Modified Sliding Mode Control Algorithm for Vibration Control of Linear and Nonlinear Civil Structures

Thesis

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By

Nengmou Wang, M.S. Graduate Program in Civil Engineering

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Thesis Committee: Hojjat Adeli, Advisor Vadim I. Utkin Shive Kumar Chaturvedi Copyright by

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Abstract

Structural control technologies have been widely accepted as effective ways to protect structures against seismic and wind hazards. Sliding mode control (SMC) is among the popular approaches for control of systems, especially for unknown linear and nonlinear civil structures. Compared with other control approaches, sliding mode control is invariant to disturbance such as wind and earthquake, and to system parameters such as the mass, stiffness, and damping ratio matrices if the uncertainties can be represented the linear combination of the control input, which is generally satisfied for most civil structures.

For known linear civil structures subjected to wind excitation, a filtered sliding mode control approach is presented in order to reduce the response of civil structures. Rather than using a Lyapunov-function based control design, an alternative way is provided to find the control force based on the *equivalent control force*. A low pass filter is properly selected to remove the high-frequency components of the control force while remaining the structural stability. Simulation results of a 76-story wind-excited highrising building show that this filtered sliding mode control method has better performance over Linear-quadratic-Gaussian (LQG), unfiltered SMC, and some other approaches with respect to maximum and root-mean-square (RMS) values of structural response.

For unknown nonlinear civil structures, an adaptive and robust control algorithm for nonlinear vibration control of large structures subjected to dynamic loading is presented through integration of a self-constructing wavelet neural network with an adaptive fuzzy sliding mode control approach. It is particularly suitable when structural properties are unknown or change during the dynamic event which is the case for civil structures subjected to dynamic loading. In other words, the proposed control model has the advantages of not requiring accurate mathematical model of the controlled structure and good adaptive ability to the changes of structural parameters and external dynamic loading. The robustness of the proposed algorithm is achieved by deriving a set of adaptive laws for determining the unknown parameters of wavelet neural networks using two Lyapunov functions. Because of these advantages, the proposed adaptive control algorithm is especially effective and implementable for vibration control of large civil structures.

The chattering in SMC is generally a problem that needs to be resolved for better control. To solve this issue, two tuning algorithms are developed for determining the sliding gain function in the SMC. The first algorithm is for systems with no noise and disturbance but with unmodeled dynamics. The second algorithm is for systems with noise, disturbance, unmodeled dynamics, or their combination. Compared with the state-dependent, equivalent-control-dependent, and hysteresis loop methods, the proposed algorithms are more straightforward and easy to implement. The performance of the algorithms is evaluated for six different cases. A 90% to 95% reduction of chattering is achieved for the first algorithm used for systems with sensor dynamics only. By using the second algorithm, the chattering is reduced by 70% to 90% for systems with noise and/or disturbance, and by nearly 25% to 50% for systems with combination of unmodeled dynamics, noise, and disturbance.

|||

Dedication

To Xiang Yi and my whole family

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Vita

November 1984	Born in Tongshan, Hubei, China
June, 2007	B.S. Civil Engineering,
	Southwest Jiaotong University
June, 2009	M.S. Bridge Engineering,
	Southwest Jiaotong University
July 2009 ~ March 2011	Graduate Research Associate,
	Department of Civil Engineer,
	Ohio State University

Fields of Study

Major Field: Civil Engineering

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1.1 Nature and Scope of Research

Structural control technologies have been widely accepted as effective ways to protect structures against seismic and wind hazards. As one of the control strategies, active vibration control has been receiving considerable attention for structural design technique for seismically-excited buildings and bridges in the past decades. Compared with passive control, active control is generally able to reduce structural response more significantly. Current approaches for active control include linear-quadratic regulator (LQR) control, linear-quadratic-Gaussian (LQG) control, H_2 control, H_{∞} control, and sliding mode control (SMC), etc.

LQR control an optimal control approach achieved by a trade-off between structural states and control forces. It is suitable for linear structures and linearized nonlinear system with all the system states available. When only part of the structural states is available, an observer is necessarily used to estimate the full states before the application of LQR control. This observer or estimator is generally a Kalman Filter. LQG is a combination of observer and LQR, and therefore is more applicable since the full states are not required to be measurable for this case. Nonlinear structures need to be linearized when LQR or LQG control are applied. This transformation from nonlinear to nonlinear structures usually causes an approximation error which may not be accepted for real implementations. H_2 and H_{∞} control have the similar optimal principle with LQR and LQG control. The main difference is that optimization for H_2 and H_∞ control are in H_2 and H_∞ norm sense, respectively. Their limitations are also similar to that of LQR and LQG control.

For nonlinear structures especially those cannot be linearized duo to unacceptable approximation errors, LQR, LQG, H_2 and H_{∞} control approaches are not suitable for vibration reduction. In this case, SMC provides an appropriate way to control the structural response. The idea of SMC consists of two parts: (1) find a desired sliding surface (s = 0) which is generally a linear combination of system states, and (2) find the control force such that the sign of the derivative of *s* is always opposite to the sign of *s*, which ensures the convergence of the s, i.e., the system states will follow the desired trajectory as it is expected in some finite time. SMC is able to control unknown nonlinear systems if the unknown factors are bounded (which is always the case in practical situations), and this control method has at least two advantages: (1) the order of controlled structures is reduced and therefore the dimension of differential equations for the structures is reduced, and (2) this method is independent of control force and unknown disturbance (Utkin, 1992). Compared with other control approaches, sliding mode control is invariant to disturbance such as wind and earthquake, and to system parameters such as the mass, stiffness, and damping ratio matrices if the uncertainties can be represented the linear combination of the control input, which is generally satisfied for most civil structures. The designs of SMC for systems with and without disturbance are similar, and thus SMC is very convenient for systems with unknown disturbance.

In fact, for linear system without any disturbance, the performance using SMC is similar with that of LQR/LQG control if the control gain (a constant or a function of time

or states for the control force) is properly selected, since in this case the control force using SMC can be expressed in a form that is same as the LQR/LQG control force.

In the past 15 years, SMC has been studied in the area of civil engineering analytically (Yang, 1996; Ning et al., 2009) and experimentally (Wu and Yang, 2004). Those research deal with the control problem for both linear and nonlinear civil structures. Those approach treat controlled structures as either a known (without any system unknown factors, or say system uncertainties) linear structures, or single-input singleoutput (SISO) known or unknown structures. However, it is not easy to find an improved control approaches based on current methods, especially for the control of multiple-inputs multiple-outputs (MIMO). In addition, it is necessary to note that the undesired chattering phenomenon caused by sapling time, high gain of the control force, sensor and actuator dynamics in SMC hinders the application in engineering areas, which remains to be resolved for control of civil structures.

1.2 Objective of Research

In this research work, first the control performance of SMC is studied for known linear civil structures, and then a self-constructing wavelet-neural-network (SCWNN) based control approach is developed for vibration control of unknown nonlinear structures, followed by the study of chattering reduction for SMC.

In chapter 2, the basic idea of SMC is present, followed by unfiltered SMC and filtered SMC approaches used to reduce the response of civil structures subjected to wind excitation. A low pass filter is properly selected to remove the high-frequency components of the control force while remain the structural stability.

In chapter 3, an improved algorithm for vibration control of large structures considering nonlinear moment-curvature behavior through adroit integration of adaptive SMC and a self-constructing wavelet neural network model. Since the controlled structure is unknown, a function estimation of the unknown differential equations which represent the structural behaviors. SCWNN is then developed for on-line functional approximation of the nonlinear structure. The wavelet basis functions used in SCWNN provide a more compact and efficient system representation over earlier neural networks based on Gaussian radial basis functions. The growing/pruning criterion is applied to construct the hidden layer in the neural network automatically. A faster learning algorithm is achieved using the recently developed PI-type adaptive law instead of a traditional I-type adaptive law. The fuzzy compensation controller achieved by using fuzzy logic is applied to ensure the stability of this SCWNN based control model.

In chapter 4, a time-varying method is proposed for determining the sliding gain function in order to solve the chattering phenomenon in SMC. Two tuning algorithms are proposed for reducing the sliding gain function for systems. The first algorithm is for systems with no noise and disturbance but with or without unmodeled dynamics (such as sensor and actuator dynamics). The second algorithm is for systems with noise, disturbance, unmodeled dynamics, or any combination of them. The performance of the algorithms is evaluated for six different cases.

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Chapter 2: A Filtered Sliding Mode Control for Vibration Control of Wind-Excited Highrising Building Structures

2.1 Introduction

Design of a robust controller with high performance for active, semi-active and hybrid control of large structures subjected to extreme dynamic loading such as earthquake or wind is a challenging research problem due to nonlinear structural behavior and many system uncertainties (unknown disturbance/excitation, sensor measurement noise, actuator dynamics, and modeling error between assumed system models and real systems). The idea of sliding mode control (SMC) consists of two parts: (1) find a sliding surface (defined as a desired linear combination of system states such as displacement, velocity, and acceleration) to stabilize the controlled system, and (2) find a control force to drive the response trajectory into the sliding surface with an exponential speed in time (Utkin, 1993). SMC is especially useful for variable structure systems (e.g., when stiffnesses vary during a dynamic event) because the sliding surface is independent of the control input and system uncertainties (Utkin, 1993). SMC has been used for control of civil structures by Yang et al. (1995) and Wu and Yang (2004). Wu (2003) presents experimental verification of SMC for vibration control of a regular 3-story building structure using a shaking table.

The main advantage of the SMC is that it is invariant to external excitation such as wind and earthquake and the variation of system parameters (such as structural stiffness and damping) during the dynamic event. The structural uncertainties can be represented by a linear combination of the control forces. Since high frequency components of the control input do not impact controlled civil structures significantly (their frequencies are substantially lower), a low pass filter can remove the high frequency part of the control input determined by SMC. Use of a low pass filter results in a reduction of maximum control input which is significant for the size of actuators and real implementation. In this article, a filtered sliding mode control approach is presented to reduce the response of civil structures subjected to wind excitation. It is applied to a 76-story wind-excited benchmark highrise building structure.

2.2 Problem formulation of Linear Wind Excited Civil Structures

2.2.1 Reduced Order Model of Physical Wind Excited Structures

The equation of the motion for a linear structure subjected to wind loading is

$$M\ddot{x} + C\dot{x} + Kx = -\eta W + B_s u \tag{2.1}$$

where $x \in \mathbb{R}^p$ is the displacement vector, $M, C, K \in \mathbb{R}^{p \times p}$ are mass, damping, and stiffness matrices, respectively, p is the number of degrees of freedom of the structure, $W \in \mathbb{R}^p$ is the vector of wind excitation, $\eta \in \mathbb{R}^{p \times p}$ is a matrix of excitation influence representing the variation of the wind over the height of the structure, $u(t) \in \mathbb{R}^q$ is the control force vector assuming the structure has q actuators, and $B_s(x, \dot{x}) \in \mathbb{R}^{p \times q}$ is the matrix related to positions of the control forces. For ease of controller design, Eq. (2.1) is rewritten as follows

$$\dot{X} = AX + Bu + EW \tag{2.2}$$

$$\dot{X} = AX + Bu + EW \tag{2.2}$$

where $X = [x^T, \dot{x}^T]^T \in \mathbb{R}^{2p}$ is the structural states. $A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix} \in \mathbb{R}^{2p \times 2p}$ is the matrix representing the properties of structural mass, stiffness, and damping, $B = \begin{bmatrix} 0 \\ B_s \end{bmatrix} \in \mathbb{R}^{2p}$ is the vector of control locations, and $E = \begin{bmatrix} 0 \\ -\eta \end{bmatrix} \in \mathbb{R}^{2p \times p}$ is a matrix of wind excitation. It is common in the control field to reduce the number of degrees of freedom (DOF) for large systems in order a) to avoid ill-conditioning of large matrices and b) reduce the number of required actuators. Consequently, the size of the model is reduced using a state order reduction technique by keeping the *dominant* eigenvalues of the matrix A (the smallest ones) while removing unimportant eigenvalues (the largest ones) (Davison, 1966). Equation (2.2) is reduced to

$$\dot{\boldsymbol{x}}_r = \boldsymbol{A}_r \boldsymbol{x}_r + \boldsymbol{B}_r \boldsymbol{u} + \boldsymbol{E}_r \boldsymbol{W} \tag{2.3}$$

where $x_r \in \mathbb{R}^n$ (the states that maintain the first *n* eigenvalues and eigenvectors of the matrix A, n < 2p), $A_r \in \mathbb{R}^{n \times n}, B_r \in \mathbb{R}^n, E_r \in \mathbb{R}^{n \times p}$. The measured states from sensors with noise are expressed as

$$y_r = C_{yr}x_r + D_{yr}u + F_{yr}W + v_r$$
(2.4)

 $y_r \in \mathbb{R}^r$, $C_{yr} \in \mathbb{R}^{r \times n}$, $D_{yr} = \mathbf{0} \in \mathbb{R}^r$, $F_{yr} \in \mathbb{R}^{r \times p}$. These matrices and vectors are obtained from the state order reduction technique. $v_r \in \mathbb{R}^r$ is assumed to be uncorrelated Gaussian white noise which is not measurable. The consideration of v_r is due to the existence of sensor noise in practical situations.

Civil structures are generally assumed to be (1) stable: the structural response is reduced if the control force is properly calculated and applied, and (2) observable: if only

part of the reduced states x_r can be measured, x_r can be determined by designing an observer to estimate the remaining states.

2.2.2 Observer Design

Since the vector of states x_r in Eq. (2.3) is only partially available by measurement, an observer is necessary to estimate the system states as follows (Skelton, 1988)

$$\hat{\boldsymbol{x}}_r = \boldsymbol{A}_r \hat{\boldsymbol{x}}_r + \boldsymbol{B}_r \boldsymbol{u} + \boldsymbol{L}(\boldsymbol{y}_r - \hat{\boldsymbol{y}}_r)$$
(2.5)

where $\hat{y}_r = C_{yr}\hat{x}_r + D_{yr}u$ and an observer gain matrix *L* is defined in the following form

$$L = (P_0 C_{yr}^T + S_0) R_0^{-1}$$
(2.6)

In Eq. (2.6), P_0 is the solution of the Riccati equation $P_0A_0 + A_0^TP_0 - P_0C_{yr}^TR_0^{-1}C_{yr}P_0^T + Q_0 - S_0R_0^{-1}S_0^T = 0$ (Saleh and Adeli, 1996, 1997; Adeli and Saleh, 1999) where $A_0 = A_r - C_{yr}^TR_0^{-1}S_0^T$, and the weight matrices Q_0 , R_0 and S_0 are given as follows (Skelton and Ikeda, 1989)

$$Q_0 = E_r \overline{S}_{ww} E_r^T; S_0 = E_r \overline{S}_{ww} F_{yr}^T; R_0 = \overline{S}_{v_r v_r} + F_{yr} \overline{S}_{ww} F_{yr}^T$$
(2.7)

In Eq. (2.7) S_{ww} and $S_{v_rv_r}$ are power spectral density matrices of W and v_r , respectively, which can be specified by the control algorithm designer. A vector of state errors is defined as $e = x_r - \hat{x}_r$. The derivative of this error vector is found by subtracting Eq. (2.5) from Eq. (2.3):

$$\dot{\boldsymbol{e}} = (\boldsymbol{A}_r - \boldsymbol{L}\boldsymbol{C}_{yr})\boldsymbol{e} + \boldsymbol{E}_r \boldsymbol{W} - \boldsymbol{L}(\boldsymbol{F}_{yr}\boldsymbol{W} + \boldsymbol{v}_r)$$
(2.8)

Since W and v_r are unknown terms, it is necessary to assume that $E_r W - L(F_{yr}W + v_r)$ is small enough to have an insignificant influence on the convergence of

the error vector e. Matrix L defined by Eq. (2.6) results in all the eigenvalues of $(A_r - LC_{yr})$ to have a negative real part. This in turn provides system stability and ensures that error vector e will converge to zero asymptotically, i.e., \hat{x}_r will reach its desired value in a finite time.

2.3 Filtered Sliding Mode Control

2.3.1 Controller Design

In SMC a sliding surface vector **s** is designed first to stabilize the controlled system followed by determination of control forces to drive the response trajectory into the discontinuous sliding surfaces with an exponential speed. Substituting Eq. (2.4) into (2.5) and noting that $D_{yr} = 0$ (because the measured states y_r is not related to the control force) yields

$$\hat{x}_r = A_r \hat{x}_r + B_r u + E\gamma \tag{2.9}$$

where $E_r = [LC_{yr}, LF_{yr}, L]$, and $\gamma^T = [e, W, v_r]$. Wu and Yang (2004) use a modified SMC approach where they find the control force based on a Lyapunov function. In this article, an alternative method is provided to find the control force based on the *equivalent control force* principle (Utkin, 1993). Estimated states are divided into two parts, without actuators (control forces) (\hat{x}_1) and with actuators (control forces) (\hat{x}_2) and Eq. (2.9) is transformed to the following two equations:

$$\hat{x}_1 = A_{11}\hat{x}_1 + A_{12}\hat{x}_2 + E_1\gamma \tag{2.10}$$

$$\hat{x}_2 = A_{21}\hat{x}_1 + A_{22}\hat{x}_2 + B_2u + E_2\gamma$$
(2.11)

where
$$\hat{x}_r = \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix}$$
, $\hat{x}_1 \in \mathbb{R}^{n-m}$, $\hat{x}_2 \in \mathbb{R}^m$, *m* is the number of actuators, $A_r = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$, $B_r = \begin{bmatrix} 0 \\ B_2 \end{bmatrix}$, $E_r = \begin{bmatrix} E_{r1} \\ E_{r2} \end{bmatrix}$, and A_{11} , A_{12} , A_{21} , A_{22} , B_2 , E_{r1} , and E_{r2} are matrices of corresponding dimensions. It should be noted that $\det(B_2) \neq 0$. The design of sliding mode control consists of two steps.

First, \hat{x}_2 in Eq. (2.10) is treated similar to a control force and Eq. (2.10) is solved like an optimal control problem using a linear-quadratic method. Assume \hat{x}_2 can be related to \hat{x}_1 as

$$\widehat{\boldsymbol{x}}_2 = -\boldsymbol{P}_1 \widehat{\boldsymbol{x}}_1 \tag{2.12}$$

where $P_1 \in \mathbb{R}^{m \times (n-m)}$ is the Riccati matrix obtained by solution of a Riccati equation similar to the previous equation. Then, based on Eq. (2.12), the equation of the sliding surface is chosen as

$$\boldsymbol{s} = \boldsymbol{P}_1 \hat{\boldsymbol{x}}_1 + \hat{\boldsymbol{x}}_2 = \boldsymbol{P} \hat{\boldsymbol{x}}_r \tag{2.13}$$

where $P = [P_1, I]$ for ease of notation. The goal in SMC is to achieve s = 0.

The second step is to find control forces such that the response trajectory will always remain along sliding surfaces s = 0. Taking the derivative of s in Eq. (2.13) and using Eq. (2.9) yields

$$\dot{s} = P\hat{x}_r = P(A_r\hat{x}_r + B_r u + E\gamma) = PA_r\hat{x}_r + PE\gamma + PB_r u$$
(2.14)

Assuming the existing of $(PB_r)^{-1}$, the following discontinuous *equivalent* control force is chosen for s to converge to zero:

$$\boldsymbol{u} = (\boldsymbol{P}\boldsymbol{B}_r)^{-1}[-\boldsymbol{P}\boldsymbol{A}_r\hat{\boldsymbol{x}}_r - Msign(\boldsymbol{s})]$$
(2.15)

where $M > ||PE_r \gamma||$ is a time-varying gain chosen to be equal to $M = a||s|| + \delta$ (a and δ

are positive constants),
$$sign(s) = \begin{bmatrix} sign(s_1) \\ \vdots \\ sign(s_m) \end{bmatrix}$$
. In this research a non-constant M is

chosen resulting in the inequality $\delta < M$ which means the undesirable chattering in SMC is reduced. Substituting Eq. (2.15) into (2.14) yields

$$\dot{s} = \boldsymbol{P}\boldsymbol{E}\boldsymbol{\gamma} - \boldsymbol{M}\boldsymbol{s}\boldsymbol{i}\boldsymbol{g}\boldsymbol{n}(\boldsymbol{s}) \tag{2.16}$$

Since $M > ||PE_r \gamma||$, -Msign(s) rather than $PE_r \gamma$ is the determining term in Eq. (2.16), which ensures that $sign(\dot{s})$ is always opposite of sign(s). As a result, the condition s = 0 for the sliding surfaces will be achieved in a finite time, which means the system states will decay with an exponential speed in accordance with Eq. (2.12).

