"A Model of Public Debt Determination:

Dynamic Implications and Empirical Evidence"

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CHAPTER I

INTRODUCTION

In an economy with a large public sector, such as that of the United States, there are three ways for the federal government to finance its expenditures: levying conventional taxes, issuing public debt and creating money. Therefore, the federal budget deficit, which is the portion of government expenditure that exceeds tax revenues, must be financed either by public debt issue or by money creation. Since year 1981 sizable federal budget deficits under the Reagan administration have gripped the public's attention and engendered predictions of disaster. However, although those who have worried about these large deficits have not been able to explain the buoyancy of the American economy during the same period; nevertheless, the fear of budget deficits still remains. Some policymakers and economists argue that the economy is prospering on money borrowed from future generations, and that the taxes required to retire this debt will enact a significant burden on those future generations. This concern triggered the Balanced Budget and Emergency Deficit Control Act (the Gramm-Rudman-Hollings Act), passed[1] in December 1985. This act sets up an arbitrary target of a zero value for the budget deficit by fiscal year 1991. In spite of questions about the feasibility[2] of implementing this Act, it is important to examine whether a zero budget deficit goal can be justified by economic theory. Because budget deficits in the United States have been largely financed by public debt issues,[3] a zero value for the budget deficit implies that there is no need for debt finance and that only tax finance is necessary. Therefore, the focus of the discussion is whether economic theory can justify the prohibition of debt finance and, if not, then how to set up the optimal mix of tax-financing and debt-financing for a given level of government spending.

The debate concerning debt burden on future generations started about two hundred years ago. Much of the literature has focused on the potential existence of a debt burden on future generations as well as the likely effectiveness of fiscal policy. However, these macroeconomic questions remain controversial. Neither economic theories nor empirical analyses provide convincing evidence to support either side of these debates. Furthermore, none of these macroeconomic arguments can be applied either to justify the balanced federal budget or to derive a model of the optimal choice between taxes and debt.

Barro (1979) establishes an optimal debt creation model based on the minimization of tax collection costs. Barro's approach assumes that the government financing decision has no first-order effect on aggregate demand. The primary purpose of debt issue is therefore to shifts the timing of tax collecting from the current period to future periods. The object of the optimal choice between debt finance and tax finance in his model is to minimize the sum of tax-collection costs over time. Therefore, the model determines the optimal timing of taxes as well as of the optimal debt issue. Further, with a specification of time paths for income and government spending, Barro derives properties of the optimal public debt policy concerning transitory income and government spending shocks.

Barro (1979, 1984a), Horrigan (1986a, 1986b) and Lee (1986) conduct numerous empirical tests in order to examine whether actual fiscal policy has been in accordance with the properties of Barro's optimal public debt model. The general results of these studies indicated that the actual U.S. fiscal policy is not inconsistent with the properties of the optimal public debt policy; however, these test procedures have several important drawbacks, and the excessive sensitivity of public debt growth, found in those studies, is in conflict with the prediction of Barro's theory.

The purpose of this work is to extend Barro's (1979) model of the optimal public debt determination. My research derives the properties of the optimal debt policy in the face of permanent income and government spending shocks. Such permanent shocks most closely resemble the effects of shocks occurring in integrated stationary processes, as opposed to transitory shocks which most closely resemble the effects of shocks occurring in trend stationary processes. This study therefore conducts empirical tests to determine the most appropriate time series specification for these two economic variables. Finally, this study compares the optimal debt policy to actual historical government policy. In particular, it reexamines Barro's recent (1984) contention that the recent deficits are not inconsistent with an optimal policy.

Notes:

1. There is a chance that the Supreme Court will rule that the Gramm-Rudman-Hollings Act is an unconstitutional blurring of executive and legislative responsibilities.

2. The mild Gramm-Rudman-Hollings cuts in the fiscal 1986 budget seem to be working. However, Business Week (Oct. 21, 1985 issue, p. 39, March 3, 1986 issue, p. 18 , and March 10, 1986 issue, p. 18.) has predicted that the massive cuts for fiscal 1987 will not be carried out "in the concrete".

3. Federal budget deficits are not reflected precisely in federal debt changes. Part of the deficits can be financed by the Federal Reserve's issuance of non-interest bearing money in exchange for federal debt instruments that are not included in the public debt.

CHAPTER II

LITERATURE SURVEY OF DEBATE ON THE DEBT BURDEN

The theoretical debate on the burden of the debt is a long standing one. The gross burden of government expenditure is the reduction in household consumption of private goods and services required by the transfer of resources from the private sector to the public sector. The main concern of the debate is that using debt finance instead of tax finance may or may not lessen the gross burden of government expenditures for this generation and postpone it to future generations. The net burden of the debt issue on the future generation, on the other hand, depends on the productivity of government expenditures.[1] The main issue in the discussion of the debt burden, however, is to analyze the gross burden of public debt on future generations: whether part of the required reduction in private consumption due to government spending can or cannot be shifted to future generations through debt finance.

The Ricardian equivalence theorem states that the reduction in resources available for the current private uses is independent of the methods of financing government spending. This view has led economists to deny that the burden of public debt can be shifted forward in time and/or to future generation by issuing public debt rather than imposing taxes. The historical aspects of the equivalence theorem was first found in Ricardo's writings edited by Piero Sraffa (1951). In his early work, Ricardo states that "A man who has 10,000£, paying him an income of 500£, out of which he has to pay 100£ per annum towards the interest of the debt, is really worth only 8,000£, and would be equally rich, whether he continued to pay 100£ per annum, or at once, and for only once, sacrificed 2,000£ " (vol. 1: pp.247-8). His reasoning indicates that tax and debt finance are equivalent. This hypothesis has been designated the "Ricardian Equivalence Theorem" by Buchanan (1976).

However, Ricardo also concedes that these two methods of financing are not in fact equivalent. He states, "people who pay the taxes never so estimate them, and therefore do not manage their private affairs accordingly. We are too apt to think, that the war is burdensome only in proportion to what we are at the moment called to pay for it in taxes, without reflecting on the probable duration of such taxes. It would be difficult to convince a man possessed of 20,000£, or any other sum, that a perpetual payment of 50£ per annum was equally burdensome with a single tax of 1,000£" (vol. 4: pp.186-7). Furthermore, he argues that "an effort is only made to save to the amount of the interest of such expenditure, and therefore the national capital is diminished in amount" (vol. 4: pp.187-8). O'Driscoll (1977), based upon this conclusion found in Ricardo, declares that Ricardo is not a Ricardian on this issue, and calls Ricardo's conclusion the "Ricardian Nonequivalence Theorem."[2]

There are some dissenters of the equivalence theorem, who give different reasons for the existence of a debt burden on future generations. First, Buchanan (1958) argues that the payment of taxes itself is a burden. Since debt finance postpones the levy of taxes, it shifts the burden from the current generation to future generations even without the reduction of capital accumulation. The justification for this statement is based on the assumption that taxes are compulsory and involuntary, while the market transactions of public debt are voluntary agreements. However, it is true that purchasers of government bonds are willing lenders, but there is a possible burden for this generation, which expects its future tax liabilities and prepares itself as well as the next generations at the present time for the future tax burden. They are unwilling borrowers-to-be or parents of borrowers-to-be. Also, Tobin (1965) points out that there is a possible burden on would-be private borrowers, if government borrowing displaces their borrowing from the funds market by raising interest rates.[3]

Besides Buchanan's notion, Modigliani (1961) gives another kind of logic of debt burden on future generations from the perspective of household consumption-saving behavior. He argues that the method of financing government expenditures does not alter the amount of the reduction in resources available for current private use, but that debt finance will displace mainly investment, and tax finance mainly consumption. On the basis of his life-cycle consumption-saving model, he contends that each generation will do only so much saving, and that the more of this which is absorbed by government debt, the less will be available for private capital formation. Following his reasoning, current consumption is higher when there is a shift from tax finance to debt finance, and consumption is lower when there is a shift from debt finance to tax finance. Therefore, there is a debt burden on future generations. There are two weaknesses in his reasoning as noted by Tobin (1965): first, following Modigliani's logic, a burden is placed on future generations whenever the government fails to run a surplus or a bigger surplus, not just when the government engages in debt finance. Secondly, the consumption-saving behavior assumed in Modigliani's notion of debt burden is questioned since it is not based on a symmetrical anticipation of the future. Modigliani assumes that the holders of government bonds think those bonds provide for their future wealth and consume more, but he fails to consider that those who will have to pay taxes in the future to service the debt may consider themselves poorer and save more accordingly.

The arguments for the debt burden and for the crowding-out of private investment hinge on the assumption that issuing debt raises perceived wealth. For instance, Patinkin (1965) includes a fraction of real government debt as a part of private wealth. However, it has been noted that the future taxes needed to finance government interest payments may offset any positive wealth effect. For example, Bailey (1962, p.77) states, "If future tax liabilities implicit in deficit financing are accurately foreseen, the level at which total tax receipts are set is immaterial; the behavior of the community will be exactly the same as if the budget were continuously balanced." Also, Tobin (1971, p.91) notes: "How is it possible that society merely by the device of incurring a debt to itself can deceive itself into believing that it is wealthier? Do not the additional taxes which are necessary to carry the interest charges reduce other component of private wealth?" In sum, a current-period tax reduction financed by issuing government debt shifts the timing of tax allocation from the current period to the future. If the future taxes implied by government debt are not fully perceived and discounted by the current generation, there will be a "net wealth effect," which increases current household consumption, thus reducing capital accumulation and growth as well. If, on the other hand, the implied future taxes are perceived and discounted by the private sector completely, the current period tax reduction will be used to increase private savings to pay for the future taxes, and government debt will be absorbed without any real effects on the economy. The latter has been called the "tax discounting hypothesis."

The debate has been carried further, and there are two major types of arguments that have been offered to defend the position that the offset of future tax liability will be only partial. One type of argument, given by Thompson (1967), is based on finite lives. It supposes that the relevant horizon for the future taxes will be shorter than that for the interest payments, so that the present value of interest payments will be larger than that of tax liability for the current generation. Blanchard (1985) also stresses the importance of finite lives of agents by incorporating the probability of death into the agent's utility maximization and derives the probability that some people current alive will not have to pay the future increase in taxes so that tax discounting can be only partial.

The second type of argument, given by Mundell (1971), is based on private capital market imperfectness. This argument supposes that for some potential borrowers with relative poor collateral, the discount rate for tax liabilities will be higher than that for interest payments. Even though the individuals subject to a low borrowing rate in the private capital market have no net wealth effect, the individuals charged with a high borrowing rate have the positive net wealth effect when the government issue bonds to postpone tax collection. Hence, even with infinite lives for tax liabilities, the present value of the interest payment will be larger than the present value of the tax liabilities. Therefore, government bonds are still net wealth to the private sector. However, as transaction costs associated with government bond sale and tax collection are included, net wealth effect for the people with a low borrowing rate becomes negative and net wealth effect for the people with a high borrowing rate may still positive if the government transaction costs are smaller than the private transaction costs. Since the government, in contrast with the private sector, has the monopoly power over legal tender and tax collection, it less costly for the government to borrow. Therefore, the government is more efficient than the private sector in arranging loans. Consequently, there still may be a net wealth effect associated with the issuance of government bonds.

Barro (1974) defends the theoretical grounds for the tax discounting hypothesis, which asserts that substitution of debt for taxes would not alter perceived private wealth. The sensitivity of this proposition is examined in relation to the finiteness of life and to imperfections of the private capital market. Barro's model, built upon the overlapping general framework of Samuelson (1958) and Diamond (1965), involves a constant population of agents each of whom lives for two periods but whose utility depends not only upon consumption in the two periods but also on the attainable utility of the agent's direct descendant. He concludes that even with finite life, neither the burden on future generations nor the crowding-out of private investment would occur in the case where the public intergenerational transfers[4] implied by debt issue are fully offset by compensating adjustments in voluntary private transfers. In other words, the current generations, having finite lives, behave as if they had infinite lives under operative intergenerational transfers. Also, the imperfection of capital markets is significant only if the government has some technical advantages over the private sector in the execution of loans. This is not likely to be the case in the developed countries, such as the United States. That is, costs to the government of intermediating funds via a deficit policy are not likely to be less than the costs of intermediating the funds in private capital market. Therefore, there is no persuasive theoretical ground for treating government debt as net perceived household wealth.

Another type of argument against the Ricardian equivalence theorem is based on the possibility of an intertemporal substitution effect of distortionary taxes on consumption behavior. McCulloch (1985) analyzes the real effects of different taxes on consumption and the equilibrium interest rate for a closed economy in a two period model of production and consumption. McCulloch's analysis illustrates that the Ricardian equivalence theorem holds only when the economy imposes lump-sum taxes. However, as a practical matter, actual taxes are never lump-sum and therefore real-world taxes are always distortionary. A cut in current taxes on output and a rise in future taxes on output will stimulate current production and consumption. Consumption taxes have similar effects on consumption as output taxes do. Income taxes have uncertain effects on consumption and the interest rate. Frenkel and Razin (1986) analyze the effects of non lump-sum taxes on consumption and interest rates in both domestic and foreign countries in a two period model. The theoretical results indicates that a reduction in taxes on capital income coupled with a rise in the future taxes on capital income increases interest rates and crowds-out both domestic and foreign consumption. On the contrary, a reduction in taxes on labor income accompanied by a rise in the future taxes on labor income lowers interest rates and stimulates investment in both domestic and foreign countries. The effects of taxes on consumption and taxes on international borrowing are ambiguous. Therefore, distortionary taxes may violate the Ricardian equivalence theorem and increase current consumption even though the future taxes are fully anticipated.

