Three Essays in Macroeconomic Dynamics

Dissertation

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ABSTRACT

This dissertation examines theoretical and empirical topics in macroeconomic dynamics. A central issue in macroeconomic dynamics is understanding the sources of business cycle fluctuations. The idea that expectations about future economic fundamentals can drive business cycles dates back to the early twentieth century. However, the standard real business cycle (RBC) model fails to generate positive comovement in output, consumption, labor-hours and investment in response to news shocks. My dissertation proposes a solution to this puzzling feature of the RBC model by developing a theoretical model that can generate positive aggregate and sectoral comovement in response to news shocks.

Another key issue in macroeconomic dynamics is gauging the performance of theoretical models by comparing them to empirical models. Some of the most widely used empirical models in macroeconomics are level vector autoregressive (VAR) models. However, estimated level VAR models may contain explosive roots, which is at odds with the widespread consensus among macroeconomists that roots are at most unity. My dissertation investigates the frequency of explosive roots in estimated level VAR models using Monte Carlo simulations. Additionally, it proposes a way to mitigate explosive roots. Finally, as macroeconomic datasets are relatively short, empirical models such as autoregressive models (i.e. AR or VAR models) may have substantial

small-sample bias. My dissertation develops a procedure that numerically corrects the bias in the roots of AR models.

This dissertation consists of three essays. The first essay develops a model based on learning-by-doing (LBD) that can generate positive covement in output, consumption, labor-hours and investment in response to news shocks. I show that the one-sector RBC model augmented by LBD can generate aggregate comovement in response to news shock about technology. Furthermore, I show that in the two-sector RBC model, LBD along with an intratemporal adjustment cost can generate sectoral comovement in response to news about three types of shocks: i) neutral technology shocks, ii) consumption technology shocks, and iii) investment technology shocks. I show that these results hold for contemporaneous technology shocks and for different specifications of LBD.

The second essay investigates the frequency of explosive roots in estimated level VAR models in the presence of stationary and nonstationary variables. Monte Carlo simulations based on datasets from the macroeconomic literature reveal that the frequency of explosive roots exceeds 40% in the presence of unit roots. Even when all the variables are stationary, the frequency of explosive roots is substantial. Furthermore, explosion increases significantly, to as much as 100% when the estimated level VAR coefficients are corrected for small-sample bias. These results suggest that researchers estimating level VAR models on macroeconomic datasets encounter explosive roots, a phenomenon that is contrary to common macroeconomic belief, with a very high frequency. Monte Carlo simulations reveal that imposing unit roots in the estimation can substantially reduce the frequency of explosion. Hence one way to mitigate explosive roots is to estimate vector error correction models.

The third essay proposes a numerical procedure to correct the small-sample bias in autoregressive roots of univariate AR(p) models. I examine the median-bias properties and variability of the bias-adjusted parameters relative to the least-squares estimates. I show that the bias correction procedure substantially reduces the median-bias in impulse response functions. Furthermore, correcting the bias in roots significantly improves the median-bias in half-life, quarter-life and up-life estimates. The procedure pays a negligible-to-small price in terms of increased standard deviation for its improved median-bias properties.

Dedicated to my family

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CHAPTER 1

INTRODUCTION

This dissertation examines theoretical and empirical topics in macroeconomic dynamics. A central issue in macroeconomic dynamics is understanding the sources of business cycle fluctuations. The idea that expectations about future economic fundamentals can drive business cycles dates back to the early twentieth century. Recently there has been a renewed interest in expectation shocks (the so-called 'news shocks') as a source of business cycle fluctuations. However, the standard real business cycle (RBC) model, which is the building block of modern macroeconomics, fails to generate positive comovement in output, consumption, labor-hours and investment in response to news shocks.

In the first essay, "News Shocks and Learning-by-doing," I propose a simple and intuitive solution to this puzzling feature of the RBC model based on learning-by-doing (LBD). I introduce LBD into the standard one-sector RBC model and show that the model is capable of generating an economic expansion in response to positive news about future technology. Such news increases the value of learning immediately, which induces the economic agents to accumulate it by increasing production as soon as the news arrives. Hence the LBD mechanism provides a countervailing force to the negative wealth effect on labor supply from positive news. The resulting increase in output is large enough to accommodate increases in both consumption and investment. As learning increases the productivity of factor-inputs, labor-hours and investment continue to rise in subsequent periods. Consequently, the model generates an expansion in response to the positive news. I also investigate the role of LBD in generating sectoral comovement in response to news shocks as several studies have emphasized the importance of sectoral comovement in developing a single unified theory of business cycles. I show that in the two-sector RBC model, LBD along with an intratemporal adjustment cost can generate sectoral comovement in response to news about three types of shocks: i) neutral technology shock, ii) consumption technology shock, and iii) investment technology shock. We show that these results hold for contemporaneous technology shocks and for different specifications of LBD.

Another key issue in macroeconomic dynamics is gauging the performance of theoretical models by comparing them to empirical models. Some of the most widely used empirical models in macroeconomics are level vector autoregressive (VAR) models. However, estimated level VAR models may contain explosive roots even if all the true autoregressive roots lie inside the unit circle. The incidence of such explosive roots is at odds with the widespread agreement among macroeconomists that roots are at most unity. Given that level VAR models are used extensively and may estimate roots greater than unity, it is important to examine how frequently researchers estimating level VAR models on macroeconomic datasets encounter explosive roots.

In the second essay, "Explosive Roots in Level Vector Autoregressive Models," I investigate the frequency of explosive roots in level VAR models using Monte Carlo simulations based on datasets that are representative of those commonly used in the macroeconomic literature. Monte Carlo results in this chapter reveal that the frequency of explosive roots exceeds 40% in the presence of unit roots. Even when all the variables are stationary, the frequency of explosive roots is substantial; it is as high as 25%. Furthermore, explosion increases significantly, to more than 90% under several specifications, when the estimated level VAR coefficients are corrected for small-sample bias. Monte Carlo results in this chapter reveal that the frequency of explosive roots exceeds 40% in the presence of unit roots. Even when all the variables are stationary, the frequency of explosive roots is substantial; it is as high as 25%. Furthermore, explosion increases significantly, to more than 90% under several specifications, when the estimated level VAR coefficients are corrected for small-sample bias. These results suggest that researchers estimating level VAR models on macroeconomic datasets encounter explosive roots, a phenomenon that is contrary to common macroeconomic belief, with a very high frequency. As per the well known evidence of nonstationarity in most macroeconomic series, one way to reduce the frequency of explosive roots is to impose unit roots in the estimation by estimating VECMs instead of level VAR models. I examine the frequency of explosive roots in estimated VECMs show that explosion occurs much less frequently in estimated VECMs. VECMs reduce the frequency of explosive roots by restricting the magnitude of some of the otherwise explosive roots to unity. Hence one way to mitigate explosive roots is to estimate vector error correction models.

Since macroeconomic datasets are relatively short, frequently used empirical models in macroeconomic literature, such as the univariate autoregressive models AR(p) models suffer from small-sample bias. In consequence, the least-squares estimator is a misleading indicator of the true values of important parameters such as the autoregressive coefficients, the autoregressive roots and, the impulse response functions. Several papers have addressed this by devising bias correction procedures to correct the bias in estimated autoregressive coefficients or the estimated coefficients in the Augmented Dickey-Fuller form. However, bias correction in coefficients may not correct the bias in roots because of the non-linear mapping between the two.

In the third essay, "Bias Correction in Autoregressive Roots," I propose a numerical procedure to correct the small-sample bias in autoregressive roots of univariate AR(p) models. I examine the median-bias properties and variability of the bias-adjusted parameters relative to the least-squares estimates. I show that the bias correction in roots (BCR) procedure substantially reduces the median-bias in impulse response functions. Furthermore, I show that correcting the bias in roots significantly improves the median-bias in half-life, quarter-life and up-life estimates. The BCR procedure pays a negligible-to-small price in terms of increased standard deviation for its improved median-bias properties.

CHAPTER 2

NEWS SHOCKS AND LEARNING-BY-DOING

2.1 Introduction

The idea that expectations about future economic fundamentals can drive business cycles dates back to the early twentieth century (e.g. Pigou (1927) and Clark (1934)). Recently there has been a renewed interest in expectation shocks (the so-called "news shocks") as a source of business cycle fluctuations. However, the standard real business cycle (RBC) model fails to generate an economic expansion in which consumption, investment and labor-hours all rise relative to their trends, in response to positive news about future technology. On the contrary, it generates a recession today in response to positive news. Good news generates a positive wealth effect today causing households to increase their consumption and leisure. Hence labor-hours and consequently output decrease. The decline in output along with an increase in consumption requires investment to decrease. Thus consumption increases while labor-hours, investment, and output decrease in response to positive news. This counterintuitive characteristic of the RBC model was first documented by Barro and King (1984) and later examined by Beaudry and Portier (2004, 2008).

This paper proposes a simple and intuitive solution to this puzzling feature of the RBC model, based on learning-by-doing (henceforth, LBD). Several micro-studies, including Bahk and Gort (1993), Benkard (1997), and Imai (2000) have estimated LBD and have found strong empirical support. Recent studies have also investigated the role of LBD in generating richer macroeconomic dynamics. Two prominent works in the macroeconomic literature that incorporate LBD into general equilibrium models are those by Chang, Gomes and Schorfheide (2002) (CGS (2002)), and Cooper and Johri (2002) (CJ (2002)). CSG (2002) model learning through skill accumulation (LBD via Skill) that captures the effects of past work experience on labor productivity. CJ (2002) model learning through the accumulation of organizational capital (LBD via Organizational Capital), which is a by-product of the production process; the idea being that production activity creates information about the organization which improves future productivity. Hence, learning in CGS (2002) is associated with labor-hours while learning in CJ (2002) depends on the overall production activity or output. These studies find empirical evidence for LBD and show that it can provide an important propagation mechanism in business cycle models. We introduce LBD along the lines of these studies into the standard one-sector RBC model and show that the model, under both these specifications of LBD, is capable of generating an economic expansion in response to positive news about future technology. Such news increases the value of LBD immediately, which induces the economic agents to accumulate it by increasing production as soon as the news arrives. Hence the LBD mechanism provides a countervailing force to the negative wealth effect on labor supply from positive news. The resulting increase in output is large enough to accommodate increases in both consumption and investment. As learning increases the productivity of factor-inputs, labor-hours and investment continue to rise in subsequent periods. Consequently, the model generates an expansion in response to the positive news.

We also investigate the role of LBD in generating sectoral comovement in response to news about three types of shock: neutral technology shocks, investment technology shocks, and consumption technology shocks. Several studies including Lucas (1977), and Burns and Mitchell (1946) emphasize the importance of sectoral comovement in developing a single unified theory of business cycles. Huffman and Wynne (1999) document that labor-hours and investment across sectors comove and are procyclical. However, the two-sector version of RBC model cannot generate sectoral or aggregate comovement in response to contemporaneous shocks or news shock about future technology. As a result of the infinite elasticity of substitution between investment across sectors and between labor in the two sectors, investment and employment across sectors are very volatile and move in opposite directions in the benchmark model. Consequently, we follow Huffman and Wynne (1999) and introduce an intratemporal investment adjustment cost, which helps in generating comovement in response to contemporaneous shocks, but not news shocks. This is because the model still lacks any propagation mechanism that can compensate for the negative wealth effect on labor supply from positive news about future technology. We show that LBD can provide a countervailing force that can offset this negative wealth effect in the two-sector model. Accordingly, LBD along with intratemporal investment adjustment cost can generate sectoral and aggregate comovement in response to contemporaneous and news shocks about technology.

Our paper is related to the emerging literature on news driven business cycles. Prominent works include Beaudry and Portier (2004), who propose a multi-sectoral durable and non-durable goods model that can produce an expansion in response to positive news about technology in the non-durable goods sector. Jaimovich and Rebelo (2008) generate news driven expansions by appending three features into the RBC model: variable capital utilization, investment adjustment cost, and special type of preferences that reduce the negative wealth effect on labor supply. Christiano et al. (2007) add habit formation and investment adjustment costs in their benchmark model, while including additional nominal frictions into their full model. Schmitt-Grohe and Uribe (2008) estimate a structural Bayesian model that incorporates both anticipated and unanticipated components of various shocks, and find that anticipated (news) shocks to technology can account for more than two-thirds of business cycle fluctuations in the U.S.[1] A recent study that is closest to our paper is by Christopher Gunn and Alok Johri (2009) (GJ (2009), henceforth).[2] They show that 'knowledge capital,' which is produced through a learning-by-doing process, can generate a boom in the aggregate economy and equity prices. While there are obvious similarities, we believe there are at least two differences in our paper. First, GJ (2009) model knowledge capital associated with labor-hours, which corresponds to the 'LBD via Skill' specification. In addition to this specification, we examine another specification of LBD that is popular in the macroeconomic literature, namely 'LBD via Organizational Capital'. Second, and more importantly, while GJ (2009)

[1]Other papers in this literature include the early works of Beveridge (1909), Pigou (1927), and Clark (1934) with more recent work done by Dupor and Mehkari (2009), Mehkari (2008), Nah (2009), Beaudry and Portier (2008), Li and Mehkari (2009), Den haan and Kaltenbrunner (2007), Eusepi (2008), and Lorenzi (2005).

[2]I had written the first draft of my paper and had presented it before I became aware of GJ (2002).

examine aggregate comovement in response to news about a neutral technology shock in a one-sector model, this paper, in addition to the one-sector model, also examines sectoral comovement in a two-sector model in response to news about three types of shocks: neutral technology shocks, consumption technology shocks, and investment technology shocks.

The rest of the paper is organized as follows. In Section 2 we explore the role of LBD in generating news driven expansions in a one-sector economy. We examine two different specifications of learning that are popular in the macroeconomic literature and show that the model with both the specifications of LBD can generate news driven booms. In section 3 we present a two-sector version of our model that can generate sectoral and aggregate comovement with respect to contemporaneous and news shocks about future technologies. The final section concludes.

2.2 The One-Sector Economy

In this section we explore the ability of learning-by-doing to generate news driven expansions in a one-sector RBC model. Several empirical studies have examined LBD and have found substantial evidence for it in micro datasets, in that production costs decrease and productivity increases with cumulative output. Some recent studies have also examined aggregate implications of LBD by incorporating it in dynamic general equilibrium models. Two prominent works in the macroeconomic literature that incorporate LBD into general equilibrium models are those by Chang, Gomes and Schorfheide (2002) (henceforth, CGS (2002)), and Cooper and Johri (2002) (henceforth, CJ (2002)).

CGS (2002) examine LBD associated with labor effort. They model a skill accumulation process that captures the effects of past work experience on labor productivity. They estimate the LBD parameters using a Bayesian approach that combines the micro-level panel data with the aggregate time-series data. They find that the LBD mechanism is capable of generating richer macroeconomic dynamics. CJ (2002) model LBD through organizational capital, which is a by-product of the production process; the idea being that production activity creates information about the organization which improves future productivity. They estimate the LBD parameters using sector and plant-level data and find that LBD can provide an important propagation mechanism in business cycle models. The key difference in the LBD mechanism of CGS (2002) and CJ (2002) is that while in the former learning is only associated with labor-hours, learning in the latter depends on the overall production activity or output.

In this section, we augment the standard one-sector RBC model with LBD along the lines of these studies. We first introduce learning through skill accumulation as outlined in CSG (2002), *LBD via Skill*. Next, we follow CJ (2002) and introduce learning through the accumulation of organizational capital, *LBD via Organizational Capital*. Subsequently, we examine the role of these LBD mechanisms in generating aggregate comovement in response to news shocks.

2.2.1 Model

The model economy is populated with many identical agents who maximize their expected discounted lifetime utility defined over consumption, c_t, and labor-hours worked, n_t:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{(1-\sigma)}}{(1-\sigma)} - \psi n_t \right] \tag{2.2.1}$$

The physical capital evolution is given by:

$$k_{t+1} = I_t + (1-\delta_k)k_t \tag{2.2.2}$$

where δ_k is the depreciation rate of the capital stock. Output is the economy can be used for production or consumption:

$$c_t + I_t = y_t \tag{2.2.3}$$

LBD via Skill

We follow CGS (2002) and assume that experience from past employment is identified with skill level, x_t. The skill accumulation process is given by:

$$ln\left(\frac{x_{t+1}}{x}\right) = \phi ln\left(\frac{x_t}{x}\right) + \mu ln\left(\frac{n_t}{n}\right) \qquad 0 \leq \phi < 1, \mu \geq 0 \tag{2.2.4}$$

where variables without the time subscript denote the steady-states. This process captures that skill level is augmented by labor-hours worked in the past and it depreciates over time ($\phi < 1$). Output in the economy is produced using constant-returns-to-scale Cobb-Douglas technology in physical capital, k_t, and labor-input, h_t:

$$y_t = k_t^{\alpha} h_t^{(1-\alpha)} a_t \tag{2.2.5}$$

where a_t is an exogenous technology shock. The labor-input in the production function consists of labor-hours worked and the skill level:

$$h_t = n_t x_t \tag{2.2.6}$$

Hence skill raises the effective unit of labor supplied. Combining (2.2.3) with (2.2.5) and (2.2.6), the recourse constraint becomes:

$$c_t + I_t = k_t^{\alpha} \left(n_t x_t\right)^{(1-\alpha)} a_t \tag{2.2.7}$$

The social planner's problem for this economy with skill accumulation is to maximize (2.2.1) subject to (2.2.2), (2.2.4), and (2.2.7).[3]

The first order conditions to the planner's problem are:

$$c_t^{-\sigma} = \lambda_t \tag{2.2.8}$$

$$\psi = \lambda_t(1-\alpha)\left(\frac{k_t}{n_t}\right)^{\alpha} x_t^{(1-\alpha)} a_t + \Lambda_t \frac{\mu}{n}\left(\frac{x_t}{x}\right)^{\phi}\left(\frac{n_t}{n}\right)^{(\mu-1)} \tag{2.2.9}$$

$$\lambda_t = \beta\lambda_{t+1}\left((1-\delta_k) + \alpha\left(\frac{x_{t+1}n_{t+1}}{k_{t+1}}\right)^{(1-\alpha)} a_{t+1}\right) \tag{2.2.10}$$

$$\frac{\Lambda_t}{x} = \beta\frac{\Lambda_{t+1}}{x}\phi\left(\frac{x_{t+1}}{x}\right)^{(\phi-1)}\left(\frac{n_{t+1}}{n}\right)^{\mu} + \beta\lambda_{t+1}(1-\alpha)\left(\frac{k_{t+1}}{x_{t+1}}\right)^{\alpha} n_{t+1}^{(1-\alpha)} a_{t+1} \tag{2.2.11}$$

where λ_t and Λ_t are the Lagrange multipliers associated with the aggregate constraint (2.2.7) and skill accumulation (2.2.4), respectively.

The first-order condition for labor-hours (2.2.9) differs from that of a standard RBC model by the second term in (2.2.9), which captures the marginal value of skill generated by an extra labor-hour. This second term, which is absent in the standard RBC model, is crucial in generating positive comovement in labor-hours and consumption in response to news shock about future technology. To see this, consider (2.2.9) without the second term and substitute out λ_t:

$$\psi c_t^{\sigma} = (1-\alpha)\left(\frac{k_t}{n_t}\right)^{\alpha} x_t^{(1-\alpha)} a_t \tag{2.2.12}$$

[3]CSG (2002) present a decentralized version of this economy. It is straightforward to verify that the solution to the planner's problem is identical to that of the decentralized economy.

This would correspond to the first-order condition for labor-hours in the RBC model, except for the skill term. The above equation shows that it is not possible to get positive comovement between labor-hours and consumption when the news shock occurs. When positive news about future productivity arrives, technology remains at steady-state. Skill and physical capital are state variables and are thus predetermined; they also remain at the steady-state level. Hence as consumption increases, labor-hours must decrease. This explains why the standard RBC model fails to generate positive comovement between labor-hours and consumption. The economics behind increase in consumption and decrease in labor-hours in response to positive news is as follows. The economic agents feel wealthier today as the good news about future technology arrives. Thus they increase their consumption and work less hours. The addition of the second term in (2.2.9) allows for the possibility of positive comovement since the shadow value of skill, Λ_t, increases in response to positive news, as we will discuss shortly. Rewriting (2.2.9) gives:

$$\frac{\psi}{\lambda_t} = (1-\alpha)\left(\frac{k_t}{n_t}\right)^{\alpha} x_t^{(1-\alpha)} a_t + \frac{\Lambda_t}{\lambda_t}\frac{\mu}{n}\left(\frac{x_t}{x}\right)^{\phi}\left(\frac{n_t}{n}\right)^{(\mu-1)} \tag{2.2.13}$$

The above equation shows that the planner equates the marginal rate of substitution between consumption and labor-hours to the sum of the marginal product of labor and the marginal value of skill (in terms of consumption) generated from increasing labor-hours by one unit.

The first-order condition for physical capital (2.2.10) is the same as that in standard RBC model, except for the skill term. First-order condition for skill (2.2.11) shows that the marginal value of skill, Λ_t, depends on next period's technology. Log linearizing (2.2.11) around the non-stochastic steady rate and rearranging shows that the shadow value of skill depends on the discounted sum of expected future technology. Consequently, marginal value of skill increases immediately in response to positive news about future technology. As we will discuss shortly, this increase in marginal value of skill induces the social planner to invest in skill when the positive news arrives, which leads to a boom in macroeconomic aggregates.

