THREE ESSAYS ON LONG RUN MOVEMENTS OF REAL EXCHANGE RATES

DISSERTATION

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ABSTRACT

My dissertation studies statistical properties of the measure that Engel uses in his 1999 paper and presents new evidence in favor of the Balassa-Samuelson theory. The Balassa-Samuelson theory implies that the importance of the traded goods component in the real exchange rate movement decreases over time. Engel's empirical results, however, indicate that the importance is very high and even increasing for the long run for some countries. The tests based on Engel's measure show no statistical evidence for the decrease in the importance of the traded goods.

My dissertation consists of three essays. The essay titled "Long-run Real Exchange Rate Changes and the Properties of the Variance of k-differences," examines the statistical properties of the measure Engel uses, the variance of k-differences. I show that the variance of k-differences tends to return to the initial value as k approaches the sample size whether the variable is stationary or unit root nonstationary. My results imply that the increasing variances for k-values close to the sample size cannot be interpreted as evidence of an increase in the importance.

In my second essay, "A Monte Carlo Investigation on the Estimator of Ratio of Long Run Variances," I investigate whether the high level of importance Engel finds should be attributed to the high persistence of the traded goods component by means of a Monte Carlo simulation. My simulation results imply that the high ratio is more likely to be attributable to the volatility of errors of the traded goods component, not

to its persistence. I also find that the power of the test based on Engel's measure is very low for given parameter values.

In my third essay, "Higher Power Tests for the Failure of Long Run Purchasing Power Parity," I apply a covariate augmented point optimal test to three different real exchange rates, each of which is constructed with CPI, PPI, and Export/Import price index, respectively. The covariate test for the real exchange rates based on PPI and Export/Import price index gives approximately 50% rejection rate, which is comparably high and higher than that for CPI, consistent with the Balassa-Samuelson theory.

To my parents and my family

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CHAPTER 1

Introduction

My primary research topic is the empirical validity of the Balassa-Samuelson theory in the long run. Since Balassa (1964) and Samuelson (1964), the disaggregation of the economy into internationally traded and nontraded sectors has been one of the main building blocks in many open economy models. In those two-good models, if the law of one price holds in the prices of traded goods in the long run, then the real exchange rate (RER) is determined by the movement of its nontraded goods component which consists of the relative prices of nontraded goods in the long run.

Engel (1999) surprisingly finds that data for U.S. RER does not support the Balassa-Samuelson theory in the long run. In order to measure the importance of the traded goods component in U.S. real exchange rate movements, Engel (1999) uses the ratio of the variance of k-differences for the traded goods component to that for the real exchange rate at each time horizon, k. The importance of traded goods should decrease as the horizon increases if the law of one price holds for traded goods in the long run. Engel finds that the importance of traded goods is over 90% in most cases. The importance decreases only initially and then increases as the time horizon approaches the sample size. He interprets the increasing variance at the longer time horizon as evidence of an increase in the importance of the traded goods component in the long run. Engel also conducts a test based on the measure and does not find any statistical evidence that the importance of the traded goods decreases over time. His findings raise a serious doubt on the validity of the law of one price for traded goods in the long run. My dissertation investigates the statistical properties of the measure used in Engel (1999) and provides new empirical evidence about the law of one price in the long run.

In Chapter 2, I demonstrate that the variance of k-differences tends to return to the initial value as k approaches the sample size whether the variable is stationary or unit root nonstationary. My result implies that the increasing variances for k-values close to the sample size cannot be interpreted as evidence of an increase in the importance of the traded goods component in the long run. In addition, after the modification of the test used in Engel (1999) which I think is more appropriate in the current context, the importance of the traded goods does fall in the long run for some countries.

In Chapter 3, I decompose the determinants of the ratio of the variances into the relative volatility of the traded goods component and its persistence. Engel (1999) attributes the high importance of the traded goods component measured in the data to its high persistence. By means of a Monte Carlo simulation, I first investigate how the mean of the ratio is determined depending on the changes in the value of the two determinants. Contrary to Engel's inference, I show that the high importance is mainly attributable to its relative volatility for given parameter values observed in the data. It implies that a high ratio should not be necessarily considered as evidence against the law of one prices in the long run. My simulation results also show that the power of the test based on Engel's measure is very low for realistic parameterization. Chapter 2 and 3 imply that the evidence presented in Engel (1999) against the law of one price in the long run is very weak.

The low power in the tests observed in Chapter 3 has been a common problem in the literature of unit root testing for the purchasing power parity (PPP). In Chapter 4, I adopt a state-of-the-art unit root testing method (a covariate augmented point optimal test, CPT) which has a stronger power than a traditional unit root test like augmented Dickey-Fuller (ADF) test. I consider twenty seven bilateral RERs with the US dollar as a base currency. I consider the real exchange rate based on PPI and Export/Import price indexes as well as CPI. Rejecting the null of a unit root in the RER implies that the law of one price holds in the long run. If the law of one price holds better for the traded good prices, the real exchange rate based on the traded goods price index should be more likely to reject the unit root in the test than that based on the general price index. While the standard ADF test does not reject the null for any country or for any price indexes, the CPT test gives approximately 50% rejection rate for the real exchange rate based on production site prices like PPI and Export/Import price indexes which are conceptually closer to traded goods prices than CPI. In case of CPI, the CPT test is not applicable in many cases and the rejection rate is about 30%, lower than the other two cases. Unlike the previous covariate unit root tests which use large data set, I use the same data set used in univariate unit root tests and succeed in getting much higher rejection rate in the test especially for the production site prices.

Chapter 5 concludes.

CHAPTER 2

LONG-RUN REAL EXCHANGE RATE CHANGES AND THE PROPERTIES OF THE VARIANCE OF K-DIFFERENCES

2.1 Introduction

Since Balassa (1964) and Samuelson (1964), the disaggregation of the economy into internationally traded and nontraded sectors has been one of the main building blocks in many open economy models. In those two-good models, if the law of one price holds in the prices of traded goods, then the real exchange rate (RER) is determined by the movement of its nontraded goods component which consists of the relative prices of nontraded goods.

Since Isard (1977), however, empirical evidence has clearly shown that, in the short run, the law of one price does not hold even for available measures of traded goods. Thus, the Balassa-Samuelson view focusing on the role of nontraded goods had been thought to better fit the long run.

Contrary to this traditional view, Engel (1999) presented empirical results which can be interpreted to imply that almost all U.S. RER movements can be accounted for by movements of the traded good component at all time horizons. Engel (1999) himself refrains from reaching a decisive conclusion about the long-run time horizon and argues that his results are mainly about short and medium horizons because of the small number of observations. Nevertheless, some authors have taken Engel's (1999) results as evidence against the relevance of the traditional dichotomy of goods in modeling long run real exchange rate movements[1]. For example, Obstfeld (2001) writes;

> This is a striking contradiction of the Harrod-Balassa-Samuelson theory. International divergences in the relative consumer price of "tradables" are so huge that the theoretical distinction between supposedly arbitraged tradables prices and completely sheltered nontradables prices offers little or no help in understanding U.S. real exchange rate movements, even at long horizons.

In his paper, Engel measures the importance of the traded goods component in accounting for U.S. real exchange rate movements by adopting the variance of k-differences used in Cochrane (1988). In this paper, we challenge this widely accepted interpretation of Engel's results about the long run movement of the RER by analyzing properties of the limit distribution of the variance of k-differences when k is close to the sample size.

The variance of k-differences of a time series z_t is denoted as $V_k(z)$ in this paper. As in Cochrane (1988), $V_k(z)$ is defined as follows:

$$V_k(z) \equiv \frac{T}{(T-k)(T-k+1)k} \sum_{t=0}^{T-k} [z_{t+k} - z_t - k\overline{\Delta z}]^2, \tag{2.1}$$

$$\text{where } \overline{\Delta z} = \frac{1}{T}(z_T - z_0).$$

According to the definition in Equation (2.1), the variance of k-differences is a variance of k-period differences centered around the sample mean of the difference.

[1] Those who discuss Engel's (1999) empirical findings in the context of the long run movements of the real exchange rate include Alexius and Nilsson (2000), Chinn (2006), Obstfeld (2001), Sarno and Taylor (2002), Sarno and Valente (2006), Schnatz, Vijselaar, and Osbat (2004), and Taylor and Taylor (2004).

Cochrane (1988) shows that $V_k(z)$ is asymptotically equivalent to the Bartlett kernel estimator of the long run variance of Δz_t. If the law of one price holds for traded goods in the long run, then the long run variance of the traded goods component of the RER is zero since it is stationary. Thus, based on Cochrane (1988), Engel (1999) expects that $V_k(z)$ for the traded goods component will converge down to zero as k increases if the traditional Balassa-Samuelson view is true for the long run[2].

However, Engel's empirical results show that $V_k(z)$ for the traded goods component decreases at first but increases towards the end of time horizons, most prominently in the case of the US-Canada RER.[3] Engel (1999, p.513) interprets the rise in the later part of the graph as an increase in the importance of the traded goods component in the long run movement of the RER[4].