However, Fremling and Lott (1986) point out that informed individuals may save not only for future taxes but also for the deadweight losses associated with distortionary taxes. They conclude that the Ricardian equivalence may still hold in the case of distortionary taxes.

The observations stressed by Barro (1974) and Fremling and Lott (1986) threaten not only the concept of the existence of debt burden to future generations but also the belief that the government can influence investment and the growth of GNP by fiscal policy. In other words, these studies deny any real consequence resulting from the shift between tax finance and debt finance for a given amount of government expenditures. Ultimately this becomes an empirical question: can it be demonstrated empirically that public debt is net wealth for this generation and thereby a burden for the next generation?

The empirical side of the debate was initiated by Kochin (1974), who attempted to test for the effects of deficits on consumption, and also by Feldstein (1974), who attempted to test for the effects of social security wealth on consumption. Some evidence[5] is found to support the tax discounting hypothesis that the current generation discounts future tax liabilities completely, and there is no real effect of debt issue on current consumption-saving behavior. However, other studies have supported the opposite conclusion.[6] Furthermore, in equilibrium, if private sector saving goes up by less than the debt issue, then real interest rates rise and "crowding-out" of private investment occurs. This "crowding-out" leads to a decline in capital accumulation and to a debt burden on future generations. Therefore, besides those studies investigating the effect of government financing decision on household consumption-saving behavior, there are other empirical studies that examine the relationship between government financing decision and interest rates by integrating the various channels through which deficits or debt can affect interest rates. The weight of evidence on these issues is therefore still inconclusive.[7]

The Ricardian equivalence theorem surely remains controversial. But in any event, until Barro (1979) both proponents and opponents of the Ricardian equivalence theorem's implication that the choice between debt and taxes does not have a first-order effect on aggregate demand were left without a theory of optimal public debt determination. It is Barro's contention that the Ricardian equivalence theorem holds as a first-order approximation and that the second-order effect, which concerns tax collection costs, determines the optimal timing of taxation and debt issue. A detailed review of Barro's optimal debt creation model is included in the next chapter.

Notes:

1. If the government spends the proceeds of bonds on a highly productive investment project that yields a return to society sufficient to pay the interest costs on the bond, then there is no future net burden of the debt issue. But if the government spends the proceeds on a less productive investment project, then there is not enough future benefit to pay for the future interest payments, leaving future generations with a net burden.

2. The term "Ricardian Equivalence Theorem" is not generally accepted in all the literature. Buiter and Tobin (1979) calls it non-Ricardian equivalence. Buiter (1979) names it as neutrality theorem. Feldstein (1982) uses the term of pre-Ricardian equivalence hypothesis. McCulloch (1985) calls it Barrovian equivalence since Barro is the major supporter to the revival of equivalence theorem in the recent debate.

3. Tobin (1971, 1978) believes that the assumptions made for the Ricardian Equivalence theorem are overly restrictive on practical grounds.

4. Barro (1974) states that the important consideration for this result is not the existence of a pecuniary bequest but rather some form of intergenerational transfer such as expenditure on the education of the descendant. The reason for that is that there are estate taxes and inheritance taxes discouraging pecuniary bequest.

5. Most empirical work, such as Tanner (1978), Kochin (1974), Kormendi (1983), Seater and Mariano (1985), etc. support tax discounting hypothesis.

6. For example, Buiter and Tobin (1979) applies slightly different version of Kochin's model (1978), and finds no supporting evidence for tax discounting hypothesis. Blinder and Solow (1973) shows the evidence that changes in government spending or taxes can have substantial effect on aggregate demand. Feldstein (1982) suggests that consumers do not regard taxes are equivalent to changes in debt, however, there are some drawbacks in this empirical study such as endogeneity problems and exclusion of a business cycle variable.

7. Feldstein and Eckstein (1970) combines liquidity preference theory with the assumption that nominal interest rates reflect the expected rate of inflation. It suggests that a small but statistically significant positive effect of government debt on nominal interest rates. Plosser (1982) just puts a list of variables likely to affect interest in the regression model and concludes that capital market is indifferent to how government finances its expenditures. Evans (1985),

based on IS-LM model, finds that large deficits have never been associated with high nominal interest rates. Evans (1987) includes the variables of the expected budget deficits and finds that the high expected budget deficits do not spell the high interest rates.

CHAPTER III

TRANSITORY AND PERMANENT SHOCKS IN THE OPTIMAL PUBLIC DEBT MODELS

This chapter is composed of two parts. The first part discusses the properties of transitory shocks in an optimal public debt creation model by reviewing Barro's (1979) optimal debt creation model and some relating empirical evidence, including Barro (1979) and Horrigan (1986a, 1986b). The second part considers permanent shocks and explores what the government's optimal response of tax-debt finance would be when it experiences permanent current shocks or predicts permanent future shocks. As a result, the properties of the optimal tax-debt finance are conditional on the specification of shocks.

3.1 An Optimal Optimal Public Debt Model for Transitory Shocks

This section includes a review of Barro's public debt model and discussions of related empirical studies to Barro's model.

3.1.1 Barro's Public Debt Model for Transitory Shocks

As the Ricardian equivalence theorem on public debt points out, shifts between debt finance and tax finance for a given amount of government spending should have no first-order effect on the real interest rate, volume of private consumption, or volume of private investment. Barro (1979) assumes that the Ricardian equivalence theorem is valid and identifies other factors that might influence the choice between debt issue and taxes. The maintained hypothesis of his study is that taxation involves not only a one-to-one transfer of purchasing power from individuals to the government, but also collection costs, including distortion costs, administration and policing costs. These costs can be considered as costs of resources for the government to produce its tax revenue. If the government issues government bonds to finance government spending,it postpones tax collection from the present to the future, and that lowers the present tax collection costs but will increase the future tax collection costs. The objective of government policy is to minimize the present value of resources consumed by the process of revenue generation over time. Under this objective, the optimal timing of taxes maintain a constant tax-income ratio. The optimal debt issues, then, are considered as a residual from the accounting budget constraint after deducting the optimal tax revenue. Futhermore, any change in the expected future income and future government spending leads to an immediate revision of the optimal mix of tax and debt finance. Thus, a set of properties of the optimal intertemporal version of fiscal policy are derived.

Although Barro's model of the optimal debt issue involves many abstractions, there are five in particular. First, Barro assumes that the economy in question is a barter economy. Thus, he rules out the possibility of money creation to finance government spending. Second, Barro assumes that the Ricardian equivalence theorem holds. This assumption has been supported for recent U.S. history by many empirical studies, as mentioned in the previous chapter. Third, in order to simplify the model, Barro assumes that the tax structure and pattern of government spending is fixed and that the government taxation policy is basically a matter of tax timing. Fourth, Barro assumes that the only one decisive factor in determining the optimal mix of tax finance and debt finance is the minimization of the present value of tax collection costs over time. Much public finance literature, such as Browning (1976), Stuart (1984), and Ballard, Shoven and Whalley (1985), has estimated the sizes of tax distortion costs[1] and has found that they are significantly large. Thus, minimizing the present value of tax collection costs over time can be considered as a reasonable objective of fiscal policy. Finally, Barro assumes that the time paths of real income and real government spending are assumed to follow deterministic time trends.

Let b_t be the level of public debt at time, t, Y_t be the taxable income in the economy at time t, G_t be government spending (not including interest payments) at time t, and T_t be the tax revenue collected at time t. Interest payments from the public debt are assumed to be excluded from taxable income. The periodic government budget constraint at every point of time t, can therefore be expressed as follows:

$$G_t+rb_{t-1} = T_t+(b_t-b_{t-1}) \tag{1}$$

In the above expression, $G_t-T_t+rb_{t-1}$ is identified as the full government deficit at time t and G_t-T_t is identified as the current government deficit at time t. The outstanding public debt at time t, b_t, is the sum of the current government deficit, G_t-T_t, plus the previous debt with accrued interest payment, $(1+r)b_{t-1}$.

The intertemporal government budget constraint states that the present value of present and future government spending plus the initial public debt must equal the present value of present and future tax revenue. This intertemporal budget constraint implies that in order to issue interest bearing debt, the government must promise to balance its budget in expected present value terms. For an infinite time horizon, the intertemporal budget constraint at present can be presented as follows:

$$\sum_{t=1}^{\infty}[G_t/(1+r)^t]+b_0 = \sum_{t=1}^{\infty}[T_t/(1+r)^t] \tag{2}$$

where b_0 is the initial public debt that the government inherits during the present policy making period. If the government is subject to the intertemporal budget constraint, then there is a necessary condition: the the economy's growth rate must be greater than the government borrowing rate so that the government does not go bankrupt. On the other hand, if the government borrows at an interest rate that equals or exceeds the economy's growth rate, then a continuing unpaid deficit implies debt must grow to become an infinite multiple of GNP, and the government will not be able to pay off its interest payment. However, a finite present value of terminal public debt is not a sufficient condition for the intertemporal budget constraint. In an infinite horizon, in order to derive the intertemporal budget constraint, we sum up the periodic budget constraint in each period up to the infinite future and set the present value of the terminal public debt to zero. Therefore, the transversality condition, $\lim_{t\to\infty} b_t/(1+r)^t = 0$, is a necessary and sufficient condition for the intertemporal budget constraint to be valid. Hamilton and Flavin (1986) applied the U. S. past-war data and accepted the hypothesis that the government did not violate the condition of balancing its budget in expected present-value terms.

Furthermore, it is reasonable to consider that the tax collection cost for period t, C_t, is an increasing function of tax revenue and a decreasing function of income. In particular, Barro assumes that this cost function is a linear homogenous function of tax revenue and income:

$$C_t = Y_t f(T_t/Y_t). \tag{3}$$

This cost function implies that tax collection cost rises more than proportionately with the tax rate. Barro also assumes this function is invariant over time.

The policy maker tries to determine current and future tax revenues, T_t, $t=1,2,...$, in order to minimize the sum of the present value of tax collection costs, C, subject to the intertemporal budget constraint, that is:

$$\underset{T_t}{\text{Min}} \sum_{t=1}^{\infty} \{[T_t f(T_t/Y_t)]/(1+r)^t\} \tag{4}$$

st. eq.(2).

The first-order condition of this set of equations is:

$$(T_{t+1}/Y_{t+1}) = (T_t/Y_t), \quad t=1,2,3,... \tag{5}$$

The second-order condition is satisfied if the bordered Hessian is positive definite. Therefore, the optimal income tax rate should be constant over time, starting from the present time period, t=1.

Given a constant income tax rate over time as the optimal taxation policy as well as the specification of time paths of real income and real government spending, it is possible to figure out the optimal tax revenue at each point in time from the intertemporal budget constraint. Once the optimal tax revenue for each period is derived, the optimal debt issue for each period can be solved from the periodic budget constraint in each period respectively. Thus, the optimal time path of public debt is determined by the cost-minimizing condition, intertemporal budget constraint, and particular specification of time paths for income and government spending.

Following this structure, Barro analyzes the determination of the optimal debt issue under three different specifications of time paths for government spending and income.

Case I. Constant Income and Government Expenditure

When income is specified to be constant over time, the optimal tax revenue is also expected to be constant over time under the cost-minimizing condition. Further, the optimal tax revenue is calculated to be $G+rb_0$ from the intertemporal budget constraint, and then with the periodic budget constraint for each period, an interesting result arises--the optimal debt is also derived to be constant and remains the same amount as the inherited debt, b_0.

Case II. Constant Rate of Growth of Income and Government Expenditure

Assume that income is growing at a rate η each period, and government spending is growing at a rate γ each period. In order to have a finite present value of income, η is less than the interest rate, r. Also, to prevent government spending from exceeding income, γ is not greater than η. Therefore, it must be that $0 \leq \gamma \leq \eta < r$.

Under this specification, the optimal tax revenue is supposed to grow at a rate η as income does to achieve a constant income tax rate over time. From the intertemporal budget constraint, and the periodic budget constraint for each period, the general result at time t is derived as follows:

$$b_t = (1+\eta)b_{t-1}+G_t(\eta-\gamma)/(r-\gamma), \quad t=1,2,... \tag{6}$$

For the case where income and government expenditure grow at a common rate, $\eta=\gamma$, the conclusion from this extension is that both government debt and tax revenue are also supposed to grow at this rate at the optimum. The model therefore retains the property that debt-income ratio is constant over time. However, this ratio is not determined within the model but is rather fixed at its historically given "initial" value.

For $\eta \neq \gamma$, an additional effect is that the current deficit rises with η and falls with γ, and debt-income ratio,

$$b_t/Y_t = b_{t-1}/Y_{t-1}+[G_t/Y_t][(\eta-\gamma)/(r-\gamma)], \quad t=1,2,... \tag{7}$$

does not remain constant over time. Since $\eta > \gamma$ is prerequisite, the future values of G/Y will be lower than the current value. Consequently, the financing of expenditure becomes easier over time, so that deferment of taxation is wanted, and debt-income ratio increases at a decreasing rate, not a constant rate; thus, higher current debt finance is suggested in this case.

Case III Transitory Income and Government Spending

This case concerns temporary departures of government spending and income from their trend values. This analysis applies especially to the role of wartime expenditures and booms or depressions--both viewed as transitory phenomena in the government debt creation process. The fixed time trends for government spending and income are assumed.