LBD via Organizational Capital

So far we have introduced LBD through skill accumulation. We now explore the second specification of LBD that is popular in the literature: LBD through the accumulation of organizational capital. CJ (2002) model organizational capital as a by-product of the production process; the idea being that production activity creates information about the organization which improves future productivity. In this specification learning depends on the overall production activity (labor-hours, physical capital and productivity) as opposed to only labor-hours in case of LBD via Skill. The organizational capital is accumulated indirectly through the production process and its evolution is given by:

$$ln(x_{t+1}) = \gamma ln(x_t) + \tau ln(y_t) \tag{2.2.14}$$

where x_t is the stock of organizational capital. The production technology converts its inputs of physical capital, labor-hours, and organizational capital into output:

$$y_t = k_t^{\theta} n_t^{\nu} x_t^{\omega} a_t \tag{2.2.15}$$

where a_t represents an exogenous technology shock. Substituting (2.2.15) into the organizational capital accumulation equation (2.2.14), we obtain:

$$x_{t+1} = x_t^{\gamma_x} n_t^{\gamma_n} k_t^{\gamma_k} a_t^{\gamma_a} \tag{2.2.16}$$

where $\gamma_x = \gamma + \tau\omega, \gamma_n = \tau\nu$, $\gamma_k = \tau\theta$, and $\gamma_a = \tau$. The aggregate constraint can be written as:

$$c_t + I_t = k_t^{\theta} n_t^{\nu} x_t^{\omega} a_t \tag{2.2.17}$$

We solve the model with organizational capital as a social planner's problem.[4] The planner maximizes (2.2.1) subject to (2.2.2), (2.2.16), and (2.2.17). The first-order conditions to the planner's problem are:

$$c_t^{-\sigma} = \lambda_t \tag{2.2.18}$$

$$\psi = \lambda_t \nu n_t^{\nu-1} k_t^{\theta} x_t^{\omega} a_t + \Lambda_t x_t^{\gamma_x} \gamma_n n_t^{\gamma_n - 1} k_t^{\gamma_k} a_t^{\gamma_a} \tag{2.2.19}$$

$$\lambda_t = \beta\lambda_{t+1} \Big((1-\delta_k) + \theta k_{t+1}^{(\theta-1)} n_{t+1}^{\nu} x_{t+1}^{\omega} a_{t+1} \Big) + \beta\Lambda_{t+1} x_{t+1}^{\gamma_x} n_{t+1}^{\gamma_n} \gamma_k k_{t+1}^{\gamma_k - 1} a_{t+1}^{\gamma_a} \tag{2.2.20}$$

$$\Lambda_t = \beta\Lambda_{t+1}\gamma_x x_{t+1}^{\gamma_x - 1} n_{t+1}^{\gamma_n} k_{t+1}^{\gamma_k} a_t^{\gamma_a} + \beta\lambda_{t+1} k_{t+1}^{\theta} n_{t+1}^{\nu} \omega x_{t+1}^{(\omega-1)} a_{t+1} \tag{2.2.21}$$

where Λ_t and λ_t are the Lagrange multipliers corresponding to (2.2.16) and (2.2.17), respectively. The first-order condition for labor-hours differs from that of a standard RBC model by the second term in (2.2.19), which captures the value of organizational capital generated by an extra labor-hour. It shows that the planner equates the marginal rate of substitution between consumption and labor-hours to the sum of the marginal product of labor and the marginal value of organizational capital (in terms of consumption) generated from increasing labor-hours by one unit.

The first-order condition with respect to physical capital can be rewritten as:

$$\frac{\lambda_t}{\lambda_{t+1}} = \beta(1-\delta_k) + \beta\theta k_{t+1}^{(\theta-1)} n_{t+1}^{\nu} x_{t+1}^{\omega} a_t + \beta \frac{\Lambda_{t+1}}{\lambda_{t+1}} x_{t+1}^{\gamma_x} n_{t+1}^{\gamma_n} \gamma_k k_{t+1}^{\gamma_k - 1} \tag{2.2.22}$$

[4]CJ (2002) solve their model as a social planner's problem since it allows them to be agnostic about the question of whether the organizational capital is firm-specific or worker-specific.

The above equation differs from that of the standard RBC model or the model with skill accumulation by the last term in (2.2.22), which captures that physical capital also contributes to the accumulation of organizational capital. Hence increasing physical capital by one unit today results in discounted undepreciated capital tomorrow, increases output, and raises the organizational capital. The planner, therefore, equates the inter-temporal marginal rate of substitution in consumption to the discounted sum of discounted undepreciated capital, the marginal product of capital, and the marginal value of organizational capital (in terms of consumption) generated from increasing physical capital by an additional unit.

The first-order condition for organizational capital is similar to that of skill discussed above, in that the marginal value of organizational capital, Λ_t, depends on future technology. Consequently, marginal value of organizational capital increases immediately in response to positive news about future technology. As we will discuss shortly, this increase in the value of organizational capital induces the economic agents to invest in it by increasing production immediately, which results in a news driven expansion.

2.2.2 Results

We now present numerical results to the one-sector economy that is calibrated to standard values found in the literature. We interpret one model economy period to be a quarter.

Structure of News Shocks

The structure of the shock to future productivity, news shock, takes the following form introduced by Christiano et al. (2007):

$$ln(a_t) = \rho_a ln(a_{t-1}) + \widetilde{e_{t-p}} - e_t \tag{2.2.23}$$

where $\widetilde{e_{t-p}}$ represents a news shock and e_t represents a contemporaneous shock. Under this specification, in period 1 the planner (unexpectedly) gets the news that productivity will change after p periods. However, depending on the value of e_{t+p}, this news may or may not turn out to be true in period *p+1*, which is the period of expected change in productivity. In the benchmark case, the news turns out to be true, $e_t = 0$; hence, the news is realized. If $e_t = \widetilde{e_{t-p}}$, then the news is false; thus the news is not realized.

Calibration

We set share of capital in the production function, α, to 0.34, and set the capital depreciation rate, δ_k, to 0.025. The subjective discount rate, β, is set to 0.99, implying an annual steady-state real interest rate of 4 percent. Following Christiano et al. (2007), we set ρ_a to 0.83 and p to 4 so that the news about technology is four quarters into the future.

The LBD parameters are based on empirical estimates in CGS (2002) and CJ (2002). In the skill accumulation specification, we set the LBD parameters to the posterior means in CGS (2002): ϕ and μ are set to 0.8 and 0.11, respectively. In the organizational capital specification, the LBD parameters ω, γ, and τ are based on empirical estimates in CJ (2002) and are set to 0.3, 0.5, and 0.5, respectively. The capital share, θ, and labor share, ν, in the production process under this specification are also 0.34 and 0.66, respectively. Setting the LBD parameters to zero under both the specifications reduces the models to the RBC model. This allows us to compare the responses of the LBD model with the RBC benchmark.

The relative risk aversion, σ, is set to 0.6, which is lower than the usual value of unity for log utility; however, it is well within the range of empirical estimates in the literature (Beaudry and Wincoop (1996), Vissing-Jorgensen and Attanasio (2003), and Mulligan (2002)). The reason why a higher inter-temporal elasticity of substitution (a lower σ) helps in generating a news driven expansion is because it dampens the recession generated by the benchmark RBC model in response to positive news, hence reducing the problem to begin with. Higher intertemporal elasticity of substitution diminishes the decrease in labor-hours, output and, investment and the increase in consumption in response to the positive news. This is because it allows for greater substitution of consumption across periods (less smooth consumption) as a result of which agents defer most of the increase in consumption until the actual technology increases. Therefore, the initial increase (decrease) in consumption (marginal utility of consumption) is relatively less with lower σ. As a result, the wealth effect on leisure is dampened through the labor-hours first-order condition. As a result the decrease in labor-hours is less, which in turn diminishes the decline in output. The relative decrease in consumption and increase in output diminishes the decline in investment through the resource constraint. Therefore, the higher intertemporal elasticity of substitution in consumption dampens the recession generated in the benchmark RBC model by positive news, and consequently helps in generating a news driven expansion. Nevertheless, the model can generate an expansion in response to positive news with log utility if the learning effect is amplified. For example, setting μ to $(1-\phi)$, so that there is CRS in the skill accumulation process, can produce a news driven expansion with log utility.

Numerical Results

We start out by examining the impulse responses to a positive news shock without any LBD mechanism. The model is calibrated to the values discussed above except that the LBD parameters are set to 0. Consequently, the model reduces to the standard RBC model. Figure 2.1 shows that the RBC model generates a recession today in response to positive news about future technology; output, investment and labor-hours all decrease until period 4 as the positive news arrives in period 1. Consumption, on the other hand, increases due to the positive wealth effect. The wealth effect also causes a decrease in labor supply. Since capital is fixed in period 1 and productivity is expected to increase in the future but does not change when news arrives in period 1, the decrease in labor-hours causes output to decline. As output decreases and consumption increases, investment must decrease. Consequently, the RBC model generates a recession in response to positive news. In period 5 if the news turns out to be true, the macroeconomic variables rise with the technology, whereas if the news turns out to be false they return to their steady-state level. This puzzling feature of the standard RBC model has been documented by Beaudry and Portier (2004, 2008).

Figure 2.2 plots the impulse responses to a news shock in the model with LBD via skill. The figure shows that the RBC model augmented by LBD can generate an expansion in response to positive news about future technology. Output, labor-hours, investment, and consumption all rise until period 4. The figure shows that the marginal value of skill, Λ_t, increases in response to the news. This induces the planner to invest in LBD immediately by increasing labor-hours. The resulting increase in output is large enough to accommodate increases in both consumption and investment. As increasing skill raises productivity of factor-inputs, labor-hours and physical capital continue to increase until period 4. In period 5 if the news turns out to be true, labor-hours, investment, consumption and output continue to increase, thus the expansion persists. If the news turns out to be false, all the variables decrease and revert to the steady-state level, hence causing a recession. This explains how introducing skill accumulation into the standard RBC can generate news driven business cycles.

Figure 2.3 shows the impulse responses in the model with organizational capital. The figure reveals that the RBC model with organizational capital can also generate an expansion in response to positive news about future productivity. Output, labor-hours, investment and consumption rise until period 4 in response to the positive news in period 1. The reason why organizational capital can generate a news driven expansion is similar to that of skill. The marginal value of organizational capital, Λ_t, increases as soon as the positive news arrives, which induces the planner to invest in it. This is accomplished by increasing labor-hours and physical capital, both of which are inputs into the organizational capital accumulation process. Increase in labor-hours raises output substantially so that both consumption and investment can increase. Consequently, labor-hours, output, consumption and investment rise until period 4. If the news turns out to be true, the expansion continues. Otherwise, all the variables decrease to the steady-state level.

While both physical capital and labor-hours are inputs into the accumulation of organizational capital, the former plays little role in the initial periods. Physical capital being predetermined does not contribute to the accumulation of organizational capital or the production process until one period after the news shock. Even after the first period it takes time for the physical capital to build up above the steady-state level. Consequently, it does not contribute much to the accumulation of organizational capital when the positive news arrives, which explains why the initial responses of the two learning mechanisms look similar. Nevertheless, as physical capital builds up, it contributes increasingly to the production of organizational capital. The low depreciation of physical capital amplifies this effect. This is evident from the more intertial responses under this specification of learning. For instance, in the model with organizational capital once the news shock is realized, aggregate variables continue to rise for a few periods even after the actual technology starts to dampen (period 6 onwards), hence displaying hump-shaped responses in output, labor-hours, consumption and investment. Learning-by-doing via skill fails to generate this hump-shaped behavior.

Next, we examine the responses to contemporaneous shock. Figure 2.4 plots the impulse responses to contemporaneous technology shocks under both the specifications of LBD. Impulse responses in the figure reveal that both the LBD specifications are capable of generating positive comovement in response to contemporaneous shock as well. While both the specifications can generate positive aggregate comovement, only the model with organizational capital can generate hump-shaped responses in labor-hours and output.[5]

[5]For a discussion on responses to contemporaneous shocks, see CSG (2002) and CJ (2002).

2.3 The Two-Sector Economy

To study sectoral comovement we consider a two-sector version of our model with a consumption sector and an investment sector. Several papers including Lucas (1977) and Burns and Mitchell (1946) have underscored the importance of sectoral comovement in developing a single unified theory of business cycles. Huffman and Wynne (1999) document that labor-hours and investment across sectors comove and are procyclical in the data. Therefore in this section we explore the ability of LBD in generating sectoral comovement in response to news shocks. We introduce learning-by-doing in both the sectors. In the interest of brevity, we focus on LBD through skill accumulation from hereon.[6]

2.3.1 Model

The model economy consists of a consumption sector and an investment sector. The production technology in the two sectors has the standard Cobb-Douglas functional form:

$$c_t = k_t^{c\alpha} h_t^{c^{1-\alpha}} a_t z_t^c \tag{2.3.1}$$

$$I_t^c + I_t^i = k_t^{i\alpha} h_t^{i^{1-\alpha}} a_t z_t^i \tag{2.3.2}$$

where the superscripts "c" and "i" denote variables specific to the consumption and investment sectors, respectively.[7] z_t^c and z_t^i are the sector-specific technology shocks while a_t is the neutral technology shock. The consumption sector produces consumption goods from capital, k_t^c, and labor-input, h_t^c, which is the product of labor-hours

[6]Our two-sector model with organizational capital can also generate sectoral and aggregate comovement in response to the three shocks considered in this paper. These results are available upon request.

[7]The above two equations replace the resource constraint (2.2.3) in the one-sector economy.

worked, n_t^c, and skill x_t^c. The investment sector produces investment goods for both the sectors using capital, k_t^i, and labor-input, h_t^i, which consists of labor-hours worked, n_t^i, and skill level x_t^i.

Following the literature, we assume that capital is not mobile across sectors. The idea here is that capital used in the production of industrial machinery cannot easily be used to produce food. This assumption is formalized by specifying separate equations for capital evolution in each sector:[8]

$$k_{t+1}^c = I_t^c + (1 - \delta_k)k_t^c \tag{2.3.3}$$

$$k_{t+1}^i = I_t^i + (1 - \delta_k)k_t^i \tag{2.3.4}$$

Similarly, we assume that skill is sector-specific and cannot easily be used in the other sector. The logic is the same; skill in producting industrial machinery cannot easily be used for producing food. Hence we specifying separate equations for the skill accumulation process in each sector:

$$ln\left(\frac{x_{t+1}^c}{x^c}\right) = \phi ln\left(\frac{x_t^c}{x^c}\right) + \mu ln\left(\frac{n_t^c}{n^c}\right) \tag{2.3.5}$$

$$ln\left(\frac{x_{t+1}^i}{x^i}\right) = \phi ln\left(\frac{x_t^i}{x^i}\right) + \mu ln\left(\frac{n_t^i}{n^i}\right) \tag{2.3.6}$$

where $0 \leq \phi < 1$ and $\mu \geq 0$. Finally, aggregate labor-hours is the sum of labor-hours in the two sectors.

$$n_t = n_t^c + n_t^i \tag{2.3.7}$$

[8]Under this specification, while capital cannot be moved across sectors in a given period, it can be moved easily after one period if there are no investment adjustment costs.

The planner solves (2.2.1) subject to the aggregate constraints, (2.3.1) and (2.3.2), and the capital and skill accumulation equations, (2.3.3) through (2.3.6). The first-order conditions to the planner's problem are:

$$c_t^{-\sigma} = \lambda_t^c \tag{2.3.8}$$

$$\psi = \lambda_t^c(1-\alpha)\left(\frac{k_t^c}{n_t^c}\right)^{\alpha} x_t^{c^{(1-\alpha)}} a_t z_t^c + \Lambda_t^c \frac{\mu}{n^c}\left(\frac{x_t^c}{x^c}\right)^{\phi}\left(\frac{n_t^c}{n^c}\right)^{(\mu-1)} \tag{2.3.9}$$

$$\psi = \lambda_t^i(1-\alpha)\left(\frac{k_t^i}{n_t^i}\right)^{\alpha} x_t^{i^{(1-\alpha)}} a_t z_t^i + \Lambda_t^i \frac{\mu}{n^c}\left(\frac{x_t^i}{x^i}\right)^{\phi}\left(\frac{n_t^i}{n^i}\right)^{(\mu-1)} \tag{2.3.10}$$

$$\frac{\Lambda_t^c}{x^c} = \beta\frac{\Lambda_{t+1}^c}{x^c}\phi\left(\frac{x_{t+1}^c}{x^c}\right)^{(\phi-1)}\left(\frac{n_{t+1}^c}{n^c}\right)^{\mu} + \beta\lambda_{t+1}^c(1-\alpha)\left(\frac{k_{t+1}^c}{x_{t+1}^c}\right)^{\alpha} n_{t+1}^{c^{(1-\alpha)}} a_{t+1} z_{t+1}^c \tag{2.3.11}$$

$$\frac{\Lambda_t^i}{x^i} = \beta\frac{\Lambda_{t+1}^i}{x^i}\phi\left(\frac{x_{t+1}^i}{x^i}\right)^{(\phi-1)}\left(\frac{n_{t+1}^i}{n^i}\right)^{\mu} + \beta\lambda_{t+1}^i(1-\alpha)\left(\frac{k_{t+1}^i}{x_{t+1}^i}\right)^{\alpha} n_{t+1}^{i^{(1-\alpha)}} a_{t+1} z_{t+1}^i \tag{2.3.12}$$

$$\lambda_t^i = \beta\lambda_{t+1}^i(1-\delta_k) + \beta\lambda_{t+1}^c\alpha\left(\frac{x_{t+1}^c n_{t+1}^c}{k_{t+1}^c}\right)^{(1-\alpha)} a_{t+1} z_{t+1}^c \tag{2.3.13}$$

$$\lambda_t^i = \beta\lambda_{t+1}^i(1-\delta_k) + \beta\lambda_{t+1}^i\alpha\left(\frac{x_{t+1}^i n_{t+1}^i}{k_{t+1}^i}\right)^{(1-\alpha)} a_{t+1} z_{t+1}^i \tag{2.3.14}$$

where λ_t^j and Λ_t^j are the Lagrange multipliers with respect to the resource-constraints and the skill accumulation equations in the two sectors ($j = c,\ i$). The first-order conditions in this two-sector economy are analogous to those in the one-sector model. For instance, the first-order conditions for labor-hours in the two sectors (2.3.9) and (2.3.10) are similar to (2.2.9) and show that the social planner equates the marginal-rate-of-substitution between labor-hours and consumption (investment) to the sum of the marginal product of labor in the consumption (investment) sector and the marginal value of skill in terms of consumption (investment) generated from increasing labor-hours by one unit in the respective sector.[9] Similarly, the first-order conditions with respect to skill in the two sectors (2.3.11) and (2.3.12) are analogous to (2.2.11), in that the marginal value of skill in the two sectors depend of future technology.[10]

Intratemporal Adjustment Cost

In the two-sector model, there is an infinite elasticity of substitution between investment across sectors, which makes it very easy to switch from the production of one type of capital good to that of another. Specifically, by cutting back the production of new capital goods for one sector by one unit, it is possible to increase production of new capital goods for the other sector by one unit without any need to increase overall production of new capital goods. Huffman and Wyne (1999) argue that while an economy can alter its capacity for producing heavy capital equipment for industrial use and alternative capital goods for service sector use, it can be costly to do so quickly in practice. Consequently, they introduce an intratemporal investment adjustment cost in a standard two-sector model and show that the this modification can generate sectoral comovement in response to contemporaneous shock. We follow Huffman and Wynne (1999) and introduce intratemporal investment adjustment cost in our model.[11] The production technology in the investment sector (2.3.2) will then

[9] As in (2.2.9), these first-order conditions differ from the standard two-sector RBC model by the second terms in (2.3.9) and (2.3.10), which capture the marginal value of skill in the respective sectors.

[10] In the same way, the first-order conditions for physical capital in the two sectors (2.3.13) and (2.3.14) are analogous to (2.2.10).

[11] We introduce the intratemporal adjustment costs since LBD by itself cannot reduce the rapid movement of factor across sectors.

be replaced by:

$$\left(I_t^{c^{-\rho}} + I_t^{i^{-\rho}}\right)^{-\frac{1}{\rho}} = k_t^{i^{\alpha}} \left(n_t^i x_t^i\right)^{1-\alpha} a_t z_t^i \tag{2.3.15}$$

The central assumption behind this specification is that it is costly to alter the composition of capital goods produced in the economy. This formulation generates a convex production possibility frontier between investment in the two sectors.[12] Setting $\rho = -1$ would result in the standard resource constraint for the capital-goods producing sector in a two-sector model. Thus, it is easy to understand the implications of introducing this adjustment cost.

2.3.2 Results

We now present numerical results to the two-sector economy. We follow Jaimovich and Rebelo (2008) and calibrate the two-sector model with the same parameter values used for the one-sector model.[13] We set the intratemporal investment adjustment cost, ρ to -1.4.[14]

Numerical Results

We now discuss the impulse responses of sector-specific and aggregate variables to news about three types of shocks. The first is a sectoral shock to technology in the investment sector, z_t^i, and the second is a sectoral shock to technology in the

[12]For a detailed motivation for this form, refer to Huffman and Wyne (1999).

[13]Jaimovich and Rebelo (2008) calibrate their two sector growth model with the same parameter values as their one sector version of the model. Huffman and Wyne(1999), on the other hand, use different depreciate rates, labor capital shares and persistent parameters in the two sector. Hence an alternative to the Jaimovich and Rebelo (2008) would be to follow Huffman and Wyne(1999).

[14]Huffman and Wyne (1999) estimated ρ in the range of -1.1 and -1.3. While ρ of -1.4 is slightly larger (in absolute value), the results are essentially the same when ρ is set to -1.3.

consumption sector, z_t^c. The third is the combination of the two sectoral shocks, which corresponds to a neutral technology shock, a_t. The timing is as follows. The economy is in the steady-state at time zero. At time one the economy learns that there is a one-percent increase in one of the three shocks after four periods.

Figure 2.5 shows that the model with LBD and intratemporal investment adjustment cost can generate both sectoral and aggregate comovement in response to news about all three shocks. The positive news increases the marginal value of skill in the two sectors, Λ_t^c, and, Λ_t^i, immediately. This induces the planner to invest in skill by increasing labor-hours in both the sectors, which raises aggregate consumption and aggregate investment. The intratemporal investment adjustment cost restricts the movement of investment across sectors and as a result investment in both the sectors increase. As skill accumulation raises the productivity of factors-inputs, labor-hours and investment continue to increase in both the sectors. Consequently, aggregate consumption, investment, labor-hours and output also continue to increase in subsequent periods. Hence the model generates both sectoral and aggregate comovement in response to positive news about neutral and sector-specific technology shocks. The next figure shows the effects of the corresponding three contemporaneous shocks. The timing is as follows. The economy is in the steady-state at time zero and the shock occurs at time one. Figure 2.6 shows that the model generates both aggregate and sectoral comovement in response to all three shocks.

