task are given immediately before that task. There is no feedback whatsoever until the end of the experiment.

In Treatment 1A, 1B, we group subjects randomly to play the 1st Game and then randomly re-group them to play the 2nd Game, which is a perfect stranger protocol.[84] In treatment 2A and 2B, each subject plays with a past player randomly selected from the subject pool. In each treatment and in both task, one game (phase) has an *obviously dominant strategy* (ODS) while the other game (phase) only has a *dominant strategy* (DS). The Treatments iA and iB, i=1,2 switch the order of the two games (phases) in Task 1 (2). So, such design allows us to compare each subject's behavior in both games (phases), controlling for sequential effect. Moreover, using the data for only the 1st Game, we are also able to compare, across subjects, how players in games with or without *an obviously dominant* strategy behave differently.

[83] The only states that the subject's vote matters are when the selected ball is pink and two computers vote different colors.

[84] The two games in Treatment 1A and 1B have symmetric payoff; while those in Treatment 2A and 2B have asymmetric payoff.

	Task 1			Task 2	
	1st Game	2nd Game	The other player	1st Decision	2nd Decision
Treatment 1A	DS	ODS	Current Player	DS	ODS
Treatment 1B	ODS	DS		ODS	DS
Treatment 2A	DS	ODS	Past Player	DS	ODS
Treatment 2B	ODS	DS		ODS	DS

Table 3.8. Treatments

3.5. The questionnaire

At the end of experiments, we add a questionnaire with items that test students' different aspects of cognitive abilities. By incorporating answers collected from the questionnaire, we can further explore if subjects' behavioral differences are related to the measurement of their cognitive abilities.[85] Our questionnaire including three parts are in Appendix E. Part I is the *Cognitive Reflection Test* (CRT), a 3-item test first introduced by Kahneman and Frederick (2002) and Frederick (2005).[86] Part II is the test of *Disjunctive Reasoning,*[87] a variant on ones used by Toplak and Stanovich (2002). In Part III, we use the 18-item *Need for Cognition Scale* (Cacioppo, Petty, and Kao 1984) to quantitatively measure the tendency for an individual to engage in and enjoy thinking (Cacioppo and Petty 1982, p.116).[88] We will also request access to student records reporting resources

[85] All the tests included in our study are parts of the Comprehensive Assessment of Rational Thinking (Stanovich 2016).

[86] CRT is broadly used in literature that measures cognitive abilities. (Oechssler, Roider, and Schmitz 2009, Campitelli and Labollita 2010, Bergman, Ellingsen, Johannesson, and Syensson 2010, Moritz, Hill, and Donohue 2013, Corgnet, Espoin and Hernan-Gonzalez 2015, Corgnet, Hernan Gonzalez, and Mateo 2015, Noori 2016)

[87] The problem of disjunctive reasoning was first introduced by Levesque (1986, 1989).

[88] The 18-item *Need for Cognition Scale* has been used in several settings. Researchers have used the scale to examine (1) how students' need for cognition relates to their academic performance (Sadowski and Gulgo,

(http://oesar.osu.edu/). By eliciting students' records, we will be able to test how our research results vary across students with different academic and geographic backgrounds. Among the records, SAT score would be used as another good proxy measure for cognitive ability (Filiz-Ozbay, Ham, and Kagel 2016, p. 8).

3.6. The pilot results

We have run two pilots so far. The experiment is presented to subjects through software programmed in zTree (Fishbacher 2007) in the Economics Experimental Lab of the Ohio State University. The experiment instruction is attached in Appendix F. The pilots allowed us to test the software and improve the instructions. More importantly, they let us realize that two important changes had to be made to the experiment design.

First, in the pilots, we used Red and Blue ball as in Esponda and Vespa (2014)'s original experiments. Surprisingly, almost all subjects voted for the Red ball in both phases. From a post-experiment survey, we found that it is just because Red is the color of Ohio State Football team and Blue is that of the archenemy Michigan. So Red and Blue tend to stimulate a strong bias in subjects who are undergraduate at Ohio State University and thus it is not a proper label of choices for the purpose of our experiment.

Secondly, in the pilots, the first session employed only Game-DS in Task 1 and Phase 1 in Task 2 with 12 subjects, and the second session used only Game-ODS in Task 1 and Phase

1992, 1996; Tolentino, Curry, and Leak, 1990); (2) how one's need for cognition and religious views influence satisfaction with one's life (Gauthier, Christopher, Walter, Mourad, and Marek, 2006); (3) the relationship between jurors' need for cognition and their legal decisions (Bornstein 2004), and (4) how college students' need for cognition impacts their self-reported satisfaction with their lives as a whole (Coutinho and Woolery, 2004).

2 in Task 2 with 12 subjects. We found that both sessions ended within 40 minutes, which suggests that we will be able to finish our crossover design (presented in 3.4) within one hour. So, we decide to use the crossover design in future experiments because we could collect more data and conduct “within-subject” comparison without much more cost.

The preliminary results provide some evidence that subjects are less willing to pay for non-instrumental information when the dominant strategy is obvious. See Figure 1: In Game DS, 8 out of 12 agreed to pay 0.5 ECU (66.67%); 6 out of 12 agreed to pay 1 ECU (50.00%); 5 out of 12 agreed to pay 1.5 ECU (41.67%); 4 out of 12 agreed to pay 2 ECU (33.33%); 3 out of 12 agreed to pay 2.5 ECU (25.00%) and 1 out of 12 agreed to pay 3 ECU (8.33%). In contrast, in Game-ODS, 5 out of 12 agreed to pay 0.5 ECU (41.67%); 3 out of 12 agreed to pay 1 ECU (25.00%); 2 out of 12 agreed to pay 1.5 ECU (16.67%) and 1 out of 12 agreed to pay 2 ECU (8.33%).

Figure 3.8. Pilot results

Ending Remark: Transdisciplinary Conversations

As a variant of Tolstoy's (1966)[89] famous quote, Economist Steve Tadelis (Presh Talwalkar 2014) says, "rational people are all alike; every irrational person is irrational in his or her own way." When we leave the paradigm of rationality defined in classic Economics, where should we go? On one hand, to model complex human behavior more closely, we need to converse with other disciplines wherein specific aspects of human behavior are researched in depth, for instance, for emotion and heuristics, with psychology; for brain and mind, with neuroscience; for ethic concern, with moral philosophy. On the other hand, to honor the elegance of classic Economic theory, we might seek a unified model that is broadly applicable, not many unsystematic models, each of which only explains a single case.[90] At the end of this dissertation, we address how our model contributes to accomplishing the two goals simultaneously: stimulating transdisciplinary conversations and providing a unified framework.

4.1. Converse with Psychology

Our approach does not necessarily contradict other *psychological* explanations for choosing a *dominated strategy*. A decision maker who has a POP might be also affected by certain "non-optimal" heuristics in scenarios where reasoning the optimal choice is beyond his or her bound of rationality. For example, Shafire, Simonson and Tversky (1993)

[89] "Happy families are all alike; every unhappy family is unhappy in its own way." (Anna Karenina, Chapter 1)

[90] As Rabin (2013) says, "For a good scientific reason, the heart of much psychological and behavioral research is to investigate possible flaws, caveats, and modifications to previous theories. But the heart of modifying existing economic theory is to formulate credible and systematic alternatives."

proposed a theory of *reason-based choice* to explain the choice of *dominated strategies* in the prisoner dilemma games and other choice tasks.[91] that leads them to cooperation. However, such a decision can also be rationalized by our POP,[92] as the player somehow believes that by cooperating, even in a single shot game, the other player is more likely to reciprocate, assigning a larger α for cooperating.[93] Explanations for the choice of *dominated strategies* such as *reason-based choice*, *magical thinking* (Arad 2014) as well as *joys of winning* (Harrison 1989), *spite-motive* (Morgan et al. 2003), and *inequality aversion* (Cooper and Kagel 2009) offer specific explanations in specific environments. Our approach provides a more unified theoretical foundation. It thus serves to bridge the gulf between the *rational* and the *psychological* narratives.

