Energetics of Human Leg-swing: Various Cost Models, Optimal Motions, and Fits to Experiments

THESIS

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ABSTRACT

The metabolic cost of leg-swing comprises (an estimated) 10% to 33% of the total cost of human walking and running. Experiments studying the swing phase of walking gait have been performed by numerous biomechanics researchers. One particular investigation by Doke, Kuo, et al. studied the cost of isolated leg-swing. Human subjects were asked to swing one leg forward and backward (in the sagittal plane) at a specified swinging amplitude and frequency. Metabolic rate was inferred by measuring maximal oxygen consumption during the leg-swinging task.

The goal of this thesis is to obtain a plausible functional form for the muscle metabolic cost that fits the Doke et al. experiments. Toward this end, we create a simple one-degree of freedom computer model of a leg, actuated by two uniarticular muscles. These muscles are modeled as having one of several functional forms for the cost, and these cost models relate the energetic expense of leg-swing to muscle torque, muscle shortening rate, etc.

Given one of the hypothesized forms of cost, optimization tools (in MATLAB) are used to find leg-swing strategies which minimize the objective function: total cost of leg-swing. The theoretical findings (from converged optimization results) are then compared with empirical data (from Doke et al.) to assess the validity of each cost model. The optimization results show that three substantially different cost functions seem to fit the experiments by Doke et al., at least approximately. This suggests that the experimental data is not rich enough to distinguish these functional forms, and that we might need more extensive experimental data to obtain a reliable model of the metabolic cost.

DEDICATION

To my father and mother, Peter and Quyen Dong.

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LIST OF SYMBOLS

Symbol	Description
<i>a</i> _{1,2,3}	Coefficient values for "Torque Rate" cost function.
α	Exponent value for "Torque" cost function.
α_{i}	Fraction of activated muscle fibers needed to generate muscle torque T_i .
β	Coefficient value for "Weighted T-squared" cost function.
F	Muscle force.
f_{swing}	Leg-swing frequency.
g	Gravitational acceleration.
Ι	Mass moment of inertia about an axis through the leg center of mass.
J	Objective value or total cost of leg-swing.
k _{eff}	Effective torsional spring stiffness of tendon.
k _i	Effective torsional spring stiffness of tendon i .
k _s	Linear spring stiffness of tendon.
L	Total length of leg.
ℓ_i	Arch length of muscle i .
$\dot{\ell}_i$	Angular shortening velocity of muscle i .
$\dot{\ell}_{\rm max}$	Maximum angular muscle shortening velocity.
М	Total body mass.
т	Leg mass.
Р	Muscle mechanical power (= $T_i \ell_i$).
Φ	Metabolic rate function.
r	Distance from hip pivot to leg center of mass.
r _m	Moment arm of the hip flexors and extensors.
Т	Muscle torque.
T_{full}	Torque produced by fully-activated muscle ($\alpha_i = 1$).

Symbol	Description
T_i	Torque generated by muscle <i>i</i> .
$\dot{T_i}$	Torque rate (first derivative) for muscle i .
$\ddot{T_i}$	Torque rate (second derivative) for muscle i .
$T_{\rm max}$	Maximum allowable muscle torque.
t _{max}	Leg-swing time period.
T_o	Maximum isometric muscle torque.
T_{total}	Total muscle torque (= $T_1 + T_2$).
θ	Angular position of swinging leg (displacement measured from vertical).
$\dot{ heta}$	Angular velocity of swinging leg.
x	Muscle length (linear).
ż	Muscle shortening velocity (linear).

CHAPTER 1: INTRODUCTION.

1.1 Animal locomotion and motives for energy optimality.

An animal's ability to move from one place to another is essential to survival. In many cases, there is an evolutionary pressure to move optimally – to minimize time, distance, energy, etc. (Srinivasan, 2006). For example, the cheetah must swiftly hunt and catch prey for sustenance, while the gazelle must tactically evade hungry predators. In preparation for the cold winter months, squirrels go in search of scarce seeds and nuts, while geese migrate toward warmer climates in remote locations.



Figure 1: Cheetah hunting gazelle, squirrel foraging for food, and geese migrating toward warmer climates.

It is commonly believed that animals move and locomote in ways that tend to minimize metabolic energy consumption. For humans and other legged animals, this inclination to

walk or run a certain distance using the least amount of metabolic energy might be a vital strategy for survival. An early human hunter-gatherer, for example, might journey long distances in search of nourishment. Given one stomach full of food (i.e. fuel), the most economical use of available food energy is desirable because the hunter-gatherer might have to travel for an indefinite amount of time (or distance) to reach the next available food source.

Studying optimal strategies for human walking and running helps researchers and bioengineers gain a better understanding of locomotion, and gives instructive information for practical applications like: building energy-efficient robots, developing prosthetic limbs, correcting walking gait disorders, and predicting how one might move in a novel situation (e.g. astronauts hopping on the moon).

1.2 Literature review and previous research works.

The metabolic cost of walking and running has been considered by various researchers in humans and other running animals. One component of the metabolic cost of walking and running relates to swinging the leg forward relative to the body. Estimates for the cost of leg-swing from past research range from 10% to 33% of the net metabolic cost of walking. Here, the term "net" refers to the metabolic rate of energy consumption which exceeds the baseline "resting" metabolic rate (typically measured while the subject stands at rest, with no leg motion).

One view suggests that the cost of moving the legs during the swing phase of walking gait is small relative to the cost of supporting body weight. Historically, the motion of swinging the legs relative to the body during walking or running has been characterized by pendulum dynamics (Mochon & McMahon, 1980). For a simple gravity pendulum, minimal effort is required for the pendulum to swing at its natural frequency. Along the same logic, research evidence has suggested that the swing phase of the human walking gait is primarily a passive motion that requires minimal metabolic energy input. Experiments in which various loads are applied to the human torso (close to the body center of mass) have shown that the stance phase accounts for the majority of energy expenditure during walking, whereas the cost of leg-swing comprises (approximately) only 10-15% of net locomotor costs (Griffin, Roberts, & Kram, 2003). Other investigations have demonstrated that animals of comparable size and weight exhibit similar walking costs, despite having different limb masses (Taylor, Shkolnik, Dmi'el, Baharav, & Borut, 1974). In other words, animals with small limbs and animals with massive limbs (but equal in size, overall) consume roughly the same amount of metabolic energy during walking, which might suggest that the cost of legswing is relatively inexpensive.

A different view on human locomotion suggests that the energetic cost of leg-swing is fairly substantial. That is, some research suggests that leg-swing during human locomotion requires active control by which muscles exert force and accordingly expend metabolic energy. Evidence has shown that metabolic energy consumption increases significantly when loads are applied to the legs, rather than the body center of mass (Soule & Gouldman, 1969). Experiments where loads were attached to the legs of humans (Myers & Stuedel, 1985) and dogs (Steudel, 1990) brought about higher locomotor costs than when the same loads were attached to the torso (close to the body center of mass). Direct in vivo measurements of muscle blood flow in guinea fowl revealed that leg-swing energy expenditure comprises 26% of the net costs of walking (Marsh, Ellerby, Carr, Henry, & Buchanan, 2004). Investigation of isolated leg-swing demonstrated that swinging the human leg faster than its natural frequency required a considerable increase in muscle force and metabolic energy input (Doke, Donelan, & Kuo, 2005). Estimates from the isolated legswing experiment suggest that the swing phase of human gait accounts for nearly 33% of net metabolic energy requirements for walking. Further studies indicate that the net metabolic rate during leg-swing might increase in proportion to peak muscle force or torque, and in inverse proportion to force duration (Doke & Kuo, 2007).

Since the metabolic cost of walking increases sharply with higher step frequencies, this might suggest that the cost of leg-swing is in fact a critical component of human locomotion. Altogether, estimates of leg-swing costs range widely from 10% to 33% of the net cost of walking. Such uncertainty exists partly because researchers are presently unable to directly measure in vivo the energy expenditure of individual muscles in human locomotion investigations. Due to these limitations, a computer modeling approach in combination with numerical optimization might be a useful technique for examining the metabolic cost of muscle function during locomotion.

1.3 Purpose of studying human leg-swing.

The aim of the present research study is to investigate the costs associated with driving the legs back-and-forth during human walking and running, and to develop reasonably accurate

models of metabolic energy expenditure. In particular, isolated human leg-swing will be the focus of the present study. Physical parameters and task constraints for the leg-swing are adopted from the experiments by Doke et al. (2005 & 2007). Empirical data obtained from these experiments is used to verify the feasibility of computer model results.



Figure 2: Experimental apparatus for isolated leg-swing experiment. Re-drawn from Doke, Donelan, & Kuo (2005).

Doke et al. (2005) built an apparatus to measure the mechanical and metabolic energy costs of isolated leg-swing for twelve healthy, adult, human subjects. As shown in Figure 2, the participant was securely fastened to the rigid metal support frame, mounted above a ground force plate. The subject rested both arms on the metal frame, stood one leg on a raised platform, and was able to swing the other leg freely. The swinging leg was constrained to be straight with the use of a knee splint. Using visual and audible cues, subjects were asked to swing their legs at an amplitude of 45° peak-to-peak, and at various swinging frequencies ranging from 0.5 to 1.1 Hz. Leg angle of rotation about the hip was measured by an optical encoder. Torque actuation due to the hip muscles was inferred by measuring the ground reaction forces of the force platform. Finally, metabolic energy expenditure was estimated by measuring the amount of oxygen consumed (VO₂) during leg-swing.

Doke et al. (2005) proposed that the rate of energy expenditure during isolated leg-swing increased approximately with the fourth power of frequency. The results of the study suggested that, due to the low muscle efficiency of exerting high forces for short periods of time, the cost of swinging the leg faster than its natural frequency increased dramatically. It was proposed that the net metabolic rate during leg-swing might be proportional to muscle force, and inversely proportionally to the duration of the applied force – called the "force/time hypothesis" (Doke, Donelan, & Kuo, 2005). A possible weakness of the force/time hypothesis, however, is that it is only applicable to swinging frequencies above the natural frequency of the leg. Perhaps a better hypothesis would involve a mathematical relationship that can be applied to all leg-swing frequencies. Hence, one purpose of this study is to examine various cost models different from the force/time hypothesis, and to see whether or not they are consistent with experimental data.

In short, some central questions that are addressed in the present research study are:

- 1. What are some plausible cost models of metabolic energy expenditure? How can these cost models be expressed in a mathematical, functional form?
- 2. Given a set of constraints (leg-swing amplitude, frequency, etc.), what is the optimal leg-swing strategy (muscle excitation and muscle shortening rate as functions of time) for each of the proposed cost functions?
- 3. Lastly, does the optimal strategy from the computer model match the leg-swing strategy that human subjects choose to do naturally?

On the whole, the leg-swing problem was chosen for this research investigation because the mechanics of the isolated leg-swing task are fairly simple. If a reasonably accurate model of metabolic energy expenditure is developed for the simple leg-swing problem, then there might be an opportunity to extend this cost model to other (perhaps more complex) motor tasks like walking, bicycling, pitching a softball, etc.

1.4 Motivation for computer modeling approach.

There are a number of advantages in using a computer modeling approach to evaluate cost functions and optimal leg-swing strategies. Computer modeling allows the researcher to run a variety of simulations by merely changing a few variables or settings in the programming. In this investigation, the swinging movement of the human leg is approximated by pendulum motion with torque actuators representing the hip muscles. The physical structure of the simulated human leg can be modified to explore different body weights, leg lengths, muscle and tendon properties, etc. Modifying various task constraints such as legswing amplitude and frequency is also possible.

With a computer modeling approach, numerical optimization may be used to determine optimal strategies for various leg-swing experiments. In Chapters 5 & 6, computer optimization results are compared with existing empirical data obtained by Doke et al. (2005 & 2007). Here, indirect evidence of energetic optimality is shown by matching the behavior predicted by the computer optimization to the behavior observed experimentally. A more direct approach to proving energetic optimality would involve purely physical experiments by which the best possible leg-swing strategy is determined by comparing *all* possible leg-swing strategies of which humans are capable – but to do this would require a sizeable number of experiments and might be time-inefficient. In a physical experiment using real human subjects, swinging the leg in a new or unusual manner might be energetically expensive just because the swinging pattern is unfamiliar (Srinivasan, 2006). For that reason, computational analysis might be a more objective tool for investigating optimal leg-swing strategies.

