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TRANSFORMATIONS BETWEEN CURRENT AND CONSTITUENT QUARKS IN THE FREE QUARK MODEL

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PUBLICATIONS

1. "A New Method for Analysis of Total Cross Sections", with H. J. Lipkin, Nucl. Physics B 27, 464 (1971).
2. "Unitarity Bounds for Inclusive Reactions", with A. Rabl, Nuovo Cimento Lett. 4, 511 (1972).
3. "Single Particle Distributions, Double Discontinuities and Triple Regge Limits", with A. Rabl, Bonn Univ. preprint PI 114 (1972).

4. 'Canonical Transformations Between "Current" and "Constituent" Quarks', with W. F. Palmer, OSU preprint COO-1545-138 (1974), to be published in Phys. Rev.
5. "Is the Pomeron a Goldstone Boson ?", with S. S. Pinsky, OSU preprint COO-1545-140.

FIELDS OF STUDY

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Multiparticle Production. Dr. A. Rabl

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INTRODUCTION

The concept of quarks, their bilinear currents and ensuing commutation relations and symmetries, has recently been clarified by Gell-Mann,[1] who pointed out the existence of and sharpened the distinction between two different $SU(6)_W$ algebras.

One, which will henceforth be called $SU(6)_{W,strong}$, is that suggested by Lipkin and Meshkov[2]. It is an approximate symmetry of the hadrons, closely related to the original nonrelativistic SU(6), and contains results associated with the naive quark model: the hadrons act for many purposes as if they were made out of two or three "strong" (constituent) quarks. The original SU(6) predicts spin conservation for collinear three point functions (hadronic decays and form factors) and thus leads to some undesirable consequences. These can be avoided by noting that collinear form factors have symmetry properties which follow only from rotation and reflection invariance and do not require other symmetry assumptions. The subgroup g_W of the improper Lorentz group, including reflections, which leaves all momenta in the z-direction invariant, has been denoted by "collinear little group" or "little - W" group[3]. The group g_W contains an arbitrary rotation about the z-axis

$$e^{i\alpha J_z} \underset{z}{=} e^{i\alpha S_z} \tag{1-1}$$

and a reflection about any plane containing the z-axis, which is written as a product of a space inversion, generated by the parity operator P, and a 180° rotation about an axis in the x-y plane

$$P\, e^{i\pi(J_x \cos\Theta + J_y \sin\Theta)} \underset{z}{=} P_{int}\, e^{i\pi(S_x \cos\Theta + S_y \sin\Theta)} . \tag{1-2}$$

The equality $\underset{z}{=}$ holds when all momenta are in the z-direction, P_{int} is the intrinsic parity and $\vec{S}$ the total quark spin. When acting on a system of spin 1/2 quarks the right hand side of Eq.(1-2) can be written as a product of single quark operators

$$e^{i\pi P_{int}(S_x \cos\Theta + S_y^{i} \sin\Theta)} . \tag{1-3}$$

If one defines an additive operator $\vec{W}$ as

$$W_x = P_{int}\, S_x , \tag{1-4a}$$

$$W_y = P_{int}\, S_y , \tag{1-4b}$$

and

$$W_z = S_z , \tag{1-4c}$$

Eqs.(1-1) and (1-2) become

$$e^{i\alpha J_z} \underset{z}{=} e^{i\alpha W_z} \tag{1-5a}$$

and

$$P\, e^{i\pi(J_x \cos\Theta + J_y \sin\Theta)} \underset{z}{=} e^{i\pi(W_x \cos\Theta + W_y \sin\Theta)} . \tag{1-5b}$$

In terms of quark and antiquark spins $\vec{S}_q$ and $\vec{S}_{\bar{q}}$ the W-spin operators read

$$W_x = S_{xq} - S_{x\bar{q}}, \tag{1-6a}$$

$$W_y = S_{yq} - S_{y\bar{q}}, \tag{1-6b}$$

and

$$W_z = S_{zq} + S_{z\bar{q}}, \tag{1-6c}$$

thus defining the W-spin classification of mesons and baryons. If the transverse momenta can be neglected W_z, (and consequently L_z) is conserved and the hadrons can be classified in multiplets of $SU(6)_W \times O(2)$ each member being specified by its $SU(3) \times SU(2)_W$ properties and the orbital excitation. The lowest lying negative parity mesons (pseudo-scalar and vector) then belong to the $\underline{1}$ and $\underline{35}$ representations and the lowest lying positive parity baryons ($1/2^+$ octet and $3/2^+$ decimet) to the $\underline{56}$ representation of $SU(6)_W$.

The second $SU(6)_W$, called $SU(6)_{W,\text{currents}}$ and introduced by Dashen and Gell-Mann[4] is based on the algebra of integrated local current densities measured in weak and electromagnetic interactions. According to the Cabibbo theory the currents form a set of eight vector and eight axial vector currents, which have the same commutation relations and Lorentz behavior as

$$V_\alpha^\mu = \bar{q}\gamma^\mu \frac{\lambda_\alpha}{2} q, \tag{1-7a}$$

and

$$A_\alpha^\mu = \bar{q}\gamma^\mu\gamma^5 \frac{\lambda_\alpha}{2} q, \tag{1-7b}$$

respectively. Here λ_α is the usual 3×3 representation of SU(3) generators and γ^μ, γ^5 are 4×4 Dirac matrices. In the limit of SU(3) invariance all vector currents are conserved (CVC). If a world scalar is added to the system (1-7) and the operators are commuted at equal times (using quark model commutation relations) the algebra closes on a U(12). Tensor currents $F^{\mu\nu}_\alpha = \frac{1}{2}\, \bar{q}\, \sigma^{\mu\nu}\, \lambda_\alpha\, q$ are found in the process and their integrated "good" components (for definition see Appendix A) can be adjoined to the chiral $SU(3) \times SU(3)$ algebra to yield a set of 35 (36) charges of $SU(6)_{W,\text{currents}}$

$$F_a = \int d^3x\, q^\dagger(x)\, \frac{\lambda_a}{2}\, q(x), \tag{1-8a}$$

$$F_a^{1,2} = \frac{1}{2}\int d^3x\, q^\dagger(x)\, \beta\sigma^{1,2}\, \frac{\lambda_a}{2}\, q(x), \tag{1-8b}$$

and

$$F_a^{3} = \frac{1}{2}\int d^3x\, q^\dagger(x)\, \sigma^{3}\, \frac{\lambda_a}{2}\, q(x). \tag{1-8c}$$

F_o is excluded from $SU(6)_{W,\text{currents}}$; its inclusion enlarges the algebra to $U(6)_{W,\text{currents}}$.

Even though the $SU(6)_{W,\text{strong}}$ charges W^i_a are taken to have the same commutation relations, charge conjugation and parity as the quark model expressions (1-8) the two algebras need not be identified. It is perhaps tempting to do so and extend the CVC hypothesis according to which the hadronic vector current which participates in weak interactions is identified with the current which generates hadron symmetries. However, predictions, based on their equality, such as $G_A/G_V = -5/3$[5] or the vanishing anomalous magnetic moments of the $1/2^+$

octet baryons[6], are not satisfied in nature. This led to a variety of phenomenological mixing schemes describing the physical baryons as complex mixtures of irreducible representations of $SU(6)_{W,\text{currents}}$. In other words, a baryon which seems to be composed of three quarks with the naive quark model wave function under $SU(6)_{W,\text{strong}}$ becomes a complex object when viewed under $SU(6)_{W,\text{currents}}$. This picture seems to be supported by the parton model, which uses an infinite number of partons in a hadron to describe the structure functions measured in deep inelastic electron scattering (see Appendix B).

It has been suggested that the two seemingly different aspects could be reconciled if a transformation between the two $SU(6)_W$ algebras was found[1]. Candidates for this mechanism have been proposed by Melosh[7] and Gomberoff, Horwitz and Ne'eman[8] whose transformations take the set of the current generators (1-8) into exact symmetries of the free quark model. As noted by Gürsey[9] the same task can also be accomplished by the Foldy - Wouthuysen transformation[10] and, as we shall show later, by other transformations. Some phenomenological implications of the transforms have also been tested[11].

In this work we inquire into the Fock space realization of these formal transformations, check whether and when they can be unitarily implemented, and in the process construct eigenstates of $SU(6)_{W,\text{currents}}$ charges and find the overlap between current and constituent quark states in the free quark model. This overlap is zero unless the theory is cut-off in momentum. Suitable remarks are made concerning nonseparable infinite tensor product spaces and the lack of unitary implementability. Certain averages and moments, however, are well defined. These are calculated and discussed in terms of a parton interpretation. The transformation we study lead to exact symmetries of the free quark model in the equal time formalism. Corresponding transforms in the light-like formalism[12-14] are unitary, do not create pairs, and produce at most a spin rotation.

The paper is organized as follows: In Sec.I we list some Foldy - Wouthuysen type transformations and discuss their uniqueness and consequences; in Sec.II, in a simplified formalism, we show how these transforms are implemented in the Hilbert space; in Sec.III we obtain the eigenstates of the $SU(6)_{W,\mathrm{strong}}$ and $SU(6)_{W,\mathrm{currents}}$ charges and calculate distributions of current quarks in a strong quark. Section IV contains the conclusions. Appendix A defines "good" and "bad" operators, Appendix B contains a description of some aspects of the parton model and Appendix C treats "exponential ordering", a technical tool necessary to study the transforms.

I CANONICAL TRANSFORMS IN THE FREE QUARK MODEL

Given a formally unitary transformation $V_i = \exp(iY_i)$, with $Y_i = Y_i^{+}$, the free quark model Hamiltonian

$$H = \int d^3x\, q^\dagger(x)[-i\vec{\alpha}\cdot\vec{\partial} + \beta m]\, q(x) \tag{2-1}$$

can be transformed into a form

$$H_i = V_i^\dagger H V_i = \sum_n \frac{(-i)^n}{n!}\{Y_i^n, H\}, \tag{2-2}$$

where $\{Y_i^n, H\} \equiv [Y_i,[Y_i,\ldots,[Y_i,H]\ldots]]$ and contains n commutators. If Y_i can be written as

$$Y_i = \int d^3x\, q^\dagger(x) \sum_j \Gamma_j f_j(\vec{\partial})\, q(x), \tag{2-3}$$

where $f_j(\vec{\partial})$ are some functions of spatial derivatives and Γ_j are Dirac matrices, the resulting form of H_i is given by

$$H_i = \int d^3x\, q^\dagger(x)\, e^{-i\sum_j \Gamma_j f_j(\vec{\partial})}[-i\vec{\alpha}\cdot\vec{\partial} + \beta m]\, e^{i\sum_j \Gamma_j f_j(\vec{\partial})}\, q(x) \tag{2-4}$$

and can therefore contain only a linear combination of the matrices β, α^k, γ^5 and $\gamma^k\gamma^5$, with k=1,2,3.

A classic example of a V_i is the Foldy - Wouthuysen transformation generated by

$$Y_{FW} = \frac{1}{2}\int d^3x\, q^\dagger(x)\, \frac{\vec{\gamma}\cdot\vec{\partial}}{|\vec{\partial}|}\, \arctan\frac{|\vec{\partial}|}{m}\, q(x), \tag{2-5}$$

where

$$|\vec{\partial}| = (-\vec{\partial}^2)^{1/2},$$

and leading to an

$$H_{FW} = \int d^3_x\, q^\dagger(x)\, \beta \sqrt{m^2 - \vec{\partial}^2}\, q(x). \tag{2-6}$$

The transformed Hamiltonian H_{FW} coincides with the Dirac Hamiltonian H only on the space of states at rest.

Another transformation, with

$$Y_U = -\frac{1}{2}\int d^3_x\, q^\dagger(x)\, \frac{\vec{\gamma}\cdot\vec{\partial}}{|\vec{\partial}|}\, \arctan \frac{m}{|\vec{\partial}|}\, q(x) \tag{2-7}$$

has been constructed by Cini and Touschek[15] and Bose, Gamba and Sudarshan[16]. Its effect is essentially to remove the mass term yielding a transformed Hamiltonian

$$H_U = \int d^3_x\, q^\dagger(x)(-i\vec{\alpha}\cdot\vec{\partial})\, \frac{\sqrt{m^2 - \vec{\partial}^2}}{|\vec{\partial}|}\, q(x) \tag{2-8}$$

with a mass dependent kinetic energy term. In this case H and H_U are equivalent when acting on states of infinite momentum.

The Melosh transformation[7] generated by

$$Y_M = \frac{1}{2}\int d^3_x\, q^\dagger(x)\, \frac{\vec{\gamma}_\perp\cdot\vec{\partial}_\perp}{|\vec{\partial}_\perp|}\, \arctan \frac{|\vec{\partial}_\perp|}{m}\, q(x), \tag{2-9}$$

where

$$|\vec{\partial}_\perp| = (-\partial_x^2 - \partial_y^2)^{1/2},$$

results in an

$$H_M = \int d^3_x\, q^\dagger(x)\left[-i\alpha^3\partial_3 + \beta\sqrt{m^2 - \vec{\partial}_\perp^2}\,\right] q(x), \tag{2-10}$$

where only the transverse components of the kinetic energy term are "rotated" into mass term. Obviously $H = H_M$ for states of zero transverse momentum.