Our theory can also be interpreted from the perspective of *procedural rationality*, similar to, but richer than, how Osborne and Rubinstein (1998) motivate their $S(K)$-*equilibrium*. We allow a decision maker to have a coarse understanding of the relationship between the other players' choice, state of the world, her own choice and the outcome. The level of coarseness is modeled by a partition of their subjective *state-space*.[94] Decision makers

[91] They claim that people's choices are guided by reasons and some players may follow the golden rule, a variant of what is mentioned in Kant's *the Categorical Imperative* (Paton 1971), "treat others as you wish to be treated," in *the Analects of Confucius* (Confucius and Watson 2007), "what you don't want done to yourself, don't do to others," and in Adam Smith's (1759, 1976) the *Theory of Moral Sentiments*, "as to love our neighbors as we love ourselves is the great law of Christianity, so it is the great precept of nature to love ourselves only as we love our neighbor, or what comes to the same thing, as our neighbor is capable of loving us."

[92] Note that in the *prisoner's dilemma* game, there is no ODS. Thus, a player who cannot reason state-by-state may fail to recognize the DS.

[93] Dawes, McTavish, and Shaklee (1977) found, in an eight-person commons dilemma game, that having to make the cooperative or defective choice did affect the expectations of what opponents would do: defectors expected much more defection than did cooperators.

[94] Osborne and Rubinstein (1998) only consider the coarsest partition. In their model, decision-makers first associate one outcome with one action, by sampling and then choosing the one that has the best outcome.

know the set of outcomes that each of their actions induces, given any event of the partition; however, they do not distinguish in detail, in which state each outcome would come out. Thus, rather than following *substantive rationality* (Simon 1976) right away, optimizing given a belief over the *state-space*, they adopt the following procedure: They first associate one outcome with each of their actions, in each event of the partition, by sampling, literately or virtually, in a certain way. They then follow *substantive rationality* in the reduced problem by optimizing a given a belief over the partition. Our approach thus retains the tight system of axioms that have dominated classical economics but also consider the actual processes of cognition that have prevailed in psychology.

4.2. Converse with Eye-tracking Technology in Neuroscience

When face the same problem, different agents might process information differently. Especially, they might partition the subjective state-space in different ways. The variations in information partitioning do not matter when there is an ODS. However, it matters when the DS is *partition obviously dominant* respect to one partition but not the other. In such scenario, if the market maker can drive agents to choose the former partition by a proper presentation,[95] then he can reduce the choice of dominated strategies and achieve a desirable outcome.

[95] In an experiment of pure coordination games, Charness and Sontuoso (2018) show that the experimenter can manipulate frames-similar to our concept of partitions-on strategic behavior by reminding subjects of various attributes by which different frames are generated.

Partition by states of nature

The State		A		The State		B	
The Other		L	R	The Other		L	R
You	A	20, 20	8, 25	You	A	22, 22	6, 27
	B	25, 8	13, 13		B	27, 6	11, 11

Partition by the strategy of the other player

The other		L		The other		R	
The State		A	B	The State		A	B
You	A	20, 20	22, 22	You	A	8, 25	6, 27
	B	25, 8	27, 6		B	13, 13	11, 11

Figure 4.1. Two ways of partitioning

For example, see two representations of the same game in Figure 4.1. There are two (exogenous) states of nature, L and R, and each player has two strategies, A and B. Payoffs are shown in the matrices. For each player, each player's subjective state space is a cross-product of $\{A, B\}$ and $\{L, R\}$. In the top case, the payoffs are presented using partitioning by states of nature; in contrast, in the bottom case, the same payoffs are presented using partitioning by the strategy of the opponent. B is *partition-ODS* based on the bottom partition but not the top. Thus, if the market maker can drive agents into the information partitioning by which the problem is presented, then empirically, we might observe fewer choice of *dominated strategies* with the bottom presentation than with the top one.

But, can the market maker influence the way how agents partition the state-space by different presentations? Choice data alone are limited in answering this question for a certain choice does not inversely infer a certain information partitioning. Can we observe attention or information searching pattern more directly? Fortunately, it has long been believed that attention is linked to foveal gaze direction in all eye-tracking studies. [96] Based on such convention, we can use eye-tracking data to enhance choice data in responding the query by recording two facets of eye-movements: *fixation*, "where" (Von Helmholtz 1925) and "what" (James 1981) of visual attention, and *scan-path* (Noton and Stark 1971), the sequence of fixation. For instance, if we observe few or no *scan-path* connecting information across two events of the partition as presented in Figure 4.1, and the subject chooses B when facing the bottom presentation while chooses A when facing the top presentation, then we have more reasons to believe that the market maker can influence the way of information partitioning by different presentations of the same problem and choose the proper one to achieve a desirable outcome.

4.3. Converse with Contract Theory

Our theory implies a tradeoff between implementing cost and the rate of truthful reporting. For instance, to facilitate bounded rational agents who reason in coarser partitions, the market marker can highlight a finer partition of the state-space; to solve the problem of mistrust, the market marker can conduct an auditing that is measurable to a finer partition. However, both highlighting and auditing can be costly. The cost of auditing and its effect on truthful-reporting have been studied in the literature of contract theory, mainly in principal-agent models (Townsend 1979, Baron and Besanko 1984, Border and Sobel 1987, Mookherjee and Png 1989, 1992; Kaplow and Shavell 1994, Rahman 2012). However, an

[96] Psychologist William James (1981) once said, "when the things are apprehended by the sense, the number of them that can be attended to at once is small, *Pluribus intentus, minor est ad singular sensus* (Many filtered into few for perception)."

appreciable study of such tradeoff in mechanism design or market design is still absent, which leaves rooms for a conversation between the literature of contract theory and mechanism design.

4.4. Converse with Moral Philosophy

Our theory shows that when there is mistrust in the market maker, a *Strategy-proof* Mechanism might not work. A mechanism with both a stronger solution concept and a relevant audit could be a remedy. But will the group of participants trust the market maker? Or, ought them trust the market maker?

To answer such questions, we refer to moral philosophy. First, we argue that to trust the market maker is plausible in an ideal civil society of high moral standard. For instance, in the west, we could find the moral foundation of trust in Adam Smith's elaboration of sympathy, i.e., both the trust giver and the trust receiver would realize the antipathy of the trust giving person being betrayed by means of sympathy, and both would feel pleasure for this mutual sympathy; and such aligned disapproval of hurting the trust given by others would end up with coordinating actions based on trust (Herrmann-Pillath 2010). In the east, the schooling of Confucianism dictates that a virtuous gentleman can be cultivated by daily reflection on three questions, "in my undertaking on others' behalf, have I been trustworthy and loyal? In my interactions with friends, have I failed to be honest and accountable? In what has been passed on to me, have I failed to carry it into practice?" (Yang and Yue 2018) Thus, it seems that in a civil society consisting of mutual sympathetic citizens or virtuous gentlemen, mistrust would not be an issue.

However, how could we be sure that trust in a market maker is rational or warranted in a more general setting? In this work, we distinguish strategic rationality from epistemic rationality (Baker 1987). In a mechanism with Partition-ODS and an audit that is measurable respect to the partition, to trust the market maker might not be epistemically rational, since the market maker could still deviate to a partition-identical game without being noticed; to trust the market maker is strategically rational, for the player has no incentive to choose an alternative dominated strategy even if he doubts that the market marker might use another partition-identical game to implement the result.[97] Thus, by providing a solution to mistrust in market makers in particular, our work also draws attention to a discussion between the effectivity of designed mechanisms and the moral foundation that the design is based on in general.