For simplicity, Barro assumes that constant positive shocks, ε's, to government spending exist for k periods from now, and constant positive shocks, u's, to income exist for n periods from now. After the shocks disappear, both government spending and income will return to their deterministic time trends at the common growth rates ρ respectively. This specification implies that both of them are assumed to follow fixed time trends. That is

$$Y_1 = (1+u)Y_0(1+\rho)$$

$$Y_t = Y_1(1+\rho)^{t-1} \text{ for } t=1,\ldots,n$$

$$Y_t = [1/(1+u)]Y_1(1+\rho)^{t-1} \text{ for } t=n+1,\ n+2,\ldots \tag{8}$$

and

$$G_1 = (1+\varepsilon)G_0(1+\rho)$$

$$G_t = G_1(1+\rho)^{t-1} \text{ for } t=1,\ldots,k$$

$$G_t = [1/(1+\varepsilon)]G_1(1+\rho)^{t-1} \text{ for } t=k+1,k+2,\ldots \tag{9}$$

The optimal tax finance still requires a constant ratio of taxes to income at all points in time, starting at t=1. Using this optimal condition, the specification of transitory income and government spending as well as intertemporal budget constraint, it is possible to determine taxes at all points in time starting at t=1. The solution can be written in the form

$$T_t = T_1(1+\rho)^{t-1} \text{ for } t=1,\ldots,n$$

$$T_t = [1/(1+u)]T_1(1+\rho)^{t-1} \text{ for } t=n+1,\ n+2,\ldots$$

$$T_1 = \frac{(1+u)}{(1+u)-u[(1+\rho)/(1+r)]^n} \{G_0(1+\rho)+(r+\rho)b_0$$

$$+\varepsilon G_0(1+\rho)(1-[(1+\rho)/(1+r)]^k)\} \quad (10)$$

The above expression for T_1 can be interpreted as follows. The term in the right-hand parentheses measures the level of required finance: the trend value of government spending, $G_0(1+\rho).$, plus the interest on the inherited debt less the part that is financed by issue of debt along with the growth rate,$(r-\rho)b_0$, plus the effect of the transitory expenditures, $\varepsilon G_0(1+\rho)(1-[(1+\rho)/(1+r)]^k)$. Generally, current taxes rise with the amount of transitory government spending and the duration of it.

The other term on the right side of eq.(10) accounts for the impact of transitory income. In general, current taxes rise with the amount of current transitory income and fall with the duration of it.

With current taxes determined from eq.(10), the current optimal debt issues can be solved as a residual from the periodic budget constraint. This solution can be rewritten in terms of its growth rate,

$$(b_1-b_0)/b_0 \simeq [(1+\rho)/(1+r)]^k(G_1-\bar{G}_1)/b_0$$
$$-[(1+\rho)/(1+r)]^n[(\bar{G}_1+rb_0)/b_0][(Y_1-\bar{Y}_1)/\bar{Y}_1]+\rho \quad (11)$$

where $\bar{G}_1=G_0(1+\rho)$, and $\bar{Y}_1=Y_0(1+\rho)$. The above approximation indicates a positive effect on the debt issue of a temporary increase in government spending and a negative effect on the debt issue of a temporary increase in income.

This study derives two special cases, which consider the effect of a transitory increase in income and the effect of a transitory increase in government spending on the optimal current tax and debt finance, separately.

If there is a transitory increase in income and no shock in government spending, i.e. $k \to 0$ and $\varepsilon \to 0$, the current tax revenue becomes

$$T_1 = \frac{(1+u)}{(1+u)-u[(1+\rho)/(1+r)]^n} [G_0(1+\rho)+(r-\rho)b_0]$$

$$>G_0(1+\rho)+(r-\rho)b_0 \qquad (12)$$

and since the first factor on the right side of eq.(12) is greater than one, the optimal current tax revenue under a positive income shock is greater than that without the shock. On the other hand, current debt growth rate under a positive transitory income shock becomes

$$(b_1-b_0)/b_0 \simeq -[(1+\rho)/(1+r)]^n\{[G_0(1+\rho)+rb_0]/b_0\}u+\rho <\rho \qquad (13)$$

and is smaller than that implied under no shock. In short, with a positive income shock alone, there is a shift from current debt finance to current tax finance suggested in Barro's model.

Another special case is to consider the condition when only a government spending shock occurs, i.e. $n \to 0$ and $u \to 0$. With a positive transitory increase in government spending, the optimal current tax revenue is as follows

$$T_1 = \{G_0(1+\rho)+(r-\rho)b_0]+\varepsilon G_0(1+\rho)\{1-[(1+\rho)/(1+r)]^k\}$$
$$> G_0(1+\rho)+(r-\rho)b_0 \qquad (14)$$

and is boosted as a positive government spending shock occurs. However, the increment in the optimal current tax revenues, $\varepsilon G_0(1+\rho)\{1-[(1+\rho)/(1+r)]^k\}$, is quite small. Also, the current public debt growth rate in this case is as follows:

$$(b_1-b_0)/b_0 \approx [(1+\rho)/(1+r)]^k\{[G_0(1+\rho)\varepsilon]b_0\}+\rho > \rho \qquad (15)$$

and is greater than that without a positive government spending shock. This indicates that the extra burden of high government spending is shared by largely by debt finance if this increase in government spending is temporary.[2]

In sum, Barro's public debt model is derived from a cost-minimizing taxation policy. This model rationalizes a system of tax laws that allows for an automatic procyclical pattern of revenues as a convenient mechanism for stabilizing the tax-income ratio. This rationale derives from efficiency in tax revenue generation and not from stabilization policy considerations. The optimal government debt behaves as a residual from the current budget constraint after deducting current optimal tax revenue. Therefore, a so-called "tax (rate) smoothing theory" of the determination of public debt is established.

3.1.2 Related Empirical Studies to Barro's Model

There are empirical studies examining the validity of Barro's tax smoothing theory. Barro (1979) analyzed the behavior of the public debt in the United States, using annual data, 1922 to 1976. Barro tested whether the public debt's behavior is consistent with the hypothesis that the government has financed its expenditures efficiently over time. The properties of the optimal tax-debt finance in Barro's model are as follows: first, anticipated inflation has an unitary effect on debt growth; second, transitory high income suggests more tax finance and less debt finance; third, transitory high government spending is largely financed with debt. Barro makes numerous assumptions[3] in his theoretical model to derive the following testable regression equation (16):

$$\begin{aligned}(B_t-B_{t-1})/B_{t-1} = {} & \alpha_0+\alpha_1\pi_t+\alpha_2[P_t(G_t-G_{p,t})/B_{t-1}] \\ & -\alpha_3[(P_tG_{p,t})/B_{t-1}]\ [(Y_t-Y_{p,t})/Y_{p,t}] \\ & +\alpha_4(P_tG_{p,t}/B_{t-1}) + \alpha_5 DUM + u_t \qquad (16)\end{aligned}$$

where B_t is the stock of nominal debt at par value at the end of the time period, π_t is anticipated inflation rate during the time period, P_t is the price level, G_t is real government spending, Y_t is real GNP, $G_{p,t}$ and $Y_{p,t}$ are the permanent values of G_t and Y_t, respectively, DUM is a dummy variable equal to one before 1941, and u_t is the error term.

Barro's theory imposes testable restrictions on the parameters in equation (16). First, the coefficient on anticipated inflation, α_1, should equal unity. The second variable in equation (16) measures transitory government spending as a percentage of debt, and the third variable measures transitory change in tax revenues as a percentage of debt caused by proportional deviations of real GNP from its trend value. The plausible values of those variables' coefficients, α_2 and α_3, correspond to $[(1+\rho)/(1+r)]^k$ and $[(1+\rho)/(1+r)]^n$, respectively, and should be below but close to 1.0. Barro found that while the data support these three propositions, debt growth is more countercyclical than predicted by his theory. (Statistical results are listed in the appendix A.) To re-test Barro's hypothesis, Horrigan (1986b) used a longer time, 1790 to 1981, for the U.S. data and slightly different methodology for constructing the independent variables. Debt growth was still found to be excessively countercyclical. (Statistical results are listed in the appendix A). Barro has suggested that the excessive sensitivity is the consequence of graduated non-indexed income tax rates. Activist fiscal policy is also suggested as another possible explanation. However, since the excessive sensitivity of the debt growth to the business cycles existed before 1905, when there was no federal income tax nor social Security System nor Keynesian fiscal policy, the explanation must lie somewhere else.

As I reexamine Barro's theoretical model, one of the major assumptions is that both real income and real government spending follow fixed time trends with transitory income and government spending shocks; however, most recent studies such as Dickey and Fuller (1981), Nelson and Plosser (1982), Campbell and Wankiw (1987), and Cochrane (1986) have rejected the traditional view[4] that shocks to macroeconomic variables have little or no permanent effects. Therefore, it is interesting to discuss permanent income shocks and permanent government spending shocks in a model of the optimal public debt determination. Based on a new specification for real income and real government spending, the new properties of the optimal debt policy are derived in the next section.

3.2 An Optimal Public Debt Model for Permanent Shocks

This section applies two cases to illustrate the properties of the optimal debt based on the assumption that both income shocks and government spending shocks behave as permanent shocks. The first case illustrates what kind of the immediate revision of the optimal tax and debt finance should be made when the income and government spending shocks occur currently and will persist indefinitely. The second case highlights the government's perfect foresight of the future income and government spending shocks, and examines their effect on the current optimal fiscal policy.

3.2.1 Case 1. Permanent Current Shocks

This case assumes that a current income shock and a current government spending shock occur currently and are expected to be permanent, and that the government does not foresee the likelihood of any future shock. Thus, the new specification of future time paths for real income and real government spending is as follows:

$$Y_1 = (1+u_1)Y_0$$

$$Y_t = Y_1, \quad t=1,2,... \tag{17}$$

and

$$G_1 = (1+e_1)G_0$$

$$G_t = G_1, \quad t=1,2,... \tag{18}$$

where u_1 and e_1 are percentage changes in income and government spending in the current time period. That is, u_1Y_0 and e_1G_0 represent permanent current shocks to income and government spending, respectively. It is assumed that there is no correlation between them.

Under the new specification of time paths, eq.(17) and eq.(18), the intertemporal budget constraint, eq.(2), and the optimal tax rate condition, eq.(5), it is possible to determine the optimal current and future tax revenues at all points in time. The new solution is as follows:

$$T_t = T_1, \; t=1,2,...$$

$$T_1 = (1+e_1)G_0+rb_0 \tag{19}$$

The above expression shows that the current optimal tax revenue, T_1, takes complete account of regular expenditures, G_0, interest payments on the initial debt, rb_0, and a current shock to government spending, e_1G_0. As a result, there should not be any increase in the public debt issue. Mathematically, with the optimal current taxes determined from eq.(19), the current optimal debt issue can be solved from eq.(1). The optimal current debt finance in this case remains the same. This solution is rewritten as follows:

$$b_1 - b_0 = 0 \tag{20}$$

From eq.(19) and eq.(20), the current government spending shock is fully absorbed by taxes under the optimal condition. Therefore, permanent high government expenditures should not be financed by debt. On the contrary, a temporary hike of government spending should be financed largely by debt, as stated in chapter III.

It is seemingly a surprise that the current shock to income, u_1Y_0, is not in the optimal current tax revenue equation, eq.(19). In fact, a positive (or negative) current shock to income will cause the optimal tax rate to fall (or rise), and that will be canceled out by the increase (or decrease) in income, leaving the volume of the optimal current tax revenue unchanged. Therefore, the optimal current tax revenue is not affected by current shock to income, i.e. $\partial T_1/\partial u_1=0$, even though the optimal tax rate is. It is noted that the optimal tax revenue equation derived in Barro, eq.(10), includes the effect of current income shock. The reason for this is based on the assumption that the shock is temporary and the government foresees the disappearance of this shock in the future, and not the occurrence of this shock at the present. Thus, the occurrence of current income shock makes no difference in determining the optimal amount of tax finance and debt finance.

3.2.2 Case 2. Permanent Future Shocks

However, recent historical data for the United States suggest that time paths for income and government spending are integrated stationary processes. The integrated stationary process has the property that most of the fluctuations in an economic variable come from the shocks which exist permanently and which accumulate over time. As a result, there is no fixed time trend and temporary disturbances fail to explain most of the variations of an economic variable. The focus in this section, therefore, turns to the properties of the public debt creation model under a new specification.

This case assumes that the government has foreseen that a future income shock and a future government spending shock will occur at the (n)th and (k)th periods, respectively, and that they are expected to last forever. Then the new specification of future time paths for income and government spending is as follows:

$$Y_t = Y_1, \quad t=1,2,\ldots n-1$$

$$Y_n = (1+u_n)Y_1$$

$$Y_t = Y_n\ , \quad t=n+1,n+2,\ldots \qquad (21)$$

and

$$G_t = G_1, \quad t=1,2,\ldots,k-1$$

$$G_k = (1+e_k)G_1$$

$$G_t = G_k, \quad t=k+1,k+2,\ldots \tag{22}$$

where u_n and e_k are percentage changes in income and government spending in the future time periods, n and k, respectively. That is, u_nY_{n-1} and e_kG_{k-1} represent future shocks to income and government spending at the (n)th and (k)th periods, respectively. It is assumed that there is no correlation between them.

Under the new specification of time paths, eq.(21) and eq.(22), the intertemporal budget constraint, eq.(2), and the optimal tax rate condition, eq.(5), it is possible to determine the stream of the optimal tax revenues at all points in time. The new solution is:

$$T_t = T_1, \quad t=1,2,\ldots,n$$

$$T_n = (1+u_n)T_1$$

$$T_t = T_n, \quad t=n+1, n+2,\ldots$$

$$T_1 = \frac{1}{1+u_n/(1+r)^n} \{G_0+rb_0+e_kG_k/(1+r)^k\} \tag{23}$$

The above expression shows that the optimal current tax revenue, T_1, takes partial account of regular expenditures, G_0, interest payments on the initial debt, rb_0, and discounted future shock to government spending, $e_kG_k/(1+r)^k$. Also, the proportion of three factors covered by current tax finance is represented by the first term on the right side of eq.(23). This term shows that the optimal current tax revenue is inversely related to discounted future income shock.

