# **VOLUNTARY PROVISION OF PUBLIC GOODS:**

# **EXPERIMENTAL EVIDENCE AND THEORETICAL ANALYSIS**

#### DISSERTATION

Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of The Ohio State University

by

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# **VITA**

# FIELDS OF STUDY

Major Field: Economics

Studies in: Public Finance, Applied Microeconomics, and Applied Macroeconomics.

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# CHAPTER I

# A BINARY CHOICE MODEL OF NON-MONETARY

#### CONTRIBUTIONS IN THE PROVISION OF PURE PUBLIC

GOODS: EXPERIMENTAL EVIDENCE

#### **SECTION 1**

#### INTRODUCTION AND LITERATURE REVIEW

In Economic theory, voluntary provision of pure public goods has a free-rider problem, due to the characteristics of pure public goods: nonrival in consumption and nonexclusion. The free-rider hypothesis has universally been accepted in theory (see Samuelson 1954, Hardin 1968, and Olson 1968). The free-rider hypothesis has also been studied in Psychology as "Social Dilemma", and in Political Science as "Irrationality of Voting". Empirical evidence shows little or no support for strong free-rider hypothesis, see Bohm (1972), Sweeney (1973), Chamberlin (1978), Smith (1979), Marwell & Ames (1979, 80, 81), Schneider & Pommerehne (1981), Kim & Walker (1984), Isaac, Walker, & Thomas (1984), Isaac, McCue, & Plott(1985), Isaac & Walker (1988), Dawes, Orbell, Simmons, & Van De Kragt(1986), Andreoni(1988),

experimental data and field data. The experiments can be generalized into two monetary contribution experiments and nonmonetary contribution categories: experiments. Two commonly used models in public good experiments are continuous choice model and binary choice model. Most of the experiments have been focused on continuous choice model with monetary contribution<sup>1</sup>. Dawes, etc designed monetary contribution experiment with binary choice model. There are some inherited problems associated with monetary contribution experiments such as transitory income, insufficient economic motivation, and lack of the experimental funds, etc. These problems can be avoided in nonmonetary contribution experiments. In the nonmonetary contribution experiment category, there has been only one continuous choice model by Sweeney(1973). Unfortunately, each individual's utility function and social payoff function were not clearly defined in Sweeney's paper. The model in this paper is a binary choice model under nonmonetary contribution mechanism. The experimental design in this paper is simple, unique, and economical.

## Monetary contribution experiment-continuous choice model

The Isaac, Walker & Thomas (IWT) experimental design is typical of most of the monetary public good experiments. This paper will use the same definition of free riding given by IWT.

See Andreoni(1988), Bohm(1972, Chamberlin(1978), Dawes(1980), Dorsey(1992), Marwell & Ames(1979,80,81), Isaac(84,85,88,89), etc.

#### IWT definition of free riding:

Consider that n individuals have selfish utility function  $U_i$  (.), i=1 ...n, the central authority requires of each individual a message  $m_i^2(m_i \in M_i)$ . The central authority uses an outcome rule  $f(m_i, ..., m_n) = f(m)$  to determine the public goods level K and tax payments  $t_1$ ,  $t_2...t_n$ . Consider the game  $\Gamma(n, [U_i(\cdot)], f(\cdot))$ , individual i chooses some  $m_i \in M_i$ , where  $M_i$  is some allowable message space. We say that the economy exhibits property  $D^*$  if and only if:

- There exists a message vector m\* such that f (m\*) yields Pareto-Lindahl optimal results<sup>3</sup>. (Assume for the moment that only one such m\* exists.)
- 2. The message vector m\* is not a dominant strategy equilibrium vector4.
- 3. For each individual i, there exists some dominant strategy  $\hat{m}_i \neq m_i^*$  (with  $\hat{m} = (\hat{m}_i \dots \hat{m}_n)$ )
- 4.  $f(m^*) \neq f(\hat{m})$
- 5. f(m) is not Pareto-Lindahl optimal.

In an economy exhibiting D\*, each individual will free ride.

#### IWT experimental design:

<sup>3</sup> Pareto-Lindahl requires that  $f(m^*) \ge f(m')$ , and  $m_i^* = t_i$ ,  $\forall m' \subseteq M$ , for all i.

 $<sup>^{2}</sup>$  we can also call  $m_{i}$  as individual i's pledge.

<sup>&</sup>lt;sup>4</sup> A dominant strategy equilibrium vector is a vector such that each individual has a dominant strategy  $\hat{m}_i \in M_i$ , for all i, or  $U_i(\hat{m}_i) \ge U_i(m_i^*)$ 

The subjects in the experiment were undergraduate students who were enrolled in lower level Economics courses at the University of Arizona. Plato computer system was used for various reasons (i.e. allow minimal experimenter-subject interaction, minimize subjects' transaction costs etc..). Each subject was given some endowment (tokens) to invest in either individual exchange (private good) or group exchange (public good). The investment in "Private Good" paid \$.01 per token. The investment in "public good" paid higher than \$.01 per token. However, the payoff from the "public good" investment was equally shared by everyone in the group. Thus, the return for each subject from investing in "public goods" was less than investing in "private goods" unless there were enough people investing in the "public good". Since no one knew whether others would invest in the "private Good."

Suppose each individual's utility function was constructed as follows:

$$U_i(Z_i - m_i + (1/N)*G(m_i + \sum_{j \neq i} m_j))$$
 .....(1)

where:

Z<sub>i</sub> = individual i's endowment of tokens

m<sub>i</sub> = individual i's contribution of tokens to the group exchange

 $\sum_{j \neq i} m_j$  = the sum of the contributions to the group exchange of all participants except individual i

(1/N)\*G(.) = the per capita return to individual i from the group exchange.

<sup>&</sup>lt;sup>5</sup> \$0.012, \$0.03, and \$0.075 were used in I-W-T experiments

Thus each individual's single period decision problem was presented as:

$$\max_{0 \le m_i \le Z_i} U_i \left\{ Z_i - m_i + (1/N) \times G(m_i + \sum_{j \ne i}^n m_j) \right\} \qquad \dots (2)$$

G'(.) > 1 (or \$0.01), but (1/N)\*G'(.) was held less than 1. The single period dominant strategy for each individual was to invest zero in public good, which satisfies the property  $D^*$  described in the definition of free riding.

In the standard IWT design, m = Z for all i is not a unique Pareto optimal outcome. In fact, the outcome that everyone but one contribute all they have is also Pareto optimal. In IWT model, monetary payoff serves as the linkage between each individual's utility function and the payoff function. The linkage becomes weaker when the monetary value is small or transitory. When the monetary incentive is not big enough, other factors like fairness concern<sup>6</sup>, social norms<sup>7</sup>, minimizing boredom<sup>8</sup> may obscure the connection between each individual's utility and the payoff function. Some experiments<sup>9</sup> guaranteed each participant a minimum pay, which should be considered as some level of assurance. It had been shown that assurance-offering resulted less free riding than without assurance<sup>10</sup>. Kim and Walker (1984)also pointed out nine

<sup>&</sup>lt;sup>6</sup> Marwell & Ames 1979, Van Dijk & Wilke 1993.

<sup>&</sup>lt;sup>7</sup> Margolis 1982, Sugden 1984, Frank 1985, Palfrey & Rosenthal 1987, and Andreoni 1987.

<sup>8</sup> Dawes 1980

<sup>&</sup>lt;sup>9</sup> Isaac, McCue, Plott(1985), VAN De Kragt, Dawes, & Orbell(1988), Van Dijk & Wilke(1993)

<sup>&</sup>lt;sup>10</sup> Bohm(1972), Isaac, Schmidtz, Walker(1989)

"invalidating factors" in experimental tests which could account for the weakness or absence of the free rider problem in the voluntary provision of a public good. Using the nine "invalidating factors as a guide, Kim and Walker designed an experiment in which the participants used their own money to make the investment. The results showed more free riding than the previous experiments. However, the results were still inconclusive in confirming the strong version of free rider hypothesis. There were also two other concerns in Kim and Walker design: First, the experiment was conducted each weekday for about four weeks, which made each individual's daily "pledge" to the common fund vulnerable to each individual's daily liquidity constraint. Secondly, all the participants were informed that they were in an experiment. This might deviate the experimental results from the true behavior of the participants.

### Sweeney's nonmonetary public good experiment:

The subjects were ninety-three female students who were selected from several introductory Psychology courses<sup>11</sup>. Each of the subjects were placed on a stationary bicycle, which was connected to an electric generator. A "group light" and also an "individual light" were attached to a light panel for each participant. They were not able to observe one another, but they were able to observe the light bulbs that would be illuminated by the pedaling. The brightness of the "group light" depended on the pedaling of all the subjects in their group. The "individual light" only reflected each

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<sup>11</sup> Each subject were told that she was one member of a six-person group

individual's own pedaling. They were told that the grade they would receive would be increased by an amount related to the average brightness of the "group light". The "group light" was considered as a "public good" since the average brightness of the "group light" could not be denied or given to any member of the group. A weak version of free rider hypothesis was supported in this experiment.

There were some questions remained unanswered in Sweeney's experiment: First, each subject's utility function was not directly discussed. The subjects were presumably supposed to maximize their grades. Did all the subjects consider pedaling as a negative utility factor? Would some subjects consider pedaling as a positive utility factor(i.e. an exercise or entertainment)? In other words, what is the dominant strategy for each individual in order to maximize her or his utility? Secondly, the group optimum was unknown<sup>12</sup>.

How serious the free-rider problem is still remains a question, but far more effort have been devoted to designing voluntary contribution mechanisms to improve the provision of public goods. Theoretical amendments have been made to find out the conditions that the public good can be provided efficient. Brubaker(1975) argued that individuals might reveal accurately their preferences for the public good under the assurance that the rest of the group would make an appropriate matching offer. Brubaker called the above "Golden Rule of Revelation". Bagnoli & Lipman (1992) showed that private provision of public goods could be efficient under certain

12 See Kim & Walker(1984)

conditions. Goetze & Galderisi(1989) tried to combine rational egoistic model with collective welfare model, but did not find conclusive experimental evidence. Schmidtz(1991), canvassed the assurance contract in the provision of public good in Prisoner's Dilemma Game. The assurance problem has been confirmed by some field data as well as controlled laboratory tests (see Schmidtz 1991). Isaac, Schmidtz, and Walker concluded in their experimental test (1989) that provision point<sup>13</sup> with money-back guarantee had "ameliorative effect in promoting provision of the public good." But Dawes, Orbell, Simmons, and Van de Kragt argued that voluntary contributions were better induced by enforced fairness rather than money-back guarantee, because "The success of the money-back guarantee can be undermined by people's expectation that it will succeed."

In section 2, a working definition of free riding is presented. Experimental designs are discussed. Individual utility function and the public good payoff function will be derived. Free rider hypothesis and related hypothesis are tested. Section 3 examines provision point and assurance contract. Experimental results are provided and analyzed. Concluding remarks are given in section 4.

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<sup>&</sup>lt;sup>13</sup> A provision point is the threshold which determines whether the public good will be provided or not.

#### **SECTION 2**

#### FREE-RIDER PROBLEM

#### **Definition**

The idea of free riding in nonmonetary public good situation is the same as in monetary public good situation. For consistency and easy analysis, this paper intends to follow the same free riding concept as in IWT experiment.

There is a group of n individuals who have selfish utility functions  $U_i$  (.), i=1, ..., n. They are involved in the provision of a public good through voluntary contributions. The public good itself requires nonmonetary resources. Each individual i is endowed with one unit of the nonmonetary resource which gives a choice set  $q^i=(0,1)$ . The choice vector of n individuals will determine the payoff function  $r(q^i ... q^n)=r(q)$ . Given a selfish utility function  $U_i$  (.), individual i has a normal form game  $I^{i+1}$   $I^{$ 

<sup>&</sup>lt;sup>14</sup> A normal form game shows what payoffs result from each possible strategy combinations.

#### Hypotheses:

**Ha:** 
$$\sum_{i=1}^{n} q^{i} = 0$$
 (i = 1, 2, ..., n)

(complete free rider hypothesis).

**Hb:** 
$$0 < \sum_{i=1}^{n} q^{i} < 50\% * n \ (i = 1, 2, ..., n)$$

(weak free rider hypothesis).

**Hc:** 
$$\sum_{i=1}^{n} q^{i} = 50\% * n \ (i = 1, 2, ..., n)$$

(random decision)

**Hd:** 
$$\sum_{i=1}^{n} q^{i} = n (i = 1, 2, ..., n)$$

(Pareto optimality).

# Subjects and Design<sup>15</sup>

The subjects of the experiments were undergraduate students who were taking lower level Economic courses. All of the subjects had taken Econ200 before as prerequisites.

The students were asked to make up the midterm questions (all multiple-choice). The midterm was a two-hour exam which counted for 50% of the final grade. They were told that they could make the questions as hard as they wished as long as the content of the questions was in the textbook which were covered by the lectures. The instructor also provided to the class a set of basic questions with answers, which

<sup>15</sup> See Appendix A

was regarded as "public goods". Their own questions were considered "private goods". Thus the students could choose either from the "public goods" or "private goods". Each student was supposed to turn in only one question for a class size of 40-60 students<sup>16</sup>. In order to make the basic set of questions truly "public goods", the students were told that they could bring the basic set of questions and answers to the exam. So if any questions were chosen from the basic set by anyone, everyone would know the right answers. Communications among the students regarding the midterm questions were lightly controlled. Two procedures were taken to block communication: one was to impose penalty for communicators; another was to give disincentive to communicate. Despite the above procedures, perfect blocking of communication was not guaranteed. However, small scale communication was not a big threat to the experimental design (this will be further discussed in the model part). Both oral and written instructions were given to the students to make sure they understood the instructions and especially the following:

- The midterm grade would be curved on their relative performance<sup>17</sup> with one exception that all get 100.
- The final exam would also be curved but the instructor would make the exam questions. The final exam would not be comprehensive.

<sup>16</sup> For a class size of 20-30 students, each student could be asked to submit two questions for a one-hour exam

-

<sup>&</sup>lt;sup>17</sup> The announced mean of the curve is 67

The final course grade would be the average of the midterm and the final grade.

The final course grade would follow an absolute scale:

A 90-100 A- 85-89 B+ 80-84 B 75-79 B- 70-74

C+ 65-69 C 60-64 C- 55-59 D 50-54 E 0-49

In order to test the robustness of free rider hypothesis, this experiment will be conducted in four separate classes with three different instructors.

## Model and Hypotheses

Two plausible variables are being considered in each individual's utility function. One is each individual's grade which is being manipulated in this experiment. Each individual's grade plays a similar role in this experiment as monetary payoff in IWT design. Another is each individual's effort. There are two kinds of efforts involved: one is the effort studying and preparing for the exam; another is the effort of making the exam questions. The second kind of effort is what we try to reduce or eliminate. In order to minimize the disutility of making the questions, the subjects are offered the opportunity to have the instructor make the questions for them privately if anyone has difficulty making the questions. Thus, the utility function of individual i can be written as  $U_i$  ( $G_i$ ,  $E_i$ ). It is assumed that  $U_i$  (.) is nondecreasing in  $G_i$ .  $E_i$  can be regarded as predetermined or constant  $^{18}$ .

 $<sup>^{18}</sup>$   $\rm E_{i}$  can also be considered as a choice variable in future model

In order not to violate the validity of the experiment, we have to make sure that all the subjects care about their grades. A set of survey questions were conducted in Four separate Economic classes to find out the general attitudes of the subjects. Three of the four classes had 100% positive response to the question of whether they care about their grades. Only one student in the other class had a negative response to the same question. But the same individual also indicated that he or she would be unhappy with any grade below C, which contradicted the attitude that this individual didn't care about the grade. This individual could either be auditing the class or just be not serious about the survey. One of the other valuable information given by the survey was that the overwhelming majority took those economics classes as a requirement, not as entertainment, because the motive for entertainment may dominate students' concern for their grades. This type of survey is very helpful not only in checking the validity of the experiment, but also in interpreting the results of the experiment.

The following describes the derivation of any individual i's payoff function  $R_i$  which depends on the relative performance of each individual:

For i = 1, 2, ..., n, where n is the total number of students,

The total number of public questions is  $Q = Q_{-i} + q^i$ , where  $Q_{-i} = \sum_{j \neq i} q^j$ .

.

<sup>19</sup> See Appendix B

let  $\beta_i$  be the probability that individual i knows the right answer for any private question chosen by another person  $j \neq i$ . Thus individual i's expected exam score can be calculated as following:

$$Q_{-i} + (n-1-Q_{-i})*\beta_i + 1 = S^i(Q_{-i},\beta_i)$$
 .....(3)

Explanation: First, individual i knows the answer to his or her own question regardless what kind of question it is, which is a guaranteed 1 point. Second, individual i knows the answer to all the public questions submitted by others, which is  $Q_{-i}$ . Last, there are total of n-1- $Q_{-i}$  private questions, thus the number of private questions that individual i knows the right answer is  $(n-1-Q_{-i})*\beta_i$ . The total score for individual i is  $1+Q_{-i}+(n-1-Q_{-i})*\beta_i$ . For convenience, we can use  $S^i(Q_{-i}, \beta_i)$  represent the above score. Now we can calculate the expected mean (average) of all the scores:

Expected Mean = 
$$\frac{1}{n} \times \sum_{i=1}^{n} S^{i} = \frac{1}{n} \times \left\{ \sum_{i=1}^{n} \left[ Q_{-i} + (n-1-Q_{-i}) \times \beta_{i} + 1 \right] \right\}$$
 .....(4)

The expected relative payoff curve can be defined as follows:

$$R_i(q^i, Q_{-i}, \beta_i, \beta_{i \neq i}) = S^i(Q_{-i}, \beta_i)$$
 - Expected Mean

$$= \left[ -1 + \frac{1}{n} + \frac{1}{n} \times \sum_{j \neq i} \beta_j \right] \times q^i + \left[ -\beta_i + \frac{1}{n} + \frac{1}{n} \times \sum_{i=1}^n \beta_i \right] \times Q_{-i}$$

$$+(n-1) \times \left[\beta_i - \frac{1}{n} \times \sum_{i=1}^n \beta_i\right] - \frac{1}{n} \times \sum_{j \neq i} q^j \beta_j$$

$$= \psi \times q^{i} + \omega \times Q_{-i} + (n-1) \times \left[\beta_{i} - \frac{1}{n} \times \sum_{i=1}^{n} \beta_{i}\right] - \frac{1}{n} \times \sum_{j \neq i} q^{j} \beta_{j} \quad \dots (5)$$

where, 
$$\psi = \left[-1 + \frac{1}{n} + \frac{1}{n} \times \sum_{j \neq i} \beta_j\right]$$

$$\omega = \left[ -\beta_i + \frac{1}{n} + \frac{1}{n} \times \sum_{i=1}^{n} \beta_i \right]$$

#### Extreme cases:

First,  $\beta_i = 1$ , and  $\beta_j = 0$  for all  $j \neq i$ ; In other words individual i is a "genius" and everyone else is "ignorant".

$$R_i(q^i, Q_{-i}, 1, 0, 0) = (-1 + \frac{1}{n}) \times q^i + (-1 + \frac{2}{n}) \times Q_{-i} + \frac{(n-2)^2}{n}$$
 .....(6)

It is obvious that  $R_i(0, Q_{-i}, 1, 0) > R_i(1, Q_{-i}, 1, 0)$ , for any  $n \ge 2$ , which shows that individual i will receive a higher payoff by submitting a private question.

