FARRELL-TATE COHOMOLOGY OF THE MAPPING CLASS GROUP

DISSERTATION

Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of The Ohio State University

By

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1990

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To my wife

Li

ACKNOWLEDGMENTS

I find myself short of words in expressing my sincere thanks to my advisors, Professor Henry Glover and Professor Guido Mislin, for their continual encouragement, excellent guidance and inspiration. This work could not have been completed without their direction throughout the past few years. Thanks go to other OSU faculty, from whom I benefitted much, especially to Professor Walter Neumann, Professor Ruth Charney and Professor Michael Davis for their valuable hours on my report of this work. I wish to thank my friend Xiaoya Zha for his help on the computer programs.I also wish to thank the department of mathematics and low dimensional topology semester at OSU for their financial supports during these four years. To my wife, Li, I offer sincere thanks for your love and understanding.

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INTRODUCTION

This thesis is devoted to a study of the Farrell-Tate cohomology of the mapping class group Γ_g, where Γ_g is defined to be the group of path components of the group of orientation preserving diffeomorphisms of the closed orientable surface, S_g, of genus $g > 1$.

The mapping class group Γ_g acts properly discontinuously on Teichmuller space T_g with quotient moduli space M_g, so the rational cohomology of the moduli space, M_g, may be identified with that of the mapping class group, Γ_g. Since the 1980's, the computation of the cohomology (or homology) of Γ_g has been studied by many people, for example, Miller [Mi], Morita [Mo], Harer $[H]_2$, Charney and Cohen [C,C], Charney and Lee [C,L], Glover and Mislin [G,M], Cohen [C], $[C]_2$ and Benson [Be].

The mapping class group Γ_g is known to be of virtual finite cohomological dimension and Harer shows that the virtual cohomological dimension(vcd) of Γ_g is 4g-5 [H].

Farrell extended Tate's cohomology theory for finite groups to groups of virtual finite cohomological dimension in 1977 [F]. Recall that the Farrell-Tate cohomology group $\hat{H}^i(\Gamma;M)$ with coefficients in the Γ-module M are always torsion groups for $i \in Z$. The groups $\hat{H}^i(\Gamma;M)$ are the same as ordinary cohomology $H^i(\Gamma;M)$ if the dimension i is greater than the virtual cohomological dimension of Γ. In fact, the Farrell-Tate cohomology groups $\hat{H}^i(\Gamma;M)$ reflect many properties related to finite subgroups of Γ and depend upon the normalizers of finite subgroups, not only upon the finite subgroups themselves.

In this thesis we give some computations of Farrell -Tate cohomology of mapping class groups Γ_g. However, the methods we use are generally suitable for the computations of Farrell-Tate cohomology of any group of finite vcd. The basic idea for most of our results is to compare the Farrell-Tate cohomology (or ordinary cohomology) groups of the mapping class group Γ_g with that of its interesting subgroups and quotient groups.

Although Γ_g is never 2-periodic for Farrell-Tate cohomology, the first observation we make is that, for fixed odd prime p, most mapping class groups Γ_g are p-periodic. We completely determine the necessary and sufficient conditions for Γ_g to be p-periodic. Of course this is equivalent to the Krull dimension of $H^*(\Gamma_g, Z_p)$ being zero or one [Bro].

The Brown decomposition theorem [Br] says that in the p-periodic case the p-primary component of the Farrell-Tate cohomology groups of Γ are given by the product of the p-primary components of the normalizers of the conjugacy classes of Z_p subgroups of Γ. A description of these conjugacy classes of Z_p subgroups and their normalizers in $\Gamma_{(p-1)/2}$ leads to a complete description of the p-torsion of the Farrell-Tate cohomology groups $\hat{H}^*(\Gamma_{(p-1)/2};Z)_{(p)}$. However, in general, we do not yet have enough information about these normalizers to compute $\hat{H}^*(\Gamma_g;Z)_{(p)}$. We ask the question: what is the p-period of Γ_g if Γ_g is p-periodic?

Burgisser and Eckmann studied the p-periodicity of arithmetic subgroups Γ of general linear groups in the 1980's [B,E]. The basic method they used is as follows: Suppose Γ is p-periodic. On the one hand, they look for some interesting finite subgroups to give a lower bound of the p-period of Γ. On the other hand, if the canonical finite quotient of Γ is p-periodic, an upper bound of the p-period of Γ is given by the p-period of the finite quotient.

In other words, the p-period of any group Γ which is of finite vcd has a strong relation with its finite subroups and finite quotients. By contrast, for finite p-periodic group G, Swan's classical result [Sw] states that for p odd the p-period of G is twice the order of $|N(\langle x\rangle)/C(\langle x\rangle)|$ where x generates the maximal p-cyclic subgroup of G, and $N(\langle x\rangle)$ (resp. $C(\langle x\rangle)$) denote the normalizer (resp. centralizer) of this cyclic subgroup.

Is it possible that the p-period of group Γ which is of finite vcd, an invariant of the homology of Γ, can be completely described in terms of "elementary" non-homological properties of the p-subgroups of Γ? As an extension of the result of Swan, we answer this question affirmatively if Γ has a finite quotient whose p-Sylow subgroup is elementary abelian or cyclic, and the kernel being torsion free. In these two cases, the p-period of Γ is exactly twice the least common multiple of $\{|N(\langle x\rangle)/C(\langle x\rangle)|\}$ where $\langle x\rangle$ ranges over the conjugacy classes of Z_p subgroups of Γ. However, we have no general answer yet.

For $g < p(p-1)/2$, choosing suitable prime q, the finite group $Sp(2g,F_q)$ has elementary abelian p-sylow subgroups. On the other hand, $Sp(2g,F_q)$ is a finite quotient of the mapping class group Γ_g with torsion-free kernel.

As an application of the previous result, we give a formula computing the p-period of Γ_g in terms of only p and g if $g < p(p-1)/2$. The key point of the proof of this formula is the observation that $|N(\langle x\rangle)/C(\langle x\rangle)|$ for Γ_g can be completely determined by the classic fixed point data introduced by Nielsen [N]. In particular, these results allows us to give a near complete description of the p-periodicity for g = 2, 3, 4 and 5. We still miss the 3-period of Γ_3 and only know that it is either 4 or 12.

A different method to get an upper bound for the p-period of a mapping class groups Γ_g is to study a homogeneous Chern class polynomial(not only Chern classes) of the canonical homology representation of the mapping class group Γ_g. More generally, this bound, even in the non-periodic case, gives an upper bound for the Yagita invariant [Y] of Γ_g. Unfortunately, this upper bound generally does not match any lower bound we can get. However, interesting inequalities for the Yagita invariants of some mapping class groups Γ_g are still obtained.

An additional byproduct of the study of subgroups and quotient groups of the mapping class group is to provide non-trivial elements in a family of the reduced projective class groups $\tilde{K}_0(Z\Gamma_{(p-1)/2})$ for p = 6k+1. These results are analogous to ones of Carter [Ca] who did this for general linear groups and special linear groups.

Now, we summarize our main results as follows:

Theorem 2.1. a) The mapping class group Γ_g is never 2-periodic.

b) The mapping class group Γ_{kp+i} is always p-periodic if $i \neq 1 \bmod(p)$ for p odd prime, $k \geq 0$.

c) The mapping class group Γ_{kp+1} is p-periodic if and only if $[(2k+3)/p, (2k+2)/(p-1)]$ does not contain an integer and $k \neq 0,-1 \bmod(p)$ for p an odd prime. In particular, Γ_{kp+1} can be p-periodic only when $k \leq (p^2-5)/2$.

Theorem 2.2. For $k \neq 0 \bmod(p)$, $p > 2$, $\Gamma_{(p-1)(kp-k-2)/2}$ is p-periodic and the p-period of $\Gamma_{(p-1)(kp-k-2)/2}$ is a multiple of $2(p-1)$. Moreover, if $k < (p-1)/2$, the p-period of $\Gamma_{(p-1)(kp-k-2)/2}$ equals $2(p-1)$.

Corollary 2.3. $\Gamma_{(p-1)(p-3)/2}$ is p-periodic and the p-period $= 2(p-1)$ for $p > 3$.

Theorem 2.4. $\Gamma_{(p-1)(d-2)/2}$ is p-periodic and the p-period of $\Gamma_{(p-1)(d-2)/2}$ is a multiple of 2d if 3 divides d and d divides p-1.

Proposition 2.5. $\Gamma_{(p-1)/2}$ is p-periodic and the p-period of $\Gamma_{(p-1)/2}$ divides p-1.

Theorem 3.1. (Farrell-Tate cohomology version)

a) $\hat{H}^i(\Gamma_{(p-1)/2};Z)_{(p)} = \prod_{1\leq n\leq k} \hat{H}^i(Z_p;Z)_{(p)}$, if $p = 6k-1$.

b) $\hat{H}^i(\Gamma_{(p-1)/2};Z)_{(p)} = \hat{H}^i(Z_p \rtimes Z_3;Z)_{(p)} \times \prod_{1\leq n\leq k} \hat{H}^i(Z_p;Z)$ if $p=6k+1$.

Corollary 3.2. (ordinary cohomology version)

a) $H^{2i}(\Gamma_{(p-1)/2};Z)_{(p)} = \prod_{1\le n\le k} H^{2i}(Z_p;Z) = \prod_{1\le n\le k} Z_p$ if $p = 6k-1$ and $2i > 2p-7$.

b) $H^{6i}(\Gamma_{(p-1)/2};Z)_{(p)} = H^{6i}(Z_p \rtimes Z_3;Z)_{(p)} \times \prod_{1\le n\le k} H^{6i}(Z_p;Z) = \prod_{1\le n\le k+1} Z_p$, if $p = 6k+1$ and $6i > 2p-7$.

c) $H^{2i}(\Gamma_{(p-1)/2};Z)_{(p)} = H^{2i}(Z_p \rtimes Z_3;Z)_{(p)} \times \prod_{1\le n\le k} H^{2i}(Z_p;Z) = \prod_{1\le n\le k} Z_p$, if $p = 6k+1$, $i \ne 0 \bmod (3)$ and $2i > 2p-7$.

d) $H^{2i}(\Gamma_{(p-1)/2};Z)_{(p)} = 0$ if $2i-1 > 2p-7$.

Theorem 4.1. Let $2g-2 = mp-i$, $0 \le i \le p-1$, p odd prime and $p^{r-1} \le m \le p^r$, $i: Z_p \to \Gamma_g$ an inclusion, $\eta: \Gamma_g \to GL(2g,C)$ the canonical homology representation and φ the Euler totient function.

a) If $[2g/(p-1)] < p^r$, then $i^*[c_{\varphi(p^r)}(\eta)]$ has order p for every Z_p in Γ_g (c_i denotes the i-th Chern class). So the Yagita invariant $p(\Gamma_g)$ divides $2p^{r-1}(p-1)$.

b) If $[2g/(p-1)] \ge p^r$, then $i^*\{[c_{\varphi(p^r)}(\eta)]^{p(p-1)} + [c_{\varphi(p^{r+1})}(\eta)]^{p-1}\}$ has order p for every Z_p in Γ_g. So the Yagita invariant $p(\Gamma_g)$ divides $2p^r(p-1)^2$.

Corollary 4.2. In addition, if Γ_g is p-periodic,

a) If $[2g/(p-1)] < p^r$, the p-period of Γ_g divides $2p^{r-1}(p-1)$.

b) If $[2g/(p-1)] \ge p^r$, the p-period of Γ_g divides $2p^r(p-1)^2$.

Theorem 4.3. Let $2^{s-1} \leq g \leq 2^s$. Then $i^*\{[c_{2^{s-1}}(\eta)]^2 + c_{2^s}(\eta)\}$ has order 2 for every Z_2 in Γ_g.

Theorem 5.1. Let Γ be a group which has a normal subgroup of finite cohomological dimension so that the associated quotient is a finite group with the elementary abelian p-Sylow subgroup. If Γ is p-periodic, then the p-period of Γ is twice the least common multiple of $\{|N(\langle x\rangle)/C(\langle x\rangle)|\}$ where $\langle x\rangle$ ranges over the conjugacy classes of Z_p subgroups of Γ.

Theorem 5.2. Let Γ be a group which has a normal subgroup of finite cohomological dimension so that the associated quotient is a finite group with the cyclic p-Sylow quotient. If Γ is p-periodic, then the p-period of Γ is twice the least common multiple of $\{|N(\langle x\rangle)/C(\langle x\rangle)|\}$ where $\langle x\rangle$ ranges over the conjugacy classes of Z_p subgroups in Γ.

Theorem 5.3. Let $2g-2=kp-i$, $0 \leq i \leq p-1$, p odd prime. Define sets

$B_{g,p} = \{i, p+i, 2p+i, \cdots\cdots ([2g/(p-1)]-k)p+i\}$, if $i \neq 1$.

$B_{g,p} = \{1+p, 2+p, \cdots\cdots ([2g/(p-1)]-k)p+1\}$, if $i = 1$.

If the mapping class group Γ_g is p-periodic and $g < p(p-1)/2$, then the p-period of $\Gamma_g = 2\mathrm{LCM}(\gcd(p-1, b_i))$, where b_i ranges over the set $B_{g,p}$.

<u>Theorem in appendix A</u>. If $p = 6k+1$ is prime, then the mapping class group $\Gamma_{(p-1)/2}$ contain metacyclic group $G = Z_p \rtimes Z_3$ (semi-direct) such that the reduced projective class group $\tilde{K}_0(Z\Gamma_{(p-1)/2}) \supset \mathrm{Ind}_G^{\Gamma}\tilde{K}_0(ZG)$ contains a cyclic group of order 3.

The remainder of this thesis is organized as follows: In chapter I, we state(without proof) the basic properties of the mapping class group Γ_g, $g > 1$, and also those of ordinary and Farrell-Tate cohomology. In chapter II, we prove theorems 2.1 through 2.5. In chapter III, we prove theorem 3.1 and corollary 3.2. In the end of this chapter, we point out a gap in the proof of lemma 5.1.[B,H]$_2$ and give our correction, proposition 3.5.1. In chapter IV, we study the homogeneous Chern class polynomials of the mapping class group needed in the proofs of theorems 4.1. through 4.3. In charpter V, we prove theorems 5.1. and 5.2. which are results about any groups of finite vcd and then apply them to the computation of the p-period of the mapping class group given in theorem 5.3. Finally, since two of our results about the mapping class group are not included in our title " Farrell-Tate cohomology of the mapping class group", we make two appendixes. The first of these provides nontrivial elements in the reduced projective class group of the mapping class group $\Gamma_{(p-1)/2}$, for p = 6k+1 prime. The second one displays the distribution of "strange" p-torsion in the cohomology of the mapping class group Γ_g in the sense of Connolly [Con].

CHAPTER I

COHOMOLOGY AND THE MAPPING CLASS GROUP

This chapter consists of two sections. In the first we review both ordinary cohomology and Farrell-Tate cohomology of groups. The standard reference for this materal is the book "Cohomology of groups" by K. Brown [Br]. In the second section, we give the definitions and basic facts about the mapping class group that we need. The references of this section will be indicated in context when necessary. We omit all nontrivial proofs and only list what we need in this thesis.

1.1. The ordinary and the Farrell-Tate cohomology of groups

Recall for any group G, there exists an aspherical space with fundamental group G. A path-connected space X is called aspherical if $\pi_i(X)$ are trivial for $i \geq 2$. Hurewicz [Hu] proved in 1936 that the homotopy type of an aspherical space X is completely determined by its fundamental group

$\pi_1(X)$. Therefore, we can consider the cohomology of the group G as the cohomology of the aspherical space with fundamental group G from the topological point of view.

Actually, the aspherical space X, whose fundamental group is $\pi_1(X) = G$, can be realized by an Eilenberg-Maclane CW-complex $K(G,1)$.

Let Y denote the covering space of the CW- complex $K(G,1)$. Then Y has a CW structure induced by $K(G,1)$. Define $C_*(Y)$ to be the cellular chain complex of Y. G acts on this covering space Y as the group of deck transformations freely and transitively, hence $C_*(Y)$ is a free ZG-module and the sequence $\rightarrow C_n(Y) \rightarrow C_{n-1}(Y) \rightarrow C_{n-2}(Y) \rightarrow \cdots\cdots C_1(Y) \rightarrow C_0(Y) \rightarrow Z \rightarrow 0$ is exact. From the definition of cohomology, $H^i(G;Z) = H^i(K(G,1);Z) = H^i(Hom(C_*(K(G,1);Z)) = H^i(Hom_{ZG}(C_*(Y);Z))$.

Now, the cohomology of a group G can be naturally defined algebraically in terms of a projective resolution.

<u>Definition 1.1.1</u>. Let ZG be the free Z-module generated by the elements of G. Any element of ZG can be write in the form $\Sigma n_g g$, with only finite many $n_g \neq 0$, and the multiplication in G extends uniquely to a Z-bilinear product $ZG \times ZG \rightarrow ZG$. This ring is called the integral group ring of group G.

<u>Definition 1.1.2</u>. Let R be an associative ring with

identity and M a R-module. An exact sequence of R-modules, $\to F_n \to \cdots\cdots \to F_2 \to F_1 \to F_0 \to M \to 0$ is called a free resolution of M if F_i are free and is called a projective resolution if F_i are projective for all i.

Definition 1.1.3. For any ZG-module M, pick a projective resolution $F=(F_i)_{i\geq 0}$, $\to F_n \to \cdots\cdots \to F_2 \to F_1 \to F_0 \to Z \to 0$.

Regard Z as a trivial module over the integral group ring ZG. Define $H^i(G;M) = H^i(\mathrm{Hom}_{ZG}(F,M))$.

One must show that the definition of the cohomology of the group does not depend on the choices of projective resolutions. In fact,

Proposition 1.1.4. Given two projective resolutions F and F' of a module M, there is an augmentation-preserving chain map f: $F \to F'$, unique up to (chain)homotopy and f is a (chain)homotopy equivalence.

Example 1.

Suppose G is the infinite cyclic group with generator t. There is a free resolution of Z over ZG-module

$$0 \to ZG \xrightarrow{t-1} ZG \to Z.$$

Hence $H^*(G;M)$ is the cohomology of $t-1: M \to M \to 0$. Thus, $H^0(G;M) = M^G$, $H^1(G;M) = M_G$, and $H^i(G;M) = 0$. Here M^G, called invariants, is equal to the submodule $\{m \in M: gm = m \text{ for all } g \in G\}$. M_G, called co-invariants, is equal to the quotient module divided by the additive subgroup generated by the elements of the form $gm-m(g \in G,\ m \in M)$.

In particular, $H^0(Z;Z) = Z$, $H^1(Z;Z) = Z$, $H^i(Z;Z) = 0$ if $i > 1$.

<u>Example 2</u>: suppose G is a cyclic group of finite order n with generator t. Then there is a free resolution of Z over ZG

$$\cdots\cdots \to ZG \xrightarrow{N} ZG \xrightarrow{t-1} ZG \xrightarrow{N} ZG \xrightarrow{t-1} Z \to 0.$$

Here $N = \Sigma_{0 \le i \le n-1} t^i$. Hence $H^*(G;M)$ is the cohomology of

$$M \xrightarrow{t-1} M \xrightarrow{N} M \xrightarrow{t-1} M \xrightarrow{N} \cdots\cdots$$

Note $Ngm = Nm$, and that $NM \subseteq M^G$; i.e. N induces a map $N^*: M_G \to M^G$, called the norm map. So $H^{2i}(G;M) = \mathrm{Coker}N^*$, $H^{2i-1}(G;M) = \mathrm{Ker}N^*$, $i > 0$. $H^0(Z_p;Z) = Z$, $H^{2i}(Z_p;Z) = Z_p$ and $H^{2i-1}(Z_p;Z) = 0$ for $i > 1$.

<u>Basic properties</u>

Let $\mathbf{A}$ be the category where an object is a pair (G,M), where G is a group and M is a ZG-module; a morphism in $\mathbf{A}$ from (G,M) to (G',M') is a pair (h,f), where $h: G \to G'$ is a homomorphism of groups and $f: M' \to M$ is a ZG-module homomorphism, i.e. f is a map of abelian groups such that $f(h(g)m') = gf(m')$ for $g \in G$, $m' \in M'$. If F and F' are resolutions for G and G' and $c: F \to F'$ is a chain map compatible with h, there is a chain map Hom(c,f): $\mathrm{Hom}_{G'}(F',M) \to \mathrm{Hom}_G(F,M)$, which induces $(h,f)^*: H^*(G',M') \to H^*(G,M)$. In this way H^* becomes a contravariant functor on A. Suppose M = M', f = Id, then the induced map $h^* = (h,Id)^*:H^*(G',M) \to H^*(G,M)$. In particular, if $h:G\to G'$ is an inclusion, $h^*:H^*(G';M) \to H^*(G;M)$ is called the restriction map.

For an inclusion $h: H \to G$ and a ZG-module M, if the index [G:H] is finite, there are maps going in the other direction, called transfer maps. In fact, let F be a projective resolution of Z over ZG. Notice that $\mathrm{Hom}_G(F,M) = \mathrm{Hom}(F,M)^G$, and $\mathrm{Hom}_H(F,M) = \mathrm{Hom}(F,M)^H$, where G acts diagonally on Hom. Define a cochain map tr: $\mathrm{Hom}(F,M)^H \to \mathrm{Hom}(F,M)^G$, $tr(f) = \Sigma_{g\in G/H} gf$, which induces tr: $H^*(H;M) \to H^*(G;M)$ on cohomology.

Given a subgroup H of the group G with finite index, the important relation between the restriction and transfer maps is as follows: $Tr\cdot Res(z)=[G:H]z$ for $z\in H^*(G;M)$.

Suppose G an arbitray group, H a subgroup of G, M a G-module. Define an element $z\in H^*(H;M)$ to be <u>stable</u> if Res(z) = Res(gz) $\in H^*(H\cap gHg^{-1};M)$ for every $g\in G$. Here $g:H^*(H;M) \to H^*(gHg^{-1};M)$ is induced by the conjugation map $g(h) = ghg^{-1}$.

<u>Theorem 1.1.5</u>. Let G be a finite group and H a p-sylow subgroup. For any G-module M and any n > 0, Res maps

$H^n(G;M)_{(p)}$ isomorphically onto the set of stable elements of $H^n(G;M)$. In particular, if H is normal in G, then $H^n(G;M)_{(p)} = H^n(H;M)_{(p)}{}^{G/H}$. Where $H^*(-)_{(p)}$ means the p-primary component of $H^*(-)$.

Cup product for the cohomology of groups

Definition 1.1.6. Let G, G' be groups, M a G-module, M' a G'-module, then $M \times M'$ is $G \times G'$-module. And let F, F' be projective resolutions of Z over ZG and ZG'. For cochains $u \in Hom_G(F,M)$ and $u' \in Hom_{G'}(F',M')$, define $u \times u' \in Hom_{G \times G'}(F \otimes F', M \otimes M')$ to be tensor product $u \otimes u'$ which satisfies $\langle u \otimes u', m \otimes m' \rangle = (-1)^{deg(u')deg(m)} \langle u,m \rangle \otimes \langle u',m' \rangle$ for $m \in M$, $m' \in M'$. It is routin to verify that $\delta(u \times u') = \delta u \times u' + (-1)^{deg(u)} u \times \delta u'$ where δ is the usual coboundary map. The induced map $H^p(G;M) \otimes H^q(G';M') \to H^{p+q}(G \times G'; M \otimes M')$ is called the cohomology cross-product.

Definition 1.1.7. For $u \in H^p(G;M)$ and $v \in H^q(G;N)$, define the cup product of u and v, denoted uv, to be the element $d^*(u \times v) \in H^{p+q}(G; M \otimes N)$, where $d: G \to G \times G$ is the diagonal map.

Properties of cup product

a) Compatibility with coboundary operators δ. Let $0 \to M' \to M \to M'' \to 0$ be a short exact sequence of G-modules and let N be a G-module such that the sequence $0 \to M' \otimes N \to M \otimes N \to M'' \otimes N \to 0$ is exact. Then the square

$$\begin{array}{ccc} H^p(G;M'') & \xrightarrow{\delta} & H^{p+1}(G;M') \\ v \downarrow & & \downarrow v \\ H^{p+q}(G;M''\otimes N) & \xrightarrow{\delta} & H^{p+q+1}(G;M'\otimes N) \end{array}$$ commutes.

b) Existence of identity. The element $1 \in H^0(G;Z) = Z$ satisfies $1u = u1 = u$ for all $u \in H^*(G;M)$.

c) Associativity. $(u_1u_2)u_3 = u_1(u_2u_3)$ holds in $H^*(G;M_1 \otimes M_2 \otimes M_3)$ for all $u_i \in H^*(G;M_i)$, $i = 1,2,3$.

d) Commutativity. $uv = (-1)^{\deg(u)\deg(v)}vu$ holds for any $u \in H^*(G;M)$, $v \in H^*(G;N)$.

e) Naturality with respect to group homomorphisms. Given a homomorphism $f: H \to G$, we have $f^*(uv) = f^*(u)f^*(v)$ for any $u \in H^*(G;M)$, $v \in H^*(G;N)$. In particular, $f^*: H^*(G;M) \to H^*(G;M)$ is a ring homomorphism.

f) Transfer formular. If $H \subsetneq G$ is a subgroup of finite index. For any $u \in H^*(G;M)$ and $v \in H^*(H;N)$, $Tr(Res(u)v) = uTr(v)$ holds.

<u>The Farrell-Tate cohomology of group</u>

<u>Definition 1.1.8.</u> Let G be a finite group. Picking a projective resolution $P = \{P_i\}_{i \geq 0}$ of Z over ZG. we can extend P to an acyclic complex of projectives modules $\hat{P}$ as follows: $\cdots\cdots \to P_1 \to P_0 \to P_{-1} \to P_{-2} \to \cdots\cdots$ Where $P_{-1} = ZG$ and $i: Z \to P_{-1}$, $i(1) = \Sigma_{g \in G} g$. Let $C = \mathrm{coker}(i)$. Take P_{-2}

$= ZG\otimes C$, $j: C \to P_{-2}$, $j(c) = \Sigma_{g\in G} g\otimes g^{-1}c$, etc.

One can show that any two such acyclic complexes of projective modules $\hat{P}_1$, $\hat{P}_2$ which are extension of P are homotopy equivalent, so we can define Tate cohomology of a finite group G as $\hat{H}^*(G;M)=H^*(Hom_{ZG}(\hat{P};M))$ for any ZG-module M.

It is not so difficult to see that:

(a) $\hat{H}^i(G;M)=H^i(G;M)$ for $i > 0$.

(b) $\hat{H}^0(G;M)$=Coker of norm map $N: H_0 \to H^0$. $\hat{H}^{-1}(G;M)$ = Ker of the norm map $N:H_0 \to H^0$.

(c) $\hat{H}^i(G;M) = H_{-i-1}(G;M)$ if $i < -1$.