Based on eq.(23), we can analyze the effects of future income shock and the future government spending shock on the optimal current tax finance. First, a positive future shock to income has a negative effect on the optimal current tax revenue, i.e. $\partial T_1/\partial u_n<0$. In other words, if a positive future income shock is expected, then this anticipation suggests deferment of taxes and a reduction of current taxes thereby. The effect of a future shock to income on the optimal current tax revenue, unlike that of the current income shock, cannot be canceled out by its effect on tax rate and on current income. Since a positive future income shock suggests a lower optimal tax rate over time, with the same current income level, a lower level of the optimal current tax revenue is derived. On the other hand, the optimal current debt finance should increase to compensate for up the decrease in tax finance.

Secondly, if the government believes that a positive future government spending shock will happen, then this expectation alone indicates that a higher current tax revenue needs to be collected in order to avoid a sudden hike of the tax rate in the future. This is a positive effect of future government spending shock on the current tax revenue.

Furthermore, with the optimal current taxes determined from eq.(23), the current optimal debt issue can be solved from eq.(1). This solution can be rewritten in terms of the increase in the debt issue:

$$b_1 - b_0 = G_0 \left[1 - \frac{1 + e_k/(1+r)^k}{1 + u_n/(1+r)^n} \right] + rb_0 \left[1 - \frac{1}{1 + u_n/(1+r)^n} \right] \quad (24)$$

The properties of the optimal debt issue model in this case are: a direct effect of a foreseen shock to future income on the current debt issue, and an inverse effect of a foreseen shock to future government spending on the current debt issue. In addition, the magnitudes of the effects of the future shocks are larger if the shocks are going to occur sooner.

Finally, the transitory income shock and government spending shock, discussed by Barro (1979), can be considered as a special case when the current permanent shocks are exactly offset by future shocks. This indicates that Barro's transitory income and government spending case, which is under the assumption of trend stationary processes, is a very special case of a general solution under the assumption that income and government spending follow the integrated stationary processes.

In conclusion, there are four propositions of the optimal public debt policy in the case of permanent shocks: no effect of a permanent current income shock on the optimal current tax-debt financing, a direct effect of a permanent future income shock on the optimal current debt financing, a complete absorbtion of a current government spending shock by the tax financing only, and an inverse effect of a permanent future government spending shock on the optimal current debt financing.

To sum up, the properties of the optimal current tax-debt finance are conditional on the nature of shocks. Therefore, it is interesting to examine the observations of real GNP and real government spending to determine either transitory shocks or permanent shocks dominate the time paths of real GNP and real government spending.

Notes:

1. Browning (1976) concludes that for 1974 the marginal welfare cost of raising taxes on labor earnings was likely to be between 9% and 15% of additional revenue raises. Ballard, Shoven and Whalley (1985) suggest that it is in the 15% to 50% range, with Stuart (1984) reporting similar results.

2. As k approaches zero, i.e. the government spending shock lasts for a very short time period, $[(1+\rho)/(1+r)]^k$ is getting close to one. Therefore, the temporary high government spending is financed largely by debt. However, if k approaches infinite, i.e. the government spending shock lasts forever, then $[(1+\rho)/(1+r)]^k$ is getting close to zero. In other words, this permanent high government spending should be financed by taxes instead of debt. Barro considers temporary shocks only.

3. In order to derive a simple form of regression model, there are three abstractions made on the coefficients: $\frac{1+u}{1+u - u[(1+\rho)/(1+r)]^n} = 1$, $u - u[(1+\rho)/(1+r)]^n = 0$, and $\frac{u[(1+\rho)/(1+r)]^n}{1+u - u[(1+\rho)/(1+r)]^n} = 0$. If we solve these three equations jointly, then the solution is u=0. Therefore, Barro assumes the transitory income shock, u, is zero in the simplification procedure. It is questionable whether this simplification is reasonable for the test that he developed.

4. This traditional view is shared by Barro and Rush (1980), Kydland and Prescott (1980), and Blanchard (1981).

CHAPTER IV

SPECIFICATION OF TIME PATHS FOR REAL GNP AND REAL GOVERNMENT SPENDING

Barro (1979, 1984) imposes an unobserved components model on real GNP and real government expenditures under the hypothesis that the growth components of real GNP and real government spending move smoothly through time and that all variations are attributed to fluctuations in the temporary components. However, it is possible that the most of the fluctuations in real GNP and real government spending may come from shocks which are permanent and accumulate over time. Consequently, there may be no fixed time trend and temporary disturbances may therefore fail to explain most of the variation in economic variables. As discussed in chapter III, temporary shocks and permanent shocks have different implications for the optimal public debt policy. To compare the importance of the policy implications for these two types of shocks, it is necessary to develop some empirical tests to examine what kind of shocks dominate the specification of the time paths for real GNP and real government spending.

This chapter starts with a basic discussion of time series analysis. This discussion outlines the differing impact of random shocks in both the integrated stationary process and in the trend stationary process. This illustration shows that shocks occurring in the trend stationary processes tend to be transient while the shocks occurring in the integrated stationary processes tend to be persistent. Finally, I empirically investigate whether the time series of real GNP and real government spending are better characterized as trend stationary processes or as integrated stationary processes.

4.1 Statistical Background

4.1.1 Stationarity and Nonstationarity:

There are strict and weak definitions of stationarity summarized as follows:

(1) Strict stationarity requires that the joint distribution be invariant with regard to a displacement in time, that is

$$F\ (Z_t,\dots,Z_{t+k}) = F\ (Z_{t+m},\dots,Z_{t+m+k})$$

where $F\ (Z_t,\dots,Z_{t+k})$ and $F\ (Z_{t+m},\dots,Z_{t+m+k})$ are joint distribution functions of $Z_t,\dots,Z_{t+k}$ and $Z_{t+m},\dots,Z_{t+m+k}$, respectively.

(2) Weaker forms of stationarity require only that moments through some specified order be invariant over time, e.g., covariance stationarity requires that mean and covariance matrix be invariant, that is:

Same mean over time:

$$E\ (Z_t) = E\ (Z_{t+m})$$

Same variance over time:

$$\text{Var}(Z_t) = \text{Var}(Z_{t+m})$$

Same covariance over time:

$$\text{Cov}(Z_t, Z_{t+j}) = \text{Cov}(Z_{t+m}, Z_{t+m+j})$$

A stationary process in the intuitive sense is a process located within a region from which it will only rarely depart. In other words, a stationary process will display the same general pattern of behavior no matter when we observe it.

Based on the meaning of stationarity, the sources of nonstationarity are classified into three categories: (a) there is a trend (fixed or stochastic) in the data, and thus the mean is not constant over time; (b) the variance, the volatility of the series, of the series is not constant over time; (c) there is a seasonal pattern. For this study, since the data is the annual data, seasonal patterns can not be a possible source of nonstationarity. Also, for the case of variant variance, there are several data transformations that can be used to induce a constant variance.[1] Therefore, the focus of investigating the nature of nonstationarity is on identifying the pattern of the non-constant mean of the time series over time; that is, whether there is a fixed time trend or a stochastic time trend in the observations over time and how to remove the trend appropriately.

4.1.2. Integrated Stationary vs. Trend Stationary Processes:

The statistical properties of the two fundamentally different classes--the integrated stationary pattern and the trend stationary pattern--are stated as follows:

The integrated stationary process is that class for which first or higher order differences is a stationary and invertible ARMA process.

$$(1-L)Z_t=\beta+d_t$$

$$\delta(L)d_t=\lambda(L)u_t; \qquad (25)$$

where (1-L) is the difference operator and $\delta(L)$ and $\lambda(L)$ are polynomials satisfying the stationarity and invertibility conditions. u_t is normally and independently distributed with a mean of 0 and a variance of σ^2.

Such behavior implies that the series lacks a fixed long-term mean, or has a tendency to drift farther away from any given initial state through time. The long-term forecast of this process will always be influenced by historical events.

The trend stationary pattern consists of those processes that can be expressed as a deterministic function of time, called a trend, plus a stationary stochastic process with a mean of zero:

$$Z_t= \alpha+\beta t+c_t$$

$$\phi(L)c_t=\theta(L)u_t, \tag{26}$$

where α and β are fixed parameters, L is the lag operator, and $\phi(L)$ and $\theta(L)$ are polynomials in L that satisfy the conditions for stationarity and invertibility. u_t is normally and independently distributed with a mean of 0 and a variance of σ^2.

The basic determinism of this process is captured in the properties of long-term forecasts. While autocorrelation in c_t can be used in short-term forecasting, it is clear that in the long run, the only information about a future Z_t is its mean, $\alpha+\beta t$. Therefore, neither current nor past events will alter long-term expectations.

4.2 Shock' Behaviors in Integrated and Trend Stationary Processes

Different time processes characterize shocks behaviors variously. This section makes a basic illustration of different shock's behaviors in the integrated stationary processes and in the trend stationary processes.

Assume that Z_t follows a random walk, which is the simplest form of the integrated stationary processes. Z_t can be written as the sum of its value in the previous period, Z_{t-1}, and disturbance, u_t, occurring at period t.

$$Z_t=\rho Z_{t-1}+u_t \quad \text{and } \rho=1, \tag{27}$$

where the disturbance term, u_t, is assumed to be a white noise. Given a disturbance occurring at period t with a value of "v", this disturbance will remain in the next period, t+1, with the same value and last in all the periods after. In other words, any shock to a variable following this simple integrated stationary process tends to be persistent. For other more complicated integrated stationary processes such as including time trends and / or autoregressive term, the value of any shock might vary over time while the shock will not disappear since ρ equals one for all the integrated stationary processes.

On the other hand, if Z_t follows a trend stationary, then Z_t can be written as the following,

$$Z_t=\mu+\gamma t+\rho Z_{t-1}+u_t \quad \text{and } |\rho|<1, \tag{28}$$

where μ is a constant term, t is time, ρZ_{t-1} is an autoregressive term and u_t is a white noise. For a shock occurring at period t with a value of "v", it will remain at the next period only with a fraction of "v" because the absolute value of ρ is less than one. If the value of ρ in a trend stationary process equals zero, the duration of shocks in this process is only one period. As the value of ρ gets close to one, the shocks in the process last longer. However, since $\lim_{n \to \infty} \rho^n = 0$ for all the trend stationary processes, shocks occurring in all the trend stationary processes cannot last forever.

Therefore, this illustration shows that the shocks occurring in the trend stationary processes tend to be transient while the shocks occurring in the integrated stationary processes tend to be persistent.

4.3 Economic Implications

Because the economic impact of random disturbances is different in the integrated stationary process and the trend stationary processes, it is important to examine which process best describes the properties of real income and real government spending. The time series properties of these two variables are therefore considered in the below.

First, an unobserved components model for aggregate income, which includes a growth component and a cyclical component, attempts to explain fluctuations in income. Since cyclical fluctuations are assumed to dissipate over time, any long-run or permanent movement is necessarily attributed to the growth component. The question then turns to whether the long-run or permanent movement follows a fixed trend. If the trend is fixed over time, i.e. a trend stationary process, then movement of the growth component follows a deterministic trend, and most fluctuations in income are explained by the shocks to the cyclical component (e.g. monetary disturbances), which are purely transitory fluctuations. On the other hand, if the trend is not fixed over time, i.e. an integrated stationary process, then shocks to the growth component, which are associated with real disturbances such as capital accumulation, population growth, and technological changes, contribute substantially to the variations in observed income. Thus permanent variations due to these real factors are essential elements of income fluctuations.

Secondly, an unobserved component model for real government spending decomposes its fluctuations into a growth component and a transitory component. A trend stationary representation assumes that the long-run or permanent component of government purchases follows a deterministic time trend, while changes in temporary spending comprise the bulk of fluctuations in real government expenditure. These fluctuations due to temporary changes such as high military expenditures during war time will disappear over time. Thus, there is a tendency for future government spending to return to a deterministic trend under the hypothesis of the trend stationary process. An integrated stationary representation for real government spending implies that the main fluctuations of real government spending come from the variations of long-run or permanent government investment such as military build up due to the competition with USSR.

4.4 The Data

Unlike Barro's empirical works (1979, 1984) which employ data from 1917 to 1976 and 1982 respectively, I extend the investigation from 1889 to 1984 in order to include a greater perspective on World War I. Annual observations of the following variables are collected: GNP deflator, nominal GNP, net federal interest-bearing debt issue, federal debt interest rate, net budget receipt and adjusted net budget expenditures. The major data manipulation is described as follows.

First, the quantity of the nominal national debt is measured as the outstanding stock of interest-bearing federal debt at par value in the hands of the "public" on June 30 of each year. In particular, the figures include both public debt issues and federal agency issues, but they net out holdings of public debt by government investment accounts, holdings of public debt by the Federal Reserve, as well as holdings of federal agency securities by the federal government and the Federal Reserve. These deductions have been made for net federal interest-bearing debt and are based on the consideration that lendings and borrowings within the federal government sector[2] entail no real change in the borrowings from the public.

Secondly, the imputed annual interest rate on the federal debt is computed as the interest payments on the public debt paid in the year divided by the total outstanding public interest-bearing debt issue in the previous year. The data of annual interest payments on federal agency debt is incomplete; therefore, it is not included in calculating the imputed annual interest rate on the federal debt. This approximation, however, would be proper if the interest rate paid on the public debt is always close to that on the federal agency debt.

Finally, annual observations of net budget receipts and adjusted net budget expenditures are collected in the following way. The definition of net budget receipts does not include the interest revenues on the holdings of government debt by federal agencies and the Federal Reserve since these interest revenues are merely transfers within the government sector. The definition of adjusted net budget expenditures is the net budget expenditures minus interest payments on the federal debt held by the public. Since the interest payments on the federal agency debt is not complete, the approximate interest payments on federal debt for each year are measured by multiplying interest-bearing federal debt by imputed interest rate on the federal debt.

