Theoretical Model and Electromagnetic Wave Propagation Characteristics for the Magnetoelectric Effect in Layered Piezoelectric and Piezomagnetic Composites

DISSERTATION

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By

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Abstract

The magnetoelectric (ME) effect is defined as the induced electric polarization of a material in an applied magnetic field, or the induced magnetization of the material in an applied electric field. The ME effect is naturally occurring in single phase crystals such as antiferromagnetic Cr_2O_3 , and can also be artificially realized through layered piezoelectric and piezomagnetic composites. This study is interested solely in the artificial ME effect realized via the mechanical interaction of piezoelectric and piezomagnetic composites. There have been several experimental and theoretical studies on magnetoelectric materials for a half century; however there remains a lack of accurate theoretical models to properly describe the media. The lack of accurate theoretical models creates limitations in accurate understanding of the material through numerical solutions. This lack of accurate models as well as small ME coefficients achieved so far are some of the reasons why after many years of research into magnetoelectric materials, few applications exist.

Here, we introduce accurate and robust theoretical models that describe the effective material parameters for the longitudinal, transverse and in-plane magnetoelectric configurations. The models characterize the composite in terms of its constitutive equations, in which the electric and magnetic fields are coupled. We show that the theoretical model obtained closely approximates results obtained from experimental measurements of the ME composite. Comparison to previous theoretical

models found in the literature shows that the current model better approximates the ME effect at points where the effect is greatest.

The theoretical model applies fundamental electromagnetic boundary conditions to fields obtained via the mechanical interaction of piezoelectric and piezomagnetic layers, under magnetic and electric field bias. Solutions to the electrical-mechanical and mechanical-magnetic interaction of the films results in the theoretical model. This model also takes into account the effects of an imperfect interface coupling that may exist between the layered composites by means of a parameter *k* whose values are between " $0 \le k \le 1$." The part of the study is completed with analyses of the different material combinations and the strength of the magnetoelectric effect possible.

Theoretical investigations into the electromagnetic propagation phenomena in magnetoelectric composites are carried out as well. Electromagnetic wave propagation is analyzed for the longitudinal, transverse, and in-plane ME configurations. This involves analytical solutions to the magnetoelectric wave equation. The magnetoelectric wave equation is derived analytically using Maxwell's equations, and solutions are constructed to gain insight on the electromagnetic wave propagation characteristics. The polarizations of the propagating electromagnetic waves, modes of propagation, and direct effect of the magnetoelectric coupling terms on the behavior of the electromagnetic waves are identified. To my parents, Ojiugo and Osaretin Uwumangbe

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I. A. Osaretin and R. G. Rojas, "Planar Bianisotropic Waveguides for Application in Microwave Devices," *European Conference on Antennas and Propagation (EuCAP)*, Barcelona, Spain, April 2010

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Fields of Study

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CHAPTER 1: INTRODUCTION

Electromagnetic (EM) wave propagation in metamaterials has received considerable attention in recent years. The interesting properties and seemingly endless possibilities available from such materials have been the driving force behind its research. In this work, we are interested in materials where the electric and magnetic fields are coupled in the constitutive relationship of the media. A more common name to EM engineers for this media is bianisotropic, and has been studied in one form or other since the late nineteenth century [1-5]. Bianisotropic media defines materials where the electric field displacement, D, and the magnetic flux, B, depend on both the electric, E, and magnetic fields, *H*. The constitutive relationships for bianisotropic materials are such that the magnetic flux density and the electric field displacement are directly related to both the magnetic and electric fields. The term bianisotropic was coined by Kong and Cheng [1] in 1968 to describe such general coupled constitutive relationship. The term magnetoelectric (ME) is used to describe the physics of the material in the bianisotropic relationship, where there is an induced polarization or magnetization of the material due to an applied magnetic or electric field. Hence, the magnetoelectric effect is defined as the induced electric polarization of a material in an applied magnetic field, or the induced magnetization of the material in an applied electric field.

Magnetoelectric, or more generally, bianisotropic materials are quite complex materials, leading to many difficulties while trying to obtain the constitutive relations by means of theoretical models, or while studying the EM wave propagation phenomena of the material in bulk and planar waveguides. Based upon the constitutive relationships of bianisotropic materials where the electromagnetic coupling terms along with the permeability and permittivity may be full 3X3 matrices; analytical solutions for the wave equation of the media can become very tedious. In many cases, numerical solutions are needed to gain insight into the phenomena held within the material. In this study, we implement analytic and numerical solutions to better understand magnetoelectric materials and gain insight into possible electromagnetic applications of this media.

Research of metamaterials is aided by accurate theoretical models that closely describe the material, which can be used in numerical investigations. There have been several experimental and theoretical studies on magnetoelectric materials for a half century [6 - 11] however there remains a lack of accurate theoretical models that properly describe the media [8]. The lack of accurate theoretical models creates limitations in accurate understanding of the material through numerical solutions. Another problem is the small magnitude of the magnetoelectric coefficient achieved, which is a hindrance to applications such as RF devices or sensors. These are two reasons why after many years of research into magnetoelectric materials, very few applications based upon the material exist.

We intend to develop accurate theoretical models for the magnetoelectric effect in composite layers to aid in the development of practical applications of the magnetoelectric effect. Although the theory on magnetoelectric materials have been around for some time, magnetoelectric thin films are still relatively new, in terms of actual physical realization [12 - 16]. Research on the physical implementation of magnetoelectric thin films is still ongoing [17].

Magnetoelectric materials are naturally occurring and can also be artificially manufactured. An example of a naturally occurring magnetoelectric material is antiferromagnetic chromium oxide (Cr_2O_3) , which exists as a single phase magnetoelectric material. In this discourse, we are interested in artificial magnetoelectric materials obtained through the use of a product property implemented in layered piezomagnetic and piezoelectric composites. Piezomagnetic materials define a media where there is a linear coupling of the magnetic polarization and mechanical strain of the material. Magnetic polarization can be induced by application of stress, and physical deformation created by application of a magnetic field. An example of a piezomagnetic material is ferric oxide (Fe_2O_3). Piezoelectricity is similar to piezomagnetism, with the difference being the electric polarization of the material rather than magnetization. The effect is also reversible, with an electric field polarization creating strain, and application of mechanical strain inducing polarization. Magnetostrictive materials rather than piezomagnetic materials have sometimes been used in magnetoelectric composites, as piezomagnetic materials are rather rare. Magnetostrictive materials have the property of magnetostriction; the materials undergo nonlinear deformation with the application of a magnetic field. However, unlike the property of piezomagnetism, magnetic polarization cannot be produced by mechanical strain alone. In this chapter, we give general

information on the magnetoelectric effect. In doing so, we will make mention of piezomagnetic materials, as that term is of a more general nature than magnetostrictive materials. Also, in subsequent chapters, we model the magnetoelectric composite in general as having piezomagnetic and piezoelectric layers. Such theoretical model gives more general information on the magnetoelectric effect, as the mechanical strain and polarization/magnetization are reversible. However, a significant number of the experimental works on magnetoelectric composites have been done using ferromagnetic materials with magnetostrictive materials, we apply the general models obtained to that for a magnetostrictive material. It is important to note that the magnetostrictive material results in the coupling of the electric and magnetic fields in the constitutive relationship for the electric field displacement only. Since strain cannot induce magnetization in the magnetostrictive material, the magnetic flux is not directly related to the electric field.

1.1 Theoretical overview

The magnetoelectric (ME) effect is the induced electrical polarization, P, of a medium due to an applied magnetic field, H, or an induced magnetization, M, of the media due to an applied electric field, E [6]. This can be generally defined in terms of a bianisotropic relationship, where the electric field and magnetic fields are coupled in the constitutive relations of the media as shown in (1.1) below

$$\mathbf{D} = \overline{\overline{\varepsilon}} \mathbf{E} + \overline{\overline{\zeta}} \mathbf{H}$$

$$\mathbf{B} = \overline{\overline{\mu}} \mathbf{H} + \overline{\overline{\zeta}} \mathbf{E}$$
(1.1)

The permittivity ($\overline{\overline{\epsilon}}$), permeability ($\overline{\overline{\mu}}$) and magnetoelectric parameters ($\overline{\overline{\xi}}$ and $\overline{\overline{\zeta}}$) are usually 3x3 matrices. This fact makes it rather difficult to obtain simple analytic solutions for electromagnetic waves propagating in the media.

We will investigate the magnetoelectric effect due to layered piezoelectric and piezomagnetic composites. But first we give some basic theory on this effect as observed in single phase materials. We assume a function, $g(\mathbf{E},\mathbf{H})$, describing the density of stored energy (free enthalpy) of a crystal. Expressing the electric and magnetic fields in a Maclaurin series of variables, we obtain [7, 8]

$$-g(\mathbf{E},\mathbf{H}) = \dots + P_{i}^{s}E_{i} + M_{i}^{s}H_{i} + \frac{1}{2}\varepsilon_{0}\varepsilon_{ik}E_{i}E_{k} + \frac{1}{2}\mu_{0}\mu_{ik}H_{i}H_{k} + \alpha_{ik}E_{i}H_{k} + \frac{1}{2}\beta_{ijk}E_{i}H_{j}H_{k} + \frac{1}{2}\gamma_{ijk}H_{i}E_{j}E_{k} + \dots \quad (1.2)$$

where P_i^s and M_i^s are the spontaneous polarization and magnetization, μ_0 and ε_0 are respectively the free space permeability and permittivity, μ_{ik} and ε_{ik} are respectively the crystal permeability and permittivity, α_{ik} represents the tensor for linear magnetoelectric effect, β_{ijk} and γ_{ijk} represent the bilinear magnetoelectric effects, and H_i and E_i , respectively, the magnetic and electric fields along the Cartesian coordinate system. From Eq. (1.2), we obtain polarization by differentiating the function $g(\mathbf{E},\mathbf{H})$ with respect to the electric field

$$P_{k}\left(\mathbf{E},\mathbf{H}\right) = -\frac{\partial g}{\partial E_{k}} = \dots + P_{k}^{s} + \varepsilon_{0}\varepsilon_{ik}E_{i} + \alpha_{ki}H_{i} + \frac{1}{2}\beta_{ijk}H_{i}H_{j} + \gamma_{ijk}H_{i}E_{j} + \dots$$
(1.3)

Further, consider Maxwell Gauss equation,

$$\nabla \cdot \mathbf{D} = \rho \tag{1.4}$$

where ρ is the electric charge density. Taking the integral of Eq. (1.4) and applying onto a crystal platelet of surface area S_k and as the polarization equals the electric displacement field ($\mathbf{P} = \mathbf{D}$), we obtain the magnetic field induced magnetoelectric effect with $\mathbf{E} = 0$ [7]

$$\frac{Q}{S_k} = D_k = P_k = \alpha_{ki}H_i + \frac{1}{2}\beta_{kij}H_iH_j$$
(1.5)

Q represents the magnetically induced charges on the surface of the crystal. The magnetoelectric susceptibility tensor α_{ij} has the units of second per meter, and is a second rank tensor. The magnetoelectric susceptibility tensor is represented in matrix form as [9]

$$\boldsymbol{\alpha}_{ij}\left(s/m\right) = \begin{bmatrix} \boldsymbol{\alpha}_{11} & \boldsymbol{\alpha}_{12} & \boldsymbol{\alpha}_{13} \\ \boldsymbol{\alpha}_{21} & \boldsymbol{\alpha}_{22} & \boldsymbol{\alpha}_{23} \\ \boldsymbol{\alpha}_{31} & \boldsymbol{\alpha}_{32} & \boldsymbol{\alpha}_{33} \end{bmatrix}$$
(1.6)

The bilinear magnetoelectric tensor β_{kij} represents the higher order magnetoelectric effect, and has units of seconds per Amperes. Special techniques are used to account for these higher order effects during experimental measurements [7]. Equation (1.5) represents the fundamental equation for the quasi-static and dynamic measuring method for the magnetic field induced magnetoelectric effect [7].

As before, from Eq. (1.2), we obtain the magnetization by differentiating the function $g(\mathbf{E},\mathbf{H})$ now with respect to the magnetic field [7]

$$M_{k}(\mathbf{E},\mathbf{H}) = -\frac{\partial g}{\partial H_{k}} = \dots + M_{k}^{s} + \mu_{0}\mu_{ik}H_{i} + \alpha_{ki}E_{i} + \frac{1}{2}\beta_{ijk}E_{i}H_{j} + \gamma_{ijk}E_{i}E_{j} + \dots$$
(1.7)

We obtain the electric field induced magnetization with by taking $\mathbf{H}=0$

$$M_{k} = \alpha_{ki}E_{i} + \frac{1}{2}\gamma_{kij}E_{i}E_{j}$$
(1.8)

Equation (1.8) represents the fundamental equation for the dynamic measuring method for the electric field induced magnetoelectric effect [7]. The bilinear magnetoelectric tensor γ_{kij} represents the higher order magnetoelectric effect, and has units of seconds per Volts. Additional information can be obtained in Refs. [7], [8], [10], and [11]. Measurement of the magnetoelectric effect usually obtains the magnetoelectric voltage coefficient, α' , which is related to the magnetoelectric susceptibility tensor as

$$\alpha' = \varepsilon_0 \varepsilon_r \alpha \tag{1.9}$$

The theory given thus far has dealt with the magnetoelectric effect as obtained in single phase crystals. Single phase materials were first researched for possible applications; however its use was hampered by the small magnitude of the induced polarization or magnetization obtained [8]. The low magnitude of the magnetoelectric effect obtained from single phase materials in the early 1970s created a need to research alternate ways to obtain greater magnetoelectric effects. The magnetoelectric effect was then obtained in composite materials through an innovative application of a product property in the interaction between layered composites. The layered films independently are devoid of the magnetoelectric effect, but in combination achieve the magnetoelectric effect. The derived effect depends on the electrical-mechanical properties of a piezoelectric media and the mechanical-magnetic properties of a piezomagnetic media. The first artificial magnetoelectric material was obtained by van Suchtelen and van

Boomgaard using ferroelectric piezoelectric $BaTiO_3$ and ferromagnetic piezomagnetic $CoFe_2O_4$ [8, 12 – 15]. The product property of the composite utilized is given as [16]

$$ME = \frac{electrical}{mechanical} \times \frac{mechanical}{magnetic}$$
(1.10)

With the product property, we obtain a new effect that is not a component of each individual phase. Hence, the magnetoelectric effect is obtained when an applied electric field on the piezoelectric material causes strain which is transferred mechanically as stress onto the piezomagnetic material and induces magnetization of the material. The reverse is also possible, with application of a magnetic field to the composite material to obtain an induced electric polarization. The magnetoelectric effect is dependent on the mechanical transfer of stresses and strains between the layers. The lossless transfer of the mechanical strain between the layers is important, and perfect mechanical coupling of the composite is recommended. Perfect interface coupling between the layers is not realistically possible; however, several bonding techniques such as annealing [47], hot press/molding [48 - 49], pulse laser deposition [50], tape casting [51 - 53], etc have been investigated with the aim to achieve good mechanical coupling. The bonding technique is to be chosen such that there is no chemical reaction between the composite phases. Such chemical reactions may alter the piezomagnetic and/or piezoelectric properties of the individual phases, thus reducing the magnetoelectric effect obtained. The mechanical coupling between the layers is usually parameterized by an interface coupling parameter, k whose values range between 0 and 1, where 0 indicates no coupling and 1 indicates perfect coupling. The interface parameter is dependent on the choice of materials used for

the piezomagnetic and piezoelectric phases. Over the years, ME composites have been designed to achieve giant magnetoelectric effects in composite materials.

In many applications of magnetoelectric composites, lead zirconate titanate (PZT), and barium titanate (BaTiO₃) are usually used as the piezoelectric phase [16 - 21]. Several metallic-doped ferrites, such as cobalt ferrite ($CoFe_2O_4$) and Nickel ferrite (NiFe₂O₄), and Terfenol-D [18] with strong magnetostrictive behavior have also been applied as the piezomagnetic phase. Using the above named materials as the layers of the composite material, the magnetoelectric effect is obtained when the induced strain in one medium from an applied magnetic or electric field is transferred to the bonded layer resulting in either electrical polarization or magnetization in the bonded layer. However, the strain produced by magnetostrictive materials is not linearly proportional to the applied magnetic field, but rather proportional to the square of the field [9, 18]. This implies that the resulting magnetoelectric effect is not a linear effect. This is unlike the case in single phase magnetoelectric materials, where the magnetoelectric effect is linear. To obtain a linear magnetoelectric effect in the composite, a DC magnetic field bias is applied across the composite. With the application of the DC magnetic field bias, the magnetoelectric effect over a short range around the bias is approximated as a linear effect [18]. Hence, this improves the possibility of the magnetoelectric composite in linear devices. The non-linear or hysteretic nature of the magnetoelectric effect can be utilized in memory devices where there is no need for linearity [18].

Linearity of the magnetoelectric effect in materials that use magnetostrictivepiezoelectric phases is achieved by applying bias magnetic field across the composite. However, the application of this bias DC magnetic field will create a secondary effect which may result in a change in the permittivity and/or permeability [18] of the ME media. The application of a DC magnetic field bias, for example in the *z* direction, $\vec{\mathbf{H}} = \hat{z}H_0$, may result in changes to the effective parameters of both phases. The changes depend on the materials used, so we make some assumption here on the types of materials used as layers of the ME media. Assume we have a combination of cobalt ferrite and PZT [9, 22], as this combination has shown very good longitudinal magnetoelectric voltage coefficients. Our investigations show we expect no changes in the permittivity tensor of each individual phase with the application of DC electric and magnetic fields. This is because both phases have very low or negligible electron mobility, $\mu_m \approx 0$, which in turn greatly reduces the collision frequency. This is unlike the case in semiconductors with high electron mobilities, leading to gyrotropic permittivity. The tensor permittivity is usually obtained in the form

$$\overline{\overline{\varepsilon}} = \varepsilon_0 \varepsilon_r \overline{\overline{I}} + \frac{\overline{\overline{\sigma}}}{j\omega}$$
(1.11)

In Eq. (1.11), \mathcal{E}_r , $\overline{\overline{I}}$, $\overline{\overline{\sigma}}$, and ω are, respectively, the relative dielectric constant, the unit tensor, the conductivity tensor, and the angular frequency. In semiconductor applications, the conductivity tensor is anisotropic due to Hall effects resulting from the presence of an applied magnetic field [23]. Using Drude's model, we observe that the equation for the motion of the electron is given by

$$m^* \frac{d\overline{\upsilon}}{dt} = q(\overline{E} + \overline{\upsilon} \times \overline{B}) - \frac{m^* \overline{\upsilon}}{\tau}$$
(1.12)

where m^* is the effective mass of the electron, τ is the momentum relaxation time, and \overline{v} is the electron velocity. The electron mobility is negligible, and directly proportional to the relaxation time. This implies that the term with the relaxation time will dominate the right-hand side of Eq. (1.12). This leads to a first order differential equation in terms of the electron velocity and relaxation time. Solution to the differential equation shows that the electron velocity decays exponentially. This leads to a zero electrical conductivity as there is a direct relationship between the electron velocity and the electrical conductivity. With zero electrical conductivity, we observe from Eq. (1.11) that the permittivity remains non-gyrotropic if ε_r is a scalar.

For the permeability of both phases to remain unchanged under the applied magnetic bias, the overall magnetic moment, m, of each phase must be negligible such that when a magnetic bias field $\hat{z}\overline{H}_0$ is applied, any torque exerted is also negligible. This is not the case for magnetostrictive materials such as ferrites, which have significant magnetic moments. A DC magnetic field bias should result in a permeability tensor for the magnetostrictive/piezomagnetic phase, but not the piezoelectric phase. For the magnetostrictive phase, we have [24, 25]

$$\frac{d\overline{m}}{dt} = -\mu_0 \gamma \overline{m} \times \vec{\mathbf{H}}$$
(1.13)

Here γ is the gyromagnetic ratio. In Eq. (1.13), we have ignored losses in the media. A more detailed form of Eq. (1.13) can be used to include losses in the system by applying the Landau-Lifshitz-Gilbert representation; however, we only wish to show the change in the shape of the permeability tensor, so we ignore losses as losses do not affect the shape

of the permittivity tensor obtained. Solution to Eq. (1.13) leads to a permeability tensor, which for the *z*-direction of the DC magnetic field is obtained as [24, 25]

$${}^{m}\mu = \begin{bmatrix} {}^{m}\mu_{11} & {}^{m}\mu_{12} & 0 \\ {}^{m}\mu_{21} & {}^{m}\mu_{22} & 0 \\ 0 & 0 & {}^{m}\mu_{33} \end{bmatrix}$$
(1.14)

The components of the permeability tensor differ and are

$${}^{m}\mu_{11} = {}^{m}\mu_{22} = \mu_{0} \left(1 + \frac{\omega_{0}\omega_{m}}{\omega_{0}^{2} - \omega^{2}} \right)$$
(1.15)

$${}^{m}\mu_{12} = -{}^{m}\mu_{21} = \mu_{0} \frac{j\omega\omega_{m}}{\omega_{0}^{2} - \omega^{2}}$$
(1.16)

$${}^{m}\mu_{33} = \mu_{0} \tag{1.17}$$

In Eq. (1.14) – (1.16), μ_0 is the free space permeability, ω_0 is the *Larmor*, or precession frequency, and is defined as

$$\omega_0 = \mu_0 \gamma H_0 \tag{1.18}$$

The constant ω_m depends on the saturation magnetization, M_s , of the material and is defined as

$$\omega_m = \mu_0 \gamma M_s \tag{1.19}$$

The permeability of the magnetostrictive phase will have a form similar to that in Eq. (1.14). The form of the matrix will change based upon the direction of the bias magnetic field. In this work, the matrix components with subscripts are used to show the position of the component in the tensor, however the basic values of the components are always computed using Eq. (1.15) – (1.17). Actual values of the components depend on

the *Larmor*, or precession frequency and magnetization saturation of the ferrite. Using the permeability as shown in Eq. (1.14) for the magnetostrictive phase, we derive the homogenized expressions. In subsequent chapters, we develop models that work with use of both magnetostrictive and piezomagnetic phases. We make use of anisotropic permeability to account for the special cases where ferromagnetic magnetostrictive materials are used in realization of the magnetoelectric composite.

The motivation for this work stems from the published reports on the realization of the magnetoelectric effect using layered composites [17, 26]. Bianisotropic materials have been researched in theory for quite some time; however, there has been little application of this phenomenon to any devices partly due to the difficulty in the physical realization of the material. Published results on the physical realization of the magnetoelectric effect imply that the possibility of implementation of magnetoelectric devices is much greater than it has ever been. With better understanding of the magnetoelectric effects realized through the novel schemes applied to layered piezoelectric and piezomagnetic/magnetostrictive composites, we can help improve the effects obtained. A theoretical model can also help researchers understand experimental results, and give insight into the factors that help increase or reduce the magnetoelectric effect in layered composites. This physical implementation of the ME effect using layered composites may bring us closer to realizing the possibilities that lay untapped in this media.

1.2 Historical overview

In 1888 Rontgen discovered that a moving dielectric became magnetized when placed in an electric field. Curie, in 1894, observed the possibility of the magnetoelectric effect in crystals based upon symmetry considerations. In 1960, Landau and Lifshitz [6] discovered that the magnetoelectric effect is only allowed in time-asymmetric media, and such violation of the time reversal symmetry can only occur through the application of an external DC magnetic field. The magnetoelectric effect was then shown as theoretically possible by Dzyaloshinkii [27] in antiferromagnetic Chromium Oxide (Cr₂O₃). Atrov [28], in 1960 proved this experimentally possible in Cr_2O_3 crystals. There were little to no advancements using the magnetoelectric effect as the induced polarization and/or magnetizations were small in magnitude. Several single phase compounds have been researched since then, including Ti₂O₃ [29], GaFeO₃ [30], etc. However, the induced magnetization and polarization derived remained too small for any practical applications. In 1973, Wood and Austin proposed several devices [31], such as optical diodes, data storage and switching devices, etc using the magnetoelectric effect. However, the small magnitude of the polarizations and magnetizations made these devices impractical.

Research then shifted to artificial materials in achieving the magnetoelectric effect, such as the use of laminate composites to achieve the desired cross coupling between the fields and polarization/magnetization. The first artificial magnetoelectric material was designed by van Suchtelen and van den Boomgaard by combining ferroelectric piezoelectric BaTiO₃ and ferromagnetic piezomagnetic CoFe₂O₄ [12 – 15]. This was the start of the magnetoelectric effect using laminate composites. Greater
magnetoelectric coefficients have been obtained since then using laminate composites as was reported by Ryu et al. [18], Cai et al. [32], and Srinivasan et al. [21]. In 1995, Krowne [33] suggested the non-reciprocal property of magnetoelectric media. Applications of this media in devices have been researched also. Recently, a magnetoelectric microwave phase shifter [34] and an attenuator [35] have also been reported using laminate composites. With the physical implementation of the ME effect, and accurate theoretical models, many more applications should be discovered.

1.3 Theoretical model for the magnetoelectric effect

An important part of the design process for magnetoelectric microwave devices is the theoretical model for the effective parameters of the magnetoelectric composite. The effective material property is then used in numerical simulations as a tool to ascertain what possibilities are available within the media. In our study, we intend to investigate the electromagnetic wave propagation characteristics of electromagnetic waves within bulk magnetoelectric materials. Several works have investigated the theoretical model for the composite material [9, 16, 17, 19, and 20]. Harshe [9] was the first to obtain analytical expressions for the magnetoelectric effect in laminate composites. His work presented the magnetoelectric voltage coefficient for the laminate composite. Bichurin et al. [22] extended the work to include the effective permeability and permittivity in the *z*direction. The work also included a study on the effect of an imperfect interface coupling between the composite layers.

The magnetoelectric effect in laminate composites is known as a product property. The effect results from the interaction of two separate properties of the two 15

layers (phases) in the composite. The magnetoelectric effect is achieved by combining layers of piezoelectric (mechanical-electrical effect) and piezomagnetic (magneticmechanical effect) materials. The magnetoelectric effect results from the mechanical connection of phases. Neither phase has the magnetoelectric effect, but their combination results in the magnetoelectric effect as shown below [16]:

$$ME = \frac{magnetic}{mechanical} \times \frac{mechanical}{electrical}$$
(1.20)

From previous studies [19], an implication of the product property is that the effective material property of the "homogenized" composite has to be obtained in two stages, using an averaging method. In the first, the constitutive equation of each composite layer is solved separately. The constitutive equations for the piezoelectric and piezomagnetic phases are given respectively as [19]:

$${}^{p}S_{i} = {}^{p}s_{ij} {}^{p}T_{j} + {}^{p}d_{ki} {}^{p}E_{k}$$

$${}^{p}D_{k} = {}^{p}d_{ki} {}^{p}T_{i} + {}^{p}\varepsilon_{kn} {}^{p}E_{n}$$
(1.21)

$${}^{m}S_{i} = {}^{m}s_{ij} {}^{m}T_{j} + {}^{m}q_{ki} {}^{m}H_{k}$$

$${}^{m}B_{k} = {}^{m}q_{ki} {}^{m}T_{i} + {}^{m}\mu_{kn} {}^{m}H_{n}$$
(1.22)

Here, *S*, *T*, *E*, *D*, *B* and *H* are the strain, stress, electric field, electric displacement, magnetic flux density and the magnetic field, respectively. Also, *s*, *d*, *q*, ε , and μ are the compliance, piezoelectric, piezomagnetic, permittivity and permeability coefficients. The superscripts, *p*, and *m*, respectively refer to the piezoelectric and magnetostrictive phases. In the second stage, the composite is considered a homogeneous material with constitutive equations given as [19]:

$$S_{i} = s_{ij}T_{j} + d_{ki}E_{k} + q_{ki}H_{k}$$

$$D_{k} = d_{ki}T_{i} + \varepsilon_{kn}E_{n} + \alpha_{kn}H_{n}$$

$$B_{k} = q_{ki}T_{i} + \alpha_{kn}E_{n} + \mu_{kn}H_{n}$$
(1.23)

Here, α is the ME coefficient. Equation (1.23) is then solved for the effective parameters, taking into account solutions to Eq. (1.21) and (1.22). The effective parameters of the media also depend on the biasing direction of the DC fields. This method uses open and closed-circuit conditions to obtain the magnetoelectric voltage coefficient for the media.

In our work, we apply fundamental electromagnetic boundary conditions to obtain effective material properties. We start by obtaining expressions for the strain transferred between the phases in terms of the electric and magnetic fields in the media. We then relate the strain transfer from one phase to the next using an interface coupling parameter, k, as a damping factor. At this point, we then apply electromagnetic boundary conditions that the fields satisfy at each composite boundary. The magnetoelectric effect obtained is dependent on the material properties of the composite layers, the interface coupling between the layers and the bias and poling directions of the magnetic and electric field respectively.

For laminate composites, there are three major orientations in terms of biasing and poling directions. These are:

• Longitudinal: In the longitudinal configuration, the media is biased and poled normal to the sample plane. That is, the media is poled and biased normal to the thickness of the composite layers.

- Transverse: In the transverse configuration, the media is poled normal to the thickness of the sample (along the sample plane) and biased along the sample plane.
- In-plane: In the in-plane configuration, the media is poled and biased along the sample plane. That is, the media is poled and biased along the length of the composite layers.



FIGURE 1.1. Bias orientations of the ME effect. (A) Transverse; (B) In-plane; (C) Longitudinal. The nature of ME effect generated depends on the orientation of the DC magnetic field bias and the poling electric field.

The descriptive diagrams of the magnetoelectric orientations are shown in figure 1.1. Proper modeling of the media for each magnetoelectric configuration will result in the same general constitutive equations of the form

$$\mathbf{D} = \overline{\overline{\alpha}}^H \mathbf{H} + \overline{\overline{\varepsilon}} \mathbf{E}$$
(1.24)

$$\mathbf{B} = \overline{\overline{\alpha}}^E \mathbf{E} + \overline{\overline{\mu}} \mathbf{H}$$
(1.25)

These constitutive equations are then solved to help understand electromagnetic wave propagation within bulk magnetoelectric media.

From the theoretical models, we obtain the effective permittivity, permeability and magnetoelectric susceptibility of the composite material. However, for most experimental measurements, the ME effect is expressed in terms of the ME voltage coefficient, α' , as defined by (1.9). Here we show the simple process to obtain the ME voltage coefficient from the magnetoelectric susceptibility tensor.

From the basic definition of the magnetoelectric effect, we express the induced polarization, **P**, as

$$\mathbf{P} = \varepsilon_0 \boldsymbol{\chi} \mathbf{E} + \boldsymbol{\alpha} \mathbf{H} \tag{1.26}$$

where ε_0 is the electric constant, χ is the electric susceptibility, **E** is the electric field and **H** is the magnetic field. From basic definitions of the electric displacement field,

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P} = \varepsilon_0 \mathbf{E} + \varepsilon_0 \chi \mathbf{E} + \alpha^H \mathbf{H}.$$
 (1.27)

Simplifying

$$\mathbf{D} = \varepsilon_0 (1 + \chi) \mathbf{E} + \alpha \mathbf{H} = \varepsilon_0 \varepsilon_r \mathbf{E} + \alpha^H \mathbf{H}.$$
(1.28)

This is the form obtained in Eq. (1.24) for the electric displacement field. To obtain the ME voltage coefficient, α' , we apply an open circuit condition [9], $\mathbf{D} = 0$, and thus we obtain the ME voltage coefficient as the *E/H* ratio.

$$\mathbf{E} = -\left(\frac{\boldsymbol{\varepsilon}_r^{-1}\boldsymbol{\alpha}^H}{\boldsymbol{\varepsilon}_0}\right)\mathbf{H} = -\boldsymbol{\alpha}'\mathbf{H}.$$
 (1.29)

From Eq. (1.29), we observe the familiar relationship between the ME susceptibility and the ME voltage coefficient

$$\alpha' = \frac{\varepsilon_r^{-1} \alpha^H}{\varepsilon_0} \implies \alpha^H = \varepsilon_0 \varepsilon_r \alpha'.$$
(1.30)

where ε_0 is the free-space permittivity constant and ε_r is the relative permittivity of the media which may be of tensor form. The ME susceptibility and ME voltage are 3 by 3 matrices hence matrix multiplication is used to convert from susceptibility to voltage coefficient.

Using Eq. (1.30), we present our theoretical model in the familiar form using the ME voltage coefficient, rather than with the ME susceptibility that was derived. This makes for easier comparison to previous models. All theoretical model obtained allows for detailed analysis of the ME effect in the composite structure.

1.4 Electromagnetic wave propagation in bulk magnetoelectric media

Electromagnetic wave propagation within this complex media has been studied for quite some time. In 1962, O'Dell [36, 37] claimed that electromagnetic waves cannot propagate in bulk magnetoelectric media due to a complex index of refraction. However, 20 in 1965, Fuchs [38] proved that lossless propagation of electromagnetic waves was possible in the media, and observed a change of propagation velocities when the direction of propagation is reversed in the magnetoelectric media. This implies a possible nonreciprocal property of electromagnetic waves in the media. Also, Birss and Shrubsall [39] confirmed this theory when they presented their work showing that the eigenvectors of the wave equation for the magnetoelectric media consisted of two circularly polarized waves with opposite senses of rotation, propagating at different velocities. This presents a similar case to the well known case of Faraday rotation which is observed in ferrites and semiconductors under a DC bias in same direction as the electromagnetic wave propagation. Hence, the application of magnetoelectric media in nonreciprocal devices is of great interest.

Solutions to wave propagation in ME media are tedious and only few special cases were considered in works by Fuch, and Birss and Shrubsall in regards to single phase ME crystals. Their work did not consider propagation in homogenized multi-layer ME composites being modeled here. Hence, to understand the electromagnetic wave propagation characteristics in bulk ME composites, we have to specifically consider the permeability, permittivity and ME tensors for bilayer ME heterostructures as modeled in this discourse. Hence, we obtain the general vector wave equation for an EM wave propagating in bulk ME composites. EM wave propagation is considered separately for each one of the three ME configurations. We only consider EM waves propagating along the sample plane (*xy* plane) of the magnetoelectric composite. Observing the ME configurations, we note that the physics of the interaction between the EM wave

propagating along the *xy* plane and the magnetic and electric bias fields will be different for each ME configuration. This difference in interaction leads to distinct propagation characteristics. From solutions to the wave equation, we look for propagation phenomena such as Faraday rotation, excitation of extraordinary waves, field displacement effects, and other propagation phenomena which can be applied to device applications.

Solutions are constructed analytically for a plane EM wave propagating along the *xy* plane of bulk (unbounded) ME composite. The propagation characteristics of the EM wave in the ME media are defined via an eigenvalue problem. In the eigenvalue problem, the propagating wave number is the eigenvalue and the electric and/or magnetic field is the eigenvector. Hence, we solve for the wave number, from which we can observe its relationship with the angular frequency, and the polarization of the propagating wave. Finally, we study the mode of propagation for each ME configuration, and investigate the role of the ME coupling coefficients in the mode of propagation. For all cases considered, we assume a loss-free magnetoelectric media. We also ignore any effects the mechanical stresses and strains within the thin film may have on the propagating fields.

Preliminary investigation into the propagation characteristics of planar magnetoelectric waveguides was carried out. Only two planar structures were considered, as an introduction to future work on ME composites. Results show that guided wave propagation is also possible in ME composites. Results of this preliminary investigation are given in Appendix A.

CHAPTER 2: THEORETICAL MODEL FOR THE LONGITUDINAL MAGNETOELECTRIC EFFECT

The Longitudinal Magnetoelectric (ME) effect is obtained when the composite material is biased and poled along the axis normal to the sample plane, as shown in Fig. 2.1. The bias/poling field induces mechanical strains and stresses in one layer of the composite layer, which in turn induces magnetic and/or electric fields within the other composite layer. The direction of the applied bias and poling fields, along with the piezoelectric and piezomagnetic coefficients controls the shape of the ME susceptibility tensor obtained. The longitudinal magnetoelectric effect has been the most researched configuration, with several theoretical models and experimental results available [9, 18, and 22]. The experimental results available in the literature are used here to validate the accuracy of the theoretical model obtained in this discourse.

Harshe, in 1991, obtained theoretical models for the longitudinal ME voltage coefficient of multilayer composites composed of lead zirconate titanate (PZT) and ferrite layers [9]. This was a significant contribution, as theoretical models had never been obtained for the ME effect in composite layers. However, there were large deviations between the ME voltage coefficient obtained via theoretical modeling, and corresponding experimental values of fabricated structures. Harshe fabricated and measured samples of ME composites to help validate his theoretical model. The experimental results had

similar characteristics to those reported in literature; however, the theoretical model produced values that were several times higher than that obtained from experimental results.



FIGURE 2.1. Longitudinal magnetoelectric composite system diagram. Diagram shows the composite layers and the directions of the poling electric field and the biasing magnetic field for the longitudinal ME effect.

Reasons for the poor agreement between theoretical and experimental results include poor interface coupling between the layers that exist in the experimental realization of the ME composite, and the inadequate application of boundary conditions on the electromagnetic fields to obtain the model. In deriving his theoretical model, Harshe did not apply appropriate boundary conditions to fields at the boundary between the piezoelectric and piezomagnetic phases. So while the interface coupling between the piezoelectric and piezomagnetic phases is a contributing factor for the discrepancy between the results from the experimental and theoretical models [8], we show here that the application of improper boundary conditions or lack thereof, makes the obtained theoretical model inaccurate [40].

The interface coupling factor is of importance and should also be included in theoretical models to help analyze the effects of imperfect bonding of the composite phases. Bichurin et al. [22] looked into obtaining more accurate theoretical models, by including a coupling factor for the mismatch at the composite's bonding interface. This was done using an interface coupling parameter that models the strain transfer relationship between the piezoelectric and piezomagnetic phases. The ME voltage coefficients obtained in these works had similar trends as the experimental data, however, there still remained similar deviation in the overall magnitude of the ME voltage coefficients, as theoretical results remained several times higher than experimental values. We believe that the difference results from the inadequate application of electromagnetic boundary conditions used to obtain the theoretical model. The boundary conditions used by Bichurin et al. involve quasi static approximations, including open circuit conditions on the piezoelectric phase, and mechanical boundary conditions on the stresses and strains induced by the bias/poling fields. Again, as was the case with the formulation by Harshe, there are no distinct conditions on the tangential and normal electromagnetic fields that lie within and along the boundary of the composite layer.

Hence, the insufficient electromagnetic boundary conditions in the Bichurin et al. models results in similar homogenized material parameters and resultant theoretical values that are much higher than experimental values.

In our theoretical modeling of the magnetoelectric composite, we use a different approach to obtain homogenized material parameters for the ME composite. We apply fundamental electromagnetic wave boundary conditions on the fields at the boundaries between the films, namely, continuity of the tangential electric and magnetic fields, and continuity of the normal magnetic flux density and electric displacement field, which follows from continuity of the tangential components. Hence, we introduce new theoretical models for the longitudinal ME effect in a piezoelectric and piezomagnetic/magnetostrictive bilayer that better approximates the experimental results. The theoretical model is initiated by solving the constitutive equations of each layer independently for the all fields present, and then applying a field averaging method [41, 42] along with electromagnetic boundary conditions on the components of the fields at the composite interface, to obtain homogenized material properties. The homogenized layer is characterized in terms of its effective permeability, effective permittivity and the effective ME susceptibility tensors.

2.1 Longitudinal magnetoelectric fields, stresses and strains

In composites, the magnetoelectric effect is obtained using piezoelectric and piezomagnetic phases in a layered structure. The effect uses a product property, which implies that each phase is solved independently and the resultant effect is obtained via a combination of the mechanical-electric effect and the mechanical-magnetic effect. Hence, we start with the solution to the fields within each phase of the composite layer. PZT and barium titanate (BTO) are examples of piezoelectric materials that have been used as a composite layer in the modeling of the longitudinal ME effect. The constitutive relationship for such piezoelectric material is

$${}^{p}S_{i} = {}^{p}S_{ij} {}^{p}T_{j} + {}^{p}d_{ki} {}^{p}E_{k}, \qquad (2.1)$$

$${}^{p}D_{k} = {}^{p}d_{ki} {}^{p}T_{i} + {}^{p}\mathcal{E}_{kn} {}^{p}E_{n}, \qquad (2.2)$$

$${}^{p}B_{k} = {}^{p}\mu_{kn} {}^{p}H_{n}.$$
(2.3)

In Eq. (2.1) – (2.3), *S*, *T*, *E*, *D*, *H*, and *B* are, respectively, the strain, stress, electric field, electric displacement field, magnetic field, and the magnetic flux density. Also, *s*, *d*, ε , and μ are, respectively, the compliance, piezoelectric, permittivity, permeability coefficients. The superscript *p* represents the piezoelectric phase. Equation (2.3) has not been utilized in previous theoretical models [9, 22] as only the electric displacement field and electric field effects are considered in this layer. This is understandable as there are no effects expected due to the magnetic flux density in the piezoelectric phase. However, we note that in order to properly model the media in terms of the fields, all field relationships for each composite phase must be obtained. Hence, one must also consider the magnetic-flux/magnetic-field relationship of the piezoelectric layer to obtain a homogeneous layer that combines properties of both layers. Hence, we include a relationship for the permeability of the piezoelectric phase in its constitutive relationship as observed in Eq. (2.3).

We also solve the piezomagnetic phase for the fields, stresses and strains. Nickel ferrite NiFe₂O₄ (NFO) and Copper ferrite CoFe₂O₄ (CFO) are examples of 27

magnetostrictive materials that have been used in ME composites for the piezomagnetic phase. Piezomagnetic materials are represented by the constitutive equations

$${}^{m}S_{i} = {}^{m}s_{ij}{}^{m}T_{j} + {}^{m}q_{ki}{}^{m}H_{k}, \qquad (2.4)$$

$${}^{m}B_{k} = {}^{m}q_{ki} {}^{m}T_{i} + {}^{m}\mu_{kn} {}^{m}H_{n}, \qquad (2.5)$$

$${}^{m}D_{k} = {}^{m}\mathcal{E}_{kn} {}^{m}E_{n}.$$
(2.6)

In Eqs. (2.4) - (2.6), *q* is the piezomagnetic coefficient, and superscript *m* represents the piezomagnetic phase. Here, we have accounted for the permittivity of the piezomagnetic phase as observed in Eq. (2.6), relating the electric field and the electric displacement field. The permittivity of the piezomagnetic phase was not taken into consideration in previous theoretical models. We have chosen to use it here for similar reasons as those given for the case of the permeability and magnetic flux density of the piezoelectric phase in Eq. (2.3)

In modeling the ME media, we assume that in addition to poling and bias fields within the composite medium, there exist time varying fields in all directions which may be time varying. As an example, the fields in the axial direction are expressed in terms of DC and AC components as

$$\overline{H}_{3} = \hat{3}H_{0} + \hat{3}H_{AC},
\overline{E}_{3} = \hat{3}E_{0} + \hat{3}E_{AC}.$$
(2.7)

The total fields in each respective phase will be represented by a vector field of the form

$${}^{m,p}\mathbf{E} = E_1\hat{1} + E_2\hat{2} + E_3\hat{3},$$

$${}^{m,p}\mathbf{H} = H_1\hat{1} + H_2\hat{2} + H_3\hat{3}.$$
(2.8)

Modeling the ME effect requires an understanding of the strain and stress transfer relationship between the layers of the structure. This is because the ME effect in composites utilizes the mechanical stresses and strains induced in each composite phase. This is the application of the product property to obtain the coupled electric and magnetic fields. Bichurin et al. introduced the use of coupling parameter, k, to model the mechanical interaction at the interface between the bilayers. The coupling parameter used in that work is defined as

$$k = \left({}^{p}S_{i} - {}^{p}S_{i0} \right) / \left({}^{m}S_{i} - {}^{p}S_{i0} \right), (i = 1, 2),$$
(2.9)

In Eq. (2.9) ${}^{p}S_{i0}$ is the strain tensor component with no friction between phases. For our theoretical model, we use a similar coupling parameter, k, as a damping factor to model the strain transfer relationship between the layers. We assume that the strain induced in one phase may not be completely transferred to the adjoining phase due to several factors, which may include some mechanical losses. We do not investigate the loss mechanism in the derivation of the theoretical model. We assume that the resulting losses will only dampen the transfer of strain from one phase to the other. So while we do not expressly indicate the factors causing the imperfect interface coupling, the factors are contained and described by the interface coupling parameter, k.