Secondly,  $\beta_i = 0$ , and  $\beta_j = 1$  for all  $j \neq i$ ; In other words, individual i is "ignorant", and everyone else is "genius".

$$R_i(q^i, Q_{-i}, 0, 1) = 0 \times q^i + Q_{-i} - (n-1)/n - (1/n)Q_{-i}$$
 .....(7)

In this case, individual i's payoff is independent of his or her choice qi.

last,  $\beta_i = \beta_j = \beta$ ,  $0 \le \beta \le 1$ ; In other words, every individual has the same probability of knowing the right answer to a private question.

$$R_{i}(q^{i},Q_{-i},\beta,\beta) = \frac{1-\beta}{n} \times Q_{-i} - \left[ (n-1) \times \frac{1-\beta}{n} \right] \times q^{i} \qquad \dots (8)$$

It is clear that  $-[(n-1)\times(1-\beta)/n]$  is negative, individual i is better off by submitting a private question than by submitting a public question.

Each individual's grade  $G_i$  is a nondecreasing function of  $R_i$ , except that  $S^j\left(Q_{-j}\,,\,\beta_i\right) = 100 \text{ for all } j.$ 

$$\label{eq:Define:Gi} Define: \ G_i = \left\{ \begin{array}{l} G_i^1 \text{, if } S^j \left(Q_{-j},\beta_j\right) = 100 \text{, for all } j; \\ \\ G_i^2 = G_i \big(R_i\big) \text{, otherwise;} \end{array} \right.$$

 $G_i^1$  is a grade that individual i assign himself or herself, most likely that  $G_i^1 = A$ . The reason  $G_i^1$  wasn't assigned directly an A is that there might be some subjects who didn't like the outcome of everybody getting an A.

One simple illustration of the relationship between the payoff function and each individual's grade can be given by assuming the payoff function follow a normal distribution:

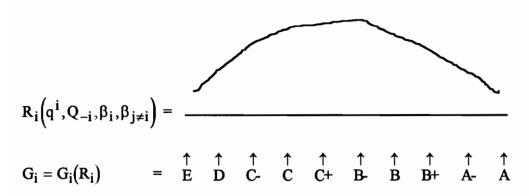


Figure 1. A normal distribution of grades.

**Exception:** if  $S^{j}(Q_{-j}, \beta_{j}) = 100$  for all j, then  $G_{i} = G_{i}^{1}$ 

let  $\epsilon$  be the probability that  $S^{j}$   $(Q_{-j}, \beta_{j}) = 100$ , Individual i is assumed to maximize the expected utility function:

Max 
$$EU(G_i) = (1-\epsilon) \times U(G_i^2(R_i)) + \epsilon \times U(G_i^1)$$
 .....(9)

For any  $\epsilon \neq 1$ , since  $\psi < 0$ ,  $q^i = 0$  gives a higher expected utility than  $q^i = 1$  for individual i;

For  $\epsilon=1$ , individual i is indifferent between  $q^i$  =0 and  $q^i$  =1; thus  $q^i$  =0 is weakly dominant.

The model above exhibits the property which is discussed in the definition: Given a selfish utility function  $U_i$  (·), individual i has a well defined game  $\Gamma = (n, (0,1), G_i$  (R<sub>i</sub>)). There exists a choice vector  $q^* = (1, ..., 1)$  such that  $G_i$  [R<sub>i</sub> ( $q^*$ )] yields Pareto optimal results. The choice vector  $q^*$  (1, ..., 1) is not a dominant strategy. But

for each individual the dominant strategy(or weakly dominant strategy) is  $\hat{q} = 0$ .  $G_i [R_i (q^*)] \neq G_i [R_i (\hat{q})]$ , and  $G_i [R_i (\hat{q})]$ , is not Pareto optimal.

# Hypotheses

Set  $\mathbb{P}_0$  = the percentage of subjects who choose "private questions". There are two sets of hypotheses testing. First,

**H0:**  $P_0 = 100\%$  (complete free-riding hypothesis)

H1:  $P_0 < 100\%$  (incomplete free-riding hypothesis)

If the null hypothesis was rejected, the results might suggest that some other factors were not considered in the subjects' utility function. The individuals' responses to one of the survey question might provide a cue.

Second,

H'0:  $P_0 = 50\%$  (Random decision hypothesis)

H'1:  $P_0 > 50\%$  (weak free-riding hypothesis)

#### Group size effect:

As n increases, the marginal return of the contribution from others ( $\omega$ ) becomes smaller and the marginal return of their own choice variable ( $\psi$ ) becomes bigger. The choice  $\stackrel{\wedge}{q} = 0$  is even more attractive as n increases. However,  $\stackrel{\wedge}{q} = 0$  is a weakly

dominant strategy regardless of the group size n. Thus, Group size might have slight adverse effect on the provision of the "public good".

# Hypothesis:

**H2:** The percentage of people who free ride will increase as n increases.

# Results from four demonstrative experiments<sup>20</sup>:

Table 1. A summary of the results from four demonstrative experiments

	total question	private question	public question	semi-public question <sup>21</sup>	P - P <sub>0</sub> Standard Error	$\frac{P_x - P_0}{\sqrt{P_0(1 - P_0)/n}}$
Class I	60	52	4	4 (6.7%)		
Class II	36	(86.7%)	(6.7%)	3 (8.3%)	2.07	6.7
Class III	46	(77.8%)	(13.9)	0	2.41	4.33
		(91.3%)	(8.7%)	Ů	1.48	3.96
Class IV	36	(77.8%)	8 (22.2%)	0	2.27	2.36

<sup>&</sup>lt;sup>20</sup> I am very grateful to Dr. Fuchun Jin and Mr. Tzu Kuang Hsu for conducting the experiments with me in their classes.

<sup>&</sup>lt;sup>21</sup> A question that was chosen from the "public questions" but a slight change was made.

Three of the experiments were conducted during the same quarter<sup>22</sup>. All the subjects in the above experiments had taken Economic Principle classes(Econ200) before. None of them participated in making up questions for their exams before. Four sets of observations were obtained. The results are summarized in table 1.

There were sixty students present at the time of the experiment in Class I. It was announced that only fifty questions would be chosen from all the questions that were handed in as midterm questions. If less than fifty students handed in questions, the instructor would make up the rest, and the instructor's questions would be very hard. This was providing the incentive to turn in questions. All sixty students turned in questions. 86.7% of the questions were "private questions"; only 6.7% of the questions were "public questions". The "semi-public questions" should be considered as "private questions" considering the motives for making the change. The "private questions" should be 93.3%.

In Class I,  $\hat{P} = 93.3\%$ . The normal approximation to the binomial distribution provides:  $\hat{P} \sim N(\hat{P}, \hat{P}(1-\hat{P}))$  We can examine the t test for  $\hat{P}$ :

$$\frac{\begin{vmatrix} \hat{P} - P_0 \end{vmatrix}}{\text{Standard Error}} = \frac{\begin{vmatrix} \hat{P} - P_0 \end{vmatrix}}{\sqrt{\frac{\hat{P} + (1 - \hat{P})}{n}}} = \frac{\begin{vmatrix} 0.933 - 1.00 \end{vmatrix}}{\sqrt{\frac{0.933 \times (1 - 0.933)}{60}}} = 2.07$$

<sup>&</sup>lt;sup>22</sup> See Appendix C, D, E and F.

Set  $\alpha = .05$ ,  $\mathbf{t}_{60,0.05} = 1.67$  Since 2.07 > 1.67, we reject **H0** at 5% significance level. However,  $\mathbf{t}_{60,0.01} = 2.39$ , we can't reject **H0** at the 1% significance level. Thus there is little or no support for complete free-riding hypothesis.

For the second set of hypothesis testing: H'0:  $P_0 = 50\%$  against H'1:  $P_0 > 50\%$ , we got  $P_x = \stackrel{\wedge}{P} = 93.3\%$ :

$$\frac{P_x - P_0}{\sqrt{P_0(1 - P_0)/n}} = \frac{0.933 - 0.5}{\sqrt{0.5(1 - 0.5)/60}} = 6.7$$

Set  $\alpha = .001$ ,  $Z_{\alpha}=3.1$  Since 6.1 is much bigger than 3.1, the null hypothesis is clearly rejected even at such a low significance level. The evidence supporting the weak free-rider hypothesis is overwhelming.

In Class II, thirty-six students showed up at the time of the experiment. Each of them was supposed to turn in one question. All of them turned in questions, twenty-eight of which turned in "private questions". Since the "semi-public questions" should be considered as "private questions", the percentage of the subjects who turned in "private questions" was  $\hat{P} = 31/36 = 86.1\%$ . Set  $\alpha = .05$ ,  $t_{36,0.05} = 1.69$ . we can reject H0 at the 5% significance level.

For the second set of hypothesis testing: H'0:  $P_0 = 50\%$  against H'1:  $P_0 > 50\%$ , we got  $P_x = \hat{P} = 86.1\%$ . Set  $\alpha = .001$ ,  $Z_\alpha = 3.1$  We can reject the null hypothesis at also a very low significance level. Thus weak free-riding hypothesis is supported.

Twenty-three students participated in the experiment in Class III. Each student was supposed to turn in two questions. Every student turned in a question, a total of forty-six questions were turned in. Nineteen students turned in "private questions", and four students turned in one "public question" and one "private question". No students turned in two "public questions. Even the "endowment" of each subject in this experiment was two questions instead of one, the dominant strategy was still to submit "private questions" If we only count half of the four students as "free riders", then  $\hat{P} = 21/23 = 91.3\%$ . Set  $\alpha = .05$ ,  $t_{23,0.05} = 1.71$ , we can't reject H0 at the 5% significance level. If we set  $\alpha = .10$ ,  $t_{23,0.10} = 1.32$ , we reject H0 at the 10% significance level.

For the second set of hypothesis testing:  $\mathbf{H'0}$ :  $P_0 = 50\%$  against  $\mathbf{H'1}$ :  $P_0 > 50\%$ , we got  $P_x = \stackrel{\wedge}{P} = 91.3\%$ . Set  $\alpha = .001$ ,  $Z_\alpha = 3.1$  The null hypothesis is rejected with another low significance level. The weak free-riding hypothesis is supported.

Eighteen students took part in the experiment in Class IV. Each student was supposed to turn in two questions just like Class III. All eighteen students turned in their questions. A total of thirty-six questions were turned in. Fourteen students turned in "private questions", and four students turned in all "public questions. No

<sup>&</sup>lt;sup>23</sup> This wasn't strictly binary-choice

students turned in a combination of "private question" and "public question". then  $\hat{P} = 14/18 = 77.8\%$ . Set  $\alpha = .05$ ,  $t_{18,0.05} = 1.73$ , H0 is rejected at the 5% significance level.

For the second set of hypothesis testing:  $\mathbf{H'0}$ :  $P_0 = 50\%$  against  $\mathbf{H'1}$ :  $P_0 > 50\%$ , we got  $P_x = \stackrel{\wedge}{P} = 77.8\%$ . Set  $\alpha = .01$ ,  $Z_\alpha = 2.33$  The null hypothesis is rejected again with another low significance level. The weak free-riding hypothesis is supported.

Weak free-riding hypothesis was supported by all four demonstrative experiments. However, there was no significant support for complete free-riding hypothesis. This result could also indicate that the model specified before was too simple compared to the tastes dimensions of all the subjects in the experiments.

To test **H2**, we can test the difference between two population proportions by using the Central Limit Theorem<sup>24</sup>.

**H20**: 
$$P_x - P_y = 0$$
 or **H0**:  $P_x - P_y \le 0$ 

against the alternative:

**H21**: 
$$P_x - P_v > 0$$

The pooled estimator  $P_0$  is given by:

$$\widehat{P}_0 = \frac{n_x * \widehat{p}_x + n_y * \widehat{p}_y}{n_x + n_y}$$

<sup>&</sup>lt;sup>24</sup> See Statistics for Business and Economics, by Paul Newbold, Page 370-371

the decision rule is:

$$\text{Rejecting H20 if: } \frac{\hat{p}_x - \hat{p}_y}{\sqrt{\hat{p}_0 \big(1 - \hat{p}_0\big) \! \bigg(\frac{n_x + n_y}{n_x n_y}\bigg)}} \ \rangle \ Z_\alpha$$

Since the students in Class III and Class IV had to turn in more than one questions, which result in some students turning in partly "public questions" and partly "private questions", It is not very clear what would be the true value of  $P_x$  and  $P_y$ . It would be inappropriate to compare Class III with Class IV.

Class I and Class II had the same experimental design as described originally. The experiments in both classes were conducted at about the same time by different instructors<sup>25</sup>. There might be difference on the teaching style of the two instructors, but that didn't seem to be a significant factor affecting the students' decisions. Both of the classes were scheduled in the afternoon. The students in both classes had basically the same background. None of the subjects were in both of the classes. It didn't appear to have any plausible factors other than the size of the class that might make the results differ in the two experiments. Although Class I and Class II were not the best candidates for testing H2, they were at least qualified.

For Class I:  $N_x = 60$ ,  $P_x = 93.3\%$ ; For Class II:  $N_y = 36$ ,  $P_y = 86.1\%$ .

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<sup>&</sup>lt;sup>25</sup> Mr. Hsu and I were the instructors

$$\frac{\hat{P}_{x} - \hat{P}_{y}}{\sqrt{\hat{P}_{0} \left(1 - \hat{P}_{0}\right) \left(\frac{N_{x} + N_{y}}{N_{x} N_{y}}\right)}} = 1.17$$

The value of  $\alpha$  corresponding to  $Z_{\alpha}=1.17$  is  $\alpha=.12$ . Hence, the null hypothesis can be rejected only at significant levels higher than 12%. The evidence supporting H2 is not very strong.

# Results from two "Role Playing" 26 experiments:

Two "Role Playing" experiments were conducted in order to test if "Role Playing" experiments could generate the same results as discussed above. One of the experiment (in Class A) was conducted at the time the midterm was given<sup>27</sup>. The instructions was attached to the midterm exam. Four bonus points were used as the incentive for the students to answer the survey questions for the experiment. The basic structure of the experiment is the same as before except that the students in this experiment were not held responsible for their choices. In other words, the students in this experiment didn't really play the game. The other experiment was conducted in Class B<sup>28</sup> at the beginning of the Quarter and it was anonymous. Both of the classes were Economic Principle classes(Econ200). Most of the students who were in

<sup>28</sup> See Appendix I and J

<sup>&</sup>lt;sup>26</sup> In these experiments, the subjects acted as if they were in the situation described in the experiments. I want to thank Ms Wei Wei Lee and Mr. Geoff Bump for conducting the experiments with me in their classes.

<sup>&</sup>lt;sup>27</sup> See Appendix G and H.

Econ200 had little or no knowledge of economics. The following are the results of the two "Role Playing" experiments:

Table 2. A summary of the results of two "role playing" experiments

	total	private	public		
	question N	question PR	question PU	$\frac{1}{PR} \bullet \sum t_{PR}$	$\frac{1}{\mathrm{PU}} \bullet \sum t_{\mathrm{PU}}$
class A	66	30	36	53.0%	83.3%
class B	61	29	32	47.4%	63.3%

- \*  $t_{PR}$ : the prediction of the percentage of students who would choose the "public question" by the students who chose "private question" themselves.
- $\frac{1}{PR} \bullet \sum t_{PR}$ : the mean of the prediction by the students who chose "private question" themselves.
- \*\*  $t_{PU}$ : the prediction of the percentage of students who could choose the "public question" by the students who chose "public question" themselves.
- $\frac{1}{PU} \bullet \sum t_{PU}$ : the mean of the prediction by the students who chose "public question" themselves.

In Class A there were 66 students who completed the survey. There were slightly less than 50% of the students chose "private question". The evidence was clearly against the weak free rider hypothesis and the strong free rider hypothesis.

The same evidence was found in Class B. The percentage of students who chose "private question" were slightly less than 50%. Again, there was no support for either weak or strong free rider hypothesis.

In the above two "Role Playing" experiments, the subjects were also asked to give a prediction of the percentage of the students who would choose "public question". The predictions were higher for the students who chose "public question" themselves<sup>29</sup>. Let's test whether the difference is significant or not.

Given two independent random samples of  $n_x$  and  $n_y$  observations from normal distributions with means  $\mu_x$  and  $\mu_y$  and variances  $\sigma^2_x$  and  $\sigma^2_y$ . If the observed sample means are  $\bar{x}$  and  $\bar{y}$ , then the tests are as follows for significance level  $\alpha$ :

**H30:** 
$$\mu_x - \mu_y = 0$$

against the alternative

**H31:** 
$$\mu_x - \mu_y > 0$$

Reject **H30** if 
$$\frac{\overline{x} - \overline{y}}{\sqrt{\frac{\sigma_x^2}{n_x} + \frac{\sigma_y^2}{n_y}}} \rangle Z_{\alpha}$$

<sup>&</sup>lt;sup>29</sup> See Appendix K, L, M, N.

For Class A,  $\bar{x} = 83.3\%$  and  $\bar{y} = 53.0\%$ ;  $n_x = 36$  and  $n_y = 30$ .

$$\frac{\bar{x} - \bar{y}}{\sqrt{\frac{\sigma_x^2}{n_x} + \frac{\sigma_y^2}{n_y}}} = \frac{0.833 - 0.53}{\sqrt{\frac{0.028}{36} + \frac{0.089}{30}}} = 4.95$$

Set  $\alpha = .01$ ,  $Z_{\alpha} = 2.33$ . Thus we reject **H30** at 1% significance level.

