CONTINUOUS SPACE FACILITY LOCATION FOR COVERING SPATIAL DEMAND OBJECTS

DISSERTATION

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ABSTRACT

Facility placement and associated service coverage are major concerns in urban and regional planning. A confounding issue in facility location models is how to represent geographic space. Often, space and associated demand for service has been conceived to be discrete points in these models. While point-based representations facilitate model construction, simplify data requirements and enable more efficient problem solutions, advances in GIScience and spatial data acquisition highlight deficiencies in using overly rudimentary feature depictions, like points, in certain modeling instances. The problem is further complicated when we aim to find optimal locations in the continuous space. In continuous siting, assuming that a facility to be located can be placed anywhere in the plane means that an infinite number of possible locations need to be considered. This research focuses on coverage optimization in the continuous plane where spatial demand is represented as objects, including points, lines or polygons. An approach is detailed for the problem of covering spatial demand objects for service, where potential facilities can be anywhere in the continuous plane. It is shown that weighted demand, represented as points, lines or polygons, can be optimally served by a finite number of potential facility locations, called the polygon intersection point set (PIPS). The developed approach extends point-based abstraction of demand to a more general representation. Empirical analysis demonstrates the applicability of the approach. Due to the limitations of existing modeling approaches for examining the coverage of space, there exist discrepancies between what is modeled and actual geographic coverage. In order to accurately reflect the mechanism of maximal coverage for spatial objects (points, lines or polygons), we introduce a new model explicitly accounting for joint service provided by multiple facilities. Application results demonstrate the reliability and efficiency that can be achieved by the proposed model. In order to solve large planning problems and generate diverse alternative solutions, a new genetic algorithm is proposed for the developed model. In particular, a new crossover operator is designed based on the characteristics of coverage problems. Application results show that the developed genetic algorithm is able to provide solutions of high-quality in a fast manner. This research proposes new modeling and spatial optimization techniques for coverage maximization and demonstrates that consideration of representational issues in the continuous plane is important in achieving efficiencies for urban and regional planning.

Dedicated to my parents

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CHAPTER 1

INTRODUCTION

1.1 Background

“Location” is an important factor for people to conduct certain activities. Some places are more preferred than others in different situations. Finding those preferred places has increasingly become part of daily life (ReVelle *et al.* 1970). Many organizations, public or private, face such important locational decisions. In the private sector, locational choices affect the ability of a firm to make use of capital investments and compete in the market place, and in the public sector such decisions involve not only monetary costs but also social benefits (White 1979). As a result, location analyses assisted by planning and design processes are important (Murray 2003).

Early location analysis dates back to Alfred Weber (1909) for the problem of siting a single warehouse so as to minimize total transportation cost, known as the Weber problem. Location theory flourished and was applied to a range of substantive contexts after the 1960’s. A survey of many applications is provided by Eiselt (1992) and Brandeau and Chiu (1998), but see also ReVelle *et al.* (1970), Toregas and ReVelle

(1971), ReVelle (1989), D'Amico *et al.* (2002), Aikens (1985), Vector and Dincer (1995), and ReVelle *et al.* (2002).

To find a solution to a locational decision problem, a model is often used (Birkin *et al.* 1996). Facility location models can be classified according to their objectives, constraints, solution approaches and other attributes (see Hamacher and Nikel 1998; Klose and Drexl 2004). Daskin (1995) classified location models into three general categories based on objective functions: covering models, *p*-center models and *p*-median models. Covering models seek to minimize the number of facilities needed for full coverage of demand, also known as set covering models, or to maximize the coverage of demand with a limited number of facilities, called maximum covering models (Schilling *et al.* 1993). *P*-center models aim to minimize the maximum distance or travel time from demand to its closest facility (Handler 1990). *P*-median models attempt to minimize the overall distance or time (or average distance or average time) between demand and its nearest facility (Hakimi 1964; ReVelle and Swain 1970).

Of interest in this research are covering location problems, which measure effectiveness through assessment of whether demand can receive service. The coverage standard, preset for either distance or time, is an important notion for covering problems, and demand is considered to be suitably served if it can be reached by a service facility within this standard. For example, fire department response time should be less than 6 minutes to an accident or structure fire after a call for service has been received (Murray and Tong 2007). So in this case, those places within a 6-minute travel time from a certain located fire station are considered to be “covered”. Coverage problems have been used in fire station location (ReVelle 1989), bus stop design (Gleason 1975), warning siren siting (Current and O’Kelly 1992) and weather monitoring (Minciardi *et al.* 2003), to name a few.

How to represent geographic space is a challenge for coverage-based modeling work. Geographical space can be represented in a variety of ways. Often, facilities and demand are represented as points (Miller 1996; Church 1999; Murray *et al.* 2006). A common formulation is: simultaneously locate a set number of point facilities to serve a set of weighted demand points. Hence, a demand point can be a sub-area of a region, such as a city or town, with associated aggregated information (Current and O’Kelly 1992). The primary reason of using the point representation instead of a geometrically complex shape (e.g., polyline, polygon, etc.) is that points can simplify data handling and solution approaches, and are easier for model construction. However, the fact is that neither demand nor facilities are necessarily points with no dimension, and the real world cannot be simplified to points of no dimension in many situations. For instance, demand, such as census tracts or planning districts, or facilities, like parks or retail outlets, are often an areal unit (Murray 2003).

In fact, the issue of how to represent geographic space, regional demand in particular, in location modeling has received considerable attention in recent years (*e.g.*, Miller 1996; Murray and O’Kelly 2002; Murray 2003, 2005). Most studies raise the question of whether or not points can effectively represent a demand region, such as neighborhoods, towns or census boundaries. Miller (1996) suggested that other spatial representations of objects, such as lines and areas (or polygons) in geographic information systems (GIS), should be considered in locational modeling, and that there is a need for location modeling capable of dealing with more complex objects. Driven by this need, this research will investigate demand represented more generally as points, lines or polygons in a GIS environment in the context of coverage location modeling.

Another important issue in coverage modeling is where potential facilities can be located. In this regard, location models can be categorized into two types: discrete or continuous problems. A continuous problem allows potential facility locations to be anywhere in the plane or a subset of the plane, whereas a discrete problem confines potential facilities to some finite set of sites specified in advance (Love *et al.* 1988). In some situations the two approaches are application oriented. For example, if there is only a finite number of feasible areas for locating service facilities due to costs, site availability or others reasons, then a discrete approach makes sense (Murray and O'Kelly 2002). However, if any place can be a potential facility site in the region, a continuous approach is more suitable, particularly with respect to optimality. In this research, a continuous approach will be studied. Thus, potential facility locations consist of all places in space and analysis needs to be carried out to find the best configuration of facilities. This necessarily means that one must consider an infinite number of possible locations at which to site. This search among an infinite number of sites is computationally challenging, and some cases require special techniques for problem solution.

1.2 Research objectives

Given the concerns indicated above, this research will focus on the analysis of covering demand objects, where facilities are to be sited in the continuous plane. In particular, three objectives will be dealt with. The first objective is to identify an approach to deal more effectively with continuous space siting where demand is represented as spatial objects. More specifically, a special set of discrete points, known as the finite dominating set (FDS) consisting of at least one optimal solution to the problem, will be identified serving as potential facility sites. By narrowing the infinite search space to those in the FDS, computational effort will be dramatically reduced. The second objective is to develop a new model for maximal coverage of spatial demand objects. The model is expected to be free of the modifiable areal unit problem (MAUP) and accurately reflects the maximal coverage mechanism for objects. The third objective is to develop a new efficient heuristic for the proposed maximal coverage model. The heuristic should be able to generate solutions of high quality within reasonable computation time.

1.3. Organization of the research

This research is organized as follows. Chapter 2 focuses on the continuous space siting problem when demand is represented as general objects. First, existing approaches for dealing with continuous space location are reviewed. Next, a FDS is identified and proved to consist of at least one optimal solution for covering spatial objects. The FDS is used in the context of warning siren evaluation in Dublin, Ohio. Results are presented and compared.

Chapter 3 proposes a method for regional maximal coverage where joint coverage for a specific demand by multiple facilities is considered. This chapter starts with a review of existing work on representation issues and modeling approaches for covering location problems. Then, a new general model is proposed when demand is represented as spatial objects. This is followed by an application of the developed model to warning siren siting in Dublin and Franklin County, Ohio. Finally, results and representational errors are examined, along with comparison to existing approaches.

Chapter 4 develops a new genetic algorithm for the model developed in Chapter 3. The chapter begins by reviewing existing heuristic techniques for maximal coverage problems. Then, existing genetic algorithms applied to similar problems are discussed. Next, a genetic algorithm with a new crossover operator structured for the characteristics of the maximal coverage problem is proposed. Finally, comparative results are presented and discussed.

The final chapter, Chapter 5, concludes the dissertation by summarizing research results and provides concluding comments. In addition, future research directions are discussed.

CHAPTER 2

COVERAGE OPTIMIZATION IN CONTINUOUS SPACE*

As noted in Chapter 1, the primary purpose of this dissertation is to investigate coverage optimization in continuous space, where demand is represented as general objects including points, lines and polygons. However, dealing with continuous space is a challenge in location problems. In this chapter, an approach is proposed for the problem of covering spatial demand for service, where potential facilities are located in the continuous plane. In particular, a special finite set is identified as potential sites for facility location. In doing so, the search space of a potential infinite number of sites is narrowed to a finite number, reducing problem complexity and computational effort dramatically.

2.1 Introduction

Facility location is a major concern in regional service analysis and planning. Of interest is making a choice about where to site a facility in order to best serve potential demand. Facility location problems arise in a variety of public and private sector contexts (Daskin 1995). For example, local governments need to decide where to locate public libraries

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* This chapter is a revised and extended version of papers published in *International Journal of Geographical Information Science* and the proceedings of the *University Consortium for Geographic Information Science Summer Assembly*, both co-authored with Dr. Alan T. Murray.

and schools to achieve maximum social benefits. In the private sector, retail outlets wish to capture market share and optimize revenue, while manufacturers are interested in reducing costs in acquiring raw materials.

A host of spatial optimization models have been proposed to support facility site selection, and there have been various attempts to categorize the range of approaches that exist (see Mirchandani and Francis 1990; Daskin 1995; Drezner and Hamacher 2002, among others). A major distinction has been discrete versus continuous (or planar) siting methods. Whereas discrete approaches assume that the locations of potential facilities are known in advance, continuous approaches allow facilities to be located anywhere in space. Thus, continuous space methods can be thought of as supporting strategic decision making. In this research we are interested in maximizing demand covered by a specified number of facilities, allowing facilities to be located in continuous space, similar to the planar coverage problem detailed in Church (1984). The motivation here is emergency warning siren siting, where the facilities are the sirens and service coverage is based on sound transmission from the siren. Thus, given a fixed budget, the question is where to place sirens in order to provide the greatest coverage of a regional population. In contrast to the approach of Church (1984), we are interested in the coverage of more complex spatial objects/features, not just points.

A confounding issue is representing geographic space in facility location models. Traditionally space and associated demand for service have been conceived to be discrete points (Bennett and Mirakhor 1973; Mirchandani and Francis 1990; ReVelle 1991; Miller

1996). In part this has been to keep representation simple, but also points serve as central locations for areas, or as towns or cities depending on the geographic scale of analysis. Miller (1996) has raised the issue, however, of whether facility location approaches relying on only point based representations of space are too simplistic given capabilities to represent more complex vector objects like lines and polygons in geographic information systems (GIS). In fact, Murray and O'Kelly (2002) highlight representation issues in coverage modeling, showing that reliance on a point based abstraction of a region could lead to an over-estimation of actual coverage provided to a region using a location model. Given this, there is a need for coverage modeling approaches capable of serving points, lines and polygons, as suggested by Miller (1996).

This chapter aims to optimally cover point, line or polygon features representing demand for service in a region, allowing facilities to be located in continuous space. In particular we look to extend the coverage problem of Church (1984) to account for line or polygon features. In the next section, continuous space coverage of demand is reviewed. The planar maximal covering problem is then discussed. This is followed by the introduction of a method for identifying a finite set of potential facility locations containing an optimal solution to the continuous space problem. An application to warning siren coverage is then detailed. Finally, a discussion and conclusions are provided.

2.2. Background

2.2.1 Continuous space location

Location problems can be classified into two types, discrete and continuous problems, based on the set of potential facility locations (see Hamacher and Nickel 1998). In continuous space, one can locate a service facility anywhere in the plane or on the network. Alternatively, in discrete space facilities can be placed only at a limited number of eligible points. It is difficult to solve continuous facility location models because of the infinite possibilities for facility placement (Plastria 2002).

Continuous space siting has been of longstanding interest. Weber (1909) was interested in finding the location in the plane for siting a single industry or firm to minimize transportation costs. A range of spatial optimization problems have followed, with a major feature being that facilities are to be located in continuous space in order to minimize or maximize objective criteria. Continuous space siting assumes that a facility to be located can be placed anywhere in the plane. This necessarily means that one must consider an infinite number of possible locations at which to site facilities. Often times this has been a foreboding challenge computationally, unless special approaches could be identified or developed.

The earliest continuous location solution algorithm was offered by the mathematician Weiszfeld (1936) for a one facility minisum Weber problem. Other variants followed (Eyster et al. 1973; Drezner and Weslowsky 1980). Geometry exploitation is another approach for solving continuous location problems. Elzinga and Hearn (1972) developed a method for a 1-center discrete demand problem by studying the geometry between point demands. Recently, a Voronoi diagram heuristic was used for solving the continuous space *p*-center problem (Suzuki and Okabe 1995; Okabe and Suzuki 1997). Medial axis was also used to delineate optimal siting locations for the single facility coverage of an area (Matisziw and Murray 2006). Other geometric solution approaches include those developed by Hershberger (1993), Hochbaum and Shmoys (1985), Khuller and Sussmann (2000), and Sharier and Welzl (1996).

Recognizing that some places are better than others under certain conditions, another method for solving a continuous problem is to identify some or all of the good places as potential facility candidates. Therefore, instead of iteratively searching for good solutions in continuous location problems, as many algorithms or heuristics do, it makes sense to identify a finite set which consists of at least one optimal solution. This finite set is known as the finite dominating set (FDS). By examining the finite set, instead of the infinite space, search effort can be reduced dramatically.

Hakimi (1964, 1965) was the first to investigate the FDS for a network location problem. He proved that nodes of the network comprise a FDS for the *p*-median problem. Inspired by this work, there have been considerable activitives in finding the FDS for different network problems. Hooker et al. (1991) surveyed the literature, unifying and generalizing FDS results for network location problems. There were also FDS endeavors for location problems in the plane. For example, Vergin and Rogers (1967) proved that the demand points were a FDS for the single-facility location problem in the plane using the sum of rectilinear distance criteria. Also, Church (1984) developed an approach for identifying the FDS for covering problems involving point-based demand for both Euclidean distance and rectilinear distance. Under the Euclidean distance measure, the FDS is a set of intersection points of the circles with radius of the service standard *S* centered on the demand locations, known as the CIPS (circle intersection point set). While under the rectilinear distance measure, the FDS is a set of intersection points of the diamonds, also known as the DIPS (diamond intersection point set). Church (1984) proved these sets contained at least one optimal solution to their respective covering problems. For some problems, there is no such FDS or it is very difficult to find. Some research identifies "good" finite sets which can provide better solutions compared with other alternatives. For example, Kuby *et al.* (2005) provided such a set as the middle points on the arcs of networks for dispersion problems.

Central to this chapter is the work on the planar maximal covering (PMC) location problem defined originally in Church (1984)[1], but also detailed in Mehrez and Stulman (1982, 1984). The PMC is important because of the planning applications it can be used to address. As an example, Mehrez and Stulman (1984) noted that it can be used for siting fire and radar stations to maximally cover demand distributed across a service region. Along these lines, Murray *et al.* (2007) detailed the need for addressing emergency warning siren placement, where sirens are to be located in a region and

[1] Richard Church originally presented this research at the ORSA/TIMS meeting in Philadelphia, PA, April 1976 ("The planar maximal covering location problem"), with a manuscript produced as well. The paper was ultimately published with the same title – Church (1984).

residents are to be covered within the audible service range of a siren. With a FDS (e.g., CIPS, DIPS), one then can rely on the original discrete space maximal covering location problem (MCLP) proposed in Church and ReVelle (1974) to solve the continuous space generalization. Again, geometrical insights make it possible to address the complexities of siting in continuous space without altering the planning problem formulation.

2.2.2 Demand representation

There has long been interest in dealing with other demand objects, like areas, in location modeling (see Wesolowsky and Love 1971; Love 1972; Bennett and Mirakhor 1974; Drezner and Wesolowsky 1980; Marucheck and Aly 1981; Aly and Marucheck 1982). However, there have been computational difficulties in addressing area (or polygon) representations of demand, limiting practical capabilities beyond the above noted work. In the context of coverage optimization, Benveniste (1982) recognized the importance of being able to represent demand as polygons in a location model, but offered no approach for doing this. Miller (1996) highlighted that much location modeling research has only addressed point-based demand. However, demand can be conceived of as line- and area-based as well. Line demand object applications include street or river segments, and polygon demand objects could be watersheds, census tracts or planning districts. A major challenge raised in Miller (1996) is to be able to address these more complex representations of space in location modeling. Driving this need is no doubt spatial information managed using GIS, where spatial features are represented as vector objects, like points, lines or polygons. Much spatial information exists to support location decision making, so there is a need to account for how demand is represented, rather than assume it is only point based.

The objective of this chapter is to extend the PMC to consider any vector-based demand objects, not just points. This begins to address the spirit of what Miller (1996) recognized as an important frontier in GIScience research, though for a particular class of spatial optimization problem. Beyond this, as will be detailed later in the chapter, the ability to address the problem of providing maximal service coverage to non-point based demand, like a polygon, is needed in its own right as it has practical use and application in urban and regional analysis (see Benveniste 1982; Li and Yeh 2005; Murray *et al.* 2007).

2.3 Problem formulation

Before proceeding, it is necessary to formally specify the planning problem of interest. As noted previously, the idea is to extend the PMC to account for other vector features, like lines and polygons. A useful beginning point is to first detail the MCLP of Church and ReVelle (1974), as the PMC naturally follows. Consider then the below notation:

i = index of demand points

j = index of potential facility locations

w_i = expected service required for demand i

$N_i = \{$ set of potential facilities capable of covering/serving demand i $\}$

p = number of facilities to site

$$x_j = \begin{cases} 1 & \text{if potential facility } j \text{ located} \\ 0 & \text{otherwise} \end{cases}$$

$$y_i = \begin{cases} 1 & \text{if demand } i \text{ suitably covered/served} \\ 0 & \text{otherwise} \end{cases}$$

It is worth mentioning that Church and ReVelle (1974) defined the cover set, N_i, in terms of a distance standard *S*, where *S* represented the distance that could be traveled to suitably serve (or cover) demand. Denote d_{ij} as the distance from demand *i* to potential facility *j*. Thus, formally the cover set is: $N_i = \{ j \mid d_{ij} \le S \}$. The model now follows.

Maximal Covering Location Problem (MCLP)

Maximize $$\sum_i w_i y_i \qquad (2.1)$$

Subject to

$$\sum_{j \in N_i} x_j \ge y_i \qquad \forall i \qquad (2.2)$$

$$\sum_j x_j = p \qquad (2.3)$$

$$x_j = \{0,1\} \qquad \forall j \qquad (2.4)$$

$$y_i = \{0,1\} \qquad \forall i$$

The MCLP requires discrete potential facility locations, *j*, to be specified in advance. The objective, (2.1), is to maximize the total demand provided suitable service, or coverage. Constraints (2.2) track whether demand is covered by sited facilities. Constraint (2.3) specifies the number of facilities to locate. Finally, integer requirements are imposed in Constraints (2.4).

What has effectively been defined above is the problem of covering as much total weighted demand as possible by positioning a given number of circles of radius *S* centered on a sub-set of potential locations *j*. The planar version of this problem relaxes the requirement that circles must be centered on the discrete, pre-specified potential facility locations, meaning that facilities can be located anywhere in the plane.

> *Planar Maximal Covering Location Problem (PMC)*: Maximize point-based demand covered by a specified number of facilities located in continuous space.

The implications of this relaxation in facility location placement are that greater efficiency in coverage is now possible, as siting locations are no longer restricted. That is, as much or more total coverage can be achieved for a given configuration of facilities using the PMC. From a planning and analysis perspective, such an insight is fundamentally important, as this establishes the greatest efficiency possible for a particular level of infrastructure investment.

Church (1984) looked at two instances of the PMC, one the PMC with Euclidean travel distances (PMCE) and the other the PMC with rectilinear travel distances (PMCR). Mehrez and Stulman (1982, 1984) assumed Euclidean distance travel, so examined the PMCE. Given the application context, extending the more generic PMC is of interest in this chapter, and travel distances based on a general distance metric are assumed.

The PMC generalization can be more formally detailed. Define *R* as the entire region of analysis, in which facilities can be located anywhere. The location of an individual facility location *j* is defined as a coordinate pair, $\left(x_j, y_j\right) \in R$, and represents decision variables in the model. That is, the optimization model is to find the coordinates of where to site the *p* facilities in order to maximize total demand covered. Given this, it should be obvious that N_i cannot be defined a priori as the location of an individual facility can be anywhere in the region *R*, in contrast to the discrete approach of the MCLP.

The model extension in this chapter is to consider demand *i* as points, lines or polygons, not just points as previously assumed. Thus, the problem of interest is:

> *Extended Planar Maximal Covering Location Problem (EPMC)*: Maximize demand objects (points, lines and/or polygons) covered by a specified number of facilities located in continuous space, assuming a general ℓ_p - distance metric.

Given this new spatial optimization problem, the task is now finding methods for solving it.

2.4 Coverage of spatial demand objects

As suggested previously, demand can be abstracted spatially as points, lines or polygons using GIS. In this section we examine coverage provision to these variants of demand representation. Furthermore, in covering models, distance as a measure of relative spatial relation of the facility to demand is often used to evaluate coverage effectiveness. In the following, coverage under several common distance measures is assessed. The coverage properties that follow provide the basis for solving the EPMC.

2.4.1 Euclidean distance metric

Euclidean (straight-line) distance is one typically used distance metric, which represents the shortest distance between two objects. Suppose we have a facility F located at (x_F, y_F) capable of providing service extending out to a distance standard S, assuming Euclidean distance travel, with demand represented by a point A located at (x_A, y_A), as shown in Figure 2.1. The circle represents the service area. Demand is covered when it is on or inside the circle. Thus, we can establish the following property.

> *Point coverage property*: Demand point A can be served by facility F if the distance between the two points is no greater than the service standard S.