In the formulation of the theoretical model, only symmetric or extensional deformations are considered, such that all flexural deformations of the layers that lead to position-dependent elastic constants are ignored [17, 22]. We make the following assumptions:

1. Shear stresses and strains are equal to zero, such that

$$^{m,p}T_i = 0, \ ^{m,p}S_i = 0, \ \text{for } i=4, 5, \text{ and } 6.$$
 (2.10)

2. The thickness of each phase is much smaller than the width and length of the phase. Hence the stress in the axial direction is approximated as zero.

$${}^{m}T_{3} = {}^{p}T_{3} = 0. (2.11)$$

3. The strain transfer between phases is related by an interface coupling parameter, k, such that

$${}^{p}S_{i} = k \cdot {}^{m}S_{i}. \tag{2.12a}$$

Equation (2.12a) relates the strain transfer between phases. In this study, we use values between zero and one for the interface coupling parameter, k. It is implied from Eq. (2.12a) that the strain is induced upon the piezoelectric layer by the piezomagnetic layer, and that all, a fraction, or none of the strain may be transferred, depending on the value of k. However, the reverse is also possible, where all, a fraction, or none of the strain from the piezoelectric layer is transferred to the piezomagnetic layer. The process uses the assumption that

$${}^{m}S_{i} = k_{r} \cdot {}^{p}S_{i}. \tag{2.12b}$$

In Eq. (2.12b), k_r is the interface coupling parameter for the reverse strain transfer. The theoretical models obtained hereafter will also work for that instance, with k_r having values between zero and one. The conversion is simple, as an example: k goes to infinity when k_r goes to zero, and k goes to one when k_r goes to one. It should be noted that this manipulation of k as just stated is to allow use of the same analytic expressions for the material parameters for both the electric and magnetic field induced ME effect. For each case, forward or reverse, the interface parameter (k or k_r) remains between 0 and 1. For a magnetostrictive media, this reverse process is not possible, as strain transferred from the piezoelectric layer cannot induce magnetization in the magnetostrictive layer. However, the assumption holds for the forward and reverse processes for piezomagnetic layers.

4. The summation of forces on the in-plane (1-2 plane) boundaries are zero, such that

$${}^{m}T_{i}{}^{m}\nu + {}^{p}T_{i}{}^{p}\nu = 0, \text{ for } i = 1, 2,$$
 (2.13)

Here ${}^{m}v$ and ${}^{p}v$, respectively, are the piezomagnetic and piezoelectric volume fractions of the magnetoelectric composite layer, and are defined as ${}^{m}v = volume^{m}/volume^{total}$ and ${}^{p}v = volume^{p}/volume^{total}$.

Solving the constitutive equations of each phase based upon the given assumptions, we obtain the electric displacement field in the piezoelectric region using the nonzero components of the permittivity, permeability, compliance and piezoelectric coefficient as shown in Table 2.1. Using Eqs. (2.10) and (2.11), and applying the coefficients as shown in Table 2.1, we obtain

$${}^{p}S_{1} = {}^{p}s_{11} {}^{p}T_{1} + {}^{p}s_{12} {}^{p}T_{2} + {}^{p}d_{31} {}^{p}E_{3}, \qquad (2.14)$$

$${}^{p}S_{2} = {}^{p}s_{21} {}^{p}T_{1} + {}^{p}s_{22} {}^{p}T_{2} + {}^{p}d_{32} {}^{p}E_{3}.$$
(2.15)

Equations (2.14) and (2.15) represent the stress-field-strain relationships for the piezoelectric phase. The relationship for the piezomagnetic phase is obtained as

$${}^{m}S_{1} = {}^{m}s_{11}{}^{m}T_{1} + {}^{m}s_{12}{}^{m}T_{2} + {}^{m}q_{31}{}^{m}H_{3},$$
(2.16)

$${}^{m}S_{2} = {}^{m}s_{21}{}^{m}T_{1} + {}^{m}s_{22}{}^{m}T_{2} + {}^{m}q_{32}{}^{m}H_{3}.$$
(2.17)

Note that ${}^{m}S_{1} = {}^{m}S_{2}$ and ${}^{p}S_{1} = {}^{p}S_{2}$, since ${}^{m}s_{11} = {}^{m}s_{22}$, ${}^{m}s_{12} = {}^{m}s_{21}$, ${}^{m}q_{31} = {}^{m}q_{32}$, ${}^{p}s_{11} = {}^{p}s_{22}$, ${}^{p}s_{12} = {}^{p}s_{21}$, and ${}^{p}d_{31} = {}^{p}d_{32}$ as observed in Table 2.1. Along similar lines, since ${}^{p}d_{31} = {}^{p}d_{32}$ and ${}^{m}q_{31} = {}^{m}q_{32}$, an assumption is made that [9]

$${}^{p}T_{1} = {}^{p}T_{2}. ag{2.18}$$

And,

$${}^{m}T_{1} = {}^{m}T_{2}. ag{2.19}$$

Thus applying Eq. (2.12),

$$\left({}^{p}s_{11} + {}^{p}s_{12}\right){}^{p}T_{1} + {}^{p}d_{31}{}^{p}E_{3} = k \cdot \left({}^{m}s_{11} + {}^{m}s_{12}\right){}^{m}T_{1} + k \cdot {}^{m}q_{31}{}^{m}H_{3}.$$
(2.20)

From Eq. (2.13) we substitute for the 1-directional stress in the piezomagnetic phase. Thus, we obtain

$$-k \cdot \left({}^{m}s_{11} + {}^{m}s_{12} \right) {}^{p}T_{1} \left({}^{p}\nu / {}^{m}\nu \right) + {}^{m}q_{31} {}^{m}H_{3} = \left({}^{p}s_{11} + {}^{p}s_{12} \right) {}^{p}T_{1} + {}^{p}d_{31} {}^{p}E_{3}.$$
(2.21)

Rewriting Eq. (2.17) in terms of the 1-directional stress ${}^{p}T_{1}$ in the piezoelectric layer, thus

$${}^{p}T_{1} = \left(\frac{{}^{m}q_{31}}{\left[k\left({}^{m}s_{11} + {}^{m}s_{12}\right)\left({}^{p}\nu/{}^{m}\nu\right) + \left({}^{p}s_{11} + {}^{p}s_{12}\right)\right]}\right){}^{m}H_{3} - \left(\frac{{}^{p}d_{31}}{\left[k\left({}^{m}s_{11} + {}^{m}s_{12}\right)\left({}^{p}\nu/{}^{m}\nu\right) + \left({}^{p}s_{11} + {}^{p}s_{12}\right)\right]}\right){}^{p}E_{3}.$$

$$(2.22)$$

TABLE 2.1NONZERO COEFFICIENTS OF BULK PIEZOELECTRIC AND PIEZOMAGNETICPHASES FOR THE LONGITUDINAL MAGNETOELECTRIC CONFIGURATION.

Piezoelectric phase					
Coefficient type	Non-zero components				
Permittivity	${}^{p}\mathcal{E}_{11} = {}^{p}\mathcal{E}_{22}, {}^{p}\mathcal{E}_{33}.$				
Permeability	${}^{p}\mu_{11} = {}^{p}\mu_{22} = {}^{p}\mu_{33}.$				
Compliance	${}^{p}s_{11} = {}^{p}s_{22}, {}^{p}s_{12} = {}^{p}s_{21}, {}^{p}s_{13} = {}^{p}s_{23} = {}^{p}s_{31} = {}^{p}s_{32},$ ${}^{p}s_{33}, {}^{p}s_{44} = {}^{p}s_{55}, {}^{p}s_{66} = 2({}^{p}s_{11} + {}^{p}s_{12}).$				
Piezoelectric	${}^{p}d_{15} = {}^{p}d_{24}, {}^{p}d_{31} = {}^{p}d_{32}, {}^{p}d_{33}.$				

Piezomagnetic phase

Coefficient type	Non-zero components			
Permittivity	${}^{m}\boldsymbol{\mathcal{E}}_{11},{}^{m}\boldsymbol{\mathcal{E}}_{22},{}^{m}\boldsymbol{\mathcal{E}}_{33}.$			
Permeability	${}^{m}\mu_{11},{}^{m}\mu_{22},{}^{m}\mu_{33}.$			
Compliance	${}^{m}s_{11} = {}^{m}s_{22} = {}^{m}s_{33}, {}^{m}s_{44} = {}^{m}s_{55} = {}^{m}s_{66},$ ${}^{m}s_{12} = {}^{m}s_{21} = {}^{m}s_{13} = {}^{m}s_{23} = {}^{m}s_{31} = {}^{m}s_{32}.$			
Piezomagnetic	${}^{m}q_{15} = {}^{m}q_{24}, {}^{m}q_{31} = {}^{m}q_{32}, {}^{m}q_{33}.$			

From the constitutive relationship as expressed in Eq. (2.2) and also applying Eq. (2.18) we obtain the components of the electric field displacement in the piezoelectric phase as

$${}^{p}D_{1} = {}^{p}\mathcal{E}_{11} {}^{p}E_{1}, \qquad (2.23)$$

$${}^{p}D_{2} = {}^{p}\mathcal{E}_{22} {}^{p}E_{2}, \qquad (2.24)$$

$${}^{p}D_{3} = 2 \cdot {}^{p}d_{31} {}^{p}T_{1} + {}^{p}\mathcal{E}_{33} {}^{p}E_{3}.$$
(2.25)

Substituting Eq. (2.22) into Eq. (2.25), we obtain the coupled 3-directional component of the electric field displacement in the piezoelectric phase as [40]

$${}^{p}D_{3} = \left(\frac{2^{p}d_{31}{}^{m}q_{31}k}{\left[k\left({}^{m}s_{11} + {}^{m}s_{12}\right)\left({}^{p}\nu/{}^{m}\nu\right) + \left({}^{p}s_{11} + {}^{p}s_{12}\right)\right]}\right){}^{m}H_{3} + \left(\frac{-2\left({}^{p}d_{31}\right)^{2}}{\left[k\left({}^{m}s_{11} + {}^{m}s_{12}\right)\left({}^{p}\nu/{}^{m}\nu\right) + \left({}^{p}s_{11} + {}^{p}s_{12}\right)\right]} + {}^{p}\varepsilon_{33}\right){}^{p}E_{3}.$$

$$(2.26)$$

In simpler terms, the electric displacement field in the 3-direction of the piezoelectric phase is

$${}^{p}D_{3} = K_{1}{}^{m}H_{3} + K_{2}{}^{p}E_{3}.$$
(2.27)

where,

$$K_{1} = \frac{2 \cdot {}^{p} d_{31} \cdot {}^{m} q_{31} \cdot k}{\left[k\left({}^{m} s_{11} + {}^{m} s_{12}\right)\left({}^{p} \nu / {}^{m} \nu\right) + \left({}^{p} s_{11} + {}^{p} s_{12}\right)\right]},$$
(2.28)

$$K_{2} = \frac{-2({}^{p}d_{31})^{2}}{\left[k({}^{m}s_{11} + {}^{m}s_{12})({}^{p}\nu/{}^{m}\nu) + ({}^{p}s_{11} + {}^{p}s_{12})\right]} + {}^{p}\varepsilon_{33}.$$
(2.29)

The magnetic flux density in the piezoelectric region is also required for formulation of the theoretical model. It has been described, and is easily obtained using Eq. (2.6). The form of the permeability tensor is obtained using the components of the permeability (piezoelectric phase) shown in Table 2.1.

2.1.1 Case 1: Non-gyrotropic composites

In most cases magnetostrictive materials such as ferrites are used for the piezomagnetic phases. Ferrites have magnetic anisotropy induced with the application of a DC magnetic field bias. However, not all materials used as the piezomagnetic phase may have this magnetic anisotropy, so it is important to derive a theoretical model for a piezomagnetic phase that lacks magnetic anisotropy. In this case-study, we assume that the application of the DC magnetic field bias does not create an anisotropic permeability tensor in the material; hence the material is non-gyrotropic. That is, the values of the permeability matrix used for the formulation of the theoretical model are as given in Table 2.1. Hence, using this assumption we derive theoretical models for the longitudinal ME configuration while using non-gyrotropic materials wherein the saturation magnetization, M_s , of the material is approximated as zero. For the non-gyrotropic composites, the material need not be isotropic; however the off-diagonal terms of the permeability matrix are zero. The composite may have uniaxial or biaxial permeability. The magnetic flux density in the piezomagnetic phase for a non-gyrotropic material is

obtained using Eq. (2.5) and applying Eq. (2.19) as

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$$B_1 = {}^m \mu_{11} {}^m H_1, \tag{2.30}$$

$${}^{m}B_{2} = {}^{m}\mu_{22} {}^{m}H_{2}, \tag{2.31}$$

 ${}^{m}B_{3} = 2 \cdot {}^{m}q_{31} {}^{m}T_{1} + {}^{m}\mu_{33} {}^{m}H_{3}.$ (2.32)

The 1-directional stress in the magnetoelectric phase is obtained from Eq. (2.20) using Eq. (2.13). This is expressed as

$${}^{m}T_{1} = \left(\frac{{}^{p}d_{31}}{\left[k\left({}^{m}s_{11} + {}^{m}s_{12}\right) + \left({}^{p}s_{11} + {}^{p}s_{12}\right)\left({}^{m}\nu/{}^{p}\nu\right)\right]}\right){}^{p}E_{3} - \left(\frac{{}^{m}q_{31} \cdot k}{\left[k\left({}^{m}s_{11} + {}^{m}s_{12}\right) + \left({}^{p}s_{11} + {}^{p}s_{12}\right)\left({}^{m}\nu/{}^{p}\nu\right)\right]}\right){}^{m}H_{3}.$$

$$(2.33)$$

Thus, substituting Eq. (2.33) into Eq. (2.32), we obtain the coupled magnetic flux density in the magnetoelectric phase as [40]

$${}^{m}B_{3} = C_{1}{}^{p}E_{3} + C_{2}{}^{m}H_{3}.$$
(2.34)

where,

$$C_{1} = \frac{2 \cdot {}^{p} d_{31} \cdot {}^{m} q_{31}}{\left[k \left({}^{m} s_{11} + {}^{m} s_{12} \right) + \left({}^{p} s_{11} + {}^{p} s_{12} \right) \left({}^{m} \nu / {}^{p} \nu \right) \right]},$$
(2.35)

$$C_{2} = \frac{-2k \left({}^{m} q_{31}\right)^{2}}{\left[k \left({}^{m} s_{11} + {}^{m} s_{12}\right) + \left({}^{p} s_{11} + {}^{p} s_{12}\right) \left({}^{m} \nu / {}^{p} \nu\right)\right]} + {}^{m} \mu_{33}.$$
(2.36)

The electric displacement field in the piezomagnetic phase is also required for the homogenization of the composite media. The electric displacement is easily obtained using Eq. (2.6) and the non-zero components of the permittivity tensor for the piezomagnetic phase. With all fields within the components obtained, we now apply boundary conditions to the field to complete the homogenization process for a non-gyrotropic material. The application of boundary conditions for the gyrotropic and non-gyrotropic material will be done later, in the Application of boundary conditions section of this chapter. First, we obtain the fields for the case of a gyrotropic material being used as the piezomagnetic phase.

2.1.2 Case 2: Gyrotropic composites

Here, we take into consideration the effect the DC magnetic field bias will have on the ferrite medium, which has an induced magnetic anisotropy on application of the bias DC magnetic field. The ME effect in composites is a non-linear effect. A DC magnetic field bias is usually applied so that the ME effect over a short range around the bias is approximated as a linear effect [18]. The applied bias however has a secondary effect when the piezomagnetic phase is a ferrite material (magnetostrictive). Ferrites have an intrinsic magnetic moment, and application of a DC magnetic field bias will lead to tensor permeability [25]. This implies a change to the nonzero permittivity values of the bulk piezomagnetic phase from what had been described in Table 2.1. The permeability becomes anisotropic, a 3 by 3 matrix, with complex off-diagonal components. For the case with the DC magnetic field bias in the axial direction as shown in Fig. 1, the permeability tensor will be of the form [24, 25]

$${}^{m}\mu = \begin{bmatrix} {}^{m}\mu_{11}^{*} & {}^{m}\mu_{12}^{*} & 0 \\ {}^{m}\mu_{21}^{*} & {}^{m}\mu_{22}^{*} & 0 \\ 0 & 0 & {}^{m}\mu_{33}^{*} \end{bmatrix}.$$
(2.37)

The values of the nonzero components of the permeability as shown in Eq. (2.37) depends on the intrinsic property of the magnetostrictive phase, such as its magnetization saturation, M_s , and the magnitude of the applied DC magnetic field bias. This change in permeability is a secondary effect from the DC magnetic field bias in the composite structure [18]. This secondary effect obtained in the ferrite layer (magnetostrictive phase) is well known and has been used in several device applications based upon the shape and

values of the permeability tensor. We do not show here the formulas for the components of the permeability tensor in Eq. (2.37), relevant formulas have been discussed briefly in the introduction chapter. Also, more detailed information is readily available in other literature [24, 25]. However, sample computations will be done in later chapters that make use of gyrotropic formulas.

The change in the shape of the permeability tensor only affects the 1- and 2directional components of the magnetic flux density. Solving the piezomagnetic constitutive equations, the magnetic flux density in the piezomagnetic phase is now derived as

$${}^{m}B_{1} = {}^{m}\mu_{11}^{*}{}^{m}H_{1} + {}^{m}\mu_{12}^{*}{}^{m}H_{2}, \qquad (2.38)$$

$${}^{m}B_{2} = {}^{m}\mu_{_{21}}^{*}{}^{m}H_{1} + {}^{m}\mu_{_{22}}^{*}{}^{m}H_{2}, \qquad (2.39)$$

$${}^{m}B_{3} = C_{1}{}^{p}E_{3} + C_{2}{}^{m}H_{3}.$$
(2.40)

Here, C_1 and C_2 are of the form shown in Eq. (2.35) and (2.36). The difference lies in the change of the permeability component in C_2 . Hence, C_1 and C_2 are expressed as

$$C_{1} = \frac{2 \cdot {}^{p} d_{31} \cdot {}^{m} q_{31}}{\left[k\left({}^{m} s_{11} + {}^{m} s_{12}\right) + \left({}^{p} s_{11} + {}^{p} s_{12}\right)\left({}^{m} \nu / {}^{p} \nu\right)\right]},$$
(2.41)

$$C_{2} = \frac{-2k \left({}^{m} q_{31}\right)^{2}}{\left[k \left({}^{m} s_{11} + {}^{m} s_{12}\right) + \left({}^{p} s_{11} + {}^{p} s_{12}\right) \left({}^{m} \nu / {}^{p} \nu\right)\right]} + {}^{m} \mu_{33}^{*}.$$
(2.42)

There are no changes to any of the expressions for the electric displacement field from what had been previously computed in the Non-gyrotropic material case. We now apply boundary conditions to the fields obtained for the gyrotropic and non-gyrotropic cases.

2.2 Application of boundary conditions

From Maxwell's equations, we deduce conditions involving the normal and tangential fields at the interface. The boundary conditions at the interface, assuming there are no applied surface currents, are:

- 1. Tangential components of the electric and magnetic fields are continuous across the interface.
- 2. Normal components of the electric displacement field and the magnetic flux density are continuous at the interface.

The above boundary conditions will be applied to obtain the effective material parameters for the composite media. We start with the case of a non-gyrotropic composite.

2.2.1 Case 1: Non-gyrotropic composites

From the first boundary condition, tangential components of the electric and magnetic fields are continuous at the boundary. Thus at the interface,

$$\bar{E}_x = {}^{m}E_1 = {}^{p}E_1, \tag{2.43}$$

$$\bar{E}_{y} = {}^{m}E_{2} = {}^{p}E_{2}, \qquad (2.44)$$

$$\bar{H}_{x} = {}^{m}H_{1} = {}^{p}H_{1}, \tag{2.45}$$

$$\bar{H}_{y} = {}^{m}H_{2} = {}^{p}H_{2}. \tag{2.46}$$

Here, $\overline{E}_x, \overline{E}_y, \overline{H}_x$, and \overline{H}_y are the homogenized tangential components of the electric and magnetic field in the ME layer. For ease of understanding, we have chosen to represent all homogenized fields using the (x, y, z) coordinate system rather than the $(1 \ 2 \ 3)$ system

used thus far. The tangential components of the electric field displacement in the piezoelectric and piezomagnetic phases are not continuous and are related as

$${}^{m}D_{1} = \frac{{}^{m}\mathcal{E}_{11}}{{}^{p}\mathcal{E}_{11}} {}^{p}D_{1}, \qquad (2.47)$$

$${}^{m}D_{2} = \frac{{}^{m}\mathcal{E}_{22}}{{}^{p}\mathcal{E}_{22}} {}^{p}D_{2}.$$
(2.48)

Similarly, the tangential components of the magnetic flux density in both phases are related as

$${}^{m}B_{1} = \frac{{}^{m}\mu_{11}}{{}^{p}\mu_{11}} {}^{p}B_{1}, \qquad (2.49)$$

$${}^{m}B_{2} = \frac{{}^{m}\mu_{22}}{{}^{p}\mu_{22}} {}^{p}B_{2}.$$
(2.50)

We assume each composite layer is electrically thin, with negligible field variation within each individual phase. Using a field averaging method [41, 42] we define the tangential components of the electric field displacement and the magnetic flux density as

$$D_{x} = {}^{m}D_{1}{}^{m}v + {}^{p}D_{1}{}^{p}v = \left[{}^{m}\varepsilon_{11}{}^{m}E_{1}{}^{m}v + {}^{p}\varepsilon_{11}{}^{p}E_{1}{}^{p}v\right] = \left[{}^{m}\varepsilon_{11}{}^{m}v + {}^{p}\varepsilon_{11}{}^{p}v\right]E_{x}, \quad (2.51)$$

$$D_{y} = {}^{m}D_{2}{}^{m}v + {}^{p}D_{2}{}^{p}v = \left[{}^{m}\varepsilon_{22}{}^{m}E_{2}{}^{m}v + {}^{p}\varepsilon_{22}{}^{p}E_{2}{}^{p}v\right] = \left[{}^{m}\varepsilon_{22}{}^{m}v + {}^{p}\varepsilon_{22}{}^{p}v\right]E_{y}, \quad (2.52)$$

$$B_{x} = {}^{m}B_{1}{}^{m}v + {}^{p}B_{1}{}^{p}v = \left[{}^{m}\mu_{11}{}^{m}H_{1}{}^{m}v + {}^{p}\mu_{11}{}^{p}H_{1}{}^{p}v\right] = \left[{}^{m}\mu_{11}{}^{m}v + {}^{p}\mu_{11}{}^{p}v\right]H_{x},$$
(2.53)

$$B_{y} = {}^{m}B_{2} {}^{m}v + {}^{p}B_{2} {}^{p}v = \left[{}^{m}\mu_{22} {}^{m}H_{2} {}^{m}v + {}^{p}\mu_{22} {}^{p}H_{2} {}^{p}v\right] = \left[{}^{m}\mu_{22} {}^{m}v + {}^{p}\mu_{22} {}^{p}v\right]H_{y}.$$
 (2.54)

In Eqs. (2.51) – (2.54), D_x , D_y , B_x , and B_y are the homogenized tangential components of the electric field displacement and magnetic flux density, respectively. The homogenized tangential components are expressed as

$$\begin{bmatrix} D_x \\ D_y \end{bmatrix} = \begin{bmatrix} {}^m \varepsilon_{11} {}^m v + {}^p \varepsilon_{11} {}^p v & 0 \\ 0 {}^m \varepsilon_{22} {}^m v + {}^p \varepsilon_{22} {}^p v \end{bmatrix} \begin{bmatrix} E_x \\ E_y \end{bmatrix},$$
(2.55)

$$\begin{bmatrix} B_{x} \\ B_{y} \end{bmatrix} = \begin{bmatrix} {}^{m}\mu_{11} {}^{m}v + {}^{p}\mu_{11} {}^{p}v & 0 \\ 0 & {}^{m}\mu_{22} {}^{m}v + {}^{p}\mu_{22} {}^{p}v \end{bmatrix} \begin{bmatrix} H_{x} \\ H_{y} \end{bmatrix}.$$
 (2.56)

The normal components of the electric field displacement and the magnetic flux density are obtained by applying boundary conditions at the interface. The normal components of the electric field displacement and the magnetic flux density are continuous across the boundary. Thus,

$${}^{m}D_{3} = {}^{p}D_{3} = D_{z}, \tag{2.57}$$

$${}^{m}B_{3} = {}^{p}B_{3} = B_{z}. ag{2.58}$$

Thus, from (2.57)

$${}^{m}\mathcal{E}_{33}{}^{m}E_{3} = K_{1}{}^{m}H_{3} + K_{2}{}^{p}E_{3}.$$
(2.59)

 K_1 and K_2 are as have been expressed in Eqs. (2.28) and (2.29).

Similarly, from (2.58)

$${}^{p}\mu_{33} {}^{p}H_{3} = C_{1} {}^{p}E_{3} + C_{2} {}^{m}H_{3}.$$
(2.60)

 C_1 and C_2 are as have been expressed in Eqs. (2.35) and (2.36).

Using a similar averaging technique, as done for the tangential component of the electric field displacement and the magnetic flux density, the homogenized normal components of the electric and magnetic field is expressed as

$$E_{z} = {}^{p}E_{3} {}^{p}v + {}^{m}E_{3} {}^{m}v, ag{2.61}$$

$$H_{z} = {}^{p}H_{3} {}^{p}v + {}^{m}H_{3} {}^{m}v.$$
(2.62)

Likewise, from Eq. (2.61)

$${}^{p}E_{3} = \frac{1}{{}^{p}v} \Big(E_{z} - {}^{m}E_{3} {}^{m}v \Big),$$
(2.63)

and from Eq. (2.62)

$${}^{m}H_{3} = \frac{1}{{}^{m}v} \Big(H_{z} - {}^{p}H_{3} {}^{p}v \Big).$$
(2.64)

Substituting Eqs. (2.63) and (2.64) into Eqs. (2.59) and (2.60), we obtain

$$B_{z} = \frac{C_{1}}{{}^{p}v} \Big(E_{z} - {}^{m}E_{3} {}^{m}v \Big) + \frac{C_{2}}{{}^{m}v} \Big(H_{z} - {}^{p}H_{3} {}^{p}v \Big),$$
(2.65)

$$D_{z} = \frac{K_{1}}{{}^{m}v} \Big(H_{z} - {}^{p}H_{3} {}^{p}v \Big) + \frac{K_{2}}{{}^{p}v} \Big(E_{z} - {}^{m}E_{3} {}^{m}v \Big).$$
(2.66)

From Eq. (2.6)

$${}^{m}E_{3} = \frac{{}^{m}D_{3}}{{}^{m}\mathcal{E}_{33}},$$
(2.67)

and from Eq. (2.3)

$${}^{p}H_{3} = \frac{{}^{p}B_{3}}{{}^{p}\mu_{33}}.$$
(2.68)

Substituting Eqs. (2.67) and (2.68) into Eqs. (2.65) and (2.66), and then applying Eqs. (2.57) and (2.58)

$$B_{z} = \frac{C_{1}}{{}^{p}_{v}} \left[E_{z} - \left(\frac{{}^{m}_{v}}{{}^{m}_{\mathcal{E}_{33}}}\right) D_{z} \right] + \frac{C_{2}}{{}^{m}_{v}} \left[H_{z} - \left(\frac{{}^{p}_{v}}{{}^{p}_{\mathcal{H}_{33}}}\right) B_{z} \right],$$
(2.69)

$$D_{z} = \frac{K_{1}}{{}^{m}v} \left[H_{z} - \left(\frac{{}^{p}v}{{}^{p}\mu_{33}}\right) B_{z} \right] + \frac{K_{2}}{{}^{p}v} \left[E_{z} - \left(\frac{{}^{m}v}{{}^{m}\varepsilon_{33}}\right) D_{z} \right].$$
(2.70)

Rewriting Eqs. (2.69) and (2.70)

$$\left[1 + \left(\frac{C_2}{m_v} \frac{p_v}{p_{33}}\right)\right] B_z = \left(\frac{C_1}{p_v}\right) E_z + \left(\frac{C_2}{m_v}\right) H_z - \left(\frac{C_1}{p_v} \frac{m_v}{m_{\mathcal{E}_{33}}}\right) D_z, \qquad (2.71)$$

$$\left[1 + \left(\frac{K_2^{\ m}v}{{}^{p}v^{m}\varepsilon_{33}}\right)\right]D_z = \left(\frac{K_1}{{}^{m}v}\right)H_z + \left(\frac{K_2}{{}^{p}v}\right)E_z - \left(\frac{K_1^{\ p}v}{{}^{m}v^{\ p}\mu_{33}}\right)B_z.$$
(2.72)

Solving Eq. (2.61) for the electric field displacement, we obtain

$$D_{z} = \left[\frac{{}^{m}\varepsilon_{33}}{{}^{m}v}\right] E_{z} + \left[\frac{C_{2}{}^{p}v^{m}\varepsilon_{33}}{C_{1}\left({}^{m}v\right)^{2}}\right] H_{z} - \left[\frac{{}^{p}v^{m}\varepsilon_{33}}{C_{1}{}^{m}v} + \left(\frac{C_{2}{}^{m}\varepsilon_{33}\left({}^{p}v\right)^{2}}{C_{1}{}^{p}\mu_{33}\left({}^{m}v\right)^{2}}\right)\right] B_{z}.$$
(2.73)

Substituting into (2.72)

$$B_{z} = \frac{\left(R_{1}R_{3} - R_{5}\right)}{\left(R_{1}R_{4} - R_{7}\right)}H_{z} + \frac{\left(R_{1}R_{2} - R_{6}\right)}{\left(R_{1}R_{4} - R_{7}\right)}E_{z},$$
(2.74)

where

$$R_{1} = \left[1 + \left(\frac{K_{2}^{\ m}v}{{}^{p}v^{m}\varepsilon_{33}}\right)\right], R_{2} = \left[\frac{{}^{m}\varepsilon_{33}}{{}^{m}v}\right], R_{3} = \left[\frac{C_{2}^{\ p}v^{m}\varepsilon_{33}}{C_{1}\left({}^{m}v\right)^{2}}\right], R_{4} = \left[\frac{{}^{p}v^{m}\varepsilon_{33}}{C_{1}^{\ m}v} + \left(\frac{C_{2}^{\ m}\varepsilon_{33}\left({}^{p}v\right)^{2}}{C_{1}^{\ p}\mu_{33}\left({}^{m}v\right)^{2}}\right)\right], R_{5} = \left(\frac{K_{1}}{{}^{m}v}\right), R_{6} = \left(\frac{K_{2}}{{}^{p}v}\right), R_{7} = \left(\frac{K_{1}^{\ p}v}{{}^{m}v^{p}\mu_{33}}\right).$$

$$(2.75)$$

Similarly, solving Eq. (2.72) for the magnetic flux density, we obtain

$$B_{z} = \left[\frac{{}^{p}\mu_{33}}{{}^{p}v}\right]H_{z} + \left[\frac{K_{2}{}^{m}v{}^{p}\mu_{33}}{K_{1}\left({}^{p}v\right)^{2}}\right]E_{z} - \left[\frac{{}^{m}v{}^{p}\mu_{33}}{K_{1}{}^{p}v} + \left(\frac{K_{2}{}^{p}\mu_{33}\left({}^{m}v\right)^{2}}{K_{1}{}^{m}\varepsilon_{33}\left({}^{p}v\right)^{2}}\right)\right]D_{z}.$$
(2.76)

Substituting into Eq. (2.71)

$$D_{z} = \frac{\left(N_{1}N_{3} - N_{5}\right)}{\left(N_{1}N_{4} - N_{7}\right)} E_{z} + \frac{\left(N_{1}N_{2} - N_{6}\right)}{\left(N_{1}N_{4} - N_{7}\right)} H_{z},$$
(2.77)

where

$$N_{1} = \left[1 + \left(\frac{C_{2}}{m_{v}} \frac{p_{v}}{p_{\mu_{33}}}\right)\right], N_{2} = \left[\frac{p_{\mu_{33}}}{p_{v}}\right], N_{3} = \left[\frac{K_{2}}{K_{1}} \frac{p_{v}}{p_{\mu_{33}}}\right], N_{4} = \left[\frac{m_{v}}{K_{1}} \frac{p_{\mu_{33}}}{p_{v}} + \left(\frac{K_{2}}{K_{1}} \frac{p_{\mu_{33}}}{p_{v}} + \frac{K_{2}}{K_{1}} \frac{p_{\mu_{33}}}{p_{v}} + \left(\frac{K_{2}}{K_{1}} \frac{p_{\mu_{33}}}{p_{v}} + \frac{K_{2}}{K_{1}} \frac{p_{\mu_{33}}}{p_{v}} +$$

Finally, we have obtained homogenized material parameters for the composite layer that describes the constitutive relationship for the media. Observing the results obtained for the tangential and normal components of the electric displacement field and the magnetic flux density, we express the constitutive equations for the homogenized composite using the form [40]

$$\begin{bmatrix} D_{x} \\ D_{y} \\ D_{z} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \alpha_{zz}^{H} \end{bmatrix} \begin{bmatrix} H_{x} \\ H_{y} \\ H_{z} \end{bmatrix} + \begin{bmatrix} \varepsilon_{xx} & 0 & 0 \\ 0 & \varepsilon_{yy} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{bmatrix} \begin{bmatrix} E_{x} \\ E_{z} \\ E_{z} \end{bmatrix}, \qquad (2.79)$$

$$\begin{bmatrix} B_{x} \\ B_{y} \\ B_{z} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \alpha_{zz}^{E} \\ 0 & 0 & \alpha_{zz}^{E} \end{bmatrix} \begin{bmatrix} E_{x} \\ E_{y} \\ E_{z} \end{bmatrix} + \begin{bmatrix} \mu_{xx} & 0 & 0 \\ 0 & \mu_{yy} & 0 \\ 0 & 0 & \mu_{zz} \end{bmatrix} \begin{bmatrix} H_{x} \\ H_{y} \\ H_{z} \end{bmatrix}. \qquad (2.80)$$

It is important to comment here that the constitutive representations in Eq. (2.79) and (2.80) apply only to magnetoelectric composites realized with piezoelectric and piezomagnetic materials. Magnetostrictive materials with only the property of magnetostriction will not have such constitutive relationship. As had been explained in the introduction chapter, piezomagnetic materials behave like piezoelectric materials where the strain to polarization/magnetization is possible, and most importantly, also reversible. For magnetoelectric composites using a magnetostrictive material, the coupling tensor α^{E} equals zero in Eq. (2.80), while Eq. (2.79) retains its form. This is

because the strain obtained from the applied electric field on the bilayer does not produce any magnetization in the magnetostrictive layer.

2.2.2 Case 2: Gyrotropic composites

The composite layer has magnetic anisotropy, but no electric anisotropy, hence changes are only observed in the tangential components of the magnetic flux density. The tangential components of the magnetic flux density in both phases are related as

$${}^{m}B_{1} = \frac{{}^{m}\mu_{11}^{*}}{{}^{p}\mu_{11}} {}^{p}B_{1} + \frac{{}^{m}\mu_{12}^{*}}{{}^{p}\mu_{22}} {}^{p}B_{2}, \qquad (2.81)$$

$${}^{m}B_{2} = \frac{{}^{m}\mu_{_{21}}^{*}}{{}^{p}\mu_{_{11}}} {}^{p}B_{1} + \frac{{}^{m}\mu_{_{22}}^{*}}{{}^{p}\mu_{_{22}}} {}^{p}B_{2}, \qquad (2.82)$$

Assuming the composite layers are electrically thin, without any field variation within each individual layer, we define the tangential components the magnetic flux density as

$$B_{x} = {}^{m}B_{1} {}^{m}v + {}^{p}B_{1} {}^{p}v = \left[{}^{m}\mu_{11} {}^{m}H_{1} {}^{m}v + {}^{m}\mu_{12} {}^{m}H_{2} {}^{m}v + {}^{p}\mu_{11} {}^{p}H_{1} {}^{p}v \right],$$

$$= \left[{}^{m}\mu_{11} {}^{m}v + {}^{p}\mu_{11} {}^{p}v \right] H_{x} + \left[{}^{m}\mu_{12} {}^{m}v \right] H_{y}.$$

$$B_{y} = {}^{m}B_{2} {}^{m}v + {}^{p}B_{2} {}^{p}v = \left[{}^{m}\mu_{21} {}^{m}H_{1} {}^{m}v + {}^{m}\mu_{22} {}^{m}H_{2} {}^{m}v + {}^{p}\mu_{22} {}^{p}H_{2} {}^{p}v \right],$$

$$= \left[{}^{m}\mu_{21} {}^{m}v \right] H_{x} + \left[{}^{m}\mu_{22} {}^{m}v + {}^{p}\mu_{22} {}^{p}v \right] H_{y}.$$

$$(2.84)$$

Notice that unlike Eqs. (2.53) and (2.54), the magnetic flux density in Eqs. (2.83) and (2.84) is related to the magnetic fields in both *x* and *y* directions. The homogenized tangential components is expressed as

$$\begin{bmatrix} B_{x} \\ B_{y} \end{bmatrix} = \begin{bmatrix} {}^{m}\mu_{11}{}^{m}v + {}^{p}\mu_{11}{}^{p}v & {}^{m}\mu_{12}{}^{m}v \\ {}^{m}\mu_{21}{}^{m}v & {}^{m}\mu_{22}{}^{m}v + {}^{p}\mu_{22}{}^{p}v \end{bmatrix} \begin{bmatrix} H_{x} \\ H_{y} \end{bmatrix}$$
(2.85)

The normal components of the electric field displacement and the magnetic flux density are obtained as done in Part I, namely,

$$B_{z} = \frac{\left(R_{1}R_{3} - R_{5}\right)}{\left(R_{1}R_{4} - R_{7}\right)}H_{z} + \frac{\left(R_{1}R_{2} - R_{6}\right)}{\left(R_{1}R_{4} - R_{7}\right)}E_{z},$$

where R_1 , R_2 , R_3 , R_4 , R_5 , R_6 , and R_7 are as have been obtained in Eq. (2.75).

The formulation for the magnetic flux density for the gyrotropic composite does not affect the electric field displacement. The electric displacement field remains the same and is expressed as

$$D_{z} = \frac{\left(N_{1}N_{3} - N_{5}\right)}{\left(N_{1}N_{4} - N_{7}\right)} E_{z} + \frac{\left(N_{1}N_{2} - N_{6}\right)}{\left(N_{1}N_{4} - N_{7}\right)} H_{z},$$

where N_1 , N_2 , N_3 , N_4 , N_5 , N_6 , and N_7 are as have been obtained in Eq. (2.78).

From the results obtained above, there is no change to the form of the magnetoelectric susceptibility tensor. The change obtained in the case of magnetic anisotropy shows in the anisotropic form of the homogenized permeability tensor. The results obtained show that the constitutive equations for the homogenized composite has the form [40]

$$\begin{bmatrix} D_{x} \\ D_{y} \\ D_{z} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \alpha_{zz}^{H} \end{bmatrix} \begin{bmatrix} H_{x} \\ H_{y} \\ H_{z} \end{bmatrix} + \begin{bmatrix} \varepsilon_{xx} & 0 & 0 \\ 0 & \varepsilon_{yy} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{bmatrix} \begin{bmatrix} E_{x} \\ E_{y} \\ E_{z} \end{bmatrix}$$

$$(2.86)$$

$$\begin{bmatrix} B_{x} \\ B_{y} \\ B_{z} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \alpha_{zz}^{E} \end{bmatrix} \begin{bmatrix} E_{x} \\ E_{y} \\ E_{z} \end{bmatrix} + \begin{bmatrix} \mu_{xx} & \mu_{xy} & 0 \\ \mu_{yx} & \mu_{yy} & 0 \\ 0 & 0 & \mu_{zz} \end{bmatrix} \begin{bmatrix} H_{x} \\ H_{y} \\ H_{z} \end{bmatrix}$$

$$(2.87)$$

As has been explained earlier, the constitutive representation here applies only to magnetoelectric media realized with the use of piezoelectric and piezomagnetic phases.

This concludes the derivation of the theoretical model for the longitudinal magnetoelectric configuration. Next, we validate the theoretical model using measured data from literature.

Material	Experimental ME Coefficient (maximum) [(V/m)/(kA/m)]	Theoretical ME coefficient $[(V/m)/(kA/m)]$
CoFe ₂ O ₄ :PZT-4	92.8	199 to 622
CoFe ₂ O ₄ :PZT-8	18.6	202 to 634
CoFe ₂ O ₄ :PZT-5H	74.4	143 to 475

 TABLE 2.2 EXPERIMENTAL AND THEORETICAL MAGNETOELECTRIC VOLTAGE

 COEFFICIENTS AS OBTAINED BY REF. 9

2.3 Validation of the theoretical longitudinal model

The theoretical model is validated via comparison to experimental results obtained by Harshe et al. The results from his experimental measurement of piezoelectric and magnetostrictive phases, as well as results from his theoretical model are shown in Table 2.2. The measured effect stems from the application of a magnetic field to the composite and the realization of electric polarization. We have obtained the magnetoelectric susceptibility of the media; however, the measurements are done for the magnetoelectric voltage coefficient. We have shown the relationship between the magnetoelectric susceptibility and the magnetoelectric voltage coefficient in the first chapter. We use that relationship to express the theoretical model in terms that make for better comparison to experimental data.

The theoretical model obtained here allows for detailed analysis of the ME effect in the composite structure. Analyses are done on the effects of the interface coupling parameter on the magnetoelectric media. Increase in volume fraction of either the piezoelectric or piezomagnetic phase is analyzed as well. Based upon such analysis, we compare our results to other published theoretical models, and observe the general trends of the ME media. In computing the magnetoelectric voltage coefficient, we use the same material characteristics of the media as given in the literature. The material characteristics of the layers are shown in Table 2.3.

Parameter (Units)	CFO	PZT-4	PZT-8	PZT-5H	PZT
$S_{11}\left(10^{-12}\frac{m^2}{N}\right)$	6.5	12.3	11.5	16.5	15.3
$S_{12}\left(10^{-12}\frac{m^2}{N}\right)$	-2.37	-4.05	-3.7	-4.78	-5
$q_{31}\left(10^{-12}\frac{m}{A}\right)$	566	-	-	-	-
$q_{33}\left(10^{-12}\frac{m}{A}\right)$	-1880	-	-	-	-
$d_{31}\left(10^{-12}\frac{C}{N}\right)$	-	-123	-90	-274	-175
$d_{33}\left(10^{-12}\frac{C}{N}\right)$	-	289	225	593	400
$rac{\mu_{_{33}}}{\mu_{_0}}$	2	1	1	1	1
$\frac{\mathcal{E}_{33}}{\mathcal{E}_{0}}$	10	1300	1000	3400	1750

 TABLE 2.3. MATERIAL PARAMETERS FOR MAGNETOSTRICTIVE AND PIEZOELECTRIC

 PHASES USED TO COMPUTE THE LONGITUDINAL MAGNETOELECTRIC VOLTAGE COEFFICIENT.

In our analysis, we compare the measured magnetoelectric voltage coefficients to results of our theoretical model using the magnetoelectric coefficient α^{H} only. We had shown in the introduction the relationship between the magnetoelectric susceptibility and the magnetoelectric voltage coefficient using Eq. (1.9). Since our analysis deals with a magnetostrictive based magnetoelectric composite rather than one with piezomagnetic material, only the ME effect stemming from magnetic field is realized. Hence, for this case, the magnetic and electric fields are not coupled in the relationship for the magnetic flux, and the magnetic flux depends directly in the magnetic field only. This is the method we intend to use in our analysis throughout this discourse, as most other models deal with the special case of magnetostrictive layers rather than the more general piezomagnetic layer. Our theoretical model is robust enough to handle such special cases.

We analyze theoretical models for four different magnetoelectric composites. The first is a composite with CFO for the magnetostrictive phase and PZT-4 for the piezoelectric phase. A plot of the theoretical results for this ME composite is shown in Fig. 2.2 The magnetoelectric voltage coefficient increases from zero (piezoelectric volume fraction also equals zero) to a maximum with an increasing volume fraction of the piezoelectric phase. This is due to the continuous increase in the elastic interaction between the piezoelectric and magnetostrictive phases. The magnetoelectric voltage coefficient then decreases to zero as the composite becomes purely piezoelectric (piezoelectric volume fraction equals 1). This result is consistent with the theoretical model obtained by Nan using Green's function method [16].


FIGURE 2.2. Longitudinal ME coefficient for the ME layer of PZT-4 and CFO. Maximum theoretical ME voltage coefficient obtained is 180 (V/m)/(kA/m). Considering an imperfect interface coupling of 0.4, the value is close to the experimental value (measured) of 92.8 (V/m)/(kA/m). Measured data is from Ref. [9].