This paper shall show that $V_k(z)$ for $k \cong T$ tends to go back to the initial value on average as k gets closer to the sample size, whether the variable of interest is mean-reverting or not. As such, Engel's (1999) observation about the long run time horizons may come simply from this statistical property of the variance of k-differences and have little to do with the long run properties of the real exchange rate.

Our findings in this paper imply that the variances of k-differences in the middle range of $k's$ are more relevant to the long run than those at $k's$ close to both ends

[2] In Engel (1999), the formula for $V_k(z)$ is a little different from Cochrane's (1988). Engel does not divide it by k. Thus, Engel says, "One expects the variance of k-differences of x_t [the traded goods component] to converge [to a finite number] as k gets large."

[3] As we shall see in the next section, what Engel (1999) actually computes is the ratio of V_k of the traded goods component to that of the real exchange rate. However, the shape of the graph is mainly determined by the numerator. It is because V_k of the nontraded goods component is expected to remain constant on average if it is random walk.

[4] Applying Engel's approach to bilateral Asian-Pacific real exchange rates, Parsley (2001, p.9) also finds the rise in the later part of the graph in the case of US-Hong Kong and interprets it as an increase in the variability of the traded goods component in that time horizon.

of the time span. Thus, the fall of the graph of the variance of k-differences in the middle range of Engel's results favors the smaller importance of the traded goods component in the longer run. However, Engel (1999) finds that the fall is not statistically meaningful from his test based on the variance of k-differences. After some adjustments in Engel's testing method, however, our results show that the fall of the graph is statistically significant for some countries, meaning that Engel's test results are not very robust. As such, arguing that the nontraded goods component plays the same minimal role for the long run movement of the US real exchange rate based on Engel's empirical results is less convincing.

The evidence in this paper is consistent with recent works. Kakkar and Ogaki (1999) run a cointegration regression of the real exchange rate on its nontraded goods component and find that the nontraded goods component can explain long run real exchange rate movements fairly well. Related evidence for the usefulness of the dichotomy of goods in understanding the real exchange movements is found in a line of studies on the half-life[5] of the real exchange rate. Crucini and Shintani (2002), Kim (2005), and Kim and Ogaki (2004) find that half-lives of the RER based on traded good prices are shorter than those of the RER based on nontraded good prices. Crucini, Telmer, and Zachariades (2005) also find that the law of one price holds better for traded goods than for nontraded goods in data for over 500 goods. Taylor and Taylor (2004) state that the Harrod-Balassa-Samuelson model of equilibrium real exchange rates is attracting renewed interest as a desirable modification [of PPP theory] after languishing for some years in relative obscurity.

[5] Half life is the time it takes for half the effects of a given shock to dissipate.

The rest of the paper is organized as follows. Section 2 will review the existing literature on the asymptotic distribution of $V_k(z)$ and provide the main theoretical result of this paper. Section 3 discusses the implication of this paper's result for Engel's findings and presents our test results based on the variance of k-differences. Section 4 concludes.

2.2 The statistical properties of the variance of k-differences

2.2.1 Existing theories on the statistical properties of $V_k(z)$

Throughout the paper, suppose that the following Assumption 1 holds for a random variable, z_t.

Assumption 2.1 For a random variable, z_t, assume that $\Delta z_t = d + \psi(L)\varepsilon_t = d + \sum_{j=0}^{\infty} \psi_j \varepsilon_{t-j}$, where $\sum_{j=0}^{\infty} j \cdot \left|\psi_j\right| < \infty$ and $\{\varepsilon_t\}$ is an *i.i.d.* sequence with mean zero, variance σ^2, and finite fourth moment[6]. Define

$$
\begin{aligned}
\gamma_j &\equiv E\left[(\Delta z_{t+j} - d)(\Delta z_t - d)\right] = \sigma^2 \sum_{s=0}^{\infty} \psi_s \psi_{s+j} \qquad for \;\; j = 0, 1, 2, \cdots \\
\lambda &\equiv \sigma \sum_{j=0}^{\infty} \psi_j = \sigma \cdot \psi(1) \\
\Omega &\equiv \sum_{j=-\infty}^{\infty} \gamma_j = \lambda^2.
\end{aligned}
$$

The variance of k-differences has been studied in the context of the variance ratio test for the random walk hypothesis. Earlier works focused on the case when k is much smaller than the sample size. As in Lo and MacKinlay(1999, p.54), the variance of k-differences can be expressed as a weighted average of sample autocovariances,

[6]We follow the assumption in Proposition 17.3 in Hamilton (1994) in order to apply the functional central limit theorem to unit root nonstationary processes with serial correlation.

only with a small difference in order of $o_p(T^{-1/2})$:

$$\begin{aligned} V_k(z) &= \sum_{\tau=-k+1}^{k-1} \frac{k-|\tau|}{k} \widehat{\gamma}_\tau + o_p(T^{-1/2}), \\ \text{where } \widehat{\gamma}_\tau &\equiv \frac{1}{T} \sum_{t=1}^{T-|\tau|} (\Delta z_t - \overline{\Delta z})(\Delta z_{t+|\tau|} - \overline{\Delta z}). \end{aligned} \tag{2.2}$$

Hence, when k is relatively small and fixed, by the law of large numbers,

$$V_k(z) \to \sum_{\tau=-k+1}^{k-1} \frac{k-|\tau|}{k} \gamma_\tau \text{ as } T \to \infty. \tag{2.3}$$

Especially when $k = 1$,

$$V_1(z) \to \gamma_0 \text{ as } T \to \infty.$$

For the variance ratio test, in which the test statistic is defined as follows:

$$VR_k(z) \equiv V_k(z)/V_1(z), \tag{2.4}$$

it is possible to show the following asymptotic distribution of $VR_k(z)$:

$$\sqrt{T}(VR_k(z) - 1) \xrightarrow{D} N(0, \sigma_k^2), \tag{2.5}$$

where σ_k^2 is a simple function of k[7].

The variance of k-differences is asymptotically equivalent to the Bartlett kernel estimator for the long-run variance of Δz_t, as pointed out in Cochrane (1988)[8]. Equation (2.2) illustrates the fact. The first term of the right hand side in equation (2.2) is the definition of the Bartlett kernel estimator with the lag length of k. Newey and

[7] For instance, if $\psi(L) = 1$ and ε_t is an iid normal random variable with variance σ^2, then $\sigma_k^2 = \dfrac{2(2k-1)(k-1)}{3k}$.

[8] Actually, what Cochrane (1988) shows is that the population variance of k-differences is exactly equal to the population counterpart of the Bartlett estimator of long run variance. After replacing the two population concepts with the sample counterparts, the equality becomes an asymptotic equivalence.

West (1987) show that the Bartlett kernel estimator converges to the long-run variance of Δz_t as $T \to \infty$ and $k \to \infty$ at a much slower growth rate[9], $O(T^{1/4})$. Thus, under certain conditions,

$$V_k(z) \to \Omega \ \text{ when } k/T \to 0 \text{ as } T \to \infty. \tag{2.6}$$

However, it turns out that the variance ratios do not converge to a point, are severely right skewed for relatively large k in a small sample, and are not asymptotically normally distributed as in equation (2.5). So it is not appropriate to apply conventional asymptotics to this case. Richardson and Stock (1989) study the limit distribution of $V_k(z)$ when $k/T \to b > 0$ under the null of a random walk, and Deo and Richardson (2003) extend Richardson and Stock's (1989) result to the process which contains both permanent and transitory components. They find that $V_k(z)$ does not converge to a limit but to a nondegenerate limiting distribution, which is a functional of a Brownian motion as follows:

$$\begin{aligned} V_k(z) \quad &\Rightarrow \quad \frac{\Omega}{(1-b)^2 b} \int_b^1 [W(r) - W(r-b) - bW(1)]^2 \, dr, \\ \text{as } k/T \quad &\to \quad b \ \text{ and } T \to \infty, \end{aligned} \tag{2.7}$$

where $W(r)$ is a standard Brownian motion.

Unlike the case when $k/T \to 0$, the limit distribution of $V_k(z)$ in this case is significantly different from that of the Bartlett kernel estimator. To see the difference,

[9] Later, Andrews (1991) shows that the Bartlett kernel estimator can attain consistency with the bandwidth at growth rate $o(T^{1/2})$.

we can rewrite the expression for $V_k(z)$ in equation (2.1) as follows[10].

$$V_k(z) = \frac{T^2}{(T-k)(T-k+1)}\left(\widehat{\Omega}_k - \frac{1}{Tk}\sum_{t=1}^{k-1} S_t^2 - \frac{1}{Tk}\sum_{t=T-k+1}^{T-1} S_t^2\right) \quad (2.8)$$

where $\widehat{\Omega}_k$ is the Bartlett kernel estimator with the bandwidth of k,

and S_t is the partial sum process, $\sum_{i=1}^{t} u_i$.