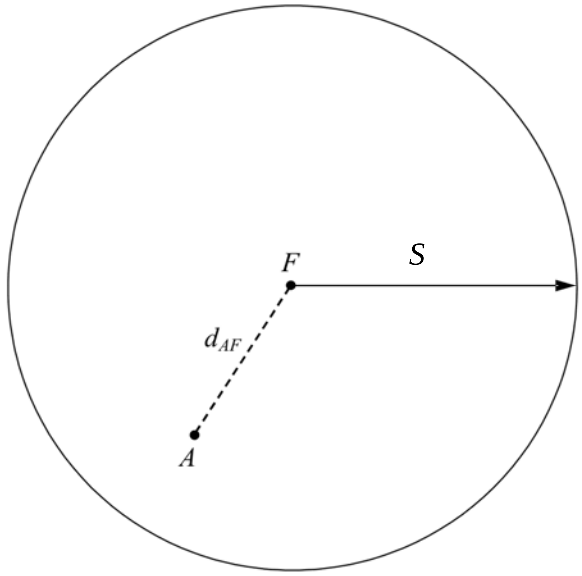


Figure 2.1. Coverage of point *A* by facility *F*.

This property can be stated mathematically when service is provided to point *A*:

$$d_{AF} = \sqrt{(x_A - x_F)^2 + (y_A - y_F)^2} \leq S \tag{2.5}$$

As a function of a circle defining service coverage, this condition is as follows:

$$d_{AF}^2 = (x_A - x_F)^2 + (y_A - y_F)^2 \leq S^2 \tag{2.6}$$

Thus, geometrically a circle with radius *S* centered on facility *F* is able to cover demand point *A* when it is on or within this circle.

Consider now a line segment $\overline{AB}$ representing linear demand. End points *A* and *B* of segment $\overline{AB}$ are located at (x_A, y_A) and (x_B, y_B) respectively. Again, facility *F* is capable of service out to a distance *S*. Coverage of this line segment is illustrated in Figure 2.2, and the following property can be stated.

Line segment coverage property: If points *A* and *B* are both covered by facility *F*, any point *C* located at (x_C, y_C) on segment $\overline{AB}$ is also covered by facility *F*.

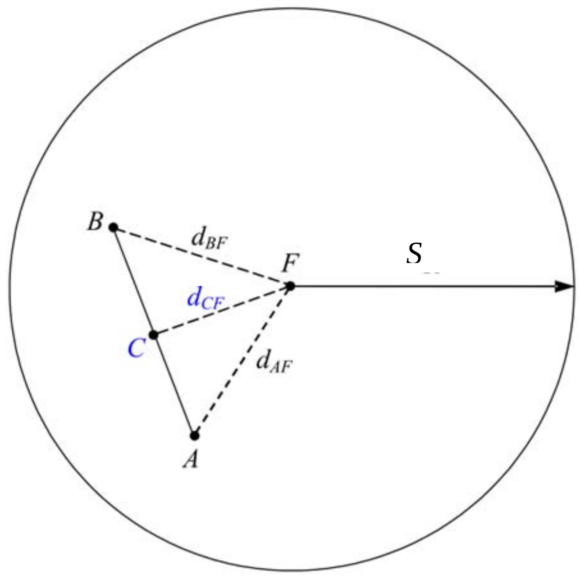


Figure 2.2. Coverage of line segment $\overline{AB}$ by facility *F*.

The line segment coverage property can be proven mathematically. Point *C* on line segment $\overline{AB}$ can be represented in terms of *A* and *B* as follows:

$$x_C = \lambda x_A + (1-\lambda)x_B \tag{2.7a}$$

$$y_C = \lambda y_A + (1-\lambda)y_B \tag{2.7b}$$

where $0 \le \lambda \le 1$. The Euclidean distance between demand point *C* and facility *F* is therefore:

$$d_{CF} = \sqrt{(x_C - x_F)^2 + (y_C - y_F)^2} \qquad (2.8)$$

If (2.7a) and (2.7b) are inserted in (2.8), the following results:

$$d_{CF} = \sqrt{(\lambda x_A + (1-\lambda)x_B - x_F)^2 + (\lambda y_A + (1-\lambda)y_B - y_F)^2} \qquad (2.9)$$

Squaring both sides of (2.9) produces:

$$d_{CF}^2 = (\lambda x_A + (1-\lambda)x_B - x_F)^2 + (\lambda y_A + (1-\lambda)y_B - y_F)^2 \qquad (2.10a)$$

Multiplication and re-arrangement of terms gives:

$$\begin{aligned} d_{CF}^2 &= \lambda^2 x_A^2 + 2\lambda(1-\lambda)x_A x_B + (1-\lambda)^2 x_B^2 - 2\lambda x_A x_F - 2(1-\lambda)x_B x_F \\ &+ x_F^2 + \lambda^2 y_A^2 + 2\lambda(1-\lambda)y_A y_B + (1-\lambda)^2 y_B^2 - 2\lambda y_A y_F - 2(1-\lambda)y_B y_F + y_F^2 \end{aligned} \qquad (2.10b)$$

Equation (2.10b) can be further re-arranged and partitioned as follows:

$$\begin{aligned} d_{CF}^2 &= \lambda\left[(x_A - x_F)^2 + (y_A - y_F)^2\right] + (1-\lambda)\left[(x_B - x_F)^2 + (y_B - y_F)^2\right] \\ &+ \lambda(\lambda - 1)\left[(x_A - x_B)^2 + (y_A - y_B)^2\right] \end{aligned} \qquad (2.10c)$$

Further simplification, as a function of distances between points, gives:

$$d_{CF}^2 = \underbrace{\lambda\, d_{AF}^2 + (1-\lambda)\, d_{BF}^2}_{\alpha} + \underbrace{\lambda(\lambda-1)\, d_{AB}^2}_{\beta} \tag{2.10d}$$

Given the assumed coverage of points *A* and *B* by facility *F*, the α component in (2.10d) relates to the service distance standard, or circle with radius *S*, as follows:

$$\alpha = \lambda\, d_{AF}^2 + (1-\lambda)\, d_{BF}^2 \leq \lambda R^2 + (1-\lambda)R^2 = R^2 \tag{2.11}$$

The β component of (2.10d) equates to:

$$\beta = \lambda(\lambda-1)\, d_{AB}^2 \leq 0 \tag{2.12}$$

The reason for this is due to the definition of λ, as $\lambda(\lambda-1)$ is at most zero. Thus, given (2.11) and (2.12), the squared distance from point *C* to facility *F* is bounded by:

$$d_{CF}^2 \leq R^2 \tag{2.13}$$

If points *A* and *B* are covered by facility *F*, any point C on line segment *AB* is also covered. Therefore, coverage of a line segment by a circle is guaranteed when both end points are covered, and the line segment coverage property is proven.

Line segment coverage can be extended for a polygon, or area object. Figure 2.3 shows polygon *ABCD* with area-based demand covered by facility *F* with service radius *S*, again assuming Euclidean distance travel. *ABCD* consists of four line segments, and the following property can be observed:

> *Polygon coverage property*: If each line segment end point in *ABCD* is covered, then any point on the boundary or within the polygon is also covered by facility *F*.

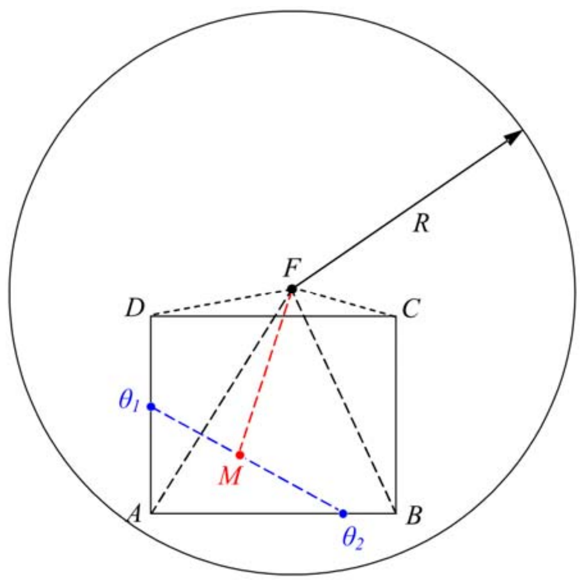


Figure 2.3. Coverage of polygon *ABCD* by facility *F*.

The proof of the polygon coverage property follows. If all vertices of polygon *ABCD* are covered by facility *F*, then according to the line segment coverage property any line segment connecting two vertices is also covered by *F*. Thus, the boundary, comprised of series of line segments, is also covered by the circle centered at *F* with radius *S*. Consider any point *M* inside *ABCD*. If a line is drawn passing through *M* in any direction, this line will intersect two or more line segments defining the boundary, *AB*, *BC*, *CD*, or *DA*. Without loss of generality, suppose the intersection points are θ_1 and θ_2. Since these two points are on the boundary of *ABCD* and the vertices (or line segment end points) are covered by facility *F*, point *M* on segment $\overline{\theta_1\theta_2}$ is also covered. Thus, the entire polygon, boundary and interior, is covered when boundary segment end points are covered, and our proof is complete.

2.4.2 General distance metric

A more general distance measure, ℓ_p - distance, is commonly used for distance evaluation. ℓ_p - distance in the plane is derived from the ℓ_p - norm, where $g(x) = \ell_p(q, r) = \sqrt[p]{|x_q - x_r|^p + |y_q - y_r|^p}$ for $1 \le p \le \infty$. (x_q, y_q) and (x_r, y_r) are points in the plane. Note, when $p = 2$, this is the Euclidean distance measure. In addition, rectilinear distance is the special case of ℓ_p - distances where $p = 1$, which is better for approximating travel along a city street grid or in a network of aisles in a factory or warehouse.

If we let $S = 1$ and the origin (0, 0) is the point measured from, the coverage ball ***B*** is then a unit ball centered on the origin. For different *p*'s, it can take different shapes. For instance, if we let $p = 1$, the coverage ball ***B*** satisfies $|x_q| + |y_q| \le 1$, which is a unit diamond as shown in Figure 2.4. If we set $p = 2$, the coverage ball satisfies $x_q^2 + y_q^2 \le 1$, and we get a unit circle as shown in Figure 2.5. For other *p*'s we may get other different unit balls. Figure 2.6 shows some examples from Love *et al*. (1988).

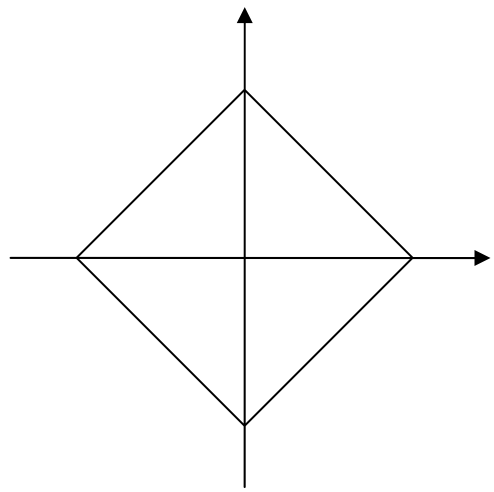


Figure 2.4. Unit coverage ball under rectilinear distance measure.

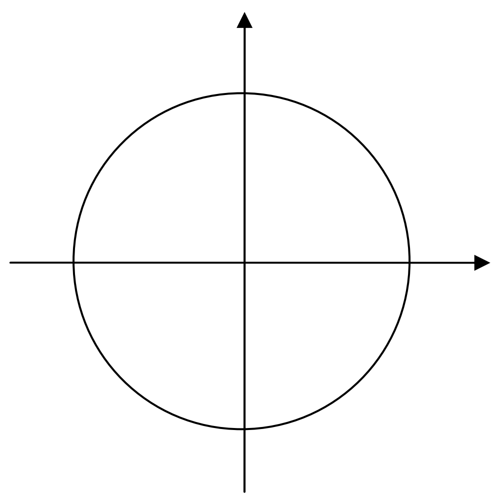


Figure 2.5. Unit coverage ball under Euclidean distance measure.

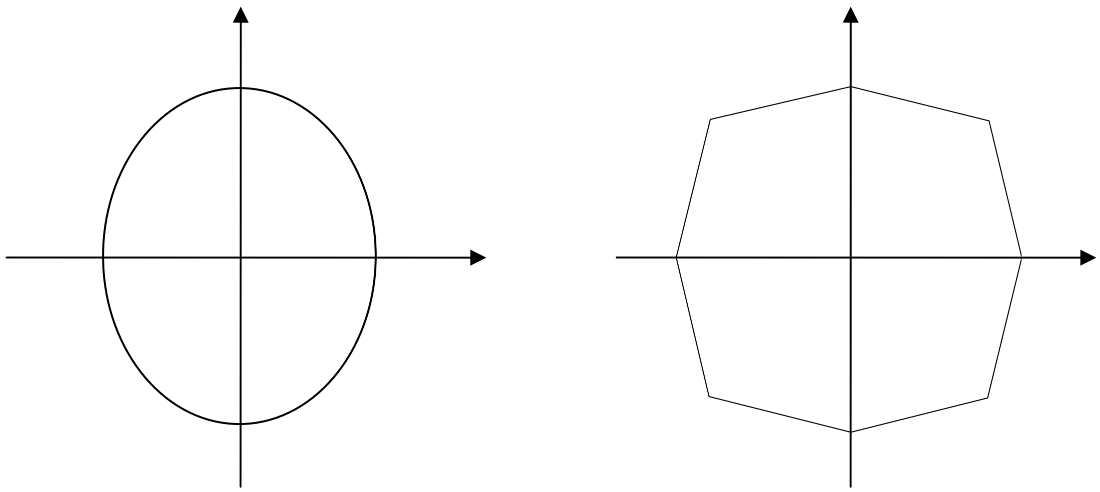


Figure 2.6. Some unit coverage balls for *p*'s other than 1 and 2.

Again, suppose we have a facility *F* located at (x_f, y_f) which can provide service within a distance standard *S*, then any point demand (x_d, y_d) falling inside or on the boundary of the coverage ball $\boldsymbol{B} = \{d \mid \ell_p(d, f) \leq S\}$ can receive service from facility *F*. For Euclidean distance, the ball ***B*** is a circle centered on (x_f, y_f) with radius *S*, whereas for rectilinear distance the ball is a diamond shape, as illustrated by Church (1984).

Consider a line segment $\overline{AB}$ representing linear demand such as a street. If each point of *A* and *B* is within the coverage ball ***B***, then any point on the line segment *AB* is also in the coverage ball ***B***. That is, the ball ***B*** is a convex set, which has been proven based on the triangle inequality and homogeneity properties of norms (see Brimberg and Love 1995). This property indicates that if two end points of a line segment receive service from facility *F*, then the entire segment also receives service from *F*. The Euclidean case shown above is a special case of this property. Based on this line segment coverage property for ℓ_p - distance, the polygon coverage property for Euclidean distance also holds for ℓ_p - distance. The proof is derived similar to that shown above for the polygon coverage property under the Euclidean distance metric.

2.5 Identifying a FDS

As noted previously, facility location is permissible anywhere in continuous space, and the intent is to identify a configuration of facilities that provides maximal coverage of demand for service using the EPMC. This necessarily means that some locations will be better than others in terms of providing coverage. The previous section established conditions under which spatial features are covered, or served, when demand is represented as points, lines or polygons. Thus, in this section these properties will be used to derive a process for identifying sufficient potential facility locations. This is done by exploiting geometrical aspects of the EPMC. Further, it will be shown that an optimal solution to the EPMC is comprised of this set of discrete points.

Before proceeding, it is worth reviewing the relationship between points, lines and polygons as GIS-based vector objects, a discussion of which can be found in Worboys and Duckham (2004) among others. A point is a coordinate pair. A line is comprised of a series of coordinate pairs, where it is typically assumed that a straight line connects consecutive coordinate pairs. Thus, a line is a series of line segments. Finally, a polygon is also a series of coordinate pairs, but the last coordinate pair corresponds to the first coordinate pair thereby enclosing the polygon. Thus, a polygon is also a series of line segments. Mathematically these definitions are:

Point: (x, y)

Line: $\{(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)\}$

Polygon: $\{(x_1, y_1), (x_2, y_2), \ldots, (x_m, y_m), (x_1, y_1)\}$

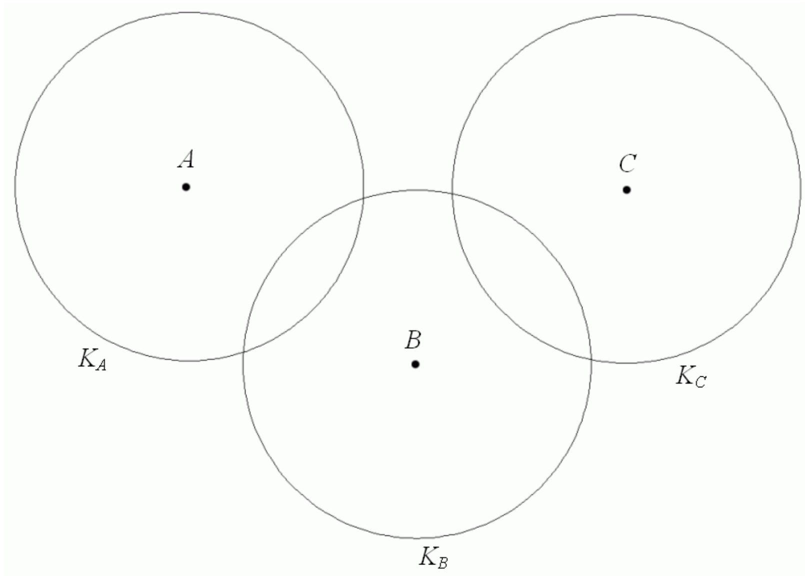
Thus, points and line segments are fundamental components of traditional vector objects. In terms of developing a solution approach for EPMC, one assumption is made regarding the relationship between the coverage distance standard *S* and object size. Here it is assumed that the object size is significantly smaller than *S*, or specifically *S* is greater than the maximum diameter of the smallest enclosing circle for any spatial demand object[2]. This assumption ensures that all objects can be suitably covered. If this is not the case, it is assumed that one can readily partition the object(s) into smaller objects as this is a standard feature of GIS.

2.5.1 Critical locations for demand objects

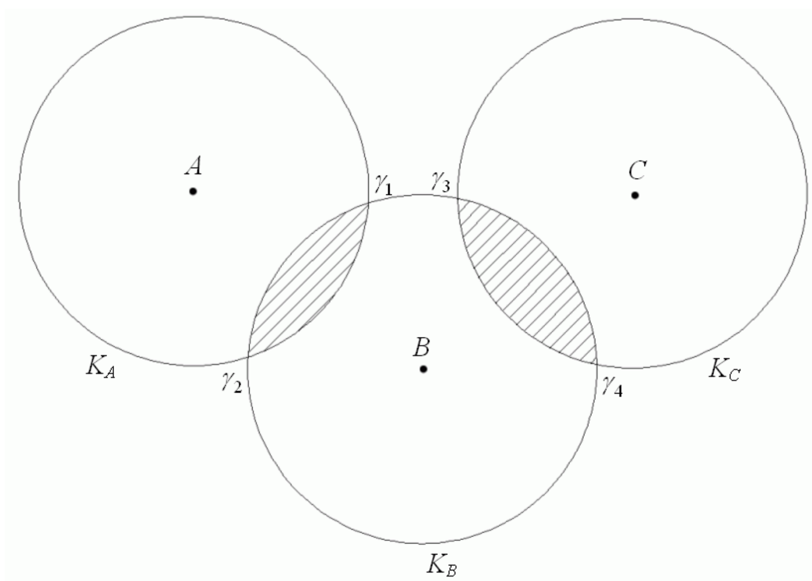
The previous section has detailed the conditions under which demand objects (points, lines or polygons) receive coverage. Expanding on this, it is possible to define the area for which placement of a facility would result in coverage of a given demand

[2] Finding the smallest enclosing circle for an object, or set of points, is itself an optimization problem. It is also referred to as a minimum circle, minimum spanning circle, smallest enclosing circle, smallest enclosing disc or a minimal covering circle in the literature. A discussion and review of this problem is given in Wei et al. (2006).

object/feature. This is, of course, dependent upon the nature of service provision, which in this case extends out a distance S. Thus, for point-based demand, a coverage ball $\boldsymbol{B} = \{f \mid \ell_p(d, f) \leq S\}$ centered on the demand point represents the area where locating a facility would ensure coverage of this associated demand. Figure 2.7a illustrates this area under the Euclidean distance metric, where three points are depicted, *A*, *B* and *C*, along with their corresponding covering boundaries, K_A, K_B and K_C respectively. Therefore, siting a facility on or in K_A would serve demand point *A*, siting a facility on or in K_B would serve demand point *B*, and siting a facility on or in K_C would serve demand point *C*. In the context of maximizing coverage, overlapping covering boundaries (e.g., $K_A \cap K_B$ and $K_B \cap K_C$ in Figure 2.7b) are important because they represent areas where efficiencies can be achieved. Specifically, if a facility is located in an overlapping area, multiple demand points can be covered. With this in mind, any point in an overlapping covering area is equally as good. Thus, we can focus on particular critical points associated with the overlapping covering boundary. These critical locations are shown in Figure 2.7b, and are referenced as γ_1, γ_2, γ_3 and γ_4. In this case we have four discrete locations where facility placement makes sense from the perspective of optimization efficiency. In fact, these critical locations are the circle intersection point set (CIPS) identified in Church (1984), and are proven to contain an optimal solution to the PMCE.



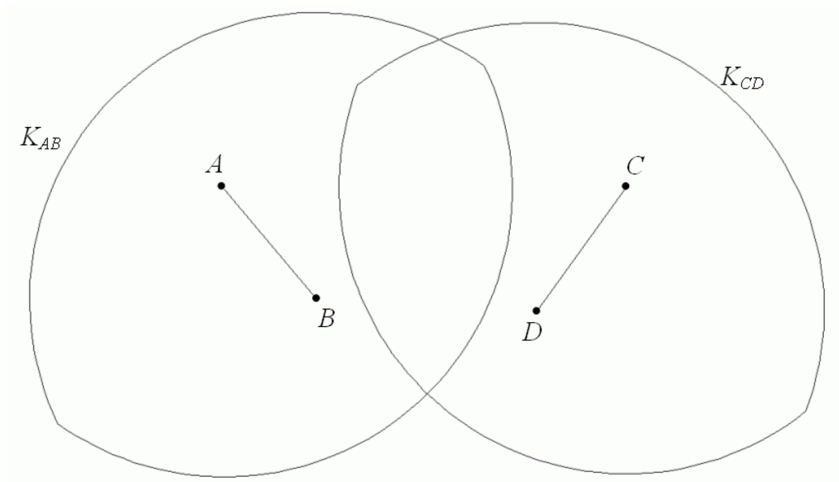
(a) Coverage boundary.



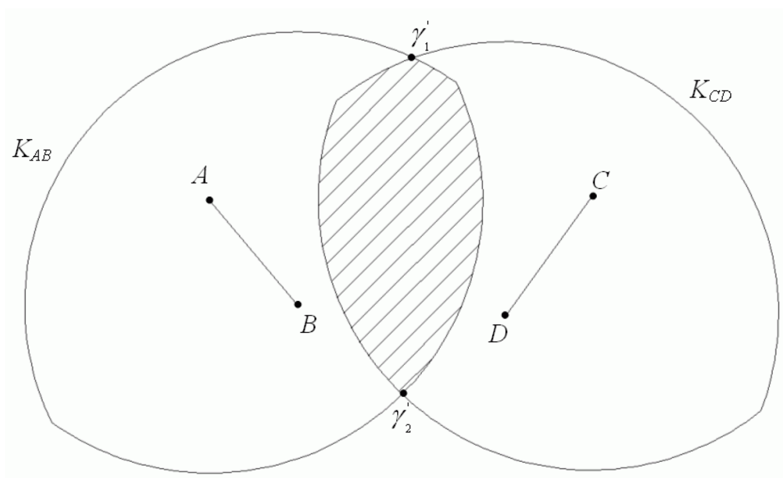
(b) Critical locations.