Yet another transformation, which shall prove relevant to the discussion below, is

$$V_{UZ} = e^{iY_{FW}} e^{-iY_{UZ}}, \tag{2-11}$$

with

$$Y_{UZ} = \frac{\pi}{4} \int d\vec{x}^3\, q^+(x) \frac{\gamma^3 \partial_3}{|\partial_3|}\, q(x), \tag{2-12}$$

transforming the Hamiltonian into

$$H_{UZ} = -i \int d\vec{x}^3\, q^+(x) \frac{\alpha^3 \partial_3}{|\partial_3|} \sqrt{m^2 - \vec{\partial}^2}\, q(x). \tag{2-13}$$

This transformation does not affect states moving at infinite momentum in the z-direction.

None of the above transformations is uniquely determined by the form of H_i ; they are arbitrary up to a unitary transformation $\tilde{V}$ which commutes with the H. In other words, if a V_i has been found such that $V_i^\dagger H V_i = H_i$ then also $V_i'^\dagger H V_i' = H_i$, where

$$V_i' = \tilde{V} V_i . \tag{2-14}$$

For example the transformation V_G used by Gomberoff, Horwitz and Ne'eman[8]

$$V_G = e^{iY_M} e^{iY_G}, \tag{2-15}$$

where

$$Y_G = \frac{i}{2}\int d^3x\, q^\dagger(x)\,\frac{\gamma^3 \partial_3}{|\partial_3|}\,\arctan\frac{|\vec{\partial}_\perp|}{\sqrt{m^2-\vec{\partial}_\perp^2}}\,q(x)\,, \tag{2-16}$$

leads to H_{FW} even though $V_G \neq V_{FW}$. Similarly, H_{UZ} can be reached by a V'_{UZ} different from V_{UZ} ,

$$V'_{UZ} = V_M\, e^{iY'_{UZ}}, \tag{2-17}$$

where

$$Y'_{UZ} = -\frac{i}{2}\int d^3x\, q^\dagger(x)\,\frac{\gamma^3\partial_3}{|\partial_3|}\,\arctan\frac{\sqrt{m^2-\vec{\partial}_\perp^2}}{|\partial_3|}\,q(x). \tag{2-18}$$

Under certain conditions the transformations can be used to generate conserved quantities out of nonconserved ones. For example, if an operator F does not commute with the H but it does commute with some H_i then

$$W_i = V_i F V_i^\dagger \tag{2-19}$$

commutes with the original Hamiltonian H. Again, the requirement that the V_i - transformed F be conserved determines the transform only up to two unitary transformations $\tilde{V}$ and $\tilde{\tilde{V}}$, such that $[\tilde{V},H] = 0$ and $[\tilde{\tilde{V}},F] = 0$; if $W_i = V_i F V_i^\dagger$ is conserved then also W_i' is conserved, with $W_i' = V_i' F V_i'^\dagger$ and

$$V_i' = \tilde{V}\, V_i\, \tilde{\tilde{V}}. \tag{2-20}$$

This technique has recently been applied in an effort to find a conserved $SU(6)_{W,\text{strong}}$ in the free quark model. The set of generators of $SU(6)_{W,\text{currents}}$ given by Eqs.(1-8) commutes with integrated bilinear densities of the type

$$\int d^3x\, q^\dagger(x) \{ 1, \alpha^3, \beta, \gamma^3 \} f(\vec{\partial}) q(x), \tag{2-21}$$

where $f(\vec{\partial})$ is some function of spatial derivatives. The set (1-8) can be enlarged to U(6) X U(6) if the operators F_o and

$$\tilde{F}_a = \int d^3x\, q^\dagger(x) \beta \frac{\lambda_a}{2} q(x), \tag{2-22a}$$

$$\tilde{F}_a^{1,2} = \frac{1}{2} \int d^3x\, q^\dagger(x) \sigma^{1,2} \frac{\lambda_a}{2} q(x), \tag{2-22b}$$

$$\tilde{F}_a^{3} = \frac{1}{2} \int d^3x\, q^\dagger(x) \beta \sigma^{3} \frac{\lambda_a}{2} q(x) \tag{2-22c}$$

are included. The full set now commutes with integrated bilinear densities of a general form

$$\int d^3x\, q^\dagger(x) \{ 1, \beta \} f(\vec{\partial}) q(x). \tag{2-23}$$

Accordingly,

$$[F_a^{1,2,3}, H_{\{M, FW, UZ\}}] = 0, \tag{2-24}$$

and

$$[\tilde{F}_a^{1,2,3}, H_{FW}] = 0 ; \tag{2-25}$$

thus V_M , V_{FW} , and V_{UZ} lead to a conserved $SU(6)_{W,strong}$, while V_G and V_{FW} transform the whole U(6) x U(6) into a symmetry of the free Hamiltonian.

The transformations which take the $SU(6)_{W,currents}$ into a conserved $SU(6)_{W,strong}$ must be SU(3) singlets in order for the conserved vector current hypothesis generalized to SU(3) to remain valid, i.e. $F_a = W_a$. The classification of states requires other quantum numbers in addition to $SU(6)_W$; one of these quantum numbers can be taken to be the spin in the z-direction. Consequently the transformation is required to be invariant under spatial rotations about the z-axis, $[V, J_z] = 0$. The transformations considered above do satisfy these conditions and thus there is a large amount of freedom left. For example $W^i_{a,FW} = V_{FW} F^i_a V^\dagger_{FW}$ can also be obtained by V_{UZ} since $[Y_{UZ}, F^i_a] = 0$. Similarly, the $SU(6)_{W,\text{ strong}}$ charges of Melosh, $W^i_{a,M} = V_M F^i_a V^\dagger_M$, result from both V_G and V'_{UZ} as Y_G and Y'_{UZ} commute with the F^i_a's. This also implies that $W^i_{a,M}$ and $W^i_{a,FW}$ are related by a unitary transformation which commutes with the Hamiltonian H and turns out to be a momentum dependent spin rotation; similar transformations can be used to construct other W^i_a's.

As explicitly shown by Melosh the structure of the "strong" charges expressed in terms of the quark fields q(x) is rather complicated and it is advantageous to view them in momentum space where they become simple. Let us write q(x) in terms of creation and annihilation operators,

$$q(x) = \frac{1}{(2\pi)^{3/2}} \sum_r \int d^3p \, \frac{M}{E} \left[a^{(r)}_{\vec{p}} u^{(r)}(\vec{p}) e^{-ipx} + b^{+(r)}_{\vec{p}} v^{(r)}(\vec{p}) e^{ipx} \right], \tag{2-26}$$

where r runs over both spin and SU(3) indices, and the operators satisfy the usual anticommutation relations

$$\{a^{+(r)}_{\vec{p}}, a^{(s)}_{\vec{k}}\} = \{b^{+(r)}_{\vec{p}}, b^{(s)}_{\vec{k}}\} = \delta_{rs} \frac{E}{M} \delta^{(3)}(\vec{p} - \vec{k}), \tag{2-27}$$

with all other anticommutators vanishing. The Melosh and FW-transformed charges are then

$$W_a = \sum_{r,s} \int d^3k \, \frac{M}{E} \left\{ a^{+(r)}_{\vec{k}} a^{(s)}_{\vec{k}} \, u^{+(r)}(0) \frac{\lambda_a}{2} u^{(s)}(0) - b^{+(s)}_{-\vec{k}} b^{(r)}_{-\vec{k}} \, v^{+(r)}(0) \frac{\lambda_a}{2} v^{(s)}(0) \right\}, \tag{2-28a}$$

$$W_a^{1,2} = \frac{1}{2} \sum_{r,s} \int d^3k \, \frac{M}{E} \left\{ a^{+(r)}_{\vec{k}} a^{(s)}_{\vec{k}} \, u^{+(r)}(0) R^{-1} \sigma^{1,2} \frac{\lambda_a}{2} R \, u^{(s)}(0) + b^{+(s)}_{-\vec{k}} b^{(r)}_{-\vec{k}} \, v^{+(r)}(0) R^{-1} \sigma^{1,2} \frac{\lambda_a}{2} R \, v^{(s)}(0) \right\}, \tag{2-28b}$$

$$W_a^{3} = \frac{1}{2} \sum_{r,s} \int d^3k \, \frac{M}{E} \left\{ a^{+(r)}_{\vec{k}} a^{(s)}_{\vec{k}} \, u^{+(r)}(0) R^{-1} \sigma^{3} \frac{\lambda_a}{2} R \, u^{(s)}(0) - b^{+(s)}_{-\vec{k}} b^{(r)}_{-\vec{k}} \, v^{+(r)}(0) R^{-1} \sigma^{3} \frac{\lambda_a}{2} R \, v^{(s)}(0) \right\}, \tag{2-28c}$$

where R=1 for the FW-transformation, while for the Melosh transform[7]

$$R = \sqrt{\frac{(\omega+E)(\omega+M)}{2\omega(E+M)}} \left[1 - \frac{i k^3 (\vec{k} \times \vec{\sigma})^3}{(\omega+E)(\omega+M)} \right], \tag{2-29}$$

with

$$\omega = (m^2 + k_\perp^2)^{1/2}. \tag{2-30}$$

Thus $W^i_{0,FW}$ are just the effective spin operators, while $W^i_{0,M}$ differ from these by a spin rotation, in accordance with what has been noted earlier. This rotation is just an anti-Wigner rotation making the charges found by Melosh invariant under boosts in the z-direction. The corresponding $SU(6)_{W,\text{strong}}$ then becomes a collinear symmetry.

On the other hand, the F's, while local functions of the fields, contain pair creating terms in momentum space. Their eigenstates are expected to be complicated, but can be nevertheless constructed using the transfotmations as a technical tool. If the eigenstates $|q_s\rangle$ (the quantum numbers are supressed in this notation) of a conserved charge $W=VFV^\dagger$ are denoted "constituent" or "strong" quarks, which are single particle Fock space states in the Dirac representation, then the corresponding eigenstates of F, "current" quarks, are given by $|q_c\rangle = V^\dagger|q_s\rangle$.

Thus the transformations have a property of shifting a kind of non-locality, in particle number as well as space, from states to operators, depending on the representation. Although not stated in a second quantized and Fock space language, this state of affairs is implicit already in the classic paper of Foldy and Wouthuysen[10] who point out that their transformation takes the naive and local position operator $\vec{x}$,

$$x^i = \int d^3x\, \underset{\sim}{q}^\dagger(x)\, x^i\, \underset{\sim}{q}(x)$$

$$= \sum_{r,s} \int d^3k\, \frac{M}{E}\Big\{ i a_{\vec{k}}^{\dagger(r)} \frac{d}{dk^i} a_{\vec{k}}^{(s)} \delta_{r,s} + i\Big(\frac{d}{dk^i} b_{\vec{k}}^{\dagger(s)}\Big) b_{\vec{k}}^{(r)} \delta_{r,s}$$

$$+ a_{\vec{k}}^{\dagger(r)} a_{\vec{k}}^{(s)} \Big[x_0 \frac{k^i}{E}\delta_{r,s} - i\frac{k^i}{2E^2}\delta_{r,s} - \frac{1}{2E(E+M)} u^{\dagger(r)}(0)(\vec{\sigma}\times\vec{k})^i u^{(s)}(0)\Big]$$

$$+ b_{\vec{k}}^{\dagger(s)} b_{\vec{k}}^{(r)} \Big[-x_0 \frac{k^i}{E}\delta_{r,s} - i\frac{k^i}{2E^2}\delta_{r,s} - \frac{1}{2E(E+M)} v^{\dagger(r)}(0)(\vec{\sigma}\times\vec{k})^i v^{(s)}(0)\Big] \tag{2-31}$$

$$+ i b_{-\vec{k}}^{(r)} a_{\vec{k}}^{(s)} \Big[-\frac{1}{2E} v^{\dagger(r)}(0)\gamma^i u^{(s)}(0) + \frac{k^i}{2E^2(E+M)} v^{\dagger(r)}(0)\vec{\gamma}\cdot\vec{k}\, u^{(s)}(0)\Big] e^{-2iEx_0}$$

$$- i a_{-\vec{k}}^{\dagger(r)} b_{\vec{k}}^{\dagger(s)} \Big[\frac{1}{2E} u^{\dagger(r)}(0)\gamma^i v^{(s)}(0) - \frac{k^i}{2E^2(E+M)} u^{\dagger(r)}(0)\vec{\gamma}\cdot\vec{k}\, v^{(s)}(0)\Big] e^{2iEx_0}\Big\},$$

intc

$$X^i = V_{FW}\, x^i\, V_{FW}^{\dagger}$$

$$= \sum_{r,s} \int d^3k \frac{M}{E} \delta_{r,s} \Big\{ \tfrac{1}{2} a_{\vec{k}}^{\dagger(r)} \big(i \tfrac{d}{dk^i} a_{\vec{k}}^{(s)}\big) - \tfrac{1}{2} \big(i \tfrac{d}{dk^i} a_{\vec{k}}^{\dagger(r)}\big) a_{\vec{k}}^{(s)} + \tfrac{1}{2} \big(i \tfrac{d}{dk^i} b_{\vec{k}}^{\dagger(s)}\big) b_{\vec{k}}^{(r)}$$

$$- \tfrac{1}{2} b_{\vec{k}}^{\dagger(s)} \big(i \tfrac{d}{dk^i} b_{\vec{k}}^{(r)}\big) + \frac{x^0 k^i}{E} \big(a_{\vec{k}}^{\dagger(r)} a_{\vec{k}}^{(s)} - b_{\vec{k}}^{\dagger(s)} b_{\vec{k}}^{(r)}\big) \Big\}, \qquad (2\text{-}32)$$

of Newton and Wigner[17] which does have localized eigenstates

$$|X\rangle = \int d^3p \sqrt{\frac{M}{E}}\, e^{ipX}\, a_{\vec{p}}^{\dagger(r)} |0\rangle. \qquad (2\text{-}33)$$

In the next two sections we set out to find the explicit form of the strong and current states, $|q_s\rangle$ and $|q_c\rangle$. We shall also calculate the distribution of the current quarks in a strong quark and determine whether this permits the identification of current quarks with partons. Since the structure of the transformations is more transparent in a simplified formalism we shall discuss that case first.