In the history of economic thought lies a dilemma for future economists: should we adopt simple models with unrealistic assumptions, or should we describe human behavior closely but give up elegant abstractions (Hausman 2013)? In this dissertation, we endeavor to crave for a middle way that synthesizes the merits in both directions. And we hope we took a modest step forward and contributed to inspiring better solutions.

[97] For more discussions about trust in institutions and government, we refer readers to Govier (1997), Hardin (2002), Potter (2002), Townley and Garfield (2013), and the references therein.

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Appendix A Mathematical Appendix for Chapter 1

A.1. Solve for λ-NLK equilibrium in the Money Request Game

We only go through the solution for $0 \leq \lambda < \frac{1}{2}$, since a similar argument follows for $\frac{1}{2} \leq \lambda < 1$.

We first claim that when $0 \leq \lambda < \frac{1}{2}$, \$20 must be played by an NLK player. Assume for contraction that \$20 will not be played, then deviation to \$20 would end up with \$20 for sure whereas choosing \$19 generates \$19+ $\lambda \times \frac{1}{10} \times 20 (< 20)$. So, \$19 will not be played by an NLK player. By induction, no strategy is valid for an NLK player. This is a contradiction.[98] So \$20 must be played by an NLK player. But \$20 couldn't be the only pure strategy of an NLK player since he has an incentive to deviate to \$19. Assume that $j < 19$ is the largest number that is played with positive probability. Hence deviating to \$19 generates strictly larger payoff. Then \$19 must be played with positive probability. Denote the probability of playing \$j in the NLK equilibrium by $\beta_j, \beta_j \in [0, 1]$ and $\sum \beta_j = 1$. The expected payoff of all strategies in equilibrium should be the same, and since playing \$20 yields \$20 for sure, it follows: $19 + (1-\lambda)\beta_{20} 20 + \lambda \frac{20}{10} = 20$. Then $\beta_{20}^* = \frac{1-2\lambda}{(1-\lambda)20} < 1$. By the same argument, $18 + (1-\lambda)\beta_{19} 20 + \lambda \frac{20}{10} = 20$. Then $\beta_{19}^* = \frac{2-2\lambda}{(1-\lambda)20}$.

[98] Since the game is finite, by Proposition 1, NLK exists.

Because $\beta_{20}^* + \beta_{19}^* < 1$, \$18 has to be played in equilibrium (otherwise there would be an incentive to deviate to \$18), so iteratively, we get $\beta_{18}^* = \frac{3-2\lambda}{(1-\lambda)20}, \beta_{17}^* = \frac{4-2\lambda}{(1-\lambda)20}, \beta_{16}^* = \frac{5-2\lambda}{(1-\lambda)20}$. Suppose \$14 is played in equilibrium too, then $14 + (1-\lambda)\beta_{15}20 + \lambda\frac{20}{10} = 20$ implies that $\beta_{15} = \frac{6-2\lambda}{(1-\lambda)20}$. But in this case, $\sum_{j=15}^{20}\beta_j > 1$. This is a contradiction. So, \$14 (and all lower numbers) would not be played by an NLK player. Then $\beta_{15}^* = 1 - \sum_{j=16}^{20}\beta_j = \frac{5-10\lambda}{(1-\lambda)20}$. In conclusion, when $0 \leq \lambda < \frac{1}{2}$, there is a unique mixed-strategy for an NLK player where

$\{\sigma_i^*\} = \{\beta_{15}^*, \beta_{16}^*, \beta_{17}^*, \beta_{18}^*, \beta_{19}^*, \beta_{20}^*\} = \{\frac{5-10\lambda}{(1-\lambda)20}, \frac{5-2\lambda}{(1-\lambda)20}, \frac{4-2\lambda}{(1-\lambda)20}, \frac{3-2\lambda}{(1-\lambda)20}, \frac{2-2\lambda}{(1-\lambda)20}, \frac{1-2\lambda}{(1-\lambda)20}\}$.

A.2. Solve for λ-NLK equilibrium in the Common Value Auction

Assume there is a linear pure strategy for a λ-NLK player, denote it as $b^\lambda(x) = b^\lambda(1) + \frac{b^\lambda(4)-b^\lambda(1)}{3}(x-1), x \in [1,4]$. Denote $d^\lambda = b^\lambda(4) - b^\lambda(1)$ .The probability that the opponent is level$_0$ conditional on a tie is $q^\lambda = \Pr(rival = level_0 | tie\ at\ bid = b) = \frac{\lambda/6}{\lambda/6+(1-\lambda)/d^\lambda}, b \in [b^\lambda(1), b^\lambda(4)] \subseteq [2,8]$.

Then by indifference in the case of Maximum Willingness to Pay conditional on a tie, denoted by $MWP(X) = b(x)$:

MWP (1)= $q^\lambda(1+2.5) + \left(1-q^\lambda\right)2 = 1.5q^\lambda + 2 = b^\lambda(1)$,

MWP (4)= $q^\lambda(4+2.5) + \left(1-q^\lambda\right)8 = 8 - 1.5q^\lambda = b^\lambda(4)$.

Then $d^\lambda = b^\lambda(4) - b^\lambda(1) = 1 - 3q^\lambda = \frac{\lambda/6}{\lambda/6+(1-\lambda)/d^\lambda}$.

Then $\left(d^\lambda\right)^2 + 3\frac{2-3\lambda}{\lambda}d^\lambda - \frac{36(1-\lambda)}{\lambda} = 0$.

So, the bidding strategy is $b^\lambda(x) = b^\lambda(1) + \frac{d^\lambda}{3}(x-1)$, where $b^\lambda(1) = 1.5q^\lambda + 2$,

$d^\lambda = \frac{3}{2\lambda}(3\lambda + \sqrt{-7\lambda^2 + 4\lambda + 4} - 2)$ and $q^\lambda = (1 - d^\lambda)/3$.

Appendix B. Mathematical Appendix for Chapter 2

B.1. Proof of Theorem 1 (POP)

The crucial part of the proof is that (i) implies (ii). First since AXIOM 1, 4, 5 implies von Neumann-Morgenstern's three axioms on lotteries, it follows directly from their theory (and the fact that $\mathcal{F}^c(\Omega)$ and $\mathcal{Z}$ are isomorphic) that there exists an affine function $u: \mathcal{Z} \to \mathbb{R}$, such that for all $p, q \in \mathcal{F}^c(\Omega)$: $p \succsim q$ iff $u(p) \geq u(q)$. Moreover, u is cardinally unique. By AXIOM2, u is not a constant function.

For the finest partition, Theorem 1 is equivalent with *Subjective Expected Utility Theorem*. We first prove it for the coarsest partition, $\Sigma = \{\Omega\}$. When the partition is the coarsest, the utility presentation reduces to

(B.1) $$V(f) = \alpha(f) \max_{p \in \mathcal{O}(f)} u(p) + [1 - \alpha(f)] \min_{q \in \mathcal{O}(f)} u(q).$$

For any $f \in \mathcal{F} \backslash \mathcal{F}^c(\Omega)$, pick measurable acts $f^{best}, f^{worst} \in \mathcal{F}^c(\Omega)$ that always generate the most and least preferred outcomes given f is chosen. Formally, $f^{best} \in \{p | p \succsim q, \forall q \in \mathcal{O}(f)\}$ and $f^{worst} \in \{h | h \precsim q, \forall q \in \mathcal{O}(f)\}$. For $f \in \mathcal{F}^e(\Omega) \backslash \mathcal{F}^c(\Omega)$, by the definition of $\mathcal{F}^e(\Omega)$, $f^{best} \sim f^{worst}$, which implies $u(f^{best}) = u(f^{worst})$, and by AXIOM 3, $f \sim f^{best} \sim f^{worst}$. So $V(f) = u(f^{best}) = u(f^{worst})$ satisfying (2.1) for any $\alpha(f) \in [0, 1]$. Hence $V(f)$ also calibrates the preference on $\mathcal{F}^e(\Omega)$.