From the plots of the magnetoelectric voltage coefficient for the CFO:PZT-4 composite in Fig. 2.2, we observe a maximum magnetoelectric voltage coefficient of 180 (V/m)/(kA/m) obtained for a perfect interface coupling between phases (*k*=1). The maximum experimental value obtained was 92.8 (V/m)/(kA/m). We assume that an imperfect interface and other material factors may be a reason for an approximate fifty percent loss in the experimental result. An interface coupling parameter of k = 0.4 results in a magnetoelectric voltage coefficient that is much closer to the experimental data. The results show similar trends to other models as the strength of the ME voltage coefficient

decreases as the coupling weakens [22]. Additionally, for weaker coupling, the ME voltage coefficient increases with increase in piezoelectric volume fraction.



FIGURE 2.3. Longitudinal ME coefficient for the ME layer of PZT-5H and CFO. Maximum theoretical ME voltage coefficient obtained is 140 (V/m)/(kA/m). Considering an imperfect interface coupling of 0.4, the value is close to the experimental value of 74.4 (V/m)/(kA/m). Measured data is obtained from Ref. [9]

Figure 2.3 shows the theoretical longitudinal ME coefficient for an ME composite consisting of CFO for the magnetostrictive phase and PZT-5H for the piezoelectric phase. Similar trends with the piezoelectric volume fraction are observed for this composition. The maximum theoretical ME voltage coefficient is 140 (V/m)/(kA/m). The experimental

value obtained was 74.4 (V/m)/(kA/m). The theoretical results show similar characteristics to available data.

We observe that the ME voltage coefficient is less than that for PZT-4. This should be expected, due to the fact that PZT-5H has a higher relative permittivity than PZT-4. For the CFO:PZT-4 composite, the ME voltage coefficient increases steadily, with a maximum value when the phases are equal for the case of a perfect interface coupling. As the interface coupling weakens, the maximum ME coefficient shifts to richer piezoelectric compositions. This trend is consistent with that obtained by Bichurin et al. [22].

The plot of the longitudinal ME voltage coefficient for an ME composite with CFO magnetostrictive phase and PZT-8 piezoelectric phase is shown in Fig. 2.4. The maximum ME voltage coefficient for this composite is 175 (V/m)/(kA/m). The experimental result was obtained as 18.6 (V/m)/(kA/m). The experimental result was obtained as 18.6 (V/m)/(kA/m). The experimental result for this composite is much lower than expected. The reason for such low experimental result is the ineffective method used to bond the layers together. Whereas the more effective tape cast method was used for the CFO:PZT-4 composite, glue was used to bond the CFO:PZT-8 layers, hence resulting in a poor interface coupling between the phases. This composition of CFO:PZT-8 is expected to have comparable ME voltage coefficient to CFO:PZT-4 as their permittivity and piezoelectric coefficients are close in value.



FIGURE 2.4. Longitudinal ME coefficient for the ME layer of PZT-8 and CFO. Maximum theoretical ME voltage coefficient obtained is 175 (V/m)/(kA/m). Relating the experimental result of 18.6 (V/m)/(kA/m) to the theoretical model, we observe that the interface coupling for the CFO/PZT-8 is very poor, with less than 10% strain transfer between phases.

The theoretical models obtained here leads to the observation that for a high longitudinal ME voltage coefficient, the piezoelectric phase should have a high piezoelectric coefficient (d_{31}), and a low permittivity. This is due to the direct relationship between the piezoelectric coefficient and the ME voltage coefficient. The ME voltage coefficient has an inverse relationship with the effective permittivity of the piezoelectric phase, hence the requirement of a piezoelectric phase with low relative permittivity.



FIGURE 2.5. Longitudinal ME coefficient for unclamped CFO-PZT layers as studied in Ref. 21. The plot uses the theoretical model discussed in this paper. The maximum coefficient obtained here (140 mV/cm Oe) is much less than that obtained in Ref. 21 (325 mV/cm Oe).

Finally, we show results of the theoretical model of an unclamped ME composite with CFO for the magnetostrictive phase, and PZT for the piezoelectric phase as studied in Ref. 22. An unclamped ME composite is one in which there are no external forces to hold the composite layers in place, besides the bond applied at the interface. The plot of the ME voltage coefficient for our theoretical model is shown in Fig. 2.5. We observed similar trends in the behavior of ME voltage coefficient, such as the direct relationship between the interface coupling parameter and the magnitude of the ME voltage coefficient. Also, the maximum ME voltage coefficient shifts to piezoelectric-rich composites for cases with less than perfect coupling.



FIGURE 2.6. Comparison of the theoretical model for the longitudinal ME voltage coefficient for the unclamped CFO-PZT layer with k = 1. The theoretical data from Ref. 22 and our work here is compared to experimental data obtained from Ref. 9.

The magnitude of the ME voltage coefficient obtained is less than 140 mV/cmOe compared to over 300 mV/cmOe in previous works, as observed in Fig. 2.6. Hence, we show that the current theoretical model better approximates the ME voltage coefficient in terms of its magnitude, while maintaining established trends for the ME composite.

We had discussed the reverse ME process, realized by inducing magnetization of the piezomagnetic layer via strain produced by the piezoelectric layer. This is the electric field induced magnetoelectric effect, and makes use of the α^{E} susceptibility tensor. As was stated while making initial assumptions to develop the theoretical model, we investigate the reverse process, simply by using an interface coupling factor, k_r , to quantify the strain transfer from the piezoelectric layer to the piezomagnetic layer. The new interface coupling factor for the reverse process was defined in Eq. (2.12b).



FIGURE 2.7. Longitudinal ME susceptibility for the electric field induced effect, ME^{E} .

The results from Fig. 2.7 are purely theoretical as we have assumed strain induced magnetization as possible in the magnetostrictive layer using same piezomagnetic coefficients. The results show that we need a minimal amount of piezomagnetic volume fraction to induce maximum magnetoelectric susceptibility. However, in cases where the interface coupling allows less than ten percent strain transfer, we will need a higher piezoelectric volume fraction. It is interesting to also observe how the piezoelectric volume fraction affects the α^{H} susceptibility obtained. We plot the results for the magnetic field induced magnetoelectric effect in Fig. 2.8.



FIGURE 2.8. Longitudinal magnetoelectric susceptibility for the magnetic field induced effect, ME^H.

For the magnetic field induced magnetoelectric effect, we need a high piezoelectric volume fraction to obtain maximum magnetoelectric susceptibility. It is also important to note that the magnetoelectric susceptibility does not follow same trends as the magnetoelectric voltage coefficient. This is because the effective permittivity of the composite layer affects the ME voltage coefficient obtained. Higher piezoelectric volume fractions results in higher effective permittivity, as the piezoelectric layer has a much higher relative permittivity than the piezomagnetic layer. The magnetoelectric susceptibility obtained for both cases is pretty comparable and is verified in Appendix B where the material parameters for the longitudinal magnetoelectric coupling tensors are the same for electric and magnetic induced effects.

The results obtained show that the theoretical model closely approximates the experimental results for magnetoelectric bilayers. The model is also very robust, as both piezomagnetic and magnetostrictive materials can be analyzed using the model. The magnetoelectric effect obtained via induced mechanical strain of the piezoelectric layer or induced mechanical strain of the piezomagnetic layer is also accounted for. However, we must state that we do not know true value of the interface coupling parameter for the cases analyzed here. Such information will lend credence to the theoretical model, for instance, should the effective interface coupling be about 40% as observed in Fig. 2.2 and 2.3. Obtaining such information will be part of future studies on the ME composite.

CHAPTER 3: THEORETICAL MODEL FOR THE TRANSVERSE MAGNETOELECTRIC EFFECT

The transverse magnetoelectric effect arises from the orientation of bias DC magnetic fields along the composite plane and poling electric fields normal to the composite plane. The stress-strain relationship obtained between the piezoelectric and piezomagnetic phases under DC magnetic and electric fields gives rise to the magnetoelectric effect. Piezoelectric and piezomagnetic phases respond differently to changes in direction of the DC magnetic field bias and the poling electric field, based upon their respective anisotropy. Hence, a DC magnetic field bias through the thickness of the sample gives a different strain magnitude and direction than a bias along the sample plane. This is quite similar to the changes in the form of the anisotropic permeability tensor of a biased magnetic material due to changes in direction of the DC magnetic field bias. This permeability tensor shape-change described above is a secondary effect observed in ME composites when a ferrite material is used as the magnetostrictive phase in the magnetoelectric composite.

The shape of the magnetoelectric susceptibility tensor obtained depends on the direction of the stresses and strains within the composite on application of bias/poling fields. Thus, the non-zero values of the piezoelectric and piezomagnetic coefficient tensor are important as they couple the stresses and fields in the media. By solving the

constitutive relationships of the piezoelectric and piezomagnetic phases we obtain relationships for the electric displacement field and the magnetic flux density.



FIGURE 3.1. Bias DC magnetic field and poling electric field for the transverse magnetoelectric configuration in a composite bilayer. The composite is poled in the axial direction while biasing along the sample plane.

3.1 Transverse magnetoelectric fields, stresses and strains

For the transverse configuration, the composite is biased in the 1-direction with a DC magnetic field and poled along the 3-direction with an electric field. Hence, we express the fields in terms of DC and time varying components as

$$\bar{H}_{1} = \hat{1}H_{0} + \hat{1}H_{AC}, \qquad (3.1)$$

$$\overline{E}_{3} = \hat{3}E_{0} + \hat{3}E_{AC}.$$
(3.2)

In Eqs. (3.1) and (3.2), H_0 and E_0 represent the DC magnetic and electric fields respectively, while H_{AC} and E_{AC} represent the time varying fields that may be present in the composite.

The theoretical formulation for the transverse magnetoelectric configuration follows a process similar to that for the longitudinal model. The constitutive equations for the piezoelectric and piezomagnetic phases are solved independently for the fields, stresses and strains. For convenience, we give the piezoelectric constitutive relationship again as

$${}^{p}S_{i} = {}^{p}S_{ij} {}^{p}T_{j} + {}^{p}d_{ki} {}^{p}E_{k},$$
(3.3)

$${}^{p}D_{k} = {}^{p}d_{ki} {}^{p}T_{i} + {}^{p}\mathcal{E}_{kn} {}^{p}E_{n}, \qquad (3.4)$$

$${}^{p}B_{k} = {}^{p}\mu_{kn} {}^{p}H_{n}.$$
(3.5)

For the piezomagnetic phase,

$${}^{m}S_{i} = {}^{m}s_{ij}{}^{m}T_{j} + {}^{m}q_{ki}{}^{m}H_{k},$$
(3.6)

$${}^{m}B_{k} = {}^{m}q_{ki} {}^{m}T_{i} + {}^{m}\mu_{kn} {}^{m}H_{n}, \qquad (3.7)$$

$${}^{m}D_{k} = {}^{m}\mathcal{E}_{kn} {}^{m}E_{n}.$$

$$(3.8)$$

For the sake of convenience, we repeat the assumptions made on the mechanical stresses and strains within the composite layer:

1. Shear stresses and strains are equal to zero, such that

$$^{m,p}T_i = 0, \, {}^{m,p}S_i = 0, \, \text{for } i=4, \, 5, \, \text{and } 6.$$
 (3.9)

2. The thickness of each phase is much smaller than the width and length of the phase. Hence the stress in the axial direction is approximated as zero

$${}^{m}T_{3} = {}^{p}T_{3} = 0. ag{3.10}$$

3. The strain transfer between phases is related by an interface coupling parameter, k, such that

$${}^{p}S_{i} = k \cdot {}^{m}S_{i}. \tag{3.11}$$

As had been discussed in chapter 2, the reverse process is also possible with a change to the strain transfer relationship from the piezoelectric layer to the piezomagnetic layer. We do not discuss that here again, nor give the expression for the reverse transfer process, as that has already been stated.

4. The summation of forces on the in-plane (1-2 plane) boundaries are zero, such that

$${}^{m}T_{i}{}^{m}\nu + {}^{p}T_{i}{}^{p}\nu = 0, \text{ for } i = 1, 2,$$
(3.12)

where ${}^{m}v$ and ${}^{p}v$ respectively, are the piezomagnetic and piezoelectric volume fractions, and are defined as ${}^{m}v = volume^{m}/volume^{total}$ and ${}^{p}v = volume^{p}/volume^{total}$.

Solving the constitutive equations of each phase based upon the given assumptions, we obtain the electric displacement field in the piezoelectric region using the nonzero components of the permittivity, permeability, compliance and piezoelectric coefficient as shown in Table 3.1. Using the nonzero coefficients from Table 3.1 along with Eqs. (3.3), (3.5), and (3.11), we obtain expressions for the strain between phases as

$$\left({}^{p}s_{11} + {}^{p}s_{12} \right) {}^{p}T_{1} + {}^{p}d_{31} {}^{p}E_{3} = k \left({}^{m}s_{11} + {}^{m}s_{12} \right) {}^{m}T_{1} + k {}^{m}q_{11} {}^{m}H_{1},$$
 (3.13)

$$\left({}^{p}s_{21} + {}^{p}s_{22} \right) {}^{p}T_{1} + {}^{p}d_{32} {}^{p}E_{3} = k \left({}^{m}s_{21} + {}^{m}s_{22} \right) {}^{m}T_{1} + k {}^{m}q_{12} {}^{m}H_{1}.$$
 (3.14)

In formulating Eqs. (3.13) and (3.14), we assumed that the *1* and 2 directional stresses are equal, i.e. ${}^{p,m}T_1 = {}^{p,m}T_2$.

Combining Eqs. (3.13) and (3.14), and using ${}^{m,p}s_{11} = {}^{m,p}s_{22}$, ${}^{m,p}s_{12} = {}^{m,p}s_{21}$ and ${}^{p}d_{31} = {}^{p}d_{32}$, we obtain

$$2\left({}^{p}s_{11} + {}^{p}s_{12}\right){}^{p}T_{1} + 2{}^{p}d_{31}{}^{p}E_{3} = 2k\left({}^{m}s_{11} + {}^{m}s_{12}\right){}^{m}T_{1} + k\left({}^{m}q_{11} + {}^{m}q_{12}\right){}^{m}H_{1}.$$
 (3.15)

Using
$${}^{m}T_{1} = -{}^{p}T_{1}\left(\frac{{}^{p}v}{{}^{m}v}\right)$$
 from Eq. (3.12), into Eq. (3.15),

$$\left[2\left({}^{p}s_{11}+{}^{p}s_{12}\right)+2k\left({}^{m}s_{11}+{}^{m}s_{12}\right)\left({}^{p}v_{m_{v}}\right)\right]{}^{p}T_{1}=k\left({}^{m}q_{11}+{}^{m}q_{12}\right){}^{m}H_{1}-2{}^{p}d_{31}{}^{p}E_{3}.$$
(3.16)

Then solving for the stress, thus rewriting (3.16) in terms of the *1*-directional stress ${}^{p}T_{1}$ in the piezoelectric layer, we obtain

$${}^{p}T_{1} = \left(\frac{k\left({}^{m}q_{11} + {}^{m}q_{12}\right)}{\left(2\left({}^{p}s_{11} + {}^{p}s_{12}\right) + 2k\left({}^{m}s_{11} + {}^{m}s_{12}\right)\left({}^{p}\nu/{}^{m}\nu\right)\right)}\right){}^{m}H_{1} - \left(\frac{2{}^{p}d_{31}}{\left(2\left({}^{p}s_{11} + {}^{p}s_{12}\right) + 2k\left({}^{m}s_{11} + {}^{m}s_{12}\right)\left({}^{p}\nu/{}^{m}\nu\right)\right)}\right){}^{p}E_{3}.$$

$$(3.17)$$

TABLE 3.1. NONZERO COEFFICIENTS OF BULK PIEZOELECTRIC AND PIEZOMAGNETIC PHASES FOR THE TRANSVERSE CONFIGURATION

Piezoelectric phase					
Coefficient type	Non-zero components				
Permittivity	${}^{p}\boldsymbol{\mathcal{E}}_{11}={}^{p}\boldsymbol{\mathcal{E}}_{22}, {}^{p}\boldsymbol{\mathcal{E}}_{33}.$				
Permeability	${}^{p}\mu_{11} = {}^{p}\mu_{22} = {}^{p}\mu_{33}.$				
Compliance	${}^{p}s_{11} = {}^{p}s_{22}, {}^{p}s_{12} = {}^{p}s_{21}, {}^{p}s_{13} = {}^{p}s_{23} = {}^{p}s_{31} = {}^{p}S_{32},$ ${}^{p}s_{33}, {}^{p}s_{44} = {}^{p}s_{66}, {}^{p}s_{66} = 2({}^{p}s_{11} + {}^{p}s_{12}).$				
Piezoelectric	${}^{p}d_{15} = {}^{p}d_{24}, {}^{p}d_{31} = {}^{p}d_{32}, {}^{p}d_{33}.$				
Piezomagnetic phase					
Coefficient type	Non-zero components				
Permittivity	${}^{m}\mathcal{E}_{11}, {}^{m}\mathcal{E}_{22}, {}^{m}\mathcal{E}_{33}.$				
Permeability	${}^{m}\mu_{11},{}^{m}\mu_{22},{}^{m}\mu_{33}.$				
Compliance	${}^{m}s_{11} = {}^{m}s_{22} = {}^{m}s_{33}, {}^{m}s_{44} = {}^{m}s_{55} = {}^{m}s_{66},$ ${}^{m}s_{12} = {}^{m}s_{21} = {}^{m}s_{13} = {}^{m}s_{23} = {}^{m}s_{31} = {}^{m}s_{32}.$				
Piezomagnetic	${}^{m}q_{35} = {}^{m}q_{26}, {}^{m}q_{12} = {}^{m}q_{13}, {}^{m}q_{11}.$				

From the constitutive relationship expressed in Eq. (3.4), we obtain the components of the electric field displacement in the piezoelectric phase as

$${}^{p}D_{1} = {}^{p}\mathcal{E}_{11} {}^{p}E_{1}, \qquad (3.18)$$

$${}^{p}D_{2} = {}^{p}\mathcal{E}_{22} {}^{p}E_{2}, \tag{3.19}$$

$${}^{p}D_{3} = 2 \cdot {}^{p}d_{31} {}^{p}T_{1} + {}^{p}\mathcal{E}_{33} {}^{p}E_{3}.$$
(3.20)

Substituting Eq. (3.17) into Eq. (3.20), we obtain the coupled *3*-directional component of the electric field displacement in the piezoelectric phase as

$${}^{p}D_{3} = \left(\frac{k^{p}d_{31}\left({}^{m}q_{11} + {}^{m}q_{12}\right)}{\left(\left({}^{p}s_{11} + {}^{p}s_{12}\right) + k\left({}^{m}s_{11} + {}^{m}s_{12}\right)\left({}^{p}\nu/{}^{m}\nu\right)\right)}\right)^{m}H_{1} + \left(\frac{-2\left({}^{p}d_{31}\right)^{2}}{\left(\left({}^{p}s_{11} + {}^{p}s_{12}\right) + k\left({}^{m}s_{11} + {}^{m}s_{12}\right)\left({}^{p}\nu/{}^{m}\nu\right)\right)} + {}^{p}\mathcal{E}_{33}\right)^{p}E_{3}.$$

$$(3.21)$$

Now, we consider the magnetic flux density of the piezomagnetic phase. Two cases are considered here as was done earlier in the previous chapter for the longitudinal configuration. For the first case, the DC magnetic field bias creates no secondary effect, such that there is no change in the form of the permeability matrix with the application of the DC magnetic field. In the second case, the piezomagnetic material can be a media with magnetic moments, such as a ferrite. In the later case, we obtain magnetic anisotropy.

3.1.1 Case 1: Non-gyrotropic composites

We will use the form of the nonzero values of the permeability as shown in Table 3.1.

$${}^{m}B_{1} = {}^{m}q_{11}{}^{m}T_{1} + {}^{m}q_{12}{}^{m}T_{2} + {}^{m}\mu_{11}{}^{m}H_{1}, \qquad (3.22)$$

$${}^{m}B_{2} = {}^{m}\mu_{22} {}^{m}H_{2}, \tag{3.23}$$

$${}^{m}B_{3} = {}^{m}\mu_{33} {}^{m}H_{3}. \tag{3.24}$$

Using ${}^{p}T_{1} = -{}^{m}T_{1}\left(\frac{{}^{m}v}{{}^{p}v}\right)$, from Eq. (3.12), we obtain the piezomagnetic stress from Eq.

(3.15). Hence the stress-field relationship is obtained as

$$\left[2k\left({}^{m}s_{11}+{}^{m}s_{12}\right)+2\left({}^{p}s_{11}+{}^{p}s_{12}\right)\left({}^{m}v_{p}\right)\right]{}^{m}T_{1}=2{}^{p}d_{31}{}^{p}E_{3}-k\left({}^{m}q_{11}+{}^{m}q_{12}\right){}^{m}H_{1},$$
(3.25)

From which we obtain the stress in the composite layer as

$${}^{m}T_{1} = \left(\frac{{}^{p}d_{31}}{\left(k\left({}^{m}s_{11} + {}^{m}s_{12}\right) + \left({}^{p}s_{11} + {}^{p}s_{12}\right)\left({}^{m}\nu/{}^{p}\nu\right)\right)}\right){}^{p}E_{3} - \left(\frac{k\left({}^{m}q_{11} + {}^{m}q_{12}\right)}{2\left(k\left({}^{m}s_{11} + {}^{m}s_{12}\right) + \left({}^{p}s_{11} + {}^{p}s_{12}\right)\left({}^{m}\nu/{}^{p}\nu\right)\right)}\right){}^{m}H_{1}.$$

$$(3.26)$$

Applying Eq. (3.26) into Eq. (3.22), the magnetic flux is expresses as

$${}^{m}B_{1} = \left(\frac{{}^{p}d_{31}\left({}^{m}q_{11} + {}^{m}q_{12}\right)}{\left(k\left({}^{m}s_{11} + {}^{m}s_{12}\right) + \left({}^{p}s_{11} + {}^{p}s_{12}\right)\left({}^{m}\nu/{}^{p}\nu\right)\right)}\right){}^{p}E_{3} + \left(\frac{-k\left({}^{m}q_{11} + {}^{m}q_{12}\right)^{2}}{2\left(k\left({}^{m}s_{11} + {}^{m}s_{12}\right) + \left({}^{p}s_{11} + {}^{p}s_{12}\right)\left({}^{m}\nu/{}^{p}\nu\right)\right)} + {}^{m}\mu_{11}\right){}^{m}H_{1}$$

$$(3.27)$$

Thus, we have the electric displacement fields in both composite layers as

$${}^{p}D_{1} = {}^{p}\varepsilon_{11}{}^{p}E_{1}$$
 (3.28a) ${}^{m}D_{1} = {}^{m}\varepsilon_{11}{}^{m}E_{1}$ (3.29a)

$${}^{p}D_{2} = {}^{p}\mathcal{E}_{22} {}^{p}E_{2}$$
 (3.28b) ${}^{m}D_{2} = {}^{m}\mathcal{E}_{22} {}^{m}E_{2}$ (3.29b)

$${}^{p}D_{3} = W_{1}{}^{m}H_{1} + W_{2}{}^{p}E_{3}$$
 (3.28c) ${}^{m}D_{3} = {}^{m}\varepsilon_{33}{}^{m}E_{3}$ (3.29c)

where,

$$W_{1} = \left(\frac{k^{p} d_{31}(^{m} q_{11} + ^{m} q_{12})}{\left(\left({^{p} s_{11} + ^{p} s_{12}}\right) + k\left({^{m} s_{11} + ^{m} s_{12}}\right)\left({^{p} \nu}/{^{m} \nu}\right)\right)}\right),$$
(3.30)

$$W_{2} = \left(\frac{-2({}^{p}d_{31})^{2}}{\left(\left({}^{p}s_{11} + {}^{p}s_{12}\right) + k\left({}^{m}s_{11} + {}^{m}s_{12}\right)\left({}^{p}\nu/{}^{m}\nu\right)\right)} + {}^{p}\mathcal{E}_{33}\right).$$
(3.31)

The magnetic flux density in both composite layers are expressed as

$${}^{p}B_{1} = {}^{p}\mu_{11} {}^{p}H_{1} \qquad (3.32a) \qquad {}^{m}B_{1} = L_{1} {}^{p}E_{3} + L_{2} {}^{m}H_{1} \qquad (3.33a)$$

$${}^{p}B_{2} = {}^{p}\mu_{22} {}^{p}H_{2}$$
 (3.32b) ${}^{m}B_{2} = {}^{m}\mu_{22} {}^{m}H_{2}$ (3.34b)

$${}^{p}B_{2} = {}^{p}\mu_{22} {}^{p}H_{2}$$
 (3.32c) ${}^{m}B_{3} = {}^{m}\mu_{33} {}^{m}H_{3}$ (3.35c)

where,

$$L_{1} = \left(\frac{{}^{p}d_{31}\left({}^{m}q_{11} + {}^{m}q_{12}\right)}{\left(k\left({}^{m}s_{11} + {}^{m}s_{12}\right) + \left({}^{p}s_{11} + {}^{p}s_{12}\right)\left({}^{m}\nu/{}^{p}\nu\right)\right)}\right),$$
(3.36)

$$L_{2} = \left(\frac{-k\left({}^{m}q_{11} + {}^{m}q_{12}\right)^{2}}{2\left(k\left({}^{m}s_{11} + {}^{m}s_{12}\right) + \left({}^{p}s_{11} + {}^{p}s_{12}\right)\left({}^{m}\nu/{}^{p}\nu\right)\right)} + {}^{m}\mu_{11}\right)$$
(3.37)

Hence, we have all the fields for a non-gyrotropic material used as the piezomagnetic phase. In subsequent sections, we will apply fundamental EM boundary conditions.

3.1.2 Case 2: Gyrotropic composites

We use a new permeability tensor for the piezomagnetic phase. Considering a DC magnetic bias in the 1-direction for a gyrotropic material such as a ferrite, the permeability tensor is given as

$${}^{m}\mu = \begin{bmatrix} {}^{m}\mu_{11} & 0 & 0 \\ 0 & {}^{m}\mu_{22} & {}^{m}\mu_{23} \\ 0 & {}^{m}\mu_{32} & {}^{m}\mu_{33} \end{bmatrix}$$
(3.38)

Thus, we rewrite the magnetic flux density in terms of the magnetic field and stresses

$${}^{m}B_{1} = {}^{m}q_{11}{}^{m}T_{1} + {}^{m}q_{12}{}^{m}T_{2} + {}^{m}\mu_{11}{}^{m}H_{1}, \qquad (3.39)$$

$${}^{m}B_{2} = {}^{m}\mu_{22} {}^{m}H_{2} + {}^{m}\mu_{23} {}^{m}H_{3}, \qquad (3.40)$$

$${}^{m}B_{3} = {}^{m}\mu_{32} {}^{m}H_{2} + {}^{m}\mu_{33} {}^{m}H_{3}, \qquad (3.41)$$

There are no changes to Eqs. (3.28) - (3.29), as the change in the form of the permeability does not affect those expressions. Thus, the expression for the electric field displacements in both regions remains the same, as does the expressions for the magnetic flux density in the piezoelectric region. Thus the magnetic flux density for the piezomagnetic region is now

$${}^{m}B_{1} = L_{1}{}^{p}E_{3} + L_{2}{}^{m}H_{1}, aga{3.42}$$

$${}^{m}B_{2} = {}^{m}\mu_{22} {}^{m}H_{2} + {}^{m}\mu_{23} {}^{m}H_{3}, \qquad (3.43)$$

$${}^{m}B_{3} = {}^{m}\mu_{32} {}^{m}H_{2} + {}^{m}\mu_{33} {}^{m}H_{3}, \qquad (3.44)$$

where,

$$L_{1} = \left(\frac{{}^{p}d_{31}\left({}^{m}q_{11} + {}^{m}q_{12}\right)}{\left(k\left({}^{m}s_{11} + {}^{m}s_{12}\right) + \left({}^{p}s_{11} + {}^{p}s_{12}\right)\left({}^{m}\nu/{}^{p}\nu\right)\right)}\right),$$
(3.45)

$$L_{2} = \left(\frac{-k\left({}^{m}q_{11} + {}^{m}q_{12}\right)^{2}}{2\left(k\left({}^{m}s_{11} + {}^{m}s_{12}\right) + \left({}^{p}s_{11} + {}^{p}s_{12}\right)\left({}^{m}\nu/{}^{p}\nu\right)\right)} + {}^{m}\mu_{_{11}}^{*}\right).$$
(3.46)

Next, we apply fundamental EM boundary conditions to the fields at the boundary of the composite layers.

3.2 Application of boundary conditions

As done for the longitudinal magnetoelectric configuration, we apply boundary conditions on the fields along the interface of the composite layers. The boundary conditions have already been introduced in Section 2.2.

3.2.1 Case 1: Non-gyrotropic composites

Tangential components of the electric and magnetic fields are continuous at the boundary, which implies that

$${}^{m}E_{1} = {}^{p}E_{1} = E_{x} \tag{3.47}$$

$${}^{m}E_{2} = {}^{p}E_{2} = E_{y} \tag{3.48}$$

$${}^{m}H_{1} = {}^{p}H_{1} = H_{x} \tag{3.49}$$

$${}^{m}H_{2} = {}^{p}H_{2} = H_{y} \tag{3.50}$$

In Eqs. (3.47) – (3.50) above, E_x , E_y , H_x , and H_y represent the homogenized components of the electric and magnetic field components in the media.

It is assumed that each composite layer is electrically thin, with negligible field variation within each individual layer, using a field averaging method [41, 42], we define the tangential components of the electric field displacement and the magnetic flux density as

$$D_{x} = {}^{m}D_{1}{}^{m}v + {}^{p}D_{1}{}^{p}v = \left[{}^{m}\varepsilon_{11}{}^{m}E_{1}{}^{m}v + {}^{p}\varepsilon_{11}{}^{p}E_{1}{}^{p}v\right] = \left[{}^{m}\varepsilon_{11}{}^{m}v + {}^{p}\varepsilon_{11}{}^{p}v\right]E_{x},$$
(3.51)

$$D_{y} = {}^{m}D_{2} {}^{m}v + {}^{p}D_{2} {}^{p}v = \left[{}^{m}\varepsilon_{22} {}^{m}E_{2} {}^{m}v + {}^{p}\varepsilon_{22} {}^{p}E_{2} {}^{p}v \right] = \left[{}^{m}\varepsilon_{22} {}^{m}v + {}^{p}\varepsilon_{22} {}^{p}v \right] E_{y},$$
(3.52)

$$B_{x} = {}^{m}B_{1}{}^{m}v + {}^{p}B_{1}{}^{p}v = \left[\left(L_{1}{}^{p}E_{3} + L_{2}{}^{m}H_{1} \right){}^{m}v + {}^{p}\mu_{11}{}^{p}H_{1}{}^{p}v \right]$$
$$= L_{1}{}^{p}E_{3}{}^{m}v + \left(L_{2}{}^{m}v + {}^{p}\mu_{11}{}^{p}v \right) H_{x}, \qquad (3.53)$$

$$B_{y} = {}^{m}B_{2} {}^{m}v + {}^{p}B_{2} {}^{p}v = \left[{}^{m}\mu_{22} {}^{m}H_{2} {}^{m}v + {}^{p}\mu_{22} {}^{p}H_{2} {}^{p}v\right] = \left[{}^{m}\mu_{22} {}^{m}v + {}^{p}\mu_{22} {}^{p}v\right]H_{y}$$
(3.54)

As should be expected, the homogenized magnetic flux density in the x-direction will depend on the piezoelectric electric field in the 3-direction. We will simplify Eq. (3.53) eliminate the 3-directional term when we obtain homogenized expressions for the electric and magnetic fields in the 3-direction.

Applying a second boundary condition, normal components of the magnetic flux density and the electric field displacement are continuous across the boundary, we obtain,

$${}^{m}D_{3} = {}^{p}D_{3} = D_{z}$$
(3.55)

$${}^{m}B_{3} = {}^{p}B_{3} = B_{z} \tag{3.56}$$

Using the field expressions for the 3-directed magnetic flux density and electric displacement fields,

$${}^{m}\mathcal{E}_{33}{}^{m}E_{3} = W_{1}{}^{m}H_{1} + W_{2}{}^{p}E_{3}$$
(3.57)

$${}^{m}\mu_{33}{}^{m}H_{3} = {}^{p}\mu_{33}{}^{p}H_{3}$$
(3.58)

Assuming the each layer is electrically thin, with no field variation within each individual composite layer, we obtain homogenized field components

$$E_{z} = {}^{p}E_{3} {}^{p}v + {}^{m}E_{3} {}^{m}v$$
(3.59)

$$H_{z} = {}^{p}H_{3} {}^{p}v + {}^{m}H_{3} {}^{m}v$$
(3.60)

Thus,

$${}^{p}E_{3} = \frac{1}{{}^{p}v} \left(E_{z} - {}^{m}E_{3} {}^{m}v \right)$$
(3.61)

Using Eq. (3.51) and Eq. (3.55), we express the z-component of the electric displacement as

$$D_{z} = W_{1}^{m} H_{1} + \frac{W_{2}}{m_{v}} \left(E_{z} + {}^{m} E_{3}^{m} v \right)$$
(3.62)

From Eq. (3.57), ${}^{m}E_{3} = D_{z} / {}^{m}\mathcal{E}_{33}$, and thus

$$D_{z} = W_{1}^{m} H_{1} + \frac{W_{2}}{p_{v}} E_{z} - \frac{W_{2}^{m} v}{p_{v}^{m} \mathcal{E}_{33}} D_{z}$$
(3.63)

Thus, the homogenized normal electric field displacement is obtained as

$$\left(1 + \frac{W_2^{m}v}{{}^{p}v^{m}\varepsilon_{33}}\right)D_z = W_1^{m}H_1 + \frac{W_2}{{}^{p}v}E_z$$

Simplifying further, we obtain the z-component of the electric displacement field as

$$D_{z} = \left(\frac{W_{1}^{p} v^{m} \varepsilon_{33}}{W_{2}^{m} v + {}^{m} \varepsilon_{33}^{p} v}\right) H_{x} + \left(\frac{W_{2}^{m} \varepsilon_{33}}{W_{2}^{m} v + {}^{m} \varepsilon_{33}^{p} v}\right) E_{z}$$
(3.64)

The homogenized normal magnetic flux density is obtained as follows:

$$B_{z} = {}^{m}\mu_{33}{}^{m}H_{3} \tag{3.65}$$

Using the boundary condition as represented in Eq. (3.58), we have

$${}^{p}H_{3} = \frac{{}^{m}\mu_{33}}{{}^{p}\mu_{33}}{}^{m}H_{3}$$
(3.66)

Applying Eq. (3.66) in Eq. (3.60), then we obtain the magnetic field as

$$H_{z} = \left({}^{p} v \frac{{}^{m} \mu_{33}}{{}^{p} \mu_{33}} + {}^{m} v \right) {}^{m} H_{3}$$
(3.67)

Solving Eq. (3.67) for ${}^{m}H_{3}$, we have

$${}^{m}H_{3} = \left(\frac{{}^{p}\mu_{33}}{{}^{p}v^{m}\mu_{33} + {}^{m}v^{p}\mu_{33}}\right)H_{z}$$
(3.68)

Thus into Eq. (3.65), we obtain the z-component of the magnetic flux density as

$$B_{z} = \left(\frac{{}^{m}\mu_{33}{}^{p}\mu_{33}}{{}^{p}\nu^{m}\mu_{33} + {}^{m}v^{p}\mu_{33}}\right)H_{z}$$
(3.69)

We have obtained the homogenized z-components of the magnetic flux density and electric displacement field. Now, we simplify the homogenized magnetic flux density for the *x*-components. The magnetic flux density in Eq. (3.47) depends on the *3*-directional electric field in the piezoelectric phase, with the expression

$$B_{x} = L_{1}^{p} E_{3}^{m} v + (L_{2}^{m} v + {}^{p} \mu_{11}^{p} v) H_{x}$$

From Eq. (3.61), we have that

$${}^{p}E_{3} = \frac{1}{{}^{p}v} \left(E_{z} - {}^{m}E_{3} {}^{m}v \right)$$

Applying Eq. (3.55), ${}^{m}E_{3} = D_{z} / {}^{m}\mathcal{E}_{33}$ thus

$${}^{p}E_{3} = \frac{1}{{}^{p}v} \left(E_{z} - \frac{D_{z}}{{}^{m}\mathcal{E}_{33}} {}^{m}v \right) = \left[\frac{E_{z}}{{}^{p}v} - \left(\frac{W_{1}{}^{p}v{}^{m}v}{{}^{p}v\left(W_{2}{}^{m}v + {}^{m}\mathcal{E}_{33}{}^{p}v\right)} \right) H_{x} - \left(\frac{W_{2}{}^{m}v}{{}^{p}v\left(W_{2}{}^{m}v + {}^{m}\mathcal{E}_{33}{}^{p}v\right)} \right) E_{z} \right].$$

$$(3.70)$$

Simplifying the expression in Eq. (3.70), we then obtain

$${}^{p}E_{3} = \left[\left(\frac{1}{{}^{p}v} - \frac{W_{2}{}^{m}v}{{}^{p}v \left(W_{2}{}^{m}v + {}^{m}\varepsilon_{33}{}^{p}v \right)} \right) E_{z} - \left(\frac{W_{1}{}^{m}v}{\left(W_{2}{}^{m}v + {}^{m}\varepsilon_{33}{}^{p}v \right)} \right) H_{x} \right].$$
(3.71)

Simplifying further, we have

$${}^{p}E_{3} = \left[\left(\frac{W_{2}^{m}v + {}^{m}\mathcal{E}_{33}^{p}v - W_{2}^{m}v}{{}^{p}v \left(W_{2}^{m}v + {}^{m}\mathcal{E}_{33}^{p}v \right)} \right) E_{z} - \left(\frac{W_{1}^{m}v}{\left(W_{2}^{m}v + {}^{m}\mathcal{E}_{33}^{p}v \right)} \right) H_{x} \right].$$
(3.72)

Factorizing the terms of Eq. (3.72), we obtain an expression for the *3*-directional electric field in the piezoelectric region as

$${}^{p}E_{3} = \left[\left(\frac{{}^{m}\varepsilon_{33}}{\left(W_{2} {}^{m}v + {}^{m}\varepsilon_{33} {}^{p}v \right)} \right) E_{z} - \left(\frac{W_{1} {}^{m}v}{\left(W_{2} {}^{m}v + {}^{m}\varepsilon_{33} {}^{p}v \right)} \right) H_{x} \right]$$
(3.73)

Finally, we express the homogenized x-components of the magnetic flux density as

$$B_{x} = L_{1}^{m} v \left[\left(\frac{{}^{m} \varepsilon_{33}}{\left(W_{2}^{m} v + {}^{m} \varepsilon_{33}^{p} v \right)} \right) E_{z} - \left(\frac{W_{1}^{m} v}{\left(W_{2}^{m} v + {}^{m} \varepsilon_{33}^{p} v \right)} \right) H_{x} \right] + \left(L_{2}^{m} v + {}^{p} \mu_{11}^{p} v \right) H_{x}$$

Simplifying the expression for the x-component magnetic flux above, we have

$$B_{x} = \left(\frac{L_{1}^{m} v^{m} \varepsilon_{33}}{W_{2}^{m} v + {}^{m} \varepsilon_{33}^{p} v}\right) E_{z} + \left(L_{2}^{m} v + {}^{p} \mu_{11}^{p} v - \frac{L_{1}^{m} v W_{1}^{m} v}{W_{2}^{m} v + {}^{m} \varepsilon_{33}^{p} v}\right) H_{x}$$
(3.74)

Hence, we have obtained all the material parameters for the homogenized composite media for the case where the piezomagnetic phase is a non-gyrotropic material. The constitutive relationship for the homogenized media in terms of the electromagnetic fields is given as

$$\begin{bmatrix} D_{x} \\ D_{y} \\ D_{z} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \alpha_{zx}^{H} & 0 & 0 \end{bmatrix} \begin{bmatrix} H_{x} \\ H_{y} \\ H_{z} \end{bmatrix} + \begin{bmatrix} \varepsilon_{xx} & 0 & 0 \\ 0 & \varepsilon_{yy} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{bmatrix} \begin{bmatrix} E_{x} \\ E_{z} \\ E_{z} \end{bmatrix},$$
(3.75)
$$\begin{bmatrix} B_{x} \\ B_{y} \\ B_{z} \end{bmatrix} = \begin{bmatrix} 0 & 0 & \alpha_{xz}^{E} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} E_{x} \\ E_{y} \\ E_{z} \end{bmatrix} + \begin{bmatrix} \mu_{xx} & 0 & 0 \\ 0 & \mu_{yy} & 0 \\ 0 & 0 & \mu_{zz} \end{bmatrix} \begin{bmatrix} H_{x} \\ H_{y} \\ H_{z} \end{bmatrix}.$$
(3.76)

As we have explained in chapter 2, the above constitutive relationships apply only to magnetoelectric composites using piezoelectric and piezomagnetic layers. The magnetoelectric effect arising from the use of piezoelectric and magnetostrictive layers will only be observed in the magnetic field induced magnetoelectric susceptibility α^{H} only.