In equation (2.8), the main differences between the Bartlett kernel estimator of the long run variance and the variance of k-differences are the two partial sum processes in the parenthesis. This difference indicates the fact that the variance of k-differences underweights observations around both endpoints as mentioned in Cochrane (1988).

To see the difference between the variance of k-differences and the Bartlett kernel estimator for large k's, we compute the mean of each term in equation (2.8). As in equation (2.9), Kiefer and Vogelsang (2005) provide the analytical form of the limit distribution of the Bartlett kernel estimator and its mean when $k/T \to b > 0$.[11]

$$\begin{aligned} \widehat{\Omega}_k &\Rightarrow Q(b) \equiv \frac{2\Omega}{b}\left[\int_0^1 \widetilde{W}(r)^2 dr - \int_0^{1-b} \widetilde{W}(r+b)\widetilde{W}(r)dr\right], && (2.9) \\ \text{where } \widetilde{W}(r) &\equiv W(r) - \lambda W(1) && \\ E\left(Q(b)\right) &= \Omega\left(1 - b + \frac{b^2}{3}\right). && (2.10) \end{aligned}$$

From the functional central limit theorem and the continuous mapping theorem,

$$\frac{1}{Tk}\sum_{t=1}^{k-1} S_t^2 \Rightarrow \frac{\Omega}{b}\int_0^b \widetilde{W}(r)^2 dr, \quad (2.11)$$

$$\frac{1}{Tk}\sum_{t=T-k+1}^{T-1} S_t^2 \Rightarrow \frac{\Omega}{b}\int_{1-b}^1 \widetilde{W}(r)^2 dr, \quad \text{as } k/T \to b \text{ and } T \to \infty. \quad (2.12)$$

[10] This expression is inspired by Cai and Shintani (2006) and Kiefer and Vogelsang (2002a).

[11] $\widetilde{W}(r)$ is called a Brownian bridge. Davidson (1994, p.445) explains this as a Brownian motion tied down at both ends.

The expectations of the limit distributions for the two partial sum processes are as follows.

$$E\left(\frac{\Omega}{b}\int_0^b \widetilde{W}(r)^2 dr + \frac{\Omega}{b}\int_{1-b}^1 \widetilde{W}(r)^2 dr\right) = \Omega\left(b - \frac{2}{3}b^2\right)^{12}. \tag{2.13}$$

Equation (2.10) shows that the mean of the Bartlett kernel estimator when $b > 0$ is proportional to, but different from the long run variance, Ω. Equation (2.13) shows that the variance of k-differences before the small sample correction may be even further away from long run variance on average than the Bartlett kernel estimator. The small correction term adjusts the mean of the variance of k-differences to the level of the long run variance[13]: That is,

$$\frac{T^2}{(T-k)(T-k+1)} \to \frac{1}{(1-b)^2} \quad \text{as } k/T \to b \text{ and } T \to \infty.$$

Meanwhile,

$$\begin{aligned} E\left(Q(b)\right) - E\left(\frac{\Omega}{b}\int_0^b \widetilde{W}(r)^2 dr + \frac{\Omega}{b}\int_{1-b}^1 \widetilde{W}(r)^2 dr\right) &= \Omega\left(1 - b + \frac{b^2}{3}\right) - \Omega\left(b - \frac{2}{3}b^2\right) \\ &= \Omega(1-b)^2. \end{aligned}$$

To sum up, when $k/T \to b > 0$, the limit distribution of the variance of k-differences is significantly different from that of the Bartlett kernel estimator. On the other hand, both the Bartlett kernel estimator and the variance of k-differences converge to a limit distribution which is the multiple of the long run variance and a nuisance-parameter-free distribution. The nuisance-parameter-free distributions depend only on the value of b and are invariant to the distribution of each variable.[14]

[12] The following formula on p.445 in Davidson (1994) is used to compute the expectation: $E(\widetilde{W}(t)\widetilde{W}(s)) = \min(t,s) - ts$.

[13] It can be shown that the three limit distributions in equations (2.9), (2.11), and (2.12) are consistent with Deo and Richardson's (2003) limit distribution as in equation (2.7).

[14] Using this property, Kiefer and Vogelsang (2005) are able to construct a test with this inconsistent Bartlett kernel estimator. They call their approach "fixed-b asymptotics" and the conventional

Since the variance of k-differences is proportional to the long run variance when $k/T \rightarrow b > 0$, the ratio of the variance of k-differences at large k's may contain some information about the ratio of long run variances even though the variance of k-differences is no longer consistent.

Engel's (1999) inference about the relative importance of the traded goods component in the RER movement relies on the statistical properties of the variance of k-differences at $b = 0$ even when k is relatively big. The fact that the variance of k-differences has different limit distributions depending on the value of b raises doubt about Engel's inference. However, there seems to be some hope for Engel's argument about large $k's$ because of the proportionality of the limit distribution of the variance of k-differences to the long run variance at $b > 0$. Even so, it should also be noted that there exists a noticeable difference between the limit distribution of the Bartlett kernel estimator and that of the variance of k-differences at $b > 0$[15].

2.2.2 Statistical properties of $V_k(z)$ when k is close to the sample size

The limit distribution of the Bartlett kernel estimator in Kiefer and Vogelsang (2005) is applicable for $k/T \rightarrow b$ in the interval of (0,1], including the case when $b = 1$. On the other hand, the limit distribution of the variance of k-differences in equation (2.7) is applicable only for b in (0,1). In equation (2.7), the limit distribution is not defined when $b = 1$ since both numerator and denominator in the limit distribution

approach "small-b asymptotics". Sun, Phillips, and Jin (2006) show another way to utilize "fixed-b asymptotics".

[15] Later in this paper, we shall see how different the variance of k-differences can be from the Bartlett kernel estimator for a given b in Figure 2.

become zero in this case[16]. So we cannot say that the variance of k-differences is proportional to the long run variance when $b = 1$. In other words, while there is continuity in the limit distribution of the Bartlett kernel estimator at $b = 1$, such continuity does not exist for the limit distribution of the variance of k-differences. Thus, at this point, the difference between the Bartlett kernel estimator and the variance of k-differences is so huge that the two are not even close to each other.

Unlike the previous cases when $b < 1$, only a small of number of observations are used to compute the variance of k-differences at $b = 1$ regardless of the sample size. For example, when $k = T - 1$, there are only two observations available for any given sample size. As a result, conventional asymptotic theory is not applicable. Due to this restriction, we characterize the statistical properties of the variance of k-differences with the mean of its limit distribution instead of the analytical expression for the limit distribution itself.

The exact analytical solution for the mean of the limit distribution can be computed for the case when $k = T - 1$, the largest possible value of k. It turns out that there exists a symmetric relationship between the two extreme cases when $k = 1$ and when $k = T - 1$. For the case when $k < T - 1$, the symmetry is not exact but approximate. The following proposition establishes a statistical property of the variance of k-differences when $k = T - 1$, the largest possible k.

Proposition 2.1 Under Assumption 2.1, the limit of the mean of $V_k(z)$ when k is the largest possible, i.e. $T - 1$, is equal to the variance of the change, which is equal

[16] In particular, the fact that the whole term in the parenthesis in the equality (2.8) is zero when $k = T$ is consistent with Kiefer and Vogelsang's (2002a) proof.

to the limit of $V_1(z)$. That is,

$$\lim_{T\to\infty} E(V_{T-1}(z)) = \gamma_0 = \lim_{T\to\infty} V_1(z). \tag{2.14}$$

Proof of Proposition 2.1. First, without loss of generality, let's assume that the drift term, d, in Assumption 1 is zero[17].

Next, let's transform equation (2.1) into the following:

$$\begin{aligned}
V_k(z) &\equiv \frac{T}{(T-k)(T-k+1)k}\sum_{t=0}^{T-k}[z_{t+k}-z_t-k\overline{\Delta z}]^2 \\
&= \frac{T}{(T-k)(T-k+1)k}\sum_{j=0}^{T-k}[\sum_{t=1}^{k}\left\{\Delta z_{t+j}-\overline{\Delta z}\right\}]^2 \\
&= \frac{T}{(T-k)(T-k+1)k}\sum_{j=0}^{T-k}[\sum_{t=1}^{k}u_{t+j}]^2, \text{ where } u_t\equiv \Delta z_t-\overline{\Delta z} \qquad (2.15)
\end{aligned}$$

To deal more easily with the case when k is close to the sample size, let $m \equiv T-k$. Then

$$\begin{aligned}
V_k(z) &= V_{T-m}(z) \\
&= \frac{T}{m(m+1)(T-m)}\sum_{j=0}^{m}[\sum_{t=1}^{T-m}u_{t+j}]^2 \\
&= \frac{T}{m(m+1)(T-m)}\sum_{j=0}^{m}[\sum_{i=1}^{j}u_i+\sum_{s=T-m+j+1}^{T}u_s]^2 \qquad (2.16)
\end{aligned}$$

The last equality holds because $\sum_{t=1}^{T}u_t=\sum_{t=1}^{T}\left(\Delta z_t-\overline{\Delta z}\right)=0:$

$$\begin{aligned}
0 &= \sum_{t=1}^{T}u_t=\sum_{i=1}^{j}u_i+\sum_{t=1}^{T-m}u_{t+j}+\sum_{s=T-m+j+1}^{T}u_s \qquad (2.17)\\
&\Rightarrow\ -\sum_{t=1}^{T-m}u_{t+j}=\sum_{i=1}^{j}u_i+\sum_{s=T-m+j+1}^{T}u_s \\
&\Rightarrow\ [\sum_{t=1}^{T-m}u_{t+j}]^2=[\sum_{i=1}^{j}u_i+\sum_{s=T-m+j+1}^{T}u_s]^2.
\end{aligned}$$

[17] When $d\neq 0$, all the following steps in the proof hold true for $\Delta\tilde{z}_t\equiv\Delta z_t-d$ after $\Delta\tilde{z}_t$ replaces Δz_t.

