# A PSYCHOPHYSICAL STUDY OF THE JOINT EXTRAPOLATION OF TWO INTERSECTING STRAIGHT LINES AS A FUNCTION OF DISTANCE, VELOCITY, AND ANGLE

DISSERTATION

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## INTRODUCTION

Many life situation judgments concern the extrapolation of pairs of converging objects such as approaching pedestrians, approaching vehicles at an intersection, approaching targets on a radar display. Common to each is information as to (a) speeds of the objects, (b) distances of the objects from their intersection, and (c) their angle of intersection. This choice of categories, however, is not unique, because no single classification scheme appears dictated on _a priori_ grounds. Although certain minimum amounts of information are necessary for a strictly "mathematical" (as versus "judgmental") solution, the form this information takes may be quite varied. For example, equally correct mathematical solutions may be obtained using cartesian coordinates, polar coordinates, vectors, or any of several relations between sides and angles of any plane triangle. Presumably some mathematical systems are more applicable than others. If the couching of the variables according to one system failed to yield a satisfactory relation to human performance, another system might legitimately be tried. There is another way of posing the problem: What sort of mathematical system might the human be said to employ? When the investigator has been able to construct, according to some system, a mathematical model that predicts empirically observed behavior, he is inclined to believe he has at least a partial answer to the question.

A typical extrapolation judgment is the prediction as to whether the objects (pedestrians, vehicles, targets, etc.) will eventually collide as a result of simultaneous arrival at their paths' point of intersection. To provide some immediate background it might be pointed out that the present research grew out of the use of radar displays in the control of air traffic. Here one of the typical tasks of the operator is to recognize, from his radar display, when two aircraft are in danger of colliding and to take corrective steps. In a characteristic situation the operator sees a pair of converging dots of light (or "blips") on his radar scope. The leading dots represent the present positions of the aircraft. The other dots represent former positions of the aircraft. Since the dots represent successive time samples, the speed of the aircraft is given by the distance between successive dots. The direction (or course) of the aircraft is represented by the direction in which the dots are pointing. Although aircraft in flight are in continuous motion, their display on a radar scope is discrete, and when an operator glances at a radar scope he sees essentially a static display.

In the present study the collision situation in radar has been abstracted in general terms to the point where the problem is a much broader one and is related theoretically to several areas of general experimental psychology as discussed below.

Already raised is the question as to the analogy between a mathematical formula and the human's mode of performance. Subsumed under this are other questions. What kind of errors are made in such a perceptual task? How do the errors relate to the stimulus dimensions of the task? What part of the total error related to any single dimension? What are the dimensions? How do the errors combine?

The present study is also related to the work on visual illusions, within the area of perception. The general findings and theory in this area are discussed elsewhere (for example, see 2 and 10). In the studies of geometric illusions it is not common to find a single mathematical formula for the results associated with any one figure and its variants. However, in 1895 Heymans (6) took a step in this direction for the Muller-Lyer figure when he found that the amount of illusion was proportional to the cosine of the angle between the oblique and horizontal sections.

A limited amount of research has been conducted in the area of immediate interest for the present problem. Bowen and Woodhead (3) in a paper and pencil test had Os (observers) judge single radar-type trails approaching a line. Error in judging the number of dots required to fill in the distance between the leading blip and the line were found to increase with distance, but were not influenced by trail length. In another study ( ) using a somewhat more realistic display, pairs of trails, and collision judgments, trail length did not have a significant effect, but angle of intersection and distance to intersection did. The latter study was primarily concerned with the effects of the number of dots in the trail (i.e., phosphor persistence) and the remaining variables were employed only to lend generality to the major-variable finding. In a task requiring a prediction as to which of two parallel targets would reach a line first, Schipper and Versace (8) found significant effects associated with distances from the line and speeds, but no effect due to dot size, dot sharpness, or scope size.

Out of some of these studies grew the suspicion that several variables not previously investigated in their own right might account for many of the observed findings. These variables were angle, distances, and speeds (or their equivalents expressed in some other system). When the present research was in the planning stage various mathematical models were constructed for the purpose of predicting man's performance in extrapolating pairs of lines (i.e., judging whether objects would collide). The practice followed in constructing these models was first to write some mathematical equation which was selected on an intuitive and best-judgment basis. Such an equation represented the precise mathematical relationship among the variables concerned. Since the human system, like other systems, is subject to errors in operating, ways were explored for subjecting to variability (or error) the separate terms or members in the equation already written. The models obtained were then examined in terms of the ease and kind of research required to verify and modify them. The value of such a practice was thus heuristic. Instead of merely throwing some values together ("blind empiricism", if it indeed really exists), by using the models it was possible to exercise more judgment in the experimental design. For example, the experimenter could arrange to obtain more data at points along a curve where rapid acceleration was expected.

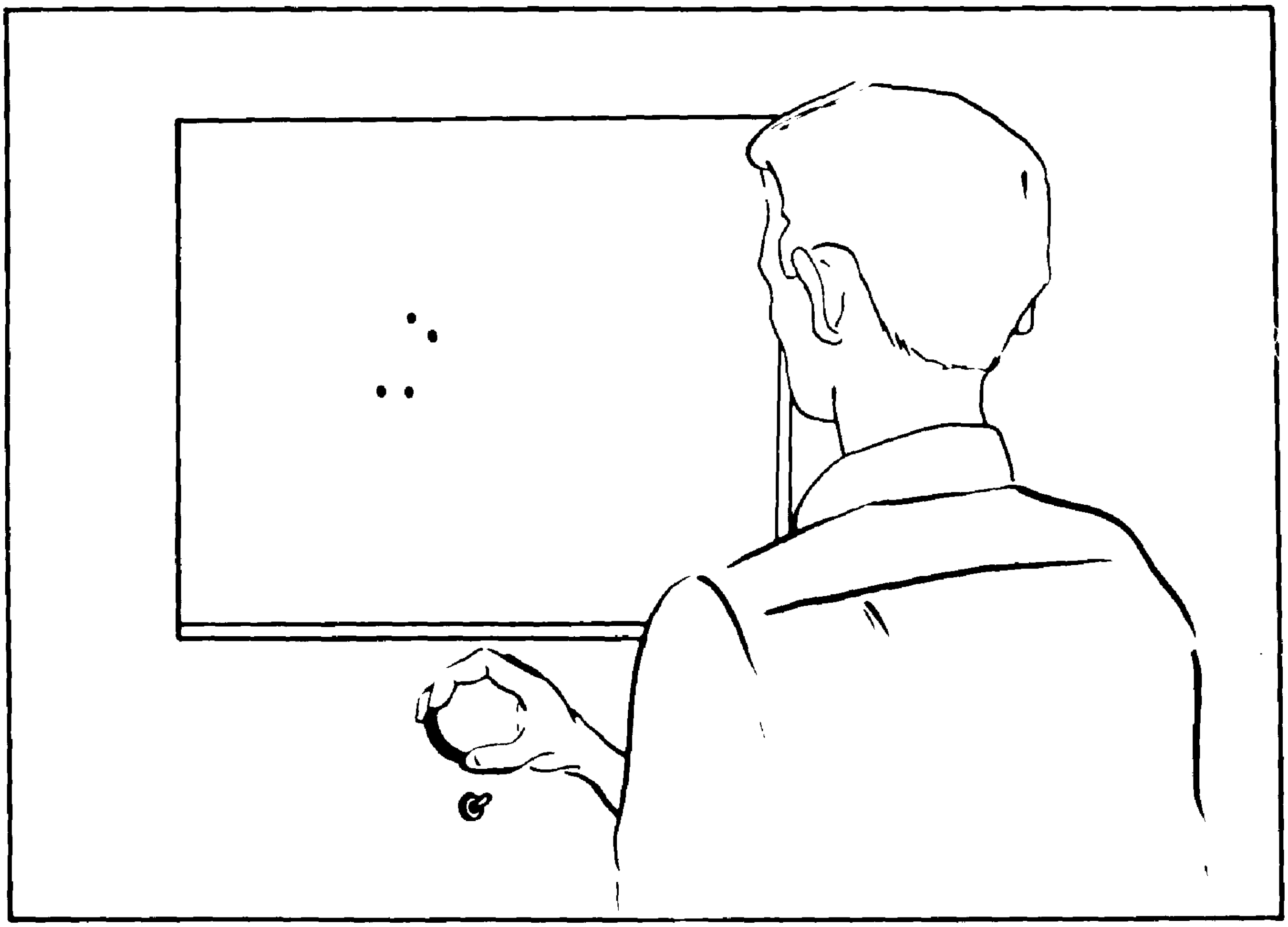
Three performance scores traditional to psychophysics were selected: constant error, average error, and variable error. Although traditional, they are also capable of sophisticated mathematical manipulation. Variable error, expressed as the standard deviation, was of special interest in the present study, and for several reasons. For science in general, the uncertainty in predicting phenomena has led to probabilistic approaches such as statistical mechanics in physics, statistical prediction of progeny characteristics in genetics, as well as the statistical learning and statistical perceptual theories in psychology. Dealing frequently as it does with man-machine systems, engineering psychology is interested in randomly occurring errors. Such errors are apt to be the most difficult kind to eliminate and are inclined to have the most serious consequences.

The study thus set forth to investigate man's ability to extrapolate pairs of straight lines (i.e., judge collision), with particular emphasis on variability, as affected by distance, velocity, and angle of intersection.

## METHOD

*Apparatus*.—Two sound insulated booths were used. The observer's (*O*'s) booth shown in Fig. 1 contained a 24 in. x 24 in. square opal glass screen directly below which was a 2-1/4 in. diameter hand-crank and a toggle switch. The hand-crank was used by *O* to make his responses. The switch was used by *O* to turn off the display and thus signal that he had completed his adjustment. Ambient illumination was provided by a Lite-Mite fluorescent lamp located behind *O* and equipped with two 4-watt Sylvania "cool white" standard tubes. The experimenter's (*E*'s) booth shown in Fig. 2 contained two Optical Target Generators which served as the stimulus-producing component. The Optical Target Generator (OTG) is an apparatus developed by the Laboratory of Aviation Psychology, The Ohio State University (1). Essentially the apparatus is a photographic 2 in. x 2 in. square slide projector and its unique feature is that any photographic image projected on the screen can be moved continuously and evenly over a wide area without distortion. The OTG which projected the variable object was connected with *O*'s hand-crank. Both OTG's were mounted on a sliding platform which could be moved parallel with the screen. Work light for the experimenter was provided by a Lite-Mite with blue filter attached. An orange filter against *E*'s side of the opal glass screen prohibited transmission of any visible work-light into *O*'s booth.

*Observers*.—Four male students at The Ohio State University



[illegible] …tch of [illegible] display.

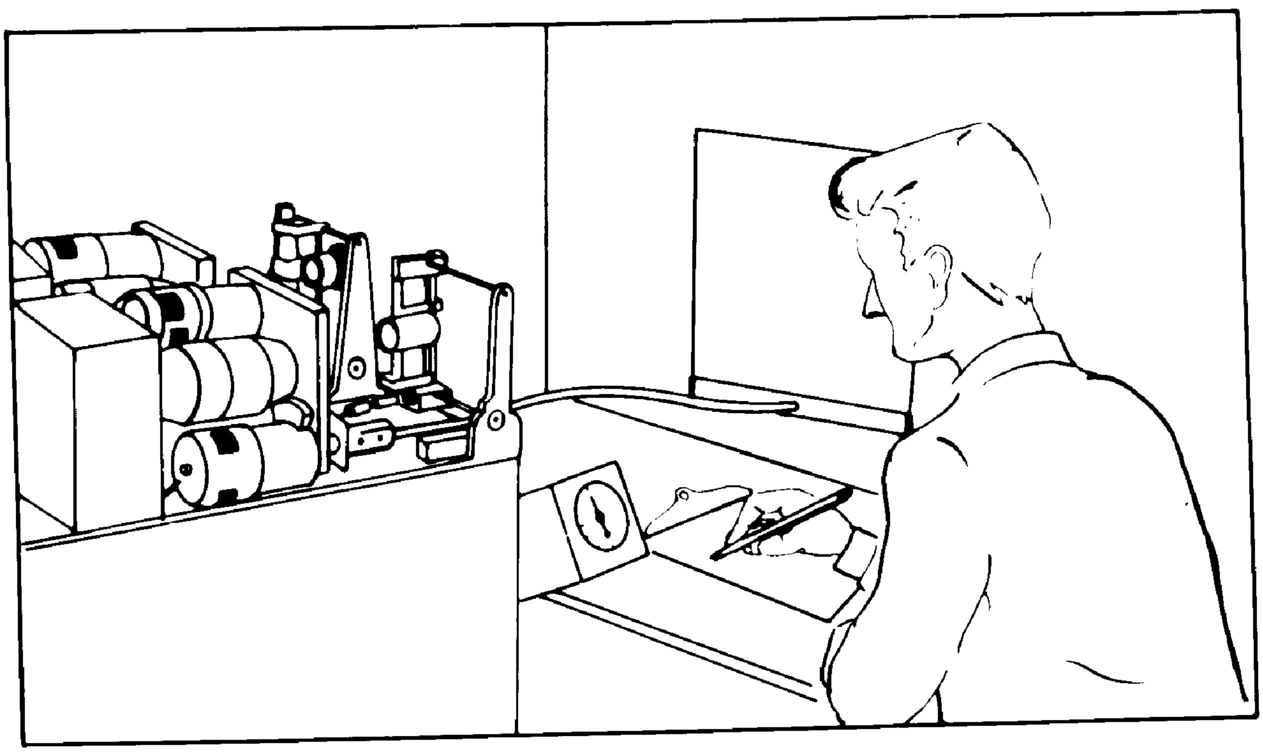


Fig. 2. Sketch of experimenter's booth.

served as Os. Three were naive to the specific task employed; one had served in an earlier study employing the same task. Since the study was conceived as being essentially psychophysical, the number of Os employed was selected as falling within the customary range.