2.3.2 First Order Low Pass Filter for the Control Force

The control force proposed in Eq. (2.15) generally contains high-frequency components due to fast and frequent switching of the sliding mode control force. The high-frequency components have an insignificant influence on the system response since most civil structures have a low frequency compared with that of control force and it is unlikely that a resonant phenomenon will happen (Adeli and Kim, 2004; Kim and Adeli, 2004). Removing the high frequency components of the control force, however, results in a smaller force and actuator size without any loss of response reduction. In this research, a first order low pass filter is properly selected to remove the high-frequency components of the control force in Eq. (2.15) while maintaining structural stability. A filtered control force u_f is selected to satisfy

$$\tau \dot{\boldsymbol{u}}_f + \boldsymbol{u}_f = \boldsymbol{u} \tag{2.17}$$

where the time constant $\tau \ll 1$, $\tau \gg T_s$ and T_s is the sampling time interval. It is necessary to choose a proper τ that is not too large to cause an unstable situation for the controlled structure. After applying a Laplace transformation to Eq. (2.17) the filtered control force u_f can be rewritten as

$$\boldsymbol{u}_f = \frac{\boldsymbol{u}}{\tau\omega+1} = \frac{(\boldsymbol{P}\boldsymbol{B}_r)^{-1}[-\boldsymbol{P}\boldsymbol{A}_r\hat{\boldsymbol{x}}_r - \boldsymbol{M}sign(\boldsymbol{s})]}{\tau\omega+1}$$
(2.18)

where ω is a variable in frequency domain.

2.4 Example of A 76-story Wind Excited Building

2.4.1 Physical Model of the 76-Story Wind Excited Building

The filtered SMC method presented in this chapter is applied to a benchmark control problem developed based on a 76-story, 306-m office tower proposed for the city of Melbourne, Australia (Yang et al., 2004). This reinforced concrete building consists of a central concrete core and an external concrete frame. The 153,000-metric ton slender building is sensitive to wind since its height-to-width ratio is 7.3. It is modeled as a vertical cantilever structure with rigid floors and 76 degree-of-freedom (DOF) for the translational vibration as shown in Figure 2.1 (one DOF per floor). The plan view of this building is shown in Figure 2.2. The time histories of wind excitations for the 30th, 50th, 70th, and 75th floor are shown in Figure 2.3.



Figure 2.1 2D model of 76-story wind-excited high-rising building



Figure 2.2 Plan view of the high-rising building (Unit: meter)



Figure 2.3 Wind excitations for the benchmark building

2.4.2 Coupling of the Building and An Active Tuned Mass Damper

The building is equipped with an active tuned mass damper (ATMD) on the top floor. The equation of motion of the ATMD is (Smith and Coull, 1992)

$$m(\ddot{x}_{76} + \ddot{x}_m) + c_m \dot{x}_m + k_m x_m = u \tag{2.19}$$

where m = 500 metric ton, $c_m = 0.2 \text{ kNs/m}$, and $k_m = 505.1 \text{ kN/m}$ are the mass, damping coefficient, and stiffness value of the ATMD, respectively. For the coupled ATMD-structure system, Eqs. (2.1) and (2.19) are combined and an equation similar to Eq. (2.1) is found where M, C, K, η , and B_s are replaced by the followings matrices: $M_c = \begin{bmatrix} M & 0 \\ a & m \end{bmatrix} \in \mathbb{R}^{77 \times 77}$ where $a = \begin{bmatrix} 0 & \cdots & 0 & m \end{bmatrix} \in \mathbb{R}^{76}$, $C_c = \begin{bmatrix} C & 0 \\ 0 & c_m \end{bmatrix} \in \mathbb{R}^{77 \times 77}$,

$$\boldsymbol{K}_{c} = \begin{bmatrix} \boldsymbol{K} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{k}_{m} \end{bmatrix} \in \boldsymbol{R}^{77 \times 77} , \quad \boldsymbol{\eta}_{c} = \begin{bmatrix} \boldsymbol{\eta} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} \in \boldsymbol{R}^{77 \times 77} , \quad \boldsymbol{B}_{c} = \begin{bmatrix} \boldsymbol{0} & \cdots & \boldsymbol{0} & \boldsymbol{1} \end{bmatrix}^{T} \in \boldsymbol{R}^{77} ,$$

respectively. The displacement vector is also expanded to $\mathbf{x} = [x_1, x_2, ..., x_{76}, x_m]^T$ where x_i is the displacement of the *i*th floor and x_m is the displacement of the ATMD relative to the top floor.

A reduced order system is constructed as follows: The state vector is $\mathbf{x}_r = [x_{16}, x_{30}, x_{46}, x_{60}, x_{76}, x_m, \dot{x}_{16}, \dot{x}_{30}, \dot{x}_{46}, \dot{x}_{60}, \dot{x}_{76}, \dot{x}_m]^T \in \mathbb{R}^{12}$, the measured output vector is $\mathbf{y}_r = [\ddot{x}_{50}, \ddot{x}_{76}, \ddot{x}_m]^T$, $\mathbf{A}_r \in \mathbb{R}^{12 \times 12}$, $\mathbf{B}_r \in \mathbb{R}^{12}$, $\mathbf{E}_r \in \mathbb{R}^{12 \times 77}$, $C_{yr} \in \mathbb{R}^{3 \times 12}$, $\mathbf{D}_{yr} = \mathbf{0} \in \mathbb{R}^3$, $\mathbf{F}_{yr} \in \mathbb{R}^{3 \times 77}$, and $\mathbf{v}_r \in \mathbb{R}^3$. Yang et al. (2004) show that the peak and RMS values of the structural response of the reduced order system are very close to that of the full order system. Figure 2.4 shows the schematic architecture of the control approach for the 76-story building. For simplicity, the rotational DOF is removed by static condensation and hence not considered.



Figure 2.4 Schematic architecture for vibration control of the 76-story building

2.4.3 Selection of Parameters

The parameters used in this simulation are given as follows:

 force. For the purpose of practical implementation, the following hardware limitations are chosen for the actuator: the peak control input/force $|u|_{max} \leq 300kN$, RMS value of the control input (related to energy consumption) $\sigma_u \leq 100kN$, peak relative displacement of the ATMD $|x_m|_{max} \leq 95cm$, and RMS value of the relative displacement of the ATMD $\sigma_{xm} \leq 30cm$.

The time constant in Eq. (2.17), $\tau = 0.1$, is chosen by trial-and-error to avoid an unstable situation. A value of $\delta = 0.001$ is chosen by trial-and-error. The sampling time is $T_s = 0.001$ sec. Three different values are chosen for the constant a = 1.065, 1.265, and 1.865: The first two are chosen for comparison with LQG presented in Yang et al. (2004) and the SMC approach presented by Wu and Yang (2004), respectively. The last value is chosen to show that the structural vibrations can be reduced significantly. For the sake of comparison, only one actuator is used on the top floor of the structure similar to Yang et al. (2004).

2.4.4 Comparison of Simulation Results

Yang et al. (2004) defined 16 criteria for the benchmark problem summarized in the Appendix. The lower the value of each criterion the more effective the control algorithm.

Table 2.1 shows a comparison of 16 evaluation criteria for the proposed filtered SMC method with the LQG algorithm presented in Yang et al., (2004) as well as the unfiltered SMC using a = 1.065. To test the robustness of the proposed method, a $\pm 15\%$ stiffness uncertainty is applied in the simulation. It is found that criteria J_1 to J_4 and J_7 to J_{10} of unfiltered SMC are slightly smaller than those of the LQG control, but the

other 8 criteria are slightly higher for the unfiltered SMC. Therefore, in general the performance of the unfiltered SMC approach is similar to that of LQG control for vibration control of this building. Next, for the filtered SMC, it is found that most of the criteria are much or slightly smaller than that of both unfiltered SMC and LQG control, especially in maximum control force and actuator displacement which are significant due to the limitation of commercially available actuators. Also, the proposed filtered SMC with $\pm 15\%$ stiffness uncertainty is less sensitive compared with the LQG algorithm.

Table 2.2 shows a comparison of the same 16 evaluation criteria for the proposed filtered SMC method with the SMC technique presented in Wu and Yang (2004) as well as the unfiltered SMC using 1.265. This table shows that the performance of unfiltered SMC is similar to that of Wu and Yang (2004). Without stiffness uncertainty, performance of the proposed filtered SMC in 12 criteria is better compared with the unfiltered SMC and the SMC method of Wu and Yang (2004). For all three different stiffness values the maximum control force of filtered SMC is less than that for the SMC method of Wu and Yang (2004). For all three different stiffness values the maximum control force of filtered SMC is less than that for the SMC method of Wu and Yang (2004). For the $\pm 15\%$ stiffness uncertainty cases, the performance of filtered SMC is better than the other two control approaches with the exception of the values of J_7 and J_8 for the $\pm 15\%$ stiffness uncertainty case which are slightly higher than the corresponding values in Wu and Yang (2004).

The time histories of the structural response of the 75th floor and control force on the top of the structure are shown in Figure 2.5 and Figure 2.6 for filtered SMC with a = 1.865 and a = 1.265, respectively. It is found that the response has been reduced by using the filtered SMC compared with the case without control.
Filter SMC ($\alpha = 1.065, \tau = 0.1$)				Unfiltered SMC ($\alpha = 1.065$)				LQG			
Criteria	$\begin{array}{l} \Delta K \\ = 0 \end{array}$	$\Delta K = 15\%$	$\Delta K = -15\%$	Criteria	$\begin{array}{l} \Delta K \\ = 0 \end{array}$	$\Delta K = 15\%$	$\Delta K = -15\%$	Criteria	$\begin{array}{l} \Delta K \\ = 0 \end{array}$	$\Delta K = 15\%$	$\Delta K = -15\%$
J_1	0.372	0.379	0.381	J_1	0.367	0.362	0.385	J_1	0.369	0.365	0.387
J_2	0.420	0.422	0.431	J_2	0.415	0.407	0.435	J_2	0.417	0.409	0.438
J_3	0.578	0.492	0.703	J_3	0.577	0.486	0.709	J_3	0.578	0.487	0.711
J_4	0.580	0.494	0.705	J_4	0.579	0.488	0.711	J_4	0.580	0.489	0.712
J_5	2.243	1.808	2.658	J_5	2.297	1.836	2.746	J_5	2.271	1.812	2.709
J_6	11.594	8.480	15.914	J_6	12.399	8.786	17.198	J_6	11.99	8.463	16.61
$\sigma_u(kN)$	34.15	28.49	44.25	$\sigma_u(kN)$	34.96	29.09	45.51	$\sigma_u(kN)$	34.07	28.29	44.32
$\sigma_{x_m}(\text{cm})$	22.74	18.32	26.95	$\sigma_{x_m}(\text{cm})$	23.29	18.61	27.83	$\sigma_{x_m}(\text{cm})$	23.03	18.37	27.46
J_7	0.382	0.434	0.482	J_7	0.379	0.407	0.485	J_7	0.381	0.411	0.488
J ₈	0.431	0.457	0.534	J_8	0.432	0.442	0.537	J_8	0.432	0.443	0.539
J_9	0.710	0.617	0.781	J_9	0.716	0.608	0.766	J_9	0.717	0.607	0.770
J_{10}	0.718	0.624	0.791	J_{10}	0.724	0.615	0.775	J_{10}	0.725	0.614	0.779
J ₁₁	2.244	1.875	2.846	J_{11}	2.320	1.870	2.865	J ₁₁	2.300	1.852	2.836
J ₁₂	66.690	53.326	117.713	<i>J</i> ₁₂	73.920	54.241	121.864	J ₁₂	71.96	52.68	118.33
$\max u $ (kN)	118.44	107.15	167.09	$\max u $ (kN)	122.38	109.15	168.74	$\max u $ (kN)	118.24	105.58	164.33
$\frac{\max x_m }{(\text{cm})}$	72.48	60.55	91.93	$\frac{\max x_m }{(\text{cm})}$	74.94	60.39	92.53	$\frac{\max x_m }{(\text{cm})}$	74.29	59.83	91.60

Table 2.1 A Comparison of Evaluation Criteria with LQG control

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Filter SMC ($\alpha = 1.265, \tau = 0.1$)				Unfiltered SMC ($\alpha = 1.265$)				MSMC (Wu & Yang's Method)			
Criteria	$\begin{array}{l} \Delta K \\ = 0 \end{array}$	$\Delta K = 15\%$	$\Delta K = -15\%$	Criteria	$\begin{array}{l} \Delta K \\ = 0 \end{array}$	$\Delta K = 15\%$	$\Delta K = -15\%$	Criteria	$\Delta K = 0$	$\Delta K = 15\%$	$\Delta K = -15\%$
J_1	0.359	0.367	0.368	J_1	0.353	0.351	0.371	J_1	0.354	0.351	0.371
J_2	0.407	0.414	0.417	J_2	0.400	0.397	0.420	J_2	0.401	0.397	0.421
J_3	0.571	0.489	0.694	J_3	0.569	0.482	0.700	J_3	0.569	0.482	0.701
J_4	0.573	0.491	0.696	J_4	0.571	0.484	0.702	J_4	0.571	0.484	0.703
J_5	2.370	1.930	2.835	J_5	2.436	1.964	2.944	J_5	2.439	1.962	2.954
J_6	13.650	10.216	18.865	J_6	14.715	10.626	20.525	J_6	14.762	10.586	20.696
$\sigma_u(kN)$	39.00	32.86	50.52	$\sigma_u(kN)$	39.90	33.51	51.99	$\sigma_u(kN)$	40.07	33.55	52.34
$\sigma_{x_m}(\text{cm})$	24.03	19.565	28.74	$\sigma_{x_m}(\text{cm})$	24.70	19.91	29.84	$\sigma_{x_m}(\text{cm})$	24.73	19.89	29.95
J ₇	0.364	0.420	0.464	J_7	0.368	0.387	0.466	J ₇	0.369	0.387	0.469
J_8	0.424	0.455	0.524	J_8	0.428	0.437	0.527	J ₈	0.428	0.437	0.527
J_9	0.703	0.624	0.767	J_9	0.710	0.614	0.745	J_9	0.711	0.613	0.744
J ₁₀	0.711	0.631	0.776	J ₁₀	0.718	0.621	0.754	J_{10}	0.719	0.620	0.753
J ₁₁	2.338	1.963	3.011	<i>J</i> ₁₁	2.431	1.958	3.017	J ₁₁	2.436	1.958	3.021
J ₁₂	75.664	61.346	137.350	J ₁₂	85.011	62.509	141.061	J ₁₂	85.644	62.453	141.606
$\max u $ (kN)	140.55	129.47	189.61	$\max u $ (kN)	145.63	129.19	192.39	$\max u $ (kN)	145.98	129.00	193.61
$\frac{\max x_m }{(\text{cm})}$	75.50	63.41	97.24	$\frac{\max x_m }{(\text{cm})}$	78.52	63.25	97.45	$\frac{\max x_m }{(\text{cm})}$	78.68	63.24	97.57

Table 2.2 A Comparison of Evaluation Criteria with MSMC control

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Figure 2.5 Time series of structural response and control force for filtered SMC with $\alpha = 1.865$: (a) x_{75} ; (b) \ddot{x}_{75} ; and (c) control force



Figure 2.6 Time series of structural response and control force using filtered SMC with $\alpha = 1.265$: (a) x_{75} ; (b) \ddot{x}_{75} ; and (c) control force

2.5 Conclusion

A filtered SMC approach is presented in the article for active vibration control of wind-excited highrise building structures and its performance is evaluated by application to a 76-story building benchmark problem equipped with an ATMD on the roof. Assuming rigid floors the 77-degree-of-fredom (DOF) structure is reduced to a 12-DOF model. An asymptotic observer is employed to estimate the system states since only 3 out of the 12 states are measured directly. Due to the elimination of high-frequency part of the control force, the structure, sensors, actuators, and dampers are all less excited, and consequently their response is reduced compared with the unfiltered SMC approach. In addition, the required control forces are reduced which means a reduction in the size of actuators making their implementation more practical. Compared with LQG and another implementation of SMC (Yang et al., 2004; Wu and Yang, 2004), the proposed filtered SMC has in general better performance, especially in reducing the maximum control force and control power. Furthermore, the proposed method is more robust to structural stiffness variations and uncertainties. A proper selection of the low pass filter is necessary to ensure the stability of the controlled structure. Further research includes the consideration of the actuator dynamics and actuator-structure interaction for practical implementation.

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Chapter 3: Self-Constructing Wavelet Neural Network Algorithm for Nonlinear Adaptive Control of Civil Structures

3.1 Introduction

A large number of articles have been published on linear vibration control of civil structures over the past three decades (Yao, 1972; Soong, 1990; Saleh and Adeli, 1994; 1996, 1998a&b; Yang et al., 1996; Yang et al., 2004) using a number of different algorithms developed in the vibration control community such as LQR (for example, Adeli and Saleh; 1997, 1998; Agrawal et al., 1997), LQG (Soong, 1990; Vasques, and Dias Rodrigues, 2006), and H_{∞} (Chase and Smith, 1996). The great majority of these papers deal with small academic problems where the structure is modeled as a twodimensional (2D) structure with a few degrees-of-freedom or large structures assuming the controlled structure behaves linearly.

Vibration control of large nonlinear structures remains a challenging problem because of a) unknown time-varying properties of structural systems and b) uncertainties existing in both structural system identification and external excitations such as those due to an earthquake. Sliding mode control (SMC) has been used as a competitive control approach in civil structures (Yang et al., 1996; Singh et al., 1997; Kim and Yun, 2000; Wu, 2003; Wu and Yang, 2004; Lee et al., 2004; Lee et al., 2004; Wang and Lin, 2006; Ning et al., 2009).

Yang et al. (1996) applied the sliding mode control to a seismically-excited 3story building isolated by a frictional sliding-isolation system and reported its effectiveness based on experimental test results. Singh et al. (1997) applied the SMC approach to a seismically-excited 10-story two-dimensional (2D) frame. Sarbjeet and Datta (2000) apply the sliding mode control strategy to a 20-story 2D frame subjected to a narrow band ground excitation and report more reduction in displacements compared with conventional linear control strategies such as LQR. Kim and Yun (2000) proposed a fuzzy sliding mode control (FSMC) for a three-story benchmark building considering actuator-structure interaction, sensor noise, actuator time delay, precision of the analogto-digital (A/D) and digital-to-analog (D/A) converters, control force saturation range, and order of the control model. They report improved performance for FSMC compared with other control algorithms such as $H_{2/\infty}$ control, optimal polynomial control, neural networks-based control, and SMC. Using a distributed parameter system equipped with active tuned mass dampers (ATMDs), Wang and Lin (2006) indicate that FSMC is more economical and practical than a variable control algorithm such as SMC (a variable highfrequently switching feedback control where the control gains in each feedback path switch between two values according to some rule) in terms of controlling force and control energy use when applied to a seismically excited three story reinforced-concrete building. Wu (2003) and Wu and Yang (2004) use a pre-filtered sliding mode control method to reduce the response of a seismically-excited three-story building, and demonstrate its performance through shaking table experimental tests of a full-scale building equipped with active bracing systems. They also applied it to wind-induced vibrations of a 76-story high rising building. Lee et al. (2004) apply SMC to a 3-story

frame considering controller saturation. Ning et al., (2009) propose a FSMC control to the seismically excited nonlinear benchmark bridge in Yang et al. (2009), and show that the most of the performance including displacement, shear, and moment has been reduced, but with a slightly increase of the mid-span acceleration.