Especially when $m = 1$ or $k = T - 1$, from equation (2.16),

$$V_{T-1}(z) = \frac{T}{(1+1)(T-1)} \sum_{j=0}^{1} [\sum_{i=1}^{j} u_i + \sum_{s=T+j}^{T} u_s]^2 = \frac{T}{(T-1)} \frac{1}{2} \left[u_T^2 + u_1^2\right]. \qquad (2.18)$$

The first term on the right hand side of equation (2.18) is

$$\begin{aligned} \frac{T}{T-1} u_1^2 &= \frac{T}{T-1} \left(\Delta z_1 - \frac{1}{T} \sum_{s=1}^{T} \Delta z_s\right)^2 \\ &= \frac{T}{T-1} \left[(\Delta z_1)^2 - \frac{2}{T} \sum_{s=1}^{T} \Delta z_1 \Delta z_s + \frac{1}{T^2} \left(\sum_{s=1}^{T} \Delta z_s\right)^2\right]. \qquad (2.19) \end{aligned}$$

By taking the unconditional expectation of equation (2.19),

$$\begin{aligned} E\left(\frac{T}{T-1} u_1^2\right) &= \frac{T}{T-1} \left(\gamma_0 - \frac{2}{T} \sum_{j=0}^{T-1} \gamma_j + \frac{1}{T} \sum_{j=-T+1}^{T-1} \frac{T-j}{T} \gamma_j\right) \\ &= \frac{T}{T-1} \left(\frac{T-1}{T} \gamma_0 - \frac{1}{T} \sum_{j=-T+1}^{T-1} \gamma_j + \frac{1}{T} \sum_{j=-T+1}^{T-1} \frac{T-j}{T} \gamma_j\right) \\ &= \gamma_0 - \frac{1}{T-1} \sum_{j=-T+1}^{T-1} \gamma_j + \frac{1}{T-1} \sum_{j=-T+1}^{T-1} \frac{T-j}{T} \gamma_j \\ &\to \gamma_0 - \frac{1}{T-1} \Omega + \frac{1}{T-1} \Omega \\ &\to \gamma_0 \quad \text{as } T \to \infty. \qquad (2.20) \end{aligned}$$

Finally, from equations (2.18) and (2.20),

$$\begin{aligned} E\left(V_{T-1}(z)\right) &= \frac{1}{2} \left[E\left(\frac{T}{T-1} u_T^2\right) + E\left(\frac{T}{T-1} u_1^2\right)\right] \\ &\to \frac{1}{2} (\gamma_0 + \gamma_0) = \gamma_0 \quad \text{as } T \to \infty. \qquad (2.21) \end{aligned}$$

In equation (2.18), $V_{T-1}(z)$ is a function of u_t^2 but not a function of any $u_{t+j} u_t$, $j \neq 0$. $V_1(z)$ is also a function of u_t^2 as follows:

$$V_1(z) = \frac{T}{(T-1)T} \sum_{j=0}^{T-1} [\sum_{t=1}^{1} u_{t+j}]^2 = \frac{T}{(T-1)} \frac{1}{T} \sum_{t=1}^{T} {u_t}^2. \qquad (2.22)$$

By the law of large numbers,

$$V_1(z) \to \gamma_0 \quad \text{as } T \to \infty. \qquad \blacksquare \tag{2.23}$$

Proposition 2.1 shows that the final value of $E\left(V_k(z)\right)$ goes back to its initial value as k varies from 1 to $T-1$. The proposition indicates that $E\left(V_k(z)\right)$ for the largest k has little to do with the long run movement of the variable since its limit is γ_0, which represents the shortest run movement of the variable. So, if we treat it as an estimator of the long run variance, $V_{T-1}(z)$ has a severe bias.

Equation (2.18) shows that $V_{T-1}(z)$ can be expressed only by ${u_t}^2$. No higher order sample autocovariance terms, $u_t u_{t+\tau}$ ($\tau \neq 0$), appear in equation (2.18). It indicates that $E\left(V_{T-1}(z)\right)$ is associated only with the shortest run movement of the variable. In the proof of the proposition, equation (2.17) is the key to derive equation (2.18). Equation (2.17) holds because the mean of the change is unknown and estimated by $\overline{\Delta z}$. So estimated unknown drift is a source of bias. The Bartlett kernel estimator also has such bias due to the estimated unknown drift term. Equation (2.10) shows the bias from the long run variance[18]. The bias grows bigger as τ increases. The Bartlett kernel dampens the bias by assigning smaller weights to higher order sample autocovariances, but the variance of k-differences reverses the effect by underweighting observations near both endpoints.[19]

[18]The Bartlett kernel estimator is a weighted sum of $\widehat{\gamma_\tau}$. The estimate of autocovariance, $\widehat{\gamma_\tau}$, is biased when the mean is unknown. See Theorem 6.2.2 in Fuller (1996) and Percival (1993) for more details.

[19]Campbell and Mankiw (1987) already warned that one must be careful not to misinterpret the behavior of $V_k(z)$ as k increases to the point where it approaches T when the sample mean is used. However, their formula goes to zero instead of γ_0 because it does not have the small sample correction term in equation (2.1).

The following linear algebraic interpretation of equation (2.1) provides another explanation of why equation (2.18) holds. Define a $(T \times T)$ matrix as follows[20]:

$$U \equiv \begin{bmatrix} u_1u_1 & u_1u_2 & \cdots & u_1u_T \\ u_2u_1 & u_2u_2 & \cdots & u_2u_T \\ \vdots & \vdots & \ddots & \vdots \\ u_Tu_1 & u_Tu_2 & \cdots & u_Tu_T \end{bmatrix}.$$

Then, $\sum_{j=0}^{T-k} [\sum_{t=1}^{k} u_{t+j}]^2$ in equation (2.1) is the sum of the elements of all $(k \times k)$ principal minors of the matrix, U. For instance, when $k = 1$, a (1×1) principal minor of U is a diagonal element of the matrix. Hence, $\sum_{j=0}^{T-1} [\sum_{t=1}^{1} u_{t+j}]^2 = \sum_{t=1}^{T} u_t{}^2$ is just the sum of all diagonal elements of the matrix. On the other hand, if $k = T - 1$, $\sum_{j=0}^{T-k} [\sum_{t=1}^{k} u_{t+j}]^2$ is the sum of the following two $(T - 1 \times T - 1)$ principal minors, $U_{T-1,1}$ and $U_{T-1,2}$:

$$U_{T-1,1} \equiv \begin{bmatrix} u_1u_1 & u_1u_2 & \cdots & u_1u_{T-1} \\ u_2u_1 & u_2u_2 & \cdots & u_2u_{T-1} \\ \vdots & \vdots & \ddots & \vdots \\ u_{T-1}u_1 & u_{T-1}u_2 & \cdots & u_{T-1}u_{T-1} \end{bmatrix},$$

$$U_{T-1,2} \equiv \begin{bmatrix} u_2u_2 & u_2u_3 & \cdots & u_2u_T \\ u_3u_2 & u_3u_3 & \cdots & u_3u_T \\ \vdots & \vdots & \ddots & \vdots \\ u_Tu_2 & u_Tu_3 & \cdots & u_Tu_T \end{bmatrix}$$

We know that

$$\begin{aligned} U &= \begin{bmatrix} U_{T-1,1} & A \\ B & u_Tu_T \end{bmatrix}, \\ where\ A &\equiv \begin{bmatrix} u_1u_T & u_2u_T & \cdots & u_{T-1}u_T \end{bmatrix}', \\ B &\equiv \begin{bmatrix} u_Tu_1 & u_Tu_2 & \cdots & u_Tu_{T-1} \end{bmatrix}. \end{aligned}$$

Note that the mean of Δz_t is unknown so estimated with its sample mean. Thus, every column sum and row sum of U is zero since $\sum_{t=1}^{T} u_t = 0$. Hence, the sum of all

[20] I thank Professor J. Huston McCulloch for giving an idea about the following explanation. The explanation is based on Percival (1993).

elements of $U_{T-1,1}$ plus that of A is zero, and so is the sum of all elements of A and $u_T u_T$. It means that the sum of all elements of $U_{T-1,1}$ is equal to $u_T u_T$. And, with the same reason, the sum of all elements of $U_{T-1,2}$ is equal to $u_1 u_1$. So when $T-k=1$, $V_k(z)$ is associated with only zero order autocovariance term, $u_1 u_1$ and $u_T u_T$, not with any higher order autocovariance terms.

