# HOW ELEMENTARY SCHOOL TEACHERS' MATHEMATICAL SELF-EFFICACY AND MATHEMATICS TEACHING SELF-EFFICACY RELATE TO CONCEPTUALLY AND PROCEDURALLY ORIENTED TEACHING PRACTICES

DISSERTATION

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By

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ABSTRACT

The purpose of this mixed method study was to explore the relationships among the variables of mathematics self-efficacy, mathematics teaching self-efficacy, and procedurally or conceptually-oriented teaching methods. The study included 75 practicing elementary teachers who teach mathematics as well as other subjects. These teachers completed the Mathematics Teaching and Mathematics Self-Efficacy survey, designed as part of the study and based on the Mathematics Self-Efficacy Scale - Revised (MSES-R) and the Mathematics Teaching Efficacy Beliefs Instrument (MTEBI). Sixteen of the teachers also participated in an interview probing teaching methods for two mathematics topics the teachers believed they are most confident or least confident teaching. Interviews were assessed using the Conceptually and Procedurally Oriented Teaching Method Frequency Chart, designed as part of the study. Quantitative data analysis methods include descriptive statistics, Pearson's Product Moment correlation, and chi-square tests. Qualitative data analysis includes case study anecdotes for two of the interviewed teachers. Results indicate a strong relationship between mathematics self-efficacy and mathematics teaching self-efficacy and suggest that mathematics self-efficacy may be a precursor to mathematics teaching self-efficacy. Additionally, results indicate that when teaching their most confident mathematics topic teachers are more likely to use conceptually oriented teaching methods and when teaching their least

confident mathematics topic teachers are more likely to use procedurally oriented teaching methods. This study offers findings to mathematics teacher educators and elementary mathematics teachers about the importance of developing mathematics self-efficacy and mathematics teaching self-efficacy because of their relationship to teachers' choices of instructional methods. Additionally the two instruments developed in the study will help future researchers assess these variables.

DEDICATION

Dedicated to Rick, Ryan, Kelli and Adam

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## FIELDS OF STUDY

Major Field: Mathematics Education

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# CHAPTER 1

# STATEMENT OF THE PROBLEM

Consider an elementary classroom where parents take turns helping the teacher once a month for an hour with miscellaneous tasks. One parent, who the teacher knew was a mathematics educator, usually was asked to listen to the children as they read independently. One day, the teacher asked her to assist three students in catching up on their mathematics lessons after being ill for a week. The parent happily accepted and soon had the three children, their workbooks, and a pile of manipulatives set up in the hallway. The children were soon grasping the concepts.

After a few minutes the neighboring teacher stepped into the hallway. She looked over and said, “Oh no, you got stuck doing math today.” The children’s faces changed expression. Their hands stopped moving the manipulatives so quickly. Immediately the children began stating their opinions of liking or disliking mathematics and feeling successful or unsuccessful with mathematics. The comments of the teacher changed the students’ focus from mathematical engagement to questioning their mathematical self-efficacy. This incident raises many questions. Why did the neighboring teacher make the comment? Did she dislike mathematics? Did she lack confidence with her own mathematics ability? Did she verbalize an acceptance of disliking mathematics to her own students? What was the strength of her mathematics self-efficacy and her mathematics teaching self-efficacy?

**Math phobia.** When the topic of mathematics arises in a conversation, many people will tell you passionately that they are inadequate at mathematics, fear mathematics, or simply dislike mathematics. People generally do not make similar statements about literacy or writing. In America, it is socially acceptable to verbalize a disliking or fear of mathematics (Burns, 1998; Paulos, 1988). A noted mathematics educator, Marilyn Burns (1998), stated in her book *Math: Facing the American Phobia*, that people believe, "Math is right up there with snakes, public speaking, and heights" (p. ix). Many believe that only some people are good at mathematics, or you are only good in mathematics if you have some special mathematics gene. In contrast, books have also been written about humans' natural ability to do mathematics including algebra and calculus (Devlin, 2000). Despite efforts to stifle the stereotypes related to learning mathematics, "Math phobia is a widespread national problem" (Burns, 1998, p. ix).

Mathematical illiteracy is often flaunted, whereas people would not admit other failures (Paulos, 1988). Frequently comments are heard such as "I can't even balance my checkbook"; "I'm a people person, not a numbers person"; or "I always hated math." Paulos asked, "Why is innumeracy so widespread even among otherwise educated people?" (p. 72). His answer, "The reasons, to be a little simplistic, are poor education, psychological blocks, and romantic misconceptions about the nature of mathematics" (pp.72-73).

Many people claim to have had a bad experience with mathematics and later accept this *American phobia*. They have children whom they expect to dislike mathematics or do poorly in mathematics, and the vicious cycle continues. In *Everybody Counts*, a national publication reporting on mathematics education, the National Research Council (1989) stated, “Parents often accept – and sometimes even expect – their children’s poor performance in mathematics” (p. 9). The *American phobia* is prevalent in some homes, but also in some schools. Fennell (2007), the current president of the National Council of Teachers of Mathematics (NCTM), related a common experience of holding a parent conference where the parent expresses that he or she was “never good at math either (p. 3).” This belief acknowledges and reinforces his or her child’s weak mathematics performance by explaining that it is a result of the parent’s own weak mathematics ability.

In general, in America it is socially acceptable to fear or dislike mathematics, and this attitude toward mathematics affects our schoolchildren. This socially acceptable but poor attitude toward mathematics may be reflected in our schools by various individuals who tell their stories of mathematical woe. Certainly this poor attitude toward mathematics is not expected among our mathematics teachers at the university and high school levels who have been specially trained in mathematics and who likely selected this career due to their appreciation for the subject, but what about the teachers at the elementary level? Have they entered the profession due to their appreciation for mathematics or for some other subject or for their love of young children? Are too many elementary teachers typical of Americans who embrace the acceptance of the *American phobia* – mathematics? In elementary teachers, the American Phobia toward mathematics likely would be exhibited as low self-efficacy in mathematics or mathematics teaching.

**Self-efficacy.** The need for self-efficacy, or a person's belief in his or her capabilities, toward mathematics and mathematics teaching within individual students and teachers of mathematics has been highlighted in the NCTM *Standards* (1989, 1991, 2000). Confidence was addressed in the *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989), which states that one goal for students is "that they become confident in their ability to do mathematics" (p. 5). Again in the NCTM *Professional Standards for Teaching Mathematics* (1991) confidence was addressed stating, "Mathematical power encompasses the ability to explore, conjecture, and reason logically, as well as the ability to use a variety of mathematical methods effectively to solve non-routine problems and the self-confidence and disposition to do so" (p. 19). Again in *Principles and Standards for School Mathematics* (NCTM, 2000), confidence was addressed as part of NCTM's vision for school mathematics anticipating that "students confidently engage in complex mathematical tasks chosen carefully by teachers" (p. 2).

Researchers have shown the need for teacher self-efficacy and how that self-efficacy affects the types of learning environment in a classroom. "Few would argue that the beliefs teachers hold influence their perceptions and judgments, which, in turn, affect their behavior in the classroom, or that understanding the belief structures of teachers and teacher candidates is essential to improving their professional preparation and teaching practices" (Pajares, 1992, p. 307). Bandura (1993) related teacher self-efficacy with both classroom environment and student learning as he said, "Teachers' beliefs in their personal efficacy to motivate and promote learning affect the types of learning environments they create and the level of academic progress their students achieve" (p.

117). Teacher self-efficacy has also been shown to affect student self-efficacy. Siegle & McCoach (2007) stated, “Teachers can modify their instructional strategies with minimal training and effort, and this can result in increases in their students’ self-efficacy” (p. 279). It is a premise of the present study that teacher self-efficacy affects teacher choice of instructional methods and classroom environment which affect both student learning and student self-efficacy.

**Teaching methods.** Conceptual and procedural approaches to teaching mathematics were also addressed in the NCTM Standards documents. One of four assumptions about teaching mathematics (NCTM, 1991) suggests that, “WHAT students learn is fundamentally connected with HOW they learn it…What students learn about particular concepts and procedures as well as about thinking mathematically depends on the ways in which they engage in mathematical activity in their classroom” (NCTM, 1991, p. 21). Furthermore, NCTM’s Learning Principle states that, “Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge” (NCTM, 2000, p. 20). This principle goes on to say that conceptual understanding along with factual knowledge and procedural facility are important parts of becoming proficient in mathematics.

NCTM linked confidence with conceptual and procedural learning in another one of the four assumptions about mathematics teaching (NCTM, 1991) stating, “Teachers must help every student develop conceptual and procedural understandings of number, operations, geometry, measurement, statistics, probability, functions, and algebra and the connections among ideas…and to develop the self-confidence and interest to do so” (NCTM, 1991, p. 21). Additionally, the *Professional Standards* (NCTM, 1991) report that student "dispositions toward mathematics are also shaped by such experiences" (p. 21) with conceptual understanding.

Despite the strong support by NCTM regarding teaching for mathematical understanding, Marshall (2003) states that the current mathematics standards fall short in the guidance they offer to teachers who lack the experience and confidence to teach in a way that they themselves were not taught. This argument aims to provoke improved mathematical confidence and to move a balance between conceptual and procedural teaching methods from being simply a goal to becoming an issue that is being researched, improved, and implemented in the classroom.

**Curriculum**. In 2006 NCTM issued its *Curriculum Focal Points* which were intended to be "the next step in the implementation of *Principles and Standards for School Mathematics*" (NCTM, 2006a, p. 1). These focal points were identified by a combined group of mathematicians and mathematics educators and have been described as a common ground in the U.S. "math wars" (American Association for the Advancement of Science, 2006). These focal points "comprise related ideas, concepts, skills, and procedures that form the foundation for understanding and lasting learning. They are building blocks that students must thoroughly understand to progress to more advanced mathematics" (NCTM, 2006b). The *Curriculum Focal Points* include both concepts and procedures that reinforce the idea of balancing conceptual and procedural learning. The focal points inspired some of the interview questions used in this study as they focused on a few key concepts that need to be taught at each grade level. When asked whether standard algorithms must be mastered by all students, NCTM responded stating:

> One grade 4 focal point recommends the quick recall of multiplication facts and fluency with efficient procedures, including the standard algorithm for multiplying whole numbers. Again, it is most important that fluency emerge through deep understanding of the multiplication process – including how to represent multiplication and how to use properties when multiplying. (NCTM, 2006a, p. 6)

NCTM also addressed the concern that these *Curriculum Focal Points* were an attempt to revert back to the basics, reiterating their "longstanding position on teaching students to learn critical foundational topics (e.g. multiplication) with conceptual understanding" (NCTM, 2006a, p. 6). NCTM stated the *Curriculum Focal Points* were not a retreat to basics, but rather the next step in implementation.

American elementary teachers likely have a wide range of the presence or absence of mathematics phobia which affects their mathematics self-efficacy and mathematics teaching self-efficacy. Their self-efficacy affects their approach to teaching mathematics and their choice of teaching practices which affects student learning and student self-efficacy. NCTM connects the constructs of balancing procedural and conceptual teaching and supporting mathematics confidence in students as goals for all mathematics educators. The goal of the present study was to examine more closely the relationship between these constructs in hopes of helping teachers, who in turn help students, to become more confident in their ability to understand mathematics conceptually and perform it with procedural fluency.

## Practical Rationale

**Fostering self-efficacy.** "The negative attitude toward mathematics, unfortunately so common even among otherwise highly-educated people, is surely the greatest measure of our failure and a real danger to our society" (Bondi, 1976, p. 8). People's attitudes and confidence in their mathematical abilities has long been and still is a growing concern of many who strive for quality American education. In the TIMSS report (U.S. Department of Education, 1997) former U.S. President Clinton recognized the significance of confidence on performance as he encouraged American fourth graders on their performance on the TIMSS test, saying that the results showed that Americans can be the best in the world if we simply believe it and then organize ourselves to achieve it. Recently, NCTM President Fennell (2007) indicated that the problem is still prevalent as he expressed concern for the national problem with self-concept and mathematics learning.

The National Council of Teachers of Mathematics, worldwide the largest professional organization in the field of mathematics education states, "Educational goals for students must reflect the importance of mathematical literacy. The K-12 national standards articulate five general goals for all students, including: … that they become confident in their ability to do mathematics" (NCTM, 1989, p. 5). Student confidence is related to their self-efficacy, or perceived ability.

Teachers are a key instrument in fostering self-efficacy in students. Their classroom experiences and social interactions with other students influence students' mathematics self-efficacy. This self-efficacy formed in school will likely be carried with them for a lifetime. The mathematics self-efficacy of the teacher is visible to the students in the class and the mathematics teaching self-efficacy likely affects the choices of instructional strategies, time spent on mathematics, emphasis on the importance of mathematics, and so on.

In many elementary classrooms, the teacher is responsible for teaching all content areas despite his or her feelings toward any subject. Many elementary teachers, however, enjoy teaching some academic subjects more than others (Bandura, 1997b). People with both positive and negative beliefs about mathematics influence children's lives; some of those with negative beliefs may include many elementary teachers (Ball, 1990a). Results from Ball's (1990b) study about feelings toward mathematics showed that only one-half of elementary teacher candidates indicated they enjoyed mathematics and over one-third of them indicated they were not good at mathematics. What if the individuals who do not believe they are good at mathematics are our children's mathematics teachers? Will these teachers teach differently than those who have higher self-efficacy? Will these teachers teach in a way that fosters students' self-efficacy?

**Teaching for mathematical proficiency**. According to the National Research Council (NRC) (2001) mathematical proficiency was constructed of conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition as defined here. These five constructs are intertwined and interdependent and have implications on student acquisition of mathematical proficiency, how teachers promote mathematical proficiency and how teachers are educated to achieve the goal of mathematical proficiency. Conceptual understanding was defined as "comprehension of mathematical concepts, operations, and relations" (p. 5). Procedural fluency was defined as "skill in carrying out procedures flexibly, accurately, efficiently, and appropriately" (p. 5). Strategic competence was defined as "ability to formulate, represent, and solve mathematical problems" (p. 5). Adaptive reasoning was defined as "capacity for logical thought, reflection, explanation, and justification" (p. 5). Productive disposition was defined as the “habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy” (p. 5). The current study explored productive disposition, specifically self-efficacy, and how it relates to teaching methods which develop conceptual understanding and procedural fluency.

The career of teaching often allows professional flexibility in choosing the appropriate methods to teach the required curriculum. Current trends in mathematics education, spurred from the NCTM Standards, emphasize conceptual teaching of mathematics as a forerunner to procedural development of skills. Certainly, choices are influenced by availability of resources, training, interest, and self-efficacy (Raymond, 1997). Will these teachers likely teach our children using conceptual building blocks aimed at understanding or using algorithms, memorization, and rules?

This study constitutes an important step in understanding how teacher mathematics self-efficacy and mathematics teaching self-efficacy impact his or her teaching behaviors, including the relationship between mathematics self-efficacy and mathematics teaching self-efficacy and the inclination toward procedural or conceptual teaching methods.

## Research Questions

This mixed-methodology study addressed the following three research questions:

1. How does mathematics self-efficacy relate to mathematics teaching self-efficacy?
2. How does elementary teachers’ mathematical self-efficacy relate to their tendency to teach conceptually or procedurally?

3. How does elementary teachers' mathematics teaching self-efficacy relate to their tendency to teach conceptually or procedurally?

**Variables.** Independent variables in this study included mathematics self-efficacy and mathematics teaching self-efficacy. Dependent variables were conceptual and procedural teaching methods as measured by teacher responses to questions about teaching mathematics topics in which they believe they are least and most confident teaching. It was hypothesized that each teacher possessed varying degrees of mathematics self-efficacy and mathematics teaching self-efficacy depending on the topic and that some teachers taught more procedurally or more conceptually based on their self-efficacy relative to each topic.

| | Mathematics Self-Efficacy | | |
|---|---|---|---|
| Mathematics Teaching Self-Efficacy | High MSE<br><br>High MTSE | Medium MSE<br><br>High MTSE | Low MSE<br><br>High MTSE |
| | High MSE<br><br>Medium MTSE | Medium MSE<br><br>Medium MTSE | Low MSE<br><br>Medium MTSE |
| | High MSE<br><br>Low MTSE | Medium MSE<br><br>Low MTSE | Low MSE<br><br>Low MTSE |

Figure 1.1: Levels of mathematics and mathematics teaching self-efficacy

**Conjectures.** The author conjectured that teachers fit in one of the nine categories in Figure 1.1 depending on the topic. The extreme four corner categories were examined more closely to look for differences among the variables. While it was anticipated that more teachers may have both high mathematics self-efficacy and high mathematics teaching self-efficacy or both low mathematics self-efficacy and low mathematics teaching self-efficacy, the author also desired to find teachers who exhibited high mathematics self-efficacy and low mathematics teaching self-efficacy or low mathematics self-efficacy and high mathematics teaching self-efficacy. The author conjectured that teachers with a lower mathematics self-efficacy and a lower mathematics teaching self-efficacy teach mainly procedurally and those who have a higher mathematics self-efficacy and a higher mathematics teaching self-efficacy mainly teach using methods which are more conceptually oriented or which attempt to balance conceptual understanding coupled with procedural fluency.

## Definitions

Many mathematics education researchers have offered definitions related to self-efficacy, confidence, attitudes, and beliefs. Mathematics beliefs have been defined as "personal judgments about mathematics formulated from experiences in mathematics, including beliefs about the nature of mathematics, learning mathematics, and teaching mathematics" (Raymond, 1997, p. 552). Attitudes have been defined as learned predispositions to respond in a particular way (Richardson, 1996) and as a "learned internal state, whose function is to influence choices of personal action" (Gagne, 1977, p. 249).

### Self-Efficacy

Pajares (1996a) has explained the difference between self-concept and self-efficacy in that self-efficacy is context-specific. Self-concept, or confidence, is broader and includes such things as self-efficacy and self-worth. Consider the difference between a student who claims "I am good at math" versus a student who claims "I am confident that I can accurately perform two-digit subtraction." The first displays overall confidence and the latter displays self-efficacy toward a specific task.

According to Albert Bandura (1994), efficacy is a belief in one's personal capabilities. Teacher efficacy is described in one study as a "variable accounting for individual differences in teaching effectiveness" (Gibson & Dembo, 1984, p. 569). These definitions offered by the efficacy researchers above have been synthesized to focus in on the particular definition of self-efficacy used in this study.

Self-efficacy is a perceived ability. Definitions of mathematics self-efficacy and mathematics teaching self-efficacy do not state an attitude that the teacher likes or dislikes mathematics nor a belief that mathematics is difficult or easy. Self-efficacy, therefore, is related to, but different from, beliefs and attitudes. *Mathematics self-efficacy* is a person's perception of his or her own mathematical ability and *mathematics teaching self-efficacy* is a person's perception of his or her ability to teach others mathematics. Consider the following four examples which aim to illustrate mathematics self-efficacy and mathematics teaching self-efficacy for fictional teachers in each of the four corners in Figure 1.1.

**Sue.** Sue is an example of a teacher with low mathematics self-efficacy but high mathematics teaching self-efficacy. Sue is a teacher who was hired to teach elementary mathematics even though she preferred to teach language arts or social studies. She has a low mathematics self-efficacy, meaning she believes she is not good at mathematics, despite the fact that she has often been successful in mathematics. She has a high mathematics teaching self-efficacy, meaning she believes that she can help students learn mathematics, especially those skills in which she herself has overcome difficulties. Sue prepares lessons diligently and is sure to have each detail properly noted and each example problem thoroughly worked through before she presents the lesson to children. She likely teaches new topics procedurally until she is very comfortable with the topic, when her mathematics self-efficacy becomes higher. She is hesitant but willing to try new teaching methods. Sue enjoys working closely with other mathematics teachers to share ideas and frustrations.

**Ray.** As an example of a teacher in another corner of Figure 1.1, consider Ray, a teacher who had a previous career as an engineer. He knows the subject of mathematics very well and thus has a solid mathematical self-efficacy. However, his mathematics teaching self-efficacy is low because even though he understands the concepts, he feels unsure how to explain the concepts to children. Ray spends more of his planning time on his implementation strategies than on the mathematics. Ray understands the application and usefulness of mathematics and shares it with his students. Ray teaches primarily procedurally because he understands mathematics procedurally and, as a teacher, is being forced to think conceptually about mathematics for the first time.

**Pat.** Pat is a teacher who is confident in her mathematical ability and confident in her teaching ability. She anticipates students' questions and quickly interprets students' mathematical errors. She can make up a multiple choice test with all the incorrect choices that students will make when erroneously solving the problem. Her success lies in her ability to explain these errors to students so they learn from their mistakes and understand the underlying concepts. Pat dislikes using a textbook because she would rather create the lesson herself. She presents topics conceptually and her classroom features a community of discourse that allows students to propose multiple strategies for solving problems. Pat is a teacher who fits the category of high mathematical self-efficacy and high mathematics teaching self-efficacy.

**Ann.** As the final example, consider Ann, a teacher who values literacy and fears mathematics. Each year her students excel in improving their reading and writing abilities. Her school allows flexible time scheduling for the academic subjects that must be taught in the elementary classroom. Ann is experimenting with block scheduling and has decided that she should teach mathematics and science twice a week, social studies and language arts three days a week, and reading and writing daily. Frequently the activities in the other subjects are so involved that she shortens the mathematics time. She tries to avoid mathematics in her personal life by denying that she uses it. In the classroom she teaches what is required and rarely ventures from the worksheets that go with the textbook. She deems answers as either right or wrong and she accepts one right way to do each problem. She does not particularly encourage questions from students because she is afraid she may not know how to answer them. Ann is an example of a teacher with low mathematics self-efficacy and low mathematics teaching self-efficacy.

**Comparison.** These four teachers, while only examples, illustrate some possible differences in individual teachers and how they vary in mathematics self-efficacy and mathematics teaching self-efficacy. These same teachers may also vary significantly in their teaching methods. Some teachers believe there is one right way to solve every problem while others view multiple solutions as a rich inquiry basis for class discussions. Some teachers have manipulatives in their classroom available for students to use at any time while others view the drill of quick basic facts as of foremost importance. Some teachers want students simply to know how to do a problem and others want students to ask why the mathematics works. The current study strived to define these varying teaching methods as primarily procedurally or conceptually oriented teaching methods and analyzed differences in the relative emphasis on procedurally oriented versus conceptually oriented methods when teaching various mathematical content topics.

## Procedurally Versus Conceptually Oriented Teaching

Educational researchers and theorists have offered several definitions for procedural and conceptual knowledge. Procedural knowledge has been described as the "formal language, or symbol representation system, of mathematics" (Hiebert, 1986, p. 6) and the "rules, algorithms, or procedures used to solve mathematical tasks" (Hiebert, 1986, p. 6) or as the "ability to execute action sequences to solve problems" (Rittle-Johnson, Siegler, & Alibali, 2001, p. 346). Conceptual knowledge has been described as "knowledge that is rich in relationships" (Hiebert, 1986, p. 3) or as flexible "implicit or explicit understanding of the principles that govern a domain and of the interrelations between units of knowledge in a domain" (Rittle-Johnson, Siegler, & Alibali, 2001, p. 346-7). Skemp (1976) has described a similar pair of understanding types. He described relational understanding as "knowing both what to do and say" (p. 153) and instrumental understanding as "rules without reason" (p. 153). These definitions relate to types of knowledge and student understanding while the current study addressed teaching methods that promote these types of understanding.

*Procedurally oriented teaching* is defined by the author as using teaching methods that focus on developing students' procedural understanding of mathematics. Procedurally oriented teaching can include teaching rote memorization of basic division facts, presenting a formula for calculating areas, and providing definitions without activities to explore the definitions. On the other hand, *conceptually oriented teaching* is defined by the author as using teaching methods that focus on developing students' conceptual understanding of mathematics. Conceptually oriented teaching can include asking students to determine how to divide a class of children into equal groups, exploring the areas of various sized rectangles on a geoboard, and asking students to compare and contrast different kinds of 2-dimensional shapes. Any teacher could likely approach some lessons with a procedural orientation and others with a conceptual orientation. Likewise a teacher may exhibit both types of teaching methods in one lesson.

**Jane.** In the current study, teachers were classified as either conceptually oriented or procedurally oriented toward a specific mathematical topic based upon whether they use a greater number of procedurally or conceptually oriented methods. For example, Jane began an algebra lesson involving the order of operations by asking students to type $3 + 2 \times 5$ into various calculators. She led a class discussion about the different answers that scientific and non-scientific calculators produced and speculated why different calculators got different answers. Next the teacher indicated the convention that has been agreed upon for the order of operations and taught students a mnemonic to remember the order of operations. The students practiced five problems and Jane then taught them a song to remember the mnemonic. Ten more problems were then assigned. Jane is categorized as a procedurally oriented teacher for order of operations. She began the lesson conceptually by using technology to explore a problem and leading a discussion. However she used several more procedurally oriented methods, including demonstrating a definition, showing examples of the order of operations, teaching a mnemonic and a song, and assigning drill problems.

**Jeff.** Jeff, on the other hand, is a more conceptually oriented teacher for order of operations. Jeff began the lesson as Jane did by asking students to type $3 + 2 \times 5$ into various calculators to discover different answers. Next he presented the students with 4 numerical expressions involving two operations and three numbers each. The students were asked to determine the value of each expression, and different students obtained different values. Then Jeff led a class discussion about the reasons for the different answers. He posed questions such as, which answer is right, and how should mathematicians know which operation to do first. Jeff then explained that mathematicians have developed rules to insure that everyone gets the same answer to problems like these. He then described the rules and asked the students to use those rules to evaluate the four expressions they looked at before. Jeff finishes class by asking students to explain in a sentence how to figure out the correct solution to $3 + 2 \times 5$. During this example lesson, Jeff used more conceptually oriented methods than procedurally oriented methods. He procedurally presented the meaning of the order of operations term. The conceptually oriented methods included using technology to explore, encouraging decision making about which operation to perform first, leading a class discussion, and asking students to write about their thinking.

## Theoretical Framework

Many would agree that teachers exhibit individual characteristics that are reflected in their classroom setup, teaching strategies, and even the way they interact with children. Previous self-efficacy studies have shown that teachers have varying degrees of mathematics self-efficacy (Kranzler & Pajares, 1997), and varying degrees of mathematics teaching self-efficacy (Enochs, Smith & Huinker, 2000). Additionally, research has shown that teachers use teaching strategies that include varying degrees of conceptual and procedural teaching methods (Ma, 1999). These variables contribute to the unique teaching styles of individual elementary teachers.

The current study focused on the relationship between mathematics self-efficacy and mathematics teaching self-efficacy versus procedural or conceptual teaching. The underlying premises of the study included that the researcher believes that teacher self-efficacy toward mathematics varies among individuals and across topics and that math phobia is prevalent in America. The study was framed in Bandura's Social Cognitive Theory, particularly related to self-efficacy, and Hiebert's theories of procedural and conceptual knowledge, including that procedural knowledge may evolve from conceptual understanding and that a balance between procedural and conceptual knowledge is ideal. Conceptual and procedural understanding was further supported by Skemp's theory of instrumental and relational understanding. Three principles of these theories investigated in this study include: (1) that there is a relationship among mathematics teaching self-efficacy, mathematics self-efficacy, and conceptual or procedural teaching practices, (2) that teachers tend to teach certain topics more procedurally and others more conceptually, and (3) that the tilt toward more conceptual or more procedural teaching is related to self-efficacy toward each topic, among other factors.

Constructs, or components of the study that could not be directly observed included mathematics self-efficacy, mathematics teaching self-efficacy, procedurally based instruction, and conceptually based instruction. Mathematics teaching self-efficacy was broken down into teaching self-efficacy related to various NCTM content strands, while mathematics self-efficacy was broken down into efficacy related to using mathematics in daily tasks and solving mathematics problems. Levels of mathematics self-efficacy and mathematics teaching self-efficacy were inferred from responses to instruments that have been validated as measuring mathematics self-efficacy and mathematics teaching self-efficacy. Procedurally or conceptually based instructional approaches were inferred from responses to interview questions that were validated by a panel of experts through a pilot study.

**Self-Efficacy**

Much of the recent research related to self-efficacy is based on the social cognitive theory of Bandura (1986, 1993, 1994, 1997a, 1997b). In his theory, Bandura (1986) states that of all the different aspects of self-knowledge, none is more influential in people's everyday lives than their personal self-efficacy. Efficacy fosters the relationship between knowledge and action. Bandura's research on self-efficacy has shed light on how humans use their personal confidence related to specific tasks. People who have high self-efficacy expect favorable outcomes, while those who doubt themselves expect mediocre performances, which result in negative outcomes (Bandura, 1986). This basic premise of self-efficacy directs a person's beliefs in his or her ability.

The reasons for various levels of belief in one's ability toward a particular task vary among individuals. Bandura (1994, 1997b) identified four main sources of influence on self-efficacy:

1. *Mastery experiences* – a person achieves success and as a result becomes more confident in his or her abilities;
2. *Vicarious experiences provided by social models* – a person strengthens his or her self-beliefs by observing someone similar who finds success;
3. *Social persuasion* – a person is encouraged or verbally persuaded that he or she possesses the capabilities to master a given activity; and

4. *Stress reduction* – a person's negative emotional state is altered to adjust his or her judgment of personal self-efficacy.