(d) As in the ordinary cohomology theory, there are restriction and transfer maps and cup products.

Definition 1.1.9. For any group Γ, recall that the cohomological dimension of Γ is defined as the minimal length of a projective resolution of Z over $Z\Gamma$(possible ∞).

Definition 1.1.10. The group Γ is virtually torsion-free if there is a torsion-free subgroup H of finite index. the virtual cohomological dimension of Γ, denoted vcd(Γ), is defined as the cohomological dimension of H. (It does not depend on the choice of H by Serre's theorem)

Definition 1.1.11. let Γ be a group such that vcd(Γ) = n finite, $P = \{P_i\}_{i\geq 0}$ a projective resolution of Z over $Z\Gamma$.

We can construct an acyclic complex $\hat{P}$ of projectives which agrees with P in dimension $i > n$.

Let $K = \mathrm{Im}\{P_n \to P_{n-1}\}$, Γ' be a torsion free subgroup of finite index. Then K is $Z\Gamma'$-projective. For example, take $\hat{P}_{n-1} = Z\Gamma \otimes_{Z\Gamma'} K$ and $i(x) = \Sigma\gamma\otimes\gamma^{-1}x$, where γ ranges over a set of represenatives for the cosets Γ/Γ'. Applying the same process to coker(i) and inductively, we obtain a "completion $\hat{P} = \{\hat{P}_i\}$" of P: $\hat{P}_i = P_i$ if $i > n$.

Again one shows that any two such acyclic complexes of projectives are canonically homotopy equivalent. Define the Farrell-Tate cohomology groups by $\hat{H}^*(\Gamma;M) = H^*(\mathrm{Hom}_{Z\Gamma}(\hat{P},M))$.

There is a chain map $\hat{P} \to P$, whence a map $g^*: H^*(\Gamma;M) \to \hat{H}^*(\Gamma;M)$ which is the identity map in dimensions higher than $\mathrm{vcd}(\Gamma)$. So

(a) $\hat{H}^i(\Gamma;M) = H^i(\Gamma;M)$ for $i > n = \mathrm{vcd}(\Gamma)$.

(b) $\hat{H}^n(\Gamma;M)$ = Coker of the tr: $H^n(\Gamma';M) \to H^n(\Gamma;M)$. Where Γ' is a torsion-free subgroup of finite index.

(c) $\hat{H}^*(\Gamma;M) = 0$ if Γ is torsion-free.

(d) There are the restriction and transfer maps and cup products.

In addition, there is a Hochschild-Serre spectral sequence associated to a short exact sequence $1 \to \Gamma' \to \Gamma \to \Gamma'' \to 1$ of group of finite vcd.

(e) If Γ'' is torsion-free this takes the form $E_2^{pq} = H^p(\Gamma''; \hat{H}^q(\Gamma')) \Rightarrow \hat{H}^{p+q}(\Gamma)$, and if Γ' is torsion-free then it takes the form $E_2^{pq} = \hat{H}^p(\Gamma'; H^q(\Gamma'')) \Rightarrow \hat{H}^{p+q}(\Gamma)$.

(f) $\hat{H}^*(\Gamma;M)$ are torsion groups.

As the case of ordinary cohomology, The Farrell-Tate cohomology $\hat{H}^*(\Gamma;M)$ has ring structure with identity element.

Definition 1.1.12. We say that a group Γ which is of finite vcd has periodic cohomology if for some integer $d > 0$ there is an element of $\hat{H}^d(\Gamma;M)$ which is invertible in the ring $\hat{H}^*(\Gamma;M)$. Similarly, for a fixed prime p, if there is an invertible element of positive degree d in the ring $\hat{H}^*(\Gamma;M)_{(p)}$, the p-primary component of $\hat{H}^*(\Gamma;M)$, we say that group Γ is p-periodic. The p-period of Γ is defined as the minimum value of d.

Theorem 1.1.13.(Brown) Let Γ be a group of finite vcd, p a fixed prime. The following conditions are equivalent:

(1) Γ has p-periodic cohomology.

(2) There exist integers i and $d > 0$, such that $\hat{H}^i(\Gamma;M)_{(p)} = \hat{H}^{i+d}(\Gamma;M)_{(p)}$ for all Γ-modues M.

(3) Γ does not contain any subgroup isomorphic to $Z_p \times Z_p$.

(4) Every finite p-subgroup of Γ is a cyclic or generalized quaternion group.

Theorem 1.1.14.(Brown) If Γ contains no subgroup isomorphic to $Z_p \times Z_p$, then $\hat{H}^i(\Gamma;M)_{(p)} = \Pi \hat{H}^i(N(P);M)_{(p)}$, where P ranges over the conjugacy classes of Z_p subgroups.

Theorem 1.1.15. (Brown-Venkov) If there exists an element $u \in \hat{H}^d(\Gamma;Z)_{(p)}$ $(d > 0)$ whose restriction to $\hat{H}^d(G;M)_{(p)}$ is invertible for every finite subgroup G, then u is invertible and hence Γ is p-periodic and the p-period of Γ divides d.

1.2. The definition and some basic facts about the mapping class group.

Definition 1.2.1. The mapping class group, Γ_g, is defined to be the group of path components of orientation preserving diffeomorphisms of the orientable closed surface S_g of genus g. We always assume $g > 1$ in this thesis.

Similarly, the mapping class group can be defined for a nonclosed orientable surface. Unless specially indicated, we restrict our study to the mapping class group of closed orientable surface.

Let S_g denote the closed orientable surface with genus g.

Let R_g denote a Riemann Surface of genus g, i.e.

S_g equipped with an complex structure.

Let C_g denote the space of complex structures on S_g. A complex structure on S_g can be considered as an automorphism A of tangent bundle TS_g satisfying $A^2 = -Id$. Conversely, every automorphism $A: TS_g \to TS_g$ satisfying $A^2 = -Id$ can be realized by a complex structure on S_g. This is essentially the classical result of Gauss and Riemann. Therefore, the space of complex structures C_g can be identify with a subspace of smooth sections of the vector bundle $End(TS_g)$. The topology on C_g is defined induced from topology on the space of sections.

Let $T_g = C_g/Diff_0^+(S_g)$ denote Teichmuller space of S_g. Two complex structures on S_g represent the same point in T_g iff there exists an orientation preserving diffeomorphism, which is isotopic to identity, taking one structure into another.

Let $M_g = C_g/Diff^+(S_g)$ denote moduli space of S_g. Two complex structures on S_g represent the same point in M_g iff there exists an orientation preserving diffeomorphism taking one structure into another.

Recall $\Gamma_g = Diff^+(S_g)/Diff_0^+(S_g)$. Obviously, $M_g = T_g/\Gamma_g$.

It is also a classical result "called the uniformization theorem" which states that every Riemann surface R_g is conformally equivalent to one which admits a hyperbolic metric, and this metric is uniquely determined up to isometry by the conformal equivalence class of R_g.

Namely, we can also view T_g, M_g as the Techimuller space and Moduli space of a hyperbolic surface. By a hyperbolic surface we will mean a smooth surface S_g equipped with a complete Riemann metric of constant curvature -1.

The remarkable fact [F,N] is that T_g actually is homeomorphic to R^{6g-6}, and the action of Γ_g on T_g is properly discontinuous. i.e. for every compact set $K \subset T_g$, the collection of $\phi \in \Gamma_g$ such that $\phi(K) \cap K \neq \emptyset$ is finite.

Theorem 1.2.1 (Heegaard Splitting) Any orientable closed 3-manifold M can be split into two handle bodies with the same genus g gluing together by an orientation preserving diffeomorphism of their boundaries.

Namely, the orientable closed 3-manifold M is nothing but two handle bodies and an element of some mapping class group since two isotopic diffemorphisms of the boundaries of the handle bodies make the diffeomorphic 3-manifolds. The mapping class groups seem to play a fundamental role in 3-dimension manifold theory from this point of view.

Every homeomorphism $\phi: S_g \to S_g$ is isotopic to a diffeomorphism and if two diffeomorphisms ϕ_1, ϕ_2: $S_g \to S_g$

can be homotopic by a path of homeomorphisms, then they can be connected by a path of diffeomorphisms. Therefore, the definition of mapping class groups Γ_g can be defined as the groups of path component of orientation preserving homeomorphisms of S_g.

Furthermore, it is again a classical result by Dehn and Nielsen that every homotopy class which trivially acts on $H_2(S_g;Z)$ contains a homeomorphism $\phi: S_g \to S_g$ and if any two homeomorphisms $\phi_1, \phi_2 : S_g \to S_g$ are homotopic, then ϕ_1 and ϕ_2 are isotopic. Therefore Γ_g = subgroup of $[S_g,S_g]$ = subgroup of $[K(\pi_1(S_g),1),K(\pi_1(S_g),1)]$.

There exists a purely algebraic definition of the mapping class group Γ_g, i.e. $\Gamma_g = Out^+(\pi_1(S_g)) = Aut^+(\pi_1(S_g))/Inn^+(\pi_1(S_g))$. It is easy to see this description agrees with an definition. It is remarkable result [H] by Harer that the mapping class groups Γ_g are all groups of finite vcd and $vcd(\Gamma_g) = 4g-5$. Therefore the Farrell-Tate cohomology groups of Γ_g are well-defined.

In fact, in the last ten years, a number of people have worked on the computations of cohomology(or homology) of mapping class groups Γ_g.

For example, Miller and Morita [Mi] [Mo] showed independently in the 1980's that:

<u>Theorem 1.2.2.</u> (Miller-Morita) Let $Q[k_1,k_2 \cdots\cdots]$ be the polynomial algebra on indeterminate k_i of degree 2i.

Then there is a map I: $Q[k_1, k_2 \cdots\cdots\cdots] \rightarrow H^*(\Gamma_g;Q)$ which is injective in dimension less than g/3.

As one of many properties analogous to arithmetic groups, a stability theorem of mapping class groups was shown by Harer in 1985 $[H]_2$.

<u>Theorem 1.2.3.</u> (Harer) $H^i(\Gamma_g;Z)$ is independent of g when g >> i.

Glover and Mislin obtained some torsion in the stable cohomology of the infinite mapping class group in 1987 [G,M].

<u>Theorem 1.2.4.</u> (Glover and Mislin) The stable cohomology group $H^{4i}(\Gamma_g;Z)$ (g >> i) contains an element of order E_{2i} = denominator of $B_{2i}/2i$, where B_{2i} is the 2i-th Bernoulli number.

Charney and Cohen [C,C] proved a stable splitting theorem for the infinite mapping class group Γ.

<u>Theorem 1.2.5.</u> (Charney and Cohen) The map $B\tau$: $B\Gamma^+ \rightarrow ImJ_{(1/2)}$ has a stable section. That is, there is a stable map $\theta \in \{ImJ_{(1/2)}, B\Gamma^+\}$ such that $\Sigma^\infty B\tau\theta$ is an equivalence. Here $B\Gamma^+$ is the Quillen's plus-construction of $B\Gamma$, $ImJ_{(1/2)} = \prod BGL(F_q)^+{}_{(p)}$ for suitable q, p ranges over all odd prime, {X, Y} denotes the group of homotopy classes of stable maps $\Sigma^\infty X \rightarrow \Sigma^\infty Y$.

Cohen, Bensen and others have been working recently on the torsion in mapping class groups of low genus, and prove many beautiful theorems [Be] [C] $[C]_2$ etc.

It is classical that there is a homology representation of the mapping class group Γ_g with image the sympletic group Sp(2g,Z) and kernel, called Torreli group, which is torsion free. We will frequently make use of this fact.

CHAPTER II

THE P-PERIODICITY OF THE MAPPING CLASS GROUP

In this chapter, we will determine completely the primes p for which Γ_g is p-periodic. Also we compute the exact p-period of Γ_g for g equal to certain multiples of (p-1)/2. As an application of these results we tabulate some information about the p-periodicity of Γ_g for g = 2, 3, 4 and 5. A near complete tabulation of these p-periodicity results appears at the appendix C in table C.3. The results in this chapter are obtained by using the p-periodicity of metacyclic subgroups as lower bounds and the non-vanishing of Chern classes of the canonical homology representation when restricted to every Z_p subgroup as an upper bound. We note Burgisser studied the p-periodicity of arithmetic subgroups of general linear groups in a similar way [Bu].

Assume g > 1. The main results are as follows:

Theorem 2.1. a) the mapping class group Γ_g is never 2-periodic.

b) The mapping class group Γ_{kp+i} is always p-periodic if $i \neq 1 \bmod(p)$ for p odd prime, $k \geq 0$.

c) The mapping class group Γ_{kp+1} is p-periodic if and only if $[(2k+3)/p, (2k+2)/(p-1)]$ does not contain an integer and $k \neq 0, -1 \bmod(p)$ for p odd prime. In particular Γ_{kp+1} can be p-periodic only when $k \leq (p^2-5)/2$.

Theorem 2.2. If $k \neq 0 \bmod(p)$, $p > 2$, $\Gamma_{(p-1)(kp-k-2)/2}$ is p-periodic and the p-period of $\Gamma_{(p-1)(kp-k-2)/2}$ is a multiple of $2(p-1)$. Moreover, if $k < (p-1)/2$, the p-period of $\Gamma_{(p-1)(kp-k-2)/2}$ equals $2(p-1)$.

Corollary 2.3. $\Gamma_{(p-1)(p-3)/2}$ is p-periodic and the period equals $2(p-1)$ for $p > 3$.

Theorem 2.4. $\Gamma_{(p-1)(d-2)/2}$ is p-periodic and the p-period of $\Gamma_{(p-1)(d-2)/2}$ is a multiple of 2d if 3 divides d and d divides p-1.

Proposition 2.5. $\Gamma_{(p-1)/2}$ is p-periodic and the period of $\Gamma_{(p-1)/2}$ divides p-1.

Two well-known theorems are consequently employed to reach the upper and lower bounds of mapping class groups Γ_g in this chapter:

1) Brown-Venkov theorem $[Br]_2$. If $a \in \hat{H}^m(\Gamma;Z)$ exists such that $Res_S{}^{\Gamma}(a)$ is a maximal generator in $\hat{H}^m(S_p;Z)$ for every p-Sylow subgroup S_p of Γ, then Γ is p-periodic, and the p-period of Γ divides m. Moreover, the cup-product

with a is an isomorphism $\hat{H}^{i}(G;A)_{(p)} = \hat{H}^{i+m}(G;A)_{(p)}$ for all i and A.

2) If Γ is p-periodic, and H is a finite subgroup, then the p-period of H divides the p-period of Γ [Br].

The rest of this chapter is organized as follows: In section 2.1 we prove theorem 2.1. In section 2.2 we find a metacyclic subgroup of Γ_g that we use to give a lower bound for the p-period of Γ_g and in section 2.3 we identify the Chern classes of the canonical homology representation of Γ_g which we use to give an upper bound for this p-period. In section 2.4 we present the Euler class of the homology representation which allows us to improve the upper bound in certain cases. Finally in section 2.5 we apply the previous results to Γ_g, g = 2, 3, 4 and 5.

2.1. The proof of theorem 2.1

For a group Γ of finite vcd, recall the property that Γ has p-periodic cohomology is equivalent to the property that every elementary abelian p-subgroup of Γ has rank ≤ 1 [Br]. In other words, Γ is p-periodic if and only if Γ does not contain $Z_p \times Z_p$.

Let S_g be an orientable closed surface of genus g. As a conclusion of the positive solution of the Nielsen conjecture by Kerchoff [K], F is a finite subgroup of Γ_g if and only if F is contained in $Homeo^+(S_g)$, the group of orientation preserving homeomorphisms of S_g.

The following is a necessary and sufficient condition for a finite subgroup F to be contained in $Homeo^+(S_g)$.

Proposition 2.1.1. The finite group F is isomorphic to a subgroup of $Homeo^+(S_g)$ with branching data $(h; n_1 \cdots\cdots n_b)$ if and only if F satisfies the following conditions:

1) $F=<a_1, \cdots\cdots, a_h, b_1, \cdots\cdots, b_h, c_1, \cdots\cdots, c_b>$.
2) $\prod_{1\leq i\leq h}[a_i, b_i]\prod_{1\leq j\leq b} c_j = 1$.
3) $Order(c_i) = n_i$.
4) Riemann-Hurwitz equation

$$2g-2 = |F|(2h-2) + |F|\sum_{1\leq i\leq b}(1-1/n_i).$$

Proof: See[Tu].

We need an essentially geometrical lemma.

Lemma 2.1.2. Let G be a finite subgroup of $Homeo^+(S_g)$, then G is also a finite subgroup of $Homeo^+(S_{g+k|G|})$. Here k is a non-negative integer.

Proof: By induction. If k = 1, For any $g \in G$, note that g can be represented as an orientation preserving homeomorphism $T_g: S_g \to S_g$ such that T_g has only finite many fixed points. Picking a point $x \in S_g$ which is not a singular point for every $g \in G$, a neighberhood N of x can be found satisfing $g_1 N \cap g_2 N = \emptyset$ for every $g_1 \neq g_2 \in G$. Remove $|G|$ disjoint neighborhoods $\{gN\}$, $g \in G$, and connect sum $|G|$

tori to $S_g - \{gN\}$, $g \in G$. We obtain the closed orientable surface $S_{g+|G|}$ with G acting so that the $|G|$ tori are permuted.

If G acts on $S_{g+(k-1)|G|}$, using the construction above again, G can also act on $S_{g+k|G|}$.

Remark: The lemma above can be also proved in algebraic way simply using the proposition 2.1.1.

The proof of theorem 2.1: a) Let D_8 be the dihedral group of order 8. We will show that every mapping class group Γ_g contains D_8, not only $Z_2 \times Z_2$. Therefore, Γ_g is not 2-periodic for $g \geq 2$.

In fact, we only need use proposition 2.1.1 to check $\Gamma_i \supset D_8 \supset Z_2 \times Z_2$, $2 \leq i \leq 9$, by lemma 2.1.2.

a) For g = 2:

1) $D_8 = \langle x, y \mid x^2 = y^4 = 1, xyx^{-1} = y^{-1} \rangle = \langle x, y, y^2, yx \rangle$

2) $xyy^2yx = 1$.

3) $O(x) = 2$, $O(y) = 4$, $O(y^2) = 2$, $O(yx) = 2$. Branch data $(2^3, 4)$.

4) Riemann-Hurwitz: $2(2)-2 = 8(2(0)-2)+8(1-1/2)3+8(1-1/4)$.

b) For g = 3:

1) $D_8 = \langle x, y \mid x^2 = y^4 = 1, xyx^{-1} = y^{-1} \rangle = \langle x, xy^2, y, y \rangle$

2) $xxy^2yy = 1$.

3) $O(x) = 2$, $O(xy^2) = 2$, $O(y) = 4$, $O(y) = 4$. Branch data $(2^2, 4^2)$.

4) Riemann-Hurwitz: $2(3)-2 = 8(2(0)-2)+8(1-1/2)2+8(1-1/4)2$.

c) For $g = 4$:

1) $D_8 = <x, y \mid x^2 = y^4 = 1, xyx^{-1} = y^{-1}> = <x, y, xy, x, x>$

2) $xyxyxx = 1$.

3) $O(x) = 2, O(y) = 4, O(xy) = 2$, $O(x) = 2$, $O(x) = 2$. Branch data $(2^4, 4)$.

4) Riemann-Hurwitz: $2(4)-2 = 8(2(0)-2)+8(1-1/2)4+8(1-1/4)1$.

d) For $g = 5$:

1) $D_8 = <x, y \mid x^2 = y^4 = 1, xyx^{-1} = y^{-1}> = <x, x, y^2, y, y>$

2) $xxy^2yy = 1$.

3) $O(x) = 2, O(x) = 2, O(y^2) = 2, O(y) = 4, O(y) = 4$. Branch data $(2^3, 4^2)$.

4) Riemann-Hurwitz: $2(5)-2 = 8(2(0)-2)+8(1-1/2)3+8(1-1/4)2$.

e) For $g = 6$:

1) $D_8 = <x, y \mid x^2 = y^4 = 1, xyx^{-1} = y^{-1}> = <x, xy, y, y, y>$

2) $xxyyyy = 1$.

3) $O(x) = 2, O(xy) = 2, O(y) = 4, O(y) = 4, O(y) = 4$. Branch data $(2^2, 4^3)$.

4) Riemann-Hurwitz: $2(6)-2 = 8(2(0)-2)+8(1-1/2)2+8(1-1/4)3$.

f) For $g = 7$:

1) $D_8 = <x, y \mid x^2 = y^4 = 1, xyx^{-1} = y^{-1}> = <x, y^2, xy^2, y^2, y, y>$

2) $xy^2xy^2y^2yy = 1$

3) $O(x) = 2, O(y^2) = 2, O(xy^2) = 2, O(y^2) = 2, O(y) = 4$. Branch data $(2^4, 4^2)$.

4) Riemann-Hurwitz: $2(7)-2 = 8(2(0)-2)+8(1-1/2)4+8((1-1/4)2$.

g) For $g = 8$:

1) $D_8 = <x, y \mid x^2 = y^4 = 1, xyx^{-1} = y^{-1}> = <x, xy, y^2, y, y, y^3>$.

2) $xxyy^2yyy^3 = 1$.

3) $O(x) = 2$, $O(xy) = 2$, $O(y2) = 2$, $O(y) = 4$, $O(y) = 4$, $O(y^3) = 4$. Branch data $(2^3, 4^3)$.

4) Riemann-Hurwitz: $2(8)-2 = 8(2(0)-2)+8(1-1/2)3+8(1-1/4)3$.

h) For $g = 9$:

1) $D_8 = <x, y \mid x^2 = y^4 = 1, xyx^{-1} = y-1> = <x, x, x, x, y^2, y, y>$.

2) $xxxxy^2yy = 1$.

3) $O(x) = 2$, $O(x) = 2$, $O(x) = 2$, $O(x) = 2$, $O(y^2) = 2$, $O(y) = 4$, $O(y) = 4$. Branch data $(2^5, 4^2)$.

4) Riemann-Hurwitz: $2(9)-2 = 8(2(0)-2)+8(1-1/2)5+8(1-1/4)2$.

b) Γ_{kp+i} is p-periodic for $i \neq 1$, here p is an odd prime, $k \geq 0$, and $0 \leq i \leq p-1$.

In fact, if $\Gamma_{kp+i} \supset Z_p \times Z_p$, the Riemann-Hurwitz formula must hold: $2(kp+i)-2 = p^2(2h-2)+p^2(1-1/p)b$, $2k+(2i-2)/p = p(2h-2) +(p-1)b$ implies $2i-2 = 0 \bmod(p)$. Forcing $i = 1 \bmod(p)$. This is a contradiction!

c) Claim 1: If $k = 0, -1 \bmod(p)$ or $[(2k+3)/p, (2k+2)/(p-1)]$ contains an integer, then $\Gamma_{kp+1} \supset Zp\times Zp$. Therefore, Γ_{kp+1} is not p-periodic.

Case 1: If $k = 0 \bmod(p)$.

Let $Z_p \times Z_p = \langle a, b \mid a^p = b^p = 1, ab = ba\rangle$. We show a $Z_p \times Z_p$ free action on S_{kp+1} by proposition 2.1.1. Suppose $k = np$, here n is a non-negative integer.

1) $Z_p \times Z_p = \langle a_1, b_1, a_2, b_2 \cdots\cdots a_{(n+1)}, b_{(n+1)}\rangle$. $a_i = a$, $b_i = b$, $1 \leq i \leq n+1$.

2) $\Pi_{1\leq i\leq n+1}[a_i, b_i] = 1$.

3) Riemann-Hurwitz: $2(kp+1)-2 = p^2(2(n+1)-2)$.

Case 2: If $k = -1 \bmod(p)$.

We show that there is a $Zp \times Zp$ action with two singular points on S_{kp+1}.

1) $Zp \times Zp = \langle a_1, b_1, a_2, b_2 \cdots\cdots a_n, b_n, a, a^{-1}\rangle$, $n = (k+1)/p \geq 1$. $a_i = a$, $b_i = b$, $1 \leq i \leq n$.

2) $\prod_{1\leq i\leq n}[a_i, b_i]aa^{-1} = 1$.

3) $O(a) = p$, $O(a^{-1}) = p$.

4) Riemann-Hurwitz: $2(kp+1)-2 = p^2 2(n-1)+p^2(1-1/p)2$.

Case 3: The interval $[(2k+3)/p, (2k+2)/(p-1)]$ contains an integer n.

We show that there is a $Z_p \times Z_p$ action with $t = np-2k$ singular points on S_{kp-1}. Notice $t = np-2k \geq 3$.

Let $h = k+1-n(p-1)/2$. Notice $h \geq k+1-(2k+2)/2 = 0$.

1) $Z_p \times Z_p = \langle a_1, b_1, a_2, b_2 \cdots\cdots a_h, b_h, b, a_1, a_2 \cdots \cdots a_{t-2}, (\prod_{1\leq j\leq t-2} a_j)^{-1}b^{-1} \rangle$. Here, $a_i = a$, $b_i = b$, $a_j = a$, $b = b$, $1 \leq i \leq h$, $1 \leq j \leq b-2$.

2) $\prod_{1\leq i\leq h}[a_i, b_i]b\prod_{1\leq j\leq t-2}a_j(\prod_{1\leq j\leq t-2}a_j)^{-1}b^{-1} = 1$.

3) $O(b) = p$, $O(a^j) = p$, $O((\prod_{i\leq j\leq t-2}a_j)^{-1}b^{-1}) = p$.

4) Riemann-Hurwitz: $2(kp+1)-2 = p^2(2h-2)+p^2(1-1/p)t$, i.e. $2kp = p^2(2k-n(p-1))+p(p-1)(np-2k) = 2kp^2-np^2(p-1) +np^2(p-1)-2kp(p-1)$.

Claim 2: Conversely, if Γ_{kp+1} is not p-periodic, then $k = 0 \mod (p)$, $k = -1 \mod(p)$ or $[(2k+3)/p, (2k+2)/(p-1)]$ contains an integer.

In fact, suppose $\Gamma_{kp+1} \supset Z_p \times Z_p$. i.e. there exists a $Z_p \times Z_p$ action on S_{kp+1}.

Case 1: $Z_p \times Z_p$ acts freely on S_{kp+1}. Riemann-Hurwitz formula $2(kp+1)-2 = p^2(2h-2)$ implies $k = p(h-1)$, i.e. $k = 0 \mod(p)$.

Case 2: $Z_p \times Z_p$ acts on S_{kp+1} with two singular points. Riemann-Hurwitz formula $2(kp+1)-2 = p^2(2h-2)+ p^2(1-1/p)2$ implies $k = p(h-1)+p-1$. i.e, $k = -1 \mod(p)$.