To better understand the dynamics of the model, we first examine the responses to contemporaneous and news shocks about investment-specific technology in the two-sector version of standard RBC model. Subsequently, we will add the intratemporal investment adjustment cost and LBD one at a time to examine their relative contribution in generating a news driven expansion. Figure 2.7 shows the response to contemporaneous shock and news shock in the benchmark model without skill accumulation or intratemporal adjustment cost. The figure shows that in response to contemporaneous shock, aggregate output and investment rise immediately and in subsequent periods, while consumption falls for several periods. This is because as investment productivity increases, investment (and subsequently capital) in the investment sector will increase to take advantage of the increased productivity. Later, as investment technology decreases to the steady-state level, capital and investment will also decrease. Since investment in the investment sector has increased by so much, the corresponding investment in the consumption sector will fall immediately upon the rise in technology, and consequently consumption falls in the following periods. As more capital goods are accumulated, capital in the investment sector falls and capital in the consumption sector grows as agents desire more consumption. The figure also plots responses to positive news about investment technology. In response to this positive news, the planner increases labor-hours and capital in the consumption sector immediately in order to build consumption before the investment-specific technology arrives. However, due to the negative wealth effect on labor supply there are more than offsetting decreases in the investment sector, which cause aggregate labor-hours, output and investment to decline. Subsequently, the planner reallocates the factors to the investment sector in order to take advantage of the increased productivity when the actual investment technology arrives.[15] After the shock, the planner reallocates the factor to the consumption sector to increase consumption. As

[15]Since capital is predetermined, the planner increases investment one period in advance to ensure that capital in the investment sector is at a higher level in the next period when the investment technology arrives.

the technology subsequently reverts to its steady-state level, so do the investments and labor-hours in the two sectors. It is clear from the figure that the benchmark two-sector model fails to generate sectoral or aggregate comovement in response news and contemporaneous shocks.

Next we examine the impulse responses when the two key elements are introduced to the benchmark two-sector model: skill accumulation and intratemporal investment adjustment cost. Figure 2.8 shows that introducing intratemporal adjustment costs substantially reduces the volatility in the factors as there is no longer an infinite elasticity of substitution between the two types of investment goods. The figure confirms that introducing this adjustment cost leads to positive sectoral and aggregate comovement in response to contemporaneous shock. However, the adjustment cost by itself cannot produce an expansion in response to positive news about future investment technology. Labor and investment decrease in the consumption sector, causing aggregate consumption to decline. While investment increases slightly in the investment sector, the decrease in labor causes aggregate investment to decrease. The figure shows that all the sectoral variables (except for investment in the investment sector) and all aggregate variables decline, hence causing a recession in response to the positive news. The reason why the two-sector model with only intratemporal investment adjustment cost fails to generate comovement in response to news shock is because there are no forces in the model that can compensate for the negative wealth effect on the labor supply from news about future productivity.

We now examine the impulse responses when the model is augmented with LBD. Introducing LBD via skill in the two sectors increases the marginal value of skill when the positive news arrives. This induces the planner to invest in skill, which is accomplished by an increase in labor-hours. Hence the LBD mechanism provides a countervailing force to the negative wealth effect on labor supply. The figure shows that when skill accumulation is added into the model both the sector-specific variables and the aggregate variables rise in response to the positive news.[16] Hence skill accumulation combined with intratemporal adjustment can produce both sectoral and aggregate comovement in response to news shock.

Figure 2.9 shows the response in the benchmark two-sector model to news and contemporaneous shocks in the consumption sector. Once again, the responses are volatile as the factors are moved freely across sectors to where their marginal products are higher. Introducing intratemporal investment adjustment cost leads to comovement in response to contemporaneous shock. While in this case adding the adjustment cost can also generate comovement in response to news about consumption technology, initial increase in labor in the consumption sector and aggregate consumption is negligible. Introducing LBD substantially increases the size of this initial boom.

Finally, we examine responses to news and contemporaneous shocks to neutral technology, which is a combination of the two sectoral shocks. Figures 2.11 and 2.12 show that the benchmark two-sector model fails to generate sectoral or aggregate comovement and introducing intratemporal investment adjustment cost helps in case of contemporaneous shock. However, adjustment cost by itself fails to produce an expansion in response to positive news about neutral technology. Investment and labor shrink in the consumption sector, resulting in a decrease in aggregate consumption. While investment increases in the investment sector, the corresponding decrease in

[16] The impulse responses when only skill is added to the benchmark two-sector model are still volatile because of the infinite elasticity of substitution between investment and labor in the two sector. Hence learning-by-doing by itself is not sufficient to generate an expansion in response to positive news.

labor-hours cause aggregate investment to shrink. As a result aggregate output also decreases. The figure shows that except for investment in the investment sector all the aggregate and sector-specific variables decline, thus causing a recession in response to positive news. Introducing learning-by-doing via skill in the two sectors induces the planner to invest in it by increasing labor-hours, which leads to increases in both sectoral and aggregate variables.

2.4 Conclusion

It is well documented that the standard RBC model fails to generate positive comovement in output, consumption, investment, and labor-hours in response to news about future technology. This paper proposes a solution to this puzzling feature of the RBC model based on learning-by-doing. We examine two specifications of LBD that are popular in the literature and show that both these specifications can generate aggregate comovement in response to news shocks about technology. Furthermore, we show that LBD plays a crucial role in generating sectoral comovement in response to news shocks. While several other recent studies have added features to the RBC model to account for aggregate comovement in response to news shocks, we believe that the primary virtue of our approach is that it provides a simple and intuitive solution based on a mechanism that has strong empirical support. In addition, we show that our model can generate sectoral comovement in response to news about three types of shocks: neutral technology shocks, consumption technology shocks, and investment technology shocks.