For Class B,  $\bar{x} = 63.3\%$  and  $\bar{y} = 47.4\%$ ;  $n_x = 30$  and  $n_y = 28$ .

$$\frac{\overline{x} - \overline{y}}{\sqrt{\frac{\sigma_x^2}{n_x} + \frac{\sigma_y^2}{n_y}}} = \frac{0.633 - 0.474}{\sqrt{\frac{0.024}{30} + \frac{0.031}{28}}} = 3.64$$

Set  $\alpha$  = .01,  $Z_{\alpha}$ = 2.33. Thus we reject H30 at 1% significance level. From the above results, we can conclude that the contributors tend to give a higher prediction that others will contribute than the noncontributors. This suggests that there is a positive relationship between the contributions of a decision maker and the predictions about others' contributions. However it is hard to draw any conclusions about the direction of the causality. On one hand, why the decision maker chooses to contribute is because his or her prediction of the contributions by others are high; On the other hand, if the decision maker believes contribution is the best choice, s/he would like others to do the same, thus s/he gives a higher prediction on others' contributions. It is obvious that a higher prediction gives the decision maker a higher assurance about the contributions by others.

Provision point and assurance problem will be examined in the next section.

# **SECTION 3**

### PROVISION POINT AND ASSURANCE PROBLEM

In the standard I-W-T design, the provision point can be provided by setting G(.) function in the following forms:

$$G(\cdot) = \begin{cases} \alpha \bullet \sum m_i & \text{for } \sum m_i \ge m^+ \\ 0 & \text{for } \sum m_i \le m^+ \end{cases}$$

where  $\alpha > 1$ .

m<sup>+</sup> is called the provision point. The introduction of the provision point makes zero-contribution not a dominant strategy anymore. It creates multiple Nash Equilibria<sup>30</sup>.

Isaac, Schmidtz, & Walker modified the provision point environment by adding an assurance contract imposed by Schmidtz(1987). The assurance contract provides a money-back guarantee if the provision point is not met. Thus, G(.) function changes to:

$$G(\cdot) = \begin{cases} \alpha \cdot \sum m_i & \text{for } \sum m_i \ge m^+ \\ m_i & \text{for } \sum m_i \le m^+ \end{cases}$$

where  $\alpha > 1$ .

Isaac, Schmidtz, & Walker concluded, "the results showed that this combined institution did have an ameliorative effect in promoting provision of the public good."

<sup>&</sup>lt;sup>30</sup> See details in Isaac, Schmidtz, & Walker (1989).

Bagnoli and McKee (1987) also reported that provision point along with money-back guarantee greatly improved the provision of the public good. Dawes et al.(1986) found no significant effect.

# An operationalization of the idea of provision point plus assurance contract:

The experimental design in section 2 can be modified by adding a provision point and an assurance contract. Suppose m is the provision point, if m or more of the subjects choose the public question, the grades will be curved by their relative performance plus each subject will be guaranteed a minimum grade of X. If fewer than m subjects choose the public question, then the "public good" will not be provided, which means that the instructor will make up the exam questions. Let's look at individual i's payoff matrices with the presence of the provision point plus the assurance contract:

Table 3. individual i's payoff matrices

	The Number of Subjects Who Choose Public Question				
	m-2 or fewer	m-1	m or more		
$q^i = 1$	$G_i(R_i^3)$	$\operatorname{Max}\left\{X, G_{i}\left(R_{i}^{1}\right)\right\}$	$\operatorname{Max}\left\{X, G_{i}\left(R_{i}^{1}\right)\right\}$		
$q^i = 0$	$G_i(R_i^3)$	$G_i(R_i^3)$	$Max\{X,G_i(R_i^2)\}$		

Where:  $R_i^1 = \psi + \omega * Q_{-i} + \left(n-1\right) * \left(\beta_i - \frac{1}{n} * \Sigma_i^n \beta_i\right) - \frac{1}{n} * \Sigma_{j \neq i} \, q^j \beta^j$ 

$$R_i^2 = \omega * Q_{-i} + \left(n-1\right) * \left(\beta_i - \frac{1}{n} * \Sigma_i^n \beta_i\right) - \frac{1}{n} * \Sigma_{j \neq i} q^j \beta^j$$

$$R_i^3 = n*\left(\beta_i - \frac{1}{n}*\sum_{i=1}^{n}\beta_i\right), j = 1, ... n.$$

 $\psi$ < 0 for any  $\beta_i \neq 0$ , and  $0 \leq \beta_i \leq 1$ ;

 $R_i^1$  is individual i's relative payoff when individual i chooses "public question";  $R_i^2$  is individual i's relative payoff when individual i chooses "private question". It is obvious that  $R_i^1 \setminus R_i^2$ .

 $R_i^3$  is individual i's relative payoff when the instructor makes the exam questions. In this case, all the questions are "private questions". Out of n questions, individual i knows the answers to  $n*\beta_i$ , the mean of all the scores is  $\Sigma_i^n\beta_i$ . Thus  $R_i^3 = n*\left(\beta_i - \frac{1}{n}*\Sigma_i^n\beta_i\right).$ 

Let  $P_2$  be individual i's probability that m-2 or fewer other subjects will choose the public question,  $P_1$  the probability that exactly m-1 other subjects will choose the public question, P the probability that m or more other subjects will choose the public question. Thus the expected value of choosing the public question is:

$$EV(1) = P_2 * G_i(R_i^3) + P_1 * Max \left\{ X, G_i(R_i^1) \right\} + P * Max \left\{ X, G_i(R_i^1) \right\} \qquad ......(10)$$

The expected value of choosing the private question is:

$$EV(0) = P_2 * G_i(R_i^3) + P_1 * G_i(R_i^3) + P * Max \{X, G_i(R_i^2)\} \qquad ......(11)$$

$$D = EV(1) - EV(0)$$

$$G_i\!\!\left(R_i^1\!\right) \! < G_i\!\!\left(R_i^2\!\right)\!; \quad \ \, \text{Max}\!\!\left\{\!X, G_i\!\!\left(R_{il}\right)\!\right\} \! < \text{Max}\!\!\left\{\!X, G_i\!\!\left(R_i^2\right)\!\right\}$$

( given that  $G_{i}$  (.) is nondecreasing in  $R_{i}$  , and  $R_{i}^{1}$   $\langle$   $R_{i}^{2}$  ).

If  $Max\left\{X,G_i\left(R_i^1\right)\right\}$   $\langle G_i\left(R_i^3\right)$ , which means that individual i thinks that he or she can do better on the instructor-made exam, or individual i has a strong preference on instructor-make exam, the dominate strategy is to choose private question.

If  $Max\left\{X,G_i\left(R_i^1\right)\right\} > G_i\left(R_i^3\right)$ , which means that individual i can do no better in instructor-made exam than either in students-made exams or the offer X, then choosing the private question is no longer the dominate strategy, at least for some individuals.

### **Experimental design:**

Four provision points were considered in this paper: 100%, 90%, 80%, and 50%. First, m<sup>+</sup> = 100% plus assurance contract, which guaranteed an A for everyone if unanimous vote was obtained. It was obvious that the dominant strategy for each of the individual was to choose the "public question". The incentive to free ride was completely eliminated. Anyone in the group had complete veto power. If any other factors like "fairness concern" might be dominant in some individual's decisions, the experiment would fail to get unanimous consent. Thus the results of this experiment would provide a benchmark for testing the accuracy of the simplified model as described in section 2. Some survey questions were designed to disclose other factors. The same survey questions would be used in the following experiments.

Second, m<sup>+</sup> = 90% plus Assurance Contract, which guaranteed a minimum grade A-. This gave a slight incentive for free-riding. Choosing "private question" remained a dominate strategy only for those who were sure that they could get A from the instructor-made exam. For most of the students, choosing "private question" was no longer a dominant strategy. However, the incentive to free ride was not eliminated. The answers to the survey questions as mentioned above might shed some light on that matter.

Third,  $m^+$  = 80% plus Assurance Contract, which guaranteed a minimum grade of A- or B. The reason for using two different minimum guaranteed grade was to find out if there was any difference in the free-riding behavior. In both cases, choosing a

<sup>&</sup>lt;sup>31</sup> Dawes, Orbell, Simmons, and Van De Kragt argue that voluntary contributions are better induced by enforced fairness rather than money-back guarantee

"private question" was not a dominant strategy for individuals who were satisfied with B or A-. The results would be different if there were many individuals who were satisfied with A- but dissatisfied with B. Otherwise, the results should be close.

Last, m<sup>+</sup> = 50% plus Assurance Contract, which guaranteed a minimum grade of C-. The incentive to free ride under this set up was higher than all the ones mentioned above. Choosing "private question" was no longer a dominant strategy for individuals who would be satisfied with C-. We should expect more subjects choosing "private question" than all the above designs with higher provision points.

### **Results:**

 $m^+ = 100\%$ :

Table 4. Experimental results given provision point 100%

	Instructor	exam	total	votes for	votes for
			votes	traditional exam	conditional exam
Class100a <sup>32</sup>	Jian Chen	final	59	3 (5%)	56 (95%)
Class100b <sup>33</sup>	Jian Chen	midterm	35	4 (11%)	31 (89%)
Class100c <sup>34</sup>	Lin Ba	final	42	4 (10%)	38 (90%)

<sup>32</sup> This experiment was conducted in class Econ560N, Winter 1994.

<sup>&</sup>lt;sup>33</sup> This experiment was conducted in class Econ444, Spring, 1994

<sup>&</sup>lt;sup>34</sup> This experiment was conducted in class Econ 400, Winter, 1994

First, none of the classes reached unanimous consent. The provision of "public good" failed even though 100% assurance contract was given. Second, other concerns do exist and appeared to be dominant in some individuals' decisions. In other words, some individuals' utility function were not as simple as described by the model given in section 2. The answers to a specific survey question provided on the instruction sheet could give us some explanations for their choices.

The overwhelming majority stated that it would be easier for everyone to get an A by choosing the conditional exam. Some of them showed altruism by claiming that even though they were very confident that they could get an A in either exam, but choosing the traditional exam would be easier for everyone else.

There was concerns about the quality of the "Public Good"<sup>35</sup> as well as the credibility of the instructor giving everyone an A<sup>36</sup>. Thus a broader concept of assurance problem should include not only the promise of the contributions from others but also the quality of the public goods and the credibility of the provider.

There were some "moral constraints" that some students imposed on themselves<sup>37</sup>. There were also evidence for "fairness concerns"<sup>38</sup> as well as indications for confusion and misunderstanding<sup>39</sup>.

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One out of the three who chose traditional exam in Class100a and three in Class100c had doubt about the "conditional exam".

One in Class100b who chose traditional exam expressed concern about whether the instructor would give everyone an A for the midterm.

<sup>&</sup>lt;sup>37</sup> In Class 100b, two of the four who chose traditional exam claimed that they would rather force themselves to learn something rather than getting an easy A, so that they wouldn't regret later.

One of four who chose traditional exam in Class100c claimed that everyone should work hard for his or her grade.

Three of the four who chose traditional exam stated that they wanted to gain relative advantage by doing well on the "traditional exam" not realizing that the final course grade would be the average

 $m^+ = 90\%$ :

Table 5. Experimental results given provision point 90%

	total	private	public		
	question N	question PR	question PU	$\frac{1}{PR} \bullet \sum t_{PR}$	$\frac{1}{PU} \bullet \sum t_{PU}$
class 91 <sup>40</sup>	23	5 (22%)	18 (78%)	36%	88.2%
class 92 <sup>41</sup>	33	11 (33%)	22 (67%)	55.1%	68.5%

Class91 and Class92 were both Economic principle classes (Econ200). The same survey questions were asked in both classes during the experiment <sup>42</sup>. According to the survey, all the subjects in both experiment did care about their grade for the course. Almost all of the subjects had never had Economics classes before. The instructions were basically the same for both classes except that the experiment carried out in Class92 was a "role playing" experiment, because the subjects were not held responsible for their choices and it was anonymous. The subjects in Class91 were told that the questions would be collected according to their choices if the "condition" was satisfied. They were asked to report their social security numbers on the survey forms. None of the experiments succeeded in the provision of the "public good".

All of the subjects were asked to give a reason for their choices. In Class91, two of the five subjects who chose "private questions" claimed that they preferred to

of the midterm and the final regardless how well others did.

<sup>&</sup>lt;sup>40</sup> This experiment was conducted by Mr. Cespedes-Torres in his Econ200 class, Winter, 1994

<sup>&</sup>lt;sup>41</sup> This experiment was conducted by Mr. Cespedes-Torres in his Econ200 class, Spring, 1994

<sup>&</sup>lt;sup>42</sup> See Appendix B

submit "private questions". The other three were confused<sup>43</sup>. If the three confused subjects understood the "condition", they might have chosen the "Basic Set" instead. Then, we would have got 21/23 = 91.3%, which would satisfy the provision point for the "conditional exam".

All the subjects were asked to give a prediction on the percentage of subjects who would choose from the "Basic Set". Column 5 and 6 in table 5 shows the average predictions by the "noncotributors" and "contributors" respectively<sup>44</sup>. In general, the average prediction by the contributors were higher than that by the noncontributors.

The contribution levels in both classes were much higher than the results reported in section 2, which confirmed the results in Isaac, Schmidtz, Walker, etc. that assurance contract did have ameliorative effect in the provision of public goods.

 $m^+ = 80\%$ :

Table 6. Experimental results given provision point 80%

	total	private	public		
	question N	question PR	question PU	$\frac{1}{PR} \bullet \sum t_{PR}$	$\frac{1}{PU} \bullet \sum t_{PU}$
class 81 <sup>45</sup>	38	6 (15.8%)	32 (84.2%)	45%	79.9%
class 82 <sup>46</sup>	58	9 (15.5%)	49 (84.5%)	65.0%	82.2%

<sup>&</sup>lt;sup>43</sup> They stated that it would be easier for all the students to choose their own questions for the final, which showed that they didn't really understand that the "conditional exam" would not be provided if the "condition" was not met.

<sup>&</sup>lt;sup>44</sup> Detailed predictions are in Appendix O, P, Q, R.

<sup>&</sup>lt;sup>45</sup> The experiment was conducted by Bill Muma during Winter Quarter 1994

<sup>&</sup>lt;sup>46</sup> Matt Klopfstein conducted this experiment during Winter Quarter 1994

Class81 and Class82 were both economic principles classes. Both experiments were conducted at about the same time during Winter Quarter 1994. All the subjects surveyed were taking the class for credit and all of them responded positively whether they care about their grades or not. None of the subject was in both experiment. The provision level were both set at 80%, but there was a slight difference in the guaranteed grade if the "conditional exams" were provided (Class81 guaranteed A- and Class82 guaranteed B.) The results of the two experiments were very close. The difference in the guaranteed grade did not make significant difference. Both of the experiments succeeded in the provision of the "public good".

The "contributors" in general tend to give a higher prediction that others would contribute, and the "noncontributors" tend to give a lower prediction that others would contribute<sup>47</sup>. All of the prediction s except one by the "noncontributors" in Class81 were less than 80%. Eight out nine "noncontributors" gave predictions in Class82. Five of the predictions were less than 80%. Here are the reasons given by the noncontributors: 1, to make the question harder. 2, to get a better grade 3, feel safer to take the exam. 4. it is more of a challenge to all the students. 5. students themselves could make good and reasonable questions. 6. the "traditional exam" was more fair. Two pertinent comments we could make about the reasons listed above: one is that free riding behavior still exist even assurance was given; the other is that "fairness

<sup>&</sup>lt;sup>47</sup> See Appendix S, T, U, V.

concern" was a factor in some individuals' decisions. However, "fairness concern" was not as compelling as claimed by Dawes, et al.

 $m^+ = 50\%$ :

Table 7. Experimental results given provision point 50%

	total	private	public		
	question N	question PR	question PU	$\frac{1}{PR} \bullet \sum t_{PR}$	$\frac{1}{PU} \bullet \sum t_{PU}$
class 51	59	8 (13.6%)	51 (86.4%)	53.8%	79.0%
class 52	44	12 (27.3%)	32 (72.7%)	66.0%	74.2%

Class51 and Class52 had exactly the same experimental set up. They were both conducted by the same instructor<sup>48</sup>. The only differences were that Class52 was a higher level economics class than class51<sup>49</sup> and they were conducted at different time<sup>50</sup>. However, none of the subject was in both experiments. All the subjects in both classes did care about their grade for the course. Only one of the subjects in Class51 had never taken Economics courses before. Both of the experiments succeeded in the provision of the "public good".

The percentage of subjects free riding was considerably higher in Class52 than in Class51. One plausible explanation was that subjects with more economics

<sup>&</sup>lt;sup>48</sup> Colin Feng was the instructor for both classes

<sup>&</sup>lt;sup>49</sup> Class51 was Econ400 and Class 52 was Econ530

<sup>&</sup>lt;sup>50</sup> Class51 was conducted during Winter Quarter 1994 and Class52 Spring Quarter 1994

knowledge were more likely to free ride than the ones with little or no knowledge of economics<sup>51</sup>.

The average prediction on the percentage of subjects who would "contribute" by the "contributors" was much higher than that by the "noncontributors" in both classes and all the predictions by the "contributors" were higher than 50%<sup>52</sup>.. It was clear that the "contributors" were aware of the opportunities to free ride, but they decided not to. The survey questions showed that there were "fairness concern" as well other concerns.

<sup>52</sup> See details in Appendix W, X, Y, Z.

<sup>&</sup>lt;sup>51</sup> Marwell and Ames had similar findings in their experiments. See Marwell & Ames (1981).

### **SECTION 4**

### CONCLUDING REMARKS

Free rider problem and Assurance problem in the voluntary provision of public good were examined in this dissertation paper through a binary-choice model with nonmonetary contributions.

Four demonstrative experiments showed very low contribution level (6.7%-22.2%) in the voluntary provision of public goods when there was no assurance offered. Free rider hypothesis was supported, but there was no complete free-riding scenario as suggested by the model. There was evidence showing the adverse effect of group size on the provision level of public goods, but the effect was not significant. Actually all the results from the four experiments turned out to be quite robust.

Two "role playing" experiments were conducted. The results did not support the free rider hypothesis, which showed that the behavior of individuals was quite different in the "role playing" environments as compared to the "real environments".

All the contribution levels were significantly increased when different provision levels and assurance contracts were provided.