The data set and detailed descriptions of data sources are presented in appendix B.

4.5 Empirical Results

The first step in the time series analysis is to plot the available observations against time. Examining the plots of the annual data of real GNP and real government spending, we observe the growing variance over time. To reduce the concern of non-constant variance, we take the natural logs of the series.

Then, we examine the pattern of their mean over time by testing whether the time series of real GNP and real government spending are trend stationary processes or integrated stationary processes. There are two parts of the empirical evidence concerning the nature of the trends of the time series: the sample autocorrelations and the results of the Dickey-Fuller test.

4.5.1. Sample Autocorrelations Evidence:

First of all, nonstationary series would produce sample autocorrelations that remain large even at long lags. This is because in a realization, the series tends to be on one or the other side of the sample mean of the series for many periods and reveals a great tendency to wander away persistently from its historical mean over time. The sample autocorrelations of the levels for each time series, tabulated in table 1, typically start at around 0.96 at lag one and decay slowly with increasing lags. This indicates nonstationarity in the data, and is also similar to the behavior of sample autocorrelations from random walk, which is the simplest form of integrated stationary representation, as indicated by the value calculated from a formula derived from Wichern (1973).

Table 1

Sample Autocorrelations of the Level (Natural Logs)

Series[b]	Sample autocorrelations[a]						
	T	r_1	r_2	r_3	r_4	r_5	r_6
GNP	96	0.96	0.92	0.89	0.86	0.82	0.79
Government spending	96	0.96	0.91	0.87	0.83	0.80	0.77
Random walk[c]	100	0.95	0.90	0.85	0.81	0.76	0.70

[a] T is the sample size and r_i is the ith order autocorrelation coefficient.

[b] The data in these series are deflated by GNP deflator.

[c] Computed by Nelson and Plosser (1982) from the approximation due to Wichern (1973).

Further, sample autocorrelations of first differences, presented in table 2, show stationarity in each first-order differenced series. It is important to stress that sample autocorrelations in each series are positive and significant at lag one but negative or insignificant at longer lags. However, Chan et al. (1977) have proven that if the true model of a time series is a linear trend stationary model, the use of first differences induces stationarity but creates a spurious first lag negative autocorrelation in the differences as well. The common characteristic in the first differences of these series, found in table 2, is not consistent with that of the trend stationary representation.

This is evidence against the trend stationary representation for these time series.

Table 2

Sample Autocorrelations of the First Differences of the Natural Logs of Data

Series[b]	Sample autocorrelations[a]						
	T	r_1	r_2	r_3	r_4	r_5	r_6
GNP	96	0.21	0.01	-0.17	-0.17	-0.16	0.03
Government Spending	96	0.33	-0.15	-0.19	-0.20	-0.21	-0.11

[a]See footnote to table 1.
[b]See footnote to table 1.

This evidence against the trend stationary model is reinforced by sample autocorrelations of the deviations from fitted trend lines presented in table 3. Chan et al. (1977) and Nelson and Kang (1981) state that when the true model of a time series is an integrated stationary model, the use of linear least squares to eliminate a suspected time trend creates large spurious positive autocorrelations in the first few lags. In table 3, the pattern of sample autocorrelations of the deviations from the time trend is similar across series starting at close to 0.90 at lag one and declining roughly exponentially. This finding again suggests the consistency of the data with a form of the integrated stationary process.

Table 3

Sample Autocorrelations of the Deviations From Time Trend

Series[b]	Sample autocorrelations[a]						
	T	r_1	r_2	r_3	r_4	r_5	r_6
GNP	96	0.86	0.66	0.47	0.32	0.22	0.16
Government spending	96	0.82	0.53	0.29	0.12	0.02	0.00
Detrended random walk	101	0.91	0.82	0.74	0.66	0.58	0.51

[a]See footnote to table 1.

[b]See footnote to table 1.

[c]Approximate expected sample autocorrelations based on Nelson and Kang (1981).

The above preliminary results of sample autocorrelations lean toward integrated stationary processes rather than trend stationary processes. However, since the standard deviations of sample deviations are relatively large, further formal tests are necessary to provide more convincing evidence.

4.5.2. Dickey-Fuller Tests:

If the time series are nonstationary, then the roots of their characteristic equations would lie outside one. This imposes certain restrictions of the estimators of the parameters of the time series. Thus, we can test the restrictions of estimators in order to examine nonstationarity. The formal testing procedures include three parts. First, we used the Dickey (1975) and Fuller (1976) test to insure the existence of nonstationarity in the time series of real GNP and real government spending. Secondly, we employed the same test to examine whether the nonstationarity still remains after detrending time series of these two variables and after differencing time series of these two variables, respectively. Finally, we included the time variable into the testable regression equation, and examined the properties of the estimators of the parameters of the time series in order to investigate whether macroeconomic time series are best characterized as trend stationary processes or integrated stationary processes.

In the first application of the Dickey-Fuller test, we examined nonstationarity by estimating the following first-order regression equation:

$$Z_t=\mu+\rho_1 Z_{t-1}+u_t, \tag{29}$$

and the following higher-order regression equations:

$$Z_t=\mu+\rho_1 Z_{t-1}+\sum_{j=1}^{k}\rho_j(Z_{t+1-j}-Z_{t-j})+u_t, \tag{30}$$

where μ, ρ_j, $j=1,2,\ldots,k$ are fixed parameters, and Z_t is the natural log of any time series at time t. u_t is assumed to be normally and independently distributed with a mean of 0 and variance of σ^2.

The testing hypotheses are as follows:

Null hypothesis H_0: $\rho_1=1$, Z_t is nonstationary.

Alternative hypothesis H_1: $\rho_1<1$, Z_t is stationarity.

Using historical time series of real GNP and real government expenditures for the U.S. during 1889 and 1984, we estimated regression models, eq.(29) and eq.(30)[3], for real GNP and real government spending by the ordinary least squares estimation[4]. The results are tabulated in table 4.

Table 4

Dickey-Fuller Test on $\rho_1=1$ in eqs. (29) and (30) for the Series of Real GNP and Real Government Spending (Ordinary Least Squares Estimation)

Equation		Coefficient	Estimate of Coefficient	Standard Error	Studentized[a] Coefficient	test H_0: Parameter
$\ln Y_t$	k=1	$\hat{\rho}_1$	0.9958	0.0072	-0.5833	1
	k=2	$\hat{\rho}_1$	0.9965	0.0073	-0.4794	1
$\ln G_t$	k=1	$\hat{\rho}_1$	0.9821	0.0180	-0.9944	1
	k=2	$\hat{\rho}_1$	0.9776	0.0174	-1.2874	1

[a]Studentized coefficients for $\hat{\rho}_1$ are computed as $\hat{\rho}_1-1$ divided by their own standard errors.

The distribution of the estimator of the studentized coefficient,

$$\hat{t}_\rho = \frac{\hat{\rho}_1 - 1}{[s^2(\sum_{t=2}^{n} Y_{t-1}^2)^{-1}]^{1/2}}, \text{ where } s^2 = \frac{1}{n-2}\sum_{t=2}^{n} \hat{e}_t^2,$$

is a biased t-distribution. Dickey (1975) tabulated a Monte-Carlo distribution for $\hat{t}_\rho$, when the true value of the parameter, ρ_1, is equal to one. The distribution of $\hat{t}_\rho$ is listed in the table 5.

Table 5

Empirical Cumulative Distribution of

Studentized Coefficient[a] for $\rho=1$ in eqs, (29) and (30)

Sample Size	Probability of a smaller value							
	0.01	0.025	0.05	0.10	0.90	0.95	0.975	0.99
100	-3.51	-3.17	-2.89	-2.58	-0.42	-0.05	0.26	0.63

[a]This table was compiled by David A. Dickey using the Monte Carlo method. Details are given in Dickey (1975). Standard errors of the estimates vary, but most are less than 0.02.

Comparing the estimates of studentized coefficient in table 4 with the empirical cumulative distribution for the studentized coefficient for $\rho_1=1$, in table 5, the null hypothesis, nonstationarity, can not be rejected at the significant level= 10% in all cases. Therefore, the time series of real GNP and real government spending are likely to be nonstationary.

In the second application of the Dickey-Fuller test, we examines the nonstationarity of the detrended data for real GNP and real government spending. Using the detrended data to estimate eq.(29) and eq.(30)[3], the OLS estimation[4] results are listed in table 6.

Table 6

Dickey-Fuller Test on ρ_1=1 in eqs. (29) and (30)
for the Detrended Series of Real GNP and Real Government Spending
(Ordinary Least Squares Estimation)

Equation		Coefficient	Estimate of Coefficient	Standard Error	Studentized[a] Coefficient	Test H_0: Parameter
$\ln Y_t$	k=1	$\hat{\rho}_1$	0.8587	0.0525	-2.6914	1
	k=2	$\hat{\rho}_1$	0.8205	0.0526	-3.4125	1
$\ln G_t$	k=1	$\hat{\rho}_1$	0.8280	0.0588	-2.9251	1
	k=2	$\hat{\rho}_1$	0.7460	0.0554	-4.5848	1

[a]See footnote to table 4a.

Comparing the results in table 6 with the empirical cumulative distribution for the studentized coefficient for ρ=1, in table 5, the null hypothesis, nonstationarity, can not be rejected at the significant level=2.5% for both detrended series as k=1, and can not be rejected at the significant level= 1% for the detrended series of real GNP as k=2. But the null hypothesis can be rejected at the significant level= 1% for the detrended series of real government spending as k=2. In general, these results throw doubt upon the method of detrend to stationarize the series of real GNP and real government spending.

If we take first-order differences of the series of real GNP and real government spending, then examine the nonstationarity of the differenced data, we produce the results listed in table 7.

Table 7

Dickey-Fuller Test on $\rho_1=1$ in eqs. (29) and (30) for the First-Ordered Differenced Series of Real GNP and Real Government Spending (Ordinary Least Squares Estimation)

Equation		Coefficient	Estimate of Coefficient	Standard Error	Studentized[a] Coefficient	Test H_0: Parameter
$\ln Y_t$	k=1	$\hat{\rho}_1$	0.2064	0.1020	-7.7804	1
	k=2[b]	$\hat{\rho}_1$	0.1775	0.1328	-6.1935	1
$\ln G_t$	k=1	$\hat{\rho}_1$	0.3304	0.0984	-6.8048	1
	k=2	$\hat{\rho}_1$	0.1295	0.1163	-7.4850	1

[a]See footnote to table 4a.

[b]The coefficient of the second explanatory variable is not statistically significant.

Comparing the results in table 7 with the empirical cumulative distribution for the studentized coefficient for $\rho_1=1$, in table 5, we can reject the null hypothesis at the significant level=1% for all cases. Therefore, taking the first-order difference can make these series stationary.

The third step of the testing procedure was to investigate whether these macroeconomic time series are best characterized as trend stationary processes or integrated stationary processes. It postulates the regression equation by allowing the possibility that the mean of the time series is a linear function of time. The first-order regression equation, eq.(31), is listed below:

$$Z_t=\mu+\gamma t+\rho_1 Z_{t-1}+u_t, \tag{31}$$

The higher-order regression, eq.(32), is generalized as follows:

$$Z_t=\mu+\gamma t+\rho_1 Z_{t-1}+\sum_{j=2}^{k} \rho_j(Z_{t+1-j}-Z_{t-j})+u_t, \tag{32}$$

where μ, $\gamma, \rho_j, j=1,2,\ldots,k$ are fixed parameters, and Z_t is the natural log of any time series at time t. u_t is a disturbance term, and is normally and independently normally distributed with a mean of 0 and a variance of σ^2.

The testing hypotheses are the following:

Null hypothesis, H_0: $\rho_1 = 1$, integrated stationary process.

Alternative hypothesis, H_1: $\rho_1 < 1$, trend stationary process.

Using the historical time series of real GNP and real government expenditures for the U.S. during 1889 and, we estimated the following first-order regression model, eq.(31), and higher-order regression model, eq.(32),[3] for real GNP, real government spending. The results of OLS estimation[4] are summarized in table 8.

Table 8

Dickey-Fuller Test on ρ_1=1 in eqs.(31) and (32)
for the Series of Real GNP and Real Government Spending
(Ordinary Least Squares Estimation)

Equation		Coefficient	Estimate of Coefficient	Standard Error	Studentized Coefficient[a]	Test H_0: Parameter
$\ln Y_t$	k=1	$\hat{\rho}_1$	0.8587	0.0528	-2.6787	1
	k=2	$\hat{\rho}_1$	0.8205	0.0529	-3.3945	1
$\ln G_t$	k=1	$\hat{\rho}_1$	0.8281	0.0591	-2.9086	1
	k=2	$\hat{\rho}_1$	0.7460	0.0557	-4.5601	1

[a]Studentized coefficient for $\hat{\rho}_1$ is computed as $\hat{\rho}_1-1$ divided by its own standard error, and studentized coefficient for $\hat{\gamma}$ is computed as $\hat{\gamma}$ divided by its own standard error.

Both Fuller (1976) and Nelson and Plosser (1982) state that the studentized coefficient for testing the hypothesis that $\rho_1=1$ is biased towards indicating a trend stationary process. Therefore, it is necessary to conduct Monte Carlo experiments to illustrate the sampling distributions for the estimator of studentized coefficient, $\hat{t}_\rho$, in the regression model, eq.(31) and eq.(32). The table of the sampling distributions for the studentized coefficient of $\hat{\rho}_1$ in this regression model for $\rho=1$, done by Dickey (1975), is listed below:

Table 9

Empirical Cumulative Distribution of Studentized Coefficient for $\rho=1$

Sample Size	Probability of a smaller value							
	0.01	0.025	0.05	0.10	0.90	0.95	0.975	0.99
100	-4.04	-3.73	-3.45	-3.15	-1.22	-0.90	-0.62	-0.28

[a]See footnote to table 5.