Instructions.—Two one-hour practice sessions were given each O. The practice problems were similar but not identical to those in the experiment proper. Knowledge of results was given only during training sessions. To do this, E moved the variable target from O's response position to the correct position while O watched the correction on the screen. No further training or knowledge of results were given. Our experience (7) and that of others (3) had suggested relatively little learning effect as measured by variable and average errors, and the present study (see Section III) bore out this belief. As an aid to rapport and orientation, the Laboratory of Aviation Psychology of the Ohio State University, and the goals of the study were described in general terms. The specific variables and values employed in the present study were omitted, primarily because it was believed that knowledge of them would contribute nothing helpful to judgments and by their irrelevancy might even be confusing. The instructions quoted below were read before each of the two practice sessions.

> One of the tasks of the air traffic controller is to predict whether or not the objects he sees on his radar display will collide. We are interested in how well he can make these judgments under various conditions.
>
> Here we have a simulated radar display with these two objects on converging courses such that they will cross through their intersection (E points) at sometime in the

future. The leading dots indicate the present position of the two objects. The lengths of the configurations indicate their speed. Your task is to slide this object back and forth along its course to a position such that the two objects will reach the intersection simultaneously if they both were to continue at their displayed speeds and headings. We call this object the variable and you adjust its position by means of this crank. The other object is not controlled by you and is called the standard.

The experimenter will close a switch which will light up the objects in an off position. You will manipulate the knob and position the variable object so as to produce a collision. As soon as you make your setting, press this switch. Your accuracy will then be recorded, and the process repeated until 10 observations are obtained. The correct answer will be shown after the fifth and tenth setting during your two practice hours. No further correct answers will be shown in the sessions which follow the practice hours. Be on your guard against using irrelevant cues. It is our experience that these usually lead to a poorer performance. Strive primarily for accuracy. We find that after a few hours practice, people are making their observations at the rate of ten seconds per observation, and this is adequate.

Before starting the first practice problem, let's take three hypothetical examples to help give you the feel of the task.

1. If the speeds of the standard and variable are both 1 unit, how far should the variable be from the intersection? (Answer: the same distance as the standard.)

2. If the speed of the standard is 20 units, and the speed of the variable is 40 units, how far should the variable be from the intersection? (Answer: twice as far.)

3. If the speed of the standard is 30 units, and the speed of the variable is 31 units, how far should the variable be from the intersection? (Answer: taking the standard's distance from the intersection as 30 units, the variable should be 31 units.)

Do you have any questions before we begin the practice problem?

In the event of a question, E was prepared to reread the pertinent section of the instructions. However, none was asked.

<u>Experimental design</u>.—As a preface to the experimental design, an illustration of terms appears desirable. Figures 3 and 4 show a typical problem correctly positioned to give a collision. The variable object always had a compass heading of 90° (i.e., a horizontal course moving from left to right). The angle of intersection was defined as the angle formed by the two objects; in the illustration the angle is 45°. The speeds of the objects are represented pictorially by the length of the configuration measured between corresponding parts of the dots (e.g., center-to-center separation). A configuration twice the length of another would have twice the speed. These speeds were assigned relative to the blip diameter which was 1/16 in. Thus the configuration of speed 4 was of a length such that 4 additional dots, exactly tangent to themselves and the configuration dots, might have been interposed. The "distance-to-go" or distance extrapolated is shown in the figures and is the distance from the center of the leading dot to the intersection. The figures show the variable, which has the same speed as the standard, correctly positioned equidistant from the intersection.

A 4 x 4 x 7 factorial design was used in which there were four values for the speed variable, four for distance, and seven for angle. The speed of the variable object had relative values of 1, 2, 4, and 5, which corresponded to the absolute values shown in Table 1. The standard was always speed 5, i.e., .625 in. distance between dots measured center-to-center.

The values selected for the speed of the variable object repre-

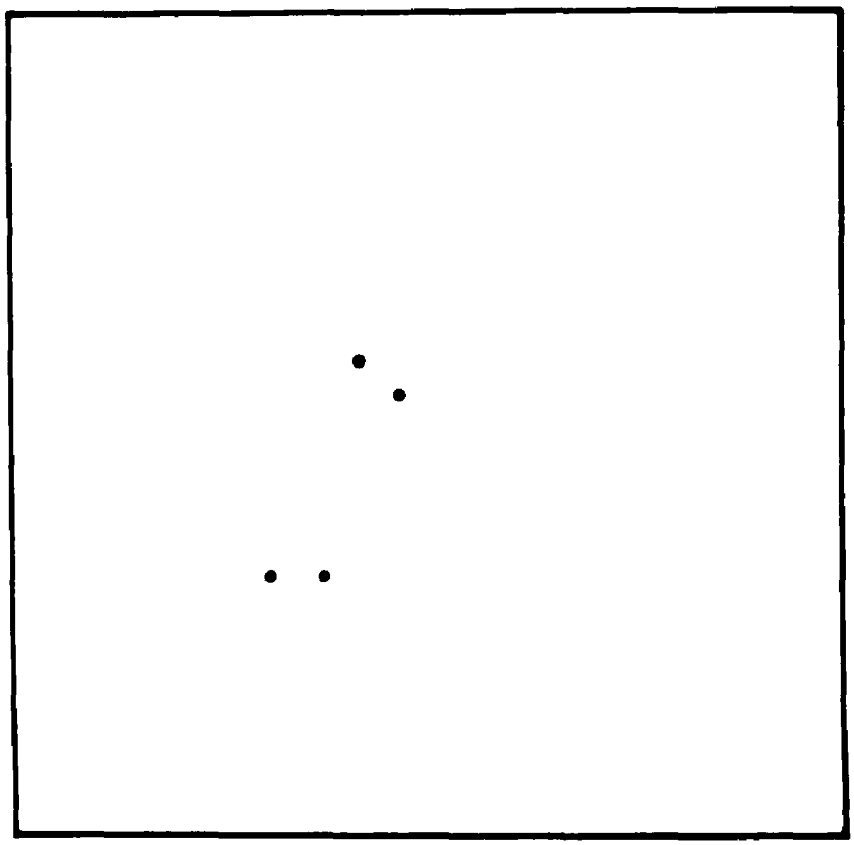


Fig. 3. Observer's view of a typical problem. (The actual display showed light blips on a dark background.)

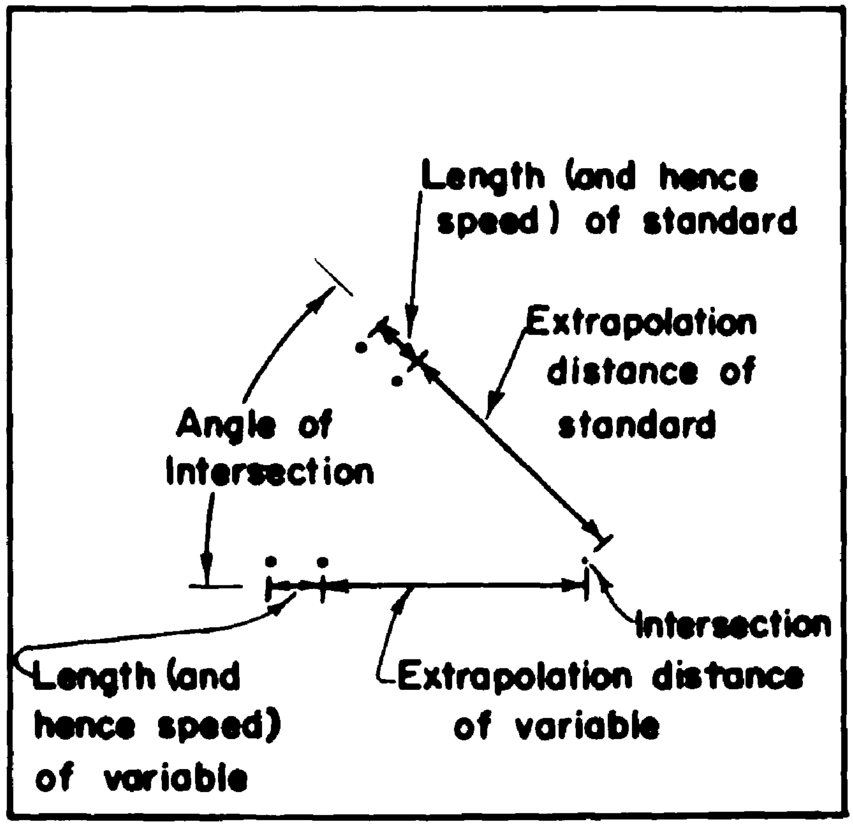



Fig. 4. Observer's view of a typical problem with illustration of terms.

Table 1

Values of Speed Studied

| Speed of Variable Object | Center-to-Center Separation of Dots |
|---|---|
| 1 | .125" |
| 2 | .250" |
| 4 | .500" |
| 5 | .625" |

Table 2

Values of Distance Studied
(in inches)

| Distance Extrapolated, Variable Object | Distance Extrapolated, Standard Object | | | |
|---|---|---|---|---|
| | .75" | 1.5" | 3" | 6" |
| Speed 1 | .15 | .30 | .60 | 1.20 |
| Speed 2 | .30 | .60 | 1.20 | 2.40 |
| Speed 4 | .60 | 1.20 | 2.40 | 4.80 |
| Speed 5 | .75 | 1.50 | 3.00 | 6.00 |

sented a compromise between a desire to study the effects resulting from a geometric series on the one hand, and on the other to include one situation in which the variable and standard had nearly equal speeds.

The four values for distance extrapolated refer to the standard, and were .75 in, 1.5 in., 3 in., and 6 in. The distance extrapolated for the variable depended on both the speed of the variable and the distance of the standard as shown in Table 2.

The seven values for angle of intersection and the corresponding headings of the standard object are shown in Table 3. The variable object always headed 90° as mentioned above.

The 112 separate problems resulting from the 4 x 4 x 7 design were organized into a 4 x 4 square in which each cell was a unique

Table 3

Values of Angle of Intersection Studied

| Angle of Intersection | Compass Heading of Standard Object |
|---|---|
| 10° | 100° |
| 20° | 110° |
| 45° | 135° |
| 90° | 180° |
| 135° | 225° |
| 160° | 250° |
| 170° | 260° |

combination of distance and speed, but contained all seven angles. A Latin square of four blocks and four cells and four O's was formed so that the four O's had the same four cells within any block, but in different order. The seven angles were assigned randomly to each cell. A single experimental session consisted of the seven angles and one of the distance-speed combinations. A total of 16 sessions, of approximately one hour per session, was required per O. Quotidian variability was thus averaged out in each session for angle, and counterbalanced within observer-blocks for distance and speed.

The psychophysical method of adjustment was employed with Os making ten observations per condition. Of the ten observations, half were ascending and half descending, in random order. When E turned on the display, O saw a pair of converging two-dot configurations. The variable was adjusted by O by cranking it to a position closer to (or farther from) the intersection until the objects appeared to be in a colliding attitude. The O then switched off the display, and E recorded the setting to the nearest 1/37 in. The E then displaced the variable again, and shifted the platform mounting the OTGs to a new position. The shifting was done to help overcome any tendency Os might have to make successive dependent position responses as a result of any specks or imperfections that might have been on the display surface.