II. STRUCTURE OF THE TRANSFORMATIONS IN A SIMPLIFIED FORMALISM

The $SU(6)_{W,\text{strong}}$ operators annihilate the vacuum, are conserved and do not create pairs, while the charges of $SU(6)_{W,\text{current}}$ (apart from the common SU(3) subgroup) do not annihilate the vacuum, are not conserved and do create pairs. Thus the transformation between them has to contain pair creating terms, and it turns out that this occurs in all momentum states. As a consequence, the transformations we are considering are not unitary on Fock space and their implementation is naturally discussed in the larger infinite tensor product spaces of Von Neumann[18]. To explore their structure we shall restrict ourselves to a simplified formalism by neglecting the spin and SU(3) degrees of freedom. Let us consider an infinite set of creation and annihilation operators labeled by a discrete index k and obeying the anticommutation relations

$$\{a_k^+, a_{k'}\} = \delta_{k,k'}, \tag{3-1a}$$

$$\{b_k^+, b_{k'}\} = \delta_{k,k'}, \tag{3-1b}$$

all other anticommutators vanishing. Apart from counting terms like $a_k^\dagger a_k$ the operators of interest have the form

$$V = \exp\{i \sum_k (y_k A_k^+ + y_k^* A_k)\} \tag{3-2}$$

where

$$A_k = b_{-k} a_k , \tag{3-3a}$$

$$A_k^\dagger = a_k^\dagger b_{-k}^\dagger , \tag{3-3b}$$

and γ_k is some function of k. Since

$$[A_k , A_{k'}^\dagger] = \delta_{k,k'} (1 - a_k^\dagger a_k - b_{-k}^\dagger b_{-k}), \tag{3-4}$$

V can be written in a product form

$$V = \prod_k V_k , \tag{3-5}$$

where

$$V_k = \exp\{ i(\gamma_k A_k^\dagger + \gamma_k^* A_k)\}. \tag{3-6}$$

In the nonseparable infinite tensor product space, in which V acts, a general state $|f\rangle$ is written as

$$|f\rangle = \prod_k |f_k\rangle \tag{3-7}$$

and thus

$$V|f\rangle = \prod_k V_k |f_k\rangle . \tag{3-8}$$

The inner product is defined by

$$\langle f|g\rangle = \prod_k \langle f_k | g_k\rangle . \tag{3-9}$$

The k-th mode, $|f_k\rangle$, is built on the common vacuum, $|0_k\rangle$, of a_k and b_{-k} , and is in general a normed linear combination of the states $|0_k\rangle$,

$|1_k\rangle = a_k^\dagger|0_k\rangle$, $|\bar{1}_k\rangle = b_{-k}^\dagger|0_k\rangle$ and $|2_k\rangle = A_k^\dagger|0_k\rangle$. (The fermion commutation relations do not permit any other state.) By a parameter differentiation method described in the Appendix C it can be shown that

$$V_k = e^{f_k A_k^+} e^{g_k A_k} e^{m_k [A_k, A_k^+]}, \tag{3-10}$$

where

$$f_k = \frac{i\gamma_k}{|\gamma_k|} \, tg\,|\gamma_k|, \tag{3-11a}$$

$$g_k = \frac{i\gamma_k^*}{|\gamma_k|} \sin|\gamma_k| \cos|\gamma_k|, \tag{3-11b}$$

and

$$m_k = \lg \cos|\gamma_k|. \tag{3-11c}$$

Thus

$$V_k |0_k\rangle = \cos|\gamma_k|\,|0_k\rangle + \frac{i\gamma_k}{|\gamma_k|} \sin|\gamma_k|\,|2_k\rangle, \tag{3-12a}$$

$$V_k |1_k\rangle = |1_k\rangle, \tag{3-12b}$$

$$V_k |\bar{1}_k\rangle = |\bar{1}_k\rangle, \tag{3-12c}$$

and

$$V_k |2_k\rangle = \cos|\gamma_k|\,|2_k\rangle + \frac{i\gamma_k^*}{|\gamma_k|} \sin|\gamma_k|\,|0_k\rangle. \tag{3-12d}$$

The eigenstates of V_k are $|1_k\rangle$ and $|\bar{1}_k\rangle$, both with eigenvalue one, and

$$\frac{1}{\sqrt{2}}\left(\pm \frac{\gamma_k^*}{|\gamma_k|}|0_k\rangle + |2_k\rangle\right) \tag{3-13}$$

with eigenvalues $\exp(\pm i|\gamma_k|)$.

Let us now define an equivalence class corresponding to a state $|f\rangle = \Pi_k|f_k\rangle$ as a set of states which differ from $|f\rangle$ in at most a finite number of modes. If the Fock vacuum, $|0\rangle = \Pi_k|0_k\rangle$, belongs to this class the class spans the Fock space. In general γ_k is nonzero for an infinite number of k's and consequently, with the exception of the eigenstates of V and their equivalence classes, $|f\rangle$ and $V|f\rangle$ lie in different spaces corresponding to distinct equivalence classes. For example

$$V|0\rangle = \Pi_k \left(\cos|\gamma_k||0_k\rangle + \frac{i\gamma_k}{|\gamma_k|}\sin|\gamma_k||2_k\rangle\right) \tag{3-14}$$

is orthogonal not only to the vacuum,

$$\langle 0|V|0\rangle = \Pi_k \cos|\gamma_k|, \tag{3-15}$$

but$\langle 0|V|0\rangle = \Pi_k \cos|\gamma_k|,$te particle number, whether finite or infinite. Thus V is not unitarily implementable on a separable space unless a cut-off procedure is introduced which limits the degrees of freedom to a finite number.

Nevertheless, expectation values of Fock space operators in V-transformed Fock states do not vanish . Certain "inclusive" probabilities are also well defined, such as the probability $P(n_p)$ of finding n-pairs in the p-th mode and anything else in the other modes

$$P(n_p) = \sum_X |\langle n_p, X|V|0\rangle|^2, \tag{3-16}$$

giving

$$P(0_p) = \cos^2|\gamma_p|, \tag{3-17a}$$

and

$$P(1_p) = \sin^2|\gamma_p|. \tag{3-17b}$$

The average number of pairs in $V|0\rangle$ can also be evaluated by taking the expectation value of the pair number operator $N = \Sigma_k A_k^\dagger A_k$ in the $V|0\rangle$ state

$$\langle N\rangle = \langle 0|V^\dagger N V|0\rangle = \sum_k \sin^2|\gamma_k|. \tag{3-18}$$

We can also show that V, even though not strictly unitary, does not lead to states of infinite norm and, in fact, V has a unit norm. Let us take an arbitrary state $|f\rangle = \Pi_k|f_k\rangle$, where

$$|f_k\rangle = \alpha_k|0_k\rangle + \beta_k|1_k\rangle + \mu_k|\bar{1}_k\rangle + \nu_k|2_k\rangle, \tag{3-19}$$

whose norm is

$$\|f\| = \Pi_k\left(|\alpha_k|^2 + |\beta_k|^2 + |\mu_k|^2 + |\nu_k|^2\right). \tag{3-20}$$

The norm of $V|f\rangle$,

$$V|f\rangle = \Pi_k\Big\{\alpha_k \cos|\gamma_k|\,|0_k\rangle + \frac{i\gamma_k}{|\gamma_k|}\alpha_k \sin|\gamma_k|\,|2_k\rangle$$

$$+\beta_k|1_k\rangle + \mu_k|\bar{1}_k\rangle \tag{3-21}$$

$$+ \nu_k \cos|\gamma_k| \, |2_k\rangle + \frac{i\gamma_k^*}{|\gamma_k|} \nu_k \sin|\gamma_k| \, |0_k\rangle,$$

is given by

$$\| V|f\rangle \| = \Pi_k \left(|\alpha_k|^2 + |\beta_k|^2 + |\mu_k|^2 + |\nu_k|^2 \right)$$

$$= \| f \| \tag{3-22}$$

and thus the norm of V is one. V also has an inverse since to each vector $|g\rangle = \Pi_k |g_k\rangle$,

$$|g_k\rangle = c_k |0_k\rangle + d_k |1_k\rangle + e_k |\bar{1}_k\rangle + h_k |2_k\rangle, \tag{3-23}$$

corresponds one and only one vector $|f\rangle$ given by Eq.(3-19), where

$$\alpha_k = c_k \cos|\gamma_k| - \frac{i\gamma_k^*}{|\gamma_k|} h_k \sin|\gamma_k|, \tag{3-24a}$$

$$\beta_k = d_k, \tag{3-24b}$$

$$\mu_k = e_k, \tag{3-24c}$$

and

$$\nu_k = h_k \cos|\gamma_k| - \frac{i\gamma_k}{|\gamma_k|} c_k \sin|\gamma_k|. \tag{3-24d}$$

This inverse V^{-1},

$$V^{-1} = \exp\{-i \sum_k (\gamma_k A_k^+ + \gamma_k^* A_k)\}, \tag{3-25}$$

is seen to be equal to $V^\dagger$, the adjoint of V.

Thus when in the next section we calculate the probability of finding a specified configuration of current quarks in a constituent quark it should come as no surprise that these probabilities vanish unless the high momenta are cut-off, whereas inclusive probabilities associated with these distributions, such as averages and moments, are well defined.

III. EIGENSTATES, DISTRIBUTIONS, AND OVERLAPS

In this section we implement the Foldy-Wouthuysen and Melosh transforms on Fock or larger spaces. Some results will also be given for the V'_{UZ} transformation. We shall find the eigenstates of the strong and current charges, and calculate the distribution of current quarks in a strong quark as well as some expectation values of current quark operators in strong quark states. To avoid cumbersome SU(3) indices, which can be easily incorporated, we limit the discussion to the W-spin subalgebra of the $SU(6)_W$ charges, writing $F^{1,2,3}$ for $F_o^{1,2,3}$ and $W_i^{1,2,3}$ for $V_i F_o^{1,2,3} V_i^\dagger$, where i = FW or M.

As mentioned in Sec. I, the W_i commute with the Dirac Hamiltonian and create no pairs; they also commute with the momentum operator $\vec{P}$, the quark number operator

$$N_q = \sum_r \int d\vec{p}^{\,3}\, \frac{M}{E}\, a^{\dagger(r)}_{\vec{p}}\, a^{(r)}_{\vec{p}} \tag{4-1}$$

and its charge conjugate, the antiquark number operator $N_{\bar{q}}$. In addition, the W_i^3 commute with J_z, the z-component of the angular momentum. Let us denote by $a^{\dagger(r)}_{s,i}(\vec{p})$ a creation operator for a single strong quark, which is a simmultaneous eigenstate of the classifying charges $(\vec{W}_i)^2$, W_i^3, H, $\vec{P}$, J_z, N_q and $N_{\bar{q}}$, and belongs to the W_i^3 eigenvalue $r = \pm 1/2$, i = FW or M, provided

$$[W_i^3, a_{s,i}^{\dagger(\pm)}(\vec{p})] = \pm \frac{1}{2} a_{s,i}^{\dagger(\pm)}(\vec{p}). \tag{4-2}$$