These four influences are prevalent in the mathematics classroom, which affects dynamics of school structure, curriculum, teacher, students, and classmates. Furthermore these influences shape an individual's self-efficacy toward mathematics throughout life.

Bandura's (1997a) theory of self-efficacy includes three kinds of efficacy related to schools: (a) the self-efficacy of students, (b) the self-efficacy of teachers, and (c) the collective efficacy of schools. The current study focused only on teacher self-efficacy related to mathematics and mathematics teaching. Self-efficacy is an important part of shaping students' lives so it is essential for mathematics teachers and educators to foster positive self-efficacy in their classrooms. "A major goal of formal education should be to equip students with the intellectual tools, self-beliefs, and self-regulatory capabilities to educate themselves throughout their lifetime" (Bandura, 1993, p.136).

The challenge of creating classroom environments conducive to learning relies significantly on the skills and self-efficacy of teachers. Elementary teachers' self-efficacy fosters students' knowledge through their actions, including how they encourage and motivate their students, and through their choices of presentation methods. "Teachers' beliefs in their efficacy affect their general orientation toward the educational process as well as their specific instructional activities" (Bandura, 1997a, p.241). The first studies on self-efficacy in education based their research on simply this:

> Teachers with a high sense of instruction efficacy operate on the belief that difficult students are teachable through extra effort and appropriate techniques and that they can enlist family supports and overcome negative community influences through effective teaching. In contrast, teachers who have a low sense of instructional efficacy believe there is little they can do if students are

> unmotivated and that the influence teachers can exert on students' intellectual development is severely limited by unsupportive or oppositional influences from the home and neighborhood environment. (Bandura, 1997a, p.240)

Bandura (1997a) stated that "teachers' sense of collective efficacy varies across grade level and subjects" and "teachers judge themselves more efficacious to promote language skills than mathematical skills" (p. 249). Specifically, as the grade level increases, perceived efficacy declines and the self-efficacy gap between language and mathematics increases.

In 1986, Bandura explained his *Social Cognitive Theory,* which was rooted in the belief that humans are agents of their own development and actions. This theory proposes that three factors or influences determine human functioning: (1) personal factors including cognitive, affective, and biological factors, (2) behavioral factors, and (3) environmental factors. Bandura called this *triadic reciprocality*. Thus in a classroom, a teacher could improve personal confidence of the students (personal factors), challenge the performance level (behavior), and alter the class environment which includes the types of instruction they receive (environment) (Pajares, 2007).

Teacher mathematics self-efficacy, according to Bandura's (1986) social cognitive learning theory, is related to many factors including teacher knowledge, teacher preparation, student achievement, personal efficacy, and vicarious experiences. A web of variables seems to all play a part in various types of self-efficacy, including student-, teacher-, and mathematics-self-efficacy. There is a need for research investigating the relationship between teacher mathematics self-efficacy and mathematics teaching self-efficacy and the use of various teaching methods that can influence the personal, behavioral, and environmental factors that help students learn.

## Procedurally and Conceptually Oriented Instruction

Teaching methods may include those identified by the mathematics education community as either procedurally or conceptually based instruction. Hiebert (1986) describes procedural knowledge in two parts: (1) "the formal language, or symbol representation system, of mathematics" (p. 6) and (2) "rules, algorithms, or procedures used to solve mathematical tasks" (p. 6). Conceptual knowledge is described as "knowledge that is rich in relationships" (p. 3). These two types of knowledge are contrasting viewpoints related to the teaching and learning of mathematics, yet contain important connections. Most mathematics educators would agree that we desire our students to understand mathematical concepts but that they also benefit from having efficient procedures to solve problems.

For decades the pendulum has swung back and forth emphasizing either procedural or conceptual teaching. McLellan and Dewey (1895), Brownell (1935), Bruner (1960), Skemp (1971, 1987), and NCTM (1989, 2000) have emphasized the importance of conceptual understanding while Thorndike (1923), Gagne (1977), and back-to-the-basics movements, which followed both the 'new math' era of the 1960s and the introduction of the NCTM Standards (1989, 2000), have emphasized procedural understanding. Do children benefit from rote skills and memorized algorithms or from building understanding and making connections? The challenge is for teachers to find the proper balance of procedural and conceptual teaching methods to help children to understand and perform mathematics efficiently, accurately and appropriately.

## Relational and Instrumental Understanding

Educational theorist Richard Skemp (1971) impressed upon the mathematics education community that the teaching of concepts is very important. Skemp defined a concept as “a way of processing data which enables the user to bring past experience usefully to bear on the present situation” (p. 28). He believed that when a person learns a new concept, that person links it to previous concepts in his or her own unique way. Skemp offered these two principles of learning mathematics:

> 1) Concepts of a higher order than those which a person already has cannot be communicated to him by a definition, but only by arranging for him to encounter a suitable collection of examples. 2) Since in mathematics these examples are almost invariably other concepts, it must first be ensured that these are already formed in the mind of the learner. (Skemp, 1971, p. 32)

Skemp (1976) also presented the ideas of relational and instrumental understanding. He described relational understanding as, “knowing both what to do and say” (p. 153) and instrumental understanding as “rules without reason” (p. 153). These are very similar to Hiebert’s (1986) descriptions of conceptual and procedural knowledge, respectively. Skemp noted three advantages of instrumental mathematics: (1) it was easier to understand, (2) the rewards were more immediate, and (3) the right answer could be obtained more quickly. He also noted four advantages of relational mathematics: (1) it adapted easier to new tasks, (2) it was easier to remember, (3) it motivated students so fewer rewards and punishments were needed, and (4) it encouraged students to learn more since they could appreciate and understand the mathematics. Skemp concluded that the strong long-term effects formed by the advantages of relational mathematics may produce a stronger case than the strong short-term effects formed by the advantages of instrumental mathematics. He stated that relational understanding is different from instrumental understanding in that the means become independent of the ends, building up a schema becomes a satisfying goal, the "more complete a pupil's schema, the greater his feeling of confidence in his own ability to find new ways of 'getting there' without outside help" (Skemp, 1987, p. 163), and as our never-complete schemas enlarge, our awareness of possibilities is also enlarged.

While Hiebert and Skemp have theorized about student knowledge and understanding, teachers are a significant factor in building understanding and fostering knowledge development. The various teaching methods that teachers may choose can promote procedural facility or conceptual understanding in the classroom. For example, if a teacher favors Hiebert's idea of procedural understanding or Skemp's idea of instrumental understanding, the teacher may choose to teach students a standard algorithm to add fractions. However, if a teacher favors Hiebert's idea of conceptual understanding or Skemp's idea of relational understanding, the teacher may choose to provide activities using manipulatives to help students invent their own method to add fractions. A third possibility is the teacher who values both of Hiebert's ideas of procedural and conceptual understanding or both of Skemp's ideas of instrumental and relational understanding and teaches by balancing both methods. This teacher might allow students to use manipulatives to invent their own method of adding fractions followed by comparing this method to the standard algorithm. Practice problems might allow students to experiment with both their invented algorithm and the standard algorithm until they are proficient in adding fractions.

Hiebert and Carpenter (1992) emphasized that rather than arguing over which approach is superior, educators need to examine how conceptual and procedural understanding interact with each other. Both approaches are needed for success in mathematics as one who understands procedural approaches only would not be able to apply and explain the mathematics of practical applications and problem situations and one who understands conceptual approaches only will understand the problem but not be able to solve it using mathematical procedures recognized by others. Hiebert and Carpenter note that current research stressed “understanding before skill proficiency” (p. 79). A key to understanding mathematical concepts is knowing how and knowing why (Ma, 1999).

**Model**

This theoretical visual, in Figure 1.2, shows how the author views mathematics self-efficacy and mathematics teaching self-efficacy as independent constructs that may significantly vary in degree by individual. Conceptual and procedural teaching methods, while also constructs, tend to vary inversely with each other. Most mathematics problems entail some degree of both procedural and conceptual thinking (Engelbrecht, Harding, & Potgieter, 2005) and similarly the teacher likely presents some topics or concepts more conceptually and some more procedurally.

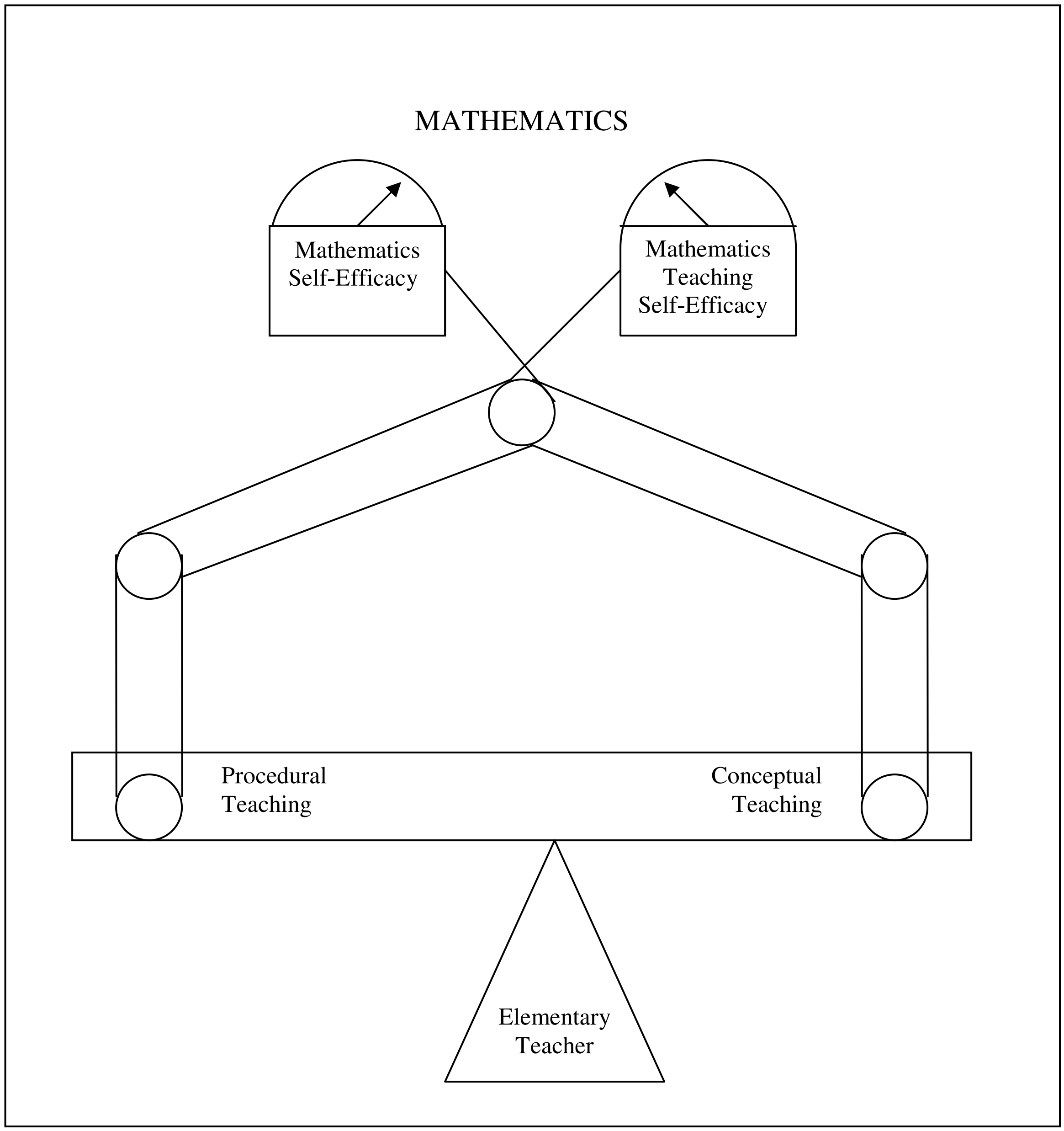


Figure 1.2: Proposed diagram depicting the supposed relationship between teacher mathematics self-efficacy and mathematics teaching self-efficacy relative to procedurally and conceptually oriented teaching

The triangle represents the elementary teacher who is somewhere along a balance tilted toward either procedurally or conceptually oriented teaching, or perhaps well balanced. This balance is pulled upon by two pulleys linked to a meter measuring mathematics self-efficacy and mathematics teaching self-efficacy. As suggested by Raymond (1997) numerous variables affect teaching practice. Thus mathematics self-efficacy and mathematics teaching self-efficacy are only two factors of many that characterize the teacher. Each teacher's collection of presentation methods which may be based on mathematics self-efficacy and mathematics teaching self-efficacy would make them a teacher who leans toward one end of the balance, toward either conceptual methods or procedural methods, or one who balances the two types of methods. Keep in mind that these four constructs, mathematics self-efficacy, mathematics teaching self-efficacy, conceptually oriented teaching methods, and procedurally oriented teaching methods, are not static but can vary over time. Additionally, these variables may affect the balance differently within the same teacher depending on the different mathematical concept, skill, or problem.

## Conclusion

Pajares (1996a) states that:

> Although self-efficacy research has made notable contributions to the understanding of self-regulatory practices and academic motivation, the connection from theory to practice has been slow. Classroom teachers and policymakers may well be impressed by the force of research findings arguing that self-efficacy beliefs are important determinants of performance and mediators of other self-beliefs, but they are apt to be more interested in useful educational implications, sensible intervention strategies, and practical ways to alter self-efficacy beliefs when they are inaccurate and debilitating to children. (p. 568)

So as the author began this study, the constructs of mathematics self-efficacy and mathematics teaching self-efficacy were known to vary significantly in elementary teachers, and it was desired to investigate the role that these constructs play in the conceptually and procedurally oriented teaching methods teachers use across various mathematics topics. The current study aimed to illustrate useful educational implications of self-efficacy that may further inform the mathematics educational research field. Ultimately, the researcher hopes to help fulfill the common goals of mathematical understanding, proficiency, and strong mathematics self-efficacy held by NCTM and government agencies, teachers and educators, parents, students, and Americans.

# CHAPTER 2

# LITERATURE REVIEW

This chapter features existing educational research literature that involves self-efficacy and conceptual and procedural teaching of mathematics. The research has been synthesized and organized to focus first on self-efficacy in general and then more specifically on both mathematics self-efficacy and mathematics teaching self-efficacy. Following the self-efficacy sections is a summary of related literature on conceptual and procedural teaching. Finally a section of literature is presented which brings together self-efficacy with conceptual and procedural teaching and highlights the importance of the current study in the field of mathematics education.

## Self-Efficacy

Educational researchers have been studying self-efficacy since at least the mid-1970s (Hoy & Spero, 2005). This self-efficacy research delves into numerous areas of study including psychology and education and has become regarded as a respectable area of study due to the influences of self-efficacy on such topics as performance and success among students. As seen in Chapter 1, much of this educational research on self-efficacy is based on Albert Bandura's research (1977, 1986, 1993, 1994, 1997a, 1997b) on self-efficacy, which has shed light on how humans use their personal confidence related to specific tasks. Today the importance of self-efficacy is recognized so much that in mathematics education the NCTM Curriculum and Evaluation Standards for School Mathematics (1989) stated that one goal for students is “that they become confident in their ability to do mathematics” (p. 5).

**Premise.** This basic premise of self-efficacy directs a person’s beliefs in his or her ability and can relate to any area, including mathematics, reading ability, job attainment, and college course selection. In the educational setting, self-efficacy may play a role in academic goals, motivation, effort, interest, and self-concept in both student and teacher.

**Goals.** Self-efficacy studies have examined some relationships but broader and more integrated views of beliefs such as self-efficacy should be studied to find its meaning in mathematics education (McLeod, 1992). A relationship between self-efficacy and goals was studied and indicated that people with higher perceived self-efficacy tend to set higher goals and have a firmer commitment to them (Bandura & Wood, 1989). Allinder (1995) confirmed this finding in his study which reported that by raising student self-efficacy, students raise their end-of-year goals. Allinder studied special education teachers’ use of curriculum-based measurement on student achievement and determined that teachers with high personal and teaching efficacy had a greater chance of increasing students’ end-of-year goals. Allinder also concluded from this study that teachers with high personal efficacy effected significantly greater growth in students’ computational skills.

**Motivation.** Bandura’s cognitive social learning theory suggests that motivation is also affected by self-efficacy. Specifically, motivation is affected both by outcome expectations and efficacy expectations. A person has efficacy expectations about a behavior and an outcome expectation that the behavior will have a particular outcome

(Bandura, 1977). Woolfolk and Hoy (1990) describe outcome expectations as “judgments about the likely consequences of specific behaviors in a particular situation” (p. 82) and efficacy expectations as “the individual’s belief that he or she is capable of achieving a certain level of performance in that situation” (p.82). In mathematics, as in general, students and teachers have varying levels of efficacy expectations and outcome expectations.

As self-efficacy has become an important aspect of society, the world of mathematics education can examine it from numerous directions. This study took a closer look at mathematics self-efficacy and mathematics teaching self-efficacy.

## Mathematics Self-Efficacy

Confidence and self-efficacy toward mathematics are closely related, as are attitudes and beliefs about mathematics. Attitudes include an individual’s confidence related to mathematics and an individual’s anxiety level toward mathematics. Mathematics self-efficacy defines confidence further by indicating one’s belief that he or she can perform mathematical tasks successfully. The difference is that self-efficacy is specific to an individual’s capabilities in a particular area rather than in general. In related research, authors have related self-efficacy to attitude, achievement, and sources that affect mathematics self-efficacy.

**Fear of mathematics.** Ufuktepe and Ozel (2002) reported that students acquire a general fear of mathematics from the society around them. Anxiety and fear of mathematics impedes a student’s success with mathematics. Their study from Turkey involved a survey of 500 elementary students who attended a mathematics show encouraging students to understand that the mathematical process is more important than the correct answer. Their study showed that teaching styles and learning styles do not always match up, which affects student confidence toward mathematics.

**Anxiety.** Taylor and Brooks (1986) studied the relationship between confidence and anxiety in mathematics students. Their study with adults in basic education courses indicated that students must first build mathematics confidence by overcoming mathematics anxiety before they are able to find success.

**Confidence.** Kloosterman and Cougan (1994) conducted student interviews that posed questions about school and mathematics, including whether they were confident in their mathematical abilities. Students in grades 3-6 were more able to articulate their beliefs than younger students. Whether students in grades 1-2 were less confident or simply unable to articulate their beliefs effectively is uncertain. However, in both grade groups, students who enjoyed mathematics were more confident of their abilities.

**Mathematics Self-Efficacy Scale.** Kloosterman and Cougan (1994), Taylor and Brooks (1986), and Ufuktepe and Ozel (2002) all examined the interplay between attitudes, anxiety, confidence, or fear of mathematics. As stated in Chapter 1, self-efficacy is more specific than general confidence and focuses on a person's perceived ability to perform a task. Hackett and Betz (1989) define mathematics self-efficacy as, "a situational or problem-specific assessment of an individual's confidence in her or his ability to successfully perform or accomplish a particular [mathematics] task or problem" (p.262). Hackett and Betz (1989) created the Mathematics Self-Efficacy Scale (MSES) to specifically measure mathematics self-efficacy among problems, tasks, and college courses. A revised version of the MSES was used in the current study.

**MSES-Revised.** Pajares has delved more deeply into specific areas within mathematics self-efficacy. Pajares and Miller (1995) conducted a study that asked students to make three types of mathematics self-efficacy judgments. The three self-efficacy judgments included: (a) students' reported confidence about answering problems they were about to solve, (b) their confidence to perform in general on mathematics-related tasks, and (c) their confidence to succeed in mathematics-related courses. These three judgments were the foundation of Pajares and Miller's MSES-R instrument, which was a modified version of the MSES. A total of 391 undergraduates took the MSES-R mathematics self-efficacy scale followed by the same set of problems from the MSES-R of which they just stated their self-efficacy level. Expected outcome and performance measures indicated that their confidence in answering problems they were about to solve was the most significant factor of the three.

**Studies using MSES-R.** In a previous study, Pajares (1994) used path analysis to rate self-efficacy beliefs related to mathematical problem solving of middle school gifted students mainstreamed in regular education classes. Gifted students reported higher mathematics self-efficacy and lower mathematics anxiety than regular education students. Interestingly, Pajares found that other factors, such as parental encouragement, may also provide a key role in a student's self-confidence. Ferry, Fouad, and Smith (2000) also addressed parental encouragement as a social persuasion and concluded that parental encouragement in mathematics and science not only influences achievement but also influences self-efficacy and grade expectation.

Grades, success, and achievement also are frequent topics of research in mathematic education and this holds true in mathematics self-efficacy research.

Mathematics self-efficacy of college students has been shown to correlate positively with achievement (Hackett & Betz, 1989). This study involved the authors' own self-efficacy instrument, the MSES. Lent, Lopez and Bieschke (1993) examined undergraduate college students' mathematics self-efficacy as related to achievement, interest, grades, and enrollment intentions. Mathematics self-efficacy and achievement predicted mathematics grades while mathematics self-efficacy and outcome expectations predicted interest and enrollment intentions.

**Meta-Analysis.** To investigate the multitude of research relating attitudes and achievement, Ma and Kishor (1997) conducted a meta-analysis study of 113 primary studies. The 113 studies were coded according to source, scale size, type of study, sampling method, and date. The comparing factor was effect size, estimated by Pearson product-moment correlation. Exploratory data analysis indicated effect sizes were close to a normal distribution and the overall mean effect size was 0.12 for the general relationship between attitude toward mathematics and achievement in mathematics. Thus, little consensus was found in existing research literature concerning the relationship between students' mathematics attitude and achievement in mathematics. This meta-study, however, focused on all mathematics related attitudes of which self-efficacy is one specific type. Perhaps other factors were involved that could not be analyzed in this study, such as teaching methods used in the study classrooms and the sources of self-efficacy.

**Bandura's Theory.** Recall that Bandura (1994, 1997a) stated four main sources of influence on self-efficacy including mastery, vicarious experiences provided by social models, social persuasion, and stress reduction. Lent, Lopez, Brown, and Gore (1996) conducted two studies testing four- and five-factor models of self-efficacy among either high school or college mathematics students. One study involved factor analysis of responses from 295 college students using a four-factor structure (performance, vicarious learning, social persuasion, and emotional arousal); the other study analyzed responses from 481 students in a five-factor structure (performance, adult modeling, peer modeling, social persuasion, and emotional arousal). The four-factor model fit best for college students and the five-factor model fit best for high school students, indicating that, apparently because of age and maturity differences, high school students may react differently to adult and peer modeling, whereas the age of the model seems less important among college students. Theoretically the confirmatory factor analysis fit indices supported discrete factors, but practically there existed a strong inter-correlation among personal performance, social persuasion, and emotional arousal. Vicarious learning did not fit as well as the other three factors, which indicates that watching others succeed mathematically may or may not affect an individual's self-efficacy toward mathematics.

**Summary.** In summary, students and teachers both have varying degrees of mathematical self-efficacy which affect several aspects of education, such as goals and achievement. These differences are supported by Bandura's definition and sources for self-efficacy. One source, mastery, may be interpreted as achievement. Bandura's other three sources—vicarious experiences, social persuasion, and stress reduction—may be interpreted as the influence from a teacher, either through good teaching practices or encouragement.

**Teaching Self-Efficacy**

How do teachers and educators influence students positively to foster success in mathematics and reduce anxiety and negative beliefs about mathematics? This question has been examined at least as far back as a famous study from the Rand Corporation in 1976 (Armor et al.). The Rand study found that teacher efficacy was a strong predictor of student performance, project goals completed, and teacher change. This study defined teachers' sense of efficacy as "teachers' judgments about their abilities to promote students' learning" (Wolfolk Hoy & Spero, 2005, p. 343) and featured two questions that appear frequently in the literature in teacher self-efficacy studies. These questions are: (a) "When it comes right down to it a teacher really can't do much because most of a student's motivation and performance depends on his or her home environment," and (b) "If I really try hard, I can get through to even the most difficult or unmotivated students" (Ashton & Webb, 1986, pp. 189-190). These two questions were based on the work of Rotter (1966) and viewed self-efficacy as control of reinforcements that were believed to influence achievement and motivation of students (Tschannen-Moran, Woolfolk Hoy, & Hoy, 1998).

**Bandura's Theory.** Bandura (1977) viewed teacher self-efficacy differently, however, believing that people construct their own beliefs about their ability to perform a specific task at a particular level. These two contrasting viewpoints remain important future research topics but the current study relied on Bandura's interpretation and thus looked, not at the influence of teacher self-efficacy on students, but rather on the relationship between teacher self-efficacy and teaching methods.

**Instruments.** Many teacher self-efficacy instruments have been made in the last three decades to assist in the research about teacher self-efficacy, including the Efficacy Scale (Ashton and Webb, 1986), the Teacher Efficacy Scale, (Gibson & Dembo, 1984), the Science Teaching Efficacy Beliefs Instrument (Riggs & Enochs, 1990), and Bandura's Teacher Efficacy Scale (1997).

**Efficacy Scale.** Ashton and Webb (1986) examined teacher self-efficacy by creating an instrument that included the two Rand efficacy questions, eight teaching efficacy questions, two stress questions and 15 personal teaching efficacy vignettes. This instrument, along with student achievement scores and classroom observations, were used with 48 basic skills mathematics and communication high school teachers. Results indicated that teachers' beliefs about their instructional efficacy predicted their students' levels of achievement in language and mathematics. The study also focused on attitudes and behaviors related to teacher relationships with students, classroom management strategies, and instructional methods. The instructional methods of low efficacy teachers were less likely to go much beyond teach-and-drill methods which provided no fostering of advanced critical thinking skills. High efficacy teachers tended to control their classroom in a warm manner in which the teacher was attentive to each student. Students' academic achievement was cumulatively impacted by teachers' instructional self-efficacy.

**Teacher Efficacy Scale.** Gibson and Dembo (1984) examined teacher self-efficacy by conducting a study to validate a teacher efficacy measurement instrument called the Teacher Efficacy Scale. This instrument was based on Bandura's construct of self-efficacy which included outcome expectancy and outcome efficacy and examined the relationship between teacher efficacy and observable teacher behaviors. Factor analysis was performed on data from 208 elementary school teachers to create the instrument. Fifty-five teachers enrolled in graduate level education courses then completed the Teacher Efficacy Scale and additional open-ended items. Multitrait-multimethod analysis was conducted on data along with classroom observations. The only significant results were that teachers with low self-efficacy spent almost 50% of their observed time in small group instruction whereas high-efficacy teachers spent only 28% of their time in small groups. Also, when students gave incorrect responses to low-efficacy teachers, 4% of the teacher responses were criticism which is only significant in comparison to high-efficacy teachers, who never criticized students.

**Factors of Self-Efficacy.** Examining teacher self-efficacy more closely resulted in two dimensions of self-efficacy: (a) personal efficacy and (b) teaching efficacy (Ashton & Webb, 1986; Gibson & Dembo, 1984; Woolfolk & Hoy, 1990). A modified version of Gibson and Dembo's Teacher Efficacy Scale (1984) showed that personal efficacy and teaching efficacy were not correlated (Woolfolk & Hoy, 1990). Multiple regression was used to assess the relation between personal efficacy and teaching efficacy in interaction with pupil control ideology, bureaucratic orientation, and motivational style. Teaching efficacy was negatively correlated with pupil control ideology and bureaucratic orientation whereas personal efficacy showed little or no correlation with these factors. Motivational orientation was not significantly correlated with either personal or teaching efficacy. Teachers with low efficacy favored custodial orientation that relied heavily on rewards and negative reinforcements in order to encourage students to study. Teachers with high efficacy, however, supported the development of students' interests and academic independence.

Woolfolk and Hoy (1990) question the accuracy of these dimensions since efficacy may be determined by the way questions are answered. It was noted that the instruments contain mostly negative statements about teaching efficacy and all positive statements about personal efficacy. Their study of 182 preservice teachers examined the structure of efficacy for preservice versus experienced teachers and how preservice teacher efficacy related to discipline, order, control, and motivation (Woolfolk & Hoy, 1990). Teachers' sense of personal efficacy affects their general orientation toward the educational process and their specific instructional practices. This study featured factor analysis on four instruments to analyze relationships on teaching efficacy variables. While examining relationships among different characteristics of self-efficacy, Woolfolk and Hoy have further clarified the concept of self-efficacy.

Brown (2005) conducted a correlational study hypothesizing that early childhood teachers high in efficacy would rate the importance of mathematics higher than teachers with low efficacy but the correlation was weak. Additionally, Brown hypothesized that high efficacy combined with high teacher mathematics beliefs would show alignment with current standards-based mathematics instructional practices but no results were significant.

Guskey (1988) suggested that teacher efficacy further divides into responsibility for positive student outcomes and responsibility for negative outcomes. Achievement is only one factor influenced by teacher efficacy, however. Teacher efficacy influences the student learning environment. Teacher efficacy may also impact the amount of time spent on the subject, the choice of teaching strategies, and comments the teacher makes that support or deflate a student's self-efficacy.