Figure 2.7. Coverage properties for point-based demand under Euclidean distance.

For line-based demand, the covering region is more complex given the need to cover an entire segment, yet remains a function of the service distance standard S. Thus, the covering boundary is a spatial object, likely comprised of line segments, arcs and curves. An example for two line segments is shown in Figure 2.8a under Euclidean distance measure. Demand along line segment $\overline{AB}$ can be suitably covered if a facility is located in or on K_{AB} and demand along line segment $\overline{CD}$ can be suitably covered if a facility is located in or on K_{CD}. No point on AB is further than S from any point on K_{AB}. The same is true for segment CD and K_{CD}. As in the case of demand points, overlapping covering boundaries (*e.g.*, $K_{AB} \cap K_{CD}$ in Figure 2.8b) represent potential optimization efficiencies. Not surprisingly, critical locations associated with intersecting covering boundaries also exist in this case, as shown in Figure 2.8b. These locations are referred to as γ_1' and γ_2'.



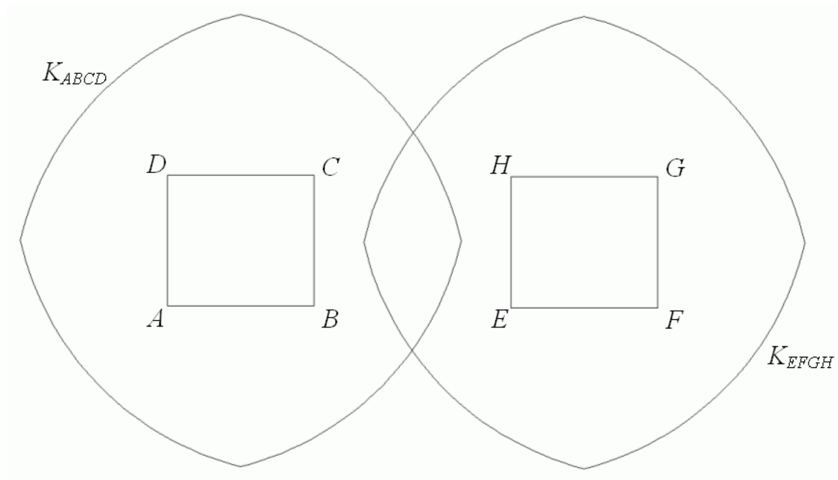
(a) Coverage boundary.



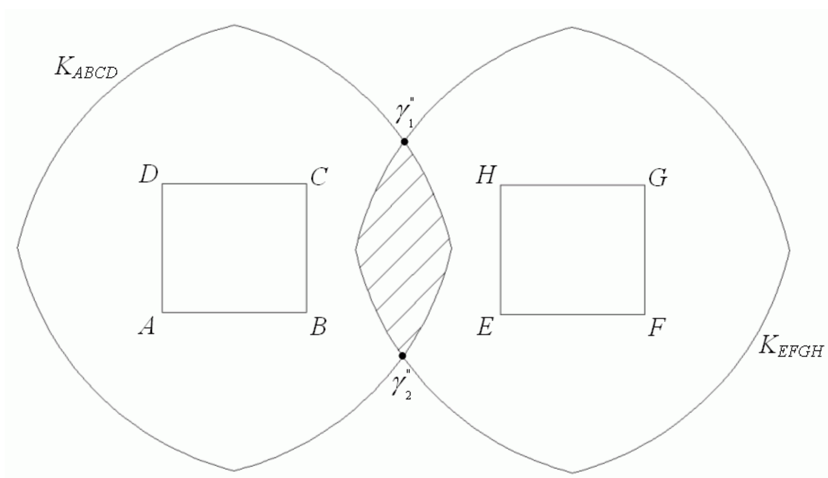
(b) Critical locations.

Figure 2.8. Coverage properties for segment-based demand under Euclidean distance.

Finally, for polygon-based demand, the covering boundary again is more complex than a circle given the need to cover the entire polygon, but remains a function of the service distance standard *S*. Thus, again the covering boundary is a spatial object, likely comprised of line segments, arcs and curves. An example for two polygons is shown in Figure 2.9a. Demand in polygon *ABCD* can be suitably covered if a facility is located in or on K_{ABCD} and demand in polygon *EFGH* can be suitably covered if a facility is located in or on K_{EFGH}. As in the case of demand points and line segments, overlapping covering boundaries (*e.g.*, $K_{ABCD} \cap K_{EFGH}$ in Figure 9b) represent efficiencies for optimizing coverage. Not surprisingly, critical locations associated with intersecting covering boundaries exist here as well, as shown in Figure 2.9b. These locations are referred to as γ_1'' and γ_2''.



(a) Coverage boundary.



(b) Critical locations.

Figure 2.9. Coverage properties for polygon-based demand under Euclidean distance.

Coverage boundaries and corresponding critical locations under general ℓ_p - distances can be defined similarly. For instance, Figures 2.10 and 2.11 show the coverage boundaries and the critical locations under the rectilinear distance, for line-based and polygon-based objects, respectively.

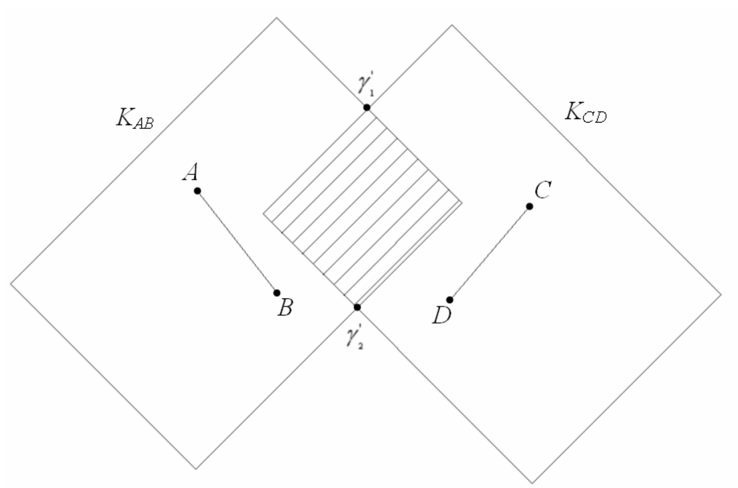


Figure 2.10. Coverage boundary and critical locations for segment-based demand under the rectilinear metric.

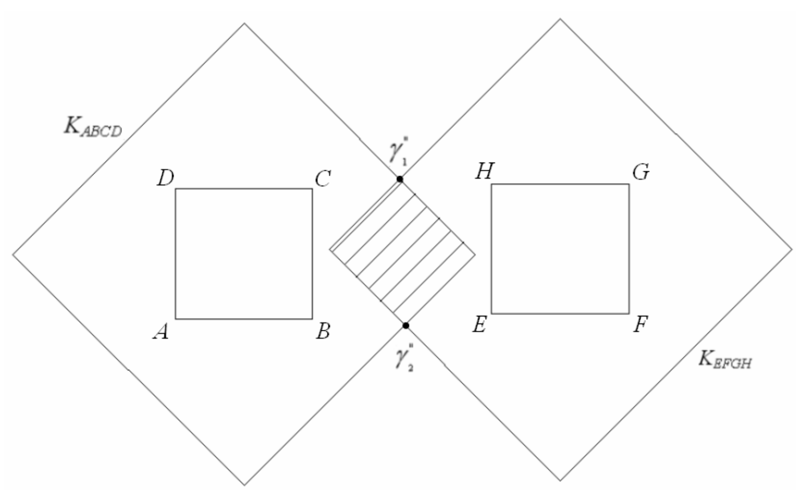


Figure 2.11. Coverage boundary and critical locations for polygon-based demand under the rectilinear metric.

2.5.2 Identification process

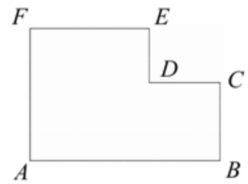
Given these insights regarding covering boundaries, an approach can be structured to identify a discrete set of critical locations to consider as facility sites in solving the EPMC, thereby reducing an infinite number of potential facility locations to a finite and discrete set of locations. Of course, with such a discrete set of locations, one can then use the MCLP to solve the EPMC. Formally, the approach for identifying critical locations is as follows:

Polygon Intersection Point Set (PIPS) Approach

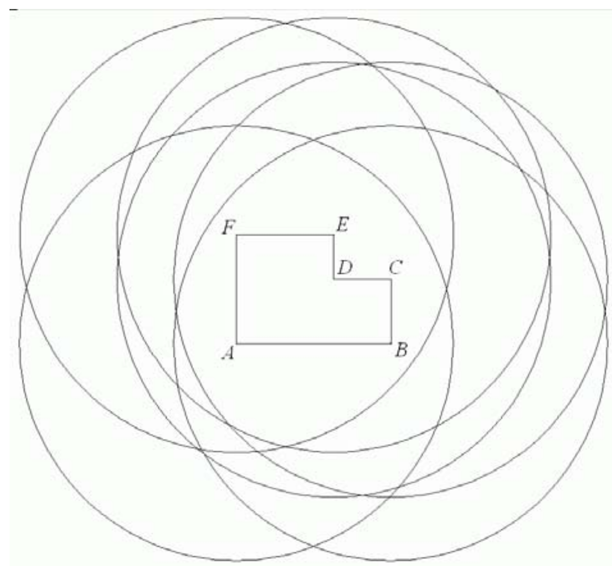
1. Identify spatial demand objects (points, lines and/or polygons) in need of coverage
2. Extract object vertices as potential facility locations
3. Derive covering boundaries (areas) for each demand object
4. Identify the intersection points of covering boundaries as potential facility locations
5. (Optional) Remove dominated critical locations

The first step of PIPS is straightforward. Step 2, along with the assumption regarding object size relative to covering distance S, ensures that a feasible covering solution exists. Steps 3 and 4 are obviously central, but are readily supported by standard GIS functionality. The covering boundary in Step 3 is effectively structured using an overlay process, from which the associated spatial object can be extracted. Given the assumption that S is greater than the maximum diameter of the smallest enclosing circle of any object, the covering boundary can be obtained by establishing a circle of radius S centered on each point, or vertex, of the object to be covered (*e.g.*, $(x_1, y_1), (x_2, y_2), (x_3, y_3), ..., (x_m, y_m)$) under Euclidean distance. The covering boundary results, as it is the area of overlap of all object edge circles. Figure 2.12 depicts this process for a polygon under Euclidean distance, where Figure 2.12a shows the object, Figure 2.12b illustrates the circles of radius S on each object defining vertex, and the overlapping area is highlighted in Figure 2.12c. This overlapping area is the covering boundary for this object. Given covering boundaries identified in Step 3, Step 4 involves finding the intersection points for boundary pairs, if they exist[3]. Extracting these locations is supported by standard commercial GIS functionality, but involves solving for intersecting points for spatial objects. Step 5 is optional, but recognizes two issues. First, the number of critical locations identified may be rather large. Second, some locations may in fact be better than others in the sense that one location can cover everything that another location can cover and more. As an example, if γ_1 covers all demand objects covered by γ_2 plus additional objects, γ_1 dominates γ_2 as it would always be better to utilize γ_1 over γ_2 given the coverage superiority. This domination principle has long been recognized in coverage modeling (see Toregas and ReVelle 1973), and was detailed in the context of CIPS as well by Church (1984).

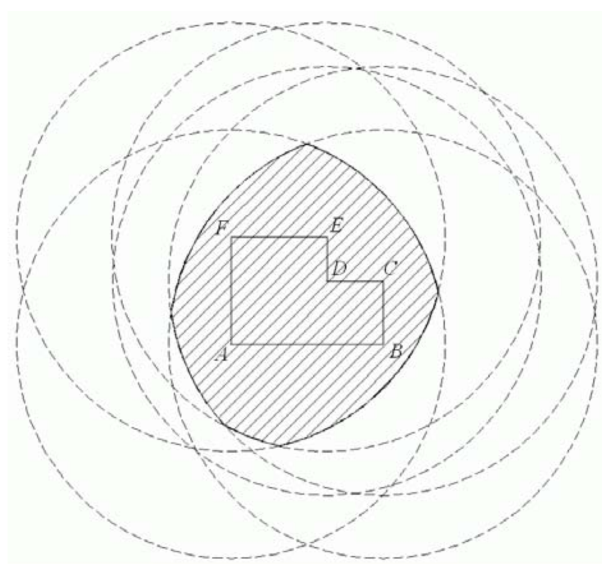
[3] If an intersection point does not exist for an object *i*, the object vertices (Step 2) are necessary critical points.



(a) Polygon object.



(b) Covering circles centered on object vertices.



(c) Covering boundary.

Figure 2.12. Process of generating object covering boundary.

What remains is to prove that the PIPS contain an optimal solution to the EPMC. This proof is similar to that given in Church (1984) for the CIPS applied to the PMCE. Consider any solution to the EPMC and it can be shown there is another solution, at least as good, with all facility locations in the PIPS. If some facility location F of the solution lies outside boundary K_i, then F cannot cover object i. Replacing F by a critical point γ_t on K_i, and in the PIPS, gives a solution at least as good. If some facility location F of the solution lies inside or on exactly one K_i, then it covers object i but no other object l. Replacing F by any critical point on K_i yields a solution equally as good. If some facility F of the solution lies inside two or more covering boundaries K_i, but not on any K_i, move it in any direction until it intersects a K_i containing its original position, *e.g.*, $F \subset K_i$. Replacing F by its new position, F', yields a solution at least as good. Finally, if some facility location F of the solution lies exactly on K_i and inside one or more K_l, move it along the boundary of K_i until it intersects one of the K_l's or until a complete rotation is made about K_i without encountering another boundary K_l. If a move along K_i meets a K_l, replacing F with this intersection point, γ, yields a solution at least as good. If a complete rotation about K_i is made without encountering another boundary K_l, then K_i is contained in K_l, and replacing F with a vertex γ of object i yields a solution at least as good. Assume that all such replacements have been made. Then, the current solution has all facility locations in the PIPS and the proof is complete.

2.6 Application and results

The EPMC is applied to site emergency warning sirens in Dublin, Ohio (Figure 2.13). The city comprises parts of three counties, Franklin, Delaware and Union, with a total area of approximately 60 km^2 (23 square miles) and approximately 29,000 residents (1998). Emergency warning sirens are used to alert the public of an impending danger, *e.g.*, tornado, severe thunderstorm, hazardous material spill, a national threat, *etc*. To site warning sires, two factors are considered: one is that there is a nontrivial cost associated with purchasing and maintaining a warning siren as noted in Current and O'Kelly (1992) and Murray *et al.* (2007); another is that individuals who fail to hear a warning during an emergency period are in danger. The service coverage in this area is provided by omni-directional sirens that have a maximum effective range of 976 meters. Rated performance standards indicate that a siren's electromagnetic (acoustic) waves propagate in straight lines uniformly in all directions, so Euclidean distance travel is appropriate for representing service provision. Then our problem becomes EPMC under Euclidean as EPMCE. All areas of the Dublin region are viewed as being important to provide service to, if possible given budget limitations. Figure 2.14 depicts the region as a "regular" tessellation of 255 polygons, where mostly square polygons 500x500 m in size reflect potential demand for service. By delineating the city into 256 polygons most with 500m x 500m, the problem becomes one to site sirens in continuous space to provide service to these polygons. Demand is assumed uniformly distributed to avoid altering the importance of covering any particular polygon.

The analysis is carried out on a Pentium Xeon 3.0 GHz personal computer with 2 GB RAM. ArcGIS was utilized for spatial data analysis, manipulation and processing. The PIPS approach was structured using ArcObjects, the Visual Basic interface for accessing ArcGIS functions, and the associated EPMCE problem was structured using PIPS and the MCLP and exported as a text file. This text file was imported into CPLEX 8.1 and solved, with solution results exported to ArcGIS for subsequent analysis and display.

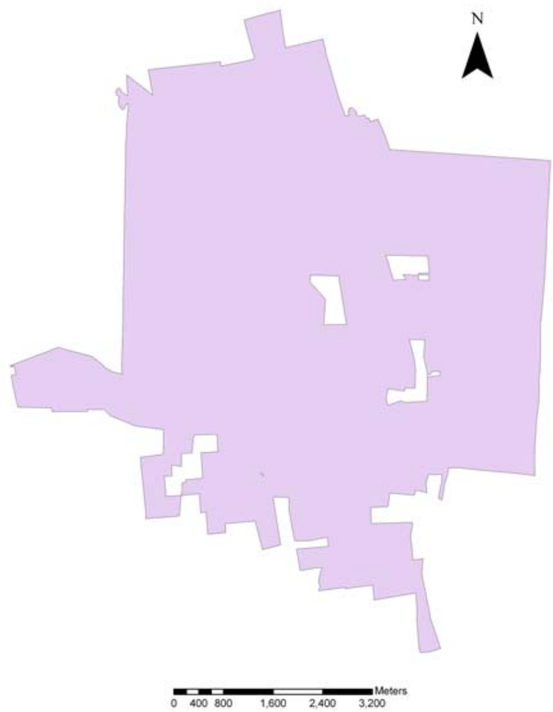


Figure 2.13. Study area of Dublin, Ohio.

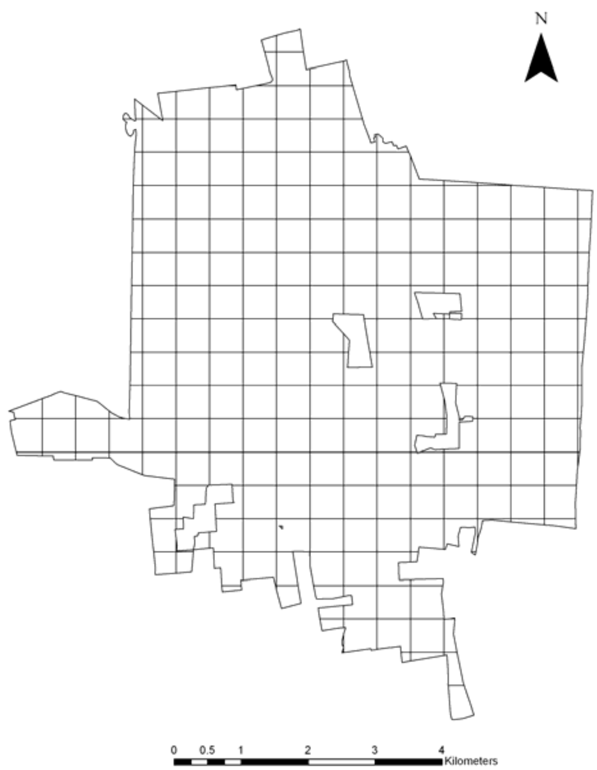


Figure 2.14. A delineation for the region of interest.

Application of the PIPS approach to identify sufficient potential facility locations for the EPMCE resulted in 944 sites with 622 valid sites inside the region in order to cover the associated 255 polygons. Required processing time was 187.95 seconds. If Step 5 of the PIPS is not included to reduce the number of identified potential facility locations, some 6288 sites would be needed with 4,232 valid sites inside the region. Only 6.98 seconds would be necessary to identify the 6,288 sites, but there are almost seven times more sites that would need to be considered. As these additional sites are dominated, they need not be assessed, and if included would likely require significantly more computational effort in problem solution as we have encountered instances where 83 times more processing was needed.

The EPMCE is evaluated for a range of investment options in providing warning siren services, varying *p* from 1 to 30. Solution details and coverage provided for each level of investment are summarized in Table 2.1. Table 2.1 shows that siren coverage ranges from 4.21% when *p*=1 (one siren sited) to 100% when *p*=30 (thirty sirens sited). Model solution time is minimal, requiring at most 118.44 seconds in the case of *p*=29. The coverage tradeoff is depicted in Figure 2.15, based upon coverage for value of *p* considered in Table 2.1. An interesting tradeoff solution is when only 20 sirens could be acquired (*p*=20), and is shown in Figure 2.16. The model finds that 78.45% of demand is suitably covered by this configuration of emergency warning sirens.

p	Demand coverage (%)	Branches	Iterations	Solution time*
1	4.21	0	393	0.16
2	8.38	0	450	0.16
3	12.46	0	486	0.16
4	16.50	0	633	0.19
5	20.52	0	603	0.17
6	24.53	0	686	0.19
7	28.52	0	685	0.20
8	32.50	0	677	0.19
9	36.47	0	817	0.24
10	40.44	0	745	0.23
11	44.34	0	852	0.24
12	48.23	0	883	0.25
13	52.09	0	791	0.23
14	55.92	0	866	0.30
15	59.73	0	887	0.30
16	63.54	0	959	0.33
17	67.36	0	975	0.34
18	71.17	0	1021	0.39
19	74.90	0	1060	0.39
20	78.45	2	1279	1.20
21	81.92	6	1287	2.86
22	85.25	0	1359	0.81
23	88.04	96	3549	9.63
24	90.75	56	3555	10.20
25	93.04	1505	53716	47.84
26	95.28	590	27033	27.92
27	96.92	1018	39457	41.16
28	98.19	3573	127044	90.95
29	99.21	4873	192618	118.44
30	100.00	18	3160	6.99

* Solution time in seconds on Pentium III dual processor 733MHz Windows NT server with 1 GB RAM.

Table 2.1. EPMCE solution details.

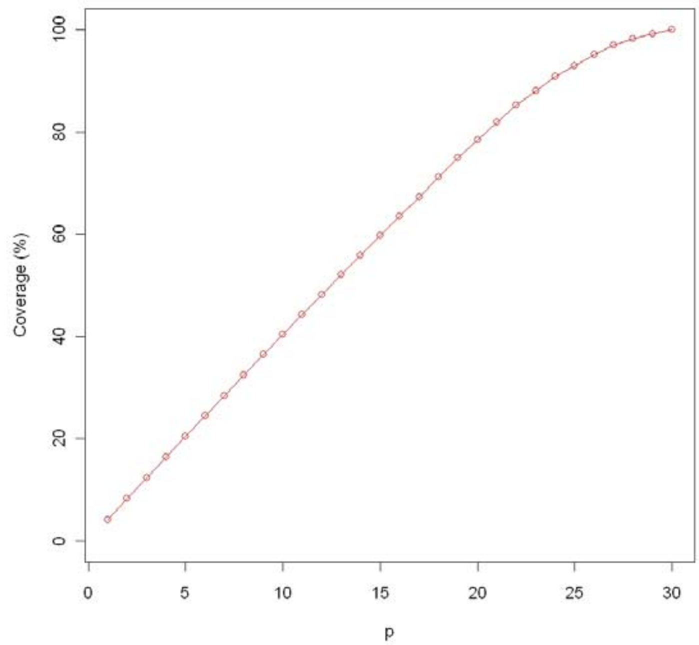


Figure 2.15. Investment tradeoff curve.