Finally, for $f \in \mathcal{F} \backslash \mathcal{F}^e(\Omega)$, by the definition of $\mathcal{F}^e(\Omega)$, $f^{worst} \prec f^{best}$. And by AXIOM 3, $f^{worst} \precsim f \precsim f^{best}$.

Lemma B.1. for $f \in \mathcal{F} \backslash \mathcal{F}^e(\Omega)$, AXIOM 1-5 imply that there exists a unique $\beta^* \in [0, 1]$ such that $f \sim \beta^* f^{best} + (1 - \beta^*)\, f^{worst}$.

Proof. First since $u[\beta f^{best} + (1-\beta)\, f^{worst}] = \beta u(f^{best}) + (1-\beta) u(f^{worst})$, so for $0 \le a < b \le 1$, $b f^{best} + (1-b)\, f^{worst} \succ\ a f^{best} + (1-a)\, f^{worst}$. Then it ensures that if β^* exists, it is unique.

If $f \sim f^{best}$, then $\beta^* = 1$ works. The same way around, if $f \sim f^{worst}$, then $\beta^* = 0$ works. Otherwise, $f^{worst} \prec f \prec f^{best}$. Define

$$\beta^* = \sup\{\beta \in [0, 1]: f \succsim \beta f^{best} + (1-\beta) f^{worst}\}.$$

Since $\beta = 0$ is in the set, we aren't taking a sup over an empty set. By the definition of β^*, if $1 \ge\ \beta > \beta^*$, then $f \prec\ \beta f^{best} + (1-\beta) f^{worst}$. Moreover, by the same argument to prove uniqueness above, if $0 \le\ \beta < \beta^*$, then $f \succ \beta f^{best} + (1-\beta) f^{worst}$. To see this, note that if $0 \le\ \beta < \beta^*$, then there exists β' such that $0 \le\ \beta <\ \beta' < \beta^*$ and $f \succsim \beta' f^{best} + (1-\beta') f^{worst}$ by the definition of β^*. And $\beta <\ \beta'$ implies that $f \succ \beta f^{best} + (1-\beta) f^{worst}$.

There are three possibilities to consider.

(1) Suppose $\beta^* f^{best} + (1-\beta^*)\, f^{worst} \succ\ f \succ f^{worst}$, then by AXIOM 4, there exists $b \in (0, 1)$ such that $b[\beta^* f^{best} + (1-\beta^*) f^{worst}] + (1-b) f^{worst} = b\beta^* f^{best} + (1 - b\beta^*) f^{worst} \succ\ f$. But $b\beta^* < \beta^*$, so by the previous argument $f \succ\ b\beta^* f^{best} + (1 - b\beta^*) f^{worst} \succ\ f$. Contradiction.

(2) Suppose instead that $f^{best} \succ f \succ \beta^* f^{best} + (1-\beta^*)\, f^{worst}$. Then by AXIOM 4, There exists $a \in (0,1)$ such that $f \succ a[\beta^* f^{best} + (1-\beta^*) f^{worst}] + (1-a) f^{best} = \left(1 - a(1-\beta^*)\right) f^{best} + a(1-\beta^*) f^{worst}$. Since $\left(1 - a(1-\beta^*)\right) > \beta^*$, we have from above that $\left(1 - a(1-\beta^*)\right) f^{best} + a(1-\beta^*) f^{worst} \succ f$. Contradiction.

(3) This leaves us with the third possibility (which is what we are supposed to proof), namely $f \sim \beta^* f^{best} + (1-\beta^*)\, f^{worst}$.

Proof of Lemma B.1. ends.

Follows the argument of Lemma B.1, then $V(f) = V[\beta^* f^{best} + (1-\beta^*) f^{worst}]$. Since $[\beta^* f^{best} + (1-\beta^*) f^{worst}] \in \mathcal{F}^c(\Omega)$.

$$\mathrm{V}[\beta^* f^{best} + (1-\beta^*) f^{worst}] = u[\beta^* f^{best} + (1-\beta^*) f^{worst}].$$

Moreover, since u is affine,

$$u[\beta^* f^{best} + (1-\beta^*) f^{worst}] = \beta^* u(f^{best}) + (1-\beta^*) u(f^{worst}).$$

Then by the definition of f^{best}, f^{worst}, and $u(\cdot)$,

$$\min_{q \in \mathcal{O}^{\mathcal{B}}(f)} u(q) = u(f^{worst}) < u(f^{best}) = \max_{p \in \mathcal{O}^{\mathcal{B}}(f)} u(p).$$

So

$$u[\beta^* f^{best} + (1-\beta^*) f^{worst}] = \beta^* \max_{p \in \mathcal{O}^{\mathcal{B}}(f)} u(p) + (1-\beta^*) \min_{q \in \mathcal{O}^{\mathcal{B}}(f)} u(q).$$

So

$a(f) = \beta^*$ works and is uniquely determined.

Now for a partition that is neither the finest nor the coarsest, $\Sigma = (\mathcal{B}_k)_{k=1}^{n}$. First, for partition indifferent acts, $f \in \mathcal{F}^e(\Sigma)$, $u(f(s))$ is constant, given $s \in \mathcal{B}$ for any $\mathcal{B} \in \Sigma$.

Define $V(f|\mathcal{B}) = u\big(f(s)\big), s \in \mathcal{B}$, which satisfies Equation 2.2. Then it follows from the proof of *Subjective Expected Utility Theorem* (Fishburn 1970) that there exists a unique probability function $P: \Sigma \rightarrow [0,1]$, such that the utility functional for f is

$$V(f) = \sum_{k=1}^{n} V(f|\mathcal{B}_k)P(\mathcal{B}_k).$$

For any $f \in \mathcal{F} \backslash \mathcal{F}^e(\Sigma)$, define *partition measurable acts* f^{best}, $f^{worst} \in \mathcal{F}^c(\Sigma)$ that generate the best and the least preferred outcomes in every event B of partition Σ when f is chosen. Formally, $f^{best}(\mathcal{B}) \in \{p|p \gtrsim q, \forall q \in \mathcal{O}^B(f)\}$ and $f^{worst}(\mathcal{B}) \in \{h|h \lesssim q, \forall q \in \mathcal{O}^B(f)\}, \forall \mathcal{B} \in \Sigma$. Then by AXIOM 3, $f^{worst} \lesssim f \lesssim f^{best}$.

Lemma B.2. For $\mathcal{F} \backslash \mathcal{F}^e(\Sigma)$, AXIOM 1-5 imply that there exists a unique $\beta^* \in [0, 1]$ such that $f \sim \beta^* f^{best} + (1-\beta^*)\, f^{worst}$. (Note that $\beta^* f^{best} + (1-\beta^*)\, f^{worst} \in \mathcal{F}^c(\Sigma)$.

Proof. The same argument follows as in the proof of Lemma B.1.

Following the argument of Lemma B.2., then $V(f) = V[\beta^* f^{best} + (1-\beta^*)\, f^{worst}] = \sum_{k=1}^{n} V(\beta^* f^{best} + (1-\beta^*)\, f^{worst}|\mathcal{B}_k)P(\mathcal{B}_k)$.