## RESULTS

Constant error (CE), average error (AE), and variable error (VE or SD) were computed for the ten observations in each O's 112 problems. (Because there is occasional lack of agreement as to the definition of these terms, the computational formulas that were employed are given in footnotes 1-3.) The subsequent analysis was based primarily on these three statistics. The present section presents the results of an analysis by block (which measured learning effects), and is followed by a presentation in terms of the factors affecting the variable and average errors. Separate consideration has not been afforded constant errors for the reason that they tended to be positive in value and would therefore approximately duplicate the functions obtained for average error.

Analysis by block.—The problems contained in the 16 sessions were divided into four blocks of four sessions each, such that upon

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[1] An error (e) was defined as the difference between the correct and the observed response (X). The computational formula for constant error is

$$CE = \frac{\Sigma e}{N} .$$

[2] The computational formula for average error is

$$AE = \frac{\Sigma |e|}{N} .$$

[3] The variable error was based on deviations (d) of responses from O's own mean, i.e., d = X-M. The computational formula for the variable error is

$$SD = \sqrt{\frac{\Sigma d^2}{N-1}}$$

completion of each block, all Os had done the same series of problems although in different orders. Figure 5 shows constant, average, and variable errors as analyzed by block. Variable error was consistently smaller than either average or constant errors, and showed virtually no learning effect. Both constant and average errors became smaller in the course of the experiment, and the reduction in the case of average error was approximately .2 in. or 25 per cent.

Factors affecting the variable error.—Angle of intersection had a pronounced and systematic effect on variable error as shown in Fig. 6 and 7. Figure 6 shows an increasing degree of difficulty, when measured by the SD, as the angle approached 180°. It is believed (see section V) that this finding is related to the increasing difficulty in determining the intersection point toward 180°, whereas the reduced variability near 0° is related to the increasing nearness of the objects (which is turn provides both an anchor point and greater ease in comparing the two configuration lengths). Figure 6 also shows four distinct curves which appear to be members of the same family and which were generated from the four distances extrapolated. The greater variability was consistently associated with the greater distance extrapolated. In Fig. 7 a family of curves is again apparent for the four speeds, with a crossover point at 45°. It appears that O's work-method may differ at the smaller angles of 10° and 20° where, for example, the speed 5 problems (same speed) merely require placing the two objects equidistant from the intersection (or for that matter, equidistant from any point on the perpendicular

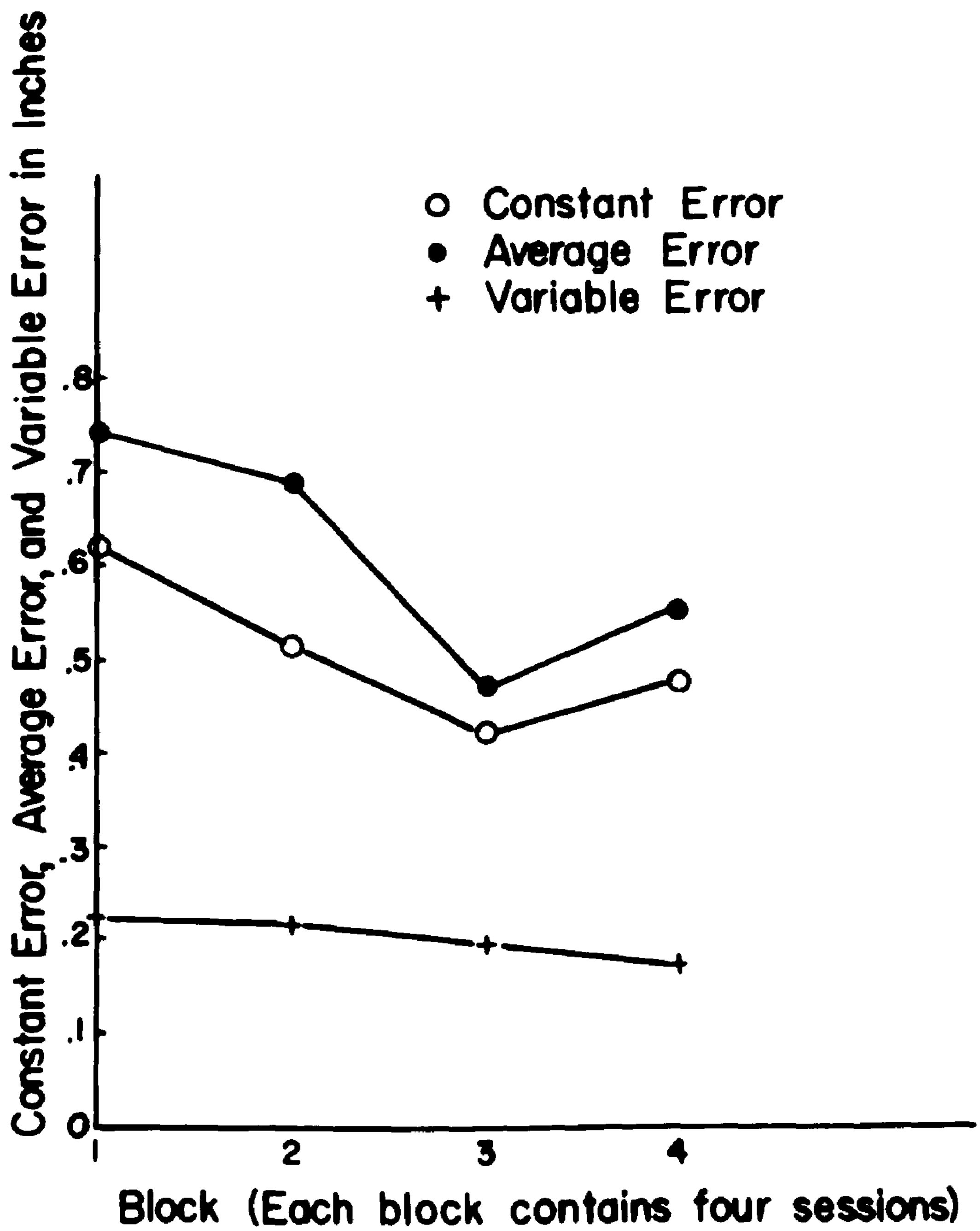


Fig. 5. Constant error, average error, and variable error as a function of block.

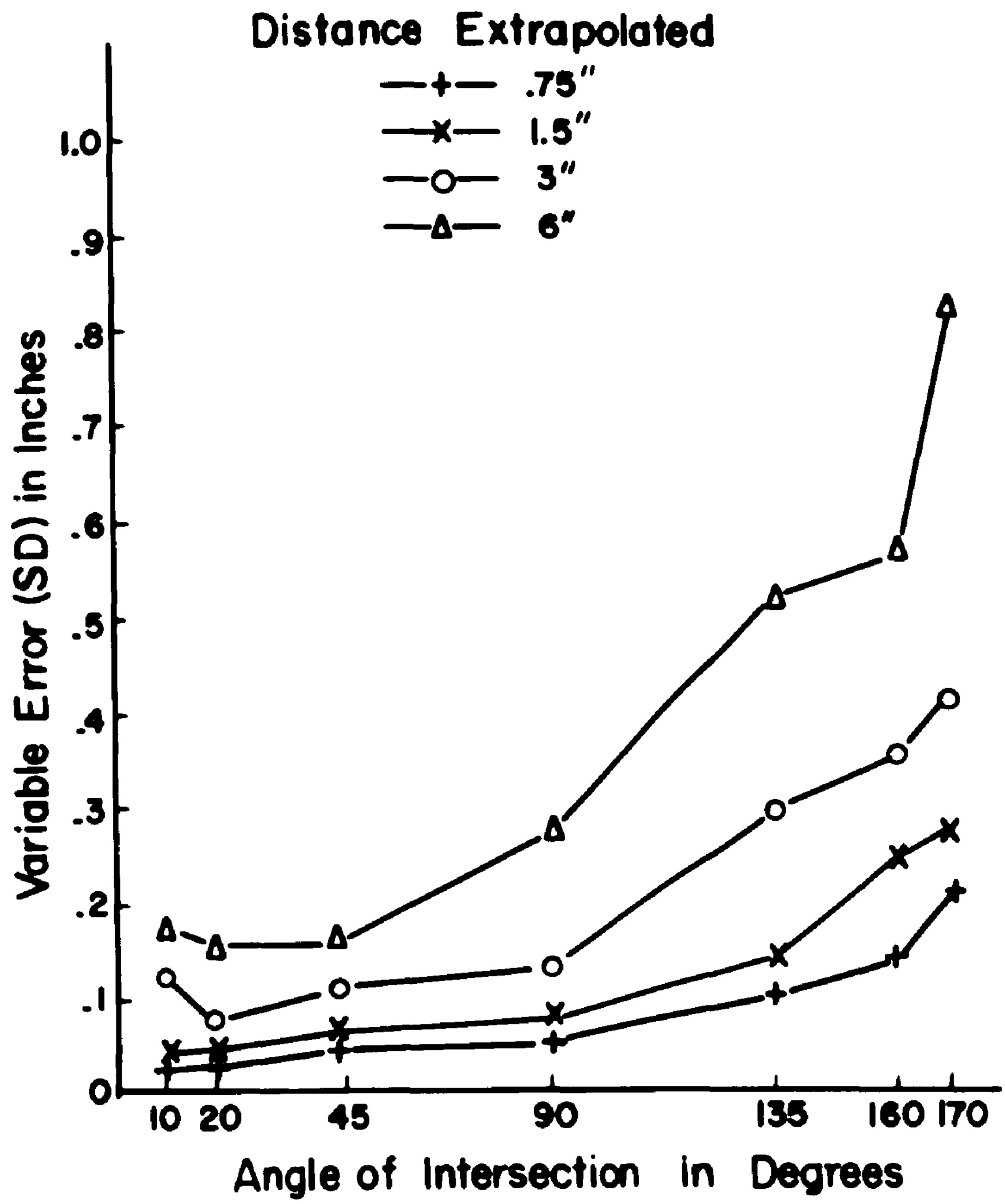


Fig. 6. Variable error for four distances extrapolated as a function of angle of intersection.

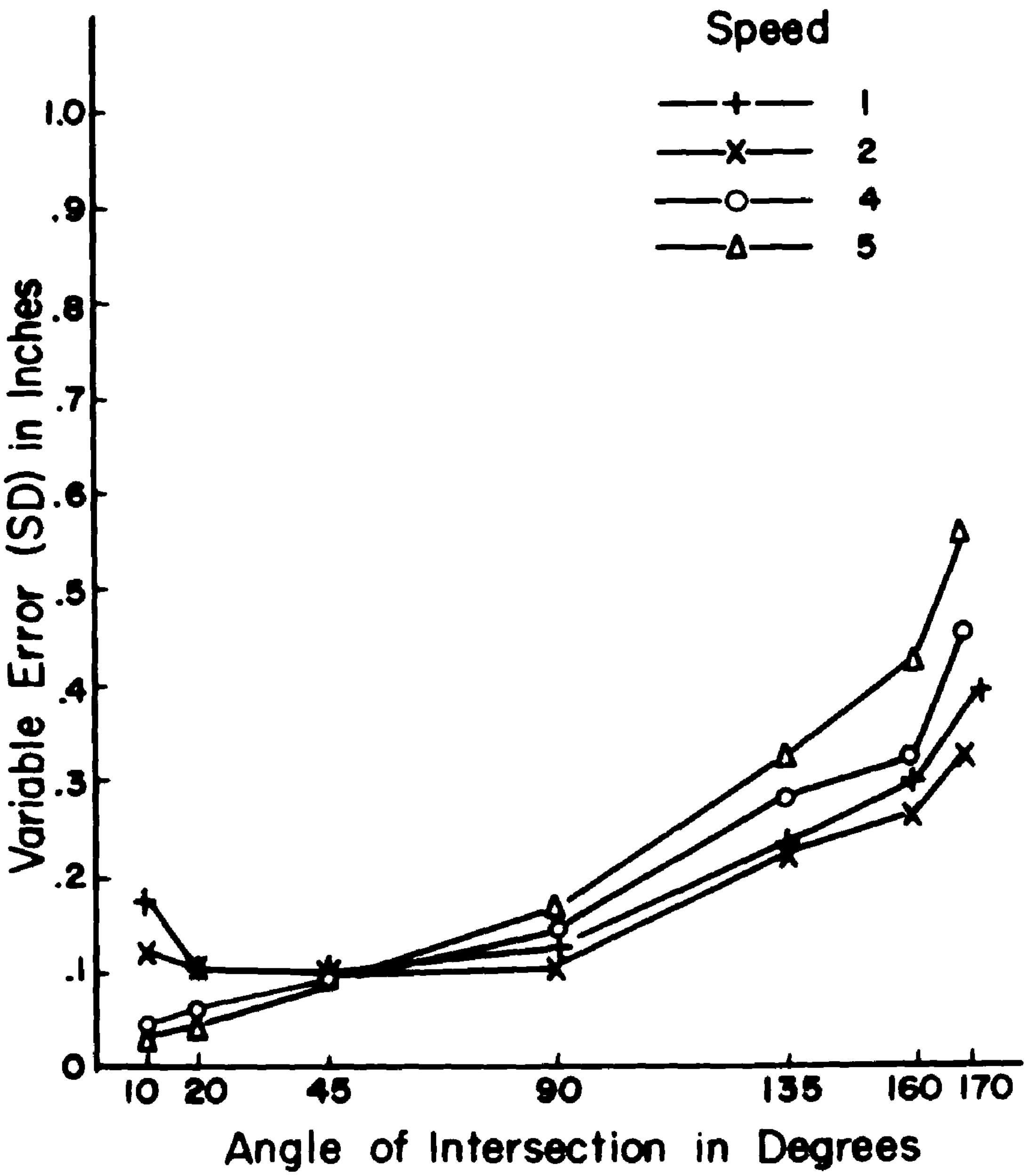


Fig. 7. Variable error for four speeds as a function of angle of intersection.