The incentive to free ride was completely eliminated when 100% assurance level was imposed. However, none of the three experiments reached unanimous votes, which showed the limitation of a very simplified model. The answers to some of the survey questions indicated that other concerns appeared to be dominant in a few

individuals' decisions. It also raised two other kinds of assurance problems: one is the assurance of the quality of the public good and another is the assurance of the credibility of the provider of the public goods. These two assurance problems can be very influential in the real world environment instead of the lab environment.

There was still incentive to free ride when different assurances were offered (except the 100% assurance level). However, the effect of offering assurance plus provision level was very significant. All the experiments with 80% and 50% assurance level resulted in sufficient contribution. The experiments with 90% assurance level failed to reach the provision level due to misunderstanding and "role playing". The results of the contribution among all the experiments with different assurance levels turned out to be fairly close (except the 100% assurance level).

Another finding of this paper is that all survey results of some of the experiments confirmed that the contributors tend to give a higher prediction that others will contribute; likewise the noncontributors tend to do the opposite.

The experimental design in this research is very simple, yet it has generated fruitful results. Further research is needed in extending the binary choice model to continuous choice model.

# CHAPTER II

# **VOLUNTARY PROVISION OF PURE PUBLIC GOODS:**

## SUPPLY-SIDE INCENTIVES AS WELL AS

### DEMAND REVEALING MECHANISMS

### **SECTION 1**

### INTRODUCTION

There has been a growing interest in private provision of pure public goods. Private provision normally relies on voluntary contribution mechanism(VCM). In economic theory, VCM has a free rider problem, due to the characteristics of pure public goods: nonrival in consumption and nonexclusion. Palfrey and Rosenthal pointed out that the free rider problem lead to two theoretical issues in the analysis of the provision of public goods: demand revelation and participation<sup>53</sup>. Many mechanisms have been explored to eliminate the free-rider problems<sup>54</sup>. Some claim that voluntary provision of public goods can be efficient<sup>55</sup>. Complete information has

<sup>53</sup> See Palrey and Rosenthal (1984)

<sup>&</sup>lt;sup>54</sup> Brubaker's golden rule, and Vernon Smith's induced value auction mechanism, etc.

<sup>55</sup> See Bagnoli and Lipman(1992).

been assumed in most of the models. Incomplete information has also been introduced<sup>56</sup>. All previous studies shared one common feature: demand-side analysis. In this paper, I propose a third theoretical issue: supply side incentives. The supply side analysis is just as important as the demand side analysis. Even if there exists a "super mechanism" that guarantees true demand revelation and total participation, the analysis of private provision of public goods is still not complete without analyzing the supply side incentives. Is the supplier necessarily a monopoly? How do suppliers compete? We also have to understand that there are constitutional constraints on the supply of some public goods, like the national defense. The suppliers of public goods can be one of the following three: the government, nonprofit organizations, and private profit-maximizing firms. One advantage that the government has is the legal power to use taxation. Private provision under VCM may have a problem collecting all the pledges if someone refuses to pay according to their original pledges. Thus even though a VCM may be able to eliminate free rider problems, private provision can be discouraged if it lacks law enforcement in the collection of all the pledges. In this chapter, I will analyze the supply side incentives as well as the demand revealing mechanisms.

In order to simplify the above theoretical endeavor, it is necessary to study public goods in two separate categories: discrete public goods and continuous public goods. Examples of discrete public goods are: a community playground of a certain

<sup>56</sup> See D'aspremont and Gerard-varet(1979)

standard size, community streetlights, or a community swimming pool, etc. The values to represent the discrete public goods are [0,1]. 0 means the public good is not provided, 1 is when the public good is provided. The continuous public goods are: the purity of the community drinking water, the quantity of the mosquito spray for the community, or the number of the community security guards<sup>57</sup>, etc. The values of the continuous public goods are infinite.

Discrete public goods model are discussed in section 2. The analysis begins with one public good with one supplier. Then it is extended to multiple suppliers with multiple public goods. In section 3, earlier results are extended in the continuous public goods models. Concluding remarks will be provided in section 4.

<sup>&</sup>lt;sup>57</sup> This is usually viewed as continuous choice even though the numbers are discrete.

### **SECTION 2**

### DISCRETE PUBLIC GOODS MODEL

Let's start with one public good and one supplier. Suppose there are n individuals in a community, each of the individual is indexed by  $i \in \mathbb{N}$ ,  $\mathbb{N} = \{1, ..., n\}$ . The endowment of individual i is denoted wi and the vector of endowments is w, where  $w \in \mathbb{R}^n_{++}$ . The decision g is made from a decision set G={0, 1}; g=1 represents the decision to provide the public good, and g=0 represents the decision not to provide the good. Individual i's utility function can be expressed as  $U_i(x_i, g)$ , where  $x_i = w_i - \sigma_i$ ,  $\sigma_i$ represents individual i's contribution for the public good. U<sub>i</sub> is assumed to be increasing in g and  $x_i$  for all i. A simple cost function c(g) has two values: c(0)=0 and c(1)>0. For simplicity, set c(1)=c. First, we assume complete information which makes all the above common knowledge among the n individuals. We further assume that individual i knows his or her own true maximum valuation of the public good as v<sub>i</sub>, so that  $U_i(w_i, 0) = U_i(w_i - v_i)$ . Assume also no one is willing to contribute more than his or her maximum valuation, which can be expressed by  $0 \le \sigma_i \le v_i < w_i$  for all i. Finally, we assume that  $\sum_i w_i > c$ . Let us examine the efficiency aspects of providing the public good under the following cases:

# CASE 1. $\sum_{i} v_{i} < c$ :

In this case, the sum of the maximum valuations is less than the cost of providing the public good. Since the public good is discrete, the marginal values are equal to the total values. It is Pareto inferior to provide the public good.

Under any voluntary contribution mechanisms(VCM), it is obvious that the public good will not be provided because no one is willing to contribute more than his/her maximum valuation of the public good. Any VCM is efficient in the sense that the public good will not be provided. Of course, it is obvious that there is no incentive for private provision of the public goods. However, This is not always the case under taxation<sup>58</sup>.

# CASE 2. $\sum_{i} v_{i} \ge c$ :

It is Pareto improvement under any VCM if the public good is provided. It is clear that the public good will be provided if we have any  $v_i > c$  for all i. To make our analysis interesting, let us assume that  $v_i < c$  for all i. Two mechanisms have been heavily studied and compared: one is voluntary contribution mechanism with no assurance contract(VCMNA), in which individual i looses his or her contribution if the total contribution is not enough; the other is voluntary contribution with assurance

For example, assume equal lump-sum taxation and simple majority rule. Set n=3, t = c/3,  $v_1 = v_2 = (4/9)c > t$ , and  $v_3 = 0 < t$ . Thus, the public good is provided. But,  $v_1 + v_2 + v_3 = (8/9)c < c$ .

contract(VCMA), which offers a money-back guarantee if the total contribution is less than a required amount R (or proposed amount by the supplier.)

# 2a. Pure Nash equilibrium:

A vector of contributions (strategies) is a pure Nash equilibrium if for each individual i,  $\sigma_i$  is a best strategy for i against  $\sigma_{-i}$  ( $\sigma_{-i} = \sum_{j \neq i} \sigma_j$ )<sup>59</sup>. We can further illustrate this as follows:

Individual i will choose  $\sigma_i$  to maximize his or her utility taking the contributions of others  $\sigma_{-i}$  as given.

$$MaxU_i(x_i,g)$$

subject to 
$$x_i = w_i - \sigma_i$$
;  $g=0$  if  $\sum_i \sigma_i < c$  and  $g=1$  if  $\sum_i \sigma_i \ge c$ 

Under VCMNA, the following is individual i's contribution reaction function to the total of contributions by others:

$$\sigma_i = \begin{cases} 0 & \text{if } \sigma_{-i} > R \\ R - \sigma_{-i} & \text{if } 0 \le R - \sigma_{-i} \le v_i \\ 0 & \text{if } R - \sigma_{-i} > v_i \end{cases}$$

where, R = the provision point;  $\sigma_{-i} = \sum_{j \neq i} \sigma_j$ 

and individual i's utility based on the outcomes of the total contribution is depicted as:

$$U_i = \begin{cases} U_i(w_i - \sigma_i, \ 0) & \text{if } \sum_j \sigma_j < c \\ U_i(w_i - \sigma_i, \ l) & \text{if } \sum_j \sigma_j \geq c \end{cases}$$

<sup>59</sup> See Bagnoli and Lipman (1992)

There are many Nash equilibria for this game. Any vector of contributions  $\sigma$  with the outcomes of  $\Sigma_i \sigma_i = c$  must be a Nash equilibrium. Since  $0 \le \sigma_i \le v_i$ , individual i can not be better off by decreasing his or her contribution and cause the public good not being provided. Similarly, any increase in individual i's contribution will make him or her worse off. Another equilibrium is that  $\sigma_i = 0$  for all i and  $\Sigma_i \sigma_i = 0$ . It is obvious that no one has incentive to deviate given that the others do not deviate. Any Nash equilibrium with the outcome of  $\Sigma_i \sigma_i = c$  Pareto-dominate the Nash equilibrium with the outcome of  $\Sigma_i \sigma_i = c$  Pareto-dominate that we will get the Pareto optimal outcome. In a repeated game, there is more experimental support for zero-contribution equilibrium<sup>60</sup>.

Under VCMA, individual i has the same reaction function to the total contributions by others as shown above. The only change is individual i's total utility which is described as follows:

$$U_i = \begin{cases} U_i(w_i, 0) & \text{if } \sum_j \sigma_j < c \\ U_i(w_i - \sigma_i, 1) & \text{if } \sum_j \sigma_j \ge c \end{cases}$$

In addition to the Nash equilibria under VCMNA, there are also Nash equilibria with the outcome of  $\sum_i \sigma_i < c$ . For example, any vector of  $\sigma$  satisfying  $0 \le \sigma_i \le v_i$ ,  $\sum_i \sigma_i < c$  and  $\sum_{j \ne i} \sigma_j + v_i < c$  for all i will be a Nash equilibrium. In this case, no one can be better off by changing his or her contributions because his or her effort is

<sup>60</sup> See Isaac, et. al(1989)

not enough to make the public good provided. Thus there is a continuous Nash equilibrium outcomes. Clearly, VCMA does not make Pareto-optimal outcome of  $\sum_i \sigma_i = c$  more plausible than the outcome of  $\sum_i \sigma_i < c$ . However, under VCMA, individual i can not be worse off by giving a small contribution  $\sigma_i$ , so long as  $0 \le \sigma_i \le v_i$ . It is still not clear how much individual i will contribute, since her or his optional choice set is an interval rather than an unique point. The contributions have been found to be higher under VCMA than that under VCMNA<sup>61</sup>. In the rest sections, I shall introduce individual i's uncertainty about  $\sigma_{-i}$  and analyze the equilibrium. My starting point is the following Rapoport's model (1987).

# 2b. Rapoport's Expected Utility Model:

There is a group of n players participating the provision of public goods game. Each player has a monetary endowment of e units(e>0) and must decide independently and anonymously whether to contribute it to the benefit of the group. If m or more players contribute, each player, i ( $i \in N$ ), receives a reward(public good) of r units(r>e). If m-1 or fewer players contribute, the contributors lose their endowments, whereas the noncontributors keep theirs.

Assurance contract can be added by making all players keep their endowments if m-1 or fewer players contribute.

<sup>61</sup> See Bagnoli & Mckee(1991), Isaac, Schmitz, & Walker(1989), etc.

Let  $P_{m-1}$  be player i's probability that exactly m-1 other players will contribute,  $P_{m-}$  be the probability that m-2 or fewer other players will contribute, and  $P_{m+}$  the probability that m or more other players will contribute before the decision to contribute or not is privately made. We have:  $P_{m-1} + P_{m-} + P_{m+} = 1$ .

When no assurance (NA) is provided, the expected value of contributing for player i is  $EV(C) = r(P_{m-1} + P_{m+})$ , and the expected value of not contributing is  $EV(\overline{C}) = rP_m + e$ . Defining  $D_{NA} = EV(C) - EV(\overline{C})$  for the NA paradigm, we get:

$$D_{NA} = r(P_{m-1} + P_{m+}) - e - rP_{m+} = rP_{m-1} - e$$
 (1)

Equation (1) means that in order to contribute, the expected value of making a difference must exceed player i's cost of contributing to the public good.

When the assurance contract is provided,  $EV(C) = eP_{m-} + r(P_{m-1} + P_{m+})$  and  $EV(\overline{C}) = rP_{m+} + e$ . Defining  $D_A = EV(C) - EV(\overline{C})$  for the Assurance paradigm, we have:

$$D_A = eP_{m-} + r(P_{m-1} + P_{m+}) - rP_{m+} - e = rP_{m-1} - e(1 - P_{m-})$$
 (2)

It is obvious that  $D_A \ge D_{NA}$ , which means that given the same probabilities, contributing is at least not worse off with the assurance contract than without. However, the sign of equation (2) in indeterminate, which means that contributing is not always a better strategy than not contributing.

Let's set the provision point at m=n. Under the assurance contract,  $EV(C) = eP_{n-} + rP_{n-1} \text{ and } EV(\overline{C}) = e. \text{ Thus equation (2) becomes:}$ 

$$D_A = eP_{n-} + rP_{n-1} - e > e(P_{n-} + P_{n-1}) - e = 0$$
, since  $r > e$ .

The above shows that only when there is no incentive to free ride, contributing becomes a superior strategy than not contributing.

Each individual in the above model has a binary choice. People often face a continuous choice. Let's extend the above model to a continuous choice model.

## 2c. Continuous Choice Model In the Provision of A Discrete Public Good

Suppose individual i has a prior believe about the probability of the total contributions by others, which can be written as  $F(g_i)$ , and it has a density function of  $f(g_{-i})$ ,  $g_{-i} \in [\underline{g}_{-i} \ \overline{g}_{-i}]$ . For simplicity, we can set  $\underline{g} = \underline{g}_{-i}$  and  $\overline{g} = \overline{g}_{-i}$ . Thus,  $F(\underline{g}) = 0$  and  $F(\overline{g}) = 1$ . Let R be the provision point and  $R \ge c$ . Individual i will choose  $\sigma_i$  to maximize his or her expected utility given the above belief. We can analyze this game under both VCMNA and VCMA:

# **VCMNA:**

Suppose individual i is risk neutral, then we can write:  $U_i(w_i - \sigma_i, 0) = -\sigma_i$  and  $U_i(w_i - \sigma_i, 1) = v_i - \sigma_i$ . Assuming also  $R \le \overline{g} + \overline{\sigma}_i$ , where  $\overline{\sigma}_i$  is the maximum contribution by individual i, and  $\overline{\sigma}_i \le v_i$ . It is clear that if  $R > \overline{g} + \overline{\sigma}_i$ , Individual i will

be better off not contributing. Thus individual i will choose  $\sigma_i$  to maximize the following expected utility function:

$$EU_{i} = \int_{\underline{g}}^{R-\sigma_{i}} U(w_{i} - \sigma_{i}, 0) f(g_{-i}) dg_{-i} + \int_{R-\sigma_{i}}^{\overline{g}} U(w_{i} - \sigma_{i}, 1) f(g_{-i}) dg_{-i}$$

$$= \int_{\underline{g}}^{R-\sigma_{i}} (-\sigma_{i}) f(g_{-i}) dg_{-i} + \int_{R-\sigma_{i}}^{\overline{g}} (v_{i} - \sigma_{i}) f(g_{-i}) dg_{-i} \dots (3)$$

$$\frac{dEU_i}{d\sigma_i} = -1 + v_i f(R - \sigma_i) \cdots (4)$$

$$\frac{d^2EU_i}{d\sigma_i^2} = -v_i f'(R - \sigma_i) \quad \cdots (5)$$

For transparency and easy analysis, assume  $f(g_{-i})$  is an uniform distribution as shown in figure 2:

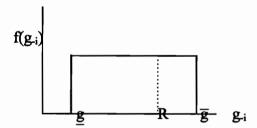


Figure 2. A uniform distribution function

Equation (4) becomes  $\frac{dEU_i}{d\sigma_i} = -1 + \frac{v_i}{\overline{g} - \underline{g}} = \frac{v_i + \underline{g} - \overline{g}}{\overline{g} - \underline{g}}$  and equation (5) is

obviously zero. Thus, there are two corner solutions for the above maximization problem, which depend on the sign of  $v_i + \underline{g} - \overline{g}$ . First,  $v_i + \underline{g} - \overline{g} > 0$  or  $v_i + \underline{g} > \overline{g} > R$ , which means that individual i's valuation of the public good is so high that his or her own contribution will be enough to provide the public good even given the minimum effort by others. Individual i is going to contribute  $\sigma_i = R - \underline{g}$ , as long as  $R - \underline{g} \leq \overline{\sigma}_i$ . In this case, individual i assures himself or herself that the public good be provided<sup>62</sup>. Secondly, if  $v_i + \underline{g} - \overline{g} < 0$ , the maximization condition for EU<sub>i</sub> is  $\sigma_i = 0$ .

Thus, unless individual i is able and willing to assure herself or himself that the public good be provided. She or he is not going to contribute.

### VCMA:

Suppose  $F(g_{\cdot i})$  has the same density function as discussed above and  $R \leq \overline{g} + \overline{\sigma}_i$ . Individual i's expected utility function can be written as(assuming individual is risk neutral):

<sup>&</sup>lt;sup>62</sup> This result is limited to a case with small group. For example, providing an answering machine in a busy office for 12 graduate students who are also part-time tutors. The person who gets the most tutoring appointments normally ends up providing the answering machine.

$$EU_{i} = \int_{\underline{g}}^{R-\sigma_{i}} U(w_{i}, 0) f(g_{-i}) dg_{-i} + \int_{R-\sigma_{i}}^{\overline{g}} U(w_{i} - \sigma_{i}, 1) f(g_{-i}) dg_{-i}$$

$$= \int_{R-\sigma_{i}}^{\overline{g}} (v_{i} - \sigma_{i}) f(g_{-i}) dg_{-i} \qquad \dots (6)$$

$$\frac{dEU_i}{d\sigma_i} = -1 + F(R - \sigma_i) + (v_i - \sigma_i) f(R - \sigma_i) \qquad \cdots (7)$$

$$\frac{d^2EU_i}{d\sigma_i^2} = -2 f(R - \sigma_i) - (v_i - \sigma_i) f'(R - \sigma_i) \quad \dots (8)$$

Again for easy analysis, assuming  $F(g_i)$  has an uniform distribution density function, equation (8) is automatically satisfied since  $\frac{-2}{\overline{g}-\underline{g}} < 0$ . This indicates that the expected utility function is concave and there is a maximum. Set equation (7) equal to zero, we get:  $\sigma_i = \frac{R - \overline{g} + v_i}{2}$ . The following gives individual i's contribution function:

$$\sigma_i = \begin{cases} 0 \text{ to } \overline{\sigma}_i & \text{if } R > \overline{g} + v_i \\ \frac{R - \overline{g} + v_i}{2} & \text{if } \overline{g} - v_i < R \le \overline{g} + v_i \\ 0 & \text{if } R < \overline{g} - v_i \end{cases}$$

Notice that individual i's contribution depends on not only the absolute values of R,  $\overline{g}$ , and  $v_i$  but also on the relationships of R,  $\overline{g}$ , and  $v_i$ . Given that  $R \leq \overline{g} + v_i$ , the higher the provision level R, ceteris paribus, the higher individual i's contribution.

the higher the maximum contributions by others, ceteris paribus, the lower the contribution by individual i. the higher the maximum valuation of the public good by individual i, ceteris paribus, the higher the contribution by individual i.