1.5 Thesis outline.

A brief outline of the rest of the thesis follows: First, Chapter 2 introduces muscle contraction dynamics, and describes the force-length and force-velocity properties of muscle. Chapter 3 explains how pendulum dynamics are used to model the mechanics of leg-swing, and how constraints are defined to set up the nonlinear optimization problem. Chapter 4 proposes different ways to model the energetics of leg-swing, and gives details about each of the different cost models under investigation. Chapters 5 & 6 examine theoretical results (from the computer model) against existing empirical data (Doke et al., 2005 & 2007). Lastly, Chapter 7 presents a summary of research findings and offers some concluding remarks.

CHAPTER 2: MODELING MUSCLE BEHAVIOR.

2.1 Contraction dynamics.

Performing a motor task is a complex process that involves interactions between different physiological systems within the body. Motor function begins with the central nervous system of the brain and spinal cord. Neurons are fired and excite the muscles to generate a force, which is then transmitted from muscle to tendon, and from tendon to the skeletal structure. This process is what enables an animal to move and to impart mechanical energy to the surrounding environment. Generally speaking, a broad research goal in the field of biomechanics is to develop dynamic equations that express how the body's structural, muscular, and central nervous systems are related and how they interact.

Examining the mechanics and energetics of locomotion requires modeling of muscle behavior – how muscles generate force and how metabolic energy is expended when exerting such a force. The diagrams shown in Figure 3 and Figure 4 illustrate different levels of the muscle structural hierarchy. The basic units of muscle tissue are called sarcomeres – a collection of which form muscle fiber. Similarly, a collection of muscle fibers form a motor unit, and a collection of motor units form a whole muscle. The properties of the whole muscle are assumed to be homogenous, scaled-up versions of the muscle motor units, fibers, and sarcomeres. That is, the properties of muscle tissue are assumed to be the same for any level of the muscle structural hierarchy.



Figure 3: Repeating sarcomere units form a muscle fiber. Adapted from Zajac (1989).



Figure 4: A collection of muscle fiber forms a motor unit. Repeating motor units form a whole muscle. Adapted from Zajac (1989).

So, what properties do muscles possess? Muscle contraction dynamics are characterized by force-length and force-velocity properties. The force-length curve illustrated in Figure 5

shows that the muscle is capable of actively producing a maximum force F_o at an optimal muscle length x_o . The active muscle force falls to zero when the muscle is stretched to 1.5 times its optimal length, or is shortened to 0.5 times its optimal length. Passively, the muscle generates tension when stretched beyond the optimal length (like stretching a spring beyond its equilibrium length).

The force-velocity curve illustrated in Figure 6 shows that the muscle produces the maximum isometric force F_o when muscle shortening rate is zero. Muscle force decreases as the muscle shortening rate increases, and ultimately, force drops to zero at a maximum shortening velocity v_{max} . A lengthening muscle, on the other hand, exerts a force that is higher than the maximum isometric force F_o . At high muscle lengthening rates, the muscle is able to generate a force close to 1.8 times that of F_o .

Note that the movement characterized by a shortening muscle is called **concentric contraction**, whereas the movement of a lengthening muscle is called **eccentric contraction**. Static effort where the muscle produces a pulling force but does not change in length is called **isometric contraction**.



Figure 6: Force-velocity property of muscle. Re-drawn from Zajac (1989).

2.2 Mechanical power and metabolic cost rate.

Power is the rate at which work is performed. For a working muscle, mechanical power (or rate of mechanical work) is calculated as follows:

Mechanical power =
$$F \dot{x}$$
 (1)

where F is muscle force and \dot{x} is (linear) muscle shortening rate. Note that muscles can only generate a pulling (not pushing) force. For an analogous system in which muscle produces torque rather than force (assuming a constant moment arm for the muscle), the equation for mechanical power becomes:

Mechanical power =
$$T\ell$$
 (2)

where T is muscle torque and $\dot{\ell}$ is (angular) muscle shortening rate. Rate of mechanical work is considered positive when the muscle is shortening, and negative when the muscle is lengthening. Based on the definitions of mechanical power expressed in Equation (1) and Equation (2), a muscle that exerts an isometric force (static effort) does zero mechanical work. It is clear from experience, however, that metabolic energy is required to exert such a force and to keep the muscle at a constant length. In other words, metabolic energy is expended when muscles are engaged and producing force or torque, regardless if the muscle experiences a change in length. Since one major goal of this research is to develop various models that accurately predict legswing costs, understanding metabolic power might be a key part of this investigation. Metabolism refers to the chemical reactions that take place in the body to convert food energy to useful mechanical work. The efficiency of converting metabolic energy to mechanical energy, however, is not constant (Minetti & Alexander, 1997). As previously explained in Section 2.1, the amount of force (or torque) that muscles produce is dependent on both muscle length and shortening rate. Accordingly, metabolic cost rate may be a function of muscle force (and its derivatives), and muscle length (and its derivatives). One plausible generalized expression for rate of metabolic energy expenditure could be:

$$\text{Cost rate} = f\left(F, \dot{F}, \ddot{F}, \dots, x, \dot{x}, \ddot{x}, \dots\right)$$
(3)

where F is muscle force and x is muscle length. For an analogous system in which muscle produces torque rather than force, the corresponding generalized expression for rate of metabolic energy expenditure becomes:

$$\text{Cost rate} = f\left(T, \dot{T}, \ddot{T}, \dots, \ell, \dot{\ell}, \ddot{\ell}, \dots\right)$$
(4)

where *T* is muscle torque and ℓ is muscle arch length. Here, metabolic cost rate may be a function of muscle torque (and its derivatives), and muscle arch length (and its derivatives).

In this research investigation, a number of different models of metabolic cost will be evaluated. These cost functions will be introduced in Chapter 4, and be further discussed in subsequent chapters.

CHAPTER 3: MODELING MECHANICS OF LEG-SWING.

By modeling the swinging leg as a pendulum, the mechanics of isolated leg-swing can be roughly simulated. This chapter describes how pendulum dynamics are used to model the swinging human leg, and how constraints are defined to set up the nonlinear optimization problem.

3.1 Pendulum dynamics.

Assuming that the knee and the ankle are locked, the leg is modeled as a single planar rigid body (see Figure 7) with mass m, moment of inertia I about an axis through the center of mass, and the center of mass located a distance r from the hip pivot. The upper body is ignored as being infinitely massive, so the motion of the leg does not affect the upper body – modeled as the rigid ceiling in Figure 7. The leg is assumed to be actuated by two uniarticular muscles (the main flexor and extensor muscles at the hip). Total torque T_{total} applied to the leg is the summation of torques T_1 and T_2 generated by the agonist and antagonist hip muscles. Tendons are represented as zero-length (torsional) springs added in series with the muscles.


Figure 7: Leg-swing schematic with muscles and tendons.

The following differential equation is used to model the pendulum-like dynamics of the swinging leg:

$$\left(I + mr^2\right)\ddot{\theta} + mgr\cdot\sin\theta = T_1 + T_2 \tag{5}$$

where θ is the angular displacement of the leg (measured from vertical), T_1 and T_2 are the torques applied from the agonist and antagonist hip muscles, m is the total leg mass, I is the centroidal mass moment of inertia, and r is the distance between the hip pivot and leg center of mass. Height and weight measurements of the twelve healthy, adult, human subjects (Doke, Donelan, & Kuo, 2005) were used as a guideline for defining the physical

parameters of the swinging leg in the computer model. The assumed physical parameters are listed in Table 1.

Variable	Physical description	Parameter value
М	Total body mass.	70 kg
т	Leg mass (16% of total body mass).	11.2 kg
L	Total leg length.	0.90 m
r	Distance from hip to leg center of mass.	0.45 m
Ι	Centroidal mass moment of inertia.	1.0 kg-m ²

Table 1: Parameter values for isolated leg-swing simulation model.

The mass of the leg m was taken as 16% of the subject's total body mass M (Doke, Donelan, & Kuo, 2005) and the distance r from the hip joint to the leg center of mass was estimated to be half the total length of the leg L. Assuming the leg to be a cylinder of length L and mass m, an estimate for the centroidal mass moment of inertia could also be calculated:

$$m = 0.16M = 0.16(70 \ kg) = 11.2 \ kg \tag{6}$$

$$r = 0.5L = 0.5(0.90 \text{ meters}) = 0.45 \text{ meters}$$
(7)

$$I = \frac{mL^2}{12} = \frac{(11.2 \ kg)(0.90 \ meters)^2}{12} \approx 1 \ kg \cdot meter^2 \tag{8}$$

The pendulum-like motion of isolated leg-swing may be modeled with or without the presence of tendons. For the case where tendons are present, each of the hip muscles are connected in series with tendons, as illustrated in Figure 7.

Tendons are bands of connective tissue that connect muscle to bone and function much like springs – they have elastic properties and are capable to storing and releasing elastic energy. When tendon "springs" are added in series with muscle "motors", muscle shortening rate can be computed as follows:

$$\dot{\ell}_1 = -\frac{\dot{T}_1}{k_1} - \dot{\theta}$$

$$\dot{\ell}_2 = +\frac{\dot{T}_2}{k_2} + \dot{\theta}$$
(9)

where $\dot{\ell}_1$ and $\dot{\ell}_2$ are the angular shortening velocities of the two muscles, k_1 and k_2 are the torsional stiffnesses of the two tendons (again, assuming constant moment arms), and $\dot{\theta}$ is the angular velocity of the leg.

In order to determine the stiffness of tendon, empirical data was used as an estimate for average linear spring stiffness. Assuming a moment arm of 5 cm (0.05 m) for the hip flexors and extensors, torsional spring stiffness might be estimated by the following:

$$F = k_s \ \Delta x = k_s \left(r_m \ \Delta \ell \right) \tag{10}$$

$$T = F r_m = k_s r_m^2 \Delta \ell = k_{eff} \Delta \ell$$
(11)

$$k_{eff} = k_s r_m^2 \tag{12}$$

where k_s is the linear spring stiffness of tendon (in N/m), r_m is the moment arm (assumed to be 5 cm), and k_{eff} is the effective torsional spring stiffness. Additionally, F is muscle force, T is muscle torsion, Δx is linear muscle displacement, and $\Delta \ell$ is angular muscle displacement.

The elastic properties of tendons play an important role in muscle function, but the level of tendon stiffness varies for different parts of the human body. The tendons connected to hip flexor and extensor muscles in the human body are relatively short and, as a result, offer high stiffness (Minetti & Alexander, 1997). In order to model infinitely-high spring stiffness, tendons are excluded from the hip muscle complex as shown in Figure 8. Using this model, the angular displacement (position) of the leg is directly related to the change in length of the hip muscles.



Figure 8: Leg-swing schematic with muscles only (no tendons).

Consequently, the shortening velocity of each muscle is directly related to the velocity of the swinging leg:

$$\dot{\ell}_1 = -\dot{\theta}$$

$$\dot{\ell}_2 = +\dot{\theta}$$
(13)

where $\dot{\ell}_1$ and $\dot{\ell}_2$ are the shortening velocities of the two muscles, and $\dot{\theta}$ is the angular velocity of the leg. Both cases – with and without tendon in series with muscle – are examined in this research study, but excluding tendons from the muscle complex (simulating infinitely-high spring stiffness) yields a leg-swing model that better fits experimental data (see

Appendix A). For this reason, the proposed cost functions are evaluated with the assumption that tendon stiffness is very high.

3.2 Constrained nonlinear optimization.

The two muscles can move and swing the leg in essentially an infinite number of ways. Given a specific amplitude and frequency, it is assumed that a person swings their leg in a manner that minimizes something (e.g. mechanical work, metabolic cost, etc.). So correspondingly, the *optimal* leg-swing motion for the model is always compared with experiment.

