Affordant Chord Transitions in Selected Guitar-Driven Popular Music

Thesis

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By

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Abstract

It is proposed that two different harmonic systems govern the sequences of chords in popular music: affordant harmony and functional harmony. Affordant chord transitions favor chords and chord transitions that minimize technical difficulty when performed on the guitar, while functional chord transitions favor chords and chord transitions based on a chord's harmonic function within a key. A corpus analysis is used to compare the two harmonic systems in influencing chord transitions, by encoding each song in two different ways. Songs in the corpus are encoded with their absolute chord names (such as "Cm") to best represent affordant factors in the chord transitions. These same songs are also encoded with their Roman numerals to represent functional factors in the chord transitions. The total entropy within the corpus for both encodings are calculated, and it is argued that the encoding with the lower entropy value corresponds with a harmonic system that more greatly influences the chord transitions. It is predicted that affordant chord transitions play a greater role than functional harmony, and therefore a lower entropy value for the letter-name encoding is expected. But, contrary to expectations, a lower entropy value for the Roman numeral encoding was found. Thus, the results are not consistent with the hypothesis that affordant chord transitions play a greater role than functional chord transitions. However, post hoc analyses yield significant results consistent with two other more moderate claims: first, affordant chord transitions do influence the chord sequences of popular music, although not as much as functional chord transitions; second, the relative influence of affordant versus functional factors on chord transitions has increased since the 1950s. Lastly, the corpus was refined to include only songs composed by guitarists and performed by its composers, and it was found that affordant chord transitions played a greater role in these songs.

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Introduction

In a 2011 study, de Clerq and Temperley analyzed aggregate chords and chord transitions of popular music, constructing a 99-song corpus from Rolling Stone magazine's "500 Greatest Songs of All Time" (2004). One notable observation made by de Clerq and Temperley was that chord transitions appear to be mostly symmetrical in popular music: for example, a I chord is immediately followed by a IV chord with probability of approximately 0.4, while a IV chord is followed by a I chord with the same probability. This chord transition symmetry found in the harmony of popular music ("pop harmony") is not found in common-practice harmony, where chord transitions are typically asymmetrical.¹ For example, while ii chords are immediately followed by V chords with a high probability in common-practice harmony, V chords are immediately followed by ii chords with a much lower probability (Huron 2006; p.251).

Functional / Affordant Harmony

This asymmetry can be illustrated by playing the chords of a Baroque chorale backwards. The resulting sequence sounds strange because chord transitions that were probable and normal in the forward direction become improbable and abnormal in the reverse direction. This directionality can be expressed simply by saying that commonpractice chord progressions are governed by *functional harmony*. Functional harmony is usually understood to rest on two foundations: First, chords have different functions in relation to a key; and second, there are preferred kinds movements amongst these functions. That is, every chord belongs to one of three functional classes relative to a key,

¹ The comparisons with common-practice harmonic theory in this paper are not intended to suggest that such a system is normative. Instead, it is merely hoped that it will be a point of reference that is helpful and familiar, at least to those coming from that background.

either tonic, dominant or pre-dominant. A functional progression tends to move from tonics to pre-dominants to dominants then back to tonics. That is, a functional progression is "directed" towards the goal of tonic. The symmetry of popular music means that it is not directed in the same way as Baroque harmony. In this paper, I argue that this is because the progressions are not primarily governed by functional harmony. Instead, I suggest a different organizing principle, that of *affordant harmony*.

In design, *affordance* is the concept that an object's attributes and its perception constrain and suggest the use of that object (Gibson 1977). For example, a horizontal bar on a door suggests that it should be pushed, whereas a loop suggests that it should be pulled. In the same way, the construction of an instrument can make it more or less suitable for playing in certain ways. For example, that the trumpet is used to allude to the neighing of a horse in Leroy Anderson's *Sleigh Ride* (rather than, say, the clarinet) is not an arbitrary assignment; it is attributable to the construction of the trumpet itself and its affordance for that particular sound. Similarly, a *glissando* performed on the trombone differs in pitch content from one performed on the piano because the continuous nature of a trombone slide affords a different sort of *glissando* than that afforded by the adjacency of white notes on a piano.

Guitar Affordances

To suggest that chord progressions in popular music are governed by affordant harmony, then, is to suggest that the musical instrument influences how chords and chord transitions are used in a song. Note that the following discussion assumes that the guitar is the primary instrument used in popular music, although the corpus used in this study includes many songs that are not composed with a guitar. The problems resulting from this sweeping generalization will be addressed in the last study of this paper. For now, we assume that in popular music, the construction and tuning of the guitar constrains and suggests the sorts of chord sequences that are used. Note also that only chord-to-chord transitions are investigated in this paper, although the scope of the word "harmony" is of course much broader than this. For example, no concept of chord heirarchy is included here, so there is no notion of passing chords or other embellishing chords. And since the larger organization of successive chords into phrases is not included either, there is therefore no notion of cadence or interruption.² Because so many elements of "harmony" are not taken into consideration, it is more correct to use the term "affordant chord transitions" instead of "affordant harmony." Specifically, "affordant chord transitions" are those chord sequences which may be influenced by the fact that certain chords are easier to play relative to other chords, and certain chord transitions are easier to play relative to other chords.

On a standard-tuning guitar, chords are formed by fingering a combination of six strings over frets. The six strings are tuned at E2, A2, D3, G3, B3, and E4, and each fret position raises the pitch of the scale by half-step. Thus, a C-major chord might be played as follows:

	(E4)			
	(B3)	-C4-		
(TOWARD	(G3)			(TOWARD
GUITAR	(D3)	-E3-		GUITAR
HEAD)	(A2)	-C3-		BODY)
	(E2)		Х	

Figure 1. A C-major chord. Each row of hyphens represents one of six guitar strings, with frets separated by the vertical bars. An "X" indicates that the string is not sounded, and a pitch name in place of hyphens indicates that the string is depressed at that fret.

The lowest string E2 is not played. The ring finger is used to depress the third fret of the A2 string, which sounds as C3. The middle finger depresses the second fret of D3,

² Although only pairs of adjacent chords are examined, triples of adjacent chords could be studied in the same way. The principle and methods would be the same, although this is not done in this paper.

which sounds as E3, and the index finger depresses the first fret of B3, which sounds as C4. The strings G3 and E4 are open, and so the pitches C3, E3, G3, C4, and E4 all sound to construct the C-major chord.

This fingering can be more concisely expressed as the vector (x32010). In this notation, each position represents a different guitar string, with the lowest, E2, on the left, and the highest, E4 on the right. An "x" indicates that the string is not played. A "0" indicates that the string is open (thus, this is an *open chord*). Any other number indicates the fret that is depressed for that string. Notice that this notation does not specify which fingers are used in the chord.

As another example, a C-minor chord could be played in the following way:

	(E4)	-G4-		
	(B3)	-**- -Eb-		
(TOWARD	(G3)	-C4-		(TOWARD
GUITAR	(D3)	-**- -G3-		GUITAR
HEAD)	(A2)	-C3-		BODY)
	(E2)		Х	

Figure 2. A C-minor chord. Asterisks indicate that the index finger forms a barre over those strings.

This could be expressed as the vector (x35543). To play this chord, the index finger must be placed on its side along the third fret (across the space indicated by asterisks) in order to play both the C3 and the G4 simultaneously. This technique is called *barring*, and the resultant chord is a *barre chord*. The remaining three fingers depress the remaining three notes.

Note that although a C-major chord and a C-minor chord are theoretically similar,

differing in only one pitch class, their physical implementation is quite different. The Cmajor voicing (x32010) can be played with three fingers, and has two open strings, while the C-minor voicing (x35543) uses four fingers, requires barring and has no open strings. While there are C-major voicings that use a barre, such as (x35553), there is no C-minor voicing that is open.³ Thus it can be argued that the easiest C-major chord is easier to play than than the easiest C-minor chord. To express this differently, one can say that the guitar affords C-major chords more so than it affords C-minor chords. Note that this claim is specific to these two chords, and cannot be generalized to all major and minor chords.

There are certainly other factors which can influence how difficult it is to play a chord. Physical factors include how many notes are barred, the total number of fingers used, the particular configuration of fingers, the position on the guitar's neck, and so on. There are also individual differences: difficulty may vary based on a performer's training or experience, and the particular model and condition of the guitar may be factors as well. Additionally, the manner of playing a chord – whether it is plucked or strummed, for example – may also be a factor in its difficulty of performance. Whatever factors there are, suffice it to say that there exists some gradient of chord difficulty, based simply on the easiest fingering of that chord on the guitar in the given context. That is, the guitar in standard tuning affords certain chords over others – not to say that certain chords are impossible or inappropriate.

Similarly, certain chord transitions may be easier than others. For example, consider the motion from a C-major chord (x32010) to G-dominant-seventh chord (320001), compared to the motion from a G-major chord (320003) to a D-dominant seventh chord (xx0212):

³ Practically speaking, anyway. For example, (x31013) is technically a C-minor voicing but very awkward to play.

	С				G7	
(E4)				(E4)	-F4-	
(B3)	-C4-			(B3)		
(G3)			->	(G3)		
(D3)	-E3-		->	(D3)		
(A2)	-C3-			(A2)	-B2-	
(E2)		Х		(E2)	-G2-	

Figure 3. A C-major chord and a G-dominant-seventh chord.

	G			D7	
(E4)	-G4-		(E4)	-F#-	
(B3)			(B3)	-C4-	
(G3)		 ->	(G3)	-A3-	
(D3)		 ->	(D3)		
(A2)	-B2-		(A2)		Х
(E2)	-G3-		(E2)		Х

Figure 4. A G-major chord and a D-dominant-seventh chord.

In the first case, the distance moved by each finger is minimal – to the same fret on adjacent strings. In the second case, however, the fingers must move to non-adjacent strings and to different frets. As above, there are certainly other factors that influence chord transition difficulty, but one could suppose that there exists a gradient of difficulty for chord transitions as well. Thus, the guitar affords certain chord transitions over others, with some transitions being relatively easy or hard compared to others.

Affordance, Idiomaticity and Transposition

So far, I have postulated an inverse relationship between affordance and technical difficulty – an instrument affords musical constructs that are technically easy to play, relative to others. Affordant harmony, then, governs chord sequences based on the technical difficulty (or ease) of playing certain chords or chord transitions, independent of key. This is proposed as an alternative to functional harmony, which governs chord sequences based on the directed motion of chord functions, within a key. As a caveat, it should be made clear that the concept of affordance does not prescribe what one should do with an instrument, or that certain things are impossible. Furthermore, something that is easy to play does not mean that is musically unsophisticated; for example, it may be an efficient way to achieve a desired musical effect. In addition, it is important to note that affordances can vary from musician to musician (for example, large hands may accommodate certain transitions over others).

Nonetheless, the minimization of technical difficulty in musical passages may reflect an instrument-specific conception, or an awareness of that instrument's affordances. This point is illustrated in a study by Huron and Berec (2009), who first made this connection between affordance and technical difficulty in trumpet music – although they refer to this concept as *idiomaticity*. Musical passages are idiomatic if they rely on resources specific to that instrument. In the study, a model was first constructed based on subjective difficulty ratings by trumpet players, considering a variety of factors such as valve transitions and note duration. Then, the model was used to predict the technical difficulty of passages of trumpet music. The model's predictions were largely consistent with human judgments of difficulty. The model was then used to calculate the difficulty of transposed versions of trumpet pieces: these pieces were transposed up to all twelve keys within an octave, and also transposed down to all twelve keys within an octave, for a total of 24 transpositions. Thus, for a given trumpet piece, 25 difficulty ratings were obtained at 25 different transposition levels.

Huron and Berec found that for works composed by non-trumpet players, the difficulty ratings were similar across all transposition levels. However, for works composed by trumpet players, the difficulty level tended to be lowest in the original key (that is, without transposition). This is consistent with the hypothesis that trumpet-playing composers wrote music with the capabilities of the instrument in mind. Or in other words, an idiomatic composition – that is, one readily afforded by the given instrument – minimized technical difficulty compared to alternative keys, but an unidiomatic composition was equally difficult in any key.

There are similarities in the present study and with the study by Huron and Berec, and a clear understanding of how key context and transposition interact with the two systems of chord transitions is necessary before proceeding. Chords in *affordant chord transitions* are viewed relative to the instrument, whereas chords in *functional chord transitions* are viewed relative to the key context. Because affordant chord transitions are based on the specific location of chords on the physical instrument, it is blind to the key context. So, in affordant chord transitions, a G-major chord in the key of G major is identical to a G-major chord in the key of F-sharp major. By contrast, key is central to functional chord transitions, and a G-major chord means something very different in G major versus in F-sharp major. In other words, two chords are identical in terms of affordances if and only if they have the same physical implementation on the instrument. When the key context is different, however, two physically identical chords are not identical in function.