bisector of the angle). Observers commented to the effect that this was their approach to problems involving the small angles and equal or nearly equal speeds. The judgment of the intersection may be considerably in error without differentially affecting the judged distances of the two objects from that intersection. However, for more discrepant speed combinations the uncertainty of intersection would have a differential effect. For example, the problems in which the variable's speed was 2 required that the variable be placed 2/5 that of the standard from the intersection. In such a case the judged distance between the standard and point of intersection matters considerably. For angles greater than 45°, the greater magnitude of error is associated with the greater speed, except for the inversion of the curves for speeds 1 and 2. This may simply be related to the fact that objects displaying greater speeds must be placed at greater distances from the intersection, and at greater distances the judgment as to the location of the intersection becomes increasingly difficult. The possible significance of the inversion is discussed in Section V.

Figure 8 shows that variable error as a function of distance extrapolated is essentially a straight-line relationship over the range investigated. This finding may be interpreted as saying that the Weber ratio gave nearly perfect accounting of the obtained data, where one defines the Weber ratio as SD of judgments divided by extrapolated distance of the standard. No systematic effect was observed when the SD-distance relationship was analyzed separately by speed.


Fig. 8. Variable error as a function of distance extrapolated.

Figure 9, variable error as a function of speed of variable object for the four distances extrapolated, shows that virtually no influence was exerted by speed <u>per se</u>. As noted earlier, however, speed and angle appear to interact. Figure 9 also shows that the distance extrapolated curves maintain their identity and do not interact with speed.

<u>Factors affecting average error</u>.—Figure 10 shows what appears to be a U-shaped relation for average error as a function of angle, with minimum error observed at the cardinal angle position of 90°. The over-all effect, however, was the result of the separate and differential effect of the four speeds employed. As shown in Fig. 11, there is a tendency for the more discrepant speeds to be associated with greater average error at smaller angles. Figure 12, average error as a function of angle for four distances extrapolated, again shows a differential effect due to distance extrapolated. Whereas equal or nearly-equal distances appear to give average errors which increase exponentially as the angle approaches 180°, discrepant speeds give rise to a U-shaped function. As shown in Fig. 13, average error increases with distance extrapolated, in a linear fashion. Average error also increases with increasingly discrepant speed ratios as shown in Fig. 14. Apparently a discrepant speed ratio was handled reasonably well if extrapolation distance was quite small (note curve for .75 in.), but poorly handled in problems with large extrapolation distances.

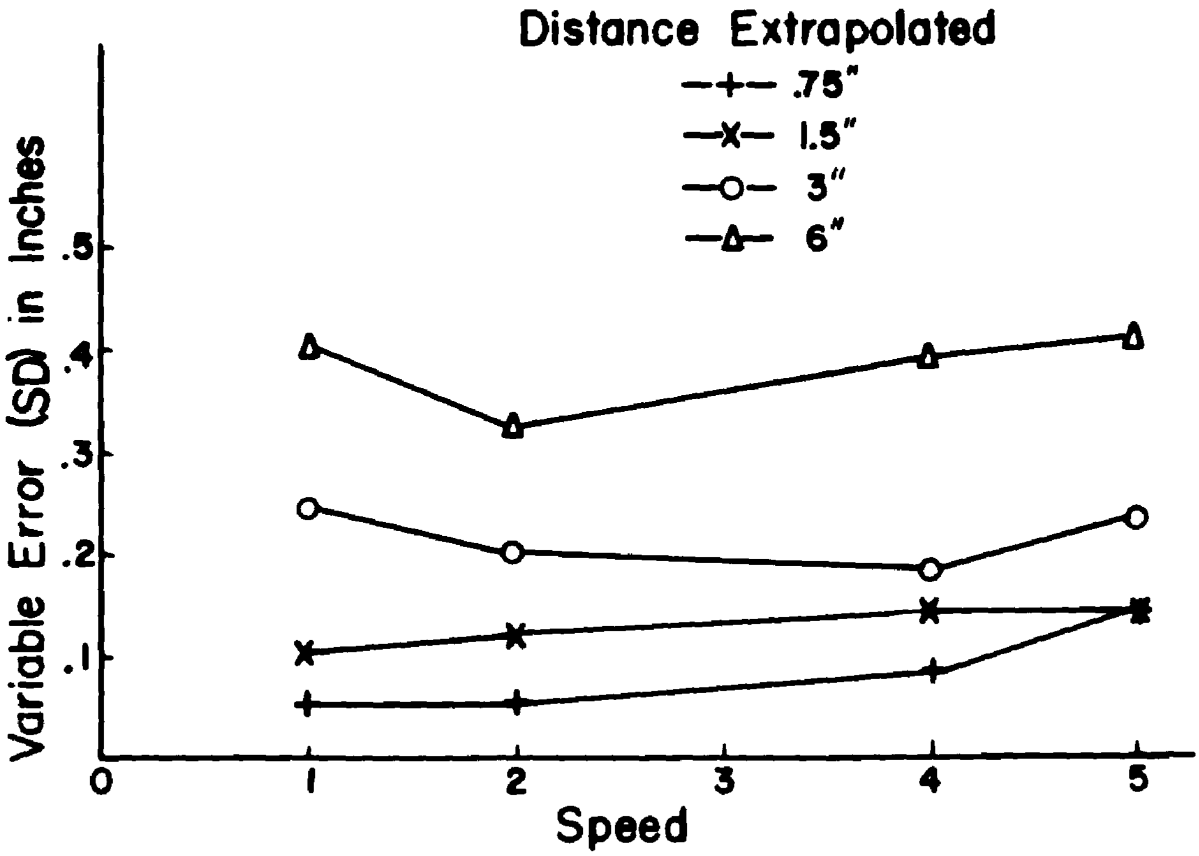


Fig. 9. Variable error for four distances extrapolated as a function of speed.

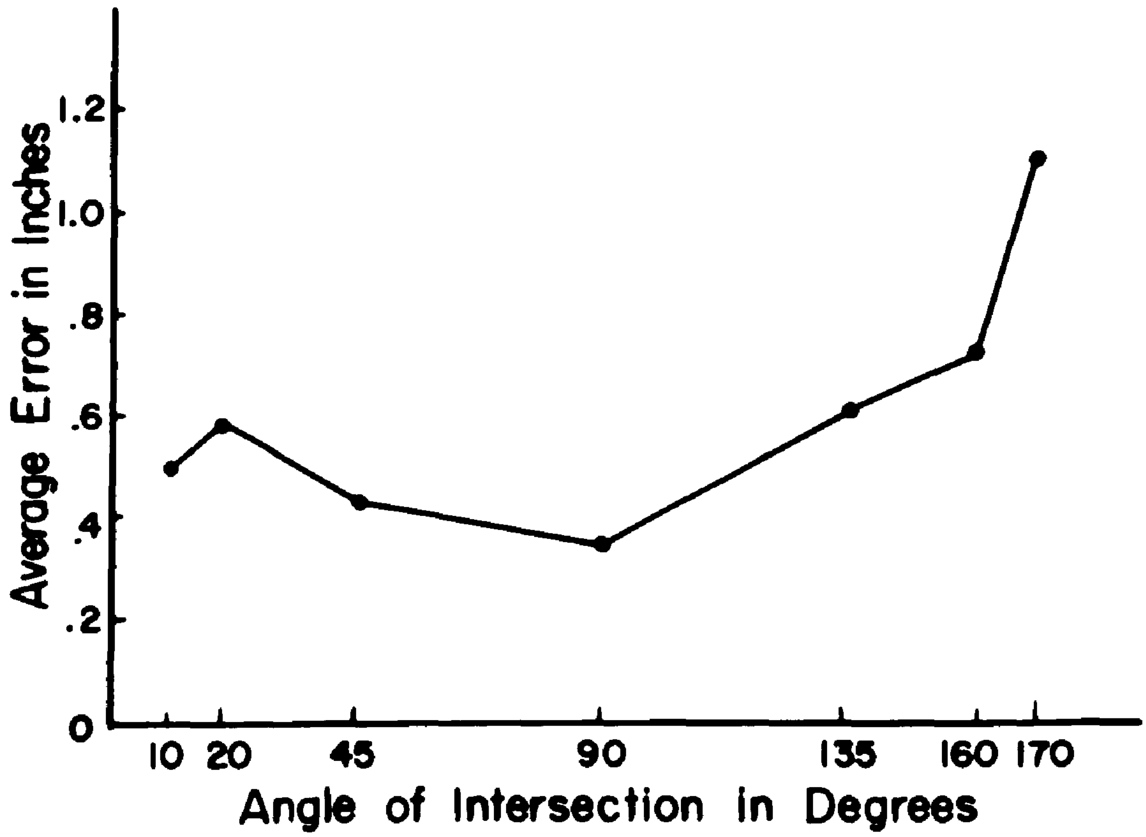


Fig. 10. Average error as a function of angle of intersection.