Case 3: $Z_p \times Z_p$ acts on S_{kp+1} with more than three singular points. Riemann-Hurwitz formula: $2(kp+1)-2 = p^2(2h-2)+p^2(1-1/p)t$ implies $(2k+t)p = (2h-2+t)p^2$. Suppose $n = 2h-2+t$, $np = (2h-2+t)p = 2k+t \geq 2k+3$, since $t \geq 3$. i.e, $n \geq (2k+3)/p$. In addition, $2kp-(2h-2)p^2 = p(p-1)t$, $t = (2k-2hp+2p)/(p-1)$ implies $n = 2h-2+t = [(2h-2)(p-1)+2k-2hp+2p]/(p-1) = (2k+2-2h)/(p-1) \leq (2k+2)/(p-1)$ since $h \geq 0$.

So, $(2k+3)/p \leq n \leq (2k+2)/(p-1)$. n is an integer.

Claim 3: Γ_{kp+1} is not p-periodic if $k \geq (p^2-3)/2$.

Recall Γ_g is never 2-periodic for $g > 1$. For p an odd prime, $(2k+2)/(p-1)-(2k+3)/p = (2kp+2p-2kp+2k-3p+3)/[p(p-1)] = (2k-p+3)/[p(p-1)] \geq (p^2-p)/[p(p-1)] = 1$ implies that there exists at least a integer $n \in [(2k+3)/p, (2k+2)/(p-1)]$.

<u>Remark</u>: No finite group acts on S_g with only one singular point.

<u>Remark</u>: As a supplement of theorem 2.1, we give table C.1 and table C.2 in the appendix C by working on the computer programs. In table C.1, the p-torsion gaps of

mapping class groups Γ_g are displayed, i.e. fixing odd prime p, list the all genus g of mapping class groups Γ_g which do not have p-torsion. In table C.2, corresponding to theorem 2.1.b), we list all genus g = kp+1 of mapping class groups Γ_{kp+1} which have p-periodicity for odd prime p.

2.2. The metacyclic subgroups of the mapping class group

Define metacyclic groups $M_{p,k} = \langle a, b \mid a^{k(p-1)} = 1, b^p=1, aba^{-1} = b^m \rangle$, where p is an odd prime, k is a positive integer, m is an integer such that the order of m in the multiple group $(Z/pZ)^*$ is equal to $\varphi(p) = p-1$, then $|M_{p,k}| = kp(p-1)$.

Lemma 2.2.1. $\Gamma_{(p-1)(kp-2-k)/2} \supset M_{p,k}$ except for k = 1 and p = 3.

Proof: We need to show $Homeo^+(S_{(p-1)(kp-2-k)/2}) \supset M_{p,k}$.

1) $M_{p,k} = \langle b^{-1}, a, a^{-1}b \rangle$.

2) $b^{-1}aa^{-1}b = 1$.

3) $O(b^{-1}) = p$, $O(a) = k(p-1)$, $O(a^{-1}b) = k(p-1)$.

Actually, $(a^{-1}b)^n = a^{-n}b^{1+m+\cdots+m^{n-1}}$

4) Riemann-Hurwitz: $2(p-1)(kp-2-k)/2-2 = kp(p-1)(2h-2)+kp(p-1)(1-1/p)t_1+kp(p-1)(1-1/[k(p-1)])t_2$, taking h = 0, $t_1 = 1$, $t_2=2$, i.e. $kp^2-kp-2p+2-kp+k-2 = -2kp^2+2kp+kp^2-2kp+k+2kp^2-2kp-2p$. Define metacyclic groups $N_{p,d} = \langle a,b \mid a^d = 1, b^p = 1, aba^{-1} = b^m \rangle$, where p is an odd

prime, $3 \le d$, d divides $(p-1)$, m is an integer such that the order of m in the multiple group $(Z/pZ)^*$ is equal to d, then $|N_{p,d}| = pd$.

Lemma 2.2.2. $\Gamma_{(p-1)(d-2)/2} \supset N_{p,d}$.

Proof: We only need to show $Homeo^+(S_{(p-1)(d-2)/2}) \supset N_{p,d}$.

1) $N_{p,d} = < b^{-1}, a, a^{-1}b >$.

2) $b^{-1}aa^{-1}b = 1$.

3) $O(b^{-1}) = p$, $O(a) = d$, $O(a^{-1}b) = d$, since $(a^{-1}b)^n = a^{-n}b^{1+m+\cdots+m^{n-1}}$.

4) Riemann-Hurwitz: $2(p-1)(d-1)/2-2 = pd(2h-2)+ pd(1-1/p)t_1 + pd(1-1/d)t_2$. Taking $h = 0, t_1 = 1, t_2 = 2$, $pd-d-2p = -2pd+pd-d+2pd-2p$.

Corollary 2.2.3. $\Gamma_{(p-1)(p-3)/2} \supset N_{p,p-1} = <a,b \mid a^{p-1} = 1, b^p = 1\ aba^{-1} = b^m >$.

Corollary 2.2.4. $\Gamma_{(p-1)/2} \supset N_{p,3} = <a, b \mid a^3 = 1, b^p = 1, aba^{-1} = b^m >$, if 3 divides $(p-1)$.

Suppose G a p-periodic finite group for p odd prime. The main result in [Sw] by Swan is that the p-period of G is equal to $2|N(Z_{p^n})/C(Z_{p^n})|$, where N and C denote the normalizer and centralizer of order p^n cyclic p-sylow subgroup in G.

Lemma 2.2.5. The finite group $M_{p,k}$ [resp $N_{p,d}$] is p-periodic with the p-period is $2(p-1)$ [resp 2d] for $k \ne 0$ mod(p).

Proof: $|M_{p,k}| = kp(p-1)$, $|N_{p,d}| = pd$ imply that the $M_{p,k}$ and $N_{p,d}$ can not contain $Z_p \times Z_p$. i.e, These groups are p-periodic.

Now, consider the group $M_{p,k} = \langle a, b \mid a^{k(p-1)} = 1, b^p = 1, aba^{-1} = b^m \rangle$, let $Z_p = \langle b \rangle$, then $N(Z_p) = M_{p,k}$, $C(Z_p) = \langle a^{p-1}, b \rangle$, $|N(Z_p)| = kp(p-1)$, $|C(Z_p)| = kp$, therefore, $|N(Z_p)/C(Z_p)| = p-1 = \varphi(p)$. The p-period of $M_{p,k} = 2(p-1)$.

Similarly, consider the group $N_{p,d} = \langle a, b \mid a^d = 1, b^p = 1, aba^{-1} = b^m \rangle$, Let $Z_p = \langle b \rangle$ again, then $N(Z_p) = N_{p,d}$, $C(Z_p) = \langle b \rangle$, $|N(Z_p)| = pd$, $|C(Z_p)| = p$, therefore, $|N(Z_p)/C(Z_p)| = d$. The p-period of $N_{p,d} = 2d$.

In particular, the p-period of $N_{p,p-1} = 2(p-1)$.

Actually, The p-periodicity of $\Gamma_{(p-1)(kp-2-k)/2}$ and $\Gamma_{(p-1)(d-2)/2}$ are obviously by b) of theorem 2.1. The low bounds of the p-period of these mapping class groups are obtained by combining the results of this section.

<u>Proposition 2.2.6</u>

a) The p-period of $\Gamma_{(p-1)(kp-2-k)/2}$ is a multiple of $2(p-1)$.

b) The p-period of $\Gamma_{(p-1)(d-2)/2}$ is a multiple of 2d, if 3 divides d, d divides (p-1).

c) In particular, The p-period of $\Gamma_{(p-1)(p-3)/2}$ is a multiple of $2(p-1)$.

2.3. The Chern classes of the canonical homology representation of the mapping class group

Recall that for a complex representation $f: G \to GL_k(C)$ of the discrete group G the Chern classes $c_i(f) \in H^{2i}(G;Z)$ are defined as Chern classes of the flat C^k-bundle over $K(G,1)$ classfied by $Bf: K(G,1) \to BGL_k(C)$.

Q is a subring in C, if $f: G \to GL_k(Q)$ is a representation over Q, we will write $c_i(f)$ for the i-th Chern class of the associated complex representation $G \to GL_k(Q) \to GL_k(C)$ $[E,M]_2$.

It is well-known that over Q the group Z/nZ has a unique faithful irreducible representation σ_n: $Z/nZ \to GL_{\varphi(n)}(Q)$, where $\varphi(n)$ is the Euler function [Se].

Glover and Mislin [G,M] showed the proposition as following:

<u>Proposition</u> [Glover, Mislin]: Let $r: Z/p^\alpha \to GL_k(Q)$ be a Q-representation. Suppose that in the decomposition of r into Q-irreducible representation σ_{p^α} occurs with multiplicity m, where m is not divisible by p, then, for every $j > 0$, $(C_{\varphi(p^\alpha)}(r))^j \in H^{2j\varphi(p^\alpha)}(Z/p^\alpha; Z)$ has order p^α.

Let $\mu: \Gamma_g \to Sp(2g,Z)$ be the map obtained by allowing a homeomorphism h of S_g to act on $H_1(S_g;Z)$, $i: Sp(2g,Z) \to GL(2g,Z) \to GL(2g,Q)$ be canonical inclusion, then $\varepsilon = i\mu: \Gamma_g \to GL(2g,Q)$ is a representation over Q.

If $\rho: Z_p \to \Gamma_g \to GL(2g,Q)$ is composite of inclusion and ε, since the representation ρ is faithful, $\chi_\rho = m_\rho\chi_{tr} + n_\rho\chi_\sigma$, where χ stands for character of the representation; tr is the trivial representation, σ is the unique irreducible representation of Z_p, the integers m_ρ and n_ρ only depend on ρ [Se].

Proposition 2.3.1. Suppose Γ_g is p-periodic for p odd prime, $\rho = \varepsilon i: Z_p \to \Gamma_g \to GL(2g,Q)$ is a representation of Z_p over Q for any inclusion $i: Z_p \to \Gamma_g$, $\chi_\rho = m_\rho\chi_{tr} + n_\rho\chi_\sigma$. If n_ρ is never divisible by p where i varies over all inclusions, then Γ_g has the p-period m_p, which divides $2\varphi(p)$.

Proof: It is well-known that Γ_g is p-periodic if and only if every p-Sylow subgroup S_p of Γ_g is cyclic. We only need to show that there exists an element $a \in \hat{H}^{2\varphi(p)}(\Gamma_g; Z)$ such that $Res(a) \in \hat{H}^{2\varphi(p)}(Z_p;Z)$ is nontrivial for every Z_p inclusion by Brown-Venkov theorem.

Let $g^*: H^{2\varphi(p)}(\Gamma_g; Z) \to \hat{H}^{2\varphi(p)}(\Gamma_g; Z)$ be the canonical map from the ordinary cohomology to the Farrell-Tate cohomology [Br]. The following diagram is commutative.

$$\begin{array}{ccc} & \text{Res} & \\ \hat{H}^{2\varphi(p)}(\Gamma_g;\ Z) & \rightarrow & \hat{H}^{2\varphi(p)}(Z_p;Z) \\ \uparrow g^* & & \uparrow g^* \\ H^{2\varphi(p)}(\Gamma_g;\ Z) & \rightarrow & H^{2\varphi(p)}(Z_p;Z) \\ & \text{Res} & \end{array}$$

Now, let $a = g^*[c_{p-1}(\varepsilon)] \in \hat{H}^{2\varphi(p)}(\Gamma_g;\ Z)$, $\varepsilon: \Gamma_g \rightarrow GL(2g,Q)$, $i: Z_p \rightarrow \Gamma_p$. Then $Res(a) = Resg^*[c_{p-1}(\varepsilon)] = g^*Res[c_{p-1}(\varepsilon)] = g^*[c_{p-1}(\rho)]$ has nontrival order in $\hat{H}^{2\varphi(p)}(Z_p;Z) \approx H^{2\varphi(p)}(Z_p;Z)$ for every inclusion $\rho = \varepsilon i$ since the irreducible representation σ occurs with multiplicity n which is not divisible by p. i.e. the cup-product with $g^*[c_{p-1}(\varepsilon)]$ is an isomorphism for all integer i and Γ_g-module A: $\hat{H}^{i}(\Gamma_g;A)_{(p)} \approx \hat{H}^{i+2\varphi(p)}(\Gamma_g;A)_{(p)}$.

Consider any inclusion $i: Z_p \rightarrow \Gamma_{(p-1)(kp-2-k)/2}$. Riemann-Hurwitz holds : $2(p-1)(kp-2-k)/2-2 = p(2h-2)+(p-1)t$ implies $t = kp-k-2ph/(p-1) = k(p-1)-sp$, here $s = 2h/(p-1)$ must be an integer. The Lefschetz-Hopf trace formula also gives us $\chi_\rho(T) = 2-t = 2-k(p-1)+sp$, here T is a generator of Z_p, $\rho = \varepsilon i: Z_p \rightarrow GL((p-1)(kp-k-2),Q)$.

Suppose $\chi_\rho = m_\rho\chi_{tr}+n_\rho\chi_\sigma$, since $\chi_{tr}(1) = 1$, $\chi_{tr}(T) = 1$, $\chi_\sigma(1) = p-1$, $\chi_\sigma(T) = -1$, $\chi_\rho(1) = m+n(p-1) = (p-1)(kp-k-2)$, $\chi_\rho(T) = m-n = 2-k(p-1)+sp$. Then $np = (p-1)(kp-k-2)-2 +k(p-1)-sp = kp^2-kp-kp+k-2p+2-2+kp-k-sp = kp^2-kp-2p-sp$, implies $n = kp-k-2-s$. Recall $0 \le s \le k-1$, if $k < (p-1)/2$, then, $(k-1)p < kp-k-k+1-2 \le kp-k-s-2 = n < kp$, or say, n is not divisible by p. So the p-period of $\Gamma_{(p-1)(kp-2-k)/2}$ divides $2(p-1)$ if $k < (p-1)/2$ and $k \neq 0 \bmod(p)$. We already proved theorem 2.2 by combining the lower bounds of section 2 this chapter.

In particular, take $k = 1$, $p \ge 5$, we obtain that $\Gamma_{(p-1)(p-3)/2}$ is p-periodic and the period equals $2(p-1)$.

2.4. The Euler class of the canonical homology representation of the mapping class group

Recall that for a real orientable representation $\varphi: G \to GL_k(R)$ of the discrete group G the Euler class $e(\varphi) \in H^k(G;Z)$ is defined as Euler class of the flat R^k-bundle over $K(G,1)$ classified by $B_j: K(G,1) \to BGL_k(R)$.

If $\varphi: G \to GL_k(A)$ is an orientable representation over A, A is a subring of R, we can write $e(\varphi)$ for the Euler class of real representation $G \to GL_k(A) \to GL_k(R)$. An important relation between the Euler class and the Chern class is as follows: $c_k(\varphi)=(-1)^{k(k-1)/2}e^2(\varphi)\in H^{2k}(G;Z)$ [E,M].

<u>Proposition 2.4.1</u>. Let $\rho = \varepsilon i: Z_p \to \Gamma_{(p-1)/2} \to GL(p-1,Q)$, then $e(\varepsilon)\in \hat{H}^{p-1}(\Gamma_{(p-1)/2}; Z)$ are restricted to order p elements in $\hat{H}^{p-1}(Z_p;Z)$ for any i. The p-period of $\Gamma_{(p-1)/2}$ divides p-1.

Proof: Consider ρ: $Z_p \to \Gamma_{(p-1)/2} \to GL(p-1,Q)$. The Riemann-Hurwitz $2(p-1)/2-2 = p(2h-2)+(p-1)t$ implies $t = 3-2ph/(p-1)$, forcing $t = 3$. By the Lefschetz fixed point theorem again, $\chi(T) = 2-t = -1$, but $\chi = m\chi_{tr}+n\chi_s$, $m+n(p-1) = p-1$, $m-n = -1$, solving for $n = 1 \neq 0 \bmod(p)$. The proposition by Glover and Mislin provides the element $Res(g^*c_{p-1}(\varepsilon))$ has order p for every ρ. Furthermore, $c_{p-1}(\varepsilon) = (-1)^{(p-1)(p-2)/2}e(\varepsilon)^2$, $Res(g^*e(\varepsilon))$ has order p for every i. So the p-period of $\Gamma_{(p-1)/2}$ divides p-1.

2.5. The p-period of low genus mapping class groups

As the examples of application of the theorems above, in the rest of this chapter, we start to check the p-period of low genus mapping class groups Γ_g for g = 2,3,4 and 5.

a) Γ_2. There are only 2, 3 and 5 torsions in Γ_2.

1) Γ_2 is not 2-periodic.

2) Γ_2 is 3-periodic. The 3-period of Γ_2 is equal to 4.

The 3-period of Γ_2 is multiple of 4 by theorem 2.2. $(k = 2, p = 3.)$

The 3-period of Γ_2 divides 4 by proposition 2.3.1. In fact, Riemann-Hurwitz formular: $2(2)-2 = 3(2h-2)+2t$ implies $t = 4$, $\chi(T) = -2$, $n = 2 \neq 0 \bmod(3)$.

3) Γ_2 is 5-periodic.

The possible 5-period of Γ_2 = 2 or 4 by prop.2.4.1.

b) Γ_3. There are only 2, 3 and 7 torsions in Γ_3.

1) Γ_3 is not 2-periodic.

2) Γ_3 is 3-periodic and the 3-period of Γ_3 is a multiple of 4.

In fact, $\Gamma_{3k} \supset D_6$, since $\Gamma_3 \supset D_6$ and $\Gamma_6 \supset D_6$, by Lemma 2.1.1 in section 1. Here D_6 is order 6 dihedral group, k is a positive integer.

3) Γ_3 is 7-periodic and the 7-period of Γ_3 is 6.

In fact, the 7-period of Γ_3 is multiple of 6 by theorem 2.4, the 7-period of Γ_3 divides 6 by prop. 2.4.1.

c) Γ_4. There are only 2, 3 and 5 torsions in Γ_4.

1) Γ_4 is not 2-periodic.

2) Γ_4 is not 3-periodic by c) of theorem 2.1.

In fact, $4 = 3(1)+1$ and $2 \in [(2k+3)/p, (2k+2)/(p-1)] = [5/3, 2]$.

3) Γ_4 is 5-periodic and the 5-period of $\Gamma_4 = 8$.

In fact, this is exactly theorem 2.2.for $k = 1$, $p = 5$.

d) Γ_5 has only 2, 3, 5 and 11 torsions.

1) Γ_5 is not 2-periodic.

2) Γ_5 is 3-periodic and the 3-period of $\Gamma_5 = 4$. In fact, $D_6 = \Gamma_{5,2} = \langle a, b \mid a^3 = b^2 = 1, bab^{-1} = a^2 \rangle = \langle b, b, b, b, a, a^2 \rangle$ $bbbbaa^2 = 1$, $O(b) = 2$, $O(a) = 3$, $O(a^2) = 3$. Branch data $(2^4, 3^2)$. Riemann-Hurwitz: $2(5)-2 = 6(2(0)-2) +6(1-1/2)4+6(1-1/3)2$. So, $\Gamma_5 \supset D_6$, the 3-period of Γ_5 is a

multiple of 4.

Consider $\rho = \varepsilon i: Z_3 \to \Gamma_5 \to GL(10, Q)$, $Z_3 = < T >$. Riemann-Hurwitz formula: $2(5)-2 = 3(2h-2)+3(1-1/3)t$ implies $t = 4$ or 7. i.e. $\chi(T) = -2$ or -5, therefore, $\chi = m\chi_{tr}+n\chi_{\sigma}$ leads $n = 4$ or 5, both 4 and 5 are not multiples of 3, by proposition 2.3.1, the 3-period of Γ_5 divides 4.

(3) Γ_5 is 5-periodic and the 5-period of $\Gamma_5 = 2$, 4 or 8.

In fact, Riemann-Hurwitz: $2(5)-2 = 5(2h-2)+4t$ implies $t = 2$, $\chi(T) = 0$, therefore, $n = 2$ is prime to 3. The 5-period of Γ_5 divides 8.

(4) Γ_5 is 11-periodic and the possibilities of the 11-period of $\Gamma_5 = 2$ or 10.

This is simply proposition 2.4.1.

CHAPTER III

THE p-TORSION OF THE FARRELL-TATE COHOMOLOGY OF THE MAPPING CLASS GROUP $\Gamma_{(p-1)/2}$

In this chapter we give a complete calculation of the p-torsion of the Farrell-Tate cohomology of the mapping class group $\Gamma_{(p-1)/2}$. Therefore, the p-torsion of the ordinary cohomology of mapping class group $\Gamma_{(p-1)/2}$ is also determined for dimensions greater than 2p-7.

Theorem 3.1. (the main result)

a) $\hat{H}^i(\Gamma_{(p-1)/2};Z)_{(p)} = \prod_{1\leq n\leq k}\hat{H}^i(Z_p;Z)$, if p = 6k-1.

b) $\hat{H}^i(\Gamma_{(p-1)/2};Z)_{(p)} = \hat{H}^i(Z_p \rtimes Z_3;Z)_{(p)} \times \prod_{1\leq n\leq k}\hat{H}^i(Z_p;Z)$ if p = 6k+1.

Corollary 2. (ordinary cohomology version)

a) $H^{2i}(\Gamma_{(p-1)/2};Z)_{(p)} = \prod_{1\leq n\leq k}H^{2i}(Z_p;Z) = \prod_{1\leq n\leq k}Z_p$ if p = 6k-1 and 2i > 2p-7.

b) $H^{6i}(\Gamma_{(p-1)/2};Z)_{(p)} = H^{6i}(Z_p \rtimes Z_3;Z)_{(p)} \times \prod_{1\leq n\leq k}H^{6i}(Z_p;Z) = \prod_{1\leq n\leq k+1}Z_p$, if p=6k+1 and 6i > 2p-7.

c) $H^{2i}(\Gamma_{(p-1)/2};Z)_{(p)} = H^{2i}(Z_p \rtimes Z_3;Z)_{(p)} \times \prod_{1\leq n\leq k} H^{2i}(Z_p;Z)$ $= \prod_{1\leq n\leq k} Z_p$, if $p = 6k+1$, $i \neq 0 \bmod(3)$ and $2i > 2p-7$.

d) $H^{2i-1}(\Gamma_{(p-1)/2};Z)_{(p)} = 0$ if $2i-1 > 2p-7$.

3.1. The number of conjugacy classes of order p elements in the mapping class group $\Gamma_{(p-1)/2}$

The odd, prime order, cyclic group Z_p acts on the closed surface $S_{(p-1)/2}$ with 3 fixed points, and $Z_p \times Z_p$ can not act on $S_{(p-1)/2}$ since the Riemann-Hurwitz formula fails, i.e. the mapping class group $\Gamma_{(p-1)/2}$ is p-periodic [Br].

The Brown theorem for the p-primary components of Farrell-Tate cohomology groups can be used since $\Gamma_{(p-1)/2}$ has p-periodicity: $\hat{H}^i(\Gamma_{(p-1)/2};Z)_{(p)} = \prod_{P_j \in S} \hat{H}^i(N(P_j);Z)_{(p)}$, where S is the set of conjugacy classes of Z_p in $\Gamma_{(p-1)/2}$, and $N(P_j)$ is the normalizer P_j in $\Gamma_{(p-1)/2}$ [Br].

<u>Lemma 3.1.1.</u> The number of conjugacy classes of elements of order p in $\Gamma_{(p-1)/2}$ equals $(p-1)(p+1)/6$.

Proof: The set of conjugacy classes of elements of order p in $\Gamma_{(p-1)/2}$ is in one to one correspondence with all possible fixed point data ($\beta_1, \beta_2, \beta_3$), $\beta_i \in Z_p-\{0\}$, $\beta_1+\beta_2+\beta_3 = 0 \bmod(p)$ [Sy]. Recall for x, an orientation-preserving diffeomorphism of closed orientable surface S_g of prime period p, the fixed point datum of x is an (unordered) set $\sigma(x) = (\beta_1, \beta_2, \cdots\cdots \beta_q)$, where q is the number of fixed points of x and β_i is the integer mod(p) such that x^{β_i} acts as multiplication by $e^{2\pi/p}$ in the local invariant complex structure at the i-th fixed point.

Arrange all fixed point data as follows:

Table 3.1

[1]	[2]	[3]	[4]	....
(1,1,p-2)				
(1,2,p-3)	(2,2,p-4)			
(1,3,p-4)	(2,3,p-5)	(3,3,p-6)		
(1,4,p-5)	(2,4,p-6)	(3,4,p-7)	(4,4,p-8)	
:	:	:	:	
(1,p-2,1)	EMPTY	(3,p-2,p-1)	(4,p-2,p-2)	....
EMPTY	(2,p-1,p-1)	(3,p-1,p-2)	(4,p-1,p-3)	....

[(p-3)/2]	[(p-1)/2]	[(p+1)/2]
((p-3)/2,(p-3)/2,3)		
((p-3)/2,(p-1)/2,2)	((p-1)/2,(p-1)/2,1)	
((p-3)/2,(p+1)/2,1)	EMPTY	((p+1)/2,(p+1)/2, 1)
EMPTY	((p-1)/2,(p+3)/2,p-1)	((p+1)/2,(p+3)/2,p-2)
:	:	:
(p-3)/2,p-1,(p+5)/2)	(p-1)/2,p-1,(p+3)/2)	(p+1)/2,p-1(p+1)/2)

table 3.1 (continued)

[(p+3)/2]	....	[p-2]	[p-1]
((p+3)/2,(p+3)/2,p-3)			
:			
((p+3)/2,p-2,(p+1)/2)	....	(p-2,p-2,4)	
((p+3)/2,p-1,(p-1)/2)	....	(p-2,p-1,3)	(p-1,p-1,2)

Notice:

1) Given two numbers β_1 and β_2, the third number β_3 is uniquely determined.

2) In the i-th column the first number $\beta_1 = i$, $i = 1, 2 \dots\dots p-1$.

3) The inequality $\beta_2 \leq \beta_1$ is always true.

We observe the following facts:

(a) The i-th column has an "empty box" when $i \leq (p-1)/2$ the i-th column has no "empty box" when $i > (p-1)/2$.

(b) If we count the number of fixed point data in table 3.1 ignoring order, $(\beta_1, \beta_2, \beta_3)$ occurs twice if $\beta_i = \beta_j$ for some $1 \leq i < j \leq 3$, otherwise it occurs three times. For example, (1, 1, 5) occurs twice and (1, 2, 4) occurs three times.

(c) Each fixed point datum must appear in some column.

(d) The number of fixed point data in the i-th column $= p-1-i$ for $i \leq (p-1)/2$.

(e) The number of fixed point data in the i-th column $= p-i$ for $i > (p-1)/2$.

The number of conjugacy classes of elements of order p

$= 1/3[(p-2)(p-1)/2+(p-1)/2+p-1] = (p+1)(p-1)/6.$

Remark, $(p+1)(p-1)/6$ is always an integer.