SMC, however, has a shortcoming for application to large civil structures rarely discussed in the literature. Controlled responses from sliding mode control are highly sensitive to the bounds of structural system uncertainties and weighing matrices of the sliding surface. A small sliding bound may cause instability in vibration control of structures, while a large sliding bound will lead to the so-called chattering effect which means the sign of the control force changes rapidly and frequently within a short time period caused by a discontinuous switching function (usually a discontinuous sign function). This chattering effect can be reduced by using a continuous approximation of the discontinuous sliding mode controller, but it may cause system instability (Leu et al., 2009). To overcome the chattering phenomena while maintaining system stability, a fuzzy compensation controller is sometimes designed to model system uncertainties and function approximation errors (Hsu et al., 2009).

Neural networks (NNs) can be used for universal approximation of both linear and nonlinear functions (Hornik et al., 1989; Cybenko, 1989; Adeli and Hung, 1994; Hung and Adeli, 1994; Senouci and Adeli, 2001; Adeli and Jiang, 2003). Traditional NNs consist of multiple layers with a sufficient number of nodes in each hidden layer and adjustable weights. They suffer from some common drawbacks such as lack of an efficient constructive model resulting in an arbitrary selection of the number of hidden nodes, slow convergence rate, and entrapment in a local minimum. Control algorithms based on this type of NN require extensive off-line training.

To overcome the aforementioned drawbacks of classical NNs used for system control and/or identifications, radial basis function (RBF) neural networks have been used to simplify the network structure and reduce computational burden where Gaussian functions are generally used as the basis functions (Chen, 1990; Adeli and Karim, 2000; Karim and Adeli, 2002, 2003; Ghosh-Dastidar, 2008). These offline RBF-based NNs are further improved by using a resource allocating network (RAN) algorithm (Platt, 1991), which adds new hidden neurons depending on the input characteristics and output errors, where the weights connecting hidden layer and output layer are updated based on a least mean square (LMS) criterion. Two modifications of RAN are: (1) the replacement of the LMS criterion with extended Kalman Filter (EKF) (Kadirkamanathan and Niranjan, 1993) which improves the network compactness, and (2) the pruning criterion which is able to remove hidden neurons that are less influential to the output in order to make the network more compact (Lu et al., 1997, 1998). A network based on these two improvements is generally referred to as minimal resource allocation network (MRAN).

As an extension of MRAN, the extended MRAN (EMRAN) was introduced subsequently (Irwin et al., 1995; Li et al., 2000; Wang et al., 2002). Rather than updating the parameters of all hidden neurons in each time step in MRAN, EMRAN allocates new hidden nodes (called Gaussian nodes) using a growing/pruning criterion, which means the number of nodes is reduced if the network can accurately approximate the unknown system given an allowed error range, and is increased if the error is outside the range. Gaussian nodes are able to store characteristic information of the unknown system. Each Gaussian node responds only to the local region of the input space. Only those parameters of a given node closest to the selected *winner node* are updated. As such, the learning patterns are not fully repeated as a result of the local updating process. The EMRAN algorithm reduces the computational time compared with MRAN (Li et al., 2000) and therefore is more suitable for online adaptation of high order unknown nonlinear systems. However, the EMRAN algorithm cannot ensure the stability of control models. A learning algorithm based on the Lyapunov function may be used to guarantee system stability (Gao and Er, 2003; Hsu, 2007).

Compared with the Gaussian radial basis functions, wavelet basis functions yield more compact and efficient system representations while preserving global closed-loop stability if a proper adaptive law is used to train the neural network (Cannon and Slotine, 1995). A wavelet neural network (WNN) model was proposed by Zhang and Benveniste (1992) for signal processing. Hung et al. (2003) applied WNN to system identification of civil structures. Adeli and Jiang (2006) modified WNN using fuzzy logic to achieve a more efficient constructive model and higher identification accuracy. Their modified fuzzy WNN model is based on adroit integration of four different computing concepts: dynamic time delay neural network, wavelet as the basis function, fuzzy logic, and the state space reconstruction based on the chaos theory. They used a Mexican hat wavelet in their WNN model because a) its analytical expression makes it amenable for both differentiation of multiple dimensional time series, and b) it provides computational efficiency (Zhou et al, 2003; Jiang and Adeli, 2003; Jiang and Adeli, 2005). They employ chaos theory to model the complicated and unknown nonlinear dynamics of structureearthquake system which requires determining an appropriate embedding dimension for

which they use the false nearest neighbor method. The input dimension of a time series can be obtained using the embedding theorem (Takens, 1981) for structural identification (Adeli and Jiang, 2006). The number of wavelet neurons in the hidden layer of their WNN model is determined by a self-constructing method using the Akaike's final prediction error (AFPE) criterion. Their model works well when the time series for training data is available.

The selection of the number of nodes in the hidden layer is crucial for obtaining consistently accurate approximations with a reasonable computational cost. A trial-anderror method was generally used in earlier approaches to obtain the most suitable value for the number of nodes in the hidden layer using the NARMAX approach (Zhang 1997; Hung et al. 2003). That approach is time-consuming, does not provide a rational basis for the selection of the number of nodes in the hidden layer, and cannot guarantee accurate approximations. In order to determine the number of nodes in a neural network model for real-time control of nonlinear dynamic systems, a self-constructing method without a *priori* knowledge may be used. Researchers have introduced self-organizing/self-constructing algorithms to dynamically adapt the Gaussian basis neurons in the hidden layer. So far, these methods have been used for control of simple problems such as control of first order circuit systems (Hsu, 2007).

But the stability of self-constructing NN-based control algorithms should also be guaranteed. A number of learning algorithms have been proposed based on the Lyapunov function to guaranteed system stability. Hsu (2007) applied the self-constructing fuzzy neural controller for a simple first order chaotic circuit system. Further, to overcome the chattering effect noted in traditional SMC (Wu and Er, 2000; Wu et al., 2001; Gao and Er, 2003; Hsu, 2007), Hsu et al. (2009) apply a fuzzy compensation controller to replace the traditional signal-switched compensation controller in a wavelet-based adaptive control approach. They proved the stability for the same first order chaotic circuit problem and present a faster online learning algorithm using the more recently developed proportional integral (PI)-type adaptive law instead of a traditional integral (I)-type adaptive law (Golea et al., 2002).

In this article, an improved control algorithm is presented for nonlinear vibration control of large structures subjected to dynamic loading such as earthquake or wind loading. It is based on integration of a self-constructing wavelet neural network (SCWNN) developed specifically for structural system identification with an adaptive fuzzy sliding mode control approach. The algorithm is particularly suitable when the physical properties such as the stiffnesses and damping ratios of the structural system are unknown or known only within an approximate range which is the case when a structure is subjected to an extreme dynamic event such as an earthquake or wind as the structural properties change during the event. Further, no off-line training of the neural network is required thus removing a major hurdle in application of the vibration control technology to real structures. Compared with traditional linear control algorithms such as LQR and LQG, SMC is more effective in handling system uncertainties such as model and approximation errors in system identification for adaptive control design. SCWNN is developed for functional approximation of the nonlinear behavior of large structures using neural networks and wavelets. Adeli and Jiang (2006) present a dynamic timedelay fuzzy wavelet NN for nonparametric identification of structures using the nonlinear autoregressive moving average with exogenous inputs approach. In that work the

identification process is conducted by training data off-line. In this chapter, the identification and control are processed simultaneously which makes the resulting adaptive control more applicable to real life situations. A two-part growing and pruning criterion is developed to construct the hidden layer in the neural network automatically. The wavelet basis functions used in SCWNN provide a more compact and efficient system representations over earlier neural networks based on Gaussian radial basis functions. A fuzzy compensation controller is developed to overcome the chattering phenomenon. The model is applied to vibration control of a benchmark seismically excited highway bridge.

3.2 Problem Formulation of Unknown Nonlinear Civil Structures

The equations of dynamic equilibrium for a nonlinear multiple-degree-of-freedom (MDOF) structure with controllers excited by ground accelerations are expressed as:

$$\boldsymbol{M}\ddot{\boldsymbol{x}}(t) + \boldsymbol{C}(\boldsymbol{x}, \dot{\boldsymbol{x}})\dot{\boldsymbol{x}}(t) + \boldsymbol{K}(\boldsymbol{x}, \dot{\boldsymbol{x}})\boldsymbol{x}(t) = -\boldsymbol{M}\ddot{\boldsymbol{x}}_{a}(t) + \boldsymbol{B}_{s}(\boldsymbol{x}, \dot{\boldsymbol{x}})\boldsymbol{u}(t)$$
(3.1)

where $M \in R^{p \times p}$, $C(x, \dot{x}) \in R^{p \times p}$, and $K(x, \dot{x}) \in R^{p \times p}$ are the mass, damping, and stiffness matrices, respectively, $x(t) \in R^p$ is the displacement vector, p is the number of degrees of freedom (DOF) of the structure, $\ddot{x_g}(t)$ is the ground acceleration due to an earthquake, $u(t) \in R^q$ is the control force vector assuming the structure has q actuators, and $B_s(x, \dot{x}) \in R^{p \times q}$ is the matrix related to positions of the control forces. In this research, matrices $K(x, \dot{x})$, $C(x, \dot{x})$ and $B_s(x, \dot{x})$ are considered to be unknown timevariant nonlinear functions of displacements and velocities. Equation (3.1) is transformed to its canonical form as follows:

$$\ddot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{X}) + \mathbf{g}(\mathbf{X})\mathbf{u}(t) + \mathbf{d}(t)$$
(3.2)

where $X = \begin{bmatrix} x \\ \dot{x} \end{bmatrix} \in \mathbb{R}^n$ (n = 2p) is the state vector consisting of *n* system states (displacements and velocities) that are assumed to be measurable,

 $f(X) = -M^{-1}[C(x, \dot{x})\dot{x}(t) + K(x, \dot{x})x(t)] \in R^p \text{ and } g(X) = -M^{-1}B_s(x, \dot{x}) \in R^{p \times p}$ are unknown continuous functions, $d(t) = -\ddot{x}_g(t) \in R^p$ is the unknown ground disturbance with an upper bound *D*, that is, $||d(t)|| \le D$. SCWNN is a model to estimate functions f(X) and g(X) approximately in real time or online.

The difference between the measured displacement output \mathbf{x} and desired displacement output \mathbf{x}_d (generally $\mathbf{x}_d = \mathbf{0}$) is defined as an error vector:

$$\boldsymbol{e}(t) = \boldsymbol{x} - \boldsymbol{x}_d \tag{3.3}$$

A 2-D error space is defined with e and \dot{e} as its two axes. The main idea behind SMC is to develop a controller to drive the output response to the desired trajectory by forcing the equations of the sliding surfaces in the error space into zero:

$$\mathbf{s}(\mathbf{e}) = \mathbf{0} \tag{3.4}$$

within some given time and keep the point (e, \dot{e}) within the surface after that. That is to make s(e) approach **0** after some finite time and then to ensure that s(e) remains **0** and stable within a small tolerance. A sufficient condition for stability of SMC is that a selected Lyapunov function $V = \frac{1}{2}s^Ts$ must satisfy the follow inequality (Slotine and Li, 1991; Utkin, 1992; Khalil, 1996)

$$\dot{V} = \mathbf{s}^T \dot{\mathbf{s}} \le \mathbf{0} \tag{3.5}$$

where over-dot indicates differentiation in time and the sliding surface is defined as

$$\boldsymbol{s} = \dot{\boldsymbol{e}} + k_1 \boldsymbol{e} + k_2 \int_0^t \boldsymbol{e}(\tau) \, d\tau \tag{3.6}$$

in which the coefficient k_1 and k_2 are positive constants. The right-hand side of equations of the sliding surfaces, Eq. (3.6), consists of three terms referred to as D part for the first derivative term, P part for the second proportional term, and I part for the last integral term. A combination of two terms would be sufficient for an SMC algorithm. The corresponding control algorithms are dubbed as PD, PI, and PID algorithms. All three terms are included in this research to achieve a faster convergence. Differentiation of Eq. (3.6) with respect to time and using Eq. (3.2) yields:

$$\dot{\boldsymbol{s}} = \ddot{\boldsymbol{e}} + k_1 \dot{\boldsymbol{e}} + k_2 \boldsymbol{e} = (\ddot{\boldsymbol{x}} - \ddot{\boldsymbol{x}}_d) + k_1 \dot{\boldsymbol{e}} + k_2 \boldsymbol{e}$$
$$= [k_1 \dot{\boldsymbol{e}} + k_2 \boldsymbol{e} + \boldsymbol{f}(\boldsymbol{X}) - \ddot{\boldsymbol{x}}_d + \boldsymbol{g}(\boldsymbol{X})\boldsymbol{u}] + \boldsymbol{d}(t)$$
(3.7)

The goal of the control is $\lim_{n\to\infty} e = 0$. To achieve this, **s** is made equal to **0**. This is done by choosing a control force vector **u** such that Eq. (3.7) can be presented in a form similar to $\dot{s} = -Msign(s)$ where *M* is a positive scalar (different from the matrix **M** in Eq. 1). Such a form will make **s** decay exponentially to zero. Thus by assuming the existence of nonsingular matrix g(X), the control force vector **u** is selected to be

$$u_w = g(X)^{-1}[-f(X) + \ddot{x_d} - k_1 \dot{e} - k_2 e + u_a]$$
(3.8)

where

$$\boldsymbol{u}_{a} = -\eta_{s} \begin{bmatrix} sgn(s_{1}) \\ \vdots \\ sgn(s_{p}) \end{bmatrix}$$
(3.9)

The coefficient $\eta_s > ||\mathbf{d}(t)||$, $(1 \le i \le p)$ is a positive constant representing the bounds on the amplitude of external excitation and chosen based on experience, and sgn(s) is the sign function defined as

$$sgn(s) = \begin{cases} 1, & if \ s > 0 \\ -1, & if \ s < 0 \end{cases}$$
(3.10)

The vector \boldsymbol{u}_a is generally called the compensator control input. Substituting Eq. (3.9) into Eq. (3.8) yields

$$\dot{\boldsymbol{s}} = \boldsymbol{u}_a + \boldsymbol{d}(t) \tag{3.11}$$

Since $\eta_s > ||\boldsymbol{d}(t)||$, \boldsymbol{u}_a determinates the sign of $\dot{\boldsymbol{s}}$. That is, the sign of \boldsymbol{s} will depend on the term \boldsymbol{u}_a rather than $\boldsymbol{d}(t)$. Moreover, because the sign of $\dot{\boldsymbol{s}}$ is always opposite of the sign of \boldsymbol{s} based on Eq. (3.11), $\lim_{t\to\infty} \boldsymbol{s} = \boldsymbol{0}$ as desired.

Solution of Eq. (3.8) cannot be found directly because f(X) and g(X) are unknown. In this research a fuzzy WNN is developed to approximate them.

$$f(X) = \hat{f}(X) + \tilde{f}(X)$$
(3.12)

$$\boldsymbol{g}(\boldsymbol{X}) = \boldsymbol{\widehat{g}}(\boldsymbol{X}) + \boldsymbol{\widetilde{g}}(\boldsymbol{X}) \tag{3.13}$$

where $\hat{f}(\mathbf{X})$ and $\hat{g}(\mathbf{X})$ are approximate values of the unknown dynamic functions $f(\mathbf{X})$ and $g(\mathbf{X})$, respectively, and $\tilde{f}(\mathbf{X})$ and $\tilde{f}(\mathbf{X})$ are the model error due to function approximation. Control force vector, u_w , can be found from Eq. (3.8) using the approximate values $\hat{f}(\mathbf{X})$ and $\hat{g}(\mathbf{X})$.

An unknown external disturbance due to earthquake or wind generally requires a relatively large upper bound (η), and this generally causes a chattering phenomenon in the sliding mode control especially when the sampling time is large, for example, in the order of 0.01 sec or larger. A possible solution for the chattering problem using the fuzzy logic approach will be presented in the following section.

3.3 Self-Constructing Wavelet Neural Network Model

In this section a self-constructing wavelet neural network is presented for estimation of the unknown dynamic functions f(X) and g(X) where the number of nodes in the hidden layer and the corresponding wavelet basis functions are determined automatically without any prior knowledge of the structure or external excitation. SCWNN is an extension of the WNN model developed by (Adeli and Jiang, 2006). That model consists of two distinct phases of identification and control. In contrast, there is no separation of these two steps in the SCWNN control algorithm presented in this chapter and both are performed simultaneously. The WNN model consists of an input layer, a hidden layer, and an output layer. The hidden layer consists of wavelet nodes, used as activation functions and characterized by their localization since each unit responds only to a certain region of input space. The unknown function is approximated as the linear combination of the basis units.

3.3.1 Selection of the Wavelet Function

Following Adeli and Jiang (2006) wavelet functions $\boldsymbol{\varphi} = [\varphi_1, \varphi_2, ..., \varphi_h]^T$ used in this research are Mexican hat wavelets. A Mexican hat wavelet function is the negative normalized second derivative of Gaussian function defined by

$$\varphi_i(\zeta) = \frac{2}{\sqrt{3\pi^{\frac{1}{4}}}} (1 - \zeta^2) \exp\left(-\zeta^2/2\right), 1 \le i \le h$$
(3.14)

where $\zeta(i) = \left\|\frac{x-c_i}{\sigma_i}\right\|$ and $\|.\|$ denotes the Euclidean norm, $c_i = [c_{i1}, c_{i2}, ..., c_{in}]^T \in \mathbb{R}^n$ is the center, and $\sigma_i = [\sigma_{i1}, \sigma_{i2}, ..., \sigma_{in}]^T \in \mathbb{R}^n$ is the width of the *i*th wavelet node; *h* is the total number of wavelet nodes in the hidden layer in each time step (its value is changed automatically over the duration of excitation). An example Mexican hat wavelet is shown in Figure 3.1. This wavelet is chosen because it has a differentiable analytical expression.