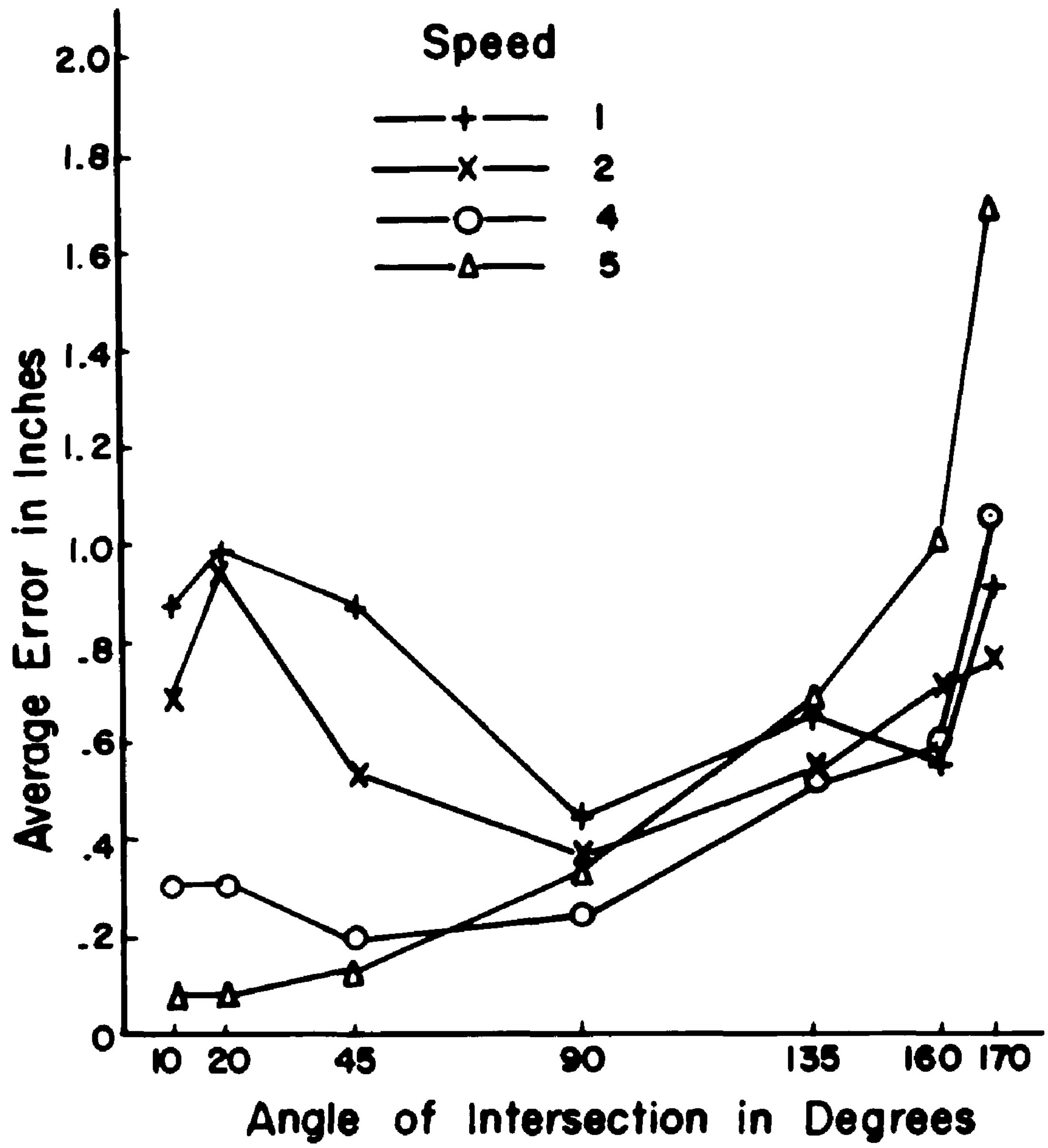


Fig. 11. Average error for four speeds as a function of angle of intersection.

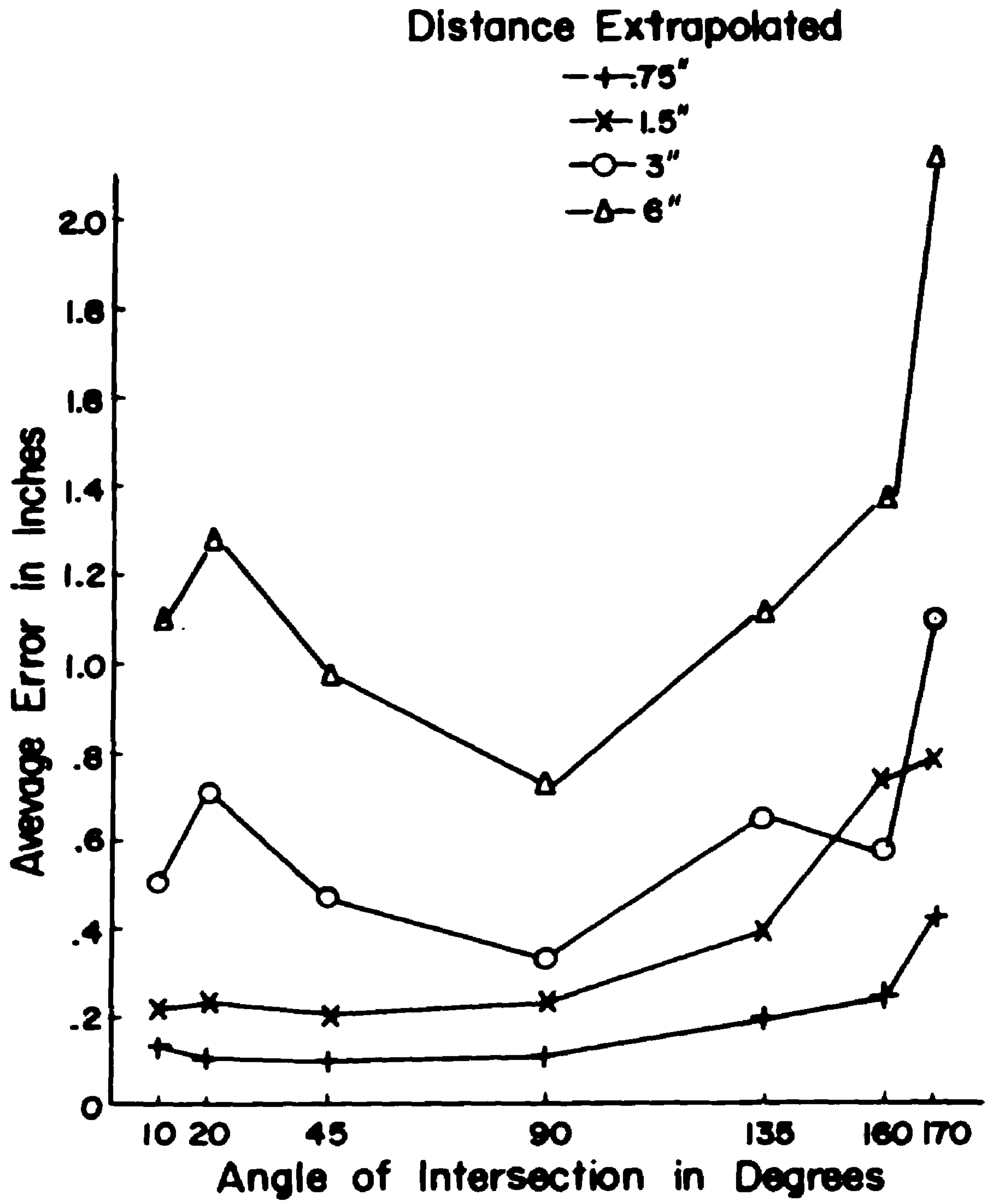


Fig. 12. Average error for four distances extrapolated as a function of angle of intersection.

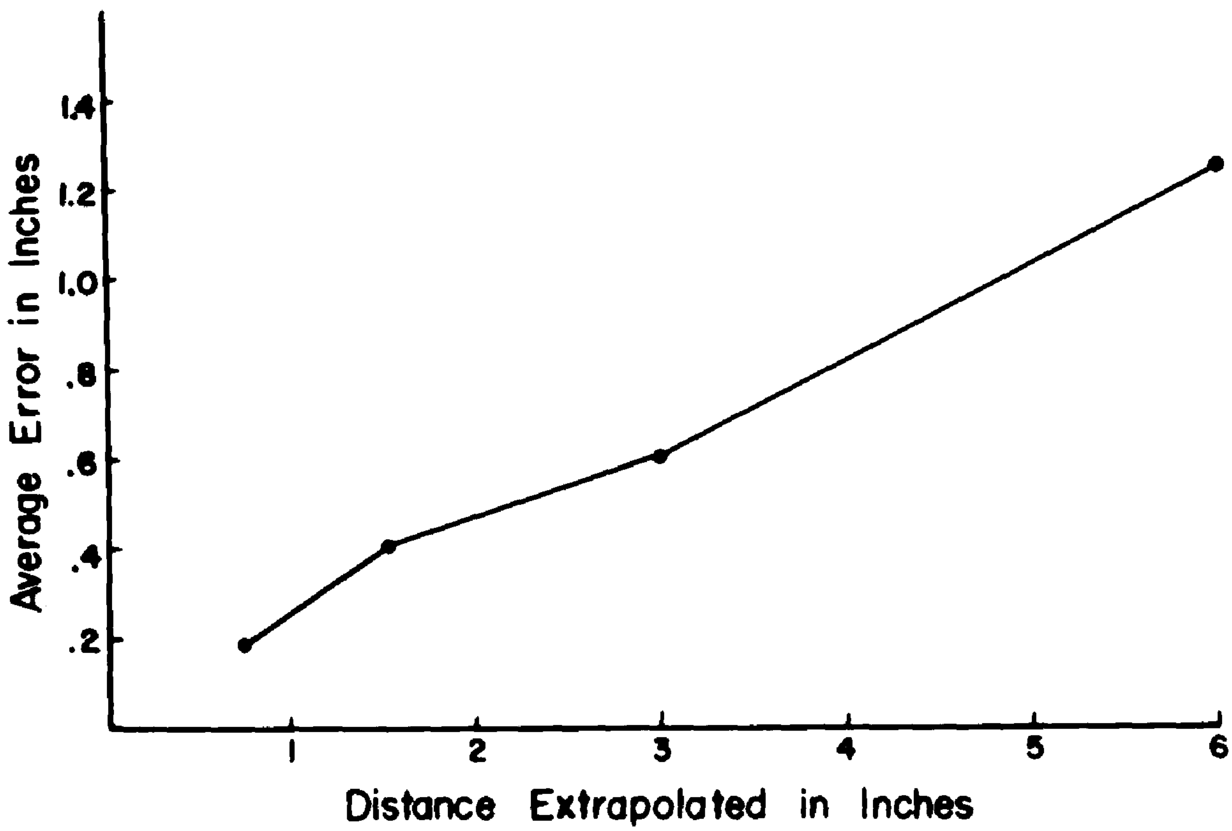


Fig. 13. Average error as a function of distance extrapolated.

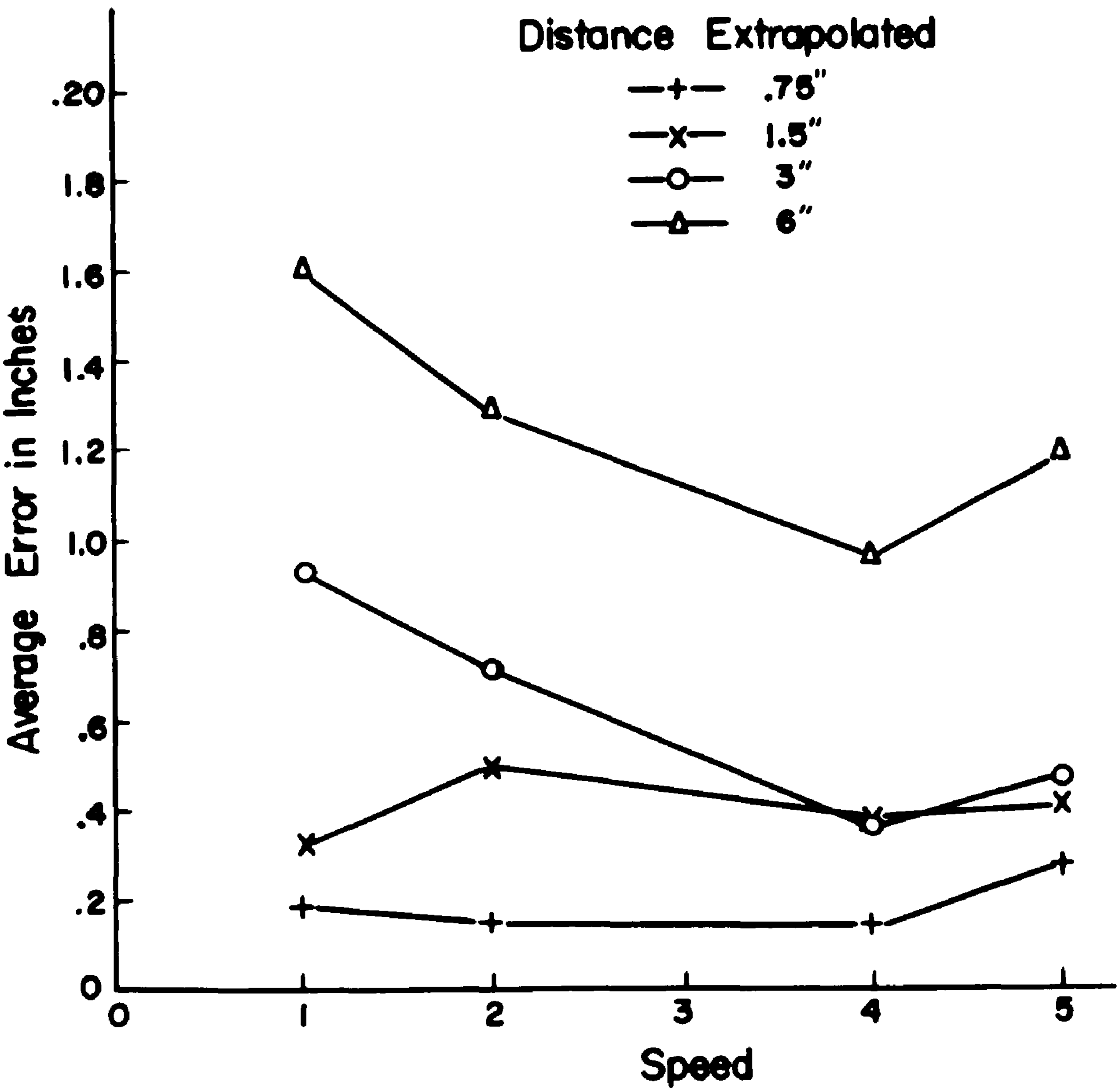


Fig. [illegible]. Average error for four distances extrapolated as a function of speed.