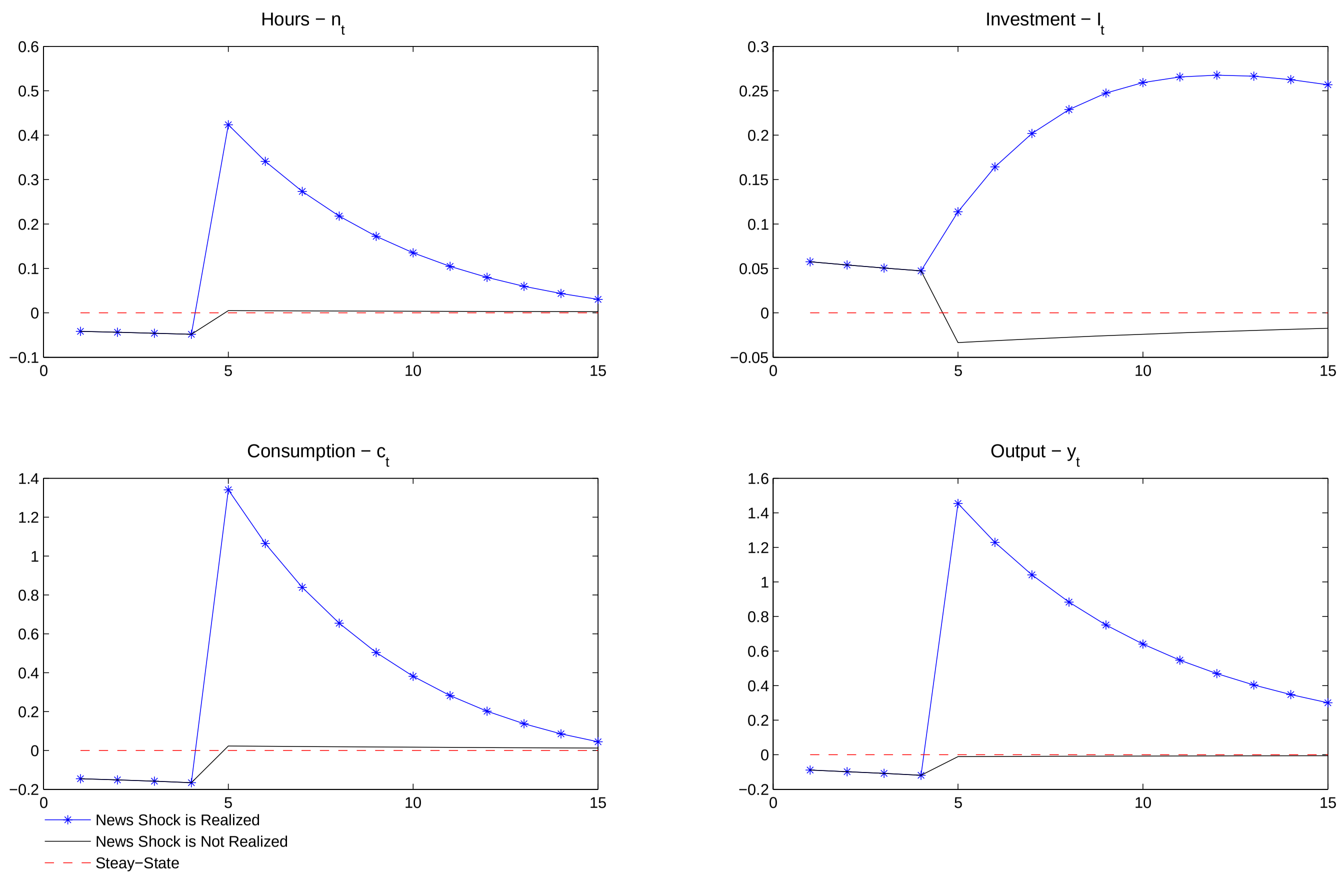

Figure 2.1: RBC Model - News Shock

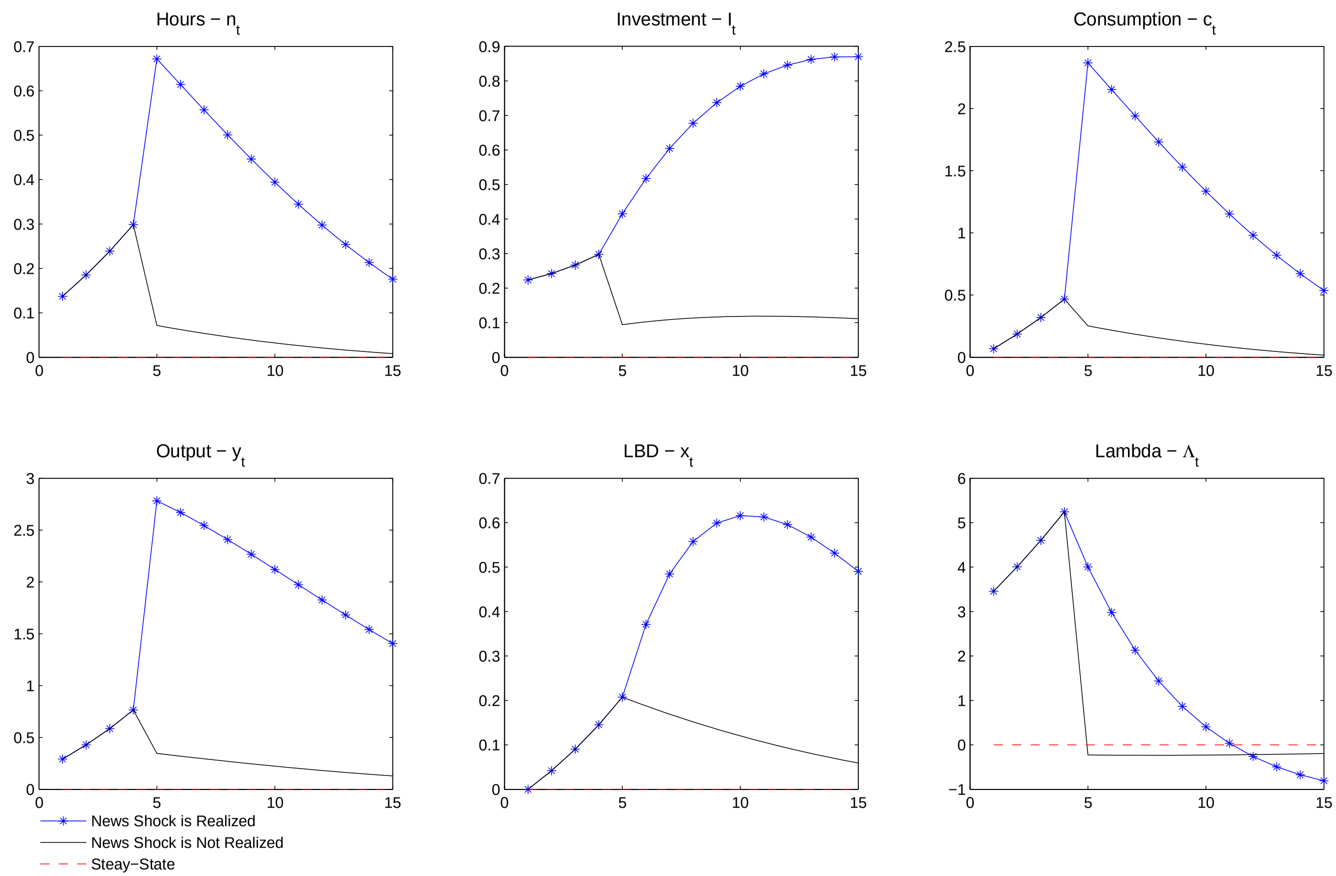

Figure 2.2: LBD via Skill - News Shock

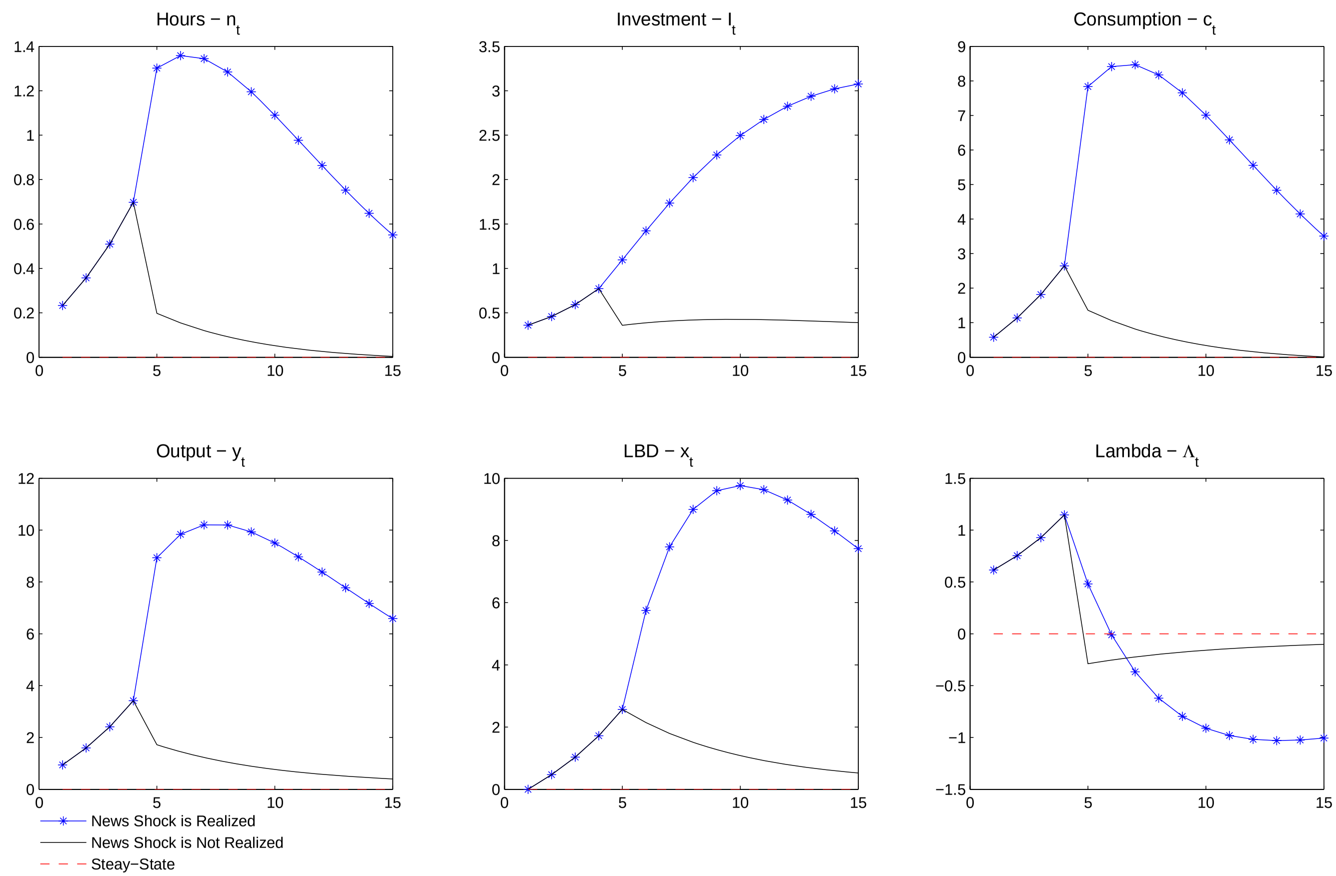

Figure 2.3: LBD via Organizational Capital - News Shock

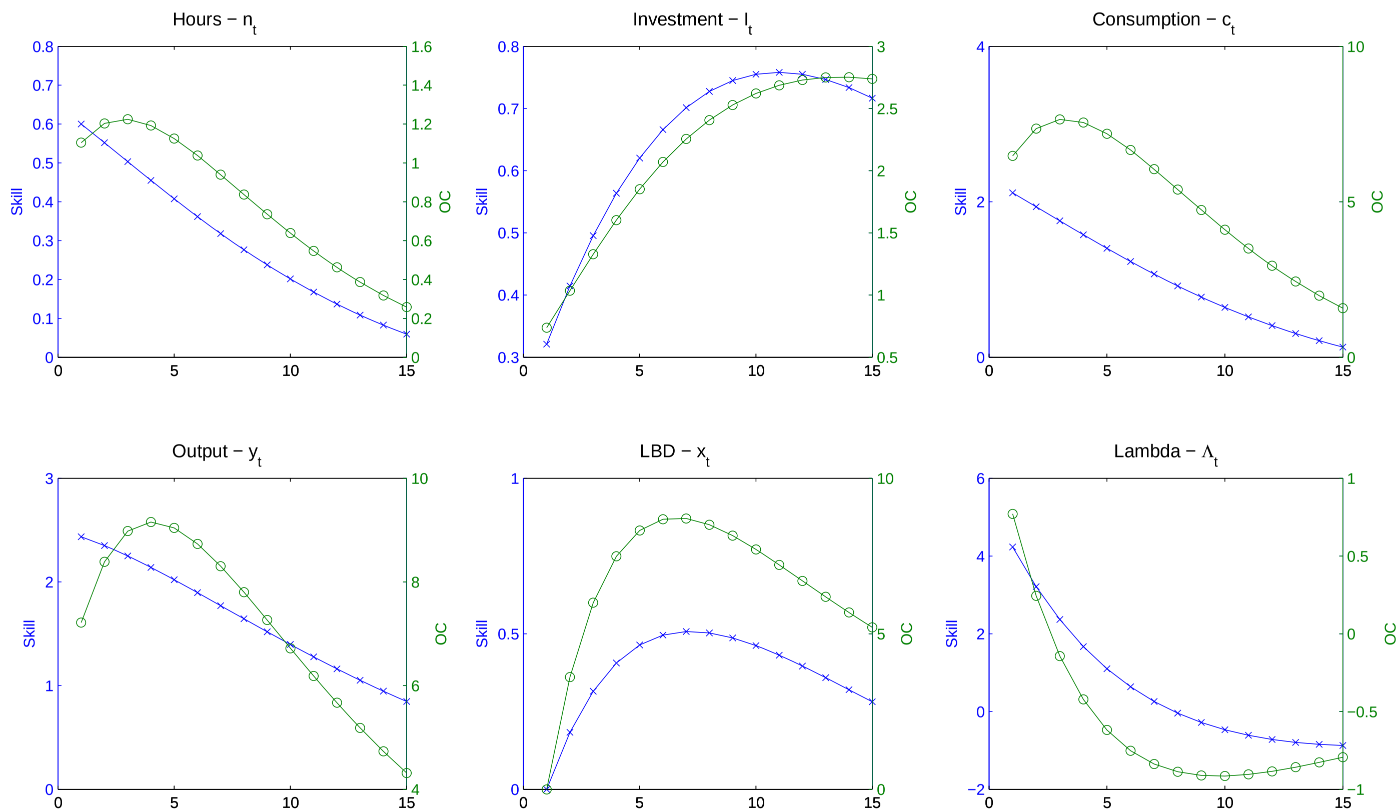

Figure 2.4: LBD - Contemporaneous Shock

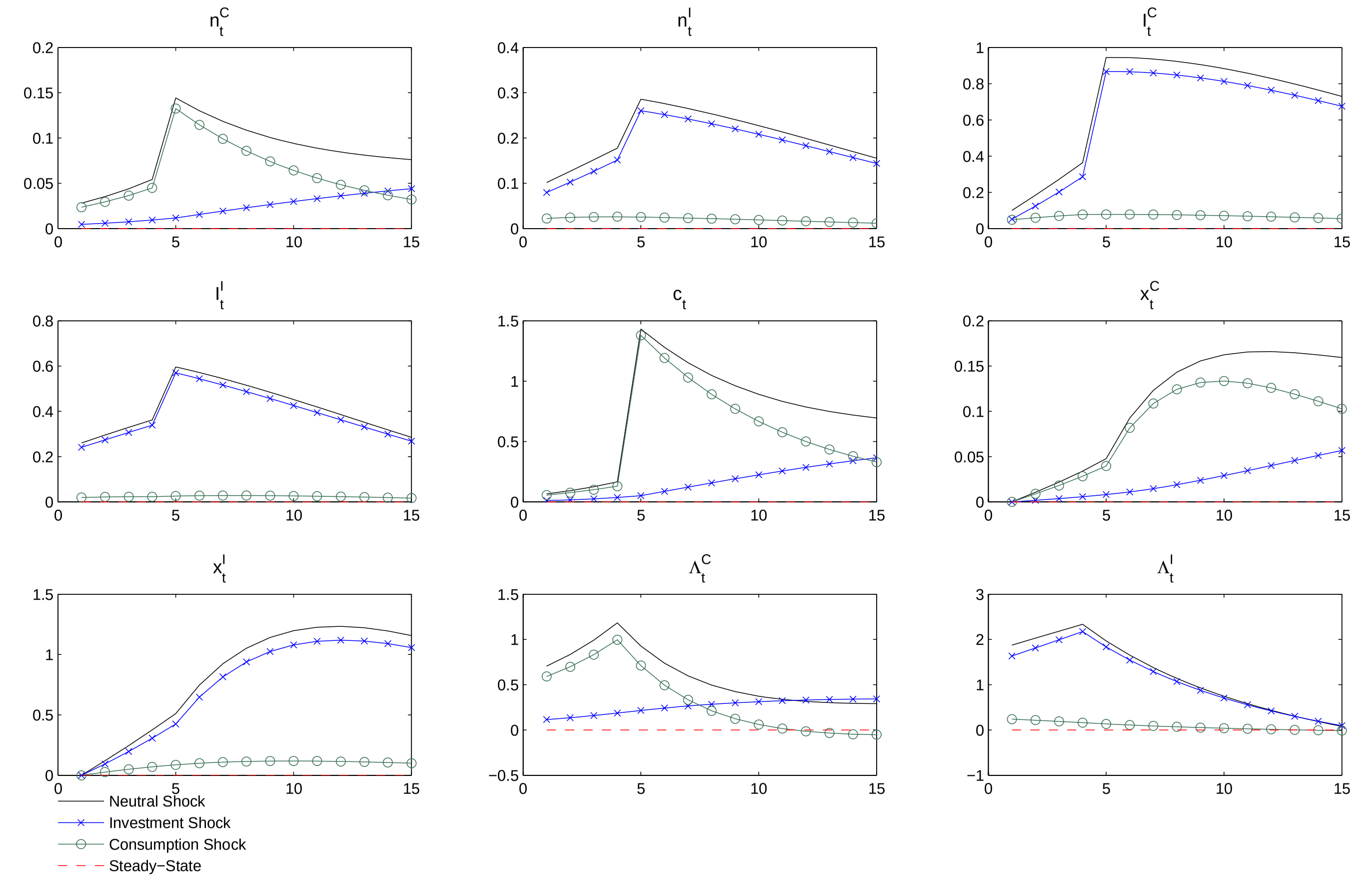

Figure 2.5: News Shocks in Our Two-Sector Model

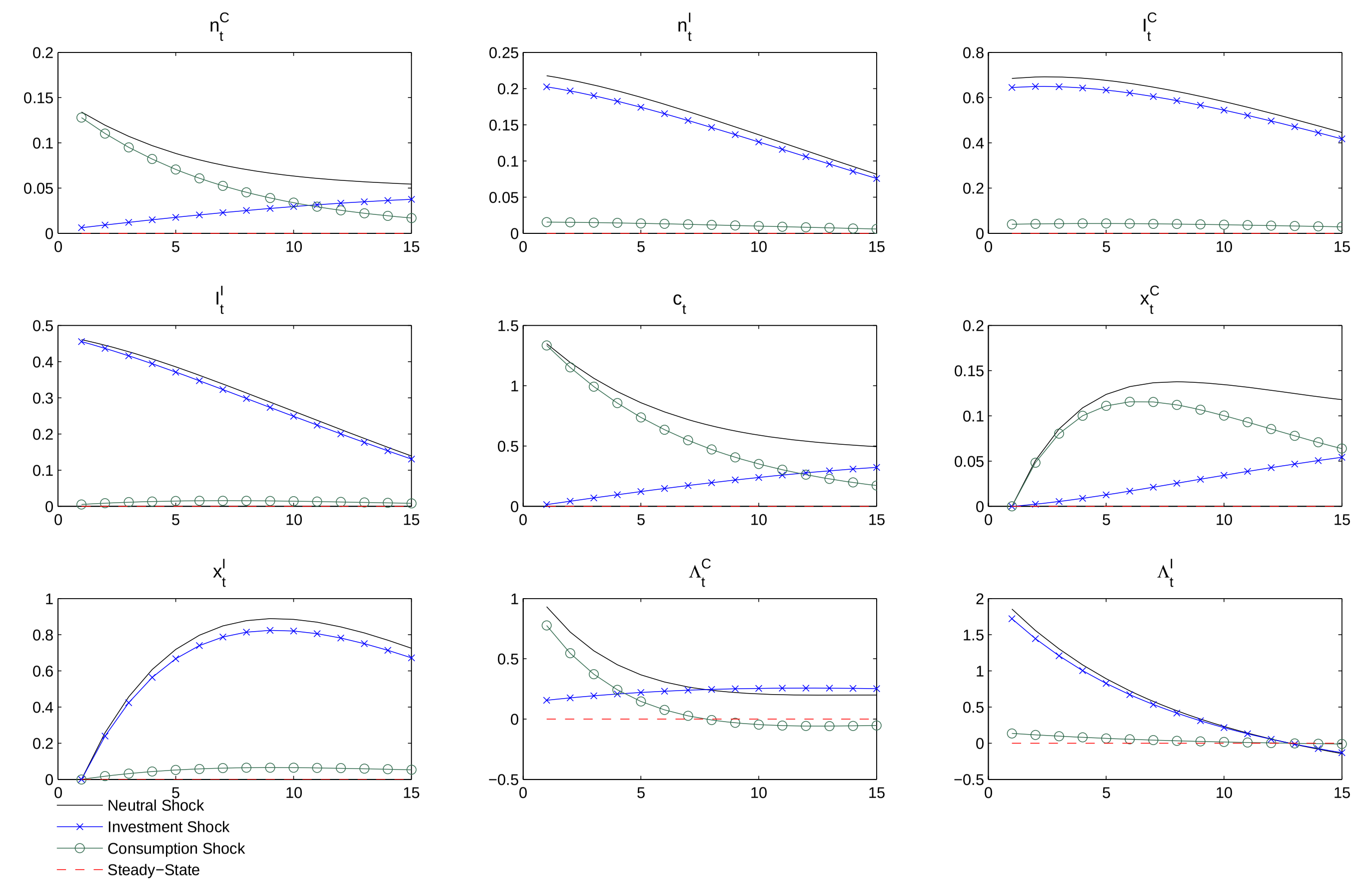

Figure 2.6: Contemporaneous Shocks in Our Two-Sector Model

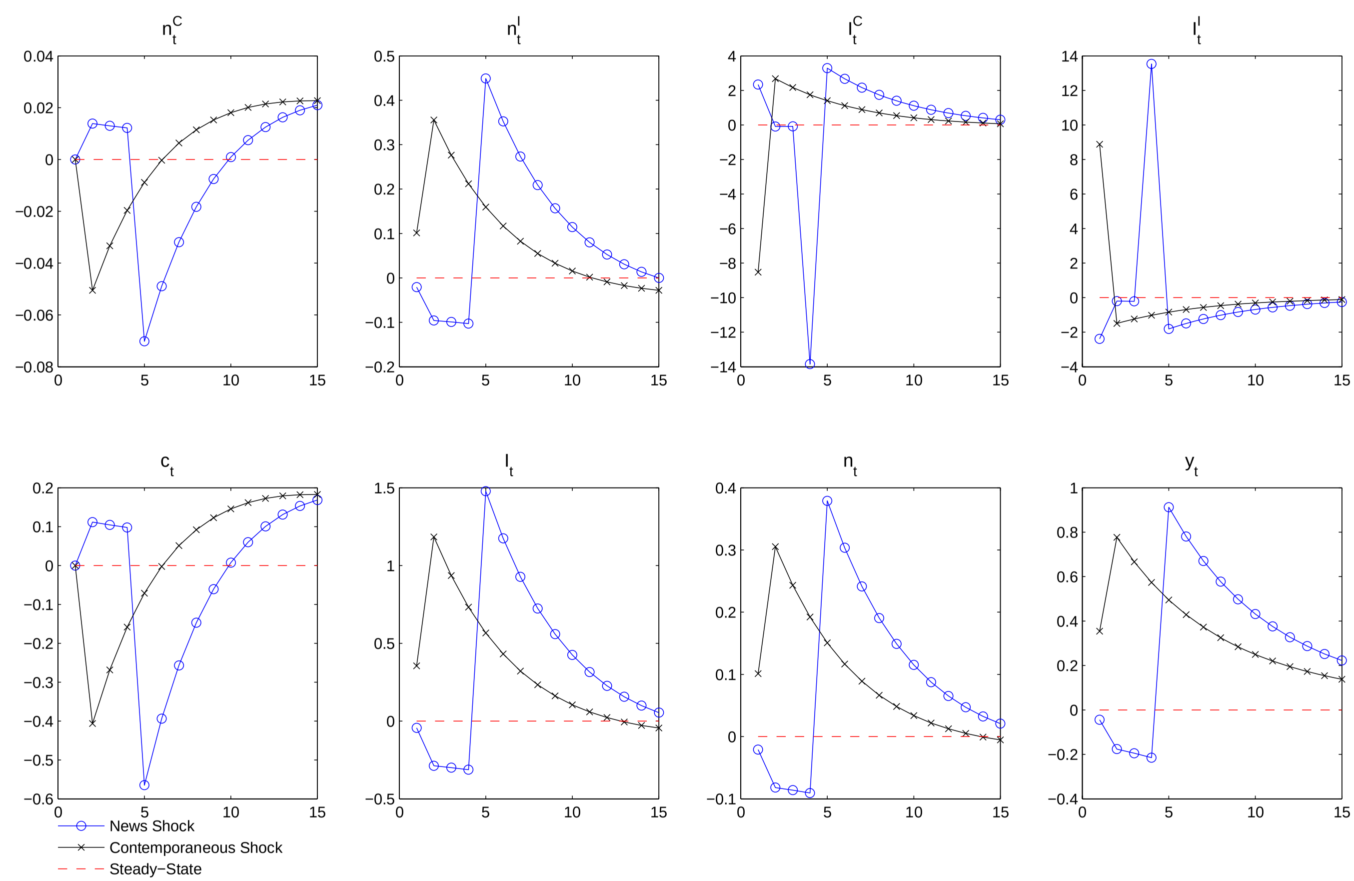

Figure 2.7: Investment Technology Shock in the BENCHMARK Two-Sector Model

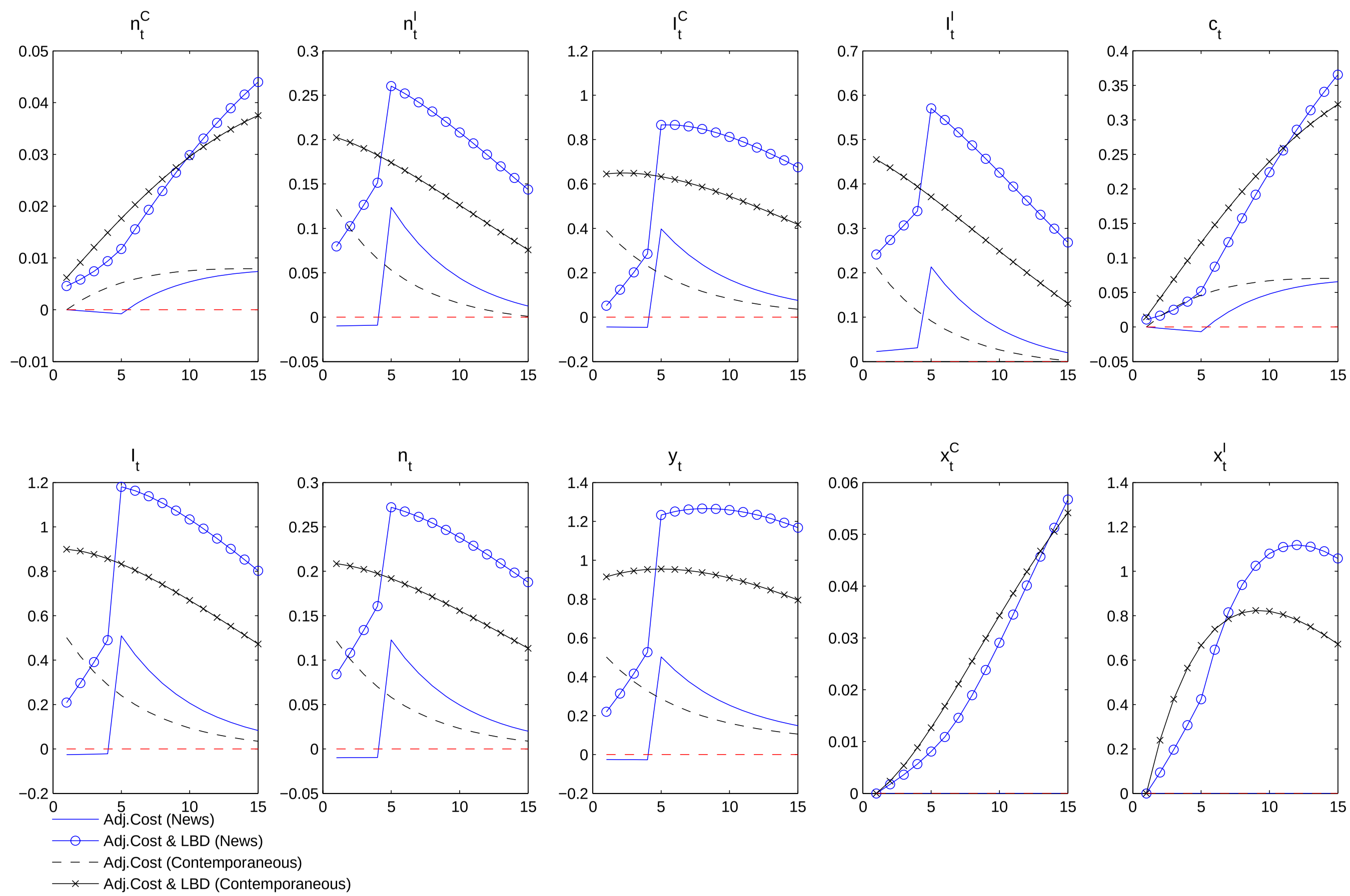

Figure 2.8: Investment Technology Shock in OUR Two-Sector Model

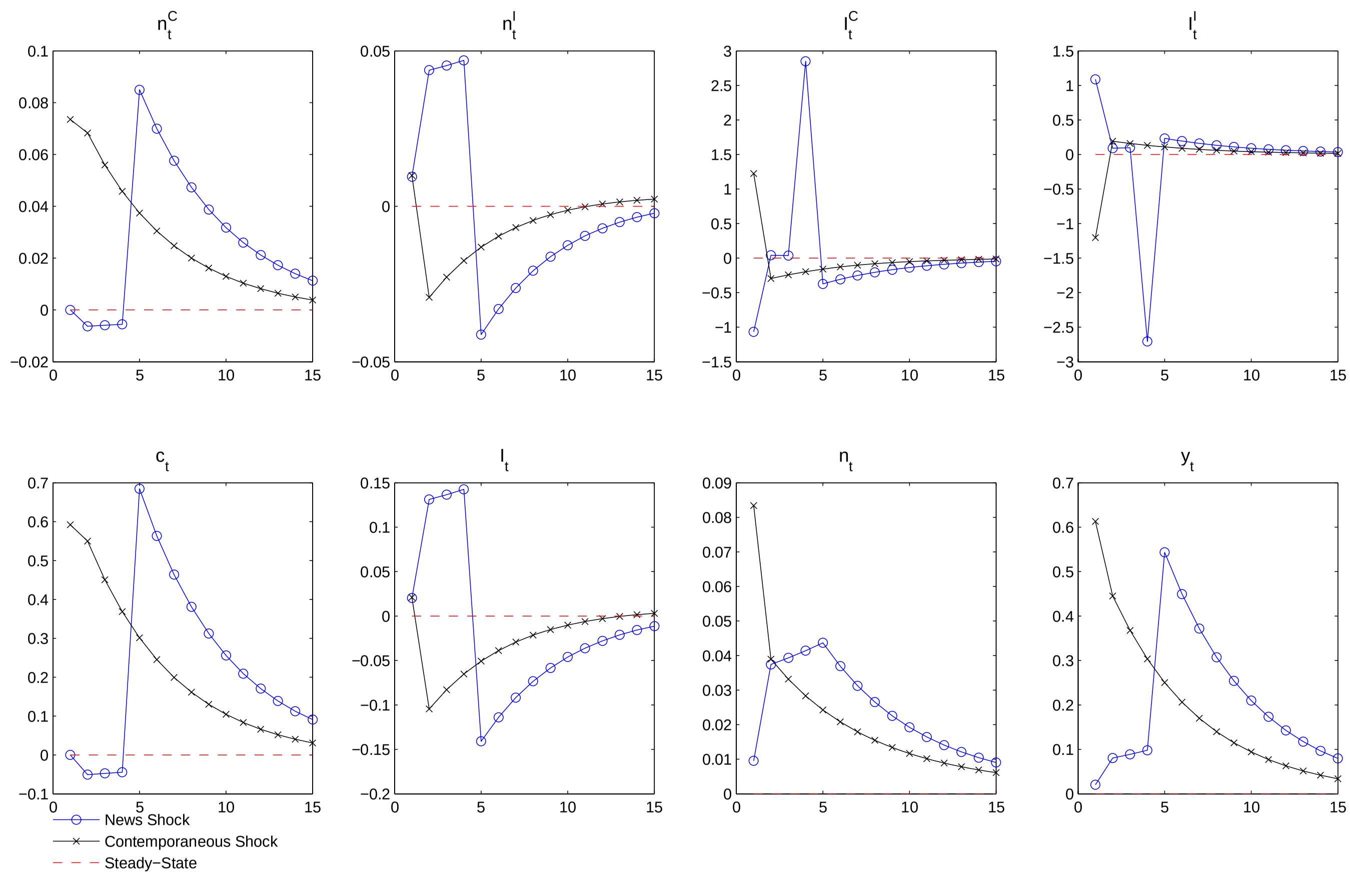

Figure 2.9: Consumption Technology Shocks in the BENCHMARK Two-Sector Model

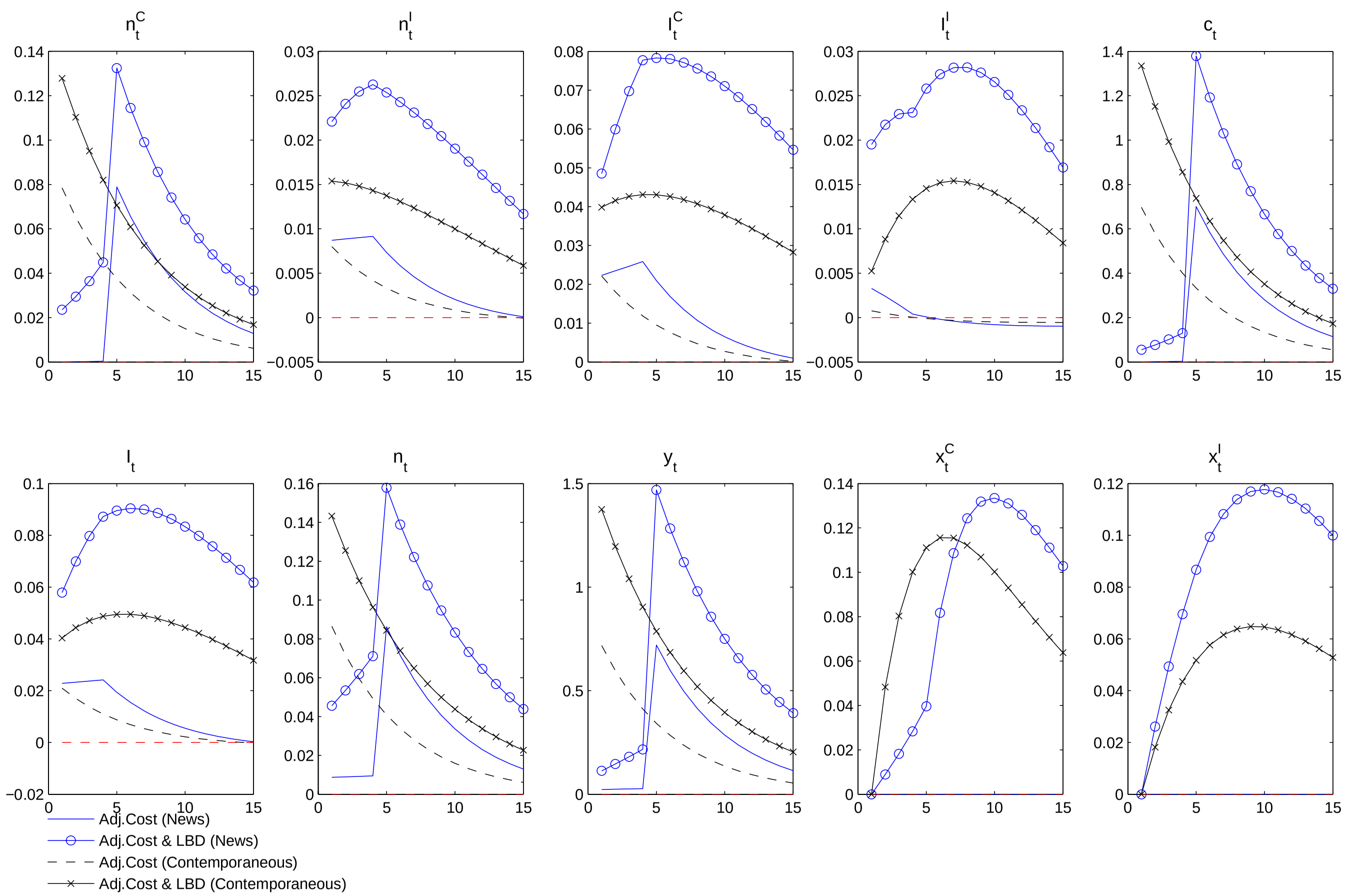

Figure 2.10: Consumption Technology Shock in OUR Two-Sector Model

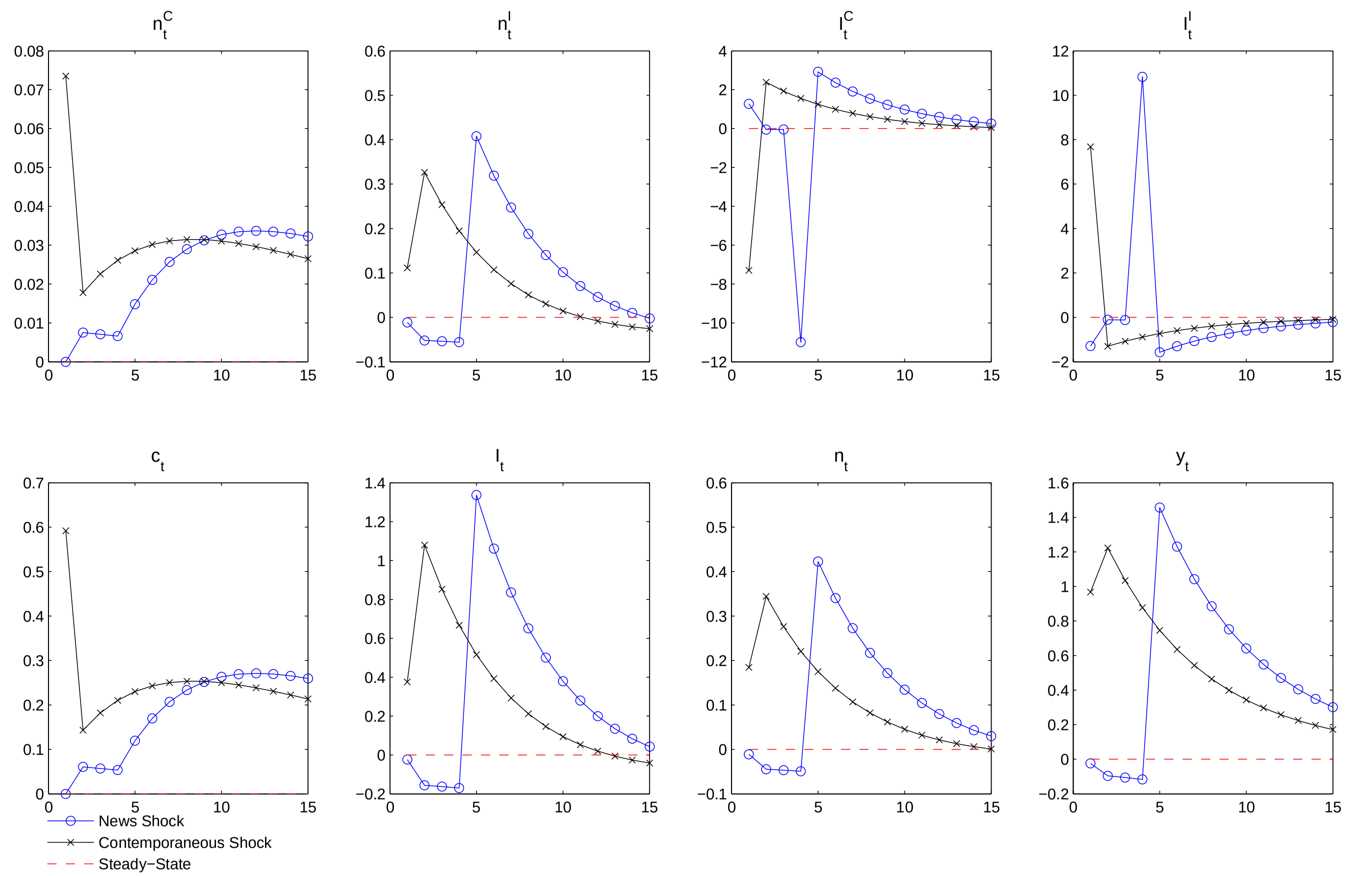

Figure 2.11: Neutral Technology Shock in the BENCHMARK Two-Sector Model

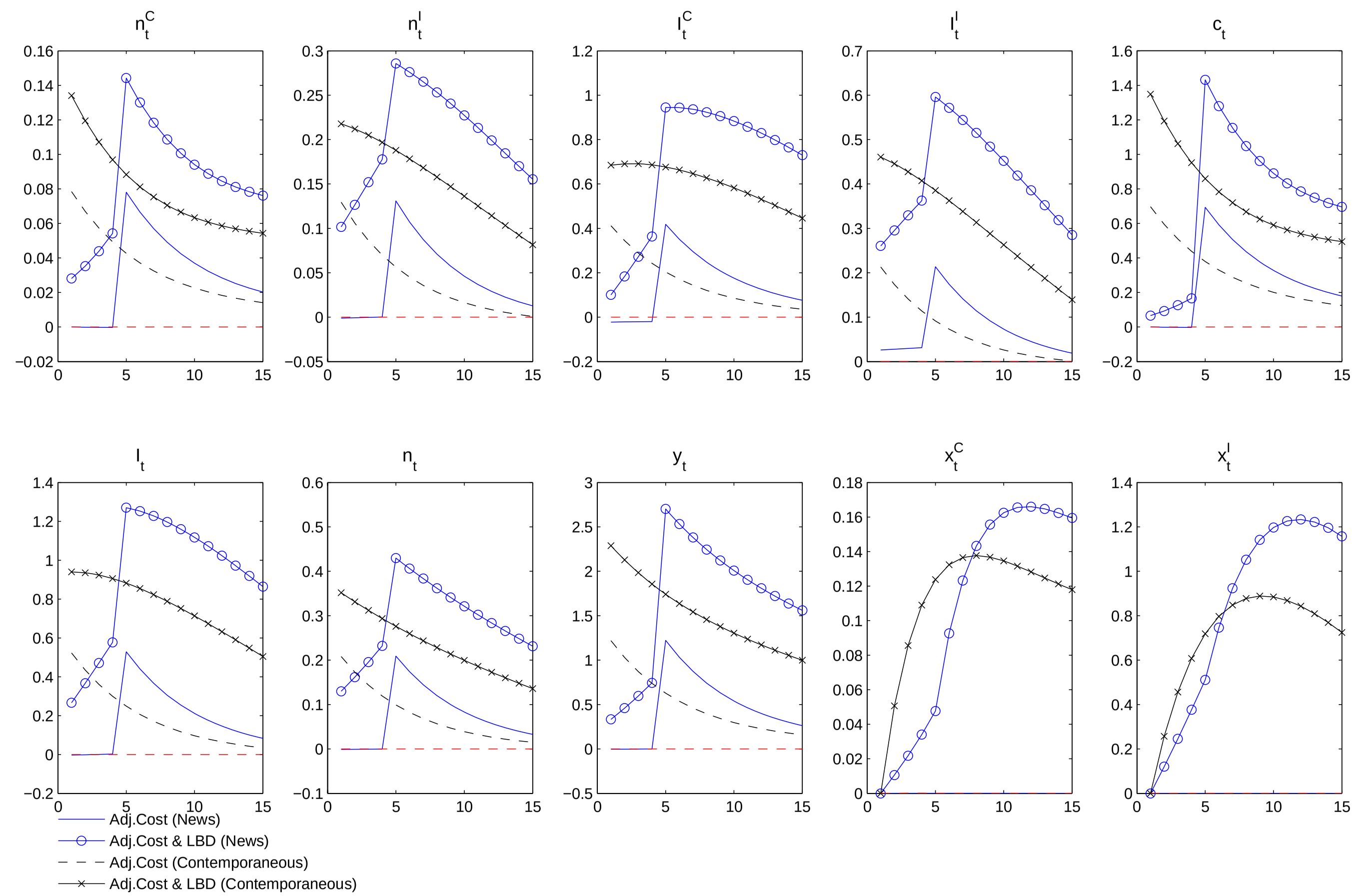

Figure 2.12: Neutral Technology Shock in OUR Two-Sector Model

CHAPTER 3

EXPLOSIVE ROOTS IN LEVEL VECTOR AUTOREGRESSIVE MODELS

3.1 Introduction

Following the work of Sims (1980), impulse response analysis based on level vector autoregressive (VAR) models has been utilized in numerous studies and plays an important role in contemporary macroeconomic research. One advantage of level VAR models over alternatives such as the vector error correction models is that the former are robust to the number of unit roots in the system. This robustness is one of the reasons why level VAR models are used extensively in applied macroeconomic research. However, estimated level VAR models may contain explosive roots even if all the true autoregressive roots lie inside the unit circle. The incidence of such explosive roots is at odds with the widespread agreement among macroeconomists that roots are at most unity.[17] Given that level VAR models are used extensively and may estimate roots greater than unity, it is important to examine how frequently researchers estimating level VAR models on macroeconomic datasets encounter explosive roots.

[17]Macroeconomists may model few phenomenon such as hyperinflations as explosive processes (see Neilsen (2005) and Juselius (2002)). These are important but very specific cases and in general most macroeconomic variables are modeled as non-explosive processes.

This paper investigates this frequency using Monte Carlo simulations based on datasets that are representative of those commonly used in the macroeconomic literature. In specific, datasets from three highly cited papers in the literature, Christiano, Eichenbaum, & Evans (1999, 2005), CEE henceforth, and Eichenbaum & Evans (1995), EE henceforth, are employed to examine the frequency of explosive roots (explosion) in estimated level VAR models.[18] Monte Carlo samples are generated under two specifications of the data-generating process (DGP). The first specification of the DGP imposes unit roots in the simulated data, while the second specification is based on a stationary process. Subsequently, level VAR models are estimated on the simulated data to compute the frequency of explosive roots. Under both these specifications, this paper also examines the frequency of explosion after correcting for the small-sample bias in estimated level VAR coefficients.

Monte Carlo results in this study reveal that the frequency of explosive roots exceeds 40% in the presence of unit roots. Even when all the variables are stationary, the frequency of explosive roots is substantial; it is as high as 25%. Furthermore, explosion increases significantly, to more than 90% under several specifications, when the estimated level VAR coefficients are corrected for small-sample bias.

To understand why the frequency of explosive roots is very high in level VAR models, it is useful to consider a hypothetical estimator that yields median-unbiased autoregressive roots (MUAR estimator). If there are unit root(s) in the system and thus the magnitude of the largest autoregressive root (λ_{max}) is unity, we would expect

[18] CEE (1999), CEE (2005) and EE (1995) are among the most highly cited papers in the applied macroeconomic research. Differences in VAR order, data frequency and variables used in these papers facilitate the assessment of explosive roots under a variety of specifications.

to encounter explosive roots with a very high frequency of 50% with MUAR estimator. However, since the least-squares estimator used for level VAR estimations is not median-unbiased, we can expect much lower frequency of explosive roots if there is a substantial downward median-bias in λ_{max}, whereas in the case of an upward median-bias it would be even higher than 50%. Results in this paper suggest that the least-squares bias in λ_{max} is not substantially downward: there is either only a slight downward median-bias or the median-bias is upward. Consequently, the frequency of explosive roots is close to (or even exceeds) 50% in the presence of unit roots and is very high even if all the variables are stationary.

The reason why bias correction in level VAR models leads to even higher frequency of explosive roots is because the commonly used bias correction procedures correct the bias in the coefficients and not the roots. Bias correction in coefficients may not correct the bias in the roots because of the non-linear mapping between the two. Median-unbiasedness is preserved under monotone transformation whereas mean-unbiasedness is persevered under linear combinations. Since the autoregressive roots are neither a monotone transformation nor a linear combination of the autoregressive coefficients, median or mean bias corrections in coefficients will not in general correct the bias in the roots. Results in the paper show that the commonly used bias correction procedures often increase the bias in the roots, resulting in substantial upward bias. Consequently, the frequency of explosive roots is as high as 100% after bias correction under several specifications.

These results suggest that researchers estimating level VAR models on macroeconomic datasets encounter explosive roots, a phenomenon that is contrary to common macroeconomic belief, with a very high frequency. Considering the consensus among macroeconomists that roots are at most unity, applied macroeconomists may discard explosive VAR draws in *simulated* data used for constructing confidence intervals for the impulse responses. For instance, Ditmar, Gavin & Kyland (2005) and Altig, Christiano, Eichenbaum & Linde (2004) discard explosive VAR draws in the *simulated* data used for constructing error bands for their impulse responses. However, discarding explosive VAR specifications when estimating level VAR models on the *actual* datasets is problematic because it may lead to *data mining biases*. Data mining can be a serious problem since it invalidates statistical theory. The high frequency of encountering explosive roots in estimated level VAR models suggests that this data mining problem can be severe. Additionally, the sharp increase in explosion after bias correction in estimated level VAR coefficients indicates that researchers correcting for the small-sample bias in these coefficients may encounter explosive roots with an even higher probability.

As per the well known evidence of nonstationarity in most macroeconomic series, one way to reduce the frequency of explosive roots is to impose unit roots in the estimation by estimating VECMs instead of level VAR models. I examine the frequency of explosive roots in estimated VECMs under the same specifications of the DGPs. Monte Carlo simulations reveal that explosion occurs much less frequently in estimated VECMs. VECMs reduce the frequency of explosive roots by restricting the magnitude of some of the otherwise explosive roots to unity.

The rest of the paper is organized as follows. Section II examines the frequency of explosive roots in estimated level VAR models in the presence of nonstationary variables. Section III focuses on explosive roots in estimated level VAR models when all the variables are stationary. Section IV examines the frequency of explosive roots in estimated VECMs. Section V concludes.

3.2 DGP with NonStationary Variables

Many macroeconomists model highly persistent time series, such as inflation, interest rate, exchange rate and money demand, as unit root processes since empirical studies that estimate these series have mostly failed to reject the null hypothesis of unit root nonstationarity. Given this evidence for nonstationarity of several macroeconomic variables and that macroeconomic theory predicts that some of these series have long-run equilibrium relationships, this paper tests for unit roots and cointegration in CEE (1995), CEE (2005) and EE (1995) datasets.[19] Several unit root tests are implemented to test stationarity of macroeconomic variables in CEE (1999), CEE (2005) and EE (1995). These tests fail to reject the null of unit root for most macroeconomics series. Johansen's (1988) tests are used to estimate the cointegration ranks in the datasets. Based on these tests, cointegration ranks of five, four and two are used for the DGPs based on CEE (1999), CEE (2005) and EE (1995) respectively.[20] However, Podivinsky's (1998) results suggest that Johansen's cointegration test may not be very reliable, especially in shorter samples due to severe size distortions.[21] I therefore examine the sensitivity of the results to varying cointegration ranks in the

[19]Unit root tests and cointegration tests are reported in Tables 3.8 and 3.9

[20]These cointegration ranks are chosen based on trace tests. The maximum eigenvalue tests, on the other hand, yield cointegration ranks of four for CEE (1999), and one for CEE (2005) and EE (1995). Given these mixed results, sensitivity of results to different cointegration ranks is also examined.