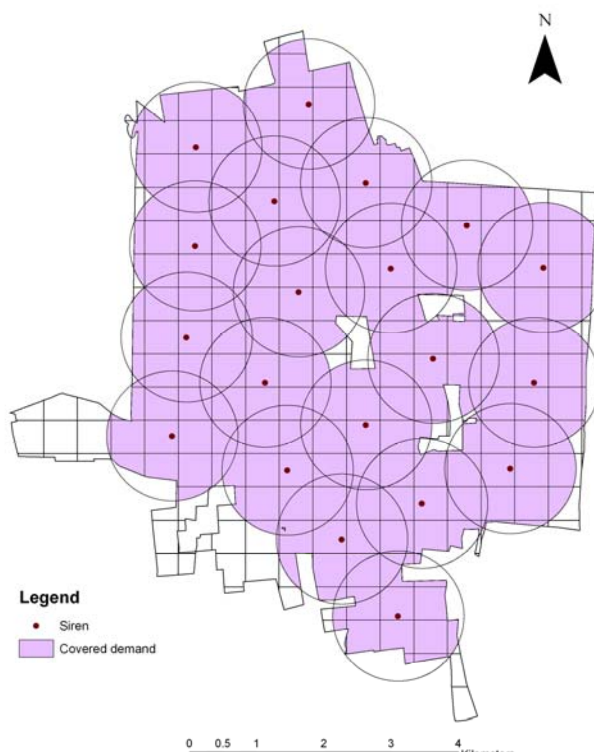


Figure 2.16. EPMCE solution (*p*=20).

2.7 Discussion and conclusions

The application results demonstrate that it is possible to solve the EPMC using PIPS combined with the MCLP (maximal covering location problem). In particular, identifying the PIPS is computationally feasible and the associated MCLP instance is relatively easy to solve using commercial optimization software.

The implications are that this continuous space problem, the EPMC, can take advantage of geometrical insights to derive a tractable solution approach. Thus, PIPS exploits the nature of service coverage to identify suitable potential facility locations.

For the maximal coverage problem, one question then is whether another approach is possible. Certainly one can use the MCLP to cover spatial objects, as this is done here. However, a discrete set of potential facility sites is needed to apply the MCLP. One approach taken in the literature is to generate a uniform pattern of points to represent potential facility locations, as done in Current and O'Kelly (1992) for siting warning. Consider then allowing sirens to be placed at polygon vertices in Figure 2.14. There are 186 such vertices in this case. If an associated MCLP is solved, allowing sirens to be placed at polygon vertices and coverage provided to polygons, one finds that the range of *p* values to consider actually goes from 1 to 51. Thus, it is not until 51 sirens that all demand areas finally are covered. This is an increase of 70% over what the EPMCE found to be necessary for complete coverage. It should be no surprise then that achieving 78% coverage of total demand in this case would require 34 sirens. Not only is this more than what the EPMCE found necessary for complete coverage, but it is 70% more sirens than the EPMCE identified to provide this level of service (i.e., p=20 shown in Figure 2.16). Thus, one cannot expect to reasonably identify sufficient potential facility locations when facilities can be sited anywhere in continuous space.

2.8 Summary

The extended planar maximal covering location problem (EPMC) was applied to address planning situations where coverage maximization is desired assuming that demand is represented as a vector object (point, line or polygon). An approach was proposed to deal with continuous space siting, exploiting geometric insights and demand representation. The polygon intersection point set (PIPS) was proven to contain an optimal solution to the EPMC. Application results demonstrated the computational capabilities for addressing important planning problems. Approaching this complex spatial optimization problem from a GIScience perspective leads to a tractable solution, highlighting the importance of geographical insights.

CHAPTER 3

MAXIMIZING COVERAGE OF SPATIAL DEMAND FOR SERVICE*

Chapter 2 addressed the continuous coverage optimization problem by introducing a finite dominating set (FDS) serving as potential sites on which to locate. The FDS is then used in the maximal coverage location problem (MCLP) for facility selection. The MCLP was initially designed for covering demand represented as points with no dimension. However, when applying the MCLP to spatial objects, there exist significant discrepancies between what is modeled and actual geographic coverage. In order to accurately reflect the mechanism of maximal coverage for spatial objects (points, lines and polygons), in this chapter we propose new models to maximize coverage of spatial demand.

3.1 Introduction

Coverage location problems have attracted significant attention in recent years (Schilling *et al.* 1993; Murray *et al.* 2007). A primary focus has been the maximal coverage location problem (MCLP). Due to limited resources, this problem seeks to locate a fixed number

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of facilities so as to provide service to as much spatial demand as possible. The problem was first introduced by Church and ReVelle (1974). Subsequent work has applied (and/or extended) the MCLP to a variety of problems, including locating health centers (Bennett *et al.* 1982), emergency service planning (Current and O'Kelly 1992; Eaton *et al.* 1986), selection of mailing lists (Dwyer and Evans 1981), resource conservation (Church *et al.* 1996), siting bank branches (Sweeney *et al.* 1979; Pastor 1994), data management (Chung 1986), and air pollution control (Hougland and Stephens 1976) to name a few.

An important challenge in location coverage modeling is how to represent geographic space. Traditionally, due to limited geometric and spatial data handling capabilities, a demand region has been represented as a collection of points, such as centroids of census units (Miller 1996; Church 1999). A coverage goal is then to simultaneously locate facilities to serve a set of weighted demand points. This process involves simplifying and aggregating a continuous area, such as a city or a town, into discrete points. This simplification, and aggregation, could lead to potential measurement and coverage errors in location problems, however (Daskin *et al*. 1989; Current and Schilling 1990; Murray and O'Kelly 2002). Murray and O'Kelly (2002) highlighted that point representation of regional demand generally results in an over-estimation of the actual coverage provided by a configuration of service facilities.

The assumption that space be represented as aggregated points is potentially very problematic, which is why research has broadened to include other spatial entities (e.g., points, lines and polygons). Consider that a region typically exists as an area with varying geometric shape, instead of a collection of points with no dimension. Given this, areas have long been recognized as an important regional representation (see Wesolowsky and Love 1971; Drezner and Wesolowsky 1980; Benveniste 1982; Murray *et al.* 2007). Reinforcing this importance is the work of Miller (1996), highlighting the need for research into alternative representations of space in location modeling arising from the proliferation of geographic information systems (GIS) and associated digital spatial information. That is, GIS make spatial information in the form of points, lines, polygons and raster cells readily available, so analytical methods must be capable of appropriately dealing with alternative representations of space. To this end, more recent research has focused on complete coverage of a region, and explicit treatment of areal units in particular (e.g., Murray 2005; Murray *et al.* 2007).

Given the realities of limited budgets in the provision of public or private services, this chapter examines maximal coverage of areal units of demand. The aim is to investigate approaches for maximal service coverage of a region when demand is represented as an areal unit (or polygon). In the next section, representation issues of regional demand are reviewed. This is followed by a discussion of existing models applicable for maximizing regional coverage. A new approach is then introduced, taking into account areal unit representation. Next, models are tested in the context of warning siren siting. Finally, a discussion and conclusions are provided.

3.2 Background

In coverage modeling, continuously distributed demand in a region has historically been represented as a collection of points (Murray and O'Kelly 2002; Murray *et al.* 2007). These points can be centroids of areal units, such as districts, census units, sectors, *etc.*, or towns or cities at a broader regional scale. They can also be points regularly spaced in a region (Current and Schilling 1990; Murray and O'Kelly 2002). Point representation has long been used due to the fact that simplification makes data handling and solution procedures tractable in many situations (Miller 1996; Church 2002).

Point representation can lead to inaccuracies or errors, however. Using points to represent a region or sub-area can be viewed as an aggregation of an infinite number of locations into a finite number of demand points (Francis and Lowe 1992). Aggregation can introduce a number of potential errors, including inaccurate distance measurement and uncertainty in assessing coverage. As noted by Current and Schilling (1990), aggregation error in coverage models could be dramatic. An area partially covered by a facility can be misclassified as completely covered or not covered at all based on the distance calculated using aggregated locations. Daskin *et al.* (1989) examined aggregation effects specifically for maximum covering models and found that with an increase in aggregation level, optimality and location errors increase accordingly. Murray and O'Kelly (2002) summarized that these inaccuracies or errors are the result of representing geographic space inappropriately through aggregation, when such points are an approximation of continuously distributed demand.

The fact is that demand is not actually point based with no dimension. Rather, demand is often continuously distributed in the form of an areal unit with a specific geometric shape (Murray 2005). Recognizing this, some studies have focused on using areas for regional demand representation. Wesolowsky and Love (1971) explicitly incorporated rectangular areas as demands in a Weber location problem under a rectilinear distance measure. Other shapes and distance metrics were explored for variants of the Weber problem (see Love 1972; Aly and Marucheck 1982; Drezner and Weslowsky 1980; Carrizosa *et al* 1998; Fekete *et al.* 2005, among others). For coverage problems, Benveniste (1982) discussed the evaluation of coverage for located facilities and suggested that analysis results would depend heavily on how a region is partitioned based on existing models. Current and Schilling (1990) and Daskin *et al.* (1989) discussed the issue of spatial representation in covering models. However, demand was evaluated based on aggregated points. Recently, Murray (2005) developed a new model specifically addressing set covering problems where demand is represented as spatial objects, including points, lines or areas.

While issues of aggregation and scale for point representations have been examined in the context of maximal coverage, there has been no investigation of whether spatial objects present challenges to modeling efforts. To address this, we detail new approaches for maximal coverage modeling when demand is represented as spatial units with non-zero dimension.

3.3 Model development

As with any intended planning or analysis, there are often alternative ways to formalize the problem. A direct approach for modeling regional coverage maximization in the siting of service facilities is to use the MCLP (Church and ReVelle 1974) as discussed in Chapter 2. While points are generally relied upon in the application of the MCLP, GIS enables lines or polygons to be evaluated (see Murray 2005). Here we focus on the analysis of polygons (areal units), though more general application to spatial objects is discussed later in the chapter.

We now detail the MCLP, but first consider the following notation:

i = index for demand objects (entire set denoted as I)

j = index for potential facility sites (entire set denoted as J)

S = maximum acceptable service distance (or time)

d_{ij} = distance from demand i to candidate facility location j

$N_i = \{ j \mid d_{ij} \leq S\}$, *i.e.* set of facility sites capable of serving demand i

w_i = service demand at i

p = number of facilities to be located

$$x_j = \begin{cases} 1 & \text{if site } j \text{ is selected} \\ 0 & \text{otherwise} \end{cases}$$

$$y_i = \begin{cases} 1 & \text{if demand } i \text{ is covered (or served)} \\ 0 & \text{otherwise} \end{cases}$$

The maximal covering location problem (MCLP) is formulated as follows:

$$Maximize \quad \sum_{i \in I} w_i y_i \tag{3.1}$$

subject to:

$$\sum_{j \in N_i} x_j \geq y_i \qquad \forall i \tag{3.2}$$

$$\sum_{j \in J} x_j = p \tag{3.3}$$

$$x_j \in \{0,1\} \qquad \forall j \tag{3.4}$$

$$y_i \in \{0,1\} \qquad \forall i \tag{3.5}$$

The objective, (3.1), is to maximize the amount of weighted demand covered. Constraints (3.2) track whether demand *i* is covered by at least one facility located capable of providing service. Constraint (3.3) specifies the number of facilities to be located. Constraints (3.4) and (3.5) impose integrality conditions on decision variables.

In the MCLP, demand *i* is either covered or not covered, as structured using y_i. This is conceptually appropriate when demand is represented a point.[1] However, in the case where demand is represented as an area (or polygon), if only part of an area is covered (within the service standard *S* of the closest facility) the area is not recognized as partially covered in the model. This becomes an important issue in two ways. First, service coverage is underestimated, as shown in Murray (2005) and Murray *et al.* (2007) for set covering. Second, complete coverage provided by multiple facilities cannot be accounted

[1] Murray & O'Kelly (2002) show that coverage models (set covering) using point based abstractions of continuous demand generally are overly optimistic in service provided.

for, an issue highlighted by Murray (2005). Previous research, therefore, suggests that optimizing coverage using the MCLP will be challenging, as there are problems in accurately portraying demand for service due to spatial representation.

The PMP aims to locate a certain number of facilities so that total demand-weighted travel distance from demand to the closest facility is minimized. The PMP was introduced by Hakimi (1964, 1965) and mathematically specified in ReVelle and Swain (1970). Consider the following additional notation:

$$\lambda_{ij} = \begin{cases} 1 \text{ if demand } i \text{ is assigned to facility located at point } j \\ 0 \text{ otherwise} \end{cases}$$

The *p*-median problem (PMP) is formulated as:

$$Minimize \quad \sum_{i \in I} \sum_{j \in J} w_i d_{ij} \lambda_{ij} \qquad (3.6)$$

subject to:

$$\lambda_{ij} \le x_j \qquad \forall i, j \qquad (3.7)$$

$$\sum_{j \in J} \lambda_{ij} = 1 \qquad \forall i \qquad (3.8)$$

$$\sum_{j \in J} x_j = p \qquad (3.9)$$

$$x_j \in \{0,1\} \qquad \forall j \qquad (3.10)$$

$$\lambda_{ij} \in \{0,1\} \qquad \forall i, j \qquad (3.11)$$

Objective (3.6) aims to minimize total demand-weighted distance. Constraints (3.7) require that demand can only be assigned to a sited facility. Constraints (3.8) specify that such assignment for each demand is at most once. Constraint (3.9) dictates the number of facilities to be sited. Constraints (3.10) and (3.11) impose binary integrity on decision variables.

The PMP establishes the assignment relationship of each demand to its corresponding closest facility. In fact, the MCLP and PMP are theoretically linked in that d_{ij} in the PMP can be modified to reflect binary service coverage (see Church and Weaver 1986; ReVelle 1986; Berman and Krass 2002). Thus, one could use either the MCLP or the PMP to achieve coverage modeling goals. However, the PMP would still be sensitive to spatial representation in the same manner as MCLP as discussed above. Thus, neither the MCLP nor the PMP adequately model actual coverage of spatial objects.

In the PMP, the binary variable λ_{ij} can be relaxed to incorporate fractional assignment. Then, a modified PMP adopting this relaxation, taking into account partial area coverage, is an alternative to alleviate representational issues arising from the use of the MCLP or PMP due to their "all" or "nothing" assessment. The underlying idea is to track and optimize the amount of coverage a demand object can receive from a potential facility. PMP variants have been widely applied. Most related to this chapter and the concept of partial coverage is the service quality decay approaches of Church and Roberts (1983), Farhan and Murray (2006) and Berman and Krass (2002). Consider the following additional notation:

c_{ij} = fraction of coverage demand object i receives from facility j

$\Omega_i = \{ j \mid c_{ij} > 0\}$, *i.e.* the set of facilities that can fully or partially cover demand object i

$$y_{ij} = \begin{cases} 1 \text{ if demand } i \text{ is fully or partially served by a facility located at } j \\ 0 \text{ otherwise} \end{cases}$$

With this notation, a straightforward modified version of the PMP can be introduced. This modification makes use of actual coverage provided by a facility, c_{ij}.

P-Median Problem – Single-facility Coverage (PMP-SC)

$$Maximize \quad \sum_{i \in I} \sum_{j \in \Omega_i} w_i c_{ij} y_{ij} \tag{3.12}$$

subject to:

$$y_{ij} \le x_j \qquad \forall i, j \in \Omega_i \tag{3.13}$$

$$\sum_{j \in \Omega_i} y_{ij} \le 1 \qquad \forall i \tag{3.14}$$

$$\sum_{j \in J} x_j = p \tag{3.15}$$

$$x_j \in \{0,1\} \qquad \forall j \tag{3.16}$$

$$y_{ij} \in \{0,1\} \qquad \forall i, j \in \Omega_i \tag{3.17}$$

The objective, (3.12), aims to maximize overall coverage. Demand area i cannot receive coverage from facility j unless facility j is selected in Constraints (3.13). Constraint (3.14) specifies that at most one facility can be considered as providing coverage for each demand object. Accordingly, if there are multiple facilities within the service standard *S* from a demand, Constraints (3.14) along with the objective (3.12) dictate that only the greatest amount of coverage provided by one of the sites will be counted. Constraint (3.15) establishes that *p* facilities be sited. Constraints (3.16) and (3.17) are integrality conditions for decision variables.

Compared to the MCLP, the PMP-SC improves coverage evaluation by introducing partial coverage. However, close inspection reveals that the PMP-SC under-estimates actual coverage in the case where an area is covered by two or more sited facilities. Such a situation is depicted in Figure 3.1, where there is one shown areal unit and two sited facilities. Here Facility 1 covers 66.2% of this areal unit and Facility 2 covers 39.1% of the areal unit. Jointly, some 94.4% of the areal unit is covered. Since neither of the two facilities provides complete coverage of the areal unit, the MCLP would not account for any coverage in the model solution because the unit is not completely covered and there is no mechanism to consider fractional coverage. As a result, coverage is always under-estimated. The implications are that more facilities would likely be required to provide a desired level of coverage using the MCLP due to this under-estimation of coverage. Alternatively, if the PMP-SC is relied upon, only coverage provided by Facility 1 would be accounted for in the model as this is the greater percentage covered (as opposed to 31.9% by Facility 2). As with the MCLP, the PMP-SC also suffers from actual coverage under-estimation when areal units are served by multiple facilities. While the PMP-SC is an improvement over the MCLP in this regard, a representational issue remains. Such a representational issue underlies the modifiable areal unit problem (MAUP) concerns raised in Murray and O'Kelly (2002), Murray (2005) and Murray *et al.* (2007) for set coverage modeling.

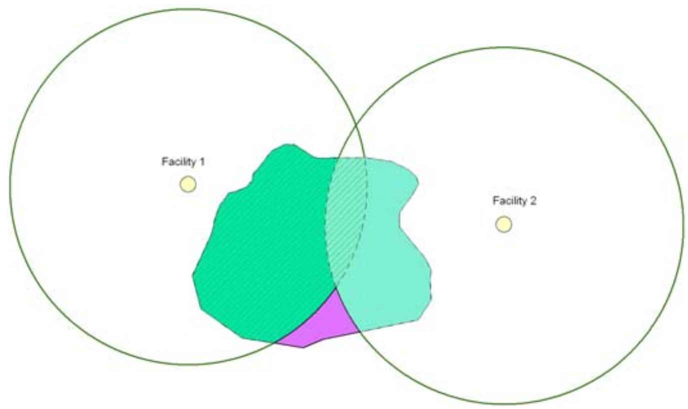


Figure 3.1 Coverage assessment.

Driven by the need to account for the shortcomings of the MCLP and PMP-SC, a more general structure is proposed for the coverage of areal units (or spatial objects). The basic idea is that if multiple sited facilities can provide coverage to a specific demand area simultaneously, the overall joint coverage should be accounted for, if representational issues are to be addressed. We assume here that a maximum of k ($k \geq 1$) facilities can simultaneously provide coverage to a demand area. The single coverage and joint coverage of two or more (up to k) facilities could then be enumerated a priori, provided that potential facility sites are known in advance. This information on joint coverage could then be incorporated in a model for coverage optimization. Consider the following additional notation:

$l \in L = \{1, 2, ..., k\}$, index of the level of multi-facility coverage

h_{il} : number of configurations where there are l facilities simultaneously providing joint coverage to demand i

$m \in M_{il} = \{1, 2, ..., h_{il}\}$, index of configurations of l-facility coverage

Π_{ilm} = set of facilities in the m^{th} configuration of l-facility coverage

$\hat{c}_{ilm}$ = fraction of coverage provided to demand i by facility configuration m at level l

$$z_{ilm} = \begin{cases} 1 \text{ if the demand } i \text{ is either partially or fully covered by configuration } m \text{ at level } l \\ 0 \text{ otherwise} \end{cases}$$

Here, l can equal one to k, and dictates the number of facilities jointly providing simultaneous coverage to a demand i. The potential combinations of such l facilities are enumerated in advance. There are h_{il} such configurations for demand i. Π_{ilm}, then, is the set of facilities, l in total, jointly covering demand i. Incorporating such an extension into the PMP structure can be accomplished as follows.

P-Median Problem – Multi-facility Coverage (PMP-MC)

$$Maximize \sum_{i \in I} \sum_{l \in L} \sum_{m \in M_{il}} w_i \hat{c}_{ilm} z_{ilm} \tag{3.18}$$

subject to

$$z_{ilm} \leq x_j \qquad \forall i, \forall l, \forall m \in M_{il}, \forall j \in \Pi_{ilm} \tag{3.19}$$

$$\sum_{l \in L} \sum_{m \in M_{il}} z_{ilm} \leq 1 \qquad \forall i \tag{3.20}$$

$$\sum_{j \in J} x_j = p \tag{3.21}$$

$$x_j = \{0,1\} \qquad \forall j \in J \tag{3.22}$$

$$z_{ilm} = \{0,1\} \qquad \forall i, \forall l, \forall m \in M_{il} \tag{3.23}$$

The objective, (3.18), maximizes overall coverage. Constraints (3.19) prevent assignment (or coverage) of a demand area by a group of facilities unless the associated facilities are sited. Constraints (3.20) specify that a demand area is covered by at most one group of facilities among all the coverage configurations. Constraint (3.21) specifies the number of facilities to be sited. Constraints (3.22) and (3.23) impose integrality on decision variables.

The PMP-MC is an extension of the PMP-SC (and the PMP) that explicitly tracks combinations of afforded coverage. By taking into account coverage provided by multiple facilities, the PMP-MC reflects the actual mechanism of coverage for areal units. For example, if $k = 2$ in the PMP-MC for the coverage shown in Figure 3.1, there exists at most two facilities that can provide coverage to the one demand area. In total, there are two possible configurations for single level coverage and one configuration for joint coverage by these two facilities. The following constraints would therefore be structured in the associated PMP-MC:

$$\left.\begin{matrix} z_{111} \le x_1 \\ z_{112} \le x_2 \end{matrix}\right\} \text{two single coverage}$$

$$\left.\begin{array}{l} z_{121} \leq x_1 \\ z_{121} \leq x_2 \end{array}\right\} \text{ one joint coverage}$$

$$z_{111} + z_{112} + z_{121} \leq 1$$

In the objective, there would be the individual coverage of 66.2% ($\hat{c}_{111}$ = 0.662) and 39.1% ($\hat{c}_{112}$ = 0.391) of the demand unit by each individual facility, 1 and 2 respectively. For joint coverage of the demand unit, 94.4% is covered ($\hat{c}_{121}$ = 0.944). Thus the objective would be structured as (assuming w_1 = 1):

$$0.662\, z_{111} + 0.391\, z_{112} + 0.944\, z_{121}$$

The objective includes the joint combination of coverage, something not accounted for in the PMP-SC. It should be noted that for k = 1, the PMP-MC is equivalent to the PMP-SC. This means that the PMP-SC is a special case of PMP-MC. As k increases, the number of combinations increases as do the corresponding decision variables. This would no doubt add to model complexity in data processing and problem solution. Suppose there are n demand objects and on average there are m_l combinations for an l-level coverage. Obviously m_l increases with l. The PMP-SC would have $(m_1 -1)n$ more decision variables and $m_1 n$ more constraints than the MCLP. The PMP-MC, for each additional combination level l, requires $m_l n$ more decision variables and $m_l n\, l$ constraints.