These studies have provided evidence that teacher mathematics self-efficacy, according to Bandura's (1986) social cognitive learning theory, is related to many factors including teacher knowledge, teacher-preparation, student achievement, personal efficacy, and vicarious experiences. Reliable instruments have been developed and fine tuned to measure mathematics self-efficacy.

## Mathematics Teaching Self-Efficacy

**Beliefs versus practice.** Specifically related to mathematics teachers, self-efficacy has been examined for relationships with teaching practices as well as effect on students. Raymond (1997) investigated the relationship between the beginning elementary school teacher's beliefs and mathematics teaching practices. Over a 10-month period she used audio-taped interviews, observations, document analysis, and a beliefs survey with 6 first- and second-year teachers. Raymond constructed a model of mathematical beliefs and practices that showed how practice is more closely related to beliefs about mathematics content than to beliefs about mathematics pedagogy. This model relates how past school experiences, teacher education programs, social teaching norms, and the teacher's and students' lives outside school influence mathematics beliefs and mathematics teaching practices. Additionally, the model shows that early family experiences, the classroom situation, including the characteristics of the particular students, time constraints, current mathematics topic to teach, and teacher personality traits, including confidence, creativity, humor, and openness to change, influence mathematics beliefs and mathematics teaching practices. Raymond's model shows the most significant influence on mathematics beliefs is past school experience. The most significant influences on mathematics teaching practices are mathematics beliefs, the classroom situation, and current mathematics teaching practices themselves. Given all these factors, Raymond concluded that beliefs about teaching mathematics are not always consistent with teachers' teaching practice, and beliefs about mathematics content are typically less traditional than their actual teaching practice.

**Frustrated mathematics teachers.** Cornell (1999) stated in an article titled "I Hate Math? I Couldn't Learn It, and I Can't Teach It!" that nearly half of a group of graduate students in an elementary teacher certification program indicated they disliked mathematics. Teachers in the study indicated their frustrations were due to obscure vocabulary, incomplete instruction, too much drill and practice, not keeping up with the class, the overemphasis on rote memory, learning mathematics in isolation, and teachers assuming they, as students, could learn computational procedures easily. Furthermore, Cornell stated that instructional activities and materials should be incorporated into a mathematics lesson in addition to other methods, including teaching mathematics in practical contexts and considering various forms of assessment.

**Mathematical content knowledge.** Ball (1990b) determined that, as prospective teacher candidates begin their college courses, they bring an understanding of mathematics which is rule-bound and thin. Ball's study involved over 250 prospective teacher candidates in a longitudinal study that addressed the prospective teachers' subject knowledge of mathematics, mathematical ways of knowing, and feelings toward mathematics. Their ideas, beliefs, and understandings were explored using both questionnaires and interviews. Teachers' feelings about mathematics were approached through a questionnaire, paying attention to affective dimensions such as giggles and sighs, and through interviews that asked about their recollections of their own school mathematics experiences. Teachers' understandings of mathematics were interrelated with how they felt about themselves and about mathematics. The majority of the teachers demonstrated a weak ability to appropriately represent a division problem and their responses were affected by knowledge, ways of thinking, beliefs, and self-confidence. Results from the items about feelings toward mathematics showed that only one-half of the elementary teacher candidates indicated they enjoyed mathematics and over one-third of them indicated they were not good at mathematics. Teachers are one key instrument in helping to develop self-confidence in students. Many people influence children's lives who have negative beliefs about mathematics, including many elementary teachers (Ball, 1990a).

**Mathematics Teaching Efficacy Beliefs Instrument.** A study by Bursal and Paznokas (2006) investigated the relationship between teachers' mathematics anxiety levels and their confidence levels to teach elementary mathematics and science. Sixty-five preservice elementary teachers were given the Revised-Mathematics Anxiety Survey (R-MANX) (Bursal & Paznokas, 2006) along with nine selected questions from each of the Mathematics Teaching Efficacy Beliefs Instrument (MTEBI) (Enochs, Smith, & Huinker, 2000) and the Science Teaching Efficacy Beliefs Instrument (STEBI-B) (Riggs & Enochs, 1990). Results indicated a negative correlation between preservice teachers' mathematics anxiety and their confidence in teaching elementary mathematics. Furthermore, nearly half of the teachers in the study who had higher mathematics anxiety than their colleagues “believe that they will not be able to teach mathematics effectively” (Bursal & Paznokas, 2006, p. 177).

**Teacher preparation.** Studies have shown that involvement in mathematics teaching methods courses raises the self-efficacy of teachers (Darling-Hammond, 2000; Hart, 2002; and Huinker & Madison, 1997). Previous studies have shown that teachers become more confident in their ability to teach mathematics when involved in mathematics teacher preparation programs. “Teachers who have had more preparation for teaching are more confident and successful with students than those who have had little or none” (Darling-Hammond, 2000, p. 166).

A study of preservice elementary teachers concurrently enrolled in semester-long mathematics and science methods courses showed that these methods courses improved the teachers’ beliefs in their ability to teach mathematics effectively (Huinker & Madison, 1997). The goal of the mathematics methods course was to help teachers understand that mathematics should be a sense-making experience and to develop a capacity to teach for understanding. The purpose of the study was to determine whether methods courses in science and mathematics influence the personal teaching efficacy and outcome expectancy beliefs of preservice teachers, and the consistency of the influence. Pre- and post- efficacy tests were given to two cohorts of preservice teachers with interviews following for those with significant changes in their self-efficacy. Results showed that the mathematics methods courses consistently improved preservice elementary teachers’ beliefs in their ability to teach mathematics effectively.

In a similar study, preservice elementary teachers’ beliefs about mathematics improved after participating in a mathematics methods course which focused on the reforms suggested by the NCTM Standards (Hart, 2002). This study measured the consistency of an individual's beliefs with the NCTM Curriculum and Evaluation Standards (1989), the change in beliefs about teaching and learning within and outside the school setting, and teacher efficacy toward learning and teaching mathematics. Descriptive statistics indicated a significant increase in all three measures indicating their program had a positive impact on the mathematical teaching beliefs of preservice elementary teachers. Hart stated that there is substantial evidence that teachers' beliefs about mathematics became more consistent with the current reform philosophy after the methods courses.

**Reform.** Several researchers have studied the changes in beliefs and efficacy teachers face when involved in current mathematics reform movements (Battista, 1994; DeMesquita & Drake, 1994; Smith, 1996). Smith (1996) states that a teacher's sense of efficacy is rooted in the ability to state facts about mathematics and provide direct demonstration of mathematics. Reform efforts, however, challenge this behavior of *telling* mathematics due to the increased interest in mathematical activities that foster learning. Although a teachers' strong sense of efficacy supports their efforts when faced with challenges, Smith states that more research is needed in the area of changes in teacher efficacy amidst reform.

## How Teachers Affect Student Mathematics Self-Efficacy

Many research studies in the literature focus on student mathematics self-efficacy. A few key studies however reinforce the idea that student self-efficacy is affected by teacher choice of activities, implementation strategies, and attitudes.

Siegle & McCoach (2007) conducted a study of 40 fifth grade teachers and their 872 students which explored the impact of teacher training on student mathematics self-efficacy. Participating teachers were divided into a control group and an experimental group. Both groups received instruction on how to teach a 4-week unit on measurement. The experimental group also received instruction on self-efficacy constructs and strategies for improving student self-efficacy. The strategies included were based on three of Bandura's (1986) four self-efficacy sources: an individual's part performance, vicarious experiences of others, and verbal persuasion. Data was generated by student pre- and post-tests assessing their mathematics self-efficacy and their achievement in measurement. Results "demonstrated that teachers can modify their instructional strategies with minimal training, and this can result in increases in students' self-efficacy" (p. 301-302).

Researchers suggest that low student self-efficacy causes motivational problems and if students believe they cannot succeed on a specific task they give up or avoid the task (Margolis & McCabe, 2006). "Motivations toward mathematics are developed early, are highly stable over time, and are influenced greatly by teacher actions and attitudes" (Middleton & Spanias, 1999, p. 80). By focusing on self-efficacy and specific strategies which support student self-efficacy, "teachers can help struggling learners develop a more accurate, optimistic, *can do* attitude" (Margolis & McCabe, 2006, p. 226).

**Summary**

Research has shown the effect of self-efficacy in one's life. It is important to individual success whether you are a student, a teacher, or enjoying another walk of life. The present research is just a beginning. The need exists for research that relates teacher efficacy and instructional practices. For example, Woolfolk & Hoy (1990) have identified the need to establish the categories of high and low efficacy, particularly when the measure of efficacy is not one-dimensional. Furthermore, their study tested a few relationships among independent dimensions of efficacy, thus opening the door to explore these and other relationships. These studies have more carefully defined the attributes related to self-efficacy.

Many studies of self-efficacy in teachers involve preservice teachers. It is also important to view how mathematics efficacy changes over time. Also, how do veteran teacher self-efficacies and teaching practices compare with preservice teacher efficacies? Furthermore, are there differences in self-efficacy and/or teaching practice demonstrated by a given teacher relative to different topics in mathematics?

Teacher mathematics self-efficacy, based on Bandura's (1986) cognitive social learning theory has been related to many factors, including teacher knowledge, teacher-preparation, student achievement, personal efficacy, and vicarious experiences. A web of variables all seem to play a part in various types of self-efficacy, including student, teacher, and mathematics self-efficacy. Different teachers therefore have different teaching strategies and techniques to teach different mathematical topics. Teachers affect the environment of their classrooms and influence students' learning. The need lies for research investigating the influence of teacher mathematics and mathematics teaching self-efficacy on their behaviors in the classroom and selection of instructional strategies.

## Conceptual and Procedural Learning and Teaching

The second area of interest in the current study is teachers' tendency to teach various topics in mathematics either conceptually or procedurally. Student understanding, teacher understanding, and conceptually and procedurally oriented teaching will all be examined as crucial puzzle pieces.

Numerous theories in mathematics education have addressed different approaches to teaching mathematics and the type of student learning that is most important. The debate has continued throughout the past century and has emphasized rote learning (Thorndike, 1923) and meaningful learning (Brownell, 1935), discovery learning (Bruner, 1960) and guided learning (Gagne, 1977), relational versus instrumental understanding (Skemp, 1976), and conceptual versus procedural knowledge (Hiebert, 1986), to name just a few. Historically these theories have swayed the focus of educational curriculum from rote processes in the 1920s to practical mathematics in the 1940s. New Math in the 1960s led to a back-to-basics movement in the 1970s, and the introduction of the NCTM Standards (1989) spurred a reform movement in the 1990s.

After Skemp's (1976) and Hiebert's (1986) theories on instrumental versus relational understanding and conceptual versus procedural knowledge, teacher educators debated whether to teach procedurally or conceptually. Much of America's current *math wars* stem from the controversy over the importance of teaching conceptually versus procedurally. "The question of whether developing skills with symbols leads to conceptual understanding, or whether the presence of basic understanding should precede symbolic representation and skill practice, is one of the basic disagreements between the behaviorist and cognitivist approaches to learning mathematics" (Sowder, 1998, p. 5). Like a mathematics curriculum pendulum, educators are still debating what the essential skills and concepts are for American students to learn and what are the most effective methods for teaching these skills and concepts.

## Procedural and Conceptual Student Understanding

Many researchers, however, tend to support the argument that both procedural skills and conceptual understanding are desired, important, and necessary in the mathematics classroom. Hiebert's (1986) description of procedural understanding includes learning steps to an algorithm, learning definitions, and memorizing multiplication facts. These skills are important in the well-developed mathematics student but not without conceptual understanding, which includes using multiple representations to express an answer, multiple solution strategies, or constructing an algorithm. In a similar theory, Skemp (1976) had already described relational and instrumental understanding of concepts. Relational understanding was described as "knowing both what to do and say" (p. 153) and instrumental understanding was described as "rules without reason" (p. 153).

In response to Skemp's original idea of instrumental knowledge equating to *rules without reasons*, Sfard (1991) suggests that understanding could also involve *reasons without rules*. This type of understanding involves purely intuitive understanding when vague structural concepts are achieved before operational processes are fully developed. For example, when a mathematician introduces a new concept or theorem without the full knowledge of its proof or process, he is exemplifying *reasons without rules*. Reason (2003) also recognizes the importance of both instrumental and relational understanding and adds a third type of understanding which she calls *creative understanding*. Creative understanding involves understanding two concepts and their related procedures and seeking an understanding of their relationship. She describes an example as having two completely different maps of the same city featuring different characteristics with only a set of instructions on how the two relate.

**Other types of knowledge.** Other researchers have suggested additional types of knowledge beyond procedural and conceptual knowledge. Leinhardt (1988) recognized four types of knowledge that children may have or may learn. *Intuitive knowledge* is applied circumstantial knowledge which may or may not be accurate. *Concrete knowledge* uses nonalgorithmic systems such as pictures to represent concepts. *Computational knowledge* equates to Hiebert's procedural knowledge and *principled conceptual knowledge* is an underlying knowledge of mathematical procedures and constraints.

De Jong and Ferguson-Hessler (1996) describe types and qualities of knowledge relating to problem solving in science. Four types of knowledge include situational, conceptual, procedural, and strategic. *Situational knowledge* is sifting relevant features from a problem. *Conceptual* or *declarative knowledge* is static knowledge of facts, concepts, and principles. *Procedural knowledge* is knowledge of valid actions and manipulations necessary to the problem. *Strategic knowledge* is knowledge of the correct stages to progress through to complete a problem. Qualities include levels (surface to deep), generality of knowledge (general to domain specific), level of automation of knowledge (declarative to compiled), modality of knowledge (verbal to pictorial), and structure of knowledge (isolated elements to structured knowledge). De Jong and Ferguson-Hessler describe each of the five qualities for each of the four types of knowledge. Mathematics education researchers have further explored the relationships of the types and qualities related to mathematics instruction (Baroody, Feil, and Johnson, 2007; Star, 2005, 2007)

**Levels of knowledge.** Star (2005) defined conceptual knowledge as “not only what is known, knowledge of concepts, but also one way that concepts can be known” and procedural knowledge as “not only what is known, knowledge of procedures, but also one way that procedures, algorithms, can be known” (p.408). He also defined deep procedural knowledge as “knowledge of procedures that is associated with comprehension, flexibility, and critical judgment and distinct from, but possibly related to, knowledge of concepts” (p. 408).

Star (2005, 2007) called for a renewed attention to procedural knowledge stemming from de Jong and Ferguson-Hessler’s (1996) theory of both surface, or superficial, and deep levels of knowledge. He says that Hiebert’s (1986) definitions of procedural and conceptual knowledge equate to a superficial procedural knowledge and a deep conceptual knowledge. Star suggests that the qualities of deep and superficial should be related to both procedural and conceptual knowledge forming four different types of knowledge. Contrary to Hiebert (1986) and Baroody, Feil, and Johnson (2007), Star also claims that students can have a deep procedural knowledge independent of conceptual knowledge. For example, Star suggests that individuals with a deep procedural understanding would solve the following three algebra problems differently:

$2(x+1) + 3(x+1) = 10$,

$2(x+1) + 3(x + 1) = 11$, and

$2(x+1) + 3(x + 2) = 10$.

In the first, 2(x+1) + 3(x+1) = 10, 5(x + 1) = 10. Next, divide both sides by 5 and subtract 1. In 2(x+1) + 3(x + 1) = 11, first combine like terms to get 5(x + 1) = 11. Then distribute the 5, subtract 1, and divide by 5. To solve 2(x+1) + 3(x + 2) = 10, first distribute the 2 and the 3, combine like terms and solve the two-step problem. Without a deep procedural knowledge, Star suggests that mathematics students may not be able to identify the most efficient method to solve these problems.

Star's (2005) article discusses the lack of recent research on how students learn procedurally and ways to study and assess procedural knowledge. While also recognizing the importance of conceptual knowledge, he stresses that deep procedural knowledge should be an instructional goal for every age of schooling.

In addition to Star, many researchers state the importance of balance between conceptual and procedural understanding. Davis (2005) examined students' conceptual and procedural knowledge of functions. He described conceptual understanding of functions as the ability to translate among different representations including tabular, graphical, symbolic, or real-world situation of a function (O'Callaghan, 1998) and procedural understanding of functions as learning developed through skill worksheets. While emphasizing the importance of both procedural and conceptual understanding, many researchers agree that not all knowledge can be divided into conceptual or procedural knowledge (Davis, 2005; Silver, 1986). Davis notes three concerns related to solving equations which are detached from a real-world context. First, students believe these procedures work only in an abstract context. Second, by using only procedural knowledge they may not be using sense-making strategies. Third, students' procedural and conceptual knowledge of solving equations may become separated.

**Related studies.** Heywood (1999) conducted a study involving high school chemistry students to examine the differences between novice and expert students' procedural and conceptual understanding of a chemistry problem. Results indicated that expert students tended to have both – a strong conceptual understanding that was linked to underlying procedural skills. Novice students had misconceptions and poor use of formulas. These novice students differed from the expert students in conceptual understanding, use of procedures, and types of strategies chosen.

LeFevre et al. (2006) found low correlations between conceptual and procedural knowledge of basic counting in their study with K-2 students. Students were asked to watch an animated frog count tiles correctly from left to right, incorrectly, and correctly using unusual orders. Conceptual knowledge scores were given from students' judgments of the frog's accuracy. Procedural knowledge was assessed using speed and accuracy of their own counting of objects. Results indicated that although procedural counting skills are well developed by grade 2, conceptual knowledge is still developing; therefore their relationship is not necessarily linear and intercorrelations are not strong. Another study concluded that conceptual understanding and procedural learning are iterative processes (Rittle-Johnson, Siegler, & Alibali, 2001). Thus, as one develops, it causes the other to develop, which causes the first to develop, and so on.

## Which Should Come First – Procedural Fluency or Conceptual Understanding?

Many mathematics education researchers (Hiebert, 1986; Hiebert & Carpenter, 1992) emphasize that both procedural and conceptual knowledge is important. The manner and order, however, in which they are taught is of key significance.

Several researchers have seen benefits in developing conceptual understanding before procedural understanding. Leinhardt (1988) interviewed second graders about subtraction and fractions to better understand their types of knowledge. One observation Leinhardt noted was that a strong procedural knowledge impeded conceptual knowledge.

Students in grades 1-3 who used invented strategies to perform multidigit addition and subtraction before learning standard algorithms were more successful in extending their knowledge to new situations and demonstrated a better understanding of number concepts than students who first learned the standard algorithms (Carpenter, Franke, Jacobs, Fennema, and Empson, 1997).

Mack (1990) found that students who first learned fractions procedurally had difficulty with the conceptual understanding of fraction problems. Mack (1990) conducted a study involving sixth-graders' understanding of fractions. Results showed that knowledge of rote procedures interferes with student invention of meaningful algorithms. Furthermore, Mack argues in favor of teaching concepts before procedures, as in Hiebert & Wearne (1988) and Resnick, Nesher, Leonard, Magone, Omanson and Peled (1989), in that by building on informal knowledge students can construct meaningful algorithms.

Byrnes and Wasik (1991) conducted two experiments to explore the relationship between conceptual and procedural knowledge in fourth, fifth, and sixth graders. Their study compared two perspectives which they called *simultaneous activation* and *dynamic interaction*. Based on Bruner's (1966) work, the *simultaneous activation* perspective stated that "the source of children's computational errors lies in the fact that mathematical symbols are meaningless to them" (Byrnes & Wasik, 1991, p. 777) and recommended using concrete referents for symbols. This perspective stated that a strong conceptual knowledge was both necessary and sufficient for procedural understanding. The *dynamic interaction* perspective stated that conceptual knowledge was the basis for new acquisition of procedures and that conceptual and procedural knowledge interacted diachronically over time rather than simultaneously. The dynamic interaction perspective implied that conceptual and procedural knowledge interact and that conceptual knowledge was necessary but not sufficient for procedural understanding. The dynamic interaction perspective was based on Inhelder and Piaget's (1980) work which distinguished between conceptual and procedural knowledge and argued that, when children attempted to understand the procedures, the conceptual knowledge is strengthened. Results from Byrnes and Wasik's first experiment indicated that when students learned conceptually they were still likely to make computational errors. Results from their second experiment indicated that students mastered conceptual knowledge before procedural knowledge.

## Teachers' Mathematical Understanding

We desire conceptual and procedural understanding from our students, but it is essential that the teacher has both a deep conceptual and deep procedural understanding of mathematics. The teacher is expected to possess the knowledge and the ability to construct lessons that develop conceptual and procedural understanding in students.

**Content knowledge.** Shulman (1986) describes three categories of content knowledge: (a) subject matter content knowledge, (b) pedagogical content knowledge, and (c) curricular knowledge. It would seem that subject matter knowledge may be related to mathematics self-efficacy, and pedagogical content knowledge and curricular knowledge may both be related to mathematics teaching self-efficacy.

Thompson and Thompson (1994) discuss the need for a conceptual curriculum in mathematics classes that involves discourse and communication if curricular reform is the goal. Their study involved two instructional sessions between one teacher and one student related to the concept of rate. During the study it became evident that the teacher's language choices were a challenge for him as he had mostly developed a procedural understanding of mathematics. This study suggests the importance of internalizing mathematics conceptually in order to teach conceptual concepts effectively.

In a second study, Thompson and Thompson (1996) state that a teacher with conceptual orientation is driven by:

> an image of a system of ideas and ways of thinking that he or she intends the students to develop, an image of how these ideas and ways of thinking can develop, ideas about features of materials, activities, expositions, and students' engagement with them that can orient students' attention in productive ways, and an expectation and insistence that students be intellectually engaged in tasks and activities. (Thompson and Thompson, 1996, pp. 20-21)

Teachers with a conceptual orientation focus student attention away from thoughtless procedures and toward situations, ideas, and relationships among ideas. Thompson and Thompson recommend additional research to understand how teachers come to understand mathematics and mathematics teaching in order to teach conceptually.

**Profound Understanding of Fundamental Mathematics:** Ma (1999) examined mathematical content knowledge among elementary teachers. She initially wanted to explore the difference in achievement of American versus Asian students, but her interest sharply turned to comparing the mathematical understanding and teaching of teachers from both countries. She conducted a study involving beginning and experienced elementary teachers and included 23 teachers from the United States and 72 teachers from China. Ma interviewed teachers regarding their understanding of fundamental mathematics and teaching strategies with four concepts: explanations of subtraction with regrouping, error detection in multi-digit subtraction, modeling division of fractions, and responding to a student who has just presented a discovery about the relationship between area and perimeter. Her work illuminates the profound understanding of mathematics that the majority of the Chinese elementary teachers possess in relation to the weak understandings held by the U.S. elementary teachers. Furthermore, Ma describes how this profound understanding on the part of Chinese teachers relates to their mathematics teaching pedagogy, stating that no teacher in her study taught beyond their own level of understanding. Results of Ma's study indicated that the teachers possessed procedural understanding or both procedural and conceptual understanding, but not just conceptual understanding. Those teachers with both procedural and conceptual understanding used the mathematical procedures as a supplement to the conceptual mathematical explanation in both their own understanding and in their teaching strategies.

Ma (1999) labeled some teachers as having a profound understanding of fundamental mathematics (PUFM). She defined fundamental mathematics as elementary, foundational, and primary. PUFM teaching and learning included the properties of connectedness, multiple perspectives, basic ideas, and longitudinal coherence. In other words, PUFM teachers can connect concepts and procedures among various operations, appreciate multiple approaches to solutions, display positive mathematical attitudes, uphold the power of basic concepts and principles in mathematics, and possess a fundamental understanding of the whole elementary mathematics curriculum.

**Structure of the Observed Learning Outcome (SOLO) Taxonomy:** In another study, Groth and Bergner (2006) used both Ma's (1999) PUFM and Biggs & Collins (1982, 1991) SOLO Taxonomy to examine 46 preservice elementary teachers' conceptual and procedural knowledge of the statistical measures of mean, median, and mode. Their study involved written responses to only one item, "How are the statistical concepts of mean, median, and mode different? How are they similar?" (Groth & Bergner, 2006, p.48). Their research focused both on the level of thinking related to these concepts and definitions of these statistical terms. The SOLO Taxonomy involves concrete-symbolic and formal modes of representation. Biggs and Collins (1982, 1991) offered the SOLO taxonomy to the body of educational research theorizing that four levels of thinking were situated within various modes of representation: *unistructural*, *multistructural*, *relational*, and *extended abstract*. In this study, *unistructural* involves process-telling while *multistructural* adds the understanding of a vague purpose – that mean, median, and mode are statistical tools – as well as process-telling. *Relational* involves process-telling and the understanding that these tools measure central tendency. *Extended abstract* still includes process-telling but goes beyond to include why one of the three may be a better measure of central tendency than the others.

Results indicated that the researchers identified responses in each of the four distinct levels of thinking which matched the SOLO Taxonomy levels, but Groth and Bergner did not believe the responses that were identified at the extended abstract SOLO level reached Ma's PUFM status since those responses either did not include well-developed definitions or only included one illustration of the effectiveness of one measure of central tendency over another. Groth and Bergner concluded that teachers would need to have a SOLO relational understanding of the definitions of mean, median, and mode as a prerequisite to PUFM thinking about them.

## Procedurally and Conceptually Oriented Teaching

The question now is what are different teaching methods that aim for procedural understanding or aim for conceptual understanding? While many studies give examples of their author's interpretation of procedural or conceptual teaching, no research was found that directly examines which teaching methods lead to conceptual understanding and which lead to procedural understanding. The current study defined *procedurally oriented teaching* as using teaching methods that focus on developing student procedural understanding of mathematics and *conceptually oriented teaching* as using teaching methods that focus on developing student conceptual understanding of mathematics.

**Procedural teaching.** Some researchers are fine tuning the definition of procedural teaching. After studying how college students solve an integral calculus problem, Eley and Norton (2004) describe the learning advantages of embedding solution steps in an explicit hierarchical goal structure that makes the goal the instruction of systematic steps. Both children and adults may be capable of achieving procedural competence in mathematical operations but this procedural understanding may be coupled with incomplete conceptual understanding (Laupa & Becker, 2004). One example of this is that some students fail to use their conceptual understanding to verify the reasonableness of the results of their mathematical calculations using algorithms.

**Conceptual teaching.** Others emphasize mathematics curriculum reform that includes teaching conceptually (Raymond, 1997; Ross, McDougall, Hogaboam-Gray, & LeSage, 2003). Tracy & Gibbons (1999) list conceptually oriented teaching methods for measurement which include using measuring tools, number lines, manipulatives, videos, calculators, and websites.

A study involving fourth and fifth graders examined conceptual understanding and procedures used in solving equivalence problems (Rittle-Johnson & Alibali, 1999). The study focused on how instruction could influence students' problem solving behaviors. Children in a conceptual instruction group were instructed by explaining with gestures to show the meaning of equivalence in a specific problem. Children in a procedural instruction group were instructed by explaining the numerical manipulation necessary to solve the problem. Findings indicated that conceptual instruction led to conceptual understanding and transfer of the correct procedure, whereas procedural instruction led to conceptual understanding but limited transfer of the correct procedure.

Kazemi and Stipek (2001) used video-tapes of lessons in 4 fourth- and fifth-grade classes to analyze students' conceptual understanding of fractions. A problem was presented to the class that involved 12 brownies being shared by 8 friends. Following this the 8 friends shared 9 more brownies. Results from the study, which focused on what the authors called "high press for conceptual thinking" (p. 59), indicated four teaching characteristics: (1) explanations consist of mathematical arguments, (2) understanding involves multiple strategies, (3) errors offer opportunities to investigate the problem further, and (4) work involves collaboration, accountability, consensus and mathematical argumentation.

**Balanced strategies.** Eisenhart, Borko, Underhill, Brown, Jones, & Agard (1993) followed one student teacher through her final year in a middle school mathematics teacher preparation program. Researchers examined her ideas and practices related to both procedural and conceptual teaching that appeared in her own mathematical experiences, in her classroom placement with a cooperating teacher, and with her university methods course. The teacher equated procedural understanding with performing arithmetic accurately and conceptual understanding with the ability to think. Although the student teacher stated that teaching procedurally and conceptually were both important, which was also reflected by her cooperating teacher and her university instructor, she had an easier time implementing procedural lessons than conceptual lessons.

Eisenhart et al. (1993) also suggested that at the school district level there was a conflict between procedural and conceptual teaching in that procedural teaching is supported by formal assessments despite the fact that district leaders state the importance of conceptual teaching. This conflict again raises the question of whether this teacher's first experiences with conceptual and procedural teaching will change with experience, social persuasion, or other factors.

Miller and Hudson (2007) emphasize the importance of using a balanced curriculum to teach mathematics to students with learning disabilities that includes the five NCTM content standards as well as three knowledge areas of conceptual, procedural, and declarative knowledge. They recommend instructional guidelines for developing the three knowledge areas. To develop conceptual knowledge Miller and Hudson recommend the concrete-representation-abstract (CRA) teaching sequence that involves three concrete lessons using manipulative devices and three representational lessons using pictorial representations to teach a mathematics concept to a learning disabled student. Additionally Miller and Hudson suggest using either a compare and contrast, example and non-example, or step-by-step structure to illustrate the concept. To develop procedural knowledge, Miller and Hudson suggest using a strategy that has sequential steps that can be generalized, prompt the student for action, are simple in use, and offer a mnemonic device. Declarative knowledge is, "information that students retrieve from memory without hesitation" (Miller & Hudson, 2007, p.53). To develop declarative knowledge, they recommend using controlled response times while monitoring accuracy and to consider individual needs while selecting implementation strategies.