Proposition 2.1 implies that, when k is close to the sample size, there is a central tendency for $V_k(z)$ to go back toward the initial value of $V_k(z)$ no matter what DGP z_t follows. To get an idea about the quasi-symmetry of $E\left(V_k(z)\right)$ near both ends of the time horizon, let's find a similar expression to equations (2.21) and (2.22) for $V_k(z)$ when $k=2$ and $T-2$. From equation (2.9), when $k=2$,

$$
\begin{aligned}
V_2(z) &= \frac{T}{(T-1)}\frac{1}{(T-1)}\sum_{j=0}^{T-k}[\sum_{t=1}^{k} u_{t+j}]^2 \\
&= \frac{T}{(T-1)}\frac{1}{(T-1)}\sum_{j=1}^{T-1}[u_j + u_{j+1}]^2
\end{aligned} \tag{2.24}
$$

while, by equation (2.16) when $k=T-2$,

$$
\begin{aligned}
V_{T-2}(z) &= \frac{T}{(T-1)}\frac{1}{3}\sum_{j=0}^{T-k}[\sum_{t=1}^{k} u_{t+j}]^2 \\
&= \frac{T}{(T-1)}\frac{1}{3}\left\{[u_1+u_2]^2 + [u_{T-1}+u_T]^2 + [u_1+u_T]^2\right\}.
\end{aligned} \tag{2.25}
$$

Hence, $V_2(z)$ is associated with zero and the first order sample autocovariance term-namely, u_t^2 and $u_t u_{t+1}$. On the other hand, $V_{T-2}(z)$ is affected by u_t^2, $u_t u_{t+1}$ and $u_1 u_T$. Under the summability condition in Assumption 1, $\sum_{j=0}^{\infty} j \cdot \left|\psi_j\right| < \infty$, the high order autocovariance term, $u_1 u_T$, should be negligible on average. It hints that, for fixed and small m, $V_{T-m}(z)$ is mainly associated with the $(m-1)^{th}$ or lower order autocovariance terms as is $V_m(z)$.

Engel (1999) infers the importance of the traded goods component in the long run movement of the US RER based on the asymptotics in equation (2.6). According to equation (2.6), for small and fixed $k's$, the larger k is, the longer run movements of z_t $V_k(z)$ represents. Contrary to what equation (2.6) indicates, however, when k is close to the sample size, $V_k(z)$ seems to get associated with lower order autocovariances as k increases up to the sample size.

2.2.3 Simulation results for the distribution of $V_k(z)$

Table 2.1 recapitulates our discussion so far on the limit of $V_k(z)$ or the limit of its mean over various time horizons. Under Assumption 1, $V_1(z)$ converges to γ_0. As k increases, $V_k(z)$ converges to the long run variance of Δz_t when $k/T \to 0$. However, if k is big enough compared with the sample size resulting in $k/T \to b > 0$, then $V_k(z)$ converges to a limit distribution and not to a number. In this case, we can show that, from equation (2.7), the mean of the limit distribution is the long run variance. Finally, as k gets close to the sample size, the mean of the limit distribution of $V_k(z)$ goes back to its initial value, γ_0.

If z_t is a random walk, the limit of $V_k(z)$ or the limit of its mean continues to be γ_0 irrespective of k. This follows from the fact that $\gamma_\tau = 0$ for each $\tau \neq 0$, implying that the long run variance of Δz_t is equal to the variance, γ_0. If z_t is stationary, on the other hand, although the graph of the mean of $V_k(z)$ starts from the same point ($V_1(z) \to \gamma_0$), it will go down toward zero as k grows. When the variable is stationary, its long run variance of the first difference is zero. Thus, the limit distribution in this case degenerates to zero. Thus, even in case of $k/T \to \delta > 0$, $V_k(z)$ converges to zero.

Later, as k grows close to the sample size, the mean of $V_k(z)$ goes back toward the initial value, γ_0.

By means of a Monte Carlo simulation, we get the mean and 90% confidence intervals of V_k for each $k = 1, 2, \cdots, T-1$ from 5,000 simulated series of pure random walks and a stationary AR(1) as in Figure 2.1.[21] In the graph, bold lines are the means of $V_k(z)$ in the simulation while normal lines represent 90% two-sided confidence intervals. The solid lines are for the stationary AR(1) process, and the dotted lines are for the random walk process.

Figure 2.1 illustrates our findings in Proposition 2.1. The graph for the mean of $V_k(z)$ for each DGP starts at its variance of the change and ends at the same value. Note especially that the mean of $V_k(z)$ for random walk does not change much as we see in Table 2.1. On the other hand, the mean of $V_k(z)$ for the stationary AR(1) shows as a U-shaped graph.

Then, next observation from Figure 2.1 is that the mean of $V_k(z)$ for the stationary AR(1) has the closest value to its long run variance, zero, in the middle range of time horizons. The minimum of the mean of $V_k(z)$ over different $k's$ is 0.2 at $k = 179$, a little less than half of the sample size. Hence, $V_k(z)$ in the middle range of time horizons seems more relevant to the long run movement of the variable than that at the time horizons close to the sample size

Another observation is that the mean of $V_k(z)$ for the stationary AR(1) process even in the middle range of time horizons is clearly above its long run variance, i.e. zero. As an estimate of the long run variance, $V_k(z)$ has a severe upward bias when

[21] In the simulation, the number of observation is 408 ($t = 0, 1, 2, \ldots, 407$) as in the first data set in Engel (1999). The AR(1) coefficient for the stationary process is set to be .94387 which implies that the half life is one year in a monthly data. The variance of the difference in each process is set to be one. The error terms in each series are assumed to be normal.

the variable is stationary AR(1) even at the most relevant time horizons. Since $V_k(z)$ in equation (2.1) is defined as a sum of squared terms, the value of $V_k(z)$ in a finite sample should be always positive. Thus, the issue here is how close the value of $V_k(z)$ is to the true long run variance. The simulation result shows that the 34 year time-span in Engel (1999) even with fairly short half life is not enough to get an estimate close to the true long run variance. This matter will be considered in details in Chapter 3.

In terms of the width of the confidence intervals, the confidence interval for the stationary AR(1) process is much narrower than that for the random walk in the middle range of time horizons. Intuitively, the narrower confidence interval of the stationary AR(1) may be related to the fact that the limit distribution of $V_k(x)$ does not converges to a nondegenerate distribution but goes to a number even when $k/T \to b \geq 0$.

The next observation for the confidence intervals is that the two distributions become indiscernible as k gets close to the sample size. Hence, the test based on $V_k(z)$ for k close to the sample size will suffer from very low power with the null hypothesis of a random walk against the alternative hypothesis of a stationary AR(1).

We can compare this with simulation results for the Bartlett kernel estimator. As is apparent in equation (2.10), the mean of the Bartlett kernel estimator is getting smaller as b increases when the variable follows a random walk while the mean of the variance of k-differences remains constant because of the small sample correction term. To compare the two statistics, we divide the Bartlett kernel estimator by the terms in parentheses in equation (2.10). After the adjustment, we find no difference between the mean of the Bartlett kernel estimator and that of the variance of k-differences when the variable follows a pure random walk. On the other hand, the two are very different for large $k's$ in the case of a stationary AR(1).

Figure 2.2 shows the mean of the simulation results for both the Bartlett kernel estimator after the adjustment and that for the variance of k-differences in the case of a stationary AR(1). In the figure, the bold solid line is the mean of the Bartlett kernel estimator, and the normal solid line is for the variance of k-differences. The dotted line is the mean of the population counterpart of the variance of k-differences[22]. Both the Bartlett kernel estimator and the variance of k-differences are above the population counterpart on average. There is not much difference between the Bartlett kernel estimator and the variance of k-differences for the first half of the time horizons. However, for the second half, the two statistics are very different. The Bartlett kernel estimator does not change much in this region while the variance of k-differences goes back to the initial level.

Another model for our simulation is an integrated AR(1) which is considered as a possible DGP of log stock price in Lo and MacKinlay (1988). An integrated AR(1) process with a positive AR coefficient is more persistent than a pure random walk. The integrated AR(1) model in Lo and MacKinlay is

$$\Delta z_t = \kappa \cdot \Delta z_{t-1} + \epsilon_t, \qquad \text{where } \epsilon \sim i.i.d.N(0, \sigma_\epsilon^2) \text{ and } |\kappa| < 1. \tag{2.26}$$

In the simulation, $\kappa = .2$, $\sigma_{\Delta z}^2 = 1$, and the sample size is set to be 408 for comparison with the previous simulation. Figure 2.3 represents the simulation result.