3.4 Application

3.4.1 Siren coverage in Dublin, Ohio

The comparative analysis for the reviewed maximal covering modeling approaches examines warning siren siting in Dublin, Ohio (Figure 2.12). Warning sirens are used to alert the public of an impending danger, such as a tornado, severe thunderstorm, hazardous material spill or a national threat. Given that a warning siren is a significant investment, models are essential for identifying optimal siren placement with respect to maximal coverage and efficiency. Consistent with previous research omnidirectional sirens (Whelen WPS-2750) with a maximum effective service range of 976m are considered.

The demand region is partitioned into 256 polygons, most 500m x 500m in size as is done in Chapter 2. The problem is to provide coverage to as much demand as possible, given p sirens to be located. For strategic purposes, sirens may be located anywhere in the region. This infinite set of potential siren locations is then reduced using the polygon intersection point set (PIPS) derived in Chapter 2. It has been shown that PIPS provide at least one optimal solution if demand objects are to be completely covered. There are 622 reduced valid PIPS locations, representing potential facility sites to be evaluated.

The number of sirens, p, evaluated ranges from 1 to 25 (except for the MCLP, which goes up to 29). For the PMP-MC, it is assumed that $k = 2$. The VB ArcObjects interface in ArcGIS is used to structure each optimization model. The mixed-integer problems are solved using a commercial optimization package, CPLEX employing a branch-and-bound technique. Results for the three models are shown in Tables 3.1-3.3. Optimal solutions have been reached except for PMP-MC when $p = 22$. “Model coverage” is the coverage percentage of the entire region derived from the model objective function. “Actual coverage” is the coverage percentage achieved by evaluating the facility sites derived from the model solution in ArcGIS. “Error” is defined as “Actual coverage” less “Model coverage”, and quantifies the discrepancy between the model objective and the actual coverage obtained when evaluating the identified siren locations (model solution). This may be interpreted as an indicator of model correctness. A positive error indicates that the model solution provides more actual coverage, and a negative value would correspond to less coverage than modeled.

p	Model coverage (%)	Actual coverage (%)	Error (%)	Solution time (s)
1	4.17	5.78	1.60	0.16
2	8.32	12.19	3.87	0.30
3	12.34	18.28	5.95	0.45
4	16.35	23.68	7.33	0.59
5	20.34	29.87	9.53	0.75
6	24.34	35.00	10.66	0.91
7	28.32	39.37	11.05	1.05
8	32.29	45.21	12.92	1.19
9	36.26	50.95	14.69	1.34
10	40.15	55.46	15.31	1.48
11	44.01	59.87	15.87	1.64
12	47.84	63.23	15.40	1.78
13	51.65	68.10	16.45	1.92
14	55.47	72.28	16.82	2.06
15	59.28	74.92	15.64	2.20
16	63.10	76.91	13.82	2.36
17	66.89	79.02	12.12	2.52
18	70.69	84.26	13.58	2.67
19	74.33	88.51	14.18	2.81
20	77.66	88.90	11.24	2.97
21	81.05	90.86	9.81	3.13
22	83.84	91.38	7.54	3.27
23	86.55	94.03	7.47	3.41
24	88.91	95.12	6.21	3.56
25	91.07	95.83	4.76	3.72
26	92.98	96.81	3.83	4.53
27	94.07	97.05	2.97	12.00
28	94.94	97.40	2.46	34.95
29	95.63	98.87	3.24	13.00

Table 3.1. Warning siren evaluation using MCLP.

p	Model coverage (%)	Actual coverage (%)	Error (%)	Solution time (s)
1	6.52	6.52	0.00	9.34
2	13.05	13.05	0.00	20.22
3	19.57	19.57	0.00	33.38
4	26.09	26.09	0.00	51.89
5	32.61	32.61	0.00	69.56
6	39.03	39.07	0.04	89.78
7	45.38	45.42	0.04	110.45
8	51.37	51.42	0.04	134.23
9	56.89	57.34	0.45	165.55
10	62.01	63.03	1.02	767.56
11	66.98	68.15	1.17	1444.52
12	71.85	73.20	1.35	1482.47
13	76.25	77.77	1.53	1515.06
14	80.16	82.02	1.86	1541.91
15	83.13	85.39	2.26	1573.59
16	85.83	88.61	2.78	1602.84
17	88.33	91.32	2.99	55.72
18	90.48	93.35	2.87	111.41
19	92.43	94.93	2.49	139.42
20	93.70	96.08	2.38	205.61
21	94.96	97.24	2.27	692.98
22	96.26	98.15	1.89	1196.17
23	97.11	98.62	1.51	1410.25
24	97.88	98.95	1.07	1730.30
25	98.43	99.07	0.64	1861.66

Table 3.2. Warning siren evaluation using PMP-SC.

p	Model coverage (%)	Actual coverage (%)	Error (%)	Solution time (s)
1	6.52	6.52	0.00	35.45
2	13.05	13.05	0.00	35.45
3	19.57	19.57	0.00	52.13
4	26.09	26.09	0.00	48.38
5	32.62	32.62	0.00	54.56
6	39.14	39.14	0.00	58.55
7	45.55	45.55	0.00	75.81
8	51.81	51.81	0.00	87.41
9	57.84	57.86	0.01	119.78
10	63.55	63.56	0.01	344.08
11	68.91	68.95	0.04	852.39
12	74.21	74.23	0.02	899.56
13	79.21	79.28	0.06	390.73
14	83.56	83.95	0.39	352.33
15	86.59	86.63	0.04	9100.11
16	89.51	89.66	0.15	4610.328
17	92.43	92.69	0.27	5264.421
18	94.33	94.68	0.35	8525.64
19	95.79	95.98	0.19	28563.52
20	97.04	97.25	0.21	38039.48
21	98.10	98.41	0.31	43777.22
22*	98.67	98.84	0.17	33159.38
23	99.27	99.37	0.10	11429.38
24	99.64	99.69	0.05	6492.69
25	99.89	99.93	0.03	3006.69

*"Out of memory" was encountered and a suboptimal solution was reported with maximum optimality gap of 0.22%.

Table 3.3. Warning siren evaluation using PMP-MC.

Coverage errors are also displayed in Figure 3.2. All errors are non-negative for the three models. This is due to the fact that the models never over-estimate coverage. The MCLP simply accounts for complete coverage provided by a facility, ignoring any partial coverage. The PMP-SC only accounts for the greatest coverage provided by a single sited facility. The PMP-MC accounts for coverage provided by up to *k* (2 in this case) selected facilities, if joint coverage exists. In Figure 3.2, the MCLP has the largest errors and the PMP-MC has the smallest, as expected. For the MCLP, errors average 9.87% and the highest is 16.82% (p = 14), which implies that substantially more facilities (or investment) are required by the MCLP than actually needed for any specified level of coverage. For the PMP-SC, coverage errors average 1.23%, with the highest 2.99%. As expected, for small p, the selected siren configurations are spatially dispersed and no coverage errors exist. In this situation the coverage for each demand object is correctly accounted for in the objective function because each demand is only served by one sited warning siren. When the service areas of the selected facilities start to overlap, however, coverage provided by multiple facilities is under-counted in the PMP-SC. Thus, errors appear for the PMP-SC for $p > 5$. For the PMP-MC, coverage errors are reduced significantly to a maximum of 0.39% (p = 14). Comparatively, by introducing a higher level coverage, the errors in the PMP-SC are reduced from an average of 1.23% to 0.09% across the examined ranges.

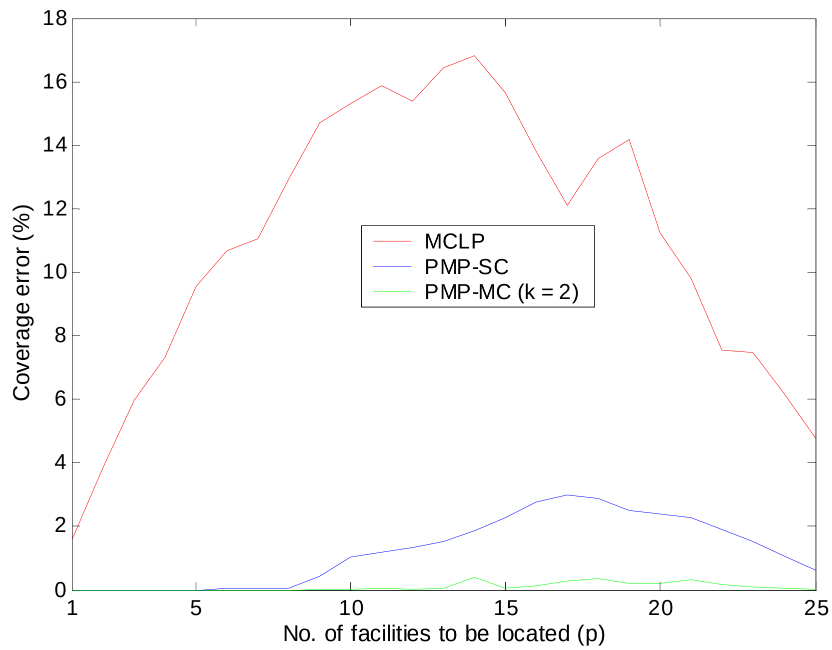
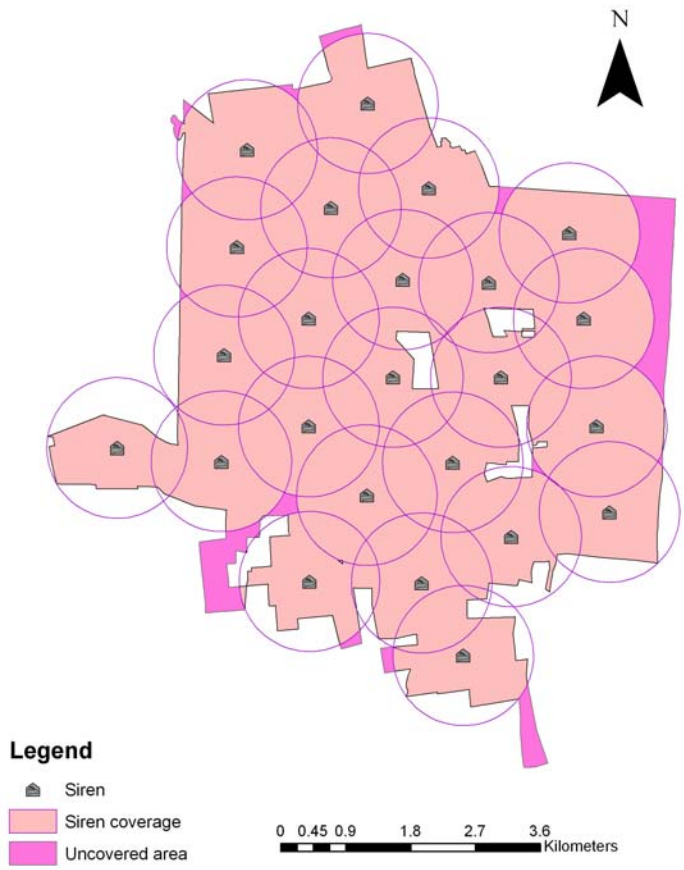


Figure 3.2. Coverage errors for the three models.

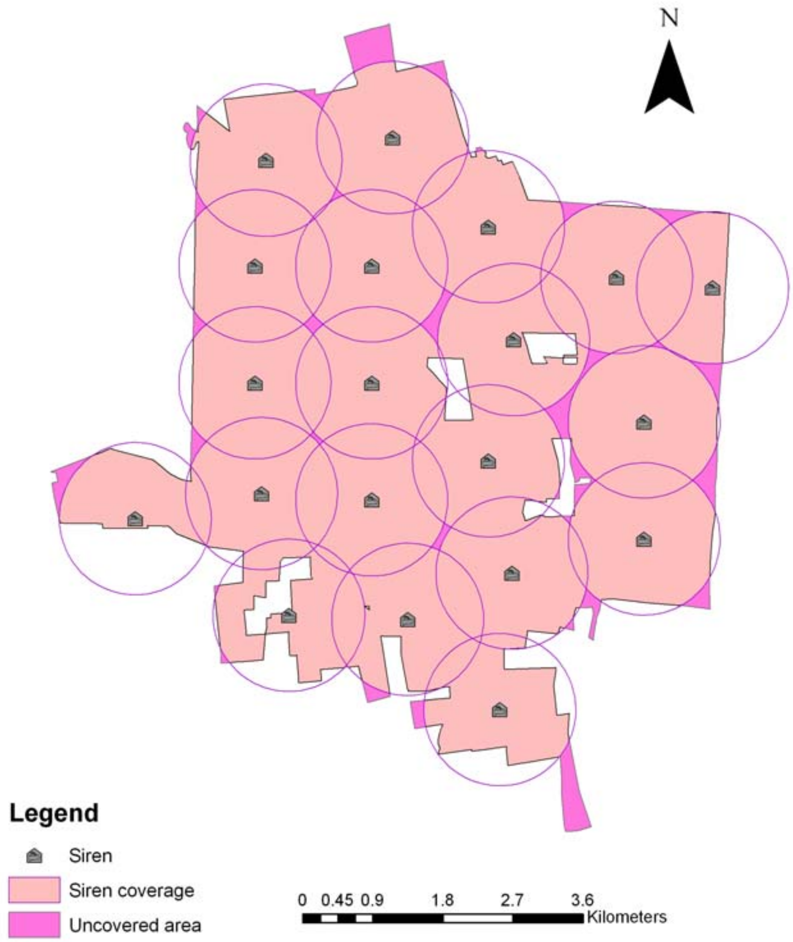
Comparing the coverage achieved by the three models, for a given *p*, the PMP-MC always identifies a configuration giving the greatest coverage and the MCLP the least. This means that obtaining a certain level of coverage requires the MCLP to need more sirens (or monetary investment), while the PMP-MC the least investment. For example, if 95% coverage is necessary for the region, the MCLP finds that twenty nine sirens (p = 29) are required, or twenty four (p = 24) if actual coverage is considered. The PMP-SC identifies twenty two sirens as necessary, or twenty considering actual coverage. On the other hand, only nineteen sirens (p = 19) would be needed to achieve the required level of coverage using the PMP-MC, irrespective of whether modeled or actual coverage is considered. The three configurations achieving 95% coverage (actual) are shown in Figure 3.3. The discrepancy between the number of sirens required by the three models to achieve this service level illustrates the representation issues inherent to both MCLP and PMP-SC. Considering the non-trivial investment in facilities (sirens in this case), the budget implications are potentially significant (24 in the case of the MCLP, 20 in the case of the PMP-SC and 19 in the case of the PMP-MC).



(a)

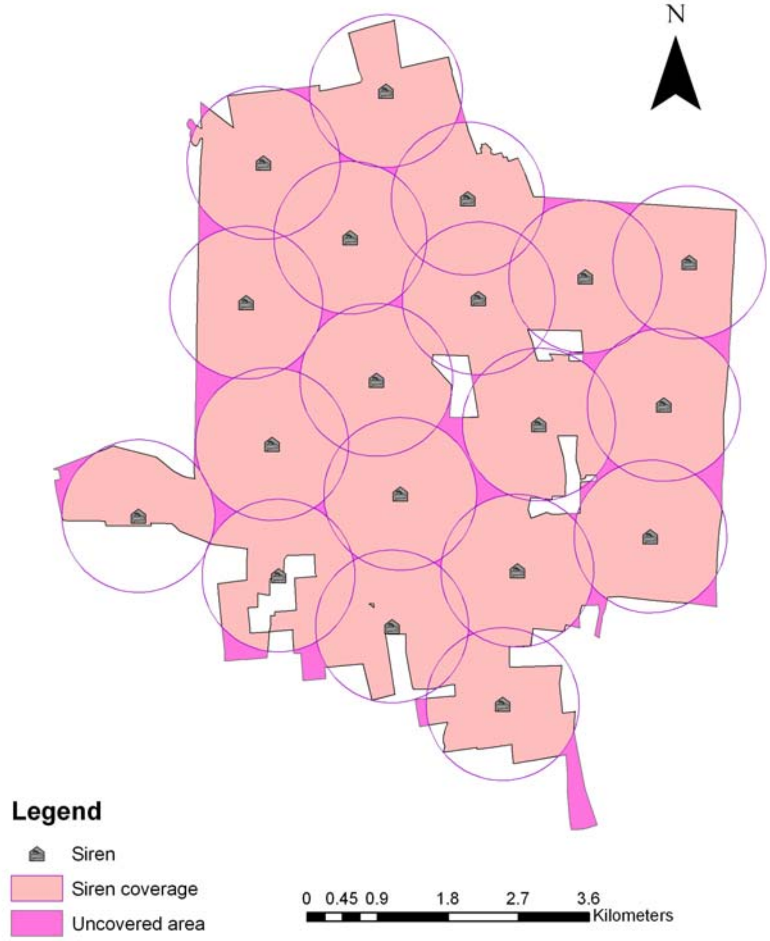
Figure 3.3. Siren configurations covering at least 95% of the region for: (a) MCLP (p=24, actual coverage), (b) PMP-SC (p = 20, actual coverage), and (c) PMP-MC (p = 19).

Figure 3.3 continued



(b)

Figure 3.3 continued



(c)

Computationally, the PMP-MC requires more effort, both in pre-processing of data and problem solution using CPLEX. The results reported here using CPLEX required an average of 3.88 seconds to solve the MCLP, about 12 minutes to solve the PMP-SC and 2.17 hours to solve the PMP-MC (k=2). Of course these times do not include data pre-processing. It is important to note that for the PMP-SC and the PMP-MC when p is large, substantive computational time has been invested. For instance when p=22, twenty minutes are required for the PMP-SC, and over nine hours are needed just to find an approximate solution for the PMP-MC (potentially suboptimal).

3.4.2 Siren coverage in Franklin, Ohio

The siren coverage analysis was also carried out for Franklin County, Ohio. Currently, there are 200 existing sirens covering 58.6% area of the region as shown in Figure 3.4. The objective is to evaluate coverage provided by additional sirens. The remaining uncovered area is derived using GIS and displayed in Figure 3.5.

The uncovered area is delineated into 1506 polygons, most 800m x 800 m in size. 2156 regularly distributed points are chosen as potential siren sites. The number of sirens to be sited ranges from 1 to 100. The three models (MCLP, PMP-SC, PMP-MC (k =2)) are applied to solve the coverage problems. Note that the sirens in Franklin County differ from those in Dublin, having a service standard of 1585 m.

Figure 3.6 provides the derived overall coverage with additional sirens located. By considering the partial coverage and joint coverage in the PMP-MC, more coverage can be achieved than if the MCLP or the PMP-SC are used. For instance, with an additional 86 sirens, 77.33% or 92.59% of the region is covered for MCLP and PMP-SC, respectively. However, an overall coverage of 93.14% can be obtained if the PMP-MC (k = 2) is used.

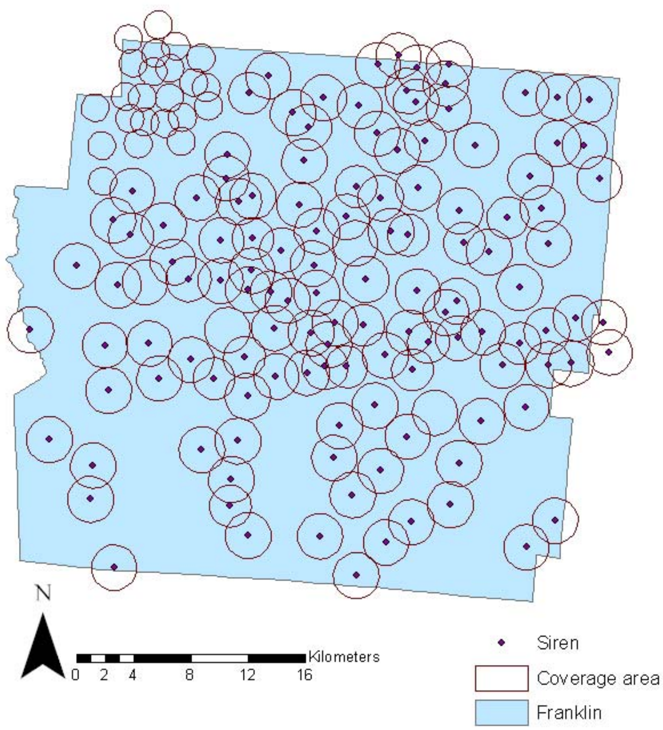


Figure 3.4. Existing warning siren coverage in Franklin Co., Ohio.

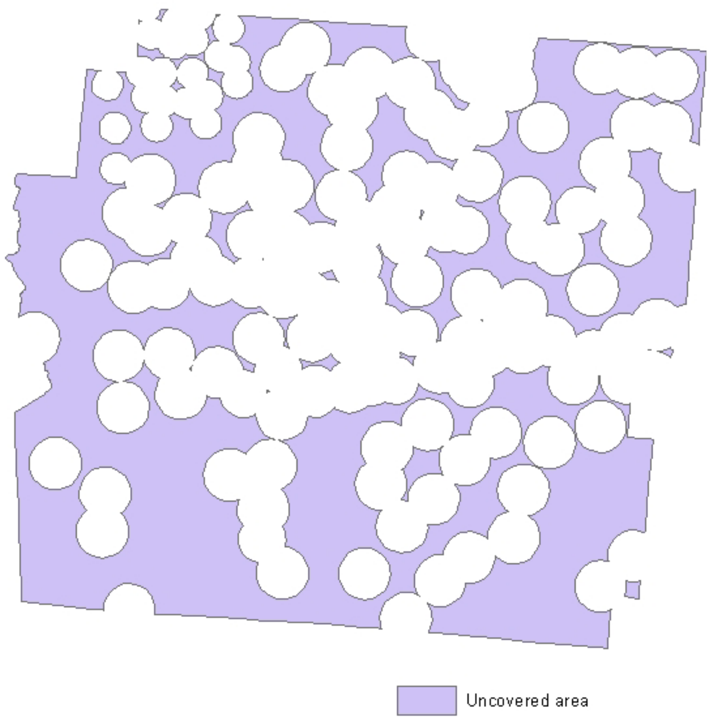


Figure 3.5. Uncovered area in Franklin Co.