3.2.2 Case 2: Gyrotropic composites

The change in the derivation from case 1 stems from the application of a DC magnetic bias to a media with magnetic moments, and a significant saturation magnetization to induce magnetic anisotropy. For the transverse orientation, with the DC magnetic bias in the 1-direction, we obtain a permeability of the form [24, 25]

$${}^{m}\mu = \begin{bmatrix} {}^{m}\mu_{11} & 0 & 0 \\ 0 & {}^{m}\mu_{22} & {}^{m}\mu_{23} \\ 0 & {}^{m}\mu_{32} & {}^{m}\mu_{33} \end{bmatrix}$$
(3.38)

We apply boundary conditions to obtain homogenized fields for the structure as was done in case 1. Keeping in mind that, tangential components of the electric and magnetic fields are continuous at the boundary, we obtain

$$D_{x} = {}^{m}D_{1}{}^{m}v + {}^{p}D_{1}{}^{p}v = \left[{}^{m}\varepsilon_{11}{}^{m}E_{1}{}^{m}v + {}^{p}\varepsilon_{11}{}^{p}E_{1}{}^{p}v\right] = \left[{}^{m}\varepsilon_{11}{}^{m}v + {}^{p}\varepsilon_{11}{}^{p}v\right]E_{x},$$
(3.51)
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$$D_{y} = {}^{m}D_{2} {}^{m}v + {}^{p}D_{2} {}^{p}v = \left[{}^{m}\varepsilon_{22} {}^{m}E_{2} {}^{m}v + {}^{p}\varepsilon_{22} {}^{p}E_{2} {}^{p}v \right] = \left[{}^{m}\varepsilon_{22} {}^{m}v + {}^{p}\varepsilon_{22} {}^{p}v \right] E_{y}, \qquad (3.52)$$
$$B_{x} = {}^{m}B_{1} {}^{m}v + {}^{p}B_{1} {}^{p}v = \left[\left(L_{1} {}^{p}E_{3} + L_{2} {}^{m}H_{1} \right) {}^{m}v + {}^{p}\mu_{11} {}^{p}H_{1} {}^{p}v \right]$$
$$= L_{1} {}^{p}E_{3} {}^{m}v + \left(L_{2} {}^{m}v + {}^{p}\mu_{11} {}^{p}v \right) H_{x}, \qquad (3.53)$$

However, the y-component of the magnetic flux density changes in form, and is given as

$$B_{y} = {}^{m}B_{2} {}^{m}v + {}^{p}B_{2} {}^{p}v = \left[{}^{m}\mu_{22}^{*} {}^{m}H_{2} {}^{m}v + {}^{m}\mu_{23}^{*} {}^{m}H_{3} {}^{m}v + {}^{p}\mu_{22} {}^{p}H_{2} {}^{p}v \right]$$
$$= \left[{}^{m}\mu_{22}^{*} {}^{m}v + {}^{p}\mu_{22} {}^{p}v \right]H_{y} + {}^{m}\mu_{23}^{*} {}^{m}v {}^{m}H_{3}$$
(3.77)

Applying a second boundary condition, normal components of the magnetic flux density and the electric field displacement are continuous across the boundary. Thus,

$${}^{m}D_{3} = {}^{p}D_{3} = D_{z}, \tag{3.55}$$

$${}^{m}B_{3} = {}^{p}B_{3} = B_{z}. aga{3.56}$$

From Eq. (3.55) and (3.56), we obtain the expressions,

$${}^{m}\mathcal{E}_{33}{}^{m}E_{3} = W_{1}{}^{m}H_{1} + W_{2}{}^{p}E_{3} = D_{z}$$
(3.78)

$${}^{m}\mu_{_{32}}^{*}{}^{m}H_{2} + {}^{m}\mu_{_{33}}^{*}{}^{m}H_{3} = {}^{p}\mu_{_{33}}{}^{p}H_{3} = B_{z}$$
(3.79)

Assuming each composite layer is electrically thin, without any field variation within each individual composite phase, we obtain homogenized field components

$$E_{z} = {}^{p}E_{3} {}^{p}v + {}^{m}E_{3} {}^{m}v, ag{3.59}$$

$$H_{z} = {}^{p}H_{3} {}^{p}v + {}^{m}H_{3} {}^{m}v.$$
(3.60)

The homogenization process for the electric displacement field is similar to that for the non-gyrotropic ME media case, and results in a similar homogenized electric displacement field in the normal direction

$$D_{z} = \left(\frac{W_{1}^{m} v^{m} \varepsilon_{33}}{W_{2}^{m} v + {}^{m} \varepsilon_{33}^{p} v}\right) H_{x} + \left(\frac{W_{2}^{m} \varepsilon_{33}}{W_{2}^{m} v + {}^{m} \varepsilon_{33}^{p} v}\right) E_{z}.$$
 (3.64)

We expect changes in the form of the magnetic flux density obtained, and therefore show some steps. The homogenized magnetic flux density in the normal direction is obtained using Eqs. (3.56) and (3.79)

Hence,

$${}^{p}H_{3} = \frac{{}^{m}\mu_{32}^{*}}{{}^{p}\mu_{33}}{}^{m}H_{2} + \frac{{}^{m}\mu_{33}^{*}}{{}^{p}\mu_{33}}{}^{m}H_{3}$$
(3.80)

Applying Eq. (3.80) into Eq. (3.79), also noting ${}^{m}H_{2} = {}^{p}H_{2} = H_{y}$ from Eq. (3.50)

$$H_{z} = {}^{p} v \frac{{}^{m} \mu_{_{32}}^{*}}{{}^{p} \mu_{_{33}}} H_{y} + \left({}^{p} v \frac{{}^{m} \mu_{_{33}}^{*}}{{}^{p} \mu_{_{33}}} + {}^{m} v \right) {}^{m} H_{3}.$$
(3.81)

Solving for ${}^{m}H_{3}$

$${}^{m}H_{3} = \left(\frac{{}^{p}\mu_{33}}{{}^{p}v^{m}\mu_{33}^{*} + {}^{m}v^{p}\mu_{33}}\right)H_{z} - \left(\frac{{}^{p}\mu_{33}}{{}^{p}v^{m}\mu_{33}^{*} + {}^{m}v^{p}\mu_{33}} \cdot \frac{{}^{p}v^{m}\mu_{32}^{*}}{{}^{p}\mu_{33}}\right)H_{y},$$

simplifying further,

$${}^{m}H_{3} = \left(\frac{{}^{p}\mu_{33}}{{}^{p}v^{m}\mu_{33}^{*} + {}^{m}v^{p}\mu_{33}}\right)H_{z} - \left(\frac{{}^{p}v^{m}\mu_{32}^{*}}{{}^{p}v^{m}\mu_{33}^{*} + {}^{m}v^{p}\mu_{33}}\right)H_{y}$$
(3.82)

Thus into Eq. (3.79), we obtain the z-component of the magnetic flux density as

$$B_{z} = {}^{m}\mu_{32}^{*}{}^{m}H_{2} + {}^{m}\mu_{33}^{*}{}^{m}H_{3}$$

Substituting for the 2 and 3 directional components of the magnetic field, we have

$$B_{z} = \left({}^{m}\mu_{32}^{*} - \frac{{}^{p}v \cdot {}^{m}\mu_{33}^{*} \cdot {}^{m}\mu_{33}}{{}^{p}v \cdot {}^{m}\mu_{33}^{*} + {}^{m}v \cdot {}^{p}\mu_{33}}\right) H_{y} + \left(\frac{{}^{m}\mu_{33}^{*} \cdot {}^{p}\mu_{33}}{{}^{p}v^{m} \cdot \mu_{33}^{*} + {}^{m}v \cdot {}^{p}\mu_{33}}\right) H_{z}$$
(3.83)
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As done in case 1, we rewrite Eq. (3.53) for the magnetic anisotropy case as

$$B_{x} = \left(\frac{L_{1}^{m} v^{m} \varepsilon_{33}}{W_{2}^{m} v + {}^{m} \varepsilon_{33}{}^{p} v}\right) E_{z} + \left(L_{2}^{m} v + {}^{p} \mu_{11}{}^{p} v - \frac{L_{1}^{m} v W_{1}{}^{m} v}{W_{2}^{m} v + {}^{m} \varepsilon_{33}{}^{p} v}\right) H_{x}$$
(3.84)

We also rewrite Eq. (3.77) in terms of the homogenized fields,

$$B_{y} = \left({}^{m}v^{m}\mu_{22}^{*} + {}^{p}v^{p}\mu_{22} - \frac{{}^{m}v^{p}v^{m}\mu_{23}^{*} {}^{m}\mu_{32}}{{}^{p}v^{m}\mu_{33}^{*} + {}^{m}v^{p}\mu_{33}}\right)H_{y} + \left(\frac{{}^{m}v^{m}\mu_{23}^{*} {}^{p}\mu_{33}}{{}^{p}v^{m}\mu_{33}^{*} + {}^{m}v^{p}\mu_{33}}\right)H_{z}$$
(3.85)

The constitutive equations for the composites under the transverse ME orientation is given as

$$\begin{bmatrix} D_{x} \\ D_{y} \\ D_{z} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \alpha_{zx}^{H} & 0 & 0 \end{bmatrix} \begin{bmatrix} H_{x} \\ H_{y} \\ H_{z} \end{bmatrix} + \begin{bmatrix} \varepsilon_{xx} & 0 & 0 \\ 0 & \varepsilon_{yy} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{bmatrix} \begin{bmatrix} E_{x} \\ E_{z} \\ E_{z} \end{bmatrix}$$
(3.86)
$$\begin{bmatrix} B_{x} \\ B_{y} \\ B_{z} \end{bmatrix} = \begin{bmatrix} 0 & 0 & \alpha_{zz}^{E} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} E_{x} \\ E_{y} \\ E_{z} \end{bmatrix} + \begin{bmatrix} \mu_{xx} & 0 & 0 \\ 0 & \mu_{yy} & \mu_{yz} \\ 0 & \mu_{zy} & \mu_{zz} \end{bmatrix} \begin{bmatrix} H_{x} \\ H_{y} \\ H_{z} \end{bmatrix}$$
(3.87)

This finalizes the theoretical model for the transverse magnetoelectric effect in piezoelectric and piezomagnetic layers. Equations (3.86) and (3.87) only apply to piezoelectric and piezomagnetic bilayers, where the magnetoelectric effect is obtained in either direction. We analyze the effects as done previously, and also compare to other published theoretical models using specific materials for each phase.

3.3 Analysis of theoretical transverse magnetoelectric model

As previously mentioned, the ME effect is expressed in terms of the ME voltage coefficient, α' , for most experimental measurements, rather than as the magnetoelectric susceptibility. The magnetoelectric susceptibility and magnetoelectric voltage are 3 by 3 matrixes; hence simple matrix multiplication is used to convert from susceptibility to voltage coefficient. Comparison to other theoretical models in the literature is then carried out to ascertain the differences and similarities between the work here and other results.

The material properties available in the literature are for magnetostrictive materials. Hence, we analyze the theoretical model for the special case of piezoelectric and magnetostrictive layers. The material characteristics of the piezoelectric and magnetostrictive phases are shown in Table 3.2. Again, as was done in Chapter 2, we only analyze the magnetoelectric effect arising from the coupling of the magnetic and electric fields in the relationship for the electric displacement field. For magnetostrictive materials, the electric field is not coupled to the magnetic field in the expression for the magnetic flux density. Similarly to the analyses in Chapter 2, we present our theoretical model for the transverse ME configuration using the magnetoelectric voltage coefficient, rather than with the magnetoelectric susceptibility that was derived. This makes for easier comparison to previous theoretical models in the literature. The theoretical model obtained allows for detailed analysis of the ME effect in the composite structure. We show the magnetoelectric voltage coefficients for several piezoelectric and magnetostrictive layers.

Parameter (Units)	CFO	NFO	PZT-4	PZT-5H
$S_{11}\left(10^{-12}\frac{m^2}{N}\right)$	6.5	6.5	12.3	16.5
$S_{12}\left(10^{-12}\frac{m^2}{N}\right)$	-2.37	-2.37	-4.05	-4.78
$q_{12}\left(10^{-12}\frac{m}{A}\right)$	566	125	-	-
$q_{11}\left(10^{-12}\frac{m}{A}\right)$	-1880	-680	-	-
$d_{31}\left(10^{-12}\frac{C}{N}\right)$	-	-	-123	-274
$d_{33}\left(10^{-12}\frac{C}{N}\right)$	-	-	289	593
$\frac{\mu_{_{33}}}{\mu_{_0}}$	2	3	1	1
$rac{{oldsymbol{\mathcal{E}}}_{33}}{{oldsymbol{\mathcal{E}}}_{0}}$	10	10	1300	3400

TABLE 3.2. MATERIAL PARAMETERS FOR MAGNETOSTRICTIVE AND PIEZOELECTRICPHASES USED TO ANALYZE THE TRANSVERSE MAGNETOELECTRIC VOLTAGE COEFFICIENT.



FIGURE 3.2. Result for the Transverse ME effect for CFO/PZT-4 bilayer. The transverse ME effect for the CFO/PZT-4 bilayer is much higher for the transverse orientation than for the longitudinal orientation. Less than perfect interface coupling parameter requires more volume of the piezoelectric phase to achieve maximum ME voltage coefficient.

We start with the analysis of the ME voltage coefficient for a CFO/PZT-4 bilayer. The result for the bilayer is shown in Fig. 3.2. We observe a shift towards higher piezoelectric volume fractions as the coupling between layers deteriorates. From the result, we observe that piezoelectric-rich volumes should be used in composites where the interface coupling is much less than perfect. We obtain a maximum ME voltage coefficient of 460 mV/cmOe, with a perfect coupling at the interface. The ME voltage coefficient obtained for the CFO-PZT-4 bilayer using the transverse orientation is much greater than 144 mV/cmOe, as obtained for the same bilayer using the longitudinal orientation [40]. This is due to the enhanced piezomagnetic coupling in the magnetostrictive phase where there is a combination of q_{11} and q_{12} while deriving the ME susceptibility. This result follows same trends that have been established in the literature [22] with the only difference being that the magnitude of the ME voltage coefficients are not as large as had been obtained.



FIGURE 3.3. Transverse ME effect for a NFO/PZT-4 bilayer. Maximum ME voltage coefficient is 155 mV/cmOe for a perfect interface coupling. Richer piezoelectric composites are required if the interface coupling factor is less than perfect, as we observe the shift of the maximum ME voltage coefficient to PZT-rich compositions.

Using the theoretical models obtained, we also investigate the ME effect with changes to the materials that constitute the piezoelectric and magnetostrictive phases. We start with a change to the magnetostrictive phase of the ME bilayer. We replace the CFO layer with NFO. With a change to the magnetostrictive phase, from CFO to NFO, we observe a reduction in the ME voltage coefficient obtained. The ME voltage coefficient for the NFO/PZT-4 bilayer is shown in Fig. 3.3.

The maximum ME voltage coefficient is 155 mV/cmOe for a perfect interface coupling parameter, k = 1. This is much less than that obtained for the CFO/PZT-4 bilayer (460 mV/cmOe). The reason for this reduction in the ME voltage coefficient is the reduced piezomagnetic coefficient of NFO compared to that of CFO. NFO has an increased permeability, but we note that the magnetoelectric susceptibility does not depend on the magnetic property of the media.

High ME susceptibilities/voltage coefficients are desired for practical applications. It is important to know what combination of piezoelectric and magnetostrictive phases will produce the greatest magnetoelectric coefficient. For the transverse ME field orientation, we have compared the ME voltage coefficient for four PZT/ferrite combinations, and the results are shown in Fig. 3.4.

From Table 3.2, one may assume that the PZT-5/CFO combination should have the highest ME voltage coefficient. However, the high permittivity of the PZT-5H leads to a lower ME voltage coefficient than for PZT-4. This is as should be expected given the inverse relationship between the ME voltage coefficient and the permittivity of the media. In both NFO cases in Fig. 3.4, there is a lower ME voltage coefficient obtained. This result thus implies that NFO is not an ideal material in the design of magnetoelectric composites. The results help prove that the magneto-mechanical coupling expected in NFO and piezoelectric composites is not large enough to warrant its choice over CFO in the design of ME composites.



FIGURE 3.4. Comparison of the transverse ME effect for different PZT/ferrite combinations. Perfect interface coupling, k=1, is assumed at the interface for each bilayer. CFO has the best performance for the magnetostrictive phase, and PZT-4 has the best performance for the piezoelectric phase. Hence, the PZT-4/CFO combination has best performance.

The transverse magnetoelectric effect has a greater magnitude than the longitudinal effect, in terms of the magnetoelectric voltage coefficient. The magnetoelectric voltage coefficient obtained for the transverse case of a CFO/PZT-4 bilayer is 450 mV/cmOe, which is much higher than that for the longitudinal case (180 mV/cmOe). This is due to the fact that the combination of piezomagnetic coefficients in the direction of the applied magnetic field for the transverse case is much greater than that for the longitudinal case.

We analyze the difference starting from the derivation of the theoretical models. For the longitudinal model, from Eq. (2.27), we have

$${}^{p}D_{3} = K_{1}{}^{m}H_{3} + K_{2}{}^{p}E_{3}.$$
(3.88)

where,

$$K_{1} = \frac{2 \cdot {}^{p} d_{31} \cdot {}^{m} q_{31} \cdot k}{\left[k\left({}^{m} s_{11} + {}^{m} s_{12}\right)\left({}^{p} \nu / {}^{m} \nu\right) + \left({}^{p} s_{11} + {}^{p} s_{12}\right)\right]},$$
(3.89)

$$K_{2} = \frac{-2({}^{p}d_{31})^{2}}{\left[k({}^{m}s_{11} + {}^{m}s_{12})({}^{p}\nu/{}^{m}\nu) + ({}^{p}s_{11} + {}^{p}s_{12})\right]} + {}^{p}\mathcal{E}_{33}.$$
(3.90)

 K_I represents the coupling for the fields within the magnetoelectric effect. For the theoretical transverse magnetoelectric model,

$${}^{p}D_{3} = W_{1}{}^{m}H_{1} + W_{2}{}^{p}E_{3}.$$
(3.28c)

where,

$$W_{1} = \left(\frac{k^{p}d_{31}(^{m}q_{11} + ^{m}q_{12})}{\left(\left({^{p}s_{11} + ^{p}s_{12}}\right) + k\left({^{m}s_{11} + ^{m}s_{12}}\right)\left({^{p}\nu}/{^{m}\nu}\right)\right)}\right),$$
(3.30)

$$W_{2} = \left(\frac{-2({}^{p}d_{31})^{2}}{\left(\left({}^{p}s_{11} + {}^{p}s_{12}\right) + k({}^{m}s_{11} + {}^{m}s_{12})({}^{p}\nu/{}^{m}\nu)\right)} + {}^{p}\mathcal{E}_{33}\right).$$
(3.31)

 W_I represents the coupling term for the piezoelectric phase of the magnetoelectric composite. Observing Eq. (3.89) and (3.30), we see that the coupling is much greater for the transverse model when we compare W_I to K_I as $\binom{m}{q_{11}} + \binom{m}{q_{12}} > 2 \cdot \binom{m}{q_{31}}$. The greater magnetoelectric coupling for the transverse model makes it a more desirable configuration than the longitudinal model. In chapter 5, we will consider the electromagnetic wave propagation characteristics of the three magnetoelectric coupling plays a role in the phenomena obtained.

We compare results of our model to theoretical models obtained in the literature. The comparison is observed in Figs. 3.5 and 3.6. In both figures, the models follow similar trends in terms of the observed characteristics of the ME voltage coefficient in relation to the piezoelectric volume fraction, however our model obtains lower magnetoelectric voltage coefficients than obtained by Refs. [22] and [46]. The reason for the difference in the magnitude of the ME voltage coefficient obtained is due to application of electromagnetic wave boundary conditions as shown in our derivations.


FIGURE 3.5. Comparison of the theoretical model for the transverse magnetoelectric configuration to that obtained by Bichurin et al. [22] for a perfect interface coupling parameter (k=1). This figure compares the results for a PZT-nickel ferrite composite.

We have previously shown that the ME voltage coefficient obtained from the PZT/ferrite composites should be lower [40], using experimental data for the longitudinal magnetoelectric configuration. This information has been previously discussed in Chapter 2. However, we lack experimental data for the transverse and in-plane configurations with which to compare our theoretical model. We do observe that our current model follows similar characteristics to previous models with magnitude mismatches along the curve for the ME voltage coefficient.



FIGURE 3.6. Comparison of the theoretical model for the transverse magnetoelectric configuration to that obtained by Bichurin et al. [46] for a perfect interface coupling parameter (k=1). This figure compares the results for a PZT-cobalt ferrite composite.

We observe that the mismatch is greatest at the points of maximum magnetoelectric voltage coefficient for the comparisons shown in figs. 3.5 and 3.6. This was already observed in Chapter 2 and is so since the application of the boundary condition is most critical at the points of maximum magnetoelectric coupling between the layers. As the strength of the magnetoelectric effect diminishes with changes to the piezoelectric volume fraction, the agreement between the models improves as is observed in Figs. 3.5 and 3.6.

The magnetoelectric susceptibility is investigated for two PZT-ferrite combinations. The magnetoelectric susceptibilities for PZT-cobalt ferrite and PZT-nickel

ferrite bilayers are shown in Figs. 3.7 and 3.8, respectively. Observe that as was the case with the ME voltage coefficient, the PZT-cobalt ferrite combination produces a higher ME susceptibility than PZT-nickel ferrite.



FIGURE 3.7. Magnetic field induced magnetoelectric susceptibility for a PZT-Cobalt ferrite bilayer. The susceptibility increases with increase in the piezoelectric volume fraction.

Both bilayers show an increase in the ME susceptibility as the piezoelectric volume fraction increases. This is easily explained due to the volume ratio $\binom{p}{v} \binom{m}{v}$ which is obtained in Eq. (3.30) and (3.31). Since the volume ratio is in the denominator of Eq. (3.30) and (3.31), it creates an inverse relationship with the ME susceptibility in Eq. (3.64). We note that increase in the piezomagnetic volume fraction results in a decreased magnetoelectric coupling. Also, as the piezomagnetic volume fraction goes to zero, the

ME susceptibility also goes to zero. In reality, the best combination should be at about equal volume fractions for both phases, so as to cause a large enough strain to be produced by the piezomagnetic fraction. A very small piezomagnetic volume fraction may not produce enough strain to affect the piezoelectric layer. This is important has the bianisotropic application of the media will be better suited by an ME media with equally high electric and magnetic field induced ME susceptibilities. Our theoretical model represents a tool for the analysis of the best combination of volume fractions, and the resultant ME effect obtained.



FIGURE 3.8. Magnetic field induced magnetoelectric susceptibility for a PZT-Nickel ferrite bilayer. The susceptibility increases with increase in the piezoelectric volume fraction.

Results from the theoretical model in terms of the magnetoelectric susceptibilities show that the volume fractions required for maximum magnetoelectric voltage coefficients and susceptibilities differ. This is important since most of the experimental measurements in ME composites are presented in terms of the magnetoelectric voltage coefficient. However, that does not give any information on the maximum ME coupling obtained for the media. The reason for the discrepancy is the effective permeability which also changes with the piezoelectric volume fraction and is an important factor in the measurement and derivation of the ME voltage coefficient. The choice of a ME voltage coefficient or ME susceptibility depends on what application is required. For example, for electromagnetic wave propagation, one is interested in the ME susceptibility as the electric and magnetic field coupling in the constitutive relationship of the media depends on directly on the ME susceptibility. The ME voltage coefficient is really required more for its application in the experimental measurement and determination of the ME effect. It is important that the theoretical model also presents information on the ME composite in terms of its ME susceptibility, as that information is vital to conducting research on the electromagnetic wave propagation characteristics of the composite. The results obtained from the theoretical model show that same trends are obtained when compared to other theoretical models in the literature, however, the magnitude of the ME voltage coefficient obtained is much less than that from theoretical models in the literature as is the case with experimental measurements.

We have obtained the theoretical model for the magnetoelectric effect for the transverse magnetoelectric configuration. The theoretical model takes into account the

effects of an imperfect interface, and also deals with the resultant change in the permeability of the composite due to application of a DC magnetic field bias. This concludes the theoretical modeling for the transverse magnetoelectric configuration.

CHAPTER 4: THEORETICAL MODEL FOR THE IN-PLANE LONGITUDINAL MAGNETOELECTRIC EFFECT

In the in-plane ME configuration, the composite media has a DC magnetic field bias in the *1*-direction and poling electric field in the *1*-direction as shown in Fig. 4.1. This orientation has been investigated in the literature, and the resultant theoretical models produced the largest magnetoelectric voltage coefficient of all three ME configurations.



FIGURE 4.1. In-plane ME configuration, showing the orientation of bias fields. The DC magnetic field bias and the poling electric field are in the same direction, lying along the surface plane of the composite.

We express the fields within the composite in terms of DC and time varying components as

$$\bar{H}_1 = \hat{1}H_0 + \hat{1}H_{AC}, \tag{4.1}$$

$$\overline{E}_{1} = \hat{1}E_{0} + \hat{1}E_{AC}.$$
(4.2)

In Eq. (4.1) and (4.2), H₀ and E₀ represent the DC magnetic and electric fields respectively, while H_{AC} and E_{AC} represent the time varying fields that may be present in the composite.

4.1 In-plane fields, Stresses, and strains

The electromagnetic fields in each respective phase are represented by vector fields of the form

$${}^{m,p}\mathbf{E} = E_1 \hat{1} + E_2 \hat{2} + E_3 \hat{3}, \tag{4.3}$$

$${}^{m,p}\mathbf{H} = H_1 \hat{1} + H_2 \hat{2} + H_3 \hat{3}. \tag{4.4}$$

In Eq. (4.3) and (4.4), the total fields are a combination of time varying fields that may exist within the composite in all directions, and the bias/poling fields in fixed directions along the sample plane (1-direction) as has been described in Eq. (4.1) and (4.2).

The theoretical formulation for the in-plane magnetoelectric configuration follows a similar process that has been laid out for the previous models. The constitutive equations for the piezoelectric and piezomagnetic phases are solved independently for the fields, stresses and strains, and then related in terms of the induced strain in each layer. The piezoelectric constitutive relationship is given as

$${}^{p}S_{i} = {}^{p}s_{ij} {}^{p}T_{j} + {}^{p}d_{ki} {}^{p}E_{k},$$
(4.5)

$${}^{p}D_{k} = {}^{p}d_{ki} {}^{p}T_{i} + {}^{p}\mathcal{E}_{kn} {}^{p}E_{n}, \qquad (4.6)$$

$${}^{p}B_{k} = {}^{p}\mu_{kn} {}^{p}H_{n}.$$
(4.7)

For the piezomagnetic phase, the constitutive relationship is

$${}^{m}S_{i} = {}^{m}s_{ij}{}^{m}T_{j} + {}^{m}q_{ki}{}^{m}H_{k},$$
(4.8)

$${}^{m}B_{k} = {}^{m}q_{ki} {}^{m}T_{i} + {}^{m}\mu_{kn} {}^{m}H_{n},$$
(4.9)

$${}^{m}D_{k} = {}^{m}\mathcal{E}_{kn} {}^{m}E_{n}.$$

$$(4.10)$$

We repeat the assumptions made on the mechanical stresses and strains within the composite layer:

1. Shear stresses and strains are equal to zero, such that

$$^{m,p}T_i = 0, \,^{m,p}S_i = 0, \,\text{for } i=4, 5, \,\text{and } 6.$$
 (4.11)

2. The thickness of each phase is much smaller than the width and length of the phase. Hence the stress in the axial direction is approximated as zero

$${}^{m}T_{3} = {}^{p}T_{3} = 0. ag{4.12}$$

3. The strain transfer between phases is related by an interface coupling parameter, *k*, such that

$${}^{p}S_{i} = k \cdot {}^{m}S_{i}. \tag{4.13}$$

4. The summation of forces on the in-plane (1-2 plane) boundaries are zero, such that

$${}^{m}T_{i}{}^{m}\nu + {}^{p}T_{i}{}^{p}\nu = 0, \text{ for } i = 1, 2,$$
 (4.14)

Solving the constitutive equations of each phase based upon the given assumptions, we obtain the electric displacement field in the piezoelectric region using the nonzero components of the permittivity, permeability, compliance and piezoelectric coefficient as shown in Table 4.1.

Applying the nonzero coefficients from Table 4.1 into the constitutive relationship for each phase, we obtain a stress-field relationship for the composite by relating the strain between phases. Considering the strains on the composite using Eq. (4.13),

$$\left({}^{p}s_{11} + {}^{p}s_{12}\right){}^{p}T_{1} + {}^{p}d_{11}{}^{p}E_{1} = k\left({}^{m}s_{11} + {}^{m}s_{12}\right){}^{m}T_{1} + k{}^{m}q_{11}{}^{m}H_{1}$$
(4.15)

$$\left({}^{p}s_{21} + {}^{p}s_{22}\right){}^{p}T_{1} + {}^{p}d_{12}{}^{p}E_{1} = k\left({}^{m}s_{21} + {}^{m}s_{22}\right){}^{m}T_{1} + k{}^{m}q_{12}{}^{m}H_{1}$$
(4.16)

Combining Eq. (4.15) and (4.16), using ${}^{m,p}s_{11} = {}^{m,p}s_{22}$, and ${}^{m,p}s_{12} = {}^{m,p}s_{21}$

$$2\left({}^{p}s_{11}+{}^{p}s_{12}\right){}^{p}T_{1}+\left({}^{p}d_{11}+{}^{p}d_{12}\right){}^{p}E_{1}=2k\left({}^{m}s_{11}+{}^{m}s_{12}\right){}^{m}T_{1}+k\left({}^{m}q_{11}+{}^{m}q_{12}\right){}^{m}H_{1}$$
(4.17)

Using the relationship between the stresses in each phase, ${}^{m}T_{1} = -{}^{p}T_{1}\left(\frac{{}^{p}v}{{}^{m}v}\right)$, thus

$$\left[2\left({}^{p}s_{11}+{}^{p}s_{12}\right)+2k\left({}^{m}s_{11}+{}^{m}s_{12}\right)\left({}^{p}v_{w}\right)\right]{}^{p}T_{1}=k\left({}^{m}q_{11}+{}^{m}q_{12}\right){}^{m}H_{1}-\left({}^{p}d_{11}+{}^{p}d_{12}\right){}^{p}E_{1} \quad (4.18)$$

Then solving for the stress, rewriting (4.18) in terms of the 1-directional stress ${}^{p}T_{1}$ in the piezoelectric layer, we obtain

$${}^{p}T_{1} = Y_{1}^{m}H_{1} + Y_{2}^{p}E_{1}$$
(4.19)

where

$$Y_{1} = \left(\frac{k\left({}^{m}q_{11} + {}^{m}q_{12}\right)}{\left(2\left({}^{p}s_{11} + {}^{p}s_{12}\right) + 2k\left({}^{m}s_{11} + {}^{m}s_{12}\right)\left({}^{p}\nu/{}^{m}\nu\right)\right)}\right)$$
(4.20)

TABLE 4.1. NONZERO COEFFICIENTS OF BULK PIEZOELECTRIC AND PIEZOMAGNETIC PHASES FOR THE IN-PLANE ME CONFIGURATION

Piezoelectric phase					
Coefficient type	Non-zero components				
Permittivity	${}^{p}\boldsymbol{\varepsilon}_{11} = {}^{p}\boldsymbol{\varepsilon}_{22}, {}^{p}\boldsymbol{\varepsilon}_{33}.$				
Permeability	${}^{p}\mu_{11} = {}^{p}\mu_{22} = {}^{p}\mu_{33}.$				
Compliance	${}^{p}s_{33} = {}^{p}s_{22}, {}^{p}s_{32} = {}^{p}s_{23}, {}^{p}s_{13} = {}^{p}s_{23} = {}^{p}s_{31} = {}^{p}S_{32},$ ${}^{p}s_{11}, {}^{p}s_{55} = {}^{p}s_{66}, {}^{p}s_{44} = 2({}^{p}s_{33} + {}^{p}s_{32}).$				
Piezoelectric	${}^{p}d_{35} = {}^{p}d_{26}, {}^{p}d_{13} = {}^{p}d_{12}, {}^{p}d_{11}.$				
Piezomagnetic phase					
Coefficient type	Non-zero components				
Permittivity	${}^{m}\boldsymbol{\mathcal{E}}_{11}, {}^{m}\boldsymbol{\mathcal{E}}_{22}, {}^{m}\boldsymbol{\mathcal{E}}_{33}.$				
Permeability	${}^{m}\mu_{11}, {}^{m}\mu_{22}, {}^{m}\mu_{33}.$				
Compliance	${}^{m}s_{11} = {}^{m}s_{22} = {}^{m}s_{33}, {}^{m}s_{44} = {}^{m}s_{55} = {}^{m}s_{66},$ ${}^{m}s_{12} = {}^{m}s_{21} = {}^{m}s_{13} = {}^{m}s_{23} = {}^{m}s_{31} = {}^{m}s_{32}.$				
Piezomagnetic	${}^{m}q_{35} = {}^{m}q_{26}, {}^{m}q_{12} = {}^{m}q_{13}, {}^{m}q_{11}.$				

$$Y_{2} = \left(\frac{-\binom{p}{d_{11}} + \binom{p}{d_{12}}}{\left(2\binom{p}{s_{11}} + \binom{p}{s_{12}} + 2k\binom{m}{s_{11}} + \binom{m}{s_{12}}\binom{p}{\nu}\binom{m}{m}\right)}\right)$$
(4.21)

From the constitutive relationships in Eq. (4.6), we obtain components of the electric field displacement in the piezoelectric phase as

$${}^{p}D_{1} = \left({}^{p}d_{11} + {}^{p}d_{12}\right){}^{p}T_{1} + {}^{p}\mathcal{E}_{11}{}^{p}E_{1}$$
(4.22)

$${}^{p}D_{2} = {}^{p}\mathcal{E}_{22} {}^{p}E_{2} \tag{4.23}$$

$${}^{p}D_{3} = {}^{p}\mathcal{E}_{33} {}^{p}E_{3} \tag{4.24}$$

Substituting Eq. (4.19) into Eq. (4.22), we obtain the coupled 1-directional component of the electric field displacement in the piezoelectric phase as

$${}^{p}D_{1} = G_{1} {}^{m}H_{1} + G_{2} {}^{p}E_{1}$$

$$(4.25)$$

where

$$G_{1} = \left(\frac{k\left({}^{p}d_{11} + {}^{p}d_{12}\right)\left({}^{m}q_{11} + {}^{m}q_{12}\right)}{2\left[\left({}^{p}s_{11} + {}^{p}s_{12}\right) + k\left({}^{m}s_{11} + {}^{m}s_{12}\right)\left({}^{p}\nu/{}^{m}\nu\right)\right]}\right)$$
(4.26)

$$G_{2} = \left(\frac{-\left({}^{p}d_{11} + {}^{p}d_{12}\right)^{2}}{2\left[\left({}^{p}s_{11} + {}^{p}s_{12}\right) + k\left({}^{m}s_{11} + {}^{m}s_{12}\right)\left({}^{p}\nu/{}^{m}\nu\right)\right]} + {}^{p}\varepsilon_{33}\right)$$
(4.27)

Now, we consider the magnetic flux density of the piezomagnetic phase. Two cases are considered here. For the first case, the DC magnetic field bias creates no secondary effect, such that there is no change in the form of the permeability with the application of the DC magnetic field. In the second case, the piezomagnetic material is a media with magnetic moments, such as a ferrite.

4.1.1 Case 1: Non-gyrotropic composites

We use the non-zero values of the permeability as obtained from Table 4.1. From the constitutive relationship for the magnetoelectric phase,

$${}^{m}B_{1} = {}^{m}q_{11}{}^{m}T_{1} + {}^{m}q_{12}{}^{m}T_{2} + {}^{m}\mu_{11}{}^{m}H_{1}$$
(4.28)

$${}^{m}B_{2} = {}^{m}\mu_{22} {}^{m}H_{2} \tag{4.29}$$

$${}^{m}B_{3} = {}^{m}\mu_{33} {}^{m}H_{3} \tag{4.30}$$

Using ${}^{p}T_{1} = -{}^{m}T_{1}\left(\frac{{}^{m}v}{{}^{p}v}\right)$ into Eq. (4.17), we obtain

$$\left[2k\binom{m}{s_{11}+m}s_{12}+2\binom{p}{s_{11}+p}s_{12}\binom{m}{p}\right]^{m}T_{1} = \binom{p}{d_{11}+p}d_{12}^{p}E_{1} - k\binom{m}{q_{11}+m}q_{12}^{m}H_{1}$$
(4.31)

Thus, we obtain the 1-directional stress in the piezomagnetic layer as

$${}^{m}T_{1} = X_{1}{}^{p}E_{1} - X_{2}{}^{m}H_{1}$$
(4.32)

where

$$X_{1} = \frac{\binom{p}{d_{11}} + \binom{p}{d_{12}}}{2\left[k\binom{m}{s_{11}} + \binom{m}{s_{12}} + \binom{p}{s_{11}} + \binom{p}{s_{12}}\binom{m}{\nu} + \binom{p}{\nu}\right]}$$
(4.33)

$$X_{2} = \frac{k \left({}^{m} q_{11} + {}^{m} q_{12}\right)}{2 \left[k \left({}^{m} s_{11} + {}^{m} s_{12}\right) + \left({}^{p} s_{11} + {}^{p} s_{12}\right) \left({}^{m} \nu / {}^{p} \nu\right)\right]}$$
(4.34)

Applying Eq. (4.32) into Eq. (4.28), and using ${}^{m}T_{1} = {}^{m}T_{2}$ we have

$${}^{m}B_{1} = R_{1}{}^{p}E_{1} + R_{2}{}^{m}H_{1}$$
(4.35)

where

$$R_{1} = \frac{\left({}^{p}d_{11} + {}^{p}d_{12}\right)\left({}^{m}q_{11} + {}^{m}q_{12}\right)}{\left[k\left({}^{m}s_{11} + {}^{m}s_{12}\right) + \left({}^{p}s_{11} + {}^{p}s_{12}\right)\left({}^{m}\nu/{}^{p}\nu\right)\right]},$$
(4.36)

$$R_{2} = \frac{-k \left({}^{m} q_{11} + {}^{m} q_{12}\right)^{2}}{2 \left[k \left({}^{m} s_{11} + {}^{m} s_{12}\right) + \left({}^{p} s_{11} + {}^{p} s_{12}\right) \left({}^{m} \nu / {}^{p} \nu\right)\right]} + {}^{m} \mu_{11}$$
(4.37)

Thus, we have obtained the electric displacement fields in both composite layers as

$${}^{p}D_{1} = G_{1}{}^{m}H_{1} + G_{2}{}^{p}E_{1}$$
(4.25)
$${}^{m}D_{1} = {}^{m}\mathcal{E}_{11}{}^{m}E_{1}$$
(4.38)

$${}^{p}D_{2} = {}^{p}\mathcal{E}_{22} {}^{p}E_{2}$$
(4.23)
$${}^{m}D_{2} = {}^{m}\mathcal{E}_{22} {}^{m}E_{2}$$
(4.39)

$${}^{p}D_{3} = {}^{p}\mathcal{E}_{33} {}^{p}E_{3}$$
(4.24)
$${}^{m}D_{3} = {}^{m}\mathcal{E}_{33} {}^{m}E_{3}$$
(4.40)

where G_1 and G_2 are as given in Eq. (4.26) and Eq. (4.27)

The magnetic flux in both composite layers has also been obtained and are expressed as

$${}^{p}B_{1} = {}^{p}\mu_{11} {}^{p}H_{1}$$
(4.41)
$${}^{m}B_{1} = R_{1} {}^{p}E_{1} + R_{2} {}^{m}H_{1}$$
(4.35)

$${}^{p}B_{2} = {}^{p}\mu_{22} {}^{p}H_{2} \qquad (4.42) \qquad {}^{m}B_{2} = {}^{m}\mu_{22} {}^{m}H_{2} \qquad (4.29)$$

$${}^{p}B_{2} = {}^{p}\mu_{22} {}^{p}H_{2} \qquad (4.43) \qquad {}^{m}B_{3} = {}^{m}\mu_{33} {}^{m}H_{3} \qquad (4.30)$$

where R_1 and R_2 are as given in Eq. (4.36) and Eq. (4.37)

Now we apply boundary conditions to obtain homogenized fields for the structure.

4.1.2 Case 2: Gyrotropic composites

We will use a new permeability tensor for a DC magnetic bias in the 1-direction for a ferrite media. The permeability tensor is given as

$${}^{m}\mu = \begin{bmatrix} {}^{m}\mu_{11} & 0 & 0 \\ 0 & {}^{m}\mu_{22} & {}^{m}\mu_{23} \\ 0 & {}^{m}\mu_{32} & {}^{m}\mu_{33} \end{bmatrix}$$
(4.43)

Thus, we rewrite the magnetic flux density as

$${}^{m}B_{1} = R_{1}{}^{p}E_{1} + R_{2}{}^{m}H_{1}$$
(4.44)

$${}^{m}B_{2} = {}^{m}\mu_{22} {}^{m}H_{2} + {}^{m}\mu_{23} {}^{m}H_{3}$$
(4.45)

$${}^{m}B_{3} = {}^{m}\mu_{32} {}^{m}H_{2} + {}^{m}\mu_{33} {}^{m}H_{3}$$
(4.46)

where

$$R_{1} = \frac{\binom{p}{d_{11}} + \binom{p}{d_{12}}\binom{m}{q_{11}} + \binom{m}{q_{12}}}{\left[k\binom{m}{s_{11}} + \binom{m}{s_{12}} + \binom{p}{s_{11}} + \binom{p}{s_{12}}\binom{m}{v}\binom{p}{v}\right]},$$
(4.47)

$$R_{2} = \frac{-k \left({}^{m} q_{11} + {}^{m} q_{12}\right)^{2}}{2 \left[k \left({}^{m} s_{11} + {}^{m} s_{12}\right) + \left({}^{p} s_{11} + {}^{p} s_{12}\right) \left({}^{m} \nu / {}^{p} \nu\right)\right]} + {}^{m} \mu_{11}^{*}$$
(4.48)

There are changes to the value of R_2 based upon a change to the permeability tensor. In terms of the general shape of the magnetic flux density, the changes are only observed in Eq. (4.45) and Eq. (4.46). There are no changes to the equations involving the electric displacement field in both phases of the magnetoelectric composite. Hence we apply boundary conditions to obtain homogenized material parameters for the composite.

4.2 Application of boundary conditions

We apply boundary conditions on the fields along the interface of the composite layers. The boundary conditions are same as those applied in sec 2.2.

4.2.1 Case 1: Non-gyrotropic composites

Tangential components of the electric and magnetic fields are continuous at the boundary between both media, therefore we obtain the relationships

$${}^{m}E_{1} = {}^{p}E_{1} = E_{x}$$
(4.49)

$${}^{m}E_{2} = {}^{p}E_{2} = E_{y} \tag{4.50}$$

$${}^{m}H_{1} = {}^{p}H_{1} = H_{x} \tag{4.51}$$

$${}^{m}H_{2} = {}^{p}H_{2} = H_{y} \tag{4.52}$$

In Eq. (4.49) – (4.52) above E_x , E_y , H_x , and H_y represent the homogenized components of the electric and magnetic fields in the media. It is assumed that each composite layer is electrically thin, with no field variation within each individual composite layer. Using a field averaging method [41, 42], we define the tangential components of the electric field displacement as

$$D_{x} = {}^{m}D_{1}{}^{m}v + {}^{p}D_{1}{}^{p}v = \left[{}^{m}\mathcal{E}_{11}{}^{m}E_{1}{}^{m}v + \left(G_{1}{}^{m}H_{1} + G_{2}{}^{p}E_{1}\right){}^{p}v\right].$$
(4.53)

Simplifying Eq. (4.53), we obtain the *x*-component of the electric displacement field as

$$D_{x} = (G_{1}^{p}v)H_{x} + ({}^{m}\varepsilon_{11}^{m}v + G_{2}^{p}v)E_{x}.$$
(4.54)

Similarly, the y-component of the electric displacement field is obtained as

$$D_{y} = {}^{m}D_{2}{}^{m}v + {}^{p}D_{2}{}^{p}v = \left[{}^{m}\varepsilon_{22}{}^{m}E_{2}{}^{m}v + {}^{p}\varepsilon_{22}{}^{p}E_{2}{}^{p}v\right] = \left[{}^{m}\varepsilon_{22}{}^{m}v + {}^{p}\varepsilon_{22}{}^{p}v\right]E_{y}.$$
(4.55)

Also, the magnetic flux density is obtained as

$$B_{x} = {}^{m}B_{1} {}^{m}v + {}^{p}B_{1} {}^{p}v = \left[\left(R_{1} {}^{p}E_{1} + R_{2} {}^{m}H_{1} \right) {}^{m}v + {}^{p}\mu_{11} {}^{p}H_{1} {}^{p}v \right].$$
(4.56)

Simplifying Eq. (4.56), we obtain an expression for the *x*-component of the magnetic flux density as

$$B_{x} = R_{1}^{m} v E_{x} + \left(R_{2}^{m} v + {}^{p} \mu_{11} {}^{p} v\right) H_{x}$$
(4.57)

Finally, the y-component of the magnetic flux density is obtained as

$$B_{y} = {}^{m}B_{2} {}^{m}v + {}^{p}B_{2} {}^{p}v = \left[{}^{m}\mu_{22} {}^{m}H_{2} {}^{m}v + {}^{p}\mu_{22} {}^{p}H_{2} {}^{p}v \right] = \left[{}^{m}\mu_{22} {}^{m}v + {}^{p}\mu_{22} {}^{p}v \right] H_{y}$$
(4.58)

Applying the second boundary condition that normal components of the magnetic flux density and the electric field displacement are continuous across the boundary, which implies that

$${}^{m}D_{3} = {}^{p}D_{3} = D_{z} \tag{4.59}$$

$${}^{m}B_{3} = {}^{p}B_{3} = B_{z} \tag{4.60}$$

Using the expression for the electric displacement field and magnetic flux density in each region, we have the relationships

$${}^{m}\mathcal{E}_{33}{}^{m}E_{3} = {}^{p}\mathcal{E}_{33}{}^{p}E_{3} = D_{z}$$
(4.61)

$${}^{m}\mu_{33}{}^{m}H_{3} = {}^{p}\mu_{33}{}^{p}H_{3} = B_{z}$$
(4.62)

We assume that each composite layer is electrically thin, with negligible field variation within each individual composite layer. We use a field averaging method to obtain field components as

$$E_{z} = {}^{p}E_{3} {}^{p}v + {}^{m}E_{3} {}^{m}v \tag{4.63}$$

$$H_{z} = {}^{p}H_{3} {}^{p}v + {}^{m}H_{3} {}^{m}v$$
(4.64)

Using the boundary condition as represented in Eq. (4.61),

$${}^{p}E_{3} = \frac{{}^{m}\mathcal{E}_{33}}{{}^{p}\mathcal{E}_{33}} {}^{m}E_{3}$$
(4.65)

Applying Eq. (4.65) in Eq. (4.63)

$$E_{z} = \left({}^{p} v \frac{{}^{m} \mathcal{E}_{33}}{{}^{p} \mathcal{E}_{33}} + {}^{m} v \right) {}^{m} E_{3}$$

$$(4.66)$$

Solving for ${}^{m}E_{3}$ we obtain

$${}^{m}E_{3} = \left(\frac{{}^{p}\boldsymbol{\varepsilon}_{33}}{{}^{p}\boldsymbol{v}^{m}\boldsymbol{\varepsilon}_{33} + {}^{m}\boldsymbol{v}^{p}\boldsymbol{\varepsilon}_{33}}\right)E_{z}$$
(4.67)

Thus the z-component of the electric displacement field is obtained as

$$D_{z} = \left(\frac{{}^{m} \mathcal{E}_{33} {}^{p} \mathcal{E}_{33}}{{}^{p} v^{m} \mathcal{E}_{33} + {}^{m} v^{p} \mathcal{E}_{33}}\right) E_{z}$$
(4.68)

For the magnetic flux density, from Eq. (4.62),

$$B_{z} = {}^{m}\mu_{33}{}^{m}H_{3} \tag{4.69}$$

Using the boundary condition as represented in (4.64), w obtain

$${}^{p}H_{3} = \frac{{}^{m}\mu_{33}}{{}^{p}\mu_{33}}{}^{m}H_{3}$$
(4.70)

Applying Eq. (4.70) in Eq. (4.64), we obtain the *z* component of the magnetic field as

$$H_{z} = \left({}^{p} v \frac{{}^{m} \mu_{33}}{{}^{p} \mu_{33}} + {}^{m} v \right) {}^{m} H_{3}$$
(4.71)

Solving for ${}^{m}H_{3}$ we obtain

$${}^{m}H_{3} = \left(\frac{{}^{p}\mu_{33}}{{}^{p}v^{m}\mu_{33} + {}^{m}v^{p}\mu_{33}}\right)H_{z}$$
(4.72)

Finally, applying Eq. (4.72) into Eq. (4.69), we obtain the magnetic flux as

$$B_{z} = \left(\frac{{}^{m}\mu_{33} {}^{p}\mu_{33}}{{}^{p}v^{m}\mu_{33} + {}^{m}v^{p}\mu_{33}}\right)H_{z}$$
(4.73)

Hence, the constitutive equations for the composites in the in-plane ME orientation is given as

$$\begin{bmatrix} D_{x} \\ D_{y} \\ D_{z} \end{bmatrix} = \begin{bmatrix} \alpha_{xx}^{H} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} H_{x} \\ H_{y} \\ H_{z} \end{bmatrix} + \begin{bmatrix} \varepsilon_{xx} & 0 & 0 \\ 0 & \varepsilon_{yy} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{bmatrix} \begin{bmatrix} E_{x} \\ E_{y} \\ E_{z} \end{bmatrix}, \quad (4.74)$$

$$\begin{bmatrix} B_{x} \\ B_{y} \\ B_{z} \end{bmatrix} = \begin{bmatrix} \alpha_{xx}^{E} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} E_{x} \\ E_{y} \\ E_{z} \end{bmatrix} + \begin{bmatrix} \mu_{xx} & 0 & 0 \\ 0 & \mu_{yy} & 0 \\ 0 & 0 & \mu_{zz} \end{bmatrix} \begin{bmatrix} H_{x} \\ H_{y} \\ H_{z} \end{bmatrix}.$$
 (4.75)

4.2.2 Case 2: Gyrotropic composites

The change in the homogenized material parameters for the case of magnetic anisotropy is seen in the y- and z-directed magnetic flux density. The electric displacement fields are essentially of same form as obtained for the non-gyrotropic model. Hence, we will not expressly show the derivation of the homogenized electric displacement fields for this case. Also, the x-directed magnetic flux density stays the same and will not be reformulated here. However, it is important to note that while the form of the x-directed magnetic flux density will be the same, there will be a change in its magnitude due to the change in the values of the components in the permeability matrix.