Pesek and Kirschner (2000) conducted a study with fifth graders to explore the order of teaching for relational and instrumental learning. One group received five days of instrumental instruction on area and perimeter followed by three days of relational instruction on area and perimeter. A second group received only three days of relational instruction on area and perimeter. Instrumental instruction facilitated the memorization and routine application of formulas. Specifically, students were asked to write new formulas multiple times, teachers demonstrated, and students practiced. Relational instruction facilitated constructing relationships. Specifically, students were asked to compare and contrast and then construct their own methods to calculate the perimeter and area measures. Teachers encouraged the use of concrete materials, posed questions, and encouraged student communication and problem solving. Results indicated that students in the relational instruction group outperformed the group that received instrumental followed by relational instruction. Pesek and Kirschner conclude that there is a negative effect on students' learning when instrumental instruction precedes relational instruction.

Many mathematics education researchers support the ideas that procedural knowledge and conceptual knowledge are both important in the curriculum. However, research suggests that students have a greater understanding when they learn concepts before they learn procedures. Teaching conceptually first leads to acquisition of procedural skills, but the reverse is not necessarily true (Brown, n.d.).

**Summary**

As described by Skemp (1976) and Hiebert (1986), much has been learned about instrumental and relational learning and procedural and conceptual understanding in mathematics education. The debate over their importance is widely accepted to include both types of skills (Hiebert, 1986; Hiebert & Carpenter, 1992; Miller and Hudson, 2007; Skemp, 1987), and current reform efforts and other efforts strive for conceptual understanding among students before procedural understanding (Brown, n.d.; Hiebert & Wearne, 1988; Leinhardt, 1988; Mack, 1990; Pesek and Kirschner, 2000, Resnick, Nesher, Leonard, Magone, Omanson and Peled, 1989). Teachers' mathematical understanding has been shown to have significant importance (Ma, 1999; Shulman, 1986; Thompson and Thompson, 1994). While specific teaching methods have been associated with procedurally and conceptually oriented teaching, only informal research methods have been found that assess or examine the factors that affect teachers' choices of specific teaching methods.

### Self-Efficacy and Procedurally or Conceptually Oriented Teaching

One possible factor affecting teachers' choice of specific procedurally or conceptually oriented teaching methods may be mathematics self-efficacy and mathematics teaching self-efficacy. The relationships among these variables need to be examined further and were the focus of the current study.

**Teacher efficacy.** Few authors have linked confidence or self-efficacy to studies involving conceptually or procedurally oriented teaching. Ashton and Webb (1986) examined teaching efficacy among high school basic mathematics teachers and communications teachers. Teaching self-efficacy, student achievement scores, attitudes, and instructional methods were analyzed using their teaching self-efficacy instrument and classroom observations. Results showed that low self-efficacy leads to teaching practices involving procedural teach-and-drill methods that provide no fostering of advance critical thinking skills.

In an article addressing teaching self-efficacy and procedural telling as mathematics instruction, Smith (1996) found that high teacher self-efficacy is evident in teachers who can clearly state facts and give direct demonstration by telling, also a procedural teaching method. He recognizes that reform efforts challenge teaching by telling and encourage more active involvement by students in their learning. Smith recognizes that more research is needed in the area of changes in teacher efficacy amidst reform.

**Teacher conceptual understanding.** In Ma's (1999) study involving U.S. and Chinese elementary teaching, she indicated that teachers needed a deep conceptual understanding of mathematical concepts prior to teaching. In a portion of her study examining the teaching of area and perimeter concepts, it became evident that confidence in the teachers' own understanding of area and perimeter was also a key factor in encouraging discussion of the topic with students.

Raymond's (1997) study involving beginning elementary teachers concluded that practice is more closely related to beliefs about mathematics content than to beliefs about mathematics pedagogy. Raymond reported that numerous variables are involved and indicated that teaching practices and beliefs are not always consistent, thus they may vary among mathematical topics. Her study showed the most significant influence on mathematics beliefs is past school experience, while the most significant influences on mathematics teaching practices are mathematics beliefs, the classroom situation, and current mathematics teaching practices themselves.

## Significance of the Study

While self-efficacy has been related to conceptual and procedural understanding in only a few studies, the current study went beyond the current emphasis of both procedural and conceptual understanding among students and delved into how teachers teach differently based on these variables. The need lies for research investigating how teachers with different mathematics self-efficacy and mathematics teaching self-efficacy levels may teach differently. The current study offers an instrument that measures both mathematics self-efficacy and mathematics teaching self-efficacy, as well as assessment strategies to evaluate procedural and conceptual teaching methods. The data may show a relationship between these variables and more specifically to identify conceptual or procedural teaching methods common among teachers with high or low mathematics self-efficacy and mathematics teaching self-efficacy. By clarifying the relationships among these variables, teachers and teacher educators may understand how their own self-efficacy affects their teaching practices.

# CHAPTER 3

# METHODOLOGY

To examine the relationships among mathematics self-efficacy, mathematics teaching self-efficacy, and conceptually and procedurally oriented teaching methods within and across various mathematics topics, a mixed-method study was conducted involving current third through sixth grade teachers. This study involved a survey and an interview. Primarily quantitative methods were used to address the independent variables of mathematics self-efficacy and mathematics teaching self-efficacy, and primarily qualitative methods were used to address the dependent variables of conceptually or procedurally oriented teaching methods. Thus the different levels of self-efficacy, or independent variables, were viewed in relation to the resulting teaching methods, or dependent variables. This chapter describes details about the sample selection, instrumentation, pilot studies, data collection, and data analysis plan for the study.

## Participants

This study focused on elementary teachers assigned to teach grades 3 through 6. Eighty (80) teachers involved in summer workshops to enhance mathematics teaching skills completed the MTMSE survey, developed as part of the current study. This workshop involved practicing teachers from central Ohio school districts who wanted to learn innovative approaches to teaching mathematics. The workshop included a two-week summer session and five follow-up sessions throughout the next school year. The researcher learned of the workshop through a fellow graduate student and selected it by convenience since it involved the targeted type of sample for the study. Five (5) teachers provided incomplete surveys and were eliminated from the study. From the remaining 75 survey participants, 22 agreed to be interviewed. Sixteen (16) of these teachers completed the interview and were included in the interview portion of the study. The teachers in the study all had three or more years teaching experience and were from a variety of school districts including urban, suburban, and rural. They each taught all academic subjects in a self-contained classroom or multiple academic subjects in a team format. A variety of certification and licensure types including grades 1-8, K-8, 4-8, mathematics specialists, middle grades, and mathematics concentration were included. The six teachers who were interviewed but whose data were excluded from the interview portion of the study did not meet one or more qualifications above. Teachers were chosen via a convenience sample (Patton, 1990). Teachers were located for the study via summer courses for teachers. All participants had the necessary consent forms on file prior to the study.

## Data Collection: Survey

### Survey Instrumentation

The goals of the survey included: (1) to identify teachers' mathematics self-efficacy and mathematics teaching self-efficacy levels, (2) to identify mathematics topic preferences, and (3) to situate teachers in a nine-section mathematics self-efficacy/mathematics teaching self-efficacy grid in order to select participants for the interview portion. The study measured mathematical self-efficacy and mathematics teaching self-efficacy using the Mathematics Teaching and Mathematics Self-Efficacy (MTMSE) survey, as seen in Appendix A, that was developed specifically for the current study based on Pajares and Kranzler's (1997) Mathematics Self-Efficacy Scale Revised (MSES-R) and Enochs, Smith, and Huinker's (2000) Mathematics Teaching Efficacy Beliefs Instrument (MTEBI). The MTMSE survey was divided into six parts as follows: parts 1 and 3 assessed teacher mathematics self-efficacy, parts 2 and 4 assessed teacher mathematics teaching self-efficacy, part 5 assessed conceptual and procedural teaching orientation and part 6 contained demographic questions.

**Mathematics self-efficacy.** The mathematics self-efficacy portions, parts 1 and 3, were based on Pajares and Kranzler's (1997) Mathematics Self-Efficacy Scale Revised (MSES-R). Nielsen and Moore (2003) conducted a study with 302 high school freshmen which validated that the MSES can be administered in different contexts by tailoring questions toward the target group. As done in Nielsen and Moore's study, the current study tailored the MSES-R toward the target group of practicing elementary classroom teachers.

Part 1 is the problem subscale from the MSES-R. All 18 questions from the original study were included, but questions 8 and 15 were reworded slightly and question 18 was replaced by another question 18 found on a more recent version of the MSES-R (Pajares, 2007). Part 3 is a revised subscale based on the MSES-R tasks subscale. The original 18 questions from the MSES-R were revised to include more current mathematical tasks and a wider variety of mathematical content topics from the five NCTM (2000) content standards, number and operations, geometry, algebra, data analysis and probability, and measurement. This study also excluded question 5 from the tasks subscale of the MSES-R about teacher confidence using a scientific calculator, which Kranzler and Pajares (1997) found did not load to the tasks factor. Pajares and Kranzler (1997) found a reliability of .95 for the MSES-R instrument during their study. Factor analysis identified four factors in the MSES-R: mathematics problems, mathematics tasks, mathematics courses, and science courses.

**Mathematics teaching self-efficacy.** The courses subscale of the MSES-R, confidence in various college level courses such as Zoology and Economics, was not included in the present study but rather served as motivation to create part 4 of the MTMSE survey. The instrument asked teachers to rate their level of confidence in teaching various NCTM content standard topics, such as area and perimeter or fractions. This part now qualifies as a measure of mathematics teaching self-efficacy rather than a measure of mathematics self-efficacy.

Part 2 of the MTMSE also measures mathematics teaching self-efficacy and was based on Enochs, Smith, and Huinker's (2000) Mathematics Teaching Efficacy Beliefs Instrument (MTEBI). Enochs, Smith and Huinker found two significant subscales while testing reliability for the MTEBI: (1) the personal mathematics teaching efficacy subscale and (2) the mathematics teaching outcome expectancy subscale. Only the questions on the personal mathematics teaching efficacy subscale were used in the current study since they dealt directly with mathematics teaching self-efficacy. The personal mathematics teaching efficacy subscale in the original MTEBI showed a Cronbach's alpha coefficient of internal consistency of .88.

**Permission.** Before using the MSES-R and MTEBI instruments, the five authors were contacted via e-mail to request permission to use the instrument in the current study. Responses were received from all five authors (L. Enochs, personal communication, January 8, 2007; D. Huinker, personal communication, January 20, 2008; J. H. Kranzler, personal communication, January 5, 2007; F. Pajares, personal communication, January 6, 2007; P. Smith, personal communication, January 5, 2007), all of whom were supportive in the use of the instruments.

**Conceptual and procedural teaching.** Part 5 of the MTMSE instrument consisted of twelve questions assessing a teacher's tendency toward the use of conceptually oriented teaching methods or procedurally oriented teaching methods. The questions were inspired by Hiebert (1989), Skemp (1987), and numerous other researchers noted in chapter 2. One example of a procedurally oriented question is, "Formulas and rules should be presented first when introducing new topics." One example of a conceptually oriented question is, "I frequently ask my students to explain *why* something works." Questions were written, critiqued by a panel of experts, and field tested to arrive at the twelve questions to be included in part 5.

**Demographics.** Part 6 included demographic questions probing the teacher's position in mathematics education, such as what subject do you enjoy teaching most and least, which NCTM content strand are you most and least confident teaching, and how many years have you been a teacher. The entire MTMSE instrument thus contains five subscales which assess: (1) mathematics self-efficacy problems, (2) personal mathematics teaching self-efficacy, (3) mathematics self-efficacy tasks, and (4) mathematics teaching self-efficacy on specific mathematics topics and (5) procedurally oriented teaching versus conceptually oriented teaching. The MTMSE can be found in Appendix A. Table 3.1 summarizes the sources and purposes of the different survey parts.

| Survey Part | Purpose | Source |
|---|---|---|
| 1 | Mathematics Self-Efficacy Problems | MSES-R (Kranzler & Pajares, 1997) Problems Subscale |
| 2 | Overall Mathematics Teaching Self-Efficacy | MTEBI (Enochs, Smith, & Huinker, 2000) Personal Efficacy Subscale |
| 3 | Mathematics Self-Efficacy Tasks | MSES-R (Kranzler & Pajares, 1997) Tasks Subscale and NCTM (2000) Content Standards |
| 4 | Mathematics Content Teaching Self-Efficacy | NCTM (2000) Content Standards |
| 5 | Conceptually or Procedurally Oriented Teaching | See Table 3.6 |
| 6 | Demographic Questions | |

Table 3.1: Sources and Purpose of Mathematics Teaching and Mathematics Self-Efficacy Parts.

## Validity and Reliability of the MTMSE

Prior to the administration of the survey, the MTMSE Instrument was tested for face validity and content validity by a panel of experts in the field of mathematics education. Four individual mathematics educators were asked to give feedback both on face validity and content validity for the instrument. These four included a high school mathematics teacher, a high school principal who was a former mathematics teacher, a former district mathematics curriculum director and middle school mathematics teacher, and a graduate student in a mathematics education doctoral program. To select the panel of experts from potential colleagues, the author first asked various mathematics educators if they could define self-efficacy. If they were able to state an accurate meaning clearly, they then were asked if they were willing to offer feedback for the instrument. This panel of experts was asked to critique the overall instrument, to suggest missing items, and to point out items that do not measure what they were intended to measure. Additionally the panel of experts judged face validity of the instrument by indicating that it appeared professional and non-threatening. Changes were made to the format of the instrument, typographical errors were corrected, and clarity of instructions was improved before its administration. No items were changed.

**Survey Pilot Study**

To pilot test the MTMSE survey, 52 students taking a master's level early childhood/elementary mathematics pre-service teaching methods course volunteered to complete the survey. The survey took approximately ten to fifteen minutes to complete.

Data were entered into SPSS statistical computer software for analysis. Questions 2, 4, 5, 7, 9, 10, 11, and 13 of part 2, the mathematics teaching self-efficacy subscale, were directionally recoded. Reliability of the complete instrument was computed at .916. The reliability of individual subscales were all computed above 0.7.

**Levels of self-efficacy:** New variables were created showing the sum of the two mathematics self-efficacy subscales and the sum of the mathematics teaching self-efficacy subscales. In an efficacy study by Brown (2005) teachers were grouped by high or low efficacy and high or low beliefs which situated them into four quadrants. Similarly the data in this study positioned each subject into a placement of high, medium, or low both for mathematics self-efficacy and mathematics teaching self-efficacy. Nine categories of teachers emerged from this grouping as seen in Table 3.2. Examining the extreme four corners more closely, it was found that no teachers fell into the low mathematics self-efficacy – high mathematics teaching self-efficacy category, which did not cause alarm since the pilot study was conducted with preservice teachers who have little teaching experience. Pearson's Product Moment Correlation between mathematics self-efficacy and mathematics teaching self-efficacy was calculated at .565 which indicates a fairly strong relationship between the two variables.

| | | Mathematics Self-Efficacy | | |
|---|---|---|---|---|
| | | High | Medium | Low |
| Mathematics Teaching Self-Efficacy | High | 8 | 6 | 0 |
| | Medium | 7 | 14 | 4 |
| | Low | 2 | 1 | 10 |

Table 3.2: Self-efficacy groupings of pilot study teachers (n=52).

**NCTM content strands.** Questions in parts 1, 3, and 4 had previously been aligned to the appropriate NCTM content strand. A content self-efficacy variable was calculated for each participant by averaging their responses within each strand. This information was used to help determine the appropriate areas to probe during interviews. The alignment of survey questions to the NCTM content strands can be seen in Table 3.3. Note that no specific strands were present in part 2 of the MTMSE since this section addresses self-efficacy related to general teaching practices relating to teaching any mathematics topic.

| NCTM Content Strand | MTMSE Part 1: Mathematics Self-Efficacy Problems | MTMSE Part 2: Mathematics Teaching Self-Efficacy Personal Teaching Efficacy | MTMSE Part 3: Mathematics Self-Efficacy Tasks | MTMSE Part 4: Mathematics Content Teaching Self-Efficacy |
|---|---|---|---|---|
| Arithmetic | 1, 2, 6, 9, 10, 11, 14 | | 1, 2, 3, 6, 7, 8, 9 | 2, 5, 8, 11 |
| Algebra | 3, 5, 7, 12, 15, 17 | | 4 | 3, 9 |
| Geometry | 4, 18 | | | 4 |
| Measurement | 8, 16 | | 10, 11, 12 | 6, 10, 12 |
| Data Analysis | 13 | | 5, 13 | 1, 7, 13 |

Table 3.3: Alignment of MTMSE questions to NCTM Content Strands.

**Readability:** In addition to evaluating the validity and reliability of the MTMSE, the readability of the instrument was tested. Readability of the survey questions was desired to be no higher than an eighth grade reading level as suggested as appropriate for elementary teachers by Miller (2005). The readability was calculated using the Gunning fog index (Gunning, 1952) on each of the four subscales and then averaged for the whole instrument.

The readability index can be calculated using 100 words from the passage. Divide the number of words by the number of sentences to find an average sentence length. Count complex words with three or more syllables and calculate a percentage of complex words by dividing the number of complex words by the total number of words. Add the average sentence length and the percentage of complex words and multiply the sum by 0.4.

The results indicated an average readability of 7.75 which was not higher than an eighth grade reading level and therefore was deemed as an appropriate level for elementary teachers.

## Data Collection: Interview

### Pilot Interview Instrumentation

The pilot interview protocol contains 13 questions, as seen in Figure 3.1. Questions include those about mathematics self-efficacy and mathematics teaching self-efficacy, various mathematics topic preferences, types of instructional methods used to teach various mathematical topics, and demographic questions.

To address mathematics self-efficacy, teachers were asked whether they believe they are good at mathematics outside school. To address mathematics teaching self-efficacy, teachers were asked which subject, from language arts, mathematics, reading, science, or social studies, was their favorite and least favorite to teach. Also, teachers were asked which content area, from among arithmetic, algebra, geometry, measurement, or data analysis and probability, they were most and least confident teaching. These questions were used to confirm the results of the MTMSE survey.

1. Why did you choose to teach 4th (or 5th) grade?
2. How many years have you been teaching?
3. What type of teaching certificate/license do you hold?
4. What was your major in college?
5. What subjects do you teach? Is there any subject you do not teach?
6. Of the following subjects, which of the following are your favorite and least favorite to teach?

   Language Arts, Math, Reading, Science, Social Studies
7. Within mathematics content areas, which of the following are you most confident and least confident teaching?

   Arithmetic, Algebra, Geometry, Measurement, Data Analysis & Probability
8. How confident are you when teaching math?
9. Outside of school, do you believe you are good at mathematics?
10. How have your teaching methods changed over the years?
11. *Pick 2 of 5 that relate to the answers to number 7.*
    a. <u>Arithmetic:</u> Describe a lesson on making the connection between fractions and decimals.
    b. <u>Algebra:</u> Describe a lesson which introduces to students the order of operations.
    c. <u>Geometry:</u> Describe a lesson which introduces to students the difference between a prism and a pyramid.

Continued

Figure 3.1: Pilot Interview Protocol

Figure 3.1 continued

d. <u>Measurement:</u> Describe a lesson that explores what happens to the area of a rectangle as the perimeter increases.

e. <u>Statistics:</u> Describe a lesson that introduces to students the differences between the statistical concepts of mean and median.

12. For each of the following two terms, quickly state which is your focus when teaching math:

    a. Algorithms: learn and memorize or create your own
    b. Solution Process: One right way or many right ways
    c. Goal for Students: Understanding or speed and accuracy
    d. Wrong Answers: Wrong answers should be corrected or wrong answers should lead to discussion
    e. Calculators: for problem solving or for computations
    f. Teaching Math: Asking students why or asking how
    g. Focus: Procedures or concepts
    h. Lesson Planning: be thorough or be creative
    i. Math: confident or hesitant
    j. Teaching Math: confident or hesitant

13. Do you have any comments about the interview questions?

    a. Were there any questions during this interview that were confusing or difficult to answer?
    b. Were there any that made you uncomfortable?

**Content questions.** Based on the answer to the content question and the content scores calculated from the MTMSE data, two questions were chosen from the five content questions, one in the area of the teacher's most confidence and one in the area of least confidence. The goal of these two questions was to identify teaching methods that could be categorized as procedural and/or conceptual. By selecting the teacher's areas in which he or she feels most and least confident, the goal was to find differences in teaching methods according to level of confidence.

The five content questions were developed after review of mathematics education literature and were carefully aligned with the NCTM Standards (2000) and the NCTM Focal Points (2006b) and are summarized in Table 3.4.

1. Arithmetic: Describe a lesson on making the connection between fractions and decimals.
2. Algebra: Describe a lesson that introduces to students the order of operations.
3. Geometry: Describe a lesson that introduces prisms and pyramids.
4. Measurement: Describe a lesson that introduces area and perimeter of a rectangle.
5. Statistics: Describe a lesson that introduces statistical mean and median.

Ma's (1999) research influenced the question relating to measurement. One question in Ma's study asked:

> Imagine that one of your students comes to class very excited. She tells you that she has figured out a theory that you never told the class. She explains that she has discovered that as the perimeter of a closed figure increases, the area also increases. …. She shows the teacher a picture to prove what she was doing, which showed a 4 cm by 4 cm square which has a perimeter of 16 cm and an area of 16 square cm followed by a rectangle which measures 4 cm by 8 cm and has a perimeter of 24 cm and an area of 32 square cm. … How would you respond to this student? (p.84)

The measurement question above is a modified version of Ma's question on perimeter and area.

Groth and Bergner's (2006) study on preservice elementary teachers focused on conceptual and procedural understanding of the concepts of mean, median, and mode. Their study included one question, "How are the statistical concepts of mean, median, and mode different? How are they similar?" (p. 28). The statistics question above is a modified version influenced by that question.

Three questions align with the NCTM Focal Points (2006b) in the areas of arithmetic, measurement and geometry. The arithmetic question aligns to the grade 4 Focal Point on number and operations that requires, "Developing an understanding of decimals, including the connections between fractions and decimals" (p.16). The measurement question aligns with the grade 4 Focal Point on measurement which requires, "Developing an understanding of area and determining the areas of two-dimensional shapes" (p.16). The geometry question aligns with the grade 5 Focal Point on geometry and measurement and algebra, which requires, "Describing three-dimensional shapes and analyze their properties, including volume and surface area" (p. 17).

The remaining two areas, algebra and data analysis and probability, align with the 3-5 grade band expectations in the NCTM *Standards* (2000), as do the questions that align with the NCTM Focal Points.

The arithmetic question aligns with the expectation that all students should "understand numbers, ways of representing numbers, relationships among numbers, and number systems" with specific goals to "recognize equivalent representations for the same number and generate them by decomposing and composing numbers" and "recognize and generate equivalent forms of commonly used fractions, decimals, and percents" (NCTM, 2000, p.148).

The algebra question aligns with the expectation that all students should "represent and analyze mathematical situations and structures using algebraic symbols" with the specific goal to "identify such properties as commutativity, associativity, and distributivity and use them to compute with whole numbers" (NCTM, 2000, p.158). The geometry question aligns with the expectation that all students should "analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships" including specific goals to "identify, compare, and analyze attributes of two- and three-dimensional shapes and develop vocabulary to describe the attributes" and to "classify two- and three-dimensional shapes according to their properties and develop definitions of classes of shapes such as triangles and pyramids" (NCTM, 2000, p.164).

The measurement question aligns with the expectation that all students should "understand measurable attributes of objects and the units, systems, and processes of measurement" including the specific goal to "explore what happens to measurements of a two-dimensional shape such as its perimeter and area when the shape is changed in some way" (NCTM, 2000, p.170).

Finally, the statistics question aligns with the expectation that all students should "select and use appropriate statistical methods to analyze data" with specific goals to "describe the shape and important features of a set of data and compare related data sets, with an emphasis on how the data are distributed" and "use measures of center, focusing on the median, and understand what each does and does not indicate about the data set" (NCTM, 2000, p.177). Table 3.4 summarizes the alignment of the interview questions with the NCTM Focal Points (2006), and NCTM Standards (2000). Table 3.5 shows sample answers for each content question related to either conceptual or procedural teaching.

| Interview Question | NCTM Focal Point (2006a, pp.16-17) | NCTM (2000) Expectation (Grades 3-5) | Specific NCTM (2000) Goal |
|---|---|---|---|
| ARITHMETIC | | | |
| Describe a lesson on making the connection between fractions and decimals. | "Developing an understanding of decimals, including the connections between fractions and decimals" | "Understand numbers, ways of representing numbers, relationships among numbers, and number systems" | "Recognize equivalent representations for the same number and generate them by decomposing and composing numbers" and "recognize and generate equivalent forms of commonly used fractions, decimals, and percents" (p.148) |
| ALGEBRA | | | |
| Describe a lesson which introduces to students the order of operations. | | "Represent and analyze mathematical situations and structures using algebraic symbols" (p.158). | "Identify such properties as commutativity, associativity, and distributivity and use them to compute with whole numbers" (p.158) |
| GEOMETRY | | | |
| Describe a lesson which introduces to students the difference between a prism and a pyramid. | "Describing three-dimensional shapes and analyzing their properties, including volume and surface area" | "Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships" | "Identify, compare, and analyze attributes of two- and three-dimensional shapes and develop vocabulary to describe the attributes" and "classify two- and three-dimensional shapes according to their properties and develop definitions of classes of shapes such as triangles and pyramids" (p.164) |
| MEASUREMENT | | | |
| Describe a lesson which explores what happens to the area of a rectangle as the perimeter increases. | "Developing an understanding of area and determining the areas of two-dimensional shapes" | "Understand measurable attributes of objects and the units, systems, and processes of measurement" | "Explore what happens to measurements of a two-dimensional shape such as its perimeter and area when the shape is changed in some way" (p.170). |
| DATA ANALYSIS | | | |
| Describe a lesson which introduces to students the differences between the statistical concepts of mean and median. | | "Select and use appropriate statistical methods to analyze data" (p.177) | "Describe the shape and important features of a set of data and compare related data sets, with an emphasis on how the data are distributed" and, "use measures of center, focusing on the median, and understand what each does and does not indicate about the data set" (p.177). |

Table 3.4: Alignment of Interview Questions with NCTM Standards and Focal Points.

| Question | Procedural Teaching | Conceptual Teaching |
|---|---|---|
| ARITHMETIC | | |
| Describe a lesson on making the connection between fractions and decimals. | The teacher demonstrates to students how to divide a numerator by a denominator to convert fractions to decimals. | The teacher uses base ten blocks to show how .3 is the same as 3/10. |
| ALGEBRA | | |
| Describe a lesson that introduces to students the order of operations. | The teacher encourages students to memorize PEMDAS and practice with problems. | The teacher asks students to explore how inserting parentheses into mathematical expressions may change the value. |
| GEOMETRY | | |
| Describe a lesson that introduces prisms and pyramids. | The teacher shows the class a physical model of both shapes, defines their properties, and asks students to find other prisms and pyramids. | The teacher shows the class a physical model of both shapes and asks students to describe and compare attributes of each. |
| MEASUREMENT | | |
| Describe a lesson that introduces area and perimeter of a rectangle. | The teacher presents formulas for perimeter and area of rectangles. The students substitute numbers to find the measure. | The teacher asks students to draw rectangles on grid paper and counts the perimeter and area. |
| DATA ANALYSIS | | |
| Describe a lesson that introduces statistical mean and median. | The teacher presents definitions of mean and median. Students compute both values for a data set. | The teacher asks students to physically line up data items to count to the center item to find the median and balance distances from a projected target number to find a mean. |

Table 3.5: Examples of Procedural and Conceptual Teaching for each content strand.