Dynamic optimization methods are used to determine the control parameters $(T_1(t), \dot{T}_1(t), ..., T_2(t), \dot{T}_2(t), ...)$ that simulate optimal leg-swing behavior (please see Srinivasan (2006 & 2011) for a detailed description of the numerical methods used). This is an "infinite-dimensional" optimization (optimal control problem), because infinitely-many numbers need to be specified to define a general function for torques $T_1(t)$ and $T_2(t)$ such that some objective function is minimized. The computer, however, is unable to solve an infinite-dimensional optimization problem. Hence, the model must be made into a finite-dimensional problem by constraining the control variables to be finite, well-defined functions (e.g. piece-wise function, Fourier series, Taylor series expansion, polynomial expansion, etc.). In this case $\ddot{T}_1(t)$ and $\ddot{T}_2(t)$ are specified as piece-wise linear functions of time. The corresponding torques $T_1(t)$ and $T_2(t)$, and torque rates $\dot{T}_1(t)$ and $\dot{T}_2(t)$ are then smooth, continuous functions of time.

Linear and nonlinear constraints are established to help bound the solution space. For instance, the leg-swing simulation is constrained to be cyclic such that starting and ending conditions – leg position, leg velocity, muscle torque, and muscle torque rates – are the same.

$$\begin{aligned} \theta(t_{start}) &= \theta(t_{end}) \\ \dot{\theta}(t_{start}) &= \dot{\theta}(t_{end}) \end{aligned}$$

$$T_1(t_{start}) &= T_1(t_{end}) \\ \dot{T}_1(t_{start}) &= \dot{T}_1(t_{end}) \\ \ddot{T}_1(t_{start}) &= \ddot{T}_1(t_{end}) \end{aligned}$$

$$T_2(t_{start}) &= T_2(t_{end}) \\ \dot{T}_2(t_{start}) &= \ddot{T}_2(t_{end}) \\ \ddot{T}_2(t_{start}) &= \ddot{T}_2(t_{end}) \end{aligned}$$

$$(14)$$

In addition, symmetry about mid-cycle is imposed such that the forward swing motion is the reverse of the backward swing motion. The position and velocity of the leg at mid-cycle is constrained to be equal and opposite to the position and velocity of the leg at the start (and end) of the leg-swing cycle.

$$\theta(t_{start}) = \theta(t_{end}) = -\theta(t_{mid \ cycle})$$

$$\dot{\theta}(t_{start}) = \dot{\theta}(t_{end}) = -\dot{\theta}(t_{mid \ cycle})$$
(15)

Further constraints are added to ensure that muscle torque T does not exceed a maximum value of 100 N-m, and to limit muscle shortening (or stretching) rate $\dot{\ell}$ to a maximum velocity of 8 rad/sec (Minetti & Alexander, 1997).

$$\left|T_{1}\right| and \left|T_{2}\right| < T_{\max} = 100 \ N \cdot m \tag{16}$$

$$\left|\dot{\ell}_{1}\right|$$
 and $\left|\dot{\ell}_{2}\right| < \dot{\ell}_{\max} = 8 \ rad/sec$ (17)

Note that the agonist and antagonist hip muscles tend to move the leg in opposite directions (see Figure 8). As previously noted, muscles can only generate a pulling (not pushing) force or torque. Therefore, the torques exerted by the opposing hip muscles are constrained such that $T_1 \ge 0$ and $T_2 \le 0$.

The last missing piece of the leg-swing optimization puzzle is the objective function. What scalar value should be minimized (or maximized) in the leg-swing optimization problem? A reasonable objective pertaining to human locomotion (as discussed in Section 1.1) is to minimize energy expenditure. Using this hypothesis, the goal of the optimization search might be to determine the leg-swing strategy which minimizes total cost. The objective value J (total cost of leg-swing) would then be the time-integral of cost rate:

$$t_{\max} = \frac{1}{f_{swing}} \tag{18}$$

$$J = \int_{0}^{t_{\text{max}}} f\left(T, \dot{T}, \ddot{T}, \dots, \ell, \dot{\ell}, \ddot{\ell}, \dots\right) dt$$
(19)

where f_{swing} is the swinging frequency of the leg, and t_{max} is the time period of one full legswing cycle. Equation (19) suggests that the total cost of leg-swing might be a function of muscle torque (and its derivatives) and muscle arch length (and its derivatives).

Figure 9 shows a flowchart describing the structure of the numerical optimization procedure. Appendix B provides a listing of the typical MATLAB code used to solve the highdimensional leg-swing optimization problem described here. Note that each different cost function required some modification of this MATLAB code, as also the addition of tendons (described in Appendix A).



Figure 9: Flowchart describing the structure of the numerical optimization procedure. The functions to be optimized (i.e. $T_1(t)$ and $T_2(t)$) are discretized to make the leg-swing problem finite-dimensional, and constraints are added to narrow the solution space. Nonlinear programming software (such as "fmincon" in MATLAB's in optimization toolbox) is used to determine the optimal variable values such that the objective function (in this case, cost of leg-swing) is minimized.

The following chapter presents various simple cost functions $f(T, \dot{T}, \ddot{T}, ..., \ell, \dot{\ell}, \ddot{\ell}, ...)$ for

use in Equation (19).

CHAPTER 4: POSSIBLE COST MODELS.

In this chapter, a variety of objective functions J are considered. The objective of the legswing optimization is to minimize total cost – which might be related to metabolic energy, mechanical work, or some other function of muscle torque, muscle length, and their derivatives. General expressions for the cost rate functions considered in this study are listed below:

$$f_{Mechanical Work} = \begin{bmatrix} T_1 \dot{\ell}_1 \end{bmatrix}^+ + \begin{bmatrix} T_2 \dot{\ell}_2 \end{bmatrix}^+$$

$$f_{Minetti-Alexander} = \alpha_1 T_o \dot{\ell}_{max} \cdot \Phi\left(\frac{\dot{\ell}_1}{\dot{\ell}_{max}}\right) + \alpha_2 T_o \dot{\ell}_{max} \cdot \Phi\left(\frac{\dot{\ell}_2}{\dot{\ell}_{max}}\right)$$

$$f_{Torque Rates} = a_1 \cdot f_{Minetti-Alexander} + a_2 \cdot \left(\left|\ddot{T}_1\right| + \left|\ddot{T}_2\right|\right) + a_3 \cdot \left(\left|\dot{T}_1\right| + \left|\dot{T}_2\right|\right)$$

$$f_{Torque} = \left|T_1\right|^{\alpha} + \left|T_2\right|^{\alpha}$$

$$f_{Weighted T-squared} = (1-\beta) \left|T_1\right|^2 + \beta \left|T_2\right|^2$$

In each case, the objective function J is the time-integral of cost rate function f over the duration of one full leg-swing cycle (see Equation (19), Section 3.2). Hence, the cost function $J_{Mechanical Work}$ quantifies the total positive mechanical work exerted by the hip actuator muscles, whereas $J_{Minetti-Alexander}$ estimates metabolic energy consumption. $J_{Torque Rates}$

is a cost function which (in addition to $f_{Minetti-Alexander}$) incorporates the rate at which muscle torque (or force) is applied. J_{Torque} is related to the magnitude of torque raised to some power of α . Lastly, $J_{Weighted T-squared}$ presents a cost model that distinguishes the muscle efficiencies of the hip flexors and extensors. Not all of these cost expressions are biophysically motivated. Nevertheless, it seems potentially useful to understand how well such (perhaps unphysical) cost models fit the cost data and what optimal strategies arise when they are optimized.

4.1 Positive Mechanical Work cost model.

The mechanical work cost model measures total positive work performed by the hip actuator muscles. Here, mechanical work refers to the energy required for a muscle to exert a torque T through an angular displacement ℓ . The rate of positive mechanical work is of the form:

$$f_{Mechanical Work} = \left[T_1 \dot{\ell}_1\right]^+ + \left[T_2 \dot{\ell}_2\right]^+$$
(20)

where T_1 and T_2 are torques generated by the agonist and antagonist hip muscles, and $\dot{\ell}_1$ and $\dot{\ell}_2$ are muscle angular shortening rates. If $P = T \dot{\ell}$ is the rate of mechanical work, then the positive part of work rate $[P]^+$ is defined such that: $[P]^+ = P$ when $P \ge 0$, and $[P]^+ = 0$ when P < 0. The negative part of work rate $[P]^-$ is defined as: $[P]^- = [-P]^+$. Recall that muscle can only exert a pulling (not pushing) force. Thus, positive work is performed when muscle is shortening ($\ell > 0$) and exerting a pulling torque (i.e. torque and angular displacement are in the same direction).

Note that the slope of $[P]^+$ changes abruptly at P = 0 (i.e. the first derivative is discontinuous). In order to smooth out this kink and thereby help the smooth optimization methods converge to a solution, the functional form of positive mechanical work rate is modified as follows:

$$\left[P\right]^{+} = \frac{P + \sqrt{P^2 + \varepsilon^2}}{2} \tag{21}$$

where constant \mathcal{E} is a small value (relative to the magnitude of P), and $|P| \approx \sqrt{P^2 + \varepsilon^2}$. Equation (21) is consistent with the definition of $[P]^+$ described previously: $[P]^+ \approx P$ when $P \ge 0$, and $[P]^+ \approx 0$ when P < 0

The total positive mechanical work of a full leg-swing cycle is then the time-integral of Equation (20):

$$J_{Mechanical Work} = \int_0^{t_{max}} \left[T_1 \dot{\ell}_1 \right]^+ + \left[T_2 \dot{\ell}_2 \right]^+ dt$$
(22)

Note that, for this cost model, the energetic expense of static effort $(\dot{\ell}_i = 0)$ is zero.

4.2 Minetti-Alexander cost model.

The Minetti-Alexander cost model is one of many models of muscle energy expenditure. The metabolic rate cost function developed by Minetti & Alexander (1997) is of the form:

$$f_{Minetti-Alexander} = \alpha_1 T_o \dot{\ell}_{\max} \cdot \Phi\left(\frac{\dot{\ell}_1}{\dot{\ell}_{\max}}\right) + \alpha_2 T_o \dot{\ell}_{\max} \cdot \Phi\left(\frac{\dot{\ell}_2}{\dot{\ell}_{\max}}\right)$$
(23)

where $f_{Minetti-Alexander}$ is rate of metabolic energy expenditure, α_i is the fraction of activated muscle fibers needed to exert muscle torque T_i ($\alpha_i = 1$ implying full activation), T_o is maximum isometric muscle torque, and $\dot{\ell}_i$ is the angular shortening velocity of muscle (Minetti & Alexander, 1997). Here, $\dot{\ell}_i$ is taken to be positive when the muscle is shortening and performing positive work, whereas $\dot{\ell}_i$ is negative when the muscle is stretching and performing negative work. The maximum shortening rate of the hip muscles, $\dot{\ell}_{max}$, is assumed to be 8 rad/sec for both flexors and extensors (Minetti & Alexander, 1997). Lastly, Φ is a function of relative muscle shortening rate $\dot{\ell}_i / \dot{\ell}_{max}$. Adapted from experimental data obtained from Ma & Zahalak (1991), Figure 10 shows function Φ plotted against the relative angular velocity ratio $\dot{\ell}_i / \dot{\ell}_{max}$.

The metabolic rate function Φ shown in Figure 10 illustrates that the metabolic cost of static effort (isometric contraction, muscle shortening rate $\dot{\ell}_i = 0$) has been taken into account. That is, an isometric muscle force or torque requires some amount of metabolic energy expenditure.

If muscle shortening rate $\dot{\ell}_i$ is known, the muscle torque T_{full} exerted by a fully-activated muscle ($\alpha_i = 1$) can be predicted by the force-velocity property of muscle. Figure 11 shows the ratio T_{full} / T_o plotted against relative angular velocity $\dot{\ell}_i / \dot{\ell}_{max}$. Note that Figure 11 resembles (and is essentially the same as) the force-velocity curve shown in Figure 6 (see Section 2.1), except that muscle force F is replaced by muscle torque T.