Figure 3.1 Maxican hat wavelet

3.3.2 Wavelet Neural Network for Function Approximation

A schematic architecture of SWCNN model for vibration control of structures is presented in Figure 3.2. Let f = f(X) for ease of notation. The function $f = [f_1, f_2, ..., f_p]$ is approximated by the Mexican hat wavelet:

$$f_j = \hat{f}_j + \tilde{f}_j, \ 1 \le j \le p$$
 (3.15)

$$\hat{f}_j = \sum_{i=1}^h w_i \,\varphi_i \left(\left\| \frac{X - c_i}{\sigma_i} \right\| \right), \, 1 \le i \le h, \tag{3.16}$$

where \hat{f}_j is the *j*th element of \hat{f} , \tilde{f}_j is the *j*th element of the error function \tilde{f} , w_i is the weight from the *i*th wavelet node to the output node. For ease of notation, $\varphi_i\left(\|\frac{X-c_i}{\sigma_i}\|\right)$ is rewritten as



Figure 3.2 Schematic architecture of SWCNN model for vibration control of structures

$$\varphi_i\left(\left\|\frac{\mathbf{X}-\mathbf{c}_i}{\sigma_i}\right\|\right) = \varphi_i(\mathbf{X}, \mathbf{c}_i, \boldsymbol{\sigma}_i) = \varphi_i$$
(3.17)

Therefore, \hat{f} can be rewritten in matrix form as follows:

$$\hat{\boldsymbol{f}} = \boldsymbol{w}^T \boldsymbol{\varphi}(\boldsymbol{X}, \boldsymbol{c}, \boldsymbol{\sigma}) \tag{3.18}$$

where $\boldsymbol{w}^T \in R^{p \times h}$ is the weight matrix for $\hat{\boldsymbol{f}}, \boldsymbol{c} = [\boldsymbol{c}_1, \boldsymbol{c}_2, ..., \boldsymbol{c}_h]^T \in R^{nh}, \boldsymbol{\sigma} =$

 $[\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2, \dots, \boldsymbol{\sigma}_h]^T \in \mathbb{R}^{nh}, \boldsymbol{\varphi} = [\varphi_1, \varphi_2, \dots, \varphi_h]^T \in \mathbb{R}^h.$ Similarly,

$$\widehat{\boldsymbol{g}} = \boldsymbol{\theta}^T diag[\boldsymbol{\varphi}(\boldsymbol{X}, \boldsymbol{c}, \boldsymbol{\sigma})]$$
(3.19)

where $\boldsymbol{\theta}^T \in R^{p \times qh}$ is the weight matrix for function $\hat{\boldsymbol{g}}_i$, and $diag[\boldsymbol{\varphi}(\boldsymbol{X}, \boldsymbol{c}, \boldsymbol{\sigma})] =$

 $\begin{bmatrix} \boldsymbol{\varphi} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\varphi} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \boldsymbol{\varphi} \end{bmatrix} \in R^{qh \times q}.$ In order to capture the linear characteristics of the nonlinear

system, similar to Adeli and Jiang (2006) a linear term, $\boldsymbol{b}^T \boldsymbol{X}$, is added to Eq. (3.18): $\hat{\boldsymbol{f}} = \boldsymbol{w}^T \boldsymbol{\varphi}(\boldsymbol{X}, \boldsymbol{c}, \boldsymbol{\sigma}) + \boldsymbol{b}^T \boldsymbol{X}$. The *j* th element of $\hat{\boldsymbol{f}}$ is written as

$$\hat{f}_j = \boldsymbol{w}_j^T \boldsymbol{\varphi}(\boldsymbol{X}, \boldsymbol{c}, \boldsymbol{\sigma}) + \boldsymbol{b}_j^T \boldsymbol{X}, \ 1 \le j \le p$$
(3.20)

where $\boldsymbol{w}_j^T \in R^h$ and $\boldsymbol{b}_j^T \in R^n$ are the weights of the nonlinear and linear terms, respectively.

The estimated values of the unknown parameters w, c, σ , and b are denoted by \hat{w} , \hat{c} , $\hat{\sigma}$, and \hat{b} . In each time step nodes are classified into active and inactive nodes to be described later in this section. The values of w, c and σ of active nodes are updated automatically in any given time step while those values for inactive nodes are discarded. After this classification the parameters are divided as follows

$$\boldsymbol{w}_{j} = [\boldsymbol{w}_{j_{a}}], \boldsymbol{\varphi} = [\boldsymbol{\varphi}_{a}], = [\boldsymbol{c}_{a}], \boldsymbol{\sigma} = [\boldsymbol{\sigma}_{a}]$$
(3.21)

where w_{j_a} , c_a , σ_a are the parameters of the active nodes, and $w_{j_{ia}}$, c_{ia} , σ_{ia} are the parameters of the inactive nodes.

The j th element of \hat{f} is

$$\hat{f}_j = \hat{\boldsymbol{w}}_{j_a}^T \hat{\boldsymbol{\varphi}}_a + \hat{\boldsymbol{b}}_j^T \boldsymbol{X}$$
(3.22)

where $\widehat{\boldsymbol{\varphi}}_a = \boldsymbol{\varphi}(\boldsymbol{X}, \widehat{\boldsymbol{c}}_a, \widehat{\boldsymbol{\sigma}}_a).$

The approximation error is computed using Eqs. (3.20) and (3.22):

$$\tilde{f}_{j} = \tilde{\boldsymbol{w}}_{j_{a}}^{T} \boldsymbol{\widehat{\varphi}}_{a} + \tilde{\boldsymbol{w}}_{j_{a}}^{T} \boldsymbol{\widetilde{\varphi}}_{a} + \boldsymbol{\widehat{w}}_{j_{a}}^{T} \boldsymbol{\widetilde{\varphi}}_{a} + \boldsymbol{\widetilde{b}}_{j}^{T} \boldsymbol{X} + \boldsymbol{w}_{j_{ia}}^{T} \boldsymbol{\varphi}_{ia}$$
(3.23)

where $\widetilde{w}_{j_a} = w_{j_a} - \widehat{w}_{j_a}$, $\widetilde{\varphi}_a = \varphi_a - \widehat{\varphi}_a$, $\widetilde{b}_j = b_j - \widehat{b}_j$. The detailed formulation can be found in Appendix B.1.

Function $\boldsymbol{\varphi}_a$ is a nonlinear function of $\tilde{\boldsymbol{c}}_a = \boldsymbol{c}_a - \hat{\boldsymbol{c}}_a$ and $\tilde{\boldsymbol{\sigma}}_a = \boldsymbol{\sigma}_a - \hat{\boldsymbol{\sigma}}_a$. In order to formulate a proper control law it is necessary to linearize it using Taylor's expansion as follows (Hsu, 2009):

$$\widetilde{\boldsymbol{\varphi}}_a = \boldsymbol{A}^T \widetilde{\boldsymbol{\sigma}}_a + \boldsymbol{B}^T \widetilde{\boldsymbol{c}}_a + \boldsymbol{H}$$
(3.24)

where $\boldsymbol{A} = \frac{\partial \varphi_a}{\partial \sigma_a} |_{\boldsymbol{\sigma} = \hat{\boldsymbol{\sigma}}_a} = \begin{bmatrix} \frac{\partial \varphi_1}{\partial \sigma_a}, \frac{\partial \varphi_2}{\partial \sigma_a}, \dots, \frac{\partial \varphi_h}{\partial \sigma_a} \end{bmatrix} |_{\boldsymbol{\sigma}_a = \hat{\boldsymbol{\sigma}}_a}$

$$\boldsymbol{B} = \frac{\partial \varphi_a}{\partial c_a} |_{\boldsymbol{c}_a = \hat{\boldsymbol{c}}_a} = \left[\frac{\partial \varphi_1}{\partial c_a}, \frac{\partial \varphi_2}{\partial c_a}, \dots, \frac{\partial \varphi_h}{\partial c_a} \right] |_{\boldsymbol{c}_a = \hat{\boldsymbol{c}}_a}, \boldsymbol{A}^T, \boldsymbol{B}^T \in \mathbb{R}^{h \times (nh)} \text{ and } \boldsymbol{H} \text{ represents the higher}$$

order terms of Taylor expansion which is ignored in this research.

Substituting Eq. (3.24) into Eq. (3.23) yields

$$\tilde{f}_{j} = \tilde{\boldsymbol{w}}_{j_{a}}^{T} \hat{\boldsymbol{\varphi}}_{a} + \tilde{\boldsymbol{\sigma}}_{a}^{T} \boldsymbol{A} \hat{\boldsymbol{w}}_{j_{a}} + \tilde{\boldsymbol{c}}_{a}^{T} \boldsymbol{B} \hat{\boldsymbol{w}}_{j_{a}} + \hat{\boldsymbol{w}}_{j_{a}}^{T} \boldsymbol{H} + \tilde{\boldsymbol{w}}_{j_{a}}^{T} \tilde{\boldsymbol{\varphi}}_{a} + \tilde{\boldsymbol{b}}_{j}^{T} \boldsymbol{X} + \boldsymbol{w}_{j_{ia}}^{T} \boldsymbol{\varphi}_{ia}$$
(3.25)

where relations $\tilde{\boldsymbol{\sigma}}_{a}^{T} A \hat{\boldsymbol{w}}_{j_{a}} = \hat{\boldsymbol{w}}_{j_{a}}^{T} A^{T} \tilde{\boldsymbol{\sigma}}_{a}$ and $\tilde{\boldsymbol{c}}_{a}^{T} B \hat{\boldsymbol{w}}_{j_{a}} = \hat{\boldsymbol{w}}_{j_{a}}^{T} B^{T} \tilde{\boldsymbol{c}}_{a}$ are employed because they are scalars. A detailed formulation of Eq. (3.25) is available in Appendix B.2.

In order to speed up the computation of weight parameters w_a and b, instead of using the traditional I-type adaptive algorithm, a PI-type algorithm is used in this nonlinear control model as follows (Lin, 2002):

$$\boldsymbol{w}_{j_a} = \eta_{pw} \boldsymbol{w}_{j_{pw}} + \eta_{iw} \boldsymbol{w}_{j_{iw}}$$
(3.26)

$$\boldsymbol{b}_{j} = \eta_{pb} \boldsymbol{b}_{j_{pb}} + \eta_{ib} \boldsymbol{b}_{j_{ib}} \tag{3.27}$$

where η_{pw} , η_{iw} , η_{pb} , η_{ib} are positive constants, $\boldsymbol{w}_{j_{pw}}$ and $\boldsymbol{w}_{j_{iw}} = \int_0^t \boldsymbol{w}_{j_{pw}} d\tau$ are proportional and integral terms of \boldsymbol{w}_{j_a} , respectively; $\boldsymbol{b}_{j_{pb}}$ and $\boldsymbol{b}_{j_{ib}} = \int_0^t \boldsymbol{b}_{j_{pb}} d\tau$ are proportional and integral terms of \boldsymbol{b}_j , respectively.

Similarly, \boldsymbol{w}_a and \boldsymbol{b} are estimated as

$$\widehat{\boldsymbol{w}}_{j_a} = \eta_{pw} \widehat{\boldsymbol{w}}_{j_{pw}} + \eta_{iw} \widehat{\boldsymbol{w}}_{j_{iw}} \tag{3.28}$$

$$\widehat{\boldsymbol{b}}_{j} = \eta_{pb} \widehat{\boldsymbol{b}}_{j_{pb}} + \eta_{ib} \widehat{\boldsymbol{b}}_{j_{ib}}$$
(3.29)

where $\widehat{\boldsymbol{w}}_{j_{pw}}$ and $\widehat{\boldsymbol{w}}_{j_{iw}} = \int_0^t \widehat{\boldsymbol{w}}_{j_{pw}} d\tau$ are proportional and integral terms of $\widehat{\boldsymbol{w}}_{j_a}$, respectively, and $\widehat{\boldsymbol{b}}_{j_{pb}}$ and $\widehat{\boldsymbol{b}}_{j_{ib}} = \int_0^t \widehat{\boldsymbol{b}}_{j_{pb}} d\tau$ are proportional and integral terms of $\widehat{\boldsymbol{b}}_j$, respectively.

Error matrices are obtained by subtracting Eq. (3.28) from Eq. (3.26), and Eq. (3.29) from (3.27):

$$\widetilde{\boldsymbol{w}}_{j_a} = \eta_{iw} \widetilde{\boldsymbol{w}}_{j_{iw}} - \eta_{pw} \widehat{\boldsymbol{w}}_{j_{pw}} + \eta_{pw} \boldsymbol{w}_{j_{pw}}$$
(3.30)

$$\widetilde{\boldsymbol{b}}_{j} = \eta_{ib} \widetilde{\boldsymbol{b}}_{j_{ib}} - \eta_{pb} \widehat{\boldsymbol{b}}_{j_{pb}} + \eta_{pb} \boldsymbol{b}_{j_{pb}}$$
(3.31)

where $\widetilde{\boldsymbol{w}}_{j_{iw}} = \boldsymbol{w}_{j_{iw}} - \widehat{\boldsymbol{w}}_{j_{iw}}, \widetilde{\boldsymbol{b}}_{j_{ib}} = \boldsymbol{b}_{j_{ib}} - \widehat{\boldsymbol{b}}_{j_{ib}}.$

Substituting Eqs. (3.30) and (3.31) into Eq. (3.25) yields

$$\tilde{f}_{j} = \eta_{iw} \tilde{\boldsymbol{w}}_{j_{iw}}^{T} \hat{\boldsymbol{\varphi}}_{a} - \eta_{pw} \hat{\boldsymbol{w}}_{j_{pw}}^{T} \hat{\boldsymbol{\varphi}}_{a} + \eta_{ib} \tilde{\boldsymbol{b}}_{j_{ib}}^{T} \boldsymbol{X} - \eta_{pb} \hat{\boldsymbol{b}}_{j_{pb}}^{T} \boldsymbol{X} + \tilde{\boldsymbol{\sigma}}_{a}^{T} \boldsymbol{A} \hat{\boldsymbol{w}}_{j_{a}} + \tilde{\boldsymbol{c}}_{a}^{T} \boldsymbol{B} \hat{\boldsymbol{w}}_{j_{a}} + \varepsilon_{j_{f}}$$
(3.32)

where the term ε_{j_f}

$$\varepsilon_{j_f} = \eta_{pw} \boldsymbol{w}_{j_{pw}}^T \widehat{\boldsymbol{\varphi}}_a + \eta_{pb} \boldsymbol{b}_{j_{pb}}^T \boldsymbol{X} + \widehat{\boldsymbol{w}}_{j_a}^T \boldsymbol{H} + \widehat{\boldsymbol{w}}_{j_a}^T \widetilde{\boldsymbol{\varphi}}_a + \boldsymbol{w}_{j_{ia}}^T \boldsymbol{\varphi}_{ia}$$
(3.33)

is unknown since $\boldsymbol{w}_{j_{pw}}^{T}$, $\boldsymbol{b}_{j_{pb}}^{T}$, \boldsymbol{H} , $\boldsymbol{\tilde{\varphi}}_{a}$, $\boldsymbol{w}_{j_{ia}}^{T}$, $\boldsymbol{\varphi}_{ia}$ are all unknown. The detailed formulation of Eq. (3.32) and Eq. (3.33) is in Appendix B.3.

3.3.3 Self-Constructing Criteria

In this research a modification of the self-constructing criterion of EMRAN based on the principle of the growing and pruning criterion is developed. The growing part of the criterion is from the literature given in terms of two defined errors as follows (Er et al., 2010)

$$d_i = \|\boldsymbol{X}_i - \boldsymbol{c}_{win}\| > E_1 \tag{3.34}$$

$$e_{rmsi} = \sqrt{\sum_{j=i-(m-1)}^{i} (\frac{\|e_j\|}{m})^2} > E_2$$
 (3.35)

where d_i is the Euclidean distance (norm) between X_i and c_{win} , E_1 , E_2 are thresholds to be defined shortly. Equation (3.34) is to determine if the existing nodes are close to the observed center of c. Equation (3.35) compares the accumulated error of the past m outputs within a given threshold value with the goal of preventing over-fitting of the hidden nodes caused by noise. Values of E_1 and, E_2 are selected in the following manner following other researchers (Lu et al. 1998; Li et al. 2000; Er et al., 2010):

$$E_1 = \max[e_{max}\beta^i, e_{min}], 0 < \beta < 1$$
(3.36)

$$E_2 = \max[d_{max}\gamma^i, d_{min}], 0 < \gamma < 1$$
(3.37)

where *i* is the time index, e_{max} and e_{min} are the largest and smallest output error, respectively; d_{max} and d_{min} are the largest and smallest error distance from input to the observed center, respectively; β and γ are constants representing the scale of resolution.

A new node is added to the hidden layer when Eqs. (3.34) to (3.35) are both satisfied. The number of hidden nodes will increase to

$$h(i+1) = h(i) + 1 \tag{3.38}$$

The parameters of the new node are chosen:

$$\boldsymbol{w}_{new}^{i+1} = \boldsymbol{e}_{i}, \ \boldsymbol{c}_{new}^{i+1} = \boldsymbol{X}_{i}, \ \boldsymbol{\sigma}_{new}^{i+1} = \rho \| \boldsymbol{X}_{i} - \boldsymbol{c}_{win} \|$$
(3.39)

where the ρ is a positive constant.

The node pruning criterion developed in this research is based on the fact that a node far away from the input contributes little to the response because $(\mathbf{X} - \mathbf{c}_j)$ will be large and the j th wavelet function φ_j will be small per Eq. (3.14). The pruning strategy is defined as

$$\frac{\min\left(\varphi_{j}\right)}{\max\left(\varphi_{j}\right)} < \mathcal{E}_{3}, 1 \le j \le h(i)$$
(3.40)

where E_3 is the threshold for the pruning criterion. When Eq. (3.40) is satisfied, then the node with minimum φ_i will be discarded in the following time step.

3.4 Adaptive Law

In order to find the weights of active nodes an adaptive law or learning rule is needed. An adaptive law is developed in this research by a) obtaining the second derivative of the error vector \ddot{e} , b) determining \dot{s} , and c) selecting a Lyapunov function to ensure the stability of the control algorithm. The optimal control input is defined by u_w , and u_s is the compensator controller to compensate for the control input error. Substituting Eq. (3.12) and Eq. (3.13) into Eq. (3.2) yields

$$\ddot{\boldsymbol{e}} = \ddot{\boldsymbol{x}_d} - k_1 \dot{\boldsymbol{e}} - k_2 \boldsymbol{e} + \tilde{\boldsymbol{f}} + \widetilde{\boldsymbol{g}} \boldsymbol{u}_w + \boldsymbol{u}_s + \boldsymbol{d}$$
(3.41)

The complete formulation of Eq. (3.41) is available in Appendix B.4. The excitation vector is rewritten as $\boldsymbol{d}(t) = \boldsymbol{d}$ for ease of notation. Using Eqs. (3.41) and (3.32) and the definitions $\tilde{f}_j = f_j - \hat{f}_j$, $\tilde{\boldsymbol{g}}_j = \boldsymbol{g}_j - \hat{\boldsymbol{g}}_j$, the *j*th elements of the derivative

of the sliding surface in Eq. (3.6) can be rewritten as (see Appendix B.5. for detailed formulation):

$$\dot{s}_{j} = (\eta_{iw}\widetilde{w}_{j_{iw}}^{T}\widehat{\varphi}_{a} - \eta_{pw}\widehat{w}_{j_{pw}}^{T}\widehat{\varphi}_{a} + \eta_{ib}\widetilde{b}_{j_{ib}}^{T}X - \eta_{pb}\widehat{b}_{j_{pb}}^{T}X + \widetilde{\sigma}_{a}^{T}A\widehat{w}_{ja} + \widetilde{c}_{a}^{T}B\widehat{w}_{ja} + \varepsilon_{ja}^{T}A\widehat{y}_{ja} + \widetilde{c}_{a}^{T}B\widehat{y}_{ja} + \varepsilon_{ja}^{T}A\widehat{y}_{ja} + \varepsilon_{ja}^{T}A\widehat{y}_{j$$

A Lyapunov function is defined as follows to ensure the stability of the SCWNN model:

$$V_{1j} = \frac{1}{2} s_j^2 + \frac{\eta_{iw}}{2} \widetilde{w}_{j_{iw}}^T \widetilde{w}_{j_{iw}} + \sum_{k=1}^q (\frac{\eta_{i\theta}}{2} \widetilde{\theta}_{kj_{i\theta}} \widetilde{\theta}_{kj_{i\theta}}^T + \frac{1}{2\eta_\sigma} \widetilde{\sigma}_a^T \widetilde{\sigma}_a + \frac{1}{2\eta_c} \widetilde{c}_a^T \widetilde{c}_a) + \frac{1}{2\eta_\sigma} \widetilde{\sigma}_a^T \widetilde{\sigma}_a + \frac{1}{2\eta_c} \widetilde{\sigma}_a^T \widetilde{\sigma}_a + \frac{1}{2\eta_c} \widetilde{c}_a^T \widetilde{c}_a + \frac{\eta_{ib}}{2} \widetilde{b}_j^T \widetilde{b}_j$$
(3.43)