The total contributions by all the agents can be expressed as follows:

$$\Sigma_{i}\sigma_{i} = \begin{cases} \text{uncertain} & R > \max\{\overline{g} + v_{i}\} \\ \frac{n}{2}R - \frac{n-1}{2}\hat{g} + \frac{1}{2}\Sigma_{i}v_{i} & \min\{\overline{g} - v_{i}\} \leq R \leq \max\{\overline{g} + v_{i}\} \\ 0 & R < \min\{\overline{g} - v_{i}\} \end{cases}$$

where:  $\hat{g} = \overline{g} + \overline{\sigma}_i$  for all i.

Since the cost of providing the public good is pre-determined exogenous variable, it is not hard to see that the total contribution may or may not be enough to cover the cost of providing the good depending on the parameterizations of the total contribution function.

The above results are obviously valuable information for the supplier of the public good. Let's examine the supply-side incentives under the following situations:

# 2d. Supply-Side Incentives

We can start with one supplier, it can either be a non-profit organization or a profit-maximizer. Assuming that the supplier has the same beliefs as the agents and knows the above contribution functions under both VCMNA and VCMA. First of all the supplier has to decide whether to offer an assurance contract. If the supplier knows that at least one agent is able and willing to assure himself or herself by contributing

just enough given the minimum effort by others, the supplier does not have to offer an assurance contract, because the supplier can simply go after that agent. Otherwise, the supplier may have to offer an assurance contract and choose the right provision point R.

Under VCMA, the total contributions as a function of the provision point can be depicted in the following situations:

# Situation 1:

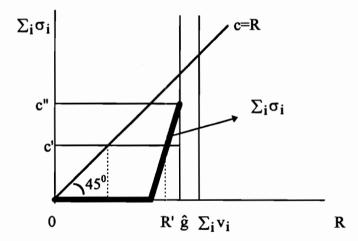


Figure 3. The total contribution function is below the 45 degree line.

As shown in figure 3, the total contribution function is always below the 45 degree line. If the supplier is a benevolent nonprofit organization<sup>63</sup> and has to propose

<sup>&</sup>lt;sup>63</sup> The supplier devotes all the contributions to the supply of the public good and does not make a profit.

R=c, the total contribution will always fall short of the cost. For example, assuming all the individuals are identical, In other words,  $w_i = w_j = w$  and  $v_i = v_j = v$  for  $i \neq j$ . Assume that  $c = c' = \frac{1}{8} \sum_i v_i = \frac{n}{8} v$  and  $\overline{g} = \frac{1}{2} \sum_{j \neq i} v_j = \frac{n-1}{2} v$ . For any n>4, we have  $R = c' < \overline{g} - v$ . From the contribution function above, We know that no one is going to contribute. Thus the public good will not be provided. Given a cost c', the only way the supplier can cover the cost is to propose a provision point R">R' in the above example. However, This may violate the assurance contract because the total contributions will always be short of the provision point R" even though it may exceed

the cost c'. If the cost for any supplier is greater than c", it is almost impossible for the

supplier to cover the cost no matter what level of R he or she proposes.

# Situation 2:

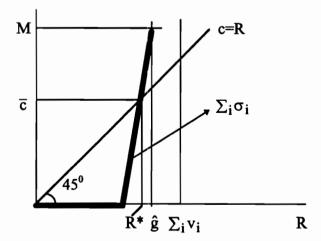


Figure 4. The total contribution function crosses the 45 degree line.

In figure 4, for any  $c < \overline{c}$ , the situation is the same as in situation 1. For any c in between  $\overline{c}$  and M, the total contributions exceed the provision point. Under this situation, the provision point  $R = \hat{g}$  gives the highest possible contributions. A profit-maximizer will certainly choose this provision point to maximize the profit. A nonprofit organization may choose this provision point too if it wants to maximize the budget. If the nonprofit organization is a benevolent nonprofit organization, the excess contribution can be simply refunded.

# 2e. Competition

Assuming that all the suppliers have the same cost function, we can set up the competition mechanism in such a way that the suppliers submit their provision points which can be treated as "bids", and the lowest "bid" will get the business. Thus, the focus of their competition is the provision point. Our intuition tells us that the provision point will be driven down to R\* under situation 2. From the last section, we understand that the lower the provision point, the lower the contribution level. So, competition may lower the chances for the supply of a public good.

# 2f. Two public goods

For simplicity, we assume that the two public goods are unrelated but being considered at the same time<sup>64</sup>. We could think of the two public goods as community

<sup>&</sup>lt;sup>64</sup> If the two public goods are considered during different time frame, the analysis becomes one public good case as discussed before.

streetlights and community pond. The costs of providing the above two public goods are  $c_1$  and  $c_2$ . We also assume complete information and individual i's valuation of the two public goods are  $v_i^1$  and  $v_i^2$  respectively.  $\sigma_i^1$  and  $\sigma_i^2$  are individual i's contributions for the two public goods. We have the following restrictions for individual i's contribution:  $o \le \sigma_i^1 \le v_i^1 \le w_i$  and  $o \le \sigma_i^2 \le v_i^2 \le w_i$ , for all i. It is again practical to assume that  $\sum_i w_i > c_1 + c_2$ . Let's discuss three different conditions:

Condition 1:  $\sum_i v_i^1 < c_1$  and  $\sum_i v_i^2 < c_2$ . Let's start with one supplier. It is Pareto inferior to provide any of the public good. Obviously, under any VCM, it is impossible to supply any of the public good, because no one is supposed to contribute more than her or his maximum valuation of the public good. From the above, we obtain:  $\sum_i (v_i^1 + v_i^2) < c_1 + c_2$ . Thus, unifying the two public goods shares the same fate as treating each one of them separately. It is also true that it is impossible to supply any of the public good when there exist multiple suppliers. Any VCM is efficient in the sense that none of the public goods will be provided. It is certainly not the case under taxation<sup>65</sup>.

Condition 2:  $\sum_1 v_i^1 \ge c_1$  and  $\sum_1 v_i^2 < c_2$ . As usual, we start with a single supplier. The supplier would not be able to collect enough money for the second

<sup>&</sup>lt;sup>65</sup> Putting the two public goods on the same ballot may succeed in providing the public goods even when both fail under separation. This is known as implicit logrolling, which is defined as when political interests succeed in pairing, on the same ballot, two(or more) issues of strong interest to divergent groups.

public good if the two public goods were considered separately. It is clear that there is no incentive for a private supplier to tie the two public goods under VCM; Otherwise, even the supplier may succeed in providing both public goods and the profit will be less since the second public good has to be subsidized by the first one. This result would still hold if there were multiple suppliers. However, there is a strong incentive for the government to pair the public goods under taxation.

Condition 3:  $\sum_1 v_i^1 \ge c_1$  and  $\sum_1 v_i^2 \ge c_2$ . If there is only one private supplier, under symmetric information, there doesn't seem to be any obvious advantage to separate or to tie the two public goods. On one hand, the monopoly may be able to reduce some of the costs of eliciting contributions by pairing the two public goods. One the other hand, pairing the two public goods may have adverse effect on the total contribution if there is an income effect<sup>66</sup>, especially, if  $w_i$  is considered current income. In this case, it is more likely for the public goods to be provided by the government.

<sup>66</sup> assuming decreasing marginal utility of public goods in general.

### **SECTION 3**

### CONTINUOUS PUBLIC GOODS MODEL

## 3a. No provision point and no assurance contract

Suppose the supplier is a benevolent nonprofit organization and it has a cost function c(g) which is assumed to be strictly increasing and convex:  $c(0)=\overline{c}$ ,

c'(g) > 0, and c''(g) > 0. We further assume that the supplier's cost function c(g) also common knowledge. The following is a standard Cournot-Nash model for individual i's maximization problem:

Individual i will choose  $\sigma_i$  to maximize her utility taking the contributions of others as given.

$$MaxU_i(x_i,g)$$

subject to  $x_i = w_i - \sigma_i$ 

$$\sum_{i} \sigma_{i} = c(g)$$
.

where: g is the amount the public good which totally depends on the total contribution level  $\sum_i \sigma_i$ .

The first order condition is: 
$$\frac{\frac{\partial}{\partial g}U_{i}(\cdot)}{\frac{\partial}{x_{i}}U_{i}(\cdot)} = c'(g) \qquad .....(9)$$

The above is known as the Cournot-Nash equilibrium.

However, Samuelsonion condition for Pareto Optimality is:

$$\sum_{i} \frac{\frac{\partial}{\partial g} U_{i}(\cdot)}{\frac{\partial}{\partial x_{i}} U_{i}(\cdot)} = c'(g) \qquad \dots (10)$$

We can rewrite (10) as: 
$$\frac{\frac{\partial}{\partial g}U_{i}}{\frac{\partial}{\partial x_{i}}U_{i}} = c'(g) - \sum_{j \neq i} \frac{\frac{\partial}{\partial g}U_{j}(\cdot)}{\frac{\partial}{\partial x_{j}}U_{j}(\cdot)}$$

If g and  $x_i$  are both normal goods, then  $\sum_{j\neq i}\frac{\frac{\partial}{\partial g}U_j(\cdot)}{\frac{\partial}{\partial x_i}U_j(\cdot)}>0$ . Thus, more g and

less x are consumed under (10) than  $(9)^{67}$ . This indicates that there is usually an underprovision of public good.

### Example:

Suppose individual i has a relative-risk aversion utility function and c'(g) = 1.

Individual i has the following maximization problem:

Max 
$$U_i = x_i^{1-\alpha} g^{1-\beta}$$
  $0 \le \alpha, \beta \le 1$ 

subject to  $x_i = w_i - \sigma_i$ 

The first order condition is: 
$$\frac{\frac{\partial}{\partial g}U_{i}(\cdot)}{\frac{\partial}{x_{i}}U_{i}(\cdot)} = \frac{(w_{i} - \sigma_{i})(1 - \beta)}{g(1 - \alpha)} = 1 \qquad .....(11)$$

Thus individual i's contribution is  $\sigma_i = w_i - \frac{1-\alpha}{1-\beta}g$  .....(12)

<sup>&</sup>lt;sup>67</sup> For detailed discussion, see Dennis C. Mueller, Public Choice II.

The total contribution 
$$\sum_{i} \sigma_{i} = \sum_{i} v_{i} - \frac{1-\alpha}{1-\beta}$$
 n g = g .....(13)

Solve the above equation we get the Cournot-Nash equilibrium level of public good:

$$g_{cn} = \frac{\sum_{i} w_{i}}{1 + \frac{(1 - \alpha)n}{1 - \beta}}$$
 .....(14)

From the Samuelsonion condition for Pareto Optimality, we have:

Rewrite the above, we get the Pareto-optimal supply of the public good:

$$g_{p} = \frac{\sum_{i} w_{i}}{1 + \frac{(1 - \alpha)}{1 - \beta}}$$
 .....(16)

 $g_{cn} < g_p$  for any n > 1, thus, there is an underprovision of the public good under the mechanism of no provision point and no assurance contract.

### **Expected Utility Model:**

Suppose individual i has a prior believe about the probability of the total contributions by others, which can be written as F(g.i), and it has a density function of

 $f(g_{-i})$ ,  $g_{-i} \in [\underline{g} \ \overline{g}]$ .  $F(\underline{g}) = 0$ . To make it transparent, we assume that  $f(g_{-i})$  is an uniform distribution.

For consistency and easy comparison, we also assume that c'(g) = 1, and individual i has a relative-risk aversion utility function:

$$U_i(x_i, g) = x_i^{1-\alpha} g^{1-\beta}$$
  $0 \le \alpha \le 1, 0 \le \beta \le 1$ 

Individual i choose  $\sigma_i$  to maximize his or her following expected utility:

$$\frac{d^{2}EU_{i}}{d\sigma_{i}^{2}} = \frac{-\alpha(1-\alpha)(w_{i}-\sigma_{i})^{-\alpha}\left[\left(\overline{g}+\sigma_{i}\right)^{2-\beta}-\left(\underline{g}+\sigma_{i}\right)^{2-\beta}\right]}{2-\beta}$$

$$+ \frac{-(1-\alpha)(w_{i} - \sigma_{i})^{-\alpha} \left[ (\overline{g} + \sigma_{i})^{1-\beta} - (\underline{g} + \sigma_{i})^{1-\beta} \right]}{2-\beta}$$

$$+ \frac{-(1-\alpha)(w_{i} - \sigma_{i})^{1-\alpha} (2-\beta) \left[ (\overline{g} + \sigma_{i})^{1-\beta} - (\underline{g} + \sigma_{i})^{1-\beta} \right]}{2-\beta}$$

$$+ \frac{(w_{i} - \sigma_{i})^{1-\alpha} (2-\beta)(1-\beta) \left[ (\overline{g} + \sigma_{i})^{-\beta} - (\underline{g} + \sigma_{i})^{-\beta} \right]}{2-\beta} \qquad ......(19)$$

The second-order derivative of the above is negative, which indicates that there exist a maximum for EU<sub>i</sub>. From the first-order condition we get the following:

$$\frac{(\overline{g} + \sigma_i)^{2-\beta} - (\underline{g} + \sigma_i)^{2-\beta}}{(\overline{g} + \sigma_i)^{1-\beta} - (\underline{g} + \sigma_i)^{1-\beta}} = \frac{(w_i - \sigma_i)(2-\beta)}{1-\alpha} \qquad \dots (20)$$

Let us look at a special case when  $\beta = 0$  and  $\alpha = 1/2$ , the solution becomes:

$$\sigma_{i} = \begin{cases} \frac{4w_{i} - (\overline{g} + \underline{g})}{6} & \text{if } w_{i} \ge \frac{1}{2} \left( \frac{\overline{g} + \underline{g}}{2} \right) \\ 0 & \text{if } w_{i} < \frac{1}{2} \left( \frac{\overline{g} + \underline{g}}{2} \right) \end{cases}$$
(21)

The above solution tells us that individual i will give a positive contribution if his or her endowment is greater than half of the average expected total contributions by others. Otherwise, individual i will choose not to contribute.

The maximum quantity of the public good given by (21) assuming everybody contributes is:

$$g_{eu} = \sum_{i} \sigma_{i} = \frac{2}{3} \sum_{i} w_{i} - \frac{n}{6} (\overline{g} + \underline{g}) \qquad ......(22)$$

However, from equation (16), when  $\beta=0$  and  $\alpha=1/2$ , the Pareto-optimal quantity is:  $g_p=\frac{2}{3}\sum_i w_i$ . Obviously,  $g_{eu}< g_p$ . This result further confirms the Cournot-Nash equilibrium under the same mechanism when there is no provision point and no assurance contract.

#### 3b. Provision point but no assurance contract:

Assuming that there is a minimum provision point R. If the total contribution is less than R, the public good will not be provided and the contributors will loose their contributions. If the total contributions is more than R, the quantity of the public good g will be determined by the level of the total contributions. Assuming c'(g) = 1,  $U_i(x_i, g) = x_i^{1/2} (1+g)$ . Given the same beliefs as in the expected utility model above, individual i will maximize his or her following expected utility:

$$\frac{dEU_{i}}{d\sigma_{i}} = \frac{-(w_{i} - \sigma_{i})^{1/2}}{2(\overline{g} - \underline{g})} \left[ R - \sigma_{i} - \underline{g} + (1 + \sigma_{i})(\overline{g} - R + \sigma_{i}) + \frac{\overline{g}^{2} - (R - \sigma_{i})^{2}}{2} \right] + \frac{(w_{i} - \sigma_{i})^{1/2}}{(\overline{g} - g)} \left[ \overline{g} + \sigma_{i} \right] \qquad \dots (24)$$

The second-order condition is satisfied for  $R - \sigma_i - \underline{g} \ge 0$ , and  $\overline{g} - R + \sigma_i \ge 0$ .

Set 
$$\frac{dEU_i}{d\sigma_i} = 0$$
, we obtain:

$$5\sigma_i^2 + (2R + 6\overline{g} - 4w_i)\sigma_i + \overline{g}^2 - R^2 + 2(\overline{g} - g) - 4w_i\overline{g} = 0$$
 .....(25)

If  $(2R+6\overline{g}-4w_i)^2-20[\overline{g}^2-R^2+2(\overline{g}-\underline{g})-4w_i\overline{g}]>0$ , there are two solutions to the above quadratic equation. Only the positive solution is significant. We can express (25) as  $\sigma_i=f(\overline{g},\ \underline{g},\ R,\ w_i)$ . Differentiating (25) with respect to R holding  $\overline{g}$ ,  $\underline{g}$ , and  $w_i$  constant, we get  $\frac{d\sigma_i}{dR}>0$ , and  $\frac{d^2\sigma_i}{dR^2}<0$ . In other words,

individual i's contribution is increasing in R and concave. It is reasonable to assume that the total contribution function is increasing in R and concave too. We can analyze the supply-side incentives by studying three different situations given the above results.

## Supply-side incentives:

Assuming that the supplier has the same beliefs as the agents do and the supplier knows the total contribution function. Let's examine the supplier's strategies given the following three different situations:

## Situation 1:

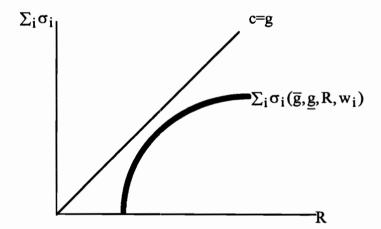


Figure 5. The total contribution function is below the cost function

In the above situation, If the supplier is a benevolent nonprofit organization and has to set R = c, it is almost impossible for the supplier to get enough contributions to

cover the cost. If the supplier can propose R > c, the supplier may be able to get enough contributions to cover the cost. Otherwise, the supplier will not be able to supply the public good.