The expression for metabolic cost rate $f_{Minetti-Alexander}$ presented in Equation (23) may be rearranged in such a manner that eliminates the muscle activation term α :

$$\alpha_1 = \frac{T_1}{T_{full}} and \ \alpha_2 = \frac{T_2}{T_{full}}$$
(24)

$$f_{Minetti-Alexander} = T_1 \frac{T_o}{T_{full}} \dot{\ell}_{\max} \cdot \Phi\left(\frac{\dot{\ell}_1}{\dot{\ell}_{\max}}\right) + T_2 \frac{T_o}{T_{full}} \dot{\ell}_{\max} \cdot \Phi\left(\frac{\dot{\ell}_2}{\dot{\ell}_{\max}}\right)$$
(25)

Note that the relative muscle torque ratio T_o/T_{full} is the reciprocal of the torque function expressed by Figure 11. So, by fitting algebraic functions to Figure 10 and Figure 11, Equation (25) becomes an expression for computing metabolic power at any instance, given only four inputs: muscle torques T_1 and T_2 , and muscle shortening rates $\dot{\ell}_1$ and $\dot{\ell}_2$.



Figure 10: Metabolic rate function Φ versus relative angular velocity. Re-drawn from Minetti & Alexander (1997).



Figure 11: Relative muscle torque versus relative angular velocity. Re-drawn from Minetti & Alexander (1997).

The specific functional form used by Minetti & Alexander (1997) does not satisfy certain desirable convexity and smoothness properties (see Srinivasan (2011)). An approximation to the Minetti-Alexander cost rate function (albeit a cumbersome one) that has these desirable properties is presented below. The product of relative torque function T_o/T_{full} multiplied by metabolic rate function $\Phi(\dot{\ell}_i/\dot{\ell}_{max})$ might be replaced by the following algebraic expression:

$$c_1 f_1 + \frac{c_2 f_2}{f_3} \tag{26}$$

where c_1 , c_2 , f_1 , f_2 , and f_3 are defined as follows:

$$c_{1} = 0.075, c_{2} = 2$$

$$f_{1} = \frac{-q + \sqrt{q^{2} + \varepsilon_{1}^{2}}}{2}$$

$$f_{2} = \frac{q + \sqrt{q^{2} + \varepsilon_{1}^{2}}}{2}$$

$$f_{3} = \frac{s + \sqrt{s_{2} + \varepsilon_{2}^{2}}}{2}$$
(27)

with $q = \dot{\ell}_i / \dot{\ell}_{max} + 0.01$, $s = 1 - \dot{\ell}_i / \dot{\ell}_{max}$, $\varepsilon_1 = 0.04$, and $\varepsilon_2 = 0.1$. In essence, the expression $c_1 f_1 + (c_2 f_2) / f_3$ is an approximation to Figure 10 divided by Figure 11. Substituting this expression into Equation (25), the new metabolic cost rate function becomes:

$$f_{Minetti-Alexander} = T_1 \dot{\ell}_{max} \cdot \left(c_1 f_1 + \frac{c_2 f_2}{f_3} \right)_{\dot{\ell}_1} + T_2 \dot{\ell}_{max} \cdot \left(c_1 f_1 + \frac{c_2 f_2}{f_3} \right)_{\dot{\ell}_2}$$
(28)

Curve fitting to the relative torque function T_o/T_{full} and metabolic rate function $\Phi(\dot{\ell}_i/\dot{\ell}_{max})$ may be slightly adjusted by changing the constants c_1, c_2, ε_1 , and ε_2 . Figure 12 plots metabolic power per unit torque $(f_{Minetti-Alexander}/T_i)$ against relative muscle shortening rate. Figure 13 plots muscle efficiency ("output" mechanical work rate divided by "input" metabolic power, $T_i \dot{\ell}_i/f_{Minetti-Alexander}$) against relative muscle shortening rate.

Figure 12 shows that metabolic rate (per unit torque) increases as the value of c_2 increases. Figure 13 illustrates how muscle efficiency (ratio of mechanical work rate to metabolic rate) decreases as the value of c_2 increases. For $c_2 = 2$ the peak efficiency is 0.4, whereas for $c_2 = 4$ the peak efficiency is only 0.2. Thus, c_2 is essentially inversely proportional to efficiency.

After selecting the appropriate positive scalar values for constants c_1, c_2, ε_1 , and ε_2 , the total metabolic cost of one full leg-swing cycle may be estimated by computing the time-integral of Equation (28):

$$J_{Minetti-Alexander} = \int_{0}^{t_{\text{max}}} T_{1} \dot{\ell}_{\text{max}} \cdot \left(c_{1} f_{1} + \frac{c_{2} f_{2}}{f_{3}} \right)_{\dot{\ell}_{1}} + T_{2} \dot{\ell}_{\text{max}} \cdot \left(c_{1} f_{1} + \frac{c_{2} f_{2}}{f_{3}} \right)_{\dot{\ell}_{2}} dt \qquad (29)$$



Figure 12: Metabolic rate (per unit torque) versus relative muscle shortening rate, using different values of c_2 in the metabolic cost rate function $f_{\text{Minneti-Alexander}}$.



Figure 13: Muscle efficiency versus relative muscle shortening rate, using different values of c_2 in the metabolic cost rate function $f_{\text{Minneti-Alexander}}$.

4.3 Torque Rate cost model.

Investigations by Doke et al. (2005 & 2007) suggest that the metabolic cost of isolated legswing might be related to muscle force (or torque) and the duration which the force is applied. For speeds higher than the natural frequency of the swinging leg, the force/time hypothesis (Doke, Donelan, & Kuo, 2005) predicts that metabolic cost is proportional to muscle force (or torque) and inversely proportional to force duration. Using the force/time hypothesis as the basis for a new cost model, an appropriate cost function might include torque and torque rate:

$$f_{Torque Rate} = a_1 \cdot f_{Minetti-Alexander} + a_2 \cdot \left(\left| \ddot{T}_1 \right| + \left| \ddot{T}_2 \right| \right) + a_3 \cdot \left(\left| \dot{T}_1 \right| + \left| \dot{T}_2 \right| \right)$$
(30)

where $f_{Minetti-Alexander}$ is the cost function described in Section 4.2, \dot{T} and \ddot{T} are derivatives of muscle torque, and a_1 , a_2 , and a_3 are non-negative constant coefficients that weigh the contributions by each of the cost terms. The Torque Rate cost functions examined in this research investigation use the following coefficient values:

Experiment	a_1	<i>a</i> ₂	<i>a</i> ₃
А	1	1	1
В	1	1	0
С	1	0	1
D	0	1	0
Е	0	0	1

Table 2: Coefficient values for Torque Rate cost function.

Table 2 proposes five different cost rate functions, each having cost terms which are either one or zero (i.e. "on" or "off"). The total cost of one full leg-swing cycle is then the time-integral of Equation (30):

$$J_{Torque Rate} = \int_{0}^{t_{max}} a_1 \cdot f_{Minetti-Alexander} + a_2 \cdot \left(\left|\ddot{T}_1\right| + \left|\ddot{T}_2\right|\right) + a_3 \cdot \left(\left|\dot{T}_1\right| + \left|\dot{T}_2\right|\right) dt$$
(31)

4.4 Torque cost model.

Another commonly used model for metabolic rate is torque magnitude raised to some power:

$$f_{Torque} = \left|T_1\right|^{\alpha} + \left|T_2\right|^{\alpha} \tag{32}$$

where $\alpha \ge 1$. The set of α values examined for this cost function are:

$$\alpha = \{1, 2, 3, 4, 5\} \tag{33}$$

Here, the set of exponent values are all greater than or equal to one. For exponent values less than one ($\alpha < 1$), $|T|^{\alpha}$ yields a concave function of |T| rather than a convex function. Such concave functions are ignored as they tend to give rise to optimal solutions with potentially wildly fluctuating (and therefore unphysical) torque functions. In any case, the total cost of one full leg-swing cycle is the time-integral of Equation (32):

$$J_{Torque} = \int_{0}^{t_{max}} |T_{1}|^{\alpha} + |T_{2}|^{\alpha} dt$$
(34)

4.5 Weighted T-squared cost model.

A variation of the $|T|^{\alpha}$ family of cost functions might include different weights for the two muscles. In this cost model, the exponent is taken as $\alpha = 2$ because $|T|^2$ (or simply T^2) is commonly used by other researchers as a substitute for metabolic cost. Thus, the general expression for the cost rate function described above would be:

$$f_{Weighted T-squared} = (1 - \beta) \left| T_1 \right|^2 + \beta \left| T_2 \right|^2$$
(35)

where $0 \le \beta \le 1$. Note that summing the two coefficients β and $1-\beta$ adds up to unity. The set of β values examined for this cost function are:

$$\beta = \{0.1, 0.2, 0.3, 0.4, 0.5\}$$
(36)

The size difference between coefficients β and $1-\beta$ denote the relative contribution by each muscle to the overall cost of leg-swing. For example, $\beta = 0.1$ and $1-\beta = 0.9$ implies that the total cost contribution by the muscle exerting T_1 is significantly larger than the cost contribution by the complementary muscle exerting T_2 . Hence, the muscle exerting T_1 is considerably less cost efficient than the muscle exerting T_2 for the same amount of torque. Finally, the total cost of one full leg-swing cycle is the time-integral of Equation (35):

$$J_{Weighted \ T-squared} = \int_{0}^{t_{max}} (1-\beta) |T_1|^2 + \beta |T_2|^2 \ dt \tag{37}$$

CHAPTER 5: OPTIMAL LEG MOTIONS, OPTIMAL COSTS, AND COMPARISON TO DOKE ET AL. (2005).

The optimization results are compared with empirical data to check the validity of the proposed cost models and to explore ways to better fit the model results to the work and energy-related costs measured experimentally by Doke et al (2005 & 2007). In this chapter, the amplitude of leg-swing is held constant while frequency of leg-swing is varied. In the next chapter, the amplitude and frequency of leg-swing are both varied such that the rate of mechanical work is (more or less) held constant. For all cases, tendon springs are excluded from the hip muscle complex to imply infinitely-high tendon stiffness (see Section 3.1).

5.1 Positive Mechanical Work cost model.

Recall the "Positive Mechanical Work" cost function (Equation (22), Section 4.1):

$$J_{Mechanical Work} = \int_0^{t_{max}} \left[T_1 \cdot \dot{\ell}_1 \right]^+ + \left[T_2 \cdot \dot{\ell}_2 \right]^+ dt$$

Minimizing this cost function (for a 1.0 Hz leg-swing frequency) results in leg-swing motion that is shown in Figure 14, which plots leg position and velocity as functions of time. The plot shows symmetry about mid-swing and shows the leg achieving a swinging amplitude of 45° peak-to-peak (from -22.5° to +22.5°). Figure 14 illustrates one full period of leg-swing which repeats in a continuous, cyclical fashion.



Figure 14: Leg position and velocity versus time for "Positive Mechanical Work" cost model (1.0 Hz leg-swing frequency).

The torque-histories of the two hip muscles (for a 1.0 Hz leg-swing frequency) are shown in Figure 15. The plot shows periodic behavior and symmetry about mid-swing. The leg seems to be driven in a "bang-coast-bang" in which the hip muscles quickly apply a large torque (or force) on the leg, and then relax for an extensive amount of time. Hence, "bang" refers to the torque impulses, and "coast" refers to the interludes where the hip muscles are not engaged. Figure 15 shows that the "coast" duration is relatively long in comparison with the "bang" duration. One possible way to augment the bang-coast-bang behavior (longer "coasts" and sharper, more rapid "bangs") in the model would be to remove bounds or

constraints on torque rate, which limit how sharply muscle torque can rise and fall. A formal mathematical proof of why a bang-coast-bang-like strategy is optimal is provided in Srinivasan (2006) (see also appendix of Srinivasan (2011)). Such a strategy is also consistent with human data – where it is observed that the swinging leg is powered by muscle activations (measured via EMG) at the beginning and the end of the swing, with very little muscle activation in between (see Doke et al. (2005) for a plot of the EMG data).



Figure 15: Muscle torque versus time for "Positive Mechanical Work" cost model (1.0 Hz leg-swing frequency).