[21]Johansen (2002) proposes a small sample Barlett correction that improves the finite-sample performance of his test. However, Juselius (2006) points out that these corrections do not solve the power problem and in some cases the size of the test and the power of alternative hypotheses close to the unit circle are almost of the same magnitude.

DGPs.

Given the evidence for the existence of nonstationarity and cointegration, common stochastic trends are imposed in the Monte Carlo samples by estimating vector error correction models (VECMs) on the datasets and using the estimated regression coefficients for the DGP. Subsequently, the frequency of explosive roots in estimated level VAR models is computed. Unrestricted level VAR models are robust to the number of unit roots in the system and hence are not misspecified in the presence of unit roots and cointegration, as is the case in the simulated data under this specification. However, estimating VAR in levels in the presence of cointegration involves a loss of efficiency because some restrictions, namely the reduced rank of ζ_0 in (2) below, are not imposed.

3.2.1 Estimation Procedure

Monte Carlo experiments in the paper can be summarized into the following steps:

1. First I estimate reduced form VECMs using Johansen's maximum likelihood method on the datasets.[22]

$$\Delta Y_t = c + \zeta_1 \Delta Y_{t-1} + \zeta_2 \Delta Y_{t-2} + ... + \zeta_{p-1} \Delta Y_{t-p+1} + \zeta_0 Y_{t-1} + \epsilon_t \qquad (3.2.1)$$

Assuming normal errors, VECM coefficients can be estimated by maximizing the following likelihood function:

$$L\left(\Omega, \zeta_1, ..., \zeta_{p-1}, c, \zeta_0\right) = \left(-Tn/2\right) log\left(2\pi\right) - \left(T/2\right) log\left|\Omega\right| \qquad (3.2.2)$$

[22]VECM(4) is estimated on the CEE (1999) and CEE (2005) datasets, and VECM(6) is estimated on the EE (1995) dataset, since CEE (1999, 2005) used level VAR(4) and EE (1995) used VAR(6) specifications for their reduced form estimations.

$$-\frac{1}{2}\sum_{t=1}^{T}[(\Delta Y_t - c - \zeta_1 \Delta Y_{t-1} - ... - \zeta_{p-1}\Delta Y_{t-p+1} - \zeta_0 Y_{t-1})'$$

$$\Omega^{-1}(\Delta Y_t - c - \zeta_1 \Delta Y_{t-1} - ... - \zeta_{p-1}\Delta Y_{t-p+1} - \zeta_0 Y_{t-1})]$$

subject to $\zeta_0 = -BA'$

where Y_t is a n-dimensional vector of variables, Ω is the covariance matrix of ϵ_t, B is an (n x h) matrix, A' is an (h x n) matrix of cointegrating vectors, and h is the cointegration rank based on Johansen's test.[23]

2. Next I use the estimated VECM coefficients to generate 10,000 Monte Carlo samples.[24]

3. Finally I estimate level VAR models on each of these samples to get the reduced form coefficients:

$$Y_t = c^i + \theta_1^i Y_{t-1} + \theta_2^i Y_{t-2} + ... + \theta_p^i Y_{t-p} + \epsilon_t \qquad for\ i = 1, 2, ...10,000 \quad (3.2.3)$$

and subsequently check their stability to compute the frequency of explosive roots.[25]

[23] The likelihood function is maximized by implementing the step by step procedure proposed by Johansen (1988, 1991) as outlined in Hamilton (1994).

[24] Initial values from the datasets are used as the starting values for the Monte Carlo samples.

[25] Stability of a VAR(p) model can be checked by calculating λ_{max}, the modulus of the largest root of its companion matrix. If λ_{max} of an estimated VAR model lies outside the unit circle in a given sample, the VAR model would be unstable for that Monte Carlo sample. Frequency of explosive roots corresponds to the proportion of unstable VAR draws in the Monte Carlo samples. In order to allow for rounding off errors, I consider a VAR model to be explosive only if its λ_{max} exceeds a threshold value of 1.00001 (instead of exactly one). Results are essentially the same for other thresholds such as 1.0001 or 1.0005.

3.2.2 Theoretical Predictions

Consider an estimator that yields median-unbiased estimates of autoregressive roots in multivariate time series models. I refer to this imaginary estimator as 'median unbiased autoregressive roots estimator' (MUAR).[26] If the true data-generating process is a VECM and the magnitude of the largest autoregressive root, λ_{max}, is exactly one, we would expect to encounter explosive roots with a probability of 0.5 with MUAR. Needless to say, a 50% likelihood of explosion is extremely high. However, since the least-squares estimator used for level VAR estimations is not median-unbiased, we can expect lower frequency of explosive roots if the least-squares bias in λ_{max} is downward, whereas in the case of an upward bias it would be even higher.

Least-squares bias in autoregressive roots can be downward or upward. Andrews (1993) shows that the least-squares estimator is significantly downward biased in AR(1)/unit root models. Similarly Andrews and Chen (1994) show that least-squares estimates of α, the sum of autoregressive coefficients in AR(p) models, are substantially downward biased in small samples. However, since the mapping from autoregressive coefficients to autoregressive roots is nonlinear, the bias in autoregressive roots can go either way even if the autoregressive coefficients are downward biased. For instance, Andrews and Chen (1994, Table 2) report upward least-squares bias in most autoregressive roots.[27] Given that the least-squares estimator can be significantly biased in small samples and the bias in autoregressive roots can go in either

[26]It must be emphasized that no such estimator exists and MUAR is just an imaginary estimator, mentioned solely for expository purpose.

[27]Andrews and Chen (1994, Table 2) report results for three autoregressive models, which have upward bias in the magnitude of most autoregressive roots other than that of the largest one. Monte Carlo simulations (available upon request) based on their models with slightly different coefficient values yield upward bias in the magnitude of the largest root.

direction, it is hard to predict how often estimated level VAR models may contain explosive roots. Consequently, Monte Carlo simulations are used to estimate the frequency of explosive roots in estimated level VAR models with and without correcting for the small-sample bias using standard bias correction procedures.

Andrews (1993) and Andrews & Chen (1994) among others have proposed bias-corrected estimators for univariate autoregressive models. Kilian (1998) proposes a bias correction approach for multivariate time series models such as VAR. His approach relies on calculating the mean-bias using nonparametric bootstrapping. Nicholls and Pope (1988), on the other hand, provide a closed-form expression for the bias in stationary multivariate Gaussian autoregressions. Pope (1990) extends these results by relaxing the assumption of Gaussian innovations. It must be emphasized that common bias correction procedures, including those by Kilian (1998) and Pope (1990), are designed to correct the small-sample bias in the autoregressive *coefficients*, which may not correct the bias in autoregressive *roots* due to the nonlinear relationship between the two. Median-unbiasedness is preserved under monotone transformation whereas mean-unbiasedness is presevered under linear combinations. Since the autoregressive roots are neither a monotone transformation nor a linear combination of the autoregressive coefficients, median or mean bias corrections in coefficients will not in general correct the bias in the roots.[28] This paper uses bias correction procedures based on Kilian (1998) and Pope (1990) to correct for the small-sample bias in estimated level VAR coefficients.

Kilian's bias correction procedure involves estimating VAR models and generating

[28]To my knowledge, there does not exist any bias correction procedure that is designed to correct the bias in autoregressive roots of VAR models. Qureshi (2008) proposes a method to numerically correct the median-bias in autoregressive roots.

N replications of the estimated coefficients using standard nonparametric bootstrap techniques. Subsequently, the mean-bias is estimated as the difference between the average of the N replications of coefficients and the initial estimate of coefficients used in the DGP. This procedure is computationally demanding since it requires generating N replications on each Monte Carlo sample. Therefore, this paper uses a modest number of Monte Carlo samples: it generates 1000 Monte Carlo samples for the bias correction simulations and estimates the bias using 1000 replications of the estimated coefficients on each Monte Carlo sample.[29]

Kilian implements a stationarity correction after correcting the bias in coefficients to avoid pushing stationary impulse response estimates into the nonstationary region. Kilian's bias correction with stationarity correction would ensure that explosive roots in estimated VAR models are eliminated. However, Sims and Zha (1995) criticize Kilian's stationarity correction as '*ad hoc*'. This paper implements Kilian's bias correction method without the stationarity correction. Hence results in this paper reveal how frequently Kilian's method relies on stationarity correction to avoid explosive roots in estimated level VAR models.

Pope's expression for the mean-bias in VAR coefficients is defined for demeaned stationary VAR(1) models. In order to implement bias correction based on this expression, VAR(p)s are estimated on demeaned simulated data and then reformulated as VAR(1)s.[30] Subsequently, the mean-bias is calculated using Pope's expression.

[29]This would result in 1000^2 or one million simulations which take considerable time even with the fast processors available to date. Sensitivity of results to increasing the number of simulations to 2000^2 is examined for simulations reported in Table 3.1. Frequency of explosive roots essentially remains the same.