**Teaching methods.** Ideas from Groth & Bergner (2006), Hiebert (1986), Ma (1999), NCTM (2000, 2006), Skemp (1976) and others were synthesized to generate the list of conceptual and procedural teaching methods that pertained in general to mathematics teaching and specifically to the five NCTM content strands, as seen in Table 3.6. For example, if a teacher indicated there was one right way to obtain any answer, the statement would be considered to use a procedural teaching method. On the contrary, if a teacher indicated there were many ways to obtain an answer or that the reasoning along the way to the answer was most important, the statement would be considered to be a conceptual teaching method. Similarly, if a teacher emphasized how something worked, the statement would be considered procedural and if a teacher emphasized why something worked, the statement would be considered conceptual.

| Procedurally Oriented Teaching | | | Conceptually Oriented Teaching | | |
|---|---|---|---|---|---|
| **Teaching Practice** | **Reference** | **Teaching Examples** | **Teaching Practice** | **Reference** | **Teaching Examples** |
| | | | **Reasoning/Communication** | | |
| Explaining "how to". | (Miller & Hudson, 2007; Skemp, 1987) | First line up the decimal points, … | Explaining "why"; Asking "why not". | (Skemp, 1987) | Asking a student why they should divide by 100 to change a percent into a decimal. |
| | | | **Content** | | |
| Teaching definitions and symbols; Skill drill. | (Brown, n.d.; Hiebert, 1986) | Reading ten thermometers on a worksheet. | Posing situations which explore mathematics. | (Brown, n.d.; Hiebert 1986) | Looking at real thermometers over several hours and make predictions for tomorrow. |
| | | | **Algorithms** | | |
| Learning steps or practicing. | (Eley & Norton, 2003; Hiebert, 1986; Pesek & Kirshner, 2000; Thorndike, 1922) | Teaching the algorithm for long division or to "flip and multiply" to divide fractions. | Discovering or explaining steps to algorithms. | (Pesek & Kirshner, 2000) | Students generate their own strategy or algorithm. |
| | | | **Calculators/Technology** | | |
| Learning how to use technology to perform mathematical skills. | | Students are given key sequences to enter in a calculator. | Using technology for problem solving or explorations. | (Davis, 2005) | Exploring a certain feature on the calculator to figure out how it works. |

Continued

Table 3.6 Procedurally and Conceptually Oriented Teaching Methods.

Table 3.6 continued

| Procedurally Oriented Teaching | | | Conceptually Oriented Teaching | | |
|---|---|---|---|---|---|
| **Teaching Practice** | **Reference** | **Teaching Examples** | **Teaching Practice** | **Reference** | **Teaching Examples** |
| | | **Solution Strategies** | | | |
| There is "one right way" to solve a problem. The right answer is most important. | | Statements that this is or is not the right way to solve this problem when another equivalent method is used. | Multiple strategies to get to a solution; Problem solving process is valued over the solution; Flexible reasoning. | (Engelbrecht, Harding & Poteieter, 2005) | Acknowledging a student's answer and asking for other ways to solve the same problem. |
| | | **Speed and Accuracy** | | | |
| Statement that "speed" and "accuracy" are the goal. | (Thorndike, 1922) | Timed tests. Graded for accuracy only. Answers only are shared. | Understanding of fewer problems emphasized. | (LeFevre et al, 2006) | Asking students to explain their answer in writing or verbally. |
| | | **Connectivity** | | | |
| Focusing on a single skill with limited connection to related concepts, skills or applications. Isolated skills. Teaching definitions. Sequential. | (Engelbrecht, Harding & Poteieter, 2005) (Brown, n.d.) | Teaching addition of decimals using rules. | Connecting ideas or concepts in math Connecting math to other academic subjects or the real world; Intertwined. | (Brown, n.d.; Brownell, 1935; Bruner, 1960) | Teaching addition of decimals in the context of money or baseball averages. |

Continued

Table 3.6 continued

| Procedurally Oriented Teaching | | | Conceptually Oriented Teaching | | |
|---|---|---|---|---|---|
| **Teaching Practice** | **Reference** | **Teaching Examples** | **Teaching Practice** | **Reference** | **Teaching Examples** |
| | | | **Problem Solving** | | |
| Selecting word problems to model the current skill; Following steps to solve a word problem. | (Miller & Hudson, 2007) | Sue has 3 rows of 4 brownies, how many are there? | Posing a real problem. Creative problem solving. | (Brown, n.d.) | How many different ways can you arrange 12 brownies? |
| | | | **Representation** | | |
| Algorithm is most important. | Hiebert (1986) | $\frac{is}{of} = \frac{\%}{100}$ | Multiple representation Proof or reasoning emphasized; Encouraging verbal or written explanations. | (Davis, 2005; Miller & Hudson, 2007) | Discovering a formula. |
| | | | **Correcting Wrong Answers** | | |
| Wrong answers not acknowled ged as a rich source for discourse. | (Reason, 2003) | Teacher validates answers. | Wrong answers lead to discourse. | (Reason, 2003) | Students validate answers. |
| | | | **Questioning** | | |
| Mostly lower order questioning ; Ask students to recite or repeat. | (Pesek & Kirshner, 2000) | What is … How many … | Frequent use of higher order questioning; Ask students to compare and contrast. | (LeFevre et al, 2006; Pesek & Kirshner, 2000) | Why … Can you tell me another way to … |

Continued

Table 3.6 continued

| Procedurally Oriented Teaching | | | Conceptually Oriented Teaching | | |
|---|---|---|---|---|---|
| **Teaching Practice** | **Reference** | **Teaching Examples** | **Teaching Practice** | **Reference** | **Teaching Examples** |
| | | | **Role of Teacher** | | |
| Teacher demonstrates. | (Pesek & Kirshner, 2000) | Teacher shows how to find the mean and median. | Teacher facilitates. | (Pesek & Kirshner, 2000) | Teacher asks students to explore how to find the middle of a set of numbers. |
| | | | **Assessment** | | |
| Computational problems only. Students list steps. | | Given a rectangle, calculate its perimeter and area. Timed fact tests. | Open ended questions. Students explain thoughts. | | Explain the difference between perimeter and area. Explain mental computation. |
| | | | **Manipulatives** | | |
| Using manipulatives to model a problem. | (Miller & Hudson, 2007) | Student learns steps to follow to add fraction pieces. | Using manipulatives to explore a concept. | (Tracy & Gibbons, 1999) | Exploring areas and perimeters on a geoboard. |
| | | | **Types of Problems** | | |
| Focuses on different “types” of problems or “cases”. | (Raymond, 1997) | There are three types of percent problems to solve: What % of 60 is 12? 20% of what is 60? 20% of 60 is what? | Focuses on problems to encourage thinking or student created problems. | (Raymond, 1997) | Problem is presented without one obvious strategy to attempt the problem. |

**Frequency Chart:** Key ideas were taken from Table 3.6 to make an interview assessment frequency chart as seen in Table 3.7.

| Procedurally Oriented Teaching | | Conceptually Oriented Teaching | |
|---|---|---|---|
| **Teaching Practice** | **Tally** | **Teaching Practice** | **Tally** |
| **Reasoning/Communication** | | | |
| Explaining "how to". | | Explaining "why"; Asking "why not" | |
| **Content** | | | |
| Teaching definitions and symbols; Skill drill. | | Posing situations which explore mathematics. | |
| **Algorithms** | | | |
| Learning steps or practicing. | | Discovering or explaining steps to algorithms. | |
| **Calculators/Technology** | | | |
| Learning how to use technology to perform mathematical skills. | | Using technology for problem solving or explorations. | |
| **Solution Strategies** | | | |
| There is "one right way" to solve a problem. The right answer is most important. | | Multiple strategies to get to a solution. Problem solving process is valued over the solution. Flexible reasoning. | |
| **Speed and Accuracy** | | | |
| Statement that "speed" and "accuracy" are the goal. | | Understanding of fewer problems emphasized. | |
| **Connectivity** | | | |
| Focusing on a single skill with limited connection to related concepts, skills or applications; isolated skills; Teaching definitions; Sequential. | | Connecting ideas or concepts in math; Connecting math to other academic subjects or the real world; Intertwined. | |
| **Problem Solving** | | | |
| Selecting word problems to model the current skill; Following steps to solve a word problem. | | Posing a real problem. Creative problem solving. | |

Continued

Table 3.7: Conceptually and Procedurally Oriented Teaching Methods Frequency Chart.

Table 3.7 Continued

| Procedurally Oriented Teaching | | Conceptually Oriented Teaching | |
|---|---|---|---|
| **Teaching Practice** | **Tally** | **Teaching Practice** | **Tally** |
| **Representation** | | | |
| Algorithm is most important. | | Multiple representations; Proof or reasoning emphasized; Encouraging verbal or written explanations. | |
| **Correcting Wrong Answers** | | | |
| Wrong answers not acknowledged as a rich source for discourse. | | Wrong answers lead to discourse. | |
| **Questioning** | | | |
| Mostly lower order questioning; Ask students to recite/repeat. | | Frequent use of higher order questioning; Ask students to compare and contrast. | |
| **Role of Teacher** | | | |
| Teacher demonstrates. | | Teacher facilitates. | |
| **Assessment** | | | |
| Computational problems only. Students list steps. | | Open ended questions. Students explain thoughts. | |
| **Manipulatives** | | | |
| Using manipulatives to model a problem. | | Using manipulatives to explore a concept. | |
| **Types of Problems** | | | |
| Focuses on different “types” of problems or “cases”. | | Focuses on problems to encourage thinking or student created problems. | |

**Dichotomous Questions:** One section of the interview asked teachers to quickly respond to dichotomous statements about approaches to a particular task. These statements addressed procedural and conceptual teaching as well as two statements which addressed mathematics self-efficacy and mathematics teaching self-efficacy. For example, one statement asked whether teachers should explain “how” or explain “why.” Another asked whether teachers believe calculators should be used for problem solving or for computation.

Key points from the interview assessment frequency chart also formed the dichotomous statement choices. The dichotomous questions of the interview explored teachers’ first instinct of whether procedural or conceptual teaching is most important when introducing a lesson. Table 3.8 shows coding of the possible answers as either procedurally oriented or conceptually oriented teaching.

| Aspect of Teaching Mathematics | Procedural | Conceptual |
|---|---|---|
| Teaching Goal | Explaining “how” | Explaining “why” |
| Focus | Skill Drill | Concept development |
| Algorithms | Memorize steps | Discover steps |
| Calculators | for computations | for problem solving |
| Problem Types | Word Problems | Problem Solving |
| Goal for Students | speed and accuracy | understanding |
| Number of Skills | Sequential, isolated skills | Mixing concepts |
| Solution Process | one right way | many right ways |
| Wrong Answers | should be corrected | should lead to discussion |
| Questioning | Recite solution | Justify reasons |
| Manipulatives | To model | To explore |
| Focus | Procedures | concepts |

Table 3.8. Interview Question 12 coding.

After the interview assessment frequency chart was generated, it was given to a panel of experts to evaluate content validity. The four individuals who served on the panel of experts consisted of two high school mathematics teachers, one science education doctoral student, and one mathematics education doctoral student. Clarifications to the frequency chart were made in response to the panel's feedback.

**Interview Pilot Study**

Pilot interviews were conducted with four elementary inservice teachers. Field notes were taken immediately following the interview. Three interviews, which were about 20 minutes in length, were conducted in the teacher's classroom, audio-recorded, and transcribed. The fourth was conducted long-distance via a series of e-mails and was therefore already in a digital format. The researcher made additional field notes and each interview was evaluated using the assessment frequency chart.

During the four pilot interviews, responses that indicated a balance of procedurally and conceptually oriented teaching evolved from each of the two content strand interview questions. Additionally, teachers were quite frank about their mathematics self-efficacy and mathematics teaching self-efficacy levels. Results from the interview portion of the pilot study reinforced the idea that teachers could indeed be placed into groups based on high, medium, or low mathematics self-efficacy and mathematics teaching self-efficacy. Additionally, these four teachers offered varied types of responses related to specific content questions according to the interview assessment frequency chart.

**Pilot interview highlights.** Results from the four pilot interviews provided some interesting and valuable data of the sort that was also desired from the dissertation study.

All names of participants have been changed to protect the innocent. One participant, Carmen, clearly admitted her discomfort, or low mathematics self-efficacy, in the area of algebra. When asked the algebra question about order of operations, she indicated that she would teach them the jingle but she would have to look up the jingle first. This response, clearly procedural in nature, seemed to reflect her low mathematics self-efficacy in algebra. In contrast, Carmen described a conceptual lesson on her most self-efficacious content area, measurement, which involved using pictures, numbers, and words to help students discover their thinking.

Carmen and Alison both indicated strength in their self-efficacy related to arithmetic and also indicated that algorithms should be invented by students rather than simply memorized. Betty, however, stated a low self-efficacy with arithmetic and responded that algorithms should be memorized, a procedurally oriented teaching method. In the arithmetic portion of the interview, a relationship between high mathematics self-efficacy and conceptual learning of arithmetic was evident. A prime example was Carmen's procedural lesson on algebra but her conceptual approach to teaching arithmetic. The dissertation study was designed to examine this and other relationships further.

Tim indicated that although his mathematics self-efficacy was high in algebra, he chose this area as his least confident area to teach. Interestingly, the reverse was true about statistics in that he indicated moderate to low mathematics self-efficacy with statistics but chose it as his area of most confidence to teach. Additionally he stated that "statistics lent itself to good teaching better than algebra did." Tim was more likely to drill algebra and explore statistics. He also was more likely to teach isolated skills in algebra versus teaching by posing problems in statistics. Tim's responses were evidence that each teacher's teaching methods may differ by mathematics content areas as well as by level of self-efficacy.

The four pilot interviews reinforced the hypothesis that teachers with low mathematics self-efficacy tend to use more procedural teaching methods and teachers with high mathematics self-efficacy tend to use more conceptual teaching methods.

**Changes made prior to the dissertation study:** After administering the pilot surveys and conducting the pilot interviews, few changes were made to the study instruments. The only change to the MTMSE survey was to shorten part 4 so it did not appear too overwhelming.

Clarifications were made during the pilot interviews, and as a result, some questions were tweaked to offer greater clarity and generate deeper responses from future dissertation study interviews. The main differences between the pilot interview protocol and the dissertation interview protocol were to change the content specific questions and to restructure the dichotomous questions. In the pilot interviews teachers were asked about two specific NCTM content areas chosen from their survey results. In the dissertation interviews teachers were asked to identify the mathematics topics that they were most confident and least confident teaching. The participant was then asked to describe in detail an introductory lesson on each of these topics. The interviewer probed these specific topics using ideas from the former dichotomous question list to expose procedural and conceptual teaching methods that after analysis would categorize the participant as a teacher who used mainly procedurally or conceptually oriented teaching methods for that specific mathematics topic. The dissertation MTMSE survey instrument can be found in Appendix A and the dissertation interview protocol can be found in Appendix B.

**Researcher as Instrument**

One significant aspect of a study with a qualitative component is the perspective of the researcher. The perspective is like a stone that has changed over time. The stone becomes unique as it ages depending on its locations, the temperatures, and the conditions of its surroundings. Some end up smooth and small, others large and rigid; each has its own journey and history which make it unique. The researcher, too, is unique. Some have years of teaching experience and are just entering the research world. Others have only a small amount of teaching experience and a wider research experience. Like the stone, each educational researcher has traveled a unique journey to gain the perspective s/he reflects today. For this reason, I believe it is pertinent to share my personal history in the field of mathematics education.

Coming from a family of teachers, I could not fathom a career more interesting, more rewarding, or more purposeful than that of a teacher. I chose mathematics because it was the subject that had challenged me the most during my school years. I earned my Bachelor's Degree in 1987 and was inspired by a professor, Dr. Johnny Hill at Miami University, who was passionate about helping teachers and children understand the beauty of mathematics. I entered the teaching profession as a middle school mathematics teacher just before the NCTM Standards (1989) began the current reform movement in mathematics education. I remained a classroom teacher mainly in middle grades mathematics for the next fifteen years. During this time, I enjoyed learning new teaching methods and began speaking about such topics as problem solving and mental mathematics at local, state, and national conferences. Also during this time, I earned my teaching certificate in Computer Science and a Master of Arts Degree in Mathematics Education, and I served as mathematics department chair and on curriculum committees. As a teacher and as a parent I observed and interacted with a variety of teachers whose passions, self-efficacies, teaching methods, effectiveness, and even interests in teaching mathematics were very different. I entered the doctoral program with a quest for more knowledge that would help me to become a better mathematics teacher of children and future teachers. My philosophy of teaching mathematics evolves from a basic premise that all students can learn mathematics, can understand the value and usefulness of mathematics, and should feel confident at their current level of understanding. Also, I believe that the American phobia, mathematics fear, needs to be addressed and battled in American schools. My perspective therefore is one of a wealth of classroom experience, a passion for mathematics, and an interest in improving others' mathematics and mathematics teaching self-efficacies.

## Dissertation Study Procedures

**Survey participants.** Teachers taking a summer workshop for elementary teachers of mathematics were invited to complete the MTMSE survey. The survey script, located in Appendix C, was used before administration of the survey, that took approximately 15 minutes to complete. Workshop participants were involved in one of four summer sessions of two-week duration. A total of 75 participants from the four sessions were surveyed approximately mid-way through their workshop.

It was planned that participants completing the MTMSE would be divided among the self-efficacy levels shown in Figure 1.1. By eliminating all teachers with medium levels of mathematics self-efficacy or mathematics teaching self-efficacy, only those teachers in the four extreme categories were targeted to become cases for the interview phase of the study. In the end, however, to gain a greater interview sample, all survey participants were invited to participate in an interview.

**Interview participants:** Approximately two months after the survey, all survey participants were invited by email or personal contact to participate in a follow-up interview using the interview script found in Appendix C. Those accepting were interviewed individually in their classrooms. Each teacher's most and least confident mathematical topic to teach were probed further during the interview. Teachers were each contacted at least twice to participate in an interview. Twenty-five (25) agreed to be interviewed. Of the 25 only 22 interviews were completed and 16 were usable in the study. Two of the three teachers who initially agreed to be interviewed but did not complete an interview set up a time and did not show up or respond to my later contacts. Another agreed to be interviewed but was eliminated since she did not want to answer all the questions on the survey. The six teachers who completed interviews but did not become participants of the study were eliminated due to recording/transcription difficulties (3), incorrect grade level (1), incomplete survey (1), and incorrect teaching assignment (1) – this last mentioned was a special education teacher instead of regular elementary classroom teacher.

**Field notes and transcription.** A journal was kept noting dated, narrative descriptions of all research activities. Field notes contained a calendar of events, overall impressions of interview participants, questions to be clarified by participants, ideas for data analysis, and possible conclusions. The interviews were audio-taped and transcribed.

Following an interview, a summary was formed for each interview participant. The transcribed interview and interview summary were emailed to each participant for corrections, comments, or additions. Twelve of the 16 interviewees replied with minor or no adjustments. One gentleman called me to emphasize a particular point. The other three chose not to reply after two requests. Statements of self-efficacy and questions validating or extending the interview responses were assessed using the audio-tapes and field notes. A panel of two expert colleagues looked at a representative sample of interview summaries and transcripts to ensure consistency and accuracy.

**Qualitative data.** The qualitative portions of the study were evaluated for trustworthiness and credibility as described by Lincoln and Guba (1985). Trustworthiness was measured by asking participants to read the transcripts of their interviews for accuracy. Member checking extends trustworthiness by asking participants to verify the statements and conclusions made concerning their mathematical self-efficacy, mathematics teaching self-efficacy, and teaching method choices summarized in their individual interview summaries.

Credibility was ensured by persistent observation, triangulation, peer debriefing, and member checking. Persistent observation in this study involved conducting an interview that exhausted the need for clarification of beliefs and involved looking for truth by questioning inconsistencies found, revisiting existing premises, and further delving into emerging issues. Triangulation was achieved by looking for the same outcomes through various sources including the MTMSE survey, the interview, and field notes. Peer debriefing was conducted through consulting a panel of experts to look at the transcripts and scales for issues missed. Member checking was conducted as described in the preceding section.

The qualitative portion was analyzed using phenomenology and ethnographic interpretive research methods. Interpretive methods involve gaining insight into the way that people think about something, in this case the phenomenon of teacher mathematics confidence in both their mathematics ability and their mathematics teaching ability. Ethnographic methods help the researcher hear the voices of participants and offer meaning to the phenomenon of how self-efficacy toward mathematics and mathematics teaching affects teachers' choices of procedurally or conceptually oriented teaching methods. By combining quantitative and qualitative methods, the subjective became objectified, and vice versa. The combination of quantitative and qualitative paradigms was classified as Brannen's (2004) triangulation method whereby mathematics self-efficacy and mathematics teaching self-efficacy were evaluated using data analysis of the MTMSE survey, interpretations of interviews, and member checks of participant's interpretations of the interview transcripts. Procedurally and conceptually oriented teaching was also triangulated through the interview, field notes, and part 5 of the survey.

The interview transcriptions and the journal of events and reflections were analyzed using inductive methods, a form of reasoning where an argument leads up to a supported conclusion. The interview transcripts were analyzed using *a priori* coding, or coding categories which were designed in advance of the interviews. The processes of sorting and tagging (James-Brown, 1995) were used to examine statements, words, and phrases used to describe mathematics, mathematics self-efficacy, and mathematics teaching. Comments from transcripts and narrative comments during the interviews were analyzed using cross-case analysis by grouping the participants into categories according to their scaled orientation toward mathematics and mathematics teaching.

The researcher looked for statements related to mathematics teaching self-efficacy and mathematics self-efficacy through this analysis to validate the quantitative results. Also, the interview transcripts were searched for key words and statements that matched the Interview Assessment Frequency Chart (see Table 3.7). Teachers were categorized as conceptually oriented, procedurally oriented, or mixed for each mathematical topic. These findings expanded upon the quantitative findings in order to explore relationships between variables.

**Security.** The researcher kept all information organized, labeled, and secured. Participant numbers for identification were used instead of names. Pseudonyms were created for publication in this dissertation. In the future, the study will be authenticated by submitting an article for publication in a scholarly educational research journal making this work available to the public.

## Data Analysis

**Quantitative Data Analysis:** The MTMSE self-efficacy scale was analyzed using deductive methods. The survey data as a whole and within and across subscales were analyzed using SPSS statistical computer software. Additionally, portions of the interview were quantified for analysis using SPSS. The following statistical tests were performed:

1. Cronbach's alpha was calculated to determine the reliability of the instrument and its subscales.

2. Pearson-product moment correlations were found to identify the strength of relationships between the independent and dependent variables.
3. Chi-square tests were performed to examine relationships among categorical data such as the numbers of teachers who were conceptually versus procedurally oriented in their teaching of their most or least confident mathematics topic.

**Qualitative Data Analysis:** The two research questions that can be analyzed qualitatively include: (a) how does mathematical self-efficacy relate to the elementary classroom teacher's tendency to teach conceptually or procedurally, and (b) how does elementary teachers' mathematics teaching self-efficacy relate to their tendency to teach conceptually or procedurally?

In order to address the research questions, specifically, the following analyses were made using the qualitative data:

1) Do the mathematics self-efficacy and mathematics teaching self-efficacy interview statements match the MTMSE results?
2) Is the teacher primarily stating that s/he teaches procedurally, conceptually, or both for each mathematics topic?
3) What conceptual and procedural teaching methods are evident?
4) Are similarities or differences evident within each teacher between procedural or conceptual teaching methods and the least and most confident mathematical topics?
5) How do mathematics self-efficacy and mathematics teaching self-efficacy relate to the use of conceptual and procedural teaching methods?

**Ethics and politics.** Following are some concerns regarding ethics and politics in this study. First, as I have such a passion for teaching the world the beauty of mathematics, I believe I want to change the world to help all teachers and students develop higher self-efficacy toward mathematics and toward mathematics teaching. To address this possible breech of neutrality, I conducted this study looking for interpretations using only positivist and interpretivist methods to try to remain as objective as possible.

Second, by researching a phenomenon uncomfortable to some, I may seem like the omnipotent researcher. This is not desirable because I am not omnipotent, rather an individual concerned about the welfare of our students and teachers and the impact that teachers may have on students. My goal is to shed light on the implications of how self-efficacy toward mathematics may affect teaching practices. To monitor my bias against those who fear mathematics, I have presented myself as friendly, neutral, and professional and have recorded respondents' answers to questions accurately and asked for clarification as needed.

Third, it is important that the words in my interview and checklist are not perceived by my participants as loaded, but rather that they are nondirectional, fair, and able to paint an accurate picture. To insure a fair questionnaire, I have asked a panel of experts for feedback and conducted pilot studies before implementing my study.

## Summary

In summary, this study added to the research base of studies investigating teacher self-efficacy related to mathematics self-efficacy and mathematics teaching self-efficacy as well as those of conceptual and procedural teaching. This study investigated third through sixth grade teachers using both a survey and an interview that examined the relationship between the mathematics and mathematics teaching self-efficacy with conceptual and procedural teaching methods across mathematics topics. By focusing on upper elementary teachers, where confidence in mathematical ability and in mathematics teaching ability are extremely important, consistencies and inconsistencies in preferred teaching methods were illuminated relative to various mathematical topics.

# CHAPTER 4

# DATA ANALYSIS

The current mixed methods study explored the relationship between mathematics self-efficacy and mathematics teaching self-efficacy as well as the relationships between those variables and conceptually or procedurally oriented teaching methods among elementary mathematics teachers. The study involved the design of the Mathematics Teaching and Mathematics Self-Efficacy (MTMSE) survey, found in Appendix A, and the design of the Conceptually and Procedurally Oriented Teaching Methods Frequency Chart, found in Table 3.7. The primary focus of this chapter is the data analysis of the survey and interview responses related to the research questions. The results of the pilot study are also included in this chapter since the development of the MTMSE survey and the Conceptually and Procedurally Oriented Teaching Methods Frequency Chart instruments were significant pieces of the study. This chapter details the analyses used to find relationships between and among the variables in the study.

## Instrument Reliability

The Mathematics Teaching and Mathematics Self-Efficacy (MTMSE) survey was developed based on the Mathematics Self-Efficacy Survey – Revised (MSES-R) (Kranzler & Pajares, 1997), Mathematics Teaching Efficacy Beliefs Instrument (MTEBI)

(Enochs, Smith, & Huinker, 2000), National Council of Teachers of Mathematics (NCTM) Standards (2000), and other current mathematics education research literature. Table 4.1 shows reliability using Cronbach's alpha coefficient of internal consistency for each part of the MTMSE compared with the reliability from the unrevised source instrument, where applicable. It is evident from these results that the MTMSE is a very reliable instrument overall with an alpha level of 0.942 and very reliable for the mathematics self-efficacy and mathematics teaching self-efficacy parts, all with alpha levels over 0.850. The conceptual/procedural part is less reliable with an alpha level of 0.554, which may be explained by the broadness of the questions in contrast to the specific mathematics topics in the other parts of the survey. Teachers' orientations toward procedural or conceptual teaching were clarified during the qualitative interview portion of the study. Further research should explore the use of quantitative measures to assess conceptually and procedurally oriented teaching practices.

| Survey Part | Purpose | Source | Source Reliability | Current Study Reliability |
|---|---|---|---|---|
| 1 | Mathematics Self-Efficacy Problems | MSES-R (Kranzler & Pajares, 1997) Problems Subscale | 0.900 | 0.900 |
| 2 | mathematics teaching self-efficacy Efficacy | MTEBI (Enochs, Smith, & Huinker, 2000) Personal Efficacy Subscale | 0.880 | 0.855 |
| 3 | Mathematics Self-Efficacy Tasks | MSES-R (Kranzler & Pajares, 1997) Tasks Subscale and NCTM (2000) Content Standards | 0.910 | 0.862 |
| 4 | Mathematics Teaching Self-Efficacy Content | NCTM (2000) Content Standards | N/A | 0.880 |
| 5 | Conceptually or Procedurally Oriented Teaching Methods | See Table 3.6 | N/A | 0.554 |
| 6 | Demographic Questions | N/A | N/A | N/A |
| 1-5 | Full Instrument | N/A | N/A | .942 |

Table 4.1: Cronbach's alpha coefficient for each part of the MTMSE compared to source instruments.

First studies involving the MSES-R or MTEBI instruments used different samples than this study. The MSES-R (Kranzler & Pajares, 1997) was conducted with a sample of college students. The MTEBI (Enochs, Smith, & Huinker, 2000) was conducted with a sample of preservice elementary teachers. Self-efficacy research is prevalent for these two populations, whereas few studies on self-efficacy related to mathematics or mathematics education were found that used practicing elementary teachers for their sample, and none were found that administered the MSES or MTEBI. As mentioned in chapter 3, Nielsen and Moore (2003) validated that the MSES-R could be reliably administered in different contexts and has been used with college students (Pajares & Kranzler, 1997), with high school freshmen (Nielsen & Moore, 2003), and with practicing elementary teachers in the current study, as well as in other studies. Overall, the consistent reliability results compared to the source studies' reliabilities suggests that MTMSE results are reliable for the population of the current study involving practicing elementary teachers.

## Participants

Eighty (80) teachers involved in summer workshops to enhance mathematics teaching skills completed the MTMSE survey. All teachers worked with third through sixth grade students. Five (5) teachers provided incomplete surveys and were eliminated from the study. The remaining 75 surveys were analyzed according to the goals set forth in Chapter 3.

The 75 participants were employed by a variety of types of school districts, as seen in table 4.2. Sixty-five (65) teachers indicated that teaching was their first career and 10 indicated it was not. Six (6) males and 69 females completed the survey and 59 of the

75 were parents. Seventy-three (73) categorized their race as white while 2 gave no response.

| School District Type | Frequency |
|---|---|
| Urban | 50 |
| Suburban | 23 |
| Rural | 2 |

Table 4.2: Survey participants' school district type (n = 75).