Since the torsional stiffnesses of the tendon "springs" are assumed to be infinitely-high, the muscle shortening velocity $\dot{\ell}$ is directly related to the leg-swing velocity $\dot{\theta}$. That is, $\dot{\ell}_1 = -\dot{\theta}$ and $\dot{\ell}_2 = +\dot{\theta}$ as previously shown in Equation (13). Consequently, the rate of mechanical work exerted by the muscles is equivalent to the rate of work performed on the swinging leg:

$$T_1 \cdot \dot{\ell}_1 = T_1 \cdot \dot{\theta}$$

$$(-T_2) \cdot \dot{\ell}_2 = T_2 \cdot \dot{\theta}$$
(38)

Rate of positive mechanical work (i.e. positive mechanical power) versus time is plotted in Figure 16. Again, periodic behavior is illustrated.



Figure 16: Rate of positive mechanical work versus time for "Positive Mechanical Work" cost model (1.0 Hz leg-swing frequency).

Figure 17 plots average rate of positive mechanical work against leg-swing frequency, and compares the optimization results (from model) with existing empirical data obtained by Doke et al. (2005). For both model and experiment, positive mechanical work rate is minimal at an approximate leg-swing frequency of 0.6 Hz – corresponding to the theoretical natural frequency of a gravity pendulum. Although the two curves are shaped similarly, there is a visible vertical offset between the optimization results and the empirical data.

Since the optimal leg-swing strategy performs zero work at the natural frequency, it may be impossible for the model results to match the experimental results (which appear to have non-zero work at the natural frequency). Thus, an offset difference between model and experiment at the natural frequency is to be expected. There are a couple of possible explanations for this disparity. First, the human leg is *not* completely isolated from the rest of the body – as was assumed in the leg-swing model. Rather, the leg is connected to the human subject's body which might move a little during leg-swing experiments. Second, the procedure of inferring hip torque from the experimentally measured ground reaction forces might be a reason why average rate of work is non-zero at the natural frequency of leg-swing.



Figure 17: Average rate of positive mechanical work (per unit weight) versus leg-swing frequency for "Positive Mechanical Work" cost model, compared with empirical data (Doke, Donelan, & Kuo, 2005).

In Figure 17, the theoretical and experimental results appear to be in reasonable agreement, but the cost model might be improved by accounting for both positive *and* negative work performed by the muscles. Such a model might weigh the price of positive work differently from the price of negative work to distinguish their individual contributions to the total cost of leg-swing.



Figure 18: Average rate of cost (per unit weight) versus leg-swing frequency for "Positive Mechanical Work" cost model, compared with empirical data (Doke, Donelan, & Kuo, 2005).

Although the Positive Mechanical Work model does not directly measure metabolic energy consumption, metabolic rate might be estimated by applying some constant "inverse efficiency" or "pre-multiplier" term to the cost rate function $f_{Mechanical Work}$. Figure 18 plots average rate of cost (i.e. average rate of positive mechanical work) against leg-swing frequency for pre-multiplier values ranging from 1 to 32. Using a pre-multiplier value of 16,

the cost model roughly matches the plot of net metabolic rate (empirical data from Doke et al., 2005) particularly for frequencies higher than the natural frequency of leg-swing. Muscle efficiency (ratio of mechanical work rate to metabolic rate) may be computed by taking the inverse of the pre-multiplier value as shown in Equation (39).

Thus, the muscle efficiency (a constant) corresponding to the pre-multiplier value of 16 is 0.0625 or 6.25%, as shown below:

Muscle Efficiency =
$$(premultiplier)^{-1} = \frac{1}{16} = 0.0625$$
 (39)

Perhaps a weakness of using an "inverse efficiency" or "pre-multiplier" term is that muscle efficiency (i.e. efficiency of transforming metabolic energy into useful, mechanical work) is assumed to be constant. However, this is not always a valid assumption (see Section 2.2) and therefore a different cost model might be needed to more accurately estimate metabolic energy expenditure. In the following section, the Minetti-Alexander cost model of muscle energy expenditure is evaluated.

5.2 Minetti-Alexander cost model.

Recall the "Minetti-Alexander" cost function (Equation (29), Section 4.2):

$$J_{Minetti-Alexander} = \int_{0}^{t_{\text{max}}} T_{1} \dot{\ell}_{\text{max}} \cdot \left(c_{1} f_{1} + \frac{c_{2} f_{2}}{f_{3}} \right) \Big|_{\dot{\ell}_{1}} + T_{2} \dot{\ell}_{\text{max}} \cdot \left(c_{1} f_{1} + \frac{c_{2} f_{2}}{f_{3}} \right) \Big|_{\dot{\ell}_{2}} dt$$

Minimizing this cost function (for a 1.0 Hz leg-swing frequency) results in leg-swing motion that is shown in Figure 19, which plots leg position and velocity as functions of time for various c_2 values ranging from 2 to 11. The plots are nearly identical to the position and velocity graphs of the Positive Mechanical Work cost model (see Figure 14, Section 5.1).

Figure 19 also shows that the optimal motion appear to be essentially independent of the value of c_2 (see Srinivasan (2011) for proof of this observation in the context of a simplified problem). The torque-histories of both muscles (for a 1.0 Hz leg-swing frequency) are shown in Figure 20. The plots have the expected symmetry and periodic behavior, and shows how the leg is driven in a "bang-coast-bang" manner when optimal.



Figure 19: Leg position and velocity versus time for "Minetti-Alexander" cost model (1.0 Hz leg-swing frequency).



Figure 20: Muscle torques T_1 and T_2 versus time for "Minetti-Alexander" cost model (1.0 Hz leg-swing frequency).

Figure 21 plots average rate of positive mechanical work against leg-swing frequency, and compares the optimization results with existing empirical data by Doke et al. (2005). Aside from minor fluctuations due to optimization convergence issues, modifying the value of c_2 has little effect on the average rate of positive mechanical work (obviously because c_2 has no effect on the optimal motion). Figure 21 shows that the theoretical and experimental curves are shaped similarly, yet there is a visible offset difference between the optimization results and the empirical data. As discussed in Section 5.1, adjustments to the model can change the shape of the work rate curve, but the offset difference at the leg-swing natural frequency is not likely to change.



Figure 21: Average rate of positive mechanical work (per unit weight) versus leg-swing frequency for "Minetti-Alexander" cost model, compared with experimental data (Doke, Donelan, & Kuo, 2005).

Figure 22 plots average rate of cost (i.e. average rate of metabolic energy expenditure) against leg-swing frequency, and compares the optimization results with existing empirical data (net metabolic rate) by Doke et al. (2005). The plot shows how the value of c_2 affects the shape of the cost rate curves. Theoretical and experimental results seem to be in good agreement when $c_2 = 11$. The efficiency plot shown in Figure 13 (see Section 4.2) shows that $c_2 = 11$ corresponds to muscle efficiency that is rather low (less than 8%).

Although modifications to the cost rate function might change the shape of the cost rate curve, there still exists an offset difference (between model and experiment) near the natural frequency of legs-wing. There is a possibility that the human subjects expended significant amounts of energy to stabilize their bodies (standing on one leg, holding the armrests, activating core muscles, etc.) during the leg-swing experiment. Therefore, the costs of stabilization, balance, and support were likely adjoined to the cost of isolated leg-swing when measured in the experiment. There is also a possibility that the leg-swing model underestimated the cost of muscle energy expenditure – thus, the disparity between model and experiment might be reduced if the cost of muscle activity were more heavily penalized in the model.



Figure 22: Average rate of cost (per unit weight) versus leg-swing frequency for "Minetti-Alexander" cost model, compared with experimental data (Doke, Donelan, & Kuo, 2005).

Another possible way to improve the leg-swing model might be to take into account the rate at which torque is applied to the leg. As suggested by the experimental findings by Doke et al. (2005 & 2007), the amount of force (or torque) and duration for which muscle force (or torque) is applied might contribute to overall costs of leg-swing. For swinging speeds higher than the leg-swing natural frequency, it was suggested that higher forces applied for short periods of time might drastically increase metabolic rate. In the following section, different variations of torque rate-related costs (in combination with the Minetti-Alexander cost function) are evaluated.

5.3 Torque Rate cost model.

Recall the "Torque Rate" cost function (Equation (31), Section 4.3):

$$J_{Torque Rate} = \int_0^{t_{max}} a_1 \left(f_{Minetti-Alexander} \right) + a_2 \left(\left| \ddot{T}_1 \right| + \left| \ddot{T}_2 \right| \right) + a_3 \left(\left| \dot{T}_1 \right| + \left| \dot{T}_2 \right| \right) dt$$

Minimizing this cost function (for a 1.0 Hz leg-swing frequency, and for various values of a_1 , a_2 , and a_3) results in leg-swing motion that is nearly identical to the position and velocity graphs of the Positive Mechanical Work and the Minetti-Alexander cost models. Figure 23 plots leg position and velocity versus time for the Torque Rate cost functions described in Section 4.3. Note that there exist only minor variations between the five different cost functions.



Figure 23: Leg position, velocity versus time for "Torque Rate" cost model (1.0 Hz leg-swing frequency).

Figure 24 plots muscle torques T_1 and T_2 as functions of time (for a 1.0 Hz leg-swing frequency) whereas Figure 25 plots total torque T_{total} (summation of T_1 and T_2) against time. The plots show qualities of symmetry and cyclic-behavior, and appear to be wave-like. For two of the cost functions, torque flattens out to a constant value such that T (first derivative of torque) is minimized (nearly zero). Note that T_{total} as a function of time would likely be a square wave if torque rate (T, T, etc.) were unbounded and permitted to be as large as infinity. The other three cost functions show triangle-like waveforms where torque steadily rises then steadily falls in a repeating manner. These three cost functions each contain the term T (second derivative of torque), therefore both torque and torque rate change gradually to minimize the overall cost of leg-swing.



Figure 24: Muscle torques T_1 and T_2 versus time for "Torque Rate" cost model (1.0 Hz leg-swing frequency).


Figure 25: Total muscle torque T_{total} versus time for "Torque Rate" cost model (1.0 Hz leg-swing frequency).

Figure 26 plots average rate of positive mechanical work against leg-swing frequency for each of the Torque Rate cost functions. Perhaps the model which best fits the empirical data (Doke, Donelan, & Kuo, 2005) has the cost function that sums $f_{Minetti-Alexander}$ and \dot{T} . In general, the other cost models predict average rate of mechanical work to be higher than what was measured experimentally.

In Figure 26, the cost function modeled as \dot{T} (only) appears to be in large disagreement with the experimental data. However, this does not indicate that the computer model is mistaken or incorrect. Since cost depends only on \dot{T}_1 and \dot{T}_2 (first derivative of torque) and the optimization algorithm seeks to minimize this objective value (making T_1 and T_2 almost constant), muscle torques T_1 and T_2 can be selected in an infinite number of ways (the solution is non-unique) and achieve the same minimal cost of leg-swing. The size or magnitude of the individual muscle torques is of little concern and, consequently, rate of mechanical work appears to fluctuate wildly when plotted against leg-swing frequency.



Figure 26: Average rate of positive mechanical work (per unit weight) versus leg-swing frequency for "Torque Rate" cost model, compared with experimental data (Doke, Donelan, & Kuo, 2005).

Figure 27 plots average rate of cost against leg-swing frequency, and illustrates how the cost function modeled as \dot{T} (only) offers the best match to the experimental, net metabolic rate curve. This finding might be in agreement with the "force/time hypothesis" (Doke, Donelan, & Kuo, 2005) which suggest that net metabolic rate during leg-swing is proportional to muscle force (or torque), and is inversely proportional to the duration of the applied force (or torque).



Figure 27: Average rate of cost (per unit weight) versus leg-swing frequency for "Torque Rate" cost model, compared with experimental data (Doke, Donelan, & Kuo, 2005).



Figure 28: Scaled version of the cost rate curve for "Torque Rate" cost model, compared with experimental data (Doke, Donelan, & Kuo, 2005).

Since average rate of cost varies in magnitude and the cost function coefficients a_1 , a_2 , and a_3 can be chosen somewhat arbitrarily (see Section 4.3), perhaps another useful comparison between model and experiment would be to evaluate the *shapes* of the cost curves. Figure 28 plots a modified version of the cost rate curve where the theoretical results are scaled such that the peak rate of cost (from the model) matches the peak metabolic rate (measured experimentally). Using this method of comparison, all five Torque Rate cost functions seem viable and are in good agreement with the experimental data obtained by Doke et al. (2005).