[22] If $x_t = \rho x_{t-1} + \varepsilon_t$ with $0 < \rho < 1$, $\varepsilon_t \sim iid(0, \sigma_\varepsilon^2)$, $E(V_k(x)) = \frac{2(1-\rho^k)}{k(1-\rho^2)}\sigma_\varepsilon^2 = \frac{(1-\rho^k)}{k(1-\rho)}\sigma_{\Delta x}^2$.

Figure 2.3 also illustrates the result in Proposition 2.1. For k close to the sample size, the mean goes back to the variance of the change as k increases[23]. Unlike in our first simulation results, the mean of the variance of k-differences soon reaches the level of its long run variance and stays around this level throughout the middle time horizons.

So far, the error terms in the DGP are assumed to be normally distributed. We also performed the same simulation above assuming that the error terms follow t-distribution with 3 degrees of freedom. In this case, the mean of the graph is the same as in Figures 2.1 and 2.3. On the other hand, the confidence intervals for the short run time horizons are wider than those for the normal distribution case. However, as k increases, the confidence intervals converge to those in Figures 2.1 and 2.3.

In conclusion, first, our simulation results in Figures 2.1 and 2.3 show that the mean of $V_k(z)$ does go back to γ_0 as Proposition 2.1 states, irrespective of the DGP of z_t under Assumption 2.1. Second, in terms of its mean, $V_k(z)$ reaches the closest point to the long run variance not in the end but in the middle of the time horizons. Third, while the Bartlett kernel estimator and $V_k(z)$ are very close to each other in the first half of the time horizon, the two are quite different in the second half. Finally, there are two differences between the case of a stationary AR(1) and the case of an integrated AR(1). First, when k is around the middle of the sample size, the mean of $V_k(z)$ for an integrated AR(1) is very close to the long run variance while that for a stationary AR(1) has a severe upward bias. Second, the slope of the graph of the

[23]In this case, the long run variance of Δz_t, Ω is $\sigma_\epsilon^2/(1-\kappa)^2$, whereas $\gamma_0 = \sigma_\epsilon^2/(1-\kappa^2)$. Thus, $\Omega/\gamma_0 = 1.5$. In this example, for comparison with Figure 2.1, γ_0 is set to be one. Then $\Omega = 1.5$.

mean of $V_k(z)$ for a stationary AR(1) is much less steep at both ends of the graph than that for an integrated AR(1).

2.3 Implication of Proposition 2.1 for Engel's ratio of $V_k(z)$

2.3.1 Ratio of the variances of k-differences in Engel (1999)

As in Engel (1999), we define the real exchange rate, q_t, as

$$q_t \equiv s_t + p_t^* - p_t. \tag{2.27}$$

where s_t is the logarithm of the nominal exchange rate, p_t is the logarithm of the general price index of the home country, and p_t^* is the logarithm of the general price index of the foreign country.

Engel (1999) regards the logarithm of the general price index as a weighted average of traded- and nontraded-goods prices:

$$p_t = (1-\alpha)p_t^T + \alpha p_t^N, \tag{2.28}$$

$$p_t^* = (1-\beta)p_t^{T*} + \beta p_t^{N*} \tag{2.29}$$

Superscripts T and N indicate traded and nontraded goods each. An asterisk represents the foreign country. α and β are the shares of nontraded goods in each country's price index.

The RER can be decomposed by

$$q_t = x_t + y_t, \tag{2.30}$$

where

$$x_t \equiv s_t + p_t^{T*} - p_t^T, \tag{2.31}$$

$$y_t \equiv \beta(p_t^{N*} - p_t^{T*}) - \alpha(p_t^N - p_t^T). \tag{2.32}$$

x_t, the traded goods component, is the relative price of traded goods between the two countries while y_t, the nontraded goods component, is a weighted difference of the relative prices of nontraded goods in each country.

Engel (1999) measures the importance of the traded goods component in explaining US RER movements with the ratio of the variance of k-differences of x_t over that of q_t, RV_k : [24]

$$RV_k = \frac{Var(x_t - x_{t-k})}{Var(q_t - q_{t-k})} = \frac{V_k(x)}{V_k(x) + V_k(y)} \tag{2.33}$$

assuming x and y are uncorrelated

2.3.2 An illustration with highly tradable goods

In order to illustrate the implication of Proposition 1 for Engel's method, we first apply his method to data involving highly tradable goods for which the law of one price is likely to hold. Our purpose in this exercise is to show that the ratio of the variances of k-differences for stationary x_t is likely to have a U-shaped graph.

Burstein, Neves and Rebelo (2003) point out that distribution costs are so large for consumer goods that the law of one price may not hold at the retail price level. For this reason, Burstein, Eichenbaum and Rebelo (2005 and 2006) use the prices of pure-traded goods at the dock[25]. Following Burstein, Eichenbaum and Rebelo (2006), we measure the prices of traded goods using a geometric average of import and export prices and compute x_t in equation (2.31) with those prices. Then we construct data

[24] Engel (1999) mainly uses the ratio of the mean-squared errors (MSE), the sum of the squared drift and the variance of k-differences, in order to measure the movement comprehensively. However, he states that the results based on the variance of k-differences are not very different from the results based on MSE's for US RER. His inference in the paper is based on the properties of the variance of k-differences. For simplicity we only consider the ratio of the variance of k-differences in this paper.

[25] Betts and Kehoe (2006) also show that the choice of the price series significantly affects the statistical measure of the relative importance of the traded goods component in the real exchange rate movement.

for y_t as the difference between the RER and x_t.[26] For the RER, the CPI general indexes for both countries are used.

The data are collected from the IFS CD ROM. The sample period is 1973:01-2002:12. Among the ten bilateral real exchange rates with the US in Burstein, Eichenbaum and Rebelo (2006), graphs for the US-Italy RER are presented in Figure 2.4 since our unit root test and stationarity test results consistently indicate that its traded goods component is likely to be stationary.

Figure 2.4 is a graph for $V_k(x)$ of the US-Italy RER, and Figure 2.5 is for RV_k. As far as the long run movement of RER is concerned, V_k can be interpreted as an estimator of the long-run variance following the asymptotics in equation (2.6). According to the traditional Balassa-Samuelson theory, the numerator of RV_k should converge to zero since the long run variance of the traded good component, which is stationary, is zero. On the other hand, the denominator of RV_k is more likely to have a positive value because the nontraded good component is more persistent and could even be unit-root nonstationary. As a whole, therefore, RV_k is likely to converge to zero as k increases at an appropriate rate as the sample size increases. In other words, the importance of the traded goods component should be small in the long run.

Given this result, it is tempting to interpret the rise of V_k and RV_k for large k in Figures 2.4 and 2.5 as evidence against the Balassa-Samuelson theory in the long-run. However, Proposition 1 shows that, for large k, the asymptotics in equation (2.6) are not applicable and that V_k has a tendency to go back to the initial level as k gets closer to the sample size irrespective of whether the variable is stationary or difference stationary. Due to the statistical properties of its numerator and denominator, RV_k

[26]So the decomposition of the RER is based on equation (3) in Engel (1999) rather than equation (1) in Engel's paper. In other words, $y_t = (p_t^* - p_t^{T*}) - (p_t - p_t^T)$.

also has a tendency to go back to the neighborhood of the initial level. Therefore, the rise of V_k and RV_k for large k is more likely due to the property of V_k specified in Proposition 2.1 than to the properties of US RER long run movements.

If we focus on the fall of V_k and RV_k for the first half of the graph in these figures, the importance of the traded goods component in explaining the movement of the real exchange rate becomes smaller in the longer run. The graph in Figure 2.5 is clearly in favor of stationarity of x_t since the solid line, RV_k, is, in the longer periods, under the lower dotted line which is the critical value of the null that x_t follows a random walk[27]. Thus, it is important not to interpret the rise of the ratio in the second half of Figure 2.5 as evidence against the Balassa-Samuelson theory in the long-run.

2.3.3 Reexamination of Engel's Empirical Results

We now reexamine Engel's results in light of our findings in this paper. Solid lines in Figure 2.6 plot the graphs of RV_k for the US RER computed from Engel's (1999) first data set in his paper[28]. For short time horizons, the ratios are all over 90%. For the middle range of time horizons, the ratios go down except for the US-Italy RER, although the magnitude of change varies from country to country. And finally, for long time horizons, the ratios move back to higher levels. The most prominent case is for the US-Canada RER.

Although, Engel (1999) refrains from reaching a decisive conclusion because of the small number of independent observations for large $k's$, he interprets the rise

[27] How we construct the critical value will be explained in detail in the next subsection.