We obtain new relationships for the homogenized flux density in the y-direction using the boundary condition on the tangential electric and magnetic fields. Thus,

$$B_{y} = {}^{m}B_{2} {}^{m}v + {}^{p}B_{2} {}^{p}v = \left[{}^{m}\mu_{22}^{*} {}^{m}H_{2} {}^{m}v + {}^{m}\mu_{23}^{*} {}^{m}H_{3} {}^{m}v + {}^{p}\mu_{22} {}^{p}H_{2} {}^{p}v \right].$$
(4.76)

Simplifying the expression in Eq. (4.76), we have

$$B_{y} = \left[{}^{m}\mu_{22}^{*} {}^{m}v + {}^{p}\mu_{22} {}^{p}v\right]H_{y} + {}^{m}\mu_{23}^{*} {}^{m}v^{m}H_{3}.$$
(4.77)

The magnetic flux density in the *z*-direction is obtained as follows:

Normal components of the magnetic flux density and the electric field displacement are continuous across the boundary. Thus,

$${}^{m}D_{3} = {}^{p}D_{3} = D_{z}. ag{4.78}$$

$${}^{m}B_{3} = {}^{p}B_{3} = B_{z}. ag{4.79}$$

From Eq. (4.79), we obtain the relationship

$${}^{m}\mu_{_{32}}^{*}{}^{m}H_{2} + {}^{m}\mu_{_{33}}^{*}{}^{m}H_{3} = {}^{p}\mu_{_{33}}{}^{p}H_{3} = B_{z}.$$
(4.80)

Hence, we have that

$${}^{p}H_{3} = \frac{{}^{m}\mu_{32}^{*}}{{}^{p}\mu_{33}} {}^{m}H_{2} + \frac{{}^{m}\mu_{33}^{*}}{{}^{p}\mu_{33}} {}^{m}H_{3}.$$
(4.81)

Applying Eq. (4.81) into Eq. (4.64), also noting ${}^{m}H_{2} = {}^{p}H_{2} = H_{y}$ from Eq. (4.52), we have

$$H_{z} = {}^{p} v \frac{{}^{m} \mu_{_{32}}^{*}}{{}^{p} \mu_{_{33}}} H_{y} + \left({}^{p} v \frac{{}^{m} \mu_{_{33}}^{*}}{{}^{p} \mu_{_{33}}} + {}^{m} v \right) {}^{m} H_{3}.$$
(4.82)

Solving for ${}^{m}H_{3}$ in Eq. (4.82), we obtain

$${}^{m}H_{3} = \left(\frac{{}^{p}\mu_{33}}{{}^{p}v^{m}\mu_{33}^{*} + {}^{m}v^{p}\mu_{33}}\right)H_{z} - \left(\frac{{}^{p}\mu_{33}}{{}^{p}v^{m}\mu_{33}^{*} + {}^{m}v^{p}\mu_{33}} \cdot \frac{{}^{p}v^{m}\mu_{32}^{*}}{{}^{p}\mu_{33}}\right)H_{y}.$$

Simplifying the expression for ${}^{m}H_{3}$, we obtain

$${}^{m}H_{3} = \left(\frac{{}^{p}\mu_{33}}{{}^{p}v^{m}\mu_{33}^{*} + {}^{m}v^{p}\mu_{33}}\right)H_{z} - \left(\frac{{}^{p}v^{m}\mu_{32}^{*}}{{}^{p}v^{m}\mu_{33}^{*} + {}^{m}v^{p}\mu_{33}}\right)H_{y}.$$
(4.83)

Then applying Eq. (4.83) into Eq. (4.80), we have

$$B_{z} = \left({}^{m}\mu_{32} - \frac{{}^{p}v^{m}\mu_{32} {}^{m}\mu_{33}}{{}^{p}v^{m}\mu_{33} + {}^{m}v^{p}\mu_{33}}\right)H_{y} + \left(\frac{{}^{m}\mu_{33} {}^{p}\mu_{33}}{{}^{p}v^{m}\mu_{33} + {}^{m}v^{p}\mu_{33}}\right)H_{z}.$$
(4.84)

We simplify Eq. (4.84), and obtain

$$B_{z} = \left[{}^{m}\mu_{32}^{*} \left(1 - \frac{{}^{p}v^{m}\mu_{33}^{*}}{{}^{p}v^{m}\mu_{33}^{*}} + {}^{m}v^{p}\mu_{33}} \right) \right] H_{y} + \left(\frac{{}^{m}\mu_{33}^{*} {}^{p}\mu_{33}}{{}^{p}v^{m}\mu_{33}^{*}} + {}^{m}v^{p}\mu_{33}} \right) H_{z}.$$
(4.85)

Hence we rewrite Eq. (4.77) in terms of the homogenized fields,

$$B_{y} = U_{1}H_{y} + U_{2}H_{z} \tag{4.86}$$

where

$$U_{1} = \left({}^{m} v^{m} \mu_{22}^{*} + {}^{p} v^{p} \mu_{22} - \frac{{}^{m} v \cdot {}^{p} v \cdot {}^{m} \mu_{23}^{*} \cdot {}^{m} \mu_{32}^{*}}{{}^{p} v \cdot {}^{m} \mu_{33}^{*} + {}^{m} v \cdot {}^{p} \mu_{33}}\right).$$
(4.87)

$$U_{2} = \left(\frac{{}^{m}v^{m}\mu_{23}^{*}{}^{p}\mu_{33}}{{}^{p}v^{m}\mu_{33}^{*} + {}^{m}v^{p}\mu_{33}}\right).$$
(4.88)

The constitutive equations for the composites under the transverse ME orientation is given in a straight forward way as

$$\begin{bmatrix} D_{x} \\ D_{y} \\ D_{z} \end{bmatrix} = \begin{bmatrix} \alpha_{xx}^{H} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} H_{x} \\ H_{y} \\ H_{z} \end{bmatrix} + \begin{bmatrix} \varepsilon_{xx} & 0 & 0 \\ 0 & \varepsilon_{yy} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{bmatrix} \begin{bmatrix} E_{x} \\ E_{y} \\ E_{z} \end{bmatrix}$$
(4.89)

$$\begin{bmatrix} B_{x} \\ B_{y} \\ B_{z} \end{bmatrix} = \begin{bmatrix} \alpha_{xx}^{E} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} E_{x} \\ E_{y} \\ E_{z} \end{bmatrix} + \begin{bmatrix} \mu_{xx} & 0 & 0 \\ 0 & \mu_{yy} & \mu_{yz} \\ 0 & \mu_{zy} & \mu_{zz} \end{bmatrix} \begin{bmatrix} H_{x} \\ H_{y} \\ H_{z} \end{bmatrix}$$
(4.90)

This concludes the derivations of the theoretical model for the in-plane magnetoelectric model.

With the constitutive relationship for the in-plane magnetoelectric effect now obtained, we are able to investigate the electromagnetic wave propagation characteristics for all three magnetoelectric configurations. First, we analyze the in-plane magnetoelectric model, and make comparisons to the longitudinal and transverse models. As was done in previous models, we investigate the special case of magnetostrictive-piezoelectric layer combination. Material parameters for the piezoelectric and magnetostrictive media are shown in Table 4.2. Using the material parameters, we compute the magnetoelectric voltage coefficient for the in-plane configuration. Also, as done in the previous two chapters, all analyses are done using the magnetoelectric susceptibility α^{H} only.

Parameter (Units)	CFO	NFO	PZT-4	PZT-5H
$S_{11}\left(10^{-12}\frac{m^2}{N}\right)$	6.5	6.5	12.3	16.5
$S_{12}\left(10^{-12}\frac{m^2}{N}\right)$	-2.37	-2.37	-4.05	-4.78
$q_{12}\left(10^{-12}\frac{m}{A}\right)$	566	125	-	-
$q_{11}\left(10^{-12}\frac{m}{A}\right)$	-1880	-680	-	-
$d_{31}\left(10^{-12}\frac{C}{N}\right)$	-	-	-123	-274
$d_{33}\left(10^{-12}\frac{C}{N}\right)$	-	-	289	593
$rac{\mu_{33}}{\mu_0}$	2	3	1	1
$rac{{oldsymbol{\mathcal{E}}}_{33}}{{oldsymbol{\mathcal{E}}}_{0}}$	10	10	1300	3400

TABLE 4.2. MATERIAL PARAMETERS FOR MAGNETOSTRICTIVE AND PIEZOELECTRICPHASES USED TO ANALYZE THE IN-PLANE MAGNETOELECTRIC VOLTAGE COEFFICIENT.

4.3 Analysis of the theoretical in-plane magnetoelectric model

We analyze the in-plane ME effect in piezoelectric and magnetostrictive bilayers. The ME voltage coefficient for CFO/PZT-4 and NFO/PZT-4 bilayers are shown in Fig. 4.2 and 4.3 respectively. The current theoretical model agrees with previous models in the literature [22] that the in-plane orientation produces the largest ME voltage coefficient of all three magnetoelectric configurations.



FIGURE 4.2. ME voltage coefficient for the in-plane orientation for CFO/PZT-4 bilayers. The maximum ME voltage coefficient is obtained as 3140 mV/cmOe.

The maximum ME voltage coefficient is obtained for a 10% piezoelectric volume fraction. This means that the maximum coupling results when small piezoelectric volume is under stress due to the strain from a larger magnetostrictive volume. This is in agreement with the results by Ryu et al. [18], that greater ME effect can be obtained when the stress on the PZT layer is high, and this requires a thinner PZT layer. As the piezoelectric volume fraction increases, the magnetoelectric coupling strength is greatly reduced, so much that the interface coupling parameter barely has any effect on the ME voltage coefficient obtained.



FIGURE 4.3. ME voltage coefficient for the in-plane orientation for NFO/PZT-4 bilayers. The maximum ME voltage coefficient is obtained as 1020 mV/cmOe.

Using Figs. 4.2 and 4.3, we compare the use of NFO to CFO. The maximum ME voltage coefficient is obtained for a 10% piezoelectric volume fraction for both material combinations. Hence, we can infer that the materials used do not affect the required volume ratio to obtain maximum ME voltage coefficient. We observe that with the change in orientation, CFO remains a better ME composite material in comparison to NFO in terms of the maximum ME voltage coefficient. With the use of NFO, the ME voltage coefficient drops from 3140 mV/cmOe to 1020 mV/cmOe. This is simply due to the fact that CFO has a greater magnetostriction than does NFO. The maximum achievable ME voltage coefficient is not the only factor in the choice of the composite materials. Another important factor in the choice of material is the strength of the interface coupling in both cases. For this reason NFO has been used in ME composites due to its efficient magneto-mechanical coupling [22]. Such analysis is strongly aided by the current theoretical model should information on the effective interface coupling parameter be available.

Comparisons are made to other theoretical models in the literature. In Fig. 4.4, we compare the current model to one obtained by Bichurin et al [22] for a CFO-PZT bilayer. We observe that the ME voltage coefficient in both cases follow the same trends in terms of the rise and fall of its magnitude in relation to the piezoelectric volume fraction. However, the magnitude of the realized ME voltage coefficient is different. The difference is much greater in the region of the maximum ME effect, as the application of boundary conditions are much more important at points of maximum coupling. This is same as with the cases for the longitudinal and transverse configurations.



FIGURE 4.4. Comparison of the in-plane magnetoelectric model obtained in this work to that obtained by Bichurin et al. [22] for a CFO-PZT bilayer. We observe that the magnetoelectric voltage coefficient follows similar trends; however our model produces a lower magnitude.

The theoretical model for the in-plane ME configuration is robust; such that we can also analyze the magnetoelectric effect obtained using the magnetoelectric susceptibility, α^{H} , as shown in Fig. 4.5. We observe that the ME susceptibility obtained for both transverse and in-plane configurations have different characteristics in relation to the piezoelectric volume fraction than the ME voltage coefficient. This is of importance to researchers, as this theoretical model gives an insight to both the susceptibility and ME voltage coefficient expected than has been previously obtained from other published theoretical models. The ME susceptibility is of importance when investigating

characteristics of the ME composite such as the electromagnetic wave propagation as will be done in Chapter 5. Hence, through the current theoretical model, we obtain the magnetoelectric voltage coefficient for experimentalist to use as a guide in verification of measurement results. Also, theorists can obtain the magnetoelectric susceptibility to conduct analysis of the characteristics or properties of the ME composite.



FIGURE 4.5. Magnetic field induced ME susceptibility for the In-plane configuration. The susceptibility for the bilayer is highest for piezoelectric rich compositions.

The maximum ME susceptibility for the in-plane ME configuration is obtained with close to equal volume fractions as shown in Fig 4.5. The maximum ME susceptibility is obtained as 10 ns/m with 60% piezoelectric volume fraction. It interesting to know how the piezoelectric volume fraction affects the electric field induced ME susceptibility α^{E} . As a check, we assumed the strain-magnetization relationship on the magnetostrictive material as reversible such that $\alpha^{E} \neq 0$, and we can then refer to the phase as piezomagnetic. Hence, we obtained the electric field induced ME susceptibility for the bilayer as shown in Fig. 4.6. It should be noted that this is purely theoretical and used only due to the lack of data for actual piezomagnetic materials. The result obtained is interesting as we also observe that the maximum ME susceptibility is obtained for close to equal volume fractions.



FIGURE 4.6. Electric field induced In-plane magnetoelectric susceptibility.

In Fig. 4.6, k_r is used to show that the ME effect is obtained from the electric field, which is the reverse to the cases we have studied thus far where the ME effect has been generated solely via magnetostriction. The interface coupling parameter, k_r , does not differ from k that has been used thus far and still ranges between 0 and 1 as explained in Section 2.1. The need for a different interface coupling parameter for this case stems from the fact that the strain transfer from the piezoelectric phase to the piezomagnetic phase may not always be equal as the transfer from the piezomagnetic phase to the piezoelectric phase. This may be because the method used to bond the composite layers may more effective in one layer than the other due to the characteristics of one composite surface (rough, smooth, etc) compared to the other. The information from Figs. 4.5 and 4.6 reflect that the magnetoelectric susceptibility is greatest using close to equal volume fractions for both the electric and magnetic field induced effect. This is not the case for the ME voltage coefficient where 10% volume fraction of the piezoelectric layer is ideal. Results from the theoretical model in terms of the magnetoelectric susceptibilities showing that the volume fractions required for maximum magnetoelectric voltage coefficients and susceptibilities differ implies that one cannot assume a high ME voltage coefficient at a certain piezoelectric volume fraction implies a high ME susceptibility. This is important since most of the experimental measurements in ME composites are presented in terms of the magnetoelectric voltage coefficient rather than with the ME susceptibility. Thus, a theoretical complete model as is introduced here, which allows the researcher to obtain results using the material parameter that best suits his/her work. Hence, the experimentalist can verify results using the voltage coefficient and the theorist can investigate the media's characteristics via the susceptibility. This completes the theoretical model for the magnetoelectric composite.

CHAPTER 5: ELECTROMAGNETIC WAVE PROPAGATION IN BULK MAGNETOELECTRIC COMPOSITES

Several studies have investigated the propagation of electromagnetic waves through magnetoelectric media. In 1962, O'Dell [36, 37] studied electromagnetic wave propagation in lossless magnetoelectric materials, and concluded that lossless propagation will not be possible in the media. O'Dell arrived at that conclusion due to the complex index of refraction he obtained in solution to the wave equations of the media. However, in 1965, Fuchs [38] showed that electromagnetic waves can indeed propagate in lossless magnetoelectric media without attenuation for any value of the magnetoelectric coupling tensor. Fuchs solved the problem and discovered sign errors in the derivation obtained by O'Dell. Fuchs observed a change in the propagation velocities in magnetoelectric media with changes to the direction of propagation (forward or backward). Fuchs, however, considered only certain special cases of the ME media. It is important to note that the electromagnetic wave propagation characteristics in magnetoelectric materials will vary with changes in the nonzero coefficients of the magnetoelectric tensor. Depending upon the coefficients of the magnetoelectric tensor, we may obtain circularly polarized waves as was observed by Birss and Shrubsall [39] or linearly polarized waves.

We intend to observe the electromagnetic wave propagation phenomena for bulk magnetoelectric composites, utilizing the effective material parameters already obtained through the theoretical modeling of the magnetoelectric composite. Propagation of the electromagnetic wave is described via an eigenvalue equation where the wave number is the eigenvalue and the electric or magnetic field is the eigenvector. We solve for the wave number, from which we observe the relationship with angular frequency, and the polarization state of the propagating wave. We start with solutions to Maxwell's equation for electromagnetic wave propagating along the sample plane of the magnetoelectric media. We obtain a wave equation for the magnetoelectric media, and solve for the eigenvalues and eigenvectors. We assume a $e^{j\omega t}$ time dependence in all the following analysis. Starting with electromagnetic waves in a source free media of the form

$$\mathbf{E} = \overline{E}_0 e^{-j\overline{k}\cdot\overline{r}} \tag{5.1}$$

$$\mathbf{H} = \bar{H}_0 e^{-j\bar{k}\cdot\bar{r}} \tag{5.2}$$

Applying the electromagnetic wave into Maxwell's equation, we obtain [43, 44]

$$\overline{k} \times \mathbf{E} = \omega \mathbf{B} \tag{5.3}$$

$$\overline{k} \times \mathbf{H} = -\omega \mathbf{D} \tag{5.4}$$

$$\overline{k} \cdot \mathbf{D} = 0 \tag{5.5}$$

$$\overline{k} \cdot \mathbf{B} = 0 \tag{5.6}$$

The electric displacement field and the magnetic flux density are defined as

$$\mathbf{B} = \overline{\boldsymbol{\alpha}}^{E} \mathbf{E} + \overline{\boldsymbol{\mu}} \mathbf{H}$$
(5.7)

$$\mathbf{D} = \overline{\overline{\varepsilon}} \mathbf{E} + \overline{\overline{\alpha}}^H \mathbf{H}$$
(5.8)

From Maxwell-faraday equation, we obtain

$$\mathbf{B} = \frac{1}{\omega} \left(\overline{k} \times \mathbf{E} \right) \tag{5.9}$$

From the constitutive relationship for the magnetic flux density, Eq. (5.3), we obtain

$$\mathbf{H} = \overline{\overline{\mu}}^{-1} \left(\mathbf{B} - \overline{\overline{\alpha}}^{E} \mathbf{E} \right)$$
(5.10)

Hence, we express the magnetic field as

$$\mathbf{H} = \overline{\overline{\mu}}^{-1} \left[\frac{1}{\omega} \left(\overline{k} \times \mathbf{E} \right) - \overline{\overline{\alpha}}^{E} \mathbf{E} \right]$$
(5.11)

We relate Ampere's law to the constitutive equation for the electric displacement field, thus, we have the electric displacement field as

$$\mathbf{D} = -\frac{1}{\omega} \left(\overline{k} \times \mathbf{H} \right) = \overline{\overline{\varepsilon}} \mathbf{E} + \overline{\overline{\alpha}}^H \mathbf{H}$$
(5.12)

Substituting for the magnetic field in Eq. (5.12), by using Eq. (5.11), we have

$$-\frac{1}{\omega} \left\{ \overline{k} \times \overline{\overline{\mu}}^{-1} \left[\frac{1}{\omega} (\overline{k} \times \mathbf{E}) - \overline{\overline{\alpha}}^{E} \mathbf{E} \right] \right\} = \overline{\overline{\varepsilon}} \mathbf{E} + \overline{\overline{\alpha}}^{H} \overline{\overline{\mu}}^{-1} \left[\frac{1}{\omega} (\overline{k} \times \mathbf{E}) - \overline{\overline{\alpha}}^{E} \mathbf{E} \right]$$
(5.13)

Simplifying Eq. (5.13), we obtain

$$-\left\{\overline{k}\times\overline{\overline{\mu}}^{-1}\left[\left(\overline{k}\times\mathbf{E}\right)-\omega\overline{\overline{\alpha}}^{E}\mathbf{E}\right]\right\}=\omega^{2}\overline{\overline{\varepsilon}}\mathbf{E}+\overline{\overline{\alpha}}^{H}\overline{\overline{\mu}}^{-1}\left[\omega(\overline{k}\times\mathbf{E})-\omega^{2}\overline{\overline{\alpha}}^{E}\mathbf{E}\right]$$
(5.14)

We simplify further using an identity, $\overline{\overline{g}}$, such that

$$\overline{\overline{g}} \cdot \mathbf{E} = \overline{k} \times \mathbf{E} \tag{5.15}$$

Then, Eq. (5.14) becomes

$$-\overline{k} \times \left[\overline{\overline{\mu}}^{-1} \left(\overline{\overline{g}} \cdot \mathbf{E}\right) - \omega \overline{\overline{\mu}}^{-1} \overline{\overline{\alpha}}^{E} \mathbf{E}\right] = \omega^{2} \overline{\overline{\varepsilon}} \mathbf{E} + \omega \overline{\overline{\alpha}}^{H} \overline{\overline{\mu}}^{-1} \left(\overline{\overline{g}} \cdot \mathbf{E}\right) - \omega^{2} \overline{\overline{\alpha}}^{H} \overline{\overline{\mu}}^{-1} \overline{\overline{\alpha}}^{E} \mathbf{E}$$
(5.16)

Thus we obtain a simplified wave equation for the ME media as

$$\overline{k} \times \left[\overline{\overline{\mu}}^{-1} \left(\overline{\overline{g}} \cdot \mathbf{E}\right) - \omega \overline{\overline{\mu}}^{-1} \overline{\overline{\alpha}}^{E} \mathbf{E}\right] + \omega \overline{\overline{\alpha}}^{H} \overline{\overline{\mu}}^{-1} \left(\overline{\overline{g}} \cdot \mathbf{E}\right) - \omega^{2} \overline{\overline{\alpha}}^{H} \overline{\overline{\mu}}^{-1} \overline{\overline{\alpha}}^{E} \mathbf{E} + \omega^{2} \overline{\overline{\varepsilon}} \mathbf{E} = 0 \quad (5.17)$$

Equation (5.17) is used to examine the electromagnetic wave propagation phenomena in the bulk magnetoelectric media. Solving Eq. (5.17), we obtain the eigenvalue which is represented by the wave number of the media. We then solve for the eigenvector of the problem which is the electric field. Using the obtained results, we do a complete analysis of electromagnetic wave propagation in a bulk ME media.

We must obtain the operator, $\overline{\overline{g}}$, which is used to simplify the ME wave equation. The operator has been defined in Eq. (5.15) in relation to the wave number. First, we express the wave number and electric field as

$$k = \hat{x}k_x + \hat{y}k_y + \hat{z}k_z \tag{5.18}$$

$$\mathbf{E} = \hat{x}E_x + \hat{y}E_y + \hat{z}E_z \tag{5.19}$$

Hence, the curl operation for the wave number and electric field is given as

$$\overline{k} \times \mathbf{E} = \hat{x} \left(k_y E_z - k_z E_y \right) + \hat{y} \left(k_z E_x - k_x E_z \right) + \hat{z} \left(k_x E_y - k_y E_x \right)$$
(5.20)

Thus, if we express the operator, $\overline{\overline{g}}$, as a matrix of the form

$$\overline{\overline{g}} = \begin{bmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{31} & g_{32} & g_{33} \end{bmatrix}$$
(5.21)

Then, we obtain the components of $\overline{\overline{g}}$, by equating the components of Eq. (5.20) and (5.15). For the *x*-components of Eq. (5.20), we have

$$g_{11}E_x + g_{12}E_y + g_{13}E_z = k_y E_z - k_z E_y$$
(5.22)

Then, the components of the operator are

$$g_{11} = 0, g_{12} = -k_z, g_{13} = k_y$$
(5.23)

For the y-component of Eq. (5.20),

$$g_{21}E_x + g_{22}E_y + g_{23}E_z = k_z E_x - k_x E_z$$
(5.24)

Then, the components of the operator are

$$g_{21} = k_z, g_{22} = 0, g_{23} = -k_x \tag{5.25}$$

Finally for the *z*-component of Eq. (5.20),

$$g_{31}E_x + g_{32}E_y + g_{33}E_z = k_x E_y - k_y E_x$$
(5.26)

Then, the components of the operator are

$$g_{31} = -k_y, g_{32} = k_x, g_{33} = 0$$
(5.27)

Thus, the operator $\overline{\overline{g}}$, is expressed in matrix form as

$$\overline{\overline{g}} = \begin{bmatrix} 0 & -k_z & k_y \\ k_z & 0 & -k_x \\ -k_y & k_x & 0 \end{bmatrix}$$
(5.28)

Using the wave equation for the media we obtain expressions for the wave numbers in each of the magnetoelectric configurations. We will only consider electromagnetic wave propagation along the sample plane (*xy*-plane) for the bulk magnetoelectric composite.

5.1 Wave numbers for bulk magnetoelectric composites

5.1.1 Longitudinal ME Electromagnetic wave Propagation

For the longitudinal magnetoelectric configuration, we can consider electromagnetic wave propagation along the sample plane as shown in Fig. 5.1. However,
we consider only propagation along the *x*-axis, as the results will be the same as that for propagation along the *y*-axis. For propagation along the *x*-axis, we consider cases of gyrotropic and non-gyrotropic ME media.



FIGURE 5.1. Homogenized ME medium for the longitudinal ME configuration showing the possible propagation directions along the *xy* plane.

5.1.1.1 Gyrotropic ME media: Propagation along the x-axis

We consider propagation along the *x*-axis, such that the wave number for the propagating wave is given as

$$\overline{k} = \hat{x}k_x \tag{5.29}$$

Thus, the operator, $\frac{\overline{g}}{g}$, is expressed as

$$\overline{\overline{g}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -k_x \\ 0 & k_x & 0 \end{bmatrix}$$
(5.30)

For the gyrotropic ME media, the permeability tensor for the longitudinal configuration is given as

$$\overline{\overline{\mu}} = \begin{bmatrix} \mu_{xx} & \mu_{xy} & 0\\ \mu_{yx} & \mu_{yy} & 0\\ 0 & 0 & \mu_{zz} \end{bmatrix}$$
(5.31)

The inverse of the permeability tensor is

$$\overline{\overline{\mu}}^{-1} = \begin{bmatrix} -\mu_{yy} / (\mu_{yx}\mu_{xy} - \mu_{xx}\mu_{yy}) & \mu_{xy} / (\mu_{yx}\mu_{xy} - \mu_{xx}\mu_{yy}) & 0 \\ \mu_{yx} / (\mu_{yx}\mu_{xy} - \mu_{xx}\mu_{yy}) & -\mu_{xx} / (\mu_{yx}\mu_{xy} - \mu_{xx}\mu_{yy}) & 0 \\ 0 & 0 & 1/\mu_{zz} \end{bmatrix}$$
(5.32)

For simplicity, we will express the inverse of the permeability tensor as

$$\overline{\overline{\mu}}^{-1} = \begin{bmatrix} u_{xx} & u_{xy} & 0\\ u_{yx} & u_{yy} & 0\\ 0 & 0 & u_{zz} \end{bmatrix}$$
(5.33)

Additional material parameters for the longitudinal ME configuration are given as

$$\overline{\overline{\varepsilon}} = \begin{bmatrix} \varepsilon_{xx} & 0 & 0 \\ 0 & \varepsilon_{yy} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{bmatrix}$$
(5.35)
$$\overline{\overline{\alpha}}^{H} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \alpha_{zz}^{H} \end{bmatrix}$$
(5.36)
$$\overline{\overline{\alpha}}^{E} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \alpha_{zz}^{E} \end{bmatrix}$$
(5.37)

Solving for the dispersion relationship, as obtained in Eq. (5.17), we obtain

$$\begin{bmatrix} \omega^{2} \varepsilon_{xx} & 0 & 0\\ 0 & \omega^{2} \varepsilon_{yy} - k_{x}^{2} u_{zz} & \omega k_{x} u_{zz} \alpha_{zz}^{E} \\ 0 & \omega k_{x} u_{zz} \alpha_{zz}^{H} & \omega^{2} \varepsilon_{zz} - k_{x}^{2} u_{yy} - \omega^{2} u_{zz} \alpha_{zz}^{H} \alpha_{zz}^{E} \end{bmatrix} \cdot \begin{bmatrix} E_{x} \\ E_{y} \\ E_{z} \end{bmatrix} = 0 \quad (5.36)$$

For non-trivial solutions, the determinant of the matrix operating on the electric field goes to zero. Thus, we obtain

$$\begin{vmatrix} \omega^{2} \varepsilon_{xx} & 0 & 0 \\ 0 & \omega^{2} \varepsilon_{yy} - k_{x}^{2} u_{zz} & \omega k_{x} u_{zz} \alpha_{zz}^{E} \\ 0 & \omega k_{x} u_{zz} \alpha_{zz}^{H} & \omega^{2} \varepsilon_{zz} - k_{x}^{2} u_{yy} - \omega^{2} u_{zz} \alpha_{zz}^{H} \alpha_{zz}^{E} \end{vmatrix} = 0$$
(5.37)

Hence, we obtain the equation,

$$\left(\omega^{2}u_{yy}u_{zz}\varepsilon_{xx}\right)k_{x}^{4} + \omega^{2}\varepsilon_{xx}\left(-\omega^{2}u_{yy}\varepsilon_{yy} - \omega^{2}u_{zz}\varepsilon_{zz}\right)k_{x}^{2} + \omega^{2}\varepsilon_{xx}\left(\omega^{4}\varepsilon_{yy}\varepsilon_{zz} - \omega^{4}u_{zz}\varepsilon_{yy}\alpha_{zz}^{H}\alpha_{zz}^{E}\right) = 0$$
(5.38)

We obtain a bi-quadratic equation in Eq. (5.38). We solve Eq. (5.38) to obtain expressions for the propagating wave number as

$$k_{x_{1}} = \pm \frac{\omega \left[u_{yy} u_{zz} \left(u_{yy} \varepsilon_{yy} + u_{zz} \varepsilon_{zz} + \sqrt{u_{yy}^{2} \varepsilon_{yy}^{2} - 2u_{yy} \varepsilon_{yy}} u_{zz} \varepsilon_{zz} + u_{zz}^{2} \varepsilon_{zz}^{2} + 4u_{zz}^{2} u_{yy} \varepsilon_{yy} \alpha_{zz}^{H} \alpha_{zz}^{E} \right) \right]^{1/2}}{\sqrt{2} u_{yy} u_{zz}}$$

$$k_{x_{2}} = \pm \frac{\omega \left[u_{yy} u_{zz} \left(u_{yy} \varepsilon_{yy} + u_{zz} \varepsilon_{zz} - \sqrt{u_{yy}^{2} \varepsilon_{yy}^{2} - 2u_{yy} \varepsilon_{yy} u_{zz} \varepsilon_{zz} + u_{zz}^{2} \varepsilon_{zz}^{2} + 4u_{zz}^{2} u_{yy} \varepsilon_{yy} \alpha_{zz}^{H} \alpha_{zz}^{E} \right) \right]^{1/2}}{\sqrt{2} u_{yy} u_{zz}}$$
(5.40)

For electromagnetic wave propagation in a bulk ME media under the longitudinal configuration, we obtain four waves propagating with two distinct speeds. The solution also shows that propagation is reversible, as the wave propagates in the forward or

backward direction with same speed. The propagating fields have no cut-off frequency based upon the direct relationship with the angular frequency; hence the propagating mode may be transverse electromagnetic (TEM). TEM mode of propagation implies lack of field components in the direction of propagation. This cannot be confirmed now, but we will confirm the mode of propagation for the longitudinal ME media in later sections when we obtain the field relationships and polarization of the propagating EM waves.

We obtain the wave number for the special case of a magnetostrictive layer. For this case, the electric field induced magnetoelectric susceptibility $\alpha^{E} = 0$. The wave numbers for this case are given as

$$k_{x_{1}} = \pm \omega \sqrt{\frac{\mathcal{E}_{yy}}{u_{zz}}}$$
(5.41)

$$k_{x_2} = \pm \omega \sqrt{\frac{\mathcal{E}_{zz}}{u_{yy}}}$$
(5.42)

We obtain four propagating waves with two distinct speeds. As observed from Eq. (5.41) and (5.42), the magnetoelectric susceptibility components do not factor into the electromagnetic wave propagation characteristics for the longitudinal configuration with magnetostrictive and piezoelectric layers. This is important information, as electromagnetic wave propagation in this orientation does not present any new propagation phenomena due to its lack of dependence on the ME effect. Hence, this orientation should not be a candidate for device applications that will make use of any EM wave propagation phenomena.

5.1.1.2 Non-gyrotropic ME media: Propagation along the x-axis

The permeability of the media changes, as we assume a non-gyrotropic ME media. The media need not be isotropic in terms of its permeability or permittivity, and may be uniaxial or biaxial. The general form of the permeability is given as

$$\overline{\overline{\mu}} = \begin{bmatrix} \mu_{xx} & 0 & 0 \\ 0 & \mu_{yy} & 0 \\ 0 & 0 & \mu_{zz} \end{bmatrix}$$
(5.43)

The inverse of the permeability tensor is

$$\overline{\mu}^{-1} = \begin{bmatrix} 1/\mu_{xx} & 0 & 0\\ 0 & 1/\mu_{yy} & 0\\ 0 & 0 & 1/\mu_{zz} \end{bmatrix}$$
(5.44)

For simplicity, we express the inverse of the permeability tensor as

$$\overline{\overline{\mu}}^{-1} = \begin{bmatrix} u_{xx} & 0 & 0 \\ 0 & u_{yy} & 0 \\ 0 & 0 & u_{zz} \end{bmatrix}$$
(5.45)

All other material parameters remain the same. Solving the wave equation for the media,

$$\begin{bmatrix} \omega^{2} \varepsilon_{xx} & 0 & 0\\ 0 & \omega^{2} \varepsilon_{yy} - k_{x}^{2} u_{zz} & \omega k_{x} u_{zz} \alpha_{zz}^{E} \\ 0 & \omega k_{x} u_{zz} \alpha_{zz}^{H} & \omega^{2} \varepsilon_{zz} - k_{x}^{2} u_{yy} - \omega^{2} u_{zz} \alpha_{zz}^{H} \alpha_{zz}^{E} \end{bmatrix} \cdot \begin{bmatrix} E_{x} \\ E_{y} \\ E_{z} \end{bmatrix} = 0 \quad (5.46)$$

For non-trivial solutions, we set the determinant of the matrix operating on the electric field to zero. Thus, we have

$$\begin{array}{cccc} \omega^{2} \varepsilon_{xx} & 0 & 0\\ 0 & \omega^{2} \varepsilon_{yy} - k_{x}^{2} u_{zz} & \omega k_{x} u_{zz} \alpha_{zz}^{E} \\ 0 & \omega k_{x} u_{zz} \alpha_{zz}^{H} & \omega^{2} \varepsilon_{zz} - k_{x}^{2} u_{yy} - \omega^{2} u_{zz} \alpha_{zz}^{H} \alpha_{zz}^{E} \end{array} = 0$$
 (5.47)

Hence, we obtain a bi-quadratic equation given as

$$\left(\omega^{2}u_{yy}u_{zz}\varepsilon_{xx}\right)k_{x}^{4} + \omega^{2}\varepsilon_{xx}\left(-\omega^{2}u_{yy}\varepsilon_{yy} - \omega^{2}u_{zz}\varepsilon_{zz}\right)k_{x}^{2} + \omega^{2}\varepsilon_{xx}\left(\omega^{4}\varepsilon_{yy}\varepsilon_{zz} - \omega^{4}u_{zz}\varepsilon_{yy}\alpha_{zz}^{H}\alpha_{zz}^{E}\right) = 0$$
(5.48)

Observe that this is the same expression obtained for the case of the gyrotropic ME media in Eq. (5.38). The gyrotropic case is similar to this case because the off-diagonal components of the permeability matrix (gyrotropic) do not play a role in the eigenvalue equations due to the mostly zero components of the ME susceptibility tensors. This implies that the change to the shape of the permeability tensor does not affect the electromagnetic wave propagation along the sample plane for the longitudinal configuration. However, the actual speed of propagation will change since the values of the diagonal terms of the permeability for gyrotropic and non-gyrotropic cases are different. Hence the wave numbers obtained are not the same

$$k_{x_{1}} = \pm \frac{\omega \left[u_{yy}u_{zz} \left(u_{yy}\varepsilon_{yy} + u_{zz}\varepsilon_{zz} + \sqrt{u_{yy}^{2}\varepsilon_{yy}^{2} - 2u_{yy}\varepsilon_{yy}}u_{zz}\varepsilon_{zz} + u_{zz}^{2}\varepsilon_{zz}^{2} + 4u_{zz}^{2}u_{yy}\varepsilon_{yy}\alpha_{zz}^{H}\alpha_{zz}^{E} \right) \right]^{1/2}}{\sqrt{2}u_{yy}u_{zz}}$$

$$(5.49)$$

$$k_{x_{1}} = \pm \frac{\omega \left[u_{yy}u_{zz} \left(u_{yy}\varepsilon_{yy} + u_{zz}\varepsilon_{zz} - \sqrt{u_{yy}^{2}\varepsilon_{yy}^{2} - 2u_{yy}\varepsilon_{yy}}u_{zz}\varepsilon_{zz} + u_{zz}^{2}\varepsilon_{zz}^{2} + 4u_{zz}^{2}u_{yy}\varepsilon_{yy}\alpha_{zz}^{H}\alpha_{zz}^{E} \right) \right]^{1/2}}{\sqrt{2}u_{yy}u_{zz}}$$

$$(5.50)$$

Again, we obtain the wave number for the special case of a magnetostrictive layer. For this case, the electric field induced magnetoelectric susceptibility $\alpha^{E} = 0$. The

wave numbers are same, in expression, as obtained for a gyrotropic media composed of a magnetostrictive phase as expressed in Eq. (5.41) and (5.42). We obtain four waves propagating in the media with two distinct speeds. The waves propagate in the forward and backward directions with same speed. The magnetoelectric susceptibility components do not factor into the electromagnetic wave propagation characteristics for the longitudinal configuration with non-gyrotropic magnetostrictive and piezoelectric layers.



FIGURE 5.2. Propagation directions for the transverse ME configuration in a homogenized ME medium. Propagation in the x direction results in propagation parallel to the magnetic field bias, while propagation in the y direction results in propagation perpendicular to the magnetic bias.

5.1.2 Transverse ME Electromagnetic wave Propagation

For the transverse configuration, we consider propagation parallel and perpendicular to the DC magnetic field bias as shown in Fig. 5.2. We expect a change in the propagation direction to affect the resultant electromagnetic wave propagation 129 characteristics. The direction of propagation along the *xy* plane plays affects propagation in the transverse ME configuration unlike the case for the longitudinal ME configuration. Propagation along the *x* axis results in the propagation being parallel with the magnetic bias. This orientation usually results in Faraday rotation in gyrotropic materials such as ferrites. Propagation in the *y* implies that the propagation direction is perpendicular to the magnetic bias. In ferrites, such orientation results in the propagation of ordinary and extraordinary waves in the media. Solutions to the wave equation for the transverse ME configuration will give clear insights on how the ME coupling affects the electromagnetic wave propagation phenomena in the media.