Comparing the estimates for the studentized coefficient for $\rho_1=1$ in table 8 with the distribution of the studentized coefficient for $\rho_1=1$ in table 9, the null hypothesis, integrated stationary process, can not be rejected at the significant level=5% for real GNP as k=1, 2. As a result, including the time variable does not make real GNP time series stationary. In other words, we cannot reject the hypothesis that real GNP follows a integrated stationary process. However, the results for real government spending are mixed. The null hypothesis cannot be rejected at significant level=5% as k=1, but can be rejected as k=2.

Based on all the empirical results presented in this chapter, we may conclude that both real GNP and real government spending are more likely to follow integrated stationary processes rather than trend stationary processes, even though the empirical evidence for real government spending is not robust.

Notes:

1. The logarithmic transformation can be employed effectively when the variance of the series is proportional to the mean level of the series, and the mean level of the series increase or decrease at a constant percentage rate. Another similar but milder correction is the square root transformation of the data.

2. Federal government sector, in this study, includes federal government, federal agencies, and the Federal Reserve.

3. The error terms in these two time series might not be normally and independently distributed with a mean of 0 and variance of σ^2. If the error terms are autocorrelated, the OLS estimates of parameters are unbiased and consistent, but the OLS estimates of the standard errors are biased and inconsistent; therefore, it affects the significance of tests for studentized coefficients. Due to this fact, it is necessary to examine the existence of autocorrelation among error terms. Since Durbin-Watson test in this case is biased, I conduct h test and find that we can not reject the null hypothesis, nonautocorrelated error terms.

4. The coefficients for the higher order ($k>2$) are statistically insignificant: therefore, only the results from the estimation of the regression model, $k=1, 2$ are shown.

CHAPTER V

EXAMINING THE PUBLIC DEBT POLICY UNDER THE REAGAN ADMINISTRATION

The purpose of this chapter is to examine Barro's (1984) contention that the high deficits under the Reagan administration have been consistent with the optimal debt policy during 1981 and 1984. In order to make such a comparison, we attempt to derive the optimal amount of tax finance and debt finance for these four years individually.

In the first part of this chapter, we identify and estimate the ARIMA models for the time series of real GNP and real government spending. Applying the observations between 1889 and 1981, we may forecast the future values of real GNP and real government spending annually, starting with 1982. It is noted that income shocks and government spending shocks tend to persist over time in an integrated stationary process. When new information is observed, the long-run forecasting in an integrated stationary process needs to be revised. Therefore, as new observations for the years of 1982, 1983 and 1984 pour in, we update the estimation and forecast of real GNP and real government spending annually. In the second part of this chapter, we incorporate the different sets of the forecasting values of income and government spending into the modified theoretical model of the optimal public debt, and we approximately compute the optimal tax finance and debt finance for each year during 1981 and 1984. Then we compare the actual tax and debt finance with the optimal amount of tax and debt finance. Based on the comparison, we conclude that there has been too much debt finance and not enough tax finance during 1982 and 1984. In other words, the high deficits in these three years under the Reagan administration are not justified by the theory.

5.1 Estimating ARIMA Models

The empirical evidence in chapter IV indicates that real GNP and real government spending are likely to be non-stationary processes with stochastic time trends, and this finding suggests that we take first-order differences of these time series to induce stationarity. The empirical work in this chapter will go one step further to identify and estimate the ARIMA models for the time paths of these two economic variables.

There are three steps as we proceed in identification and estimation: first, identifying tentative Box-Jenkins ARIMA models for each time series according to their autocorrelation functions and partial autocorrelation functions; second, estimating the parameters of tentative models and checking whether the estimates lie within the bounds of stationarity and invertibility; finally, applying diagnostic checks to test the validity of the models.

We start with the U.S. data of 1889-1981 and follow these three steps to identify and estimate the past behavior of real income and real government spending, and then later we forecast future values for these two variables.

In step one, in order to identify potential stationary models, autocorrelations and partial autocorrelations are estimated for the first-order differenced data. Figures 1 and 2 contain the autocorrelation and partial autocorrelation functions of the first-order differenced series of real GNP and real government spending, respectively. It is clear that first-order differencing makes the two series stationary since the values of the autocorrelation die out rapidly. Besides that, the values of partial autocorrelation are high at lag 1 for GNP series and at both lag 1 and lag 2 for the series of government spending. Therefore, based on these results, we tentatively identify the model ARIMA (1, 1, 0) for real GNP and the model ARIMA (2, 1, 0) for real government spending[1].

```
      Autocorrelation Function            Partial Autocorrelation Function
--------------------------------+-------------------------------------------
-.8     -.4     0.0     .4    .8|-.8     -.4     0.0     .4     .8
 . . . . . . . . . . . . . . . .| . . . . . . . . . . . . . . . . .
           a(   :****           |            (   :****
            (   :   )           |            (  *:   )
            ****:   )           |            ****:   )
            (***:   )           |            ( **:   )
           ( ***:    )          |            ( **:   )
           (    :*   )          |            (   :*  )
           (    :*   )          |            (   :   )
           (    :    )          |            ( **:   )
           (  **:    )          |            ( **:   )
           (****:    )          |            ****:   )
           (    :    )          |            (   :*  )
           (    :    )          |            ( **:   )
           (    :*   )          |            (  *:   )
           (    :**  )          |            (   :   )
           (    :    )          |            ( **:   )
           (    :*   )          |            (   :** )
           (    :*** )          |            (   :***)
           (    :    )          |            ( **:   )
           (    :*   )          |            (   :*  )
           (  **:    )          |            (***:   )
```

[a]The parentheses mark the standard errors.

Figure 1

Autocorrelation and Partial Autocorrelation Function
of First Differences of the Natural Logs of Real GNP
(1889-1981 annual data)

```
      Autocorrelation Function     |    Partial Autocorrelation Function
-.8     -.4     0.0      .4      .8|-.8     -.4     0.0      .4      .8
 .  .  .  .  .  .  .  .  .  .  .  .| .  .  .  .  .  .  .  .  .  .  .  .
           a(    :*******          |            (   :*******
            ( ***:    )            |          ******:   )
            (****:    )            |            (  *:   )
            (****:    )            |            ****:   )
            (****:    )            |            (***:   )
            (  **:    )            |            ( **:   )
            (    :    )            |            ( **:   )
            (    :    )            |            ( **:   )
            (    :*   )            |            (   :   )
            (    :**  )            |            (  *:   )
            (    :    )            |            (  *:   )
            (  **:    )            |            (***:   )
            (    :*   )            |            (   :***)
            (    :    )            |            (***:   )
            (    :    )            |            (   :** )
            (   *:    )            |            (***:   )
            (    :    )            |            (   :*  )
            (    :*   )            |            (  *:   )
            (   *:    )            |            ( **:   )
            (   *:    )            |            (   :   )
```

[a]See footnote to figure 1.

Figure 2

Autocorrelation and Partial Autocorrelation Function
of First Differences of the Natural Logs of Real Government Spending
(1889-1981 annual data)

The next step is to estimate the parameters of the tentative models. The results of ordinary least squares estimation are listed in table 10.

Table 10

Estimation of ARIMA Models for
Real GNP and Real Government Spending
(1889-1981 annual data)

Series	Parameter (lag)	Estimate	t-ratio
$\ln Y_t$	AR(1)	0.3793	3.91
$\ln G_t$	AR(1)	0.4494	4.44
	AR(2)	-0.2811	-2.78

All the estimates of parameters lie within the bounds of stationarity, and there is no overfitting phenomenon in this estimation since all the estimates are statistically significant at a 5% significance level.

In step three, we need to check the properties of the residuals in these tentative models. If the estimated models are adequate, their residuals should be approximately white noise. The autocorrelation plots of residuals for these models are presented in figures 3 and 4.

```
  Autocorrelation Function            Partial Autocorrelation Function

-.8     -.4     0.0      .4      .8|-.8     -.4     0.0      .4      .8
 .  .  .  .  .  .  .  .  .  .  .  . | .  .  .  .  .  .  .  .  .  .  .  .
            a(  R:   )             |              ( R:   )
             (   :RR )             |              (  :RR )
             (  R:   )             |              ( R:   )
             (   :R  )             |              (  :R  )
             (  R:   )             |              (  :   )
             (   :RRR)             |              (  :RRR)
             (   :RRR)             |              (  :RRR)
             (   :RR )             |              (  :RR )
             (   :R  )             |              (  :R  )
             ( RR:   )             |              (RR:   )
             (   :RRR)             |              (  :RRR)
            (    :R    )           |              (  :R  )
            (    :RR   )           |              (  :R  )
            (    :RRR  )           |              (  :RRR)
            (    :R    )           |              (  :   )
            (    :R    )           |              (  :RR )
            (    :RRRR )           |              (  :RRRR
            (    :     )           |              (  :   )
            (    :RRR  )           |              (  :RRR)
            (   R:     )           |              (RR:   )
```

[a]See footnote to figure 1.

Figure 3

Autocorrelation and Partial Autocorrelation Function
of Residuals of ARIMA (1,1,0) for Natural Logs of Real GNP
(1889-1981 annual data)

```
     Autocorrelation Function       |   Partial Autocorrelation Function
-.8    -.4    0.0     .4     .8     | -.8    -.4    0.0     .4     .8
 .  .  .  .  .  .  .  .  .  .  .  . |  .  .  .  .  .  .  .  .  .  .  .  .
            a(   :   )              |            (   :   )
             ( R:   )               |            ( R:   )
             (   :   )              |            (   :   )
             ( RR:   )              |            ( RR:   )
             (RRR:   )              |            (RRR:   )
             ( R:   )               |            ( R:   )
             (   :R  )              |            (   :   )
             (   :   )              |            (   :   )
             (   :R  )              |            (   :R  )
             (   :R  )              |            (   :R  )
             (   :RR )              |            (   :RR )
             (RRR:   )              |            (RRR:   )
             (   :RRR)              |            (   :RRRR
            (    R:    )            |            ( R:    )
            (     :RR  )            |            (   :RRR)
            (    R:    )            |            ( R:    )
            (     :    )            |            (   :R  )
            (     :RR  )            |            (   :RR )
            (   RR:    )            |            ( R:    )
            (     :R   )            |            (   :R  )
```

[a]See footnote to figure 1.

Figure 4

Autocorrelation and Partial Autocorrelation Function Residuals of ARIMA (2,1,0) for Natural Logs of Real Government Spending (1889-1981 annual data)

In both cases, there is no spike in the autocorrelation plots of residuals at key lags; that is, the residuals are well within the white noise confidence band at a 95% confidence level. The overall adequacy of the model can be tested by the Ljung-Box residual portmanteau test of model adequacy. The resulting Q statistics can be found in table 11. Since the Q statistics are not significant, they do not provide any startling evidence of model inadequacy.

Table 11

Ljung-Box Residual Portmanteau Test of Model Adequacy
(1889-1981 annual data)

Series	lag	Q statistic	Degree of Freedom	P-value
Y_t	6	4.25	5	0.514
	12	10.93	11	0.449
$\ln G_t$	6	3.78	4	0.437
	12	8.08	10	0.621

In an integrated stationary process, all the shocks exist forever and accumulate over time and affect the future values of the series. In order to decide the optimal current finance in the years 1982, 1983 and 1984, respectively, we incorporate new current variations in real GNP and real government spending during these three years and update the estimation and the forecast for these two variables each year. We apply the data of real GNP and real government spending in the periods of 1889-1982, 1889-1983, and 1889-1984 to repeat the three steps of identifying and estimating ARIMA models for them.

In step one, in order to identify potential stationary models, autocorrelations and partial autocorrelations are estimated for the three data sets, 1889-1982, 1889-1983 and 1889-1984. The autocorrelation and partial autocorrelation functions of these three data sets for the differenced series of real GNP and real government spending are very similar to those of 1889-1981. In other words, including the new observations does not alter the characteristic of their past behavior much. Therefore, based on these results, we retain the same tentative models for real GNP and real government spending.

The next step is to estimate the autoregressive parameters of the tentative models for the first-order differenced series of real GNP and real government spending. The ordinary least squares estimates for three data sets are compiled in table 12. All the estimates of autoregressive parameters lie within the bounds of stationarity, and there is no overfitting phenomenon in this estimation since all the estimates are statistically significant at a 5% significance level.

Table 12

Estimation of ARIMA Models for
Real GNP and Real Government Spending
(1889-1982, 1889-1983 and 1889-1984)

Period	Series	ARIMA Model	Parameter (lag)	Estimate	t-ratio
1889-1982	$\ln Y_t$	(1,1,0)	AR(1)	0.3775	3.91
	$\ln G_t$	(2,1,0)	AR(1)	0.4494	4.47
			AR(2)	-0.2811	-2.79
1889-1983	$\ln Y_t$	(1,1,0)	AR(1)	0.3753	3.90
	$\ln G_t$	(2,1,0)	AR(1)	0.4493	4.49
			AR(2)	-0.2806	-2.80
1889-1984	$\ln Y_t$	(1,1,0)	AR(1)	0.3796	3.96
	$\ln G_t$	(2,1,0)	AR(1)	0.4491	4.51
			AR(2)	-0.2806	-2.82

In step three, we need to check the properties of the residuals in these tentative models. If the estimated models are adequate, their residuals should be approximately white noise. The autocorrelation and partial autocorrelation functions are very similar to those of 1889-1981 in figures 3 and 4. In all three cases, there is no spike in the autocorrelation plots of residuals at key lags; that is, the residuals are well within the white noise confidence band at a 95% confidence level. The overall adequacy of the model can be tested by the Ljung-Box residual portmanteau test of model adequacy. The resulting Q statistics can be found in table 13. Since the Q statistics are not significant, they do not provide any startling evidence of model inadequacy.