3.2. The normalizer of Z_p in $\Gamma_{(p-1)/2}$

Lemma 3.2.1. Let Z_p be in $\Gamma_{(p-1)/2}$. Then there exists an exact sequence $1 \to Z_p \to N(Z_p) \to \Gamma^3$, where Γ^3 denotes $\pi_0 \mathrm{Diff}^+(S^2;3)$, the group of path components of orientation preserving diffeomorphisms of S^2 which permute three distinguished points. (In fact, $\Gamma^3=\Sigma_3$, the symmetry group of 3 letters [Bi].)

Proof: We try to construct an injective homomorphism $I: N(Z_p)/Z_p \to \Gamma^3$.

a) Let $Z_p=\langle x\rangle$, $h \in N(\langle x\rangle)$, $hxh^{-1}=x^k$ in $\Gamma_{(p-1)/2}$. By the result of Birman and Hilden (1972)[B,H]$_2$, we can represent x and h be elements of $\mathrm{Diff}^+(S_{(p-1)/2})$ (which we also denote x and h) such that $hxh^{-1}=x^k$.

Consider the following diagram:

$$\begin{array}{ccccc} & h & & h^{-1} & \\ S_{(p-1)/2} & \rightarrow & S_{(p-1)/2} & \rightarrow & S_{(p-1)/2} \\ \downarrow p & & \downarrow p & & \downarrow p \\ S^2=S_{(p-1)/2}/Z_p & \rightarrow & S^2=S_{(p-1)/2}/Z_p & \rightarrow & S^2=S_{(p-1)/2}/Z_p \\ & f & & f^{-1} & \end{array}$$

The map h takes fiber to fiber, so h induces a map f from S^2 to S^2. Obviously f is an orientation preserving diffeomorphism since h is.

For the fixed points b_1, b_2, b_3 of diffeomorphism x, since $x^k(h(b_i)) = hx(b_i) = h(x(b_i)) = h(b_i)$, $h(b_i)$ is also a fixed point of x, i.e. $h(b_i) = b_j$ $1 \leq i,j \leq 3$. Therefore f permutes the three points $p(b_1)$, $p(b_2)$ and $p(b_3)$ in S^2. This implies $f \in \Gamma^3$.

b) I: $N(<x>)/Z_p \rightarrow \Gamma^3$ is well-defined.

If $h_1 = h_2$ in $N(<x>)$, i.e. h_1 is isotopic to h_2 in $Diff^+(-)$ and $h_1 x h_2^{-1} = x^k$, $h_2 x h_2^{-1} = x^k$, then by Birman and Hilden's result again [B,H]$_2$, there exists an isotopy $H: S_{(p-1)/2} \times I \rightarrow S_{(p-1)/2}$ such that $H_0 = h_1$, $H_1 = h_2$, $H_s x H_s^{-1} = x^k$. So, the diffeomorphisms f_1 and f_2 of S^2 induced by h_1 and h_2 are isotopic in $Diff^+(S^2,3)$. This implies $f_1 = f_2$ in Γ^3. If $h = x^k$, then f induced by h must be the identity map from S^2 to S^2.

c) I is a homomorphism by the definition of I.

d) I is an one to one.

Suppose f, induced by h, is isotopic to the identity map in $Diff^+(S^2,3)$. The homotopy lifting theorem shows that h is isotopic to the identity map up to the choice of initial point, i.e. I is a one to one.

e) In fact, $\Gamma^3 = < w_1, w_2 | w_1w_2w_1 = w_2w_1w_2, w_1w_2{}^2w_1 = 1, (w_1w_2)^3 = 1 > = < w_1, w_2 | w_1{}^2 = 1, w_2{}^2 = 1, (w_1w_2)^3 = 1 > = \Sigma_3$ = the symmetric group of 3 letters [Bi].

3.3. The action of $N(Z_p)$ on Z_p

Lemma 3.3.1. For each Z_p of $\Gamma_{(p-1)/2}$, consider the subgroup $Im(I) \subseteq \Gamma_3$ of lemma 3.2.1.

a) If $p = 6k-1$, $Im(I)$ acts trivially on Z_p.

b) If $p = 6k+1$, there exists exactly one conjugacy class of Z_p in $\Gamma_{(p-1)/2}$ such that the order 3 elements in $Im(I)$ act nontrivially on that Z_p. For other Z_p's in $\Gamma_{(p-1)/2}$, $Im(I)$ acts trivially on Z_p.

A basic number theory sublemma is needed before we complete the proof of lemma 3.3.1.

<u>Sublemma</u>. There is no integer m such that $m^2+m+1 = 0$ mod(p) for p = 6k-1, 1 < m < p. There are two integers m_1 and m_2 such that $m_i{}^2+m_i+1 = 0$ mod(p) for p = 6k+1 and 1 < m_i < p, and these satisfy $m_2 = m_1{}^2$.

Proof: Suppose m is a solution of $m^2+m+1 = 0$ mod(p), then $m \neq 1$ mod(p), otherwise, m =1 mod(p) and $m^2+m+1=0$ mod(p) implies 3 = 0 mod(p), a contradiction.

The fact that $m \neq 1$ mod(p) and $m^2+m+1 = 0$ mod(p) also imply $m^3 = 0$ mod(p), i.e. 3 divides p-1. So, for p = 6k-1, there is no solution; for p = 6k+1, there are exactly two solutions of the quadratic equation. In fact, if m is a solution, m^2 is a solution too.

<u>The proof of lemma 3.3.1</u>: Let us rearrange all the fixed point data $(\beta_1, \beta_2, \beta_3)$ for $\Gamma_{(p-1)/2}$ as follows:

Table 3.2

(1)	(2)	(3)	....	(p-1)
(1, 1, p-2)	(1, 2, p-3)	(1, 3, p-4)	....	(1, p-2,1)
(2, 2, p-4)	(2, 4, p-6)	(2, 6, p-8)	....	(2, 2p-4,2)
(3, 3, p-6)	(3, 6, p-9)	(3, 9, p-12)	....	(3, 3p-6,3)
:	:	:		:
(p-1,p-1,2)	(p-1,2p-2,3)	(p-1,3p-3,4)	...	(p-1,(p-2)(p-1),p-1)

Note that the i-th row of the table is i times the first row mod(p).

We observe two facts:

(1) Any fixed point datum is contained in table 3.2.

(2) If $(\alpha_1, \beta_1, \gamma_1) = (\alpha_2, \beta_2, \gamma_2)$ as unordered triple, where $(\alpha_1, \beta_1, \gamma_1)$ and $(\alpha_2, \beta_2, \gamma_2)$ are in different columns, then these two columns must be the same up to permutation.

In fact, for (1), any fixed point data (α, β, γ) can be multipled by m such that $m\alpha = 1$ mod(p), so (α, β, γ) must be in the $m\beta$-th column.

For (2), as unordered triple, $(\alpha_1, \beta_1, \gamma_1) = (\alpha_2, \beta_2, \gamma_2)$ implies $m(\alpha_1, \beta_1, \gamma_1) = m(\alpha_2, \beta_2, \gamma_2)$ i.e. $(m\alpha_1, m\beta_1, m\gamma_1) = (m\alpha_2, m\beta_2, m\gamma_2)$ for any m, $1 \le m \le p-1$. Therefore, these two columns are the same.

Claim: a) Assume $p > 3$. Then any two fixed point data in the m-th column are different if $m^2+m+1 \neq 0$ mod(p).

b) Three sets of fixed point data are the same in the m-th column in table 2 if $m^2+m+1 = 0$ mod(p).

Suppose $(1, m, n) = (h, hm, hn)$, $1 \neq h$ mod(p).

Case 1: $1 = hm$, $m = hn$, $n = h$ mod(p) implies $1 = n^3$, $m = n^2$ mod(p) or $m^3 = 1$ mod(p), so $m^2+m+1 = 0$ mod(p).

Case 2: $1 = hn$, $m = h$, $n = hm$ mod(p) implies $1 = m^3$ mod(p) or $m^2+m+1 = 0$ mod(p).

Note: in fact, $n = m^2$ mod(p).

Conversely, if there is a m such that $m^2+m+1 = 0 \bmod(p)$ $1 < m \leq p-1$, then $(1, m, m^2)$ is a fixed point datum in the m-th column and $(1, m, m^2) = m(1, m, m^2) = m^2(1, m, m^2)$

Therefore, three sets in the m-th column are the same.

Let x be an element of order p in $\Gamma_{(p-1)/2}$. If $\sigma(x)$, the fixed point datum of x, lies in some column of table 3.2, then all the sets in the i-th column of table 3.2 represent the fixed points data $\sigma(x^k)$, $1 \leq k \leq p-1$. Therefore, by using the claim above, if $\sigma(x)$ is in the m-column, where $m^2+m+1 = 0 \bmod(p)$, then $\sigma(x) = \sigma(x^m) = \sigma(x^{m^2})$. Otherwise, $\sigma(x^i) \neq \sigma(x^j)$ for $i \neq j$. It is well-known that $|N(\langle x \rangle)/C(\langle x \rangle)|$ = the number of i such that x^i is conjugate with x, where $1 \leq i \leq p-1$. Here $N(\langle x \rangle)$ (resp. $C(\langle x \rangle)$) denotes the normalizer (resp. centralizer) of the cyclic subgroup generated by x.

The sublemma and argument above imply that:

a) If $p = 6k-1$, $|N(Z_p)/C(Z_p)| = 1$ for all Z_p in $\Gamma_{(p-1)/2}$.

b) If $p = 6k+1$, there exists exactly one conjugacy class of Z_p in $\Gamma_{(p-1)/2}$ such that $|N(Zp)/C(Zp)| = 3$. For other Z_p in $\Gamma_{(p-1)/2}$, $|N(Zp)/C(Zp)| = 1$.

The lemma 3.2.1, a) and b) above imply lemma 3.3.1.

Corollary 3.3.2. There are k different conjugacy classes of Z_p in $\Gamma_{(p-1)/2}$ for $p = 6k-1$. There are $k + 1$ different conjugacy classes of Z_p in $\Gamma_{(p-1)/2}$ for $p = 6k+1$.

Proof: Lemmas 3.1.1 and 3.3.1 imply this corollary.

3.4. The proof of theorem 3.1

There is a short exact sequence for an odd prime p by lemma 3.2.1.

$$1 \to Z_p \to N(Z_p) \to Im(I) \to 1,$$

where $Im(I)$ is a finite subgroup of Σ_3.

Case 1: $Im(I)$ acts trivially on Z_p. By [Br,p.84], for $i > 0$, $\hat{H}^i(N(Z_p);Z)_{(p)} = H^i(N(Z_p);Z)_{(p)} = H^i(Z_p;Z)^{Im(I)} = H^i(Z_p;Z)$. Since the p-period of $N(Z_p) = 2|N(Z_p)/C(Z_p)| = 2$ [Sw], $\hat{H}^i(N(Z_p);Z)_{(p)} = \hat{H}^i(Z_p;Z)$ for all integer i.

Case 2: $Im(I)$ acts nontrivially on Z_p. By [Br,p.84] and lemma 3.3.1, for $i > 0$, $\hat{H}^i(N(Z_p);Z)_{(p)} = H^i(N(Z_p);Z)_{(p)} = H^i(Z_p;Z)^{Im(I)} = H^i(Z_p;Z)^{Z_3} = H^i(Z_p \rtimes Z_3;Z)_{(p)}$. Since the p-period of $N(Z_p) = 2|N(Z_p)/C(Z_p)| = 6$ = the p-period of $Z_p \rtimes Z_3$ (semi-direct), $\hat{H}^i(N(Z_p);Z)_{(p)} = \hat{H}^i(Z_p \rtimes Z_3;Z)_{(p)}$ for all integers i.

Finally, Brown's decomposition theorem, lemma 3.3.1 and corollary 3.3.2 imply theorem 3.1.

3.5. Birman-Hilden theory

Lemma 3.2.1 is a special case of the statement by Birman and Hilden that $N(Z_p)/Z_p$ is isomorphic to $\Gamma^n = \pi_0 \mathrm{Diff}^+(S^2;n)$ if the orbit space $S_g/Z_p = S^2$, where n is the number of fixed points of Z_p acting on S_g.

However, it seems that there exists a gap on the proof of lemma 5.1.[B,H]$_2$. In particular, the homomorphism I may not be a surjection of $N(Z_p)/Z_p$ onto Γ^n.

The claim in lemma 5.1. [B,H]$_2$ that a closed curve lifts to a closed curve if and only if it encircles a multiple of p branch points is false. Actually,

Proposition 3.5.1. For a prime p, let $p: S_g \to S^2$ be a p-sheeted branched covering map with ramification points $b_1, b_2 \cdots\cdots b_n \in S_g$, $Z_p = < T >$, T the deck transformation of p with n fixed points in S_g. Denote the fixed point datum of T as $\sigma(T) = (\beta_1, \beta_2, \beta_3, \cdots\cdots\cdots \beta_n)$. Let γ be a closed curve in $S^2 - \{p(b_1), p(b_2) \cdots\cdots\cdots p(b_n)\}$, then γ lifts to a closed curve in $S_g - \{b_1, b_2, \cdots\cdots\cdots b_n\}$ if and only if the homotopy class $[\gamma] \in \pi_1(S^2 - \{p(b_1), p(b_2), \cdots p(b_n)\})$ $= < x_1, x_2, \cdots\cdots\cdots x_n \mid x_1 x_2 \cdots\cdots\cdots x_n = 1 >$ can be written in the form $[\gamma] = \prod x_i^{n_i} \prod x_i^{m_i} \cdots\cdots \prod x_i^{k_i}$ such that $\Sigma(n_i\beta_i + m_i\beta i + \cdots\cdots + k_i\beta_i) = 0 \bmod(p)$ where x_i is represented by a simple closed curve c_i such that $p(b_i)$ is in one complementary component of c_i and $p(b_j)$ $j \neq i$, are in the other.

Proof: In fact, the covering map $p: S_g - \{b_1, b_2, \cdots\cdots b_n\} \to S^2 - \{p(b_1), p(b_2), \cdots\cdots\cdots p(b_n)\}$ is classfied by the homomorphism $f: \pi_1(S^2 - \{p(b_1), p(b_2), \cdots\cdots\cdots p(b_n)\}) = < x_1, x_2, \cdots\cdots\cdots x_n \mid x_1x_2\cdots\cdots\cdots x_n = 1 >$ into $Z_p = < T >$, where $f(x_i) = T^{\beta_i}$, $1 \le i \le n$ [Ew].

Recall that a closed curve γ lifts to a closed curve if and only if $[\gamma] \in p^*[\pi_1(S_g - \{b_1, b_2, \cdots\cdots\cdots b_n\})]$, i.e. $f([\gamma]) = 0$ in Z_p, or $T^{\Sigma(n_i\beta_i + m_i\beta_i + \cdots\cdots + k_i\beta_i)} = 0$ if $[\gamma] = \Pi x_i^{n_i} \Pi x_i^{m_i} \cdots\cdots \Pi x_i^{k_i}$. This implies $\Sigma(n_i\beta_i + m_i\beta_i + \ldots\ldots + k_i\beta_i) = 0 \bmod(p)$.

<u>Example 1</u>. Let $p: S_2 \to S^2$ be a 5-sheeted branched covering map with 3 branch points b_1, b_2 and b_3, T the deck transformation of p with the fixed points b_1, b_2 and b_3 in S_2, and the fixed point datum $\sigma(T) = (1,3,1)$. If γ, an orientable closed curve in $S^2 - \{p(b_1), p(b_2), p(b_3)\}$, goes around $p(b_1)$ twice, around $p(b_2)$ once and not around $p(b_3)$, then γ lifts to $S_2 - \{b_1, b_2, b_3\}$, although γ encircles the two branched points(or three if counted with multiplicity).

Consider the diffeomorphism w of S^2, which switches $p(b_1)$ and $p(b_2)$ in a disk D containing $p(b_1)$ and $p(b_2)$, and $w = Id$ in $S^2 - D$. Then $w(\gamma)$ can not lift to $S_2 - \{b_1, b_2, b_3\}$ since $w(\gamma)$ goes around $p(b_1)$ once, around $p(b_2)$ twice and not around p(b3), $n_1\beta_1 + n_2\beta_2 + n_3\beta_3 = 1+6 = 7 \ne 0 \bmod(5)$, i.e. the w can not lift to a diffeomorphism of S_2. So the w, as an element of Γ^3, can not be in the image Im(I) of lemma 3.2.1.

Example 2. The special case when p = 2 $[B,H]_1$ is valid. Consider the 2-sheeted covering map $p: S_g \to S^2$ with n branch points. Now $\beta_1 = \beta_2 = \cdots\cdots = \beta_n = 1$ and a closed curve γ in $S^2 - \{p(b_1), p(b_2), \cdots\cdots p(b_n)\}$ lifts if and only if $[\gamma] = \prod x_i^{n_i} \prod x_i^{m_i} \cdots\cdots \prod x_i^{k_i}$, $\Sigma(n_i+m_i+\cdots\cdots k_i) = 0$ mod(2). For any diffeomorphism w of S^2 permuting the set $\{p(b_1), p(b_2), \cdots\cdots p(b_n)\}$, the closed curve $w(\gamma)$ lifts if and only if γ lifts. So w lifts to a diffeomorphism of S_g permuting the set $\{b_1, b_2, \cdots\cdots b_n\}$. This shows that $N(Z_2)/Z_2 = C(Z_2)/Z_2 = \Gamma^n$ by an argument similar to lemma 3.2.1.

CHAPTER IV

A FAMILY OF HOMOGENEOUS CHERN CLASS POLYNOMIALS OF MAPPING CLASS GROUPS

Some torsion for $H^i(\Gamma_g;Z)$ ($g >> i$) has been worked out by Glover and Mislin in terms of the Chern classes of the canonical homology representation $\eta\colon \Gamma_g \to GL(2g,Z)$ [G,M].

In this chapter, we construct a family of homogeneous Chern class polynomials of the canonical homology representation $\Gamma_g \to GL(2g,Z)$, which depend only upon the genus g and the prime p so that the restrictions of the homogeneous Chern class polynomial to all Z_p inclusions in Γ_g are nontrivial. Therefore the upper bounds of the Yagita's invariant $p(\Gamma_g)$ [Y], in particular, the upper bounds of the p-period for p-periodic mapping class groups Γ_g, may be obtained.

The main results are as follows:

Theorem 4.1. Let $2g-2 = mp-i$, $0 \le i \le p-1$, p an odd prime, and $p^{r-1} \le m \le p^r$, $i\colon Z_p \to \Gamma_g$ an inclusion,

$\eta: \Gamma_g \to GL(2g,C)$ the canonical homology representation, $\varphi(p^r) = p^{r-1}(p-1)$ the Euler totient function.

a) If $[2g/(p-1)] < p^r$, $i^*[c_{\varphi(p^r)}(\eta)]$ has order p for every Z_p in Γ_g. As a result, the Yagita invarant $p(\Gamma_g)$ divides $2p^{r-1}(p-1)$.

b) If $[2g/(p-1)] \geq p^r$, $i^*\{[c_{\varphi(p^r)}(\eta)]^{p(p-1)}+[c_{\varphi(p^{r+1})}(\eta)]^{p-1}\}$ has order p for every Z_p inclusion in Γ_g. As a result, the Yagita invarant $p(\Gamma_g)$ divides $2p^r(p-1)^2$.

Corollary 4.2. In addition, Γ_g is p-periodic and

a) If $[2g/(p-1)] < p^r$, the p-period of Γ_g divides $2p^{r-1}(p-1)$

b) If $[2g/(p-1)] \geq p^r$, the p-period of Γ_g divides $2p^r(p-1)^2$.

Theorem 4.3. Let $2^{s-1} \leq g \leq 2^s$. Then $i^*\{[c_{2^{s-1}}(\eta)]^2 + c_{2^s}(\eta)\}$ has order 2 for every Z_2 in Γ_g.

4.1. The Yagita invariant and the p-period of a group of finite vcd

Recall that Yagita defined an invariant in 1985 [Y], denoted p(G), for a finite group G as follows:

Let $i: Z_p \to G$ be an inclusion of an order p subgroup in a finite group G, then $Im(i^*: \bar{H}^*(G;Z) \to \bar{H}^*(Z_p;Z)) \neq 0$. Recall the fact that $H^*(Z_p;Z) = Z[u]/\langle pu \rangle$, where $u \in H^2(Z_p;Z)$. Consider the reduction map $P: Z[u]/\langle pu \rangle \to$

$Z_p[u]$ by P: $Z \to Z_p$. These implies that there is a maximum value m such that the $Im(H^*(G;Z)) \subseteq Z_p[u^m]$. Define p(G) as the least common multiple of the values 2m for all inclusions $Z_p \to G$.

Thomas [Th] comments that the Yagita invariant can be extended to the Farrell-Tate cohomology [F] for group Γ of finite vcd with finitely many conjugacy classes of order p subgroups by the same defintion for p an odd prime. In fact, there exists a torsion free, normal and finite index subgroup N such that $\Gamma/N = M$. Consider the following diagram, for the finite group M, choosing a representation η: $M \to GL(n,C)$ so that $\eta(Z_p)$ is nontrivial for subgroup Z_p [Se], then $i^*(c_m(\eta)) \neq 0$ in $H^*(Z_p;Z)$ for at least one $m > 0$ where the element $c_m(\eta) \in H^{2m}(M;Z)$ is the Chern class associated to the representation η.

Note $Im(\bar{H}^*(M;Z)) \neq 0 \in \bar{H}^*(Z_p;Z)$ implies $Im(\hat{H}^*(\Gamma;Z)) \neq 0 \in \hat{H}^*(Z_p;Z)$ for some $* > 0$.

$$\begin{array}{ccccc}
 & & \hat{H}^*(\Gamma;Z) & \xrightarrow{i^*} & \hat{H}^*(Z_p;Z) \\
 & & \uparrow \pi^* & & \uparrow || \\
 & & \hat{H}^*(M;Z) & \xrightarrow{i^*} & \hat{H}^*(Z_p;Z) \\
 & & \uparrow g^* & & \uparrow g^* \\
H^*(GL(n,C);Z) & \xrightarrow{\eta} & H^*(M;Z) & \xrightarrow{i^*} & H^*(Z_p;Z)
\end{array}$$

Note $\hat{H}^*(Z_p;Z) = Z_p[u,u^{-1}]$. Similarly, there exists a maximum value m such that $\mathrm{Im}(i^*:\hat{H}^*(\Gamma;Z) \to \hat{H}^*(Z_p;Z)) \subsetneq Z_p[u^m,u^{-m}]$, where $0 \neq u \in \hat{H}^2(Z_p;Z)$. The Yagita invariant of group Γ of finite vcd, denoted $p(\Gamma)$, is defined as the least common multiple of values 2m for all inclusions $Z_p \to \Gamma$, where p is an odd prime.

The Yagita invariant $p(\Gamma)$ shares many nice properties of the p-period of the groups [Y], [Th]. For example,

a) p(abelian group) = 2.

b) p(H) divides p(G), if H is a subgroup of G.

Proposition 4.1.1. If Γ is a p-periodic group of finite vcd, $p(\Gamma)$ = the p-period of Γ.

If Γ is a p-periodic group of finite vcd, by Brown's theorem, $\hat{H}^*(\Gamma;Z)_{(p)} = \prod_{P_i \in S} \hat{H}^*(N(P_i);Z)_{(p)}$ where S denotes the set of conjugacy classes of Z_p in Γ, $N(P_i)$ denotes the normalizer of the subgroup P_i. Obviously, the p-period of Γ equals $\mathrm{LCM}_{P_j \in S}$\{the p-period of $N(P_i)$\}; the Yagita invariant $p(\Gamma)$ equals $\mathrm{LCM}_{P_j \in S}\{p(N(P_i))\}$. So we only need to show the p-period of Γ equals $p(\Gamma)$ in the case Γ containing only one conjugacy class of Z_p.

Lemma 4.1.2. $p(\Gamma)$ = the greatest common divisor of dimensions of $\hat{H}^*(Z_p;Z)$ whose elements are hit by $i^*: \hat{H}^*(\Gamma;Z) \to \hat{H}^*(Z_p;Z)$.

Proof: In fact, let $2m = p(\Gamma)$, if $u^n \in \hat{H}^{2n}(Z_p;Z)$ is hit by the element $x \in \hat{H}^*(\Gamma;Z)$, i.e. $i^*(x) = u^n$, $u^n \in Im(\hat{H}^*(\Gamma;Z)) \subseteq Z_p[u^m,u^{-m}]$, then m divides n.

Conversely, if d divides all dimensions of $\hat{H}^*(Z_p;Z)$ whose elements are hit by $i^*: \hat{H}^*(\Gamma;Z) \to \hat{H}^*(Z_p;Z)$, then $i^*: \hat{H}^*(\Gamma;Z) \to Z_p[u^d,u^{-d}]$.

<u>The proof of proposition 4.1.1</u>: a) The p-period of Γ divides $p(\Gamma)$. We want to show that $2m = p(\Gamma)$ is a p-period of Γ. In fact, if the element in dimension 2n of $\hat{H}^*(Z_p;Z)$ is hit by $i^*: \hat{H}^*(\Gamma;Z) \to \hat{H}^*(Z_p;Z)$, then 2n is a p-period of Γ by Brown-Venkov theorem [B]. We can suppose $2m = 2a_1n_1+2a_2n_2+\cdots\cdots 2a_kn_k$ by lemma 4.1.2, where $2n_i$ are the dimensions where there exist elements $x_i \in \hat{H}^{2n_i}(\Gamma;Z)$ such that $i^*(x_i) \neq 0$ in $\hat{H}^*(Z_p;Z)$, a_i are integers, $1 \leq i \leq k$, and x_i are invertable in $\hat{H}^*(\Gamma;Z)$. Consider the element $\Pi_{1\leq i\leq k} x_i^{a_i} \in \hat{H}^{2m}(\Gamma;Z)$, $i^*(\Pi_{1\leq i\leq k} x_i^{a_i}) = \Pi_{1\leq i\leq k} i^*(x_i)^{a_i} \neq 0$ in $\hat{H}^{2m}(Z_p;Z)$. i.e. 2m is a p-period of Γ.

b) $p(\Gamma)$ divides the p-period of Γ. Let 2d = the p-period of Γ. Because if there exists an element $x \in \hat{H}^{2d}(\Gamma;Z)$ such that $i^*(x) \neq 0$ in $\hat{H}^*(Z_p;Z)$. then 2m divides 2d by lemma 4.1.2.

4.2. The fixed points number set of Z_p action on the surface S_g

For $g > 1$, p odd prime, let $2g-2 = mp-i$, $0 \le i \le p-1$. The m and i are determined uniquely by g and p. Also, define sets $B_{g,p} = \{ i, i+p, i+2p, \cdots\cdots i+([2g/(p-1)]-m)p\}$ if $i \ne 1$. $B_{g,p} = \{ 1+p, 1+2p, \cdots\cdots 1+([2g/(p-1)]-m)p\}$ if $i=1$.