Equation (3.43) includes deliberately the error terms of all the WNN weights and parameters so that they all will converge to zero with no instability. The purpose of including the first term in the Lyapunov function is to ensure s_j converges to zero. Differentiating Eq. (3.43) and using Eq. (3.42) yields

$$\dot{V}_{1j} =$$

$$\eta_{iw}\widetilde{\boldsymbol{w}}_{iw}^{T}\left(s\widehat{\boldsymbol{\varphi}}_{a}+\dot{\widetilde{\boldsymbol{w}}}_{j_{iw}}\right)+\sum_{k=1}^{q}\left[\eta_{i\theta}\widetilde{\boldsymbol{\theta}}_{kj_{i\theta}}^{T}\left(s_{j}u_{k_{wnn}}\widehat{\boldsymbol{\varphi}}_{a}+\dot{\widetilde{\boldsymbol{\theta}}}_{kj_{i\theta}}\right)\right]+\eta_{ib}\widetilde{\boldsymbol{b}}_{j_{ib}}^{T}\left(s_{j}\boldsymbol{X}+\dot{\widetilde{\boldsymbol{b}}}_{j_{ib}}\right)+\\ \widetilde{\boldsymbol{\sigma}}_{a}^{T}\left[s_{j}\boldsymbol{A}\left(\widehat{\boldsymbol{w}}_{j_{a}}+\sum_{k=1}^{q}u_{k_{w}}\widehat{\boldsymbol{\theta}}_{kj_{a}}\right)+\frac{1}{\eta_{\sigma}}\dot{\widetilde{\boldsymbol{\sigma}}}_{a}\right]+\widetilde{\boldsymbol{c}}_{a}^{T}\left[s_{j}\boldsymbol{B}\left(\widehat{\boldsymbol{w}}_{j_{a}}+\sum_{k=1}^{q}u_{k_{wnn}}\widehat{\boldsymbol{\theta}}_{kj_{a}}\right)+\frac{1}{\eta_{c}}\dot{\widetilde{\boldsymbol{c}}}_{a}\right]-\\ s_{j}\eta_{pw}\widehat{\boldsymbol{w}}_{j_{pw}}^{T}\widehat{\boldsymbol{\varphi}}_{a}-s_{j}\eta_{pb}\widehat{\boldsymbol{b}}_{pb}^{T}\boldsymbol{X}-s_{j}\sum_{k=1}^{q}\left(\eta_{p\theta}\widehat{\boldsymbol{\theta}}_{kj_{p\theta}}^{T}\widehat{\boldsymbol{\varphi}}_{a}u_{k_{w}}\right)+s_{j}\varepsilon_{j}+s_{j}u_{j_{a}} \qquad (3.44)\\ \text{where }\varepsilon_{j}=\varepsilon_{j_{f}}+\sum_{k=1}^{q}\left(\varepsilon_{kj_{g}}u_{k_{w}}\right)+d_{j}, \text{ and the detailed formulation of Eq. (3.44) is\\ \text{available in Appendix B.6. Based on (3.44), the adaptive law can be obtained as follows}$$

$$\widehat{\boldsymbol{w}}_{j_{pw}} = s_j \widehat{\boldsymbol{\varphi}}_a, \quad \dot{\boldsymbol{w}}_{j_{iw}} = -\dot{\boldsymbol{w}}_{j_{iw}} = s_j \widehat{\boldsymbol{\varphi}}_a \tag{3.45}$$

$$\widehat{\boldsymbol{\theta}}_{kj_{p\theta}} = s_j u_{k_w} \widehat{\boldsymbol{\varphi}}_a, \ \dot{\widehat{\boldsymbol{\theta}}}_{kj_{i\theta}} = -\dot{\widetilde{\boldsymbol{\theta}}}_{kj_{i\theta}} = s_j u_{k_w} \widehat{\boldsymbol{\varphi}}_a \tag{3.46}$$

$$\widehat{\boldsymbol{b}}_{j_{pb}} = s_j \boldsymbol{X}, \quad \dot{\widehat{\boldsymbol{b}}}_{j_{ib}} = -\dot{\widehat{\boldsymbol{b}}}_{j_{ib}} = s_j \boldsymbol{X}$$
(3.47)

$$\dot{\widehat{\sigma}}_{a} = -\dot{\widetilde{\sigma}}_{a} = \eta_{\sigma} A \sum_{j=1}^{p} s_{j} \left(\widehat{w}_{j_{a}} + \sum_{k=1}^{p} u_{k_{w}} \widehat{\theta}_{k j_{a}}^{T} \right)$$
(3.48)

$$\dot{\hat{\boldsymbol{c}}}_{a} = -\dot{\tilde{\boldsymbol{c}}}_{a} = \eta_{c}\boldsymbol{B}\sum_{j=1}^{p}s_{j}\left(\hat{\boldsymbol{w}}_{j_{a}} + \sum_{k=1}^{p}u_{k_{w}}\hat{\boldsymbol{\theta}}_{kj_{a}}^{T}\right)$$
(3.49)

where
$$\boldsymbol{\theta}_{kj} \in \mathbb{R}^h$$
 from $\boldsymbol{\theta} = \begin{bmatrix} \boldsymbol{\theta}_{11} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\theta}_{22} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \boldsymbol{\theta}_{qp} \end{bmatrix} \in \mathbb{R}^{qh \times p}$ is a column vector with

length *h* and this vector is in the *k* th row and the $(jh - h + 1) \sim jh$ th column of matrix θ . The five Eqs (3.45) to (3.49) collectively define the adaptive law or learning rule developed in this research.

The derivative of the Lyapunov function is expressed as

$$\dot{V}_{1j} = s_j \eta_{pw} \widehat{\boldsymbol{w}}_{j_{pw}}^T \widehat{\boldsymbol{\varphi}}_a - s_j \eta_{pb} \widehat{\boldsymbol{b}}_{pb}^T \boldsymbol{X} - s_j \sum_{k=1}^p \left(\eta_{p\theta} \widehat{\boldsymbol{\theta}}_{jk_{p\theta}} \widehat{\boldsymbol{\varphi}}_a \boldsymbol{u}_{k_w} \right) + s_j \varepsilon_j + s_j \boldsymbol{u}_{a_j}$$
(3.50)

In Eq. (3.50), u_{a_j} is divided into two parts: $u_{a_j} = u_{a_{j_1}} + u_{a_{j_2}}$ where $u_{a_{j_1}}$ is

chosen to cancel the first three terms (the known part) on the right hand side of Eq. (3.50):

$$u_{a_{j_1}} = -\eta_{pw} \widehat{\boldsymbol{w}}_{j_{pw}}^T \widehat{\boldsymbol{\varphi}}_a + \eta_{pb} \widehat{\boldsymbol{b}}_{pb}^T \boldsymbol{X} + \sum_{k=1}^q \left(\eta_{p\theta} \widehat{\boldsymbol{\theta}}_{kj_{p\theta}} \widehat{\boldsymbol{\varphi}}_a u_{k_w} \right)$$
(3.51)

In that case the derivative of the Lyapunov function becomes $\dot{V}_{1j} = s_j(\varepsilon_j + u_{a_{j2}})$ where ε_j is unknown, and the inequity $\dot{V}_{1j} \leq 0$ still needs to be satisfied to ensure system stability. To achieve this, a compensation control term, $u_{a_{j2}}$, is obtained using fuzzy logic in the in next section.

3.5 Fuzzy Compensation Controller

Fuzzy logic (Zadeh, 1988) is used to develop a stable compensator controller or determine $u_{a_{j2}}$. In this research the sliding surface variable s_j is treated as a fuzzy variable. A triangular membership function with seven if-then rules are employed as shown in Figure 3.3 (this number of rules was observed to be sufficient). During the fuzzification these rules map s_j to the 7 × 1 vector of membership function μ_j .



Figure 3.3 The membership function of s_i

During the subsequent defuzzification membership function μ_j are mapped to the compensator control force $u_{a_{j2}}$. The defuzzification of the output $u_{a_{j2}}$ is obtained by the center-of-gravity method as follows:

$$u_{a_{j2}} = \sum_{i=1}^{7} r_{ji} \,\mu_{ji} = \mathbf{r}_{j}^{T} \,\boldsymbol{\mu}_{j} \tag{3.52}$$

where $\boldsymbol{\mu}_j = [\mu_{j1}, \mu_{j2}, ..., \mu_{j7}]^T$, the *i* th membership function $\mu_{ji} \ge 0$, $\sum_{i=1}^7 \mu_{ji} = 1$, r_{ji} is the coefficient of the *i* th membership function (i=1,7), and

 $\boldsymbol{r}_{j} = [-r_{j}, -\frac{2}{3}r_{j}, -\frac{1}{3}r_{j}, 0, r_{j}, \frac{2}{3}r_{j}, r_{j}]^{\mathrm{T}}.$ For ease of notation, define $\mu_{sum_{j}} = -\mu_{j1} - \frac{2}{3}\mu_{j2} - \frac{1}{3}\mu_{j3} + \frac{1}{3}\mu_{j5} + \frac{2}{3}\mu_{j6} + \mu_{j7}.$ Then, Eq. (3.52) can be rewritten as follows

$$u_{a_{j2}} = r_j \mu_{sum_j} \tag{3.53}$$

Note from Fig. 3 that s_j and μ_{sum_j} have the same sign, then $s_j \mu_{sum_j} =$

 $|s_j||\mu_{sum_j}|$, and Eq. (3.50) can be rewritten as follows

$$\dot{V}_{1j} = s_j \left(\varepsilon_j + r_j \mu_{sum_j} \right) \le \left| s_j \right| \left| \varepsilon_j \right| + s_j r_j \mu_{sum_j} = -\left| s_j \right| \left| \mu_{sum_j} \right| \left(-r_j - \left| \frac{\varepsilon_j}{\mu_{sum_j}} \right| \right)$$
(3.54)

From Eq. (3.54) it is concluded that the stability condition $\dot{V}_{1j} \leq 0$ is guaranteed

for any value of *r* that satisfies $r_j < -\left|\frac{\varepsilon_j}{\mu_{sum_j}}\right|$. Thus, the value of *r* is chosen as

$$r_j = -\left|\frac{\varepsilon_j}{\mu_{sum_j}}\right| - \delta \tag{3.55}$$

where δ is a positive constant. Since the value of r_j is unknown, Eq. (3.53) for compensator control force is written as

$$u_{a_{j2}} = \hat{r}_j \mu_{sum_j} \tag{3.56}$$

where \hat{r}_i is an estimated value of r_i and the estimation error is

$$\tilde{r}_j = r_j - \hat{r}_j \tag{3.57}$$

In order to find an adaptive law for estimation of r_j another Lyapunov function

including the estimation error \tilde{r} is defined as

$$V_{2j} = V_{1j} + \frac{1}{2\eta_r} \tilde{r}_j^2 \tag{3.58}$$

where η_r is a positive constant. Differentiating (3.58) with respect to time yield

$$\dot{V}_{2j} \le \tilde{r}_j \left(-s_j \mu_{sum_j} + \frac{1}{\eta_r} \dot{\tilde{r}}_j \right) + |s_j| |\varepsilon_j| + r_j s_j \mu_{sum_j}$$
(3.59)

To cancel the two terms in the bracket on right hand side of Eq. (3.59), the adaptive law of r_j is chosen as

$$\dot{\hat{r}}_j = -\dot{\tilde{r}}_j = -\eta_r s_j \mu_{sum_j} \tag{3.60}$$

Substituting Eqs. (3.55) and (3.60) and the equation of $s_j \mu_{sum_j} = |s_j| |\mu_{sum_j}|$ into Eq. (3.59) yields

$$\dot{V}_{2j} = |s_j||\varepsilon_j| + \left(-\left|\frac{\varepsilon_j}{\mu_{sum}}\right| - \delta\right)|s_j||\mu_{sum_j}| = -\delta|s_j||\mu_{sum_j}| \le 0$$
(3.61)

Inequality (3.61) together with Eq. (3.50) show the bounded property of the function parameters of \tilde{w}_{iw} , \tilde{b} , $\tilde{\sigma}_a$, \tilde{c}_a , \tilde{r} , and the *s* satisfies

$$\lim_{t \to \infty} (s_j) = 0, \ 1 \le j \le p \tag{3.62}$$

Equation (3.62) ensures the stability of the proposed SCWNN based control

approach. So from (3.51) and (3.53), the compensator control input u_{a_j} is further written as

$$u_{a_{j}} = u_{a_{j1}} + u_{a_{j2}} = -\eta_{pw} \widehat{\boldsymbol{w}}_{j_{pw}}^{T} \widehat{\boldsymbol{\varphi}}_{a} + \eta_{pb} \widehat{\boldsymbol{b}}_{j_{pb}}^{T} \boldsymbol{X} + \sum_{k=1}^{q} \left(\eta_{p\theta} \widehat{\boldsymbol{\theta}}_{k j_{p\theta}} \widehat{\boldsymbol{\varphi}}_{a} u_{k_{w}} \right) + \widehat{r}_{j} \mu_{sum_{j}}$$

$$(3.63)$$

3.6 Example of An Unknown Nonlinear Highway Bridge

3.6.1 Physical Model of Constructed 91/5 Overcrossing Bridge

The proposed nonlinear control approach is studied through a benchmark control problem created by Agrawal et. al, (2009) for the newly constructed 91/5 overcrossing bridge located in Orange County in Southern California, a continuous cast-in-place prestressed concrete box-girder bridge as shown in Figure 3.4.



Figure 3.4 View of the 91/5 highway bridge

The four-lane highway bridge has two spans, each 58.5m (192 ft) long, with two abutments skewed at 33°. The width of the deck is 12.95m (42.5 ft) along the east direction and 15m (49.2 ft) along the west direction. The cross section of the deck consists of three concrete box cells. The deck is supported by a 31.4m (103 ft) long and 6.9m (22.5 ft) high prestressed outrigger, which rests on two pile groups, each consisting of 49 driven concrete friction piles. The columns are approximately 6.9m (22.5 ft) high. Eight bearings between bridge deck and abutments are used to isolate the bridge superstructure at both abutment-ends.

3.6.2 3D Finite-Element Model of the Bridge

Agrawal et. al, (2009) developed a 3D finite-element model of the full-scale highway over-crossing in MATLAB plotted in Figure 3.5. They model the soil–structure interaction by equivalent springs and dashpots. The bilinear hysteresis force–deformation relationship is considered by modeling the nonlinear behavior of center columns and the eight isolators. Ground motions from six different earthquakes are applied in two directions simultaneously.



Figure 3.5 3-D finite element model of the bridge

The whole nonlinear model of the bridge has 430 DOF. A nonlinear structural analysis tool of MATLAB has been developed and made available for nonlinear dynamic analysis. Control devices are installed between the deck and the end abutments of the nonlinear bridge. Evaluation criteria and control constraints are specified for the design of controllers (Agrawal et. al, 2009).

3.6.3 Placement of Sensors and Actuators

In this example, 16 control devices are placed at the two ends of the deck between the abutments and the bridge deck, 8 at each end (4 in the horizontal and 4 in the vertical direction), the same as the sample control problem presented by Tan and Agrawal et al. (2008) and shown in Figure 3.6.



Figure 3.6 Distribution of actuators and sensors of the bridge

3.6.4 Numerical Results

Agrawal et al. (2009) define 21 criteria for the benchmark problem. Table 3.1 shows a comparison of the evaluation criteria using the control algorithm presented in this chapter and the LQG algorithm provided by Agrawal et al. (2009) for six different earthquake records. The smaller the number the more effective the control algorithm. Parameters $e_{max} = 0.05$, $e_{min} = 0.001$, $d_{max} = 0.05$, $d_{min} = 0.005$, $\beta = 0.97$, and $\gamma = 0.97$ are selected following Er et al. (2010). The other parameters are chosen as follows: sampling time $T_s = 0.005$ sec, simulation period $T_t = 100$ sec, initial number of hidden nodes h(0) = 2, m = 5, $E_3 = 0.01$, $\rho = 0.8$, $k_1 = 100$, $k_2 = 100$, $\eta_s = 0.001$, $\eta_{\sigma} = 0.001$, $\eta_c = 0.001$, $\eta_{pw} = 0.001$, $\eta_{iw} = 0.01$, $\eta_{pb} = 0.001$, $\eta_{ib} = 0.01$, $\eta_{pv} =$ 0.001, $\eta_{iv} = 0.01$, $\eta_r = 10$.

Criteria	Control Approach	N. Palm	Chi-Chi	EI Centro	North- ridge	Turkey	Kobe
I	LQG	0.9502	0.8776	0.7905	0.8965	0.9121	0.7888
J_1	SCWNN	0.9659	0.6963	0.8716	0.8945	0.8816	0.8248
T	LQG	0.7699	0.9666	0.7425	0.9782	0.9779	0.7040
J ₂	SCWNN	0.7382	0.9632	0.7981	0.9772	0.9737	0.6200
T	LQG	0.8231	0.7993	0.7791	0.8669	0.746	0.7045
J_3	SCWNN	0.8124	0.6531	0.8768	0.8652	0.704	0.6699
L	LQG	0.7941	0.8753	0.8829	0.8435	0.7983	0.8986
J_4	SCWNN	0.8783	0.9557	0.9828	0.9795	0.8988	0.9238
T	LQG	0.9370	0.8027	0.6433	0.8826	0.7144	0.5862
J_5	SCWNN	0.6015	0.6046	0.5112	0.8449	0.6601	0.3539
T	LQG	0.7699	0.7433	0.7425	0.8516	0.4626	0.7040
J ₆	SCWNN	0.7382	0.5114	0.7981	0.8200	0.3192	0.6200
J ₇	LQG	0	0.5119	0	0.6244	0.3317	0
	SCWNN	0	0.1114	0	0.4981	0.1655	0
I.	LQG	0	0.6667	0	1	0.3333	0
J ₈	SCWNN	0	0.5000	0	nidge Turkey 0 0.8965 0.9121 0 0.8945 0.8816 0 0.9782 0.9779 0 0.9782 0.9779 0 0.9772 0.9737 0 0.8669 0.746 0 0.8652 0.704 0 0.8435 0.7983 0 0.8435 0.7983 0 0.8435 0.7983 0 0.8435 0.7983 0 0.8435 0.7983 0 0.8435 0.7983 0 0.8449 0.6601 5 0.8516 0.4626 1 0.8200 0.3192 0.6244 0.3317 0.4981 0.4981 0.1655 1 1 0.3333 1 0.8673 0.8937 0 0.7764 0.8171 3 0.8780 0.5316 2 0.7672 0.4737	0	
T	LQG	0.7426	0.8856	0.6757	0.8673	0.8937	0.7389
J ₉	SCWNN	0.8440	0.6653	0.6039	0.7764	0.8171	0.6880
I	LQG	0.6964	0.8329	0.6433	0.8780	0.5316	0.7127
J_{10}	SCWNN	0.7229	0.6804	0.5782	0.7672	0.4737	0.6334
I	LQG	0.7033	0.7833	0.6563	0.8047	0.6071	0.7293
J ₁₁	SCWNN	0.7770	0.5629	0.5958	0.7610	0.5569	0.6674
I	LQG	0.7233	0.7910	0.6852	0.7956	0.7946	0.7976
J ₁₂	SCWNN	0.7535	0.7416	0.8617	0.8343	0.9189	0.9229

Continued

Table 3.1 A Comparison of SCWNN based control and sampled LQG control of the benchmark bridge

Criteria	Approach	N. Palm	Chi-Chi	EI Centro	North- ridge	Turkey	Kobe
T	LQG	0.4829	0.7821	0.4844	0.8211	0.5210	0.4720
J ₁₃	SCWNN	0.3653	0.5323	0.4148	0.7536	0.3748	0.2566
7	LQG	0.6964	0.6270	0.6433	0.8274	0.2388	0.7127
J_{14}	SCWNN	0.7229	0.6763	0.5782	0.9552	0.1652	0.6334
T	LQG	0.0101	0.0238	0.0057	0.0230	0.0147	0.0079
J_{15}	SCWNN	0.0241	0.0241	0.0241	0.0241	0.0241	0.0241
T	LQG	0.9019	0.7686	0.5916	0.8039	0.7082	0.5779
J_{16}	SCWNN	0.5790	0.5789	0.4701	0.7696	0.6544	0.3489
Ŧ	LQG	0.0512	0.1092	0.0213	0.1105	0.0664	0.0356
J ₁₇	SCWNN	0.0506	0.1153	0.0440	0.1119	0.0770	0.0392
I	LQG	0.0119	0.0150	0.0032	0.0150	0.0136	0.0064
J ₁₈	SCWNN	0.0118	0.0159	0.0066	CentroridgeTurkey0.48440.82110.52100.41480.75360.37480.64330.82740.23880.57820.95520.16520.00570.02300.01470.02410.02410.02410.59160.80390.70820.47010.76960.65440.02130.11050.06640.00320.01500.01360.00660.01520.0158161616121212323232282828N/AN/AN/A	0.0158	0.0064
T	LQG	16	16	16	16	16	16
J ₁₉	SCWNN	16	16	16	16	16	16
T	LQG	12	12	12	12	12	12
J ₂₀	SCWNN	32	32	32	32	32	32
I	LQG	28	28	28	28	28	28
J21	SCWNN	N/A	N/A	N/A	N/A	N/A	N/A

Table 3.1 continued

It is observed that the maximum base shear J_1 has been reduced in case Chi-Chi, Northridge and Turkey earthquake. The performance of SCWNN based control shows a better performance in the aspect of J_2 , J_3 , J_5 , J_6 , J_8 , J_{10} , J_{11} , J_{13} , J_{14} , and J_{16} in reducing structural displacement, moment, shear, and device stroke consistently in most of the five different earthquakes (see Appendix C for detail description of those criteria). Figure 3.7 and Figure 3.8 show the displacement at the position of the second and seventh sensors
(in both x- and y- direction), respectively. It is found that the displacement has been reduced significantly compared with that without control or with LQG control. Maximum displacement, moment, and shear in the structure and maximum displacement and stroke of the device are substantially smaller than the corresponding values for the LQG controller. The acceleration in the middle of the span is slightly higher than that of the sample LGG control. Two explanations are provided. First, there is no control device in the middle span of the bridge in the benchmark problem. Second, given structural properties including mass, stiffness, and damping ratios are used in the benchmark problem which are required in classical control algorithms such as LQG. The proposed model does not rely on known structural properties and does not use any of those properties. In addition, the time history of the wavelet node is shown in Figure 3.9.