5.1.2.1 Gyrotropic ME media: Propagation along the x-axis

We consider propagation along the *x*-axis, thus the wave number vector is given as

$$\overline{k} = \hat{x}k_x \tag{5.51}$$

Thus, the operator, $\frac{\overline{g}}{g}$, is expressed as

$$\overline{\overline{g}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -k_x \\ 0 & k_x & 0 \end{bmatrix}$$
(5.52)

For the gyrotropic ME media, the permeability tensor for the transverse configuration is given as

$$\overline{\overline{\mu}} = \begin{bmatrix} \mu_{xx} & 0 & 0 \\ 0 & \mu_{yy} & \mu_{yz} \\ 0 & \mu_{yz} & \mu_{zz} \end{bmatrix}$$
(5.53)

For simplicity, as was done previously, we will express the inverse of the permeability tensor as

$$\overline{\overline{\mu}}^{-1} = \begin{bmatrix} u_{xx} & 0 & 0\\ 0 & u_{yy} & u_{yz}\\ 0 & u_{zy} & u_{zz} \end{bmatrix}$$
(5.54)

The values of the matrix elements for the inverse of the permeability are easily obtained and are expressed in matrix form as

$$\overline{\overline{\mu}}^{-1} = \begin{bmatrix} 1/\mu_{xx} & 0 & 0\\ 0 & \mu_{zz} / (\mu_{yy}\mu_{zz} - \mu_{yz}\mu_{zy}) & -\mu_{yz} / (\mu_{yy}\mu_{zz} - \mu_{yz}\mu_{zy})\\ 0 & -\mu_{zy} / (\mu_{yy}\mu_{zz} - \mu_{yz}\mu_{zy}) & \mu_{yy} / (\mu_{yy}\mu_{zz} - \mu_{yz}\mu_{zy}) \end{bmatrix}$$
(5.55)

Additional material parameters for the transverse orientation are given as

$$\overline{\overline{\varepsilon}} = \begin{bmatrix} \varepsilon_{xx} & 0 & 0 \\ 0 & \varepsilon_{yy} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{bmatrix}$$
(5.56)

$$\overline{\overline{\alpha}}^{H} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \alpha^{H}_{zx} & 0 & 0 \end{bmatrix}$$
(5.57)

$$\overline{\overline{\alpha}}^{E} = \begin{bmatrix} 0 & 0 & \alpha_{xz}^{E} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$
(5.58)

Solving for the dispersion relationship, as obtained in Eq. (5.17), we obtain

$$\begin{bmatrix} \omega^{2} \varepsilon_{xx} & 0 & 0\\ 0 & \omega^{2} \varepsilon_{yy} - k_{x}^{2} u_{zz} & k_{x}^{2} u_{zy}\\ 0 & k_{x}^{2} u_{yz} & \omega^{2} \varepsilon_{zz} - k_{x}^{2} u_{yy} - \omega^{2} u_{xx} \alpha_{zx}^{H} \alpha_{zz}^{E} \end{bmatrix} \cdot \begin{bmatrix} E_{x} \\ E_{y} \\ E_{z} \end{bmatrix} = 0 \quad (5.59)$$

For non-trivial solutions, the determinant of the matrix is set to zero. We have,

$$\begin{vmatrix} \omega^{2} \varepsilon_{xx} & 0 & 0\\ 0 & \omega^{2} \varepsilon_{yy} - k_{x}^{2} u_{zz} & k_{x}^{2} u_{zy} \\ 0 & k_{x}^{2} u_{yz} & \omega^{2} \varepsilon_{zz} - k_{x}^{2} u_{yy} - \omega^{2} u_{xx} \alpha_{zx}^{H} \alpha_{xz}^{E} \end{vmatrix} = 0$$
(5.60)

Hence, we obtain the equation

$$-\omega^{2}\varepsilon_{xx}\left(-u_{yy}u_{zz}+u_{yz}u_{zy}\right)k_{x}^{4}-\omega^{2}\varepsilon_{xx}\left(\omega^{2}u_{yy}\varepsilon_{yy}+\omega^{2}u_{zz}\varepsilon_{zz}-\omega^{2}u_{zz}u_{xx}\alpha_{zx}^{H}\alpha_{xz}^{E}\right)k_{x}^{2}$$
$$-\omega^{2}\varepsilon_{xx}\left(-\omega^{4}\varepsilon_{yy}\varepsilon_{zz}+\omega^{4}u_{xx}\varepsilon_{yy}\alpha_{zx}^{H}\alpha_{xz}^{E}\right)=0$$
(5.61)

We obtain solutions to the bi-quadratic equation above, and obtain the first set of propagating wave numbers as

$$k_{x_{1}} = \pm \frac{\omega \left[\left(u_{yy} u_{zz} - u_{yz} u_{zy} \right) \left(u_{yy} \varepsilon_{yy} + u_{zz} \varepsilon_{zz} - u_{xx} u_{zz} \alpha_{zx}^{H} \alpha_{xz}^{E} + \sqrt{RootTerm} \right) \right]^{1/2}}{\sqrt{2} \left(u_{yy} u_{zz} - u_{yz} u_{zy} \right)}$$
(5.62)

In Eq. (5.62)

$$RootTerm = u_{yy}^{2} \varepsilon_{yy}^{2} - 2u_{yy} \varepsilon_{yy} u_{zz} \varepsilon_{zz} + 2u_{zz} u_{yy} u_{xx} \varepsilon_{yy} \alpha_{zx}^{H} \alpha_{xz}^{E} + u_{zz}^{2} \varepsilon_{zz}^{2}$$
$$-2u_{zz}^{2} u_{xx} \varepsilon_{zz} \alpha_{zx}^{H} \alpha_{xz}^{E} + u_{zz}^{2} u_{xx}^{2} \left(\alpha_{zx}^{H}\right)^{2} \left(\alpha_{xz}^{E}\right)^{2} + 4u_{yz} u_{zy} \varepsilon_{yy} \varepsilon_{zz}$$
$$-4u_{yz} u_{zy} u_{xx} \varepsilon_{yy} \alpha_{zx}^{H} \alpha_{xz}^{E}$$
(5.63)

The second set of roots describing the wave number is obtained as

$$k_{x_{2}} = \pm \frac{\omega \left[\left(u_{yy} u_{zz} - u_{yz} u_{zy} \right) \left(u_{yy} \varepsilon_{yy} + u_{zz} \varepsilon_{zz} - u_{xx} u_{zz} \alpha_{zx}^{H} \alpha_{xz}^{E} - \sqrt{RootTerm} \right) \right]^{1/2}}{\sqrt{2} \left(u_{yy} u_{zz} - u_{yz} u_{zy} \right)}$$
(5.64)

Here, the *RootTerm* is same as has been given in Eq. (5.63).

We obtain four propagating EM waves, two in the forward direction and two in the reverse direction. The waves propagate in the forward direction with two distinct speeds. This is also the case for waves propagating in the reverse direction. We observe that the wave has no cut-off frequency and may be TEM in nature.

We obtain the wave numbers for the special case of a magnetostrictive layer, by setting the electric field induced magnetoelectric susceptibility $\alpha^{E} = 0$. The wave numbers for this case are given as

$$k_{x_{1}} = \pm \frac{\omega \left[\left(u_{zz} u_{yy} - u_{zy} u_{yz} \right) \left(\varepsilon_{yy} u_{yy} + u_{zz} \varepsilon_{zz} + \sqrt{\varepsilon_{yy}^{2} u_{yy}^{2} - 2\varepsilon_{yy} u_{yy} u_{zz} \varepsilon_{zz} + u_{zz}^{2} \varepsilon_{zz}^{2} + 4\varepsilon_{yy} \varepsilon_{zz} u_{zy} u_{yz} \right) \right]^{0.5}}{\sqrt{2} \left(u_{zz} u_{yy} - u_{zy} u_{yz} \right)}$$

$$k_{x_{2}} = \pm \frac{\omega \left[\left(u_{zy} u_{yz} - u_{zz} u_{yy} \right) \left(\varepsilon_{yy} u_{yy} + u_{zz} \varepsilon_{zz} - \sqrt{\varepsilon_{yy}^{2} u_{yy}^{2} - 2\varepsilon_{yy} u_{yy} u_{zz} \varepsilon_{zz} + u_{zz}^{2} \varepsilon_{zz}^{2} + 4\varepsilon_{yy} \varepsilon_{zz} u_{zy} u_{yz} \right) \right]^{0.5}}{\sqrt{2} \left(u_{zz} u_{yy} - u_{zy} u_{yz} \right)}$$
(5.66)

We obtain four waves propagating with two distinct speeds. The waves are reversible, propagating in the forward and reverse directions with same speed. It is important to note that the magnetoelectric susceptibility components still do not factor into the propagation characteristics when $\alpha^{E} = 0$. Hence, we have observed for the cases studied thus far that the propagation characteristics are devoid of any ME effects.

5.1.2.2 Gyrotropic ME media: Propagation along the y-axis

Now we consider propagation in the *y*-direction, such that the direction of propagation is transverse to the direction of the DC magnetic field bias. Here, the wave number is then expressed as

$$\overline{k} = \hat{y}k_{y} \tag{5.67}$$

Thus, the operator, $\overline{\overline{g}}$, is expressed as

$$\overline{\overline{g}} = \begin{bmatrix} 0 & 0 & k_y \\ 0 & 0 & 0 \\ -k_y & 0 & 0 \end{bmatrix}$$
(5.68)

The material parameters are as have been used in the *x*-directed propagation and are defined by Eq. (5.53) - (5.58). Solving for the dispersion relationship, we obtain

$$\begin{bmatrix} \omega^{2} \varepsilon_{xx} - k_{y}^{2} u_{zz} & 0 & 0 \\ 0 & \omega^{2} \varepsilon_{yy} & 0 \\ 0 & 0 & \omega^{2} \varepsilon_{zz} + \left(-k_{y} + \omega \alpha_{zx}^{H}\right) u_{xx} \left(k_{y} - \omega \alpha_{xz}^{E}\right) \end{bmatrix} \cdot \begin{bmatrix} E_{x} \\ E_{y} \\ E_{z} \end{bmatrix} = 0 \quad (5.69)$$

For non-trivial solutions, we set the determinant of the matrix operating on the electric field to zero. Thus, we have

$$\begin{vmatrix} \omega^{2} \varepsilon_{xx} - k_{y}^{2} u_{zz} & 0 & 0 \\ 0 & \omega^{2} \varepsilon_{yy} & 0 \\ 0 & 0 & \omega^{2} \varepsilon_{zz} + \left(-k_{y} + \omega \alpha_{zx}^{H} \right) u_{xx} \left(k_{y} - \omega \alpha_{xz}^{E} \right) \end{vmatrix} = 0 \quad (5.70)$$

Hence, we obtain the equation

$$\left(\omega^{2}u_{xx}u_{zz}\varepsilon_{yy}\right)k_{y}^{4}-\omega^{2}u_{zz}\varepsilon_{yy}\left(\omega u_{xx}\alpha_{xz}^{E}+\omega u_{xx}\alpha_{zx}^{H}\right)k_{y}^{3}$$
$$+\left[-\omega^{4}u_{xx}\varepsilon_{xx}\varepsilon_{yy}+\omega^{2}u_{zz}\varepsilon_{yy}\left(-\omega^{2}\varepsilon_{zz}+\omega^{2}u_{xx}\alpha_{zx}^{H}\alpha_{xz}^{E}\right)\right]k_{y}^{2}$$
$$134$$

$$+\omega^{4}\varepsilon_{xx}\varepsilon_{yy}\left(\omega u_{xx}\alpha_{xz}^{E}+\omega u_{xx}\alpha_{zx}^{H}\right)k_{y}-\omega^{4}\varepsilon_{xx}\varepsilon_{yy}\left(-\omega^{2}\varepsilon_{zz}+\omega^{2}u_{xx}\alpha_{zx}^{H}\alpha_{xz}^{E}\right)=0$$
(5.71)

Solution to the equation is obtained as

$$k_{y_1} = \pm \frac{\omega \sqrt{u_{zz}} \varepsilon_{xx}}{u_{zz}}$$
(5.72)

$$k_{y_{2}} = \frac{\omega \left[u_{xx} \left(\alpha_{xz}^{E} + \alpha_{zx}^{H} \right) \pm \sqrt{u_{xx}^{2} \left(\alpha_{xz}^{E} \right)^{2} - 2u_{xx}^{2} \alpha_{zx}^{E} \alpha_{zx}^{H} + u_{xx}^{2} \left(\alpha_{zx}^{H} \right)^{2} + 4u_{xx} \varepsilon_{zz}}{2u_{xx}} \right]}{2u_{xx}}$$
(5.73)

We obtain a set of ordinary waves as shown in Eq. (5.72) and a set of extraordinary waves as expressed in Eq. (5.73). The waves are reversible, propagating with the same speed in the forward and reverse directions. The wave number in Eq. (5.73) is viewed as extraordinary as it contains and is dependent on the magnetoelectric susceptibility parameters. We will see a little later how this affects the polarization of the wave.

We obtain the wave number for the special case of a magnetostrictive layer. For this case, the electric field induced magnetoelectric susceptibility $\alpha^{E} = 0$. The wave numbers for this case are given as

$$k_{y_1} = \pm \omega \sqrt{\frac{\varepsilon_{xx}}{u_{zz}}}$$
(5.74a)

$$k_{y_2} = \frac{\omega \left(\alpha_{zx}^H u_{xx} \pm \sqrt{\left(\alpha_{zx}^H u_{xx}\right)^2 + 4u_{xx}\varepsilon_{zz}}\right)}{2u_{xx}}$$
(5.74b)

We also obtain a set of ordinary waves and another set of extraordinary waves propagating in the forward and reverse directions with two distinct speeds. The extraordinary waves are influenced by the magnetoelectric susceptibility tensor. The 135 result is similar to that obtained for the piezoelectric/piezomagnetic composite media. Additional details on the mode of propagation, and polarization state of the eigenvectors will be discussed in subsequent sections.

5.1.2.3 Non-gyrotropic ME media: Propagation along the y-axis

We observed ordinary and extraordinary waves for the magnetoelectric composite with magnetic anisotropy. We keep the propagation along the *y*-axis, and observe the effects of a non-gyrotropic permeability matrix. Here, the wave number is expressed in vector form as

$$\overline{k} = \hat{y}k_{y} \tag{5.75}$$

Thus, the operator, $\overline{\overline{g}}$, is expressed as

$$\overline{\overline{g}} = \begin{bmatrix} 0 & 0 & k_y \\ 0 & 0 & 0 \\ -k_y & 0 & 0 \end{bmatrix}$$
(5.76)

The permeability for the non-gyrotropic ME media is

$$\overline{\overline{\mu}} = \begin{bmatrix} \mu_{xx} & 0 & 0 \\ 0 & \mu_{yy} & 0 \\ 0 & 0 & \mu_{zz} \end{bmatrix}$$
(5.77)

The inverse of the permeability tensor is

$$\overline{\overline{\mu}}^{-1} = \begin{bmatrix} 1/\mu_{xx} & 0 & 0\\ 0 & 1/\mu_{yy} & 0\\ 0 & 0 & 1/\mu_{zz} \end{bmatrix}$$
(5.78)

For simplicity, as has been done previously, we will express the inverse of the permeability tensor as

$$\overline{\overline{\mu}}^{-1} = \begin{bmatrix} u_{xx} & 0 & 0 \\ 0 & u_{yy} & 0 \\ 0 & 0 & u_{zz} \end{bmatrix}$$
(5.79)

Solving for the dispersion relationship, we obtain

$$\begin{bmatrix} \omega^{2} \varepsilon_{xx} - k_{y}^{2} u_{zz} & 0 & 0 \\ 0 & \omega^{2} \varepsilon_{yy} & 0 \\ 0 & 0 & \omega^{2} \varepsilon_{zz} + \left(-k_{y} + \omega \alpha_{zx}^{H}\right) u_{xx} \left(k_{y} - \omega \alpha_{xz}^{E}\right) \end{bmatrix} \cdot \begin{bmatrix} E_{x} \\ E_{y} \\ E_{z} \end{bmatrix} = 0 \quad (5.80)$$

For non-trivial solutions, the determinant of the matrix goes to zero. Thus, we have

$$\begin{vmatrix} \omega^{2} \varepsilon_{xx} - k_{y}^{2} u_{zz} & 0 & 0 \\ 0 & \omega^{2} \varepsilon_{yy} & 0 \\ 0 & 0 & \omega^{2} \varepsilon_{zz} + \left(-k_{y} + \omega \alpha_{zx}^{H} \right) u_{xx} \left(k_{y} - \omega \alpha_{xz}^{E} \right) \end{vmatrix} = 0 \quad (5.81)$$

Hence, we obtain the equation

$$\left(\omega^{2} u_{xx} u_{zz} \varepsilon_{yy} \right) k_{y}^{4} - \omega^{2} u_{zz} \varepsilon_{yy} \left(\omega u_{xx} \alpha_{xz}^{E} + \omega u_{xx} \alpha_{zx}^{H} \right) k_{y}^{3}$$

$$+ \left[-\omega^{4} u_{xx} \varepsilon_{xx} \varepsilon_{yy} + \omega^{2} u_{zz} \varepsilon_{yy} \left(-\omega^{2} \varepsilon_{zz} + \omega^{2} u_{xx} \alpha_{zx}^{H} \alpha_{zz}^{E} \right) \right] k_{y}^{2}$$

$$+ \omega^{4} \varepsilon_{xx} \varepsilon_{yy} \left(\omega u_{xx} \alpha_{xz}^{E} + \omega u_{xx} \alpha_{zx}^{H} \right) k_{y} - \omega^{4} \varepsilon_{xx} \varepsilon_{yy} \left(-\omega^{2} \varepsilon_{zz} + \omega^{2} u_{xx} \alpha_{zx}^{H} \alpha_{zz}^{E} \right) = 0$$

$$(5.82)$$

Solution to the equation is obtained as

$$k_{y_1} = \pm \frac{\omega \sqrt{u_{zz}} \mathcal{E}_{xx}}{u_{zz}}$$
(5.83a)

$$k_{y_{2}} = \frac{\omega \left[u_{xx} \left(\alpha_{xz}^{E} + \alpha_{zx}^{H} \right) \pm \sqrt{u_{xx}^{2} \left(\alpha_{xz}^{E} \right)^{2} - 2u_{xx}^{2} \alpha_{xz}^{E} \alpha_{zx}^{H} + u_{xx}^{2} \left(\alpha_{zx}^{H} \right)^{2} + 4u_{xx} \varepsilon_{zz}}{2u_{xx}} \right]}{2u_{xx}}$$
(5.83a)

We obtain a set of ordinary waves, and a set of extraordinary waves propagating in the medium. Observe that this is the same solution obtained for propagation in the *y* direction for a gyrotropic magnetoelectric media. Hence, we state that the electromagnetic wave propagation characteristics for a propagation direction transverse to the magnetic field bias in a magnetoelectric media remains the same whether the media is gyrotropic or non-gyrotropic. The actual speeds of the wave will differ, however the eigenvectors should have same modes of propagation and polarization states.

We now consider the case of a magnetostrictive material used as the piezomagnetic layer. For this case, as defined for the case of magnetostrictive layers, the electric field induced magnetoelectric susceptibility $\alpha^{E} = 0$. The wave numbers for this case are given as

$$k_{y_1} = \pm \omega \sqrt{\frac{\varepsilon_{xx}}{u_{zz}}}$$
(5.84a)

$$k_{y_2} = \frac{\omega \left(\alpha_{zx}^H u_{xx} \pm \sqrt{\left(\alpha_{zx}^H u_{xx}\right)^2 + 4u_{xx} \varepsilon_{zz}}\right)}{2u_{xx}}$$
(5.84b)

This is same result as that obtained for *y*-directed propagation with magnetic anisotropy. We obtain a set of ordinary waves and another set of extraordinary waves propagating in the forward and reverse directions.

5.1.2.4 Non-gyrotropic ME media: Propagation along the x-axis

We consider a non-gyrotropic magnetoelectric composite for propagation along the bias direction. Hence, we assume propagation in the *x*-direction, thus,

$$\overline{k} = \hat{x}k_x \tag{5.85}$$

Thus, the operator, $\overline{\overline{g}}$, is expressed as

$$\overline{\overline{g}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -k_x \\ 0 & k_x & 0 \end{bmatrix}$$
(5.86)

The magnetoelectric susceptibility and effective permittivity are of the form as have already been used in obtaining the wave numbers for propagation in a gyrotropic magnetoelectric composite. The permeability of the media remains the same as that in the case of the non-gyrotropic magnetoelectric composite, and propagation along the *y*direction. Solving the dispersion relationship, we obtain

$$\begin{bmatrix} \omega^{2} \varepsilon_{xx} & 0 & 0\\ 0 & \omega^{2} \varepsilon_{yy} - k_{x}^{2} u_{zz} & 0\\ 0 & 0 & \omega^{2} \varepsilon_{zz} - \omega^{2} u_{xx} \alpha_{zx}^{H} \alpha_{xz}^{E} - k_{x}^{2} u_{yy} \end{bmatrix} \cdot \begin{bmatrix} E_{x} \\ E_{y} \\ E_{z} \end{bmatrix} = 0$$
(5.87)

For non-trivial solutions,

$$\begin{vmatrix} \omega^2 \varepsilon_{xx} & 0 & 0\\ 0 & \omega^2 \varepsilon_{yy} - k_x^2 u_{zz} & 0\\ 0 & 0 & \omega^2 \varepsilon_{zz} - \omega^2 u_{xx} \alpha_{zx}^H \alpha_{xz}^E - k_x^2 u_{yy} \end{vmatrix} = 0$$
(5.88)

Then we obtain the equation,

$$\left(\omega^{2}u_{yy}u_{zz}\varepsilon_{xx}\right)k_{x}^{4}+\left[-\omega^{4}u_{yy}\varepsilon_{xx}\varepsilon_{yy}-\omega^{2}u_{zz}\varepsilon_{xx}\left(\omega^{2}\varepsilon_{zz}-\omega^{2}u_{xx}\alpha_{zx}^{H}\alpha_{xz}^{E}\right)\right]k_{x}^{2}$$
$$+\omega^{4}\varepsilon_{xx}\varepsilon_{yy}\left(\omega^{2}\varepsilon_{zz}-\omega^{2}u_{xx}\alpha_{zx}^{H}\alpha_{xz}^{E}\right)=0$$
(5.89)

Solution to the problem is obtained as

$$k_{x_{\rm l}} = \pm \frac{\omega \sqrt{u_{zz} \varepsilon_{yy}}}{u_{zz}}$$
(5.90a)

$$k_{x_2} = \pm \frac{\omega \sqrt{u_{yy} \left(\varepsilon_{zz} - u_{xx} \alpha_{zx}^H \alpha_{xz}^E\right)}}{u_{yy}}$$
(5.90b)

We obtain two distinct wave numbers, one for a set of ordinary waves propagating in the positive and negative *x*-directions, and another for a set of extraordinary waves also propagating in the positive and negative *x*-directions. The ordinary and extraordinary waves propagate with different speeds. Effects of the wave numbers on the polarization of the fields will be discussed later when we derive the polarization state for each propagating wave in the media.

Finally for the transverse configuration, we obtain the wave numbers for the special case of a magnetostrictive layer. The wave numbers for this case are obtained as

$$k_{x_1} = \pm \omega \sqrt{\frac{\varepsilon_{yy}}{u_{zz}}}$$
(5.91a)

$$k_{x_2} = \pm \omega \sqrt{\frac{\varepsilon_{zz}}{u_{yy}}}$$
(5.91b)

Hence, we obtain two sets of ordinary wave propagating with two distinct speeds. As observed from Eq. (5.91), the magnetoelectric susceptibility components do not affect the electromagnetic wave propagation characteristics of the magnetoelectric composite. Additional information on the mode of propagation and polarization will be given in subsequent sections.

5.1.3 In-plane ME Electromagnetic wave Propagation

The effective material properties for computing solutions to the electromagnetic wave propagation in the in-plane configuration are similar to that for the transverse ME wave propagation. This is due to the fact that the change in direction of the poling electric field does not directly affect the shape of the permeability or permittivity tensor. This is due to the low electron mobility in the piezoelectric media, as has already been explained in the first chapter. The change in the poling electric field only changes the shape of the magnetoelectric susceptibility tensors. In this section, we will investigate the effects of the changes to the magnetoelectric susceptibility tensors to the electromagnetic wave propagation characteristics in the magnetoelectric composite. We will also consider the effects of gyrotropy on the electromagnetic wave propagation characteristics, as have been done in preceding sections. A figurative description of electromagnetic wave propagation in the in-plane configuration is shown in Fig. 5.3.

5.1.3.1 Gyrotropic ME media: Propagation along the x-axis

We follow similar steps as has been done for previous magnetoelectric configurations. We start with propagation in the *x*-direction such that

$$\overline{k} = \hat{x}k_x \tag{5.85}$$

Thus, the operator, $\overline{\overline{g}}$, is expressed as

$$\overline{\overline{g}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -k_x \\ 0 & k_x & 0 \end{bmatrix}$$
(5.86)



FIGURE 5.3. Possible propagation directions in a homogenized ME medium under the in-plane ME configuration. Propagation in the x-direction is parallel to both bias and poling field, while in the y-direction result in propagation direction perpendicular to the bias and poling fields.

All material properties but the magnetoelectric susceptibilities are same as that for the case of the transverse magnetoelectric configuration. The magnetoelectric susceptibility is given as

$$\overline{\overline{\alpha}}^{H} = \begin{bmatrix} \alpha_{xx}^{H} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$
(5.92)
$$\overline{\overline{\alpha}}^{E} = \begin{bmatrix} \alpha_{xx}^{E} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$
(5.93)

Solving for the dispersion relationship using Eq. (5.17), we obtain

$$\begin{bmatrix} \omega^{2} \varepsilon_{xx} - \omega^{2} u_{xx} \alpha_{xx}^{H} \alpha_{xx}^{E} & 0 & 0\\ 0 & \omega^{2} \varepsilon_{yy} - k_{x}^{2} u_{zz} & k_{x}^{2} u_{zy}\\ 0 & k_{x}^{2} u_{yz} & \omega^{2} \varepsilon_{zz} - k_{x}^{2} u_{yy} \end{bmatrix} \cdot \begin{bmatrix} E_{x} \\ E_{y} \\ E_{z} \end{bmatrix} = 0$$
(5.94)

For non-trivial solutions, the determinant of the matrix operating on the electric field goes to zero. Thus, we have

$$\begin{vmatrix} \omega^{2} \varepsilon_{xx} - \omega^{2} u_{xx} \alpha_{xx}^{H} \alpha_{xx}^{E} & 0 & 0 \\ 0 & \omega^{2} \varepsilon_{yy} - k_{x}^{2} u_{zz} & k_{x}^{2} u_{zy} \\ 0 & k_{x}^{2} u_{yz} & \omega^{2} \varepsilon_{zz} - k_{x}^{2} u_{yy} \end{vmatrix} = 0$$
(5.95)

Thus, we obtain a bi-quadratic equation given as

$$\left[\left(\omega^{2}\varepsilon_{xx}-\omega^{2}u_{xx}\alpha_{xx}^{H}\alpha_{xx}^{E}\right)\left(u_{yy}u_{zz}-u_{zy}u_{yz}\right)\right]k_{x}^{4}+\left[\left(\omega^{2}\varepsilon_{xx}-\omega^{2}u_{xx}\alpha_{xx}^{H}\alpha_{xx}^{E}\right)\left(-\omega^{2}u_{yy}\varepsilon_{yy}-\omega^{2}u_{zz}\varepsilon_{zz}\right)\right]k_{x}^{2}$$
$$+\omega^{4}\varepsilon_{zz}\varepsilon_{yy}\left(\omega^{2}\varepsilon_{zz}-\omega^{2}u_{xx}\alpha_{zx}^{H}\alpha_{xz}^{E}\right)=0$$
(5.96)

Solutions to the problem are then obtained as

$$k_{x_{1}} = \pm \frac{\omega \left[\left(u_{yy} u_{zz} - u_{yz} u_{zy} \right) \left(u_{yy} \varepsilon_{yy} + u_{zz} \varepsilon_{zz} + \sqrt{RootTerm_{2}} \right) \right]^{1/2}}{\sqrt{2} \left(u_{yy} u_{zz} - u_{yz} u_{zy} \right)}$$
(5.97a)

In Eq. (5.97a)

$$RootTerm_2 = u_{yy}^2 \varepsilon_{yy}^2 - 2u_{yy} \varepsilon_{yy} u_{zz} \varepsilon_{zz} + u_{zz}^2 \varepsilon_{zz}^2 + 4u_{yz} u_{zy} \varepsilon_{yy} \varepsilon_{zz}$$
(5.98)

The second set of roots describing the wave number is obtained as

$$k_{x_{2}} = \pm \frac{\omega \left[\left(u_{yy} u_{zz} - u_{yz} u_{zy} \right) \left(u_{yy} \varepsilon_{yy} + u_{zz} \varepsilon_{zz} - \sqrt{RootTerm_{2}} \right) \right]^{1/2}}{\sqrt{2} \left(u_{yy} u_{zz} - u_{yz} u_{zy} \right)}$$
(5.97a)

Here, the *RootTerm*₂ is same as has been given in Eq. (5.98).

We obtain four possible propagating waves in the medium, two propagating in the forward direction and two in the reverse direction. The results we obtained here confirm the results obtained by Birss and Shrubsall [39] that the electromagnetic wave propagation in ME materials is reversible with a constant speed. That has been the case with all the wave numbers obtained thus far for all magnetoelectric configurations. Observe that with the direction of propagation parallel to the magnetoelectric coupling axis, the wave only sees the transverse components of the permeability and permittivity which confirms the study conducted by Lindell [5], that the propagating wave will not see the magnetoelectric coupling when the propagation direction lies along the direction of the magnetoelectric effect. This may also imply that the magnetoelectric effect in the in-plane configuration lies along the optical axis of the material. Hence, the ME coupling tensors do not affect the wave propagating along that direction.

As have been done for the previous two magnetoelectric configurations, we obtain the wave number for the special case of a magnetostrictive material for the piezomagnetic layer. Setting the electric field induced magnetoelectric susceptibility $\alpha^{E} = 0$. The wave numbers for this case are given as

$$k_{x_{1}} = \pm \frac{\omega \left[\left(u_{zz} u_{yy} - u_{zy} u_{yz} \right) \left(\varepsilon_{yy} u_{yy} + u_{zz} \varepsilon_{zz} + \sqrt{\varepsilon_{yy}^{2} u_{yy}^{2} - 2\varepsilon_{yy} u_{yy} u_{zz} \varepsilon_{zz} + u_{zz}^{2} \varepsilon_{zz}^{2} + 4\varepsilon_{yy} \varepsilon_{zz} u_{zy} u_{yz} \right) \right]^{0.5}}{\sqrt{2} \left(u_{zz} u_{yy} - u_{zy} u_{yz} \right)}$$

$$k_{x_{2}} = \pm \frac{\omega \left[\left(u_{zy} u_{yz} - u_{zz} u_{yy} \right) \left(\varepsilon_{yy} u_{yy} + u_{zz} \varepsilon_{zz} - \sqrt{\varepsilon_{yy}^{2}} u_{yy}^{2} - 2\varepsilon_{yy} u_{yy} u_{zz} \varepsilon_{zz} + u_{zz}^{2} \varepsilon_{zz}^{2} + 4\varepsilon_{yy} \varepsilon_{zz} u_{zy} u_{yz} \right) \right]^{0.5}}{\sqrt{2} \left(u_{zz} u_{yy} - u_{zy} u_{yz} \right)}$$
(5.99b)

We obtain four propagating waves, with two propagating in the forward direction and two in the backward direction. The results show that the magnetoelectric coupling tensor does not affect the propagation characteristics of the wave number in the in-plane configuration with magnetostrictive/piezoelectric composite layers.

5.1.3.2 Gyrotropic ME media: Propagation in the y-direction

In the *y*-directed propagation for the in-plane configuration, the wave number is perpendicular to the magnetic bias and poling electric field. The wave number is defined as

$$k = \hat{y}k_x \tag{5.75}$$

Thus, the operator, $\frac{\overline{g}}{\overline{g}}$, is expressed as

$$\overline{\overline{g}} = \begin{bmatrix} 0 & 0 & k_y \\ 0 & 0 & 0 \\ -k_y & 0 & 0 \end{bmatrix}$$
(5.76)

All material properties are same as that used for the propagation in the *x*-direction. We directly solve for the dispersion relationship.

Solving for the dispersion relationship, we obtain

$$\begin{bmatrix} \omega^{2} \varepsilon_{xx} - \omega^{2} u_{xx} \alpha_{xx}^{H} \alpha_{xx}^{E} - k_{y}^{2} u_{zz} & 0 & \omega u_{xx} \alpha_{xx}^{H} k_{y} \\ 0 & \omega^{2} \varepsilon_{yy} & 0 \\ \omega u_{xx} \alpha_{xx}^{E} k_{y} & 0 & \omega^{2} \varepsilon_{zz} - k_{y}^{2} u_{xx} \end{bmatrix} \cdot \begin{bmatrix} E_{x} \\ E_{y} \\ E_{z} \end{bmatrix} = 0$$
(5.100)

For non-trivial solutions, we set the determinant of the matrix to zero. Hence we have

$$\begin{vmatrix} \omega^{2} \varepsilon_{xx} - \omega^{2} u_{xx} \alpha_{xx}^{H} \alpha_{xx}^{E} - k_{y}^{2} u_{zz} & 0 & \omega u_{xx} \alpha_{xx}^{H} k_{y} \\ 0 & \omega^{2} \varepsilon_{yy} & 0 \\ \omega u_{xx} \alpha_{xx}^{E} k_{y} & 0 & \omega^{2} \varepsilon_{zz} - k_{y}^{2} u_{xx} \end{vmatrix} = 0$$
(5.101)
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Thus, we obtain the bi-quadratic equation

$$\left(\omega^{2}u_{xx}u_{zz}\varepsilon_{yy}\right)k_{x}^{4}-\left(\omega^{4}u_{xx}\varepsilon_{xx}\varepsilon_{yy}+\omega^{4}u_{zz}\varepsilon_{xx}\varepsilon_{yy}\right)k_{x}^{2}+\omega^{6}\varepsilon_{zz}\varepsilon_{yy}\varepsilon_{xx}-\omega^{6}u_{xx}\varepsilon_{yy}\varepsilon_{zz}\alpha_{xx}^{H}\alpha_{xx}^{E}=0$$
 (5.102)

Solutions to the problem are obtained as

$$k_{y_1} = \pm \frac{\omega \left[u_{xx} u_{zz} \left(u_{xx} \varepsilon_{xx} + u_{zz} \varepsilon_{zz} + \sqrt{RootTerm_3} \right) \right]^{1/2}}{\sqrt{2} u_{xx} u_{zz}}$$
(5.103a)

In Eq. (5.103)

$$RootTerm_3 = u_{xx}^2 \varepsilon_{xx}^2 - 2u_{xx}\varepsilon_{xx}u_{zz}\varepsilon_{zz} + u_{zz}^2 \varepsilon_{zz}^2 + 4u_{xx}^2 u_{zz}\varepsilon_{zz} \alpha_{xx}^H \alpha_{xx}^E$$
(5.104)

The second set of roots describing the wave number is obtained as

$$k_{y_2} = \pm \frac{\omega \left[u_{xx} u_{zz} \left(u_{xx} \varepsilon_{xx} + u_{zz} \varepsilon_{zz} - \sqrt{RootTerm_3} \right) \right]^{1/2}}{\sqrt{2} u_{xx} u_{zz}}$$
(5.103b)

Here, the *RootTerm*₃ is same as has been given in Eq. (5.104).

We obtain four waves propagating with two distinct speeds in the magnetoelectric composite. The propagation characteristics of the waves are affected by the magnetoelectric susceptibility components. The wave numbers obtained are reversible as the waves propagate in the forward and backward direction with the same speed. More information on the polarization of the fields and modes of propagation will be discussed in subsequent sections.

We now obtain the wave number for the special case of a magnetostrictive material being used for the piezomagnetic layer. The wave numbers for this case are given as

$$k_{y_1} = \pm \omega \sqrt{\frac{\varepsilon_{xx}}{u_{zz}}}$$
(5.105a)

$$k_{y_2} = \pm \omega \sqrt{\frac{\mathcal{E}_{zz}}{u_{xx}}}$$
(5.105b)

We obtain four waves propagating with two distinct speeds. The waves are reversible with same speeds in the forward and reverse directions. The magnetoelectric coupling components do not affect the propagation characteristics for this case.

5.1.3.3 Non-gyrotropic ME media: propagation in the x-direction

We consider propagation in the *x*-direction; the wave number is expressed in vector form as

$$k = \hat{x}k_x \tag{5.85}$$

Thus, the operator, $\overline{\overline{g}}$, is expressed as

$$\overline{\overline{g}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -k_x \\ 0 & k_x & 0 \end{bmatrix}$$
(5.86)

The magnetoelectric susceptibility and effective permittivity are of the form as have already been used in obtaining the wave numbers for propagation in a gyrotropic ME media. The permeability for the non-gyrotropic ME media has been previously given as well. Solving for the dispersion relationship, we obtain

$$\begin{bmatrix} \omega^{2} \varepsilon_{xx} - \omega^{2} u_{xx} \alpha_{xx}^{H} \alpha_{xx}^{E} & 0 & 0\\ 0 & \omega^{2} \varepsilon_{yy} - k_{x}^{2} u_{zz} & 0\\ 0 & 0 & \omega^{2} \varepsilon_{zz} - k_{x}^{2} u_{yy} \end{bmatrix} \cdot \begin{bmatrix} E_{x} \\ E_{y} \\ E_{z} \end{bmatrix} = 0$$
(5.106)

For non-trivial solutions, we have

$$\begin{vmatrix} \omega^{2} \varepsilon_{xx} - \omega^{2} u_{xx} \alpha_{xx}^{H} \alpha_{xx}^{E} & 0 & 0 \\ 0 & \omega^{2} \varepsilon_{yy} - k_{x}^{2} u_{zz} & 0 \\ 0 & 0 & \omega^{2} \varepsilon_{zz} - k_{x}^{2} u_{yy} \end{vmatrix} = 0$$
(5.107)

Then we obtain the equation,

$$u_{yy}u_{zz}\left(\omega^{2}\varepsilon_{zz}-\omega^{2}u_{xx}\alpha_{xx}^{H}\alpha_{xx}^{E}\right)k_{x}^{4}+\left[\left(-u_{yy}\varepsilon_{yy}-u_{zz}\varepsilon_{zz}\right)\left(\omega^{4}\varepsilon_{xx}-\omega^{4}u_{xx}\alpha_{xx}^{H}\alpha_{xx}^{E}\right)\right]k_{x}^{2}$$
$$+\omega^{4}\varepsilon_{yy}\varepsilon_{zz}\left(\omega^{2}\varepsilon_{xx}-\omega^{2}u_{xx}\alpha_{xx}^{H}\alpha_{xx}^{E}\right)=0$$
(5.108)

Solution to the problem is obtained as

$$k_{x_1} = \pm \frac{\omega \sqrt{u_{zz}} \mathcal{E}_{yy}}{u_{zz}}$$
(5.109a)

$$k_{x_2} = \pm \frac{\omega \sqrt{u_{yy}} \varepsilon_{zz}}{u_{yy}}$$
(5.109b)

We obtain four ordinary waves propagating with two distinct speeds.

We now solve for the wave number for the special case of a magnetostrictive layer. The wave numbers for this case are obtained as

$$k_{x_1} = \pm \omega \sqrt{\frac{\varepsilon_{yy}}{u_{zz}}}$$
(5.110a)

$$k_{x_2} = \pm \omega \sqrt{\frac{\mathcal{E}_{zz}}{u_{yy}}}$$
(5.110b)

We obtain two sets ordinary waves propagating in the forward and reverse directions with two distinct speeds. The magnetoelectric effect does not affect the propagation characteristics of the waves in this case. Observe that the wavenumbers are same as that for the piezomagnetic/piezoelectric composite, as the waves are not affected by the ME effect.

5.2 Polarization of propagating waves in ME composites

We have obtained all propagating wave numbers for the magnetoelectric composite, in the three basic configurations with attention paid to the gyrotropy of the bulk homogenized ME composite. The wave numbers are the eigenvalues of the dispersion relationship equation that has been solved. The electric field is the eigenvector, and its field relationships and polarization state can be obtained using the dispersion relationship. More insight into propagation of electromagnetic waves in bulk magnetoelectric composites is obtained by investigating the polarization of the propagating waves. The wave numbers obtained thus far represent the eigenvalues of the propagation problem. The associated fields represent the eigenvectors and help give a complete understanding of the electromagnetic wave propagation characteristics. Analytical expressions for the eigenvectors in terms of the material parameters are quite tedious, and do not always break down into simple forms that are easily analyzed for polarization and transmission mode information. Hence, we solve the problem and obtain the eigenvectors numerically using expressions for the wave number and wave equation. In obtaining the numerical solutions, we make assumptions on the piezomagnetic material. In creating a theoretical material parameter for the electric field induced ME effect, we simply assume that the piezomagnetic coefficients as used in the magnetostrictive media also relates the strain to the induced magnetization in the piezomagnetic phase. This allows for a little more insight as we obtain solutions using the numerical values computed for the composite. The values for the material parameters used in the computations are given in Appendix B.

5.2.1 Longitudinal ME Electromagnetic wave Polarization

5.2.1.1 Gyrotropic ME media: Propagation along the x-axis

For the case of the longitudinal magnetoelectric configuration, with magnetic anisotropy, the wave numbers were obtained as

$$k_{x_{1}} = \pm \frac{\omega \left[u_{yy} u_{zz} \left(u_{yy} \varepsilon_{yy} + u_{zz} \varepsilon_{zz} + \sqrt{u_{yy}^{2} \varepsilon_{yy}^{2} - 2u_{yy} \varepsilon_{yy} u_{zz} \varepsilon_{zz} + u_{zz}^{2} \varepsilon_{zz}^{2} + 4u_{zz}^{2} u_{yy} \varepsilon_{yy} \alpha_{zz}^{H} \alpha_{zz}^{E} \right) \right]^{1/2}}{\sqrt{2} u_{yy} u_{zz}}$$

(5.44)

$$k_{x_{2}} = \pm \frac{\omega \left[u_{yy} u_{zz} \left(u_{yy} \varepsilon_{yy} + u_{zz} \varepsilon_{zz} - \sqrt{u_{yy}^{2} \varepsilon_{yy}^{2} - 2u_{yy} \varepsilon_{yy} u_{zz} \varepsilon_{zz} + u_{zz}^{2} \varepsilon_{zz}^{2} + 4u_{zz}^{2} u_{yy} \varepsilon_{yy} \alpha_{zz}^{H} \alpha_{zz}^{E} \right) \right]^{1/2}}{\sqrt{2} u_{yy} u_{zz}}$$

(5.45)

Piezomagnetic materials are very uncommon and its material parameters are not readily available. We have modified the material parameters of a magnetostrictive composite by introducing a strain-magnetization relationship to obtain a theoretical layer with complete piezomagnetic characteristics (such that $\alpha^E \neq 0$). This is necessary as the analytical solutions to the propagation problem are rather complex and do not breakdown into simple forms when solved. For propagation along the *x*-axis, we obtain eigenvector relationships of the form

$$E_x = 0$$

$$E_y = a \cdot E_z$$
(5.111)

In Eq. (5.111), *a* is a constant that depends on the material characteristic of the homogenized composite, and is easily computed from Eq. (5.41). We observe that we obtain linearly polarized fields tilted between the *y* and *z* axes for electromagnetic wave propagation in the media. Easier understanding of the media and its propagation characteristics is possible using the theoretical material parameters obtained in Appendix B. Using the effective material parameters for the longitudinal magnetoelectric configuration, at an operating frequency of 500 MHz, we numerically obtain the wave numbers as

$$k_{x_1} = \pm 309.3/m \tag{5.112}$$

$$k_{x_2} = \pm 46.64/m \tag{5.113}$$

We compute the polarization of the waves using Eq. (5.41). Hence, for k_{x_1} we obtain

$$\begin{bmatrix} 7.69 \times 10^{10} & 0 & 0\\ 0 & -1.65 \times 10^4 & 3.5 \times 10^7\\ 0 & 3.5 \times 10^7 & -7.46 \times 10^{10} \end{bmatrix} \cdot \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = 0$$
(5.114)

For k_{x_2} , we obtain

$$\begin{bmatrix} 7.69 \times 10^{10} & 0 & 0\\ 0 & 7.52 \times 10^{10} & 5.3 \times 10^{6}\\ 0 & 5.3 \times 10^{6} & 3.74 \times 10^{2} \end{bmatrix} \cdot \begin{bmatrix} E_{x} \\ E_{y} \\ E_{z} \end{bmatrix} = 0$$
(5.115)

Now we determine the polarization state for the waves propagating in this media. From Eq. (5.114), we obtain

$$E_x = 0$$

$$E_y = 2123E_z$$
(5.116)

We obtain the polarization state of the propagating wave by computing the sets of angles (γ, δ) and (ε, τ) . By definition,

$$\gamma = \tan^{-1}\left(\frac{E_{z_0}^+}{E_{y_0}^+}\right), \text{ or } \tan^{-1}\left(\frac{E_{y_0}^+}{E_{z_0}^+}\right) \qquad 0^\circ \le \gamma \le 90^\circ$$
 (5.117)

 δ = Phase difference between y and z field components $-180^{\circ} \le \delta \le 180^{\circ}$

(5.118)

$$\mathcal{E} = \cot^{-1}(AR) \qquad -45^{\circ} \le \mathcal{E} \le +45^{\circ} \tag{5.119}$$

$$\tau = \text{Tilt angle} \qquad 0^{\circ} \le \tau \le 180^{\circ} \tag{5.120}$$

Utilizing the Poincare sphere, the two sets of angles (γ, δ) and (ε, τ) are related by [45]

$$\cos(2\gamma) = \cos(2\varepsilon)\cos(2\tau) \tag{5.121}$$

$$\tan(\delta) = \frac{\tan(2\varepsilon)}{\sin(2\tau)} \tag{5.122}$$

or

$$\sin(2\varepsilon) = \sin(2\gamma)\sin(\delta) \tag{5.123}$$

$$\tan(2\tau) = \tan(2\gamma)\cos(\delta) \tag{5.124}$$

Thus, using Eq. (5.116), we obtain

$$\gamma = \tan^{-1} \left(\frac{E_{z_0}^+}{E_{y_0}^+} \right) = 89.97^{\circ}$$
(5.125)

From the field relationship in Eq. (5.116), δ is 0°, and we easily compute the values of ϵ and τ using Eq. (5.123) and (5.124)

$$\varepsilon = 0.5 \sin^{-1} \left[\sin(2\gamma) \sin(\delta) \right] = 0^{\circ} \tag{5.126}$$

$$\tau = 0.5 \tan^{-1} \left[\tan(2\gamma) \cos(\delta) \right] = 179.97^{\circ}$$
(5.127)

Hence, we obtain a linearly polarized wave for the case of propagation in the *x* direction with magnetic anisotropy. Similarly, for the second set of wave numbers, k_{x_2} . From Eq. (5.115), we obtain

$$E_x = 0$$

$$E_z = -1.42 \times 10^4 E_y$$
(5.128)

Thus, using Eq. (5.128), we obtain

$$\gamma = \tan^{-1} \left(\frac{E_{z_0}^+}{E_{y_0}^+} \right) = 0.004^{\circ}$$
(5.129)

From the field relationship in Eq. (5.128), δ is 0°, and we easily compute the values of ϵ and τ using Eq. (5.123) and (5.124)

$$\varepsilon = 0.5 \sin^{-1} \left[\sin(2\gamma) \sin(\delta) \right] = 0^{\circ} \tag{5.130}$$

$$\tau = 0.5 \tan^{-1} \left[\tan(2\gamma) \cos(\delta) \right] = 0.004^{\circ}$$
(5.131)

Thus we obtain two linearly polarized waves propagating with different speeds in the positive and negative x directions. There is a difference in the tilt angle of the linearly polarized waves obtained from the wave numbers.