5.4 Torque cost model.

Recall the "Torque" cost function (Equation (34), Section 4.4):

$$J_{Torque} = \int_0^{t_{max}} \left| T_1 \right|^{\alpha} + \left| T_2 \right|^{\alpha} dt$$

Minimizing this cost function (for a 1.0 Hz leg-swing frequency) results in leg-swing motion that is shown in Figure 29, which plots leg position and velocity as functions of time. The plot is nearly identical to the position and velocity graphs of the Positive Mechanical Work, Minetti-Alexander, and Torque Rate cost models. Using values of α ranging from 1 to 5, Figure 29 plots leg position and velocity versus time for each of the different Torque cost functions. The figure shows that there exist only minor differences between the various cost models. The maximum swinging velocity increases as α increases from 1 to 5. For $\alpha = 1$, the velocity profile seems to flatten out, signifying a more steady and uniform leg-swing motion than for cost models with higher values of α .



Figure 29: Leg position, velocity versus time for "Torque" cost model (1.0 Hz leg-swing frequency).

For each of the Torque cost functions, Figure 30 plots muscle torques T_1 and T_2 against time and Figure 31 plots total muscle torque T_{total} (= $T_1 + T_2$) as a function of time. The plots exhibit the expected symmetry and periodic behavior. For $\alpha = 1$, the leg seems to be driven in a bang-coast-bang manner with sharp rises in torque followed by long timeintervals where the muscles are not engaged. For $\alpha = 2$, plotting T_{total} as a function of time shows that muscle torque tends to rise and fall in a (roughly) triangle-wave fashion. In contrast, for $\alpha = 6$, plotting T_{total} as a function of time shows that muscle torque tends to rise and fall in a (roughly) square-wave manner. Peak muscle torque dictates the total cost of leg-swing because generating a large torque comes at a great penalty when α is very large. Consequently, for $\alpha \gg 1$, it is expected that T_{total} plotted against time would take the form of a square-wave so that the maximum torque value is diminished.



Figure 30: Muscle torques T_1 and T_2 versus time for "Torque" cost model (1.0 Hz leg-swing frequency).



Figure 31: Total muscle torque T_{total} versus time for "Torque" cost model (1.0 Hz leg-swing frequency).

Figure 32 plots average rate of positive mechanical work versus leg-swing frequency for each of the Torque cost rate functions. For the cost function $|T_1| + |T_2|$ (i.e. $\alpha = 1$), the plot is close to the minimum possible mechanical work (see Figure 17). But when the objective function is very different from mechanical work (e.g. rate of cost is defined as $|T_1|^4 + |T_2|^4$ or $|T_1|^5 + |T_2|^5$) the mechanical work for the optimal motions can be much larger than the minimum possible mechanical work. Figure 32 shows that the cost function with $\alpha = 3$ offers perhaps the best fit to the experimental data (from Doke et al., 2005). The theoretical and experimental results are in particularly good agreement for swinging frequencies higher than the natural frequency of leg-swing. Aside from the (more or less) constant offset difference, the cost function with $\alpha = 2$ seems to mimic the shape and curvature of the experimental plot.



Figure 32: Average rate of positive mechanical work (per unit weight) versus leg-swing frequency for "Torque" cost model, compared with experimental data (Doke, Donelan, & Kuo, 2005).



Figure 33: Scaled and shifted cost rate curve for "Torque" cost model, compared with experimental data (Doke, Donelan, & Kuo, 2005).

Since the computed cost of leg-swing drastically varies in magnitude between the different Torque cost functions, perhaps it would instead be useful to compare the *shapes* of the cost curves. In Figure 33, theoretical costs are scaled and shifted such that the computed rate of cost matches metabolic rate (measured experimentally) for two of the examined leg-swing frequencies: 1.0833 Hz and 0.7500 Hz. Because $|T|^{\alpha}$ is not directly motivated by biological measurements, it is acceptable to multiply this rate of cost by any scalar value. In addition, it is acceptable to shift the theoretical cost curves up or down because (as discussed in Sections 5.1 and 0) there are a number of possible reasons to explain the vertical offset between model and experiment (human subjects might expend substantial amounts of energy to stabilize their bodies, the swinging leg is *not* completely isolated from the rest of the body, etc.). Using this scaling and shifting approach, Figure 33 shows that the cost curves for $\alpha = 1$ and $\alpha = 2$ best fit the experimental data obtained by Doke et al. (2005).

5.5 Weighted T-squared cost model.

Recall the "Weighted T-squared" cost function (Equation (37), Section 4.5):

$$J_{Weighted T-squared} = \int_0^{t_{max}} (1-\beta) |T_1|^2 + \beta |T_2|^2 dt$$

As explained in Section 4.5, the coefficients β and $1-\beta$ denote the weighted contribution by each muscle to the overall cost of leg-swing. The Weighted T-squared cost function described here is nearly identical to the Torque cost function with $\alpha = 2$, which is commonly used by other researchers as a proxy for metabolic cost. Figure 32 and Figure 33 shows that model and experiment are in strong agreement, giving confidence that the square of muscle torque is a reasonable proxy for the cost of leg-swing.

Unlike previously explored cost models, the aim of the Weighted T-squared cost model is *not* to match theoretical findings with experimental data. Rather, the goal is to see how torque actuation is affected when the cost of leg-swing is weighted differently for each of the two muscles. If the hip flexors and extensors are not the same size or shape, for instance, then the muscles might not equally contribute to total metabolic cost. In this sense, this section might be treated as a digression, and the reader may skip it without loss of continuity.

Minimizing the Weighted T-squared cost function (for a 1.0 Hz leg-swing frequency) results in leg-swing motion that is shown in Figure 34, which shows leg position and velocity versus time for coefficient values ranging from $\beta = 0.1$ to $\beta = 0.5$. The plot shows that there exist only minor variations between the optimal motions for different cost functions.



Figure 34: Leg position, velocity versus time for "Weighted T-squared" cost model (1.0 Hz leg-swing frequency).

Figure 35 plots muscle torques T_1 and T_2 as functions of time, while Figure 36 plots total muscle torque T_{total} (= T_1 + T_2) against time. For β =0.5 (the two hip muscles *equally* contribute to the total cost of leg-swing) the plot appears to be cyclic and symmetrical, and may be approximated as a triangle-like waveform. This is identical to the results found in Section 5.4, Figure 31. Conversely, for β =0.1 (one muscle is more energetically expensive than the other muscle) the peak torque generated by the cost-heavy muscle is significantly smaller than the peak torque generated by the low-cost muscle. Lastly, the case where the total cost of leg-swing depends only on one of the two muscles (β = 0) is a degenerate special case. True flatness of the cost landscape (i.e. non-uniqueness) in some directions would be expected and therefore the optimization search might find multiple optima and result in bad convergence.



Figure 35: Muscle torques T_1 and T_2 versus time for "Weighted T-squared" cost model (1.0 Hz leg-swing frequency).



Figure 36: Total muscle torque T_{total} versus time for "Weighted T-squared" cost model (1.0 Hz leg-swing frequency).

5.6 Summary of results (part 1 of 2).

In this chapter, optimal leg-swing strategies were examined and evaluated against empirical data (Doke, Donelan, & Kuo, 2005) to check the validity of the proposed cost models and to seek ways for improvement. The following list identifies some of the cost models that best fit the experimental data:

- "Positive Mechanical Work" cost model with pre-multiplier =16
 (i.e. constant muscle efficiency =1/16=.0625).
- "Minetti-Alexander" cost model with $c_2 = 11$.
- "Torque Rate" cost model with $a_1 = 0$, $a_2 = 0$, and $a_3 = 1$
 - (i.e. $J_{Torque Rate} = \int_0^{t_{max}} |\dot{T}_1| + |\dot{T}_2| dt$), and with some extra scaling.
- "Torque" cost model with $\alpha = 1$ and $\alpha = 2$, and with some extra shifting and scaling.

The proposed cost functions identified in the list above are re-examined in the next chapter, using a different set of leg-swing amplitude and frequency conditions. Findings from this new optimization problem are evaluated against empirical data from Doke & Kuo (2007) to determine whether or not these cost models are applicable to an entirely different leg-swing experiment.

CHAPTER 6: OPTIMAL COSTS AND COMPARISON TO DOKE & KUO (2007).

A good leg-swing model should be applicable to any leg-swing experiment. Several of the best cost models identified in the previous chapter (see Section 5.6) are re-examined in the present chapter to assess their accuracy, using a new set of leg-swing conditions.

6.1 A different leg-swing experiment.

Last chapter, the amplitude of leg-swing was held constant while frequency of leg-swing was varied. In this chapter, the amplitude and frequency of leg-swing are both varied such that the rate of mechanical work is (more or less) held constant. For the leg-swing experiments conducted by Doke & Kuo (2007), subjects were asked to swing their legs at increasing frequency, and with decreasing amplitude as shown in Figure 37. The plot illustrates how swinging amplitude varies as a function of frequency.

Again, rate of mechanical work and (metabolic) cost are examined in contrast to empirical information obtained by Doke & Kuo (2007). Using the new set of leg-swing conditions (decreasing amplitude with increasing frequency), the following list of cost functions are evaluated:

- "Positive Mechanical Work" cost model with pre-multiplier =16.
- "Minetti-Alexander" cost model with $c_2 = 11$.
- "Torque Rate" cost model with $a_1 = 0$, $a_2 = 0$, and $a_3 = 1$.
- "Torque" cost model with $\alpha = 1$ and $\alpha = 2$.



Figure 37: Swinging amplitude versus swinging frequency. Plot adapted from Doke & Kuo (2007).

6.2 Rate of positive mechanical work.

For the cost functions listed in Section 6.1, the optimization search determines the leg-swing strategy which minimizes total cost. Figure 38 through Figure 42 plot average rate of mechanical work against swinging frequency for each of the proposed cost functions. All of the plots illustrate that rate of mechanical work stays (more or less) constant and appear to be in reasonable agreement with the experimental measurements, despite the visible offset

difference. This disparity between model and experiment is to be expected for a couple of possible reasons. First, (as explained in Section 5.1) the human leg is *not* completely isolated from the rest of the body – as was assumed in the leg-swing model. Instead, the leg is connected to the human subject's body which might move a little during leg-swing experiments. Second, the procedure of inferring hip torque from the experimentally measured ground reaction forces might be a possible source of error.

The only trace of inconsistency is seen in Figure 40, where the plot of mechanical work rate seems to deviate from a constant value. However, this is the result of flatness in the cost landscape due to the fact that cost (for this particular "Torque Rate" cost model) only depends on the rate at which torque is applied, \vec{T} (see Section 5.3).



Figure 38: Average rate of positive mechanical work (per unit weight) versus legswing frequency for "Positive Mechanical Work" cost model with pre-mult = 16, compared with experimental data (Doke & Kuo, 2007). Rate of mechanical work is roughly constant for both model and experiment.



Figure 39: Average rate of positive mechanical work (per unit weight) versus legswing frequency for "Minetti-Alexander" cost model with $c_2 = 11$, compared with experimental data (Doke & Kuo, 2007). Rate of mechanical work is roughly constant for both model and experiment.



Figure 40: Average rate of positive mechanical work (per unit weight) versus legswing frequency for "Torque Rate" cost model with $a_1 = 0$, $a_2 = 0$, and $a_3 = 1$; compared with experimental data (Doke & Kuo, 2007). The computed mechanical work curve deviates from a constant value, due to flatness in the cost landscape.



Figure 41: Average rate of positive mechanical work (per unit weight) versus legswing frequency for "Torque" cost model with $\alpha = 1$, compared with experimental data (Doke & Kuo, 2007). Rate of mechanical work is roughly constant for both model and experiment.



Figure 42: Average rate of positive mechanical work (per unit weight) versus legswing frequency for "Torque" cost model with $\alpha = 2$, compared with experimental data (Doke & Kuo, 2007). Rate of mechanical work is roughly constant for both model and experiment.