Figure 3.7 Bridge displacements at the position of the second sensor under Kobe Earthquake: (a) x-direction; and (b) y-direction.



Figure 3.8 Bridge displacements at the position of the seventh sensor under Kobe Earthquake: (a) x-direction; and (b) y-direction.



Figure 3.9 Variance of wavelet node number h(t)

3.7 Conclusion

An adaptive and robust control algorithm for nonlinear vibration control of large structures subjected to dynamic loading was presented through integration of a selfconstructing wavelet neural network with an adaptive fuzzy sliding mode control approach. It is particularly suitable when structural properties are unknown or change during the dynamic event which is the case for civil structures subjected to dynamic loading. In other words, the proposed control model has the advantages of not requiring accurate mathematical model of the controlled structure and good adaptive ability to the changes of structural parameters and external dynamic loading. The robustness of the proposed algorithm is achieved by deriving a set of adaptive laws for determining the unknown parameters of wavelet neural networks using two Lyapunov functions. No offline training of neural network is necessary for the system identification process. In addition, the earthquake signals are considered as unidentified. This is particularly important for on-line vibration control of large civil structures since the external dynamic loading due to earthquake is not available in advance. Because of these advantages, the proposed adaptive control algorithm is especially effective and implementable for vibration control of large civil structures.

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Chapter 4: Tuning Algorithms for Chattering Reduction in System Control

4.1 Introduction

Sliding model control (SMC) has been used widely for control of both linear and nonlinear systems, especially when the system is unknown or partially known. Consider a simple system defined by the differential equation $m\ddot{x} + c\dot{x} + kx = -m\ddot{x_g} + u$ where x is the system state, m, k, and c are physical parameters of the system that are functions of the state variables, $\ddot{x_g}$ is the disturbance, and u is control input which aims to drive the state to some desired trajectory (usually zero). The state variables are $x_1 = x$, $x_2 = \dot{x}$. Their derivatives make typical second order steady-space equations:

$$\dot{\mathbf{x}}_1 = \mathbf{x}_2 \tag{4.1}$$

$$\dot{\mathbf{x}}_2 = \mathbf{f}(\mathbf{x}_1, \mathbf{x}_2) + \mathbf{u}$$
 (4.2)

where function $f(x, \dot{x}) = m^{-1}(-c\dot{x} - kx - m\ddot{x}_g)$. Assume the sliding surface to be $s = x_2 + ax_1=0$ where the constant is required to be positive, i.e., a > 0. In that case, $x_2 = -ax_1$. Substituting this value in Eq. (4.1) yields $\dot{x}_1 = -ax_1$, with solution $x_1 = e^{-at}$, which means $\lim_{t\to\infty} x_1 = 0$, and $\lim_{t\to\infty} x_2 = \lim_{t\to\infty} \dot{x}_1 = 0$. Consequently, $\lim_{t\to\infty} {x_1 = 0}$. The larger the value of a the factor the convergence of the system.

 $\lim_{t\to\infty} {X_1 \brack X_2} = 0$. The larger the value of a the faster the convergence of the system response to zero.

Taking the time derivative of the sliding surface yields

$$\dot{s} = \dot{x}_2 + a\dot{x}_1 = f(x_1, x_2) + ax_2 + u$$
 (4.3)

In order to achieve a sliding surface defined by $s = x_2 + ax_1 = 0$, the control force, u, is chosen such that the sign of \dot{s} is always the opposite of the sign of s:

$$\mathbf{u} = -\mathbf{M}(\mathbf{x}_1, \mathbf{x}_2)\mathbf{sgn}(\mathbf{s}) \tag{4.4}$$

where the sliding gain function M is chosen to satisfy $M(x_1, x_2) > |f(x_1, x_2) + ax_2|$, and sgn(s) is a sign function of s. Substituting for u from Eq. (4.4) into Eq. (4.3) yields:

$$\dot{s} = -M(x_1, x_2)sgn(s) + f(x_1, x_2) + ax_2$$
 (4.5)

Since $M(x_1, x_2) > |f(x_1, x_2) + ax_2|$, the term $-M(x_1, x_2)$ sgn(s) determines the

sign of \dot{s} and ultimately the value of s. Equation (4.5) guarantees the sign of \dot{s} is always opposite of the sign of s, and $\lim_{t\to\infty} s = 0$ as it is desired, and the discontinuous Eq. (4.5) ensures the exponential decay of |s|.

4.2 A Tuning Algorithm for the Sliding Gain Function

Traditionally, a constant value has been selected for the sliding gain function $M(x_1, x_2)$ (Utkin, 1992; Khalil, 1996), usually the absolute value of the lower and upper boundaries of the unknown system factors including external disturbance, noise, and sensor and actuator dynamics. Such a constant M is normally large and will force the trajectory in the phase plane (x_1, x_2) into the desired sliding surface (s = 0). However, an unavoidable oscillation, called chattering, occurs after the trajectory reaches the sliding surface. This chattering in SMC is undesirable, for example, it may cause the loss of power in the control of DC-DC convertors (Lee et al., 2009). This chattering generally depends on the sampling time (T_s) , sliding gain function (M), external disturbance, noise, and sensor or actuator dynamics (the last two are referred to as unmodeled dynamics).

Several approaches have been proposed in the literature in order to reduce the chattering. They are mentioned here briefly.

- 1) Choose a small sampling time such as $T_s = 0.01$ or 0.001sec. However, T_s cannot be too small due to hardware limitation and excessive computational time requirement.
- 2) Reduce the effect of system uncertainty through system identification, noise estimation, and disturbance measurement, etc. These methods help reduce the M value and consequently the chattering since the boundary value of the system uncertainty is reduced. However, this method is indirect and requires data training in the identification process.
- 3) Replace the discontinuous sgn(s) function with a discontinuous saturation

function sat(s) = $\begin{cases} s, & \text{if } |s| < \gamma \\ sgn(s), & \text{if } |s| \ge \gamma \end{cases}$ (Slotine and Sastry, 1983) where γ is a small positive constant. However, this strategy may lead to unstable solutions (Utkin et al., 1999). Another replacement includes hysteresis loop which also improves system stability (Nguyen and Lee, 1995; Aroudi et al., 2005) but its use is more complicated than the saturation function.

4) Replace the constant M with function $M(x_1, x_2)$ or M(s). Since $M(x_1, x_2)$ in Eq. (4.4) is a function of x_1 and x_2 , and s is a function of (x_1, x_2) , $M(x_1, x_2)$ and M(s) are related and their tuning methods are similar. The idea of this method is to adjust the time-varying gain $M(x_1, x_2)$ or M(s) such that it will respond to the varying boundary of unknown factors in the system properly. A tuning algorithm based on this idea is especially useful when disturbance and unmodeled dynamics are included in the system.

Using the idea of the last approach, Lee and Utkin (2007) proposed a "statedependent" and "equivalent-control-dependent gain" approach in the form $M(\mathbf{x}, \mathbf{s}) = M_0(\sigma + \delta)$ where M_0 and δ are positive constants, and the function $\sigma(\mathbf{x}) = ||\mathbf{x}||$ for state-dependent method and $\sigma(\mathbf{s}) = \operatorname{ave}[\operatorname{sgn}(\mathbf{s})]$ [the average value of the time series $\operatorname{sgn}(\mathbf{s})$ achieved through a low pass filter] for equivalent-control-dependent gain approach. These two approaches usually work since δ is much smaller than the boundary value of M, i.e., the sliding gain has been reduced to a smaller value which will always reduce chattering.

In this article, a time-varying method is proposed for determining the sliding gain function inspired by the aforementioned state-varying approach. Since x is a function of time it is more straightforward to use a time-dependent rather than a state-dependent gain function. The idea is to adjust M(t) rather that $M(x_1, x_2)$ or M(s) such that M(t) will respond to the varying boundary of the unknown factors in the system. Two alternative tuning algorithms are proposed for reducing M(t) for systems with and without sensor dynamics, noise, and external disturbance. The first algorithm is for systems with no noise and disturbance but with or without unmodeled dynamics. The second algorithm is for systems with noise, disturbance, unmodeled dynamics, or any combination of them. Compared with the state-dependent, equivalent-control-dependent, and hysteresis loop methods, the proposed algorithms are more straightforward and easy to implement. They will be introduced for five different cases.

4.3 Algorithm I for Systems with Unmodeled Dynamics Only

For a system without any sensor dynamics, noise, and disturbance, the selected value of M in SMC will be too large after the sliding surface (s = 0) is reached. Hence, the value of M should be reduced after the sliding surface is reached in order to reduce the chattering and ensure that a sliding mode always exists along the sliding surface. To achieve this, the following tuning algorithm is proposed for M(t):

1) If sign[s(i)] = sign[s(i - 1)], then choose M(i) = M(i - 1); else select

$$M(i) = \frac{M(i-1)}{rate};$$

- 2) Select sign[s(i)] = sign[s(i 1)];
- 3) Go to the next time step and go to step 1.

where the index i indicates the time $t = iT_s$, and the coefficient rate > 1 is a constant value. It should be noted that M(i) will always decrease in time and therefore a large initial value should be selected for M(i) to ensure the existence of a sliding surface.

4.3.1 Case 1: For Systems Without Sensor Dynamics, Noise, and Disturbance

The efficacy of the proposed algorithm is shown via an example using the values: m = 1, k = 0.01, c = 0.001, and a = 1. The following arbitrary non-zero initial states are selected: $x_1(0) = 14, x_2(0) = 25$ (the same values are used for all the examples presented in this chapter).

For all examples, the sampling time is chosen as $T_s = 0.01$ sec which is small enough to keep the chattering within an acceptable range. For a constant M, it is chosen as M = 100 after several trials (a larger value creates a greater chattering problem and a smaller value cannot achieve a sliding mode). For the case of the time-varying M(i), the decreasing rate coefficient is chosen rate = 1.1, and the initial values of sign[s(i)] and M(i) are selected as sign[s(0)] = 0 and M(0) = 100.

Figure 4.1 represents the phase plane (x_1, x_2) using a constant value of M = 100. A close-up of Figure 4.1 is presented in Figure 4.2 in order to show the chattering phenomenon clearly. Figure 4.3 and Figure 4.4 are time histories of state variable x_1 and the control input u. The high-frequency switching of u leads to a condense plot in Figure 4.4 which is undesirable for practical applications because of actuator response limitation A close up of Figure 4.4 is shown in Figure 4.5, similar observations are made in Figure 4.13, Figure 4.22, Figure 4.32, Figure 4.41, and Figure 4.50 for other cases to be discussed. Figure 4.6 shows the phase plane (x_1, x_2) using a time-varying M with an initial value 100, the same as for constant M. A close-up of Figure 4.6 is presented in Figure 4.7. Figure 4.8 and Figure 4.9 show time histories of the state variable x_1 and the control input u for the case of time-varying M. A comparison of the results shown in Figure 4.2 and Figure 4.7 indicates that the proposed tuning algorithm reduces the chattering by about 95%. In addition, a comparison of Figure 4.4 and Figure 4.9 shows the frequent change of the control input is reduced significantly, and the norm value of the control input is reduced from 6335 to 672 which means the energy required by actuators has been reduced substantially. The average value of sign(s) is 0.1032 for timevarying M compared with 0.0027 for a constant M = 100. The increase for time-varying M means sign(s) does not change as frequently as in the case of constant M, which is why the chattering is reduced through the proposed tuning algorithm.

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Figure 4.1 Phase plane using a constant M = 100 (Case 1)



Figure 4.2 A close-up of the phase plane in Figure 4.1 (Case 1)



Figure 4.3 Time series of the state variable x1 using a constant M=100 (Case 1)



Figure 4.4 Time series of the control input using a constant M=100 (Case 1)



Figure 4.5 A close-up of Figure 4.4 (Case 1)



Figure 4.6 Phase plane using a time-varying M (Case 1)



Figure 4.7 A close-up of the phase plane in Figure 4.6 (Case 1)



Figure 4.8 Time series of the state x1 using a time-varying M (Case 1)



Figure 4.9 Time series of control input using a time-varying M (Case 1)

4.3.2 Case 2 For Systems with Sensor Dynamics but no Noise and Disturbance

The purpose of this exercise is to study the effect of sensor dynamics on the system performance. Consider sensor dynamics with the transfer function: $G(\omega) = \frac{1}{(\mu\omega+1)^2}$ where ω is the variable in the frequency domain. This transfer function is another form of differential equations in frequency domain where $\omega Y = \dot{Y}$ for variable Y with Y(0) = 0. The unknown time constant is selected as $\mu = 0.8 \times T_s = 0.008$ sec for simulation purpose. The same values are selected for the rate ratio and initial value of M(t) as in the previous case (without any of sensor dynamic, noise, and disturbance).

Results using constant M and time-varying M(t) are shown in Figure 4.10 to Figure 4.17. A comparison of Figs. 2 and 10 shows the chattering effect has significantly in this case which indicates that the sensor dynamics has a greater influence on the system performance than that of the sampling time. An explanation for this is that the sensor dynamics (or in general, the second order unmodeled dynamics) behaves as an undesirable filter and filters out the discontinuous control force u in Eq. (4), and transforms it into a continuous function. This will work against the key idea in SMC and against Eq. (4). The chattering in this case is reduced by nearly 90% using M(t) as shown in Figure 4.15 compared with the case with a constant M = 100. The frequent change of the control input is also reduced significantly as shown in Figure 4.17 compared with Figure 4.13 for the constant M case. The average value of sign(s) for time-varying case is 0.0745 significantly larger than 0.0075 for constant M case. The norm value of the control input is reduced from 6324 to 765.



Figure 4.10 Phase plane using a constant M = 100 (Case 2)



Figure 4.11 A close-up of the phase plane in Figure 4.10 (Case 2)



Figure 4.12 Time series of displacement using a constant M = 100 (Case 2)



Figure 4.13 Time series of control input using a constant M = 100 (Case 2)



Figure 4.14 Phase plane using a time-varying M (Case 2)



Figure 4.15 A close-up of the phase plane in Figure 4.14 (Case 2)



Figure 4.16 Time series of displacement using a time-varying M (Case 2)



Figure 4.17 Time series of control input using a time-varying M (Case 2)

4.4 Algorithm II for Systems with Noise, Disturbance, and Unmodeled Dynamics

In the previous two cases, since the mathematical function representing the sensor dynamics (and correspondingly its boundary value) will not change with time, the gain function M(t) always decreases and the sliding mode will never disappear once it appears in some finite time. However, it is not the case when noise or disturbance is included in the system. The reason is that the boundary of the unknown factors (noise or disturbance) is time-varying and of course may increase with time. As a result, the gain function M(t) should increase accordingly to maintain the sliding mode when it appears. Based on these consideration, a second algorithm for selecting M(i) is developed as follows:

1) Select $M(i) = M(i - 1) + \alpha T_s$, while satisfying $M_1 \le M(i) \le M_2$;

- 2) If sign[s(i)] = -sign[s(i-1)], then select M(i) = M(i-1) βT_s while satisfying M₁ \leq M(i) \leq M₂; else go to step 3);
- 3) Select sign[s(i)] = sign[s(i 1)];
- 4) Go to next time step and repeat step 1.

The parameters M_1 and M_2 are the positive lower and upper bounds of M(i), respectively. The coefficient α and the jump factor β for the adjustment of M(i) should satisfy $0 < \alpha < \beta$.

In all four cases 3 to 6 (to be presented shortly), M = 300 is used for constant M. For the case of time-varying M(t), $M_1 = 1$ and $M_2 = 300$, and the initial values of sign[s(0)] = 0 and M(0) = 150 are selected.

4.4.1 Case 3 For Systems with Noise Only

A Gaussian white noise with a standard normal distribution (Figure 4.18) is used in the controlled system for illustration purpose. The values of $\alpha = 10$ and $\beta = 300$ are selected for the coefficients of the algorithm in this case. Results using constant M and time-varying M(t) are shown in Figure 4.19 to Figure 4.26. Although the sliding surface is achieved for the case of M = 100, as shown Figure 4.19, there is considerable chattering (Figure 4.20) which is reduced by about 95% by the proposed algorithm as shown in Figure 4.24. In addition, the norm value of the control force has been reduced to 153, from 18976 for the case of M = 100.



Figure 4.18 Time series of noise



Figure 4.19 Phase plane using a constant M = 300 (Case 3)



Figure 4.20 A close-up of the phase plane in Figure 4.19 (Case 3)



Figure 4.21 Time series of state x1 using a constant M = 300 (Case 3)



Figure 4.22 Time series of control input using M = 300 (Case 3)



Figure 4.23 Phase plane using a time-varying M (Case 3)



Figure 4.24 A close-up of the phase plane in Figure 4.23 (Case 3)



Figure 4.25 Time series of state x1 using a time-varying M (Case 3)



Figure 4.26 Time series of control input u using a time-varying M (Case 3)



Figure 4.27 Time series of time-varying M (Case 3)

4.4.2 Case 4: For System with External Excitation

Mathematically, external excitations such as those from earthquake and wind can be treated as one kind of noise, since their average value tends to be very close to zero. For example, the average value of the North-South component of the 1940 EI Centro, California, Earthquake in (Figure 4.28) is 2.57×10^{-6} g (g is the acceleration due to gravity), very small compared with its peak value 0.215 g. Such excitations are especially important in the civil/structural engineering domain. For this reason, it is studied separately from the general noise studied in the previous section. The 1940 EI Centro accelerogram (Figure 27) is selected as an example excitation. The same values of $\alpha = 10$ and $\beta = 300$ used in Case 3 are used in this case. Results using constant M and time-varying M(t) are shown in Figure 4.29 to Figure 4.36. The chattering is reduced by more than 95% (Figure 4.34) by using the second algorithm. In addition, the norm value of the control force to 905, from 18976 for the case of M = 300.