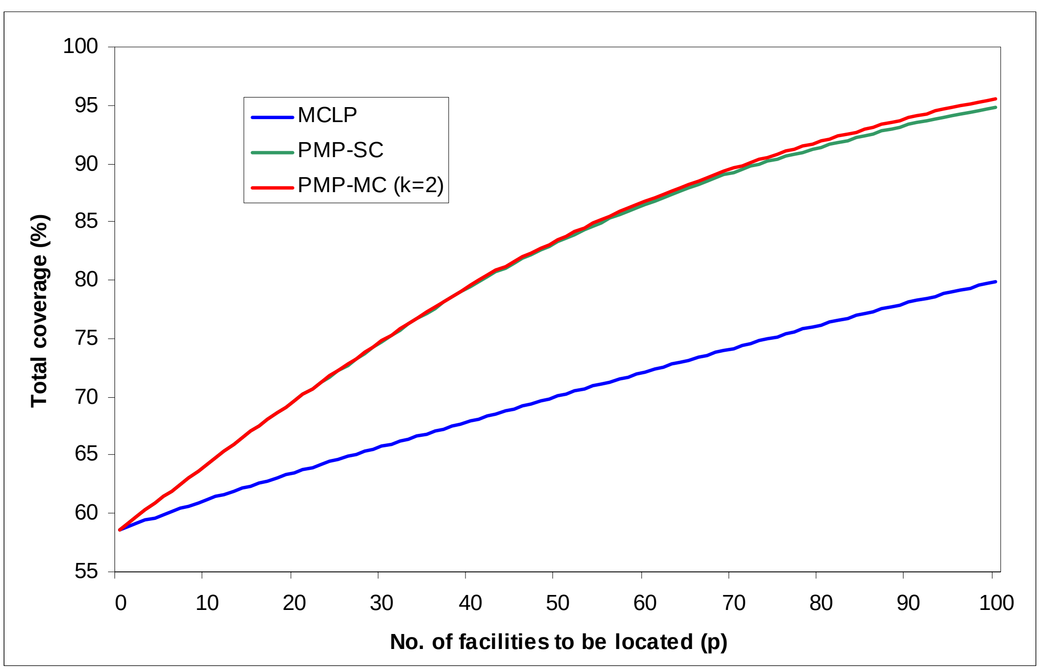


Figure 3.6. Additional warning siren coverage.

Coverage errors are shown in Figure 3.7. Again, substantial errors (an average of 71.05%) result when the MCLP is used due to its conservative modeling characteristic. In contrast, introducing partial coverage in the PMP-SC dramatically reduces errors, limiting them to under 2%. For PMP-MC (k=2), the maximum error decreases to 0.03%. In particular, coverage errors for p=1 to 91 are 0 and very small for $p > 92$. This indicates that for the current delineation of the region and the potential sites to be located, k=2 is effective.

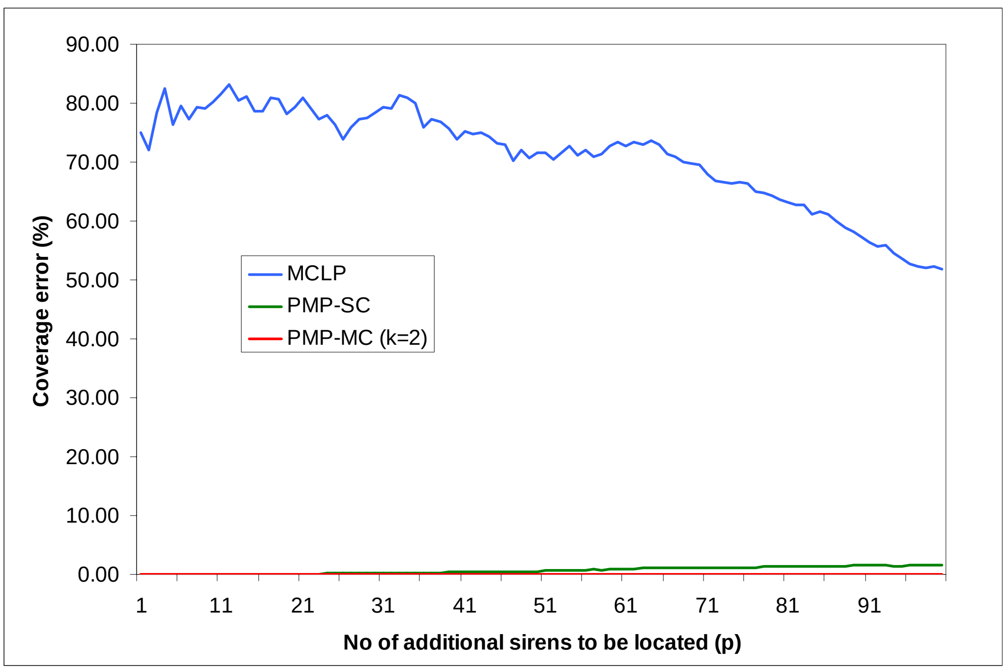


Figure 3.7. Coverage errors for the three models (Franklin Co.).

Examining the differences between results in another way, achieving 95% coverage in the region would require 101 sirens using the PMP-SC but just 96 sirens using the PMP-

MC (k=2). Using the MCLP finds that over 120 warning sirens are required, many more than necessary. Again, for the MCLP and the PMP-SC, failing to accurately model the coverage between facilities and demand objects results in unnecessary investment. The optimal configuration of 96 sirens uing the PMP-MC (k=2) is shown in Figure 3.8.

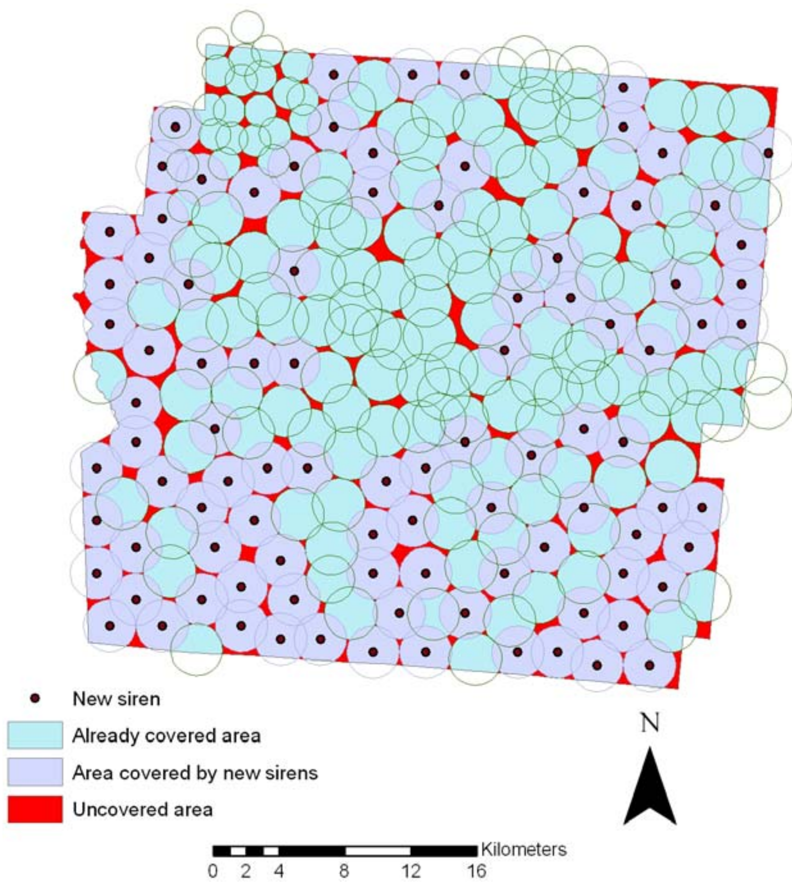


Figure 3.8. Siren configuration covering at least 95% of Franklin Co.

Computationally, an average of 8 seconds was needed to solve the MCLP, 12 minutes to solve the PMP-SC, and 20 minutes to solve the PMP-MC (k=2). Although there are 1506 demand objects and 2156 potential sites for Franklin County, as opposed to 250 demand objects and 622 potential sites for Dublin, less computational effort was required. This is due to the fact that for Franklin County, the uncovered region is scattered, dividing the region into several disconnected smaller regions. This results in fewer joint coverage combinations for the PMP-MC. In fact, for the PMP-MC (k=2) there are 111,560 constraints for Franklin County, but 258,714 constraints for Dublin, more than twice that of Franklin County.

3.5 Discussion and conclusions

The application results demonstrated that consideration of representation issues is important for achieving efficiencies in coverage maximization. Introducing partial coverage at multiple levels for areal units in the PMP-MC enables one to better model the problem. Although demand focused on areas (or polygons), the PMP-MC can also be applied to any type of object, including points, lines, curves, *etc*. GIS enables such spatial objects to be considered and evaluated in the context of coverage.

Theoretically, by incorporating sufficient potential multiple facilities to provide joint coverage, the PMP-MC is free of the modifiable areal unit problem (MAUP) and is, therefore, frame independent. For maximal coverage, it is desirable for facilities to be dispersed spatially as opposed to clustering together. Thus, it is less likely that a very high level of joint coverage is ever necessary. Empirically, the reported findings support this, as even using $k = 2$ resulted in very small coverage errors (at most 0.39% for the

Dublin application and 0.03% for the Franklin County application). Further, computational experience with alternative spatial representations supports this as well.

3.6. Summary

This chapter discussed the problem of maximizing coverage when regional demand is represented as areal objects. We introduced general model formulations to account for fractional coverage of objects. The application results demonstrated that incorporating coverage combinations effectively eliminates representation errors, making the modeling effort more reliable. Although regular circles were adopted for facility service due to the nature of siren performance and that most demand units are regular polygons, the PMP-MC is not limited to regularly shaped objects. For any regular or irregular geometric shape corresponding to service coverage or demand, such spatial objects can readily be evaluated within GIS, enabling an associated PMP-MC to be structured for subsequent analysis. Whether excessively coarse demand units combined with limited coverage parameters are a problem remains an open question. Further, computational issues are clearly an avenue for further exploration. As shown for the applications, incorporating joint coverage in the PMP-MC required substantial additional computational effort to obtain optimal solutions for large p values. The next chapter will address this computational issue by developing a heuristic that identifies solutions of high quality in a reasonable time.

CHAPTER 4

AN EFFICIENT GENETIC ALGORITHM TO MAXIMIZE REGIONAL SERVICE COVERAGE

Chapter 3 introduced a new model that accurately reflects the mechanism of maximal coverage for spatial objects by overcoming the discrepancies between what is modeled and actual geographic coverage. The model was solved using commercial optimization software, CPLEX, incorporating a branch and bound technique. Given that the solution time required was considerable, especially for large *p*, and that CPLEX is expensive, an alternative optimization technique is appealing. In this chapter, we propose an efficient evolutionary algorithm to solve the new maximal coverage model, the *p*-median problem – multi-facility coverage (PMP-MC), facilitating the applicability of the model to assist planning and decision making processes.

4.1 Introduction

Coverage in location modeling refers to customers receiving suitable service from one or more facilities (Daskin 1995). Coverage related service planning has been of interest since the 1970's and has been widely applied, including pizza delivery, bus stop design (Gleason 1975), emergency facility siting (Toregas and ReVelle 1972; Current and O'Kelly 1992), wireless communication planning (Murray and O'Kelly 2002; Murray *et* *al.* 2007), and trim placement in the apparel industry (Grinde and Daniels 1999), among others. Often times one is faced with a situation where not enough resources are available to provide complete regional coverage. This gives rise to an important type of coverage problem known as maximal covering. This problem aims to find a solution for placement of a limited number of facilities that can serve the greatest demand possible.

Church and ReVelle (1974) first formulated the maximal coverage location problem (MCLP). This model is a combinatorial optimization problem that is known to be is NP-hard[1] (Megiddo *et al.* 1983), meaning that it is computationally difficult to solve, particularly for large sized problems. As a result, heuristic procedures can be useful and/or essential. Although, heuristics do not guarantee a global optimum, if well structured, they are generally able to provide solutions of acceptable quality in a reasonable solution time.

A literature survey indicates that a few studies have been focused on developing heuristic approaches for the MCLP. The earliest was *greedy adding* and *greedy adding with substitution* heuristic procedures introduced by Church and ReVelle (1974). In these methods facilities are iteratively added to a solution in a greedy manner. To improve the simple greedy selection, Resende (1998) used a *greedy randomized adaptive search procedure* (GRASP) where the greedy search includes a parameterized randomization process. Adenso-Diaz and Fodriduez (1997) also applied a simplified *tabu* search for the MCLP. *Lagrangean relaxation* has also been used to solve the MCLP (see Weaver and Church 1984; Pirkul and Schilling 1989; Galvao and ReVelle 1996) and more recently,

[1] An NP-hard problem is a problem that no algorithm has been discovered to solve in polynomial time.

Karasakal and Karasakal (2004) presented a Lagrangean relaxation approach for a maximal coverage problem where service quality decays with distance. *Genetic algorithms* have also attracted attentions in recent years for solving MCLP (see Jaramillo *et al.* 2002). Jia *et al.* (2007) detailed a genetic algorithm, a location-allocation heuristic and a Lagrangean relaxation as heuristic approaches to solve the MCLP.

The heuristics mentioned above have been used to solve the MCLP, or a simple variant of it. However, for the PMP-MC introduced in Chapter 3, demand is represented as non-point objects and partial and/or joint coverage is considered instead of the "all" or "nothing" criteria used in the MCLP. This makes the PMP-MC more difficult to solve than the MCLP as an optimization model, because the specific assignment of coverage must be tracked. This chapter focuses on developing a new efficient and effective genetic algorithm (GA) for the PMP-MC. In particular, a new crossover operator based on the characteristic of maximal covering is introduced. The new operator accelerates the convergence of the GA compared to traditional operators. This algorithm can also be tailored to other variants of the maximal coverage problem or the *p*-median problem.

The chapter is organized as follows. First, the formal specification of the PMP-MC is presented. Then, related work on genetic algorithms is reviewed, followed by the proposed heuristic. Application results are then given. The chapter ends with discussion and summary comments.

4.2 Coverage model

Introduced in Chapter 3 was the *p*-Median problem – multi-facility coverage (PMP-MC) to address coverage optimization modeling so as to reduce/eliminate modifiable areal unit problem (MAUP) issues. The PMP-MC is an extension of the *p*-median problem (PMP), and therefore related to the MCLP. In contrary to the "all" or "nothing" criterion structured using 0's and 1's in the MCLP, partial coverage and joint coverage provided by multiple facilities are taken into count in the PMP-MC. As noted in Chapter 3, PMP-MC is a general formulation, having as special cases the PMP, the MCLP and the PMP-SC. Considering the following notation:

i = index for demand objects (entire set denoted as I)

j = index for potential facility sites (entire set denoted as J)

S = maximum acceptable service distance (or time)

w_i = service demand at i

p = number of facilities to be located

$l \in L = \{1, 2, ..., k\}$, index of the level of multi-facility coverage

h_{il} : number of configurations where there are l facilities simultaneously providing joint coverage to demand i

$m \in M_{il} = \{1, 2, ..., h_{il}\}$, index of configurations of l-facility coverage

Π_{ilm} = set of facilities in the m^{th} configuration of l-facility coverage

$\hat{c}_{ilm}$ = fraction of coverage provided to demand i by facility configuration m at level l

$$x_j = \begin{cases} 1 & \text{if site } j \text{ is selected} \\ 0 & \text{otherwise} \end{cases}$$

$$z_{ilm} = \begin{cases} 1 \text{ if the demand } i \text{ is either partially or fully covered by configuration } m \text{ at level } l \\ 0 \text{ otherwise} \end{cases}$$

As noted previously, l can equal one to k, and dictates the number of facilities jointly providing simultaneous coverage to a demand i. The potential combinations of such l facilities are enumerated in advance. There are h_{il} such configurations for demand i. Π_{ilm}, then, is the set of facilities, l in total, jointly covering demand i.

The PMP-MC is formulated as follows:

$$Maximize \sum_{i \in I} \sum_{l \in L} \sum_{m \in M_{il}} w_i \hat{c}_{ilm} z_{ilm} \tag{4.1}$$

subject to

$$z_{ilm} \le x_j \qquad \forall i, \forall l, \forall m \in M_{il}, \forall j \in \Pi_{ilm} \tag{4.2}$$

$$\sum_{l \in L} \sum_{m \in M_{il}} z_{ilm} \le 1 \qquad \forall i \tag{4.3}$$

$$\sum_{j \in J} x_j = p \tag{4.4}$$

$$x_j = \{0,1\} \qquad \forall j \in J \tag{4.5}$$

$$z_{ilm} = \{0,1\} \qquad \forall i, \forall l, \forall m \in M_{il} \tag{4.6}$$

The objective, (4.1), maximizes overall coverage. Constraints (4.2) prevent assignment (or coverage) of a demand object by a group of facilities unless the associated facilities are sited. Constraints (4.3) specify that a demand area is covered by at most one group of facilities among all the coverage configurations. Constraint (4.4) specifies the number of facilities to be sited. Constraints (4.5) and (4.6) impose integrality on decision variables.

The PMP-MC can be constructed with the strict situation modeled in the MCLP, where coverage of a demand unit is either all or nothing by one or more facilities. The PMP-MC relaxes the interpretation of coverage to be partial or complete, as provided by one or multiple facilities. Explicitly tracked are all combinations of afforded coverage. In doing so, the PMP-MC reflects the actual mechanism of coverage for demand objects. Therefore, problematic geographic representation issues inherent in the MCLP are addressed, as discussed in Chapter 3.

4.3 Solving the PMP-MC

As a mixed integer linear problem, the PMP-MC can theoretically be solved using commercial optimization software. For example, in Chapter 3, the PMP-MC was solved using CPLEX, which employs linear programming combined branch and bound. However, as a more complicated version of the NP-hard problem than the MCLP, the PMP-MC will generally be more difficult to solve exactly, especially for large sized problems. Recall the modest sized applications examined in Chapter 3, where the PMP-MC (k=2) for p = 23 required over 12 hours to identify an optimal solution. Such computational time could prevent the PMP-MC from being used in practice, but also highlights inevitable limits for large scale real-time planning. On the other hand, consider that heuristics are able to provide a high-quality (optimal or near-optimal) solution quickly, although optimal solutions are not guaranteed. To facilitate the broad applicability of the PMP-MC, our research aims to develop an effective heuristic for solving the PMP-MC quickly and ensure high-quality results.

4.4 Related genetic algorithms

Considerable success in solving location models using heuristics has been achieved (Mladenovic *et al.* 2007). Among developed heuristics, genetic algorithms (GAs) have attracted much attention recently, as they are very promising for solving combinatorial optimization problems (Blum and Roli 2003). Genetic algorithms (GAs) were proposed in the 1970s by Holland (1975). GAs are recognized for their strategy of mimicking biological evolution of natural selection - survival of the fittest (Goldberg 1989). During the evolutionary search process, fitter individuals representing better solutions are selected for reproduction. The information associated with such solutions is, therefore, more likely to survive. A population evolves over generations with an attempt to find (possibly) good solutions.

In a GA, each individual in the population is called a chromosome, representing a solution. Each element in a chromosome is called a gene. Traditionally, solutions are represented as a binary string of 0's and 1's, but other non-binary encodings, including floating point representation, integer representation, *etc.*, have also been designed and applied for specific problems (Davis 1991; Michalewicz 1992; Syswerda 1991). The quality measure of a solution is called its “fitness” and is evaluated using a devised fitness function. Generally, GAs consist of three primary operators (Mitchell 1996): selection, crossover and mutation. Selection is the process through which solutions are selected for creating offspring. Crossover is a strategy for producing new chromosomes or child solutions through a recombination of a pair of selected chromosomes or parent solutions. Mutation changes the genes randomly and is used to restore lost or unexplored genes into the population to prevent premature convergence. A general genetic algorithm is outlined in Figure 4.1.

```
Initialize population

Repeat
        Select parents
        Perform crossover and mutation operations
        Generate next generation

Until <terminating condition >
```

Figure 4.1. Structure of a general GA.

GAs have been extensively used for different optimization problems (Goldberg 1989; Davis 1991; Bäck 1993). In recent years, there has been increased interest in applying GAs to location optimization problems, including the location set covering problem (Beasly and Chu 1996; Lorena and deSouza-Lopez 1977; Aickelin 2002), the maximal coverage location problem (Jaramillo *et al.* 2002; Jia *et al.* 2007), and the *p*-median problem (Hosage and Goodchild 1986; Alp *et al.* 2002; Correa *et al.* 2004; Fathali 2006), and site search problems (Xiao 2006), among others. Jaramillo *et al.* (2002) compared and summarized the performance of genetic algorithms for several location problems, including the *p*-median problem and the maximal covering problem.

GAs applied to the location problems discussed above are mainly for the case where points are used to represent demand and only one facility is allowed to provide complete service. We aim here to develop an efficient GA for the PMP-MC, where multiple facilities are allowed to provide joint coverage to a demand object. Further, complex spatial geometry relationships between sited facilities and demand are allowed, in contrast to previous work. Therefore, a different fitness evaluation function based on spatial relationships other than the simple evaluation of coverage in the MCLP or the PMP needs to be designed. In addition, due to the complexity of the PMP-MC, a new crossover operator is also needed, as the commonly used crossover operators typically require a considerable computational time to converge.

4.5 Proposed genetic algorithm for the PMP-MC

4.5.1 Chromosome coding

The chromosomes in a GA correspond to solutions in the associated optimization problem. The one-to-one relationship between a chromosome and the corresponding solution is specified in the encoding. Individual representation is important in GAs since a gene is directly manipulated (Koza 1992). Traditionally, solutions are represented as binary strings in location problems, with all the potential facilities represented as 0s and 1s (see Hosage and Goodchild 1986; Jaramillo *et al.* 2002). As noted by Dibble and Desham (1993), this encoding scheme does not guarantee a feasible child after crossover and the use of a penalty function is needed to address violated constraints, *e.g.*, the number of required facilities. Unfortunately, this has led to poor GA performance, even for very small problems (Dibble and Desham 1993). Correa *et al.* (2004) also highlighted that using classic binary encoding wastes memory and processing time as well as evaluation time. To avoid invalid offspring, in this paper we use an integer vector encoding scheme, where the genes of facility indices, *p* in total, correspond to a potential solution. In doing this, Constraint (4.4) is automatically satisfied. This coding scheme has recently been used in GAs for *p*-median problems (e.g., Correa *et al.* 2004; Alp *et al.* 2002; Jia *et al.* 2007).

4.5.2 Fitness evaluation

The fitness function in a GA is a measure of goodness of a solution. In our GA, according to objective (4.1), the fitness for each chromosome is the overall coverage provided by the selected facilities specified in the chromosome. Although the general concept is similar to that of GA's used to solve the MCLP (see Jaramillo *et al.* 2002), the coverage evaluation for spatial demand objects is more complicated in this case. In the MCLP, the coverage evaluation for a demand unit by one sited facility is independent of other sited facilities due to the 0/1 coverage assessment. However, in the PMP-MC, both partial coverage by one facility and joint coverage involving multiple facilities are considered.

Combining Constraints (4.2) and (4.3), the coverage for each demand object is maximal among the coverage provided by all the facility configurations at all levels (l=1, 2,… k). This requires computational effort devoted to coverage evaluation for each demand object by all the potential configurations consisting of up to k facilities.

In order to efficiently evaluate the coverage for a demand object, consider:

(1) If any configuration can provide complete coverage, object i is considered covered and it is not necessary to evaluate any other configuration for this object.

(2) If a facility cannot provide any coverage for a demand object, a configuration consisting of that facility cannot achieve more coverage than that at a lower level of coverage without the facility. Therefore, there is no need to evaluate any configuration consisting of this facility for coverage of the demand object. The same applies to any combination of facilities.