[30]Demeaned data for simulations with Pope's bias correction is generated by using estimated VECM coefficients in (1) without the constant, and by setting the initial values in the Monte Carlo samples to zero.

Finally the mean-bias is subtracted from the estimated VAR coefficients to yield bias-corrected coefficients. I refer to these steps as Pope's bias correction.

In this paper the bias correction procedures are implemented only on stable VAR draws and explosive roots in unstable VARs are counted towards the frequency of explosion without bias correction. This is because Pope's solution for the bias in VAR coefficients is defined for stationary VAR models. Similarly, Kilian's approach is designed for stationary models. However, Kilian (1998) argues that based on the continuity of the finite-sample distribution of the OLS estimator, the bootstrap approximation may still be used for slightly explosive cases. In light of this argument, I estimate the frequency of explosive roots after implementing Kilian's and Pope's bias corrections on all Monte Carlo samples (including the explosive ones). Results based on this exercise are essentially the same as the benchmark case of bias correction on stable VARs only.

3.2.3 Results

Figure 2.1 presents an example illustrating the frequency of explosive roots in estimated level VAR models. It plots the distribution of λ_{max}, the modulus of the largest autoregressive root, in estimated level VAR(4) models with and without bias correction. The DGP is based on VECM estimation on the CEE (1999) dataset, with a cointegration rank of five. The frequency of explosive roots corresponds to the area under the distribution to the right of unity. The following tables report these frequencies under various specifications.[31]

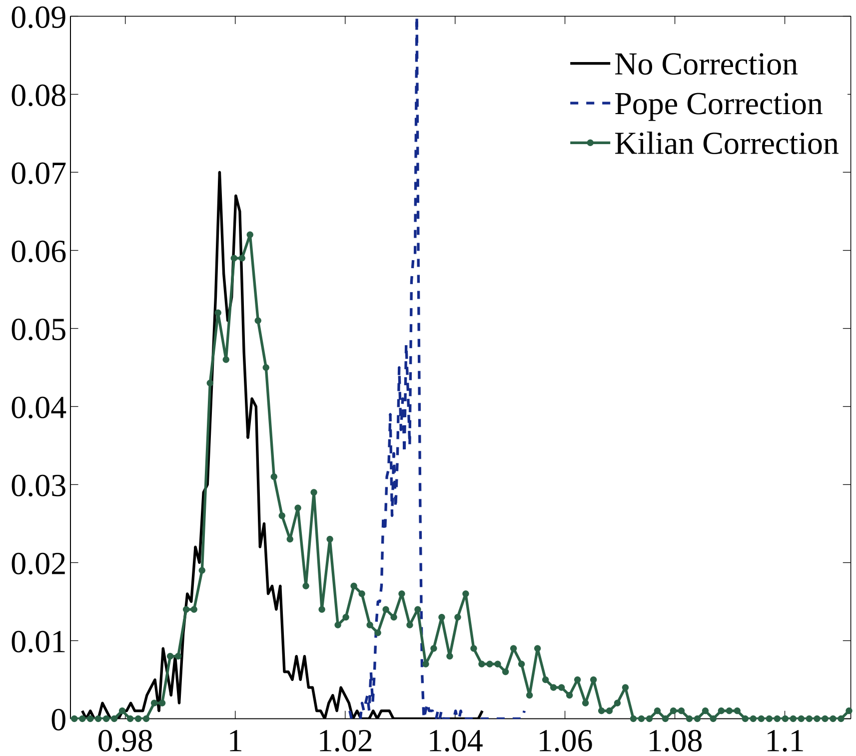

Figure 3.1: Distribution of λ_{max}

Table 3.1 reports the frequency of explosive roots in estimated level VAR models for the benchmark estimations of VAR(4) in CEE(1999) and CEE(2005), and VAR(6) in EE(1995). The frequency of explosion is considerably high. Estimated level VAR models have explosive roots in 46.4%, 47.7% and 41.9% of the Monte Carlo samples

[31] EE (1995) estimate level VAR models with five, seven and eight variables and examine five different nominal and real exchange rates. In the interest of brevity, this paper only presents results for their nominal \$/Franc exchange rate model with five variables. Results for other specifications and exchange rates are essentially the same. CEE (1999) report results with both $M1$ and $M2$ in their benchmark specification. This paper only presents results with $M1$. Once again, results are almost the same if $M1$ is replaced by $M2$.

based on CEE (1999), CEE (2005) and EE (1995) respectively. Furthermore, the frequency of explosion increases substantially after correcting for the small-sample bias. Results for both Pope (1990) and Kilian (1998) bias correction procedures, denoted by 'Pope' and 'Kilian' respectively, are reported. Estimated level VAR models have explosive roots more that 75% of the time after Kilian's bias correction and 100% of the time after Pope's bias correction. Table 3.1 reveals that λ_{max} has downward median-bias because the frequency of explosive roots is less than 50% in the benchmark specifications. Additionally, Kilian's and Pope's bias corrections on level VAR coefficients overcorrect this bias in λ_{max}, consequently resulting in upward median-bias.

These results suggest that researchers estimating level VAR on macroeconomic datasets, which include some nonstationary I(1) variables, encounter explosive roots very frequently, and even more so if they correct for the finite-sample bias in their estimation.

The following subsection examines the sensitivity of these results to varying cointegration ranks in the DGP, and to shorter samples and different lag orders in the estimated VAR models. Results from table 3.1 are reproduced (*in italics*) in the following tables to facilitate comparison with these *benchmark* specifications.

3.2.4 Sensitivity Analysis

Considering that cointegration rank tests may not be reliable in small samples, table 3.2 examines the sensitivity of results to different cointegration ranks in the DGP.[32] In most cases as the cointegration rank, h, increases, and hence the number

[32]Number of variables in the system, n, equals 7, 9 and 5 for CEE (1999), CEE (2005) and EE (1995) respectively. Any remaining cointegration ranks that are not reported yield very similar results.

of unit roots in the DGP decreases, the frequency of explosion goes down. For instance, as h increases from 3 to 6 in CEE (1999), the frequency of explosion decreases from 41.7% to 36.7%. However, explosion still remains high; in most simulations estimated level VAR models have explosive roots in more than 40% of the Monte Carlo samples. Once again, explosion increases substantially, to more than 90% in several cases, after bias correction. These results confirm that the high frequency of explosive roots is robust to varying cointegration rank in the DGP. The next table assesses the sensitivity of results to different subsamples and lag orders in estimated level VAR models.

Macroeconomic datasets for the post-Bretton Woods or post-Volcker eras are relatively short, which may exacerbate explosion in estimated level VAR models.[33] Level VAR models are estimated on truncated Monte Carlo samples, namely 'post-Bretton Woods' and 'Volcker & post-Volcker', to estimate the frequency of explosion in shorter samples.[34]

Different lag orders in estimated level VAR models may also affect the frequency of explosive roots. I therefore examine the sensitivity of results to varying orders in level VAR models in the full-sample as well as the two subsamples. CEE (1999) and CEE (2005) use level VAR(4) models while EE (1995) use level VAR(6) model for their reduced form estimation. Since these subsamples are fairly short, degrees of freedom would be low for the benchmark specifications of for four lags in CEE (1999,

[33]For example, if λ_{max} is biased downward explosion may rise due to an increase in the variance of λ_{max} in shorter samples. However, it must be emphasized that median-bias as well as other characteristics of the distribution (skewness, kurtosis, etc.) would also in general change in smaller samples making it hard to predict how the frequency of explosion would be affected.

[34]'post-Bretton Woods' and 'Volcker & post-Volcker' subsamples correspond to the following sample periods: *CEE (1999, 2005)* quarterly - 'post-Bretton Woods' (1974:1 to 1995:2) and 'Volcker & post-Volcker' (1979:3 to 1995:2). *EE (1995)* monthly - 'post-Bretton Woods' (1974:1 to 1991:12) and 'Volcker & post-Volcker' (1979:8 to 1991:12).

2005) and six lags in EE (1995). Hence, the frequency of explosion is also reported for lower lag orders in level VAR models.

Table 3.3 reveals that the frequency of explosive roots remains high for different lag orders in estimated models. Moreover, explosion increases further in shorter samples in several simulations. For instance, the frequency of explosion in the benchmark cases increases to 56.1%, 70.2% and 44.9% in the 'Volcker & post-Volcker' subsamples. Once more, explosion increases appreciably after correcting for the small-sample bias in estimated level VAR coefficients. The frequency of explosive roots is more than 75% under all specifications after Kilian's bias correction and increases to 100% in all cases after Pope's correction.

3.3 DGP with Stationary Variables

The previous section examined the frequency of explosive roots in estimated level VAR models in the presence of unit root nonstationary variables. This section focuses on explosive roots in estimated level VAR models when all the variables are stationary. In this case the data-generating processes are based on level VAR models, as opposed to VECMs.

3.3.1 Estimation Procedure

The procedure for conducting Monte Carlo experiments is the same as that in the previous section except for the first two steps in which level VAR(p) models are estimated on demeaned datasets and the corresponding coefficients are used to generate 10,000 Monte Carlo samples.[35] Starting values for the Monte Carlo samples

[35] p equals 4 for the DGP based on CEE (1999, 2005) datasets and 6 for EE (1995) dataset. Since the DGP is based on demeaned data, I estimate level VAR models (without constant) on the

are drawn from the stable VAR distribution. Subsequently, level VAR models are estimated on these samples to compute the frequency of explosive roots.[36]

3.3.2 Results

Table 3.4 summarizes results for the frequency of explosive roots in estimated level VAR models when the DGP is stationary. It presents results under the same specifications of estimated level VAR models as those reported in the table 3.3. Results in table 3.4 reveal that even in the absence of any unit roots, the frequency of explosive roots is considerable. Estimated level VAR models on full-samples have explosive roots in 25.9%, 12.9% and 19.6% of the simulations based on the benchmark specifications in CEE (1999), CEE (2005) and EE (1995) respectively. Furthermore, explosion increases substantially in shorter subsamples. For instance, the frequency of explosive roots in these benchmark cases increases to 56.0%, 61.4% and 31.7% respectively in the 'Volcker & post-Volcker' subsamples. Results for different lag orders in estimated models show that the high frequency of explosive roots is robust to varying order in level VAR estimation. As before, explosion increases substantially after bias correction. In most cases, explosive roots are encountered in more that 70% simulations after Kilian's bias correction and in more than 90% simulations after Pope's bias correction.

These results indicate that macroeconomists estimating level VAR model on datasets encounter explosive roots very frequently even if all the variables in their dataset are

Monte Carlo samples. This is useful for Pope's bias correction since Pope's expression is defined for demeaned stationary VARs.

[36]Since λ_{max} in the stationary DGP is less than unity, it is hard to predict the frequency of explosion even for the imaginary MUAR estimator.

stationary. Moreover, they may almost always estimate explosive roots on macroeconomic datasets if they correct for the small-sample bias in level VAR coefficients.

Considering that the frequency of explosive roots in estimated level VAR models is very high, the next section explores alternatives to level VAR models and examines the frequency of explosion in one such alternative, namely the vector error correction models.

3.4 Explosive Roots in VECMs

As per the well known evidence of nonstationarity in most macroeconomic series, one way to reduce the frequency of explosive roots is to impose unit roots in the estimation by estimating VECMs instead of level VAR models. Imposing unit roots in the estimation would restrict the magnitude of some of the otherwise explosive roots to unity, hence reducing the frequency of explosive roots. This section examines the frequency of explosive roots in estimated VECMs under the same specifications of the DGPs as the previous sections.

3.4.1 Estimation Procedure

The procedure for conducting Monte Carlo experiments is identical to that in the previous sections, except for the third step in which VECMs are estimated on the simulated datasets instead of level VAR models. Cointegration ranks of five, four and two are imposed on each simulated dataset for CEE (1999), CEE (2005) and EE (1995) respectively. These cointergation ranks are the same as those imposed in the nonstationary DGP.[37]

[37] An alternative approach would be to estimate cointergration rank for each simulated dataset and impose the corresponding number of unit roots in the estimation.

3.4.2 Results

Table 3.5 reports the frequency of explosive roots in estimated VECMs when the DGP is nonstationary. Since the standard bias correction procedures for level VAR models can not be applied to VECMs, the following tables only report the cases without bias correction. These results reveal that the frequency of explosion reduces dramatically once VECMs are estimated on the simulated datasets. Imposing unit roots in the estimation restricts the magnitude of some of the explosive roots to unity, hence reducing the frequency of explosion. Based on the benchmark specifications in CEE (1999), CEE (2005) and EE (1995), estimated VECMs on full-samples have explosive roots in only 2.3%, 0.6% and 0.4% of the simulations respectively, compared to 46.4%, 47.7% and 41.9% in estimated level VAR models. Frequency of explosive roots in estimated VECMs increases to some extent in shorter subsamples. Explosion increases to 9.4% in the 'Volker & post-Volker' subsamples in the benchmark specification of CEE (1999). However, the frequency of explosive roots in estimated VECMs is still much lower than estimated level VAR models.

Table 3.6 reports the frequency of explosive roots when the DGP is stationary. It presents results under the same specifications of estimated VECMs as those reported in table 3.5. Once again, the frequency of explosive roots is very low. It is less that 1% under all specifications on full-samples. Estimated VECMs on full-samples have explosive roots in only 0.9%, 0.0% and 0.1% of the simulations based on the benchmark specifications in CEE (1999), CEE (2005) and EE (1995) respectively. Even in shorter samples, the frequency of explosive is less than 5% under most specifications.

The last table presents the sensitivity of these results to varying cointegration ranks in estimated VECMs. Results in table 3.7 reveal that the frequency of explosion decreases as more unit roots are imposed in the VECM estimation. For instance, explosion decreases from 47.7% to 5.1% in CEE (2005) simulations if the cointegration rank, h, is reduced from 9 to 7. These results show that the frequency of explosive roots can be reduced substantially by estimating VECMs instead of level VAR models.

3.5 Conclusion

Level VAR models are used extensively in applied macroeconomic research. However, estimating VAR in levels may result in explosive roots even if all the true roots lie strictly inside the unit circle. The occurence of such explosive roots is inconsistent with the prevalent agreement among macroeconomists that roots are at most unity. Given that level VAR models are used extensively and may estimate roots greater than unity, this paper examines how frequently researchers estimating level VAR models on macroeconomic datasets may encounter explosive roots. Monte Carlo simulations based on datasets from the macroeconomic literature reveal that the frequency of explosive roots exceeds 40% in the presence of unit roots and is substantial even if all the variables are stationary. Furthermore, explosion increases substantially, to as much as 100%, after correcting for the small-sample bias in estimated level VAR coefficients.

These results suggest that researchers estimating level VAR models on macroeconomic datasets encounter explosive roots with a very high frequency. Considering the consensus among macroeconomists that roots are at most unity, if applied macroeconomists discard explosive VAR specifications when VAR models are estimated on the datasets, it may lead to biases in the estimation or can even result in data mining. Data mining can be a serious problem since it invalidates statistical theory. The high frequency of encountering explosive roots in level VAR models suggests that this data mining problem can be severe. Additionally, the sharp increase in explosion after bias correction indicates that researchers, who correct for the small-sample bias in level VAR coefficients, may almost always estimate explosive roots on macroeconomic datasets.

As per the well known evidence of nonstationarity in most macroeconomic series, one way to reduce the frequency of explosive roots is to impose unit roots in the estimation by estimating VECMs instead of level VAR models. Simulation results suggest that VECMs can substantially reduce the frequency of explosive roots. Another alternative could be imposing cointegrating relationships among variables in the VAR as in Shapiro & Watson (1988). Based on the continuity of the finite sample distribution of least-squares estimator, applied macroeconomists may ignore explosive roots with magnitudes artibrarily close to unity. Hence depending on the objective of the analysis, ignoring *slightly* explosive roots may be another alternative. However, it is not clear as to what would be a reasonable cutoff for categorizing an autoregressive root as *slightly* explosive. Moreover, such a cutoff would vary with the system and the purpose of the research.

Evaluating these alternatives in terms of the accuracy of estimated impulse responses, variance decompositions and robustness to various specifications such as the number of unit roots in the system would be an interesting topic for future research.

Table 3.1: Explosive Roots in Estimated Level VAR Models

Explosive Roots (percent)	CEE 99	CEE 05	EE 95
VAR(p)	46.4	47.7	41.9
VAR(p) - Pope	100	100	100
VAR(p) - Kilian	77.2	92.4	85.3

DGP: VECM(p*) with cointegration rank h. **Estimated Model: VAR(p)**. p^* and p equal 4 for CEE (1999, 2005) and 6 for EE (1995). h equals 5, 4 and 2 for CEE(1999), CEE (2005) and EE(1995) respectively.

Table 3.2: Sensitivity Analysis of Cointegration Rank h in the DGP

Explosive Roots (percent)	CEE 99				CEE 05				EE 95			
h	3	4	*5*	n-1	3	*4*	5	n-1	1	*2*	3	n-1
VAR(p)	41.7	44.5	*46.4*	36.7	48.7	*47.7*	39.9	28.4	45.2	*41.9*	39.9	35.7
VAR(p) - Pope	100	100	*100*	100	100	*100*	100	100	100	*100*	100	100
VAR(p) - Kilian	89.4	83.7	*77.2*	67.9	96.5	*92.4*	88.9	62.4	89.1	*85.3*	75.4	59.4

DGP: VECM(p*) with cointegration rank h. **Estimated Model: VAR(p)**. p^* and p equal 4 for CEE (1999, 2005) and 6 for EE (1995). Benchmark cases from Table 1 are in *italics*. n is the number of variables in the system: n equals 7, 9 and 5 for CEE-(1999), CEE (2005) and EE(1995) respectively.

Table 3.3: Explosive Roots in Estimated Level VAR Models - **DGP: VECM**

Explosive Roots (percent)	CEE 99			CEE 05			EE 95		
p	3	*4*	5	3	*4*	5	5	*6*	7
FULL SAMPLE									
VAR(p)	44.7	*46.4*	47.1	46.3	*47.7*	50.3	42.4	*41.9*	42.5
VAR(p) - Pope	100	*100*	100	100	*100*	100	100	*100*	100
VAR(p) - Kilian	75.0	*77.2*	77.0	95.3	*92.4*	93.3	83.7	*85.3*	85.5
p	2	3	*4*	2	3	*4*	4	5	*6*
POST-BRETTON WOODS									
VAR(p)	47.3	47.9	49.7	46.9	50.4	55.9	42.4	43.9	44.0
VAR(p) - Pope	100	100	100	100	100	100	100	100	100
VAR(p) - Kilian	79.3	77.7	79.5	92.2	92.8	93.2	76.9	83.2	84.2
VOLCKER & POST-VOLCKER									
VAR(p)	48.8	50.3	56.1	48.2	56.5	70.2	42.6	44.7	44.9
VAR(p) - Pope	100	100	100	100	100	100	100	100	100
VAR(p) - Kilian	82.6	81.5	82.5	92.4	91.5	94.2	82.9	85.1	87.1

DGP: VECM(p*) with cointegration rank h. **Estimated Model: VAR(p)**. p^* equals 4 for CEE (1999, 2005) and 6 for EE (1995). Benchmark cases from Table 1 are in *italics*. h equals 5, 4 and 2 for CEE(1999), CEE (2005) and EE(1995) respectively.

Table 3.4: Explosive Roots in Estimated Level VAR Models - **DGP: VAR**

Explosive Roots (percent)	CEE 99			CEE 05			EE 95		
p	3	*4*	5	3	*4*	5	5	*6*	7
FULL SAMPLE									
VAR(p)	24.9	*25.9*	29.6	12.4	*12.9*	16.8	21.4	*19.6*	17.7
VAR(p) - Pope	95.5	*95.2*	95.0	82.3	*79.4*	80.3	92.0	*88.6*	87.0
VAR(p) - Kilian	74.1	*72.8*	72.6	64.0	*64.2*	64.3	56.4	*53.3*	51.3
p	2	3	*4*	2	3	*4*	4	5	*6*
POST-BRETTON WOODS									
VAR(p)	33.0	35.1	41.3	24.2	26.9	35.9	24.1	22.5	20.4
VAR(p) - Pope	98.9	98.5	98.5	95.3	92.7	93.5	94.5	92.5	90.3
VAR(p) - Kilian	80.8	81.7	81.4	82.3	85.3	87.2	55.4	59.7	60.2
VOLCKER & POST-VOLCKER									
VAR(p)	41.5	46.3	56.0	33.6	44.1	61.4	30.9	31.6	31.7
VAR(p) - Pope	99.6	99.5	99.5	97.5	97.1	98.1	97.3	96.5	96.1
VAR(p) - Kilian	83.8	86.4	89.5	88.1	92.0	93.3	70.0	75.4	78.4

DGP: VAR(p*). **Estimated Model: VAR(p)**. p^* equals 4 for CEE (1999, 2005) and 6 for EE (1995).

Table 3.5: Explosive Roots in Estimated VECM - **DGP: VECM**

Explosive Roots (percent)	CEE 99			CEE 05			EE 95		
p	3	*4*	5	3	*4*	5	5	*6*	7
FULL SAMPLE									
VECM(p)	1.8	*2.3*	2.3	0.0	*0.6*	0.2	0.1	*0.4*	0.3
p	2	3	*4*	2	3	*4*	4	5	*6*
POST-BRETTON WOODS									
VECM(p)	4.4	3.4	4.8	0.2	0.2	0.8	0.3	0.8	2.5
VOLCKER & POST-VOLCKER									
VECM(p)	5.6	6.6	9.4	0.4	1.0	5.6	0.4	1.1	3.2

DGP: VECM(p*) with cointegration rank h. **Estimated Model: VECM(p)** with cointegration rank h. p^* equals 4 for CEE (1999, 2005) and 6 for EE (1995). Benchmark cases are in *italics*. h equals 5, 4 and 2 for CEE(1999), CEE (2005) and EE(1995) respectively.

Table 3.6: Explosive Roots in Estimated VECMs - **DGP: VAR**

Explosive Roots (percent)	CEE 99			CEE 05			EE 95		
p	3	*4*	5	3	*4*	5	5	*6*	7
FULL SAMPLE									
VECM(p)	0.2	*0.9*	1.1	0.1	*0.0*	0.1	0.0	*0.1*	0.1
p	2	3	*4*	2	3	*4*	4	5	*6*
POST-BRETTON WOODS									
VECM(p)	1.0	1.3	3.1	0.0	0.1	0.2	0.3	0.5	1.1
VOLCKER & POST-VOLCKER									
VECM(p)	2.0	3.4	8.5	1.2	5.0	3.9	0.9	0.8	1.3

DGP: VAR(p*). **Estimated Model: VECM(p)** with cointegration rank h. p^* equals 4 for CEE (1999, 2005) and 6 for EE (1995). h equals 5, 4 and 2 for CEE(1999), CEE (2005) and EE(1995) respectively. Benchmark cases are in *italics*.