6.3 Cost rate (or net metabolic rate).

Figure 43 through Figure 47 plot average rate of cost against swinging frequency for each of the cost functions listed in Section 6.1. Generally speaking, all of the plots seem to be in reasonable agreement with the experimental measurements obtained by Doke & Kuo (2007).

In particular, the Minetti-Alexander cost curve shown in Figure 44 matches the shape of the experimental curve very well. The (more or less) constant offset difference between model and experiment might be explained by possible error in experimental procedure. Recall that there is a possibility that human subjects expend significant amounts of energy to stabilize their bodies (standing on one leg, holding the armrests, activating core muscles, etc.) during leg-swing experiments (see Section 5.2). Therefore, the costs of stabilization, balance, and support are likely adjoined to the cost of isolated leg-swing – perhaps explaining why the experimental cost curve is higher than the Minetti-Alexander cost curve.

The modified (scaled and shifted) cost curves shown in Figure 45, Figure 46, and Figure 47 intersect with the experimental curve. Though the shape and bend of the cost curves might differ between model and experiment, they appear to be in good accord with one another.



Figure 43: Average rate of cost (per unit weight) versus leg-swing frequency for "Positive Mechanical Work" cost model with pre-multiplier = 16, compared with experimental data (Doke & Kuo, 2007). The costs of body stabilization (during experiment) might account for the vertical offset between model and experiment.



Figure 44: Average rate of cost (per unit weight) versus leg-swing frequency for "Minetti-Alexander" cost model with $c_2 = 11$, compared with experimental data (Doke & Kuo, 2007). Except for the vertical offset, the curves match well.



Figure 45: Scaled cost rate curve for "Torque Rate" cost model with $a_1 = 0$, $a_2 = 0$, and $a_3 = 1$; compared with experimental data (Doke & Kuo, 2007). Except for the difference in slope, the curves match well.



Figure 46: Scaled and shifted cost rate curve for "Torque" cost model with $\alpha = 1$, compared with experimental data (Doke & Kuo, 2007). Except for the difference in slope, the curves match well.



Figure 47: Scaled and shifted cost rate curve for "Torque" cost model with $\alpha = 2$, compared with experimental data (Doke & Kuo, 2007). Except for the difference in slope, the curves match well.

6.4 Summary of results (part 2 of 2).

In this chapter, optimal leg-swing strategies are examined and evaluated against empirical data (Doke & Kuo, 2007) to determine whether or not the proposed cost models are applicable to a new leg-swing experiment with varying swinging amplitudes and frequencies. In examining rate of mechanical work and (metabolic) cost, the findings suggest that most (or all) of the cost models identified in Section 6.1 are in agreement with the experimentally measured data. Hence, there are various possible ways to express the cost of leg-swing in a functional form which, when minimized, results in leg-swing activity similar to the behavior observed experimentally with real, human subjects.

CHAPTER 7: CONCLUSIONS.

7.1 Discussion, summary, and concluding remarks.

By approximating the mechanics of isolated leg-swing as a pendulum actuated by torque "motors", a variety of simple cost functions (estimating the energetic expense of leg-swing) were examined. Different cost functions related to muscle torque, rate of muscle length change, mechanical work, and metabolic energy were considered. For this nonlinear, constrained optimization problem, total cost of leg-swing was minimized to find optimal leg-swing strategies (muscle excitation and muscle shortening rate as functions of time). Rate of mechanical work and (metabolic) cost were then examined in contrast to empirical data obtained by Doke et al. (2005 & 2007). We found that, up to an affine transformation (scaling and shifting), several of the proposed cost functions agreed reasonably with experimental measurements. Also, some of the minimizations produced bang-coast-bang optimal solutions, consistent with what is qualitatively observed in human data.

The fact that multiple models fit the experimental curves suggests that the experimental data we fit our models to might not be rich enough to distinguish categorically between the various models. One option is to obtain more extensive experimental data – for instance, perform leg swing experiments at a 10-by-10 grid of frequency and swing amplitude, and then attempt to fit the models to these 100 data points. Another option would be to use

biophysically motivated "molecular" models of muscle mechanics to restrict the metabolic cost functional forms explored to only those that are consistent with these molecular models.

Other cost functions of the form $\int F^{\alpha} g(\dot{\ell}) dt$ or $\int h(F,\dot{\ell}) dt$ might also be considered to see if they can fit the data better. Also, to be sure, even though several of our cost functions agreed reasonably with experimental data, the best-fit cost function often implied very low efficiencies – perhaps unrealistically low for mammalian muscles. Ideally, we would like cost functions that both have plausible efficiency values (about 20%) for steady state isotonic stretch experiments, and also fit the experimental data. Perhaps this requires use of terms related to force derivatives as is the case in some of our cost functions and as also posited implicitly by Doke et al. Perhaps the molecular-based models alluded above will provide some insights in this regard.

Moreover, additional confidence might be gained if the proposed cost functions can be applied to any human movement experiment – for instance, to other motor tasks like legswing with bending at the knee, or swinging the arm. Another research direction might be to incorporate activation dynamics (i.e. the action of transmitting neural signals to excite the muscles) into the leg-swing model with the aim of simulating all the physiological interactions in the human body. Overall, we believe that the problem of finding a cost function that is both plausible and fits a variety of metabolic cost data satisfactorily remains open – even in the limited context of leg-swing – and an extensive program of more carefully controlled experiments and associated theory would be required to fill this gap in our understanding. Once a good model of metabolic cost is developed, it can be used in the context of whole body tasks such as walking and running.

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APPENDIX A: THE EFFECTS OF INCLUDING TENDONS IN THE LEG-SWING MODEL.

Minimizing the Minetti-Alexander cost function for various values of tendon stiffness, the resulting optimal leg-swing strategies (from model) are evaluated against the measured experimental data from Doke et al. (2005). Average rate of cost versus swinging frequency is shown in Figure 48. The plot illustrates that the leg-swing model best fits the experimental data when tendon stiffness is very large. This supports the notion that human hip muscles have short tendons and, accordingly, exhibit low compliance (Minetti & Alexander, 1997). For these reasons, the swinging human leg is modeled with the assumption that stiffness is infinitely-high and the tendons are excluded from the muscle complex, as shown in Figure 8.



Figure 48: Average rate of cost using various tendon stiffnesses for "Minetti-Alexander" cost model, compared with experimental data (Doke, Donelan, & Kuo, 2005).

APPENDIX B: SAMPLE MATLAB CODE.

Solve function:

```
8
% The following code analyzes the swinging leg problem. The dynamics of
% a swinging leg are modeled by a simple pendulum with an applied hip
% torque. ddTorque is represented as a piecewise linear function of time.
% This code uses "fmincon" to determine the piecewise linear torque rate
% function that minimizes the objective function "J" defined in
% "pendulumObjective.m", where "J" is some form of leg-swing cost.
۶ _____
clear all; close all; clc;
§ _____
% DEFINE PARAMETERS AND INITIAL CONDITIONS
8 -----
% Initialize global iteration counter, for in-progress saving purposes.
global iterationCounter;
iterationCounter = 0;
% Specify parameter values in struct called "param".
param.m = 70*(.16); % leg mass (~16% of total body mass) [kg]
param.g = 9.8; % gravitational acceleration [m/sec^2]
param.g
                          % gravitational acceleration [m/sec^2]
% distance to center of leg mass [m]
               = 0.45;
param.r
param.I= 1;% leg mass moment of inertia [kg-m^2]param.torqueMax= 100;% maximum hip torque [N-m]param.N= 24;% number of piecewise segments [unitless]param.epsilon= .01;% square root approximationparam.Wmax= 8:% movement of above
param.Wmax = 8; % max muscle shortening rate [rad/sec]
param.savedelay = 500; % delay counter for saving [unitless]
% Define initial leg position, leg velocity, and cost.
pos_initial = -pi/8; % position [rad]
vel_initial = 0;
                             % velocity [rad/sec]
cost initial= 0;
                            % cost []
% Make an initial guess for muscle torques and torque rates.
dT1_initial = 0;
                           % initial guess for torque rate [N-m/sec]
dT2_initial = 0;
                             % initial guess for torque rate [N-m/sec]
T1_{initial} = 0;
T2_{initial} = 0;
                             % initial guess for torque [N-m]
                             % initial guess for torque [N-m]
% Define initial values for piecewise linear ddTorque function (C0),
% and initial values for initial state variables (x0).
C0 = zeros(2*param.N+2,1); % piece-wise values for ddTorque [N-m/sec]
x0 = [pos_initial; vel_initial; cost_initial; ...
    dT1_initial; dT2_initial; T1_initial; T2_initial];
% Define "p0" the set of parameters to be SOLVED by the fmincon optimizer.
p0 = [C0; dT1_initial; dT2_initial; T1_initial; T2_initial];
% Specify desired leg-swing frequencies, ranging from 0.5000Hz to
% 1.0833Hz. Specify shape factor constant c2.
frequency = 13/12 : -1/12 : 1/2;
```

= [2 3 5 7 9 11];

c2

```
for c2Counter = 1:1:length(c2);
   for frequencyCounter = 1:1:length(frequency);
   ٥٥ ------
   % MINIMIZE FUNCTION "J"
   8 _____
   % Specify parameter values that change.
   param.c2 = c2(c2Counter);
                                           % shape factor c2.
   param.tmax = 1/frequency(frequencyCounter); % simulation time [sec]
   \ Use TOMLAB's "snopt" to determine the piecewise linear ddTorque
   % function that minimizes the objective function "J" (total cost).
   options = optimset('display','iter','MaxFunEvals',100000, ...
      'MaxIter',100000);
   LB = -50000 * ones(size(p0));
                                           % ddTorque lowerbound
   UB = +50000*ones(size(p0));
                                           % ddTorque upperbound
   Prob = ProbDef; Prob.Solver.Tomlab ='snopt';
   [presult,objval] = fmincon('pendulumObjective',p0,[],[],[],[], ...
      LB,UB,'pendulumConstraint',options,Prob,x0,param);
   % Make initial guess p0 equal to presult (the converged result of
   % the previous iteration).
   p0 = presult;
   %
   % SAVE CONVERGED RESULTS
   %
   save(['MinettiAlexander_c2val',num2str(c2(c2Counter)),'_',...
      num2str(frequency(frequencyCounter),'%0.4f'),'Hz.mat']);
         % End "frequencyCounter" for-loop.
   end
end
      % End "c2Counter" for-loop.
```

ODE file:

```
96
% This pendulum ODE file solves the differential equation for a gravity
% pendulum with applied hip torque.
% Motion occurs in a 2-D plane. ddTorque is represented as a function
% of time in terms of a piecewise linear function. The applied hip torque
% results from integrating the torque rate function.
06 ______
function xdot = pendulumODE(t,x,C,param,i)
                     _____
% DEFINE STATE VARIABLES
% Introduce state variables x1, x2, x3, x4, x5.
          % leg angular position [rad]
x1 = x(1);
              % leg angular velocity [rad/sec]
x^2 = x(2);
x3 = x(3);
             % function "J", metabolic energy cost to be minimized
             % muscle1 torque rate [N-m/sec]
x4 = x(4);
            % muscle2 torque rate [N-m/sec]
x5 = x(5);
x6 = x(6);
              % muscle1 torque [N-m]
            % muscle2 torque [N-m]
x7 = x(7);
2 ____
                                   _____
% WRITE 1ST ORDER ODE'S FOR LEG SWING DYNAMICS
۶
_____۶
% Unpack struct "param".
m = param.m; % leg mass [kg]
g = param.g;
                 % gravitational acceleration [m/sec^2]
                % distance ot center of leg mass [m]
% leg mass moment of inertia [kg-m^2]
r = param.r;
I = param.I;
% Compute total torque, the summation of hip torques developed by muscle1
% and muscle2.
torqueTotal = x6 + x7;
% Write pendulum equations as first order ODEs by appropriately using the
% variables x1,x2. x1dot is the angular velocity and x2dot is the angular
% acceleration of the leg mass.
x1dot = x2;
                                           % velocity [rad/sec]
x2dot = (torgueTotal-m*g*r*sin(x1))/(I+m*r^2); % acceleration [rad/sec^2]
% Define "tstep" the time interval for each segment of the piecewise linear
% ddTorque function.
tstep = param.tmax/param.N;
tInitial = -param.tmax/2+(i-1)*tstep;
% Determine the value of ddTorque for all instances of time. ddTorque
% is a time-dependent piecewise linear function. Hip torque (the
% anti-derivative of torque rate) is driven by two opposing muscles --
% agonist and attogonist (torque1 and torque2).
x4dot = C(i)+(C(i+1)-C(i))/tstep*(t-tInitial);
                                              % muscle1 ddTorque
x5dot = C((param.N+1)+i)+(C((param.N+1)+i+1)- \dots)
                                              % muscle2 ddTorgue
   C((param.N+1)+i))/tstep*(t-tInitial);
x6dot = x4;
                                               % muscle1 torque rate
x7dot = x5;
                                               % muscle2 torque rate
& DEFINE RATE OF MUSCLE CONTRACTION/EXTENSION
8 -----
% Determine the muscle shortening rate for muscles 1 and 2. Note that W1
% and W2 are positive when muscle is shortening. Conversely, W1 and W2 are
% negative when muscle is lengtening.
W1 = +x2;
                                               % NO tendons
W2 = -x2;
                                               % NO tendons
```