The second situation is illustrated in Figure 4.2, depicting three facilities and one demand object. Here, Facility 1 is not able to provide any coverage to the demand area since the demand is located beyond the effective range of the facility. Consider the coverage provided by the configuration of level l=3 consisting of all the three facilities. The overall coverage does not exceed the joint coverage provided the configuration of level l=2 consisting of facilities 2 and 3. Hence, it is not necessary to evaluate the configuration of level l=3 in this case in coverage evaluation.

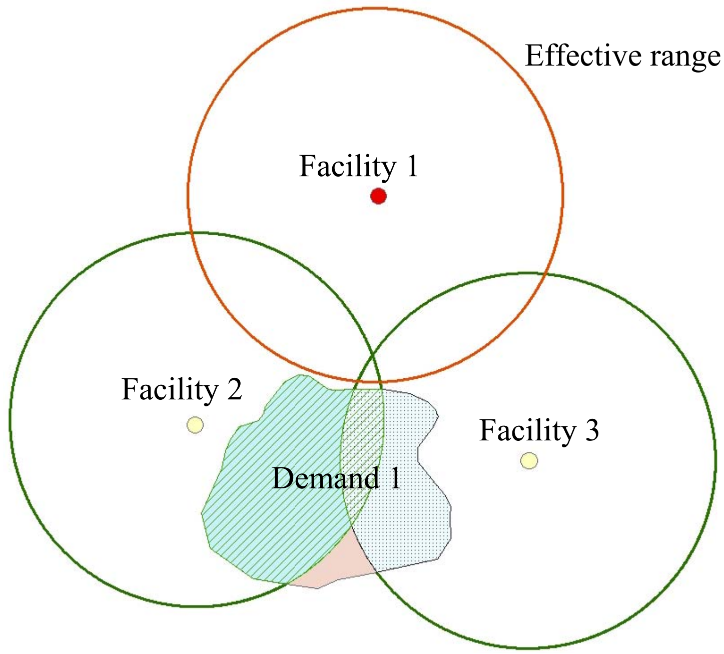


Figure 4.2. Coverage evaluation.

After evaluating the configurations of all levels for a demand object, the maximal value is chosen as the coverage the demand object receives. The fitness for an individual is then the overall coverage for all demand objects. A pseudo-code detailing the fitness calculation for an individual is constructed as follows:

```
procedure fitness_calculation(individual)
1. ind_fitness ← 0;
2. for each demand object i
3.     best_fitness ← 0;
4.     for all configurations of each level l in 1: k
5.         cur_fitness ← 0;
6.         if every single facility can provide some coverage to object i
7.             cur_fitness ← coverage provided by a configuration of l
               facilities;
8.             if cur_fitness > best_fitness, best_fitness ← cur_fitness;
9.             if a complete coverage of object i is achieved, break the
               evaluation for demand object i;
10.    ind_fitness ← ind_fitness + best_fitness;
11. return (ind__fitness)
```

4.5.3 A new crossover operator: nearby-swap

The crossover operator, one of the GA's defining characteristics, plays a vital role in a GA (Herrera *et al.* 1998). It is one of the components that is essential to improve GA performance (Liepins and Vose 1992). Many different forms of crossover operators exist. One-point (Holland 1975; Goldberg 1989), two-point (De Jong 1985), *n*-point crossover (Eshelman *et al.* 1989) and uniform crossover operators (Syswerda 1989) are commonly used (De Jong 1985; Spears and De Jong 1991). For example, Correa *et al.* (2004) used the one-point crossover for the *p*-median problem, where the common genes in both parent chromosomes are kept and the rest of the genes subject to the one-point crossover. On one hand, the arrangement of the same set of facilities has no effect on individual fitness evaluation. However, when carrying out the crossover operator, the order of facility sites in an encoded chromosome plays a important role in determining which set of genes are to be exchanged to produce new individuals. Therefore, depending on the arrangement of the genes in the parents, a substantial number of possible exchanges exist using this type of crossover operator. Consider Figure 4.3 where two parents each with 5 genes are to be recombined. Of 100 possible children through recombination, two of them are displayed here, for the case where a random number indicates the first two genes to be exchanged. With this blind combination, individuals of low-quality are very likely to be produced. This can slow down the GA's convergence process, especially for large sized problems. To facilitate the convergence, Alp *et al.* (2002) used a greedy heuristic to generate a child with good quality by selecting *p* genes from the parent gene pool. The heuristic is implemented by deleting genes that contribute least to fitness evaluation one by one until *p* genes are left. Fathali (2006) (also see Jia *et al.* 2007) applied the same idea to the *p*-median problem with a greedy add procedure and reported that greedy add produced better solutions than the greedy drop employed in Alp *et al.* (2002). Consider that when *p* is large, this greedy heuristic operator will require intensive computation. This particularly complicates the fitness evaluation process in our problem where coverage provided by each additional facility involves the consideration of the joint coverage of demand objects with other facilities that are already included in the chromosome. In addition, premature convergence can easily occur due to the inability of the greedy heuristic to identify the optimal solution. Further discussion will be given later in the chapter.

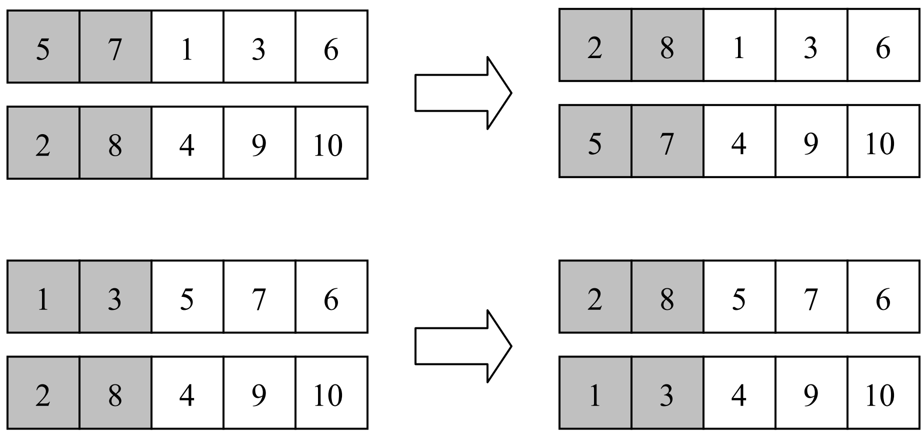
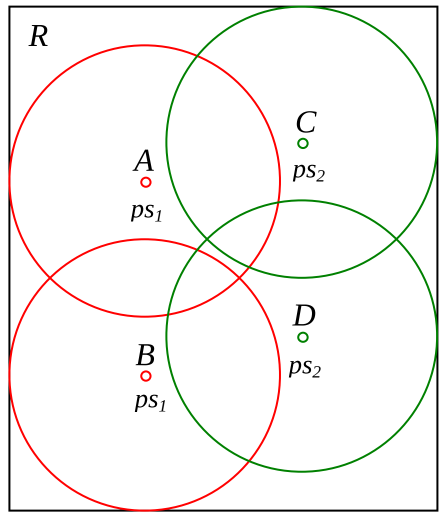
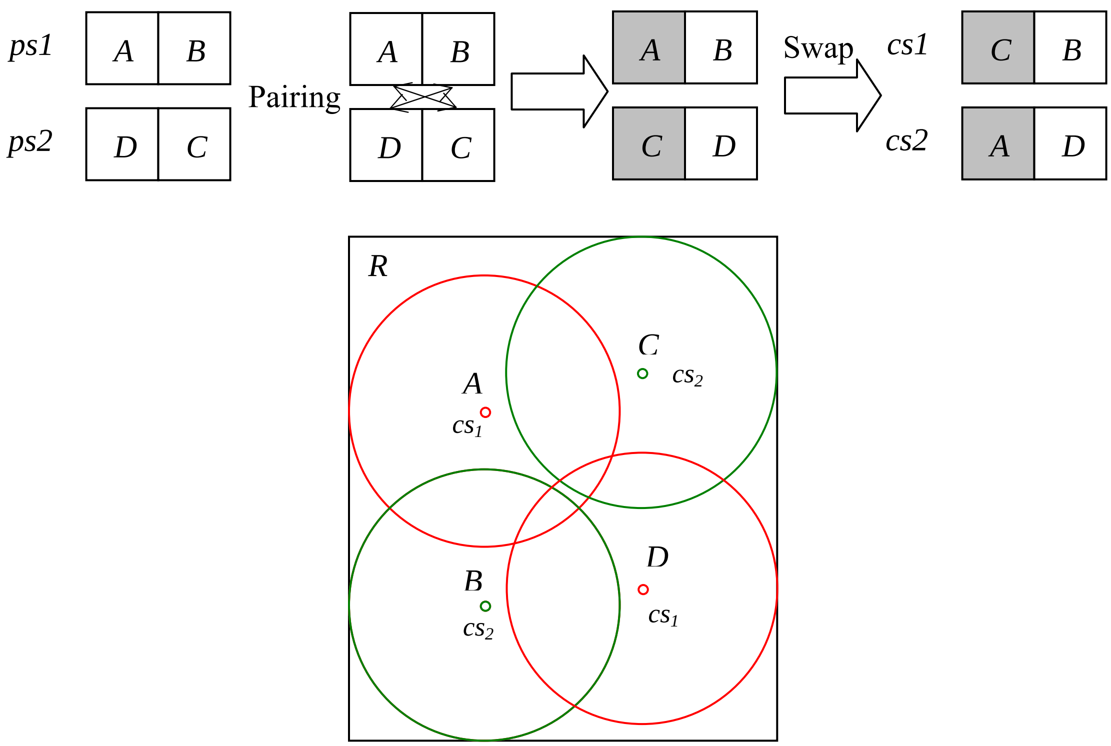


Figure 4.3. Possible children produced by a one-point crossover on two parent chromosomes.

Consider the characteristics of good solutions in coverage problems: facilities tend to disperse in space so as to cover more demand (see Murray 2005; Tong and Murray 2007). That is, the redundant coverage in the region is minimized. Correspondingly, in the GA operator, solutions that use unnecessarily redundant facilities to cover the same demand areas should be avoided. To partially incorporate this, we propose a new GA crossover operator: nearby-swap crossover. This crossover operates by pairing genes based on their spatial information. Genes representing facility sites that are closer together have a higher chance of being paired for exchange. In doing so, facilities that are close in space will have a smaller chance to co-exist in the same solution, avoiding excessive overlapping coverage. This reduces the chance of inferior children and, therefore, poor solution production. A simple example is shown in Figure 4.4 where two facilities are to be selected out of four potential facilities. Each circle displays the facility's effective coverage range. Initially as shown in Figure 4.4a, there are two parent solutions containing potential locations of *A* and *B* for one solution, *ps*1, and *C* and *D* for the other solution, *ps*2. For site *A* in *ps*1, site *C* is closer among the two potential sites in *ps*2. The same goes for *B* in *ps*1 and *D* in *ps*2. According to the proposed algorithm, *A* and *C*, *B* and *D* can be swapped when conducting the crossover operator. As a result, either none of the parents change or two new solutions *cs1* and *cs2* are created, as is shown in Figure 4.3b. In Figure 4.3b the optimal solution consisting of sites of *B* and *C* is identified. Using the random swap procedure, it is also very likely to create the solutions as shown in Figure 4.3c that are even worse than the parent solutions. Hence, using the new crossover operator, it is expected that quicker convergence to a high-quality solution can be achieved.



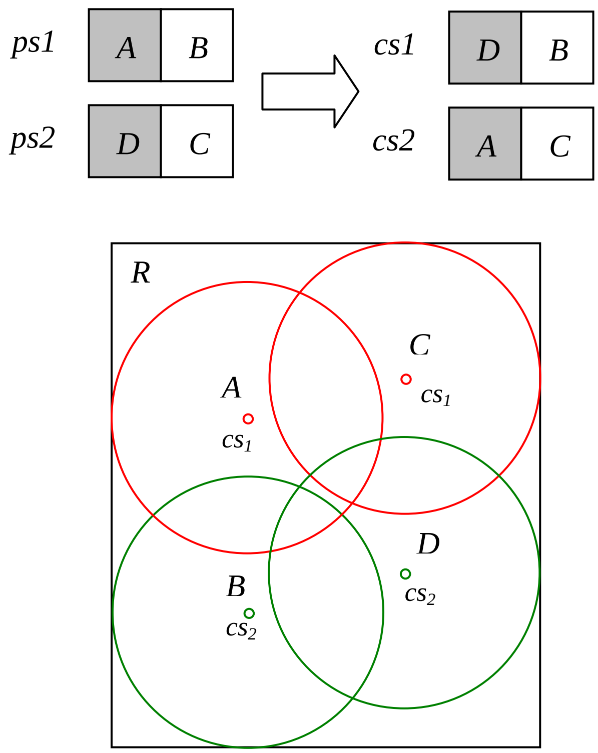
(a)



(b)

Figure 4.4 Crossover illustrations: (a) Parent solutions, (b) Child solutions with the nearby-swap crossover operator, and (c) Possible child solutions with a uniform crossover operator.

Figure 4.4 Continued



(c)

For the pairing process, first the common genes in both parents are automatically selected for pairing. For the rest of the genes, one gene (A) in one parent selects the one (B) in another parent that has the smallest distance between them among those that have not been paired. To promote diversity, a uniform crossover is carried out for the exchange of the paired genes. The pseudo-code is constructed as follows:

procedure crossover (parent1, parent2)

1. **for** each gene *g* in 1: *p* in parent1
2. shortest_dist ← a big number;
3. **for** each unpaired gene *h* in parent2
4. **if** gene *g* and gene *h* are the same, *h* is labeled as paired, break the paring process for gene *g* and go to step 1;
   elseif distance between *g* and *h* dist(*g*,*h*) < shortest_dist, shortest_dist ← dist(*g*,*h*) and shortest_gene ← *h*;
5. pair *g* with shortest_gene in parent2 and shortest_gene is labeled as paired
6. *f* ← random(0,1);
7. **if** *f* > 0.5, swap *g* *with* shortest_gene*;*
8. **return** two recombined child individuals

4.5.4 Other operations

For the strategy of selecting parents for offspring production, Alp *et al.* (2003) and Jia *et al.* (2007) reported little difference among biased selection mechanisms. In this chapter, we use the binary tournament selection method as it is easy to implement. For the offspring to be selected for the next generation, we use a steady-state update *elitist* strategy similar to the CHC selection strategy (Eshelman 1991). Among the 2*N* individuals with parents (*N*) and children (*N*), the best *N* distinct individuals are selected as the next generation and the rest discarded. After accepting the offspring generation, mutation is applied to the updated parent population. For the gene representing a facility selected for mutation, another facility site that is not included in the solution is randomly selected and substituted. To ensure elitism, the best solution is kept and not subjected to the mutation. Although we try to apply the mutation only to the offspring, as a conventional GA genetic operation does, there is little difference from no mutation, and has been discussed in Alp *et al.* (2002) for the *p*-median problem. Applying mutation to the updated population produces better solutions, generally, for our problems. The overall pseudo-code genetic algorithm is as follows:

1. Initial population through randomization
2. Repeat
   a. Use binary tournament selection to select parents
   b. Produce offspring through nearby-swap crossover
   c. Select best *N* distinctive individuals from parent and offspring population as a new population
   d. Apply mutation to the new population and create a new parent population for the next generation
3. Until <terminating condition>

4.6 Computational results

The new genetic algorithm is applied to solve the PMP-MC (k=2) for warning siren siting in Dublin, Ohio, as utilized in Chapter 3. In Chapter 3, the commercial optimization software CPLEX is relied upon for solutions. Due to the complexity of the problem, a considerable amount of time is invested to obtain the optimal solution. In addition, when p = 22, no optimal solution could be identified due to memory limits, and a maximum optimality gap of 0.22% is reported. In this chapter, the proposed genetic algorithm is used to solve the same set of problems. Results are compared with those obtained from CPLEX. In addition, other crossover operators, including one-point, two-point, uniform crossover and the greedy add heuristic crossover, are also tested and compared. Given that for small p the problems are relatively trivial, problems only for $p \geq 8$ are examined. For each p, ten runs are carried out for the GA, except for the GA with heuristic crossover where five runs are performed due to the considerable computational time involved. GAs are coded in C++ and run on a Pentium Xeon 3.0 GHz personal computer with 2 GB RAM. The program terminates if the best solution does not change after 400 iterations or when 1,500 total iterations have been reached.

Table 4.1 presents the computational results using the proposed GA. Here, the gap (%) was the relative quality of the solution obtained from the GA compared to that obtained from CPLEX as follows:

$$\text{gap} = \frac{\text{best coverage using CPLEX - best coverage using GA}}{\text{best coverage using CPLEX}} \times 100\%$$

As indicated in the table, out of the ten runs, optimal solutions are reached when $p \leq 21$ if the new GA is used. There was a negative gap for p =22, which indicates that a better solution was identified in about 3 minutes compared to the sub-optimal solution obtained from CPLEX in 9 hours using branch and bound. Even for $p > 22$, the gaps are relatively small. The average gap for all runs is 0.21%, indicating that even the non-optimal solutions obtained by the GA are very close to optimal solutions. In addition, computationally, an average of 2 minutes is needed for each problem using the proposed GA as opposed to an average of 3 hours if CPLEX is used.

	GA with nearby-swap crossover				CPLEX
p	Best gap (%)	Worst gap (%)	Average gap (%)	Average solution time (s)	solution time (s)
8	0.00	0.23	0.13	39.87	87.41
9	0.00	0.06	0.02	53.10	119.78
10	0.00	0.29	0.09	76.42	344.08
11	0.00	0.51	0.18	80.47	852.39
12	0.00	0.50	0.16	85.99	899.56
13	0.00	0.22	0.12	83.57	390.73
14	0.00	0.53	0.12	85.99	352.33
15	0.00	0.59	0.27	91.39	9100.11
16	0.00	0.58	0.22	109.59	4610.33
17	0.00	0.68	0.18	126.62	5264.42
18	0.00	0.85	0.44	125.71	8525.64
19	0.00	0.60	0.27	136.92	28563.52
20	0.00	0.26	0.13	207.70	38039.48
21	0.00	0.58	0.24	174.11	43777.22
22*	-0.01	0.40	0.27	168.94	33159.38
23	0.25	0.60	0.42	177.17	11429.38
24	0.12	0.43	0.31	238.59	6492.69
25	0.11	0.56	0.30	216.94	3006.69

*"Out of memory" was encountered and a suboptimal solution was reported with maximum optimality gap of 0.22%.

Table 4.1. Computational results of the new GA compared to that from CPLEX.

Computational results for crossovers of one-point, two-point, uniform and greedy heuristic are shown in Tables 4.2-4.5. Generally speaking, it is relatively difficult for a crossover operator to identify the optimal solution compared with the proposed nearby-swap crossover operator. For instance, out of ten runs, no optimal solution can be obtained when $p > 14$ for one-point crossover, $p > 12$ for two-point crossover and $p > 10$ for uniform crossover. No optimal solution has been reached using the greedy heuristic crossover operator for our problem. The average gaps are 0.89% for the one-point crossover, 1.07% for the two-point crossover, 0.84% for the uniform crossover, 1.06% for the greedy heuristic and are all larger than the average gap of 0.21% for the nearby-swap crossover. Comparing the worst solutions for each GA, those obtained by the nearby-swap crossover operator are also relatively smaller than those using other crossover operators. Furthermore, the nearby-swap crossover also performed the best in regard to computing time, averaging 2 minutes. The greedy add heuristic is much more computationally intensive due to the complicated fitness function evaluation required, taking an average of over 40 minutes. Hence, the greedy heuristic crossover is not a good approach for the PMP-MC.

	GA with one-point crossover operator			
p	Best gap (%)	Worst gap (%)	Average gap (%)	Average solution time (s)
8	0.00	0.53	0.19	62.43
9	0.00	0.06	0.04	85.35
10	0.09	1.01	0.30	83.11
11	0.20	1.38	0.53	90.18
12	0.00	1.13	0.44	104.96
13	0.00	1.31	0.56	105.79
14	0.00	1.96	0.65	136.05
15	0.61	1.34	0.78	164.61
16	0.44	2.73	1.06	172.14
17	0.16	2.19	1.19	194.61
18	0.49	2.27	1.36	235.51
19	0.43	3.03	1.41	215.40
20	0.63	2.21	1.36	247.73
21	1.19	2.11	1.61	239.83
22	0.63	1.64	1.04	315.62
23	0.82	2.05	1.31	317.79
24	0.76	2.19	1.32	344.62
25	0.55	1.46	0.86	314.20

Table 4.2. Computational results of the GA with a one-point crossover.

	GA with two-point crossover operator			
p	Best gap (%)	Worst gap (%)	Average gap (%)	Average solution time (s)
8	0.00	0.23	0.07	77.93
9	0.00	0.61	0.20	92.08
10	0.16	0.63	0.35	92.90
11	0.00	0.80	0.50	113.16
12	0.00	1.35	0.68	121.45
13	0.14	1.13	0.52	148.65
14	0.11	1.80	0.87	160.36
15	0.13	1.78	0.88	189.41
16	0.18	1.71	1.03	184.66
17	0.68	2.22	1.73	198.98
18	0.80	2.79	1.83	217.96
19	1.12	2.84	1.82	233.15
20	0.74	2.63	1.75	248.42
21	1.12	2.08	1.59	320.94
22	0.88	2.66	1.55	335.61
23	1.22	1.93	1.51	377.18
24	0.91	2.09	1.40	357.20
25	0.66	1.40	1.02	363.79

Table 4.3. Computational results of the GA with a two-point crossover.

	GA with uniform crossover operator			
p	Best gap (%)	Worst gap (%)	Average gap (%)	Average solution time (s)
8	0.00	0.23	0.10	57.06
9	0.00	0.06	0.03	92.08
10	0.08	0.94	0.32	87.93
11	0.05	0.75	0.34	76.97
12	0.02	1.50	0.49	93.02
13	0.11	1.49	0.71	86.43
14	0.00	2.60	0.54	169.21
15	0.00	1.18	0.53	195.76
16	0.12	2.63	0.91	196.11
17	0.66	1.95	1.00	289.61
18	0.88	2.84	1.59	301.24
19	0.65	1.83	1.14	258.55
20	0.59	1.92	1.02	234.11
21	0.67	1.64	1.18	248.79
22	0.81	1.63	1.16	283.17
23	0.87	1.55	1.18	263.98
24	0.58	1.36	1.03	285.36
25	0.78	1.44	1.11	311.23

Table 4.4. Computational results of the GA with a uniform crossover.

	GA with greedy heuristic crossover operator			
p	Best gap (%)	Worst gap (%)	Average gap (%)	Average solution time (s)
8	0.23	0.34	0.28	422.91
9	0.39	0.40	0.40	352.88
10	0.50	0.58	0.54	465.78
11	0.46	1.27	0.92	858.14
12	0.72	1.27	1.01	1145.96
13	0.84	1.03	0.92	1439.26
14	0.89	1.20	1.06	1261.25
15	1.12	1.29	1.18	1410.06
16	0.61	0.93	0.82	2235.39
17	0.96	1.75	1.22	2217.64
18	0.94	1.87	1.42	2665.53
19	1.37	1.90	1.50	3495.00
20	1.40	1.85	1.67	2526.91
21	1.47	2.06	1.74	4561.09
22	0.98	1.63	1.25	4973.98
23	0.87	1.62	1.21	4346.56
24	0.66	1.33	1.07	5906.57
25	0.38	1.09	0.72	5580.85

Table 4.5. Computational results of the GA with the greedy add heuristic crossover.