Table 3.7: Sensitivity to Cointegration Rank in Estimated VECMs

Cointergration Rank	EE 95	CEE 99	CEE 05
1	0.0	0.4	0.0
2	0.2	0.6	0.0
3	1.2	1.2	0.0
4	9.5	2.1	0.0
5	41.9	2.3	1.2
6	-	11.2	2.0
7	-	46.4	5.1
8	-	-	12.4
9	-	-	47.7

DGP: VECM(p) with cointegration ranks of 5, 4 and 2 for CEE(1999), CEE (2005) and EE(1995) respectively. **Estimated Model: VECM(p)**. p equals 4 for CEE (1999, 2005) and 6 for EE (1995). h equals 5, 4 and

Table 3.8: Unit Root Tests

CEE(1999)	SDF	PP	MPP	MSB	PT	MPT	DF-GLS	MPP-GLS
Y	(0.82)	1.37	1.47	1.42	189.3	146.6	1.52	1.38
P_{def}	(1.32)	(10.93)*	(10.85)*	0.20	3.52	2.80*	(0.55)	(1.27)
P_{com}	(3.38)*	(21.32)*	(19.48)*	(0.16)*	(1.39)*	(1.39)*	(3.37)*	(19.48)*
FFR	(2.24)	(3.11)	(5.97)	0.29	4.82	4.13	(1.57)	(5.17)
NBR	(0.49)	1.61	1.70	1.06	106.9	88.21	1.12	1.24
$TOTR$	(0.59)	1.29	1.38	0.82	64.80	53.00	0.51	1.24
$M1$	(1.08)	0.08	0.17	0.54	27.40	22.06	0.08	(1.32)
CEE(2005)	**SDF**	**PP**	**MPP**	**MSB**	**PT**	**MPT**	**DF-GLS**	**MPP-GLS**
Y	(0.82)	1.37	1.47	1.42	189.3	146.6	1.52	1.38
Inf	(1.57)	(4.17)	(3.31)	0.39	8.63	7.39	(1.30)	(2.79)
C	(0.52)	1.19	1.29	1.20	134.46	102.97	1.41	1.02
I	(1.46)	0.18	0.28	0.56	30.02	23.43	0.14	0.18
w	(2.08)	(2.54)	(2.48)	0.44	12.50	9.76	(0.95)	(2.53)
$Prod$	(0.65)	1.48	1.60	1.32	170.02	130.57	1.92	1.44
FFR	(2.24)	(3.11)	(5.97)	0.29	4.82	4.13	(1.57)	(5.17)
$M2_{growth}$	(0.73)	(6.75)	2.51	0.43	9.43	9.57	(0.82)	(2.46)
π	(1.25)	0.49	0.56	1.05	86.92	69.72	0.54	0.49
EE(1995)	**SDF**	**PP**	**MPP**	**MSB**	**PT**	**MPT**	**DF-GLS**	**MPP-GLS**
Y_{ind}	(1.53)	0.85	0.87	1.10	93.61	80.63	0.85	0.86
CPI	(1.38)	(2.90)	(2.87)	0.34	9.25	8.03	(0.18)	(0.72)
$NBRX$	(2.51)	(14.00)*	(13.02)*	0.19*	1.96*	1.97*	(2.49)*	(13.25)*
$R_{US} - R_{France}$	(2.21)	(9.64)*	(9.07)*	0.22*	3.13*	3.13*	(2.11)*	(9.88)*
$e_{\$/Franc}$	(1.55)	(5.15)	(5.14)	0.31	4.73	4.77	(1.55)	(5.14)

* denotes the rejection of the hypothesis at 5%significance level. SDF, PP, MPP, MSB, PT, MPT, DF-GLS, MPP-GLS denote Said-Dickey-Fuller, Phillips-Perron, Modified-Phillips-Perron, Modified-Sargan Bhargava, ERS feasibale point test, Modified feasible point test, DF with GLS detrending and Modified Philips-Perrron with GLS detrending respectively.

Table 3.9: Johansen's Tests

Eigenvalue	Eig_{max}	$Trace$	Rank (h)	Eig_{max}(5% $c.v$)	$Trace$(5% $c.v$)
CEE(1999)					
0.4388	69.9**	217.1**	0	45.28	124.24
0.3455	51.3**	147.2**	1	39.37	94.15
0.2653	37.3*	95.9**	2	33.46	68.52
0.2066	28.0*	58.6**	3	27.07	47.21
0.1560	20.53	30.6*	4	20.97	29.68
0.0540	6.72	10.09	5	14.07	15.41
0.0275	3.37	3.37	6	3.76	3.76
CEE(2005)					
0.4445	71.12**	251.72**	0	57.12	192.89
0.3274	47.99	180.59**	1	51.42	156.00
0.2480	34.49	132.60*	2	45.28	124.24
0.2244	30.75	98.12*	3	39.37	94.15
0.2040	27.61	67.37	4	33.46	68.52
0.1266	16.38	39.76	5	27.07	47.21
0.1009	12.68	23.37	6	20.97	29.68
0.0451	5.59	10.51	7	14.07	15.41
0.0399	4.92	4.92	8	3.76	3.76
EE(1995)					
0.1453	38.63*	87.08**	0	33.46	68.52
0.0827	21.22	48.45*	1	27.07	47.21
0.0696	17.75	27.23	2	20.97	29.68
0.0274	6.83	9.48	3	14.07	15.41
0.0107	2.65	2.65	4	3.76	3.76

*(**) denotes the rejection of the hypothesis at 5%(1%) significance level. Testing Assumption: Linear trend in data

CHAPTER 4

BIAS CORRECTION IN AUTOREGRESSIVE ROOTS

4.1 Introduction

It is well documented that the least-squares estimators of key parameters in autoregressive models are significantly biased in small samples, especially when the dominant root is close to unity. In consequence, the least-squares estimator is a misleading indicator of the true values of important parameters such as the autoregressive coefficients, the autoregressive roots and, the impulse response functions.

Several papers have addressed this by devising bias correction procedures for the least-squares estimator in univariate models.[38] Andrews (1993) proposed an 'exactly' median-unbiased estimator for AR(1)/unit-root models which was later extended by Andrews and Chen (1994) who 'approximately' median-unbiased AR(p) models. McCulloch (2007) generalized median bias correction in Andrews and Chen (1994) by simultaneously correcting the bias in all the coefficients of a 'recursive' ADF form, hence proposing an 'exactly' median-unbiased estimation of AR(p) models. Hansen (1999) developed a grid bootstrap method for univariate autoregressive processes,

[38]Pesavento and Rossi (2006), Kilian (1998), and Pope (1990) address bias in multivariate autoregressive models.

which provides correct asymptotic coverage regardless of whether the autoregressive model is near-integrated or exactly integrated. Other procedures have been proposed by Wright (2000), Gospodinov (2004), and McCulloch (2009).

Most of the bias correction methods are designed to correct the bias in estimated autoregressive coefficients or the estimated coefficients in the Augmented Dickey-Fuller form. However, bias correction in coefficients may not correct the bias in roots because of the non-linear mapping between the two. Median-unbiasedness is preserved under monotone transformations whereas mean-unbiasedness is preserved under linear combinations. Since the autoregressive roots are neither a monotone transformation nor a linear combination of the autoregressive coefficients, median- or mean-bias corrections in coefficients will not in general correct the bias in roots. For instance, Qureshi (2008) shows that mean bias correction in autoregressive coefficients can increase the bias in autoregressive roots.

Applied researchers are interested in bias-corrected roots for several reasons. For instance, empirical economists frequently estimate the half-life of real exchange rates, which depend critically on the magnitude of dominant root(s). Bias correction on roots should enhance the accuracy of half-life estimates. Similarly, macroeconomists widely use impulse response functions to evaluate the effect of shocks on macroeconomic time-series. Bias correction on roots should also reduce the bias in impulse response functions.

This paper develops a procedure to correct the small-sample bias in autoregressive roots of AR(p) models. The procedure simultaneously corrects the bias in the p autoregressive roots using numerical procedures. We estimate the median-bias in estimated roots using Monte Carlo simulations and use standard optimization algorithms to minimize it. The objective is to estimate autoregressive roots whose Monte Carlo median converges to the roots estimated from the dataset.

We evaluate the median-bias properties and variability of the bias-adjusted parameters by comparing them to the least-squares estimates. Considering that applied macroeconomists are primarily interested in impulse response functions (IRFs), we focus on examining the accuracy of bias-adjusted impulse responses. We also examine measures such as half-life, quarter-life, and up-life, which are used in the literature to capture the dynamics of impulse response functions.

Our simulation results reveal that bias correction in roots achieves a substantial reduction in the median-bias of IRFs relative to the least-squares estimates. Furthermore, correcting the bias in roots significantly improves the median-bias in half-life, quarter-life and up-life estimates. The procedure pays a negligible-to-small price in terms of increased standard deviation for its improved median-bias properties.

The rest of the paper is organized as follows. Section 2 discusses the implementation of the bias correction procedure. Section 3 describes the simulation design to examine the small-sample properties of bias-adjusted parameters, and subsequently presents the results. The final section concludes.

4.2 The Bias Correction in Roots Method

In this section, we provide implementation details of our bias correction method. The method is designed to correct the small-sample bias in autoregressive roots of AR(p) models. It simultaneously corrects the bias in all the p roots using numerical procedures. Median-bias in autoregressive roots is estimated using Monte Carlo simulations and standard optimization routines are utilized to minimize it. The objective is to estimate autoregressive roots whose Monte Carlo median converges to the roots estimated from the dataset. The bias correction procedure can be summarized in the following steps:

1. Calculate the median-bias in roots using Monte Carlo simulations:

   (a) Estimate an AR(p) model in levels on the dataset and use the estimated coefficients to calculate the corresponding autoregressive roots

   (b) Use the estimated AR(p) coefficients as the data-generating process (DGP) to generate N Monte Carlo samples. In each Monte Carlo sample, estimate an AR(p) model by least-squares and calculate the autoregressive roots. Subsequently, estimate the Monte Carlo medians of the p roots. Median-bias in autoregressive roots is defined as the difference between the absolute roots used as the DGP and the corresponding Monte Carlo medians of the p roots.[39]

2. Find roots whose absolute values minimize the median-bias using a minimization routine:

   (a) Find roots, $\lambda_1^*, \lambda_2^*, ..., \lambda_p^*$, such that the Monte Carlo median of these roots converges to the roots estimated from the dataset. This is achieved by setting it up as the following constrained minimization problem.

$$min_{\lambda_1,\lambda_2,....,\lambda_p} \sum_{j=1}^{p} |bias_{med}(\lambda_j)| \tag{4.2.1}$$

[39]The autoregressive roots in each Monte Carlo samples are sorted in ascending order to get the roots corresponding to the sorted roots in the DGP

subject to $|\lambda_j| \leq 1, \ \forall j \ \ (j = 1, 2, ..p)$

(b) For a given iteration, $k-1$, in the minimization routine:

i. Using $\lambda_1^{k-1}, \lambda_2^{k-1}, ..., \lambda_p^{k-1}$ as the initial roots, calculate the sum of median-bias in these roots using Step 1. Subsequently, the minimization search routine estimates the next iteration of roots $\lambda_1^k, \lambda_2^k, ..., \lambda_p^k$.

(c) Let the minimization routine iterate for a large number of iterations to get $\lambda_1^*, \lambda_2^*, ..., \lambda_p^*$.

The procedure can also be implemented in the presence of complex or repeated roots. We now describe how to modify the above steps to account for a) complex roots and b) repeated roots.

a) Complex Roots

If an autoregressive root in a given iteration, k, is complex, then the real and imaginary components are adjusted to yield an absolute value corresponding to the magnitude of the root for next iteration, $k+1$, while preserving the sign and the angle θ (ratio of real to imaginary component) in the Argand plane.[40] For example, if $\lambda^k = \lambda_{real}^k + \lambda_{imag}^k.i$ and the magnitude of the root for iteration $k+1$ is $|\lambda|$, then the adjusted values of real and imaginary components of the root would be:

$$\lambda_{real}^{k+1} = sign(\lambda_{real}^k).\sqrt{\frac{|\lambda|^2}{1 + \left(\frac{\lambda_{imag}^k}{\lambda_{real}^k}\right)^2}}$$

$$\lambda_{imag}^{k+1} = \lambda_{real}^k.\left(\frac{\lambda_{imag}^k}{\lambda_{real}^k}\right)$$

[40]Since we only correct the bias in the magnitude of complex root(s) and not in θ, the method only ‘approximately’ corrects the bias in roots.

Preserving the angle θ in the real-imaginary plane ensures that the corresponding bias-corrected coefficients will still be real.[41] b) Repeated Roots

For matrices with repeated roots, the eigenvalue-eigenvector decomposition does not exist. In this case Jordan Canonical form or Schur Decomposition form can be used.[42]

4.3 Evaluating Bias Correction in Roots

We now evaluate the small-sample properties of our bias correction method. The main features that are of interest are the median-bias properties and variability of the bias-adjusted parameters relative to the least-squares estimates.

Considering that applied macroeconomists are primarily interested in impulse response functions, we focus on examining the accuracy of bias-adjusted impulse responses. While impulse response functions are mostly considered in the context of multivariate models, there are several empirical applications in univariate models as well. Typical examples include the evaluation of the persistence of shocks on aggregate output (Campbell and Mankiw (1987)) or examining the effects of shocks on real exchange rates (Murray and Papell (2002); Kilian and Zha (2002)). Bias in impulse response functions (IRFs) arises from two distinct sources: the small-sample bias in estimated AR coefficients or roots and the additional bias induced by the non-linear

[41]Since complex eigenvalues occur in conjugate pairs, absolute values of both conjugates should be the same after bias correction to ensure that the bias-corrected coefficients are real. If the estimated values of absolute bias in the simulations are different, an average of the biases is used for bias correcting the conjugate pair. It is important to make sure that the coefficients after bias correction are real because otherwise impulse responses generated from a complex coefficients may exhibit oscillatory behavior, even though the true impulse responses are smooth.

[42]Matrices with repeated roots are rarely encountered in practice. Even if the true model has repeated roots, rounding-off errors in the estimation would most likely produce slightly different magnitudes of the roots.

transformations of the estimated parameters.[43] In practice, even small amounts of bias in the autoregressive roots or coefficients can translate into dramatic changes in the impulse response functions. To the extent that least-squares bias in autoregressive roots is responsible for the bias in the IRF estimator, replacing the biased estimates of autoregressive roots by bias-corrected estimates prior to constructing the impulse response functions should reduce the bias. Given the nonlinearity mapping between the roots and the IRFs, bias correction in roots will not in general produce unbiased IRF estimates (and can potentially even increase the bias in IRFs), but as long as the resulting impulse response estimators are approximately unbiased, the implied impulse responses are likely to be good approximations.

We also examine measures such as 'half-life', 'quarter-life' and 'up-life', which are used to capture the dynamics of impulse response functions in univariate models. Half-life of real exchange rate is one of the most commonly used measure in international economics. It captures the time it takes for the impulse response to fall below half the size of the impulse. It is defined as the largest time t such that $IRF(t-1) \geq 0.5$ and $IRF(t) < 0.5$, where $IRF(t)$ denotes the impulse response function at time t. Studies on the dynamics of the real exchange rate interpret half-life of real exchange rate as its rate of mean reversion. However, Steinsson (2008) points out that only looking at half-life estimates can be misleading because the rate of mean-reversion of real exchange rate is far from being constant. Thus, he examines additional scalar measures that capture the dynamics of impulse responses, such as

[43]It may seem that a simple correction for median-bias in the Monte Carlo distribution of the impulse estimator would suffice to produce accurate intervals. However, as Kilian (1998) points out treating bias as a pure location shift problem ignores the fact that the distribution of the IRF estimator is not scale-invariant.

‘up-life’, ‘quarter-life’ and the difference between quarter-life and half-life.[44] Consequently, we examine four scalar measures: i) half-life (HL), ii) quarter-life (QL), iii) up-life (UP), and iv) the difference between quarter-life and half-life (QL - HL).

In order to assess the small-sample properties of the bias-adjusted estimates relative to the least-squares estimates, we generate N Monte Carlo samples and estimate AR(p) with and without bias correction on each sample. This would give N draws from the distribution of the least-squares estimators and the corresponding bias-adjusted parameters.

4.3.1 Monte Carlo Design

The Monte Carlo design is as follows. Let the DGP be:

$$\prod_{j=1}^{p}(1 - \lambda_j L)y_t = e_t$$

where $e_t \sim iidN(0, 1)$, λ_j are the possible roots of the process. We assume p is known to abstract from small-sample problems associated with the choice of lag length. We consider a variety of representative AR(2) processes in which we vary the persistence of the process by changing the largest root, λ_1. We choose four values of λ_1 (0.99, 0.95, 0.9, and 0.8) and fix the smaller root, λ_2, at 0.5. The number of Monte Carlo replications, N, is set to 500 and we consider a sample size of 80.[45]

We estimate autoregressive models with and without bias correction on each of these samples to get draws from the distribution of the least-squares estimators and

[44] The up-life is the largest time t such that $IRF(t-1) \geq 1$ and $IRF(t) < 1$. It measures the extent of hump-shaped response in impulse responses. Quarter-life is the largest time t such that $IRF(t-1) \geq 0.25$ and $IRF(t) < 0.25$. It is meant to measure the time it takes for the impulse response to fall below a quarter. The difference between half-life and quarter-life measures the time it takes for the impulse response to fall from 0.5 to 0.25.

[45] Results for larger sample-size (e.g. 100) were similar. In smaller samples (e.g. 60), the least-squares bias was larger and the improvement in median-bias of IRFs was more significant.

the corresponding bias-adjusted estimates for i) autoregressive roots, ii) autoregressive coefficients, iii) impulse response estimates at different horizons, and iv) measures that capture dynamics of impulse responses. In the interest of brevity, we only report impulse response functions at (t = 1, 5, 10, 15, 20, 25, 30, 35, 40, 25, and 50) in the tables below. However, to get a more complete picture we also subsequently plot the impulse responses for the first 50 periods.

4.3.2 Results

We now examine the properties of the bias-adjusted estimates of autoregressive coefficients, autoregressive roots, impulse response functions, and the four measures, relative to their least-squares estimates. Table 4.1 reports results for the first specification of the DGP with true largest root, λ_1, set to 0.99. The second column of the table shows the values of the true parameters. The next four columns report the median-bias, standard deviation, and the 90% range of the least-squares estimators. The last four columns report the corresponding statistics after correcting the bias in roots.

Table 4.1 shows that when the AR(2) process is highly persistent, there is a downward median-bias in the least-squares estimator of the largest root.[46] The second largest root has an upward median-bias. The bias correction procedure removes the bias in roots without increasing the standard deviation. The 90% range for the roots after bias correction are almost the same as that of least-squares.[47] Median-bias in

[46]Median-bias is the difference between the median of the least-squares estimator for λ_1 and its true value of 0.99.

[47]The standard deviation for the largest root is actually lower after bias correction because of the upper bound of unity in the optimization routine. Consequently, the upper limit of the range is estimated to be 1.

coefficients increases slightly after bias correction while the standard deviations are esentially the same.[48] There are large median-biases in the IRFs corresponding to the least-squares estimator, especially at long horizons. The corresponding biases are much lower after bias correction in roots. Additionally, the substantial bias at longer horizon is essentially reduced to 0. The standard deviations are also lower after bias correction in roots because of the upper bound of unity imposed in the procedure. The least-squares estimates for half-life (HL), quarter-life (QL), up-life (UL), and the difference between quarter-life and half-life (QL-HL) are substantially downward biased. The corresponding median-biases in all the four measures are much lower after bias correction in roots. The 90% range is very wide for both least-squares and bias-adjusted estimates.[49] This is consistent with several papers in international economics that have estimated similar wide confidence intervals for half-life of real exchange rates (see Murray and Papell (2002), Steinsson (2008) and the references therein.) Standard deviations of these measures are not reported in Table 4.1 and other tables because half-life, quarter-life and up-life estimates in some simulations have infinite magnitudes. As a result, standard-deviation would not be meaningful. To sum up Table 4.1 shows that bias correction in roots substantially reduces the bias in IRFs without increasing the standard deviation by much. In addition, the bias correction procedure results in much lower bias in half-life, quarter-life and up-life estimates relative to the least-squares estimator.

Table 4.2 shows the corresponding results for the second specification of the DGP with λ_1 of 0.95. The least-squares estimator for the largest root is downward biased

[48]This should not be a surprising considering that median bias correction in roots will not be preserved under the non-monotone mapping between the autoregressive roots and coefficients.

[49]This is because impulse responses decay very slowly and consequently the estimated half-life, quarter-life and up-life are very large (infinity).

while the smaller roots has an upward bias. The bias correction procedure removes the bias in roots at a negligible cost in terms of increased standard deviation in roots. Median-bias and standard deviation in coefficients are almost the same in least-squares and the bias-adjusted estimates. Once again, the least-squares estimator has a large bias in IRFs, which is reduced substantially after bias correction in roots. Additionally, least-squares estimator for half-life and other measures have significant downward bias. Median-bias of the bias-adjusted estimates are much lower. The standard deviations for all the estimands are only slightly higher after bias correction relative to that of least-squares.

Table 4.3 and 4.4 report the last two specifications of the DGP with λ_1 set to 0.9 and 0.8, respectively. It shows that results for these two specifications are similar to those of the first two. Bias correction procedure removes the bias in roots without increasing the standard deviation by much. Bias in coefficients essentially remains the same. Bias correction in roots substantially improves the median-bias of IRFs as well as half-life and other measures.