The corresponding state is then given by

$$|q_s, \vec{p}, r, i\rangle = N_i a_{s,i}^{\dagger(r)}(\vec{p})|0\rangle, \tag{4-3}$$

where N_i is a normalization factor. In general r would include the SU(3) indices. Since $a_{s,i}^{\dagger(\pm)}(\vec{p})$ is a single particle creation operator, its most general form is a linear combination of the usual fermion operators $a_{\vec{p}}^{\dagger(r)}$,

$$a_{s,i}^{\dagger(\pm)}(\vec{p}) = \mu_i^{\pm}(\vec{p}) a_{\vec{p}}^{\dagger(+)} + \nu_i^{\pm}(\vec{p}) a_{\vec{p}}^{\dagger(-)} \tag{4-4}$$

and similarly for the strong antiquark

$$b_{s,i}^{\dagger(\pm)}(\vec{p}) = \lambda_i^{\pm}(\vec{p}) b_{\vec{p}}^{\dagger(+)} + \kappa_i^{\pm}(\vec{p}) b_{\vec{p}}^{\dagger(-)}, \tag{4-5}$$

where μ, ν, λ and κ are complex functions of the momentum. Since $a_{s,i}^{\dagger(+)}$ and $a_{s,i}^{\dagger(-)}$ belong to distinct W_i^3 eigenvalues the corresponding states have to be orthogonal to each other and we obtain

$$\mu_i^+ (\mu_i^-)^* + \nu_i^+ (\nu_i^-)^* = 0 \tag{4-6}$$

and

$$\lambda_i^+ (\lambda_i^-)^* + \kappa_i^+ (\kappa_i^-)^* = 0 . \tag{4-7}$$

for the antiquarks. Furthermore, if

$$|\mu_i^{\pm}|^2 + |\nu_i^{\pm}|^2 = |\lambda_i^{\pm}|^2 + |\kappa_i^{\pm}|^2 = 1 \tag{4-8}$$

the strong operators will obey the usual anticommutation relations. The linear combinations (4-4) and (4-5) are determined from Eq.(4-2) and a similar relation for the strong antiquark operators combined with Eq.(4-8). This yields

$$a^{+(r)}_{S,FW}(\vec{p}) = a^{+(r)}_{\vec{p}} , \tag{4-9}$$

$$b^{+(r)}_{S,FW}(\vec{p}) = b^{+(r)}_{\vec{p}} , \tag{4-10}$$

and

$$a^{+(+)}_{S,M}(\vec{p}) = \left[\frac{(E+\omega)(M+\omega)}{2\omega(E+M)}\right]^{1/2}\left(a^{+(+)}_{\vec{p}} - \frac{p^3 p_+}{(E+\omega)(M+\omega)}\, a^{+(-)}_{\vec{p}}\right), \tag{4-11}$$

$$b^{+(+)}_{S,M}(\vec{p}) = \left[\frac{(E+\omega)(M+\omega)}{2\omega(E+M)}\right]^{1/2}\left(b^{+(+)}_{\vec{p}} + \frac{p^3 p_+}{(E+\omega)(M+\omega)}\, b^{+(-)}_{\vec{p}}\right), \tag{4-12}$$

where $\omega = (M^2 + p_\perp^2)^{1/2}$ and $p_\pm = p^1 \pm ip^2$. Thus the eigenstates of the FW-transformed charges $W^3_{FW} = V_{FW}F^3V^\dagger_{FW}$ are simply the naive spin states created by the usual fermion operators, whereas the eigenstates of W^3_M differ from these by a momentum dependent spin rotation which does not affect particles moving in the z-direction or in the transverse plane. As pointed out in Sec. I this is just a Wigner rotation arising when a state of a given transverse momentum is boosted in the z-direction; the Wigner rotation is necessary if the W_M-spin classification is to be z-boost invariant.

We also introduce creation and annihilation operators for "current quarks" and antiquarks, related to the strong operators by

$$a^{+(r)}_{c,i}(\vec{p}) = V_i^{+} a^{+(r)}_{s,i}(\vec{p}) V_i \tag{4-13}$$

with an analogous relation for $b^{\dagger(r)}_{c,i}(\vec{p})$. The corresponding single current quark states created by these operators

$$\begin{aligned} |q_c, \vec{p}, r, i\rangle &= V_i^{+} |q_s, \vec{p}, r, i\rangle \\ &= N_i\, a^{+(r)}_{c,i}(\vec{p}) |0, i\rangle_c \end{aligned} \tag{4-14}$$

are time dependent eigenstates of $(\vec{F})^2$, F^3, $V_i^\dagger H V_i$, $\vec{P}$, J_z and of the current quark number operators

$$V_i^{+} N_q V_i = \sum_r \int d^3p \frac{M}{E} a^{+(r)}_{c,i}(\vec{p})\, a^{(r)}_{c,i}(\vec{p}) \tag{4-15}$$

and $V_i^\dagger N_{\bar{q}} V_i$. The current vacuum

$$|0, i\rangle_c = V_i^{+} |0\rangle \tag{4-16}$$

is annihilated by $a^{(r)}_{c,i}(\vec{p})$, $b^{(r)}_{c,i}(\vec{p})$, $F^{1,2,3}$ and $V_i^\dagger H V_i$, and carries no momentum. Since the V_i act as homogeneous operator transforms, the $a^{\dagger(r)}_{c,i}$ are again linear combinations of ordinary creation/annihilation operators, with all the complexity, and possible subtleties of the transforms contained in the transformed vacuum $|0,i\rangle_c$.

Before obtaining these linear combinations we must introduce some notation. If we define

$$A(g) \equiv \int d^3p \frac{M}{E} g(\vec{p}) A_{\vec{p}} \tag{4-17}$$

with $A_{\vec{p}}$ an operator on a $\vec{p}$ mode subspace and $g(\vec{p})$ a complex function of momentum, then the generators Y_i have the form

$$Y_i = -iC^{\dagger}(\alpha_i) - iD^{\dagger}(\beta_i) + iC(\alpha_i) + iD(\beta_i) + F(\gamma_i), \tag{4-18}$$

where

$$C^{\dagger}_{\vec{p}} = \left(a^{+(-)}_{\vec{p}} b^{+(+)}_{-\vec{p}} - a^{+(+)}_{\vec{p}} b^{+(-)}_{-\vec{p}}\right) e^{2iEx_0}, \tag{4-19}$$

$$D^{\dagger}_{\vec{p}} = \frac{1}{|p_\perp|}\left(p_+ a^{+(-)}_{\vec{p}} b^{+(-)}_{-\vec{p}} + p_- a^{+(+)}_{\vec{p}} b^{+(+)}_{-\vec{p}}\right) e^{2iEx_0} \tag{4-20}$$

$$F_{\vec{p}} = F^{+}_{\vec{p}} = \frac{i}{|p_\perp|}\left[p_+ \left(a^{+(-)}_{\vec{p}} a^{(+)}_{\vec{p}} - b^{+(-)}_{-\vec{p}} b^{(+)}_{-\vec{p}}\right) - p_- \left(a^{+(+)}_{\vec{p}} a^{(-)}_{\vec{p}} - b^{+(+)}_{-\vec{p}} b^{(-)}_{-\vec{p}}\right)\right], \tag{4-21}$$

$C_{\vec{p}}$ and $D_{\vec{p}}$ are hermitian conjugates of $C^{\dagger}_{\vec{p}}$ and $D^{\dagger}_{\vec{p}}$, respectively, and the functions α, β and γ are

$$\alpha_{FW}(\vec{p}) = \frac{1}{2}\frac{p^3}{|p|} \arctan \frac{|p|}{M}, \tag{4-22}$$

$$\beta_{FW}(\vec{p}) = -\frac{1}{2}\frac{|p_\perp|}{|p|} \arctan \frac{|p|}{M}, \tag{4-23}$$

$$\gamma_{FW}(\vec{p}) = 0 \tag{4-24}$$

for the FW transform, and

$$\alpha_M(\vec{p}) = \frac{1}{2}\frac{p^3|p_\perp|}{E(E+M)} \arctan \frac{|p_\perp|}{M}, \tag{4-25}$$

$$\beta_M(\vec{p}) = -\frac{1}{2}\frac{ME+\omega^2}{E(E+M)} \arctan \frac{|p_\perp|}{M}, \tag{4-26}$$

$$\gamma_M(\vec{p}) = \frac{1}{2}\frac{p^3}{E} \arctan \frac{|p_\perp|}{M} \tag{4-27}$$

for the Melosh transform. It is then straightforward to show that

$$V_i a_{\vec{p}}^{+(r)} V_i^{+} = e^{iY_i} a_{\vec{p}}^{+(r)} e^{-iY_i}$$

$$= \cos\Gamma_i \{ a_{\vec{p}}^{+(+)} - \frac{\gamma_i p_+}{\Gamma_i |p_\perp|} \, tg\,\Gamma_i \, a_{\vec{p}}^{+(-)}$$

$$+ e^{-2iEx_0} \left(\frac{\alpha_i}{\Gamma_i} b_{-\vec{p}}^{(-)} - \frac{\beta_i p_+}{\Gamma_i |p_\perp|} b_{-\vec{p}}^{(+)} \right) tg\,\Gamma_i \} \delta_{r,+} \tag{4-28}$$

$$+ \cos\Gamma_i \{ a_{\vec{p}}^{+(-)} + \frac{\gamma_i p_-}{\Gamma_i |p_\perp|} \, tg\,\Gamma_i \, a_{\vec{p}}^{+(+)}$$

$$- e^{-2iEx_0} \left(\frac{\alpha_i}{\Gamma_i} b_{-\vec{p}}^{(+)} + \frac{\beta_i p_-}{\Gamma_i |p_\perp|} b_{-\vec{p}}^{(-)} \right) tg\,\Gamma_i \} \delta_{r,-} ,$$

where $\Gamma_i = (\alpha_i^2 + \beta_i^2 + \gamma_i^2)^{1/2}$, and hence

$$a_{c,FW}^{+(+)}(\vec{p}) = \left(\frac{E+M}{2E}\right)^{1/2} \left[a_{\vec{p}}^{+(+)} - \left(\frac{p_+}{E+M} b_{-\vec{p}}^{(+)} + \frac{p^3}{E+M} b_{-\vec{p}}^{(-)} \right) e^{-2iEx_0} \right], \tag{4-29}$$

and

$$a_{c,M}^{+(+)}(\vec{p}) = \frac{1}{2E} \left(\frac{E+\omega}{E+M}\right)^{1/2} \left[(2E+M-\omega) a_{\vec{p}}^{+(+)} + \frac{p^3 p_+}{\omega+E} a_{\vec{p}}^{+(-)} \right.$$

$$\left. - \left(p_+ b_{-\vec{p}}^{(+)} + p^3 \frac{\omega - M}{\omega+E} b_{-\vec{p}}^{(-)} \right) e^{-2iEx_0} \right]. \tag{4-30}$$

Note that at this point we have three, in general different, kinds of creation and annihilation operators: the ordinary fermion operators as $a_{\vec{p}}^{\dagger(r)}$, strong operators as $a_{s,i}^{\dagger(r)}(\vec{p})$ and current quark operators as $a_{c,i}^{\dagger(r)}(\vec{p})$.

It is often helpful to write V_i as a product of exponential operators with all operators which annihilate the vacuum appearing to the right of those which do not annihilate the vacuum. This ordering procedure is described in Appendix C. The result is

$$
\begin{aligned}
V_i &= \exp\{C^+(\alpha_i) + D^+(\beta_i) - C(\alpha_i) - D(\beta_i) + iF(\gamma_i)\} \\
&= \exp\{C^+(f_i^{(1)}) + D^+(f_i^{(2)})\}\exp\{C(g_i^{(1)}) + D(g_i^{(2)})\} \qquad (4\text{-}31) \\
&\cdot \exp\{iF(h_i)\}\exp\{K(m_i)\},
\end{aligned}
$$

where

$$K(fg) \equiv [C(f), C^+(g)], \qquad (4\text{-}32)$$

$$K_{\vec p} = \frac{2E}{M}\delta(0) - a_{\vec p}^{+(+)}a_{\vec p}^{(+)} - a_{\vec p}^{+(-)}a_{\vec p}^{(-)} - b_{\vec p}^{+(+)}b_{\vec p}^{(+)} - b_{\vec p}^{+(-)}b_{-\vec p}^{(-)}, \qquad (4\text{-}33)$$

$$f_i^{(1)} = \left[1 + \frac{\gamma_i^2}{\Gamma_i^2}\,tg^2\Gamma_i\right]^{-1} tg\,\Gamma_i \left(\frac{\alpha_i}{\Gamma_i} - \frac{\beta_i\gamma_i}{\Gamma_i^2}\,tg\,\Gamma_i\right), \qquad (4\text{-}34)$$

$$f_i^{(2)} = \left[1 + \frac{\gamma_i^2}{\Gamma_i^2}\,tg^2\Gamma_i\right]^{-1}\left(\frac{\beta_i}{\Gamma_i} + \frac{\alpha_i\gamma_i}{\Gamma_i^2}\,tg\,\Gamma_i\right) tg\,\Gamma_i, \qquad (4\text{-}35)$$

$$g_i^{(1)} = -\left[1 + tg^2\Gamma_i\right]^{-1}\left(\frac{\alpha_i}{\Gamma_i} - \frac{\beta_i\gamma_i}{\Gamma_i^2}\,tg\,\Gamma_i\right) tg\,\Gamma_i, \qquad (4\text{-}36)$$

$$g_i^{(2)} = -\left[1 + tg^2\Gamma_i\right]^{-1}\left(\frac{\beta_i}{\Gamma_i} + \frac{\alpha_i\gamma_i}{\Gamma_i^2}\,tg\,\Gamma_i\right) tg\,\Gamma_i, \qquad (4\text{-}37)$$

$$h_i = \arctan\left(\frac{\gamma_i}{\Gamma_i}\,tg\,\Gamma_i\right), \qquad (4\text{-}38)$$

and

$$m_i = -\frac{1}{2}\lg\left[1 + (f_i^{(1)})^2 + (f_i^{(2)})^2\right]. \qquad (4\text{-}39)$$