Table 13

Ljung-Box Residual Portmanteau Test of Model Adequacy
(1889-1982, 1889-1983 and 1889-1984)

Periods	Series	Q statistic	Degree of Freedom	P-value
1889-1982	$\ln Y_t$	4.16	5	0.526
		10.91	11	0.451
	$\ln G_t$	3.82	4	0.431
		8.16	10	0.613
1889-1983	$\ln Y_t$	4.39	5	0.495
		11.43	11	0.408
	$\ln G_t$	3.83	4	0.429
		8.21	10	0.608
1889-1984	$\ln Y_t$	4.32	5	0.504
		11.85	11	0.375
	$\ln G_t$	3.88	4	0.423
		8.29	10	0.600

5.2 Computing the optimal debt issues for 1981-1984

According to empirical evidence in chapter IV and section 5.1 of this chapter, real GNP and real government spending follow integrated stationary processes; more specifically, they are identified as the ARIMA (1,1,0) process and the ARIMA (2,1,0) process, respectively. Based on this estimation, we trace shock behaviors in both time series. Shocks occurring in these two specific integrated stationary processes last forever and have specific autoregressive tendencies. The estimation of shock behaviors in these two time series is computed and listed in tables 14 and 15.

Table 14

Income shock behavior in the ARIMA (1,1,0) process, 1889-1984

Time	Income Shock	% change in shock
1	u	--
2	(1.37960)u	37.96
3	(1.52370)u	10.44
4	(1.57840)u	3.59
5	(1.59916)u	1.32
6	(1.60704)u	0.49
7	(1.61003)u	0.19
8	(1.61117)u	0.07
9	(1.61160)u	0.03
10	(1.61176)u	0.01
11	(1.61182)u	0.00
12	(1.61185)u	0.00
13[a]	(1.61186)u	0.00

[a]After 13 time periods, an income shock converges to (1.61186)u

Table 15

Government Spending Shock Behavior in the ARIMA (2,1,0) process, 1889-1984

Time	Gov't Spending Shock	% change in Shock
1	v	--
2	(1.44910)v	44.91
3	(1.37019)v	- 5.45
4	(1.20874)v	-11.78
5	(1.15837)v	- 4.17
6	(1.18105)v	1.96
7	(1.20537)v	2.06
8	(1.20993)v	0.38
9	(1.20515)v	-0.37
10	(1.20173)v	-0.28
11	(1.20153)v	-0.02
12	(1.20240)v	0.07
13	(1.20285)v	0.04
14	(1.20281)v	-0.00
15	(1.20266)v	-0.01
16	(1.20261)v	-0.00
17	(1.20262)v	0.00
18[a]	(1.20265)v	0.00

[a]After 18 time periods, a government spending shock converges to (1.20265)v.

Let's assume that the government perceives the current real GNP and real government spending, and that the government forecasts GNP and its spending according to past behaviors, and renews its forecasting each year based on all the available information. Tables 16 and 17 summarize the available analyzed information of GNP and government spending behaviors in each year from 1981 to 1984.

Table 16

Estimated Processes for Real GNP , Estimated During 1981 and 1984

Year estimated	Real GNP: ARIMA (1,1,0)
1981:	$\ln Y_t - \ln Y_{t-1} = .0326 + .2073(\ln Y_{t-1} - \ln Y_{t-2}) + e_t$
	(4.13) (2.01)
1982:	$\ln Y_t - \ln Y_{t-1} = .0319 + .2084(\ln Y_{t-1} - \ln Y_{t-2}) + e_t$
	(4.06) (2.02)
1983:	$\ln Y_t - \ln Y_{t-1} = .0321 + .2060(\ln Y_{t-1} - \ln Y_{t-2}) + e_t$
	(4.14) (2.02)
1984:	$\ln Y_t - \ln Y_{t-1} = .0325 + .2064(\ln Y_{t-1} - \ln Y_{t-2}) + e_t$
	(4.24) (2.03)

The numbers in the parentheses are t-ratios.

Table 17

Estimated Processes for Real Government Spending
Estimated During 1981 and 1984

Year estimated	Real government spending: ARIMA(2,1,0)
1981:	$\ln G_t - \ln G_{t-1} = .0557 + .4293(\ln G_{t-1} - \ln G_{t-2}) - .2994(\ln G_{t-2} - G_{t-3}) + e_t$
	(1.64) (4.31) (-3.01)
1982:	$\ln G_t - \ln G_{t-1} = .0567 + .4295(\ln G_{t-1} - \ln G_{t-2}) - .2999(\ln G_{t-2} - G_{t-3}) + e_t$
	(1.65) (4.29) (-3.00)
1983:	$\ln G_t - \ln G_{t-1} = .0563 + .4297(\ln G_{t-1} - \ln G_{t-2}) - .2999(\ln G_{t-2} - G_{t-3}) + e_t$
	(1.63) (4.27) (-2.98)
1984:	$\ln G_t - \ln G_{t-1} = .0569 + .4297(\ln G_{t-1} - \ln G_{t-2}) - .3000(\ln G_{t-2} - G_{t-3}) + e_t$
	(1.63) (4.25) (-2.97)

The numbers in the parentheses are t-ratios.

Furthermore, according to the estimated ARIMA models for these two economic variables, we can forecast their future values at different points of time. Based on the available information for each year, future values of annual changes in real GNP and real government spending forecasted in the years 1981, 1982, 1983 and 1984 are listed in the table 18.

Table 18

Changes in Future Values in Real GNP and
Real Government Spending Forecasted in 1981, 1982, 1983 and 1984
(in billion dollars)

Year	Forecasted in 1981		Forecasted in 1982		Forecasted in 1983		Forecasted in 1984	
	GNP	Spending	GNP	Spending	GNP	Spending	GNP	Spending
1982	1.0315	1.0662						
1983	1.0328	1.0598	1.0210	1.0384				
1984	1.0330	1.0576	1.0705	1.0630	1.0336	1.0756		
1985	1.0331[a]	1.0573	1.0319	1.0661	1.0328	1.0632	1.0402	1.0260
1986		1.0583	1.0323	1.0599	1.0326[a]	1.0552	1.0346	1.0637
1987		1.0589	1.0324[a]	1.0563		1.0558	1.0333	1.0696
1988		1.0588		1.0566		1.0580	1.0331	1.0606
1989		1.0585[a]		1.0581		1.0590	1.0330[a]	1.0550
1990				1.0589[a]		1.0586		1.0553
1991						1.0582		1.0571
1992						1.0581		1.0578
1993						1.0583[a]		1.0575
1994								1.0572[a]

[a]The future values beyond this year converge to the value of this year.

With the future values and a modified model of the optimal public debt, we can compute the optimal public debt issue for the past four years, 1981-1984, under the Reagan administration. Incorporating future values of real GNP and real government spending into the model of the optimal public debt, eq. (1), eq. (2) and eq. (5), the optimal amounts of annual debt issue from the year 1981 to 1985 can be calculated approximately and are listed in table 19[2].

Table 19

Comparison of the Optimal Policy and the Actual Policy
(in billion dollars)

Year	Optimal tax Finance	Actual tax Finance	Optimal debt[a] Finance	Actual debt[a] Finance
1981	525.3446	297.0598	-193.9755	34.3113
1982	482.1022	301.8973	-140.0986	40.1063
1983	521.2825	275.2243	-156.3893	89.6689
1984	505.1370	290.2023	-135.5376	79.3971

[a]Conceptually, the amount of annual debt finance is the annual increase in the outstanding stock of public debt. The optimal debt finance is calculated by subtracting the optimal debt finance from the periodic budget constraint. The actual debt finance is calculated by subtracting the actual taxes finance from the periodic budget constraint.

Based on the assumed properties of the optimal public debt model for permanent shocks, table 19 provides estimates of the optimal amounts of tax finance. The suggested levels of taxation in table 19 are much higher than any recent actual levels of taxation because of the permanent increases in government spending during 1981 and 1983. These permanent government spending shocks increase not only the current level but also the future trend for government spending, and so high current tax finance is necessary. The permanent current income shocks in a random walk stochastic process, however, as explained in chapter III, do not affect the optimal amounts of tax and debt finance. With an autoregressive tendency, the positive permanent income shocks occurring in these years have a weak but positive impact on the optimal tax finance. The negative optimal debt finance is also attributed to the expectation that the future growing trend for real government spending is higher than the future growing trend for real GNP as indicated by the historical data. Therefore, suggests high current taxes are needed to retire the outstanding public debt as well as to accumulate funds to finance the extremely high government spending expected in the future.

If we compare the actual tax and debt finance with the optimal amounts of tax and debt finance in table 19, we find that there has been too much debt finance and not enough tax finance during 1981 and 1984. Since the standard deviations of these estimates are too difficult to derive, we have not been able to develop a strict statistical test for this claim. Nevertheless, the gap between the actual policy and the optimal policy, indicated in table 19, has remained observably large in these four years. In other words, the high deficits under the Reagan administration are very likely to be inconsistent with the the optimal policy suggested by the theory presented in chapter III.

Barro (1984a) assumes that future GNP follows the same fixed time trend as future government spending does and assumes that all shocks are temporary. Based on these assumptions, he argues that the large deficits under the Reagan Administration have not been inconsistent with his tax smoothing theory. However, even if the ARIMA model is not the correct model of future GNP and government spending, the trend stationary models for real GNP and real government spending still imply that the trend for government spending will eventually exceed the trend for real GNP. In this case, the government's optimal mix of finance still requires budget surpluses even though the magnitude of such surpluses may be smaller than in case of permanent government spending shocks. Barro's incorrect assumption that real GNP and real government spending follow the same future trend results in his inappropriate conclusion that high deficits are justified.

There are several reasons why the government may not have financed optimally. First, the government may have failed to estimate the future time paths for real GNP and real government spending. Either underestimating future government spending shocks or overestimating future income may cause overly high budget deficits. Second, the specific case in question may not correspond to all of the assumptions built into the theoretical model. For example, individuals in the private sector may not behave according to the assumptions of the Ricardian equivalence theorem due to finite lives, imperfect capital market or distortionary taxes; or the private economy is Ricardian, but the government may have acted in the erroneous belief that deficit finance might stimulate economic activity. Another possible reason for the non-optimal government behavior is that the policymakers may have behaved as to maximize their personal political popularity with voters and have not attempted to minimize tax collection costs over time.

NOTES

1. Other tentative ARIMA models have been tried, but they do not fit as well as ARIMA (1, 1, 0) and ARIMA (2, 1, 0) for real GNP and real government spending, respectively.

2. Interest rates used in calculation are the imputed interest rates on the federal debt. The definition of imputed interest rates on the federal debt is described in the appendix B.

CHAPTER VI

CONCLUSIONS

There are three major contributions of my dissertation. First, I derive the properties of the optimal mix of tax finance and debt finance when income shocks and government spending shocks are permanent. Second, since the properties of the optimal debt policy are conditional on the specification of time paths for real GNP and real government spending, I conduct numerous tests to determine the prefered specification of time paths for these two variables. Finally, I examine the propriety of high deficits under the Reagan administration and find that high deficits during 1981 and 1984 are not justified by the optimal public debt theory.

The theoretical model of the optimal public debt employed in my dissertation is an extension of Barro (1979). The analysis of fiscal policy is grounded in the microeconomics of intertemporal choice. In Barro's optimal debt theory, the dependence of total tax collection costs on the timing of taxation determines an optimal time path for current and future debt issues. The central result of his analysis is that debt issues should be optimally distributed over time in order to maintain expected constancy in tax rates. With a specification of future time paths for real GNP and real government spending, Barro derives the following properties of the optimal tax-debt finance: transitory high income suggests more tax finance and less debt finance; transitory high government spending is supposed to be largely financed with debt.

However, permanent changes in income and in government spending due to technology advance, long-term military build-up,... etc. may be important in determining the optimal tax-debt finance. Therefore, I formulate new properties of the optimal debt model for the case of permanent shocks. The results of this analysis include the lack of an effect of permanent current income shocks on the optimal current tax-debt financing, a direct effect of permanent future income shocks on the optimal current debt financing, and an inverse effect of permanent future government spending shocks on the optimal current debt finance. Also, current government spending shocks are absorbed by the tax finance only.

Since the properties of the optimal public debt policy are conditional on the specification of shocks, it is interesting to examine empirically whether transitory shocks or permanent shocks dominate the time paths for real GNP and real government spending. As noted, shocks occurring in trend stationary processes are transitory, while shocks occurring in integrated stationary processes are persistent. Thus, in order to determine the comparative importance of transitory shocks vis-a-vis permanent shocks, we examine the time series properties of real GNP and real government spending for representative periods of U.S. history as an aid to identifying appropriate properties of these time series. We present that real GNP and real government spending are better characterized as a non-stationary process that has no tendency to return to a deterministic time trend, i.e. an integrated stationary process. As a result, the permanent rather than transitory income and government spending shocks dominate the time paths for real GNP and real government spending.