[28] The data are monthly from January 1962 to December 1995 for Canada, France, Germany, Italy, Japan, and the United States. Thus it has 408 observations (so $T = 407$). CPI's for goods are used for traded goods prices, and CPI's for services are used for nontraded goods prices. See Appendix A of Engel (1999) for more details.

of the graph for the US-Canada RER in longer time horizons as an increase in the importance of the traded goods component (p.513), implying that the traditional Balassa-Samuelson theory does not work even in the long run.

However, previous discussions in this paper show that $V_k's$ for $k's$ near the sample size have little to do with the long run movement of the variables. Thus, the rise of the graph for the US-Canada $RV_k's$ at large $k's$ may not be interpreted as an increase in the importance of the traded goods component for long run time horizons.

Instead, our simulation results indicate that $V_k's$ in the middle range of time horizons are more relevant to the long run movement of the variable while $V_k's$ at both ends of the time horizon are associated with the short run movement. If x_t is AR(1) and y_t is a random walk, the graph for $RV_k's$ is likely to be U-shaped on average while the graph should be close to a flat line if both x_t and y_t are random walks. The graphs in Figure 2.6 show a U-shape except for US-Italy so that $RV_k's$ in the middle range have smaller value than those at both ends. It may imply that the traded good component becomes less important in the longer run in accounting for the movement of the US RER.

Although the graphs look U-shaped, $RV_k's$ in the middle range may not be statistically different from those at both ends. With $RV_k's$ computed from the data, Engel (1999) tries to test his null hypothesis that the law of one price for the traded goods does not hold. Since there is no standardized asymptotic distribution of RV_k, Engel uses a parametric bootstrap method to compute the confidence intervals of RV_k. Under his null, he supposes that both x_t and y_t are random walks with drift. Engel, then, shows that the RV_k at every time horizon in the data is within the two-sided 95% confidence interval of RV_k. As such, Engel does not find any evidence for a less important role for the traded goods component in the longer run movement of RER. Engel compares his results with Kakkar and Ogaki's (1999) which are in favor of an important role for the nontraded goods component in the long run. He attributes the difference to the low power of the tests to distinguish between unit roots and stationarity in relatively short time spans.

We believe that Engel's confidence interval needs some adjustments. Those adjustments lead to a different conclusion from that in Engel (1999). We find that Engel's empirical results are not very robust. Our adjustments to Engel's testing method include the following.

First, we perform a one-sided test, as opposed to the two-sided test in Engel (1999). Our main interest in this paper is the long run movement of the RER. In the long run, V_k is the estimator of long run variance. If the law of one price for traded goods does not hold in the long run, then x_t is nonstationary and the long run variance of Δx_t will have a positive value. On the other hand, if the law of one price holds in the long run, then x_t is stationary and the long run variance of Δx_t is zero. Therefore, RV_k under the null that both x_t and y_t are random walks should be statistically larger than that under the alternative hypothesis. Thus, lower dotted line in Figure 2.6 is the critical value under the null that both x_t and y_t are random walk. That is, if RV_k from the data is lower than the confidence intervals, then the test rejects the null.

Second, we report the confidence interval only up to half of the sample size while Engel (1999) reports up to the largest possible k. Since Cochrane (1988), it has been known that the variance of k-differences for large k is not reliable. Admitting the inaccuracy of the statistics in his paper, Engel reports it for the entire time horizon probably because he believes that a small piece of information about the long run is better than no information. However, since our findings indicate that RV_k for large k has little to do with the long run, there is not much gain from reporting inaccurate test results for large $k's$.

Third, we do not allow drift either in x_t or in y_t. While the literature on the Balassa-Samuelson theory has given some models which allow drift in y_t, it is difficult to find a model which explains why x_t may have drift. Engel does not provide a theoretical explanation for it either.

Fourth, with the same testing method, it is easy to flip the null. We can compute the confidence interval under the null that x_t is stationary and that y_t is still a random walk. When RV_k from the data is above the confidence interval, the test rejects the null. An additional problem in this case is how to specify the data generating process of x_t. Apparently, there are many different kinds of stationary processes. We report the simplest case in which x_t is $AR(1)$ in this paper.

The dotted lines in Figure 2.6 are critical values for one-sided tests with 5% size after the changes we make. The lower dotted line is the critical value under the null that x_t is a random walk without drift while the upper dotted line is the critical value under the null that x_t is stationary $AR(1)$. Overall, the graphs at many time horizons are within the two dotted lines, indicating the low power problem pointed out by Engel (1999). However, unlike the results in Engel (1999), $RV_k's$ for long time horizons are below the lower dotted line, rejecting the null that both x_t and y_t are random walk for Canada, France, and Germany. In the case of Italy and Japan, on the other hand, $RV_k's$ for short time horizons are above the upper dotted line, rejecting the null that x_t is $AR(1)$ and y_t is a random walk.

To sum up, the test based on $RV_k's$ computed from the data and bootstrap critical values does not necessarily support Engel's null hypothesis but provides some evidence for smaller importance of the traded goods component in accounting for longer run RER movement.

2.4 Conclusion

According to the traditional Balassa-Samuelson view, the traded goods component of the real exchange rate is stationary while the nontraded goods component is more persistent and could even have a unit root. The long run variance of the traded goods component is zero while the real exchange rate itself has a positive long run variance if the nontraded goods component has an autoregressive unit root.

Cochrane (1988) shows that the variance of k-differences (V_k) is asymptotically equivalent to the Bartlett kernel estimator of long run variance. Engel (1999) computes V_k for the real exchange rate and for its traded goods component. Based on Cochrane (1988), Engel expected that the ratio of V_k of the traded goods component to V_k of the real exchange rate would converge to zero as k increases if the traditional theory were true.

In contrast to the traditional view, Engel finds that the ratios decrease at first but increase at the end of the time horizons, most prominently in case of the US-Canada RER. Engel interprets this as an increase in the importance of the traded goods component at longer run time horizons. Based on the empirical results, Engel concludes that the behavior of the traded goods component is indistinguishable from the behavior of a random walk.

This paper, however, shows that the mean of the variance of k-differences for the largest k, V_{T-1}, converges to the limit of the variance of the first difference, V_1. Therefore, if V_k falls as k increases, V_k tends to rise as k approaches $T-1$ irrespective of whether the variable of interest is stationary or unit root nonstationary. This means that the rise of the graph at k close to the sample size in Engel (1999) cannot be interpreted as evidence for unit root nonstationarity of the traded goods component in the real exchange rate.

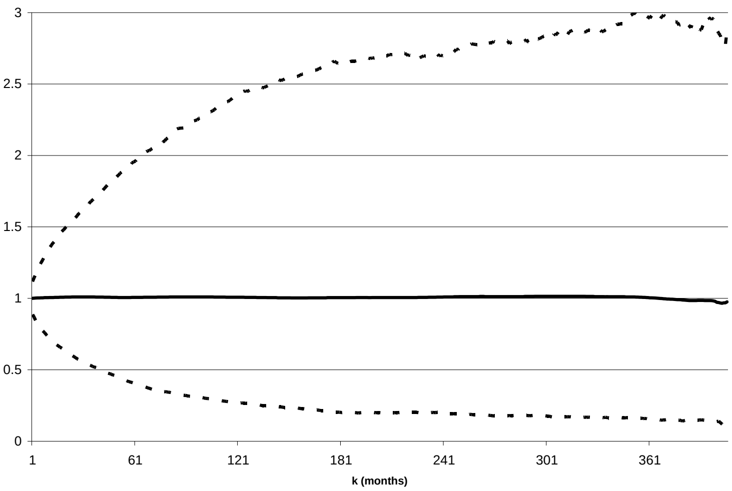
While V_k for k close to the sample size does not reflect the long run properties of the variable, the simulation results in the paper show that V_k will get closer to the long run variance as k increases from one to time horizons in the middle range. The ratio of V_k in Engel (1999) decreases in the first half of time horizons, which indicates that the nontraded goods component plays a more important role in the longer run.

Engel (1999) show that the $RV_k's$ from the data are all within the confidence intervals he constructs under the null that there is no change in the importance of the nontraded goods component over different time horizons. On the contrary, after some adjustments of the testing method, our test results provide some evidence consistent with a more important role for the nontraded goods component at longer time horizons for some countries.

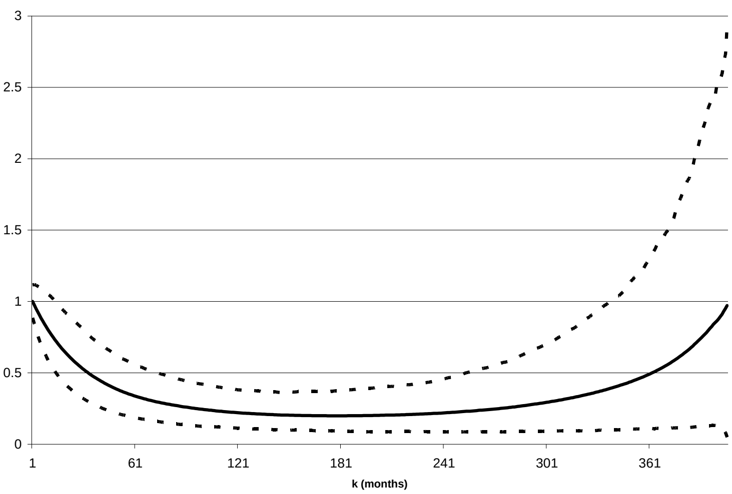
Cochrane (1988) pointed out that the variance of k-differences for large k is less reliable. He explains that the degrees of freedom of V_k are roughly equal to the number of nonoverlapping long runs, which is less than two when k is more than half of the sample size. Considering the inaccuracy due to low degrees of freedom, Cochrane (1988) reports his results at time horizons only up to one fourth of the sample size. Lo and MacKinlay (1988) also report their simulation results at time horizons up to half of the sample size. It is exceptional in the literature to report up to the longest time horizon as in Engel (1999), and Engel admits that his longer run horizon numbers are less reliable, probably based on Cochrane's (1988) argument.