Denote by $V(f|\mathcal{B}) = V((\beta^* f^{best} + (1-\beta^*)\, f^{worst}|\mathcal{B})$. Then $V(f|\mathcal{B}) = \beta^* u(f^{best}) + (1-\beta^*)u(f^{worst}) = \beta^* \max_{p \in \mathcal{O}^{\mathcal{B}}(f)} u(p) + (1-\beta^*) \min_{q \in \mathcal{O}^{\mathcal{B}}(f)} u(q)$ satisfying Equation 2.2.

So $a(f) = \beta^*$ works and is uniquely determined when $f \notin \mathcal{F}^e(\Sigma)$.

B.2. Proof of Lemma 2.

When the partition is the finest, $\Sigma = \{\Omega_i\}$, it is straightforward that Definition 2.9 implies Definition 2.8.

Now we show that Definition 2.8 implies definition 2.9. Assume by contradiction that Equation 2.3 is satisfied but Equation 2.4 is not. Then there exists a state $({s_{-i}}', {\omega_n}') \in \Omega_i$

such that $u_i(s_i^*, s_{-i}', \omega_{n'}) < u_i(s_i', s_{-i}', \omega_n')$. Then the terminal history $(s_i^*, s_{-i}', \omega_n') \neq (s_i', s_{-i}', \omega_n')$. So there exists a sub-history $h \in I \in \mathcal{I}(s_i^*)$ such that h is a sub-history of both $(s_i^*, s_{-i}', \omega_n')$ and $(s_i', s_{-i}', \omega_n')$, $(s_{-i}', \omega_n') \in [I]$ and that $s_i' \in \mathcal{S}_i(I)[s_i^*(I)]^c$. Then for event $\mathcal{B}(I)$ such that $(s_{-i}', \omega_n') \in \mathcal{B}(I)$,

$$\inf_{(s_{-i},\omega_n)\in\mathcal{B}(I)} u_i(s_i^*, s_{-i}, \omega_n) \leq u(s_i^*, s_{-i}', \omega_n') < u(s_i', s_{-i}', \omega_n') \sup_{(s_{-i},\omega_n)\in\mathcal{B}(I)} u_i(s_i', s_{-i}, \omega_n).$$

Contradiction.

B.3. Proof of Proposition 2.1.

($\Rightarrow$) If s_i^* is Σ-ODS, then by Equation (2.1), (2.2) and (2.3), (2.5) is satisfied.

($\Leftarrow$)If (2.5) holds, assume by contradiction that s_i^* is not Σ-ODS. Then there exists an information set $I \in \mathcal{I}(s_i^*)$, a deviating strategy $s_i' \in \mathcal{S}_i(I)[s_i^*(I)]^c$, and an event $\mathcal{B} \in \Sigma(I)$ such that

$$\inf_{(s_{-i},\omega_n)\in\mathcal{B}(I)} u_i(s_i^*, s_{-i}, \omega_n) < \sup_{(s_{-i},\omega_n)\in\mathcal{B}(I)} u_i(s_i', s_{-i}, \omega_n).$$

Consider Partition Obvious Preference at $[I]$, where $\alpha(s_i^*) = 0$ and $\alpha(s_i') = 1$ with $P(B)$ close enough to 1 such that, by the continuity of real number, $V(s_i^*) < V(s_i')$. Then $s_i^* \prec_{[I]} s_i')$. Contradiction.

B.4. Ausubel Auction vs Static Vickrey Auction.

We illustrate the argument by a simple example, consider the case when there are only two bidders and two units on sale. We denote bidder i's marginal value for unit j by $v_i^j \in$

$[\$0, \$100], i, j = 1, 2.$[99] In *Ausubel* auction, bidders can choose any cut-off strategy for their quantity demanded at any price of the following form.

$$s_i(v_i^1, v_i^2) = \begin{cases} 2, 0 \le p < p_i^1 \\ 1, p_i^1 \le p < p_i^2 \\ 0, p_i^2 < p \le 100 \end{cases}$$

Where, $0 \le p_i^1 \le p_i^2 \le 100, i = 1,2.$[100] Player i clinches a unit when the other bidder reduces his demand by one unit, given Player i being active in bidding that unit. Then bidders pay each clinched unit at the *clinching price*−the clock price at which the item is clinched. Formally, Player i wins and pays one unit at p_{-i}^1 if $p_i^2 > p_{-i}^1$ and another at p_{-i}^2 if $p_i^1 > p_{-i}^2$. We define sincere bidding in *Ausubel* auction as follows

$$s_i(v_i^1, v_i^2) = \begin{cases} 2, 0 \le p < v_i^2 \\ 1, v_i^2 \le p < v_i^1 \\ 0, v_i^1 < p \le 100 \end{cases}.$$

Its corresponding strategy in the static *Vickrey* auction is simply reporting the values truthfully, $s_i(v_i^1, v_i^2) = (v_i^1, v_i^2)$. Sincere bidding is DS but not ODS in both multi-unit dynamic and static *Vickrey* Auctions.

First, we claim that if sincere bidding is a $\Sigma - \text{ODS}$ in static Vickrey auction, then it is also $\Sigma - \text{ODS}$ in *Ausubel* auction. The intuition follows directly from the definition of $\Sigma - \text{ODS}$: since any alternative strategy is given up “later” in the dynamic mechanism when more information is updated, Equation (2.3) is a weaker condition at any information set in static *Ausubel* than at the only information set in static *Vickrey*. Now, we consider a partition Σ^*

[99] We assume without loss of generality that $v_i^1 \geq v_i^2$.

[100] We assume that the prices and values are discrete.

based on the first clinching price p_{-i}^1 in *Ausubel* (or equivalently, the lower value reported by the other player) and show sincere bidding is $\Sigma^* - ODS$ in Ausubel, however, it is not $\Sigma^* - ODS$ in the static *Vickrey* auction.

$$\mathcal{B}_p = \{(s_{-i}, \omega_n) \in \Omega_i | p_{-i}^1 = p\},$$

$$\Sigma^* = \left(\mathcal{B}_p\right)_{p \in [0,100]}.$$

In the static *Vickrey* auction, consider the event $\mathcal{B}_p$ where $p < v_i^2 < v_i^1$. The worst case of sincere bidding for bidder i is to win one unit at price p. However, the best case of overbidding is to win both units at p. Thus, the best case of overbidding is strictly better than the worst case of bidding my value$-$ Equation (2.3) does not hold. Hence, sincere bidding is not $\Sigma^* - ODS$, in the static *Vickrey* auction. Now in *Ausubel* auction, consider, for example, the information set I_i when the clock price reaches bidder i's value for the second unit at v_i^2, and none item has been clinched. For event $\mathcal{B}_p$ where $v_i^2 < p \leq v_i^1$, the worst case of sincere bidding is to win one item at p and to win none item for event $\mathcal{B}_p$, $p > v_i^1$, while the best case of any deviating strategy (to drop out later, $p_i^1 > v_i^2$) can't generate higher payoff. Thus Equation 2.3 is satisfied. A similar argument follows for any other information set in the *Ausubel* auction.[101] Thus, sincere bidding $\Sigma^* - ODS$ in the *Ausubel* auction.

[101] At any clock price when none item has been clinched, we only need to evaluate the events $\mathcal{B}_p \in \Sigma^*$ where p is higher than the current price. For the event $\mathcal{B}_p \in \Sigma^*$ where p is lower than the current price, $\mathcal{B}_p \cap [I] = \emptyset$.