### Situation 2:

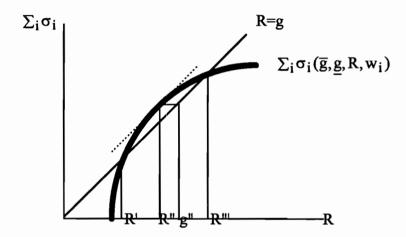


Figure 6. The total contribution function intersects the cost function.

The supplier will be able to get excess contributions if the supplier propose any provision point in between R' and R'". If the supplier is a profit-maximizer, R" should be chosen as the provision point and supply g = R". If the supplier is a benevolent nonprofit organization, the provision point can be chosen anywhere in between R' and R"; the supply of the public good equal to the total contributions. For example, if the benevolent nonprofit organization chooses R" as the provision point, the supply of the public good will be g" as depicted on figure 6. Another alternative is to refund the

excess contributions. Thus, the supply of the public good may be overprovision as well as underprovision depending on not only the parameterizations of the total contribution function but also on the incentives of the supplier.

#### Situation 3:

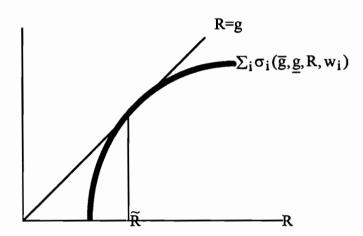


Figure 7. The total contribution function is tangent to the cost function

In figure 7, if the supplier propose R = g, the only provision point that covers the cost is  $\widetilde{R}$ . Obviously, a profit-maximizer has no incentive to enter this market unless the supplier is able to supply less than the proposed provision point.

## 3c. Provision point plus assurance contract:

Under the assurance contract, if the total contribution is less than the provision point R, the public good will not be provided and all the contributions will be refunded.

If the total contributions is more than R, the quantity of the public good g will be determined by the level of the total contributions. Assuming also c'(g) = 1,  $U_i(x_i, g) = x_i^{1/2} (1+g)$ . Given the same beliefs as in the expected utility model above, individual i will maximize his or her following expected utility:

$$\begin{split} EU_{i} &= \int_{\underline{g}}^{R-\sigma_{i}} U(w_{i}, \ 0) \ f(g_{-i}) \ dg_{-i} \ + \int_{R-\sigma_{i}}^{\overline{g}} U(w_{i}-\sigma_{i}, \ \sigma_{i}+g) \ f(g_{-i}) \ dg_{-i} \\ &= \int_{\underline{g}}^{R-\sigma_{i}} w_{i}^{1/2} \ \frac{1}{\overline{g}-\underline{g}} dg_{-i} \ + \int_{R-\sigma_{i}}^{\overline{g}} (w_{i}-\sigma_{i})^{1/2} (1+\sigma_{i}+g_{-i}) \frac{1}{\overline{g}-\underline{g}} \ dg_{-i} \\ &= \frac{w_{i}^{1/2}}{\overline{g}-\underline{g}} \Big[ R-\sigma_{i}-\underline{g} \Big] \ + \frac{(w_{i}-\sigma_{i})^{1/2}}{\overline{g}-\underline{g}} \Bigg[ (1+\sigma_{i})(\overline{g}-R+\sigma_{i}) + \frac{\overline{g}^{2}-(R-\sigma_{i})^{2}}{2} \Bigg] \\ &\qquad \qquad \dots \dots (26) \end{split}$$

$$\frac{dEU_{i}}{d\sigma_{i}} = \frac{-w_{i}^{1/2}}{\overline{g} - \underline{g}} + \frac{-(w_{i} - \sigma_{i})^{-1/2}}{2(\overline{g} - \underline{g})} \left[ (1 + \sigma_{i})(\overline{g} - R + \sigma_{i}) + \frac{\overline{g}^{2} - (R - \sigma_{i})^{2}}{2} \right] + \frac{(w_{i} - \sigma_{i})^{1/2}}{\overline{g} - \underline{g}} [\overline{g} + \sigma_{i} + 1] \qquad \dots (27)$$

$$\frac{d^{2}EU_{i}}{d\sigma_{i}^{2}} = \frac{-(w_{i} - \sigma_{i})^{-3/2}}{4(\overline{g} - \underline{g})} \left[ (1 + \sigma_{i})(\overline{g} - R + \sigma_{i}) + \frac{\overline{g}^{2} - (R - \sigma_{i})^{2}}{2} \right]$$
$$+ \frac{-(w_{i} - \sigma_{i})^{-1/2}}{\overline{g} - \underline{g}} [\overline{g} + \sigma_{i} + 1] + \frac{(w_{i} - \sigma_{i})^{1/2}}{\overline{g} - \underline{g}}$$

$$= \frac{(w_{i} - \sigma_{i})^{-3/2}}{-4(\overline{g} - \underline{g})} \left[ (1 + \sigma_{i})(\overline{g} - R + \sigma_{i}) + \frac{\overline{g}^{2} - (R - \sigma_{i})^{2}}{2} + 4(w_{i} - \sigma_{i})(\overline{g} + 2\sigma_{i} - w_{i} + 1) \right]$$
.....(28)

For 
$$\overline{g} - R + \sigma_i \ge 0$$
 and  $\overline{g} + 2\sigma_i - w_i + 1 \ge 0$ , we have  $\frac{d^2EU_i}{d\sigma_i^2} < 0$ .

Set 
$$\frac{dEU_i}{d\sigma_i} = 0$$
, we get:

$$4w_i^{1/2}(w_i - \sigma_i)^{1/2} = -5\sigma_i^2 + (4w_i - 6\overline{g} - 6)\sigma_i - \overline{g}^2 + R^2 - 2\overline{g} + 2R + 4w_i\overline{g} + 4w_i$$
.....(29)

The solution to equation (29) can be expressed as:  $\sigma_i = f(\overline{g}, R, w_i)$ . This result is very close to the result obtained in section 3b except that individual i's contribution does no longer depend on  $\underline{g}$ . It makes perfect sense! Under no assurance contract, individual i may loose his or her contribution if R is not met. The higher  $\underline{g}$ , the more likely that R being met, thus the more individual i is willing to contribute. This intuition can be verified by the fact that  $\frac{d\sigma_i}{d\underline{g}} = \frac{2}{10\sigma_i + 2R + 6\overline{g} - 4w_i} > 0$ , which can be obtained from equation (25). However, under assurance contract, all the contributions will be refunded if the provision point R is not met. The only influencing factors in individual i's decision are R,  $\overline{g}$  and  $w_i$ .

Let's examine the two contribution functions given by equation (25) and (29).

Set 
$$\sigma_i = 0$$
, from equation (25), we obtain:  $R_1 = \sqrt{\overline{g}^2 + 2\overline{g} - 2\underline{g} - 4w_i\overline{g}}$ ; from (29)

we obtain:  $R_2 = \sqrt{\overline{g}^2 + 2\overline{g} - 2R - 4w_i\overline{g}}$ .  $R_2 \ge R_1$ , for any  $R \ge \underline{g}$ . The slopes of the two contribution function are given as the following:

Equation (25) gives: 
$$\frac{d\sigma_i}{dR} = \frac{2R - 2\sigma_i}{10\sigma_i + 2R + 6\overline{g} - 4w_i}$$
 ......(30)

Equation (29) gives: 
$$\frac{d\sigma_i}{dR} = \frac{2R+2}{10\sigma_i + 6\overline{g} + 6 - 4w_i - 2w_i^{1/2}(w_i - \sigma_i)^{-1/2}} \quad .....(31)$$

For any R>3, the slope under equation (31) is greater than that given by equation (30).

Thus, given the same provision point, the mechanism with assurance contract elicits more contributions than without assurance contract. This can be illustrated as follows:

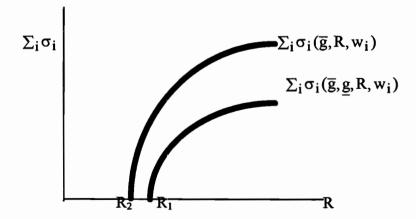


Figure 8. The total contribution functions under two mechanisms

The supply-side analysis resembles the analysis as in section 3b. Repetition is not necessary.

## 3d. Competition:

Assuming that all the suppliers have the same cost function and they all propose a provision point  $^{68}$ . Since  $\overline{g}$ ,  $\underline{g}$ , and  $w_i$  are predetermined. Any effort by any one of the suppliers to change the above will be equally beneficial to the other suppliers. Thus the focus of the competition will be the provision point and the ex post supply of the public good. Given a fixed ex post supply of the public good, a lower provision point is obviously preferred by the agents than a higher provision point. Given a fixed provision point, a higher ex post supply of public good is preferred to a lower ex post supply of the public good. However, it is not clear whether there exists a unique equilibrium.

If the suppliers do not have give a provision point and assurance contract. No provision point is obviously favored both by the supplier and the agents. For example, given the same contribution of by individual i, if the total contribution is less than R, individual i will be worse off with the assurance contract than without because individual i will at least get some provision of the public good under no assurance contract; if the total contribution is more than R, individual i is indifferent between the two mechanisms. In fact, the supplier's cost function itself offers a certain level of

<sup>&</sup>lt;sup>68</sup> One plausible reason is that the suppliers want to cover at least the fixed cost.

assurance if the cost function is common knowledge. If the suppliers are also risk averse, offering provision point is obviously inferior if some other suppliers offer no provision point.

From previous discussions we know that there will be an underprovision of public goods under one benevolent non-profit organization without offering assurance contract. Under competition, if the public good can be provided jointly by more than one suppliers and none of them offer assurance contracts, can the total supply end up being Pareto optimal or even overprovision? This needs further investigation.

### **SECTION 4**

### CONCLUDING REMARKS

When the public good is discrete, there are many pure Nash equilibria under both VCMNA and VCMA. VCMA is more appealing than VCMNA in the sense that an individual cannot be worse off by giving a small contribution under VCMA. However, VCMA does not guarantee the Pareto-optimal outcome.

Rapoport's expected utility model introduces individual's beliefs about the number of contributors. The model predicts that there is more incentive to contribute under VCMA than under VCMNA. Only when the provision point requires unanimous contributions, contributing becomes the dominate strategy for all agents. The Rapoport's model is only limited to binary choice.

This paper extends the Rapoport's model to a continuous choice model in the provision of a discrete public good. The results show that it is very likely that there will be zero contribution under VCMNA unless there exists at least one agent who is able and willing to assure the provision of the public good given the minimum effort by others. Under VCMA, individual i's contributions depend on not only the absolute values of R,  $\overline{g}$ , and  $v_i$  but also on the relationship of R,  $\overline{g}$ , and  $v_i$ .

The results from the continuous choice model of a discrete public good provide a valuable information for analyzing the supply-side incentives. The supplier

may or may not be able to elicit enough contributions to cover the cost given different parameterizations and supply-side incentives.

Competition put downward pressure on the provision point, thus lowers the probability of supplying the public good.

There is no incentives for a private supplier to tie the public goods if there exist more than one public goods. However, this is not the case under government supply through taxation because of implicit logrolling.

There is usually an underprovision of public goods in standard Cournot-Nash model when the public good is continuous. The continuous choice model under discrete public goods case is extended to the analysis of the continuous public goods model. The total contribution function provides the supplier information on choosing the right provision point. Overprovision as well as underprovision of the public good may occur under VCMA. Again, competition put downward pressure on the provision points proposed by the suppliers. The provision point will be driven to zero if the suppliers do not have a fixed cost

Future research is needed on the efficiency analysis in the continuous public goods model when the public goods can be jointly supplied by more suppliers.

#### APPENDIX A

#### INSTRUCTIONS FOR MIDTERM

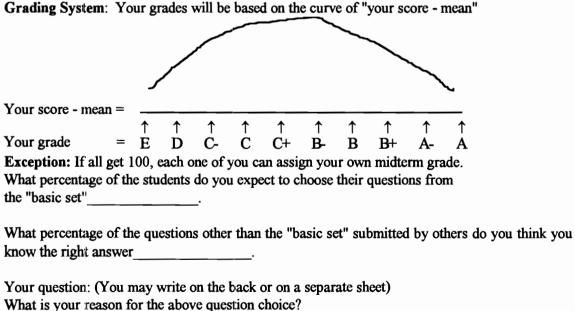
Each student is supposed to construct one and only one multiple-choice question with the answer and hand it in to me at my office by (deadline). Please make sure your question material is in (the textbook) which has been covered by my lectures. You may choose your question from the following sources:

- 1. Choose from the "basic set" which I provided to everyone in the class.
- 2. Design your own (You may choose from sources like other instructor's exams, test, etc).

Your instructor has no preference over your choices. The decision is yours. There are a total of 50 students, so the total number of questions will be 50. It will be a two-hour exam as scheduled on the syllabus.

#### Rules:

- 1. Please hand in your question to me during my office hours( ) or make appointment with me. I will see you one at a time and go over your question with you in case I have to make any changes. You may even ask me to make one question for you and explain it to you, if you want to choose your own question, but have a hard time to come up with one.
- 2. You can make the question as hard as you wish. Any question is acceptable as long as it is related to the materials in the textbook which were covered by my lectures.
- 3. Don't ask & don't tell. Each student should keep his or her question confidential. Violators will be severely punished. Exposure of your question to any other students can only be to your disadvantage, because the question any person turn in may not be the question he or she has exposed to others; and your grades will be curved on your relative performance.
- 4. You are allowed to bring the "basic set" to the exam.



#### APPENDIX B

This survey is completely voluntary and anonymous but very helpful. Please give your honest answers to the following simple questions:

- Have you ever taken Economics classes before?
   If yes, please give the course number(e.g. Econ200)
- 2. Why do you take this class?
- 3. If you didn't have to take this class, would you still take it?
- 4. Do you find Economics interesting?
- 5. What comments about Economics have you heard from other students?
- 6. How many classes are you taking this quarter?
- 7. How do you allocate your time for the courses that you are taking?
- 8. Do you care about what grade you will get at the end of the quarter?
- 9. Do you have any expectations about this class at all?
- 10. How many hours a week do you plan to study for this class?
- 11. Which of the following fits your true attitude?
  - a. Get an A with minimum effort if possible
  - b. Get an B with minimum effort if possible
  - c. I don't care about the grade as long as I can learn something useful
  - d. To be at the top of the class with whatever effort necessary
  - e. Pass the class with minimum effort
  - f. Other (specify)
- 12. With what grade do you start to feel very unhappy?

#### APPENDIX C

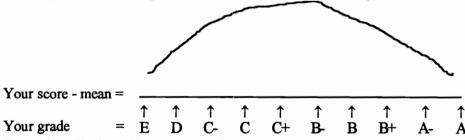
#### INSTRUCTIONS FOR MIDTERM I

Every student is supposed to construct one and only one multiple-choice question with answer and hand it in by Monday-April 19. I will choose only 50 questions from all the questions that handed in. But, if there were less than 50 students hand in questions, I would make up the rest (my questions would be extremely hard.) Every question made by the students will be on the exam exactly the same as the original. Every student's question will be kept absolutely secret. I also provide a "basic set of questions" with answers to the class (this set of questions covers only the basic knowledge). You may choose your question from the following sources:

- 1. choose from the "basic set of questions" (this is available to everyone in this class).
- 2. design your own
- 3. choose from any other sources(study guide, test bank, etc.)

- Rules: 1. Each student is required to provide one and only one multiple-choice question and the answer to it. If any one is not sure about the correct answer, talk to me privately before handing it in.
  - 2. You can make the question as hard as you wish. Any question is acceptable so long as it is related to the materials in the textbook or covered in the class.
  - 3. Each student should keep his or her question confidential. No discussion among the students is allowed regarding the midterm questions. Violators will receive 0 grade on the midterm.
  - 4. The midterm exam will be closed book and closed notes. But you may bring the "basic set of questions".

Grading System: Your grades will be based on the curve of "your score - mean"



Exception: If all get 100, each one of you can assign your own midterm grade.

Your social security number

(in case I need to correct your question)

Your Question:

#### APPENDIX D

#### INSTRUCTIONS FOR MIDTERM I

Every student except graduating seniors is supposed to construct one and only one multiple-choice question with answer and hand it in by Tuesday-April 21. Please come to my office during my office hours starting April 14. I will put all the questions that handed in on the exam in their original form (approximately 50 questions). Every student's question will be kept absolutely secret. I also provide a "basic set of questions" with answers to the class(this set of questions covers only the basic knowledge). You may choose your question from the following sources:

- 1. choose from the "basic set of questions" (this is available to everyone in this class).
- 2. design your own
- 3. choose from any other sources(study guide, test bank, etc).

Rules: 1. Each student is required to provide one and only one multiple-choice question and the answer to it. If any one is not sure about the correct answer, talk to me privately before handing it in.

- 2. You can make the question as hard as you wish. Any question is acceptable so long as it is related to the materials in the textbook or covered in the class.
- 3. Each student should keep his or her question confidential. No discussion among the students is allowed regarding the midterm questions. Violators will receive 0 grade on the midterm.
- 4. The midterm exam will be closed book and closed notes. But you may bring the "basic set of questions".

Grading System: Your grades will be based on the curve of "your score - mean"

Your score - mean =

Tour score - mean -

Your grade =  $\stackrel{\uparrow}{E}$   $\stackrel{\uparrow}{D}$   $\stackrel{\uparrow}{C}$   $\stackrel{\uparrow}{C}$   $\stackrel{\uparrow}{C}$   $\stackrel{\uparrow}{D}$   $\stackrel{\uparrow}{B}$   $\stackrel{\uparrow}{B}$   $\stackrel{\uparrow}{B}$   $\stackrel{\uparrow}{B}$   $\stackrel{\uparrow}{A}$   $\stackrel{\uparrow}{A}$ 

Exception: If all get 100, each one of you can assign your own midterm grade.

Your social security number

(in case I need to correct your question)

Your Question:

#### APPENDIX E

#### INSTRUCTIONS FOR MIDTERM I

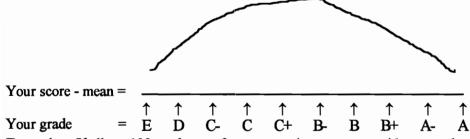
Every student is supposed to construct two and only two multiple-choice questions with answers and hand it in by Tuesday April 20. Every question made by the students will be on the exam exactly the same as the original. Thus the total number of questions on the exam will be twice as many as the total number of students in this class. If some students failed to hand in any questions, I will make up the missing questions(my questions are very hard). Every student's question will be kept absolutely secret. I also provide a "basic set of questions" with answers to every one(this set of questions covers only the basic knowledge). You may choose your question from the following sources:

- 1. choose from the "basic set of questions" (this is available to everyone in this class).
- 2. design your own
- 3. choose from any other sources(study guide, test bank, etc).