One can also observe from the Huron and Berec study that transposition affects the two harmonic systems differently. Under transposition, a difficult chord may be mapped to an easy chord and vice versa; a difficult transition may be mapped to an easy one and vice versa. Thus, affordant relationships are not necessarily preserved by transposition. On the other hand, because transposition of a piece maintains all of its internal intervallic relationships, transposition does not affect the functional relationships at all. This is summarized in Table 1 below:

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	Transposition	Point of Reference	
Affordant chord transitions	Relationships changed	Physical movements on instrument	
Functional chord transitions	Relationships unchanged	Intervallic distances from tonic	

Table 1. Summary of how affordant and functional chord transitions differ in terms of transposition and point of reference.

In the interest of thoroughness, it should be mentioned that training in a musical style can result in additional affordances beyond those arising from the physical interaction with an instrument. Suppose that the chord transitions which are directly afforded by the guitar are "first-order affordances." If these particular chord transitions are used widely and frequently in certain styles, they may become characteristic of a style. A musician experienced in that style may then have affordances for certain chord transitions that are different than those of a musican experienced in a different style. Since these affordances are built up through exposure and training to a particular style or genre, building on any "first-order affordances" that may exist, Gjerdingen (2009) calls them "second-order affordances." These affordances include those that are learned aurally, for example, and may therefore include stylistic devices that are not necessarily easy to play. While an investigation of these affordances may have interesting implications for understanding the development of styles, it is not considered further in this paper.

Statement of Main Hypothesis

So far, only two factors in the generation of chord progressions in popular music have been considered in this paper: *affordant chord transitions*, which are based on minimizing technical difficulty of chords on a specific instrument, and *functional chord transitions*, which are based on tonic-directed motion relative to a key. Of course, there are other factors that likely contribute to harmony in popular music, which are not considered in this paper. In addition to second-order affordances mentioned above, the melody of a song may also constrain the suitability of some chords over others. Another is the particular sound of the chord – clearly, the subjective value of how a chord sounds is important in its use, which again will vary depending on the individual performer and the stylistic context

Also, it is most likely that these two systems are linked, both contributing to popular music chord transitions in mutually interactive ways. For example, functional factors may constrain the chord prior to a dominant chord to be most likely either a ii chord or a IV chord. At the same time, affordant factors may constrain which of the two is more likely: in C major, a ii chord may be more likely because a D-minor chord is easier to play than an F-major chord. In A major, however, the IV chord may be more likely because a D-major chord is easier to play than a B-minor chord. One could imagine more complex interactions, but for simplicity, this study will assume that both systems do not interact at all. This assumption – and its problematic consequences – will be further detailed in Study 1.

Ultimately, this paper addresses the question, "What is the relationship between the affordant and functional factors that affect chord transitions in popular music?" Notwithstanding the above caveats – that these are not the only factors influencing the choice of successive chords, and that even these two factors are intricately linked – I will argue that affordant factors contribute to chord transitions in popular music more than functional factors do.

Encoding System

Throughout this study, the influence of affordant versus functional factors in determining chord sequences is reflected in the method of encoding each song. The same song can be encoded differently, depending on the harmonic system being used. Recall that for affordant chord transitions, it is not important to know what key the song is in; the difficulty of a chord or chord transition depends only on the exact fingering of the specific chords. However, if that chord or chord transition is transposed, the difficulty can

change. Thus, a vector describing a chord's fingering on the guitar, such as (320003), is fully sufficient to express the information of the affordant view: the notation does not refer to any key, and it will necessarily change if the song is transposed.

Conversely, for functional chord transitions, the key of the song is of central importance. However, chord function is invariant to transposition. Thus, the Roman numeral encoding, such as "V", is sufficient to express the information of the functional view: this notation describes the chord in relation to a key, and does not change if the song is transposed.

Finally, as a concession to the difficulty of notating the guitar fingering for each chord in a song, the use of letter-name encoding is proposed as an approximation of this actual guitar fingering. This is only an approximation because there is not a one-to-one mapping between the fingering of a chord and the letter-name chord. For example, "GM" (a G-major chord) may map to several different guitar fingerings – (320003), (320033), (355433) etc. – although each fingering, for the most part, will only map to one specific letter-name chord. The letter-name encoding is mostly sufficient to express the information of the affordant view: it does not specify the key of the song, and does change if the song is transposed.

Thus letter-name chords will be used to encode each song for affordant chord transitions, while Roman numerals will be used to encode each song for functional chord transitions. This is summarized below, along with the information from Table 1:

	Transposition	Point of Reference	Encoding System
Affordant chord transitions	Relationships changed	Physical movements on instrument	Letter-name chords
Functional chord transitions	Relationships unchanged	Intervallic distances from tonic	Roman numerals

Table 2. A summary of affordant versus functional chord transitions: their interaction with transposition, their points of reference, and their corresponding encoding systems.

Preview

This paper builds on the work of the study by de Clerg and Temperley (2011), and uses the same corpus of popular music. In all four studies below, the information theoretic concept of entropy is used. In brief, entropy is the measure of the unpredictability of a data set. It is assumed that the better harmonic system, affordant or functional, renders the data more predictable, and yields an overall lower entropy value. In Study 1, entropy is used as a measure of the relative contributions of affordant harmony versus functional harmony in the chord sequences of the total corpus. To anticipate the results, the Roman numeral analysis yields the lower entropy value, and the main hypothesis that affordant factors contributed more to chord transitions in popular music is therefore *not* supported. Because of the negative results obtained in Study 1, the aims of Studies 2, 3 and 4 were to determine whether affordant harmony influences chord sequences in popular music at all. In Study 2, letter-name encodings of the songs within the corpus are transposed to random keys, and it was found that the entropy of the transposed songs was significantly higher than that of the original songs, consistent with the claim that affordant harmony does influence chord sequences in popular music. In Study 3, the entropy of individual songs within the corpus are calculated for both lettername and Roman numeral encodings, and then plotted against the song's year of release. It was found that the entropy of the letter-name encodings relative to that of the Romannumeral encodings decreases with time, consistent with the claim that the influence of affordant harmony has increased over time. Finally, in Study 4, the entropy measures from Study 1 are recalculated using a corpus that excludes songs that were likely not composed on the guitar, or were likely not composed in the key of performance. Although the results were not statistically significant, it was found that this reduced corpus has a lower average entropy value for the letter-name encodings than the average entropy value found in Study 1, consistent with the claim that affordant chord transitions may account for the differences in guitar-driven popular music versus non-guitar-driven popular music.

Study 1: Average Corpus Entropy

In Study 1, the contributions of affordant chord transitions and functional chord transitions to harmony in popular music are compared. First, the corpus of popular music used in this study is defined. Then, the terms of the hypothesis are clarified and operationalized using the information theoretic concept of entropy. The methodology, results and discussion then follow.

Corpus of Study

An important issue not yet addressed is the definition of what constitutes "popular music." This study uses the corpus of "rock songs" from de Clerq and Temperley (2011) which is based on Rolling Stone magazine's "500 Greatest Songs of All Time" (2009). The magazine's list was constructed by polling 172 professional musicians and music industry leaders, asking each one for their picks for the top 50 songs of the "rock & roll era." While the list is purported to reflect "rock and roll," it is clear that the definition used is very broad, including music by Led Zeppelin, The Beatles, Eminem and R. Kelly. So, while de Clerq and Temperley's use of the label "rock" may be contentious to some, the use of the broader label "popular music" in this paper is meant to avoid having to categorize songs with more specific styles or genres.

The Top 500 list was used to create a subset of 99 songs, which was used in de Clerq and Temperley's statistical analysis. This smaller 99-song corpus was compiled from the Top 500 list by taking 20 songs from each decade from the 1950s to the 1990s. One song had to be later omitted because it was found to not contain triadic harmony. After publication, they added an additional 101 songs from the top ranked songs in the Top 500 list (that were not already in the corpus), bringing the corpus size to 200. A complete listing of this corpus is given in Appendix A. It is this expanded corpus that is used in the present study as selected "popular music."

The corpus was independently encoded by de Clerq and by Temperley using the commercial recordings of each song. Each author conducted a harmonic analysis of each song, assigning a key label and Roman numerals. Root inflections are indicated ("bIII"), as well as quality and inversion of each chord. Each author could use modulations or applied chords according to their own judgment. In addition, a grammar-like notation was used to avoid the encoding of repeated sections such as verses or choruses. They wrote a computer program that would expand each analysis into a sequential chord-by-chord listing. An example is listed below, and further examples are included in Appendix B:

```
S: (16) [C] $In $Vr $Ch $Vr $Ch I | | | $Inst [D] $Vr $Ch $Co
Co: (3) $CP $CP $CP
CP: (7) I | | ii | V |
Inst: (11) $BP $BP ii | V | iii | [D] V |
Ch: (19) I ii | IV V | I ii | IV V | I | | ii | V |
Vr: (4) $BP $BP $BP $BP
BP: (4) I | IV |
In: (5) I | | | |
```

Figure 5. Temperley's Roman numeral analyses of "My Girl" by The Temptations.

The form of the song is indicated by "S." Other sections are defined with a ":", and are referenced using a "\$". The above analysis would be expanded with a script, resulting in the output below: Figure 6. The chord listing for "My Girl." Vertical bars separate measures, and square brackets denote the key.

Approach to Hypothesis

This study investigates two different harmonic systems which may govern the chord transitions in popular music. The main hypothesis of this paper is that affordant factors contribute to pop harmony more than functional factors. Two simplifying assumptions will be made. The first is that the systems of affordant harmony and functional harmony can be approximated simply as chord-to-chord transitions, as described above. The second assumption, briefly discussed in the introduction, is that the systems do not interact on the chord-to-chord level. The focus of this investigation then, is to determine which system can describe the chord-to-chord successions more succinctly. To this end, the concept of entropy from information theory (Shannon 1948) will be used. The general overview of this concept is presented below. A lengthier justification of its application to the present work is given in Appendix C.

Elementary Information Theory

Entropy quantifies the degree of unexpectedness in a set of data (Shannon 1948). This value can also be understood as the amount of novel information, and is measured in bits. The basic unit is the event, which is associated with the probability of that event occurring. For example, in a monophonic melody, each scale degree could be viewed as an event, and the tonic would have a higher probability of occurring than the submediant. For each unique event with probability p_i , the unexpectedness of that event is given by

$$-\log_2(p_i)$$

Thus, events that are certain ($p_i = 1$) have an unexpectedness value of zero, while events that are rare (p_i is small) have large unexpectedness values. The total entropy then, is the sum unexpectedness of all events, weighted by the frequency or probability of that event happening. Thus, the formula for entropy (H) is given by

$$H = -\sum_{i=1}^{n} p_i \log_2 p_i$$

where *n* is the number of unique events. Note how $-\log_2(p_i)$ can also be used to characterize the information content provided by each kind of event.

Example of Entropy in English

Consider an English sentence in all uppercase letters, with no punctuation. As a first pass, one may make the faulty assumption that each of the 26 letters is equally probable. That is, each character has unexpectedness or information content of $-\log_2(1/26) = 4.7$ bits. This is, in fact, the maximum entropy for a domain with 26 events. However, the letters in English are not equally probable. For example, the most common letter, "e" appears with probability 0.0816, while the least common letter "z" appears with probability 0.00074 ("Letter frequency", 2011). Thus, the unexpectedness or information content of an "e" is fairly low, at $-\log_2(0.0816) = 3.62$ bits, while the information of a "z" is fairly high, at $-\log_2(0.01) = 10.4$ bits.

Since the more common letters have a lower information content, the average information content overall is lower than that with equiprobable letters. Shannon (1950) calculated the entropy of English using letter distribution probabilities to be around 4.14 bits on average per letter. This bit value is further reduced if we account for the first-order transition matrix as well – that is, using a probability table for the the letter-to-letter transitions. This table simply lists the probability for each letter being followed by any other letter. For example, this table quantifies the knowledge that "q" is most likely followed by "u", and that "y" is unlikely to be followed by a "j", and that "t" is never followed by "x." The application of this knowledge can reduce the entropy of English to

about 3.56 bits per letter. With a second-order transition matrix (that is, using transition probabilities that are based on two previous characters instead of just one) this entropy value is reduced further to 3.3 bits per letter.

Shannon also conducted experiments with human subjects involving degraded texts. Participants were required to guess the missing letters in order to reconstruct the original texts, as in the example below.

P-L-S----O---BU--L-S--O----SH-----RE--C----

POLISHED WOOD BUT LESS ON THE SHABBY RED CARPET

Figure 7. Degraded texts (above) given to participants, and original texts (below) that were to be reconstructed, in the experiments conducted by Shannon (1950).