To give a better picture of how bias correction in roots affects the accuracy of the impulse responses, we plot the bias-adjusted and the least-squares estimates of impulse response functions (IRFs) for the four specifications of the DGP. The figures plot (i) True (DGP) IRF, (ii) 'LS' (least-squares) median IRF, which is the median IRF from the Monte Carlo simulations without bias correction, (iii) 'BCR' median IRF, which corresponds to the median IRF after bias correction in roots in each sample, and (iv) the 90% range for BCR and LS denoted by 'ci-BCR' and 'ci-LS', respectively.[50] Figure 4.1 plots the IRFs for the first two specifications of the DGP (λ_1 =0.99 and 0.95), while Figure 4.2 shows IRFs for the remaining two specifications (λ_1 =0.9 and 0.8).

Figure 4.1 shows that the least-squares median IRFs are significantly below the true IRF, suggesting substantial downward bias in IRFs at all time horizons shown. Median IRFs after bias correction are much closer to the true IRFs. Additionally, while the least-squares biases in IRFs continues to increase at long horizons, that of BCR decrease and are essentially 0 as the bias-adjusted median IRFs coincides with the true IRFs at long horizons. Figure 4.2 shows the IRFs for the remaining two specifications of DGP. While the least-squares median-bias in IRFs decreases with the persistence of the DGP, BCR still significantly outperforms LS in terms of reduced bias in IRFs.

In conclusion, we find that our bias correction procedure achieves a substantial reduction in median-bias of IRFs as well as half-life and other measures examined in this paper. The procedure pays a negligible-to-small price in terms of increased standard deviations for its improved median-bias properties.

[50] Median-bias in IRFs of the BCR and least-squares estimates discussed above corresponds to the differences between the true IRFs and the median IRFs based on least-squares estimations with and without bias correction in roots, respectively.

4.4 Conclusion

It is well known that the least-squares estimates for key parameters are significantly biased in small samples, especially when the largest root is close to unity. Several papers have addressed this issue by devising bias correction methods that correct the small-sample bias in autoregressive coefficients. However, bias correction in coefficients may not correct the bias in roots because of the non-linear mapping between the two. This paper develops a procedure that numerically corrects the small-sample bias in autoregressive roots. We examine the median-bias properties and variability of our bias-adjusted parameters relative to the least-squares estimates. We find that the bias correction procedure achieves a substantial reduction in median-bias of IRFs, half-life, quarter-life, and up-life of the estimated impulse responses. The procedure pays a negligible-to-small price in terms of increased standard deviations for its improved median-bias properties.

Table 4.1: $\lambda_1 = 0.99$

	DGP	Least-Squares				Bias-Adjusted			
		Med-bias	Std	90% Range		Med-bias	Std	90% Range	
Roots									
λ_1	0.990	*-0.011*	0.057	0.821	1.012	*0.000*	0.055	0.827	1.000
λ_2	0.500	*0.008*	0.118	0.300	0.729	*-0.001*	0.117	0.297	0.706
Coefficients									
ϕ_1	1.490	*-0.014*	0.098	1.286	1.616	*-0.015*	0.099	1.285	1.615
ϕ_2	-0.495	*0.002*	0.098	-0.633	-0.305	*0.008*	0.098	-0.629	-0.299
IRFs									
1	1.490	*-0.014*	0.098	1.286	1.616	*-0.015*	0.099	1.285	1.615
5	1.890	*-0.124*	0.370	1.155	2.354	*-0.110*	0.369	1.191	2.390
10	1.826	*-0.211*	0.540	0.555	2.389	*-0.166*	0.533	0.593	2.439
15	1.738	*-0.247*	0.646	0.237	2.388	*-0.142*	0.623	0.274	2.428
20	1.653	*-0.308*	0.723	0.095	2.437	*-0.102*	0.677	0.097	2.367
25	1.572	*-0.362*	0.791	0.028	2.543	*-0.091*	0.713	0.040	2.339
30	1.495	*-0.399*	0.855	0.009	2.658	*-0.071*	0.740	0.012	2.304
35	1.421	*-0.436*	0.920	0.004	2.721	*-0.054*	0.760	0.004	2.292
40	1.352	*-0.457*	0.990	0.001	2.911	*-0.019*	0.776	0.002	2.290
45	1.285	*-0.473*	1.066	0.001	3.063	*-0.003*	0.789	0.001	2.290
50	1.222	*-0.484*	1.149	0.000	3.336	*0.007*	0.800	0.000	2.290
Measures									
HL	34.736	*-17.306*	-	2.651	∞	*1.113*	-	2.828	∞
UL	17.744	*-8.958*	-	1.750	∞	*0.899*	-	1.806	∞
QL	51.978	*-26.195*	-	3.701	∞	*1.946*	-	3.888	∞
QL-HL	17.242	*-8.945*	-	0.939	∞	*0.145*	-	0.950	∞

DGP: AR(2) with $\lambda_1 = 0.99$ and $\lambda_2 = 0.5$. HL (half-life), UL (up-life), QL (quarter-life), and QL-HL are scalar measures that capture the dynamics of impulse response functions (IRFs). Standard-deviations corresponding to these measures are not reported as in some simulations the impulse responses do not decay and hence some of these measures have infinite values.

Table 4.2: $\lambda_1 = 0.95$

	DGP	Least-Squares				Bias-Adjusted			
		Med-bias	Std	90% Range		Med-bias	Std	90% Range	
Roots									
λ_1	0.950	*-0.014*	0.073	0.756	0.994	*0.000*	0.079	0.744	1.000
λ_2	0.500	*0.012*	0.134	0.296	0.772	*0.001*	0.136	0.289	0.780
Coefficients									
ϕ_1	1.450	*-0.013*	0.099	1.250	1.578	*-0.012*	0.099	1.255	1.582
ϕ_2	-0.475	*0.001*	0.100	-0.623	-0.284	*0.005*	0.100	-0.619	-0.284
IRFs									
1	1.450	*-0.013*	0.099	1.250	1.578	*-0.012*	0.099	1.255	1.582
5	1.599	*-0.096*	0.338	0.941	2.035	*-0.044*	0.346	0.940	2.068
10	1.263	*-0.144*	0.456	0.257	1.794	*-0.026*	0.493	0.226	1.898
15	0.978	*-0.160*	0.487	0.057	1.561	*0.002*	0.547	0.053	1.768
20	0.757	*-0.170*	0.481	0.012	1.474	*0.006*	0.558	0.007	1.695
25	0.586	*-0.166*	0.465	0.001	1.428	*-0.011*	0.556	0.001	1.672
30	0.453	*-0.144*	0.448	0.000	1.350	*0.005*	0.549	0.000	1.643
35	0.351	*-0.125*	0.432	0.000	1.295	*0.015*	0.541	0.000	1.604
40	0.271	*-0.113*	0.418	0.000	1.234	*0.007*	0.533	0.000	1.594
45	0.210	*-0.097*	0.405	0.000	1.205	*0.000*	0.525	0.000	1.594
50	0.162	*-0.082*	0.394	0.000	1.194	*0.003*	0.518	0.000	1.594
Measures									
HL	7.021	*-1.434*	-	1.949	49.735	*-0.020*	-	1.927	∞
UL	3.893	*-0.675*	-	1.412	20.694	*0.009*	-	1.406	∞
QL	10.400	*-2.038*	-	2.613	76.463	*0.060*	-	2.554	∞
QL-HL	3.379	*-0.737*	-	0.630	27.880	*0.028*	-	0.608	∞

DGP: AR(2) with $\lambda_1 = 0.95$ and $\lambda_2 = 0.5$. HL (half-life), UL (up-life), QL (quarter-life), and QL-HL are scalar measures that capture the dynamics of impulse response functions (IRFs). Standard-deviations corresponding to these measures are not reported as in some simulations the impulse responses do not decay and hence some of these measures have infinite values.

Table 4.3: $\lambda_1 = 0.90$

	DGP	Least-Squares				Bias-Adjusted			
		Med-bias	Std	90% Range		Med-bias	Std	90% Range	
Roots									
λ_1	0.900	*-0.011*	0.088	0.698	0.969	*0.002*	0.092	0.700	0.981
λ_2	0.500	*0.014*	0.150	0.276	0.769	*0.001*	0.155	0.274	0.776
Coefficients									
ϕ_1	1.400	*-0.012*	0.101	1.203	1.535	*-0.008*	0.102	1.203	1.540
ϕ_2	-0.450	*-0.001*	0.101	-0.601	-0.268	*0.003*	0.104	-0.608	-0.265
IRFs									
1	1.400	*-0.012*	0.101	1.203	1.535	*-0.008*	0.102	1.203	1.540
5	1.290	*-0.062*	0.308	0.637	1.693	*-0.019*	0.315	0.672	1.735
10	0.783	*-0.074*	0.374	0.072	1.248	*-0.020*	0.414	0.076	1.384
15	0.463	*-0.065*	0.338	-0.019	1.047	*-0.006*	0.401	-0.021	1.218
20	0.274	*-0.063*	0.287	-0.008	0.851	*-0.008*	0.362	-0.009	1.078
25	0.162	*-0.040*	0.244	-0.002	0.716	*0.000*	0.326	-0.003	0.953
30	0.095	*-0.029*	0.209	0.000	0.617	*-0.001*	0.296	-0.001	0.865
35	0.056	*-0.019*	0.181	0.000	0.516	*-0.001*	0.270	0.000	0.796
40	0.033	*-0.012*	0.157	0.000	0.437	*0.000*	0.248	0.000	0.701
45	0.020	*-0.008*	0.138	0.000	0.382	*0.001*	0.229	0.000	0.646
50	0.012	*-0.005*	0.122	0.000	0.324	*0.000*	0.213	0.000	0.593
Measures									
HL	3.571	*-0.334*	-	1.446	8.938	*-0.019*	-	1.487	15.228
UL	2.162	*-0.127*	-	1.038	4.263	*0.007*	-	1.096	5.889
QL	5.215	*-0.577*	-	1.912	14.922	*-0.038*	-	1.991	25.204
QL-HL	1.644	*-0.161*	-	0.411	5.452	*0.037*	-	0.410	9.080

DGP: AR(2) with $\lambda_1 = 0.90$ and $\lambda_2 = 0.5$. HL (half-life), UL (up-life), QL (quarter-life), and QL-HL are scalar measures that capture the dynamics of impulse response functions (IRFs). Standard-deviations corresponding to these measures are not reported as in some simulations the impulse responses do not decay and hence some of these measures have infinite values.

Table 4.4: $\lambda_1 = 0.80$

	DGP	Least-Squares				Bias-Adjusted			
		Med-bias	Std	90% Range		Med-bias	Std	90% Range	
Roots									
λ_1	0.800	*-0.010*	0.093	0.630	0.923	*-0.002*	0.098	0.626	0.934
λ_2	0.500	*0.007*	0.165	0.233	0.748	*-0.001*	0.172	0.232	0.753
Coefficients									
ϕ_1	1.300	*-0.013*	0.103	1.101	1.443	*-0.007*	0.108	1.102	1.458
ϕ_2	-0.400	*0.008*	0.104	-0.559	-0.215	*0.010*	0.109	-0.568	-0.214
IRFs									
1	1.300	*-0.013*	0.103	1.101	1.443	*-0.007*	0.108	1.102	1.458
5	0.822	*-0.017*	0.260	0.304	1.174	*0.020*	0.275	0.308	1.223
10	0.285	*-0.030*	0.239	-0.063	0.688	*0.000*	0.265	-0.067	0.758
15	0.094	*-0.020*	0.157	-0.022	0.434	*-0.006*	0.186	-0.026	0.504
20	0.031	*-0.008*	0.105	-0.005	0.294	*-0.003*	0.132	-0.007	0.369
25	0.010	*-0.003*	0.073	-0.001	0.194	*-0.003*	0.099	-0.001	0.251
30	0.003	*-0.001*	0.052	0.000	0.127	*-0.001*	0.076	0.000	0.183
35	0.001	*0.000*	0.038	0.000	0.084	*0.000*	0.059	0.000	0.129
40	0.000	*0.000*	0.029	0.000	0.057	*0.000*	0.047	0.000	0.093
45	0.000	*0.000*	0.022	0.000	0.039	*0.000*	0.038	0.000	0.068
50	0.000	*0.000*	0.016	0.000	0.027	*0.000*	0.031	0.000	0.049
Measures									
HL	1.860	*-0.040*	-	1.011	3.313	*0.025*	-	0.991	3.772
UL	1.232	*-0.045*	-	0.677	1.839	*0.017*	-	0.681	1.986
QL	2.654	*-0.121*	-	1.327	5.495	*0.004*	-	1.326	6.258
QL-HL	0.794	*-0.040*	-	0.256	2.154	*-0.021*	-	0.255	2.551

DGP: AR(2) with $\lambda_1 = 0.80$ and $\lambda_2 = 0.5$. HL (half-life), UL (up-life), QL (quarter-life), and QL-HL are scalar measures that capture the dynamics of impulse response functions (IRFs). Standard-deviations corresponding to these measures are not reported as in some simulations the impulse responses do not decay and hence some of these measures have infinite values.

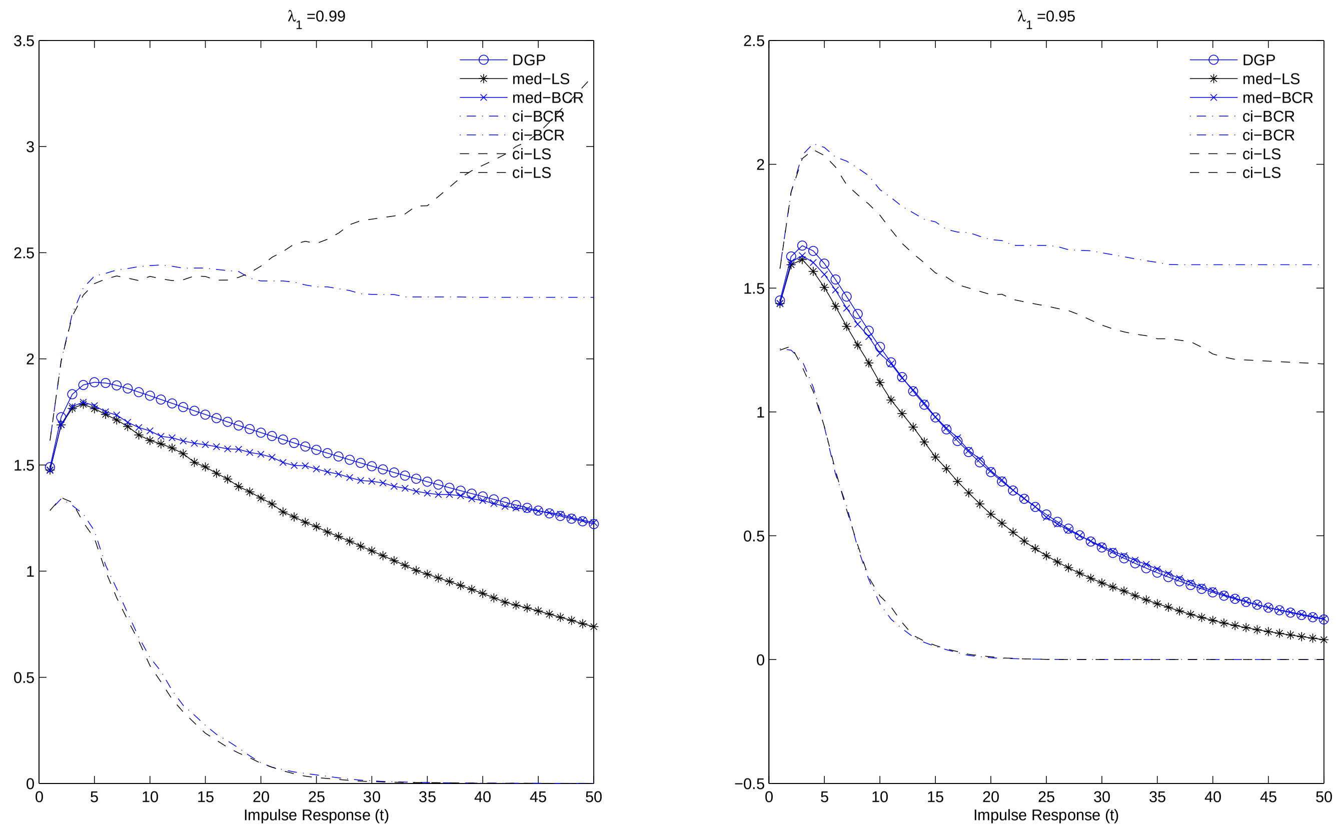

Figure 4.1: Impulse Response Functions ($\lambda_1 = 0.99,\ 0.95$)

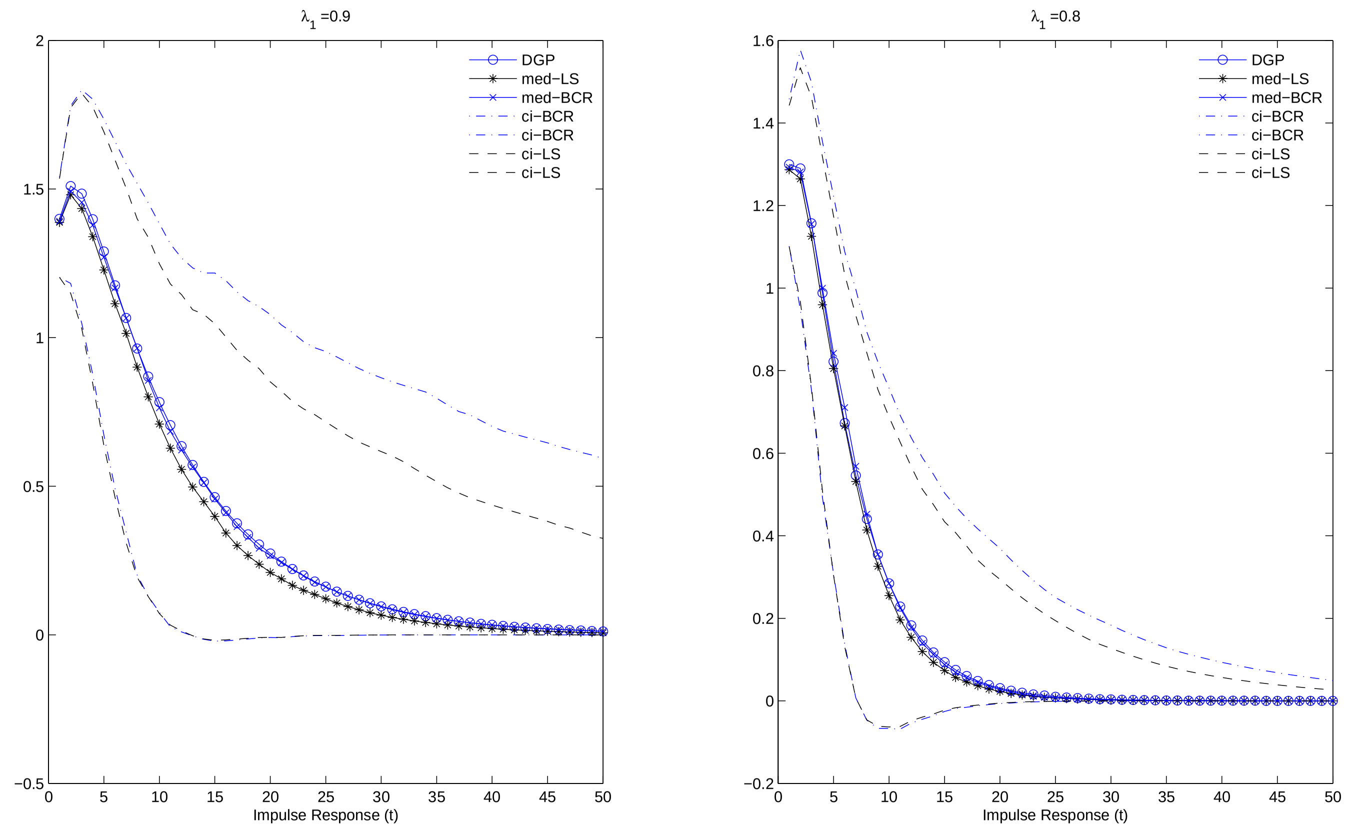

Figure 4.2: Impulse Response Functions ($\lambda_1 = 0.9,\, 0.8$)

CHAPTER 5

CONCLUSION

It is well documented that the standard RBC model fails to generate positive comovement in output, consumption, investment, and labor-hours in response to news about future technology. The first essay proposes a solution to this puzzling feature of the RBC model based on learning-by-doing. I examine two specifications of LBD that are popular in the literature and show that both these specifications can generate aggregate comovement in response to news shocks about technology. Furthermore, I show that LBD plays a crucial role in generating sectoral comovement in response to news shocks. While several other recent studies have added features to the RBC model to account for aggregate comovement in response to news shocks, I believe that the primary virtue of our approach is that it provides a simple and intuitive solution based on a mechanism that has strong empirical support. In addition, I show that our model can generate sectoral comovement in response to news about three types of shocks: neutral technology shock, consumption technology shock, and investment technology shock.

The second essay examines the frequency of explosive roots in level VAR models. Monte Carlo simulations based on datasets from the macroeconomic literature reveal that the frequency of explosive roots exceeds 40% in the presence of unit roots and is substantial even if all the variables are stationary. Furthermore, explosion increases substantially, to as much as 100%, after correcting for the small-sample bias in estimated level VAR coefficients. These results suggest that researchers estimating level VAR models on macroeconomic datasets encounter explosive roots with a very high frequency. Considering the consensus among macroeconomists that roots are at most unity, if applied macroeconomists discard explosive VAR specifications when VAR models are estimated on the datasets, it may lead to biases in the estimation or can even result in data mining. Data mining can be a serious problem since it invalidates statistical theory. The high frequency of encountering explosive roots in level VAR models suggests that this data mining problem can be severe. Additionally, the sharp increase in explosion after bias correction indicates that researchers, who correct for the small-sample bias in level VAR coefficients, may almost always estimate explosive roots on macroeconomic datasets. As per the well known evidence of nonstationarity in most macroeconomic series, one way to reduce the frequency of explosive roots is to impose unit roots in the estimation by estimating VECMs instead of level VAR models. Simulation results suggest that VECMs can substantially reduce the frequency of explosive roots.

The third essay develops a procedure that numerically corrects the small-sample bias in autoregressive roots. I examine the median-bias properties and variability of the bias-adjusted parameters relative to the least-squares estimates and show that the bias correction in roots (BCR) procedure achieves a substantial reduction in median-bias of IRFs, half-life, quarter-life, and up-life of the estimated impulse responses. The BCR procedure pays a negligible-to-small price in terms of increased standard deviations for its improved median-bias properties.

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