We observe that there are no electric fields in the direction of propagation. However, we confirm if the wave propagating through the medium is a TEM wave by obtaining the magnetic field in the media. We had previously obtained

$$\mathbf{H} = \overline{\overline{\mu}}^{-1} \left[\frac{1}{\omega} (\overline{k} \times \mathbf{E}) - \overline{\overline{\alpha}}^{E} \mathbf{E} \right]$$
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(5.11)

Hence, since we have the electric field as

$$\mathbf{E} = \hat{y}E_y + \hat{z}E_z \tag{5.132}$$

The magnetic field is expressly given as

$$\mathbf{H} = \overline{\overline{\mu}}^{-1} \left[\hat{z} \left(\frac{k_x}{\omega} E_y - \frac{\boldsymbol{\alpha}_{zz}^E}{c_0} E_z \right) - \hat{y} \frac{k_x}{\omega} E_z \right]$$
(5.133)

The propagating wave is a TEM wave as we observe no fields along the direction of propagation. This confirms the wave number obtained, as there is no cut-off frequency observed due to the direct relationship between the wave number and the angular frequency.

For the special case of a magnetostrictive material used in the realization of the magnetoelectric composite, we obtain linearly polarized waves. The mode of propagation is also realized as TEM. We observe that for the magnetostrictive material, $\alpha^{E} = 0$, hence this simplifies the form of the expression for the magnetic field in Eq. (5.11). We see that the cross product of the electric field and wave number will not result in any fields directed along the propagation direction.

5.2.1.2 Non-gyrotropic ME media: Propagation along the x-axis

The wave numbers for the non-gyrotropic ME media with propagation in the x direction were obtained as

$$k_{x_{1}} = \pm \frac{\omega \left[u_{yy} u_{zz} \left(u_{yy} \varepsilon_{yy} + u_{zz} \varepsilon_{zz} + \sqrt{u_{yy}^{2} \varepsilon_{yy}^{2} - 2u_{yy} \varepsilon_{yy} u_{zz} \varepsilon_{zz} + u_{zz}^{2} \varepsilon_{zz}^{2} + 4u_{zz}^{2} u_{yy} \varepsilon_{yy} \alpha_{zz}^{H} \alpha_{zz}^{E} \right) \right]^{1/2}}{\sqrt{2} u_{yy} u_{zz}}$$

(5.52)

$$k_{x_{2}} = \pm \frac{\omega \left[u_{yy} u_{zz} \left(u_{yy} \varepsilon_{yy} + u_{zz} \varepsilon_{zz} - \sqrt{u_{yy}^{2} \varepsilon_{yy}^{2} - 2u_{yy} \varepsilon_{yy} u_{zz}} \varepsilon_{zz} + u_{zz}^{2} \varepsilon_{zz}^{2} + 4u_{zz}^{2} u_{yy} \varepsilon_{yy} \alpha_{zz}^{H} \alpha_{zz}^{E} \right) \right]^{1/2}}{\sqrt{2} u_{yy} u_{zz}}$$

For propagation along the *x*-axis in the case of a non-gyrotropic ME media, we obtain eigenvector relationships of the form

$$E_x = 0$$

$$E_y = b \cdot E_z$$
(5.134)

(5.53)

In Eq. (5.134), b is a constant that depends on the material characteristic of the homogenized composite, and is easily computed from Eq. (5.49). We observe that we obtain linearly polarized fields tilted between the y and z axes for electromagnetic wave propagation in the media. As done before, we solve numerically using the effective material parameters from Appendix B, and an operational frequency of 500 MHz, we numerically obtain the wave numbers as

$$k_{x_1} = \pm 358.5/m \tag{5.135}$$

$$k_{x_2} = \pm 57.21/m \tag{5.136}$$

We compute the polarization of the propagating waves using Eq. (5.41). Hence, for k_{x_1} we obtain

$$\begin{bmatrix} 7.69 \times 10^{10} & 0 & 0\\ 0 & -6.01 \times 10^{3} & 2 \times 10^{7}\\ 0 & 2 \times 10^{7} & -6.64 \times 10^{10} \end{bmatrix} \cdot \begin{bmatrix} E_{x} \\ E_{y} \\ E_{z} \end{bmatrix} = 0$$
(5.137)

For k_{x_2} , we obtain

$$\begin{bmatrix} 7.69 \times 10^{10} & 0 & 0\\ 0 & 7.5 \times 10^{10} & 3.2 \times 10^{6}\\ 0 & 3.2 \times 10^{6} & 1.36 \times 10^{2} \end{bmatrix} \cdot \begin{bmatrix} E_{x} \\ E_{y} \\ E_{z} \end{bmatrix} = 0$$
(5.138)

Looking at Eq. (5.137) and (5.138), we observe that we have linearly polarized waves propagating in the media. The field is thus linearly polarized at an angle between the y, and z axes. We compute the polarization state for the fields, to help give better information on the tilt angle for the propagating wave. Using Eq. (5.137), we obtain

$$E_x = 0$$

$$E_y = 3324E_z$$
(5.139)

We solve for the polarization state of the waves in the media. First, we obtain

$$\gamma = \tan^{-1} \left(\frac{E_{z_0}^+}{E_{y_0}^+} \right) = 89.98^{\circ}$$
(5.140)

Since there is no difference in phase for the electric fields, δ is given as 0°, and we easily compute the values of ϵ and τ using Eq. (5.123) and (5.124)

$$\varepsilon = 0.5 \sin^{-1} \left[\sin(2\gamma) \sin(\delta) \right] = 0^{\circ} \tag{5.141}$$

$$\tau = 0.5 \tan^{-1} \left[\tan(2\gamma) \cos(\delta) \right] = 179.98^{\circ} \tag{5.142}$$

Hence, we obtain a linearly polarized wave for wave number.

For the second wave number, using Eq. (5.138), we obtain

$$E_x = 0$$

$$E_z = 2.35 \times 10^4 \cdot E_y$$
(5.143)

We solve for the polarization state of the fields, as has been done previously. First, we obtain

$$\gamma = \tan^{-1} \left(\frac{E_{z_0}^+}{E_{y_0}^+} \right) = 0.0024^{\circ}$$
(5.144)

Since there is no difference in phase for the electric fields, δ is given as 0°, and we easily compute the values of ϵ and τ using Eq. (5.123) and (5.124)

$$\varepsilon = 0.5 \sin^{-1} \left[\sin(2\gamma) \sin(\delta) \right] = 0^{\circ} \tag{5.145}$$

$$\tau = 0.5 \tan^{-1} \left[\tan(2\gamma) \cos(\delta) \right] = 0.003^{\circ}$$
(5.146)

Hence, just as was the case with the magnetic anisotropy, we also obtain linearly polarized waves for the case of propagation in the *x* direction in a non-gyrotropic ME media. The media contains four possible propagating waves, with two propagating in the forward direction and two in the backwards direction. For both forward and backward propagation, all fields are linearly polarized, and propagate with different speeds. Hence, we have two waves propagating in the forward direction with two distinct speeds, and also for the backward direction with two distinct speeds. Additionally, the tilt angle of one forward or backward propagating wave is different from the other. We also observe that the waves obtained are TEM in nature and is easily verified by checking the magnetic field using the relationship between the electric field and magnetic field. We had previously obtained the expression,

$$\mathbf{H} = \overline{\overline{\mu}}^{-1} \left[\frac{1}{\omega} (\overline{k} \times \mathbf{E}) - \overline{\overline{\alpha}}^{E} \mathbf{E} \right]$$
(5.11)

Hence, we observe that there will not be any magnetic fields along the direction of propagation. The expression for the magnetic field is obtained as

$$\mathbf{H} = \overline{\overline{\mu}}^{-1} \left[\hat{z} \left(\frac{k_x}{\omega} E_y + \frac{\alpha_{zz}^E}{c_0} E_z \right) - \hat{y} \frac{k_x}{\omega} E_z \right]$$
(5.147)

With the electric field as

$$\mathbf{E} = \hat{y}E_{y} - \hat{z}E_{z} \tag{5.148}$$

We observe that for the longitudinal ME electromagnetic wave propagation, we obtain linearly polarized electric field, and an accompanying linearly polarized magnetic field. This is ME coupling term for the longitudinal effect can only produce a *z*-directed electric field component.

Linearly polarized TEM waves are obtained for the case of a magnetostrictive material used for the piezomagnetic layer.

5.2.2 Transverse ME Electromagnetic wave Polarization

5.2.2.1 Gyrotropic ME media: Propagation along the x-axis

Now considering the transverse magnetoelectric configuration, the wave numbers were previously derived as

$$k_{x_{1}} = \pm \frac{\omega \left[\left(u_{yy} u_{zz} - u_{yz} u_{zy} \right) \left(u_{yy} \varepsilon_{yy} + u_{zz} \varepsilon_{zz} - u_{xx} u_{zz} \alpha_{zx}^{H} \alpha_{xz}^{E} + \sqrt{RootTerm} \right) \right]^{1/2}}{\sqrt{2} \left(u_{yy} u_{zz} - u_{yz} u_{zy} \right)}$$
(5.65)

In Eq. (5.65)

 $RootTerm = u_{yy}^2 \varepsilon_{yy}^2 - 2u_{yy}\varepsilon_{yy}u_{zz}\varepsilon_{zz} + 2u_{zz}u_{yy}u_{xx}\varepsilon_{yy}\alpha_{zx}^H\alpha_{xz}^E + u_{zz}^2\varepsilon_{zz}^2$

$$-2u_{zz}^{2}u_{xx}\varepsilon_{zz}\alpha_{zx}^{H}\alpha_{xz}^{E} + u_{zz}^{2}u_{xx}^{2}\left(\alpha_{zx}^{H}\right)^{2}\left(\alpha_{xz}^{E}\right)^{2} + 4u_{yz}u_{zy}\varepsilon_{yy}\varepsilon_{zz}$$
$$-4u_{yz}u_{zy}u_{xx}\varepsilon_{yy}\alpha_{zx}^{H}\alpha_{xz}^{E} \qquad (5.66)$$
The second set of roots describing the wave number is obtained as

$$k_{x_{2}} = \pm \frac{\omega \left[\left(u_{yy} u_{zz} - u_{yz} u_{zy} \right) \left(u_{yy} \varepsilon_{yy} + u_{zz} \varepsilon_{zz} - u_{xx} u_{zz} \alpha_{zx}^{H} \alpha_{xz}^{E} - \sqrt{RootTerm} \right) \right]^{1/2}}{\sqrt{2} \left(u_{yy} u_{zz} - u_{yz} u_{zy} \right)}$$
(5.67)

Using the effective parameters from Appendix B, and an operating frequency of 500 MHz, we obtain the wave numbers as

$$k_{x_1} = \pm 260.7/m \tag{5.149}$$

$$k_{x_2} = \pm 45.36/m \tag{5.150}$$

We compute the polarization of the waves using Eq. (5.41). Hence, for k_{x_1} we obtain

$$\begin{bmatrix} 5.72 \times 10^{10} & 0 & 0\\ 0 & -1 \times 10^7 & j7.5 \times 10^8\\ 0 & -j7.5 \times 10^7 & -5.55 \times 10^{10} \end{bmatrix} \cdot \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = 0$$
(5.151)

For k_{x_2} , we obtain

$$\begin{bmatrix} 5.72 \times 10^{10} & 0 & 0\\ 0 & 5.56 \times 10^{10} & j2.3 \times 10^{6}\\ 0 & -j2.3 \times 10^{7} & 9.2 \times 10^{3} \end{bmatrix} \cdot \begin{bmatrix} E_{x} \\ E_{y} \\ E_{z} \end{bmatrix} = 0$$
(5.152)

Looking at Eq. (5.151) and (5.152), we observe that we should have elliptically polarized waves propagating in the media. The polarization state of the propagating wave gives better insight into the polarization obtained via propagation in the media. Thus, using Eq. (5.151), we obtain

$$E_x = 0$$

$$E_y = j74.4E_z$$
(5.153)

We solve for the polarization state of the waves in the media. First, we obtain

$$\gamma = \tan^{-1} \left(\frac{E_{z_0}^+}{E_{y_0}^+} \right) = 89.2^{\circ}$$
(5.154)

Since δ is seen to be -90°, we compute the values of ε and τ using Eq. (5.123) and (5.124)

$$\varepsilon = 0.5 \sin^{-1} \left[\sin(2\gamma) \sin(\delta) \right] = -0.77^{\circ} \tag{5.155}$$

$$\tau = 0.5 \tan^{-1} \left[\tan(2\gamma) \cos(\delta) \right] = 179.2^{\circ}$$
(5.156)

Hence, we obtain a right hand elliptically polarized wave for the case of propagation in the x direction with magnetic anisotropy. We then analyze the polarization for the second set of propagating waves. Using the information from Eq. (5.152), we obtain the relationship between the fields as

$$E_{x} = 0$$

$$E_{z} = j2.46 \times 10^{3} E_{y}$$
(5.157)

We solve for the polarization state of the waves in the media. First, we obtain

$$\gamma = \tan^{-1} \left(\frac{E_{z_0}^+}{E_{y_0}^+} \right) = 0.023^{\circ}$$
(5.158)

Since δ is seen as 90°, we simply compute the values of ϵ and τ using Eq. (5.123) and (5.124)

$$\varepsilon = 0.5 \sin^{-1} \left[\sin(2\gamma) \sin(\delta) \right] = 0.023^{\circ} \tag{5.159}$$

$$\tau = 0.5 \tan^{-1} \left[\tan(2\gamma) \cos(\delta) \right] = 0.023^{\circ}$$
(5.160)

Hence, we obtain a left hand elliptically polarized wave for the second set of waves propagating with wave number k_{x_1} . This implies that we have four possible propagating waves in the media. We have two waves, one right hand elliptically

polarized and the other left hand elliptically polarized, propagating in the positive x direction. Also there are two similar waves that propagate in the negative x direction. Although we obtained elliptically polarized waves for this case, with no electric field components in the direction of propagation, the mode of propagation is not TEM in nature. We observe that since

$$\mathbf{H} = \overline{\overline{\mu}}^{-1} \left[\frac{1}{\omega} \left(\overline{k} \times \mathbf{E} \right) - \overline{\overline{\alpha}}^{E} \mathbf{E} \right]$$
(5.11)

The expression for the magnetic field is obtained as

$$\mathbf{H} = \overline{\overline{\mu}}^{-1} \left[\hat{z} \frac{k_x}{\omega} E_y + \hat{y} j \frac{k_x}{\omega} E_z - \hat{x} j \frac{\boldsymbol{\alpha}_{xz}^E}{c_0} E_z \right]$$
(5.161)

Since we have obtained the electric field as

$$\mathbf{E} = \hat{y}E_{y} - \hat{z}jE_{z} \tag{5.162}$$

We observe that the magnetoelectric coupling component introduces a field term in the direction of propagation. Hence the mode of propagation for the transverse magnetoelectric electromagnetic wave travelling in the *x*-direction is transverse electric (TE). It is important to observe how the magnetoelectric coupling affects the general characteristics of the wave. When we switch off the ME coupling terms, the polarization of the propagating wave does not change. We still observe elliptically polarized waves resulting from the solution of Eq. (5.63) or (5.64) with the ME components set to zero. However, the mode of propagation changes to TEM as there are no field components in the direction of propagation. For the case of a magnetostrictive material used in place of a piezomagnetic layer, the wave obtained is elliptically polarized. We obtain a set of right handed and left handed elliptically polarized waves propagating in the media. The mode of propagation is obtained as TEM, with no fields along the direction of propagation.

It is important to note that the fields obtained here may be circularly polarized depending on the material parameters of the magnetoelectric composite. This is especially true as the value of the piezomagnetic coefficient for the piezomagnetic layer may differ greatly from that which is used in theory to describe its piezomagnetic coefficient. It is quite possible that the polarization of the fields changes from elliptically polarized waves to circularly polarized waves with changes to the material parameters of the ME composite.

5.2.2.2 Gyrotropic ME media: Propagation along the y-axis

Now we consider propagation in the *y* direction of the sample plane. The wave numbers for this case were derived as

$$k_{y_1} = \pm \frac{\omega \sqrt{u_{zz} \varepsilon_{xx}}}{u_{zz}}$$
(5.73)

$$k_{y_{2}} = \frac{\omega \left[u_{xx} \left(\alpha_{xz}^{E} + \alpha_{zx}^{H} \right) \pm \sqrt{u_{xx}^{2} \left(\alpha_{xz}^{E} \right)^{2} - 2u_{xx}^{2} \alpha_{zx}^{E} \alpha_{zx}^{H} + u_{xx}^{2} \left(\alpha_{zx}^{H} \right)^{2} + 4u_{xx} \varepsilon_{zz}}{2u_{xx}} \right]}{2u_{xx}}$$
(5.74)

Using the effective parameters, and an operating frequency of 500 MHz, we obtain the wave numbers as

$$k_{y_1} = \pm 260.7/m \tag{5.163}$$

$$k_{y_2} = \pm 45.67/m \tag{5.164}$$

We compute the polarization of the waves using Eq. (5.70). Hence, for k_{y_1} we obtain

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 5.72 \times 10^{10} & 0 \\ 0 & 0 & -5.54 \times 10^{10} \end{bmatrix} \cdot \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = 0$$
(5.165)

For k_{y_2} , we obtain

$$\begin{bmatrix} 5.55 \times 10^{10} & 0 & 0 \\ 0 & 5.7 \times 10^{10} & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = 0$$
(5.166)

From Eq. (5.165), we observe that E_x is the only non-zero field for wave number k_{y_2} . Similarly, from Eq. (5.166), E_z is the only non-zero field for wave number k_{y_2} . For this case of propagation in the y direction, we obtain two linearly polarized waves, E_x and E_z , propagating with wave numbers k_{y_1} and k_{y_2} , respectively. Hence, we observe the birefringence of the media due to the two characteristic fields propagating with different phase velocities. This implies that propagation in the y direction for the transverse magnetoelectric configuration results in a birefringent media. We observe that the waves obtained are TEM, as there are no fields along the direction of propagation. Hence, there will be no magnetic field directed along the direction of propagation.

$$\mathbf{H}_{1} = -\hat{z}\overline{\overline{\mu}}^{-1}\frac{k_{y}}{\omega}E_{x}$$
(5.167)

For the case where the electric field is obtained as

$$\mathbf{E}_1 = \hat{x} E_x \tag{5.168}$$

And,

$$\mathbf{H}_{2} = \hat{x}\overline{\overline{\mu}}^{-1} \left(\frac{k_{y}}{\omega} - \frac{\boldsymbol{\alpha}_{xz}^{E}}{c_{0}}\right) E_{z}$$
(5.169)

For the case where the electric field is obtained as

$$\mathbf{E}_2 = \hat{z}E_z \tag{5.170}$$

Thus, the mode of propagation in this media for the case of y-directed propagation for a transverse magnetoelectric material with magnetic anisotropy is TEM. So we observe a change in the mode of propagation simply by changing the direction of propagation in the transverse ME media. Observe that the ME coupling is only active in the magnetic field for the second wave number. As was the case with the x-directed propagation, absence of the ME coupling does not result in changes to the polarization of the propagating fields. The mode of propagation also remains TEM with the ME terms set to zero in Eq. (5.74).

The results for the non-gyrotropic ME media for y-directed propagation are same as obtained for the gyrotropic ME electromagnetic wave propagation in the y-direction in terms of the polarization and the mode of propagation. We do not need to compute the polarization for the non-gyrotropic ME media with y-directed propagation as the results obtained will be exactly the same as that obtained for this current case.

For the case of a magnetostrictive media as one of the composite layers, we obtain linearly polarized ordinary and extraordinary waves. We observe a case of birefringence as we obtain characteristic waves propagating with distinct speeds. The mode of propagation of the fields in the media is TEM as there are no waves lying along the direction of propagation.

5.2.2.3 Non-gyrotropic ME media: Propagation along the x-axis

Here we consider the case of a non-gyrotropic ME media while propagating in the x direction. The wave numbers for this case have been previously obtained and are given as

$$k_{x_{1}} = \pm \frac{\omega \sqrt{u_{zz}} \varepsilon_{yy}}{u_{zz}}$$
(5.83)

$$k_{x_2} = \pm \frac{\omega \sqrt{u_{yy} \left(\varepsilon_{zz} - u_{xx} \alpha_{zx}^H \alpha_{xz}^E\right)}}{u_{yy}}$$
(5.84)

Using the effective parameters obtained from the theoretical model and an operating frequency of 500 MHz, we obtain the wave numbers as

$$k_{x_1} = \pm 309.3/m \tag{5.171}$$

$$k_{x_2} = \pm 57.15/m \tag{5.172}$$

We compute the polarization of the waves using Eq. (5.80). Hence, for k_{x_1} we obtain

$$\begin{bmatrix} 5.72 \times 10^{10} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -4.90 \times 10^{10} \end{bmatrix} \cdot \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = 0$$
(5.173)

For k_{x_2} we obtain

$$\begin{bmatrix} 5.72 \times 10^{10} & 0 & 0 \\ 0 & 5.53 \times 10^{10} & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = 0$$
(5.174)

From Eq. (5.173), we observe that E_y is the only non-zero field for wave number k_{x_1} . From Eq. (5.174), E_z is the only non-zero field for wave number k_{x_2} . This is similar to the case of propagation in the *y* direction in a gyrotropic magnetoelectric media. The difference here is the characteristic fields which are E_y and E_z , as we have no fields in the direction of propagation. Thus, we obtain two linearly polarized waves, E_y and E_z , propagating with wave numbers k_{x_1} and k_{x_2} , respectively. Once again, we observe the birefringence of the media due to the two characteristic fields propagating with different phase velocities. In conclusion, we state that birefringence property can be obtained for both gyrotropic and non-gyrotropic magnetoelectric materials in the transverse configuration.

In terms of the modes of propagation, we once again observe that one of the waves will be TEM in nature. There are no fields (electric and magnetic) directed along the direction of propagation of the ordinary wave. Hence, there are no magnetic fields directed along the direction of propagation as observed for wave number k_{x_1} .

$$\mathbf{H}_{1} = \hat{z}\overline{\overline{\mu}}^{-1}\frac{k_{x}}{\omega}E_{y}$$
(5.175)

For the case where the electric field is obtained as

$$\mathbf{E}_{1} = \hat{y}E_{y} \tag{5.176}$$

However, for the extraordinary wave, we obtain a TE mode of propagation, as the magnetoelectric coupling induces a magnetic field in the direction of propagation. Hence, we obtain the magnetic field as

$$\mathbf{H}_{2} = -\overline{\overline{\mu}}^{-1} \left(\hat{y} \frac{k_{x}}{\omega} + \hat{x} \frac{\alpha_{xz}^{E}}{c_{0}} \right) E_{z}$$
(5.177)

For the case where the electric field is obtained as

$$\mathbf{E}_2 = \hat{z}E_z \tag{5.178}$$

Hence, we have two distinct waves propagating with different speeds and also different modes of propagation within the media. The magnetoelectric coupling induces the TE mode of propagation due to the magnetic field produced in the *x*-direction via coupling to the *z*-component of the electric field.

For the case of a magnetostrictive material, we obtain two sets of characteristic linearly polarized waves propagating with different phase velocities. This is also a case of birefringence in the media. The mode of propagation in the composite media is TEM.

5.2.3 In-plane ME Electromagnetic wave Polarization

5.2.3.1 Gyrotropic ME media: Propagation along the x-axis

For the in-plane magnetoelectric configuration, we first consider the case of propagation in the x direction in a gyrotropic ME media. The wave numbers were previously obtained as

$$k_{x_{1}} = \pm \frac{\omega \left[\left(u_{yy} u_{zz} - u_{yz} u_{zy} \right) \left(u_{yy} \varepsilon_{yy} + u_{zz} \varepsilon_{zz} + \sqrt{RootTerm_{2}} \right) \right]^{1/2}}{\sqrt{2} \left(u_{yy} u_{zz} - u_{yz} u_{zy} \right)}$$
(5.92)

In Eq. (5.92)

$$RootTerm_2 = u_{yy}^2 \varepsilon_{yy}^2 - 2u_{yy}\varepsilon_{yy}u_{zz}\varepsilon_{zz} + u_{zz}^2 \varepsilon_{zz}^2 + 4u_{yz}u_{zy}\varepsilon_{yy}\varepsilon_{zz}$$
(5.93)

The second set of roots describing the wave number is obtained as

$$k_{x_{2}} = \pm \frac{\omega \left[\left(u_{yy} u_{zz} - u_{yz} u_{zy} \right) \left(u_{yy} \varepsilon_{yy} + u_{zz} \varepsilon_{zz} - \sqrt{RootTerm_{2}} \right) \right]^{1/2}}{\sqrt{2} \left(u_{yy} u_{zz} - u_{yz} u_{zy} \right)}$$
(5.94)

Using the effective parameters obtained for the in-plane magnetoelectric configuration, and an operating frequency of 500 MHz, we obtain the wave numbers as

$$k_{x_1} = \pm 2.6 \times 10^2 / m \tag{5.179}$$

$$k_{x_2} = \pm 45.50/m \tag{5.180}$$

We compute the polarization of the waves using Eq. (5.80). Hence, for k_{x_1} we obtain

$$\begin{bmatrix} 4 \times 10^{10} & 0 & 0 \\ 0 & -1 \times 10^7 & j7.4 \times 10^8 \\ 0 & -j7.4 \times 10^8 & -5.5 \times 10^{10} \end{bmatrix} \cdot \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = 0$$
(5.181)

For k_{x_2} we obtain

$$\begin{bmatrix} 4 \times 10^{10} & 0 & 0 \\ 0 & 5.5 \times 10^{10} & j2.3 \times 10^7 \\ 0 & -j2.3 \times 10^7 & 9.2 \times 10^3 \end{bmatrix} \cdot \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = 0$$
(5.182)

We expect elliptically polarized wave for the wave propagating in the media. From Eq. (5.181) we obtain

$$E_y = j74.4E_z$$
 (5.183)

We solve for the polarization state of the waves in the media. First, we obtain

$$\gamma = \tan^{-1} \left(\frac{E_{z_0}^+}{E_{y_0}^+} \right) = 89.2^{\circ}$$
(5.184)

Since δ is -90°, we then simply compute the values of ϵ and τ using Eq. (5.123) and (5.124)

$$\varepsilon = 0.5 \sin^{-1} \left[\sin(2\gamma) \sin(\delta) \right] = -0.77^{\circ} \tag{5.185}$$

$$\tau = 0.5 \tan^{-1} \left[\tan(2\gamma) \cos(\delta) \right] = 179.2^{\circ}$$
(5.186)

Hence, we obtain a right hand elliptically polarized wave for the case of propagation in the x direction with magnetic anisotropy. The z-directed electric field dominates that in the y-direction that the wave almost seems to be linearly polarized.

The case is same for the second wave number. From Eq. (5.182), we obtain the relationship for the propagating fields as

$$E_z = j2.5 \times 10^3 \cdot E_y \tag{5.187}$$

The case is same for the second wave number, as we expect elliptically polarized waves also. In this case the wave will be a left hand elliptically polarized wave. Due to the dominance of the *y*-directed electric field, the wave almost seems to be linearly polarized. We observe that since

$$\mathbf{H} = \overline{\overline{\mu}}^{-1} \left[\frac{1}{\omega} \left(\overline{k} \times \mathbf{E} \right) - \overline{\overline{\alpha}}^{E} \mathbf{E} \right]$$
(5.11)

The expression for the magnetic field is obtained as

$$\mathbf{H} = \overline{\overline{\mu}}^{-1} \left[\frac{k_x}{\omega} \left(\hat{z} E_y - \hat{y} j E_z \right) \right]$$
(5.188)

Since we have obtained the electric field with the form

$$\mathbf{E} = \hat{y}E_{y} + \hat{z}jE_{z} \tag{5.189}$$

The magnetoelectric coupling does not affect the propagation characteristics obtained for the in-plane magnetoelectric propagation. This is because the wave is propagating along the optical axis of the media. The magnetoelectric effect also lies along this axis; hence the wave only sees the transverse properties of the media.

For the special case of a magnetostrictive material used for the piezomagnetic layer, we obtain a set of left handed and right handed elliptically polarized waves propagating in the media. The right and left handed elliptically polarized waves have different speeds of propagation. The mode of propagation for the waves is TEM, as there are no electric or magnetic fields along the direction of propagation.

5.2.3.2 Gyrotropic ME media: Propagation along the y-axis

For the case of magnetic anisotropy, we consider propagation along the *y* direction. The wave numbers for this case have been previously obtained as

$$k_{y_1} = \pm \frac{\omega \left[u_{xx} u_{zz} \left(u_{xx} \varepsilon_{xx} + u_{zz} \varepsilon_{zz} + \sqrt{RootTerm_3} \right) \right]^{1/2}}{\sqrt{2} u_{xx} u_{zz}}$$
(5.100)

In Eq. (5.100)

$$RootTerm_3 = u_{xx}^2 \varepsilon_{xx}^2 - 2u_{xx}\varepsilon_{xx}u_{zz}\varepsilon_{zz} + u_{zz}^2 \varepsilon_{zz}^2 + 4u_{xx}^2 u_{zz}\varepsilon_{zz} \alpha_{xx}^H \alpha_{xx}^E$$
(5.101)

The second set of roots describing the wave number is obtained as

$$k_{y_{2}} = \pm \frac{\omega \left[u_{xx} u_{zz} \left(u_{xx} \varepsilon_{xx} + u_{zz} \varepsilon_{zz} - \sqrt{RootTerm_{3}} \right) \right]^{1/2}}{\sqrt{2} u_{xx} u_{zz}}$$
(5.102)

Using the effective parameters obtained for the in-plane magnetoelectric configuration, we obtain the wave numbers as

$$k_{y_1} = \pm 220.4/m \tag{5.190}$$

$$k_{y_2} = \pm 44.5/m \tag{5.191}$$

We compute the polarization of the waves using Eq. (5.80). Hence, for k_{y_1} we obtain

$$\begin{bmatrix} -1.7 \times 10^9 & 0 & 5.8 \times 10^9 \\ 0 & 5.7 \times 10^{10} & 0 \\ 1.15 \times 10^{10} & 0 & -3.9 \times 10^{10} \end{bmatrix} \cdot \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = 0$$
(5.192)

For k_{y_2} we obtain

$$\begin{bmatrix} 3.7 \times 10^{10} & 0 & 1.1 \times 10^{9} \\ 0 & 5.7 \times 10^{10} & 0 \\ 2.3 \times 10^{9} & 0 & 7.3 \times 10^{7} \end{bmatrix} \cdot \begin{bmatrix} E_{x} \\ E_{y} \\ E_{z} \end{bmatrix} = 0$$
(5.193)

. From Eq. (5.192), we obtain

$$E_x = 3.38E_z$$

$$E_y = 0$$
(5.194)

Thus, using Eq. (5.194), we obtain

$$\gamma = \tan^{-1} \left(\frac{E_{z_0}^+}{E_{x_0}^+} \right) = 73.52^{\circ}$$
(5.195)

From the field relationship in Eq. (5.194), δ is 0°, and we compute the values of ϵ and τ using Eq. (5.123) and (5.124)

$$\varepsilon = 0.5 \sin^{-1} \left[\sin(2\gamma) \sin(\delta) \right] = 0^{\circ} \tag{5.196}$$

$$\tau = 0.5 \tan^{-1} \left[\tan(2\gamma) \cos(\delta) \right] = 16.48^{\circ}$$
(5.197)

Hence, we obtain a linearly polarized wave for the case of propagation in the y direction with magnetic anisotropy. Similarly, a linearly polarized wave is obtained for the second set of wave numbers, k_{y_2} . From Eq. (5.193), we obtain

$$E_z = -32.1 \cdot E_x$$

$$E_y = 0$$
(5.198)

Thus, using Eq. (5.198), we obtain

$$\gamma = \tan^{-1} \left(\frac{E_{z_0}^+}{E_{x_0}^+} \right) = 1.78^{\circ}$$
(5.199)

From the field relationship in Eq. (5.198), δ is 0°, and we compute the values of ϵ and τ using Eq. (5.123) and (5.124)

$$\varepsilon = 0.5 \sin^{-1} \left[\sin(2\gamma) \sin(\delta) \right] = 0^{\circ} \tag{5.200}$$

$$\tau = 0.5 \tan^{-1} \left[\tan(2\gamma) \cos(\delta) \right] = 1.78^{\circ}$$
(5.201)

Hence, we obtain a linearly polarized wave for the case of propagation in the *y* direction with magnetic anisotropy. Thus we obtain two linearly polarized waves propagating with two distinct speeds in the positive and negative *y* directions. Although the waves are both linearly polarized, the tilt angle is not the same for each case. The waves here are also TEM waves as observed using

$$\mathbf{H} = \overline{\overline{\mu}}^{-1} \left[\hat{x} \left(\frac{k_y}{\omega} E_z - \frac{\alpha_{xx}^E}{c_0} E_x \right) - \hat{z} \left(\frac{k_y}{\omega} \right) E_x \right]$$
(5.202)

And we have obtained an electric field of the general form,

$$\mathbf{E} = \hat{x}E_x + \hat{z}E_z \tag{5.203}$$

Thus, it is relatively easy to see that there will not be any *y*-directed magnetic fields in the media. Hence, the mode of propagation is TEM, as there are no field components in the direction of propagation. There is also no change in the polarization of the magnetic field. Turning off the ME coupling terms do not change the polarization of the fields, nor the mode propagation.

For the case of magnetostrictive material used for the piezomagnetic layer, we obtain linearly polarized waves with a TEM mode of propagation. As had been obtained from the wave number, the waves propagate with different speeds.

5.2.3.3 Non-gyrotropic ME media: Propagation along the x-axis

Finally, we analyze the case of propagation along the x direction for the case of a nongyrotropic ME media. The wave numbers were previously obtained as

$$k_{x_{\rm l}} = \pm \frac{\omega \sqrt{u_{zz} \mathcal{E}_{yy}}}{u_{zz}}$$
(5.111)

$$k_{x_2} = \pm \frac{\omega \sqrt{u_{yy}} \varepsilon_{zz}}{u_{yy}}$$
(5.112)

Using the effective parameters obtained for the in-plane magnetoelectric configuration, we obtain the wave numbers as

$$k_{x_1} = \pm 61.9/m$$
 (5.204)

$$k_{x_2} = \pm 11.4/m \tag{5.205}$$

We compute the polarization of the waves using Eq. (5.80). Hence, for k_{x_1} we obtain

$$\begin{bmatrix} 1.58 \times 10^9 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1.9 \times 10^9 \end{bmatrix} \cdot \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = 0$$
(5.206)

For k_{x_2} we obtain

$$\begin{bmatrix} 1.6 \times 10^9 & 0 & 0 \\ 0 & 2.2 \times 10^9 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = 0$$
(5.207)

For each wave number, we obtain a linearly polarized wave propagating in the media. Thus, we obtain two linearly polarized waves, E_y and E_z , propagating with wave numbers k_{x_1} and k_{x_2} , respectively. Once again, we observe birefringence of the media due to the two characteristic fields propagating with different phase velocities. In conclusion, we state that birefringence property can be obtained for both non-gyrotropic magnetoelectric materials in the in-plane magnetoelectric configuration.

For the case of a magnetostrictive material used for the piezomagnetic phase, we obtain characteristic linearly polarized waves propagating with different speeds. The mode of propagation here is TEM. We obtain a case of birefringence for this configuration and propagation direction with the substitution of a magnetostrictive layer.

We have analyzed electromagnetic wave propagation in bulk magnetoelectric composites, and found propagation as possible in the media. We also discover a number of electromagnetic wave propagation phenomena in bulk magnetoelectric composites. The results agree with those obtained by other studies on similar materials. It is important to note that we have not considered the effects the mechanical stresses and strains induced in the composite will have on the propagating fields. These factors have been ignored here, but may play a substantial role in reality.

CHAPTER 6: CONCLUSION

We have obtained theoretical models for the longitudinal, transverse and in-plane magnetoelectric configurations that closely approximate experimental results and trends. The theoretical model is obtained via the application of fundamental electromagnetic boundary conditions to the fields mechanically induced in magnetoelectric composites. Application of such boundary conditions improves the accuracy of this proposed theoretical model when compared to previous models found in the literature. The magnetoelectric effect obtained is highly dependent on the strength of the interface coupling between the piezoelectric and piezomagnetic layers. Although, we do not study in detail the factors affecting the interface coupling parameter for different types of piezoelectric and piezomagnetic combinations, we believe that such study will aid in better theoretical models for the magnetoelectric composites. The theoretical models obtained here also allows for a theoretical investigation of the magnetoelectric effect using various piezoelectric and piezomagnetic combinations. We believe this ability to mix and match the piezoelectric and piezomagnetic layers will be invaluable to experimentalists, who will rather gain insights into the results possible before embarking on a physical realization of the magnetoelectric composite.

In an attempt to make the theoretical model as robust as possible, we also considered cases where the piezomagnetic phase of the ME composite is a non-ferrite material. In this case, we obtain a non-gyrotropic ME media with the application a DC magnetic bias. Hence, the results obtained here do not limit the range of piezomagnetic materials that can be used in the realization of magnetoelectric composites. Results from the theoretical models obtained here show that the in-plane magnetoelectric configuration produces the largest ME voltage coefficient. The smallest magnetoelectric voltage coefficient is obtained from the longitudinal configuration.

The results obtained from the theoretical models are used in the investigation of electromagnetic wave propagation in bulk (Chapter 5) and bounded (Appendix A) magnetoelectric media. We analyzed electromagnetic wave propagation in the longitudinal, transverse, and in-plane magnetoelectric configurations. The propagation direction is restricted to the sample plane, as propagation normal to the sample plane is not possible. The results for the bulk ME electromagnetic wave propagation are very interesting. An important result, is the fact that the polarization of the magnetoelectric media can be changed by the ME effect in the transverse configuration when propagation is along the bias direction. The mode of propagation for the transverse ME electromagnetic wave propagation along the magnetoelectric bias direction is also changed from TEM to TE due to the magnetoelectric effect. Such effect on electromagnetic wave propagation in magnetoelectric composites had not been previously reported.

Birefringence is another property observed in the propagation of electromagnetic waves through the magnetoelectric composite. Closer investigation to the wave propagation showed that the magnetoelectric coupling is not observed on the magnetic field for every propagating wave. As an example, propagation perpendicular to the DC magnetic field bias in the transverse configuration leads to two wave numbers where only one of the associated magnetic fields are affected by the magnetoelectric coupling tensor. Results on propagation along the sample plane do not show the possibility of rotation of the polarization plane, a phenomenon better known as Faraday rotation. A strong reason for this is the absence of circularly polarized waves propagating in the magnetoelectric media.

As part of future works on the ME media, we intend to consider electromagnetic wave propagation in thin films. EM wave propagation in thin films is of interest due to recent realization of on-wafer ME media [26]. The realization of ME thin films will greatly aid the application of the ME effect to integrated circuits. EM wave propagation in thin films is very much different from the EM wave propagation in bulk media as has been obtained here. In thin films, boundary conditions are enforced on the propagating waves such that they decay rapidly away from the surface of the film. Included in future works is the multi-physics modeling of the ME thin film. We have investigated the use of a commercial software COMSOL, with which we have modeled some ME waveguides in this discourse (Appendices A and C). COMSOL allows for the simulation of models that involve several coupled physics phenomena. Such ability allows for the simultaneous modeling of the mechanical and electrical properties of a material using its constitutive relationships. Hence, we are able to model propagation through the ME thin film as the films undergo stress and strain, with realization of the ME effect. Additionally, using the COMSOL multi-physics tool, we may be also able to model and investigate the interface coupling parameter for several ME composite combinations.

In summary, accurate models for the magnetoelectric effect in piezoelectric and piezomagnetic bilayers have been obtained. Three distinct models are obtained for the Longitudinal, Transverse and In-plane magnetoelectric configurations. Additional studies show propagation as possible in bulk magnetoelectric samples. Polarizations of propagating waves are obtained as either linear or elliptical polarizations. The modes of propagation obtained were either TEM or TE, based upon the magnetoelectric configuration, and the gyrotropy of the bilayer. The results contained in this work will be useful for both theorists and experimentalists working on elastic-mediated composite multiferroics, as the theoretical models are robust enough to be of value to both groups.

APPENDIX A: ELECTROMAGNETIC WAVE PROPAGATION CHARACTERISTICS OF PLANAR MAGNETOELECTRIC WAVEGUIDES

For device applications, the characteristics of guided wave propagation through magnetoelectric media are of importance. The application of nonreciprocal materials to microwave devices using planar waveguide structures has been studied for quite some time. We have developed planar waveguide structures to study the propagation of electromagnetic waves within such complex media [60]. We intend to apply knowledge nonreciprocal media and planar structures to develop new and/or similar microwave devices using magnetoelectric composites. Thus, an important part of this additional research effort was to observe and understand the electromagnetic field behavior within magnetoelectric materials in planar waveguide structures. The overall goal will be to implement possible applications based upon the observed phenomena.

The shape/form of the constitutive parameters (tensors) control the nature of the fields observed in any planar magnetoelectric waveguide structure. Some special cases of bianisotropic/magnetoelectric substrates in planar structures have been considered in [56] using the tensors for orthorhombic and tetragonal magnetic crystals. Results from [56] show that the electromagnetic wave propagation characteristics in are affected by different material parameters based upon the mode of propagation. Also, reciprocity of the modes, Transverse Electric or Transverse Magnetic, depends on the real and

imaginary parts of the magnetoelectric coupling tensors. While the results obtained in [56] are interesting, they are only hold for the special cases where the magnetoelectric coupling tensors are of the form

$$\boldsymbol{\xi} = \begin{bmatrix} 0 & \boldsymbol{\xi}_4 & 0 \\ \boldsymbol{\xi}_5 & 0 & \boldsymbol{\xi}_6 \\ 0 & \boldsymbol{\xi}_7 & 0 \end{bmatrix}, \boldsymbol{\zeta} = \begin{bmatrix} 0 & \boldsymbol{\zeta}_4 & 0 \\ \boldsymbol{\zeta}_5 & 0 & \boldsymbol{\zeta}_6 \\ 0 & \boldsymbol{\zeta}_7 & 0 \end{bmatrix}.$$
(A.1)

The form of the magnetoelectric coupling tensors in (A.1) represent orthorhombic and tetragonal magnetic crystals, and are much different from the form for the layered piezoelectric and piezomagnetic layers obtained in chapters 2, 3, and 4. Hence, we must study the specific cases for the magnetoelectric effect obtained through the longitudinal, transverse, and in-plane configurations.

In our study, observation of the electromagnetic phenomena is made possible by the use of numerical modeling, since classical analytic solutions for planar bianisotropic waveguide structures are quite tedious not easily obtainable. In this section, we summarize the results of our research efforts in understanding EM field behavior in planar waveguides that use magnetoelectric composites and the possible applications available using these structures. We investigate the electromagnetic wave propagation characteristics of microstrip and slot lines. There are two reasons for considering these structures. First, as has been mentioned, the implementation of this material is currently being done using multilayer magnetoelectric composites. Secondly, planar structures are compatible with MMIC and VLSI technologies.

In our research study, the material characteristics of the substrate are important. We intend to obtain results for material characteristics than closely bear a resemblance to those found in reality. Hence, we only use results from the theoretical model of the magnetoelectric composite as obtained in Chapters 2, 3 and 4. We used information for a PZT/cobalt ferrite composite. We would expect magnetic anisotropy for the ferrite material; hence we will use theoretical models for cases of magnetic anisotropy.