Participants were required to guess the letters in sequence, and to keep guessing until they got it correct, at which point they would then proceed to the next letter. The number of guesses required to guess each letter was then used to calculate the information content of each letter. Through this, it was found that the information carried by each letter was on average only 1.6 bits. Entropy values of the examples are summarized below:

Systems for characterizing English sentences	Entropy (bits/character)
All letters equally probable	4.7
Letter frequencies (0 th order probabilities)	4.2
1 st order transition probabilities	3.6
2 nd order transition probabilities	3.3
Human conception (experimental value)	1.6

Table 3. Entropy values for increasingly adequate accounts of English sentences.

Recalling that entropy is a measure of the unpredictability of a data set, we see therefore that unpredictability decreases as the quality of the characterizing system increases. In other words, the best system for describing a data set is the system that renders the data most predictable, with the lower entropy value. We can understand that the information content in the data set diminishes because the description of the system itself contains quality information about the data; that is, the system characterizes the data well.

Conversely, using a system that characterizes the data poorly will inflate the entropy value. For example, using the letter transition probabilities for English on a French text will result in a higher entropy value than using the transition probabilities for French. This is because the English transition probabilities cannot predict the French text better than the French transition probabilities. Common transitions in French may not be common in English, resulting in many states of high unexpectedness.

Thus, in order to compare whether affordant factors or functional factors influence chord transitions more in popular music harmony, it is proposed that the system which yields a lower entropy value may be considered to characterize the data more succintly. As an operationalization, the entropy tests will be conducted in this way: for the assumption that affordant factors govern the chord transitions entirely, the entropy should be calculated based on the tablature location of the chords. Although it is possible to collect this data, it is not included in the de Clerq and Temperley corpus. As mentioned previously, the letter name of each chord is used instead. This is not the ideal encoding, because for any given letter-name chord, say G major, there are several ways of playing it on the guitar. However, because the de Clerq and Temperley corpus already includes Roman numerals with key information, it is convenient to use letter-name chords in place of specific chord tablature. For the assumption that functional factors govern the chord transitions entirely one can simply use the Roman numerals as mentioned previously, which exactly captures the relation of a given chord to the song's key.

Recall that the main hypothesis is that affordant harmony contributes to the sequence of chords in the rock corpus more so than functional harmony. Therefore, it is expected that the normalized entropy from the letter-name encoding is lower than the normalized entropy from the Roman numeral encoding. To anticipate the results, it will be said that the results obtained were contrary to the hypothesis.

Method

This study uses the 200-song corpus by de Clerq and Temperley. Notice that since both authors performed an independent analysis, there are two analyses for each song. It was found that that there was a high degree of agreement between the two sets of analyses, although they are not identical. We will carry out the methodology for the Temperley analyses, and then repeat the method for the de Clerq analyses.

Using the expander program provided by the authors, a script was written to convert each song into a chord list. In the figure below, the first ten chords of Queen's "Bohemian Rhapsody" are shown. In the first column, labelled "RNE" is the Roman numeral encoding. In the second column is the key of the section, in pitch-class notation – this song is in B-flat. For simplicity, all extensions (including sevenths), inversions and suspensions were stripped from the Roman numeral encodings. In the third column, the chord quality is listed, with "M" for major, "m" for minor, "A" for augmented and "d" for diminished. The fourth column lists the "relative chromatic root" – this is expressed in semitones above the tonic. Thus, for the V/V chord, its root is designated "2." The fifth

column is the "absolute chromatic root", expressed as semitones above pitch-class C. Integers are used in the data table, rather than letter names, because they are easier to manipulate.

**RNE	**KEY	**CQ	**RCR	**ACR0 ⁴	**TITLE
vi	10	m	9	7	bohemian_rhapsody
V/V	10	М	2	0	bohemian_rhapsody
V	10	М	7	5	bohemian_rhapsody
I	10	М	0	10	bohemian_rhapsody
vi	10	m	9	7	bohemian_rhapsody
V/IV	10	М	0	10	bohemian_rhapsody
I	10	М	0	10	bohemian_rhapsody
ii	10	m	2	0	bohemian_rhapsody
V	10	М	7	5	bohemian_rhapsody
V/IV	10	М	0	10	bohemian_rhapsody
(etc.)					

!!!SONG: DT ANALYSES//bohemian rhapsody dt

Figure 8. An excerpt from an expanded chord listing, derived from the de Clerq and Temperley analyses. Each successive row represents a subsequent chord, and contains information about the key (in semitones above pitch class C), the quality of the chord, its root relative to the key (in semitones above tonic) and its root relative to pitch class C (in semitones above). The RNE column gives the Roman numeral for each chord, the KEY column gives the key context for each chord, the CQ column gives the quality of each chord, the RCR (relative chromatic root) column gives the semitones above tonic for each chord, the ACR0 column gives the semitones above pitch-class "C" for each chord, and the TITLE column includes the name of the song from which the chord is taken.

⁴ The "0" indicates this is the untransposed version of the roots, useful when transposing the songs later on.

Then, from the "RNE" column of all the songs, a list of transitions for Roman numeral encodings was obtained. The following is an excerpt:

!!!SONG: DT_ANALYSES//bohemian_rhapsody_dt
**RNE
vi V/V
V/V V
V/I
I
I vi
vi V/IV
V/IV
V/IV
I
I ii
ii V
V/IV
V/IV
V/IV
V/IV
(etc.)

Figure 9. An excerpt of the chord transitions listing. Each row represents an antecedent and consequent chord encoded with Roman numerals.

Likewise, using the "CQ" and "ACR0" columns, the corresponding transitions with the letter-name encoding were obtained. This is presented in the figure below. Since enharmonic equivalents are identical on the guitar, only naturals and sharps are used; never flats. This is purely a convention to ensure that enharmonic equivalents are notated in the same way.

!!!SONG: DT_ANALYSES//bohemian_rhapsody_dt
**LNE
Gm CM
CM FM
FM A#M
A#M Gm
Gm A#M
A#M A#M
A#M Cm
Cm FM
FM A#M
A#M D#M
(etc.)

Figure 10. An excerpt of the chord transitions listing. Each row represents an antecedent and consequent chord encoded with letter-name chords. The single column LNE represents the letter-name encodings of successive pairs of adjacent chords.

Thus, a listing of every single chord transition in the corpus is constructed. In the Temperley corpus, there were a total of 18,465 chord transitions; in the de Clerq corpus, there were a total of 18,903 chord transitions. The "infot -s" command from the Humdrum Toolkit was used to calculate the total entropy for the transition list for the letter-name encoding, corresponding to affordant harmony, and the Roman numeral encoding, corresponding to functional harmony.

Finally, note that in order to compare entropy scores, one should normalize them according to the number of possible transitions. This is because if one system has more possible transitions than the other, it will have a higher entropy value even if its contribution is comparable to the other. This is further explained in Appendix C. If there are n different chords, there are $(n - 1)^2$ possible transitions. Thus, the raw entropy score

for the letter-name encodings is divided by $\log_2(n_L - 1)^2$, where n_L is the number of lettername chords in the corpus. Similarly, the raw entropy score for the Roman numeral encodings is divided by $\log_2(n_R - 1)^2$, where n_R is the number of Roman numerals in the corpus.

It should be clear that if the two encoding systems were perfectly correlated, both will yield the same entropy value. That is, if every V chord in one encoding is always a G chord in the other encoding, both encodings will yield the same entropy value. An entropy difference will only be observed if a V chord under one encoding may be a G chord in one song, a D chord in another, a C chord in yet another, and so on. Thus, if some keys are more likely than others in the corpus (which is probably true), then Roman numeral encodings will be somewhat correlated with letter names. One caveat then, is that the result in the entropy difference between the two encodings will be attenuated if not all keys are equally likely. This may cause the main effect, if any, to be obscured. As it turns out, this correlation between the two systems may be problematic, and will be discussed again after the results are presented.

Results

This results from this study are summarized in the figure below.

Encoding	Letter-name		Roman numeral		
Analysis	Temperley	de Clerq	Temperley	de Clerq	
# of songs	200	200	200	200	
# of chord transitions	18465	18903	18465	18903	
# of unique chords	36	33	46	55	
Raw transition entropy (bits)	131212	134526	103699	107736	
Normalized transition entropy (bits)	12790	13453	9441	9360	

Table 4. A summary of the results for Study 1. The raw transition entropy is normalized by dividing by the logarithm of one less the number of unique chords squared.

Discussion

The mean normalized transition entropy is 13,122 bits for the letter-name encoding, and 9,401 bits for the Roman numeral encoding. This value is obtained by averaging the results for the Temperley corpus and the de Clerq corpus. Contrary to the main hypothesis, the letter-name entropy (LNE) is not lower than the Roman numeral entropy (RNE). Note that the number of chords used in the corpus under the letter-name encoding is actually less than the number of chords under the Roman numeral encoding. Therefore, all else being equal, the raw LNE should be lower than the raw RNE, even before normalization. This is not the case. In fact, the normalization operation is unnecessary to see that the results – that the RNE is lower than the LNE – are not consistent with the hypothesis that affordant harmony contributes more to rock harmony than does functional harmony. However, this should not be understood as an immediate rejection of the main hypothesis. When the results are inconsistent with a hypothesis, it is appropriate not to reject the hypothesis without first considering the other possibilities.

Alternative Sources of Negative Results

When a hypothesis is not supported by experimental results, it may be the case that the hypothesis is simply not true – the hypothesis should therefore be rejected. To anticipate the trajectory of this discussion, this hypothesis will eventually be rejected here. However, it must be restated that results may be inconsistent with the hypothesis even when the hypothesis is valid, and it is important to consider these reasons.⁵ Such reasons can broadly be grouped into two categories: random noise and methodological error.

Noise refers to any unforeseen or unaccounted disturbances in the data. Often, this refers to other factors on the phenomenon that were not controlled for in the experiment. Thus, noise is contextual – important factors are considered noise if they are not controlled in the study. For example, in a study that studies the effect of sunlight on plant growth, the effect of the plant's genes on growth may be considered noise – even though genetics may be an important factor in reality – if the plant's heredity was not controlled. If the effect of plant genes is large, the results may be dominated by the random effect of genes, even if sunlight is an important factor. Noise also includes human errors that result in data being omitted or encoded incorrectly, as well as anomalies due to poor sampling or chance. To minimize the impact of noise, one can rely on the law of large of numbers, with the expectation that truly random effects will cancel each other out in the long run. The use of large samples (or similarly, replications of the study with different samples) aims to minimize the effect of uncontrolled variables, human errors or sampling anomalies.

Methodological errors, on the other hand, are not minimized by larger samples or repetition. These are systematic problems with the way that the variables under consideration are investigated, such that the method is not a good test of the hypothesis at all. This includes invalid assumptions about the phenomena, as well as problems with measurement of the variables of interest. For example, if the scale used to measure plants in sunlight was different from the scale used to measure plants without sunlight, the

⁵ These are collectively known as Type II errors, when valid hypotheses are rejected, or false negatives. Type I errors are false positives, when invalid hypotheses are not rejected.
results would be skewed no matter how many times the experiment were repeated. Discovering such errors requires a careful examination of every aspect of the methodology, including the assumptions and execution, possibly resulting in a redesign of the experiment.

Before considering how noise and methodological error may have impacted the results of this particular study, it is also important to distinguish between *post hoc* observations, which are made after the experiment is conducted, versus those considerations that were made *a priori*, made before the experiment was conducted. In general, *post hoc* arguments are less valuable than those that are made *a priori*. *A priori* arguments are preferred because they are less likely to be rationalizations; a *post hoc* argument can be shaped to fit any result. *Post hoc* arguments are not necessarily invalid, and can be valuable considerations for planning subsequent experiments. However, it is important to acknowledge that they were formulated after viewing the results. I will be careful in making this distinction in the following discussion.

Shortcomings of the Present Study

One *a priori* factor contributing to the results concerns the songs used in the corpus, due to the study's basic assumption that guitar fingerings influence chord sequences. First, and most importantly, any songs that were not composed on the guitar adds noise to the data. Additionally, the use of a capo or alternate tunings means that the operationalization of tablature using letter-name chords becomes invalid as well. Thus, even if guitar fingerings did affect chord sequencing on standard-tuning guitar songs, the inclusion of songs in the corpus that are not composed with standard tuning guitar may dilute the observed effect.

Post hoc, it was also noted that some songs in the corpus are built from obviously functional or non-affordant harmonic elements. Firstly, songs that employ the 12-bar blues form (such as "Hound Dog", popularized by Elvis Presley) clearly use three predetermined functional chords. Secondly, songs that modulated (employing the "repeat up a step" cliché) were most likely not guided by affordant harmony in the new key, since those chords are simply transposed from the old key, according to the function of those chords. Because the operationalization of the guitar fingering relies on the assumption that the letter-name of the song corresponds to the position on a standard-tuning guitar when the song was composed, any songs that invalidate this assumption contribute to noise in the data. To address these issues, more careful control of songs in the corpus is required. This may include constructing a new corpus and setting up exclusion criteria. This is partially addressed below in Study 4 – namely, songs that are not likely composed on the guitar are excluded from the corpus.