B.5. Pay for non-instrumental information

$\alpha_R - \alpha_L$	State A	State B	State C	A, B or C each with 1/3
Strictly Prefer R	$\left(\frac{5}{12},1\right)$	$\left(\frac{5}{16},1\right)$	$\left(\frac{5}{8},1\right)$	$\left(\frac{5}{12},1\right)$
Indifferent	$\frac{5}{12}$	$\frac{5}{16}$	$\frac{5}{8}$	$\frac{5}{12}$
Strictly Prefer L	$\left(0,\frac{5}{12}\right)$	$\left(0,\frac{5}{16}\right)$	$\left(0,\frac{5}{8}\right)$	$\left(0,\frac{5}{12}\right)$

Table B.1. Decision makers' preference

Table B.1. represents the ranges of $(\alpha_R - \alpha_L)$ where the POP strictly prefer R, or L, or the POP is indifferent between them in four scenarios, upon knowing A, B, or C, or without any information about states of nature. When $(\alpha_R - \alpha_L) \in \left(\frac{5}{8}, 1\right)$, the POP strictly prefers R in all states of nature. When $(\alpha_R - \alpha_L) \in \left(0, \frac{5}{16}\right)$, the POP strictly prefers L in all states of nature. So, the information regarding states of nature is non-instrumental for $(\alpha_R - \alpha_L) \in \left(0, \frac{5}{16}\right) \cup \left(\frac{5}{8}, 1\right)$. However, when $(\alpha_R - \alpha_L) \in \left(\frac{5}{16}, \frac{5}{12}\right)$, the POP prefers L without the information but prefers R in State B; when $(\alpha_R - \alpha_L) \in \left(\frac{5}{12}, \frac{5}{8}\right)$, the POP prefers R without the information but prefers L in State C. Thus, the information regarding states of nature is instrumental when $(\alpha_R - \alpha_L) \in \left(\frac{5}{16}, \frac{5}{8}\right)$.

Appendix C. Instructions for Experiment I

C.1. Welcome

This is a study in decision-making. Money earned will be paid to you in cash at the end of the experiment. This study is about 60 minutes long. What you earn depends partly on your decisions, partly on the decision of others, and partly on chance.

We will pay you $5 for showing up. Additionally, you will be paid in cash your earnings from the experiment.

This experiment involves 4 tasks for real money. You will play each task 10 times. We will give you instructions about each task just before you begin to play it. Your choices in one task will not affect what happens in other tasks.

There is no deception in this experiment. Every game will be exactly as specified in the instructions.

Please turn off pagers, mp3 devices, and cellular phones, and close any program you may have open on the computer.

The entire session will take place through computer terminals and all interaction with other participants will take place through the computers. Please do not talk or in any way try to communicate with other participants during the session.

All payoffs (earnings) are in Experimental Currency Unit (ECU) and will be paid with exchange rate: 2 ECU = 1 U.S. dollar.

If you have any questions at any point, please raise your hand and we will answer your questions privately.

C.2. Instruction for Game S

You have been randomly assigned into a group of 4. You will play this game for 10 rounds. In each round of this game, there are four prizes labeled A, B, C, and D. Prizes will be worth between 1.00 - 9.00, and the value of each prize is the same for all players in your group.

There are two cases: L and R. At the start of each round, your group will be assigned to either Case L or Case R, each with a probability of 1/2. But your group will not know which Case you are in. At the start of each round, you will learn the value of each prize in cases L and R. You will also learn your priority score, which is a random number. Every integer between 1 and 10 is equally likely to be chosen.

The game proceeds as follows: We will ask you to list the prizes, in any order of your choice. All players will submit their lists privately and at the same time.

After all the lists have been submitted, we will assign prizes using the following rule:

1. The player with the highest priority score will be assigned the prize on the top of his/her list.

2. The player with the second-highest priority score will be assigned the prize on the top of his/her list, among the prizes that remain.

3. The player with the third-highest priority score will be assigned the prize on the top of his/her list, among the prizes that remain.

4. The player with the lowest priority score will be assigned whatever prize remains.

If two players have the same priority score, we will break the tie randomly.

You will have 90 seconds to form your list. You do this by typing a number, from 1 to 4, next to each prize, and then clicking the button says “Confirm Choices”. Each prize must be assigned a different number, from 1 (top) to 4 (bottom). Your choices will not count unless you click the button that says “Confirm Choices”.

At the end of Round 10, we will randomly select a round and add to your earnings the value of the prize you were assigned in that around.

C.3. Instruction for Game D

You have been randomly assigned into groups of 4. You will play this game for 10 rounds with the other people in the group. In each round of this game, there are four prizes labeled A, B, C, and D. Prizes will be worth between 1.00 – 9.00. For each prize, its value will be the same for all the players in your group.

There are two cases: L and R. At the start of each round, your group will be assigned to either Case L or Case R, each with a probability of 1/2. But your group will not know which Case you are in. At the start of each round, you will learn the value of each prize in cases L and R. You will also learn your priority score, which is a random number. Every integer between 1 and 10 is equally likely to be chosen.

The game proceeds as follows:

1. The player with the highest priority score will pick one prize.

2. The player with the second-highest priority score will pick one of the prizes that remain.

3. The player with the third-highest priority score will pick one of the prizes that remain.

4. The player with the fourth-highest priority score will be assigned whatever prize remains.

If two players have the same priority score, we will break the tie randomly.

When it is your turn to pick, you will have 30 seconds to make your choice. You do this by selecting a prize and then clicking the button that says “Confirm Choice”. Your choice will not count unless you click the button that says “Confirm Choice”.

At the end of Round 10, we will randomly select a round and add to your earnings the value of the prize you were assigned in that around.

C.4. Instruction for Decision S

You will play this individual decision task for 10 rounds. In each round of the task, there are six prizes worth 0, 2, 4, 6, 8, 10. They are randomly assigned, by a one-to-one matching, to be put inside box A, B, C, D, E, F, which is not seen from the outside. Another six prizes worth 0, 2, 4, 6, 8, 10 are randomly assigned, by a one-to-one matching, to six stickers A, B, C, D, E, F, which is shown on the sticker.

There are two cases: L and R. At the start of each round, you will be assigned to either Case L or Case R, each with a probability of 1/2. But you don’t know which case you are in. Your payoff depends on which case you are in and your choice.

The task proceeds as follows: we will first ask you to choose one of the stickers, and then pick one box with an unknown prize. You will see the prize value in the box once you pick it.

If you are assigned Case L, you will be awarded the lower prize among those in the box and on the sticker that you chose. If you are assigned Case R, you will be assigned the higher prize among those in the box and on the sticker that you chose.

You will have 30 seconds to make each choice. You do this by selecting a box or a sticker and then clicking the button that says “Confirm Choice”. Your choice will not count unless you click the button that says “Confirm Choice”.

If you do not make a choice by the end of 30 seconds at one choice, you will be assigned 0 in that round.

At the end of Round 10, we will randomly select a round and add to your earnings the value of the prize you were assigned in that around.

C.5. Instruction for Decision D

You will play this individual decision task for 10 rounds. In each round of the task, there are six prizes worth 0, 2, 4, 6, 8, 10. They are randomly assigned, by a one-to-one matching, to be put inside box A, B, C, D, E, F, which is not seen from the outside. Another six prizes worth 0, 2, 4, 6, 8, 10 are randomly assigned, by a one-to-one matching, to six stickers A, B, C, D, E, F, which is shown on the sticker.

There are two cases: L and R. At the start of each round, you will be assigned to either Case L or R, each with a probability of 1/2. But you don't know which case you are in. Your payoff depends on which case you are in and your choice.

The task proceeds as follows: we will first ask you to pick one box, and you see the prize value inside the box. Then we will ask you to choose a sticker.

If you are assigned Case L, you will be assigned the lower prize among those in the box and on the sticker that you chose. If you are assigned Case R, you will be assigned the higher prize among those in the box and on the sticker that you chose.

You will have 30 seconds to make each choice. You do this by selecting a box or a sticker and then clicking the button that says "Confirm Choice". Your choice will not count unless you click the button that says "Confirm Choice".

If you do not make a choice by the end of 30 seconds at one choice, you will be assigned 0 in that round.