Note, for $i \ne 1$, $2g/(p-1) < m$, define $B_{g,p} = \emptyset$, and for $i = 1$, $2g/(p-1) < m+1$, define $B_{g,p} = \emptyset$.

Lemma 4.2.1. If $\langle x\rangle = Z_p$ acts on surface S_g, and $2g-2 = mp-i$, $0 \le i \le p-1$, then the number t of fixed points of x belongs to $B_{g,p}$. Conversely, any number $t \in B_{g,p}$ can appear as the number of fixed points of a diffeomorphism x on surface S_g, $x^p = 1$.

Proof: If $Z_p = \langle x\rangle$ acts on S_g, Riemann-Hurwitz formula $2g-2 = (2\eta-2)p+tp(1-1/p)$ implies $t = 2(g-\eta)/(p-1)-2\eta-2$ $= n-(2g-n(p-1))+2 = np-2g+2$. Here $g-\eta = n(p-1)/2$. Here n is an integer and $n \le [2g/(p-1)]$ since $\eta \ge 0$. Therefore, $t \equiv -2g+2 \equiv i \mod(p)$ and $0 \le t \le ([2g/(p-1)]-m)p+i$, i.e. $t \in B_{g,p}$. Notice, if $i=1$, $t \ne 1$, since the number of fixed points of Z_p action can not be 1.

Conversely, if $t \in B_{g,p}$, i.e. $t = i+kp$, where $0 \le k \le [2g/(p-1)]-m$ if $i \ne 1$; $0 < k \le [2g/(p-1)]-m$ if $i = 1$.

Let $n = k+m \le [2g/(p-1)]$, then $\eta = g-n(p-1)/2$

$= g-(k+m)(p-1)/2 \geq g-g = 0$. Write $Z_p = \langle x\rangle = \langle x_1, x_1^{-1}, \cdots x_\eta, x_\eta^{-1}, x_{\eta+1}, \cdots\cdots x_{\eta+t-1}, x^{-(t-1)}\rangle$ for $t \neq 1 \bmod(p)$, $x_j = x$, $0 < j < \eta+t$; $Z_p = \langle x\rangle = \langle x_1, x_1^{-1}, \cdots\cdots x_\eta, x_\eta^{-1}, x^2{}_{\eta+1}, \cdots\cdots x_{\eta+t-1}, x^{-t}\rangle$ for $t = 1 \bmod(p)$. We know $(2\eta-2)p+(p-1)t = (2g-n(p-1)-2)p+(p-1)(i+kp) = mp-i = 2g-2$, since $n = k+m$, i.e. we actually construct Z_p action on surface S_g such that the number of fixed points of x is in the set $B_{g,p}$.

Lemma 4.2.2. Let Z_p act on S_g, $\rho=\eta i$: $Z_p \rightarrow \Gamma_g \rightarrow GL(2g,Q)$ is a representation for any inclusion $i: Z_p \rightarrow \Gamma_g$. Here $\eta: \Gamma_g \rightarrow GL(2g,Q)$ is the canonical homology rational representation. Then ρ is equivalent to one of the representations below up to complex representation: $\rho_k = (m+k)\sigma_p \oplus nTr$. Here $2g-2 = mp-i$, $0 \leq i \leq p-1$. If $i\neq 1$, $0 \leq k \leq [2g/(p-1)]-m$; If $i = 1$, $1 \leq k \leq [2g/(p-1)]-m$. $n = 2g-(m+k)(p-1)$. σ_p and Tr are the cyclotomic and trivial representations of Z_p. Conversely, any $\rho_k = (m+k)\sigma_p \oplus nTr$ can be equivalent to $\rho = \eta i$ for some inclusion $i: Z_p \rightarrow \Gamma g$.

Proof: On the one side, we calculate characteristic number χ of ρ_k, $\rho_k = (m+k)\sigma_p \oplus nTr$, $\chi_{\rho_k}(Id)=(m+k)(p-1)+n=2g$, $\chi_{\rho_k}(x) = (m+k)+n = 2g-(m+k)p = 2-i-kp$. Here $\langle x\rangle = Z_p$. On the other side, by using Lefschetz fixed point theorem, for any $\rho=\eta i$, $\chi_\rho(x) = 2-t \in 2-B_{g,p} = \{2-i-kp\}$, $\chi_\rho(Id) = 2g$.

lemma 4.2.1 implies $\chi_\rho = \chi_{\rho_k}$ for some k and there exists Z_p representation ρ such that $\chi_\rho = \chi_{\rho_k}$ for every k. Namely Lemma 4.2.2 holds.

4.3. Basic number theory lemmas

Lemma 4.3.1. Let $2g-2 = mp-i$, p an odd prime, $0 \le i \le p-1$. If $m < p^r$, then $[2g/(p-1)] < p^{r+1}- p^r$.

Proof: Because of $m < p^r$, we have $2g-2+i = mp < p^{r+1}$, or $2g/(p-1) < (p^{r+1}+2)/(p-1)$. $(p^{r+1}+2)/(p-1) \le p^{r+1}-p^r$ since $p^{r+1}+2 \le p^{r+2}-p^{r+1}-p^{r+1}+p^r$. i.e. $3p^{r+1}+2 \le p^{r+2}+p^r$, this is true since $p \ge 3$, $r \ge 1$.

Lemma 4.3.2. The integer $n!/p^r!(n-p^r)! \equiv k \mod(p)$ if $kp^r \le n < (k+1)p^r$, where $1 \le k \le p-1$.

Proof: We consider the integer $n!/p^r!(n-p^r)!$ as an element of F_p , then do multiplication and division operations in F_p as follows:

In fact, $n!/p^r!(n-p^r)! = [(p^r+1)/(1)][(p^r+2)/(2)]$ $[(p^r+3)/(3)] \cdots\cdots [(p^r+p)/(p)] [(p^r+p+1)/(p+1)] \cdots\cdots$ $[(p^r+p^2)/(p^2)] \cdots\cdots [(p^r+p^r)/(p^r)][(p^r+p^r+1)/(p^r+1)]$ $\cdots\cdots[(p^r+2p^r)/(2p^r)][(p^r+2p^r+1)/(2p^r+1)] \cdots\cdots$ $[(p^r+3p^r)/(3p^r)] \cdots\cdots [(kp^r)/(k-1)p^r] \cdots\cdots [(n)/(n-p^r)]$.

If $i \ne 0 \mod(p)$, then $[(p^r+i)/(i)] =1$ in the field F_p.

If $i = 0 \mod(p^{s-1})$ and $i \ne 0 \mod(p^s)$, $s \le r$, then

$[(p^r+i)/(i)] = 1$ in the field F_p.

If $i = mp^r$, $m = 0, 1, 2, \cdots\cdots, k-1$, then $[(p^r+i)/(i)] = [(m+1)/(m)]$ in F_p.

So, $n!/p^r!(n-p^r)! = [1][1]\cdots\cdots [(2)/(1)]\cdots[(k)/(k-1)] = [k]$ in F_p, i.e. the integer $n!/p^r!(n-p^r)! = k \bmod(p)$ where $kp^r \leq n < (k+1)p^r$.

4.4. The Chern class polynomials of mapping class groups for p an odd prime.

For a complex representation $\rho:G \to GL(n,C)$ of discrete group G the Chern classes $C_i(\rho) \in H^{2i}(G;Z)$ are defined as Chern classes of the flat C^n-bundle over $K(G,1)$ classified by B_ρ: $K(G,1) \to BGL(n,C)$ [E,M].

In this section, we assume that p is an odd prime. Let σ_p be the rational cyclotomic representation of Z_p. It is well-known that Chern classes $c_k(\sigma_p) = 0$ if $1 \leq k < p-1$ and $c_{p-1}(\sigma_p)$ has order p.

Proof of theorem 4.1: a) If $[2g/(p-1)] < p^r$, let ρ_k: $Z_p \to \Gamma_g \to GL(2g,C)$ be a representation corresponding to $\rho_k = (m+k)\sigma_p \oplus nTr$, here $0 \leq k \leq [2g/(p-1)]-m$.

The total Chern class C satisfies $C(\rho_k) = C(\sigma_p)^{m+k} = [1+c_{p-1}(\sigma_p)]^{m+k} = \Sigma_{0\leq t\leq m+k}(m+k)!/t!(m+k-t)![c_{p-1}(\sigma_p)]^t$.

Therefore, $c_{\varphi(p^r)}(\rho_k) = (m+k)!/p^{r-1}!(m+k-p^{r-1})! [c_{p-1}(\sigma_p)]^{p^{r-1}}$, $i^*c_{\varphi(p^r)}(\eta) = c_{\varphi(p^r)}(\rho_k) = (m+k)!/p^{r-1}!$

$(m+k-p^{r-1})!\,[c_{p-1}(\sigma_p)]^{p^{r-1}} \neq 0 \bmod(p)$ in $H^{2\varphi(p^r)}(Z_p;Z)$ for every inclusion $i: Z_p \to \Gamma_g$ since $m+k \leq [2g/(p-1)] < p^r$ by lemma 4.3.1. By using the definition of the Yagita invariant, $p(\Gamma_g)$ divides $2p^{r-1}(p-1)$. Notice the notations η^* and i^* as follows:

$$H^{2\varphi(p^r)}(GL(2g,C);Z) \xrightarrow{\eta^*} H^{2\varphi(p^r)}(\Gamma_g;Z) \xrightarrow{i^*} H^{2\varphi(p^r)}(Z_p;Z).$$

b) If $[2g/(p-1)] \geq p^r$, $i^*\{[c_{\varphi(p^r)}(\eta)]^{p(p-1)}+[c_{\varphi(p^{r+1})}(\eta)]^{p-1}\}$

$= [c_{\varphi(p^r)}(\rho_k)]^{p(p-1)}+[c_{\varphi(p^{r+1})}(\rho_k)]^{p-1}$

$= [(m+k)!/p^{r-1}!(m+k-p^{r-1})!]^{p(p-1)}[c_{p-1}(\sigma_p)]^{p^r(p-1)}$

$+[(m+k)!/p^r!(m+k-p^r)!]^{p-1}[c_{p-1}(\sigma_p)]^{p^r(p-1)}$.

Now, if $m+k < p^r$, the second term above vanishes, and the first term is nontrivial by lemma 4.3.2 since $p^{r-1} \leq m+k < p^r$.

If $m+k \geq p^r$, then $m+k \leq [2g/(p-1)] < p^{r+1} - p^r$ by lemma 4.3.1.

It implies $(m+k)!/p^r!(m+k-p^r)! \neq 0 \bmod(p)$, therefore, the second term above always equals 1 mod(p) and the first term above is 0 or 1 mod(p). i.e. $i^*\{[c_{\varphi(p^r)}(\eta)]^{p(p-1)}+[c_{\varphi(p^{r+1})}(\eta)]^{p-1}\}$ is of order p. By the definition of Yagita invariant, $p(\Gamma_g)$ divides $2p^r(p-1)^2$.

<u>The proof of corollary 4.2</u>: This follows from theorem 4.1 and proposition 4.1.1.

<u>Remark</u>: The upper bounds of the p-period in cor. 4.2.

are a litter bit rough. They can be improved by individually computing the Chern classes of the homology representation of Γ_g in same way.

Example: Consider the 3-periodic group Γ_3, $Z_3 \to \Gamma_3$ an inclusion, the number of possible fixed points are 2 or 5, i.e. the associated representations are $\rho_1 = \eta i_1 = 2\sigma_3 \oplus 2Tr$ or $\rho_2 = \eta i_2 = 3\sigma_3$. But, $i_1{}^*c_2(\eta) = c_2(\rho_1) = c_2(2\sigma_3 \oplus 2Tr)$ $= 2c_2(\sigma_3)$, $i_1{}^*c_6(\eta) = c_6(\rho_1) = c_6(2\sigma_3 \oplus 2Tr) = 0$.Therefore, $i_1{}^*\{[c_2(\eta)]^3 + c_6(\eta)\} = 2[c_2(\sigma_3)]^3$ is nontrivial. Similarly, $i_2{}^*c_2(\eta) = c_2(\rho_2) = c_2(3\sigma_3) = 3c_2(\sigma_3) = 0 \bmod(3)$, $i_2{}^*c_6(\eta)$ $= c_6(\rho_2) = c_6(3\sigma_3) = [c_2(\sigma_3)]^3$. So, $i_2{}^*\{[c_2(\eta)]^3 + c_6(\eta)\} =$ $[c_2(\sigma_3)]^3$ is nontrivial, i.e. the element $[c_2(\eta)]^3 + c_6(\eta)$ $\in H^{12}(\Gamma_g, Z)$ is nontrivial when restricted to every Z_3 subgroups. If we use canonical map from the Farrell-Tate cohomology to ordinary cohomology, we obtain the upper bound 12 of the 3-period of Γ_3 by the Brown-Venkov theorem $[Br]_2$.

Comparing the upper bound given in the corollary 4.2 and the upper bound of the computation above. Now $g = 3$, $p = 3$, $m = 2$, $i = 2$, $3^0 < 2 < 3^1$, so $r = 1$, $[2(3)/(3-1)] = 3$ satisfing b) in corollary 4.2. The upper bound 24 given by corollary 4.2 is bigger than 12 given by our computation by hand above.

4.5. The Chern class polynomials of mapping class groups for p = 2

The group Γ_g is never 2-periodic, however we still can construct Chern class polynomials of mapping class groups Γ_g which are nontrivial when restricted to every Z_2 inclusion in ordinary cohomology.

Now p = 2, if g = odd, let $B_g = \{0, 4, 8, \cdots\cdots 2g+2\}$; if g = even, let $B_g = \{2, 6, \cdots\cdots 2g+2\}$. Similar to the case p an odd prime, we have results as following:

Lemma 4.5.1. If $\langle x \rangle = Z_2$ acts on surface S_g, then the number t of fixed points of x belongs to B_g. Conversely, any number $t \in B_g$ can be realized as the number of fixed points of an order 2 orientation preserving homeomorphism x on the surface S_g.

Proof: Riemann Hurewitz formular $2g-2 = 2(2\eta-2) +2(1-1/2)t$ forces $t = 2g-4\eta+2$, which implies $t \in B_g$.

Conversely, if $t \in B_g$, then $\eta = (2g+2-t)/4$ is an integer, and $\eta \geq 0$. Write $Z_2 = \langle x \mid x^2 = 1 \rangle = \langle x_1, x_1^{-1}, \cdots \cdots x_\eta, x_\eta^{-1}, x_{\eta+1}, x_{\eta+2}, \cdots\cdots x_{\eta+t-1}, x^{-(t-1)} \rangle$. Here $x_i = x$, for $0 \leq i \leq \eta+t-1$. This shows that Z_2 acts on S_g.

Lemma 4.5.2. Let $\rho = \eta i\colon Z_2 \to \Gamma_g \to GL(2g, Q)$ be a representation for any Z_2 inclusion. Then, if g is even, ρ

is equivalent to $\rho_i = (g+2i)\sigma_2 \oplus (g-2i)Tr$; if g odd, ρ is equivalent to $\rho_i = (g+2i-1)\sigma_2 \oplus (g-2i+1)Tr$ up to complex representation. Here $0 \le i \le g/2$ or $0 \le i \le (g+1)/2$, σ_2 is the cyclotomic representation.

Conversely, ρ_i can be realized from Z_2 as a subgroup of Γ_g.

Proof: In fact, $\chi_\rho(id) = 2g$, $\chi_\rho(x) = 2-t \in 2-B_{p,g}$, the set with element 2 minus the elements of $B_{p,g}$. And $\chi_{\rho_i}(id) = 2g$, $\chi_{\rho_i}(x) = -(g+2i)+(g-2i) = -4i$ or $\chi_{\rho_i} = -(g+2i-1) +(g-2i+1) = -4i+2$, here $0 \le i \le g/2$, or $0 \le i \le (g+1)/2$, then by lemma 4.5.1.

It is well-known that $c_0(\sigma_2) = 1, c_1(\sigma_2)$ is of order 2 and $c_k(\sigma_2) = 0$, if $k > 1$. Now we suppose $2^{s-1} \le g < 2^s$.

The proof of theorem 4.3: In fact,

$(g+2i-1)!/2^{s-1}!(g+2i-1-2^{s-1})!=1 \mod(2)$ if $g+2i-1<2^s$.

$(g+2i-1)!/2^{s-1}!(g+2i-1-2^{s-1})!=0 \mod(2)$ if $g+2i-1\ge 2^s$.

$(g+2i)!/2^{s-1}!(g+2i-2^{s-1})!=1 \mod(2)$ if $g+2i<2^s$.

$(g+2i)!/2^{s-1}!(g+2i-2^{s-1})!=0 \mod(2)$ if $g+2i\ge 2^s$.

$(g+2i-1)!/2^s!(g+2i-1-2^s)!=0 \mod(2)$ if $g+2i-1<2^s$.

$(g+2i-1)!/2^s!(g+2i-1-2^s)!=1 \mod(2)$ if $g+2i-1\ge 2^s$.

$(g+2i)!/2^s!(g+2i-2^s)!=0 \mod(2)$ if $g+2i<2^s$.

$(g+2i)!/2^s!(g+2i-2^s)!=1 \mod(2)$ if $g+2i\ge 2^s$.

So, for g an odd number, $i^*\{[c_{2^{s-1}}(\eta)]^2+c_{2^s}(\eta)\}$

$= [(g+2i-1)!/2^{s-1}!(g+2i-1-2^{s-1})!]^2[c_1(\sigma_2)]^{2^s}+[(g+2i-1)!/2^s!$

$(g+2i-1-2^s)!][c_1(\sigma_2)]^{2^s} = [c_1(\sigma_2)]^{2^s} \bmod(2)$.

For g an even number, $i^*\{[c_{2^s-1}(\eta)]^2+c_{2^s}(\eta)\}$

$= [(g+2i)!/2^{s-1}!(g+2i-2^{s-1})!]^2[c_1(\sigma_2)]^{2^s}+[(g+2i)!/2^s!$

$(g+2i-2^s)!][c_1(\sigma_2)]^{2^s} = [c_1(\sigma_2)]^{2^s} \bmod(2)$.

Those imply that $[c_{2^s-1}(\eta)]^2+c_{2^s}(\eta) \in H^{2^{s+1}}(\Gamma_g)$ may be restricted nontrivially for every Z_2 inclusion in Γ_g.

4.6. The inequality on the Yagita invariant of the mapping class group

By choosing a sequence of suitable mapping class groups Γ_g which contain metacyclic subgroups with big Yagita invariants(not p-periodic) and applying for theorem 4.1 we have interesting inequalities.

Theorem 4.4: Let $g = p^{r-1}(p-1)(p^r-1)/2+1-p^r$, $r \geq 2$, $p \geq 3$. φ the Euler function. Then $2\varphi(p^r) \leq p(\Gamma_g) \leq 2\varphi(p^{2r-1})$.

Proof: Since $2g-2 = p^{r-1}(p-1)(p^r-1)-2p^r = -2p^{2r-1}(p-1) +p^{2r-1}(p-1)(1-1/p^r)+2p^{2r-1}(p-1)(1-1/p^{r-1}(p-1))$, the proposition 2.1.1 shows that there exists a $Z_{p^r} \rtimes Z_{(p-1)p^{r-1}}$ action on S_g with two order $p^{r-1}(p-1)$ and one order p^r singular points. In fact, $Z_{p^r} \rtimes Z_{(p-1)p^{r-1}} = \langle x,y \mid x^{p^r}=1, y^{(p-1)p^{r-1}}= 1, yxy^{-1} = x^r \rangle$, where $r^{(p-1)p^{r-1}} = 1 \bmod(p^r)$. Write $Z_{p^r} \rtimes Z_{(p-1)p^{r-1}} = \langle x,y,(xy)^{-1} \rangle$.

Claim: $p(Z_{p^r} \rtimes Z_{(p-1)p^{r-1}})$ is a multiple of $2\varphi(p^r)$. Consider the restriction $i^*: H^{2n}(Z_{p^r} \rtimes Z_{(p-1)p^{r-1}};Z) \to H^{2n}(Z_{p^r};Z) \to H^{2n}(Z_p;Z)$. For an element $z \in H^{2n}(Z_{p^r};Z)$, there exists an element $x \in H^{2n}(Z_{p^r} \rtimes Z_{(p-1)p^{r-1}};Z)$ such that $i^*(x) = z$ only if z is a stable element (See chapter 1, p.15). Denote $\langle x \rangle = Z_{p^r}$, however, $yxy^{-1} = x^r$ induces $y^*zy^{*-1} = r^n z$. z is stable if and only if $n = 0 \mod(\varphi(p^r))$ since $r^{\varphi(p^r)} = 1 \mod(p^r)$. Therefore, $p(Z_{p^r} \rtimes Z_{(p-1)p^{r-1}})$ is a multiple of $2\varphi(p^r)$, i.e. the Yagita invariant $p(\Gamma_g) \geq 2\varphi(p^r)$.

On the other hand, $2g-2 = p^{r-1}(p-1)(p^r-1)-2p^r = mp-i$, $0 \leq i \leq p-1$, $r \geq 2$, $p \geq 3$. This forces $m = p^{r-2}(p-1)(p^r-1) -2p^{r-1}$, $i = 0$. Then $m = -2p^{r-1}+p^{2r-1}-p^{2r-2}+p^{r-2}-p^{r-1} = -3p^{r-1}+p^{2r-1}-p^{2r-2}+p^{r-2} \geq p^{2r-2}$. The last inequality follows because of $2r-3 \geq r-1$, $p^{2r-3} \geq p^{r-1}$, $p^{2r-2} \geq 3p^{r-1}$, therefore $p^{2r-1}+p^{r-2} = pp^{2r-2}+p^{r-2} \geq 2p^{2r-2}+p^{2r-2} \geq 2p^{2r-2} +3p^{r-1}$, i.e. $p^{2r-2} \leq m \leq 2g/(p-1) = p^{r-1}(p^r-1)+2(1-p^r)/(p-1) < p^{2r-1}$. By theorem 4.1, $p(\Gamma_g) \leq 2\varphi(p^{2r-1})$.

CHAPTER V

THE p-PERIOD OF A GROUP OF VIRTUAL FINITE COHOMOLOGICAL DIMENSION

Swan showed the following result for finite groups:

Theorem (Swan) [Sw]

a) If the 2-sylow subgroup of G is cyclic, the 2-period is 2. If the 2-sylow subgroup of G is a (generalized) quaternion group, the 2-period is 4.

b) Suppose p an odd prime and the p-sylow subgroup of finite group G is cyclic. Let S_p denote a p-sylow subgroup and A_p the group of automorphisms of S_p induced by inner automorphism of G. Then the p-period of G is twice the order of A_p.

Remark: The group A_p above is isomorphic to $N(S_p)/C(S_p)$, where N and C denote the normalizer and centralizer of S_p in G.

Question: If Γ is a p-periodic group of finite vcd, are similar results still true at least with some assumption about Γ?

In other words, is it possible to describe the p-period of the group Γ of finite vcd by algebraic "non-homological" invariant of group Γ itself?

In Burgisser's thesis (1979) [Bu], he obtained the following result:

If Γ has a finite p-periodic quotient M with torsion free kernel, then the group Γ is p-periodic and the p-period of Γ divides the p-period of quotient M.

Our goal in this chapter is to generalize Swan's results for finite group to the p-periodic group Γ which has a finite quotient whose p-Sylow subgroup is elementary abelian or cyclic, and the kernel being torsion free, i.e. the p-period of group Γ, which is an homological invariant, will be completely determined as a non-homological invariant of the group Γ itself in these two cases. Finally, an application will be made for calculating the p-periods of mapping class groups.

5.1. The main results

Theorem 5.1. Let Γ be a group which has a normal subgroup of finite cohomological dimension so that the associated quotient is a finite group with the elementary abelian p-Sylow subgroup , then the p-period of Γ is twice the least common multiple of $\{|N(\langle x\rangle)/C(\langle x\rangle)|\}$ where x ranges over the conjugacy classes of Z_p subgroups of Γ.

Theorem 5.2. Let Γ be a group which has a normal subgroup of finite cohomological dimension so that the associated quotient is a finite group with the cyclic p-Sylow quotient, then the p-period of Γ is twice the least common multiple of $\{|N(\langle x\rangle)/C(\langle x\rangle)|\}$ where x ranges over the conjugacy classes of Z_p subgroups of Γ.

Recall the set $B_{g,p}$ from chapter IV,

Theorem 5.3. If the mapping class group Γ_g is p-periodic and $g < p(p-1)/2$. Then the p-period of Γ_g = $2\mathrm{LCM}\{\gcd(p-1,b_i)\}$, $b_i \in B_{g,p}$.

Lemma 5.1.1. Let $H = \langle x,y \mid x^p = 1,\ y^q = 1,\ yxy^{-1} = x^r\rangle$ where $q = 0$ or $q \neq 0 \bmod(p)$. If d is the minimal positive integer such that $r^d = 1 \bmod(p)$, then the p-period of H equals 2d.

Proof: If $q \neq 0$, H is a finite group, the proof is immediate by Swan's theorem. Otherwise, if $q = 0$, H is infinite and we look at the short exact sequence

$$1 \to Z_p \to H \to Z \to 1.$$

The spectral sequence of Farrell-Tate cohomology converges in the following way:

$$E_2^{p,q} = H^p(Z;\hat{H}^q(Z_p;Z)) \quad \to \quad \hat{H}^{p+q}(H;Z).$$

This spectral sequence collapses since $H^p(Z;\hat{H}^q(Z_p;Z)) = 0$ when $p < 0$ and $p > 1$. Therefore,

$$1 \to \hat{H}^{n-1}(Z_p;Z)_Z \to \hat{H}^n(H;Z) \to \hat{H}^n(Z_p;Z)^Z \to 1$$

is an exact sequence. By looking at the Z action given on the subgroup Z_p. Clearly $u^d \in \hat{H}^{2d}(Z_p;Z)$ is an invariant element of the Z action on $\hat{H}^{2d}(Z_p;Z)$. Here u is a generator of $\hat{H}^2(Z_p,Z)$. Therefore, there exists an element $h \in \hat{H}^{2d}(H;Z)$ such that $\mathrm{Res}^H_{Z_p}(h) = u^d \neq 0$ on $\hat{H}^{2d}(Z_p;Z)$. By Brown-Venkov theorem and caculations $\hat{H}^{2kd}(H;Z) = Z_p$, $\hat{H}^{2kd+1}(H;Z) = Z_p$, $\hat{H}^i(H;Z) = 0$ for other i's, the p-period of $H = 2d$.

Lemma 5.1.2. Let Z_p be a normal subgroup of group Γ which is of finite vcd, and let M be a finite quotient of Γ with torsion free kernel. Then $\Gamma/C_\Gamma(Z_p) = N_\Gamma(Z_p)/C_\Gamma(Z_p) = N_M(Z_p)/C_M(Z_p) = M/C_M(Z_p)$. Here we still use Z_p to stand for the image of Z_p in M.