Then, finally, we can write

$$V_i|0\rangle = \exp\{C^{\dagger}(f_i^{(1)}) + D^{\dagger}(f_i^{(2)})\}|0\rangle$$
$$\cdot \exp\{-\delta^3(0)\int d\vec{p}\, \log[1 + (f_i^{(1)})^2 + (f_i^{(2)})^2]\}, \tag{4-40}$$

a form which strikingly illustrates that $V_i|0\rangle$ is orthogonal to any state of <u>definite particle number</u>. (Crudely speaking, any such state $|\Psi\rangle$ cannot develop an infinity from $\langle\Psi|\exp\{C^{\dagger}(f_i^{(1)}) + D^{\dagger}(f_i^{(2)})\}|0\rangle$ to cancel the zero coming from the strongly divergent negative exponent.) Nonetheless, as discussed earlier, $V_i|0\rangle$ is normed, but it does not reside in Fock space. Consequently, the probability of finding n strong quarks and m strong antiquarks in a current quark,

$$|\langle n q_s, m\bar{q}_s | q_c\rangle|^2,$$

vanishes even if n and/or m are infinite. This state of affairs persists even if we quantize in a finite volume $(2L)^3$ leading to discrete momenta

$$\vec{k} = \vec{n}\pi L^{-1}, \tag{4-41}$$

where $\vec{n} = (n_x, n_y, n_z)$ are integers. With

$$\{a^{+(r)}_{\vec{k}}, a^{(s)}_{\vec{k}'}\} = \{b^{+(r)}_{\vec{k}}, b^{(s)}_{\vec{k}'}\} = \delta_{r,s}\ \delta_{\vec{k},\vec{k}'} \tag{4-42}$$

a shorthand for

$$\{a^{+(r)}(n_x, n_y, n_z), a^{(s)}(n_x, n_y, n_z)\}$$

$$= \{ b^{+(r)}(n_x, n_y, n_z), b^{(s)}(m_x, m_y, m_z) \} \tag{4-43}$$

$$= \delta_{r,s} \, \delta_{n_x, m_x} \, \delta_{n_y, m_y} \, \delta_{n_z, m_z}$$

the generators Y_i are given by

$$Y_i = \sum_{\vec{k}} Y_{i,\vec{k}}$$

$$= -i \sum_{\vec{k}} \left[\alpha_i(\vec{k}) C^+_{\vec{k}} + \beta_i(\vec{k}) D^+_{\vec{k}} - \alpha_i(\vec{k}) C_{\vec{k}} - \beta_i(\vec{k}) D_{\vec{k}} + i \gamma_i(\vec{k}) F_{\vec{k}} \right], \tag{4-44}$$

with

$$V_i = \prod_{\vec{k}} V_{i,\vec{k}} = \prod_{\vec{k}} e^{i Y_{i,\vec{k}}} \tag{4-45}$$

a bona fide infinite tensor product, each factor $V_{i,\vec{k}}$ of which can be put in "ordered" form

$$V_{i,\vec{k}} = \exp\{ f_i^{(1)}(\vec{k}) C^+_{\vec{k}} + f_i^{(2)}(\vec{k}) D^+_{\vec{k}} \} \exp\{ g_i^{(1)}(\vec{k}) C_{\vec{k}} + g_i^{(2)}(\vec{k}) D_{\vec{k}} \}$$

$$\cdot \exp\{ i h_i(\vec{k}) F_{\vec{k}} \} \exp\{ m_i(\vec{k}) K_{\vec{k}} \}, \tag{4-46}$$

where

$$K_{\vec{k}} = 2 - a^{+(+)}_{\vec{k}} a^{(+)}_{\vec{k}} - a^{+(-)}_{\vec{k}} a^{(-)}_{\vec{k}} - b^{+(+)}_{-\vec{k}} b^{(+)}_{-\vec{k}} - b^{+(-)}_{-\vec{k}} b^{(-)}_{-\vec{k}} \tag{4-47}$$

and all other functions the same as in Eqs.(4-34) - (4-39). Again,

$$V_i |0\rangle = \prod_{\vec{k}} \left[1 + (f_i^{(1)}(\vec{k}))^2 + (f_i^{(2)}(\vec{k}))^2 \right]^{-1} \exp\{ f_i^{(1)}(\vec{k}) C^+_{\vec{k}} + f_i^{(2)}(\vec{k}) D^+_{\vec{k}} \} |0\rangle \tag{4-48}$$

is orthogonal to all states of definite particle number unless a momentum cut-off, $|\vec{k}| \leqq \Lambda$, is introduced, which makes the number of degrees of freedom finite.

However, there are distributions which do not vanish even if $\Lambda \to \infty$, such as the average number of current quarks in a strong state. Even though these can be calculated directly by evaluating the appropriate matrix elements, it is more instructive to find these from the expansion of a strong quark state in terms of current quark states. Since

$$|q_s\rangle = V_i^{+} V_i |q_s\rangle \tag{4-49}$$

this expansion is found by evaluating $V_i|q_s\rangle$ and subsequently replacing each strong operator by a current operator, yielding

$$\begin{aligned} |q_s, \vec{p}, +, i\rangle &= a_{s,i}^{+(+)}(\vec{p})|0\rangle \\ &= \left[1 + (f_i^{(1)}(\vec{p}))^2 + (f_i^{(2)}(\vec{p}))^2\right]^{1/2} \left[1 + \frac{\gamma_i^2(\vec{p})}{\Gamma_i^2(\vec{p})} tg^2 \Gamma_i(\vec{p})\right]^{-1/2} \\ &\quad \left[a_{c,i}^{+(+)}(\vec{p}) - \frac{p_+ \gamma_i(\vec{p})}{|p_\perp| \Gamma_i(\vec{p})} tg\, \Gamma_i(\vec{p})\, a_{c,i}^{+(-)}(\vec{p})\right] V_i |0, i\rangle_c , \end{aligned} \tag{4-50}$$

where, by Eq. (4-48)

$$\begin{aligned} V_{FW} |0, FW\rangle_c = |0\rangle = \prod_{\vec{k}} \left[1 + (f_{FW}^{(1)}(\vec{k}))^2 + (f_{FW}^{(2)}(\vec{k}))^2\right] \Big\{ 1 \\ + f_{FW}^{(1)}(\vec{k}) \left[a_{c,FW}^{+(-)}(\vec{k}) b_{c,FW}^{+(+)}(-\vec{k}) - a_{c,FW}^{+(+)}(\vec{k}) b_{c,FW}^{+(-)}(-\vec{k})\right] e^{2iEx_0} \\ + \frac{1}{|k_\perp|} f_{FW}^{(2)}(\vec{k}) \left[k_+ a_{c,FW}^{+(-)}(\vec{k}) b_{c,FW}^{+(-)}(-\vec{k}) + k_- a_{c,FW}^{+(+)}(\vec{k}) b_{c,FW}^{+(+)}(-\vec{k})\right] e^{2iEx_0} \end{aligned} \tag{4-51}$$

$$- (f^{(1)}_{FW}(\vec{k}))^2 a^{+(-)}_{c,FW}(\vec{k}) b^{+(+)}_{c,FW}(-\vec{k}) a^{+(+)}_{c,FW}(\vec{k}) b^{+(-)}_{c,FW}(-\vec{k}) e^{4iEx_0}$$

$$+ (f^{(2)}_{FW}(\vec{k}))^2 a^{+(-)}_{c,FW}(\vec{k}) b^{+(-)}_{c,FW}(-\vec{k}) a^{+(+)}_{c,FW}(\vec{k}) b^{+(+)}_{c,FW}(-\vec{k}) e^{4iEx_0} \Big\} |0, FW\rangle_c ,$$

and

$$V_M |0, M\rangle_c = \prod_{\vec{k}} \left[1 + (f^{(1)}_M(\vec{k}))^2 + (f^{(2)}_M(\vec{k}))^2\right]^{-1} \left[1 + \frac{\gamma^2_M(\vec{k})}{\Gamma^2_M(\vec{k})} tg^2 \Gamma_M(\vec{k})\right]^{-1} \Big\{ 1$$

$$+ \frac{\omega \gamma_M(\vec{k})}{E\, \Gamma_M(\vec{k})} tg^2 \Gamma_M(\vec{k}) \left[a^{+(-)}_{c,M}(\vec{k}) b^{+(+)}_{c,M}(-\vec{k})\right.$$

$$\left. - a^{+(+)}_{c,M}(\vec{k}) b^{+(-)}_{c,M}(-\vec{k})\right] e^{2iEx_0}$$

$$- \frac{\omega}{E} \frac{1}{|k_\perp|} tg^2 \Gamma_M(\vec{k}) \left[k_+ a^{+(-)}_{c,M}(\vec{k}) b^{+(-)}_{c,M}(-\vec{k})\right. \tag{4-52}$$

$$\left. + k_- a^{+(+)}_{c,M}(\vec{k}) b^{+(+)}_{c,M}(-\vec{k})\right] e^{2iEx_0}$$

$$- \frac{\omega^2}{E^2} tg^2 \Gamma_M(\vec{k}) a^{+(-)}_{c,M}(\vec{k}) b^{+(+)}_{c,M}(-\vec{k}) a^{+(+)}_{c,M}(\vec{k}) b^{+(-)}_{c,M}(-\vec{k}) e^{4iEx_0} \Big\} |0, M\rangle_c .$$

The picture of a strong quark of momentum $\vec{p}$ as composed of current quarks, which emerges from Eq.(4-50), is that of a "leading particle" carrying the momentum $\vec{p}$ and a cloud of pairs, with momenta $\vec{k}$, $-\vec{k}$, clustering <u>about the origin</u> in momentum space (not about $\vec{p}$), with zero overlap on states containing a definite number of current quarks. This picture can be viewed from any frame, including the infinite momentum frame, and is not conspicuously amenable to a parton interpretation, if only because the "parts" do not follow the "whole". The latter effect is due to the fact that the transformation is bilinear in the fields and commutes with the momentum; for example, a transformation which is quadrilinear in the fields has a property of distributing the momentum of the strong quark among the particles it creates. It is interesting to note that the non-covariant cut-off procedure not only cures the zero overlap problem but also forces the choice of a natural frame in which to view the strong quark ($\vec{p}=0$), where the current quark pairs cluster about the "leading particle".

The probability $P(n_c, \vec{k}, 0_s, i)$ of finding in the vacuum, $|0\rangle$, n_c current quarks in the $\vec{k}$-th mode and anything else in other modes can be easily found from Eqs.(4-51) and (4-52),

$$P(0_c, \vec{k}, 0_s, i) = \left[1 + (f_i^{(1)}(\vec{k}))^2 + (f_i^{(2)}(\vec{k}))^2\right]^{-2}, \tag{4-53}$$

$$P(1_c, \vec{k}, 0_s, i) = 2\left[(f_i^{(1)}(\vec{k}))^2 + (f_i^{(2)}(\vec{k}))^2\right] P(0_c, \vec{k}, 0_s, i), \tag{4-54}$$

and

$$P(2_c, \vec{k}, 0_s, i) = \left[(f_i^{(1)}(\vec{k}))^2 + (f_i^{(2)}(\vec{k}))^2\right]^2 P(0_c, \vec{k}, 0_s, i). \tag{4-55}$$

(Due to the exclusion principle n_c cannot be larger than two.) This enables us to determine the average number of current quarks of momentum $\vec{k}$ in the vacuum

$$\langle N(c, \vec{k}, 0_s, i)\rangle = \langle 0| \sum_r a^{+(r)}_{c,i}(\vec{k})\, a^{(r)}_{c,i}(\vec{k}) |0\rangle$$

$$= \sum_{n_c} n_c P(n_c, \vec{k}, 0_s, i) \tag{4-56}$$

$$= 2\left[(f_i^{(1)}(\vec{k}))^2 + (f_i^{(2)}(\vec{k}))^2\right]\left[1 + (f_i^{(1)}(\vec{k}))^2 + (f_i^{(2)}(\vec{k}))^2\right]^{-1}.$$

Similarly, we can write down the probabilities $P(n_c, \vec{k}, 1_s, \vec{p}, i)$ of finding in a strong quark $|q_s, \vec{p}, +, i>$ n_c current quarks in the $\vec{k}$-th mode and anything else in other modes

$$P(0_c, \vec{k}, 1_s, \vec{p}, i) = (1 - \delta_{\vec{p},\vec{k}}) P(0_c, \vec{k}, 0_s, i), \tag{4-57}$$

$$P(1_c, \vec{k}, 1_s, \vec{p}, i) = \left[1 + (f_i^{(1)}(\vec{k}))^2 + (f_i^{(2)}(\vec{k}))^2\right]\delta_{\vec{p},\vec{k}} P(0_c, \vec{k}, 0_s, i)$$
$$+ (1 - \delta_{\vec{p},\vec{k}}) P(1_c, \vec{k}, 0_s, i), \tag{4-58}$$

and

$$P(2_c, \vec{k}, 1_s, \vec{p}, i) = \frac{1}{2}\left[1 + (f_i^{(1)}(\vec{k}))^2 + (f_i^{(2)}(\vec{k}))^2\right]\delta_{\vec{p},\vec{k}} P(1_c, \vec{k}, 0_s, i)$$
$$+ (1 - \delta_{\vec{p},\vec{k}}) P(2_c, \vec{k}, 0_s, i), \tag{4-59}$$

yielding the average number of current quarks of momentum $\vec{k}$ in a strong quark

$$\langle N(c, \vec{k}, 1_s, \vec{p}, i)\rangle = \sum_{n_c} n_c P(n_c, \vec{k}, 1_s, \vec{p}, i)$$
$$= \left[1 + (f_i^{(1)}(\vec{k}))^2 + (f_i^{(2)}(\vec{k}))^2\right]^{-1}\left[\delta_{\vec{p},\vec{k}} + 2(f_i^{(1)}(\vec{k}))^2 + 2(f_i^{(2)}(\vec{k}))^2\right]. \tag{4-60}$$

The fluctuations of number distributions $\mathcal{D} = \langle N^2\rangle - \langle N\rangle^2$ are given by

$$\mathcal{D}(c, \vec{k}, 0_s, i) = \mathcal{P}(1_c, \vec{k}, 0_s, i) \tag{4-61}$$

and

$$\mathcal{D}(c, \vec{k}, 1_s, \vec{p}, i) = (1 - \tfrac{1}{2}\delta_{\vec{p},\vec{k}})\mathcal{P}(1_c, \vec{k}, 0_s, i). \tag{4-62}$$

The values of the quantities (4-53), (4-54), (4-55), (4-56), (4-60), (4-61) and (4-62) for FW, Melosh and UZ transformations are given in Table 1. While the distribution of the current quarks for the FW case is spherically symmetric in momentum space, the transverse direction is prefered by the current pairs of Melosh, with none of them moving in the z-direction --- another aspect making the parton interpretation difficult, since a transverse momentum cut-off is assumed in most parton models[19,20]. Also the high momentum region is more populated by the pairs than the low one, with the maximum population reached at infinite momentum (infinite transverse momentum for Melosh and UZ). Without a cut-off, the average number of current quarks in a strong quark is infinite, even though the number of current quarks in each mode is finite. If a transverse momentum cut-off is introduced, the total number of current quarks in a strong quark becomes finite for the M- and UZ-transformations; it remains infinite for the FW-transformation.