Twenty (20) have earned a Bachelor's degree and 55 have also earned a Master's degree. Table 4.3 shows a variety of undergraduate teaching areas of the participants. Forty-eight (48) of the 75 teachers hold Ohio Teaching Licenses, 13 hold Ohio Provisional Teaching Certificates, and 14 hold Ohio Permanent Teaching Certificates. Seven (7) teachers are certified as Mathematics Specialists and 9 have a concentration in mathematics. Their teaching experience ranges from 3 to 33 years as seen in table 4.4.

| Undergraduate Degree Areas | Frequency |
|---|---|
| Education | 59 |
| Mathematics | 1 |
| Human Development | 2 |
| Other Areas | 13 |

Table 4.3: Undergraduate degree areas of survey teachers (n = 75).

| Years Experience | Frequency |
|---|---|
| 0-2 years | 0 |
| 3-5 years | 3 |
| 6-10 years | 29 |
| 11-15 years | 13 |
| 16-20 years | 15 |
| 21-30 years | 11 |
| More than 30 years | 4 |

Table 4.4: Teaching experience of survey teachers (n = 75).

After attempting to contact all 75 participants for a follow-up interview, 22 interviews were completed. Sixteen (16) of the 22 were included in the study and the remaining 6 were eliminated for various reasons including teaching in a non-traditional classroom such as a low-functioning special education class. The survey and interview data have been analyzed and results by research question follow.

## Mathematics Self-Efficacy Related to Mathematics Teaching Self-Efficacy

The first research question in the present study was: "How does mathematics self-efficacy relate to mathematics teaching self-efficacy?" To answer this question quantitative statistics were performed on both the survey data and on quantifiable interview data.

**Descriptive statistics.** Data from the surveys and interviews were entered in SPSS statistical analysis software. Survey items in part 2, measuring mathematics teaching self-efficacy were worded both positively and negatively. Negatively worded were directionally recoded to align with positively worded statements for data analysis. Both mathematics self-efficacy and mathematics teaching self-efficacy totals were calculated by converting the six Likert-scaled responses to numbers 1 through 6. Sums were found within each part as well as sums for all 31 items measuring mathematics self-efficacy (parts 1 and 3) and all 31 items measuring mathematics teaching self-efficacy (parts 2 and 4). Each set of 31 questions produced a score ranging from 31 to 186 points.

Since the two sets of questions were different, one should be cautious when comparing the two variables directly. However, it is helpful to view each variable using box-and-whisker plots as seen in Figure 4.1. Notice that mathematics teaching self-efficacy has a slightly smaller range which indicates that the teachers gave more similar responses to mathematics teaching self-efficacy questions than to mathematics self-efficacy questions. Notice also the mathematics teaching self-efficacy data has one outlier at the lower end. Outliers were calculated by finding values which were more than 1.5 times the inner quartile range above the third quartile or below the first quartile. Translated this means that one teacher's mathematics teaching self-efficacy rating was significantly lower than the rest of the sample. She was classified as low self-efficacy for both the mathematics and mathematics teaching levels. This teacher, a regular-classroom third grade teacher was included in both the survey and the interview portions of the study.

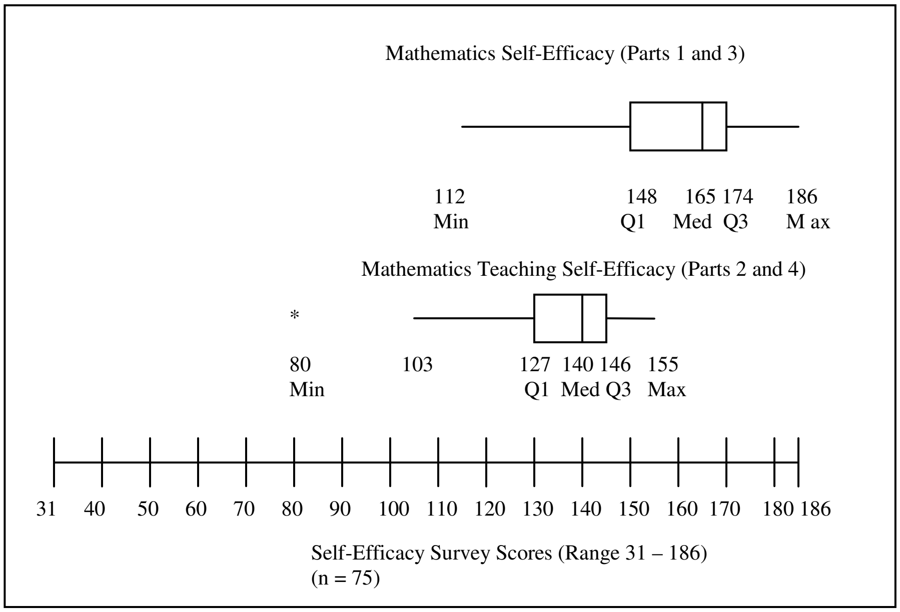


Figure 4.1: Box and whisker plots for mathematics and mathematics teaching self-efficacies (n=75)

**Box-plot.** Finally, the box-and-whisker plots show that teachers gave higher responses overall to the mathematics self-efficacy questions than they did to the mathematics teaching self-efficacy questions. This is again evident by comparing the means of 158.9 for mathematics self-efficacy and 134.8 for mathematics teaching self-efficacy. As a matter of fact, over 50% of the sample gave higher responses for mathematics self-efficacy than the highest response for mathematics teaching self-efficacy and 75% of the teachers gave higher responses for mathematics self-efficacy than the third quartile mark for mathematics teaching self-efficacy.

These descriptive statistics might be surprising to those who believe that teachers are more comfortable teaching their particular grade's mathematics concepts than in performing mathematics problems and tasks outside the classroom in general. One possible explanation for the difference between mathematics self-efficacy and mathematics teaching self-efficacy may be that mathematics concepts and skills are typically static in nature, whereas in the world of mathematics education, teaching methods are not static. As captured in Chapters 1 and 2, teaching methods are continually changing, and hopefully improving, as mathematics educators struggle to find the best ways to teach classrooms of unique students about a diverse set of mathematical skills and concepts. A higher set of mathematics self-efficacy responses compared to mathematics teaching self-efficacy scores however aligns with Ma's (1999) research stating that a teacher's confidence in her own understanding was needed before the teacher could encourage discussion among children. Thus, it is possible that a high mathematics self-efficacy is needed before a high mathematics teaching self-efficacy can develop.

When comparing each teacher's mathematics self-efficacy total with her mathematics teaching self-efficacy total, only three teachers had a lower mathematics self-efficacy score than mathematics teaching self-efficacy score and one teacher had equal mathematics and mathematics teaching self-efficacy scores. The minimum difference between the mathematics self-efficacy total and the mathematics teaching self-efficacy total was negative ten points and the maximum was 47 points with a mean difference of 24.45 points. By these descriptive statistics, combined with the box-and-whisker plots in Figure 4.1, the data showed that most elementary teachers' individual mathematics self-efficacy was higher than their mathematics teaching self-efficacy.

**Scatterplot.** Participants were then ordered from lowest to highest, first by sum for mathematics self-efficacy and second by sum for mathematics teaching self-efficacy and grouped into approximate thirds using natural breaks in the scores for each variable.

The goal of grouping into thirds was to place teachers into levels of low, medium, or high self-efficacy. “The natural breaks method is based on the assumption that data fall naturally into meaningful groups which are separated by breaks” (Smith, 1986, p. 64) and has been used in attitudinal studies such as studies by Parrott and Hewitt (1978) and Solomon, Battistich, Kim, and Watson (1997) to classify participants into group for data analysis. The scatterplot in Figure 4.2 shows the position of each participant within low, medium, or high self-efficacy groupings for each variable.

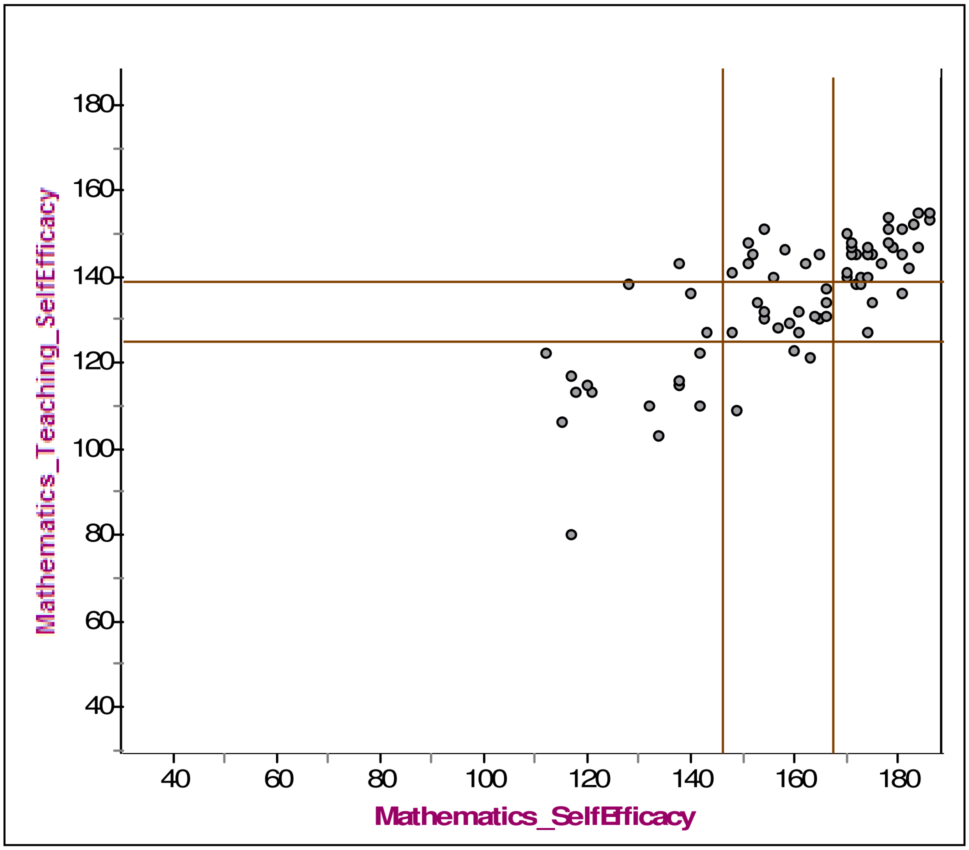


Figure 4.2: Scatterplot comparing mathematics teaching self-efficacy and mathematics self-efficacy (n=75).

**Self-efficacy levels.** Frequencies of the groupings are shown in Table 4.5. The most common group was high mathematics self-efficacy and high mathematics teaching self-efficacy, with 28 teachers in this group. Thirteen (13) teachers fell into the group of low mathematics self-efficacy and low mathematics teaching self-efficacy.

| Mathematics Teaching Self-Efficacy | | Mathematics Self-Efficacy | | |
|---|---|---|---|---|
| | | High | Medium | Low |
| | High | 28 | 9 | 1 |
| | Medium | 5 | 13 | 3 |
| | Low | 0 | 3 | 13 |

Table 4.5: Frequencies of survey teachers by self-efficacy groupings (n=75).

Participants in the extreme four corners of the table were thought to potentially offer information of interest. However, to maximize the number of participants interviewed, all teachers who responded to the survey were invited to participate in interviews. Mathematics self-efficacy and mathematics teaching self-efficacy levels of the 16 interviewed are shown in Table 4.6 indicating that 13 of the 16 interviewees fell into the extreme corner groups.

| Mathematics Teaching Self-Efficacy | | Mathematics Self-Efficacy | | |
|---|---|---|---|---|
| | | High | Medium | Low |
| | High | 8 | 1 | 1 |
| | Medium | 0 | 1 | 1 |
| | Low | 0 | 0 | 4 |

Table 4.6 Self-efficacy groupings of interview teachers (n=16).

**Correlation of self-efficacies:** The total mathematics teaching self-efficacy and total mathematics self-efficacy variables from the survey data were analyzed using Pearson's Product Moment Correlation. The correlation level was .770 indicating that there is a significant ($p < .01$) correlation between the variables of mathematics teaching self-efficacy and mathematics self-efficacy. The correlation of mathematics self-efficacy and mathematics teaching self-efficacy for the 16 interview participants was .837. Interview data also indicated (see below) that mathematics self-efficacy and mathematics teaching self-efficacy have a strong correlation. This is a significant finding for the current study, as no other research studies in mathematics education literature were found that compare the variables of mathematics self-efficacy and mathematics teaching self-efficacy.

During the interview, participants were asked about their mathematics self-efficacy, or confidence toward mathematics outside the classroom. When asked for clarification, it was suggested that they consider their ability to balance their checkbook, to calculate a tip at a restaurant, or to calculate a discount while shopping. The participants' answers varied from "terrible" to "great." Each participant was rated by the researcher as low mathematics self-efficacy, medium mathematics self-efficacy, or high mathematics self-efficacy based on their response to this one interview question. Table 4.7 illustrates the comparison between the survey mathematics self-efficacy level with the interview self-reported mathematics self-efficacy level. Recall, the survey level was assigned based on whether the sum of the participant's responses to all mathematics self-efficacy survey questions were in the highest, middle, or lowest third of all participants.

| | | Mathematics Self-Efficacy Interview Level | | | |
|---|---|---|---|---|---|
| | | Low | Medium | High | Total |
| Survey Level | Low | 3 | 3 | 0 | 6 |
| | Medium | 0 | 0 | 2 | 2 |
| | High | 0 | 4 | 4 | 8 |
| | Total | 3 | 7 | 6 | 16 |

Table 4.7: Crosstabs frequencies of mathematics self-efficacy levels between survey and interview.

Participants were also asked about their mathematics teaching confidence during the interview. Many participants indicated they were very confident teaching mathematics, but amazingly, some elementary mathematics teachers freely admitted they do not feel confident about their mathematics teaching or they only feel confident teaching mathematics up to the current grade level of their students. The interviewer categorized each teacher as low, medium, or high mathematics teaching self-efficacy after the interview based on the interpretation of the interview. Table 4.8 illustrates the comparison between the survey mathematics teaching self-efficacy level with the interview self-reported mathematics teaching self-efficacy level. Again recall, the survey level was assigned based on whether the sum of the participant's responses to all mathematics teaching self-efficacy survey questions were in the highest, middle, or lowest third of all participants.

| | | Mathematics Teaching Self-Efficacy | | | |
|---|---|---|---|---|---|
| | | Interview Level | | | |
| | | 1 | 2 | 3 | Total |
| Survey Level | low | 1 | 0 | 3 | 4 |
| | medium | 0 | 2 | 0 | 2 |
| | high | 0 | 5 | 5 | 10 |
| | Total | 1 | 7 | 8 | 16 |

Table 4.8: Crosstabs frequencies of mathematics teaching self-efficacy levels between survey and interview.

Nearly half of the participants on both frequency comparisons above remained at the same level of self-efficacy between survey and interview. Differences likely are caused by the interview portion emphasizing two specific mathematics topics, chosen by the teacher as her most and least confident mathematics topic to teach, rather than overall mathematics teaching self-efficacy as measured on the survey.

**Correlation of survey parts.** A comparison of MTMSE survey part 1, measuring self-efficacy on mathematics problems, and part 3, measuring self-efficacy on mathematics tasks, correlated to .740. This shows that there was a fairly strong relationship between the scores on parts 1 and 3 of the survey. A comparison of MTMSE survey part 2, measuring self-efficacy of general teaching of mathematics, and part 4, measuring self-efficacy toward teaching specific mathematics topics, correlated to .763 indicating another strong relationship between the scores on parts 2 and 4 of the survey.

**Correlation by favorite subject.** Participants were asked on the survey which was their favorite subject to teach from Language Arts, Reading, Math, Science, and Social Studies. As seen in Table 4.9, 52 teachers indicated mathematics was their favorite subject to teach and six indicated mathematics was their least favorite subject to teach.

Mathematics self-efficacy and mathematics teaching self-efficacy was compared using a Pearson correlation and resulted in a considerable difference between teachers who chose mathematics as their favorite or not. As expected, teachers whose favorite subject was mathematics had the highest correlation of mathematics and mathematics teaching self-efficacies at .801 and those whose least favorite subject was mathematics had the lowest correlation at .569.

| | Favorite Subject to Teach | Least Favorite Subject to Teach |
|---|---|---|
| Mathematics | .801 (n = 52) | .569 (n = 6) |
| Not Mathematics | .645 (n = 21) | .742 (n = 63) |

Table 4.9: Correlation of mathematics and mathematics teaching self-efficacies grouped by favorite subject to teach.

**Most and least confident topic.** Nearly all participants on the survey and all participants in interviews freely offered a most and least confident mathematics topic to teach. Confident teaching topics grouped by NCTM strand from the survey data are found in Table 4.10. Many participants stated their favorite topics with passion, sharing success stories and eagerness to teach the topic again. When stating their least favorite topics, many stated they lack a good teaching method for that topic or did not understand the topic fully themselves. One participant even asked to have the topic explained during the interview. Their willingness to discriminate between topics supports the hypothesis that teachers are more self-efficacious toward teaching some mathematics topics over others, a significant finding in this study.

| | Survey | | Interview | |
|---|---|---|---|---|
| Topic | Least Confident | Most Confident | Least Confident | Most Confident |
| | Number and Operation Strand | | | |
| Multiplication | 16 | 1 | 2 | 6 |
| Fractions | 5 | 8 | 1 | 1 |
| Decimals | 0 | 4 | 0 | 0 |
| Division | 1 | 3 | 0 | 0 |
| | Algebra Strand | | | |
| Number Patterns | 11 | 2 | 1 | 0 |
| Algebra | 4 | 4 | 4 | 5 |
| | Geometry Strand | | | |
| Geometry | 6 | 4 | 3 | 0 |
| | Measurement Strand | | | |
| Customary Measurement | 0 | 13 | 3 | 1 |
| Metric System | 0 | 16 | 1 | 0 |
| Perimeter & Area | 10 | 0 | 0 | 1 |
| | Data Analysis & Probability Strand | | | |
| Averages | 7 | 7 | 0 | 1 |
| Probability | 3 | 8 | 1 | 1 |
| Tables & Graphs | 5 | 2 | 0 | 1 |

Table 4.10: Frequencies of teachers who chose each topic as their most or least confident mathematics topic to teach.

**Correlation by NCTM strand:** Mathematics self-efficacy and mathematics teaching self-efficacy were also compared by NCTM content strand. Teachers were asked to identify the mathematics topic they were most and least confident teaching. Topics from part 4 of the survey were assigned to one of five NCTM content strands: 1) number and operation, 2) algebra, 3) geometry, 4) measurement, or 5) data analysis and probability, as shown in Table 3.3. As seen in Table 4.11, all correlations calculated were very strong, at $r \geq .698$. Correlations were not calculated for $n < 10$. Overall the correlation between mathematics self-efficacy and mathematics teaching self-efficacy is strong but it varies somewhat by mathematics strand.

| NCTM Strand | Most Confident Topic | Least Confident Topic |
|---|---|---|
| Arithmetic | .713<br>(n = 22) | .893<br>(n = 16) |
| Algebra | .872<br>(n = 15) | *<br>(n = 6) |
| Geometry | *<br>(n = 6) | *<br>(n = 4) |
| Measurement | .814<br>(n = 10) | .698<br>(n = 29) |
| Data Analysis and Probability | .802<br>(n = 15) | .814<br>(n = 17) |
| No topic selected | *<br>(n = 7) | *<br>(n = 3) |

Table 4.11: Correlation between mathematics self-efficacy and mathematics teaching self-efficacy by most and least confident topic grouped by mathematics content strand
* No correlation was calculated for n<10.

**Correlation with demographics.** Mathematics self-efficacy and mathematics teaching self-efficacy were correlated to certain demographic characteristics including undergraduate major, school district type (urban, suburban, or rural), gender, whether or not the teacher was a parent, and number of years of teaching experience. No significant correlations resulted from these calculations except in number of years of teaching experience. As seen in table 4.12 as the number of years of teaching experience rises, so does the correlation between mathematics and mathematics teaching self-efficacy. This result suggests self-efficacy improves over time. Furthermore, if teacher content knowledge improves through experience this result tends to reinforce the notion that stronger mathematics self-efficacy lends itself to a stronger mathematics teaching self-efficacy.

| Number of Years Teaching Experience | Correlation |
|---|---|
| 3-5 years (n = 3) | * |
| 6-10 years (n = 29) | .762 |
| 11-15 years (n = 13) | .755 |
| 16-20 years (n = 15) | .846 |
| 21-30 years (n = 11) | .838 |
| 30 or more years (n = 4) | .937 |

Table 4.12: Correlation between mathematics self-efficacy and mathematics teaching self-efficacy by number of years teaching experience. *No correlation reported.

**Summary.** In summary, the current study shows two results. First, mathematics teaching self-efficacy and mathematics self-efficacy are significantly related, which is supported by a strong correlation from the survey and interview data. The strong correlation is also visible within NCTM content strands. The willingness of participants to freely state their self-efficacy levels reinforces and strengthens the argument that the relationship is significant. Second, mathematics self-efficacy is typically higher than mathematics teaching self-efficacy. This result emerged from analyses of survey data comparing self-efficacies directly as well as by comparing self-efficacies by years of teaching experience. The fact that only one teacher fell in the extreme category of low mathematics self-efficacy and high mathematics teaching self-efficacy, while no teacher falls into the opposite category, strengthens the finding of the positive relationship between mathematics and mathematics teaching self-efficacy.

## Mathematics and Mathematics Teaching Self-Efficacy Related to Conceptually or Procedurally Oriented Teaching

The second and third research questions in the present study ask, “How does elementary teachers’ mathematics self-efficacy relate to their tendency to teach conceptually or procedurally?” and “How does elementary teachers’ mathematics teaching self-efficacy relate to their tendency to teach conceptually or procedurally?” To answer these questions, quantitative statistics were used to analyze survey data and quantifiable interview data. Anecdotes are included as case studies from the interviews to qualitatively enrich the findings.

**Survey data.** Part 5 of the MTMSE survey measured tendencies toward conceptually oriented teaching and procedurally oriented teaching. The Likert scaled answers were summed separately for conceptually and procedurally oriented questions. A low score on a procedural item did not imply conceptual orientation, and vice versa. The summed scores simply established a propensity toward conceptual or procedural orientation separately.

Correlations were computed among variables as follows. Propensity for conceptually oriented teaching correlated moderately with both mathematics teaching self-efficacy at .408 and mathematics self-efficacy at .454. Propensity for procedurally oriented teaching showed a slight negative correlation with both mathematics teaching self-efficacy at -.266 and mathematics self-efficacy at -.192. This suggested that conceptually oriented teaching was more closely related with mathematics teaching self-efficacy and mathematics self-efficacy than procedurally oriented teaching, therefore teachers with higher mathematics self-efficacy and mathematics teaching self-efficacy tended to choose more conceptual teaching methods.

Survey data showed that propensities toward conceptually or procedurally oriented teaching were slightly negatively correlated at -.239. Similarly, among only those teachers participating in the interviews, propensities toward procedurally or conceptually oriented teaching practices as measured on their corresponding survey were negatively correlated at -.100. That is, teachers who scored high on conceptual orientation had a slight tendency to score lower on procedural orientation, and vice versa. While it is safe to assume that most teachers use both conceptual and procedural teaching strategies, these results suggest that they are likely to be oriented toward one or the other.

**Interview data.** The majority of each interview consisted of the teacher describing two lessons beginning a unit on each of two mathematics topics s/he was most and least confident teaching. Because mathematics self-efficacy is a person's perception of his or her own mathematical ability, and mathematics teaching self-efficacy is a person's perception of his or her ability to teach mathematics, asking teachers to select mathematics topics they are most and least confident teaching is in essence the same as asking them to select the topics in mathematics in which they have the highest and lowest mathematics teaching self-efficacies. Probing questions shown in Appendix B were asked to clarify and extend each teacher's responses until the teaching methods were evident and the researcher felt the topic had been exhausted. Interviews were audio-recorded and transcribed.

**Interviewee self-efficacy.** Statements about self-efficacy related to mathematics and mathematics teaching can be found in Table 4.13. These statements show the variety of self-efficacy levels among participants. The table includes participant identification number, self-efficacy levels, and quotes from the participants. The self-efficacy levels, low = 1, medium = 2, and high = 3, were included for mathematics (MSE) and Mathematics Teaching (MTSE) from the interview data and assigned by the researcher based on participants' statements and compared with other interviews. Some teachers addressed their personal mathematics self-efficacy, others addressed their personal mathematics teaching self-efficacy, while some offered a philosophy toward mathematics education.

| ID | MSE | MTSE | Self-Efficacy Quotes |
|---|---|---|---|
| 1 | 3 | 2 | “Math is hard for everyone at some point. I think they need to learn how to ‘do’ math within the context of problems because that is how everyday life is presented.” |
| 7 | 3 | 3 | “I’m more confident in number sense and geometry. Algebra has made me less confident.” “The most important things for my students to learn in math is reasoning, critical thinking, problem solving, and sharing their thinking.” |
| 18 | 2 | 3 | “I am very confident teaching math. I am the school math teacher leader. Outside school I am not confident. I use my calculator always.” |
| 23 | 3 | 3 | “It cracks me up that they have little signs with the percents on it in the stores because people can’t figure it out on their own.” |
| 24 | 2 | 2 | “My favorite subject to teach is math. I am really confident teaching it.” |
| 28 | 1 | 1 | “This is my first year teaching 4$^{th}$ grade math. Before that I taught 2$^{nd}$. I’m not that confident. I found out last spring that I had to take a math course and teach 4$^{th}$ grade math or interview to teach somewhere else.” |
| 31 | 1 | 2 | I am “very comfortable with math concepts and teaching math up through fourth grade and then it gets scary.” |
| 35 | 3 | 2 | “I’m pretty confident. If I don’t know it I look it up. I can’t help my 10$^{th}$ grader with pre-Calculus but my life skills are fine.” |
| 36 | 3 | 2 | “They are learning a lot more advanced concepts now than when I was in 6$^{th}$ grade.” |
| 45 | 1 | 2 | “Because I teach 3$^{rd}$ grade I’m pretty comfortable. If I were teaching middle school I would want to review to be more sure of myself.” |
| 48 | 2 | 3 | “I’m very confident teaching math but less confident with math outside school.” |
| 50 | 2 | 2 | “Students have very poor math…number sense…grasping the big picture is very difficult. They get it better if we tell them this is how to do it. They then see the pattern and get it. If you teach the procedures first then the concepts will come.” |
| 63 | 3 | 3 | “Over the years I have incorporated a lot more writing and reading.” “I try to teach them to understand why. Parents say just teach them how to do it.” |
| 64 | 1 | 2 | “I teach all subjects and treat them all as reading. It is all about understanding what you are reading.” “I wish they taught 3$^{rd}$ grade when I was a kid like we do now. I wouldn’t be starting my sentence saying that I wasn’t good at math.” |
| 76 | 2 | 2 | “At a 6$^{th}$ grade level, I am very confident.” |
| 77 | 2 | 2 | “As I teach it I get better at it.” |

Table 4.13: Interviewee self-efficacy levels and quotes from interview.

**Interview summaries.** Different teaching strategies used to teach participants' most and least confident mathematics teaching topics were listed from each interview transcript in an individual interview summary. Distinctions between conceptually and procedurally oriented teaching methods were made using the frequency chart in Table 3.7 for each listed method. Teaching methods that were classified as procedurally oriented are followed with a (P) and those that were classified as conceptually oriented are followed with a (C).Sample interview summaries for two participants, Sally and Katy, are shown in Tables 4.14 and 4.15.

| Self-Efficacy | Mathematics | Mathematics Teaching |
|---|---|---|
| Survey | 3 | 3 |
| Interview | 2 | 3 |
| Teaching Methods | Least Confident | Most Confident |
| Topic | Geometric Shapes | Order of Operations |
| Methods | Shapes taught by abstract definition (P)<br>Steps are followed (P)<br>Procedures are memorized(P)<br>Different solution strategies explored (C)<br>One right answer (P)<br>Relate to real world (C) | Manipulatives used to explore (C)<br>Concrete representations (C)<br>Explore criteria (C)<br>Questioning analyzes organization (C)<br>Students discover properties (C)<br>Student centered (C) |
| Conceptual Orientation Ratio | 33% | 100% |
| Conceptual Procedural Tendency | Procedural | Conceptual |

Table 4.14: Interview summary for participant #31 Sally.

| Self-Efficacy | Mathematics | Mathematics Teaching |
| --- | --- | --- |
| Survey | 1 | 1 |
| Interview | 1 | 2 |
| Teaching Methods | Least Confident | Most Confident |
| Topic | Tables (algebra) | Perimeter |
| Methods | Explain how (P)<br>Learning steps to a rule (P)<br>Teacher demonstrates (P)<br>One right way (P)<br>Focus on single skill (P)<br>Algorithm (rule) is most important (P) | Explore paths (C)<br>Students invent formulas (C)<br>Discovery (C)<br>Teacher facilitates (C)<br>Focus on concept (C)<br>Reasoning emphasized (C) |
| Conceptual Orientation Ratio | 0% | 100% |
| Conceptual Procedural Tendency | Procedural | Conceptual |

Table 4.15: Interview summary for participant #48 Katy.