Proof: Let $p:\Gamma \to M$ be the projection map. The map p maps $N_\Gamma(Z_p)$ onto $N_M(Z_p)$ and $C_\Gamma(Z_p)$ to $C_M(Z_p)$, so induced map $p^*:N_\Gamma(Z_p)/C_\Gamma(Z_p) \to N_M(Z_p)/C_M(Z_p)$ is a well-defined surjective homomorphism. Let $\langle x\rangle = Z_p$, if $yxy^{-1} = x^r$, then $p(y)xp(y)^{-1} = x^r$. i.e. p^* is an injective.

Lemma 5.1.3. Suppose a group M contains a cyclic subgroup $Z_{p^n} \supset Z_p$ and $|N_M(Z_{p^n})/C_M(Z_{p^n})|$ is prime to p, then the map induced by inclusion $i^*:N_M(Z_{p^n})/C_M(Z_{p^n}) \to N_M(Z_p)/C_M(Z_p)$ is an injective homomorphism.

Proof: Notice $N_M(Z_p) \supset N_M(Z_{p^n})$ and the inclusion i maps $C_M(Z_{p^n})$ to $C_M(Z_p)$, i.e. the induced map by inclusion $i^*: N_M(Z_{p^n})/C_M(Z_{p^n}) \to N_M(Z_p)/C_M(Z_p)$ is a well-defined homomorphism. Now let $\langle x \rangle = Z_{p^n}$, then $\langle x^{p^{n-1}} \rangle = Z_p$, if $y \in C_M(Z_p)$, $yxy^{-1} = x^k$, then $yx^{p^{n-1}}y^{-1} = x^{kp^{n-1}} = x^{p^{n-1}}$, so $(k-1)p^{n-1} = 0 \bmod(p^n)$, i.e. $k = 1 \bmod(p)$. Let $k = Ap^m+1$, A is prime to p and $1 \leq m < n$, $k^d = 1 \bmod(p^n)$, d divides p-1 by assumption. Hence $k^d = (Ap^m+1)^d = B+Adp^m+1 = 1 \bmod(p^n)$, where p^{m^2} divides B. This implies $Ad = 0 \bmod(p)$, this is a contradiction unless $A = 0$.

<u>Lemma 5.1.4</u> (Swan) [Sw] Suppose the p-Sylow subgroup S_p of a finite group M is abelian. Let A_p be the group of automorphisms of S_p induced by inner automorphisms of M. Then an element $a \in H^i(S_p;Z)$ is stable if and only if it is fixed under the action of A_p on $H^i(S_p;Z)$.

Proof: See [Sw].

<u>The proof of theorem 5.1</u>: Brown's theorem states that for Γ p-periodic $\hat{H}^*(\Gamma;Z)_{(p)} = \prod_{P_j \in S} \hat{H}^*(N(P_i);Z)_{(p)}$, where S is the set of all conjugacy classes of Z_p of Γ. Therefore the p-period of Γ is nothing but the least common multiple of the p-periods of $N_\Gamma(P_i)$ if the least common multiple exists.

1) Lower bound. Let $|N_\Gamma(P_i)/C_\Gamma(P_i)| = d_i$, $\langle x \rangle = P_i$. There exists $y \in \Gamma$, such that $yxy^{-1} = x^r$, $r^{d_i} = 1 \bmod(p)$.

Let $H = \langle x,y \rangle$ be a subgroup of Γ generated by elements x and y. Then the p-period of $H = 2d_i$ by lemma 5.1.1, i.e. the p-period of $N_\Gamma(P_i)$ is a multiple of $2d_i$.

2) Upper bound. Let $p: \Gamma \to M$ be a projection onto the finite elementary abelian p-Sylow quotient M, $p_i: N_\Gamma(P_i) \to M_i$ be the restriction map of p, M_i is the image of p_i. Since the p-Sylow subgroup S_p of M_i is elementary abelian, and $M_i = \mathrm{Im}N_\Gamma(P_i) = N_M(P_i)$ normalizes P_i, the group A_p of automorphisms of S_p induced by inner automorphisms of M_i fixes P_i.

Let $u \in H^2(S_p,Z) = \mathrm{Hom}(P_i \times Z_p \times \cdots \cdots Z_p, C^*)$, $u(x) \neq 1$ and $u(y) = 1$ if $\langle x \rangle = P_i$, $\langle y \rangle = Z_p$, then $\mathrm{Res}_{Pi}^{Sp}(u) \neq 0$ in $H^2(P_i;Z)$. Now we claim that $u^{d_i} \in H^{2d_i}(S_p;Z)$ is a stable element. In fact, $d_i = N_M(P_i)/C_M(P_i)$ by lemma 5.1.2, and A_p fixes the element $u^{d_i} \in H^{2d_i}(S_p;Z)$ since $N_M(P_i)/C_M(P_i)$ fixes the element u^{d_i}. By lemma 5.1.4 [Sw], u^{d_i} is a stable element in M_i, i.e. there exists an element $v \in H^{2d_i}(M_i;Z)$ such that $\mathrm{Res}_{Pi}^{Mi}(v) = \mathrm{Res}_{Pi}^{Sp}(u^{d_i}) = \mathrm{Res}_{Pi}^{Sp}(u)^{d_i} \neq 0$. If we apply the canonical homomorphism g^* from ordinary cohomology to Farrell-Tate cohomology, we have

$\mathrm{Res}_{Pi}^{Mi}(g^*(v)) = \mathrm{Res}_{Pi}^{Sp}(g^*(u^{d_i})) = \mathrm{Res}_{Pi}^{Sp}(g^*(u))^{d_i} \neq 0$, i.e. there exists an element $p^*g^*(v) \in \hat{H}^{2d_i}(N_\Gamma(P_i);Z)$ such that

$\mathrm{Res}_{Pi}^{N(Pi)}(p^*g^*(v)) \neq 0$ in $\hat{H}^{2d_i}(P_i;Z)$, by Brown-Venkov theorem $[Br]_2$ and the fact that $N_\Gamma(P_i)$ has only a order p subgroup, the p-period of $N_\Gamma(P_i)$ divides $2d_i$.

$$
\begin{array}{ccccc}
\hat{H}^{2d_i}(N_\Gamma(P_i;Z)) & & \xrightarrow{\text{Res}} & & \hat{H}^{2d_i}(P_i;Z) \\
p^* \uparrow & & & & \uparrow \| \\
\hat{H}^{2d_i}(M_i;Z) & \xrightarrow{\text{Res}} & \hat{H}^{2d_i}(S_p;Z) & \xrightarrow{\text{Res}} & \hat{H}^{2d_i}(P_i;Z) \\
\uparrow \| g^* & & \uparrow \| g^* & & \uparrow \| g^* \\
H^{2d_i}(M_i;Z) & \xrightarrow[\text{Res}]{} & H^{2d_i}(S_p;Z) & \xrightarrow[\text{Res}]{} & H^{2d_i}(P_i;Z)
\end{array}
$$

The proof of theorem 5.2 is basically a similar argument except for the upper bound part. In fact, simply using lemma 5.1.3 and Burgisser's and Swan's theorems, we obtain the upper bound of the p-period of $N_\Gamma(P_i)$ as follows the p-period of $N_\Gamma(P_i)$ divides the p-period of M_i, which is $2|N_M(Z_{p^n})/C_M(Z_{p^n})| = 2|N_M(P_i)/C_M(P_i)| = 2|N_\Gamma(P_i)/C_\Gamma(P_i)|$.

5.2. The proof of theorem 5.3

As an application of the theorem 5.1, we can exactly obtain the p-periods of some p-periodic mapping class groups.

Lemma 5.2.1. For the mapping class group Γ_g, $LCM\{|(N(\langle x\rangle)/C(\langle x\rangle)|\} = LCM\{gcd(p-1,b_i)\}$, where x ranges over all $x \in \Gamma_g$, $x^p = 1$, b_i ranges over all $b_i \in B_{g,p}$.

Proof: 1) Let $|N(\langle x\rangle)/C(\langle x\rangle)| = d$. Then there exists r such that $x \sim x^r \sim \cdots\cdots \sim x^{r^{d-1}}$, where $r^d = 1 \mod(p)$. The d divides p-1 obviously. Let b_i = the number of fixed points of x action on Γ_g, $\sigma(x) = (\beta_1, \beta_2, \cdots \beta_b)$ fixed point datum.

Let us define the permutation r* on the (order)set $\sigma(x)$, $r^*(\beta_1, \beta_2, \cdots\cdots \beta_b) = (r\beta_1, r\beta_2, \cdots\cdots r\beta_b)$, $(r^*)^2 = (r^2)^* \cdots\cdots (r^*)^{d-1} = (r^{d-1})^*$. It is well-defined since $\sigma(x) = \sigma(x^{r^2}) = \cdots\cdots = \sigma(x^{r^{d-1}})$. We can decompose $r^* = (\beta_{i_1}, \beta_{i_2}, \cdots\cdots \beta_{i_s})(\beta_{j_1}, \beta_{j_2}, \cdots\cdots \beta_{j_t}) \cdots\cdots (\beta_{k_1}, \beta_{k_2}, \cdots \cdots \beta_{k_u})$, a product of cyclic permutations. Notice, the permutations r^*, $(r^*)^2, \cdots\cdots (r^*)^{d-1}$ do not have fixed points. otherwise, there exists β_i such that $r^j\beta_i = \beta_i \mod(p)$, $1 \le j \le d-1$. This forces $r^j = 1 \mod(p)$ and a contradiction. But, of course, $(r^*)^d = (r^d)^* = Id$. These imply $s = t = \cdots\cdots = u = d$, i.e. $|N(\langle x\rangle)/C(\langle x\rangle)| = d$ divides the number b_i of fixed points of x action on the surface S_g. We actually have shown $LCM\{|N(\langle x\rangle)/C(\langle x\rangle)|\}$ divides $LCM\{gcd(p-1, b_i)\}$, where x ranges over all $x \in \Gamma_g$, $x^p = 1$, b_i ranges over all $b_i \in B_{g,p}$.

2) Conversely, Let $gcd(p-1, b_i) = d$. Then there is an integer $r \in Z_p$ such that d is a minimal non-negative integer satisfying $r^d = 1 \mod(p)$. If $d \neq 1$, consider the b_i points unordered set $S = (1, r, r^2, \cdots\cdots r^{d-1}, 1, r, r^2, \cdots\cdots r^{d-1},$

$\cdots\cdots 1, r, r^2, \cdots\cdots, r^{d-1}$), since $(b_i/d)(1+r+r^2+\cdots\cdots r^{d-1}) = 0 \bmod(p)$. There exists an element $x \in \Gamma_g$, $x^p = 1$, and the x's fixed point datum $\sigma(x) = S$, i.e. the set S can be realized as a fixed point datum of order p element in Γ_g. Obviously, $\sigma(x) = \sigma(x^r) = \sigma(x^{r^2}) = \cdots\cdots = \sigma(x^{r^{d-1}})$ or $x \approx x^r \approx x^{r^2} \approx \cdots\cdots \approx x^{r^{d-1}}$ in Γ_g. This implies that d divides $|N(x)/C(x)|$.

If $\gcd(p-1,b_i) = d = 1$. For any order p element x in Γ_g with the number of fixed points b_i, obviously 1 divides $|N(\langle x\rangle)/C(\langle x\rangle)|$.

If $b_i = 0$, then $\gcd(p-1,b_i) = p-1$. On the other hand, x acts on S_g freely. All order p free actions are conjugate by [N] or [Ed], i.e. $|N(\langle x\rangle)/C(\langle x\rangle)| = p-1$.

So, the LCM$\{\gcd(p-1,b_i)\}$ divides LCM$\{|N(\langle x\rangle)/C(\langle x\rangle)|\}$.

<u>The proof of theorem 5.3</u>: Let $\mu: \Gamma_g \rightarrow Sp(2g,Z)$ be the canonical homology representation and $p: Sp(2g,Z) \rightarrow Sp(2g,F_q)$ be the reduction map. Here q can be chosen a primitive root of mod(p) such that $q \geq 3$, and q^{p-1} is not congruent to 1 mod(p^2)(by the Dirichlet theorem). Now $Ker(p\mu) = N$ is a torsion free, normal, finite index subgroup of Γ_g and the finite quotient $\Gamma_g/N = Sp(2g,F_q)$ has only elementary abelian p-sylow subgroup if $2g < p(p-1)$. Then we can use theorem 5.1 and lemma 5.2.1 to conclude our theorem 5.3.

Example: If $p = 47$, the 47-periods of Γ_g are obtained for $g < 1081$ if Γ_g is p-periodic.

Notice that we have a complete display of the p-period of Γ_g for all g, $2 \leq g \leq 5$, except for the 3-period of Γ_3 which is only known 4 or 12.

Finally, we give the tables C.3, C.4 and C.5 in the appendix C to list the p-periods of some mapping class groups Γ_g in terms of the computer programs. For a pair (g, p), $200 \geq g \geq 2$, $233 \geq p \geq 3$, we enter one of the four possible symbols: N = No p-torsion, X = Not p-priodic, Number = The known p-period and U = Unknown (even though p-periodical).

APPENDIX A

A NOTE ON THE PROJECTIVE CLASS GROUP OF THE MAPPING CLASS GROUP

Let $Z\Gamma_g$ denote the integer group ring of the mapping class group Γ_g. Recall that the reduced projective class group $\tilde{K}_0$ (Z-) is a covariant functor from groups to abelian groups [Bu]$_2$.

Carter [Ca] and Burgisser [Bu]$_2$ separatly found some nontrivial elements of reduced projective class groups $\tilde{K}_0$ (ZGL(n,Z)) and $\tilde{K}_0$ (ZSL(n,Z)) in the 1980's. Their basic common idea was to choose suitable finite subgroups G of GL(n,Z)(or SL(n,Z)) and finite quotients Q of GL(n,Z)(or SL(n,Z)) such that the reduced projective class groups $\tilde{K}_0$ (ZG) are nontrivially injected into $\tilde{K}_0$ (ZQ) factoring through $\tilde{K}_0$ (ZGL(n,Z))(or SL(n,Z)).

As one more example of the fact that the mapping class group shares many properties with the arithmetic group, we provide nontrivial elements of the reduced projective class group $\tilde{K}_0(Z\Gamma_{(p-1)/2})$ by studying the metacyclic subgroups and the natural finite symplectic quotient group of the mapping class group $\Gamma_{(p-1)/2}$ for $p = 6k+1$ prime.

Theorem A.1. If $p = 6k+1$ is a prime, the mapping class group $\Gamma_{(p-1)/2}$ contain metacyclic groups $G = Z_p \rtimes Z_3$ (semi-direct product) such that the reduced projective class groups $\tilde{K}_0(Z\Gamma_{(p-1)/2}) \supset \mathrm{Ind}_G^\Gamma \tilde{K}_0(ZG)$ has the cyclic group of order 3.

Denote $G = Z_p X\ Z_3$, $\Gamma = \Gamma_{(p-1)/2}$, $Q = Sp(p-1, F_q)$, where $p = 6k+1$ prime. Take $q \geq 3$ prime such that $q^{p-1} = 1 \bmod(p)$ and $q^{p-1} \neq 1 \bmod(p^2)$ (Dirichlet theorem).

Proposition A.2. If $p = 6k+1$ prime, the mapping class group $\Gamma_{(p-1)/2}$ contains subgroup $G = Z_p \rtimes Z_3 = \langle x, y \mid x^p = 1, y^3 = 1, yxy^{-1} = x^r \rangle$, $r^3 = 1 \bmod(p), r \neq 1$.

Proof: In fact, $G = \langle x, y, (xy)^{-1} \rangle$ and order$(xy) = 3$. The Riemann-Hurwitz formula shows $2(p-1)/2-2 = 3p(2(0)-2) +3p(1-1/p) +2(3p)(1-1/3)$.

Consider the inclusion $i: G \to \Gamma$, and the canonical projection $p: \Gamma \to Q$. Note that Ker(p) is a torsion-free subgroup of Γ. Therefore the composite map pi: $G \to Q$ is injective and there is an induced map

$$\mathrm{Ind}_G^Q(pi) = \mathrm{Ind}_\Gamma^Q(p)\,\mathrm{Ind}_G^\Gamma(i): \tilde{K}_0(ZG) \to \tilde{K}_0(Z\Gamma) \to \tilde{K}_0(ZQ).$$

Proposition A.3. $\mathrm{Ind}_G^Q(pi): \tilde{K}_0(ZG) \to \tilde{K}_0(ZQ)$ is injective.

Proposition A.4. $\tilde{K}_0(ZG) \supset Z_3$.

The theorem A.1 follows from propositions A.2, A.3 and A.4.

The proof of proposition A.3 basically comes from Carter's theorem 2 [Ca] which states the following: Let Q be a finite group of order pm with p prime, m not divisible by p. Let P be a sylow p-subgroup of Q, and N = $N_Q(P)$, the normalizer of P in Q. Suppose $N \supset G \supset P$ and that G is a retract of N. Then there is a subgroup H_G of G such that G = $P \rtimes H_G$ (semi-direct) and $\mathrm{Im}(\mathrm{Ind}_{H_G}^{G}) \supset \mathrm{Ker}(\mathrm{Ind}_{G}^{Q})$.

Note $|Q| = |Sp(p-1,F_q)| = q^{p(p-1)/2}\prod_{1\leq i\leq (p-1)/2}(q^{2i}-1)$ = pm, where m not divisible by p.

Lemma A.5. Let P be the Sylow p-subgroup of finite group Q = $Sp(p-1,F_q)$ and N = $N_Q(P)$, the normalizer of P in Q. Then G is a retract of N, i.e. there exists a homomorphism $r:N \to G$ such that $ri = Id:G \to N \to G$.

Proof: Let $C_Q(P)$ be the centralizer of Z_p in Q. There is a short exact sequence: $1 \to C_Q(P) \to N_Q(P) \to Z_d \to 1$, here 3 divides d and d divides p-1.

Now we claim that $C_Q(P)$ is a cyclic group. In fact, $GL(p-1,F_q) \supset Sp(p-1,F_q) = Q$, $C_{GL}(P) \supset C_Q(P)$. But $C_{GL}(P)$ is exactly the multiplicative group of the finite field $F_q[z]$ by [Ca], <z> = P, hence cyclic. So $C_Q(P) = Z_p \times C_{p'}$, $C_{p'}$ is the cyclic group of order prime to p. Therefore define the map $r : N_Q(P) = (Z_p \times C_{p'}) \rtimes Z_d \rightarrow G = Z_p \rtimes Z_3$ by $r(z,c',s) = (z,s')$, $s = s' \bmod(3)$. It is easy to check that r is a retract.

Also, $H_G = Z_3$. It is well-known that $\tilde{K}_0(ZZ_3) = 0$, $\mathrm{Im}(\mathrm{Ind}_{H_G}^{G}) = 0$ implies $\mathrm{Ker}(\mathrm{Ind}_{G}^{Q}) = 0$ by Carter, i.e. Proposition A.3 holds.

The proposition A.4 actually is due to Galovich, Reiner and Ullom [G,R,U]. For our case, their theorem states that there is an epimorphism $\tilde{K}_0(ZG) \rightarrow \tilde{K}_0(S) \oplus \tilde{K}_0(ZZ_3)$, whose kernel $D_0(ZG)$ is a finite cyclic group of order 3. Here S is the algebraic integer L, L is the unique subfield of $Q(\omega)$ such that $(Q(\omega):L) = 3$, ω is a primitive p-th root of 1.

APPENDIX B

STRANGE p-TORSION IN THE MAPPING CLASS GROUPS

Connolly posed a concept "strange p-torsion" for a group Γ in 1986.

Definition. A group Γ has strange p-torsion if

1) Γ does not contain p-torsion.

2) For some i, the cohomology group $H^i(\Gamma;Z)$ contains p-torsion.

In addition, a group Γ has very strange p-torsion if

1) Γ does not contain p-torsion.

2) There exists an integer N_p, the cohomology groups $H^i(\Gamma;Z)$ contain p-torsion if $i > N_p$.

Proposition B.1. If Γ is a vcd group (i.e. Γ has virtually finite cohomological dimension n). Then Γ has no very strange p-torsion.

Proof: Let $\hat{H}^i(\Gamma;Z)$ be Farrell-Tate cohomology groups. Then $H^i(\Gamma;Z) = \hat{H}^i(\Gamma;Z)$ if $i > n = vcd(\Gamma)$. $\hat{H}^i(\Gamma;Z)$ has p-torsion if and only if Γ has p-torsion [Br]. So Γ has no very strange p-torsion.

Obviously, a finite group G has even no strange torsion.

Proposition B.2. There are at least $(p-15)^2/4$ mapping class groups Γ_g such that $H^{2(p-1)}(\Gamma_g;Z)$ contain p-torsion, but Γ_g has strange p-torsion for $p \geq 17$ prime.

Lemma. For p an odd prime, there exist exactly $(p-3)^2/4$ gaps, here we call g as a gap of p, if Γ_g does not contain Z_p.

Proof: By [G,M], Γ_g contains an element of prime order p if and only if g is of the form $g = up+v(p-1)/2$, $(u,v) \in Z \times Z$, $u \geq 0$, $v \geq -2$ and $v \neq -1$.

List the gaps as following:

Table B.1

$v = -2$:	No.
$v = 0$:	No.
$v = 2$:	No.
$v = 4$:	$p-2$.
$v = 6$:	$p-3$, $2p-3$.
:	: :
$v = 2i$:	$p-i$, $2p-i$, $3p-i$, ········ $(i-1)p-i$.
:	: : :
$v = p-1$:	$p-(p-1)/2$, $2p-(p-1)/2$, ········ $(p-3)p/2-(p-1)/2$.

table B.1. (continued)

$v = 1$:	No.			
$v = 3$:	$(p-1)/2-1$.			
$v = 5$:	$(p-1)/2-2$,	$p+(p-1)/2-2$.		
$\vdots$	$\vdots$	$\vdots$		
$v = 2i+1$:	$(p-1)/2-i$,	$p+(p-1)/2-i$,	$\cdots\cdots\cdots$	$(i-1)p+(p-1)/2-i$.
$\vdots$	$\vdots$	$\vdots$		$\vdots$
$v = p-2$:	2,	$p+2$,	$2p+2$, $\cdots\cdots\cdots$	$(p-7)p/2+2$.

The total number of gaps $= (p-3)(p-1)/8+(p-5)(p-3)/8$ $= (p-3)^2/4$.

The proof of the proposition B.2: If $g > 3(2)(p-1)$ $= 6(p-1)$, $H^{2(p-1)}(\Gamma_g;Z)$ is independent of g by Harer's stability theorem.

On the other hand, let $g = p(p-3)/2 > (p-3)p/2-(p-1)/2$ Γ_g contains Z_p and it is easy to see by the Riemann-Hurwitz formula that $(p-3)p-2 = p(2\eta-2)+t(p-1)$ implies the fixed points number $t = p+2-2p(\eta+1)/(p-1) = 2$.

Let T be a generator of Z_p, $\rho = \eta i$ a representation of Z_p under the composite of the inclusion $i:Z_p \rightarrow \Gamma_{p(p-3)/2}$ and the canonical homology representation $\eta:\Gamma_{p(p-3)/2} \rightarrow$

$GL(p(p-3),Z)$. Note $\chi_\rho(Id) = p(p-3)$, $\chi_\rho(T) = 2-t = 2-2 = 0$. Let $\chi_\rho = m\chi_{reg} + n\chi_{red}$, then $m = p-3$, $n = 0$, $m+n = p-3$ is prime to p, by prop.2 [G,M], $c_{p-1}(\eta) \in H^{2(p-1)}(\Gamma_{(p-3)p/2};Z)$ is an element of order a multiple of p. So $H^{2(p-1)}(\Gamma_g;Z)$ contains an order p element only if $g > 6(p-1)$. Because of $p(p-1)/2 > 6(p-1)$ for $p > 17$, checking the table B.1 in the lemma above , we actually find at least $(p-15)(p-13)/8 + (p-15)(p-17)/8 = (p-15)^2/4$ mapping class groups Γ_g such that $H^{2(p-1)}(\Gamma_g;Z)$ contain p-torsion, but Γ_g has no p-torsion for $p \geq 17$, i.e. there exist at least $(p-15)^2/4$ mapping class groups Γ_g which have strange p-torsion.

APPENDIX C

TABLE C.1

THE p-TORSION GAPS OF MAPPING CLASS GROUPS

Γ_g FOR $p \leq 41$

$p = 5$:

3.							

$p = 7$:

2,	4,	5.					

$p = 11$:

2,	3,	4,	6,	7,	8,	9,	13,
14,	17,	18,	19,	24,	28,	29,	39.

$p = 13$:

2,	3,	4,	5,	7,	8,	9,	10,
11,	15,	16,	17,	20,	21,	22,	23,
28,	29,	33,	34,	35,	41,	46,	47,
59.							

$p = 17$:

2,	3,	4,	5,	6,	7,	9,	10,
11,	12,	13,	14,	15,	19,	20,	21,
22,	23,	26,	27,	28,	29,	30,	31,
36,	37,	38,	39,	43,	44,	45,	46,
47,	53,	54,	55,	60,	61,	62,	63,
70,	71,	77,	78,	79,	87,	94,	95,
111.							