Note, that since the spins are summed over, the distributions in Table 1 are insensitive to the spin rotations $\tilde{V}$ which, as discussed in Sec.I, connect the various W_i's. Thus no matter what the W_i, the form of the distribution depends only on the choice we make for the current quarks.

Distributions related to matrix elements of Fock space operators between a current and a strong state are meaningful only if Λ is finite. From Eq.(4-50), for example, we can read off the probability of finding a current quark of momentum $\vec{k}$ (and nothing else) in $|q_s, \vec{p}, +, i\rangle$

$$\sum_r |\langle q_c, \vec{k}, r, i | q_s, \vec{p}, +, i\rangle|^2$$

$$= \{1 + [f_i^{(1)}(\vec{k})]^2 + [f_i^{(2)}(\vec{k})]^2\} \delta_{\vec{p},\vec{k}} \; |\langle 0|V_i|0\rangle|^2,$$

the probability of finding a current quark of momentum $\vec{k}$ and a pair at a given momentum $\vec{k}^{(1)}$, $-\vec{k}^{(1)}$

$$\sum_{r,s,t} |{}_c\langle 0, i| a_{c,i}^{(r)}(\vec{k}) a_{c,i}^{(s)}(\vec{k}^{(1)}) b_{c,i}^{(t)}(-\vec{k}^{(1)}) | q_s, \vec{p}, +, i\rangle|^2$$

$$= \{1 + [f_i^{(1)}(\vec{k})]^2 + [f_i^{(2)}(\vec{k})]^2\} \delta_{\vec{p},\vec{k}}$$

$$\cdot 2\{[f_i^{(1)}(\vec{k}^{(1)})]^2 + [f_i^{(2)}(\vec{k}^{(1)})]^2\} \left(1 - \tfrac{1}{2}\delta_{\vec{k},\vec{k}^{(1)}}\right) |\langle 0|V_i|0\rangle|^2,$$

or the probability that $|q_s, \vec{p}, +, i\rangle$ will contain a current quark of momentum $\vec{k}$ accompanied by two current quark – antiquark pairs both at momenta $\vec{k}^{(1)}$, $-\vec{k}^{(1)}$

$$\sum_{r,s,t,l,m} |{}_c\langle 0, i| a_{c,i}^{(r)}(\vec{k}) a_{c,i}^{(s)}(\vec{k}^{(1)}) b_{c,i}^{(t)}(-\vec{k}^{(1)}) a_{c,i}^{(l)}(\vec{k}^{(1)}) b_{c,i}^{(m)}(-\vec{k}^{(1)}) | q_s, \vec{p}, +, i\rangle|^2$$

$$= \{1 + [f_i^{(1)}(\vec{k})]^2 + [f_i^{(2)}(\vec{k})]^2\} \delta_{\vec{p},\vec{k}}$$

$$\cdot \{[f_i^{(1)}(\vec{k}^{(1)})]^2 + [f_i^{(2)}(\vec{k}^{(1)})]^2\}^2 \left(1 - \delta_{\vec{k},\vec{k}^{(1)}}\right) |\langle 0|V_i|0\rangle|^2.$$

In general, there is an additional multiplicative factor of $2\{[f_i^{(1)}(\vec{k})]^2 + [f_i^{(2)}(\vec{k})]^2\}$ for each current quark - antiquark pair in the $\vec{k}$-th mode and a factor $\{[f_i^{(1)}(\vec{k})]^2 + [f_i^{(2)}(\vec{k})]^2\}^2$ for each double pair in the $\vec{k}$-th mode. The above quoted probability amplitudes strictly vanish unless a cut-off keeps the common factor

$$\langle 0|V_i|0\rangle = \prod_{\vec{k}} \{1 + [f_i^{(1)}(\vec{k})]^2 + [f_i^{(2)}(\vec{k})]^2\}^{-1}$$

from vanishing; they can be interpreted as relative probabilities, all of which vanish as $\Lambda \to \infty$, but with well defined ratios.

IV. CONCLUSIONS

We have studied a class of operators V which transform the charges of $SU(6)_{W,currents}$ into conserved quantities $W_a^i = VF_a^iV^\dagger$; the latter may be identified with the generators of an $SU(6)_{W,strong}$. The results are derived in the context of the free quark model in the equal time formulation. The transforms V, $V=\exp(iY)$, create pairs in all momentum modes and are thus not unitary in Fock space. When discussed in terms of a larger non-separable infinite tensor product space they map one separable space onto another separable space preserving the norm of the states. The V-transformed state, i.e. the image, is in general orthogonal to the object state. We have defined "strong quarks", $|q_s\rangle$, as single particle states with simple transformation properties under the W_a^i charges, and "current quarks", $|q_c\rangle$, by $|q_c\rangle = V^\dagger|q_s\rangle$ (this amounts to diagonalizing F^3 and $V^\dagger HV$ simmultaneously). Thus the current quarks, which have simple transformation properties under the $SU(6)_{W,currents}$, are orthogonal to the strong quarks unless a cut-off procedure is introduced limiting the degrees of freedom to a finite number. Such a cut-off, even though reminiscent of most parton models, is a non-covariant procedure and, in a spin 1/2 theory, it also implies a finite number of current quarks in a strong quark. The latter would be true in any reference frame since a boost in a free theory does not create any particles.

On the other hand, Fock space operators do have well defined matrix elements between members of the same equivalence class, or expressed differently, V-transformed Fock space operators remain well defined in Fock space. Consequently we find a finite non-vanishing mean number of current quarks of a given momentum in a strong quark, leading to distributions of a surprisingly rich structure and complexity. It therefore seemed tempting to speculate on their relevance to the parton model, keeping in mind that the transformations we studied induce symmetries in the absence of interactions and need not have properties more general than the framework from which they sprang.

As discussed in Sec.I the W_a^i 's we consider are related by a momentum dependent spin rotation. Since the distributions given in Tab.I are summed over the spins of the current quarks, the effect of this spin rotation is eliminated. The specific form of the distribution depends then only on the choice we make for the current quarks, or ultimately on the transformed Hamiltonian $V^\dagger HV$, which is the "energy" operator of the current quarks. No pairs are created in momentum modes not affected by the transformation; loosely speaking, pairs are produced "in proportion" to the difference within a given momentum mode between the forms of $V^\dagger HV$ and H. Thus no zero momentum pairs are created by V_{FW} , no pairs in the z-direction are created by V_M , and no $p_z \to \infty$ pairs from V_{UZ} . None of the distributions have a transverse momentum fall-off. Such a fall-off could be achieved by a transformation which does not affect large transverse momentum modes, but since the resulting transformed Hamiltonian would not commute with F_a^i , this transformation will not lead to an exact symmetry.

Another feature, which makes a parton interpretation difficult, is that the current quarks are produced in a disconnected manner, not sharing the momentum of the strong quark. This phenomenon can be traced to the bilinear character of the transformations. A quadrilinear transformation, if one could be found, would distribute the momentum of the strong quark among the current quarks, and it might still conserve momentum.

The complete hadronic distribution could be viewed as a distribution of constituent quarks in a hadron (the naive quark model wave function) convoluted with the distribution of the current quarks in a constituent quark. This approach has been suggested by Altarelli, Cabibbo, Maiani and Petronzio[21] in a different context. It turns out, however, that, due to the character of FW-type transformations, a transverse momentum cut-off for the current quarks cannot be achieved by any constituent wave function. Essentially, whatever the distribution of the strong quarks in a hadron, the number of current quarks is found to increase with transverse momentum. This result is also independent of the longitudinal momentum frame.

For these and other reasons cited in the text, the distributions we have calculated do not readily lend themselves to a parton interpretation. This is perhaps not surprising considering the lack of dynamics in the free quark model. The effect of interactions on V can be answered only if a more complete theory is at hand; one possibility, and one we investigated, was that V was closely approximated by the free quark model transformation, and that a realistic parton distribution might arise if only the convolution described above were performed. This did not turn out to be the case, which may indicate that V is particularly sensitive to dynamics. In fact, we are forced to conclude that any reasonable distribution requires that Y be at least quadrilinear in the fields, and thus even the algebraic structure of the transformations may be more complex than that suggested by the free quark model expressions.

APPENDIX A

GOOD AND BAD OPERATORS

In a discussion of amplitudes involving currents, the behavior of matrix elements at infinite momentum plays an important role; accordingly, the terms "good" and "bad" are used to classify operators. "Good" operators are those whose matrix elements do not vanish when taken between states moving at infinite momentum. Operators whose matrix elements vanish as p_z^{-1} are "bad", and those with matrix elements proportional to p_z^{-2} are sometimes called "terrible". A non-covariant normalization,

$$\langle \vec{p} \,|\, \vec{p}\,' \rangle = \delta_{\vec{p},\vec{p}\,'}, \tag{A-1}$$

is implied in these definitions.

These concepts are closely related to the saturation of commutators at infinite momentum. Let us, for example, consider an operator of the type

$$F^{\alpha} = \int d^3x \, : \underset{\sim}{q}^{+}(x) \, \Gamma^{\alpha} \underset{\sim}{q}(x) : \tag{A-2}$$

which, in the free theory, has the following form in momentum space

$$F^{\alpha} = \sum_{r,s} \sum_{\vec{k}} \frac{M}{E} \Big[a^{+(r)}_{\vec{k}} a^{(s)}_{\vec{k}} \, u^{+(r)}(\vec{k}) \, \Gamma^{\alpha} u^{(s)}(\vec{k})$$

$$- b^{+(s)}_{\vec{k}} b^{(r)}_{\vec{k}} \, v^{+(r)}(\vec{k}) \, \Gamma^{\alpha} v^{(s)}(\vec{k}) \Big) \tag{A-3}$$

$$+a^{+(r)}_{-\vec{k}}\, b^{+(s)}_{\vec{k}}\; u^{+(r)}(-\vec{k})\,\Gamma^{\alpha} v^{(s)}(\vec{k})\, e^{2iEx_0} \tag{A-3}$$

$$+b^{(r)}_{-\vec{k}}\, a^{(s)}_{\vec{k}}\; v^{+(r)}(-\vec{k})\,\Gamma^{\alpha} u^{(s)}(\vec{k})\, e^{-2iEx_0}.$$

Matrix elements of a commutator of two such operators

$$\langle\vec{p},m|[F^{\alpha}, F^{\beta}]|\vec{p}',n\rangle,$$

where m, n are spin (SU(3)) indices, are saturated by one- and three-particle intermediate states, with the latter leading to so called z-graphs. Thus

$$\langle\vec{p},m|[F^{\alpha},F^{\beta}]|\vec{p}',n\rangle = \sum_{\vec{p}'',i}\langle\vec{p},m|F^{\alpha}|\vec{p}'',i\rangle\langle\vec{p}'',i|F^{\beta}|\vec{p}',n\rangle$$

$$+\sum_{\substack{\vec{p}^{(1)},\vec{p}^{(2)},\vec{p}^{(3)}\\ i,\,j,\,k}}\langle\vec{p},m|F^{\alpha}|\vec{p}^{(1)},i;\vec{p}^{(2)},j;\vec{p}^{(3)},k\rangle\langle\vec{p}^{(1)},i;\vec{p}^{(2)},j;\vec{p}^{(3)},k|F^{\beta}|\vec{p}',n\rangle$$

$$-(\alpha\leftrightarrow\beta). \tag{A-4}$$

The one-particle contribution is given by

$$\frac{M^2}{E^2}\sum_{i} u^{+(m)}(\vec{p})\,\Gamma^{\alpha} u^{(i)}(\vec{p})\, u^{+(i)}(\vec{p})\,\Gamma^{\beta} u^{(n)}(\vec{p})\;\delta_{\vec{p},\vec{p}'} \tag{A-5}$$