However, the above notwithstanding, I suspect that the main contributing factor to the results obtained was an *a priori* methodological assumption regarding the interaction of affordant and functional harmony. Recall that the assumption made was that only one of either affordant chord transitions or functional chord transitions were applicable at any given time. It was further assumed that the two harmonic systems were uncorrelated. While this was *a priori* a simplifying assumption, *post hoc* analysis of the data reveals that the interaction is stronger than expected.

A simple tally shows that the most common transitions based on letter-names in the corpus are "GM \rightarrow DM", "DM \rightarrow AM", "AM \rightarrow EM" and "DM \rightarrow GM". Note the root motion by fourths. It was originally assumed that chords driven by affordant factors would, in a sense, "drive up" the entropy of the Roman numeral encoding, as unconventional functional transitions would result from the affordant chord transitions. However, it is clear *post hoc* that the most common affordant transitions usually will result in common Roman numeral transitions, such as "I – IV" or "V – I." Thus, if affordant harmony generates motions that are common to functional harmony, the data is confounded.⁶

Another *a priori* problem with comparing entropy values is that there is no external benchmark to compare how different they are. For example, in another genre, the ratio of the Roman-numeral entropy to the letter-name entropy ratio may be higher or lower. *A priori* the solution to this was to normalize the entropy values based on the number of transitions, but there is no way to evaluate whether this normalization is

⁶ Of course, this argument could be reversed as well. If the results instead were that letter-name entropy was lower than Roman-numeral entropy, one may have argued that this was because common Roman-numeral progressions generate the common letter-name transitions in the most common keys.

effective.7

To summarize, in this study, we did not obtain the hypothesized results. The foremost reason that this may be the case is that the hypothesis is not true. That is, affordant harmony does not contribute to rock harmony more than functional harmony. The alternative reason is that problems with the study itself led to the inconsistent results. That is, the influence of affordant harmony was not observed in the data because of noise - (mainly, problems with the chosen corpus) or methodological error (mainly, the invalid assumption that functional and harmony do not interact, and that the comparison of their transitional entropy values is meaningful.)

In either case, it is clear that functional harmony accounts for the chord transitions in rock harmony very well compared to affordant harmony. Its transitions have the lower entropy value, and the most common Roman numerals are those that would be expected based on common practice harmony. Thus, given the results, we should reject the main hypothesis of the experiment, that affordant harmony influences rock harmony sequences more than functional harmony.

Alternative Post-Hoc Hypotheses

Even so, we can propose an alternative research question that is weaker instead. As mentioned previously, because of the high correlation between the two systems of harmony, it is difficult to say which one influences the chord sequences more. Instead, one could ask "what role (if any) does affordant harmony play in rock harmony?" However, given that affordant harmony appears to be highly correlated with functional harmony, any new approaches must control for this interaction.

Below, I propose several possible approaches to this aim. The second approach will be undertaken as Studies 2, 3 and 4 in this paper; the remainder I leave as possible avenues for future research.

⁷ Although in this case, the normalization turns out to be unnecessary because there are fewer letter-name chords anyway.

Approach #1: Compare LNE / RNE to other corpora

Recall that one of the problems with interpreting the results is that there is no benchmark to compare the entropy from the LNE, versus the entropy from the RNE. This problem exists on three levels. Firstly, there is the problem of interpreting the difference in entropy between RNE and LNE. As mentioned, because of the high degree of correlation between the two (due to the uneven distribution of keys in the corpus - a Vchord is more likely to be certain chords versus other chords), it is not certain if the higher value always belongs to the system that has a higher influence on the sequences. This is because the most common functional transitions are also the most common affordant transitions. Secondly, given two entropy values, it is unclear how to interpret the difference. For example, when the entropy from RNE is half that of LNE, it does not mean that functional harmony is twice as influential. Thirdly, there is no external benchmark as to what a large RNE / LNE difference may be. For example, suppose that affordant harmony only accounts for 10% (however that is quantified) of the chord transitions in rock harmony. If affordant harmony only accounts for 1% of the chord transitions in other styles, then this would suggest that guitar fingerings play a greater role in rock harmony than in other styles. Thus, what's missing is a means of evaluating the difference in entropy for RNE and LNE.

One way to do this is to compare to other corpuses. An example would be to look at a collection of music where the physical interaction with an instrument is similar across all keys, such as choral music. That is, it seems unlikely that for a choir, chord relations in B-flat major would be different than those in B major. A Roman numeral analysis for a corpus of choral works could be performed, and the entropy value calculated. In the same way, the letter-name analysis of the same corpus can be used to calculate another entropy value. The resultant ratio, of entropy by letter name versus entropy by Roman numeral yields an estimate of the ratio of entropies – LNE / RNE – when letter-names have no effect. The calculation of the LNE / RNE ratio for several different styles will give a better context to evaluate the LNE / RNE ratio obtained in this study. The actual execution of this study is not done here, and left open for work in the future.

Approach #2: Compare LNE / RNE within corpus

As stated above, there is no good external benchmark with which to compare the present LNE / RNE ratio found in this study. However, comparisons internal to the corpus may be possible, using carefully controlled methods. In the remainder of this paper, three internal comparisons are attempted: comparing the LNE / RNE ratio of the corpus songs versus that of the same songs transposed to random keys; comparing the LNE / RNE ratio of songs that are likely to be composed using a guitar versus those that are not likely to be composed using a guitar.

It would be ideal to make comparisons with any LNE / RNE ratio where we are reasonably certain that affordant harmony is not a factor. One way of achieving this is to use the current corpus, but to transpose the songs to different keys randomly first. In this way, the Roman numeral encodings for each song remain the same, but any relationships to guitar fingerings are randomized. Calculating the LNE / RNE ratio for this random key corpus, then yields a baseline value for the entropy ratio for a corpus where affordant harmony is not a factor. Thus, this is a way to compare the LNE / RNE ratio to some internal benchmark: the entropy values for the actual keys of the songs, versus those of the songs in random keys. If the values are not different, we would suspect that there is nothing particular about the actual keys of the songs, and therefore conclude that the influence of chord fingerings is not a factor. On the other hand, if the entropy values for the actual song keys were lower, this would be consistent with the interpretation that the specific chords fingerings influenced the sequence of chords in the song in some way. In other words, the use of both Roman numerals and the key of the song accounts for the chord sequences better than just the Roman numerals alone. This is carried out in Study 2 below.

Another way to compare LNE / RNE with an internal benchmark is to calculate the LNE / RNE for each song individually. de Clerq and Temperley noted that songs before the 1960s were dominated by the use of I, IV and V chords, and that it was only later that other chords started becoming widely used. Thus, we expect the entropy for the chord transitions, for either encoding, to increase along with the year of the song's release date. As more uncommon chords are introduced, one can ask whether this was due to functional or affordant considerations.

Here, another prediction can be made: that the addition of chords beyond I, IV and V are based on affordant harmony rather than functional harmony. In Study 3, below, this prediction is tested: while both LNE and RNE are expected to increase along with a song's release date, we expect that the entropy for the letter-name encoding increases less than that of the Roman numeral encoding.

A final way to make internal comparisons to the corpus may be to exclude the songs in the corpus for which affordant factors are not likely to exist. Thus, the LNE / RNE for the total corpus may be compared to the LNE / RNE ratio for the corpus that includes only songs composed on the guitar. This ratio is expected to be lower for the guitar-composed songs. This approach addresses the issue of having non-guitar songs in the corpus at all, and is carried out in Study 4.

Approach #3: Examine distributions of chord transitions in different keys

If affordant harmony has an influence on chord progressions, then songs in different keys will tend to have different distributions of Roman numeral transitions. For example, if " $G \rightarrow D$ " is a favored transition, " $V \rightarrow II$ " might be favored in C major, but not in A major. One way to effectively carry this out may be to examine the distribution of Roman numerals in each key. However, this approach is also left open as an avenue for future research, and is not carried out in this paper.

Three Post-Hoc Studies

In the remainder of this paper, Studies 2, 3 and 4 will be presented. Study 2 addresses the question of whether affordant harmony influences chord sequences at all. Study 3 addresses the question of how the influence of affordant harmony changes over time. Study 4 is simply a replication of Study 1 using a subset of the corpus that excludes songs which do not seem to use a guitar in its composition. The results of these studies will be presented without an in-depth discussion. Then, the final section of this paper discusses the project as a whole, including some comments on these three studies.

Study 2: Randomized-Key Entropy

The results of Study 1 do not support the hypothesis that affordant factors influence chord transitions more than functional factors. Consequently, it seems prudent to test whether affordant factors influences chord progressions at all, and this is the aim of Study 2. The difficulty lies in interpreting the entropy value obtained in Study 1 for affordant chord transitions without any other reference point. How large must this value be to claim that affordant factors have no effect, or how small must it be to claim that affordant factors do have an effect?

In fact, a useful reference point can be obtained by constructing a corpus in which affordant factors have no effect, and this letter-name entropy of this corpus can then be compared to the entropy value obtained in Study 1. Recall that transposition has an unpredictable effect on affordant chord transitions. Transposing a song in the original corpus to a random key does not preserve the affordant relationships in its chord transitions. Thus, transposing each song in the corpus to a random key effectively dismantles all affordant relationships in the corpus. For example, if the "D \rightarrow G" transition arises out of affordant harmony, it will appear commonly in different songs in the corpus, irrespective of their keys. Random transposition of the corpus songs, however, will fragment this common chord transition to random keys unpredictably, such as "C# \rightarrow F#", "B \rightarrow E", and so on. The result is essentially a corpus in which affordant harmony has no effect.

Calculating the entropy value of this randomized-key corpus yields the baseline entropy level for when affordant factors are non-existent. If the entropy value of the original corpus is lower than the entropy value of the randomized-key corpus, this would be evidence that affordant factors exist for the original corpus. Note that this is analogous to the idiomaticity study of Huron and Berec (2009): when the difficulty rating for a trumpet piece was lower than the difficulty rating of the same piece in all other keys, this was taken as evidence for idiomatic considerations affecting the piece's composition.

Method

To ensure that the original letter-name entropy value can be validly compared to the the entropy value from random transpositions, the distribution of keys in the corpus must be identical after randomization. Recall that maximum entropy is achieved when all states are equally probable. In the corpus, the letter-name encoding chords are not all equally probable, because all keys are not equally probable. If randomization of the keys results in a more even distribution of keys, for example, the entropy will increase whether affordant harmony is a factor or not. Thus, it is important to ensure that the distribution of keys after randomization is the same as the distribution of keys before randomization – only then is an entropy comparison valid.

The key of each song is obtained, based on the first key indicator in each song. This set of keys reflects the distribution of keys in the corpus. Each key is then randomly assigned to a song, in a one-to-one mapping. Each song is then transposed to its new key. (If a song is randomly assigned its original key, no transposition resulted.) The entropy for the letter-name encoding of all the transposed songs is then calculated. This process was repeated twenty times to obtain a mean entropy value for randomly transposed lettername encodings.

Results

The key distribution of the 200 songs is as follows:

**Frequency	**Key (major or minor)				
4	F-sharp (or G-flat)				
5	В				
8	G-sharp (or A-flat)				
9	C-sharp (or D-flat)				
14	A-sharp (or B-flat)				
14	D-sharp (or E-flat)				
17	F				
18	D				
19	С				
26	А				
29	G				
37	E				
200 Total					

Figure 11. Tally of the keys of the songs in the corpus.

When the entropy value was calculated using the randomly-assigned keys, the mean entropy was 137,945 bits. The mean (raw) entropy value for letter-name transitions was 132,869 bits. Although this difference seems small, the entropy for the randomly-assigned keys is significantly greater than the entropy for the actual keys (P < 0.0001, t = 8.1120, df = 19). Therefore, these results are consistent with the hypothesis that affordant harmony does indeed influence chord sequences in rock music. However, the observed effect size is quite small, and it is unclear how many chord transitions are actually affected. In Study 1, it was seen than the influence of affordant factors was certainly less than fifty percent. In this study, the claim that its influence is non-zero is consistent with the results. Further research in this area may better quantify the extent to which affordant factors influences chord sequences in popular music.

Study 3: Entropy and Song Release Date

In Study 1, the entropy values for the letter-name encoding and Roman-numeral encoding of the corpus were calculated. It was found that Roman numerals yielded the lower entropy value, and this led to the conclusion that functional factors play a greater role than affordant factors in determining chord sequences. It was noted previously that one could not directly compare the two entropy values to quantity the relative influence of the two harmonic systems. It is possible, however, to directly compare the corresponding entropy values for different songs in the corpus. In Study 2, the ratio of the entropy for letter-name encodings versus Roman-numeral encodings is calculated. This measure is used to test an observation by de Clerq and Temperley (2011).