Figure 4.28 Time history of disturbance (1940 EI Centro earthquake, NS-direction)



Figure 4.29 Phase plane using M = 300 (Case 4)



Figure 4.30 A close-up of the phase plane in Figure 4.29 (Case 4)



Figure 4.31 Time series of displacement using a constant M = 300 (Case 4)



Figure 4.32 Time series of control input using M = 300 (Case 4)



Figure 4.33 Phase plane using a time-varying M (Case 4)


Figure 4.34 A close-up of the phase plane in Figure 4.33 (Case 4)



Figure 4.35 Time series of displacement using a time-varying M (Case 4)



Figure 4.36 Time series of control input using a time-varying M (Case 4)



Figure 4.37 Time series of time-varying M (Case 4)

4.4.3 Case 5: For Systems with Disturbance

General noise and the earthquake external excitation generally have zero average values (or close to zero). In order to evaluate the performance of the second tuning algorithm for systems with an external disturbance with a non-zero average value, a disturbance in the form 25[sin (t) + 1] is considered. The values of $\alpha = 10$ and $\beta = 100$ are selected for the coefficients of the algorithm in this case. Results using constant M and time-varying M(t) are shown in Figure 4.38 to Figure 4.45.

The chattering is reduced by more than 70% (Figure 4.42 and Figure 4.43) by using the second algorithm. In addition, the norm vale of the control force has been reduced to 2491, from 18976 for the case of M = 300. Similar reductions of chattering in the order of 70% are obtained when noise, external excitation, and disturbance exist. The results are not shown here due to space limitations.



Figure 4.38 Phase plane using a constant M = 300 (Case 5)



Figure 4.39 A close-up of the phase plane in Figure 4.38 (Case 5)



Figure 4.40 Time series of displacement using a constant M = 300 (Case 5)



Figure 4.41 Time series of control input using a constant M = 300 (Case 5)



Figure 4.42 Phase plane using a time-varying M (Case 5)



Figure 4.43 A close-up of the phase plane in Figure 4.42 (Case 5)



Figure 4.44 Time series of displacement using a time-varying M (Case 5)



Figure 4.45 Time series of control input using a time-varying M (Case 5)



Figure 4.46 Time series of time-varying M (Case 5)

4.4.4 Case 6: For Systems with Sensor Dynamics, Noise, and Disturbance

In this case the same sensor dynamics of Case 2, Gaussian white noise of Case 3, external excitation of Case 4 (El Centro record), and non-zero disturbance of Case 5 are used. The values of $\alpha = 10$ and $\beta = 300$ are selected for the coefficients of the algorithm in this case. Results using constant M and time-varying M(t) are shown in Figure 4.47 to Figure 4.55. In this case the amount of reduction in chattering varies from about 25% in the range $7 < x_1 < 10$ to 50% in the range $0 < x_1 < 7$ in the phase plane (Figure 4.47, Figure 4.48, Figure 4.51, and Figure 4.52) by using the second algorithm. In addition, the norm vale of the control force has been reduced to 2623, from 9487 for the case of M = 300. Similar reductions of chattering in the order of 70% are obtained when noise, external excitation, and disturbance exist.



Figure 4.47 Phase plane using M = 300 (Case 6)



Figure 4.48 A close-up of the phase plane in Figure 4.47 (Case 6)



Figure 4.49 Time series of displacement using M = 300 (Case 6)



Figure 4.50 Time series of control input using M = 300 (Case 6)



Figure 4.51 Phase plane using M(t) (Case 6)



Figure 4.52 A close-up of the phase plane using M(t) (Case 6)



Figure 4.53 Time series of displacement using M(t) (Case 6)



Figure 4.54 Time series of control input using M(t) (Case 6)



Figure 4.55 Time series of displacement using M(t) (Case 6)

A 25% to 50% reduction of chattering in this case is not as large as the 70% to 95% reduction achieved in the previous five cases. An explanation for this difference is provided here. When only unmodeled (sensor and actuator) dynamics are considered, M(t) should always have an initial value M(0) that is large enough to suppress the boundary of the unknown dynamics and then decease with time in order to reduce the mismatch between a discontinuous control force and a continuous one as explained in Case 2. M(t) will never increase in such a case. When noise and/or disturbance are considered (Cases 3 to 5) a large initial value M(0) of is not necessary and M(t) may increase or decrease depending on the boundary variation of the noise and/or disturbance. As a result, these distinct characteristics and interactions of unmodeled dynamics and noise/disturbance impair the efforts to reduce chattering.

4.5 Conclusion

SMC is among the popular approaches for control of systems, especially for unknown nonlinear systems. However, the chattering in SMC is generally a problem that needs to be resolved for better control. In this article, two tuning algorithms are proposed to reduce such chattering considering unknown sensor dynamics, general noise, external excitation, and disturbance.

For systems with unknown sensor dynamics, the boundary of sensor dynamics will not increase with time, and therefore it is not necessary to increase the gain M(t). Hence, the chattering and control force can be reduced significantly due to the decrease of M(t) and the increase of the average value of sign(s) using the first tuning algorithm. For systems with noise and disturbance, the boundary of system unknowns is time-varying and may increase or decrease with time. As a result, the first tuning algorithm does not work well for this situation. However, it is important to note that the first tuning algorithm is especially suitable for systems with unmodel dynamics only. For example, the DC motor with sensors in Samoylenko et al. (2008).

A second tuning algorithm is proposed for system with general noise, external excitation, or disturbance. The results of Cases 3 to 5 show that the chattering and control input are both reduced significantly in the order of 70 to 95%. For systems with sensor dynamics, noise, and disturbance, the reduction of chattering is not as significant as any of the previous three separated cases, but the chattering is still reduced by 25% to 50%, and the frequent changes of control force and the required energy are also reduced considerably in the range 72% to 99%.

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4.7 Reference

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5.1 Conclusion

The main advantage of the SMC is that it is invariant to disturbance (such as wind and earthquake) and the variation of system parameters (such as the mass, stiffness, and damping ratio matrices) if the uncertainties can be represented the linear combination of the control input, which is generally satisfied for most civil structures.

A filtered sliding mode control (SMC) approach is presented in Chapter 2 for vibration control of wind-excited highrise building structures. Rather than using a Lyapunov-function based control design, an alternative way is provided to find the control force based on the *equivalent control force* principle to obtain the control force. A low pass filter is properly selected to remove the high-frequency components of the control force while retaining the structural stability. The performance of the proposed filtered SMC is evaluated by application to a wind-excited 76-story building benchmark problem equipped with an active tuned mass damper (ATMD) on the roof. Due to the elimination of high-frequency part of the control force, the structure, sensors, actuators, and dampers are all less excited, and consequently their response is reduced compared with the unfiltered SMC approach. In addition, the required control forces are reduced which means a reduction in the size of actuators making their implementation more practical. It is shown the proposed method is more robust to structural stiffness

uncertainties compared with the linear quadratic Gaussian (LQG) algorithm and another implementation of SMC.

An adaptive and robust control algorithm for nonlinear vibration control of large structures subjected to dynamic loading was presented in Chapter 3 through integration of a self-constructing wavelet neural network with an adaptive fuzzy sliding mode control approach. It is particularly suitable when structural properties are unknown or change during the dynamic event which is the case for civil structures subjected to dynamic loading. In other words, the proposed control model has the advantages of not requiring accurate mathematical model of the controlled structure and good adaptive ability to the changes of structural parameters and external dynamic loading. The robustness of the proposed algorithm is achieved by deriving a set of adaptive laws for determining the unknown parameters of wavelet neural networks using two Lyapunov functions. No offline training of neural network is necessary for the system identification process. In addition, the earthquake signals are considered as unidentified. This is particularly important for on-line vibration control of large civil structures since the external dynamic loading due to earthquake is not available in advance. Because of these advantages, the proposed adaptive control algorithm is especially effective and implementable for vibration control of large civil structures.

The chattering in SMC is generally a problem that needs to be resolved for better control. A time-varying method is proposed for determining the sliding gain function in the SMC in Chapter 4. The first algorithm is for systems with no noise and disturbance but with or without unmodeled dynamics. The second algorithm is for systems with noise, disturbance, unmodeled dynamics, or any combination of them. Compared with the state-dependent, equivalent-control-dependent, and hysteresis loop methods, the proposed algorithms are more straightforward and easy to implement. The performance of the algorithms is evaluated for six different cases. A 90% to 95% reduction of chattering is achieved for the first algorithm used for systems with sensor dynamics only. By using the second algorithm, the chattering is reduced by 70% to 90% for systems with noise and/or disturbance, and by 25% to 50% for systems with combination of unmodeled dynamics, noise, and disturbance.

5.2 Further Research

Further research includes but not limit to: (1) SCWNN based observer and control design for unknown or partially known nonlinear civil structures; (2) Control of civil structural with actuator-structure and sensor-structure interactions; and (3) Chattering reduction of vibration control for real and large structures.

A central purpose of this study is to design accurate controllers to reduce structural vibration. However, the time-varying properties of structural stiffness and/or damping during dynamic events such as earthquake and wind excitations cause an adaptive control problem rather than a pure control problem for the control of timevarying civil structures. SCWNN model is employed to "identify" the controlled structures in the adaptive process. In fact, not all the structural states (such as displacement, velocity, and acceleration) are available for measurement due to structural or sensor limitations, which necessarily introduces the estimation of structural states by designing observers, for example, the asymptotic observer and sliding model observer. Therefore, one of the further researches includes both observer and control design based on SCWNN for unknown or partially known nonlinear civil structures. Moreover, it is clear that the performance of adaptive control depends heavily on the initial conditions of the control structures. For most civil structures, the initial conditions of structural states are known (even for those with time varying properties). For this reason, those conditions should be used to improve the performance of the adaptive control.

Another issue in vibration control of civil structures is to consider the actuatorstructures interactions, and unmodeled dynamics (actuator and sensor dynamics) coupled in structural models. The influence without considering the coupling unmodeled dynamics and the pure structures will impair the performance of control approaches, especially when unmodeled dynamics are with high orders (for example, second- or third-order). Hence further research for such interaction and coupling are important for building a more precise model, and ultimately, for a more accurate control.

Traditional SMC generally involve an undesired chattering, which is reduced by developing a proper time varying sliding gain function. The effectiveness of approach for chattering reduction has been proved by a simple first-order system with 6 different cases. Further work is strongly suggested to include application of this approach to large civil structures.

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Appendix

Appendix A: Performance Criteria for the 76-story Benchmark Problem

The evaluation criteria fall into three categories: peak responses, normalized responses, and control requirements. A total of 16 criteria are provided for evaluation of the performance of a control algorithm (Yang et al., 2004). They are summarized in this Appendix.

The maximum root mean squared (RMS) value of acceleration:

$$J_1 = max\{\sigma_{\vec{x}_1}, \sigma_{\vec{x}_{30}}, \sigma_{\vec{x}_{50}}, \sigma_{\vec{x}_{55}}, \sigma_{\vec{x}_{60}}, \sigma_{\vec{x}_{65}}, \sigma_{\vec{x}_{70}}, \sigma_{\vec{x}_{75}}\}/\sigma_{\vec{x}_{750}}$$

where $\sigma_{\ddot{x}_i}$ is the RMS value of acceleration of the *i* th floor, $\sigma_{\ddot{x}_{750}} = 9.142 \ cm/s^2$ is the RMS value of acceleration of the 75 th floor without control.

The average acceleration of 6 selected floors above the 49th floor:

$$J_2 = \frac{1}{6} \sum_i \frac{\sigma_{\dot{x}_i}}{\sigma_{\dot{x}_{io}}}$$

where i = 50, 55, 60, 65, 70, and 76, and $\sigma_{\vec{x}_{io}}$ is the RMS value of acceleration of the i^{th} floor without control.

The ratio of the displacements of top floor with and without control:

$$J_3 = \frac{\sigma_{\chi_{76}}}{\sigma_{\chi_{760}}}$$

where $\sigma_{x_{76}}$ and $\sigma_{x_{760}}$ are the RMS values of the displacement of the 76th floor with and without control, respectively ($\sigma_{x_{760}} = 10.137 cm$).

The average ratio of the displacements of 7 selected floors with and without control:

$$J_4 = \frac{1}{7} \sum_i \frac{\sigma_{x_i}}{\sigma_{x_{io}}}$$

where i = 50, 55, 60, 65, 70, 75, 76, and σ_{x_i} and $\sigma_{x_{io}}$ is the RMS value of the displacement of the *i*th floor with and without control, respectively.

The nondimensionalized actuator stroke (displacement) and average power

$$J_5 = \frac{\sigma_{x_m}}{\sigma_{x_{760}}}$$
$$J_6 = \sigma_P = \sqrt{\frac{1}{T} \int_0^T [\dot{x}_m(t)u(t)]^2 dt}$$

where σ_{x_m} = RMS of the actuator stroke, $\dot{x}_m(t)$ = actuator velocity, T = total time of integration (chosen as 900 seconds), and σ_P = RMS of the control power.

Criteria J_7 to J_{10} are defined in terms of maximum structural response as follows:

$$J_{7} = max\{\ddot{x}_{p1}, \ddot{x}_{p30}, \ddot{x}_{p50}, \ddot{x}_{p55}, \ddot{x}_{p60}, \ddot{x}_{p65}, \ddot{x}_{p70}, \ddot{x}_{p75}\}/\ddot{x}_{p75o}$$
$$J_{8} = \frac{1}{6} \sum_{i} \frac{\ddot{x}_{pi}}{\ddot{x}_{pio}}$$

for *i* = 50, 55, 60, 65, 70, 75, and

$$J_{9} = \frac{x_{p76}}{x_{p76o}}$$
$$-\frac{1}{2}\sum_{p} \frac{x_{p}}{x_{p76o}}$$

$$J_{10} = \frac{1}{7} \sum_{i} \frac{x_{pi}}{x_{pio}}$$

for i = 50, 55, 60, 65, 70, 75, 76, where x_{pi} and x_{pio} are the peak displacements of the i^{th} floor with and without control, respectively; \ddot{x}_{pi} and \ddot{x}_{pio} are the peak accelerations of the i^{th} floor with and without control, respectively; $x_{p76o} = 32.30cm$ and $\ddot{x}_{p76o} = 32.33cm/s^2$.

The following criteria are proposed for evaluation of the required actuators:

$$J_{11} = \frac{x_{pm}}{x_{p76o}}$$

$$J_{12} = P_{max} = max |\dot{x}_m(t)u(t)|$$

where x_{pm} = peak actuator stroke, and P_{max} = peak control power.

The remaining four (unnumbered) are the RMS value of the control force (σ_u), the RMS value of the actuator stroke (σ_{x_m}), the absolute maximum value of the control force (max|u|), and the absolute maximum value of the actuator stroke (max $|x_m|$).

Appendix B: Equation Formulations

B.1: Derivation of Eq. (3.23)

The approximation error function \tilde{f}_j is computed using Eqs. (3.20) and (3.22):

$$\begin{split} \tilde{f}_{j} &= f_{j} - \hat{f}_{j} = \left(\boldsymbol{w}_{j}^{T} \boldsymbol{\varphi} + \boldsymbol{b}_{j}^{T} \boldsymbol{X} \right) - \left(\widehat{\boldsymbol{w}}_{j_{a}}^{T} \widehat{\boldsymbol{\varphi}}_{a} + \widehat{\boldsymbol{b}}_{j}^{T} \boldsymbol{X} \right) \\ &= \boldsymbol{w}_{j_{a}}^{T} \boldsymbol{\varphi}_{a} + \boldsymbol{w}_{j_{ia}}^{T} \boldsymbol{\varphi}_{ia} + \boldsymbol{b}_{j}^{T} \boldsymbol{X} - \left(\widehat{\boldsymbol{w}}_{j_{a}}^{T} \widehat{\boldsymbol{\varphi}}_{a} + \widehat{\boldsymbol{b}}_{j}^{T} \boldsymbol{X} \right) \\ &= \boldsymbol{w}_{j_{a}}^{T} \boldsymbol{\varphi}_{a} - \widehat{\boldsymbol{w}}_{j_{a}}^{T} \widehat{\boldsymbol{\varphi}}_{a} + \widetilde{\boldsymbol{b}}_{j}^{T} \boldsymbol{X} + \boldsymbol{w}_{j_{ia}}^{T} \boldsymbol{\varphi}_{ia} \\ &= \boldsymbol{w}_{j_{a}}^{T} \boldsymbol{\varphi}_{a} - \widehat{\boldsymbol{w}}_{j_{a}}^{T} \boldsymbol{\varphi}_{a} + \widehat{\boldsymbol{w}}_{j_{a}}^{T} \boldsymbol{\varphi}_{a} - \widehat{\boldsymbol{w}}_{j_{a}}^{T} \widehat{\boldsymbol{\varphi}}_{a} + \widetilde{\boldsymbol{b}}_{j}^{T} \boldsymbol{X} + \boldsymbol{w}_{j_{ia}}^{T} \boldsymbol{\varphi}_{ia} \\ &= \widetilde{\boldsymbol{w}}_{j_{a}}^{T} \boldsymbol{\varphi}_{a} + \widehat{\boldsymbol{w}}_{j_{a}}^{T} \widetilde{\boldsymbol{\varphi}}_{a} + \widetilde{\boldsymbol{b}}_{j}^{T} \boldsymbol{X} + \boldsymbol{w}_{j_{ia}}^{T} \boldsymbol{\varphi}_{ia} \\ &= \widetilde{\boldsymbol{w}}_{j_{a}}^{T} (\widehat{\boldsymbol{\varphi}}_{a} + \widetilde{\boldsymbol{\varphi}}_{a}) + \widehat{\boldsymbol{w}}_{j_{a}}^{T} \widetilde{\boldsymbol{\varphi}}_{a} + \widetilde{\boldsymbol{b}}_{j}^{T} \boldsymbol{X} + \boldsymbol{w}_{j_{ia}}^{T} \boldsymbol{\varphi}_{ia} \\ &= \widetilde{\boldsymbol{w}}_{j_{a}}^{T} (\widehat{\boldsymbol{\varphi}}_{a} + \widetilde{\boldsymbol{\varphi}}_{a}) + \widehat{\boldsymbol{w}}_{j_{a}}^{T} \widetilde{\boldsymbol{\varphi}}_{a} + \widetilde{\boldsymbol{b}}_{j}^{T} \boldsymbol{X} + \boldsymbol{w}_{j_{ia}}^{T} \boldsymbol{\varphi}_{ia} \\ &= \widetilde{\boldsymbol{w}}_{j_{a}}^{T} \widehat{\boldsymbol{\varphi}}_{a} + \widetilde{\boldsymbol{w}}_{j_{a}}^{T} \widetilde{\boldsymbol{\varphi}}_{a} + \widehat{\boldsymbol{w}}_{j_{a}}^{T} \widetilde{\boldsymbol{\varphi}}_{a} + \widetilde{\boldsymbol{b}}_{j}^{T} \boldsymbol{X} + \boldsymbol{w}_{j_{ia}}^{T} \boldsymbol{\varphi}_{ia} \\ &= \widetilde{\boldsymbol{w}}_{j_{a}}^{T} \widehat{\boldsymbol{\varphi}}_{a} + \widetilde{\boldsymbol{w}}_{j_{a}}^{T} \widetilde{\boldsymbol{\varphi}}_{a} + \widehat{\boldsymbol{w}}_{j_{a}}^{T} \boldsymbol{\varphi}_{ia} \\ &= \widetilde{\boldsymbol{w}}_{j_{a}}^{T} \widehat{\boldsymbol{\varphi}}_{a} + \widetilde{\boldsymbol{w}}_{j_{a}}^{T} \widetilde{\boldsymbol{\varphi}}_{a} + \widehat{\boldsymbol{w}}_{j_{a}}^{T} \boldsymbol{\varphi}_{ia} \\ &= \widetilde{\boldsymbol{w}}_{j_{a}}^{T} \widehat{\boldsymbol{\varphi}}_{a} + \widetilde{\boldsymbol{w}}_{j_{a}}^{T} \widetilde{\boldsymbol{\varphi}}_{a} + \widetilde{\boldsymbol{w}}_{j_{a}}^{T} \boldsymbol{\varphi}_{a} + \widetilde{\boldsymbol{w}}_{j_{a}}^{T} \boldsymbol{\varphi}_{ia} \end{split}$$

where $\widetilde{\boldsymbol{w}}_{j_a} = \boldsymbol{w}_{j_a} - \widehat{\boldsymbol{w}}_{j_a}, \widetilde{\boldsymbol{\varphi}}_a = \boldsymbol{\varphi}_a - \widehat{\boldsymbol{\varphi}}_a, \ \widetilde{\boldsymbol{b}}_j = \boldsymbol{b}_j - \widehat{\boldsymbol{b}}_j.$ 125

B.2: Derivation of Eq. (3.25)

Substituting Eq. (3.24) into Eq. (3.23) yields

$$\tilde{f}_{j} = \hat{w}_{j_{a}}^{T} (A^{T} \tilde{\sigma}_{a} + B^{T} \tilde{c}_{a} + H) + \tilde{w}_{j_{a}}^{T} \tilde{\varphi}_{a} + \tilde{w}_{j_{a}}^{T} \hat{\varphi}_{a} + \tilde{b}_{j}^{T} X + w_{j_{ia}}^{T} \varphi_{ia}$$
$$= \tilde{w}_{j_{a}}^{T} \hat{\varphi}_{a} + \tilde{\sigma}_{a}^{T} A \hat{w}_{j_{a}} + \tilde{c}_{a}^{T} B \hat{w}_{j_{a}} + \hat{w}_{j_{a}}^{T} h + \tilde{w}_{j_{a}}^{T} \tilde{\varphi}_{a} + \tilde{b}_{j}^{T} X + w_{j_{ia}}^{T} \varphi_{ia}$$

where relations $\tilde{\boldsymbol{\sigma}}_{a}^{T} \boldsymbol{A} \hat{\boldsymbol{w}}_{j_{a}} = \hat{\boldsymbol{w}}_{j_{a}}^{T} \boldsymbol{A}^{T} \tilde{\boldsymbol{\sigma}}_{a}$ and $\tilde{\boldsymbol{c}}_{a}^{T} \boldsymbol{B} \hat{\boldsymbol{w}}_{j_{a}} = \hat{\boldsymbol{w}}_{j_{a}}^{T} \boldsymbol{B}^{T} \tilde{\boldsymbol{c}}_{a}$ are employed because they are scalars.