Rules: 1. Each student is required to provide one and only one multiple-choice question and the answer to it. If any one is not sure about the correct answer, talk to me privately before handing it in.

- 2. You can make the question as hard as you wish. Any question is acceptable so long as it is related to the materials in the textbook or covered in the class.
- 3. Each student should keep his or her question confidential. No discussion among the students is allowed regarding the midterm questions. Violators will receive 0 grade on the midterm.
- 4. The midterm exam will be closed book and closed notes. But you may bring the "basic set of questions".

Grading System: Your grades will be based on the curve of "your score - mean"



Exception: If all get 100, each one of you can assign your own midterm grade.

Your social security number

(in case I need to correct your question)

Your Question:

#### APPENDIX F

#### INSTRUCTIONS FOR FINAL EXAM

Each student is supposed to construct two and only two multiple-choice questions with the answers and hand it in to me at my office by (2:00pm Friday). I will be in my office Wednesday 1:00pm-3:00pm; Thursday 10:00-11:50am and 1:00pm-3:00pm; Friday 10:00am-2:00pm. Please make sure your question material is in (the textbook) which has been covered by my lectures. You may choose your questions from the following sources:

- 1. Choose from the "Study Guide" which I provided to everyone in the class.
- 2. Design your own (You may choose from sources like other instructor's exams, test, etc).

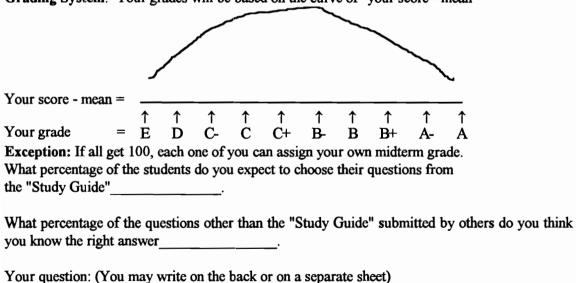
Your instructor has no preference over your choices. The decision is yours. There are a total of 25 students, so the total number of questions will be 50. It will be a two-hour exam as scheduled on the syllabus.

#### **Rules:**

- 1. Please hand in your questions to me during my office hours or make appointment with me. I will see you one at a time and go over your questions with you in case I have to make any changes. You may even ask me to make one question for you and explain it to you, if you want to choose your own question, but have a hard time to come up with one.
- 2. You can make the question as hard as you wish. Any question is acceptable as long as it is related to the materials in the textbook which were covered by my lectures.
- 3. Don't ask & don't tell. Each student should keep his or her questions confidential. Violators will be severely punished. Exposure of your question to any other students can only be to your disadvantage, because the question any person turn in may not be the question he or she has exposed to others; and your grades will be curved on your relative performance.
- 4. You are allowed to bring the Xeroxed "Study Guide" to the exam.

What is your reason for the above question choice?

Grading System: Your grades will be based on the curve of "your score - mean"



#### APPENDIX G

#### Instructor: Lee

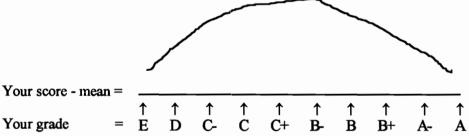
#### INSTRUCTIONS FOR THE FINAL EXAM

The results of this survey will help me determine whether I will try a new type of exam for the final. It will give the students an opportunity to choose the content of the final exam. Each student is supposed to submit one and only one multiple-choice question with the answer to the instructor. A basic set of 40 questions and the answers will be provided to everyone before the exam. Thus, every student has a choice of either choosing a question from the basic set or making up his or her own.

If you construct your own question, you may make it as hard as you wish provided the information is covered in the textbook or the lecture notes. You may discuss with the instructor about your question before you turn it in.

A copy of the basic set will be provided at the exam. If you choose a question from the basic set, everyone is entitled to the answer.

All the questions submitted will be used for the final exam. The students' grades will be curved on the relative performance (which is judged by the difference between your score and the mean). Exposure of your question to other students may be to your disadvantage. The following is a typical normal distribution curve of students' grades (the actual result may differ):



Exception: if every student gets a perfect score, everyone will get an A for the final exam.

Your course grade will be the weighted average of your midterms and the final, which is described in detail in the Syllabus.

This survey is completely voluntary and anonymous but very helpful. Please give your honest answers to the following simple questions:

What is your choice? the basic set, or your own question	
What is your reason for the above choice?	
What grade are you expecting from this class?	
What percentage of the students do you think will choose questions from the basic set	<u>_</u> .

# APPENDIX H

This survey is completely voluntary and anonymous but very helpful. Please give your
honest answers to the following simple questions:
1. Have you ever taken Economics classes before? yes; no
If yes, please give the course number(e.g. Econ200)
2. If you didn't have to take this class, would you still take it? yes ; no
3. Do you care about what grade you will get at the end of the quarter? yes ; no
4. What do you think about Economics?
a. interesting and useful
b. useful but not interesting
c. interesting but not useful
d. neither interesting nor useful
5. What comments about Economics have you heard from other students?
a. interesting and useful
b. useful but not interesting
c. interesting but not useful
d. neither interesting nor useful
6. How many classes are you taking this quarter?
a. one
b. two
c. three
d. four
e. five or more
7. How many hours a week do you plan to study for this class?
a. 0-1
b. 2-5
c. 6-9
e. 10-14
f. 15 or more
g. undecided
8. Which of the following fits your true attitude?
a. get an A with minimum effort if possible
b. get an B with minimum effort if possible
c. I don't care about the grade as long as I can learn something useful
d. to be at the top of the class with whatever effort necessary
e. pass the class with minimum effort
f. other (specify)
9. With what grade or below do you start to feel unhappy?
A A- B+ B B- C+ C C- D E
10. Are you taking this class for credit? yes; no
10. Ale you aiking and class for cleant? yes, no

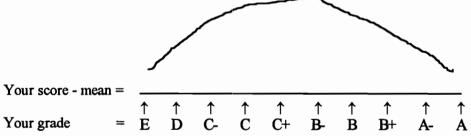
#### APPENDIX I

Instructor: Bump

Bonus question: 4 points

#### A SURVEY FOR FUTURE FINAL EXAM

Here is a plan I am considering using next Quarter. This plan will allow students to choose the content of the final exam. Suppose the students are going to make a choice for the final exam given the following information. I will provide a public set of multiple-choice questions(40) and answers before the exam. This public set of questions covers the fundamental knowledge of the course. Each student is supposed to submit one and only one multiple-choice question with the answer to the instructor. Every student has a choice of either choosing a question from the public set or making up his or her own. No communication is allowed among the students. All the questions submitted will be used on the final exam, thus the maximum number of questions on the final exam is the total number of the students in the class. The students are allowed to bring the basic set to the exam. If you choose a question from the public set, it is most likely that everyone in the class knows the answer. If you construct your own question, you may make it as hard as you wish provided the information is covered in the textbook or the lecture notes. The answer to the your own question is entitled to you only. The students' grades will be curved on the relative performance (which is judged by the differences between your score and the mean). Exposure of your question to other students will be to your disadvantage, since the grades will be curved on your relative performance. The following is a typical normal distribution curve of students' grades:



Each letter grade contains about 10% of the total students. For example, if your score is top 10%, you will get an A.

Exception: if every student gets a perfect score, everyone will get an A. Please complete:

What percentage of the students do you think will choose questions from the public set

What would be your choice? the public set,or your own question	<u>-</u> ·
Your social security number  Your answers and your identity are considered confidential in this survey.	

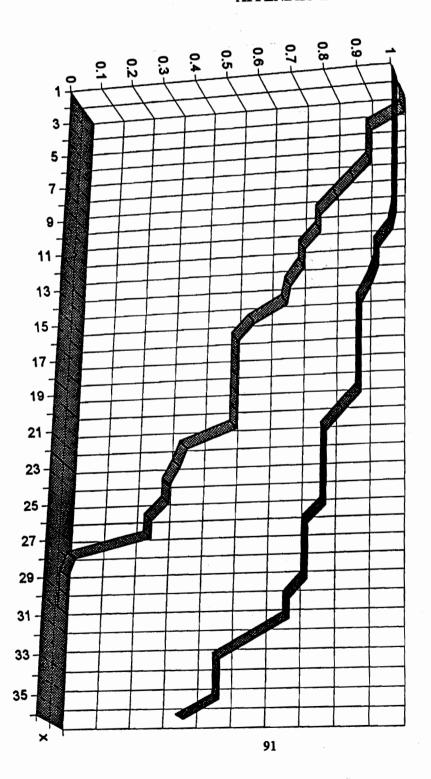
## APPENDIX J

This survey is completely voluntary and anonymous but very helpful. Please give your
honest answers to the following simple questions:
1. Have you ever taken Economics classes before? yes; no
If yes, please give the course number(e.g. Econ200)
2. If you didn't have to take this class, would you still take it? yes; no
3. Do you care about what grade you will get at the end of the quarter? yes; no
4. What do you think about Economics?
a. interesting and useful
b. useful but not interesting
c. interesting but not useful
d. neither interesting nor useful
5. What comments about Economics have you heard from other students?
a. interesting and useful
b. useful but not interesting
c. interesting but not useful
d. neither interesting nor useful
6. How many classes are you taking this quarter?
a. one
b. two
c. three
d. four
e. five or more
7. How many hours a week do you plan to study for this class?
a. 0-1
b. 2-5
c. 6-9
e. 10-14
f. 15 or more
g. undecided
8. Which of the following fits your true attitude?
a. get an A with minimum effort if possible
b. get an B with minimum effort if possible
c. I don't care about the grade as long as I can learn something useful
d. to be at the top of the class with whatever effort necessary
e. pass the class with minimum effort
f. other (specify)
9. With what grade or below do you start to feel unhappy?
A A- B+ B B- C+ C C- D E
10. Are you taking this class for credit? yes; no.
, , , , , , , , , , , , , , , , , , , ,

# APPENDIX K

Class A							
X	у	nx	ny	average x	average y	var(x)	var(y)
0.75	0.5	36	30	0.8325	0.530333	0.028071	0.088577
0.9	0.5						
0.5	0.5						
1	0.5						
1	0.5						
0.9	0.7						
0.95	0						
0.5	0.25						
0.8	0.25						
0.85	0.3						
0.5	0.33						
0.7	0						
1	0						
0.8	0.02						
0.75	0.9						
1	11						
0.75	0.9						
0.99	1						
0.9	0.9						
0.75	0.75						
1	0.75						
0.8	0.65	-					
0.9	0.7						
0.9	0.55						
1	0.3						
1	0.8						
1	0.66						
0.95	0.5						
0.93	0.85						
0.9	0.35						
1							
0.7							
0.8	1						
0.8							
0.6		·					<del>-</del>
0.4	<del></del> -	<del> </del>					
		+					-

# APPENDIX L

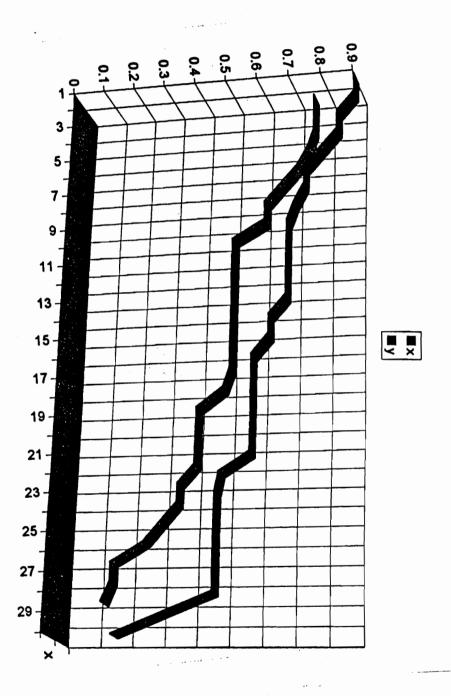


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# APPENDIX M

-	Class B								
X	у	nx	ny		average y	var(x)	var(y)		
0.72	0.125	30	28	0.632667	0.474464	0.023933	0.031363		
0.6	0.5								
0.65	0.35								
0.75	0.75								
0.6	0.6								
0.7	0.4								
0.9	0.15								
0.5	0.65								
0.5	0.35								
0.7	0.6								
0.5	0.4								
0.35	0.5								
0.5	0.48								
0.7	0.75								
0.51	0.75								
0.8	0.5								
0.65	0.5								
0.85	0.3	,							
0.85	0.4								
0.6	0.25								
0.6	0.5								
0.6	0.73								
0.5	0.5								
0.75	0.4								
0.7	0.7								
0.7	0.5								
0.2	0.15								
0.5	0.5								
0.9									
0.6									

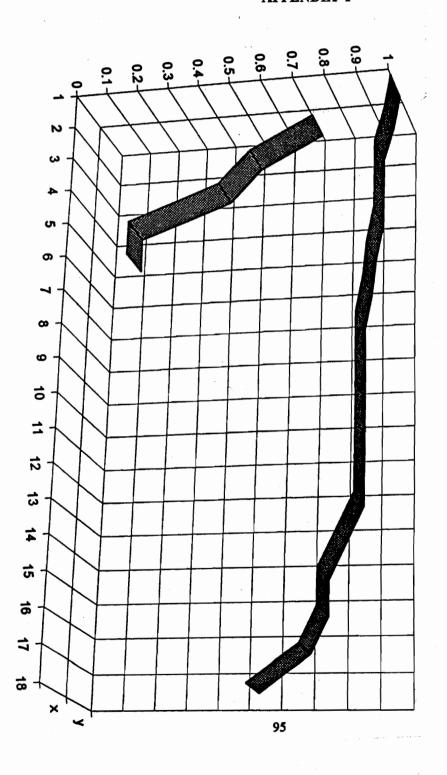
# APPENDIX N



## APPENDIX O

· · · · · · · · · · · · · · · · · · ·	Class 91								
<b>X</b> .	у	nx	ny	average x	average y	var(x)	var(y)		
0.75	0.7	18	5	0.882222	0.36	0.008584	0.0544		
8.0									
0.8									
0.6									
0.85									
0.95									
0.93									
0.9									
0.95									
0.95									
1									
0.92									
0.9									
0.9									
0.9						1			
0.9									
0.98									
0.9									

## APPENDIX P

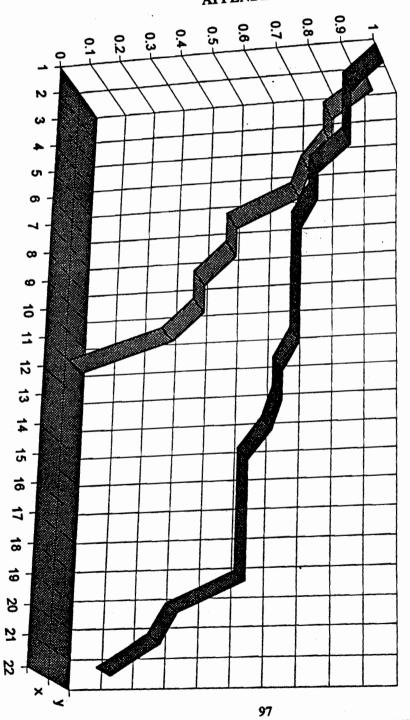


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# APPENDIX Q

	Class 92							
×	у		ny	average x	average y	var(x)	var(y)	
1	0.8	22	10	0.685	0.550909	0.034134	0.040544	
8.0	0.8							
0.67	0.73							
0.9								
0.75								
0.75								
0.75								
0.6								
0.6								
0.6								
0.2								
0.6								
0.35								
0.4								
0.9				<u> </u>				
0.75			`					
0.8		,	ξ					
0.6								
0.0						ļ <u></u>	ļ <u>.</u>	
0.7							ļ	
0.75								
0.7	1							



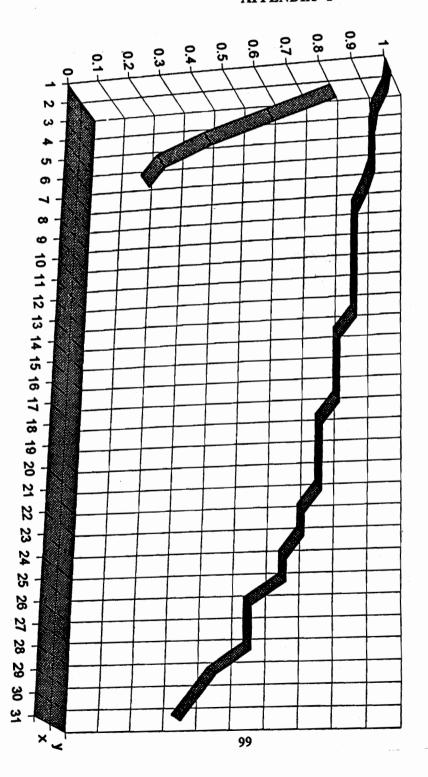


**⊠** ■ Y

# APPENDIX S

	Class 81							
×	у	nx	ny	average x	average y	var(x)	var(y)	
0.75	0.25	31	5	0.799323	0.45	0.024944	0.05	
0.7	0.4							
0.75	0.2							
0.7	0.8							
0.6	0.6							
0.6					•			
0.4								
0.6								
0.45								
0.5								
0.95								
0.93								
0.8								
0.95								
0.99								
0.85			1					
0.9		:						
0.9								
0.9								
8.0								
0.959								
0.9								
0.85	5							
0.85								
0.95								
0.8								
0.85								
0.8								
0.9								
0.9	9							