After sorting their corpus by decade, they observed that rock songs written prior to the 1960s were dominated almost entirely by the chords I, IV and V. It wasn't until after the 1960s that other chords, such as II or bVII, were used with any notable frequency. One may suppose that the new chords were a result of functional harmonic relations such as mixture or tonicization. Another possibility, however, is that the new chords are a result of affordances for certain chords on the guitar. These need not be mutually exclusive.

For example, the first chord of the chorus of David Bowie's "Heroes" (1977) was analyzed as a bVII chord, one of the harmonic functions that became more common after 1960. At the same time, the chord is specifically a C-major chord, a chord that is readily afforded by the guitar. So, is the chord sequence in the "Heroes" chorus a result of functional harmony or affordant harmony? It is impossible to say for sure, and the answer is probably "both."

But what observations, if any, would constitute evidence that affordant harmony

was a factor in the existence of this particular bVII chord? Suppose that songs in D-major were as common as songs in C-major (indeed, in the corpus, there are 18 songs in D and 19 songs in C). While the bVII chord in D major is easy to play, note that the bVII chord in C major – B-flat – is a chord that is not readily afforded by the guitar. Now, suppose that one counts the instances of bVII in D major versus bVII in C major. If bVII chords were more common in D major (as C-major chords) than in C major (as B-flat major chords), this may be taken as evidence for the influence of affordant harmony in the adoption of these less common functional harmonies.

This particular approach is not carried out in the present project, due to several methodological details that are yet to be worked out. Namely, how does one select which chord transitions to examine without biasing the results? Also, due to the relative scarcity of transitions that are not within I, IV or V, the corpus size will likely need to be increased before this analysis can be conducted.

To examine the influence of affordant harmony on the less common functional chords, this study will use a diachronic approach instead, along with entropy measures. Recall that two entropy values were calculated: one for affordant factors using letter names, and one for functional factors using Roman numerals. Although the magnitude of these values may not be comparable directly, the ratio of these two values for different songs may be compared. That is, a 2:1 ratio of letter-name entropy to Roman numeral entropy ("LNE / RNE") certainly does not mean that affordant harmony is half as influential – but if one song has a 2:1 LNE / RNE and another song has a 3:2 LNE / RNE, it suggests that in the latter song, affordant harmony was a relatively larger factor than in the former song.

So, if the prominence of chords other than I, IV and V increases over time, and if the use of these less common chords is attributable to affordant harmony, then the LNE / RNE ratio should decrease over time. This hypothesis is tested by calculating the LNE / RNE ratio of each song in the corpus.

Method

From Study 1, the probability of each chord transitions and its associated

information value can be obtained. Uncommon chord transitions will have high information or entropy, and common transitions will have low information or entropy. The information provided by each transition, is:

$$-\log_2(p_i)$$

where p_i is the probability of that transition in the corpus. Common transitions (like "I \rightarrow IV" or "G \rightarrow C") have the lowest information. This is given by the Humdrum command "infot -b". Sample output is included in the figure below. For each song then, the average information per transition can be calculated easily, for both the letter-name encoding and the Roman numeral encoding. This yields a LNE / RNE ratio for each song. Note that five songs are omitted because they consist of only of a single chord.

**Trans	**Info
I III	9.973
I VI	11.557
ii V	6.200
V IV	7.442
vi V/vi	10.973
v V/ii	11.142
I IV	12.557
I IV	3.251
(etc.)	

Figure 12. An excerpt of Roman numeral chord transitions, with the corresponding entropy value for that transition. Common transitions have low entropy, and uncommon transitions have high entropy.

Then, the correlation between the LNE / RNE ratio and the year of release of the song is calculated. The hypothesis is that affordant harmony increases as non I, IV or V

chords are used, and thus the LNE / RNE ratio decreases. The prediction, then, is that LNE / RNE ratio of a song is negatively correlated with its year of release.

Results

The LNE / RNE ratio is plotted below as RNE / LNE over date of release. The correlation between RNE / LNE and the year of release is -0.157, which is significant (p = 0.0142, t = -2.209, df = 193).



Figure 13. The ratio of letter-name entropy to Roman-numeral entropy for each song in the corpus, plotted against the year of release for that song. The negative correlation is significant (p = 0.0142) and is consistent with the hypothesis of an increasing use of affordant harmony over time.

Study 4: Entropy with Refined Corpus

It was mentioned above that problems with the present corpus may have contributed to the negative results in Study 1. That is, in investigating the effect of guitars on popular music, songs played without guitars or not written on the guitar should clearly be excluded. Songs played on guitars with alternate tunings have different affordances than those in standard tuning, and should therefore be excluded as well. Additionally, since transpositions do not preserve affordant relationships, songs that are played with a capo or those tuned down a half-step should be "untransposed" in order to reflect the actual fingerings used to play them. Special consideration may also be required for songs that modulate by step (known as a "pump-up"), and for covers that were originally written in a different key. A replication of Study 1 with a refined corpus may obtain different results. Study 4 is a replication of Study 1 using a reduced corpus, where songs that do not seem to be composed using a guitar are excluded.

Exclusion Criteria

Since the objective of this paper is to investigate how the affordances of the guitar influence chord transitions in popular music, assumptions about the composition of a song must be made. The first assumption is that the song was composed using a guitar: therefore, songs that are not composed on a guitar should not be included in Study 4. Without access to the composers of each song, however, it is imposible to know for certain whether a guitar was used in its composition. While it is true that not all guitarists will use a guitar during composition, only songs that are composed by guitar players will be included in this reduced corpus. The operational definition of a "guitar player" will be detailed in the methodology section below.

The second assumption is that the key that the song is performed in is the same as the key that it was composed in. This assumption would not be necessary if the key that the song was composed in were known. However, because the method used in this paper is based on the letter-name encoding of the chords in performance, it was necessary to assume that the chords used in the performance are in fact the same letter-name chords used in composition. Only songs for which this assumption is reasonable should be included in the reduced corpus. There are several factors which may be considered to this end. For example, songs that are in rare keys may be excluded because one assumes that composition begins in common keys. Or, songs that employ a "pump-up" (a direct modulation of an entire earlier section) may be excluded because one assumes that such a modulation is based on a the intervallic relationships of an already completed composition. For this study, these exclusions were not used. Instead, this study excludes songs that are not performed by at least one of its composers. This includes both songs that are covers and songs that are written for the artist by another songwriter. This exclusion is based on the assumption that such songs may be transposed to suit the range of the singer, and therefore the key of the song's performance may not be the key of its performance.

Methodology

First, the Wikipedia⁸ entry for each song in the corpus was used to determine the composer or composers. Every song in the corpus had an entry on Wikipedia with information on its composition. Almost always this was indicated by the words "written by" or "music by." Since many songs were written by collaboration, each collaborator is included if possible, and in some cases, the writing credit is given to the entire band. Each of these Wikipedia entries also included information about the members of the band who perform each song. Using this information, each song that was not performed by at least one of its composers was excluded.

Second, in order to determine whether any of the composers were guitar players, a

⁸ Accessed June 7, 2011. (http://www.wikipedia.org/)

Google Image Search⁹ was conducted for each composer for each song. The search results consist of several screens of image thumbnails (usually photographs, but also posters or album covers) of the composers. From the first batch of search results (that is, all the pages that appear before the "more results" button), the first ten images in a "musical context" were considered. A musical context was *a priori* defined as any image where:

- a) Any musical instrument, singing, or microphone was seen
- b) Any recording equipment was seen
- c) Any performance setting was seen (for example, an audience or stage lighting)

These criteria for "musical context" serve to filter out the numerous images of "standing around and posing" for certain more famous musicians. Then, the number of images (out of these ten) that included the composer holding a guitar was recorded. Anything resembling an acoustic or electric guitar was counted, which probably included some bass guitars. This guitar count is used because, presumably, composers that were not primarily guitar players would have many images of performing on other non-guitar instruments, or of singing, or (for many producers) of operating recording equipment. Said another way, the assumption made is that guitar-playing composers are more likely to appear in images with guitars. Then, the composer which "scored" the highest number of guitar images for each song was assumed to bring their guitar-related expertise to the composition of that song, and this score out of ten was used as a very rough "index" reflecting the extent to which a guitar may have been used in its composition.

The average guitar index for each song was 5.4 (maximum guitar-related images per composer). Therefore, every song which had a guitar index of less than 6 out of 10 was excluded. When all songs that were not performed by at least one of its composers were excluded as well, the original 200-song corpus was reduced to 98 songs (the "guitar corpus.") For every song in this reduced corpus, it was assumed that the guitar was used in its composition, and that it was performed in the same key in which it was composed. The method of entropy calculation in Study 1 was then replicated for this guitar corpus.

⁹ Accessed June 7, 2011. (http://www.google.com/imghp)

Results

The following table summarizes these results and those of Study 1:

Encoding	Letter-name				Roman numeral			
Corpus	Total		Guitar		Total		Guitar	
Author	DT	TdC	DT	TdC	DT	TdC	DT	TdC
# Transitions	18465	18903	10191	10704	18465	18903	10191	10704
# Songs	200	200	98	98	200	200	98	98
# Unique Chords	36	33	31	29	46	55	34	35
Raw transition entropy (in bits)	131212	134526	70149	73869	103699	107736	57191	60642
Entropy per transition (in bits)	7.1060	7.1166	6.8834	6.9011	5.6160	5.6994	5.6119	5.6654

Table 5. A summary of the results of Study 1 and Study 4. The columns marked "Total" indicate results when the entire corpus was used (as in Study 1), and the columns marked "Guitar" indicate results when only the guitar corpus was used (as in Study 4). In the row marked "Author", "DT" indicates that the analyses by David Temperley were used, whereas "TdC" indicates that the analyses by Trevor de Clerq were used.

Note that the number of unique chords according to the letter-name encoding is almost the same for the total corpus versus the guitar corpus. For the Roman-numeral encoding, however, there are much fewer unique chords in the guitar corpus. Also note that just as in Study 1, the raw entropy value for the Roman-numeral encoding is lower than the raw entropy value for the letter-name encoding, so the previous conclusions seem to hold.

Since the guitar corpus has fewer songs than the total corpus, it also has fewer chord transitions. When the raw transition entropy is divided by the number of transitions in the respective corpora, the entropy per transition can be obtained. The difference in entropy for the two corpora is notable. For the letter-name encodings, the mean entropy difference between the total corpus and the guitar-driven corpus was 0.2191 bits. For the Roman numeral encodings, the mean entropy between the two corpora was only 0.0191 bits. That is, reducing the sample to "guitar songs" decreased the entropy more for the letter-name encodings than for the Roman-numeral encodings. Although this is not statistically significant, these results are consistent with the conclusion that affordant factors are more suitable than functional factors in accounting for the difference of chord transitions in guitar-driven versus non-guitar-driven popular music.

Overall, the results of Study 4 are consistent with the results found in Study 1, namely that functional factors in chord transitions are greater than affordant factors. However, when the corpus was trimmed to include only those songs that were most likely to have been conceived using a guitar, the influence of affordant factors increased. This latter result is consistent with the view that the guitar affords certain chord sequences in popular music. However, because the results were not stastically significant, a larger sample is required to better support this claim.

Summary and Discussion

This project investigateted the succession of chords in a sample of popular music. Two different harmonic systems are considered to account for chord sequences. In affordant chord transitions, chord sequences are governed by the physical location of chords on the guitar, and are represented using a letter-name encoding. In functional chord transitions, chord sequences are governed by the function of chords relative to a key, and are represented by a Roman-numeral encoding. Three related studies are conducted, and in each study, entropy (a measure of the unpredictability in the data) is used: it is assumed that a low entropy value means that the encoding, and its associated harmonic system, accounts for the chord sequences well.

In Study 1, the entropy for letter-name and Roman-numeral encodings was calculated for the 200-song corpus as a whole. It was hypothesized that affordant factors have a greater influence on chord transitions and therefore it was predicted that the lettername encoding entropy would be lower than that of the Roman-numeral encoding. However, it was found that the Roman-numeral encoding entropy was actually lower. Thus, the results of Study 1 are not consistent with the original hypothesis; instead, the results are consistent with the claim that functional factors account for chord sequences in popular music better than affordant factors.

The negative results of Study 1 bring to light some of the problems with the assumptions that were made. Most importantly, the assumption that functional and affordant harmony are uncorrelated must be discarded, and methods should be devised to study this interaction. It seems especially important to take into account the dominance of I, IV and V chords when characterizing this. For even if there were chord sequences arising out of affordant factors, if those sequences fit into the basic I-IV-V context, it will

be "subsumed" by functional harmony. For example, although the " $C \rightarrow G$ " transition is well afforded on the guitar, this also happens to be " $I \rightarrow V$ " in C major and " $IV \rightarrow I$ " in G major. In this case, it seems hopeless to try to determine which system is more responsible for the chord transition.