At the end of Round 10, we will randomly select a round and add to your earnings the value of the prize you were assigned in that around.

Appendix D. Additional Empirical Analysis for Experiment I

D.1. Proportions of incorrect choices in the game task

For each group in Game S, for each round, we simulate the three choices that we would have observed under Game D. For each group, for each 5-round block, we record the proportions of choices that are incorrect. Table D.1, D.2, and D.3 display the empirical frequency of non-DS choices, by Games and by 5-round blocks in the pooled data, the within-subject, and the cross-subject comparison. Subjects deviate from the DS at lower rates in Game S than in Game D, and these differences are highly significant in both early and late rounds of the pooled data, the within-subject, and the cross-subject comparison.

	Decision S	Decision D	p-value
Rounds 1-5	12.95%	0.93%	<0.001
Rounds 6-10	13.83%	0.86%	<0.001
p-value	0.568	0.320	

Table Notes: When comparing Game S to Game D, we compute p-values using a Wilcoxon rank-sum test. When comparing early to late rounds of the same game, we compute p-values using a Wilcoxon matched-pairs signed-rank test.

Table D.1. Proportions of incorrect choices in the game task (pooled Data): Game S (simulated) vs Game D (actual)

	Treatment 1		
	Game S	Game D	p-value
Rounds 1-5	11.96%	0.48%	<0.001
Rounds 6-10	14.29%	0.63%	<0.001
p-value	0.720	0.989	

	Treatment 2		
	Game D	Game S	p-value
Rounds 1-5	1.34%	13.87%	<0.001
Rounds 6-10	1.07%	13.39%	<0.001
p-value	0.575	0.331	

Table Notes: We compute p-value using a Wilcoxon matched-pairs signed-rank test.

Table D.2. Proportions of incorrect choices in the game task (within-subject comparison)

	Game S (Treatment 1)	Game D (Treatment 2)	p-value
Rounds 1-5	11.96%	1.34%	<0.001
Rounds 6-10	14.29%	1.07%	<0.001
p-value	0.720	0.575	

Table Notes: We compute p-value using a Wilcoxon rank-sum test. When comparing early to late rounds of the same game, we compute p-values using a Wilcoxon matched-pairs signed-rank test.

Table D.3. Proportions of incorrect choice in the game task (cross-subject comparison)

D.2. Logit Regression: Decision S & Decision D

	Pooled Data	Within-Subject		Cross-Subject
		Treatment 1	Treatment 2	(The 1st Game only)
Decision S	4.428***	2.956***	6.093***	3.371***
	(0.794)	(1.124)	(1.167)	(1.149)
GPA* Decision S	-0.440**	-0.059	-0.855***	-0.589
	(0.215)	(0.313)	(0.309)	(0.313)
Female * Decision S	0.803***	0.500**	1.038***	0.500***
	(0.151)	(0.226)	(0.204)	(1.149)

Table Notes: We report Coefficient (Std. Err.).

*** Significant at the 1 percent level.

** Significant at the 5 percent level.

* Significant at the 10 percent level.

Table D.4. Logit regression on choices of dominated strategies (Decision S vs D)

Table D.4 documents higher possibility of choice of *dominated strategies* in Decision S than in Decision D by a Logit regression. It also demonstrates a positive correlation between female and the error in Decision S. Both results are significant in the pooled data, the within-subject, and the cross-subject comparison. The logit regression also shows a negative correlation between the self-reported GPA and the error in Decision S. But it is not significant for all comparisons.

D.3. Logit Regression: Deviations from DS in Game S

Table D.5. shows a negative correlation between the priority score and the error in Game S. The result is significant in the pooled data, and also significant in either Treatment 1 or 2 data. It also demonstrates a positive correlation between female and the error in Game S. The correlation is significant in the pooled data and the Treatment 2 data but not in the Treatment 1 data.

	Pooled Data	Treatment 1	Treatment 2
Priority Score	-0.598***	-0.703***	-0.525***
	(0.038)	(0.064)	(0.048)
GPA	-0.060	0.643	-0.668**
	(0.238)	(0.352)	(0.241)
Female	0.524***	0.100	0.839***
	(0.166)	(0.253)	(0.224)

Table D.5. Logit regression on deviations from the dominant strategy: Game S

Appendix E. Questionnaires for Experiment II

E.1. Part I

Below are several problems that vary in difficulty. Try to answer as many as possible:

1. A bat and a ball cost $1.10 in total. The bat costs a dollar more than the ball. How much does the ball cost? ___________cents

2. If it takes 5 machines 5 min to make 5 widgets, how long would it take 100 machine to make 100 widgets?_________min

3. In a lake, there is a patch of lily pads. Every day, the patch doubles in size. If it takes 48 days for the patch to cover the entire lake, how long would it take for the patch to cover half of the lake?______days

4. Jack is looking at Ann but Ann is looking at George. Jack is married but George is not. Is a married person looking at an unmarried person?

A) Yes B) No C) Cannot be determined

5. There are 5 blocks in a stack, where the second one from the top is green, and the fourth is not green. Is there a green block directly on top of a non-green block?

A) Yes B) No C) Cannot be determined.

E.1. Part II

Please indicate the extent to which you agree with each statement:

1. I would prefer complex to simple problems.

Strongly Disagree   Disagree   Neutral   Agree   Strongly Agree

2. I like to have the responsibility of handling a situation that requires a lot of thinking.

Strongly Disagree   Disagree   Neutral   Agree   Strongly Agree

3. Thinking is not my idea of fun.

Strongly Disagree   Disagree   Neutral   Agree   Strongly Agree

4. I would rather do something that requires little thought than something that is sure to challenge my thinking abilities.

Strongly Disagree   Disagree   Neutral   Agree   Strongly Agree

5. I try to anticipate and avoid situations where there is likely a chance I will have to think in depth about something.

Strongly Disagree   Disagree   Neutral   Agree   Strongly Agree

6. I find satisfaction in deliberating hard and for long hours.

Strongly Disagree   Disagree   Neutral   Agree   Strongly Agree

7. I only think as hard as I have to.

Strongly Disagree   Disagree   Neutral   Agree   Strongly Agree

8. I prefer to think about small, daily projects to long-term ones.

Strongly Disagree   Disagree   Neutral   Agree   Strongly Agree

9. I like tasks that require little thought once I've learned them

Strongly Disagree   Disagree   Neutral   Agree   Strongly Agree

10. The idea of relying on thought to make my way to the top appeals to me.

Strongly Disagree   Disagree   Neutral   Agree   Strongly Agree

11. I really enjoy a task that involves coming up with new solutions to problems.

Strongly Disagree   Disagree   Neutral   Agree   Strongly Agree

12. Learning new ways to think doesn't excite me very much.

Strongly Disagree   Disagree   Neutral   Agree   Strongly Agree

13. I prefer my life to be filled with puzzles that I must solve.

Strongly Disagree   Disagree   Neutral   Agree   Strongly Agree

14. The notion of thinking abstractly is appealing to me.

Strongly Disagree   Disagree   Neutral   Agree   Strongly Agree

15. I would prefer a task that is intellectual, difficult, and important to one that is somewhat important but does not require much thought.

Strongly Disagree *Disagree* *Neutral* *Agree* *Strongly Agree*

16. I feel relief rather than satisfaction after completing a task that required a lot of mental effort.

Strongly Disagree *Disagree* *Neutral* *Agree* *Strongly Agree*

17. It's enough for me that something gets the job done; I don't care how or why it works.

Strongly Disagree *Disagree* *Neutral* *Agree* *Strongly Agree*

18. I usually end up deliberating about issues even when they do not affect me personally.

Strongly Disagree *Disagree* *Neutral* *Agree* *Strongly Agree*

Appendix F. Where to go for Additional Help

F.1. Welcome

You are about to participate in a session on decision-making, and you will be paid for your participation with cash, at the end of each session. What you earn depends partly on your decisions, partly on the decision of others, and partly on chance.