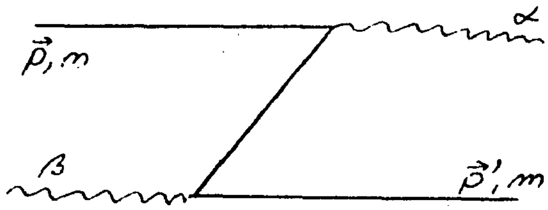
$$-(\alpha\leftrightarrow\beta),$$

while the three-particle contribution reads

$$-2\frac{M^2}{E^2}\sum_{r} u^{+(m)}(\vec{p})\,\Gamma^{\beta} v^{(r)}(-\vec{p})\, v^{+(r)}(-\vec{p})\,\Gamma^{\alpha} u^{(n)}(\vec{p})\;\delta_{\vec{p},\vec{p}'} \tag{A-6}$$

$$-(\alpha\leftrightarrow\beta)$$

and corresponds to a z-graph



At $p_z \to \infty$ $u^{(r)}(\vec{p})$ and $v^{(r)}(\vec{p})$ become eigenstates of α^3,

$$\alpha^3 u^{(r)}(\vec{p}) \sim \frac{p^3}{|p^3|} u^{(r)}(\vec{p}), \tag{A-7}$$

and

$$\alpha^3 v^{(r)}(\vec{p}) \sim \frac{p^3}{|p^3|} v^{(r)}(\vec{p}). \tag{A-8}$$

Then

$$u^{+(m)}(\vec{p})\Gamma^{\beta} v^{(r)}(-\vec{p}) \to u^{+(m)}(\vec{p})[\alpha^3, \Gamma^{\beta}] v^{(r)}(-\vec{p}), \tag{A-9}$$

$$v^{+(r)}(-\vec{p})\Gamma^{\alpha} u^{(m)}(\vec{p}) \to v^{+(r)}(-\vec{p})[\Gamma^{\alpha}, \alpha^3] u^{(m)}(\vec{p}), \tag{A-10}$$

and the contribution of the three-particle intermediate states vanishes if at least one of the operators Γ^{α}, Γ^{β} commutes with α^3. The matrix element

$$\langle \vec{p}, m | F^{\alpha} | \vec{p}', m \rangle = \frac{M}{E} u^{+(m)}(\vec{p}) \Gamma^{\alpha} u^{(m)}(\vec{p}) \delta_{\vec{p}, \vec{p}'} \tag{A-11}$$

is at $p_z \to \infty$ given by the anticommutator of Γ^{α} with α^3

$$u^{+(m)}(\vec{p})\Gamma^{\alpha} u^{(m)}(\vec{p}) \to u^{+(m)}(\vec{p})\{\Gamma^{\alpha}, \alpha^3\} u^{(m)}(\vec{p}) \tag{A-12}$$

and vanishes if $\{\Gamma^{\alpha}, \alpha^3\} = 0$. By the same reasoning, the single particle contribution to the commutator vanishes if any of the operators involved anticommute with α^3,

To summarize, operators are

a) good if $[\Gamma,\alpha^3] = 0$; their matrix elements and their contribution from one-paricle intermediate states is of order one, and their contribution from three-particle intermediate states is of order p_z^{-1},

b) bad if $\{\Gamma,\alpha^3\} = 0$; their matrix elements and their contribution from one-particle intermediate states are of order p_z^{-1}, while their contribution from three-particle intermediate states is of order one,

c) sometimes called terrible if they are given by $(1-\alpha^3)\Gamma'$, where $[\Gamma',\alpha^3] = 0$, their matrix elements are of order p_z^{-2} and thus both their one- and three-particle contributions vanish as $\Lambda \to \infty$.

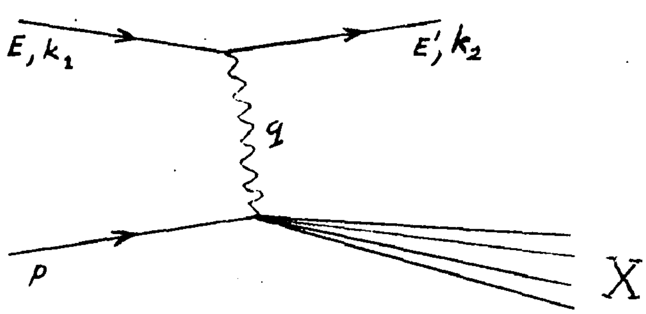
APPENDIX B

THE PARTON MODEL

One of the original motivations for this investigation was the clarification of the connection, if any, between current quarks and partons. It is therefore appropriate at this point to review and summerize those aspects of the parton model which are relevant in this context. We shall follow the presentation of Ref.19.

According to the parton model a hadron is composed of point-like constituents called partons. It is assumed that the interactions confine the partons in a finite region of the momentum space and, consequently, the larger the momentum P of the hadron, the smaller the relative importance of the transverse momenta of the partons. The relativistic transformation to large momenta P also dilates the interaction times and as $P \to \infty$ the partons behave as free particles (on mass shell).

The parton model received its greatest impetus from its success in explaining deep inelastic electron scattering, $e+p \to e+X$.



In this process the incident electron scatters off the proton target, a virtual photon of energy $\nu = E - E'$ (in the laboratory frame) and mass $q^2 < 0$ is exchanged, and in the final state only the electron is observed. The cross section is proportional to

$$M^2 = (\bar{u}_1 \gamma_\nu u_2)(\bar{u}_2 \gamma_\mu u_1)\left(\frac{4\pi e^2}{q^2}\right)^2 \cdot \sum_X \langle p | J_\nu(-q) | X \rangle \langle X | J_\mu(q) | p \rangle 2\pi \delta(M_x^2 - (p+q)^2). \tag{B-1}$$

and depends on two invariants, q^2 and $P \cdot q = M\nu$, where P is the four-momentum of the proton and M its mass. When summed over the spins the electron vertex contributes

$$\mathrm{Tr}(\bar{u}_1 \gamma_\nu u_2)(\bar{u}_2 \gamma_\mu u_1) = 2(k_{1\mu} k_{2\nu} + k_{1\nu} k_{2\mu} - g_{\mu\nu} k_1 \cdot k_2). \tag{B-2}$$

The hadron vertex

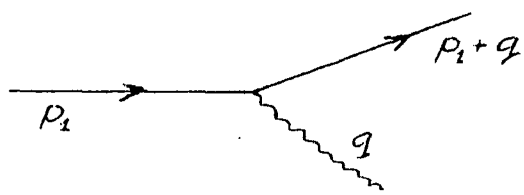
$$K_{\mu\nu} = \sum_X \langle p | J_\nu(-q) | X \rangle \langle X | J_\mu(q) | p \rangle 2\pi \delta(M_x^2 - (p+q)^2) \tag{B-3}$$

is, for unpolarized protons, symmetric in μ,ν and must have a form

$$\frac{M}{\pi} K_{\mu\nu} = 4 W_2(\nu, q^2)\left(p_\mu - q_\mu \frac{p \cdot q}{q^2}\right)\left(p_\nu - q_\nu \frac{p \cdot q}{q^2}\right) - 4 W_1(\nu, q^2) M^2 \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right). \tag{B-4}$$

Bjorken has suggested that the structure functions W_1 and νW_2 depend, for large ν and q^2, on one variable only, the scaling variable $\omega = -2M\ /q^2 = 1/x$. This hypothesis is born out by the data.

The parton model, as we shall show below, has this scaling feature "built in". When the proton is hit by the photon of momentum q, the latter interacts with a parton of some momentum p_1 , which is then scattered at a momentum p_1+q. This scattering is assumed to be



incoherent and thus the total cross section is given by the sum of cross sections of individual partons. If the mass of the parton is m^2 then

$$\frac{1}{2E} K_{\mu\nu} = \sum \frac{1}{2E_1} |M|^2\, 2\pi\, \delta\left(m^2 - (p_1+q)^2\right), \tag{B-5}$$

where E and E_1 are included because of normalization. For spin 1/2 partons $|M^2|$ is given by

$$\begin{aligned} |M|^2 &= \frac{1}{2} \cdot Tr \left\{ (\not{p}_1 + m)\gamma_\mu (\not{p}_2 + m)\gamma_\nu \right\} \\ &= 2\left(p_{1\mu} p_{2\nu} + p_{1\nu} p_{2\mu} - 2 g_{\mu\nu}\, p_1 \cdot q \right). \end{aligned} \tag{B-6}$$

If $p_1 = \xi P$, where ξ is a number between zero and one, which signifies the portion of P carried by a given parton, $|M^2|$ becomes

$$|M|^2 = 4\xi^2 P_\mu P_\nu + 2\xi (P_\mu q_\nu + P_\nu q_\mu) - 2 g_{\mu\nu}\, \xi\, P \cdot q\,. \tag{B-7}$$

(Note that the transverse momenta have been neglected.) Let us also introduce $f(x)dx$, the number of partons with momenta between x and $x+dx$, weighed by their charge squared, and use Eq.(B-4) to rewrite Eq.(B-5) as

$$
\begin{aligned}
K_{\mu\nu} &= \frac{\pi}{M}\Big\{4W_2\Big(P_\mu - q_\mu\frac{P\cdot q}{q^2}\Big)\Big(P_\nu - q_\nu\frac{P\cdot q}{q^2}\Big) \\
&\quad - 4W_1 M^2\Big(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\Big) \\
&= \int d\xi\, f(\xi)\,\frac{E}{E_1}\,|M|^2\, 2\pi\,\delta\big((\xi P+q)^2 - m^2\big)
\end{aligned}
\tag{B-8}
$$

The argument of the δ-function can be expressed in terms of the variable x introduced earlier

$$
\begin{aligned}
(\xi P+q)^2 - m^2 &= \xi^2 M^2 + 2\xi P\cdot q + q^2 - m^2 \\
&= \xi^2 M^2 - m^2 + 2M\nu(\xi - x).
\end{aligned}
\tag{B-9}
$$

As $\nu \to \infty$ and $q^2 \to -\infty$ with x fixed, the mass terms in Eq.(B-9) can be neglected and the δ-function now reads

$$
\delta(2M\nu(\xi - x)) = \frac{1}{2M\nu}\,\delta(\xi - x). \tag{B-10}
$$

Since $E/E_1 = 1/\xi$ we obtain from Eqs.(B-7) and (B-8)

$$
\begin{aligned}
&4W_2\Big(P_\mu - q_\mu\frac{P\cdot q}{q^2}\Big)\Big(P_\nu - q_\nu\frac{P\cdot q}{q^2}\Big) - 4W_1 M^2\Big(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\Big) \\
&= \frac{f(x)}{\nu}\Big\{4x P_\mu P_\nu + 2(P_\mu q_\nu + P_\nu q_\mu) - 2g_{\mu\nu} M\nu\Big\},
\end{aligned}
\tag{B-11}
$$

where we have also used the relation $P \cdot q = M\nu$. If the two sides of this equation are compared to each other one obtains the scaling predictions

$$\nu W_2(\nu, q^2) = x f(x), \tag{B-12}$$

and

$$2M W_1(\nu, q^2) = f(x) . \tag{B-13}$$

Experimentally, the νW_2 approaches a constant, .32 , as $x \to 0$, which means that $f(x)$ goes as $.32/x$ at $x \to 0$. Consequently, the number of partons diverges logarithmically as $P \to \infty$.

APPENDIX C

"EXPONENTIAL ORDERING"

As mentioned in Sec. III, all the transformations we analyze can be written as

$$\exp\{C^{\dagger}(\alpha)+D^{\dagger}(\beta)-C(\alpha)-D(\beta)+iF(\gamma)\} \tag{C-1}$$

with the symbols defined in Eqs. (4-17) - (4-21). If f_1 , f_2 , f_3 , f_4 , f_5 , f_6 are some arbitrary functions of momenta, the algebra of the set of operators $C^{\dagger}(f_1)$, $D^{\dagger}(f_2)$, $C(f_3)$, $D(f_4)$, $F(f_5)$ enlarged by $K(f_6)$, $K(fg) \equiv [C(f), C^{\dagger}(g)]$, closes and their commutation relations are as follows

$$[C(f), D(g)] = [C^{\dagger}(f), D^{\dagger}(g)] = 0, \tag{C-2}$$

$$[C(f), C^{\dagger}(g)] = [D(f), D^{\dagger}(g)] = K(fg), \tag{C-3}$$

$$[K(f), C(g)] = 2C(fg), \tag{C-4}$$

$$[K(f), C^{\dagger}(g)] = -2C^{\dagger}(fg), \tag{C-5}$$

$$[K(f), D(g)] = 2D(fg), \tag{C-6}$$

$$[K(f), D^{\dagger}(g)] = -2D^{\dagger}(fg), \tag{C-7}$$

$$[D(f), C^\dagger(g)] = iF(fg), \tag{C-8}$$

$$[C(f), D^\dagger(g)] = -iF(fg), \tag{C-9}$$

$$[F(f), C^\dagger(g)] = -2iD^\dagger(fg), \tag{C-10}$$

$$[F(f), C(g)] = -2iD(fg), \tag{C-11}$$

$$[F(f), D^\dagger(g)] = 2iC^\dagger(fg), \tag{C-12}$$

$$[F(f), D(g)] = 2iC(fg), \tag{C-13}$$

$$[K(f), F(g)] = 0. \tag{C-14}$$

The operators C, D and F annihilate the vacuum, $C^\dagger$ and $D^\dagger$ do not. K contains a c-number part and a part which annihilates the vacuum.