There is also the possibility that the construction of the guitar affords functional harmony itself. That is, it seems plausible that the instrument may be constructed and tuned to intentionally facilitate certain functional chord transitions, such as root movements by fourths. Perhaps it is better, then, to ignore the chords I, IV and V altogether – submitting that these are strictly the domain of functional harmony – and proceed with an investigation of the less common chord transitions instead. For example, determining whether "I \rightarrow bVII" is more likely to occur in keys such as D-major versus keys such as C-major is one way to study the influence of affordant harmony more directly.

Study 2 was conducted to determine whether affordant chord transitions play a role at all, since Study 1 showed that its role was less than initially hypothesized. The letter-name entropy from Study 1 was compared to the letter-name entropy of the same songs in the corpus transposed to random keys. This key randomization process ensured that the letter-name entropy of the corpus reflects the scenario where affordant chord transitions are not a factor. It was found that the entropy from Study 1 was significantly different from the randomized key corpus in Study 2. Thus, the results are consistent with the conclusion that affordant factors do in fact influence chord sequences in popular music.

In Study 3, each song's ratio of letter-name entropy to Roman-numeral entropy was plotted against that song's release date. It was found that this ratio is negatively correlated with that song's release date. Therefore, the results are consistent with the claim that the influence of affordant factors increase relative to functional factors over time in years. It is hypothesized that this is due to the increasing use of chords other than I, IV, and V after the 1960s, and that the use of these chords was driven by affordant factors of the guitar.

In Study 4, the methodology of Study 1 was repeated for a reduced corpus, where

songs were only included if they were likely to be composed on the guitar, and were also likely to be composed in the same key as its performance. It was found that the guitar corpus has a lower entropy value when using the letter-name encoding, although this was not statistically significant. It is therefore tentatively suggested that chord transitions in guitar-driven popular music are different than chord transitions in non-guitar-driven popular music, and that affordant factors, rather than functional factors, may best account for these differences.

Conclusions

In the comparison of the relative influence of affordant versus functional factors in chord transitions of popular music, no evidence was found in the main study to support the claim that affordant factors play a greater role than functional factors – contrary to the main hypothesis of this project. From the information theoretic perspective, it now seems clear that functional factors do provide a better account for chord sequences than affordant factors. However, further investigation yielded significant results consistent with two weaker claims: firstly, affordant factors do have some non-zero influence on the corpus of popular music; and secondly, this influence increases with a song's year of release. In additional, affordant factor provide a better account for chord sequences when the differences of guitar-driven music and non-guitar-driven music are compared.

Future Research

If a future study were to include entropy measures again, it would be helpful to address the difficulty of comparing entropy values in a meaningful way. First, it is difficult to determine the relative influence of two harmonic systems by comparing entropy values. Second, there is no external comparison that can be used as a benchmark to decide whether an obtained entropy value is large or small. To this end, exploratory studies that compute entropy values for corpora of various other styles of music may lead to new insights on comparing entropy values.

As mentioned above, it is advisable to limit the study of affordant harmony to chords other than I, IV or V, due to the dominance of these chords in functional harmony.

One possible approach to this is to examine the distribution of Roman numerals for songs in each key. If affordant harmony has no effect, there is no reason that different keys will have different distributions of Roman numerals. However, if the guitar does afford certain chords over others for use in chord sequences, specific chords are favored regardless of key, and songs in different keys will have different distributions of Roman numerals. A complementary method is to examine the distribution of keys for each Roman-numeral transition. While common chord transitions involving I, IV and V may show up in all keys proportionally, uncommon chord transitions (such as bVII \rightarrow I) may show up disproportionately in the keys in which they are easiest to play. Both approaches may yield evidence to support and concretize the claim that chord sequences are influenced by affordances of the guitar.

One glaring absence from this study on affordances is an actual investigation of performance difficulty. It was presumed that frequent chord transitions are those that are easiest to play, and a cursory glance at the most common chord transitions suggests that this is most likely true. However, without specifically describing on a physical level the difficulty of chords and chord transitions, any theory regarding affordant harmony falls short. One possible approach would be to construct a model that can estimate and quantify the performance difficulty of a chord sequence. Such a model may be based on pedagogical materials (for example, which chords are taught first in guitar books), or subjective ratings (for example, asking guitarists to rate difficulty of certain chords or chord transitions), or physical measures (for example, measuring the distance that fingers must move.) One use of such a model may be to quantify the idiomaticity of guitar chord sequences, in the same vein as Huron and Berec (2009).

Finally, I must admit that it is unfortunate that this study did not investigate the role of *power chords* at all. Power chords do not have thirds, employing only roots and fifths, and are almost always played with distortion. They have a simple fingering that is readily transferred to different positions via parallel motions – for this reason, it seems that power chords are ideal for studying the physical movements of fingers and hands. Many scholars, including McDonald (2000), Everett (2004) and Biamonte (2010), have suggested that the use of power chords may account for various harmonic features of

guitar-driven popular music. Perhaps this could be a starting point for constructing the performance difficulty model described above.

References

- Biamonte, N. 2010. "Triadic Modal and Pentatonic Patterns in Rock Music." *Music Theory Spectrum*, 32(2): pp. 95-110.
- de Clerq, T. & Temperley, D. (2011). "A corpus analysis of rock harmony." *Popular Music*, 30(1): pp. 47-70.
- Everett, W. 2004. "Making Sense of Rock's Tonal Systems." Music Theory Online, 10(4).
- Gibson, J. J. (1977). The theory of affordances. In R. Shaw & J. Bransford (Eds.), *Perceiving, Acting, and Knowing*. Hillsdale, New Jersey: Lawrence Erlbaum Associates, pp. 67-82.
- Gjerdingen, R. 2009. "The Price of (Perceived) Affordance: Commentary for Huron and Berec." *Empirical Musicology Review*, 4(3): 123-125.
- Huron, D. (1994). The Humdrum Toolkit; Reference Manual. (Menlo Park, Center for Computer Assisted Research in the Humanities).
- Huron, D. (2006). Sweet Anticipation: Music and the Psychology of Expectation (Cambridge, MIT Press)
- Huron, D. & Berec, J. (2009). "Characterizing Idiomatic Organization in Music: A Theory and Case Study of Musical Affordances." *Empirical Musicology Review*, 4(3): pp. 104-122.
- Letter Frequency. (n.d.). In *Wikipedia*. Retrieved May 19, 2011, from http://en.wikipedia.org/wiki/Letter_frequency
- McDonald, C. (2000). "Exploring Modal Subversions in Alternative Music." *Popular Music*, 19(3): pp. 355-363.
- Moles, A. (1966). Information Theory and Esthetic Perception (Urbana and London, University of Illinois Press).
- Neyman, J. & Pearson, E.S. (1933). "The testing of statistical hypotheses in relation to

probabilities a priori." *Joint Statistical Papers*. Cambridge University Press. pp. 186–202.

- Shannon, C. E. (1948). "A mathematical theory of communication." *The Bell System Technical Journal*, 27: pp. 623-656.
- Shannon, C. E. (1950). "Prediction and Entropy of Printed English." *The Bell System Technical Journal*, 30: pp. 50-64.

Temperley, D. (2007). Music and Probability (Cambridge, MIT Press).

Appendix A: Corpus Listing

The songs in the 200-song corpus used in this study are listed below.

2 Pac, "California Love" AC/DC, "Back in Black" Abba, "Dancing Queen" Aerosmith, "Dream On" Al Green, "Let's Stay Together" Al Green, "Love and Happiness" Al Green, "Take Me to the River" Animals, "House of the Rising Sun" Aretha Franklin, "Respect" B-52s, "Rock Lobster" Band, "The Weight" Beach Boys, "California Girls" Beach Boys, "Don't Worry Baby" Beach Boys, "God Only Knows" Beach Boys, "Good Vibrations" Beastie Boys, "Sabotage" Beatles, "A Day in the Life" Beatles, "Eleanor Rigby" Beatles, "Hard Day's Night" Beatles, "Help" Beatles, "Hey Jude" Beatles, "I Saw Her Standing There" Beatles, "I Want to Hold Your Hand" Beatles, "In My Life" Beatles, "Let it Be" Beatles, "Norwegian Wood" Beatles, "She Loves You" Beatles, "Strawberry Fields Forever" Beatles, "While My Guitar Gently Weeps" Beatles, "Yesterday" Beck, "Loser" Ben E. King, "Stand By Me" Big Joe Turner, "Shake, Rattle and Roll" Big Star, "September Gurls" Bill Haley & the Comets, "Rock Around the Clock" Bo Didley, "Who Do You Love" Bob Diddley, "Bo Diddley" Bob Dylan, "Blowin' in the Wind" Bob Dylan, "Like a Rolling Stone" Bob Dylan, "Mr. Tambourine Man" Bob Dylan, "Tangled Up in Blue"

Bob Dylan, "The Times They Are A-Changin'" Bob Marley & the Wailers, "No Woman No Cry" Bob Marley & the Wailers, "Redemption Song" Bobby Fuller Five, "I Fought the Law" Bonnie Raitt, "I Can't Make You Love Me" Bruce Springsteen, "Born to Run" Bruce Springsteen, "Thunder Road" Buddy Holly, "Rave On" Buddy Holly, "That'll Be the Day" Buffalo Springfield, "For What It's Worth" Byrds, "Eight Miles High" Byrds, "Mr. Tambourine Man" Chuck Berry, "Johnny B. Goode" Chuck Berry, "Maybellene" Chuck Berry, "Rock and Roll Music" Chuck Berry, "Roll Over Beethoven" Clash, "London Calling" Clash, "Should I Stay or Should I Go" Cream, "Sunshine of Your Love" Creedence Clearwater Revival, "Fortunate Son" Creedence Clearwater Revival, "Proud Mary" Crystals, "Da Doo Ron Ron" David Bowie, "Changes" David Bowie, "Heroes" Derek & the Dominoes, "Layla" Dionne Warwick, "Walk on By" Donna Summer, "Hot Stuff" Doors, "Light My Fire" Dr. Dre, "Nuthin' But a G Thang" Drifters, "Up on the Roof" Eagles, "Hotel California" Eddie Cochran, "Summetime Blues" Elton John, "Your Song" Elvis Presely, "Mystery Train" Elvis Presley, "Heartbreak Hotel" Elvis Presley, "Hound Dog" Elvis Presley, "Jailhouse Rock" Elvis Presley, "Suspicious Minds" Elvis Presley, "That's All Right" Eminem, "Lose Yourself" Eric Clapton, "Tears in Heaven" Everly Brothers, "All I Have to Do is Dream" Everly Brothers, "Cathy's Clown" Fats Domino, "Blueberry Hill" Five Satins, "In the Still of the Night" Flamingos, "I Only Have Eyes for You" Fleetwood Mac, "Go Your Own Way" Gene Vincent, "Be-Bop-A-Lula" Grandmaster Flash and the Furious Five, "The Message" Guns 'N Roses, "Sweet Child O' Mine" Hank Williams, "I'm So Lonesome I Could Cry" Iggy Pop & the Stooges, "Lust for Life" Ike & Tina Turner, "River Deep, Mountain High" Impressions, "People Get Ready"

Isley Brothers, "Shout" Jackson Five, "I Want You Back" James Brown, "I Got You" James Brown, "It's a Man's World" James Brown, "Papa's Got a Brand New Bag" James Brown, "Please Please" Janis Joplin, "Me and Bobby McGee" Jeff Buckley, "Hallelujah" Jerry Lee Lewis, "Blue Suede Shoes" Jerry Lee Lewis, "Great Balls of Fire" Jerry Lee Lewis, "Whole Lotta Shakin' Goin' On" Jim Hendrix, "Foxey Lady" Jimi Hendrix Experience, "Purple Haze" Jimi Hendrix, "All Along the Watchtower" Jimi Hendrix, "Voodoo Child" John Lennon, "Imagine" Johnny Cash, "Folsom Prison Blues" Johnny Cash, "I Walk the Line" Johnny Cash, "Ring of Fire" Joni Mitchell, "Both Sides Now" Joy Division, "Love Will Tear Us Apart" Kingsmen, "Louie Louie" Kinks, "Waterloo Sunset" Kinks, "You Really Got Me" Led Zeppelin, "Kashmir" Led Zeppelin, "Stairway to Heaven" Led Zeppelin, "Whole Lotta Love" Little Richard, "Good Golly Miss Molly" Little Richard, "Long Tall Sally" Little Richard, "Tutti Frutti" Mamas & Papas, "California Dreamin'" Martha & the Vandellas, "Dancing in the Street" Marvin Gaye, "I Heard it Through the Grapevine" Marvin Gaye, "Let's Get it On" Marvin Gaye, "What's Going On" Metallica, "Enter Sandman" Michael Jackson, "Billie Jean" Neil Young, "Rockin' in the Free World" New Order, "Bizarre Love Triangle" Nirvana, "All Apologies" Nirvana, "Come As You Are" Nirvana, "In Bloom" Nirvana, "Smells Like Teen Spirit" Otis Redding, "Dock of the Bay" Otis Redding, "I've Been Loving You" Patsy Cline, "Crazy" Pavement, "Summer Babe" Penguins, "Earth Angel" Percy Sledge, "When a Man Loves a Woman" Police, "Every Breath You Take" Prince, "1999" Prince, "Little Red Corvette" Prince, "Nothing Compares 2 U" Prince, "Purple Rain"