Table C.1 (continued)

p = 19:

2,	3,	4,	5,	6,	7,	8,	10,
11,	12,	13,	14,	15,	16,	17,	21,
22,	23,	24,	25,	26,	29,	30,	31,
32,	33,	34,	35,	40,	41,	42,	43,
44,	48,	49,	50,	51,	52,	53,	59,
60,	61,	62,	67,	68,	69,	70,	71,
78,	79,	80,	86,	87,	88,	89,	97,
98,	105,	106,	107,	116,	124,	125,	143.

p = 23:

2,	3,	4,	5,	6,	7,	8,	9,
10,	12,	13,	14,	15,	16,	17,	18,
19,	20,	21,	25,	26,	27,	28,	29,
30,	31,	32,	35,	36,	37,	38,	39,
40,	41,	42,	43,	48,	49,	50,	51,
52,	53,	54,	58,	59,	60,	61,	62,
63,	64,	65,	71,	72,	73,	74,	75,
76,	81,	82,	83,	84,	85,	86,	87,
94,	95,	96,	97,	98,	104,	105,	106,
107,	108,	109,	117,	118,	119,	120,	127,
128,	129,	130,	131,	140,	141,	142,	150,
151,	152,	153,	163,	164,	173,	174,	175,
186,	196,	197,	219.				

p = 29:

2,	3,	4,	5,	6,	7,	8,	9,
10,	11,	12,	13,	15,	16,	17,	18,
19,	20,	21,	22,	23,	24,	25,	26,
27,	31,	32,	33,	34,	35,	36,	37,
38,	39,	40,	41,	44,	45,	46,	47,

Table C.1 (continued)

48,	49,	50,	51,	52,	53,	54,	55,
60,	61,	62,	63,	64,	65,	66,	67,
68,	69,	73,	74,	75,	76,	77,	78,
79,	80,	81,	82,	83,	89,	90,	91,
92,	93,	94,	95,	96,	97,	102,	103,
104,	105,	106,	107,	108,	109,	110,	111,
118,	119,	120,	121,	122,	123,	124,	125,
131,	132,	133,	134,	135,	136,	137,	138,
139,	147,	148,	149,	150,	151,	152,	153,
160,	161,	162,	163,	164,	165,	166,	167,
176,	177,	178,	179,	180,	181,	189,	190,
191,	192,	193,	194,	195,	205,	206,	207,
208,	209,	218,	219,	220,	221,	222,	223,
234,	235,	236,	237,	247,	248,	249,	250,
251,	263,	264,	265,	276,	277,	278,	279,
292,	293,	305,	306,	307,	321,	334,	335,
363.							

p = 31:

2,	3,	4,	5,	6,	7,	8,	9,
10,	11,	12,	13,	14,	16,	17,	18,
19,	20,	21,	22,	23,	24,	25,	26,
27,	28,	29,	33,	34,	35,	36,	37,
38,	39,	40,	41,	42,	43,	44,	47,
48,	49,	50,	51,	52,	53,	54,	55,
56,	57,	58,	59,	72,	73,	74,	78,
79,	80,	81,	82,	83,	84,	85,	86,
87,	88,	89,	95,	96,	97,	98,	99,
100,	101,	102,	103,	104,	109,	110,	111,
112,	113,	114,	115,	116,	117,	118,	119,
126,	127,	128,	129,	130,	131,	132,	133,
134,	140,	141,	142,	143,	144,	145,	146,

Table C.1 (continued)

147,	148,	149,	157,	158,	159,	160,	161,
162,	163,	164,	171,	172,	173,	174,	175,
176,	177,	178,	179,	188,	189,	190,	191,
192,	193,	194,	202,	203,	204,	205,	206,
207,	208,	209,	219,	220,	221,	222,	223,
224,	233,	234,	235,	236,	237,	238,	239,
250,	251,	252,	253,	254,	264,	265,	266,
267,	268,	269,	281,	282,	283,	284,	295,
296,	297,	298,	299,	312,	313,	314,	326,
327,	328,	329,	343,	344,	357,	358,	359,
374,	388,	389,	419.				

p = 37:

2,	3,	4,	5,	6,	7,	8,	9,
10,	11,	12,	13,	14,	15,	16,	17,
19,	20,	21,	22,	23,	24,	25,	26,
27,	28,	29,	30,	31,	32,	33,	34,
35,	39,	40,	41,	42,	43,	44,	45,
46,	47,	48,	49,	50,	51,	52,	53,
56,	57,	58,	59,	60,	61,	62,	63,
64,	65,	66,	67,	68,	69,	70,	71,
76,	77,	78,	79,	80,	81,	82,	83,
84,	85,	86,	87,	88,	89,	93,	94,
95,	96,	97,	98,	99,	100,	101,	102,
103,	104,	105,	106,	107,	113,	114,	115,
116,	117,	118,	119,	120,	121,	122,	123,
124,	125,	130,	131,	132,	133,	134,	135,
136,	137,	138,	139,	140,	141,	142,	143,
150,	151,	152,	153,	154,	155,	156,	157,
158,	159,	160,	161,	167,	168,	169,	170,
171,	172,	173,	174,	175,	176,	177,	178,
179,	187,	188,	189,	190,	191,	192,	193,

Table C.1 (continued)

194,	195,	196,	197,	204,	205,	206,	207,
208,	209,	210,	211,	212,	213,	214,	215,
224,	225,	226,	227,	228,	229,	230,	231,
232,	233,	241,	242,	243,	244,	245,	246,
247,	248,	249,	250,	251,	261,	262,	263,
264,	265,	266,	267,	268,	269,	278,	279,
280,	281,	282,	283,	284,	285,	286,	287,
298,	299,	300,	301,	302,	303,	304,	305,
315,	316,	317,	318,	319,	320,	321,	322,
323,	335,	336,	337,	338,	339,	340,	341,
352,	353,	354,	355,	356,	357,	358,	359,
372,	373,	374,	375,	376,	377,	389,	390,
391,	392,	393,	394,	395,	409,	410,	411,
412,	413,	426,	427,	428,	429,	430,	431,
446,	447,	448,	449,	463,	464,	465,	466,
467,	483,	484,	485,	500,	501,	502,	503,
520,	521,	537,	538,	539,	557,	574,	575,
611.							

p = 41:

2,	3,	4,	5,	6,	7,	8,	9,
10,	11,	12,	13,	14,	15,	16,	17,
18,	19,	21,	22,	23,	24,	25,	26,
27,	28,	29,	30,	31,	32,	33,	34,
35,	36,	37,	38,	39,	43,	44,	45,
46,	47,	48,	49,	50,	51,	52,	53,
54,	55,	56,	57,	58,	59,	62,	63,
64,	65,	66,	67,	68,	69,	70,	71,
72,	73,	74,	75,	76,	77,	78,	79,
84,	85,	86,	87,	88,	89,	90,	91,
92,	93,	94,	95,	96,	97,	98,	99,
103,	104,	105,	106,	107,	108,	109,	110,
111,	112,	113,	114,	115,	116,	117,	118,

Table C.1 (continued)

119,	125,	126,	127,	128,	129,	130,	131,
132,	133,	134,	135,	136,	137,	138,	139,
144,	145,	146,	147,	148,	149,	150,	151,
152,	153,	154,	155,	156,	157,	158,	159,
166,	167,	168,	169,	170,	171,	172,	173,
174,	175,	176,	177,	178,	179,	185,	186,
187,	188,	189,	190,	191,	192,	193,	194,
195,	196,	197,	198,	199,	207,	208,	209,
210,	211,	212,	213,	214,	215,	216,	217,
218,	219,	226,	227,	228,	229,	230,	231,
232,	233,	234,	235,	236,	237,	238,	239,
248,	249,	250,	251,	252,	253,	254,	255,
256,	257,	258,	259,	267,	268,	269,	270,
271,	272,	273,	274,	275,	276,	277,	278,
279,	289,	290,	291,	292,	293,	294,	295,
296,	297,	298,	299,	308,	309,	310,	311,
312,	313,	314,	315,	316,	317,	318,	319,
330,	331,	332,	333,	334,	335,	336,	337,
338,	339,	349,	350,	351,	352,	353,	354,
355,	356,	357,	358,	359,	371,	372,	373,
374,	375,	376,	377,	378,	379,	390,	391,
392,	393,	394,	395,	396,	397,	398,	399,
412,	413,	414,	415,	416,	417,	418,	419,
431,	432,	433,	434,	435,	436,	437,	438,
439,	453,	454,	455,	456,	457,	458,	459,
472,	473,	474,	475,	476,	477,	478,	479,
494,	495,	496,	497,	498,	499,	513,	514,
515,	516,	517,	518,	519,	535,	536,	537,
538,	539,	554,	555,	556,	557,	558,	559,
576,	577,	578,	579,	595,	596,	597,	598,
599,	617,	618,	619,	636,	637,	638,	639,
658,	659,	677,	678,	679,	699,	718,	719,
759.							

TABLE C.2

LIST OF THE ALL GENUS g = kp+1 OF MAPPING CLASS GROUPS Γ_g WHICH HAVE p-PERIODICITY FOR $p \leq 41$

p = 5

11						

p = 7

8	22	29				

p = 11

12	23	34	56	67	78	89
133	144	177	188	199	254	298
309	419					

p = 13

14	27	40	53	79	92	105
118	131	183	196	209	248	261
274	287	352	365	417	430	443
521	586	599	755			

p = 17

18	35	52	69	86	103	137
154	171	188	205	222	239	307
324	341	358	375	426	443	460
477	494	511	596	613	630	647

Table C.2 (continued)

715	732	749	766	783	885	902
919	1004	1021	1038	1055	1174	1191
1293	1310	1327	1463	1582	1599	1871

p = 19

20	39	58	77	96	115	134
172	191	210	229	248	267	286
305	381	400	419	438	457	476
533	552	571	590	609	628	647
742	761	780	799	818	894	913
932	951	970	989	1103	1122	1141
1160	1255	1274	1293	1312	1331	1464
1483	1502	1616	1635	1654	1673	1825
1844	1977	1996	2015	2186	2338	2357
2699						

p = 23

24	47	70	93	116	139	162
185	208	254	277	300	323	346
369	392	415	438	461	553	576
599	622	645	668	691	714	783
806	829	852	875	898	921	944
967	1082	1105	1128	1151	1174	1197
1220	1312	1335	1358	1381	1404	1427
1450	1473	1611	1634	1657	1680	1703
1726	1841	1864	1887	1910	1933	1956
1979	2140	2163	2186	2209	2232	2370
2393	2416	2439	2462	2485	2669	2692
2715	2738	2899	2922	2945	2968	2991
3198	3221	3244	3428	3451	3474	3497
3727	3750	3957	3980	4003	4256	4486
4509	5015					

Table C.2 (continued)

p = 29

30	59	88	117	146	175	204
233	262	291	320	349	407	436
465	494	523	552	581	610	639
668	697	726	755	871	900	929
958	987	1016	1045	1074	1103	1132
1161	1248	1277	1306	1335	1364	1393
1422	1451	1480	1509	1538	1567	1712
1741	1770	1799	1828	1857	1886	1915
1944	1973	2089	2118	2147	2176	2205
2234	2263	2292	2321	2350	2379	2553
2582	2611	2640	2669	2698	2727	2756
2785	2930	2959	2988	3017	3046	3075
3104	3133	3162	3191	3394	3423	3452
3481	3510	3539	3568	3597	3771	3800
3829	3858	3887	3916	3945	3974	4003
4235	4264	4293	4322	4351	4380	4409
4612	4641	4670	4699	4728	4757	4786
4815	5076	5105	5134	5163	5192	5221
5453	5482	5511	5540	5569	5598	5627
5917	5946	5975	6004	6033	6294	6323
6352	6381	6410	6439	6758	6787	6816
6845	7135	7164	7193	7222	7251	7599
7628	7657	7976	8005	8034	8063	8440
8469	8817	8846	8875	9281	9658	9687
10499						

p = 31

32	63	94	125	156	187	218
249	280	311	342	373	404	466
497	528	559	590	621	652	683
714	745	776	807	838	869	993

Table C.2 (continued)

1024	1055	1086	1117	1148	1179	1210
1241	1272	1303	1334	1427	1458	1489
1520	1551	1582	1613	1644	1675	1706
1737	1768	1799	1954	1985	2016	2047
2078	2109	2140	2171	2202	2233	2264
2388	2419	2450	2481	2512	2543	2574
2605	2636	2667	2698	2729	2915	2946
2977	3008	3039	3070	3101	3132	3163
3194	3349	3380	3411	3442	3473	3504
3535	3566	3597	3628	3659	3876	3907
3938	3969	4000	4031	4062	4093	4124
4310	4341	4372	4403	4434	4465	4496
4527	4558	4589	4837	4868	4899	4930
4961	4992	5023	5054	5271	5302	5333
5364	5395	5426	5457	5488	5519	5798
5829	5860	5891	5922	5953	5984	6232
6263	6294	6325	6356	6387	6418	6449
6759	6790	6821	6852	6883	6914	7193
7224	7255	7286	7317	7348	7379	7720
7751	7782	7813	7844	8154	8185	8216
8247	8278	8309	8681	8712	8743	8774
9115	9146	9177	9208	9239	9642	9673
9704	10076	10107	10138	10169	10603	10634
11037	11068	11099	11564	11998	12029	12959

p = 37

38	75	112	149	186	223	260
297	334	371	408	445	482	519
556	593	667	704	741	778	815
852	889	926	963	1000	1037	1074
1111	1148	1185	1222	1259	1407	1444
1481	1518	1555	1592	1629	1666	1703

Table C.2 (continued)

1740	1777	1814	1851	1888	1925	2036
2073	2110	2147	2184	2221	2258	2295
2332	2369	2406	2443	2480	2517	2554
2591	2776	2813	2850	2887	2924	2961
2998	3035	3072	3109	3146	3183	3220
3257	3405	3442	3479	3516	3553	3590
3627	3664	3701	3738	3775	3812	3849
3886	3923	4145	4182	4219	4256	4293
4330	4367	4404	4441	4478	4515	4552
4589	4774	4811	4848	4885	4922	4959
4996	5033	5070	5107	5144	5181	5218
5255	5514	5551	5588	5625	5662	5699
5736	5773	5810	5847	5884	5921	6143
6180	6217	6254	6291	6328	6365	6402
6439	6476	6513	6550	6587	6883	6920
6957	6994	7031	7068	7105	7142	7179
7216	7253	7512	7549	7586	7623	7660
7697	7734	7771	7808	7845	7882	7919
8252	8289	8326	8363	8400	8437	8474
8511	8548	8585	8881	8918	8955	8992
9029	9066	9103	9140	9177	9214	9251
9621	9658	9695	9732	9769	9806	9843
9880	9917	10250	10287	10324	10361	10398
10435	10472	10509	10546	10583	10990	11027
11064	11101	11138	11175	11212	11249	11619
11656	11693	11730	11767	11804	11841	11878
11915	12359	12396	12433	12470	12507	12544
12581	12988	13025	13062	13099	13136	13173
13210	13247	13728	13765	13802	13839	13876
13913	14357	14394	14431	14468	14505	14542
14579	15097	15134	15171	15208	15245	15726
15763	15800	15837	15874	15911	16466	16503
16540	16577	17095	17132	17169	17206	17243

Table C.2 (continued)

17835	17872	17909	18464	18501	18538	18575
19204	19241	19833	19870	19907	20573	21202
21239	22571					

p = 41

42	83	124	165	206	247	288
329	370	411	452	493	534	575
616	657	698	739	821	862	903
944	985	1026	1067	1108	1149	1190
1231	1272	1313	1354	1395	1436	1477
1518	1559	1723	1764	1805	1846	1887
1928	1969	2010	2051	2092	2133	2174
2215	2256	2297	2338	2379	2502	2543
2584	2625	2666	2707	2748	2789	2830
2871	2912	2953	2994	3035	3076	3117
3158	3199	3404	3445	3486	3527	3568
3609	3650	3691	3732	3773	3814	3855
3896	3937	3978	4019	4183	4224	4265
4306	4347	4388	4429	4470	4511	4552
4593	4634	4675	4716	4757	4798	4839
5085	5126	5167	5208	5249	5290	5331
5372	5413	5454	5495	5536	5577	5618
5659	5864	5905	5946	5987	6028	6069
6110	6151	6192	6233	6274	6315	6356
6397	6438	6479	6766	6807	6848	6889
6930	6971	7012	7053	7094	7135	7176
7217	7258	7299	7545	7586	7627	7668
7709	7750	7791	7832	7873	7914	7955
7996	8037	8078	8119	8447	8488	8529
8570	8611	8652	8693	8734	8775	8816
8857	8898	8939	9226	9267	9308	9349
9390	9431	9472	9513	9554	9595	9636
9677	9718	9759	10128	10169	10210	10251

Table C.2 (continued)

10292	10333	10374	10415	10456	10497	10538
10579	10907	10948	10989	11030	11071	11112
11153	11194	11235	11276	11317	11358	11399
11809	11850	11891	11932	11973	12014	12055
12096	12137	12178	12219	12588	12629	12670
12711	12752	12793	12834	12875	12916	12957
12998	13039	13490	13531	13572	13613	13654
13695	13736	13777	13818	13859	14269	14310
14351	14392	14433	14474	14515	14556	14597
14638	14679	15171	15212	15253	15294	15335
15376	15417	15458	15499	15950	15991	16032
16073	16114	16155	16196	16237	16278	16319
16852	16893	16934	16975	17016	17057	17098
17139	17631	17672	17713	17754	17795	17836
17877	17918	17959	18533	18574	18615	18656
18697	18738	18779	19312	19353	19394	19435
19476	19517	19558	19599	20214	20255	20296
20337	20378	20419	20993	21034	21075	21116
21157	21198	21239	21895	21936	21977	22018
22059	22674	22715	22756	22797	22838	22879
23576	23617	23658	23699	24355	24396	24437
24478	24519	25257	25298	25339	26036	26077
26118	26159	26938	26979	27717	27758	27799
28619	29398	29439	31079			

TABLE C.3

THE p-PERIODS OF MAPPING CLASS GROUPS Γ_g

FOR $g \leq 200$ AND $p \leq 61$

p=	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61
g= 2 :	4	2	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g= 3 :	U	N	6	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g= 4 :	X	8	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g= 5 :	U	4	N	2	N	N	N	N	N	N	N	N	N	N	N	N	N
g= 6 :	U	X	4	N	6	N	N	N	N	N	N	N	N	N	N	N	N
g= 7 :	X	2	4	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g= 8 :	U	4	12	N	N	2	N	N	N	N	N	N	N	N	N	N	N
g= 9 :	U	8	2	N	N	N	6	N	N	N	N	N	N	N	N	N	N
g= 10 :	X	U	6	4	N	N	N	N	N	N	N	N	N	N	N	N	N

*N = No p-torsion, X = Not p-periodic, U = Unknown.

Table C.3 (continued)

p=	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61
g= 11 :	U	U	N	4	N	N	N	2	N	N	N	N	N	N	N	N	N
g= 12 :	U	8	12	20	8	N	N	N	N	N	N	N	N	N	N	N	N
g= 13 :	X	U	4	N	4	N	N	N	N	N	N	N	N	N	N	N	N
g= 14 :	U	U	4	N	24	N	N	N	2	N	N	N	N	N	N	N	N
g= 15 :	U	U	X	10	N	N	N	N	N	6	N	N	N	N	N	N	N
g= 16 :	X	X	2	2	N	8	N	N	N	N	N	N	N	N	N	N	N
g= 17 :	U	U	6	N	N	4	N	N	N	N	N	N	N	N	N	N	N
g= 18 :	U	U	4	N	2	32	4	N	N	N	6	N	N	N	N	N	N
g= 19 :	X	U	12	N	6	N	4	N	N	N	N	N	N	N	N	N	N
g= 20 :	U	U	4	4	N	N	36	N	N	N	N	2	N	N	N	N	N
g= 21 :	U	X	U	4	N	N	N	N	N	N	N	N	6	N	N	N	N
g= 22 :	X	U	U	4	N	N	N	4	N	N	N	N	N	N	N	N	N
g= 23 :	U	U	U	20	N	N	N	4	N	N	N	N	N	2	N	N	N
g= 24 :	U	U	U	N	12	2	N	44	N	N	N	N	N	N	N	N	N
g= 25 :	X	U	U	2	8	2	N	N	N	N	N	N	N	N	N	N	N

Table C.3 (continued)

p=	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61
g= 26 :	U	X	U	10	4	N	N	N	N	N	N	N	N	N	2	N	N
g= 27 :	U	U	U	2	24	N	2	N	N	N	N	N	N	N	N	N	N
g= 28 :	X	U	U	N	N	N	6	N	8	N	N	N	N	N	N	N	N
g= 29 :	U	U	U	N	N	N	N	N	4	N	N	N	N	N	N	2	N
g= 30 :	U	U	12	4	2	N	N	N	56	4	N	N	N	N	N	N	6
g= 31 :	X	X	U	4	2	N	N	N	N	4	N	N	N	N	N	N	N
g= 32 :	U	U	U	4	6	4	N	N	N	60	N	N	N	N	N	N	N
g= 33 :	U	U	U	4	N	8	N	2	N	N	N	N	N	N	N	N	N
g= 34 :	X	U	U	20	N	4	N	2	N	N	N	N	N	N	N	N	N
g= 35 :	U	U	U	2	N	32	N	N	N	N	N	N	N	N	N	N	N
g= 36 :	U	X	X	2	8	N	12	N	N	N	8	N	N	N	N	N	N
g= 37 :	X	U	U	10	12	N	4	N	N	N	4	N	N	N	N	N	N
g= 38 :	U	U	U	2	8	N	4	N	N	N	72	N	N	N	N	N	N
g= 39 :	U	U	U	N	4	N	36	N	N	N	N	N	N	N	N	N	N

Table C.3 (continued)

p=	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61
g= 40 :	X	U	U	20	24	2	N	N	N	N	N	8	N	N	N	N	N
g= 41 :	U	X	U	4	N	2	N	N	N	N	N	4	N	N	N	N	N
g= 42 :	U	U	U	4	6	2	N	N	2	N	N	80	4	N	N	N	N
g= 43 :	X	U	X	4	2	N	N	N	2	N	N	N	4	N	N	N	N
g= 44 :	U	U	U	4	2	N	N	4	N	N	N	N	84	N	N	N	N
g= 45 :	U	U	U	X	6	N	2	4	N	10	N	N	N	N	N	N	N
g= 46 :	X	X	U	2	N	N	2	4	N	6	N	N	N	4	N	N	N
g= 47 :	U	U	U	2	N	N	6	44	N	N	N	N	N	4	N	N	N
g= 48 :	U	U	12	10	4	16	N	N	N	N	N	N	N	92	N	N	N
g= 49 :	X	U	U	2	8	4	N	N	N	N	N	N	N	N	N	N	N
g= 50 :	U	U	X	4	12	8	N	N	N	N	N	N	N	N	N	N	N
g= 51 :	U	X	U	20	8	4	N	N	N	N	N	N	N	N	N	N	N
g= 52 :	X	U	U	4	4	32	N	N	N	N	N	N	N	N	8	N	N
g= 53 :	U	U	U	4	24	N	N	N	N	N	N	N	N	N	4	N	N
g= 54 :	U	U	U	4	2	N	4	N	N	N	2	N	N	N	104	N	N
g= 55 :	X	U	U	U	6	N	12	2	N	N	6	N	N	N	N	N	N

Table C.3 (continued)

p=	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61
g= 56 :	U	X	U	U	2	2	4	2	4	N	N	N	N	N	N	N	N
g= 57 :	U	U	X	U	2	2	4	2	8	N	N	N	N	N	N	N	N
g= 58 :	X	U	U	U	6	2	36	N	4	N	N	N	N	N	N	4	N
g= 59 :	U	U	U	U	N	2	N	N	56	N	N	N	N	N	N	4	N
g= 60 :	U	U	U	U	24	N	N	N	N	12	N	10	N	N	N	116	8
g= 61 :	X	X	U	U	4	N	N	N	N	4	N	2	N	N	N	N	4
g= 62 :	U	U	U	U	8	N	N	N	N	4	N	N	N	N	N	N	120
g= 63 :	U	U	U	U	12	N	18	N	N	60	N	N	2	N	N	N	N
g= 64 :	X	U	X	U	8	4	2	N	N	N	N	N	6	N	N	N	N
g= 65 :	U	U	U	U	4	16	2	N	N	N	N	N	N	N	N	N	N
g= 66 :	U	X	U	U	X	4	6	4	N	N	N	N	N	N	N	N	N
g= 67 :	X	U	U	U	2	8	N	4	N	N	N	N	N	N	N	N	N
g= 68 :	U	U	U	U	6	4	N	4	N	N	N	N	N	N	N	N	N
g= 69 :	U	U	U	U	2	32	N	4	N	N	N	N	N	2	N	N	N
g= 70 :	X	U	U	U	2	N	N	44	14	N	N	N	N	2	N	N	N

Table C.3 (continued)

p=	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61
g= 71 :	U	X	U	U	6	N	N	N	2	N	N	N	N	N	N	N	N
g= 72 :	U	U	U	U	4	2	4	N	2	N	12	N	N	N	N	N	N
g= 73 :	X	U	U	U	24	2	4	N	N	N	8	N	N	N	N	N	N
g= 74 :	U	U	U	U	4	2	12	N	N	N	4	N	N	N	N	N	N
g= 75 :	U	U	U	U	8	2	4	N	N	2	72	N	N	N	N	N	N
g= 76 :	X	X	U	U	12	2	4	N	N	10	N	N	N	N	N	N	N
g= 77 :	U	U	U	U	8	N	36	2	N	6	N	N	N	N	N	N	N
g= 78 :	U	U	X	U	U	N	N	2	N	N	N	N	N	N	2	N	N
g= 79 :	X	U	U	U	U	N	N	2	N	N	N	N	N	N	2	N	N
g= 80 :	U	U	U	U	U	8	N	2	N	N	N	4	N	N	N	N	N
g= 81 :	U	X	U	U	U	4	2	N	N	N	N	8	N	N	N	N	N
g= 82 :	X	U	U	U	U	16	18	N	N	N	N	4	N	N	N	N	N
g= 83 :	U	U	U	U	U	4	2	N	N	N	N	80	N	N	N	N	N
g= 84 :	U	U	U	U	U	8	2	N	8	N	N	N	12	N	N	N	N

Table C.3 (continued)

p=	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61
g= 85 :	X	U	X	U	U	4	6	N	4	N	N	N	4	N	N	N	N
g= 86 :	U	X	U	U	U	32	N	N	8	N	N	N	4	N	N	N	N
g= 87 :	U	U	U	U	U	N	N	N	4	N	N	N	84	N	N	2	N
g= 88 :	X	U	U	U	U	2	N	4	56	N	N	N	N	N	N	2	N
g= 89 :	U	U	U	U	U	2	N	4	N	N	N	N	N	N	N	N	N
g= 90 :	U	U	U	20	U	2	12	4	N	4	2	N	N	N	N	N	10
g= 91 :	X	X	U	U	U	2	4	4	N	12	2	N	N	N	N	N	6
g= 92 :	U	U	X	U	U	2	4	4	N	4	6	N	N	4	N	N	N
g= 93 :	U	U	U	U	U	2	12	44	N	4	N	N	N	4	N	N	N
g= 94 :	X	U	U	U	U	N	4	N	N	60	N	N	N	4	N	N	N
g= 95 :	U	U	U	U	U	N	4	N	N	N	N	N	N	92	N	N	N
g= 96 :	U	X	U	U	U	4	36	N	N	N	N	N	N	N	N	N	N
g= 97 :	X	U	U	U	U	8	N	N	N	N	N	N	N	N	N	N	N
g= 98 :	U	U	U	U	U	4	N	N	2	N	N	N	N	N	N	N	N
g= 99 :	U	U	X	U	U	16	2	22	14	N	N	N	N	N	N	N	N