## MATHEMATICAL MODEL

It is the purpose of the present section to trace the developments leading to a proposed mathematical model, i.e., a single quantitative expression, to account for the observed data as regards variable error. The requirements made of such a model are that it should be parsimonious and should accurately predict the effects associated with the three variables, i.e., angle, speed, and distance.

In constructing a model to predict variability, a logical point of departure appeared to be the positively accelerated, increasing monotonic curve obtained as a function of angle of intersection (Fig. 6), and the straight line as a function of distance extrapolated of the standard (Fig. 8). Accordingly, average SDs were divided by distance extrapolated of the standard, and plotted as a function of the angle. The resulting function is shown in Fig. 15. It was apparent that the transformation did not adequately account for differences in distance extrapolated. It appeared that some other transformation would be required to account for distance, preferably some manipulation which would involve the extrapolation distances of both standard and variable. Moreover, an accounting for speeds also appeared desirable. An attempt along these lines was therefore made.

There were four distance magnitudes which <u>O</u> must in some fashion utilize. These magnitudes are the two object lengths (which represent speed), and the two distances extrapolated, as shown in Fig. 16.

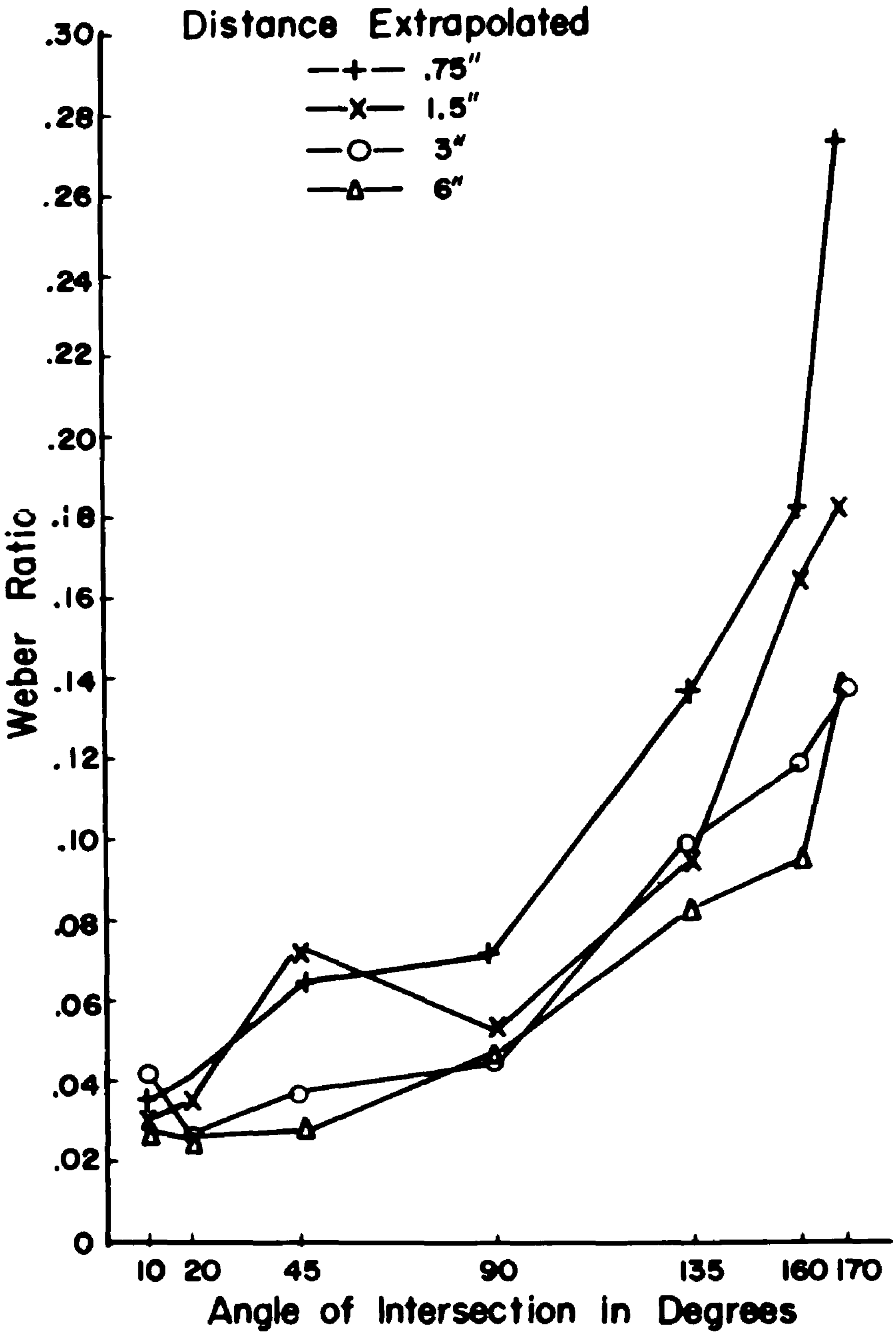


Fig. 1[illegible]. Weber ratio of four distances extrapolated as a function of angle of intersection.

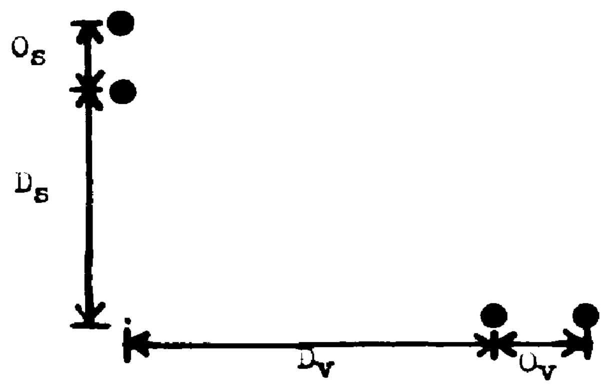


Fig. 16. An illustrative problem showing the four essential magnitudes, where

$O_v$ = length of the variable object,
$O_s$ = length of the standard object,
$D_v$ = distance extrapolated for the variable object, and
$D_s$ = distance extrapolated for the standard object.

Let us assume that there is variability associated with the perception of each of the four magnitudes ($O_v$, $O_s$, $D_v$, $D_s$) and that errors are independent, i.e., their variances are additive.

$$\sigma_{total} = \sqrt{O_v^2 + O_s^2 + D_v^2 + D_s^2}\,. \quad (1)$$

Now the Weber ratio may be written

$$\frac{\sigma_{distance}}{distance} = \frac{\sigma}{d} = \text{constant} = k \quad (2)$$

or

$$\sigma = kd \quad (3)$$

Assuming the same Weber ratio to hold, one may then use formula (3) as a predictor of variability for the four magnitudes.

$$\sigma^2_{O_v} = k^2\, O_v^2 \quad (4)$$

$$\sigma^2_{O_s} = k^2\, O_s^2 \quad (5)$$

$$\sigma^2_{D_v} = k^2\, D_v^2 \quad (6)$$

$$\sigma^2_{D_s} = k^2\, D_s^2 \quad (7)$$

Substituting (4) through (7) in (1)

$$\sigma_{total} = \sqrt{k^2 O_v^2 + k^2 O_s^2 + k^2 D_v^2 + k^2 D_s^2} \quad (8)$$

$$= k \sqrt{O_v^2 + O_s^2 + D_v^2 + D_s^2} \quad (9)$$

or

$$k = \frac{\sigma_{total}}{\sqrt{O_v^2 + O_s^2 + D_v^2 + D_s^2}} \quad (10)$$

The constant k thus derived is a kind of "adapted Weber ratio" and is defined by formula (10). It is evident from Fig. 17 that the adapted Weber ratio now accounts more completely for the effects of distance extrapolated than did the kind of Weber ratio shown in Fig. 15. The constant k as a function of speed of the variable object is shown in Fig. 18. Measures of goodness of fit and prediction were calculated to provide a precise indication of the transformations' effectiveness. The resulting standard error of estimate ($\sigma_{yx}$) was .169 in. and the corresponding coefficient of correlation was .517, both based on the 112 separate conditions pooled for the four Os. The means for the adapted Weber ratio at each of the seven values of angle were then computed, and the result was a rather smooth positively accelerated, monotic increasing function. A trigonometric function of the form shown in formula (11) was then fitted by least squares to the means of the adapted Weber ratios.

$$k = a + b \tan \frac{B}{2} \quad (11)$$

where k is the adapted Weber ratio, a, b are constants, and B is the angle of intersection.

The result was a formula (12) giving a predicted value ($\bar{k}$) for the adapted Weber ratio.

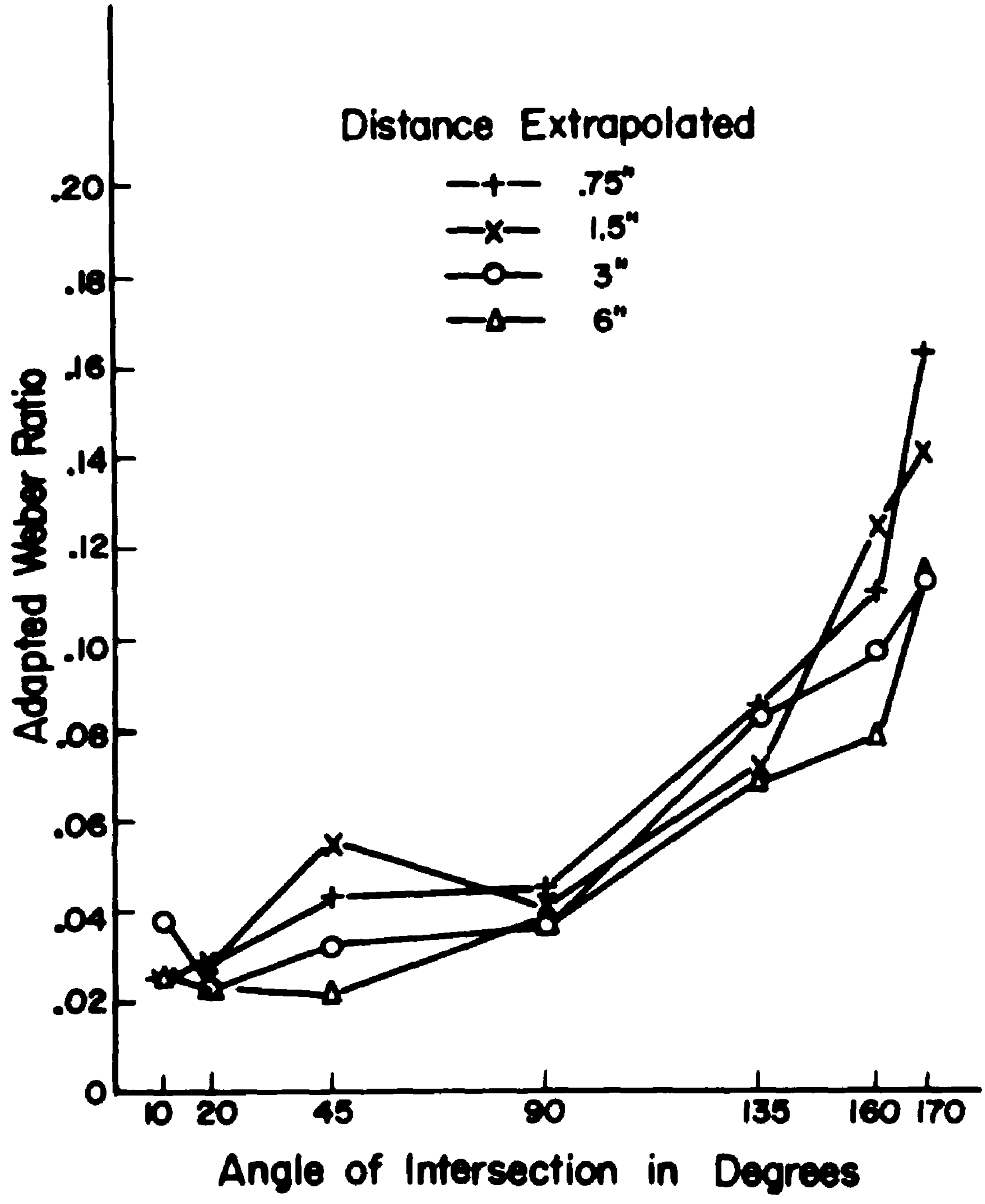


Fig. 17. Adapted Weber ratio for four distances extrapolated as a function of angle of intersection.