Prince, "When Doves Cry" Procol Harum, "Whiter Shade of Pale" Queen, "Bohemian Rhapsody" R. Kelly, "I Believe I Can Fly" REM, "Losing My Religion" Radiohead, "Fake Plastic Trees" Radiohead, "Paranoid Android" Ramones, "Blitzkrieg Bop" Ramones, "I Wanna Be Sedated" Ray Charles, "Georgia on my Mind" Ray Charles, "I Can't Stop Loving You" Ray Charles, "What'd I Say" Righteous Brothers, "You've Lost That Lovin' Feelin'" Rod Stewart, "Maggie May" Rolling Stones, "Gimme Shelter" Rolling Stones, "Honky Tonk Women" Rolling Stones, "Jumpin' Jack Flash" Rolling Stones, "Not Fade Away" Rolling Stones, "Paint it Black" Rolling Stones, "Satisfaction" Rolling Stones, "Sympathy for the Devil" Rolling Stones, "You Can't Always Get What You Want" Ronettes, "Be My Baby" Roy Orbison, "Crying" Sam Cooke, "A Change is Gonna Come" Sam Cooke, "You Send Me" Sex Pistols, "Anarchy in the UK" Sex Pistols, "God Save the Queen" Shirelles, "Will You Still Love Me Tomorrow" Simon & Garfunkel, "Bridge Over Troubled Water" Simon & Garfunkel, "The Boxer" Simon & Garfunkel, "The Sounds of Silence" Sly & the Family Stone, "Everyday People" Sly & the Family Stone, "Family Affair" Smokey Robinson, "Tracks of My Tears" Steppenwolf, "Born to be Wild" Stevie Wonder, "Living for the City" Stevie Wonder, "Superstition" Temptations, "My Girl" Temptations, "Papa Was a Rolling Stone" Tom Petty & the Heartbreakers, "Free Fallin'" Tracy Chapman, "Fast Car" U2, "I Still Haven't Found What I'm Looking For" U2, "One" U2, "With or Without You" Van Morrison, "Brown-Eyed Girl" Velvet Underground, "I'm Waiting for the Man" Verve, "Bitter Sweet Symphony" Who, "My Generation" Who, "Won't Get Fooled Again" Wilson Pickett, "In the Midnight Hour"

Appendix B: Sample Analyses

Sample David Temperley analyses:

```
% Billie Jean
In: R |*2 i |*12
BP: i | | | iv | | i | iv | | i | |
% Could also analyze as "i ii | bIII (or i7?) ii" over tonic
pedal. And in chorus, it could be i bVII7!
BP2: i | | | | iv | | i | |
Vr: $BP $BP2
Prec: bVI | i | bVI | i | bVI | i | bVI | V |
Ch: $BP
Ch2: $BP2
Solo: i |*8
Fadeout: i |*12
S: [G] $In $Vr $Prec $Ch $Vr $Prec $Ch2 $Ch $Solo iv | | i | |
$Ch $Fadeout
% Hey Jude
Vr: I | V | | I | IV | I | V | I |
BrP: V7/IV | IV I6 | ii vi6 | V6 V | I |
Br: $BrP $BrP [2/4] I | [4/4] V | |
CP: I | bVII | IV | I |
Fadeout: $CP*18
S: [F] $Vr $Vr $Br $Vr $Br $Vr I | $Fadeout
% Layla
BP: i bVI | bVII i |
In: $BP $BP $BP $BP $BP i bVI | [2/4] bVII |
Vr: [E] vi | iii | vi . bVI bVII | I | ii V | I IV | ii V | I V |
Ch: $BP $BP $BP i bVI | [2/4] bVII |
Ch2: $BP $BP $BP $BP
So: $BP*15 i bVI | bVII | |
PartI: $In $Vr $Ch $Vr $Ch $Vr $Ch2 $So
AP: I | | IV | |
A: $AP $AP bVII | | I | |
A2: $AP bVII | | I | |
B: vi | ii | V | I | vi | ii | V | |
A3: $AP $AP $AP $AP $AP bVII | | I | |
PartII: $A $A2 $A2 $B $A $A2 $A2 $B $A $A3
S: [D] $PartI [C] $PartII
```

```
% Paranoid Android
In: iv | bVII | i | vih7 | i | vih7 |
Vr: iv | bVII | i | vih7 | i | vih7 | iv | bVII | i | vih7 |
CP: i | v6 | V7/ii |
Ch: $CP $CP V7/ii |
BP1: vi bVI |
BP2: [7/8] I . . . bVI . . |
Br1: $BP1*4 $BP2*3 I | $BP1*4 $BP2*3 I | $BP1*2 ii | $BP1 $BP2*3
I | $BP1*4 $BP2*3 I | IV | % fermata on last measure
BP3: i V6 | iv6/ii V/ii | ii V/ii | ii vi6 | IV/IV IV6 | ii/IV IV
| V/vi |
Br2: $BP3 VI | $BP3 VI | $BP3 VI | $BP3 V/vi |
Br3: $BP1*2 ii | $BP1 $BP2*3 I | $BP1*4 $BP2*3 I |
S: [G] $In $Vr $Ch $Vr $Ch [C] $Br1 $Br2 $Br3
% Smells Like Teen Spirit
BP: i IV | bIII bVI |
Intro: $BP $BP $BP $BP $BP $BP $BP $BP
Vr: $BP $BP $BP $BP $BP $BP $BP $BP
Ch: $BP $BP $BP $BP $BP $CP $CP
Ch3: $BP $BP $BP $BP $BP $BP $BP $BP $BP i |
CP: i bII | i64 IV |
Vrsolo: $BP $BP $BP $BP $BP $BP $BP $BP $BP
S: [F] $Intro $Vr $Ch $BP*2 $Vr $Ch $Vrsolo $Vr $Ch3
```

Sample Trevor de Clerq analyses:

% Billie Jean A: i |*4 iv | | i | | iv | | i | | B: i |*4 iv | | i | | In: R | | i |*12 Vr: \$A \$B Ch1: \$A Ch2: \$B Pc: bVI | i | bVI | i | bVI | i | bVI | V | So: i |*4 \$B Ou: i |*12 S: [G] \$In \$Vr \$Pc \$Ch1 \$Vr \$Pc \$Ch2 \$Ch1 \$So \$Ch1 \$Ou % Hey Jude (1968) A: V7 | I I42 | vi vi42 | V6/V V/V | V | Vr: I | V | V7 | I | IV | I | V | I | Br: [Bb] \$A*2 [2/4] V7 | [4/4] [F] V | V7 | Ou: I | bVII | IV | I | S: [F] \$Vr*2 \$Br \$Vr \$Br \$Vr I | \$Ou*18

```
% Layla (1970)
A: i bVI | bVII i |
In1: [D] $A*2
Vr: [E] vi | iii | vi . bVI bVII | I | ii V | I IV | ii V | I IV|
Rf: [D] $A*3 i bVI | [2/4] bVII |
Ou: $A*19 i bVI | bVII | |
B: I | I6 | IV | |
In2: $B
So: $B bVIId7 | | I | |
Br: vi | ii | V | I | vi | II | V | |
Zz: $Rf $Vr
S: [D] $In1 $Zz*3 $Ou [C] $In2 $So*3 $Br $In2 $So*3 $Br $In2 $So
$In2*4 $So
% Paranoid Android
A: [G] iv iv42 | bVIId7 |
B: [G] i | vih7 |
C: [G] i | v6 | V7/ii |
D: [A] i V65 |
E: [C] [7/8] I . . . bVI . bVII |
In: $A $B*2
Pt1: $A $B*2 $A $B
Pt2: $C*2 V7/ii |
Pt3: $D*4 $E*3 [C] [4/4] I . . bVII |
Pt4: [F] v V6/v | v6/v V/vi | vi V/vi | vi vi42 | IV I6 | ii7 I |
IId7/vi | V/vi |
Pt4a: [F] v V6/v | v6/v V/vi | vi V/vi | vi vi42 | IV I6 | ii7 I
| IId7/vi | |
Zz1: $Pt1 $Pt2
Zz2: $Pt3*4 [A] i |
Zz3: $Pt4*3 $Pt4a
Zz4: $Pt3*2
S: [G] $In $Zz1*2 $Zz2 $Zz3 $Zz4
% Smells Like Teen Spirit (1991)
A: i iv | bIII bVI |
B: i bII | i64 . iv bIII |
In: $A*6
Vr: $A*6
Pc: $A*4
Ch: $A*6
Ta: $B*2
So: $A*8
Ou: $A*4 i |
Zz: $Vr $Pc $Ch $Ta
Zz3: $Vr $Pc $Ch
S: [F] $In $Zz*2 $So $Zz3 $Ou
```

Appendix C: Model Assumptions and Theory

This section details the underlying assumptions that are required to justify the method of entropy comparison used in this study. First, it is proposed that chord-to-chord transitions sufficiently represents the notion of "harmony." These transitions are represented using matrix notation. Second, it is assumed that the factors influencing chord transitions – either affordant or functional harmony – do not interact. This is represented by having a different transition matrix for either system.

Assume that each harmonic system can be conceived of as a $n \times n$ transition matrix **M**, where *n* is the possible number of harmonies in that harmonic system. Element **M**_{ij} is the probability that harmony *j* follows harmony *i*. Thus, the ideal transition matrix for affordant harmony, **A**, has a row and a column for every possible chord fingering in tablature notation. The ideal transition matrix for functional harmony, **F**, has a row and a column for every possible Roman numeral, including quality and inversion. Elements of the table are the probabilities of a chord transition, from the antecedent (in the rows) to the consequent chord (in the columns). This assumption is relatively benign: transition matrices have been found to be useful in characterizing harmonic systems, such as in Huron (2006) and Temperley (2007). However, note that such matrices only account for chord-to-chord transitions, and do not directly account for longer-range phenomena such as passing chords or delayed resolutions. In principle, higher-order matrices may account for such phenomena. The form of the ideal matrix for functional harmony is given below.



Figure 14. Form of the ideal transition matrix for functional harmony, F. It has a row and column for every possible chord – only the first few are shown. The value of F_{ij} is the probability that chord *i* is immediately followed by chord *j*. For example, the asterisk marks the position on the matrix which denotes the probability that a II chord is immediately by a III chord.

Secondly, assume also that functional and affordant factors in chord transition do not interact on the individual chord level. That is, for a given chord in a sequence, either functional factors affordant factors will govern the next chord transition. Although sections may be governed by one system or the other. Clearly, this is a rather naïve assumption. As mentioned in the introduction, these two factors certainly do interact. In this paper, it has been discussed that this assumption turns out to be problematic. However, the purpose of this assumption is to justify the counting of individual chord transitions as being attributable to one system or the other. With a large number of chord transitions, presumably the number of chords attributed to one system will be somewhat proportional to the overall influence of that system.

The overall approach taken by this study, then, is as follows. For a given harmonic progression, each chord can be characterized as two probabilities. The first is the probability that it is governed by affordant harmony – call this p_A (such that $0 < p_A < 1$) –

versus the probability that it is governed by functional harmony – p_F (such that $p_A + p_F =$ 1). Thus, to operationalize the hypothesis that affordant harmony contributes more than functional harmony, one can say that $p_A > 0.5 > p_F$.