Table C.3 (continued)

p=	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61
g=100 :	X	U	U	U	U	4	2	2	2	N	N	2	N	N	N	N	N
g=101 :	U	X	U	U	U	8	18	2	2	N	N	10	N	N	N	N	N
g=102 :	U	U	U	U	U	4	2	2	N	N	N	2	N	N	N	N	N
g=103 :	X	U	U	U	U	32	2	2	N	N	N	N	N	N	N	N	N
g=104 :	U	U	U	U	U	2	6	N	N	N	N	N	N	N	4	N	N
g=105 :	U	U	U	U	U	2	N	N	N	6	N	N	14	N	8	N	N
g=106 :	X	X	X	U	U	2	N	N	N	2	N	N	2	N	4	N	N
g=107 :	U	U	U	U	U	2	N	N	N	10	N	N	6	N	104	N	N
g=108 :	U	U	U	U	U	2	4	N	N	6	8	N	N	N	N	N	N
g=109 :	X	U	U	U	U	2	12	N	N	N	12	N	N	N	N	N	N
g=110 :	U	U	U	U	U	2	4	4	N	N	8	N	N	N	N	N	N
g=111 :	U	X	U	X	U	N	4	4	N	N	4	N	N	N	N	N	N
g=112 :	X	U	U	U	U	32	12	4	4	N	72	N	N	N	N	N	N
g=113 :	U	U	X	U	U	4	4	4	8	N	N	N	N	N	N	N	N
g=114 :	U	U	U	U	U	8	4	4	4	N	N	N	N	N	N	N	N

Table C.3 (continued)

p=	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61
g=115 :	X	U	U	U	U	4	36	4	8	N	N	N	N	2	N	N	N
g=116 :	U	X	U	U	U	16	N	44	4	N	N	N	N	2	N	4	N
g=117 :	U	U	U	U	U	4	6	N	56	N	N	N	N	2	N	4	N
g=118 :	X	U	U	U	U	8	2	N	N	N	N	N	N	N	N	4	N
g=119 :	U	U	U	U	U	4	2	N	N	N	N	N	N	N	N	116	N
g=120 :	U	U	X	U	U	X	18	N	N	20	N	16	N	N	N	N	12
g=121 :	X	X	U	U	U	2	2	2	N	4	N	4	N	N	N	N	8
g=122 :	U	U	U	X	U	2	2	22	N	12	N	8	N	N	N	N	4
g=123 :	U	U	U	U	U	2	6	2	N	4	N	4	N	N	N	N	120
g=124 :	X	U	U	U	U	2	N	2	N	4	N	80	N	N	N	N	N
g=125 :	U	U	U	U	U	2	N	2	N	60	N	N	N	N	N	N	N
g=126 :	U	X	U	U	U	2	4	2	2	N	18	N	4	N	N	N	N
g=127 :	X	U	X	U	U	2	4	N	2	N	2	N	12	N	N	N	N
g=128 :	U	U	U	U	U	4	12	N	14	N	2	N	4	N	N	N	N
g=129 :	U	U	U	U	U	32	4	N	2	N	6	N	4	N	N	N	N

Table C.3 (continued)

p=	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61
g=130 :	X	U	U	U	U	4	4	N	2	N	N	N	84	N	2	N	N
g=131 :	U	X	U	U	U	8	12	N	N	N	N	N	N	N	2	N	N
g=132 :	U	U	U	U	24	4	4	4	N	N	N	N	N	N	2	N	N
g=133 :	X	U	U	U	U	16	4	4	N	N	N	N	N	N	N	N	N
g=134 :	U	U	X	U	U	4	36	4	N	N	N	N	N	N	N	N	N
g=135 :	U	U	U	U	U	8	2	4	N	2	N	N	N	N	N	N	N
g=136 :	X	X	U	U	U	U	6	4	N	6	N	N	N	N	N	N	N
g=137 :	U	U	U	U	U	U	2	4	N	2	N	N	N	N	N	N	N
g=138 :	U	U	U	U	U	U	2	4	N	10	N	N	N	4	N	N	N
g=139 :	X	U	U	U	U	U	18	44	N	6	N	N	N	4	N	N	N
g=140 :	U	U	U	20	U	U	2	N	8	N	N	2	N	4	N	N	N
g=141 :	U	X	X	U	U	U	2	N	4	N	N	2	N	4	N	N	N
g=142 :	X	U	U	U	U	U	6	N	8	N	N	10	N	92	N	N	N
g=143 :	U	U	U	U	U	U	N	2	4	N	N	2	N	N	N	N	N
g=144 :	U	U	U	U	U	U	36	2	8	N	4	N	N	N	N	N	N

Table C.3 (continued)

p=	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61
g=145 :	X	U	U	U	U	U	4	22	4	N	8	N	N	N	N	2	N
g=146 :	U	X	U	U	U	U	4	2	56	N	12	N	N	N	N	2	N
g=147 :	U	U	U	U	U	U	12	2	N	N	8	N	6	N	N	2	N
g=148 :	X	U	X	U	U	U	4	2	N	N	4	N	14	N	N	N	N
g=149 :	U	U	U	U	U	U	4	2	N	N	72	N	2	N	N	N	N
g=150 :	U	U	U	U	U	U	12	N	N	12	N	N	6	N	N	N	2
g=151 :	X	X	U	U	U	U	4	N	N	20	N	N	N	N	N	N	10
g=152 :	U	U	U	U	U	U	4	N	N	4	N	N	N	N	N	N	6
g=153 :	U	U	U	U	U	U	X	N	N	12	N	N	N	N	N	N	N
g=154 :	X	U	U	U	U	U	2	4	2	4	N	N	N	N	N	N	N
g=155 :	U	U	X	U	U	U	6	4	2	4	N	N	N	N	N	N	N
g=156 :	U	X	U	U	U	U	2	4	2	60	N	N	N	N	8	N	N
g=157 :	X	U	U	U	X	U	2	4	14	N	N	N	N	N	4	N	N
g=158 :	U	U	U	U	U	U	18	4	2	N	N	N	N	N	8	N	N
g=159 :	U	U	U	U	U	U	2	4	2	N	N	N	N	N	4	N	N
g=160 :	X	U	U	U	U	U	2	4	N	N	N	20	N	N	104	N	N

Table C.3 (continued)

p=	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61
g=161 :	U	X	U	U	U	U	6	4	N	N	N	16	N	2	N	N	N
g=162 :	U	U	X	U	U	U	4	44	N	N	2	4	N	2	N	N	N
g=163 :	X	U	U	U	U	U	36	N	N	N	18	8	N	2	N	N	N
g=164 :	U	U	U	U	U	U	4	N	N	N	2	4	N	2	N	N	N
g=165 :	U	U	U	U	U	U	4	2	N	2	2	80	N	N	N	N	N
g=166 :	X	X	U	X	U	U	12	2	N	2	6	N	N	N	N	N	N
g=167 :	U	U	U	U	U	U	4	2	N	6	N	N	N	N	N	N	N
g=168 :	U	U	U	U	U	U	4	22	28	2	N	N	4	N	N	N	N
g=169 :	X	U	X	U	U	U	12	2	8	10	N	N	4	N	N	N	N
g=170 :	U	U	U	U	X	U	4	2	4	6	N	N	12	N	N	N	N
g=171 :	U	X	U	U	U	U	U	2	8	N	N	N	4	N	N	N	N
g=172 :	X	U	U	U	U	U	U	2	4	N	N	N	4	N	N	N	N
g=173 :	U	U	U	U	U	U	U	N	8	N	N	N	84	N	N	N	N
g=174 :	U	U	U	U	U	U	U	N	4	N	N	N	N	N	N	4	N
g=175 :	X	U	U	U	U	U	U	N	56	N	N	N	N	N	N	4	N

Table C.3 (continued)

p=	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61
g=176 :	U	X	X	U	U	U	U	4	N	N	N	N	N	N	N	4	N
g=177 :	U	U	U	U	U	U	U	4	N	N	N	N	N	N	N	4	N
g=178 :	X	U	U	U	U	U	U	4	N	N	N	N	N	N	N	116	N
g=179 :	U	U	U	U	U	U	U	4	N	N	N	N	N	N	N	N	N
g=180 :	U	U	U	U	U	U	U	4	N	4	24	2	N	N	N	N	8
g=181 :	X	X	U	U	U	U	U	4	N	12	4	2	N	N	N	N	12
g=182 :	U	U	U	U	U	U	U	4	2	20	8	2	N	N	2	N	8
g=183 :	U	U	X	U	U	U	U	4	2	4	12	10	N	N	2	N	4
g=184 :	X	U	U	U	U	U	U	4	2	12	8	2	N	4	2	N	120
g=185 :	U	U	U	U	U	U	U	44	2	4	4	N	N	4	2	N	N
g=186 :	U	X	U	U	U	U	U	N	14	4	72	N	N	4	N	N	N
g=187 :	X	U	U	U	U	U	U	2	2	60	N	N	N	4	N	N	N
g=188 :	U	U	U	U	U	U	U	2	2	N	N	N	N	4	N	N	N
g=189 :	U	U	U	U	U	U	U	2	N	N	N	N	2	92	N	N	N
g=190 :	X	U	X	U	U	U	U	2	N	N	N	N	6	N	N	N	N

Table C.3 (continued)

p=	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61
g=191 :	U	X	U	U	U	U	U	22	N	N	N	N	14	N	N	N	N
g=192 :	U	U	U	U	U	U	U	2	N	N	N	N	2	N	N	N	N
g=193 :	X	U	U	U	U	U	U	2	N	N	N	N	6	N	N	N	N
g=194 :	U	U	U	U	U	U	U	2	N	N	N	N	N	N	N	N	N
g=195 :	U	U	U	U	U	U	U	2	N	30	N	N	N	N	N	N	N
g=196 :	X	X	U	U	U	U	U	N	8	2	N	N	N	N	N	N	N
g=197 :	U	U	X	U	U	U	U	N	28	2	N	N	N	N	N	N	N
g=198 :	U	U	U	U	U	U	U	4	8	6	2	N	N	N	N	N	N
g=199 :	X	U	U	U	U	U	U	4	4	2	2	N	N	N	N	N	N
g=200 :	U	U	U	U	U	U	U	4	8	10	18	8	N	N	N	N	N

TABLE C.4

THE p-PERIODS OF MAPPING CLASS GROUPS Γ_g

FOR $g \leq 200$ AND $67 \leq p \leq 151$

p =	67	71	73	79	83	89	97	101	103	107	109	113	127	137	139	149	151
g = 3 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 4 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 5 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 6 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 7 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 8 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 9 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 10 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N

*N = N0 p-torsion, X = Not p-periodic, U = Unknown.

Table C.4 (continued)

p =	67	71	73	79	83	89	97	101	103	107	109	113	127	137	139	149	151
g = 11 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 12 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 13 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 14 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 15 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 16 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 17 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 18 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 19 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 20 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 21 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 22 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 23 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 24 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 25 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 26 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N

Table C.4 (continued)

p =	67	71	73	79	83	89	97	101	103	107	109	113	127	137	139	149	151
g = 27 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 28 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 29 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 30 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 31 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 32 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 33 :	6	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 34 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 35 :	N	2	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 36 :	N	N	6	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 37 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 38 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 39 :	N	N	N	6	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 40 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 41 :	N	N	N	N	2	N	N	N	N	N	N	N	N	N	N	N	N
g = 42 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N

Table C.4 (continued)

p =	67	71	73	79	83	89	97	101	103	107	109	113	127	137	139	149	151
g = 43:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 44 :	N	N	N	N	N	2	N	N	N	N	N	N	N	N	N	N	N
g = 45 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 46 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 47 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 48 :	N	N	N	N	N	N	6	N	N	N	N	N	N	N	N	N	N
g = 49 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 50 :	N	N	N	N	N	N	N	2	N	N	N	N	N	N	N	N	N
g = 51 :	N	N	N	N	N	N	N	N	6	N	N	N	N	N	N	N	N
g = 52 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 53 :	N	N	N	N	N	N	N	N	N	2	N	N	N	N	N	N	N
g = 54 :	N	N	N	N	N	N	N	N	N	N	6	N	N	N	N	N	N
g = 55 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 56 :	N	N	N	N	N	N	N	N	N	N	N	2	N	N	N	N	N
g = 57 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 58 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N

Table C.4 (continued)

p =	67	71	73	79	83	89	97	101	103	107	109	113	127	137	139	149	151
g = 59 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 60 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 61 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 62 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 63 :	N	N	N	N	N	N	N	N	N	N	N	N	6	N	N	N	N
g = 64 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 65 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 66 :	4	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 67 :	4	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 68 :	132	N	N	N	N	N	N	N	N	N	N	N	N	2	N	N	N
g = 69 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	6	N	N
g = 70 :	N	4	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 71 :	N	4	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 72 :	N	140	8	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 73 :	N	N	4	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 74 :	N	N	144	N	N	N	N	N	N	N	N	N	N	N	N	2	N

Table C.4 (continued)

p =	67	71	73	79	83	89	97	101	103	107	109	113	127	137	139	149	151
g = 75 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	6
g = 76 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 77 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 78 :	N	N	N	4	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 79 :	N	N	N	4	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 80 :	N	N	N	156	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 81 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 82 :	N	N	N	N	4	N	N	N	N	N	N	N	N	N	N	N	N
g = 83 :	N	N	N	N	4	N	N	N	N	N	N	N	N	N	N	N	N
g = 84 :	N	N	N	N	164	N	N	N	N	N	N	N	N	N	N	N	N
g = 85 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 86 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 87 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 88 :	N	N	N	N	N	8	N	N	N	N	N	N	N	N	N	N	N
g = 89 :	N	N	N	N	N	4	N	N	N	N	N	N	N	N	N	N	N
g = 90 :	N	N	N	N	N	176	N	N	N	N	N	N	N	N	N	N	N

Table C.4 (continued)

p =	67	71	73	79	83	89	97	101	103	107	109	113	127	137	139	149	151
g = 91 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 92 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 93 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 94 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 95 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 96 :	N	N	N	N	N	N	8	N	N	N	N	N	N	N	N	N	N
g = 97 :	N	N	N	N	N	N	4	N	N	N	N	N	N	N	N	N	N
g = 98 :	N	N	N	N	N	N	192	N	N	N	N	N	N	N	N	N	N
g = 99 :	2	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 100 :	6	N	N	N	N	N	N	8	N	N	N	N	N	N	N	N	N
g = 101 :	N	N	N	N	N	N	N	4	N	N	N	N	N	N	N	N	N
g = 102 :	N	N	N	N	N	N	N	200	4	N	N	N	N	N	N	N	N
g = 103 :	N	N	N	N	N	N	N	N	4	N	N	N	N	N	N	N	N
g = 104 :	N	N	N	N	N	N	N	N	204	N	N	N	N	N	N	N	N
g = 105 :	N	10	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N

Table C.4 (continued)

p =	67	71	73	79	83	89	97	101	103	107	109	113	127	137	139	149	151
g = 106 :	N	2	N	N	N	N	N	N	N	4	N	N	N	N	N	N	N
g = 107 :	N	N	N	N	N	N	N	N	N	4	N	N	N	N	N	N	N
g = 108 :	N	N	2	N	N	N	N	N	N	212	8	N	N	N	N	N	N
g = 109 :	N	N	6	N	N	N	N	N	N	N	4	N	N	N	N	N	N
g = 110 :	N	N	N	N	N	N	N	N	N	N	216	N	N	N	N	N	N
g = 111 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 112 :	N	N	N	N	N	N	N	N	N	N	N	8	N	N	N	N	N
g = 113 :	N	N	N	N	N	N	N	N	N	N	N	4	N	N	N	N	N
g = 114 :	N	N	N	N	N	N	N	N	N	N	N	224	N	N	N	N	N
g = 115 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 116 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 117 :	N	N	N	2	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 118 :	N	N	N	6	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 119 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 120 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 121 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N

Table C.4 (continued)

p =	67	71	73	79	83	89	97	101	103	107	109	113	127	137	139	149	151
g = 122 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 123 :	N	N	N	N	2	N	N	N	N	N	N	N	N	N	N	N	N
g = 124 :	N	N	N	N	2	N	N	N	N	N	N	N	N	N	N	N	N
g = 125 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 126 :	N	N	N	N	N	N	N	N	N	N	N	N	4	N	N	N	N
g = 127 :	N	N	N	N	N	N	N	N	N	N	N	N	4	N	N	N	N
g = 128 :	N	N	N	N	N	N	N	N	N	N	N	N	252	N	N	N	N
g = 129 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 130 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 131 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 132 :	12	N	N	N	N	2	N	N	N	N	N	N	N	N	N	N	N
g = 133 :	4	N	N	N	N	2	N	N	N	N	N	N	N	N	N	N	N
g = 134 :	4	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 135:	132	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 136:	N	N	N	N	N	N	N	N	N	N	N	N	N	8	N	N	N
g = 137:	N	N	N	N	N	N	N	N	N	N	N	N	N	4	N	N	N

Table C.4 (continued)

p =	67	71	73	79	83	89	97	101	103	107	109	113	127	137	139	149	151
g = 138 :	N	N	N	N	N	N	N	N	N	N	N	N	N	272	4	N	N
g = 139 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	4	N	N
g = 140 :	N	4	N	N	N	N	N	N	N	N	N	N	N	N	276	N	N
g = 141 :	N	4	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 142 :	N	4	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 143 :	N	140	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 144 :	N	N	12	N	N	N	2	N	N	N	N	N	N	N	N	N	N
g = 145 :	N	N	8	N	N	N	6	N	N	N	N	N	N	N	N	N	N
g = 146 :	N	N	4	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 147 :	N	N	144	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 148 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	8	N
g = 149 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	4	N
g = 150 :	N	N	N	N	N	N	N	10	N	N	N	N	N	N	N	296	4
g = 151 :	N	N	N	N	N	N	N	2	N	N	N	N	N	N	N	N	4
g = 152 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	300
g = 153 :	N	N	N	N	N	N	N	N	2	N	N	N	N	N	N	N	N

Table C.4 (continued)

p =	67	71	73	79	83	89	97	101	103	107	109	113	127	137	139	149	151
g = 154 :	N	N	N	N	N	N	N	N	6	N	N	N	N	N	N	N	N
g = 155 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 156 :	N	N	N	12	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 157 :	N	N	N	4	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 158 :	N	N	N	4	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 159 :	N	N	N	156	N	N	N	N	N	2	N	N	N	N	N	N	N
g = 160 :	N	N	N	N	N	N	N	N	N	2	N	N	N	N	N	N	N
g = 161 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 162 :	N	N	N	N	N	N	N	N	N	N	2	N	N	N	N	N	N
g = 163 :	N	N	N	N	N	N	N	N	N	N	6	N	N	N	N	N	N
g = 164 :	N	N	N	N	4	N	N	N	N	N	N	N	N	N	N	N	N
g = 165 :	2	N	N	N	4	N	N	N	N	N	N	N	N	N	N	N	N
g = 166 :	2	N	N	N	4	N	N	N	N	N	N	N	N	N	N	N	N
g = 167 :	6	N	N	N	164	N	N	N	N	N	N	N	N	N	N	N	N
g = 168 :	N	N	N	N	N	N	N	N	N	N	N	2	N	N	N	N	N
g = 169 :	N	N	N	N	N	N	N	N	N	N	N	2	N	N	N	N	N

Table C.4 (continued)

p =	67	71	73	79	83	89	97	101	103	107	109	113	127	137	139	149	151
g = 170 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 171 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 172 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 173 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 174 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 175 :	N	14	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 176 :	N	10	N	N	N	4	N	N	N	N	N	N	N	N	N	N	N
g = 177 :	N	2	N	N	N	8	N	N	N	N	N	N	N	N	N	N	N
g = 178 :	N	N	N	N	N	4	N	N	N	N	N	N	N	N	N	N	N
g = 179 :	N	N	N	N	N	176	N	N	N	N	N	N	N	N	N	N	N
g = 180 :	N	N	2	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 181 :	N	N	2	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 182 :	N	N	6	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 183 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 184 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 185 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N

Table C.4 (continued)

p =	67	71	73	79	83	89	97	101	103	107	109	113	127	137	139	149	151
g = 186 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 187 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 188 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 189 :	N	N	N	N	N	N	N	N	N	N	N	N	2	N	N	N	N
g = 190 :	N	N	N	N	N	N	N	N	N	N	N	N	6	N	N	N	N
g = 191 :	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 192 :	N	N	N	N	N	N	12	N	N	N	N	N	N	N	N	N	N
g = 193 :	N	N	N	N	N	N	8	N	N	N	N	N	N	N	N	N	N
g = 194 :	N	N	N	N	N	N	4	N	N	N	N	N	N	N	N	N	N
g = 195 :	N	N	N	2	N	N	192	N	N	N	N	N	N	N	N	N	N
g = 196 :	N	N	N	2	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 197 :	N	N	N	6	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 198 :	4	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 199 :	12	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g = 200 :	4	N	N	N	N	N	N	4	N	N	N	N	N	N	N	N	N

TABLE C.5

THE p-PERIODS OF MAPPING CLASS GROUPS Γ_g

FOR $g \leq 200$ AND $157 \leq p \leq 233$

p=	157	163	167	169	173	179	181	191	193	197	199	211	217	223	227	229	233
g=2:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=3:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=4:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=5:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=6:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=7:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=8:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=9:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=10:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N

*N = No p-torsion, X = Not p-periodic, U = Unknown.

Table C.5 (continued)

p=	157	163	167	169	173	179	181	191	193	197	199	211	217	223	227	229	233
g=11:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=12:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=13:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=14:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=15:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=16:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=17:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=18:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=19:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=20:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=21:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=22:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=23:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=24:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=25:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=26:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N

Table C.5 (continued)

p=	157	163	167	169	173	179	181	191	193	197	199	211	217	223	227	229	233
g=27:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=28:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=29:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=30:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=31:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=32:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=33:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=34:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=35:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=36:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=37:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=38:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=39:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=40:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=41:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=42:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N

Table C.5 (continued)

p=	157	163	167	169	173	179	181	191	193	197	199	211	217	223	227	229	233
g=43:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=44:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=45:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=46:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=47:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=48:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=49:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=50:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=51:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=52:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=53:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=54:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=55:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=56:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=57:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=58:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N

Table C.5 (continued)

p=	157	163	167	169	173	179	181	191	193	197	199	211	217	223	227	229	233
g=59:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=60:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=61:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=62:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=63:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=64:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=65:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=66:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=67:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=68:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=69:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=70:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=71:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=72:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=73:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=74:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N

Table C.5 (continued)

p=	157	163	167	169	173	179	181	191	193	197	199	211	217	223	227	229	233
g=75:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=76:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=77:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=78:	6	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=79:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=80:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=81:	N	6	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=82:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=83:	N	N	2	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=84:	N	N	N	6	N	N	N	N	N	N	N	N	N	N	N	N	N
g=85:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=86:	N	N	N	N	2	N	N	N	N	N	N	N	N	N	N	N	N
g=87:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=88:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=89:	N	N	N	N	N	2	N	N	N	N	N	N	N	N	N	N	N
g=90:	N	N	N	N	N	N	6	N	N	N	N	N	N	N	N	N	N

Table C.5 (continued)

p=	157	163	167	169	173	179	181	191	193	197	199	211	217	223	227	229	233
g=91:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=92:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=93:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=94:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=95:	N	N	N	N	N	N	N	2	N	N	N	N	N	N	N	N	N
g=96:	N	N	N	N	N	N	N	N	6	N	N	N	N	N	N	N	N
g=97:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=98:	N	N	N	N	N	N	N	N	N	2	N	N	N	N	N	N	N
g=99:	N	N	N	N	N	N	N	N	N	N	6	N	N	N	N	N	N
g=100:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=101:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=102:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=103:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=104:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=105:	N	N	N	N	N	N	N	N	N	N	N	6	N	N	N	N	N
g=106:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N

Table C.5 (continued)

p=	157	163	167	169	173	179	181	191	193	197	199	211	217	223	227	229	233
g=107:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=108:	N	N	N	N	N	N	N	N	N	N	N	N	6	N	N	N	N
g=109:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=110:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=111:	N	N	N	N	N	N	N	N	N	N	N	N	N	6	N	N	N
g=112:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=113:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	2	N	N
g=114:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	6	N
g=115:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=116:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	2
g=117:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=118:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=119:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=120:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=121:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=122:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N

Table C.5 (continued)

p=	157	163	167	169	173	179	181	191	193	197	199	211	217	223	227	229	233
g=123:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=124:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=125:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=126:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=127:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=128:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=129:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=130:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=131:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=132:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=133:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=134:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=135:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=136:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=137:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=138:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N

Table C.5 (continued)

p=	157	163	167	169	173	179	181	191	193	197	199	211	217	223	227	229	233
g=139:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=140:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=141:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=142:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=143:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=144:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=145:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=146:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=147:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=148:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=149:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=150:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=151:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=152:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=153:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=154:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N

Table C.5 (continued)

p=	157	163	167	169	173	179	181	191	193	197	199	211	217	223	227	229	233
g=155:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=156:	8	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=157:	4	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=158:	312	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=159:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=160:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=161:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=162:	N	4	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=163:	N	4	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=164:	N	324	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=165:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=166:	N	N	4	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=167:	N	N	4	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=168:	N	N	332	8	N	N	N	N	N	N	N	N	N	N	N	N	N
g=169:	N	N	N	4	N	N	N	N	N	N	N	N	N	N	N	N	N
g=170:	N	N	N	336	N	N	N	N	N	N	N	N	N	N	N	N	N

Table C.5 (continued)

p=	157	163	167	169	173	179	181	191	193	197	199	211	217	223	227	229	233
g=171:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=172:	N	N	N	N	8	N	N	N	N	N	N	N	N	N	N	N	N
g=173:	N	N	N	N	4	N	N	N	N	N	N	N	N	N	N	N	N
g=174:	N	N	N	N	344	N	N	N	N	N	N	N	N	N	N	N	N
g=175:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=176:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=177:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=178:	N	N	N	N	N	4	N	N	N	N	N	N	N	N	N	N	N
g=179:	N	N	N	N	N	4	N	N	N	N	N	N	N	N	N	N	N
g=180:	N	N	N	N	N	356	8	N	N	N	N	N	N	N	N	N	N
g=181:	N	N	N	N	N	N	4	N	N	N	N	N	N	N	N	N	N
g=182:	N	N	N	N	N	N	360	N	N	N	N	N	N	N	N	N	N
g=183:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=184:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=185:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=186:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N

Table C.5 (continued)

p=	157	163	167	169	173	179	181	191	193	197	199	211	217	223	227	229	233
g=187:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=188:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=189:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=190:	N	N	N	N	N	N	N	4	N	N	N	N	N	N	N	N	N
g=191:	N	N	N	N	N	N	N	4	N	N	N	N	N	N	N	N	N
g=192:	N	N	N	N	N	N	N	380	8	N	N	N	N	N	N	N	N
g=193:	N	N	N	N	N	N	N	N	4	N	N	N	N	N	N	N	N
g=194:	N	N	N	N	N	N	N	N	384	N	N	N	N	N	N	N	N
g=195:	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
g=196:	N	N	N	N	N	N	N	N	N	8	N	N	N	N	N	N	N
g=197:	N	N	N	N	N	N	N	N	N	4	N	N	N	N	N	N	N
g=198:	N	N	N	N	N	N	N	N	N	392	4	N	N	N	N	N	N
g=199:	N	N	N	N	N	N	N	N	N	N	4	N	N	N	N	N	N
g=200:	N	N	N	N	N	N	N	N	N	N	396	N	N	N	N	N	N

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