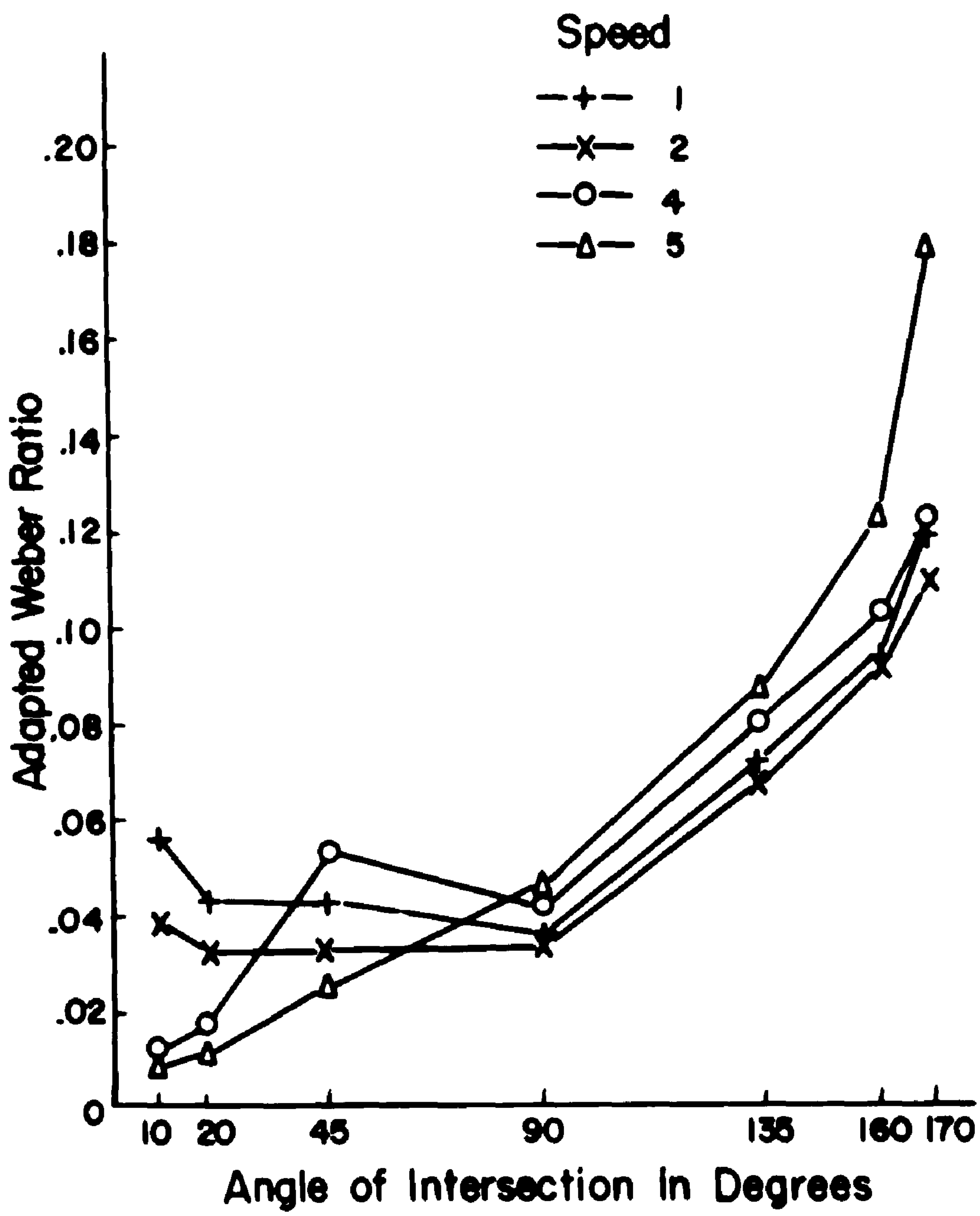


Fig. 1[illegible]. Adapted Weber ratio for four speeds as a function of angle of intersection.

$$\bar{k} = .034860 + .009477 \tan \frac{B}{2} . \qquad (12)$$

Substituting $\bar{k}$ (12) into formula (9), one obtains formula (13) which provides a predicted measure of variable error ( $\sigma$ ) as a function of all three experimental variables.

$$\sigma = (.03486 + .009477 \tan \frac{B}{2}) \sqrt{O_v^2 + O_s^2 + D_v^2 + D_s^2} \qquad (13)$$

Formula (13) as thus proposed handles the effects of angle by the expression in parentheses, and the effects of distance and speed by a kind of a Weber ratio in the expression under the square root.

In order to determine the goodness of fit of the model proposed in formula (13), the standard error of estimate ($\sigma_{yx}$) was calculated for the 112 separate conditions pooled for the four Os. The resulting standard error of estimate was .098 in. and the corresponding coefficient of correlation (Pearson product-moment r) was .87, again based on N = 112. The addition of the tangent function increased the value of r from .52 to .87 and the difference was significant ($p < .01$).

# DISCUSSION AND CONCLUSIONS

Constant, average, and variable errors were first analyzed for learning effects throughout the course of the experiment in four blocks of seven angles each (Fig. 5). No particular learning effect was evidenced in terms of variable error, although there was some learning in terms of a reduction in constant and average errors. It is believed that such learning as occurred had but minor effect on the functions obtained for the three major variables. All seven angles were presented in different random orders for each session, and such a design would thus handle any learning effects for angle. In that each block represented four pairings of the four distances and four speeds, the experimental design tended to spread out learning effects evenly. A design which provided sufficient Os to present the 16 distance-speed combinations in all possible orders would of course have achieved a completely even distribution of learning effects, although at virtually prohibitive expense. However, the regularity of the functions obtained appears to be justification for the experimental design employed.

As noted earlier in Fig. 6, variable error appears to be a positively accelerated, increasing monotonic function of angle from $10^{\circ}$ to $170^{\circ}$, the range studied. The effect of increased variability as the angle of intersection approaches $180^{\circ}$ is believed due primarily to the difficulty involved in judging the position of the intersection. Geometric analogies may be offered for this phenomenon, e.g., take any pair of converging lines which when extrapolated form an angle B, and let one of the lines be subject to some given angular error of extrapolation, then the change in the point of intersection will approach infinity as B approaches 180°. The steepness of the rise appears related to the distance extrapolated, such that with large angles and large distances the variable error is compounded. However, geometrically the same uncertainty as to intersection might be thought to apply for angles approaching 0°. The answer seems to be that it does, but an increase in variable error is only possible for particular ranges of values for speed and distance. On the one hand, near zero variability may be expected for either infinitely small extrapolation distances regardless of angle, or for objects of equal speed at angles approaching 0°. The latter effect seems due to increased ease in judging the speeds of the objects as being equal, and to the relative ease in placing the objects equidistant from the point of intersection (or from any point along the perpendicular bisector of the angle). A final variability-reducing effect is perhaps related to the proximity of the two objects such that the standard acts as a fixed reference point and thus exerts an anchoring effect on successive adjustments of the variable. The suggestion of an anchoring effect is not offered in the sense of showing causation, but rather as being phenomenalistic, i.e., variability tends to decrease when the objects are seen to occur close to one another.

Now as to circumstances which might result in an increase in variable error for angles approaching 0°, there seem to be several.

As suggested already, such an effect should not be sought for conditions associated with very small extrapolation distances or equal speeds. However, given greater extrapolation distances combined with increasingly discrepant speeds, an increase was demonstrated (Fig. 6 and 7). Discrepant speeds might be expected to have several effects. As the objects differ increasingly in speed the faster one must be located increasingly distant from the slower if collision is to result. As they consequently are increasingly separated one from the other, they become harder to compare, and at the same time anchoring decreases. Finally, it might be postulated that the human has increasing difficulty in quantitative dealings with increasingly discrepant magnitudes. The evidence in support of this last interpretation comes from Fig. 7, in which there is consistently greater variability for speed 1 (a ratio of 1:5) than for speed 2 (2:5), and from Fig. 9 in which the curves for distances 3 in. and 6 in. show a rise at speed 1. Indeed, the true effect of the discrepant speeds was probably reduced by the fact that the correct extrapolation distance for the slower object decreased in proportion to its speed, and extrapolation distance has potent effects as already noted. The experimental design thus performed a severe test of the discrepant magnitudes hypothesis.

Depending on which speeds were involved, either 10° or 90° were judged best in terms of both variable and average error. Performance was accurate when the angle was small and speeds were equal. In such a case the task reduced to simply placing one object below the other, and judged intersection was unimportant. When speeds were unequal, however, the judged intersection became important. The $90^{o}$ angle was associated with the more accurate performance in this case, presumably because less error accompanied the judgment of intersection.

When variable error was pooled over all conditions for the four extrapolation distances of the standard, the result was a markedly linear type function as shown in Fig. 8. Thus, the Weber ratio, when defined as the SD divided by the corresponding distance extrapolated of the standard, showed very little variability about the value of .072 over the range studied. Perhaps the significance of this finding lies in the fact that a measure as traditional as the Weber ratio can be applied to the complex perceptual situation such as was represented by the present study.

Since constant errors tended to be positive, the functions for both constant error (not shown) and average error (Fig. 10-14) tended to be the same. For this reason it seemed more economical and less redundant if but one set of functions were presented, and average error was selected. As was the case with variable error, angle of intersection had a strong effect on average error (Fig. 10-12). In general, the cardinal angle position of $90^{o}$ resulted in the smallest average error (Fig. 10). The general relationship tended to be U-shaped, with greater error occurring toward $0^{o}$ and $180^{o}$. However, as so often is the case, the apparently simple relationship actually represents the resolution of several separate effects. The speed curves vary from a J-shaped function for equal speeds (i.e., speed 5, Fig. 11) to a U-shaped function for increasingly discrepant speeds (e.g., speed 1, same figure). Thus, the results of any study limited to the demonstration of some over-all error-angle relationship would be dependent upon the particular value(s) of speed selected. The same situation also applies to the observed effects of distance extrapolated (Fig. 12). The distance selected would determine whether one obtained a function which was but very slightly bowed (e.g., .75-in. curve, Fig. 12) or one which had a pronounced U-shape (6-in. curve, same figure). As suggested earlier, the over-all difficulty level is low for tasks employing short extrapolation distances, and, methodologically, the shorter distances provide a region both insensitive and uneconomical for the testing of a number of variables, whether configurational (i.e., traffic variables) or display variables. The finding that average error increases with distance extrapolated corroborates the finding by Bowen and Woodhead (3) in a related extrapolation task which used one object and a line, and which required an estimation of the number of missing dots between the line and closest dot.

The mathematical model as developed (formula 13, Section IV) was designed to predict variable error on the basis of angle, speed, and distance. Human variability is of particular interest to engineering psychology, among other reasons, because of its effect on man-machine systems. Constant errors introduced by the human can often be corrected through training or equipment design. However, variable errors which remain in the trained operator or observer are particularly difficult to cope with. Such errors are thus of concern to engineering psychology in addition to their overall theoretical interest. The model developed does not give perfect prediction to the empirical data; no model ever does. With a Pearson product-moment r = .87 between observed and predicted values, approximately three-fourths of the variance was thus accounted for. Although the model showed a reasonably good fit with the empirical data, there is no way of knowing whether some other function should have been selected, since compromise must always be made between simplicity and accuracy of prediction. Perfect prediction could have been obtained at the price of considerable mathematical complexity. However, the formula thus obtained would have fitted both experimental error and main effects, and consequently would have been grossly lacking in generality. The selected tangent function is by no means a unique choice among the many simple positively accelerated, increasing monotonic functions (e.g., logarithms, power functions, parabolas, secants, etc.). In view of the earlier discussion in regard to increased variability in judgments involving increasingly discrepant speed ratios it would be correct to conclude that the proposed model fails to handle this phenomenon. Over the ranges of speed studied the effect reduces the goodness of fit, although not seriously. Greater speed ratios would have an effect progressively more serious. It is believed, however, that given additional data a still more general model than the one proposed could be developed without much addition in complexity.