The second probability is based on the first-order transition matrix of the governing system, such that the previous chord gives a set of probabilities for the next chord. Let **A** be the first-order transition matrix for affordant harmony, and **F** for that of functional harmony, such that \mathbf{A}_{ij} (with $0 < \mathbf{A}_{ij} < 1$) gives the probability of chord *j* if the previous chord was *i* under affordant harmony, and \mathbf{F}_{ij} (such that $0 < \mathbf{F}_{ij} < 1$) gives the probability of the chord *j* if the previous chord was *i* under affordant harmony, and \mathbf{F}_{ij} (such that $0 < \mathbf{F}_{ij} < 1$) gives the probability of the chord *j* if the previous chord was *i* under functional harmony. Overall, then, each chord of a sequence depends only on the previous chord, the probabilities of the governing system (p_A or p_F) and the probabilities of each transition within the relevant system (**A** and **F**). To summarize, the probability that a given chord is *j* (that is, the probability p_i), given that the previous chord is *i*, is modelled by the following equation:

$$p_j = p_A A_{ij} + p_F F_{ij}$$

The aim of Study 1 was to estimate p_A and p_F from the corpus data. Now, if one were simply given the transition matrices for affordant harmony (**A**) and functional harmony (**F**), one could easily determine whether p_A or p_F was greater. Using the transition matrix for **A**, one could calculate the total probability of all the transitions in a sequence in this way: for each transition, look up its probability in the transition matrix, then multiply all the transition probabilities together. The resultant value is the probability of the sequence. Now, assume that $p_A = 1$ (that is, the corpus was entirely governed by affordant harmony); then the calculated total probability would be equal to the true probability of the sequence. However, if the transition matrix for **F** were used instead, since the sequence actually was governed by **A**, the sequence would appear to be less probable, and would have a lower probability value than if **A** were used in the calculation. The reverse would be true when $p_A = 0$. Thus, with unknown intermediate values of p_A and p_F , one can estimate which value is larger in this way: calculate the probability of the sequence using transition matrix **A**, and then again with matrix **F**. The transition matrix, either **A** or **F**, that yields the higher total probability corresponds to the system contributing most to that system. If the probability corresponding to **A** is higher, then $p_A > p_F$, and vice versa.

However, the matrices **A** and **F** are unknown. Although transition matrices for functional harmony have been created for common-practice harmony, there is no reason *a priori* to assume that it would be the same in rock harmony. The matrix provided by de Clerq and Temperley obviously cannot be used: not only is this study based on the same corpus, but it does not "factor out" the influences of affordance on the data from which it is derived (which, of course, is part of the aims of this paper in the first place.) As for a transition matrix **A** for affordant harmony, one may propose to construct a transition matrix using, for example, subjective difficulty ratings between all chords. However, in addition to the problem of converting subjective difficulty ratings to probabilities (is something that is "twice as hard" going to be be "half as likely?"), this approach just is likely to be error-prone and arbitrary.

As an alternative approach, Study 1 used the concept of entropy from information theory (Shannon 1948). Recall that if we were given the matrices **A** and **F**, each could be used to calculate the probability of a particular sequence. If affordant harmony contributed to this sequence more, matrix **A** would deem the sequence more likely, while matrix **F** would deem the sequence as less likely. In other words, the more appropriate representation deems a given sequence as more predictable, whereas a less appropriate representation deems it as less predictable. The following section explains how entropy can be used to measure the predictability of a sequence, even when the transition matrix is not known.

The Calendar Machine Analogy

I have demonstrated that entropy can be used as a measure of the suitability of a system for characterizing a set of data. For example, first-order transition probabilities characterize English better than just letter frequencies, and therefore yield a lower entropy value. Similarly, English transition probabilities characterize French texts poorly,
and yield a high entropy value compared to French transition probabilities. It is anticipated that entropy can be used in a similar way, to compare the suitability of using affordant harmony versus functional harmony to characterize the rock corpus.

However, the reason for using entropy as a measure was because the transition probabilities for the characterizing systems – that is, the transition matrix **A** for affordant harmony and the transition matrix **F** for functional harmony – are still unknown. However, one can still calculate some measure of entropy within the data set, without a prior knowledge of transition probabilities. The following illustration is used to show how a useful entropy value can be calculated when transition probabilities are unknown.

Suppose that there is a machine that prints calendar pages. Each page has two pieces of information: the month [JAN, FEB ... DEC] and the day of the week [MON, TUE ... FRI]. However, the behavior of the machine is unusual, printing each page in the following manner: print the subsequent month to the month printed on the previous page, and then print a random day of the week. Thus, if the previous page were FEB/TUE, the next page could be MAR/SUN, or MAR/MON, MAR/TUE, and so on. Now, suppose the machine printed a hundred pages. It might look something like this:

NOV / FRI DEC / TUES JAN / MON FEB / SAT MAR / SAT APR / FRI (etc.)

Figure 15. Output of calendar machine governed only by the month.

If somebody without any knowledge of the machine operation looked at the

ordered stack of a hundred calendar pages, they will be able to deduce something about the operation of the machine. They would correctly conclude that the month on each page could predict the month on the next page fairly well, but the day of the week on each page would not predict the day on the next page at all.¹⁰ Thus, a "month-based-view" is a good system for characterizing the calendar data, while a "day-of-the-week-based-view" is a poor system for characterizing the calendar data.

Another way this could have been done would have been to calculate the entropy of the first-order transitions of the pages themselves. Thus, for each consecutive page, one would record the transition, and then tally each possible transition. The result would be something like this:

9
9
8
8
8
8
8
8
8
8
8

Figure 16. Tally by month transitions of month-governed calendar machine output.

¹⁰ This is not fail safe. Suppose the "month system" prints a random month; then it would be impossible to tell what was going on from the output. Thus, it is implicit that any system does structure the data in some systematic way.

Whereas for days of the week, it might appear like this:

MON	\rightarrow	MON	5
MON	\rightarrow	TUE	2
MON	\rightarrow	THU	2
MON	\rightarrow	FRI	2
MON	\rightarrow	SAT	1
MON	\rightarrow	SUN	2
FEB	\rightarrow	MON	3
FEB	\rightarrow	FEB	2
FEB	\rightarrow	WED	3
FEB	\rightarrow	THU	4
FEB	\rightarrow	FRI	2
FEB	\rightarrow	SAT	6
FEB	\rightarrow	SUN	1
(eta	c.)		

Figure 17. Tally by day-of-the-week transitions of month-governed calendar machine output.

Thus, for the page-to-page transitions, there is a lower entropy value when looking at the months, than when looking at days of the week. This calculation can be understood as follows. The probability of any event is related to the frequency with which it appears on consecutive pages. Since there are 99 transitions in total (i.e. there are 99 sets of consecutive pages, because there are 100 total pages), the probability of "JAN \rightarrow FEB" (which occurs 9 times) is 9 / 99 = 0.091. Each occurence of this transition has an information of $-\log_2(0.091) = 3.46$ bits, and its weighted contribution to the total entropy is 0.091 x 3.46 bits = 0.0312 bits. One could then calculate the total information of the calendar pages if we add up all the month-based transitions. In this case, there are 354.7 bits of total information. The total entropy, that is the average information conveyed per transition, is 3.58 bits.

Doing the same calculations for the day-of-the-week-based view, one arrives at a larger entropy value. For example, the transition "MON \rightarrow TUE" (which occurs twice) is 2 / 99, and has an information content of $-\log_2(2/99) = 5.63$ bits. Continuing this calculation with all the other possible states, we arrive at an entropy value of 5.14 bits, which is higher than the 3.58 bits of the month-based view.

This higher entropy value is consistent with the notion that the month-based view is more "knowledgeable" of the machine. This is because the month-based view considers the month on each page and disregards the day of the week. The days-of-theweek perspective, however, results in a high entropy value, because useful information (the month on each page) is discarded, and irrelevant information (the day of the week) is retained. In this way, it can be seen that a view (or system) that characterizes the data better consequently yields a lower entropy value.

Now, suppose that the machine has two modes: one in which it prints subsequent months and a random day of the week, and one in which it prints random months and a subsequent day of the week. That is, for every new page, the machine with probability p_M , decides whether to print according to one system of months, versus another system of days of the week. Now, given a stack of calendar outputs, shown below, could we estimate p_M ? That is, could one infer which system, months versus days, governs the operation of the machine most?

JAN / MON NOV / TUE DEC / FRI JAN / SUN JUL / SAT (etc.)

Figure 18. Output of calendar machine, sometimes governed by month and sometimes by day-of-the-week.

Once again, one can tally the transitions by months or by days:

$JAN \rightarrow$	FEB	8
$JAN \rightarrow$	SEP	1
$JAN \rightarrow$	OCT	1
FEB \rightarrow	MAR	8
MAR \rightarrow	APR	6
MAR \rightarrow	DEC	1
(etc.)		

Figure 19. Tally by month transitions of randomly-governed calendar machine output.

MON	\rightarrow	MON	3
MON	\rightarrow	TUE	3
MON	\rightarrow	WED	2
MON	\rightarrow	THU	2
MON	\rightarrow	FRI	5
MON	\rightarrow	SAT	1
MON	\rightarrow	SUN	1
TUE	\rightarrow	MON	2
TUE	\rightarrow	WED	3
TUE	\rightarrow	THU	4
TUE	\rightarrow	FRI	2
TUE	\rightarrow	SAT	1
TUE	\rightarrow	SUN	1
(eta	c.)		

Figure 20. Tally by day-of-the-week transitions of randomly-governed calendar output.

Again, one can calculate the entropy based on the month-based perspective and then the entropy on the day-of-the-week-based perspective. A lower entropy value for the month-based perspective would suggest that the machine is more likely governed by a month-based printing. On the other hand, a lower entropy value for the days-of-the-week-based perspective would suggest that the machine is most likely governed by a days-of-the-week-based printing. To illustrate this exactly, I have generated the calendars as described, and calculated the corresponding entropy value based on different values of p_{M} , the probability that the machine prints according to the month.

р _м (probability month-based)	Month entropy (bits)	Weeks entropy (bits)
1.00	3.58	5.14
0.90	3.96	5.16
0.75	4.44	5.06
0.50	5.31	4.47
0.25	5.76	3.99
0.10	6.05	3.31
0.00	6.22	2.81

Figure 21. Entropy figures for simulated calendar machine, with various probabilities of being governed by the month. Entropy for the month transitions is lowest when the machine is actually governed by months ($p_M = 1.00$); entropy for weeks entropy is lowest when the machine is actually governed by days-of-the-week ($p_M = 0.00$). Note, however, that comparison across entropy columns is difficult.

As expected, as the machine prints based on month less (as p_M goes down), the entropy based on months increases – the data becomes more and more unpredictable for a

month-based view. As the machine prints based on month more (as p_M goes up) the entropy based on month decreases. The reverse is true for the days-of-the-week-based view.

However, it should be apparent there is a problem of directly comparing entropy values across the two columns, because the overall entropy of days-of-the-week is lower than that of months. This is because there are more months than days of the week, and thus months are less predictable than days of the week, all else being equal. Notice that at $p_M = 0.5$, the entropy value for days of the week is lower, when one would expect them to be comparable, since both systems contribute equally. Thus, it is necessary to normalize the entropy values based on the complexity of the system. To this end, the entropy is then divided by the log of the number of unique transitions that are possible; log_2 144 for months (12 x 12), and log_2 49 for the days of the week (7 x 7). This is presented in the figure below. This facilitates a better way to compare the entropy values in determining whether p_M is greater than 0.5 or not.

р _м (probability	Normalized	Normalized
month-based)	Month entropy (bits)	Week entropy (bits)
1.00	0.49	0.91
0.90	0.55	0.91
0.75	0.61	0.89
0.50	0.73	0.79
0.25	0.80	0.7
0.10	0.84	0.58
0.00	0.86	0.48

Figure 22. Normalized entropy figures for simulated calendar machine, with various probabilities of being governed by the month. Comparison across normalized entropy columns is improved.

Thus, for a given set of data generated by one of two systems, with probability p_1 and p_2 , one can estimate whether $p_1 > p_2$ or $p_2 > p_1$ in this way: calculate two entropy values for the data, one according to a system that is governed by $p_1 = 1$, and one according to $p_2 = 1$. The perspective which yields the lower normalized entropy value corresponds to the higher value between either p_1 or p_2 .

A Model for Chord Sequences

The calendar machine analogy will now be extended to formulate a means for testing the comparative influence of affordant harmony and functional harmony in rock chord progressions. Recall that it was assumed that each chord in a sequence depends on the previous chord, and is characterized by two probabilities: i) the probability that it is governed by affordant harmony, p_A (or the complement, p_F , the probability it is governed by functional harmony); and ii) the transition probability based on the previous chord. Given a chord sequence, then, one can estimate whether $p_A > 0.5$ or not by calculating the

entropy for two views. That is, one can estimate whether affordant harmony influences the chord sequence most often, by calcuating the entropy for affordant harmony versus the entropy for functional harmony.

In one view, it is assumed that the system is governed entirely by affordant harmony ($p_A = 1$) and in the other, it is assumed that it is governed entirely by functional harmony ($p_F = 1$). If the system is governed by affordant harmony, the relevant information in the data is going to be the tablature, as in, the specific location of the chords on the guitar. If the system is governed by functional harmony, the relevant information in the data is going to be the relation of each chord to the key.

Recall that the main hypothesis is that affordant harmony contributes to the sequence of chords in the rock corpus more so than functional harmony, or in other words, p_A is greater than 0.5. Therefore, it is expected that the normalized entropy from the letter-name encoding is lower than the normalized entropy from the Roman numeral encoding.