We assert that Eq.(C-1) can be written in an "ordered" form

$$\begin{aligned} &\exp\{C^\dagger(\alpha) + D^\dagger(\beta) - C(\alpha) - D(\beta) + iF(\gamma)\} \\ &= \exp\{C^\dagger(f_1) + D^\dagger(f_2)\} \exp\{C(g_1) + D(g_2)\} \exp\{iF(h)\} \exp\{K(m)\}. \end{aligned} \tag{C-15}$$

This is a generalization of the well known identity

$$e^{A+B} = e^A e^B e^{-\frac{1}{2}[A,B]}, \tag{C-16}$$

which holds only if A and B commute with $[A,B]$. In order to relate the functions f, g, h and m to the known functions α, β, and γ, we the functions f, g, h and m to the known functions α, β, and γ, we introduce a real parameter λ and define

$$G(\lambda) = \exp\{K(m)\}$$

$$= \exp\{-iF(h)\}\exp\{-C(g_1)-D(g_2)\}\exp\{-C^\dagger(f_1)-D^\dagger(f_2)\} \qquad \text{(C-17)}$$

$$\cdot \exp\{\lambda[C^\dagger(\alpha)+D^\dagger(\beta)-C(\alpha)-D(\beta)+iF(\gamma)]\},$$

where f, g, h and m are now functions of both λ and the momentum. Then

$$G'(\lambda) = \frac{d}{d\lambda}G(\lambda) = K(m')G(\lambda)$$

$$= \{-iF(h') - \exp\{-iF(h)\}[C(g_1')+D(g_2')]\exp\{iF(h)\}$$

$$-\exp\{-iF(h)\}\exp\{-C(g_1)-D(g_2)\}[C^\dagger(f_1')+D^\dagger(f_2')]\exp\{C(g_1)+D(g_2)\}\exp\{iF(h)\}$$

$$+\exp\{-iF(h)\}\exp\{-C(g_1)-D(g_2)\}\exp\{-C^\dagger(f_1)-D^\dagger(f_2)\}[C^\dagger(\alpha)+D^\dagger(\beta)-C(\alpha)$$

$$-D(\beta)+iF(\gamma)]\exp\{C^\dagger(f_1)+D^\dagger(f_2)\}\exp\{C(g_1)+D(g_2)\}\exp\{iF(h)\}\}G(\lambda)$$

$$= C^\dagger\{[G_1-g_1'+g_1(f_1'g_1+f_2'g_2)-g_2(f_1'g_2-f_2'g_1)]\cos 2h$$

$$+[G_2-g_2'+g_2(f_1'g_1+f_2'g_2)+g_1(f_1'g_2-f_2'g_1)]\sin 2h \qquad \text{(C-18)}$$

$$+D^\dagger\{[G_2-g_2'+g_2(f_1'g_1+f_2'g_2)+g_1(f_1'g_2-f_2'g_1)]\cos 2h$$

$$-[G_1 - g_1' + g_1(f_1'g_1 + f_2'g_2) - g_2(f_1'g_2 - f_2'g_1)]\sin 2h\}$$

$$+ C^+[(F_1 - f_1')\cos 2h + (F_2 - f_2')\sin 2h]$$

$$+ D^+[(F_2 - f_2')\cos 2h - (F_1 - f_1')\sin 2h]$$

$$+ iF[H - h' + f_1'g_2 - f_2'g_1]$$

$$+ K[M + f_1'g_1 + f_2'g_2]\} G(\lambda),$$

where

$$F_1 = \alpha + 2f_2\gamma + f_1(f_1\alpha + f_2\beta) + f_2(f_1\beta - f_2\alpha), \qquad \text{(C-19)}$$

$$F_2 = \beta + 2f_1\gamma + f_2(f_1\alpha + f_2\beta) - f_1(f_1\beta - f_2\alpha), \qquad \text{(C-20)}$$

$$G_1 = -\alpha - g_1[(g_1\alpha + g_2\beta) + 2(f_1\alpha + f_2\beta)] - g_2[2\gamma + g_1\beta - g_2\alpha$$

$$- 2(f_1\beta - f_2\alpha)] - (f_1\alpha + f_2\beta)[g_1(f_1g_1 + f_2g_2) - g_2(f_1g_2 - f_2g_1)] \qquad \text{(C-21)}$$

$$- (2\gamma - f_1\beta - f_2\alpha)[g_1(f_1g_2 - f_2g_1) + g_2(f_1g_1 + f_2g_2)],$$

$$G_2 = -\beta + g_1[2\gamma + g_1\beta - g_2\alpha - 2(f_1\beta - f_2\alpha)] - g_2[(g_1\alpha + g_2\beta)$$

$$+ 2(f_1\alpha + f_2\beta)] - (f_1\alpha + f_2\beta)[g_2(f_1g_1 + f_2g_2) + g_1(f_1g_2 - f_2g_1)] \qquad \text{(C-22)}$$

$$+(2\gamma - f_1\beta + f_2\alpha)[g_1(f_1g_1 + f_2g_2) - g_2(f_1g_2 - f_2g_1)],$$

$$H = \gamma - (f_1\beta - f_2\alpha) + (g_1\beta - g_2\alpha) - (f_1\alpha + f_2\beta)(f_1g_2 - f_2g_1)$$

$$+ (2\gamma - f_1\beta + f_2\alpha)(f_1g_1 + f_2g_2), \qquad \text{(C-23)}$$

$$M = -(f_1\alpha + f_2\beta) - (g_1\alpha + g_2\beta) - (f_1\alpha + f_2\beta)(f_1g_1 + f_2g_2)$$

$$+ (2\gamma - f_1\beta + f_2\alpha)(f_1g_2 - f_2g_1). \qquad \text{(C-24)}$$

Because of the linear independence of the operators C, D, C , D , F and K, this leads to six first order linear differential equations

$$F_1 - f_1' = 0, \qquad \text{(C-25)}$$

$$F_2 - f_2' = 0, \qquad \text{(C-26)}$$

$$G_1 - g_1' + g_1(f_1'g_1 + f_2'g_2) - g_2(f_1'g_2 - f_2'g_1) = 0, \qquad \text{(C-27)}$$

$$G_2 - g_2' + g_2(f_1'g_1 + f_2'g_2) + g_1(f_1'g_2 - f_2'g_1) = 0, \qquad \text{(C-28)}$$

$$H - h' + f_1'g_2 - f_2'g_1 = 0, \qquad \text{(C-30)}$$

and

$$M - m' + f_1'g_1 + f_2'g_2 = 0, \qquad \text{(C-31)}$$

which have a unique solution if the six boundary conditions at $\lambda = 0$,

$$f_1(0) = f_2(0) = g_1(0) = g_2(0) = h(0) = m(0) = 0 \tag{C-32}$$

are imposed. The solutions are

$$f_1 = (\Gamma^2 + \gamma^2 \operatorname{tg}^2 \lambda\Gamma)^{-1} (\alpha\Gamma - \beta\gamma \operatorname{tg} \lambda\Gamma) \operatorname{tg} \lambda\Gamma, \tag{C-33}$$

$$f_2 = (\Gamma^2 + \gamma^2 \operatorname{tg}^2 \lambda\Gamma)^{-1} (\beta\Gamma + \alpha\gamma \operatorname{tg} \lambda\Gamma) \operatorname{tg} \lambda\Gamma, \tag{C-34}$$

$$g_1 = -\Gamma^{-2} (1 + \operatorname{tg}^2 \lambda\Gamma)^{-1} (\alpha\Gamma - \beta\gamma \operatorname{tg} \lambda\Gamma) \operatorname{tg} \lambda\Gamma, \tag{C-35}$$

$$g_2 = -\Gamma^{-2} (1 + \operatorname{tg}^2 \lambda\Gamma)^{-1} (\beta\Gamma + \alpha\gamma \operatorname{tg} \lambda\Gamma) \operatorname{tg} \lambda\Gamma, \tag{C-36}$$

$$h = \arctan\left(\frac{\gamma}{\Gamma} \operatorname{tg} \lambda\Gamma\right), \tag{C-37}$$

and

$$m = \frac{1}{2} \lg\left[\Gamma^{-2}(1 + \operatorname{tg}^2 \lambda\Gamma)^{-1}(\Gamma^2 + \gamma^2 \operatorname{tg}^2 \lambda\Gamma)\right] = -\frac{1}{2} \lg(1 + f_1^2 + f_2^2), \tag{C-38}$$

where $\Gamma = (\alpha^2+\beta^2+\gamma^2)^{1/2}$. Eq.(C-17) now holds for all values of λ, but we use it only at $\lambda = 1$ to obtain Eq.(C-15).

Let us return to the commutation relations (C-2) - (C-14). If we choose $F = 0$ and $\alpha = \beta$, the relevant operators $(C+D)/\sqrt{2}$, $(C^\dagger+D^\dagger)/\sqrt{2}$ and $K/2$ satisfy SU(2) commutation relations

$$\left[\frac{1}{\sqrt{2}}(C+D)(f), \frac{1}{\sqrt{2}}(C^\dagger+D^\dagger)(g)\right] = K(fg), \tag{C-39}$$

$$\left[\frac{1}{\sqrt{2}}(C+D)(f), \frac{1}{2}K(g)\right] = -\frac{1}{\sqrt{2}}(C+D)(fg), \tag{C-40}$$

and

$$\left[\frac{1}{\sqrt{2}}(C^+ + D^+)(f), \frac{1}{2}K(g)\right] = \frac{1}{\sqrt{2}}(C^+ + D^+)(fg). \tag{C-41}$$

These operators can thus be identified with the operators of angular momentum, for example

$$\frac{1}{\sqrt{2}}(C + D) \leftrightarrow iJ_+, \tag{C-42}$$

$$\frac{1}{\sqrt{2}}(C^+ + D^+) \leftrightarrow -iJ_-, \tag{C-43}$$

and

$$\frac{1}{2}K \longleftrightarrow J_z. \tag{C-44}$$

If we also choose $\alpha = \beta = i/\sqrt{2}$ ($\gamma=0$ is understood) we find from Eqs.(C-33) - (C-38)

$$f_1 = f_2 = \frac{i}{\sqrt{2}} \tanh \lambda, \tag{C-45}$$

$$g_1 = g_2 = -\frac{i}{\sqrt{2}} \sinh \lambda \cosh \lambda, \tag{C-46}$$

and

$$m = \lg \cosh \lambda, \tag{C-47}$$

leading to a remarkable identity

$$e^{2\lambda J_x} = e^{(\tanh \lambda) J_-} e^{(\sinh \lambda \cosh \lambda) J_+} e^{2(\lg \cosh \lambda) J_z}. \tag{C-48}$$

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quantity \ i	FW	M	UZ
$\mathcal{P}(0_c, \vec{k}, 0_s, i)$	$\left(\frac{E+M}{2E}\right)^2$	$\left(\frac{\omega(\omega+M)}{2E^2} + \frac{k_z^2}{E^2}\right)^2$	$\left(\frac{E+\lvert k_z\rvert}{2E}\right)^2$
$\mathcal{P}(1_c, \vec{k}, 0_s, i)$	$\frac{\lvert k\rvert^2}{2E^2}$	$\frac{\lvert k_\perp\rvert^2\omega}{E^2(\omega+M)}\left(\frac{\omega(\omega+M)}{2E^2} + \frac{k_z^2}{E^2}\right)$	$\frac{\omega^2}{2E^2}$
$\mathcal{P}(2_c, \vec{k}, 0_s, i)$	$\left(\frac{\lvert k\rvert^2}{2E(E+M)}\right)^2$	$\left(\frac{\lvert k_\perp\rvert^2\omega}{2E^2(\omega+M)}\right)^2$	$\frac{\omega^2}{2E(E+\lvert k_z\rvert)}$
$\langle N(c, \vec{k}, 0_s, i)\rangle$	$1-\frac{M}{E}$	$\frac{\lvert k_\perp\rvert^2\omega}{E^2(\omega+M)}$	$1-\frac{\lvert k_z\rvert}{E}$
$\mathcal{D}(c, \vec{k}, 0_s, i)$	$\frac{\lvert k\rvert^2}{2E^2}$	$\frac{\lvert k_\perp\rvert^2\omega}{E^2(\omega+M)}\left(\frac{\omega(\omega+M)}{2E^2} + \frac{k_z^2}{E^2}\right)$	$\frac{\omega^2}{2E^2}$
$\langle N(c, \vec{k}, 1_s, \vec{p}, i)\rangle$	$\frac{E+M}{2E}\left(\delta_{\vec{k},\vec{p}} + 2\frac{\lvert k\rvert^2}{(E+M)^2}\right)$	$\left(\frac{\omega(\omega+M)}{2E^2} + \frac{k_z^2}{E^2}\right)\left(\delta_{\vec{k},\vec{p}} + \frac{\lvert k_\perp\rvert^2\omega}{E^2(\omega+M)}\right)$	$\frac{E+\lvert k_z\rvert}{2E}\left(\delta_{\vec{k},\vec{p}} + \frac{2\omega^2}{(E+\lvert k_z\rvert)^2}\right)$
$\mathcal{D}(c, \vec{k}, 1_s, \vec{p}, i)$	$\frac{\lvert k\rvert^2}{2E^2}\left(1-\frac{1}{2}\delta_{\vec{k},\vec{p}}\right)$	$\frac{\lvert k_\perp\rvert^2\omega}{E^2(\omega+M)}\left(\frac{\omega(\omega+M)}{2E^2} + \frac{k_z^2}{E^2}\right)\left(1-\frac{1}{2}\delta_{\vec{k},\vec{p}}\right)$	$\frac{\omega^2}{2E^2}\left(1-\frac{1}{2}\delta_{\vec{k},\vec{p}}\right)$

Table 1. Pair Distributions As a Function of Momentum