4.7 Discussion and conclusions

Although the general principles of GAs are similar in most applications, there is no GA that works best in all cases. Users are required, and challenged, to customize for specific problems. In this chapter, we developed a new GA as a heuristic for solving the PMP-MC presented in Chapter 3. In particular, innovatively incorporate the characteristic of maximal coverage as a new crossover operator. We demonstrated the effectiveness and efficiencies of the new GA as high-quality solutions are found. In addition, test results show that the new GA using the nearby-swap crossover operator performs better than the commonly used crossover operators.

Although the new developed GA is oriented for the PMP-MC model, it could also be applied to the MCLP using a simplified fitness function evaluation. It is expected that the new GA would perform well too, considering that the PMP-MC is a generalization of the MCLP, the PMP-SC and the PMP for coverage optimization. In addition, the same idea can be applied to the *p*-median problem (PMP) given that minimal weighted travel distance is more likely as facilities disperse spatially. However, whether the nearby-swap crossover operator itself, or a variant of it, would work best for the PMP remains a question for further exploration.

CHAPTER 5

CONCLUSIONS

5.1 Summary

This research focused on coverage optimization for spatial demand objects in the plane. We tackled the continuous location problem by introducing a finite dominating set and proved that at least one optimal solution is contained in the set, assuming that complete coverage is provided to spatial demand objects. Further, a new maximal coverage model, the p-median problem – single-facility coverage (PMP-MC), was developed and oriented for the situation where demand is represented as general objects, including points, lines and polygons. In addition, in order to enhance the computational capabilities of the PMP-MC, a new genetic algorithm (GA) was proposed, and test results demonstrated its effectiveness compared with commercial optimization software (CPLEX) and other GAs.

The challenges that motivated this research are associated with representation issues of geographic space inherent to current location models, discrete and continuous. Traditionally, demand and/or the region of interest are often abstracted using points aggregated from other spatial entities. However, substantial errors can be introduced, and obtained solutions can be dependent on the degree of aggregation. This research extends demand representation beyond points only by incorporating spatial objects, including points, lines and polygons. To solve the continuous problem in the plane, instead of searching an infinite number of sites, we narrowed our search to a discrete point set, the polygon intersection point set (PIPS), as a finite dominating set. Theoretically we proved that the PIPS consist of at least one optimal solution for covering objects. Application results also demonstrated that PIPS are better than other possible approaches in terms of solution quality and efficiency in obtaining solutions.

Due to the inability of the existing maximal coverage location problem (MCLP) handle spatial demand objects appropriately, there exist significant discrepancies between what is modeled and actual geographic coverage. In order to accurately reflect the mechanism of maximal coverage for spatial objects (points, lines or polygons), we introduced a new model called the PMP-MC (*p*-median problem – single-facility coverage) to explicitly account for joint service provided by multiple facilities. Theoretically, by incorporating sufficient potential multiple facilities to provide joint coverage, the PMP-MC is free of the modifiable areal unit problem (MAUP) and is, therefore, frame independent.

As an NP-hard problem, the PMP-MC is difficult to solve, especially for large problems. In order to solve the PMP-MC efficiently and obtain diverse alternative solutions, a new genetic algorithm was proposed. In particular, built on the spatial characteristic of maximal coverage, a new crossover operator was introduced. Application results showed that the developed genetic algorithm performed well in finding solutions of high-quality very quickly.

5.2 Future work

5.2.1 Continuous coverage

In this research, PIPS are identified as a finite dominating set (FDS) assuming that the complete coverage of a spatial demand object is achieved by one facility. Although the application results found that when accounting for the joint coverage provided by multiple facilities PIPS are a good set serving as potential facilities sites, there is no theoretical evidence to show that PIPS are also a FDS for the PMP-MC. More work should be carried out to explore possibilities of such a FDS for the PMP-MC, and how to identify it, if there is one.

5.2.2 Further investigation of the PMP-MC

As indicated previously, theoretically, if incorporating sufficient multiple facilities to provide joint coverage, the PMP-MC will be free of the MAUP. However, in reality, with an increase of *k* in the model, the complexity of the problem will increase dramatically and this will add to computational burdens. Intuitively, using coarse demand units can reduce the number of variables in the model. However, this would require a higher *k* for achieving a high-quality solution. In addition, the joint coverage combination evaluation is also related to the coverage standard. Therefore, a further investigation of the balance among the resolution of demand units, coverage standard and *k* will give more insights into the model and enhance its applicability.

5.2.3 More realistic surface demand

When we represent demand as spatial objects, we assume that the demand is uniformly distributed in each object. This might not be true. On the other hand, demand can be represented as a continuous surface. For example, Figure 5.1 presents a potential demand surface for the study area in Dublin, Ohio. The surface was interpolated based on the population density. To generate the surface, geostatistical techniques are used. However, how can a coverage optimization be approached? One possible approach is to discretize the generated surface and apply the PMP-MC. Another way is to use heuristic search strategies integrated with continuous demand representation. This may suggest a new kind of continuous space coverage optimization problem, where demand is geostatistically defined.

All of these are left as potential areas of future research.

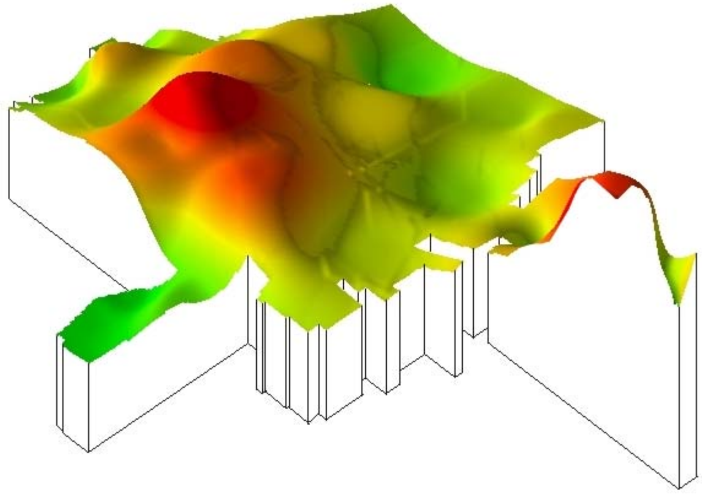


Figure 5.1 A potential demand surface for Dublin, Ohio.

BIBLIOGRAPHY

Adenso-Diaz B. and F. Rodriguez (1997) A simple search heuristic for the MCLP: application to the location of ambulance bases in a rural region. *Omega, International Journal of Management Science* 25(2): 181-187

Alp O., E. Erkut and Z. Drezner (2003) An efficient genetic algorithm for the p-median problem. *Annals of Operations Research* 122: 21-42

Aly A.A. and A.S. Marucheck (1982) Generalized Weber problem with rectangular regions. *The Journal of the Operational Research Society* 33(11):983-989

Aikens C.H. (1985) Facility location models for distribution planning. *European Journal of Operational Research* 22: 263-279

Aickelin U. (2002) An indirect genetic algorithm for set covering problems. *Journal of the Operational Research Society* 53: 1118-1126

Bäck T. (1993) *Applications of Evolutionary algorithms*. 5th edition, Dortmund, Germany.

Beasley J.E. and P.C. Chu (1996) A genetic algorithm for the set covering problem. *European Journal of Operational Research* 94: 392-404

Bennett C.D. and A. Mirakhor (1974) Optimal facility location with respect to several regions. *Journal of Regional Science* 14: 131-136

Benveniste R. (1982) A note on the set covering problem. *The Journal of the Operational Research Society* 33(3): 261-265

Berman O. and D. Krass (2002) The generalized maximal covering location problem. *Computers & Operations Research* 29: 563-581

Birkin, M., G. Clarke, M. Clarke and A. Wilson(1996). *Intelligent GIS: Location decisions and strategic planning*. Cambridge: GeoInformation International.

Blum C. and A. Roli (2003) Metaheuristics in combinatorial optimization: overview and conceptual comparison. *ACM Computing Surveys* 35(3): 268-308

Brandeau M.L. and S.S. Chiu (1989) An overview of representative problems in location research. *Management Science* 35(6): 645-674

Brimberg J. and R.F. Love (1995) Estimating distances. In Drezner Z. (eds), *Facility location: a survey of applications and methods*.

Carrizosa E., M. Muňoz-Márquez and J. Puerto (1998) The Weber problem with regional demand. *European Journal of Operational Research* 104(2):358-365

Chung C-H. (1986) Recent applications of the maximal covering location planning (M.C.L.P.) model. *The Journal of the Operational Research Society* 37(8): 735-746

Church R.L. (1984) The planar maximal covering location problem. *Journal of Regional Science* 24 (2): 185-200.

Church R.L. (1999) Location modeling and GIS. In P. Longley, M. Goodchild, D. Maguire, & D. Rhind (Eds.), *Geographical information systems* (second ed.)(pp, 293-303). New York: John Wiley.

Church R.L. (2002) Geographic information systems and location science. *Computers & Operations Research* 29 (6): 541-562

Church R.L. and K.L. Roberts (1983). Generalized coverage models and public facility location. *Papers of the Regional Science Association* 53: 117-135.

Church R.L. and C. ReVelle (1974). The maximal covering location problem. *Papers of the Regional Science Association* 32: 101-118

Church R.L., D.M. Stoms and F.W. Davis (1996) Reserve selection as a maximal covering location problem. *Biological Conservation* 76: 105-112

Church R.L. and J.R. Weaver (1986) Theoretical links between median and coverage location problems. *Annals of Operations Research 6*: 1-19

Correa E.S., M.T.A. Steiner, A.A. Freitas and C.Carnieri (2004) A genetic algorithm for solving a capacitated p-median problem. *Numerical Algorithms* 35 (2-4): 373 – 388

Current J.R., and M.E. O'Kelly (1992). Locating emergency warning sirens. *Decision Science* 14: 47-54.

Current J.R. and D.A. Schilling (1990) Analysis of errors due to demand data aggregation in the set covering and maximal covering location problems. *Geographical Analysis* 22: 116-126

D'Amico S.J., S. Wang, R. Batta and C.M. Rump (2002). A simulated annealing approach to police district design. *Computers and Operations Research* 29(6): 667-684

Daskin M.S. (1995) *Network and Discrete Location: Models, Algorithms and Application*s, John Wiley and Sons, Inc., New York.

Daskin M.S., A.E. Haghani, M. Khanal and C. Malandraki (1989) Aggregation effects in maximum covering models. *Annals of Operations Research* 18: 115-140

Davis L. (1991) *Handbook of Genetic algorithms*. Van Nostrand Reinhold, New York.

De Jong K.A. (1975). *An Analysis of the Behavior of a Class of Genetic Adaptive Systems*. Doctoral thesis, Dept. Computer and Communication Sciences, University of Michigan, Ann Arbor

Dibble C. and P.J. Densham (1993) Generating interesting alternatives in GIS and SDSS using genetic algorithms. *GIS/LIS Symposium*, University of Nebraska, Lincoln.

Drezner Z. and H.W. Hamacher (2002) *Facility location: Application and Theory* (Springer: New York)

Drezner Z. and G.O. Wsolowsky (1980) Optimal location of a facility relative to area demands. *Navel Research Logistics Quarterly* 27:199-206

Dwyer F.R. and J.R. Evans (1981) A branch and bound algorithm for the list selection problem in direct mail advertising. *Management Science* 27(6): 658-666

Eaton D., M. Hector, V. Sanchez, R. Latingua and J. Morgan (1986) Determining ambulance deployment in Santo Domingo, Dominican Republic. *Journal of the Operational Research Society* 37: 113-126

Eiselt H.A. (1992) Location modeling in practice. *American Journal of Mathematical and Management Sciences* 12: 3-18.

Elzinga J. and D.W. Hearn (1972) Geometrical solutions for some minimax location problems. *Transportation Science* 6: 379-394

Eshelman L.J. (1991) The CHC adaptive search algorithm: How to have safe search when engaging in nontraditional genetic recombination. *Foundations of Genetic Algorithms* 1: 265-283

Eshelman L.J., A. Caruana and J.D. Schaffer (1989) Biases in the crossover landscape. *Proceedings of the Third International Conference on Genetic Algorithms*. Schaffer J.D. (Ed.). Morgan Kaufmann Publishers, San Mateo: 86-91

Eyster J.W., J.A. White and W.W. Wierwille (1973) On solving multifacility location problems using a hyperboloid approximation procedure. *AIIE Transactions* 5: 1-6

Farhan B. and A.T. Murray (2006) Distance decay and coverage in facility location planning. *Annals of Regional Science* 40: 279-295

Fathali J. (2006) A genetic algorithm for the p-median problem with pos/neg weights. *Applied Mathematics and Computation* 183: 1071-1083

Fekete S.P., J.S.B. Mitchell and K. Beurer (2005) On the continuous Fermat-Weber problem. *Operations Research* 53(1):61-76

Francis R.L. and T.J. Lowe (1992) On worst-case aggregation analysis for network location problems. *Annals of Operations Research* 40: 229-246

Galvão R.D. and C. ReVelle (1996) A Lagrangean heuristic for the maximal covering location problem. *European Journal of Operational Research* 88: 114-123

Gleason J.M. (1975) A set covering approach to bus stop location. *OMEGA* 3: 605-608

Goldberg D.E. (1989) *Genetic algorithms in Search ,Optimization and Machine Learning*. Addison-Wesley, Menlo Park, CA.

Grinde and Daniels (1999) Solving an apparel trim placement problem using a maximum cover problem approach. *IIE Transactions* 31: 763-769.

Hamacher H.W. and S. Nickel (1998) Classification of location models. *Location Science* 6: 229-242

Hakimi S.L. (1964) Optimum location of switching centers and the absolute centers and medians of a graph. *Operations Research* 12:450-459.

Hakimi S.L. (1965) Optimum distribution of switching centers in a communications network and some related graph-theoretic problems. *Operations Research* 13: 462-475

Herrera F., M. Lozano and J.L. Verdegay (1998) Tackling real-coded genetic algorithms: operators and tools for behavioural analysis. *Artificial Inteelligence Review* 12: 265-319

Hershberger J. (1993) A faster algorithm for the two-center decision problem. *Inform. Process*. Lett. 47: 23–29.

Hochbaum D.S. and D. Shmoys (1985) A best possible heuristic for the k-center problem. *Mathematics of. Operations Research* 10: 180–184.

Holland, J.H. (1975) *Adaptation in Natural and Artificial Systems*. University of Michigan Press, Ann Arbor

Hooker J.N., R.S. Garfinkel and C.K. Chen (1991) Finite dominating sets for network location problems. *Operations Research* 39(1): 100-118

Hosage C.M. and M.F. Goodchild (1986) Discrete space location-allocation solutions from genetic algorithms. *Annals of Operations Research* 18: 225-244

Hougland E.S. and N.T. Stephens (1976) Air pollutant monitor siting by analytical techniques. *Journal of the Air Pollution Control Association* 26: 52-53

Jaramillo J.H., J. Bhadury and R. Batta (2002) On the use of genetic algorithms to solve location problems. *Computers & Operations Research* 29: 761-779

Jia H., O. Fernando and M.M. Dessouky (2007) Solution approaches for facility location of medical supplies for large-scale emergencies. *Computers & Industrial Engineering* 52: 257-276

Karasakal O. and E. Karasakal (2004) A maximal coverage location model in the presence of partial coverage. *Computers & Operations Research* 31: 1515-1526

Khuller S. and Y.J. Sussmann (2000). The capacitated k-center problem. *SIAM J. Discrete Math* 13(3): 403–418.

Klose A. and A. Drexl (2004) Facility location models for distribution system design. *European Journal of Operational Research* 162: 4-29

Koza J.R. (1992). *Genetic Programming*. The MIT press, Cambridge.

Kuby M., S. Lim and C. Upchuch (2005) Dispersion of nodes added to a network. *Geographical Analysis* 37: 383-409

Li X. and A.G.-O. Yeh (2005) Integration of genetic algorithms and GIS for optimal location search. *International Journal of Geographical Information Science* 19: 581-601

Liepins G.E. and M.D. Vose (1992) Characterizing crossover in genetic algorithms. *Annals of Mathematics and Artificial Intelligence* 5(1): 27-34

Lorena L. and L. deSouza-Lopez (1977) Genetica algorithms applied to computationally difficult set covering problems. *Journal of the Operational Research Society* 48: 440-445

Love R.F. (1972) A computational procedure for optimally locating a facility with respect to several rectangular regions. *Journal of Regional Science* 12: 233-242.

Love R.F., J.G. Morris and G.O. Wesolowsky (1988) *Facility location: models and methods*. NorthHolland, New York, 1988

Man K.F., K.S. Tang and S. Kwong (1999) *Genetic Algorithms: Concepts and Design*. Springer.

Marucheck A.S. and A.A Aly (1981) An efficient algorithm for the location-allocation problem with rectangular regions. *Naval Research Logistics* 28: 309-323

Matisziw T.C. and A.T. Murray (2005) Siting a facility to maximize coverage of continuously distributed demand, to appear.

Megiddo N., E. Zemel and S.L. Hakimi (1983) The maximal coverage location problem. *SIAM Journal of Algebraic and Discrete Methods* 4: 253-261

Mehrez A. and A. Stulman (1982) The maximal covering location problem with facility placement on the entire plane. *Journal of Regional Science* 22 (3): 361-365

Michalewicz Z. (1992) *Genetic Algorithms + Data Structures = Evolution Programs*. Springer-Verlag, New York

Miller H.J. (1996) GIS and geometric representation in facility location problems. *International Journal of Geographical Information Systems* 10(7): 791-816

Mirchandani P.B. and R.L. Francis (1990) *Discrete Location Theory* (John Wiley and Sons: New York)

Minciardi R., R. Sacile, and F. Siccardi (2003) Optimal planning of a weather radar network. *Journal Atmospheric and Oceanic Technology* 20: 1251-1263.

Mladenovic N., J. Brimberg, P. Hansen and J.A. Moreno-Perez (2007) The p-median problem: a survey of metaheuristic approaches. *European Journal of Operational Research* 179: 927-939

Mitchell M. (1996) *An introduction to genetic algorithms*. MIT Press, Cambridge, MA.

Murray A.T. (2003) Site placement uncertainty in location analysis. *Computer, Environment and Urban Systems* 27: 205-21

Murray A.T. (2005) Geography in coverage modeling: exploiting spatial structure to address complementary partial service of areas. *Annals of the Association of American Geographers* 95(4): 761-772

Murray A.T. and R.L. Church (1996) Applying simulated annealing to location-planning models. *Journal of Heuristics* 2: 31-53

Murray A.T., M.E. O'Kelly and R.L. Church (2007) Regional service coverage modeling. *Computers and Operations Research,* to appear (special issue)

Okabe A. and A. Suzuki (1997) Locational Optimization Problems Solved through Voronoi Diagrams. *European Journal of Operational Research* 98: 445-456.

Pirkul H. and D. Schilling (1989) The capacitated maximal covering location problem with backup service. *Annals of Operations Research* 18: 141-154

Plastria F. (2002) Continuous covering location problem. In *Facility Location: Application and Theory*

Resende M.G.C. (1998) Computing approximate solutions of the maximum covering problem with GRASP. *Journal of Heuristics* 4: 161-177

ReVelle C. (1989) Review, extension and prediction in emergency service siting models. *European Journal of Operational Research* 40:58-65.

ReVelle C. (1991) Siting ambulances and fire companies: new tools for planners. *Journal of the American Planning Association* 57: 471-485

ReVelle C., P. Marks, and J.C. Liebman (1970) An analysis of private and public sector location models. *Management Science* 16: 692-707.

ReVelle C. and R. Swain (1970) Central facilities location. *Geographical Analysis* 2: 30-34.

ReVelle C. (1986) The maximum capture or "sphere of influence" location problem: Hotelling revisited on a network. *Journal of Regional Science* 26(2): 343-385

ReVelle C., J.C. Williams and J.J. Boland (2002) Conterpart models in facility location science and reserve selection science. *Environmental Modeling and Assessment* 7: 71-80.

Schilling D.A., V. Jayaraman and R.Barkhi (1993) A review of covering problems in facility location. *Location Science* 1(1): 25-55

Sharir M. and E. Welzl (1996) Rectilinear and polygonal p-piercing and p-center problems. *Proc. 12th Annual ACM Symposium Comput. Geom*. :122–132.

Spears W.M. and K.A. De Jong (1991). On the virtues of parameterized uniform crossover. *Proceedings of the Fourth International Conference on Genetic Algorithms.* eds. R. Belew and L. Booker, San Mateo, CA: Morgan Kaufmann: 230-236
Toregas ReVelle

Suzuki A. and A. Okabe (1995) Using voronoi diagrams. In *Facility Location: A Survey of Applications and Methods*. Drezner Z (ed.). New York: Springer-Verlag: 103-118

Sweeney D.J., R.L. Mairose and R.K. Martin (1979) Strategic planning in bank location. In *AIDS Proceedings*

Syswerda G. (1989) Uniform crossover in genetic algorithms. *Proceedings of the Third International Conference on Genetic Algorithms*, ed. J.D. Schaffer, San Mateo, CA: Morgan Kaufmann 2-8

Syswerda G. (1991) Schedule optimization using genetic algorithms. *Handbook of Genetic Algorithms*. Schaffer, J.D. (Ed.). Morgan Kaufman Publishers, San Mateo: 2-9

Toregas C. (1971). Location under maximal travel time constraints. PhD. Dissertation, Cornell University

Toregas C. and C. ReVelle (1972) Optimal location under time or distance constraints. *Papers of the Regional Science Association* 28: 133-143

Toregas C. and C. ReVelle (1973) Binary logic solutions to a class of location problems. *Geographical Analysis* 5: 145-155

Vecter V. and M.C. Dincer (1995) Global manufacturing strategy. In *Facility location: A Survey of applications and methods*. Drezner, Z. (ed.). New York: Springer-Verlag: 263-282.

Vergin R.C. and J.D. Rogers (1967). An algorithm and computational procedure for locating economic facilities. *Management Science* 13: B240–B254.

Weaver J.R. and R.L. Church (1984) A comparison of direct and indirect primal heuristic/dual bounding solution procedures for the maximal covering location problem. Unpublished paper

Weber A. (1909) *Uber den Standort der* (Alfred Weber's theory of the location of industries) (1929). University of Chicago.

Wesolowsky G.O. and R.F. Love. (1971) Location of facilities with rectangular distances among point and area destinations. *Naval Research Logistics Quarterly* 18: 83-90.

Weiszfeld E. (1936) "Sur le point pour lequel la somme des distances de n points donnes est minimum. *Tohoku Mathematical Journal* 43: 355-386

Worboys M. and M. Duckham (2004) *GIS: A Computing Perspective*, second edition (CRC Press: Boca Raton)

White A.N. (1979) Accessibility and public facility location. *Economic Geography* 55(1): 18-35.

Xiao N. (2006) An evolutionary algorithm for site search problems. *Geographical Analysis* 38(3): 227-247