# APPENDIX T

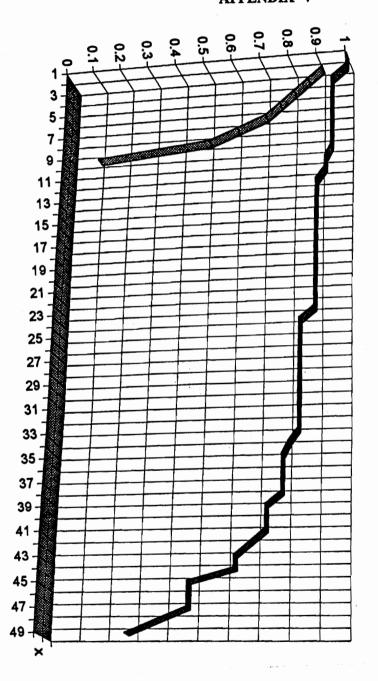


■ ×

# APPENDIX U

Class 82							
x	у	nx	ny	average x	average y	var(x)	var(y)
0.95		49	8	0.822041	0.65	0.023171	0.058125
0.82							0.000120
0.5							
0.5							
0.3	0.1						
0.65	0.6		-				
0.65	0.9						
0.4	0.5						
0.5							
0.7							
0.75							
0.75	_						
0.75							·
0.85					-		
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0.95							
0.85							
0.95			<del> </del>			-	
0.9							
1							
0.8							
1							
0.85		<b></b>	-				
0.9			<del></del>				
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0.9			-				
0.85							
0.85							
0.9							
0.93							
0.95							
0.93							
0.8		-					
0.9							
0.9							
0.8							
0.9							
0.9							
0.85							
0.85							
0.85							
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0.95							
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0.85							
0.85							
0.93							
0.95					-		
0.85							

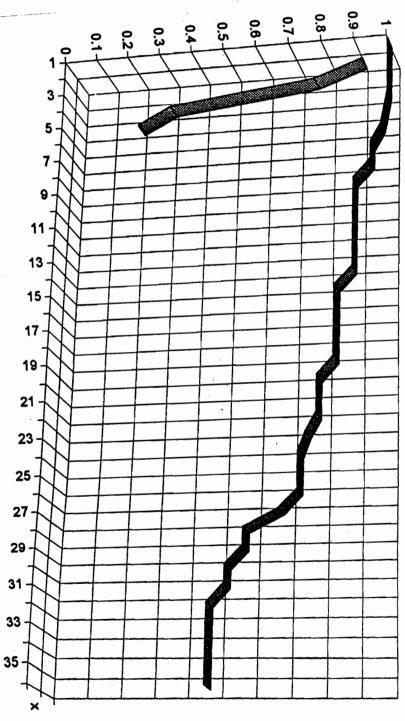
# APPENDIX V



■ ■ y

# APPENDIX W

Class51			<u> </u>				
×	у	nx	ny	average ¥	average y	var(x)	1/05/11
0.8	y 0.75	36	4	0.79	0.5375	0.028217	var(y) 0.086719
0.5	0.2				0.0070	0.020217	0.000718
0.85	0.9						
0.6	0.3						
0.85							
0.75							
0.9							
1							
0.95				-			
0.5			-	<del></del>			
0.99				-			
0.85							
1							
0.9							
0.8							
1				-			
0.9				-			
0.9	-						
1	-						
0.55							
0.5							
0.85							
0.7							
0.5				-			
0.98							
0.5				-			
0.75							
0.85							
0.75		:					
0.8							
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0.55	-						
0.9				-			
0.77							
0.6							
0.9							
0.5			<u></u>				

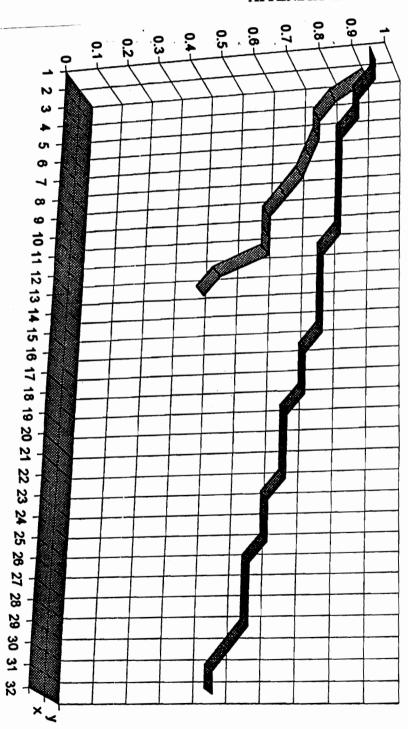


■ ■ Y X

# APPENDIX Y

Class52							
X	y	nx	ny	average x	average y	var(x)	var(y)
0.6	0.6	32	12	0.742188	0.660417	0.01533	0.018485
0.85	0.6				3.000717	0.01000	0.010403
0.75	0.7						
0.55	0.9						
0.6	0.725		1111				
0.9	8.0						
0.75	0.75					· · · · · · · · · · · · · · · · · · ·	
0.65	0.45					·	
0.75	0.6						
0.8	0.65						
0.7	0.4						
0.85	0.75						
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8.0							
0.7						· · · · · · · · · · · · · · · · · · ·	
0.5							
0.85		······································			-		
0.85							
0.9		· · · · · · · · · · · · · · · · · · ·					
0.85							







#### APPENDIX ZA

# AN ANONYMOUS SURVEY FOR FINAL EXAM

This survey result will help me to decide the kind of final exam I will give. There are two choices, one is the traditional exam which is just like the midterm; another is a "conditional exam". The "conditional exam" is that I provide you a basic set of questions and the answers before the exam. This basic set of questions covers the fundamental knowledge of the course. The final exam questions will be exactly the questions in the basic set. The condition for "conditional exam" is that everyone votes for the "conditional exam". Otherwise, I will go back to the traditional exam. If the "conditional exam" is adopted, it is very likely that all of you get an A for the final. Your course grade will be the average of the final and the midterm. However, you can still get an A in the traditional exam according to the curve described on the Syllabus. I have no preference over your choices. The decision is yours. Discussion with others is not allowed. If you are not clear about anything or have any questions, please ask me only. Your choice: traditional exam \_\_\_\_\_\_\_; "conditional exam" \_\_\_\_\_\_\_; "what is your reason for the above choice?

When you finish, please fold it up and I will collect them. I will tell you the result next time in class.

#### APPENDIX ZB

# A SURVEY FOR THE MIDTERM EXAM

The results of this survey will help me determine the type of exam I will administer for the midterm. There are two choices: The traditional exam and the conditional exam.

The traditional exam questions will be provided by the instructor at the time the exam is given. Grades will be curved as described in the Syllabus.

For the conditional exam, the instructor provides a basic set questions and the answers. The basic set of questions will be given to the students before the exam. The condition for conditional exam is that everyone has to vote for the conditional exam. Otherwise, I will use the traditional exam.

If the conditional exam is adopted, it is very likely that all of you will get an A for the midterm, unless you never attend the lectures. However, you can still get an A in the traditional exam according to the curve described in the Syllabus.

Your grade for the course will be the average of the midterm and the final regardless of other students' grades. For example, if you get an A for the midterm and an C for the final, your course grade will be B. The final exam will be traditional exam. I have no preference over your choices. The decision is yours. Discussion with others is not allowed. If you are not clear about anything or have any questions, please ask me only.

What percentage of the students do you think will choose the conditional exam?

Your choice: traditional exam; or conditional exam What is your reason for the above choice?	
What grade are you expecting from this class?  After you finish the above questions, please fold it up, and I will collect them. I w the results next time in class.	ill let you know

# APPENDIX ZC

#### A SURVEY FOR FINAL EXAM

This survey result will help me to decide the kind of final exam I will give. There are two choices, one is the traditional exam which is just like the midterms; the other is "conditional exam". The "conditional exam" is to have the students choose the questions for the final exam. All the questions will be multiple-choice. I will provide a basic set of questions (40) and answers before the exam. This basic set of questions covers the fundamental knowledge of the course. Each student is supposed to submit only one question to me. You can either choose from the basic set or make up your own. No communication is allowed among the students. condition for "conditional exam" is that at least 90% of the students choose their questions from the basic set. Otherwise, I will go back to the traditional exam. If the "conditional exam" is adopted, at least 90% of the questions will be from the basic set and you may bring the basic set of questions to the exam. All the questions submitted will be used on the final exam, thus the maximum possible number of questions on the final exam is the total number of the students in the class. If you construct your own question, the question has to be within the textbook or my lecture notes and you can make it as hard as you wish. The answers to the private questions made up by each student are only entitled to who ever made the questions. The grades for the final will be curved by your relative performance, but you are guaranteed a A- if you get 90% of the questions right. The only possibility that everyone receives an A for the final is that everyone gets perfect score. Your grades for the course will be either the final grade or the average of the midterms and the final depending on whichever is higher. I have no preference over your choices. The decision is yours. Discussion with others is not allowed. If you are not clear about anything or have any questions, please ask me only.

What percentage of the students do you think will choose the basic set of questions?										
Your choice: the basic set; my own question;										
What is your reason for the above choice?										
Vous and a consister surreless										
Your social security number										
Your answers and your identity are considered confidential.										
After you finish the above questions, please fold it up, and I will collect them. I will let you know										
he result next time in class										

# APPENDIX ZD

#### A SURVEY FOR THE MIDTERM EXAM

The results of this survey will help me determine the type of exam I will administer for the midterm. There are two choices: The traditional exam and the conditional exam. All exam questions will be multiple-choice.

The traditional exam questions will be provided by the instructor at the time the exam is given. Grades will be curved as described in the Syllabus.

For the conditional exam, the students choose the questions. They have the option to either submit one multiple choice question that they have made or choose one question from the basic set of 40 questions that the instructor provides. The basic set of questions will be given to the students before the exam. The condition for conditional exam is that at least 90% of the students choose their questions from the basic set. Otherwise, I will use the traditional exam. If the conditional exam is adopted, at least 90% of the questions will be from the basic set and a copy of the basic set of questions will be provided at the exam. All the questions submitted will be used on the exam. If you construct your own question, the question has to be within the textbook or the lecture notes and you can make it as hard as you wish. The answers to the private questions are only entitled to who ever made the questions. The grades for the midterm will depend on your relative performance, but you are guaranteed a A- if you get 90% of the questions correct. The only possibility that everyone receives an A is that everyone gets perfect score.

Your grade for the course will be the average of the midterm and the final regardless of other students' grades. For example, if you get an A for the midterm and an C for the final, your course grade will be B. The final exam will be traditional exam. I have no preference over your choices. The decision is yours. Discussion with others is not allowed. If you are not clear about anything or have any questions, please ask me only.

What percentage of the students do you think will choose the basic set of
questions?
Your choice: the basic set; my own question ;
What is your reason for the above choice?

What grade are you expecting from this class?

Your answers and your identity are considered confidential.

After you finish the above questions, please fold it up, and I will collect them. I will let you know the results next time in class.

#### APPENDIX ZE

## A SURVEY FOR FINAL EXAM

This survey result will help me to decide the kind of final exam I will give. There are two choices, one is the traditional exam which is just like the midterms; the other is "conditional exam". The "conditional exam" is to have the students choose the questions for the final exam. All the questions will be multiple-choice. I will provide a basic set of questions (40) and answers before the exam. This basic set of questions covers the fundamental knowledge of the course. Each student is supposed to submit only one question to me. You can either choose from the basic set or make up your own. No communication is allowed among the students. condition for "conditional exam" is that at least 80% of the students choose their questions from the basic set. Otherwise, I will go back to the traditional exam. If the "conditional exam" is adopted, at least 80% of the questions will be from the basic set and you may bring the basic set of questions to the exam. All the questions submitted will be used on the final exam, thus the maximum possible number of questions on the final exam is the total number of the students in the class. If you construct your own question, the question has to be within the textbook or my lecture notes and you can make it as hard as you wish. The answers to the private questions made up by each student are only entitled to who ever made the questions. The grades for the final will be curved by your relative performance, but you are guaranteed a A- if you get 80% of the questions right. The only possibility that everyone receives an A for the final is that everyone gets perfect score. Your grades for the course will be the average of the midterms and the final regardless of other students' grades. For example, if you got a C for the midterms and an A for the final, your course grade will be B. I have no preference over your choices. The decision is yours. Discussion with others is not allowed. If you are not clear about anything or have any questions, please ask me only.

What percentage of the students do you think will choose the basic set of questions?							
Your choice: the basic set own question; What is your reason for the above choice?							
Your social security number							
Your answers and your identity are considered confidential.							
After you finish the above questions, please fold it up, and I will collect them. I will let you know							
the result next time in class.							

# APPENDIX ZF

#### A SURVEY FOR FINAL EXAM

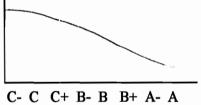
his survey result will help me to decide the kind of final exam I will give. There are two choices, one is the traditional exam which is just like the midterms; the other is "conditional exam". The "conditional exam" is to have the students choose the questions for the final exam. All the questions will be multiple-choice. I will provide a basic set of questions (50) and answers before the exam. This basic set of questions covers the fundamental knowledge of the course. Each student is supposed to submit only one question to me. You can either choose from the basic set or make up your own. The condition for "conditional exam" is that at least 80% of the students choose their questions from the basic set. Otherwise, I will go back to the traditional exam. In other words, the condition is that at least 56 out of total 70 students have to choose from the basic set. If the "conditional exam" is adopted, at least 80% of the questions will be from the basic set. The grades for the final will be curved but you are guaranteed a B even if you get only the "80%" of questions correct. The only possibility that everyone receives an A for the final is that everyone gets perfect score. Your grades for the course will be the average of the midterms and the final regardless of other students' grades. For example, if you got a C for the midterms and an A for the final, your course grade will be B. I have no preference over your choices. The decision is yours. Discussion with others is not allowed. If you are not clear about anything or have any questions, please ask me only.

What percentage of the students do you think will choose the basic set of questions?								
our choice: the basic set own question;								
What is your reason for the above choice?								
our social security number								
our answers and your identity are considered confidential.								
fter you finish the above questions, please fold it up, and I will collect them. I will let you know								
e result next time in class.								

## APPENDIX ZG

#### A SURVEY FOR FINAL EXAM

his survey result will help me to decide the kind of final exam I will give. There are two choices, one is the traditional exam which is just like the midterms; the other is "conditional exam". The "conditional exam" is to have the students choose the questions for the final exam. All the questions will be multiple-choice. I will provide you a basic set of questions (40) and answers before the exam. This basic set of questions covers the fundamental knowledge of the course. Each student is supposed to submit only one question to me. You can either choose from the basic set or make up your own question. No communication is allowed among the students. The condition for "conditional exam" is that at least 50% of the students choose their questions from the basic set. Otherwise, I will go back to the traditional exam. If the "conditional exam" is adopted, at least 50% of the questions will be from the basic set and you may bring the basic set of questions to the exam. All the questions submitted will be used on the final exam, thus the maximum possible number of questions on the final exam is the total number of the students in the class. If you construct your own question, the question has to be within the textbook or my lecture notes and you can make it as hard as you wish. The answers to the private questions made up by each student are only entitled to who ever made the questions. The grades for the final will be curved by your relative performance, but you are guaranteed a C- if you get 50% of the questions right. The following is one example of the distribution curve of students' grades assuming that there are some private questions and everyone at least gets all the basic set of questions correct:



The only possibility that everyone receives an A for the final is that everyone gets all the questions correct. Your grades for the course will be the average of the your midterms and the final regardless of other students' grades. For example, if you got a C for the midterms and an A for the final, your course grade will be B. I have no preference over your choices. The decision is yours. Discussion with others is not allowed. If you are not clear about anything or have any questions, please ask me only.

After you finish the above questions, please fold it up, and I will collect them. I will let you know the result.

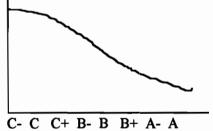
## APPENDIX ZH

#### A SURVEY FOR THE MIDTERM EXAM

he results of this survey will help me determine the type of exam I will administer for the midterm. There are two choices: The traditional exam and the conditional exam. All exam questions will be multiple-choice.

he traditional exam questions will be provided by the instructor at the time the exam is given. Grades will be curved as described in the Syllabus.

or the conditional exam, the students choose the questions. They have the option to either submit one multiple choice question that they have made or choose one question from the basic set of 40 questions that the instructor provides. The basic set of questions will be given to the students before the exam. The condition for conditional exam is that at least 50% of the students choose their questions from the basic set. Otherwise, I will use the traditional exam. If the conditional exam is adopted, at least 50% of the questions will be from the basic set and you may bring the basic set of questions to the exam. All the questions submitted will be used on the exam. If you construct your own question, the question has to be within the textbook or the lecture notes and you can make it as hard as you wish. The answers to the private questions are only entitled to who ever made the questions. The grades for the midterm will be curved by your relative performance, but you are guaranteed a C- if you get 50% of the questions right. The following is one example of the distribution curve of students' grades assuming that there are some private questions and everyone at least gets all the basic set of questions correct:



The only possibility that everyone receives an A for the midterm is that everyone gets all the questions correct.

our grade for the course will be the average of the midterm and the final regardless of other students' grades. For example, if you get an A for the midterm and an C for the final, your course grade will be B. The final exam will be traditional exam. I have no preference over your choices. The decision is yours. Discussion with others is not allowed. If you are not clear about anything or have any questions, please ask me only.

What percentage of the students do you think will choose the basic set of questions?	
Your choice: the basic set own question;	
What is your reason for the above choice?	
Your social security number	
Your answers and your identity are considered confidential.	

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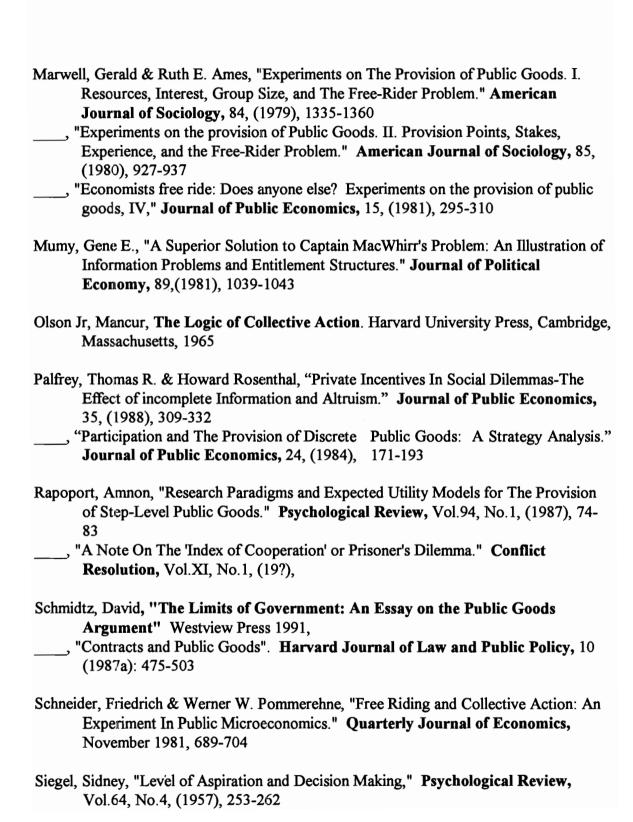
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