**Interrater reliability.** To check interrater reliability for judging teaching methods as procedural or conceptual, a fellow mathematics educator was given a subset of half of the interview summaries along with the interview frequency chart in Table 3.7. He was asked to judge each teaching method as procedurally oriented or conceptually oriented according to the frequency chart. On some items the rater asked for interpretations about the interview summaries. Full transcripts were provided for clarification. Overall, when judging 59 teaching methods he agreed with the researcher on 57 methods validating the coding with a 96% interrater reliability.

**Teaching methods.** When describing teaching methods for a lesson they were least confident teaching, the most common methods included instructing "how," offering definitions, demonstrating, offering a rule, relating to the world, and using manipulatives to model. The most common teaching methods described for a lesson of most confidence included using manipulatives to explore, students explaining solutions, pictorial representations, encouraging multiple strategies or answers, connections to the world, teachers facilitating group work, and using mathematical symbols. Table 4.16 shows quotes from the interview participants about their teaching methods for their most and least confident topic. Additionally, this table shows the overall orientation of the teacher for each topic.

| ID | Least Confident Topic C/P | Quote | Most Confident Topic C/P | Quote |
|---|---|---|---|---|
| 1 | C | Measurement: "This is my weak area! I have used liquid measure containers and had the kids pour the smaller ones into the larger ones and record how many it took." | C | Graphing: "I have the kids collect data of some sort, organize, analyze, and display that data with their small groups." |
| 7 | P | Probability: "I honestly don't remember learning about probability until I had to teach it. Flipping a coin you have a 50-50 chance but there was never more to it." | C | Proportionality: "I told a story…they had never used proportions before but they had to come up with some way to solve the problem…they had to record their plan and share it…the teacher questioning is very important." |
| 18 | P | Quadrilateral properties: "I give them the term, define it, and they go find one in everyday life." | C | Multiplication: "They don't understand the concept of multiplication. If they get the concept then they get the facts. |
| 23 | P | Measurement: "I start at the overhead projector. I am the subject and they have to tell me what to do. They have rulers at their seat." | C | Multiplication: "To introduce multiplication I do the circles and stars Marilyn Burns activity….they roll dice…write their two numbers and write for example 5 x 3 = and 3 x 5 =. Then they make stars and put 5 stars in groups of 3 circled and 3 stars in groups of 5 circled." |
| 24 | P | Geometry: "I start with the terminology then we do it with straws and twist ties. It makes for tough crowd control but it brings it home to the ones who need help seeing the abstract things. I never liked it as a kid either. That is probably why I don't like teaching it." | C | Probability: "I would start with a lot of questioning and finding their basis of knowledge. I brought out all the terms and then we would have a discussion." |

Continued

Table 4.16: Interviewee quotes about two mathematics topics.

Table 4.16 Continued

| | | | | |
|---|---|---|---|---|
| 28 | CP | Fractions: "I feel dumb asking other teachers or supervisors for help with the math." | P | Multiplication: "I introduce multiplication by modeling and they use manipulatives while I do it. I have to worry about where I supposed to be in the curriculum though." |
| 31 | P | Tables: "I instruct them how they work. This is the rule." Asked of interviewer, "Can you remind me when you multiply and when you add?" | C | Perimeter: "We explore paths around rectangles and discover different perimeter formulas." |
| 35 | P | Data Analysis: "I have them do a survey and decide which type of representation they would like to use. They display them and we discuss them so they learn about appropriate representation choices." | C | Algebra: "I don't feel confident teaching algebra topics because I believe I learned it differently. I know the answer but I have to explain it to them." |
| 36 | C | Solving Equations: "I use hands on equations. It is discovery based and helps them understand the concept. They share explanations, pictures, and symbols. There is a lot of guess and check." | P | Negative Exponents: "I start with place value. I explain examples and non-examples of how negative exponents work. I then offer the definition." |
| 45 | C | Metrics: "I tell the students that it is supposed to be a challenge. It may not be comfortable, and that's ok. That is when learning happens." | C | Multiplication: "We do a lot of visuals…how many groups of each shape, and what multiplication IS and WHY it is important. We do a lot of hands on with the introduction of multiplication. We manipulate items and document the groupings and purpose." |

Continued

Table 4.16 Continued

| | | | | |
|---|---|---|---|---|
| 48 | P | Order of Operations: "I don't feel I have as good of a grasp on how to present it. I just teach the steps. Oh, there should be a better way to teach this." | C | Shape Properties: "I have this great activity where the kids get a group of different shaped pattern blocks. They have to sort the shapes into groups by some criteria they choose. Then other groups come to see if they can figure out how they organized it. They come up with the properties. It is student centered." |
| 50 | P | Place Value: While looking at a packet her students are currently working on, "I do this packet. They have to circle groups of ten and write it as a whole number. Then they have to count by tens." | C | Subtraction with regrouping: "Students grasp money quicker than base ten blocks. They go to the bank and exchange a dime for ten pennies. Then they transfer it to paper. |
| 63 | C | Measurement: "In area and perimeter I would give them an assignment that would ask them to decorate a room. We paint the walls, count the desks and cabinets, and figure out where the blackboards are." | P | Algebra: "I typically teach them using the way that has been successful for most students in the past. Then someone usually steps up and shows us another way. I encourage that." |
| 64 | P | Algebra: "They did train us but I think there are a lot of misunderstandings in the standards and I'm not sure I'm interpreting it right. Our committee of 5 people couldn't even decide what they mean. I can't teach you if I don't know what it means." | C | Multiplication: "I start with the Marilyn Burns activity about boxes. How many ways can you make an area of 12 or 24. I let them experiment with centimeter paper and rectangles and we share our results." |
| 76 | P | Long division: "They come in being taught several methods and they don't know any well. We start back with multiplying whole numbers and it is all symbolic at that point." | C | Balancing Equations: "I do hands-on equations first. Even the lowest kids can do that. We take them through 2 or 3 levels." |

Continued

Table 4.16 Continued

| | | | | |
|---|---|---|---|---|
| 77 | P | Long division: "I don't think they are able to handle a traditional algorithm so they use lattice or partial products because it is easier." "I did a division page today with decimals. We are building them up from past experiences. We always estimate the answer first." | C | Balancing Equations: "Using hands-on equations goes from pieces to pictures to numbers on paper with inverse operations and a variable." |

**Summative frequencies of teaching methods.** The interview participants' teaching methods can be found in Table 4.17. In this table the methods have been sorted and tagged as procedurally oriented teaching or conceptually oriented teaching and categorized by teachers' most or least confident topic to teach. Counts are summative for all 16 interview participants. For example in the first row of the chart, eight interview responses described a least confident topic using teaching methods which explained "how to" while two interview responses described a most confident topic. No interview responses described a least confident topic using teaching methods which explained "why" or asked "why not" while two interview responses described a most confident topic.

| Procedurally Oriented Teaching | | | Conceptually Oriented Teaching | | |
|---|---|---|---|---|---|
| **Teaching Practice** | **Topic Confidence** | | **Teaching Practice** | **Topic Confidence** | |
| | **Least** | **Most** | | **Least** | **Most** |
| **Reasoning/Communication** | | | | | |
| Explaining "how to". | 8 | 2 | Explaining "why"; Asking "why not". | 0 | 2 |
| **Content** | | | | | |
| Teaching definitions and symbols; Skill drill. | 9 | 7 | Posing situations which explore mathematics; Building concept. | 3 | 5 |
| **Algorithms** | | | | | |
| Learning steps or practicing. | 6 | 4 | Discovering or explaining steps to algorithms. | 0 | 2 |
| **Calculators/Technology** | | | | | |
| Learning how to use technology to perform mathematical skills. | | | Using technology for problem solving or explorations. | 0 | 2 |
| **Solution Strategies** | | | | | |
| There is "one right way" to solve a problem. The right answer is most important. | 1 | | Multiple strategies to get to a solution. Problem solving process is valued over the solution. Flexible reasoning. | 2 | 6 |
| **Speed and Accuracy** | | | | | |
| Statement that "speed" and "accuracy" are the goal. | | | Understanding of fewer problems emphasized. | | |
| **Connectivity** | | | | | |
| Focusing on a single skill with limited connection to related concepts, skills or applications; isolated skills; Teaching definitions; Sequential. | 1 | | Connecting ideas or concepts in math; Connecting math to other academic subjects or the real world; Intertwined. | 5 | 8 |
| **Problem Solving** | | | | | |
| Selecting word problems to model the current skill; Following steps to solve a word problem. | | 1 | Posing a real problem. Creative problem solving. | | |

Continued

Table 4.17: Frequencies of procedural and conceptual teaching methods reported by least and most confident topic and grouped by type of teaching practice.

Table 4.17 continued

| Procedurally Oriented Teaching | | | Conceptually Oriented Teaching | | |
|---|---|---|---|---|---|
| **Teaching Practice** | **Topic Confidence** | | **Teaching Practice** | **Topic Confidence** | |
| | **Least** | **Most** | | **Least** | **Most** |
| | | | **Representation** | | |
| Algorithm is most important. | 1 | 1 | Multiple representations; Proof or reasoning emphasized; Encouraging verbal or written explanations. | 1 | 6 |
| | | | **Correcting Wrong Answers** | | |
| Wrong answers not acknowledged as a rich source for discourse. | | | Wrong answers lead to discourse. | | |
| | | | **Questioning** | | |
| Mostly lower order questioning; Ask students to recite/repeat. | 1 | | Frequent use of higher order questioning; Ask students to compare and contrast. | 1 | 1 |
| | | | **Role of Teacher** | | |
| Teacher demonstrates. | 4 | 1 | Teacher facilitates. | 3 | 10 |
| | | | **Assessment** | | |
| Computational problems only. Students list steps. | | | Open ended questions. Students explain thoughts. | | |
| | | | **Manipulatives** | | |
| Using manipulatives to model a problem. | 4 | 3 | Using manipulatives to explore a concept. | 1 | 5 |
| | | | **Types of Problems** | | |
| Focuses on different "types" of problems or "cases". | | | Focuses on problems to encourage thinking or student created problems. | 1 | |

The conceptually and procedurally oriented teaching methods were counted according to their least and most confident topic to teach. As seen in Table 4.18, the 16 interview participants used 2 ½ times as many conceptual methods as procedural methods on their most confident topics but only 2/3 as many on their least confident topic.

| | Least Confident Topic | Most Confident Topic |
|---|---|---|
| Conceptual Approaches | 22 | 57 |
| Procedural Approaches | 36 | 22 |

Table 4.18: Frequency of procedural or conceptual teaching methods reported by least and most confident topic.

**Chi-square tests:** A chi-square test for goodness of fit was calculated comparing conceptually and procedurally oriented teaching methods by most and least confident topic. To calculate the chi-square ($\chi^2$) statistic rows and columns were summed for the data in Table 4.18 and a grand total of teaching methods was found. For each cell in the table the expected number of outcomes was calculated by multiplying the row total by the column total and dividing it by the grand total. Then for each cell the square of the actual outcome subtracted from the expected outcome was divided by the expected outcome. These values were summed to result in a chi-square value of 16.04. For a 2 × 2 table the degree of freedom is 1 so the critical value at $p < .005$ is 7.88 (Gravetter & Wallnau, 2004, p. 699) or $\chi^2 = 16.04$. Therefore the results are significant ($p < .005$), showing a significant relationship between mathematics teaching self-efficacy and procedurally or conceptually oriented teaching methods for total occurrences of teaching methods by interviewed teachers.

A chi-square statistic was also calculated for interview participants' total number of teaching methods grouped by levels of mathematics self-efficacy (see Table 4.19) and mathematics teaching self-efficacy (see Table 4.20). Low and medium self-efficacy teachers were combined into one category to balance the number of teachers in each category. When comparing mathematics self-efficacy level with teaching method type, $\chi^2$ equaled 0.4, and when comparing mathematics teaching self-efficacy level with teaching method type, $\chi^2$ equaled 0.7. Neither of these statistics is significant, and they show that the self-efficacy levels are not as closely associated with the teaching method type for the entire group as they are within individual teacher, according to the confidence level of a topic, as will be shown below.

| Total Teaching Methods | Mathematics Self-Efficacy Level | |
|---|---|---|
| Orientation | Low or Medium | High |
| Conceptual | 44 | 35 |
| Procedural | 29 | 29 |

Table 4.19: Interviewee Frequency of Teaching Methods grouped by Mathematics Self-Efficacy Level and Teaching Orientation

| Total Teaching Methods | Mathematics Teaching Self-Efficacy Level | |
|---|---|---|
| Orientation | Low or Medium | High |
| Conceptual | 39 | 41 |
| Procedural | 24 | 34 |

Table 4.20: Interviewee Frequency of Teaching Methods grouped by Mathematics Teaching Self-Efficacy Level and Teaching Orientation

**Teaching orientation.** Each interviewed teacher's orientation toward procedural or conceptual teaching was judged by comparing the number of procedural teaching methods with the number of conceptual teaching methods for each topic. Each teacher was given a conceptual orientation ratio for both her least and most confident topic by reporting the number of conceptual methods over the total number of methods. A procedural orientation ratio could have been found instead, but since either one is a function of the other, it is not necessary to report both. Ratios greater than 50% were considered conceptually oriented and ratios less than 50% were considered procedurally oriented. One teacher reported an equal number of procedurally and conceptually oriented teaching methods for her least confident topic and was considered mixed.

For example, a teacher who reported two conceptual methods and four procedural methods for her least confident topic would be considered procedurally oriented for her least confident topic, reported as a ratio of 2/6 conceptually oriented, or 33%. If she also reported six conceptual methods and no procedural methods for her most confident topic then she would be considered conceptually oriented for her most confident topic, reported as a ratio of 6/6 conceptually oriented, or 100%. The individual conceptual orientation ratios converted to percents are shown in Table 4.21.

Of the 16 interview participants, 11 of them introduced their least confident topic procedurally and their most confident topic conceptually. Two teachers were opposite and introduced their least confident topic conceptually and their most confident topic procedurally. Two teachers discussed teaching conceptually for both topics and one teacher discussed an equal number of conceptual and procedural methods for her least confident topic and procedural methods for her most confident topic.

| ID | Least Confident Topic | | Most Confident Topic | |
|---|---|---|---|---|
| | Ratio | Orientation | Ratio | Orientation |
| 1 | 0% | P | 100% | C |
| 7 | 0% | P | 67% | C |
| 18 | 33% | P | 67% | C |
| 23 | 50% | CP | 0% | P |
| 24 | 20% | P | 57% | C |
| 28 | 33% | P | 100% | C |
| 31 | 100% | C | 100% | C |
| 35 | 0% | P | 100% | C |
| 36 | 33% | P | 75% | C |
| 45 | 25% | P | 60% | C |
| 48 | 25% | P | 60% | C |
| 50 | 33% | P | 100% | C |
| 63 | 100% | C | 0% | P |
| 64 | 100% | C | 0% | P |
| 76 | 0% | P | 100% | C |
| 77 | 100% | C | 100% | C |

Table 4.21: Conceptual orientation ratios and teaching orientation for interviewed teachers (n = 16).

Summative results for the group of 16 interview participants are shown in Table 4.22. A chi-square statistic was calculated for the non-mixed teachers in the table below, yielding a value of 9.3, a significant association at $p < .005$ for one degree of freedom.

| Teaching Orientation | Least Confident Topic | Most Confident Topic |
|---|---|---|
| Conceptual | 4 | 13 |
| Mixed | 1 | 0 |
| Procedural | 11 | 3 |

Table 4.22: Frequencies of teaching orientation by least or most confident teaching topic (n=16).

Interview participants Katy and Sally were representative examples of the sample of elementary teachers in the current study. Interview transcripts were summarized in the anecdotes below as well as in the interview summaries found in Tables 4.14 and 4.15. Following are anecdotes illustrating the findings of the current study.

**Sally.** Sally (#31) has been an elementary teacher for 30 years in a suburban school district. She states that she is "very comfortable with math concepts and teaching math up through fourth grade and then it gets scary". She adds that she is more confident with her mathematics abilities in the classroom than outside school. Through the NCTM Standards reform she has embraced change by accepting the challenge of teaching mathematics in ways that encourage students to work in groups on a mathematics problem and to write about mathematical thinking. Sally has been a teacher leader and enjoys offering workshops to share her mathematics teaching methods with other teachers. Sally described lessons involving measurement of perimeter, her most confident topic, and algebraic rule tables, her least confident topic.

When describing a perimeter lesson, she focused on how students should explore paths around various geometric shapes to understand the meaning of perimeter. She says, "As they explore various paths, students discover perimeter patterns for various shapes and many even come up with formulas to propose to the class." She is pleased that some of her third grade students end up using the formulas and others prefer adding the path lengths because perimeter has meaning to them. She exemplified how a teacher can develop conceptual understanding of the topic of perimeter amongst third graders. Her conceptually oriented teaching methods included focusing on the big concept of perimeter, using manipulatives to explore the concept, making conjectures about various shapes, and encouraging students to develop formulas from their thinking when they were ready.

Her discussion of algebraic rule tables was much different. She stated that she would help the student set up the table and then ask them to look for the rule. See examples in Figure 4.3.

| x | y |
|---|---|
| 1 | 4 |
| 2 | 6 |
| 3 | 8 |
| 4 | 10 |
| 5 | 12 |

| x | y |
|---|---|
| 1 | 1 |
| 2 | 2 |
| 3 | 4 |
| 4 | 8 |
| 5 | 16 |

Figure 4.3: Sally's algebraic rule tables

She asked me, during our interview, “What is the trick for finding the pattern?” She added, “When do you multiply and when do you add?” She stated her frustration in trying to teach this because she never remembered those rules. After more questioning it became evident that she never would consider presenting a table with domain values out of sequential order because it would “mess up” the pattern. Her focus when teaching these tables was to tell students a rule to memorize to help them figure out a particular problem’s pattern. She admitted wanting this to be a short unit because students just needed to know the rules of when to add and when to multiply. Sally’s approach to teaching algebraic rule tables was oriented toward procedural teaching. Sally rated as low on the survey in both mathematics and mathematics self-efficacy. During the interview her mathematics self-efficacy still rated low but her mathematics teaching self-efficacy rated in the medium level among third grade teachers. Sally clearly stated that she would not know how to teach a higher grade level of mathematics. Her low mathematics self-efficacy made her question her own understanding of the topic of algebra rule tables, and her low mathematics teaching self-efficacy toward this topic was evident by her use of procedurally oriented teaching.

**Katy.** On the other hand, Katy was rated as a teacher of high mathematics self-efficacy and high mathematics teaching self-efficacy on the survey. She is a teacher with 21 years of experience in a large urban school district. Katy believed she had tried a wide variety of mathematics teaching methods while teaching a variety of grades in elementary school. During our interview Katy (#48) was asked to describe the teaching methods she used to teach an introductory lesson on geometric shape properties and an introductory lesson on order of operations. Earlier, Katy indicated on her survey that geometric shape properties was her most confident teaching area and order of operations was her least confident teaching area. When asked to describe how she would teach an introductory lesson on each concept during the interview, she laughed and immediately responded that her lesson on geometric shapes would involve "many more concrete representations starting by exploring with manipulatives" and the order of operations lesson would be "more abstract" because she does not feel she had "as good of a grasp on how to present it."

Katy described a geometry lesson where students were presented with a wide variety of shape blocks. She would ask the children in groups to sort the shapes by any one characteristic they chose. After each group had chosen an attribute and sorted the shape blocks, the groups would then rotate to see each other group's arrangement to see if they could determine which attribute the group had used to sort. Next, she described how students would discuss the various attributes of each shape in order to discover the properties of each shape. As described by Katy, this lesson highlights conceptually oriented teaching. Her focus was the big idea of shape properties which are discovered by the students and solidified by encouraging summarization of properties by shape.

In answering the question about her teaching methods for an order of operations lesson, Katy immediately began to assess her own teaching methods. She stated that she would tell students the order of operations and let them practice. Next, she asked herself whether or not she could teach that lesson by discovery. She asked who decided that this order should be the right one. She said she would think about how she could come up with a connection that would help make it make sense. Katy was searching for a connection to mathematics or the real world which would help her make this lesson more meaningful for her students. She was uncomfortable that her students were being asked to memorize the order of operations. Procedural teaching includes Katy's methods of memorization and drill. She vocalized the desire to teach more conceptually than procedurally because she believed her students would understand the topic better. Katy rated high in both mathematics self-efficacy and mathematics teaching self-efficacy on the survey.

**Katy and Sally.** Both Sally and Katy took a sharp turn in their enthusiasm and tone when discussing their least and most confident topic. These teachers were comfortable reporting that they were strong teachers on their most confident subject and not nearly as strong on their least confident topic. This was the norm rather than the exception in the 16 interviews. The chosen teaching methods seemed to differ as drastically as the level of confidence. The interviewed teachers reverted to procedural methods when unconfident introducing a lesson and conceptual methods when confident introducing a lesson. Sally indicated, during her conceptual teaching of measurement of perimeter, that inventing formulas for perimeter was the end goal, yet when teaching her procedural lesson on algebraic tables the end goal was to learn a rule to get the right answer. Katy's end goal for her conceptual lesson on properties of geometric shapes was to discover relationships and make comparisons between shapes while her end goal for her procedural lesson on using the order of operations, part of the algebra strand, was for her students to memorize the order of operations and be able to calculate accurately. During Katy's interview, however, discussing her teaching methods made her come to realize the difference in her approaches. Likely due to her high mathematics teaching self-efficacy, she left the interview striving for a better way to teach the order of operations.

Katy and Sally both were procedurally oriented in their teaching methods for their least confident mathematics topic and conceptually oriented in their most confident teaching topic. While similar in their conceptual approach toward their most confident topic, Katy and Sally differed in their focus during the description of their least confident topic. Sally's focus was on understanding the mathematics for herself while Katy's focus was on finding a better teaching method for her students. Thus, Sally was focusing on her mathematics self-efficacy and Katy was confident in her mathematics self-efficacy but was focusing on her mathematics teaching self-efficacy. This suggests that since Katy has high mathematics self-efficacy, she was able to focus on her methods more and thus mathematics self-efficacy may be a necessary prerequisite for high mathematics teaching self-efficacy.

## Summary

In conclusion, the findings of the current study are numerous with many new research avenues to explore as a result. First, the MTMSE survey has been validated as a reliable tool for comparing mathematics self-efficacy and mathematics teaching self-efficacy. Additionally this study offers the Conceptually and Procedurally Oriented Teaching Methods Frequency Chart to gather quantitative data from interviews as one way to analyze conceptually and procedurally oriented teaching.

Mathematics self-efficacy and mathematics teaching self-efficacy had a strong correlation in this study and teacher mathematics self-efficacy was typically higher than mathematics teaching self-efficacy. This study also suggests mathematics self-efficacy may be a prerequisite for mathematics teaching self-efficacy, a relationship that should be explored further in the future. It was also found that teachers are willing to freely admit their preferences and confidence levels in discussing mathematics and mathematics teaching self-efficacy overall and toward particular topics.

Finally, there are differences within each teacher that affect her mathematics teaching self-efficacy as portrayed by the changing orientation toward teaching different mathematics topics. Teachers tend to teach conceptually the mathematics topic they are most confident teaching and procedurally the mathematics topic they are least confident teaching. This disparity suggests a relationship between mathematics teaching self-efficacy and teaching methods regardless of the overall mathematics or mathematics teaching self-efficacy levels. This conclusion suggests that when analyzing particular mathematics topics taught by individual teachers, elementary teachers tend to use conceptually oriented methods to introduce topics in which they have higher mathematics teaching self-efficacy and procedurally oriented methods to introduce topics of lower mathematics teaching self-efficacy.

# CHAPTER 5

# DISCUSSION

The purpose of the current study was to explore relationships between the constructs of mathematics self-efficacy, mathematics teaching self-efficacy, and procedurally or conceptually oriented teaching practices among practicing elementary teachers. In doing so the Mathematics Teaching and Mathematics Self-Efficacy (MTMSE) survey, as seen in Appendix A, and the Conceptually and Procedurally Oriented Teaching Interview Frequency Chart, as seen in Table 3.7, were designed, tested, and implemented. Seventy-five (75) practicing elementary teachers participated in the MTMSE survey and 16 participated in follow-up interviews. This chapter summarizes the study, its findings, connections to current mathematics education literature, and implications for future researchers and teachers.

## Research Questions Revisited

Three research questions were asked during the study. First, how did mathematics self-efficacy relate to mathematics teaching self-efficacy? Findings indicate a strong relationship between these two variables. Second and third, how did elementary teachers' mathematics self-efficacy relate to their tendency to teach conceptually or procedurally, and how did elementary teachers' mathematics teaching self-efficacy relate to their tendency to teach conceptually or procedurally? The answer to these questions resulted in a moderate relationship. But, the findings indicated that mathematics self-efficacy may be an important precursor to mathematics teaching self-efficacy, and mathematics teaching self-efficacy varies tremendously by mathematics topic within each teacher. Teachers tended to introduce topics procedurally when mathematics teaching or mathematics self-efficacy was low toward the specific mathematics topic and tended to introduce topics conceptually when mathematics teaching or mathematics self-efficacy was high toward the specific mathematics topic.

**Conjectures.** The author stated three conjectures related to the study. First, elementary teachers can be positioned in one of the nine categories in Figure 1.1 depending on the topic. In other words in addition to teachers who have both high mathematics and mathematics teaching self-efficacy or low mathematics and mathematics teaching self-efficacy, there are also teachers who are high in mathematics self-efficacy and low in mathematics teaching self-efficacy and those who are low in mathematics self-efficacy and high in mathematics teaching self-efficacy. This study did not support this conjecture, as only one teacher fell into the category of low mathematics self-efficacy and high mathematics teaching self-efficacy and no teachers fell into the opposite category. This trend supports a strong relationship between the two self-efficacy variables. With the exception of the one teacher with low mathematics self-efficacy and high mathematics teaching self-efficacy, results suggested that mathematics self-efficacy may generally be a prerequisite for mathematics teaching self-efficacy, echoing Ma's (1999) emphasis on elementary teachers developing a profound understanding of fundamental mathematics.

Overwhelmingly, participants in the study freely expressed their confidence or lack of confidence related to mathematics self-efficacy and mathematics teaching self-efficacy. Additionally, participants freely named a favorite and least favorite mathematics topic to teach on both the survey and during interviews. This openness suggests that teachers were aware of their mathematics self-efficacy and mathematics teaching self-efficacy for specific mathematics topics.

The second conjecture was that mathematics self-efficacy and mathematics teaching self-efficacy, as well as conceptual and procedural teaching methods, are not static but vary over time and by mathematical topic. The study suggested that all variables of interest varied by content topic. Variability by time for self-efficacies indicated a higher correlation between mathematics and mathematics teaching self-efficacy as years of teaching experience increased. The design of the study did not include a measure of variability of time for conceptual and procedural teaching methods. Teachers, regardless of their mathematics or mathematics teaching self-efficacy, tended to conceptually introduce topics they were highly confident teaching and tended to procedurally introduce topics they were least confident teaching. Each interviewed teacher exhibited this contrast in teaching methods.

The final conjecture was that teachers with a lower mathematics self-efficacy and a lower mathematics teaching self-efficacy teach mainly procedurally and those who have a higher mathematics self-efficacy and a higher mathematics teaching self-efficacy mainly teach using methods that are more conceptually oriented or which attempt to balance conceptual understanding with procedural fluency. The relationship between high self-efficacy and more conceptually oriented teaching was found to be positive, in comparing methods used to teach the most and least self-efficacious mathematics teaching topics.

## Model Revisited

The study model proposed in Figure 1.2 has been modified slightly to reflect the findings of the study. The entire model must be downsized to reflect specific mathematics topics as self-efficacy and teaching methods vary by topic rather than varying overall by individual. The mathematics topic is the moderator of the situation. This is noted as the title of the model changed from "mathematics" to "specific mathematics topic". Additionally, the strong line from mathematics teaching to mathematics teaching self-efficacy has been added to reflect the importance of mathematics self-efficacy over mathematics teaching self-efficacy. The finalized study model is shown in Figure 5.1.