Finally, we identified and estimated ARIMA models for real GNP and real government spending. Utilizing the forecast values of these variables in the integrated stationary processes together with the model of the optimal public debt, we calculated optimal amounts of tax finance and debt finance for the years 1981, 1982, 1983 and 1984. The optimal finance theory suggests government budget surpluses for these four years due to continuous increases in government spending and expected higher growth in government spending than that for real GNP. However, there have been record high government budget deficits during 1981 and 1984. This indicates that the high deficits under the Reagan administration are not consistent with this theory. This divergence between the optimal policy and actual policy may be explained in several ways. First, the Reagan administration's estimates of future government spending may be smaller than our estimates, and their estimates of future income may be larger than ours. Either of these assumptions could rationalize the recent budget deficits. Second, the government may not have believed that the private economy is Ricardian, and therefore used deficit finance as a tool to stimulate economy's activity. Finally, the policymakers may have behaved as to maximize their personal political popularity with voters and have not attempted to minimize tax collection costs over time.

APPENDIX A

EMPIRICAL RESULTS OF BARRO (1979) AND HORRIGAN (1986b)

The following tables 20 and 21 summarize the empirical results of Barro (1979) and Horrigan (1986b). The results indicate excessive effect of income shocks on the debt growth.

Table 20

Empirical Results of Barro (1979) for the principle time periods

Coefficient	1948-1976	1941-1976	1922-1976
α_0	0.011	0.002	-0.006
	(0.01)	(0.013)	(0.012)
α_1	1.12	1.26	1.14
	(0.22)	(0.29)	(0.28)
α_2	0.61	1.02	1.01
	(0.16)	(0.03)	(0.03)
α_3	1.75	1.78	1.62
	(0.17)	(0.19)	(0.14)
α_4	---	---	---
α_5	---	---	-0.058
			(0.009)
R^2	0.87	0.97	0.95
D-W	1.8	2.1	2.2

The numbers in the parenthesis are standard errors.

Table 21

Empirical Results of Horrigan (1986b) for the principle time periods

Coefficient	1948-1976	1948-1981	1905-1947
α_0	-0.02	-0.05	-0.14
	(0.02)	(0.01)	---
α_1	1.37	0.33	
	(0.48)	(0.32)	(0.04)
α_2	0.34	0.61	1.06
	(0.25)	(0.17)	(0.03)
α_3	1.88	1.55	2.20
	(0.37)	(0.24)	(0.48)
α_4	---	0.12	0.54
		(0.02)	(0.06)
R^2	0.66	0.97	0.98
D_W	1.3	2.1	0.9

The numbers in the parenthesis are standard errors.

APPENDIX B

U.S. DATA AND DATA DESCRIPTIONS

In the U.S. data set, the annual observations of the following six key variables during 1889-1984 are collected: GNP, GNP deflator, net federal interest-bearing debt issue, the interest payment on public debt, federal budget receipt and federal budget expenditures. This data set is constructed so that the measures of these variables are as close as possible to the measures suggested by Barro (1978, 1979, 1984).

The data for nominal GNP (A) and GNP deflator (B) come from the following sources: for 1889-1908, from Historical Statistics of the United States Colonial times to 1970, part I, series F 1-5, p. 224; for 1909-1928 and 1929-1975, from National Income and Product Accounts of the United States, 1929-1976 Statistical Tables, table 1.22, table 1.1 and table 7.1, p. 72, pp. 1-2 and p. 319, respectively; for 1976-1979, from Survey of Current Business, table 1 and table 7.1, p. 22 and p. 81, respectively, Vol. 62, No. 7 (July, 1982); for 1980-1983, ibid., p. 22 and p. 81, respectively, Vol. 64, No. 7 (July, 1984); for 1984, ibid., table 1.1-1.2 and table 7.22, p. 8 and p. 26, respectively, Vol. 65, No. 7 (July, 1985). The GNP deflators during 1889-1908, using the 1958 price as the base, can not be connected directly with deflators of the years after 1908, using the 1972 price as the base. Therefore, the observations of the deflator between 1889-1908 are rescaled by multiplying an adjusting scale factor 0.646853, which comes from a comparison of deflators between 1909-1958, the overlapping of the two series from different sources.

The level of net federal interest-bearing debt (C) includes public interest-bearing debt (D) and federal agency debt (E) but excludes both the government debt held by federal agencies and trust funds (F) or by the Federal Reserve (G) and federal agency debt held by the federal government and Federal Reserve (H). It is assumed that the debt is growing smoothly over a year; therefore, the measure of debt on June 30 is selected as the mean level of debt during the year. Ideally, federally sponsored agencies' debt should also be considered; however, the data is not available. Data sources of all these components, (D) through (H), are as follows:

The data of total public interesting-bearing debt (D) is from the following sources: for 1889-1970, from Historical Statistics of the United States Colonial Times to 1970, part II, series Y 493-504, pp. 1117-1118; for 1971-1976 and 1977-1984, from Treasury Bulletin, tables FD-1 in p. 25, May 1978; p. 24, Aug. 1978; p. 21, Sept. 1979; p. 23, Sept. 1980; p.23, Sept. 1981; p.26, Sept. 1982; p.13, 4th quarter, fiscal 1983; p. 14, 2nd quarter, fiscal 1985.

The available data for federal agency debt issue (E) begins in 1933. The figure for federal agency debt in 1933 is very small since most of the important agencies were created to deal with depression-era problems. Thus the data for federal agency debt before 1933 are assumed to be negligible. Its data sources are listed as follows: for 1933-1940, from Banking and Monetary Statistics 1941-1976, sec. 13 table 13.3, pp. 874-881; for 1971-1976 and 1977-1984, from Treasury Bulletin, table FD-6, p. 28, May 1978; table FD-6, p. 27, Aug. 1978; tables Fd-5, p. 24, p. 26 and p. 29, Sept. 1979-1982, respectively, tables FD-4, p. 15 and p. 16 for 4th quarter 1983 and 2nd quarter 1985, respectively.

The data on government debt held by federal agencies and trust funds (F) and by the Federal Reserve (G) is from the following sources: for 1916-1940, from Banking and Monetary Statistics, sec. 13, table 149, p. 512; for 1941-1970, ibid., sec. 13, table 13.4, pp. 882-883; for 1971-1976, from Treasury Bulletin, table OFS-1, p. 77, Jan. 1978; for 1977-1984, the sources are the same as (E).

The data of agency securities held by the federal government and the Federal Reserve (H) is from the following sources: before and in 1954, the data is negligible; for 1955-1960, from Treasury Bulletin, table 1, p. 53, Jan. 1962; for 1961-1970, ibid., table OFS-1, p. 65, Jan. 1971; for 1971-1984, from the same sources as (F) and (G).

Budget receipt (I), expenditures (J), and interest payment on the public debt (K) between 1889 and 1976 are from Statistical Appendix to Annual Report of the Secretary of the treasury on the State of the Finances, table 2, pp. 4-13, fiscal 1980. It is noted that the government fiscal years ended on June 30 before and in 1976. But after 1976, they end on Sept. 30. To get consistent measures for (I), (J) and (K) for 1977-1984 with those before 1976, the monthly data in the following are arranged to be annual data ending on June 30: Treasury Bulletin, table FFO-1, and FFO-3, p. 1 and p. 6 (p. 3 for '83 and '84, and p. 7 for '80, '81 and '83, and p. 8 for '84), Sept. 1977-1983, and Fall fiscal 1983-1984

The data set listed below, from left to right, includes annual observations of real GNP, real government spending, real tax revenue, real debt, and imputed interest rates on the federal debt. The observations of all variables start in 1889 and end in 1984. The observations of the first four variables are deflated by the GNP deflator and measured in billions of dollars.

U.S. Data Set

REAL GNP	REAL GOV'T SPENDING	REAL TAXES	REAL DEBT	INTEREST RATE
76.0803	1.5703	2.3554	4.9665	0.0428
81.3160	1.7507	2.5016	4.4134	0.0432
84.8523	2.0616	2.4701	3.8404	0.0521
93.6477	2.1087	2.3248	3.8310	0.0370
88.5183	2.2835	2.4759	3.7524	0.0451
86.1833	2.3256	2.0930	4.3434	0.0468
96.3939	2.2538	2.2538	4.9653	0.0477
94.7293	2.2578	2.4074	6.0328	0.0477
103.5461	2.3262	2.4681	6.0071	0.0439
106.2802	2.7950	2.7950	5.8454	0.0439
115.9227	3.7642	3.4377	6.9687	0.0461
118.9567	3.0598	3.6069	6.5076	0.0375
132.7774	3.1623	3.7716	6.3310	0.0308
134.0782	2.8305	3.4885	5.7790	0.0290
140.4908	2.9939	3.4479	5.6135	0.0307
138.8720	3.3899	3.2808	5.4275	0.0270
148.6967	3.2109	3.2227	5.3021	0.0275

U.S. Data Set (continue)

REAL GNP	REAL GOV'T SPENDING	REAL TAXES	REAL DEBT	INTEREST RATE
166.1841	3.1616	3.4453	5.1824	0.0265
169.0768	3.0868	3.7041	4.9778	0.0265
155.1821	3.5742	3.3725	5.0308	0.0232
180.5405	3.6324	3.2649	4.9351	0.0242
186.3158	3.5421	3.5579	4.8053	0.0227
191.4439	3.5829	3.7540	4.8930	0.0227
201.0204	3.4031	3.5357	4.9184	0.0248
203.0769	3.5487	3.6615	4.9538	0.0236
195.9391	3.5685	3.6802	4.9137	0.0235
193.2367	3.4928	3.2995	4.6860	0.0235
208.1897	2.9741	3.2802	3.9353	0.0234
211.1888	6.7501	3.8497	9.2483	0.0257
244.0895	39.9098	11.6454	37.9776	0.0677
228.8043	48.6135	13.9402	67.3560	0.0495
214.2857	12.5438	15.5714	55.0468	0.0396
200.0000	11.7419	16.0086	66.4397	0.0407
229.7214	7.1944	12.4644	67.2539	0.0409
253.8690	6.3390	11.4673	63.9464	0.0455
256.4955	6.0099	11.6949	60.8943	0.0419
276.0355	6.1440	10.7722	57.1834	0.0412
291.8919	6.4095	11.3964	55.0781	0.0403
293.5185	6.5196	12.3858	52.8519	0.0398
296.0366	6.9338	11.8902	49.4787	0.0393
315.6899	7.6057	11.7888	47.3352	0.0384
285.6693	8.5223	12.7811	45.1339	0.0388
263.5019	10.4821	10.7932	53.2975	0.0377
227.0354	15.9762	7.4951	65.3526	0.0356
222.1170	15.8964	7.9467	77.4771	0.0353
239.1612	21.9041	11.0440	85.3150	0.0336
260.0215	20.7133	13.2927	98.0775	0.0305
295.5107	27.4327	14.2750	117.7036	0.0267
310.1672	23.4397	16.9147	117.2526	0.0259
296.6225	20.5038	19.3929	118.9393	0.0255
319.8276	28.0127	17.5132	129.7890	0.0254
344.1766	27.8457	17.6772	139.3772	0.0258
400.3778	39.0429	22.7217	148.0628	0.0259
461.6900	95.6670	36.5589	190.8566	0.0257
531.6270	215.0566	60.7277	331.4970	0.0248
569.1381	250.4112	117.7060	455.5661	0.0191
560.3904	251.2445	117.0193	556.8214	0.0180
478.2247	128.6185	90.3601	494.0292	0.0183
470.3411	70.4626	80.0747	405.6509	0.0183
489.8150	54.4532	78.0955	365.7305	0.0202
492.1242	67.3214	71.7527	369.7180	0.0211
534.8338	65.5229	68.0022	373.1796	0.0227

U.S. Data Set (continue)

REAL GNP	REAL GOV'T SPENDING	REAL TAXES	REAL DEBT	INTEREST RATE
579.3747	69.3186	83.1669	333.1617	0.0218
600.7718	105.1376	105.8132	328.8450	0.0229
623.5838	117.8123	109.9473	327.2509	0.0250
616.0319	111.2272	117.0764	328.2267	0.0239
657.5312	104.9949	107.6085	327.2715	0.0234
671.6022	104.2952	118.7243	309.5079	0.0247
683.7502	110.1566	123.1942	296.8551	0.0265
680.9055	116.7684	120.5875	298.7447	0.0279
721.7485	128.1814	117.2322	304.9512	0.0273
737.2809	124.4662	134.6317	302.9156	0.0321
756.6046	131.5651	136.1445	303.7199	0.0311
800.2252	141.7514	141.1641	308.3175	0.0314
832.5855	145.1024	148.6815	309.2717	0.0331
876.3488	152.1979	154.8193	304.9375	0.0347
929.3316	148.2497	157.1181	298.1684	0.0362
984.8632	164.3366	170.4742	288.9226	0.0377
1011.3648	188.2607	189.1627	278.4025	0.0416
1058.1439	204.6043	186.1176	287.5915	0.0442
1087.6783	199.4639	216.3659	258.3500	0.0470
1085.5484	210.5112	211.8568	247.2335	0.0534
1122.4029	206.8366	196.2212	247.6773	0.0552
1185.9230	218.9104	208.6490	251.2680	0.0537
1254.9872	220.2650	219.7228	252.5300	0.0553
1248.0160	219.6943	230.5360	230.0792	0.0623
1233.8420	245.1888	223.7950	247.5685	0.0667
1298.1850	260.5645	226.6926	290.1065	0.0673
1369.7422	262.8107	247.9679	305.9664	0.0632
1438.5474	271.8113	259.6982	322.1181	0.0668
1479.4756	274.4349	277.1068	316.8333	0.0704
1474.9961	283.9973	282.9375	314.9910	0.0855
1512.1426	300.8485	297.0598	334.6104	0.0969
1480.0140	305.0296	301.8975	358.4921	0.1105
1534.6870	325.3874	275.2243	441.4795	0.1102
1639.3501	322.0826	290.2023	489.6446	0.1060

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