Several possible lines of attack might be suggested for future improvement of the model. Formula 13, the model proposed, does not carry any correlation terms ($\sigma_1 \sigma_2 r_{12}$, etc.), i.e., independence is assumed. It might be reasonable to ask whether the errors are correlated. If they are, prediction would improve with the introduction of additional terms. The simplest case would be one in which all the correlations were equal. Since there are four variances involved (from the two distances and two speeds), unequal correlations would add six coefficients of correlation (resulting from the four variances taken two at a time). Another possible approach would be to start with the relation $O_s / D_s = O_v/D_v$. The objects will collide when this equation is true. However, since the magnitudes are subject to perceptual errors, one might write $\sigma_{O_s} / \sigma_{D_s} = \sigma_{O_v} / \sigma_{D_v}$. This approach suggests the hypothesis that the human acts as if he were a ratio and proportion computer operating on imperfect inputs.

The author believes that the proposed model has theoretical interest on several grounds. As developed, the adapted Weber ratio (k) would predict an increase in variable error in situations where the extrapolation distance was held constant and length of object (or in radar terminology, trail length) was increased. Investigations both here (7) at the Laboratory of Aviation Psychology, The Ohio State University, and the Applied Psychology Unit, Cambridge University, England (3), have failed to show gains in performance with increased object length. It might be noted that the investigators at both laboratories, prior to obtaining their results, had expected an <u>improvement</u> of performance with increased length. While, according to the model, variability should increase with object length, failure to observe this effect seems primarily due to the use of relatively long extrapolation distances in proportion to object lengths. Consequently, the contribution of object length to total variance was very small, and made all the more so as a result of the squaring of the magnitudes under the square root sign. It should be mentioned that the two studies employed values operating in usual radar situations and, within the ranges studied, the conclusion of no effect due to object (trail) length was justified both on the basis of the studies themselves as well as the theoretical grounds just presented.

Formula 13 is, either in its final form or through its derivation, related to several older models. Equations 4 through 7 were used as substitution identities and are Weber-type functions with the standard deviation used in the place of a just noticeable difference ($\Delta I$). One aspect of formula 13 makes it different from the Fullerton-Cattell square root law (4), from Woodworth's generalized law (9), and from Guilford's power function (5). This difference arises from its use of the <u>stimulus</u> description (<u>kd</u>, formula 3) rather than a <u>response</u> description ($\Delta I$ or $\sigma$, one or the other of which is used in the other models). Were the items summed under the square root sign $\Delta I_i$ values, the model could be said to incorporate both the Fullerton-Cattell square root law and the special case of the Guilford law where $n = .5$. Were the items $\sigma_i^2$ values, the model could be said to incorporate the Woodworth law for the case where $r = 0$.

If there was justification in the earlier statement as to the significance of the application of the Weber ratio to the distance-variability function (Fig. 10), then the statement should apply even more to the development of the model derived from classical theory to fit a complex perceptual situation. Basically the situation involved five quantities (angle of intersection, two distances, and two object lengths) each of which was subject to some amount of variability or judgment error. The problem was one of determining the amount of error associated with each of the components separately and then learning their form of combining. It was necessary to accomplish this by working in the total situation with one output measure (O's variability in adjusting the variable object).

In conclusion, it is believed that the formulation developed gives prediction sufficient to justify an assertion that the methodologies of experimental psychology are equal to the task of the precise quantitative handling of some of the more complex perceptual activities in which the human engages.

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APPENDIX

Summary Data

Table 4

Constant Error in Inches Tabulated by Distance Extrapolated and Speed Pooled for Four Os

| Speed | Distance Exprapolated | | | | |
| --- | --- | --- | --- | --- | --- |
| | .75" | 1.5" | 3" | 6" | Mean |
| 1 | .179 | .313 | .774 | 1.516 | .696 |
| 2 | .104 | .496 | .404 | 1.160 | .541 |
| 4 | .121 | .296 | .301 | .910 | .407 |
| 5 | .266 | .277 | .357 | .670 | .393 |
| Mean | .167 | .346 | .459 | 1.064 | |

Table 5

Constant Error in Inches Tabulated by Speed and Angle of Intersection Pooled for Four Os

| Angle of Intersection | Speed 1 | 2 | 4 | 5 | Mean |
|---|---|---|---|---|---|
| 10° | .843 | .619 | .301 | .089 | .463 |
| 20° | .995 | .948 | .300 | .088 | .583 |
| 45° | .638 | .514 | .079 | .126 | .339 |
| 90° | .439 | .377 | .068 | .122 | .251 |
| 135° | .648 | .502 | .509 | .648 | .577 |
| 160° | .535 | .607 | .564 | .685 | .598 |
| 170° | .771 | .219 | 1.028 | .991 | .752 |
| Mean | .696 | .541 | .407 | .393 | |

Table 6

Constant Error in Inches Tabulated by Distance Extrapolated and Angle of Intersection Pooled for Four _O_s

| Angle of Intersection | Distance Extrapolated | | | | |
|---|---|---|---|---|---|
| | .75" | 1.5" | 3" | 6" | Mean |
| 10° | .129 | .220 | .438 | 1.066 | .463 |
| 20° | .093 | .237 | .707 | 1.294 | .583 |
| 45° | .092 | .141 | .174 | ..950 | .339 |
| 90° | .085 | .182 | .216 | .523 | .251 |
| 135° | .152 | .393 | .600 | 1.162 | .577 |
| 160° | .221 | .712 | .410 | 1.049 | .598 |
| 170° | .401 | .536 | .667 | 1.405 | .752 |
| Mean | .167 | .346 | .459 | 1.064 | |

Table 7

**Average Error in Inches Tabulated by Distance Extrapolated and Speed Pooled for Four *O*s**

| Speed | Distance Extrapolated .75" | 1.5" | 3" | 6" | Mean |
|---|---|---|---|---|---|
| 1 | .189 | .326 | .932 | 1.604 | .763 |
| 2 | .143 | .496 | .714 | 1.282 | .659 |
| 4 | .140 | .379 | .362 | .959 | .460 |
| 5 | .271 | .410 | .460 | 1.182 | .581 |
| Mean | .186 | .403 | .617 | 1.257 | |

Table 8

Average Error in Inches Tabulated by Distance Extrapolated and Angle of Intersection Pooled for Four Os

| Angle of Intersection | Distance Extrapolated .75" | 1.5" | 3" | 6" | Mean |
|---|---|---|---|---|---|
| 10° | .138 | .220 | .501 | 1.096 | .489 |
| 20° | .103 | .239 | .707 | 1.294 | .586 |
| 45° | .095 | .203 | .470 | .974 | .436 |
| 90° | .114 | .233 | .329 | .725 | .350 |
| 135° | .186 | .395 | .652 | 1.186 | .605 |
| 160° | .240 | .740 | .563 | 1.377 | .730 |
| 170° | .425 | .789 | 1.096 | 2.144 | 1.113 |
| Mean | .186 | .403 | .617 | 1.257 | |

Table 9

**Average Error in Inches Tabulated by Speed and Angle of Intersection Pooled for Four Os**

| Angle of Intersection | Speed 1 | 2 | 4 | 5 | Mean |
|---|---|---|---|---|---|
| 10° | .873 | .690 | .301 | .091 | .489 |
| 20° | .995 | .958 | .303 | .088 | .586 |
| 45° | .889 | .533 | .192 | .128 | .436 |
| 90° | .450 | .377 | .244 | .329 | .350 |
| 135° | .649 | .555 | .530 | .685 | .605 |
| 160° | .555 | .715 | .592 | 1.055 | .730 |
| 170° | .926 | .782 | 1.057 | 1.688 | 1.113 |
| Mean | .763 | .659 | .460 | .581 | |

Table 10

Variable Error (SD) in Inches Tabulated by Distance Extrapolated and Speed Pooled for Four Os

| Speed | Distance Extrapolated .75" | 1.5" | 3" | 6" | Mean |
|---|---|---|---|---|---|
| 1 | .057 | .101 | .247 | .404 | .202 |
| 2 | .055 | .127 | .202 | .324 | .177 |
| 4 | .084 | .143 | .183 | .393 | .201 |
| 5 | .149 | .146 | .233 | .415 | .236 |
| Mean | .086 | .129 | .216 | .384 | |

Table 11

Variable Error (SD) in Inches Tabulated By Distance Extrapolated and Angle of Intersection Pooled for Four Os

| Angle of Intersection | Distance Extrapolated .75" | 1.5" | 3" | 6" | Mean |
|---|---|---|---|---|---|
| 10° | .027 | .047 | .127 | .175 | .094 |
| 20° | .031 | .049 | .081 | .159 | .080 |
| 45° | .049 | .066 | .112 | .160 | .097 |
| 90° | .053 | .080 | .133 | .276 | .136 |
| 135° | .102 | .142 | .296 | .522 | .266 |
| 160° | .136 | .246 | .353 | .563 | .325 |
| 170° | .204 | .275 | .411 | .830 | .430 |
| Mean | .086 | .129 | .216 | .384 | |

Table 12

Variable Error (SD) in Inches Tabulated by Speed and Angle of Intersection Pooled for Four O

| Angle of Intersection | Speed 1 | 2 | 4 | 5 | Mean |
|---|---|---|---|---|---|
| 10° | .177 | .121 | .042 | .036 | .094 |
| 20° | .108 | .102 | .062 | .047 | .079 |
| 45° | .099 | .099 | .099 | .091 | .097 |
| 90° | .121 | .109 | .147 | .165 | .136 |
| 135° | .232 | .222 | .280 | .329 | .266 |
| 160° | .294 | .260 | .320 | .425 | .325 |
| 170° | .384 | .324 | .456 | .558 | .430 |
| Mean | .202 | .177 | .201 | .236 | |

Table 13

Variable Error (SD) in Inches Tabulated by Distance Extrapolated, Speed, and Angle of Intersection

| Angle of Intersection | Speed | Distance Extrapolated .75" | 1.5" | 3" | 6" |
|---|---|---|---|---|---|
| 10° | 1 | .034 | .071 | .315 | .286 |
| | 2 | .042 | .072 | .112 | .259 |
| | 4 | .017 | .028 | .037 | .086 |
| | 5 | .017 | .016 | .042 | .068 |
| 20° | 1 | .049 | .095 | .111 | .193 |
| | 2 | .041 | .045 | .109 | .213 |
| | 4 | .020 | .046 | .053 | .130 |
| | 5 | .014 | .024 | .050 | .099 |
| 45° | 1 | .065 | .071 | .107 | .151 |
| | 2 | .034 | .045 | .157 | .161 |
| | 4 | .052 | .253 | .116 | .150 |
| | 5 | .045 | .069 | .069 | .180 |
| 90° | 1 | .028 | .064 | .095 | .296 |
| | 2 | .032 | .067 | .120 | .217 |
| | 4 | .064 | .085 | .143 | .298 |
| | 5 | .090 | .103 | .173 | .295 |
| 135° | 1 | .065 | .081 | .285 | .498 |
| | 2 | .068 | .133 | .266 | .322 |
| | 4 | .110 | .173 | .331 | .505 |
| | 5 | .167 | .181 | .303 | .664 |
| 160° | 1 | .063 | .161 | .359 | .593 |
| | 2 | .084 | .222 | .326 | .407 |
| | 4 | .154 | .291 | .316 | .518 |
| | 5 | .244 | .311 | .412 | .735 |
| 170° | 1 | .093 | .177 | .457 | .809 |
| | 2 | .081 | .307 | .321 | .587 |
| | 4 | .174 | .299 | .287 | 1.063 |
| | 5 | .470 | .319 | .581 | .661 |

# AUTOBIOGRAPHY

I, John Ellis Mangelsdorf, was born in Atchison, Kansas, December 21, 1921. I received my secondary education in the public schools of Honolulu, Hawaii. My undergraduate training was obtained at Kansas State College, from which I received the degree Bachelor of Science in 1944, and the degree Master of Science in 1951. In September 1952 I received an appointment as Research Assistant in the Laboratory of Aviation Psychology under Professor Paul M. Fitts, The Ohio State University. In January 1955 my title was changed to Research Fellow. I held these positions for three years while completing the requirements for the degree Doctor of Philosophy.