Please turn off pagers, mp3 device and cellular phones now. Please close any program you may have open on the computer.

Then entire session will take place through computer terminals and all interaction with other participants will take place through the computers. Please do not talk or in any way try to communicate with other participants during the session.

We will start with a brief instruction period. During the instruction period you will be given a description of the main features of the session and will be shown how to use the computers. If you have any questions during the period, please wait until we finish reading the instructions. Then you can raise your question will be answered so everyone can hear.

All payoffs are in Experimental Currency Unit (ECU) and will be paid with exchange rate: 2 ECU=1 U.S. dollar.

F.2. Game DS

1. In this experiment you will be asked to make a decision. You will be randomly paired with another person.

2. There are three possible states: A, B, C. The computer randomly draws one state. Which is unknown to you and the person you paired with. The payoffs are as follows:

A

	The other's	
Your choice	L	R
L	20,20	8,25
R	25,8	13,13

B

	The other's	
Your choice	L	R
L	22,22	6,27
R	27,6	11,11

C

	The other's	
Your choice	L	R
L	18,18	10,23
R	23,10	15,15

Table F.1. Game DS

The first entry in each cell represents your payoff, while the second entry represents the payoff of the person you matched with.

(i). If the state is A, the choices and payoffs indicated by the above tables are as follows:

You select L and the other selects L, you each make 20 ECU

You select L and the other selects R, you make 8 ECU and the other makes 25 ECU.

You select R and the other selects L, you make 25 ECU and the other makes 8 ECU.

You select R and the other selects R, you each make 13 ECU.

(ii) If the state is B, the choices and payoffs are as follows

You select L and the other selects L, you each make 22 ECU.

You select L and the other selects R, you make 6 ECU and the other makes 27 ECU.

You select R and the other selects L, you make 25 ECU and the other makes 6 ECU.

You select R and the other selects R, you each make 11 ECU.

(iii) If the state is C, the choices and payoffs are as follows

You select L and the other selects L, you each make 18 ECU.

You select L and the other selects R, you make 10 ECU and the other makes 23 ECU.

You select R and the other selects L, you make 23 ECU and the other makes 10 ECU.

You select R and the other selects R, you each make 15 ECU.

-Once the state is randomly chosen and you and the person you are paired with have made your choices, your payoff is determined as above.

-After the state is randomly selected, we will ask both of you whether you are willing to pay some amount of ECU to know the selected state (A, B, or C). You must provide your answer for all listed prices as in the following table. After people make their choices, we will randomly draw a number form the listed prices to determine which price to be selected and implemented.

Price	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
Yes/No										

Table F.2. Price list

If you indicated that you are willing to pay for the selected price, you will be show the selected states and then asked to make your choice, L or R.

If you indicated that you are not willing to pay for the selected price, you will be asked to make your choice immediately without any more information.

-Are there any questions? If at any point during the experiment you need help, just raise your hand.

F.3. Decision DS

There is a jar with 10 balls, some of which are Red and some of which are Blue. One of these balls will be randomly drawn and will be called the "selected ball".

Your payoff will be determined by the color of the selected ball, your action, and the action of two computers. You can vote either Red or Blue. Computer 1 and 2 are programmed to behave in a specific manner:

(1) If the selected ball is red: vote Red.

(2) If the selected ball is blue: vote Blue with probability p and Red with 1-p, $0<p<1$. The exact value of p is unknown.

If the selected ball is Red and the majority voted for Red, your payoff is 2. If the selected ball is Blue and the majority votes for Blue, your payoff is 2. In all other cases, your payoff is 0. In other words, you get 2 if the vote of the majority coincides with the color of the selected ball and 0 otherwise. Note that there is a total of 3 votes (two by the computers and one by yourself), so that saying that a majority voted for a specific color means that there are 2 or 3 votes for that specific color.

The following table summarizes the payoffs:

		Color of the selected ball	
		Red ball	Blue ball
Decision of the Group	Majority votes for Red	2	0
	Majority votes for Blue	0	2

Table F.3. Payoff table

-Are there any questions? If at any point during the experiment you need help, just raise your hand.

F.4. Game ODS

1. In this experiment you will be asked to make a decision. You will be randomly paired with another person.

2. There are three possible states: A, B, C. The computer randomly draws one state. Which is unknown to you and the person you paired with. The payoffs are as follows:

A

The other's

Your choice	L	R
L	15,15	13,20
R	20,13	18,18

B

The other's

Your choice	L	R
L	16,16	12,21
R	21,12	17,17

C

The other's

Your choice	L	R
L	13,13	15,18
R	18,15	20,20

Table F.4. Game ODS

The first entry in each cell represents your payoff, while the second entry represents the payoff of the person you matched with.

(i). If the state is A, the choices and payoffs indicated by the above tables are as follows:

You select L and the other selects L, you each make 15 ECU

You select L and the other selects R, you make 13 ECU and the other makes 20 ECU.

You select R and the other selects L, you make 20 ECU and the other makes 13 ECU.

You select R and the other selects R, you each make 18 ECU.

(ii) If the state is B, the choices and payoffs are as follows

You select L and the other selects L, you each make 16 ECU.

You select L and the other selects R, you make 12 ECU and the other makes 21 ECU.

You select R and the other selects L, you make 21 ECU and the other makes 12 ECU.

You select R and the other selects R, you each make 17 ECU.

(iii) If the state is C, the choices and payoffs are as follows

You select L and the other selects L, you each make 13 ECU.

You select L and the other selects R, you make 15 ECU and the other makes 18 ECU.

You select R and the other selects L, you make 18 ECU and the other makes 15 ECU.

You select R and the other selects R, you each make 20 ECU.

-Once the state is randomly chosen and you and the person you are paired with have made your choices, your payoff is determined as above.

-After the state is randomly selected, we will ask both of you whether you are willing to pay some amount of ECU to know the selected state (A, B, or C). You must provide your answer for all listed prices as in the following table. After people make their choices, we will randomly draw a number form the listed prices to determine which price to be selected and implemented.

Price	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
Yes/No										

Table F.5. Price list

If you indicated that you are willing to pay for the selected price, you will be show the selected states and then asked to make your choice, L or R.

If you indicated that you are not willing to pay for the selected price, you will be asked to make your choice immediately without any more information.

-Are there any questions? If at any point during the experiment you need help, just raise your hand.

F.5. Decision ODS

There is a jar with 10 balls, some of which are Red and some of which are Blue. One of these balls will be randomly drawn and will be called the “selected ball”.

Your payoff will be determined by the color of the selected ball, your action, and the action of two computers. You can vote either Red or Blue. Computer 1 and 2 are programmed to behave in a specific manner:

(1) If the selected ball is red: vote Red.

(2) If the selected ball is blue: vote Blue with probability p and Red with 1-p, $0<p<1$. The exact value of p is unknown.

If the selected ball is Red and the majority voted for Red, your payoff is 2. If the selected ball is Blue and the majority votes for Blue, your payoff is 2. In all other cases, your payoff is 0. In other words, you get 2 if the vote of the majority coincides with the color of the selected ball and 0 otherwise. Note that there is a total of 3 votes (two by the computers and one by yourself), so that saying that a majority voted for a specific color means that there are 2 or 3 votes for that specific color.

The following table summarizes the payoffs:

		Color of the selected ball	
		Red ball	Blue ball
Decision of the Group	Majority votes for Red	2	0
	Majority votes for Blue	0	2

Table F.6. Payoff table

You will be informed the color of the selected ball before you make a choice.

-Are there any questions? If at any point during the experiment you need help, just raise your hand.