Constraint function:

```
96
% This code establishes the inequality and equality constraints for
% "pendulumObjective.m".
2
% The pendulum ODE file is called by the "ode45" solver in order to
% determine the angular position and velocity of the leg center of mass.
8
function [cineq,ceq] = pendulumConstraint(pinput,x0,param)
global iterationCounter;
& _____
% SOLVE USING "ode45"
% Define "tstep" the time interval for each segment of the piecewise linear
% ddTorque function.
tstep = param.tmax/param.N;
% Extract ddTorque, dTorque, and Toruge values from "pinput".
C = pinput(1:2*param.N+2); % ddTorque [N-m/sec^2]
dT1_initial = pinput(2*param.N+3); % initial dTorque1 [N-m/sec]
dT2_initial = pinput(2*param.N+4); % initial dTorque2 [N-m/sec]
T1_initial = pinput(2*param.N+5); % initial Torque1 [N-m]
T2_initial = pinput(2*param.N+6); % initial Torque2 [N-m]
% Define intial state variables (x0).
x0 = [x0(1:3); dT1_initial; dT2_initial; T1_initial; T2_initial];
% Use "ode45" solver to generate list of legmass position and velocity
% versus time. The ddTorque function is broken up into segments so that
% the optimization avoids discontinuities (and runs smoothly).
options = odeset('reltol',1e-10,'abstol',1e-10);
tliststore = -param.tmax/2; % store initial time
xliststore = x0'; % store initial position, velocity, etc.
for i = 1:param.N
   tspan = linspace(-param.tmax/2+(i-1)*tstep,-param.tmax/2+i*tstep,3);
   [tlist, xlist] = ode45(@pendulumODE,tspan,x0,options,C,param,i);
   x0 = xlist(end,:)';
   % Store tlist and xlist. These will be used to define constraints.
   tliststore = [tliststore; tlist(2:end,:)];
   xliststore = [xliststore; xlist(2:end,:)];
end
<u>۹</u>
% COMPUTE POWER
                 _____
& _____
% Extract values of torque from "xliststore".
torque1 = xliststore(:,6);
                                            % muscle1 torque
torque2 = xliststore(:,7);
                                           % muscle2 torque
% Determine the muscle shortening rate for muscles 1 and 2. Note that W1
% and W2 are positive when muscle is shortening. Conversely, W1 and W2 are
% negative when muscle is lengtening.
W1 = +xliststore(:,2);
                                            % NO tendons
W2 = -xliststore(:,2);
                                            % NO tendons
& ASSIGN INDEX NAMES FOR CLARITY
§ _____
                                 _____
% Determine the index at the end of the forward swing.
indexForward
                = (length(xliststore)+1)/2;
% For simplicity, identify the position and velocity at the end of the
% forward swing, and at the end of the backward swing.
posStart = xliststore(1,1);
```

```
posForward
             = xliststore(indexForward,1);
posBackward = xliststore(end,1);
           = xliststore(indexForward,2);
= xliststore(end,2);
velForward
velBackward
ی_____
% SET INEQUALITY AND EQUALITY CONSTRAINTS
8 _____
% Combine torque1,torque2 and W1,W2 terms for simplicity. Unpack Wmax from
% struct "param".
torque = [torque1; torque2];
       = [W1; W2];
W
% Set inequality constraints. (i.e. less than or equal to zero)
cineq = [-torque-param.torqueMax; % ALL torques >= -torqueMax
          +torque-param.torqueMax;
                                            % ALL torques <= +torqueMax</pre>
           -torquel;
                                            % torquel >= 0
           +torque2;
                                            % torque2 <= 0</pre>
           -W-param.Wmax;
                                            % W >= -Wmax
           +W-param.Wmax];
                                            % W <= +Wmax
% Set equality constraints. (i.e. equal to zero)
       % Enforce position and velocity at forward and backward positions.
cea
       = [posStart+posForward;
                                            % forward position mirrors the starting
position
          posStart-posBackward;
                                            % backward position is same as the
starting position
          velForward;
                                            % forward velocity equals zero
           velBackward;
                                            % backward velocity equals zero
       % Enforce periodicity of Torque, dTorque, and ddTorque.
           C(1)-C(param.N+1); % equal starting and ending ddTorque
C(param.N+2)-C(2*param.N+2); % equal starting and ending ddTorque
           C(1)-C(param.N+1);
          xliststore(1,4)-xliststore(end,4); % equal starting and ending dTorque
xliststore(1,5)-xliststore(end,5); % equal starting and ending dTorque
           xliststore(1,6)-xliststore(end,6); % equal starting and ending Torque
           xliststore(1,7)-xliststore(end,7); % equal starting and ending Torque
           1;
% DISPLAY GLOBAL ITERATION COUNT
& _____
% Increment global iteration counter and show count number.
iterationCounter = iterationCounter+1;
if mod(iterationCounter,param.savedelay)==0
   save inprogress24nodecalculation
   iterationCounter
end
end % End function "pendulumConstraint".
```

Objective function:

```
96
% The following code determines the piecewise linear ddTorque values "C"
% that minimize the objective function "J" (metabolic energy cost). As
% shown below, "J" is the integral of "x3dot" defined in "pendulumODE.m",
% where "x3dot" is some form of metabolic power expenditure.
2
% The pendulum ODE file is called by the "ode45" solver in order to
\ensuremath{\$} determine the angular position and velocity of the leg center of mass.
8 -----
function J = pendulumObjective(pinput,x0,param)
global iterationCounter;
% SOLVE USING "ode45"
%_____
% Define "tstep" the time interval for each segment of the piecewise linear
% ddTorque function.
tstep = param.tmax/param.N;
% Extract ddTorque, dTorque, and Toruqe values from "pinput".
 = pinput(1:2*param.N+2); % ddTorque [N-m/sec^2]
С
dT1_initial = pinput(2*param.N+3); % initial dTorque1 [N-m/sec]
dT2_initial = pinput(2*param.N+4); % initial dTorque2 [N-m/sec]
T1_initial = pinput(2*param.N+5); % initial Torque1 [N-m]
T2_initial = pinput(2*param.N+6); % initial Torque2 [N-m]
% Define intial state variables (x0).
x0 = [x0(1:3); dT1_initial; dT2_initial; T1_initial; T2_initial];
% Use "ode45" solver to generate list of legmass position and velocity
% versus time. The ddTorque function is broken up into segments so that
% the optimization avoids discontinuities (and runs smoothly).
options = odeset('reltol',1e-10,'abstol',1e-10);
for i = 1:param.N
   tspan = linspace(-param.tmax/2+(i-1)*tstep,-param.tmax/2+i*tstep,3);
   [tlist, xlist] = ode45(@pendulumODE,tspan,x0,options,C,param,i);
   x0 = xlist(end,:)';
end
÷
% DEFINE OBJECTIVE FUNCTION "J"
% _____
                           _____
% Define "J" as total cost.
J = xlist(end, 3);
% _____
                              _____
% DISPLAY GLOBAL ITERATION COUNT
8 ---
% Increment global iteration counter and show count number.
iterationCounter = iterationCounter+1;
if mod(iterationCounter,param.savedelay)==0
   save inprogress24nodecalculation
   iterationCounter
end
      % End function "pendulumObjective".
end
```
Post-processing:

```
96
% The following code illustrates the results of the swinging leg
% optimization problem. Plots include:
8
   (1) Angular position & velocity of pendulum versus time
%
  (2) Driving torque versus time
% (3) Driving power versus time
  (4) Animation of pendulum position in the xy-plane
¥ _____
function [output] = pendulumPostProcess(pinput,x0,param)
                       _____
% SOLVE USING "ode45"
% Define "tstep" the time interval for each segment of the piecewise linear
% torque function.
tstep = param.tmax/param.N;
% Unraveling pinput to get C, etc.
       = pinput(1:2*param.N+2); % piece-wise lin ddTorque [N-m/sec^2]
= pinput(2*param.N+3); % initial muscle1 dTorque [N-m/sec]
= pinput(2*param.N+4); % initial muscle2 dTorque [N-m/sec]
C
dT1 0
dT2_0 = pinput(2*param.N+4);
      = pinput(2*param.N+5); % initial muscle1 Torque [N-m]
= pinput(2*param.N+6); % initial muscle2 Torque [N-m]
т1_0
       = pinput(2*param.N+6);
Т2_0
\mathbf{x}\mathbf{0}
       = [x0(1:3); dT1_0; dT2_0; T1_0; T2_0]; % initial state variables
% Use "ode45" solver to generate list of legmass position and velocity
% versus time. The ddTorque function is broken up into segments so that
% the optimization avoids discontinuities (and runs smoothly).
options = odeset('reltol',1e-10,'abstol',1e-10);
tliststore = -param.tmax/2; % store initial time
xliststore = x0'; % store initial position, velocity, etc.
xliststore = x0';
for i = 1:param.N
   tspan = linspace(-param.tmax/2+(i-1)*tstep,-param.tmax/2+i*tstep,20);
   [tlist, xlist] = ode45(@pendulumODE,tspan,x0,options,C,param,i);
   x0 = xlist(end,:)';
    % Store tlist and xlist. These will be used for plotting purposes.
   tliststore = [tliststore; tlist(2:end,:)];
   xliststore = [xliststore; xlist(2:end,:)];
end
°
% COMPUTE METABOLIC MUSCLE POWER
     % ____
% Extract values of torque and torque rate from "xliststore".
T1 = xliststore(:,6); % muscle1 torque [N-m]
T2 = xliststore(:,7);
                        % muscle2 torque [N-m]
% Determine the muscle shortening rate for muscles 1 and 2. Note that W1
% and W2 are positive when muscle is shortening. Conversely, W1 and W2 are
% negative when muscle is lengtening.
W1 = +xliststore(:,2);
                              % NO TENDONS (infinite stiffness)
W2 = -xliststore(:, 2);
                                  % NO TENDONS (infinite stiffness)
% Using the "smooth" metabolic cost function from Manoj Srinivasan,
% compute metabolic power.
meta_power1 = computePower(T1,W1,param);
meta_power2 = computePower(T2,W2,param);
avgMetaPower = (mean(meta_power1)+mean(meta_power2));
8 -----
% COMPUTE MECHANICAL POWER
% _____
                                     _____
 Compute mechanical power due to leg swing.
```

```
mech_power1 = max(T1,0).*xliststore(:,2); % use T1 >= 0 only
mech_power2 = min(T2,0).*xliststore(:,2); % use T2 <= 0 only</pre>
mech_power1 = max(mech_power1,0); % positive mech power only
mech_power2 = max(mech_power2,0); % positive mech power only
mech_power2 = max(mech_power2,0);
                             % positive mech power only
avgMechPower = (mean(mech_power1)+mean(mech_power2));
%
% COMPUTE AVERAGE COST RATE
%
avgCostRate = xliststore(end,3)/param.tmax;
% OUTPUT RESULTS
۶ _____
% End function "pendulumPostProcess".
end
```