We make use of numerical simulations making use of a multi-physics software, COMSOL. Information on the use of the software to obtain solutions to electromagnetic wave propagation is found in Appendix C. The planar waveguide problem is solved as a 2D problem, with propagation in the *xy*-plane as was done for the case of bulk electromagnetic wave propagation. However, COMSOL solves its 2D problems with propagation only in the *z*-direction. Hence, we apply coordinate rotation to the material parameters to obtain numerical solutions with propagation in the *z*-direction, that actually represent *x* or *y* directed propagation. Information on the coordinate transformation is located in Appendix D. As seen in Appendix D, there are limitations in COMSOL on the shape of the tensors that can be solved. These limitations do not allow for the simulation of the longitudinal magnetoelectric configuration with magnetic anisotropy. However, we are able to study all other configurations in full detail.

The material parameters are computed and are found in Table A.1. The materials used here are cobalt ferrite for the piezomagnetic phase and PZT for the piezoelectric phase. The DC magnetic field bias is 1.5 Tesla (15,000 Gauss) applied to the sample to obtain the different magnetoelectric configuration. This bias is same as have been used in literature. The saturation magnetization of the cobalt ferrite is 1780 Gauss. Additional information in the material properties is available in Appendix B.

	Longitudinal	Transverse	In-plane
Property	ME	ME	ME
	Configuration	Configuration	Configuration
$\boldsymbol{\varepsilon}_{_{xx}}/\boldsymbol{\varepsilon}_{_{0}}$	880	655	465
$oldsymbol{arepsilon}_{yy}ig/oldsymbol{arepsilon}_{0}$	880	655	655
$oldsymbol{arepsilon}_{zz}/oldsymbol{arepsilon}_{0}$	19.87	19.83	19.85
$\mu_{_{xx}}/\mu_{_0}$	0.9972	0.945	0.95
$\mu_{_{xy}}/\mu_{_0}$	<i>-j</i> 0.013	-	-
$\mu_{_{yy}}/\mu_{_0}$	0.9972	0.9972	0.997
$\mu_{_{yx}}/\mu_{_0}$	<i>j</i> 0.013	-	-
$\mu_{_{yz}}/\mu_{_0}$	-	<i>j</i> 0.013	<i>j</i> 0.013
$\mu_{_{zz}}/\mu_{_0}$	0.999	0.9972	0.997
$\mu_{_{zy}}/\mu_{_0}$	-	- <i>j</i> 0.013	<i>-j</i> 0.013
$lpha_{_{XX}}^{_H}/c_0$	-	-	2.98
$lpha_{_{xx}}^{_E}/c_0$	-	-	5.96
$lpha_{zx}^{\scriptscriptstyle H}/c_{\scriptscriptstyle 0}$	-	0.03	-
$lpha_{\scriptscriptstyle xz}^{\scriptscriptstyle E}/c_{\scriptscriptstyle 0}$	-	0.03	-
$lpha_{zz}^{H}/c_{0}$	0.0135	-	-
$lpha_{_{zz}}^{_E}/c_{_0}$	0.0135	-	-

TABLE A.1. EFFECTIVE MATERIAL PARAMETERS FOR A PZT/COBALT FERRITE ME COMPOSITE IN THE THREE ME CONFIGURATIONS. E_0 , μ_0 , and C_0 are, respectively the FREE SPACE PERMITTIVITY, FREE SPACE PERMEABILITY AND SPEED OF LIGHT IN A VACUUM.

Magnetoelectric slot line

We investigate the dispersion characteristics for a magnetoelectric slot line. A representation of the magnetoelectric slot line is shown in Fig. A.1. For the substrate, we include a low-loss dielectric layer over the magnetoelectric layer. The dielectric layer is used to allow a better portion of the wave to propagate in the dielectric layer than in the bianisotropic layer where attenuation of the wave may be greater [60]. Hence, the dielectric layer serves as an insulating surface to help alleviate losses.



FIGURE A.1. Planar magnetoelectric slot line. Figure shows dielectric layer over magnetoelectric layer. Red line indicates region of metallization. The dimensions w, m, and d represent the slot width, magnetoelectric thickness, and dielectric thickness respectively.

A figurative description of the planar magnetoelectric slot line is shown in Fig. A.1. Here, we do not show the bias directions for the DC magnetic and electric fields. We do assume that the bias directions can be effectively changed without any changes to the orientation of the slot line as shown in Fig. A.1. The dimensions of the magnetoelectric slot line are given as d = 0.5mm, m = 1.2mm, and w = 2mm. As part of the investigation of the planar structure, a parametric study was done to obtain the best dimensions for the slot line. We do not show the results of that study here. Numerical simulation is done for all magnetoelectric configurations at 400MHz. The results for these simulations are discussed below.

Transverse magnetoelectric slot line

The magnetoelectric slot line is simulated for the transverse magnetoelectric configuration using the effective material parameters of the transverse magnetoelectric media as shown in Table A.1. The results from the transverse magnetoelectric slot line as observed in Figs. A.2 and A.3 respectively, show that guided wave propagation is possible in the media. The line behaves akin to a dielectric slot line. Propagation is directed along the *x*-axis for this case and all others handled here.



FIGURE A.2. Power flow for transverse magnetoelectric slot line. Uniform propagation of fields observed along both edges of the slot line. Red lines indicate metallization.



FIGURE A.3. Electric field pattern across the magnetoelectric slot line. The pattern follows similar electric field pattern for dielectric slot lines.

We study the dispersion characteristics for the transverse magnetoelectric slot line. We obtain the dispersion curve for the propagating mode, and compare it to that of a non-magnetoelectric media having similar material parameters as the magnetoelectric media. The non-magnetoelectric media has same permeability and permittivity parameters as the magnetoelectric composite. The difference lies in the fact that the magnetoelectric coupling tensors are set to zero ($\alpha^{H} = \alpha^{E} = 0$). The dispersion curve is shown for both structures are shown in Fig. A.4. We observe that there are relatively no changes in the propagation constant of the media for the magnetoelectric and non magnetoelectric structure. It is important to note that the magnetoelectric coupling for the composite may be insufficient to produce any new propagation phenomena in the media. So, we increase the value of the magnetoelectric coupling coefficients in our simulations. We observe that the speed of the propagating wave slows down with increase to the magnetoelectric components. However, this decrease in speed is relatively small. Simulating at 400 MHz, we increase the value of the magnetoelectric components to ten times the actual value, and we obtain a change from $\beta_{actual} = 9.37$, to $\beta_{increase} = 9.36$.

Hence, for the transverse case, the magnetoelectric coupling coefficients are two small to cause any relevant changes to the propagation constant, or affect the direction or polarization of the propagating fields.



FIGURE A.4. Dispersion curves for a magnetoelectric and non-magnetoelectric slot line. We observe that the magnetoelectric coupling does not affect the phase constant of the slot line.

In-plane magnetoelectric slot line

The magnetoelectric slot line is simulated for the in-plane magnetoelectric configuration using the in-plane magnetoelectric material parameters as shown in Table A.1. The simulation results, respectively for the power flow in the *x*-direction, and the electric field pattern across the slot line is shown in Figs. A.5 and A.6. The results are similar to that obtained for the transverse configuration.



FIGURE A.5. Power flow for an in-plane magnetoelectric slot line. Power flow is out of the page. Red lines indicate region of metallization.



FIGURE A.6. 2D representation of the Electric field pattern across in-plane magnetoelectric slot line.

Again, as was observed in the transverse magnetoelectric slot line, the magnetoelectric coupling does not affect the propagation constant for the planar structure. It is important to note that we are not able to observe any rotation in terms of the polarization of the propagating wave in with the 2D simulation, should that phenomena be occurring in the slot line. However, we do not expect that since the in-plane magnetoelectric composite does not produce circularly polarized waves. We also study the dispersion characteristics for the in-plane magnetoelectric slot line, and compare with

a non-magnetoelectric media having similar permeability and permittivity as the magnetoelectric media. The result is shown in Fig. A.7.



FIGURE A.7. Dispersion curves for a In-plane magnetoelectric and nonmagnetoelectric slot line. Magnetoelectric coupling is insufficient to affect the phase constant or characteristics of the slot line.

As was done for the transverse slot line, we increase the magnetoelectric coupling and simulate at 400 MHz. Increase in the magnitude of the magnetoelectric susceptibility up to ten times the actual value only caused a change of one hundredth to the value of the propagation constant of the slot line. Hence, while the coupling is observable in the bulk propagation characteristics of the media, its effects are largely ineffective in application to planar structures.

Longitudinal magnetoelectric slot line

The magnetoelectric slot line is simulated for the longitudinal magnetoelectric configuration using the longitudinal magnetoelectric material parameters as shown in Table A.1. The case of a non-gyrotropic ME media is investigated for the longitudinal case, since we are unable to numerically compute the gyrotropic case. The results for the simulations produce similar results to the transverse and in-plane cases already investigated. The results are shown in Figs. A.8 and A.9. Figure A.8 shows the power flow along the *x*-axis for the longitudinal magnetoelectric media.



FIGURE A.8. Plow flow for the longitudinal magnetoelectric slot line. Uniform power flow is observed along both sloth edges.

These result shows that the magnetoelectric effect for the media is insufficient to produce new phenomena in the composite. We also study the dispersion characteristics for the longitudinal magnetoelectric slot line. We compare all three magnetoelectric configurations in Fig A.10.



FIGURE A.9. Electric field pattern across slot line for the in-plane magnetoelectric planar structure.



FIGURE A.10. Dispersion curves, showing comparison of the different magnetoelectric configurations for the slot line.

There are noticeable differences in the dispersion curve for all three magnetoelectric configurations. However, these changes may be not as a result of the magnetoelectric coupling coefficients; rather it may arise from the differences in the permeability and permittivity of the composites. Our investigations into the magnetoelectric slot line lead us to the conclusion that unless much greater

magnetoelectric susceptibilities are realized, application of magnetoelectric composites to transform the propagation characteristics of slot lines will be unsuccessful.

Magnetoelectric microstrip line

As has been done for the slot line, we investigate the electromagnetic wave propagation characteristics of the microstrip line with magnetoelectric composite as the substrate. To handle losses due to wave attenuation within the planar structure, we use a dielectric layer as was done in the slot line case. A figurative description of the microstrip line is shown in Fig. A.11.



FIGURE A.11. Planar magnetoelectric microstrip line. Figure shows dielectric layer over magnetoelectric layer. Red lines indicate region of metallization. The dimensions w, m, and d represent the slot width, magnetoelectric thickness, and dielectric thickness respectively.

The dimensions of the magnetoelectric microstrip line are given as d = 0.4mm, m = 0.8mm, and w = 2mm. As part of our investigation of the planar structure, a parametric study was done to obtain the best dimensions for the microstrip line. We also utilize

results from previous studies [60] on magnetoelectric waveguides. Simulations were done at 400 MHz.

Transverse magnetoelectric microstrip line

The magnetoelectric microstrip line is simulated for the transverse magnetoelectric configuration using the material parameters as shown in Table A.1. The simulation results are shown in Figs. A.12 and A.13. The propagation characteristics are quite similar to the case of a non-magnetoelectric microstrip line.



FIGURE A.12. Power flow for transverse magnetoelectric microstrip line. Propagation is along the x-axis. We observe equal propagation of fields along microstrip.

As was done for the slot line, we also study the dispersion characteristics for the transverse magnetoelectric microstrip line. The propagating mode is compared to that of a non-magnetoelectric media having similar material parameters as the magnetoelectric media. The dispersion curves for the magnetoelectric and non-magnetoelectric structures are shown in Fig. A.14. We observe that there are no differences in the propagation characteristics of the media, in terms of the phase constant.



FIGURE A.13. Electric field streamline for the transverse magnetoelectric microstrip line



FIGURE A.14. Dispersion curves for a transverse magnetoelectric and nonmagnetoelectric microstrip line. Magnetoelectric coupling does not introduce changes to the phase constant of the slot line.

In-plane magnetoelectric microstrip line

The magnetoelectric microstrip line is simulated numerically for the transverse magnetoelectric configuration using the material parameters as shown in Table A.1. The
simulation results are shown in Figs. A.15 and A.16. The propagation characteristics are quite similar to the case of a dielectric microstrip line in terms of the pattern of the electric field. We study the dispersion characteristics for the transverse magnetoelectric microstrip line. The propagating mode is compared to that of a non-magnetoelectric media having similar material parameters as the magnetoelectric media.



FIGURE A.15. Power flow for the in-plane magnetoelectric microstrip line. Propagation direction is out of the page



FIGURE A.16. Electric field streamline for the in-plane magnetoelectric microstrip line



FIGURE A.17. Dispersion curves for an In-plane magnetoelectric and nonmagnetoelectric microstrip line. Magnetoelectric coupling is insufficient to affect the phase constant or characteristics of the slot line.

Longitudinal magnetoelectric microstrip line

The magnetoelectric microstrip line is simulated for the transverse magnetoelectric configuration using the material parameters as shown in Table A.1. We study the dispersion characteristics for the transverse magnetoelectric microstrip line. The propagating mode is compared to that of a non-magnetoelectric media having similar material parameters as the magnetoelectric media.



FIGURE A.18. Power flow for the longitudinal magnetoelectric microstrip line. Equal field magnitudes propagate along structure. Propagation direction if out of the page



FIGURE A.19. Electric field streamline for the in-plane magnetoelectric microstrip line

The results show guided wave propagation as possible in planar structures using magnetoelectric composites. We note that not all cases for the magnetoelectric composites were handled. The case for propagation along the y-axis which will be perpendicular to the bias direction should be an interesting case. This is because such propagation orientation usually leads to a field displacement effect in ferrite materials biased with a DC magnetic field. Obtaining results for cases such as that will be the focus of future research as we intend to investigate propagation in the magnetoelectric material.



FIGURE A.20. Dispersion curves, showing comparison of the different magnetoelectric configurations for the slot line.

The results thus far show that the magnetoelectric coupling has little magnitude to affect the propagation characteristics of the planar magnetoelectric waveguides. Increase up to ten times the normal values of the magnetoelectric coupling coefficient only affects the phase constant by one hundredths. Thus, we would expect several orders of magnitude more for the ME coupling coefficient to create any interesting characteristics on the fields or propagation constant.

Guided wave propagation with the magnetoelectric media did not yield many interesting phenomena. The magnitude of the ME coefficients obtained in all three configurations is inadequate to effect any changes to the propagation characteristics of the planar structures. Some propagation orientations could not be simulated due to limitations in the shape of the tensor that describes the material parameters of the magnetoelectric media. This is an area for future work. Solutions for the guided wave propagations can be studied using semi-analytical tools such as the Method of Lines (MoL) or the Extended Method of Lines (E-MoL). These methods may help obtain numerical results for y-directed propagation in the planar structures considered here.

APPENDIX B: MAGNETOELECTRIC MATERIAL PROPERTIES

Using the material properties of the composite materials and the results of the theoretical model, the effective parameters of the composite is obtained for each magnetoelectric bias configuration. The composite consists of a cobalt ferrite piezomagnetic phase and a PZT piezoelectric phase. Computing the effective material parameters allows for the use of numerical solvers to obtain an insight into the propagation characteristics of the media. The applied DC magnetic field bias is 1.5 Tesla (15,000 Gauss). This is applied to the sample to obtain the different magnetoelectric configuration. The saturation magnetization of the cobalt ferrite is 1780 Gauss. We assume lossless properties for both composite phases. Additional information on the cobalt ferrite can be obtained in [58]. The operating frequency is at 400 MHz. The material parameters for each configuration are given below.

Based upon the absence of material parameters for a piezomagnetic material, we have used material parameters from the magnetostrictive phase to represent parameters for the piezomagnetic layer. The assumption made here is that the piezomagnetic coefficient of the magnetostrictive layer is also coupled to the stress such that application of stress leads to magnetization. This property is absent in magnetostrictive materials, and is only done here to allow for easier understanding of how the material parameters affect

electromagnetic wave propagation in ME composites comprised of piezoelectric and piezomagnetic phases.

1. Longitudinal magnetoelectric effective parameters

Material parameters for the case of anisotropic permeability are computed using cobalt ferrite and PZT material. The permittivity and permeability are given as

$$\overline{\overline{\varepsilon}} = \varepsilon_0 \begin{bmatrix} 880 & 0 & 0 \\ 0 & 880 & 0 \\ 0 & 0 & 19.87 \end{bmatrix}, \ \overline{\overline{\mu}} = \mu_0 \begin{bmatrix} 0.9972 & -j0.013 & 0 \\ j0.013 & 0.9972 & 0 \\ 0 & 0 & 0.99 \end{bmatrix}$$
(B.1)

The magnetoelectric (ME^H) coupling term is obtained as

$$\overline{\overline{\alpha}}^{H} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 4.5E - 11 \end{bmatrix} = \frac{1}{c_0} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0.0135 \end{bmatrix},$$
(B.2)

The magnetoelectric (ME^E) coupling term is obtained as

$$\overline{\overline{\alpha}}^{E} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 4.5E - 11 \end{bmatrix} = \frac{1}{c_0} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0.0135 \end{bmatrix}.$$
 (B.3)

Material parameters for the case of non-gyrotropic permeability obtained. The effective permittivity and permeability are obtained as

$$\overline{\overline{\varepsilon}} = \varepsilon_0 \begin{bmatrix} 880 & 0 & 0 \\ 0 & 880 & 0 \\ 0 & 0 & 19.87 \end{bmatrix}, \ \overline{\overline{\mu}} = \mu_0 \begin{bmatrix} 1.5 & 0 & 0 \\ 0 & 1.5 & 0 \\ 0 & 0 & 1.33 \end{bmatrix}$$
(B.4)

The magnetoelectric (ME^H) coupling term is obtained as

$$\overline{\overline{\alpha}}^{H} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2.99E - 11 \end{bmatrix} = \frac{1}{c_0} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0.0089 \end{bmatrix},$$
(B.5)

The magnetoelectric (ME^E) coupling term is obtained as

$$\overline{\overline{\alpha}}^{E} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2.99E - 11 \end{bmatrix} = \frac{1}{c_0} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0.0089 \end{bmatrix}.$$
 (B.6)

2. Transverse magnetoelectric effective parameters

Material parameters for the case of anisotropic permeability are computed. The permittivity and permeability are obtained as

$$\overline{\overline{\varepsilon}} = \varepsilon_0 \begin{bmatrix} 655 & 0 & 0 \\ 0 & 655 & 0 \\ 0 & 0 & 19.83 \end{bmatrix}, \ \overline{\overline{\mu}} = \mu_0 \begin{bmatrix} 0.945 & 0 & 0 \\ 0 & 0.9972 & j0.013 \\ 0 & -j0.013 & 0.9972 \end{bmatrix}$$
(B.7)

The magnetoelectric (ME^H) coupling term is obtained as

$$\overline{\overline{\alpha}}^{H} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1E - 10 & 0 & 0 \end{bmatrix} = \frac{1}{c_0} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0.03 & 0 & 0 \end{bmatrix},$$
(B.8)

The magnetoelectric (ME^E) coupling term is obtained as

$$\overline{\overline{\alpha}}^{E} = \begin{bmatrix} 0 & 0 & 1E - 10 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \frac{1}{c_0} \begin{bmatrix} 0 & 0 & 0.03 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$
 (B.9)

Material parameters for the case of non-gyrotropic permeability are computed. The effective permittivity and permeability are obtained as

$$\overline{\overline{\varepsilon}} = \varepsilon_0 \begin{bmatrix} 655 & 0 & 0 \\ 0 & 655 & 0 \\ 0 & 0 & 19.83 \end{bmatrix}, \ \overline{\overline{\mu}} = \mu_0 \begin{bmatrix} 1.45 & 0 & 0 \\ 0 & 1.5 & 0 \\ 0 & 0 & 1.33 \end{bmatrix}$$
(B.10)

The magnetoelectric (ME^H) coupling term is obtained as

$$\overline{\overline{\alpha}}^{H} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1E - 10 & 0 & 0 \end{bmatrix} = \frac{1}{c_0} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0.03 & 0 & 0 \end{bmatrix},$$
(B.11)

The magnetoelectric (ME^E) coupling term is obtained as

$$\overline{\overline{\alpha}}^{E} = \begin{bmatrix} 0 & 0 & 1E - 10 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \frac{1}{c_0} \begin{bmatrix} 0 & 0 & 0.03 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$
 (B.12)

3. In-plane magnetoelectric effective parameters

Material parameters for the case of anisotropic permeability computed for the in-plane configuration. The effective permittivity and permeability as

$$\overline{\overline{\varepsilon}} = \varepsilon_0 \begin{bmatrix} 465 & 0 & 0 \\ 0 & 655 & 0 \\ 0 & 0 & 19.85 \end{bmatrix}, \ \overline{\overline{\mu}} = \mu_0 \begin{bmatrix} 0.95 & 0 & 0 \\ 0 & 0.997 & j0.013 \\ 0 & -j0.013 & 0.997 \end{bmatrix}$$
(B.13)

The magnetoelectric (ME^H) coupling term is obtained as

$$\overline{\overline{\alpha}}^{H} = \begin{bmatrix} 9.94E - 9 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \frac{1}{c_0} \begin{bmatrix} 2.98 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$
(B.14)

The magnetoelectric (ME^E) coupling term is obtained as

$$\overline{\overline{\alpha}}^{E} = \begin{bmatrix} 1.98E - 8 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \frac{1}{c_0} \begin{bmatrix} 5.96 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$
 (B.15)

Material parameters for the case of non-gyrotropic permeability computed. The effective permittivity and permeability are obtained as

$$\overline{\overline{\varepsilon}} = \varepsilon_0 \begin{bmatrix} 465 & 0 & 0 \\ 0 & 655 & 0 \\ 0 & 0 & 19.85 \end{bmatrix}, \ \overline{\overline{\mu}} = \mu_0 \begin{bmatrix} 1.45 & 0 & 0 \\ 0 & 1.5 & 0 \\ 0 & 0 & 1.33 \end{bmatrix}$$
(B.16)

The magnetoelectric (ME^H) coupling term is obtained as

$$\overline{\overline{\alpha}}^{H} = \begin{bmatrix} 9.9E - 9 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \frac{1}{c_0} \begin{bmatrix} 2.98 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$
(B.17)

The magnetoelectric (ME^E) coupling term is obtained as

$$\overline{\overline{\alpha}}^{E} = \begin{bmatrix} 1.99E - 8 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \frac{1}{c_0} \begin{bmatrix} 5.96 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$
 (B.18)

APPENDIX C: NUMERICAL MODELING OF PLANAR MAGNETOELECTRIC WAVEGUIDES

The initial step of the research effort was to obtain a means to observe the electromagnetic wave propagation in bianisotropic media. As has been stated, due to the complexity of the medium, analytic solutions are not easily obtained. Hence, we modeled the medium using numerical methods. This is done by implementing numerical solutions for Maxwell's equation for the medium. The source-free Maxwell equations are given by $(e^{j\omega t} \text{ time convention})$

$$\nabla \times \mathbf{E} = -j\omega \mathbf{B}$$

$$\nabla \times \mathbf{H} = j\omega \mathbf{D}$$
(C.1)

where ω is the angular frequency. Using

$$\mathbf{D} = \overline{\overline{\varepsilon}} \mathbf{E} + \overline{\overline{\zeta}} \mathbf{H}$$

$$\mathbf{B} = \overline{\overline{\mu}} \mathbf{H} + \overline{\overline{\zeta}} \mathbf{E}$$
(C.2)

the Helmholtz equation for the media is obtained; namely,

$$\nabla \times \overline{\overline{\mu}}^{-1} \times \mathbf{E} + j\omega \nabla \times \overline{\overline{\mu}}^{-1} \overline{\varsigma} \mathbf{E} - j\omega \overline{\overline{\xi}} \overline{\overline{\mu}}^{-1} \nabla \times \mathbf{E} + \omega^2 \overline{\overline{\xi}} \overline{\overline{\mu}}^{-1} \overline{\overline{\varsigma}} \mathbf{E} - \omega^2 \overline{\overline{\varepsilon}} \mathbf{E} = 0$$
(C.3)

The weak formulation using (C.4) is derived as shown in [57]. Numerical results are obtained by applying the relevant formulations to an FEM solver. A commercially-available multi-physics FEM solver, COMSOL, was used in this study.

After the solver had been implemented, several waveguides were investigated to assess its robustness. We present here the simulated case of a circular bianisotropic waveguide as analyzed in [57]. The circular waveguide of radius R is filled with bianisotropic material characterized by

$$\overline{\overline{\varepsilon}} = \varepsilon_0 \begin{bmatrix} 1.099 & -j0.043 & 0\\ j0.043 & 1.099 & 0\\ 0 & 0 & 1.142 \end{bmatrix}, \overline{\overline{\mu}} = \mu_0 \begin{bmatrix} 0.7 & -j0.3 & 0\\ j0.3 & 0.7 & 0\\ 0 & 0 & 1 \end{bmatrix}, \overline{\overline{\xi}} = \mu_0 \begin{bmatrix} j0.7 & 0.3 & 0\\ -0.3 & j0.7 & 0\\ 0 & 0 & j \end{bmatrix} \times 10^{-3}, \overline{\overline{\zeta}} = -\overline{\overline{\xi}}$$
(C.5)

The dispersion characteristics for the different modes propagating in the waveguide are shown in Figure 1. This figure compares favorably to a similar dispersion curve shown in [57]. A comparison of the resulting normalized propagation constant obtained using our numerical method show good agreement with results obtained in [57]. This is observed in Table C.1.

Mode	Reference 57	Our Solver
HE11	1.30	1.28
HE-11	0.62	0.55
HE21	1.00	0.94
HE-21	0.33	0.28

TABLE C.1: COMPARISON OF THE NORMALIZED PROPAGATION CONSTANT BETWEEN REF. 57 AND OUR SOLVER FOR $K_0R = 3.5$



FIGURE C.1. Dispersion characteristics for different modes propagating in a circular bianisotropic waveguide.

The numerical system is now verified and can be used to solve a variety of waveguide problems using simple or complex media.

Using this system, several planar waveguide structures were modeled, with good agreement in terms of the results obtained. We show here an example of a microstrip line on a chiral substrate. Other cases such as the coplanar chiral waveguide, coupled chiral microstrip line, and the chiral slot line were also investigated with similar accuracy. Here, we compare our chiral micro-strip line solutions to results obtained in [59]. The waveguide dimensions and properties are given below.



FIGURE C.2. Chiral microstrip line simulated for verification of numerical code

A frequency sweep is done from 1GHz to 20GHz. A summarized table of results is shown below.

belween Solutions Obtained by Ref. 37 And Our Solver.				
Frequency	Propagation Constant from	Propagation constant from	% Error	
(GHz)	[59]	our solver		
1	36.12	36.53	1.14	
5	190.5	191.41	0.48	
10	436.62	435.16	0.33	
15	766.75	766.72	0.003	
20	1147.94	1153.95	0.52	

TABLE C.2. COMPARISON OF PROPAGATION CONSTANT FOR A CHIRAL MICROSTRIP LINE BETWEEN SOLUTIONS OBTAINED BY REF. 59 AND OUR SOLVER.

The results show that the numerical system can accurately model planar chiral waveguide structures.

APPENDIX D: COORDINATE ROTATION OF MAGNETOELECTRIC MATERIAL

PARAMETERS FOR NUMERICAL SIMULATION USING COMSOL

We require electromagnetic wave propagation in the xy plane of the magnetoelectric sample. The planar waveguide structures are implemented with the assumption of propagation along the sample (xy) plane. However, the numerical software (COMSOL) only allows for 2D propagation solutions directed along the z-axis. Hence, we use this numerical solution by implementing a coordinate rotation to obtain material parameters where the numerical results, although solved with z-directed propagation, reflect the characteristics for propagation along the x or y directions.

The numerical software also has additional limitations, as COMSOL only accepts tensors with zero elements as expressed in Eq. D.1.

$$Shape = \begin{bmatrix} S_{xx} & S_{xy} & 0\\ S_{yx} & S_{yy} & 0\\ 0 & 0 & S_{zz} \end{bmatrix}.$$
 (D.1)

Hence, any tensor where the xz, yz, zx, and zy components are non zero cannot be computed. This is a limitation of the commercial software used in this study.

A. Coordinate rotation of material parameters for x-directed propagation

1. Longitudinal magnetoelectric effect

Material parameters for the case of anisotropic permeability in the longitudinal configuration were obtained as

$$\overline{\overline{\varepsilon}} = \begin{bmatrix} \varepsilon_{xx} & 0 & 0\\ 0 & \varepsilon_{yy} & 0\\ 0 & 0 & \varepsilon_{zz} \end{bmatrix}, \ \overline{\overline{\mu}} = \begin{bmatrix} \mu_{xx} & \mu_{xy} & 0\\ \mu_{yx} & \mu_{yy} & 0\\ 0 & 0 & \mu_{zz} \end{bmatrix}, \ \overline{\overline{\alpha}}^{H} = \begin{bmatrix} 0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & \alpha_{zz}^{H} \end{bmatrix}, \ \overline{\overline{\alpha}}^{E} = \begin{bmatrix} 0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & \alpha_{zz}^{E} \end{bmatrix}.$$
(D.2)

Using a coordinate rotation on the effective material parameters, we obtain material parameters for a *z*-directed propagation as

$$\overline{\overline{\varepsilon}} = \begin{bmatrix} \varepsilon_{yy} & 0 & 0\\ 0 & \varepsilon_{zz} & 0\\ 0 & 0 & \varepsilon_{xx} \end{bmatrix}, \ \overline{\overline{\mu}} = \begin{bmatrix} \mu_{yy} & 0 & \mu_{yx}\\ 0 & \mu_{zz} & 0\\ \mu_{xy} & 0 & \mu_{xx} \end{bmatrix}, \ \overline{\overline{\alpha}}^{H} = \begin{bmatrix} 0 & 0 & 0\\ 0 & \alpha_{zz}^{H} & 0\\ 0 & 0 & 0 \end{bmatrix}, \ \overline{\overline{\alpha}}^{E} = \begin{bmatrix} 0 & 0 & 0\\ 0 & \alpha_{zz}^{E} & 0\\ 0 & 0 & 0 \end{bmatrix}.$$
(D.3)

Based upon the form of the permeability obtained, we are unable to simulate this case for the longitudinal magnetoelectric configuration in COMSOL. Hence, propagation characteristics for this case are not obtained in this study.

Material parameters for the case of non-gyrotropic permeability for the longitudinal magnetoelectric configuration are given as

$$\overline{\overline{\varepsilon}} = \begin{bmatrix} \varepsilon_{xx} & 0 & 0\\ 0 & \varepsilon_{yy} & 0\\ 0 & 0 & \varepsilon_{zz} \end{bmatrix}, \ \overline{\overline{\mu}} = \begin{bmatrix} \mu_{xx} & 0 & 0\\ 0 & \mu_{yy} & 0\\ 0 & 0 & \mu_{zz} \end{bmatrix}, \ \overline{\overline{\alpha}}^{H} = \begin{bmatrix} 0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & \alpha_{zz}^{H} \end{bmatrix}, \ \overline{\overline{\alpha}}^{E} = \begin{bmatrix} 0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & \alpha_{zz}^{E} \end{bmatrix}.$$
(D.4)

Using a coordinate rotation, we obtain material parameters that show same propagation characteristics while propagating in the z-direction. The new material parameters are

$$\bar{\overline{\varepsilon}} = \begin{bmatrix} \varepsilon_{yy} & 0 & 0 \\ 0 & \varepsilon_{zz} & 0 \\ 0 & 0 & \varepsilon_{xx} \end{bmatrix}, \ \bar{\overline{\mu}} = \begin{bmatrix} \mu_{yy} & 0 & 0 \\ 0 & \mu_{zz} & 0 \\ 0 & 0 & \mu_{xx} \end{bmatrix}, \ \bar{\overline{\alpha}}^{H} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \alpha_{zz}^{H} & 0 \\ 0 & 0 & 0 \end{bmatrix}, \ \bar{\overline{\alpha}}^{E} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \alpha_{zz}^{E} & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$
(D.5)

Simulation using the material parameters as indicated will give the required results.

2. Transverse magnetoelectric effect

The effective material parameters for the case of anisotropic permeability in the transverse magnetoelectric configuration are given as:

$$\overline{\overline{\varepsilon}} = \begin{bmatrix} \varepsilon_{yy} & 0 & 0\\ 0 & \varepsilon_{zz} & 0\\ 0 & 0 & \varepsilon_{xx} \end{bmatrix}, \ \overline{\overline{\mu}} = \begin{bmatrix} \mu_{xx} & 0 & 0\\ 0 & \mu_{yy} & \mu_{yz}\\ 0 & \mu_{zy} & \mu_{zz} \end{bmatrix}, \ \overline{\overline{\alpha}}^{H} = \begin{bmatrix} 0 & 0 & 0\\ 0 & 0 & 0\\ \alpha_{xx}^{H} & 0 & 0 \end{bmatrix}, \ \overline{\overline{\alpha}}^{E} = \begin{bmatrix} 0 & 0 & \alpha_{xz}^{E}\\ 0 & 0 & 0\\ 0 & 0 & 0 \end{bmatrix}.$$
(D.6)

Using a coordinate rotation, we obtain material parameters that show same propagation characteristics while propagating in the z-direction. The new material parameters are

$$\overline{\varepsilon} = \begin{bmatrix} \varepsilon_{yy} & 0 & 0\\ 0 & \varepsilon_{zz} & 0\\ 0 & 0 & \varepsilon_{xx} \end{bmatrix}, \ \overline{\mu} = \begin{bmatrix} \mu_{yy} & \mu_{yz} & 0\\ \mu_{zy} & \mu_{zz} & 0\\ 0 & 0 & \mu_{xx} \end{bmatrix}, \ \overline{\alpha}^{H} = \begin{bmatrix} 0 & 0 & 0\\ 0 & 0 & \alpha_{zx}^{H}\\ 0 & 0 & 0 \end{bmatrix}, \ \overline{\alpha}^{E} = \begin{bmatrix} 0 & 0 & 0\\ 0 & 0 & 0\\ 0 & \alpha_{xz}^{E} & 0 \end{bmatrix}.$$
(D.7)

The effective material parameters for the case of non-gyrotropic permeability for the transverse magnetoelectric configuration are given as:

$$\bar{\overline{\varepsilon}} = \begin{bmatrix} \varepsilon_{yy} & 0 & 0\\ 0 & \varepsilon_{zz} & 0\\ 0 & 0 & \varepsilon_{xx} \end{bmatrix}, \ \bar{\overline{\mu}} = \begin{bmatrix} \mu_{xx} & 0 & 0\\ 0 & \mu_{yy} & 0\\ 0 & 0 & \mu_{zz} \end{bmatrix}, \ \bar{\overline{\alpha}}^{H} = \begin{bmatrix} 0 & 0 & 0\\ 0 & 0 & 0\\ \alpha_{zx}^{H} & 0 & 0 \end{bmatrix}, \ \bar{\overline{\alpha}}^{E} = \begin{bmatrix} 0 & 0 & \alpha_{zz}^{E}\\ 0 & 0 & 0\\ 0 & 0 & 0 \end{bmatrix}.$$
(D.8)

Using a coordinate rotation, we obtain material parameters that show same propagation characteristics while propagating in the z-direction. The new material parameters are

$$\overline{\overline{\varepsilon}} = \begin{bmatrix} \varepsilon_{yy} & 0 & 0 \\ 0 & \varepsilon_{zz} & 0 \\ 0 & 0 & \varepsilon_{xx} \end{bmatrix}, \ \overline{\overline{\mu}} = \begin{bmatrix} \mu_{yy} & 0 & 0 \\ 0 & \mu_{zz} & 0 \\ 0 & 0 & \mu_{xx} \end{bmatrix}, \ \overline{\overline{\alpha}}^{H} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \alpha_{zx}^{H} \\ 0 & 0 & 0 \end{bmatrix}, \ \overline{\overline{\alpha}}^{E} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \alpha_{xz}^{E} & 0 \end{bmatrix}.$$
(D.9)

3. In-plane magnetoelectric effect

The effective material parameters for the case of anisotropic permeability in the transverse magnetoelectric configuration are given as

$$\overline{\varepsilon} = \begin{bmatrix} \varepsilon_{xx} & 0 & 0\\ 0 & \varepsilon_{yy} & 0\\ 0 & 0 & \varepsilon_{zz} \end{bmatrix}, \ \overline{\mu} = \begin{bmatrix} \mu_{xx} & 0 & 0\\ 0 & \mu_{yy} & \mu_{yz}\\ 0 & \mu_{zy} & \mu_{zz} \end{bmatrix}, \ \overline{\alpha}^{H} = \begin{bmatrix} \alpha_{xx}^{H} & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0 \end{bmatrix}, \ \overline{\alpha}^{E} = \begin{bmatrix} \alpha_{xx}^{E} & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0 \end{bmatrix}.$$
(D.10)

Using a coordinate rotation, we obtain material parameters that show same propagation characteristics while propagating in the z-direction. The new material parameters are

$$\overline{\overline{\varepsilon}} = \begin{bmatrix} \varepsilon_{yy} & 0 & 0\\ 0 & \varepsilon_{zz} & 0\\ 0 & 0 & \varepsilon_{xx} \end{bmatrix}, \quad \overline{\overline{\mu}} = \begin{bmatrix} \mu_{yy} & \mu_{yz} & 0\\ \mu_{zy} & \mu_{zz} & 0\\ 0 & 0 & \mu_{xx} \end{bmatrix}, \quad \overline{\overline{\alpha}}^{H} = \begin{bmatrix} 0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & \alpha_{xx}^{H} \end{bmatrix}, \quad \overline{\overline{\alpha}}^{E} = \begin{bmatrix} 0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & \alpha_{xx}^{E} \end{bmatrix}.$$
(D.11)

The effective material parameters for the case of non-gyrotropic permeability for the transverse magnetoelectric configuration are given as

$$\overline{\varepsilon} = \begin{bmatrix} \varepsilon_{yy} & 0 & 0\\ 0 & \varepsilon_{zz} & 0\\ 0 & 0 & \varepsilon_{xx} \end{bmatrix}, \ \overline{\mu} = \begin{bmatrix} \mu_{xx} & 0 & 0\\ 0 & \mu_{yy} & 0\\ 0 & 0 & \mu_{zz} \end{bmatrix}, \ \overline{\alpha}^{H} = \begin{bmatrix} \alpha_{xx}^{H} & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0 \end{bmatrix}, \ \overline{\alpha}^{E} = \begin{bmatrix} \alpha_{xx}^{E} & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0 \end{bmatrix}.$$
(D.12)

Using a coordinate rotation, we obtain material parameters that show same propagation characteristics while propagating in the z-direction. The new material parameters are

$$\overline{\overline{\varepsilon}} = \begin{bmatrix} \varepsilon_{yy} & 0 & 0\\ 0 & \varepsilon_{zz} & 0\\ 0 & 0 & \varepsilon_{xx} \end{bmatrix}, \quad \overline{\overline{\mu}} = \begin{bmatrix} \mu_{yy} & 0 & 0\\ 0 & \mu_{zz} & 0\\ 0 & 0 & \mu_{xx} \end{bmatrix}, \quad \overline{\overline{\alpha}}^{H} = \begin{bmatrix} 0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & \alpha_{xx}^{H} \end{bmatrix}, \quad \overline{\overline{\alpha}}^{E} = \begin{bmatrix} 0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & \alpha_{xx}^{E} \end{bmatrix}.$$
(D.13)

B. Coordinate rotation of material parameters for y-directed propagation

1. Longitudinal magnetoelectric effect

For y-directed propagation, the rotated material parameters are obtained as:

$$\overline{\overline{\varepsilon}} = \begin{bmatrix} \varepsilon_{xx} & 0 & 0 \\ 0 & \varepsilon_{yy} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{bmatrix}, \ \overline{\overline{\mu}} = \begin{bmatrix} \mu_{zz} & 0 & 0 \\ 0 & \mu_{xx} & \mu_{xy} \\ 0 & \mu_{yx} & \mu_{yy} \end{bmatrix}, \ \overline{\overline{\alpha}}^{H} = \begin{bmatrix} \alpha_{zz}^{H} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \ \overline{\overline{\alpha}}^{E} = \begin{bmatrix} \alpha_{zz}^{E} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$
(D.14)

Rotated material parameters for the case of non-gyrotropic permeability in the longitudinal magnetoelectric configuration are given as:

$$\overline{\overline{\varepsilon}} = \begin{bmatrix} \varepsilon_{xx} & 0 & 0 \\ 0 & \varepsilon_{yy} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{bmatrix}, \ \overline{\overline{\mu}} = \begin{bmatrix} \mu_{zz} & 0 & 0 \\ 0 & \mu_{xx} & 0 \\ 0 & 0 & \mu_{yy} \end{bmatrix}, \ \overline{\overline{\alpha}}^{H} = \begin{bmatrix} \alpha_{z}^{H} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \ \overline{\overline{\alpha}}^{E} = \begin{bmatrix} \alpha_{z}^{E} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$
(D.15)

2. Transverse magnetoelectric effect

Rotated material parameters for the case of anisotropic permeability in the transverse magnetoelectric configuration are given as:

$$\overline{\overline{\varepsilon}} = \begin{bmatrix} \varepsilon_{xx} & 0 & 0 \\ 0 & \varepsilon_{yy} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{bmatrix}, \quad \overline{\overline{\mu}} = \begin{bmatrix} \mu_{zz} & 0 & \mu_{zy} \\ 0 & \mu_{xx} & 0 \\ \mu_{yz} & 0 & \mu_{yy} \end{bmatrix}, \quad \overline{\overline{\alpha}}^{H} = \begin{bmatrix} 0 & \alpha_{zx}^{H} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \overline{\overline{\alpha}}^{E} = \begin{bmatrix} 0 & 0 & 0 \\ \alpha_{zz}^{E} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \quad (D.16)$$

Rotated material parameters for the case of non-gyrotropic permeability in the transverse magnetoelectric configuration are given as:

$$\overline{\overline{\varepsilon}} = \begin{bmatrix} \varepsilon_{xx} & 0 & 0 \\ 0 & \varepsilon_{yy} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{bmatrix}, \quad \overline{\overline{\mu}} = \begin{bmatrix} \mu_{zz} & 0 & \mu_{zy} \\ 0 & \mu_{xx} & 0 \\ \mu_{yz} & 0 & \mu_{yy} \end{bmatrix}, \quad \overline{\overline{\alpha}}^{H} = \begin{bmatrix} 0 & \alpha_{zx}^{H} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \overline{\overline{\alpha}}^{E} = \begin{bmatrix} 0 & 0 & 0 \\ \alpha_{xz}^{E} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \quad (D.17)$$

3. In-plane magnetoelectric effect

Rotated material parameters for the case of anisotropic permeability in the inplane magnetoelectric configuration are given as:

$$\overline{\overline{\varepsilon}} = \begin{bmatrix} \varepsilon_{xx} & 0 & 0 \\ 0 & \varepsilon_{yy} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{bmatrix}, \quad \overline{\overline{\mu}} = \begin{bmatrix} \mu_{zz} & 0 & \mu_{zy} \\ 0 & \mu_{xx} & 0 \\ \mu_{yz} & 0 & \mu_{yy} \end{bmatrix}, \quad \overline{\overline{\alpha}}^{H} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \alpha_{xx}^{H} & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \overline{\overline{\alpha}}^{E} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \alpha_{xx}^{E} & 0 \\ 0 & 0 & 0 \end{bmatrix}. \quad (D.18)$$

Rotated material parameters for the case of non-gyrotropic permeability in the inplane magnetoelectric configuration are given as:

$$\overline{\overline{\varepsilon}} = \begin{bmatrix} \varepsilon_{xx} & 0 & 0 \\ 0 & \varepsilon_{yy} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{bmatrix}, \ \overline{\overline{\mu}} = \begin{bmatrix} \mu_{zz} & 0 & \mu_{zy} \\ 0 & \mu_{xx} & 0 \\ \mu_{yz} & 0 & \mu_{yy} \end{bmatrix}, \ \overline{\overline{\alpha}}^{H} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \alpha_{xx}^{H} & 0 \\ 0 & 0 & 0 \end{bmatrix}, \ \overline{\overline{\alpha}}^{E} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \alpha_{xx}^{E} & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$
(D.19)

These material parameters are used in the simulation of planar magnetoelectric structures. Note that cases for y-directed propagation with magnetic anisotropy cannot be simulated using COMSOL due to the requirement on the shape of tensors allowed for 2D simulations.

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