B.3: Derivation of Eq. (3.32)

Substituting Eqs. (3.30) and (3.31) into Eq. (3.25) yields

$$\begin{split} \tilde{f}_{j} &= \tilde{\boldsymbol{w}}_{ja}^{T} \hat{\boldsymbol{\varphi}}_{a} + \tilde{\boldsymbol{\sigma}}_{a}^{T} A \hat{\boldsymbol{w}}_{ja} + \tilde{\boldsymbol{c}}_{a}^{T} B \hat{\boldsymbol{w}}_{ja} + \hat{\boldsymbol{w}}_{ja}^{T} H + \tilde{\boldsymbol{w}}_{ja}^{T} \tilde{\boldsymbol{\varphi}}_{a} + \tilde{\boldsymbol{b}}_{j}^{T} X + \boldsymbol{w}_{jia}^{T} \boldsymbol{\varphi}_{ia} \\ &= \left(\eta_{iw} \tilde{\boldsymbol{w}}_{j_{iw}} - \eta_{pw} \hat{\boldsymbol{w}}_{j_{pw}} + \eta_{pw} \boldsymbol{w}_{j_{pw}} \right)^{T} \hat{\boldsymbol{\varphi}}_{a} + \tilde{\boldsymbol{\sigma}}_{a}^{T} A \hat{\boldsymbol{w}}_{ja} + \tilde{\boldsymbol{c}}_{a}^{T} B \hat{\boldsymbol{w}}_{ja} + \hat{\boldsymbol{w}}_{ja}^{T} H + \tilde{\boldsymbol{w}}_{ja}^{T} \tilde{\boldsymbol{\varphi}}_{a} \\ &+ \left(\eta_{ib} \tilde{\boldsymbol{b}}_{j_{ib}} - \eta_{pb} \hat{\boldsymbol{b}}_{j_{pb}} + \eta_{pw} \boldsymbol{b}_{j_{pb}} \right)^{T} X + \boldsymbol{w}_{jia}^{T} \boldsymbol{\varphi}_{ia} \\ &= \eta_{iw} \tilde{\boldsymbol{w}}_{j_{iw}}^{T} \hat{\boldsymbol{\varphi}}_{a} - \eta_{pw} \hat{\boldsymbol{w}}_{jpw}^{T} \hat{\boldsymbol{\varphi}}_{a} + \eta_{pw} \boldsymbol{w}_{jpw}^{T} \hat{\boldsymbol{\varphi}}_{a} + \tilde{\boldsymbol{\sigma}}_{a}^{T} A \hat{\boldsymbol{w}}_{ja} + \tilde{\boldsymbol{c}}_{a}^{T} B \hat{\boldsymbol{w}}_{ja} + \hat{\boldsymbol{w}}_{ja}^{T} H + \tilde{\boldsymbol{w}}_{ja}^{T} \tilde{\boldsymbol{\varphi}}_{a} \\ &+ \eta_{ib} \tilde{\boldsymbol{b}}_{jib}^{T} X - \eta_{pb} \hat{\boldsymbol{b}}_{jpb}^{T} X + \eta_{pb} \boldsymbol{b}_{jpb}^{T} X + \boldsymbol{w}_{jia}^{T} \boldsymbol{\varphi}_{ia} \\ &= \eta_{iw} \tilde{\boldsymbol{w}}_{jiw}^{T} \hat{\boldsymbol{\varphi}}_{a} - \eta_{pw} \hat{\boldsymbol{w}}_{jpw}^{T} \hat{\boldsymbol{\varphi}}_{a} + \eta_{ib} \tilde{\boldsymbol{b}}_{jpb}^{T} X - \eta_{pb} \hat{\boldsymbol{b}}_{jpb}^{T} X + \eta_{pb} \boldsymbol{b}_{jpb}^{T} X + \tilde{\boldsymbol{\sigma}}_{a}^{T} A \hat{\boldsymbol{w}}_{ja} + \tilde{\boldsymbol{c}}_{a}^{T} B \hat{\boldsymbol{w}}$$

where the unknown term ε_{j_f}

$$\varepsilon_{j_f} = \eta_{pw} \boldsymbol{w}_{j_{pw}}^T \widehat{\boldsymbol{\varphi}}_a + \eta_{pb} \boldsymbol{b}_{j_{pb}}^T \boldsymbol{X} + \widehat{\boldsymbol{w}}_{j_a}^T \boldsymbol{H} + \widehat{\boldsymbol{w}}_{j_a}^T \widetilde{\boldsymbol{\varphi}}_a + \boldsymbol{w}_{j_{ia}}^T \boldsymbol{\varphi}_{ia}$$

B.4: Derivation of Eq. (3.41)

Substituting Eq. (3.12) and Eq. (3.13) into Eq. (3.2) yields

$$\ddot{e} = \ddot{x} = f + gu_w + d(t)$$
$$= f + (\hat{g} + \tilde{g})u_w + d(t)$$

$$= \mathbf{f} + \hat{\mathbf{g}}\mathbf{u}_{w} + \tilde{\mathbf{g}}\mathbf{u}_{w} + \mathbf{d}(t)$$
$$= \mathbf{f} + (-\hat{\mathbf{f}} + \ddot{\mathbf{x}}_{d} - k_{1}\dot{\mathbf{e}} - k_{2}\mathbf{e} + \mathbf{u}_{a}) + \tilde{\mathbf{g}}\mathbf{u}_{w} + \mathbf{d}(t)$$
$$= \ddot{\mathbf{x}}_{d} - k_{1}\dot{\mathbf{e}} - k_{2}\mathbf{e} + \tilde{\mathbf{f}} + \tilde{\mathbf{g}}\mathbf{u}_{w} + \mathbf{u}_{s} + \mathbf{d}$$

B.5: Derivation of Eq. (3.42)

The *j* th elements of the derivative of the sliding surface in Eq. (3.6) can be rewritten as:

$$\dot{s}_{j} = \ddot{e}_{j} + k_{1}\dot{e}_{j} + k_{2}e_{j} = \tilde{f}_{j} + \tilde{g}_{j}u_{w} + u_{j_{a}} + d_{j}$$

$$= (\eta_{iw}\tilde{w}_{j_{iw}}^{T}\widehat{\varphi}_{a} - \eta_{pw}\hat{w}_{j_{pw}}^{T}\widehat{\varphi}_{a} + \eta_{ib}\tilde{b}_{j_{ib}}^{T}X - \eta_{pb}\hat{b}_{j_{pb}}^{T}X + \tilde{\sigma}_{a}^{T}A\widehat{w}_{j_{a}} + \tilde{c}_{a}^{T}B\widehat{w}_{j_{a}} + \varepsilon_{j_{a}}^{T}B\widehat{w}_{j_{a}} + \varepsilon_{j_{a}}^{T}A\widehat{w}_{j_{a}} + \tilde{c}_{a}^{T}B\widehat{w}_{j_{a}} + \varepsilon_{j_{a}}^{T}B\widehat{w}_{j_{a}} + \varepsilon_{j_{a}}^{T}A\widehat{w}_{j_{a}} + \varepsilon_{j_{a}}^{T}B\widehat{w}_{j_{a}} + \varepsilon_{j_{a}}^{T}B\widehat{w}_{j_{a}}$$

B.6: Derivation of the first Lyapunov function: Eq. (3.44)

Differentiating Eq. (3.43) and using Eq. (3.42) yields

$$\begin{split} \dot{V}_{1j} &= s_j \dot{s}_j + \eta_{iw} \widetilde{\boldsymbol{w}}_{jiw}^T \dot{\tilde{\boldsymbol{w}}}_{jiw} + \sum_{k=1}^p \left(\eta_{i\theta} \widetilde{\boldsymbol{\theta}}_{kj_{l\theta}} \dot{\boldsymbol{\theta}}_{kj_{l\theta}}^T \right) + \frac{1}{\eta_{\sigma}} \widetilde{\sigma}_a^T \dot{\widetilde{\sigma}}_a + \frac{1}{\eta_c} \widetilde{c}_a^T \dot{\widetilde{c}}_a + \eta_{ib} \widetilde{\boldsymbol{b}}_{jib}^T \dot{\widetilde{b}}_{jib} \\ &= s_j \left\{ \eta_{iw} \widetilde{\boldsymbol{w}}_{jiw}^T \widehat{\boldsymbol{\varphi}}_a - \eta_{pw} \widehat{\boldsymbol{w}}_{jpw}^T \widehat{\boldsymbol{\varphi}}_a + \eta_{ib} \widetilde{\boldsymbol{b}}_{jib}^T \boldsymbol{X} - \eta_{pb} \widehat{\boldsymbol{b}}_{jpb}^T \boldsymbol{X} + \widetilde{\sigma}_a^T \boldsymbol{A} \widehat{\boldsymbol{w}}_{ja} \\ &+ \widetilde{c}_a^T \boldsymbol{B} \widehat{\boldsymbol{w}}_{ja} + \varepsilon_{jf} + d_j \\ &+ \sum_{k=1}^q \left[\left(\eta_{i\theta} \widetilde{\boldsymbol{\theta}}_{kj_{l\theta}}^T \widehat{\boldsymbol{\varphi}}_a - \eta_{p\theta} \widehat{\boldsymbol{\theta}}_{kj_{p\theta}}^T \widehat{\boldsymbol{\varphi}}_a + \widetilde{\sigma}_a^T \boldsymbol{A} \widehat{\boldsymbol{\theta}}_{kja} + \widetilde{c}_a^T \boldsymbol{B} \widehat{\boldsymbol{\theta}}_{kja} + \varepsilon_{kjg} \right) u_{kw} \right] \\ &+ u_{ja} \right\} + \eta_{iw} \widetilde{\boldsymbol{w}}_{jiw}^T \dot{\boldsymbol{w}}_{jiw} + \sum_{k=1}^p \left(\eta_{i\theta} \widetilde{\boldsymbol{\theta}}_{kj_{l\theta}}^T \dot{\boldsymbol{\theta}}_{kj_{l\theta}} \right) + \frac{1}{\eta_{\sigma}} \widetilde{\sigma}_a^T \dot{\widetilde{\sigma}}_a + \frac{1}{\eta_c} \widetilde{c}_a^T \dot{\widetilde{c}}_a \\ &+ \eta_{ib} \widetilde{\boldsymbol{b}}_{jib}^T \dot{\widetilde{b}}_{jib} \\ &= \eta_{iw} \widetilde{\boldsymbol{w}}_{iw}^T (s \widehat{\boldsymbol{\varphi}}_a + \dot{\boldsymbol{w}}_{jiw}) + \sum_{k=1}^q \left[\eta_{i\theta} \widetilde{\boldsymbol{\theta}}_{kj_{l\theta}}^T \left(s_j u_{k_{wnn}} \widehat{\boldsymbol{\varphi}}_a + \dot{\widetilde{\boldsymbol{\theta}}}_{kj_{l\theta}} \right) \right] + \eta_{ib} \widetilde{\boldsymbol{b}}_{jib}^T \left(s_j \boldsymbol{X} + v_{k+1} \right) \end{split}$$

bjib+σaTsj**Aw**ja+k=1qukwθkja+1ησσa+caTsj**Bw**ja+k=1qukwnnθkja+1ηcca -sjηpw**w**jpwTφa-sjηpb**b**pbT**X**-sjk=1qηpθθkjpθTφaukw+sjεj+sjuja

where $\varepsilon_j = \varepsilon_{j_f} + \sum_{k=1}^q \left(\varepsilon_{kj_g} u_{k_w} \right) + d_j$.

B.7: Derivation of the second Lyapunov function: Eq. (3.59)

Differentiating (3.58) with respect to time yield

$$\begin{split} \dot{V}_{2j} &= \dot{V}_{1j} + \frac{1}{\eta_r} \tilde{r}_j \dot{\tilde{r}}_j = s_j \varepsilon_j + s_j \hat{r}_j \mu_{sum_j} + \frac{1}{\eta_r} \tilde{r}_j \dot{\tilde{r}}_j \\ &\leq |s_j| |\varepsilon_j| + \hat{r}_j s_j \mu_{sum_j} - r_j s_j \mu_{sum_j} + r_j s_j \mu_{sum_j} + \frac{1}{\eta_r} \tilde{r}_j \dot{\tilde{r}}_j \\ &= |s_j| |\varepsilon_j| - \tilde{r}_j s_j \mu_{sum_j} + r_j s_j \mu_{sum_j} + \frac{1}{\eta_r} \tilde{r}_j \dot{\tilde{r}}_j \end{split}$$
$$=\tilde{r}_{j}\left(-s_{j}\mu_{sum_{j}}+\frac{1}{\eta_{r}}\dot{r}_{j}\right)+|s_{j}||\varepsilon_{j}|+r_{j}s_{j}\mu_{sum_{j}}$$

Appendix C: Evaluation Criteria for the Benchmark Control Problem

The evaluation criteria fall into three categories: peak responses, normalized responses, and control requirements. A total of 21 criteria are used to evaluate the performance of various control devices and algorithms (Agrawal et al., 2009). The first eight evaluation criteria measure the reduction in peak response of the bridge.

 Peak base shear force in the controlled structure divided by the corresponding base shear without control

$$J_1 = max \left\{ \frac{max |F_{bi}(t)|}{F_{ob}^{max}} \right\}$$

2) Peak overturning moment in the controlled structure divided by the corresponding overturning moment of the structure without control

$$J_2 = max \left\{ \frac{max |M_{bi}(t)|}{M_{ob}^{max}} \right\}$$

3) Peak displacement at the mid-span of the controlled structure divided by the corresponding peak mid-span displacement of the uncontrolled structure

$$J_3 = max \left\{ max \left| \frac{y_{mi}(t)}{y_{om}^{max}} \right| \right\}$$

4) Peak acceleration at the mid-span of the controlled structure divided by the corresponding peak mid-span acceleration of the uncontrolled structure

$$J_4 = max \left\{ max \left| \frac{\ddot{y}_{mi}(t)}{\ddot{y}_{om}^{max}} \right| \right\}$$

5) Peak deformation of bearings in the controlled structure divided by the corresponding peak deformation of bearings in the uncontrolled structure

$$J_5 = max \left\{ max \left| \frac{y_{bi}(t)}{y_{ob}^{max}} \right| \right\}$$

6) Peak curvature at the bent column in the controlled structure divided by the corresponding curvature in the uncontrolled structure

$$J_6 = max \left\{ \frac{max \left| \phi_j(t) \right|}{\phi^{max}} \right\}$$

 Peak dissipated energy of curvature at the bent column in the controlled structure divided by the corresponding dissipated energy in the uncontrolled structure

$$J_7 = max \left\{ \frac{max \int dE_j}{E^{max}} \right\}$$

 The number of plastic connections with control divided by the corresponding number of plastic connections without control

$$J_8 = max \left| \frac{N_d^c}{N_d} \right|$$

The second set of six criteria are based on normal responses over the entire time

duration of an earthquake, which are described as follows

9) Normal base shear force in the controlled structure divided by the

corresponding normal shear in the uncontrolled structure

$$J_9 = max \left\{ \frac{max \|F_{bi}(t)\|}{\|F_{ob}^{max}\|} \right\}$$

10) Normal overturning moment in the controlled structure divided by the

corresponding overturning in the uncontrolled structure

$$J_{10} = max \left\{ \frac{max \|M_{bi}(t)\|}{\|M_{ob}^{max}\|} \right\}$$

 Normal displacement at the mid-span in the controlled structure divided by the corresponding normal displacement at the mid-span in the uncontrolled structure

$$J_{11} = max \left\{ max \; \frac{\|y_{mi}(t)\|}{\|y_{om}^{max}\|} \right\}$$

12) Normal acceleration at the mid-span in the controlled structure divided by the corresponding normal acceleration at the mid-span in the uncontrolled structure

$$J_{12} = max \left\{ max \; \frac{\|\ddot{y}_{mi}(t)\|}{\|\ddot{y}_{om}^{max}\|} \right\}$$

13) Normal deformation of bearings in the controlled structure divided by the corresponding normal displacement of bearings in the uncontrolled structure

$$J_{13} = max \left\{ max \; \frac{\|y_{bi}(t)\|}{\|y_{ob}^{max}\|} \right\}$$

14) Normal curvature at the bent column in the controlled structure divided by the corresponding normal curvature at the bent column in the uncontrolled structure

$$J_{14} = max \left\{ max \frac{\left\| \phi_j(t) \right\|}{\left\| \phi^{max} \right\|} \right\}$$

The requirement of control resource from the controller are evaluated by the following seven criteria

15) Peak control force generated by the control device(s) devided by the seismic weight of the bridge based on the mass of superstructure (excluding the foundation)

$$J_{15} = max \left\{ max \ \frac{f_l(t)}{W} \right\}$$

16) Peak stroke of the control device(s) divided by the maximum deformation of bearings in the uncontrolled structure

$$J_{16} = max \left\{ max \ \frac{f_l(t)}{x_{om}} \right\}$$

where $x_{om} = \sqrt{\sum y_{obi}^2}$

17) Peak instantaneous power required by the control device(s) divided by the product of the weight and the maximum velocity of bearing in the uncontrolled structure

$$J_{17} = max \left\{ \frac{\sum_{l} \int_{0}^{t_{f}} P_{l}(t) dt}{\dot{x}_{om}^{max} W} \right\}$$

18) Peak total power required for the control of the bridge divided by the product of weight and maximum deformation of bearings in the uncontrolled structure

$$J_{18} = max \left\{ max \; \frac{\sum_{l} P_{l}(t)}{\dot{x}_{om}^{max}} \right\}$$

- 19) Number of control devices (J_{19})
- 20) Number of sensors (J_{20})
- 21) Dimension of the discrete state vector required for the control algorithm

$$J_{21} = dim\left(x_k^c\right)$$

where |.| and ||.|| denote the absolute and norm operation, respectively; i = 1 and 2 represent x and y direction, respectively; $j = 1, ..., N_d$, is the number of plastic hinges, and *l* is the number of control devices.