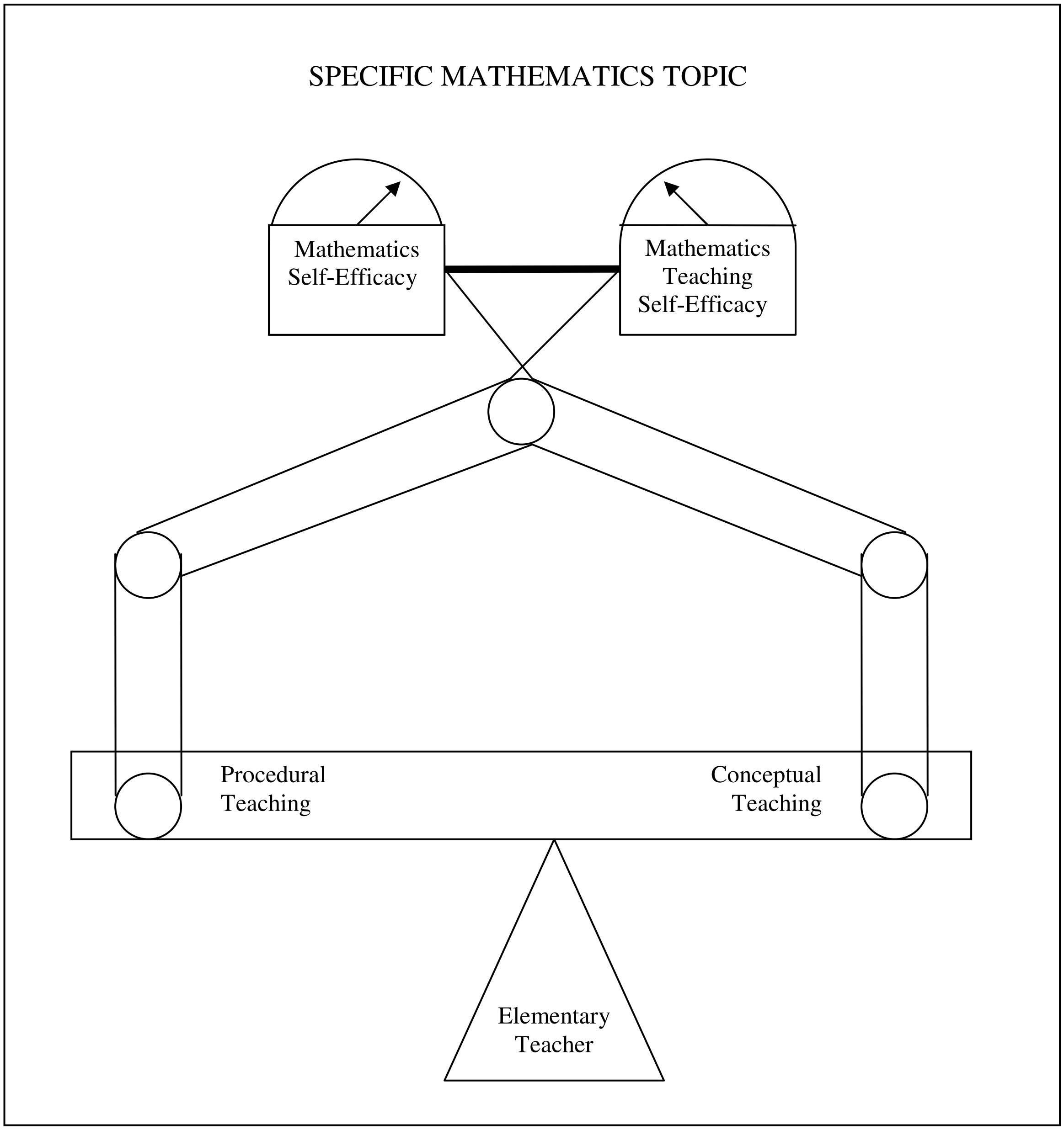


Figure 5.1: Diagram supporting the relationship between teacher mathematics self-efficacy and mathematics teaching self-efficacy with procedural and conceptual teaching

## Self-Efficacy and Teaching Practices Connected to Literature

This section relates existing mathematics education literature to the current study. Part of the theoretical framework of the current study was a specific piece of Bandura's social cognitive theory, self-efficacy. Bandura (1994, 1997b) noted four sources of influence on a person's self-efficacy: (1) mastery experiences, (2) vicarious experiences provided by social models, (3) social persuasion, and (4) stress reduction. Lent, Lopez, Brown, and Gore (1996) indicated that vicarious learning is less important to mathematics self-efficacy than other types of experience. Mastery experiences may include a teacher developing high mathematics self-efficacy and thus becoming ready to display high mathematics teaching self-efficacy. Both vicarious experiences and social persuasion may support mathematics and mathematics teaching self-efficacy as the teacher participants were involved in summer workshops to enhance their elementary mathematics teaching. The first three sources of influence may certainly cause stress reduction which in turn raises self-efficacy.

Ashton and Webb (1986) stated that low efficacy teachers do not go much beyond skill and drill type teaching methods. Skill and drill methods fall into the category of procedurally oriented teaching. This was evident in the current study as most teachers tended to use more procedurally oriented teaching methods on the mathematics topic of least confidence or lowest mathematics teaching self-efficacy.

Hiebert and Carpenter (1992) emphasize both conceptual and procedural understanding rather than argue that one is superior. (Ma, 1999) affirms the dual emphasis by suggesting that elementary mathematics teachers instruct children to know how and know why. This philosophy, also believed by the researcher, supports the entire teaching and learning of a topic. In the current study the interviews focused primarily on the introduction of two mathematics topics and the specific teaching methods used. Should a longitudinal study of teachers be conducted using the same variables, results may show how elementary teachers will use both types of methods over an extended time as they help their students learn a particular topic.

A study of early childhood teachers conducted by Brown (2005) indicated that teacher efficacy and teacher mathematics beliefs are not significantly related to observations of mathematical instructional practices. However, the current study, with its focus on specific topics in mathematics for each interview teacher revealed a different message from the interviews. Overall, self-efficacy was positively related to conceptually oriented teaching and not to procedurally oriented teaching, but when looking at how individual teachers introduced one specific mathematics topic for which they had high mathematics teaching self-efficacy, teachers tended to be conceptually oriented. When that same individual teacher introduced one specific mathematics topic for which they had low mathematics teaching self-efficacy, they tended to be procedurally oriented in their initial choice of teaching methods. It became evident through interviews that multiple factors affect teachers' choices of teaching methods for mathematical topics, as also reported by Raymond (1997). Additionally multiple factors likely affect a teacher's mathematics self-efficacy and mathematics teaching self-efficacy as these are not the only characteristics of any teacher.

## Limitations

The researcher finds a few limitations of the study. First, the instrument used to survey teachers about their current mathematical instructional materials is a limitation since, although it is broad, it certainly is not comprehensive.

Second, the research consists of self-reported data on teaching methods, which may seem optimistic. Pajares (1992) suggested that researchers would have richer data when using interviews and classroom observations to examine teacher efficacy rather than relying solely on self-reports. One interview per teacher was conducted which gave the researcher only a snapshot of the understanding of the teachers' beliefs and practices.

Third, interviews in the study were not supported by classroom observations of teachers. Other significant studies examining self-efficacy through surveys and interviews but not observations, include Hoy and Spero (2005), Huinker and Madison (1997), and Ma (1999). While potentially valuable, the researcher believes that classroom observations also serve as only a snapshot of the classroom teacher's teaching style and differences found on a daily basis may reflect more situational differences than simply differences related to the variables in the study. To conduct classroom observations of the teachers, each teacher would have to be observed on multiple occasions. As evident in the pilot study and in the research study, teachers were very candid about their teaching preferences and state only that with which they have direct experience. Observations were not conducted as part of this study but likely will be a future extension of this study. Observations will help further triangulate the survey and interview results to verify or contradict what the teachers self-reported.

Finally, the participants may not be a representative sample of the population. Teachers reflected practice in suburban, urban, and rural districts and have a broad range of teaching experience. Yet, their common thread was participation in the summer workshop which indicates they had some need or desire to obtain a mathematics teaching professional development workshop at the current time. Current topics addressed in workshop may have skewed results since teachers may have had recent exposure to a new teaching idea that sparks the teachers' interest. The workshop may have increased self-efficacy by providing both vicarious experiences offered by course leaders and social persuasion offered by colleagues also in the course (Bandura, 1994, 1997b). Additionally, more than half of the teachers interviewed indicated they use the Everyday Mathematics curriculum (University of Chicago School Mathematics Project, 2003), which is a similarity among those teachers. It is assumed, however, that nearly all elementary teachers have been exposed to some mathematics curriculum that features NCTM standards-based teaching. The researcher believes that with any sample some characteristics may be consistent so by using an adequately large number of participants these commonalities may be insignificant.

## Delimitations

This study has been narrowed to focus on the instructional methods chosen by third through sixth grade teachers based on their mathematics and mathematics teaching self-efficacy. Mathematics self-efficacy and mathematics teaching self-efficacy of other grade level mathematics teachers have not been considered. To narrow the study actual mathematical content knowledge has not been assessed as emphasized in Ball (1990a, 1990b), Shulman (1986), and Ma (1999). Considering a teacher's prior mathematical content knowledge should be a future component of this research since this study suggested that high mathematics self-efficacy may be one important prerequisite for high mathematics teaching self-efficacy. Many assumptions have been made prior to this study, such as assuming that previous research sufficiently covers the effect of mathematics teaching self-efficacy on student achievement as seen in Ashton and Webb (1986) and Ma and Kishor (1997). The impact of the mathematics self-efficacy of the parents and the school community, although an important component of the development of student mathematics self-efficacy, was also not considered in this study.

## Implication for Teachers

This research should remind teachers that their mathematics self-efficacy and mathematics teaching self-efficacy levels may influence mathematics instructional choices in the classroom. There are too many teachers who are low in either mathematics self-efficacy or mathematics teaching self-efficacy in elementary classrooms. Those in this situation need to embrace professional development opportunities in the area of mathematics education as these opportunities enhance beliefs about mathematics (Lee, 2007) and increase mathematics self-efficacy and mathematics teaching self-efficacy. All children deserve a mathematics teacher who is comfortable with the mathematics s/he is teaching.

For teachers this research should come as an eye-opener that instructional choices such as the delivery method for mathematics instruction support a teacher’s emphasis on either concepts or procedures. Teachers are very aware of their own self-efficacy related to specific mathematics topics and mathematics teaching strategies. A teacher with low self-efficacy toward the order of operations, for example such as Katy, should recognize her emphasis on procedures for this topic and work strive to better understand the mathematics topic which will help her to find teaching methods that teach the concepts of the order of operations. Conversely, a teacher who recognizes a strength in her teaching, such as Sally's lesson on perimeter, should evaluate her conceptually oriented lesson to ensure that the students not only understand the concept but are comfortable with procedures as well.

## Implications for Teacher Education

Teacher educators should take from this research the influence that mathematics and mathematics teaching self-efficacy have on teaching practices. Current emphasis in mathematics education is for elementary teachers to teach mathematics conceptually to elementary students with procedural fluency developing as a result of conceptual understanding. This research implies that if we want teachers to teach conceptually then we need teachers who have a strong mathematics and mathematics teaching self-efficacy. It is not enough to understand mathematics but at the same time not like teaching mathematics. This research shows that mathematics teaching self-efficacy is the stronger influence of conceptually oriented teaching. In teacher education courses, it is not enough to show conceptual approaches to elementary teachers that they may repeat in their classrooms. Instead teacher educators need to help future teachers first develop the conceptual understanding of the mathematics concept and develop their own procedural fluency. This foundation in mathematics self-efficacy, coupled with the instruction in conceptual teaching strategies, can lead to the mathematics teaching self-efficacy that may promote more use of conceptual approaches to teaching mathematics.

## Implications for School Administrators

Administrators need to be careful not to assign teachers to mathematics classes who have low mathematics or mathematics teaching self-efficacy. Ingersoll (1996) found that "About one-quarter of all public school students enrolled in mathematics classes in grades 7-12, or about 4,124,000 of 15,510,000 students, were taught by teachers without at least a minor in mathematics or mathematics education" (p. x). Ingersoll determined that this problem was a result of a mismatch between teachers' fields of training and their teaching assignment rather than a lack of training. American students deserve teachers who are confident teaching the topics in their classroom.

## Implications for Future Research

Literature suggests the "need for more research to explore what other personal or external factors relate to mathematics instructional practices" (Brown, 2005, p. 239). The current study offers a quantitative tool for assessing reported instances of conceptually or procedurally oriented teaching. Assessment methods such as part 5 of the MTMSE (see Appendix A) and the probing questions in interview question 10 (see Appendix B) need to be further tested for accuracy in determining teaching orientation. Furthermore the Conceptually and Procedurally Oriented Teaching Method Frequency Chart has been developed from mathematics education literature, validated, and tested for interrater reliability. This frequency chart is ready to be used in other research studies to confirm its reliability and to help answer questions about these teaching methods. Research based on this and other tools evaluating conceptually or procedurally oriented teaching should be conducted to further explore teachers' tendencies toward these methods and how they relate to a plethora of other variables.

Additional research is needed to assist in clarifying the controversy over the relationship between conceptually and procedurally oriented teaching. President George W. Bush's National Mathematics Advisory Panel (U.S. Department of Education, 2008) suggested as one of its six elements of instructional improvement needed in mathematics education "the mutually reinforcing benefits of conceptual understanding, procedural fluency, and automatic recall of facts" (p. xiii). Literature (Baroody, Feil, and Johnson, 2007; Star, 2005, 2007) suggests that perhaps higher level mathematics is being taught more procedurally with concepts developing later, whereas the current study suggests elementary mathematics is being introduced, at least by teachers with higher mathematics teaching self-efficacy, more conceptually with procedural facility developing later. There is a need for future studies which examine levels of procedural and conceptual teaching and the effect procedural and conceptual teaching approaches has on student learning.

Furthermore, mathematics and mathematics self-efficacy can be further studied to examine their relationships with student achievement and teacher content knowledge. Both connections will aid in linking self-efficacy of teachers with student learning. These studies stem from the instruments and design developed in the current study or could be expanded through studies involving observations, or multiple interviews, or studies investigating student work samples, teacher lesson plans, or presentation of topics in textbooks.

Future studies could replicate this study at other grade levels such as preschool, middle school, high school, or even the college level. Above are only a few of the many important directions for future research which could follow the current study.

## Conclusion

In closing, this dissertation study has contributed to the world of mathematics education research results that are useful to classroom teachers, teacher educators, administrators, and educational researchers. The findings are an eye-opener to elementary teachers that, no matter how high their overall mathematics or mathematics teaching self-efficacy, their teaching practices are varied within each individual teacher based on his/her highest or lowest topic of mathematics teaching self-efficacy. Each teacher strives to find the best way to present topics to students, and knowing that each teacher has areas that are weaker than others, which by this study are shown to be taught differently, teachers need to embrace their weaknesses through personal study, professional development, or working with other teachers. By understanding that high mathematics self-efficacy seems necessary before high mathematics teaching self-efficacy, and mathematics teaching self-efficacy affects choices of teaching methods, the individual teacher has control over improving the lessons presented in his/her classroom. The teacher educator not only has a significant influence on future teachers to help them understand these relationships, but also can improve teacher preparation programs to insure the highest possible mathematics self-efficacy in prospective teachers.

Finally, as the American phobia (Burns, 1989), mathematics is a topic that our students and teachers have been shown to have varying degrees of self-efficacy. As mathematics educators, our challenge remains to raise student and teacher self-efficacy and achievement through our own choices of teaching methods, and this choice is affected by our own mathematics teaching self-efficacy.

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## APPENDIX A

## Mathematics Teaching and Mathematics Self-Efficacy (MTMSE) Scale

TEACHERS BELIEFS AND PERCEPTIONS ABOUT MATHEMATICS TEACHING

This survey will take approximately ten minutes to complete. Your opinions are very important to me. Thank you in advance for participating in this study.

DIRECTIONS

**Part 1:** Suppose that you were asked the following math questions in a multiple choice form. Please indicate how confident you are that you would give the correct answer to each question *without using a calculator*.

PLEASE DO NOT ATTEMPT TO SOLVE THESE PROBLEMS.

Survey Number ______________

| 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| *Not confident at all* | | | | | *Completely confident* |

1. In a certain triangle, the shortest side is 6 inches. The longest side is twice as long as the shortest side, and the third side is 3.4 inches shorter than the longest side. What is the sum of the three sides in inches? 1 2 3 4 5 6

2. ABOUT how many times larger than 614,360 is 30,668,000? 1 2 3 4 5 6

3. There are three numbers. The second is twice the first and the first is one-third of the other number. Their sum is 48. Find the largest number. 1 2 3 4 5 6

4. Five points are on a line. T is next to G. K is next to H. C is next to T. H is next to G. Determine the positions of the points along the line. 1 2 3 4 5 6

5. If y = 9 + x/5, find x when y = 10. 1 2 3 4 5 6

6. A baseball player got two hits for three times at bat. This could be represented by 2/3. Which decimal would most closely represent this? 1 2 3 4 5 6

7. If P = M + N, then which of the following will be true? 1 2 3 4 5 6
   a. N = P – M
   b. P – N = M
   c. N + M = P
   d. All of the above

8. Find the measure of the angle that the hands of a clock form at 8 o'clock. 1 2 3 4 5 6

9. Bridget buys a packet containing 9-cent and 13-cent stamps for $2.65. If there are 25 stamps in the packet, how many are 13-cent stamps? 1 2 3 4 5 6

10. On a certain map, 7/8 inch represents 200 miles. How far apart are two towns whose distance apart on the map is 3 ½ inches? 1 2 3 4 5 6

11. Fred's bill for some household supplies was $13.64. If he paid for the items with a $20 bill, how much change should he receive? 1 2 3 4 5 6

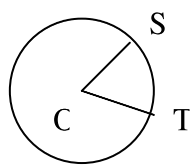
12. Some people suggest that the following formula be used to determine the average weight for boys between the ages of 1 and 7: W = 17 + 5A where W is the weight in pounds and A is the boy's age in years. According to this formula, for each year older a boy gets, should his weight become more or less, and by how much? 1 2 3 4 5 6

13. Five spelling tests are to be given to Mary's class. Each test has a value of 25 points. Mary's average for the first four tests is 15. What is the highest possible average she can have on all five tests? 1 2 3 4 5 6

14. $3\frac{4}{5} - \frac{1}{2} =$ ________. 1 2 3 4 5 6

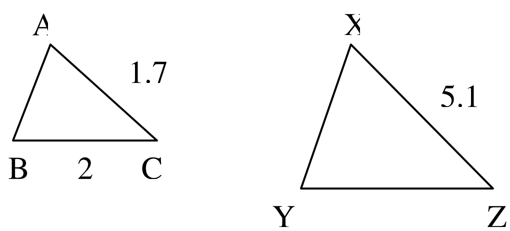
15. In an auditorium, the chairs are usually arranged so that there are x rows and y seats in a row. For a popular speaker, an extra row is added, and an extra seat is added to every row. Thus, there are x + 1 rows and y + 1 seats in each row. Write a mathematical expression to show how many people the new arrangement will hold. 1 2 3 4 5 6

16. A Ferris wheel measures 80 feet in circumference. The distance on the circle between two of the seats S and T, is 10 feet. See figure below. Find the measure in degrees of the central angle SOT whose rays support the two seats. 1 2 3 4 5 6




17. Write an expression for "six less than twice 4 5/6"? 1 2 3 4 5 6

18. The two triangles shown below are similar. Thus, the corresponding sides are proportional, and AC/BC = XZ/YZ. If AC = 1.7, BC = 2, and XZ = 5.1, find YZ. 1 2 3 4 5 6



**Part 2:** Directions: Please use the following scale to answer each question.

| 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| *Strongly Disagree* | | | | | *Strongly Agree* |

| | |
|---|---|
| 1. I will continually find better ways to teach mathematics. | 1 2 3 4 5 6 |
| 2. Even if I try very hard, I will not teach mathematics as well as I will most subjects. | 1 2 3 4 5 6 |
| 3. I know how to teach mathematics concepts effectively. | 1 2 3 4 5 6 |
| 4. I will not be very effective in monitoring mathematics activities. | 1 2 3 4 5 6 |
| 5. I will generally teach mathematics ineffectively. | 1 2 3 4 5 6 |
| 6. I understand mathematics concepts well enough to be effective in teaching elementary mathematics. | 1 2 3 4 5 6 |
| 7. I will find it difficult to use manipulatives to explain to students why mathematics works. | 1 2 3 4 5 6 |
| 8. I will typically be able to answer students' questions. | 1 2 3 4 5 6 |
| 9. I wonder if I will have the necessary skills to teach mathematics. | 1 2 3 4 5 6 |
| 10. Given a choice, I will not invite the principal to evaluate my mathematics teaching. | 1 2 3 4 5 6 |
| 11. When a student has difficulty understanding a mathematics concept, I will usually be at a loss as to how to help the student understand it better. | 1 2 3 4 5 6 |
| 12. When teaching mathematics, I will usually welcome student questions. | 1 2 3 4 5 6 |
| 13. I do not know what to do to turn students on to mathematics. | 1 2 3 4 5 6 |

**Part 3:** Directions: How much confidence do you have that you are able to successfully perform each of the following tasks?

Please use the following scale:

| 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| *Not confident at all* | | | | | *Completely confident* |

| | |
|---|---|
| 1. Add two large numbers in your head. | 1 2 3 4 5 6 |
| 2. Multiply quantities in a recipe to feed a larger group. | 1 2 3 4 5 6 |
| 3. Balance your checkbook. | 1 2 3 4 5 6 |
| 4. Figure out how long it will take to travel from City A to City B driving x mph. | 1 2 3 4 5 6 |
| 5. Understand a graph accompanying an article on business profits. | 1 2 3 4 5 6 |
| 6. Figure out how much you would save if there is a 15% markdown on an item you wish to buy. | 1 2 3 4 5 6 |
| 7. Estimate your grocery bill in your head as you pick up items. | 1 2 3 4 5 6 |
| 8. Figure out which of two summer jobs is the better offer; one with a higher salary but no benefits, the other with a lower salary plus room, board, and travel expenses. | 1 2 3 4 5 6 |
| 9. Figure out the tip on your part of a dinner bill. | 1 2 3 4 5 6 |
| 10. Figure out how much lumber you need to buy in order to build a set of bookshelves. | 1 2 3 4 5 6 |
| 11. Measure your height in centimeters. | 1 2 3 4 5 6 |
| 12. Determine how many boxes of a certain size will fit into a closet. | 1 2 3 4 5 6 |
| 13. Explain your chances of flipping tails on both of two coins. | 1 2 3 4 5 6 |

**Part 4:** Directions: Please rate the following mathematics topics according to how confident you would be teaching elementary students each topic.

Please use the following scale:

| 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| *Not confident at all* | | | | | *Completely confident* |

1. Averages, Mean, Median & Mode 1 2 3 4 5 6
2. Multiplication 1 2 3 4 5 6
3. Number Patterns 1 2 3 4 5 6
4. Shape Properties 1 2 3 4 5 6
5. Fractions 1 2 3 4 5 6
6. U.S. Customary Measurement System (e.g. feet, pounds, gallons) 1 2 3 4 5 6
7. Probability 1 2 3 4 5 6
8. Decimals 1 2 3 4 5 6
9. Order of Operations 1 2 3 4 5 6
10. Metric System (e.g. meters, liters, grams) 1 2 3 4 5 6
11. Division 1 2 3 4 5 6
12. Perimeter & Area 1 2 3 4 5 6
13. Tables & Graphs 1 2 3 4 5 6

**Part 5**: Directions: Please rate the following statements according to how much you agree or disagree. Please use the following scale:

| *1* | *2* | *3* | *4* | *5* | *6* |
|---|---|---|---|---|---|
| *Strongly Disagree* | *Moderately Disagree* | *Disagree* | *Agree* | *Moderately Agree* | *Strongly Agree* |

| | |
|---|---|
| 1. Developing speed and accuracy of math skills improves understanding. | 1 2 3 4 5 6 |
| 2. I encourage students to use manipulatives to explain their mathematical ideas to other students. | 1 2 3 4 5 6 |
| 3. I put more emphasis on getting the correct answer than on the process followed. | 1 2 3 4 5 6 |
| 4. The teacher's primary role is to carefully demonstrate new math problems to students. | 1 2 3 4 5 6 |
| 5. When introducing math topics which I am confident teaching, it is important to first build understanding of a concept before focusing on algorithms. | 1 2 3 4 5 6 |
| 6. I like my students to master basic mathematical operations before they tackle complex problems. | 1 2 3 4 5 6 |
| 7. When two students solve the same problem correctly using two different strategies I have them share the steps they went through with each other. | 1 2 3 4 5 6 |
| 8. I frequently ask my students to explain *why* something works. | 1 2 3 4 5 6 |
| 9. Formulas and rules should be presented first when introducing new topics. | 1 2 3 4 5 6 |
| 10. A lot of things about mathematics must simply be accepted as true and remembered. | 1 2 3 4 5 6 |
| 11. When teaching a topic which I am less confident teaching, if I start with the process students will come to understand the concept. | 1 2 3 4 5 6 |
| 12. With topics I am more confident teaching, I am more likely to explore alternative teaching strategies. | 1 2 3 4 5 6 |

**Part 6: Demographic Questions**

1. Which type of school do you work in? (Circle one on each line.)

   Urban Suburban Rural

   Parochial Public Private Charter

2. Is teaching your first career? Yes No

3. What is your gender? Male Female

4. Are you a parent? Yes No

5. What is your race? (optional)

   African-American Asian Hispanic

   White Mixed Other _______________

6. What is your highest level of degree earned?

   Associate Bachelor's Master's Doctorate

7. What was your major in college? ____________________________________

8. How many years have you been teaching?

   0-2 3-5 6-10 11-15 16-20 21-30 30+

9. What type of teaching certificate/license do you hold? Circle all that apply

   Type: teaching license provisional certificate permanent certificate

   Grades: PreK K 1 2 3 4 5 6 7 8

   Math specialist middle grade validation math concentration

   Other ____________________________________________________________

Turn page to continue…

10. Circle all subjects that you teach.

Language Arts Math Reading Science Social Studies

Others ____________________________________________________

11. What subject are you most confident teaching in an elementary school?

Language Arts Math Reading Science Social Studies

12. What subject are you least confident teaching in an elementary school?

Language Arts Math Reading Science Social Studies

13. Which one of the 13 mathematics topics listed on page 6 of the survey are you most confident teaching? ____________

14. Which one of the 13 mathematics topics listed on page 6 of the survey are you least confident teaching?

Please keep in mind that all answers will be kept strictly confidential. So that I may contact you for a possible follow-up interview, please provide this information:

Name ___________________________________

School __________________________________ Grade ________

School District __________________________________

e-mail address _____________________ Phone number _____________________

*Thank you very much for your participation in my study!*

Survey Number __________

## APPENDIX B

### Interview Protocol

Survey Number ____________

1. Why did you choose to teach 3rd (4th, 5th, or 6th) grade?
2. What subjects do you teach?
3. Of the following subjects, which of the following are your favorite and least favorite to teach?   Language Arts, Math, Reading, Science, Social Studies
4. Generally, how confident are you when teaching math?
5. Which math topics are you most confident and least confident teaching?
6. Outside of school, do you believe you are good at mathematics?
7. How have your teaching methods changed over the years?
8. *Using the survey answers, ask the participant to* ______________________

   Describe a lesson which introduces (<u>most</u> confident mathematics teaching topic).
9. *Using the survey answers, ask the participant to* ______________________

   Describe a lesson which introduces (<u>least</u> confident mathematics teaching topic).
10. *For questions 8& 9 use a sampling of the following probing questions:*:
    a. Teaching Goal: Explaining "why" or explaining "how".

       When explaining <u>why</u> a math procedure works, how much can students understand?

    b. Algorithms: memorize steps or discover steps

       How does a student come to understand the steps of an algorithm?

c. Number of Skills: Teaching sequential isolated skills or mixing math concepts.

   Should skills be taught in isolation or mixed?

d. Calculators: for problem solving or for computations

   What is the role of calculators in your classroom?

e. Wrong Answers: Should be corrected or should lead to discussion

   What do you do when a student gives a wrong answer?

f. Goal for Students: Understanding or speed and accuracy

   How important is developing students' speed and accuracy of getting answers?

g. Focus: Concept development or skill drill

   What is the most important thing for students to learn in mathematics?

h. Solution Process: One right way or many right ways

   How important is it to learn a solution process for a particular type of problem?

i. Problem Solving or Solving Word Problems

   What types of problem solving do students experience in your class?

j. Questioning: Justify reasons or recite facts

   What types of questions are important to ask in your mathematics class?

k. Manipulatives: to explore or to model

   How and when do your students use manipulatives?

l. Role of Teacher: Teacher demonstrates or teacher facilitates

   What is the role of the teacher in your math class?

Do you have any comments to add about the interview?

Thank you very much for your participation.

# APPENDIX C

## Survey and Interview Recruitment Script

### Survey Script

"Hello! My name is Diane Kahle and I am a doctoral student in Mathematics Education at Ohio State. I am conducing research about teachers' beliefs and preferences about mathematics education. Today I am here to ask you to participate in a survey of your opinions. You will be asked to first sign a consent form allowing me to use your answers in my project, and then you will be asked to complete a survey which will take approximately 15 minutes to complete. Some participants will be contacted again in the near future to participate in a short interview. For this reason I have asked for contact information on the last page. Please be assured that your answers will be kept confidential. Only the principal investigator and I will see your answers. You may quit and leave at any time during the study. If you have any questions during the survey, please ask. Thank you in advance for your participation. Your opinions are very important to the success of my project. Are there any questions?"

**Interview Script**

"Thank you for completing my dissertation survey on teacher beliefs about mathematics at the COMET workshop this summer. I would like to interview you to find out more about your perspective and ideas about teaching math. I am learning so many interesting things by hearing the ideas of practicing teachers! The interview would take only about 15-20 minutes and I would be happy to come to you. For most interviews I am meeting at the teacher's classroom or at a public library. I am only available after school or on weekends as I also am a teacher. As with the survey, information provided will be kept confidential. Please let me know if you are available any of the dates below at or after 3:45. If two or more of you are at the same building I can do consecutive interviews. Thank you in advance and I hope you are off to a great school year!"