What is new in this paper, though, is that V_k for k close to the sample size not only has a big variance due to its low degrees of freedom but also has little to do with the long run movement of the variable.

(a) A random walk without a drift

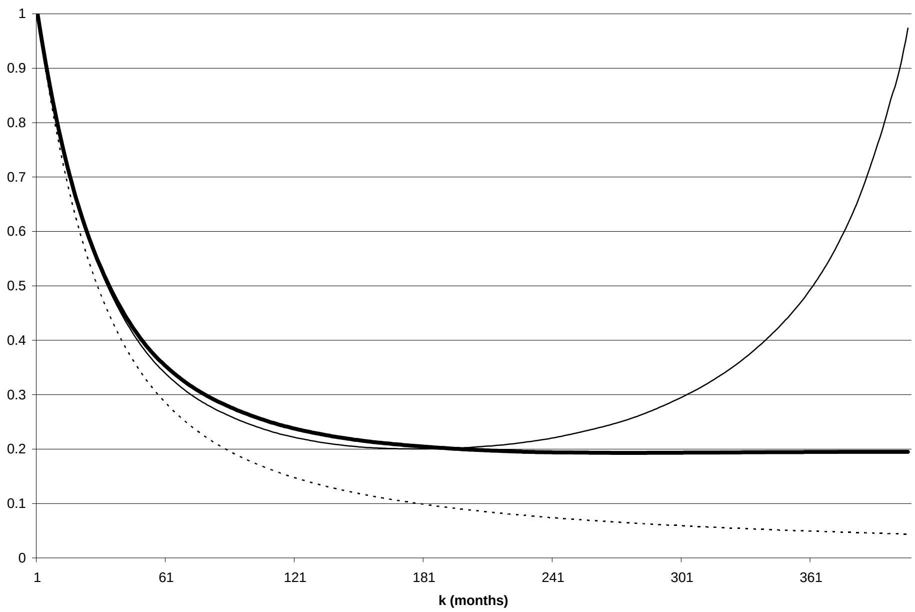


(b) A stationary AR(1) with half life of one year



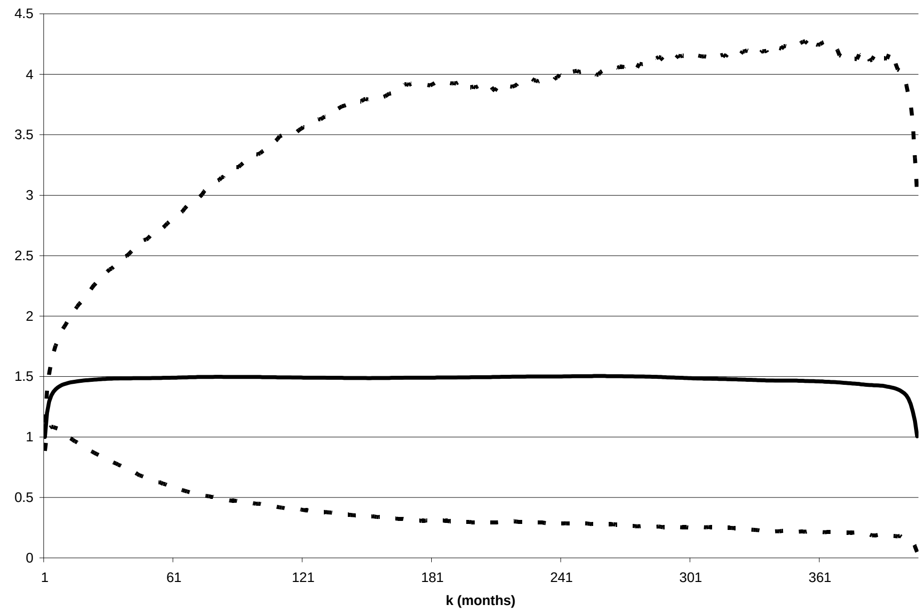
Notes: i) A solid line is the mean of the V_k.
ii) Dotted lines are the 90% confidence intervals of V_k.

Figure 2.1: The distribution of the variance of k-differences



Notes: i) The normal solid line is for V_k, the bold solid line is for the Bartlett kernel estimator, and the dotted line is for the population counterpart.

Figure 2.2: Mean of V_k and mean of the Bartlett kernel estimator for a stationary AR(1)



Notes: i) A solid line is the mean of the V_k.
ii) Dotted lines are the 90% confidence intervals of V_k.

Figure 2.3: The distribution of V_k (ARIMA(1,1,0) Model)

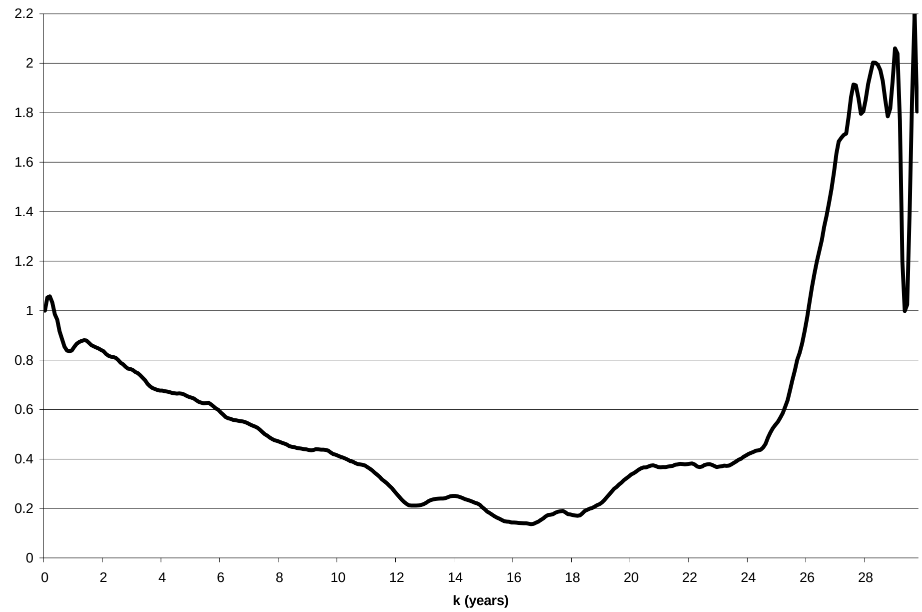
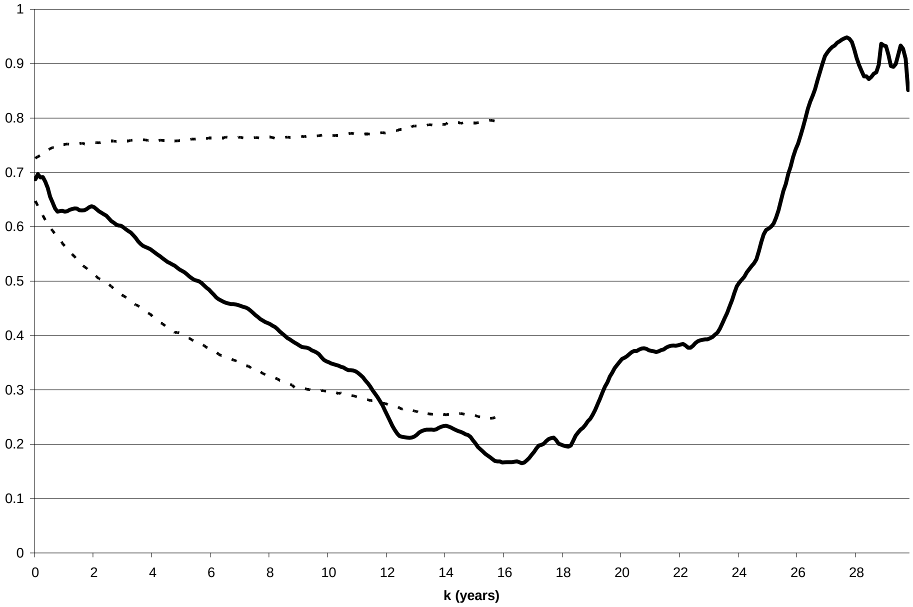
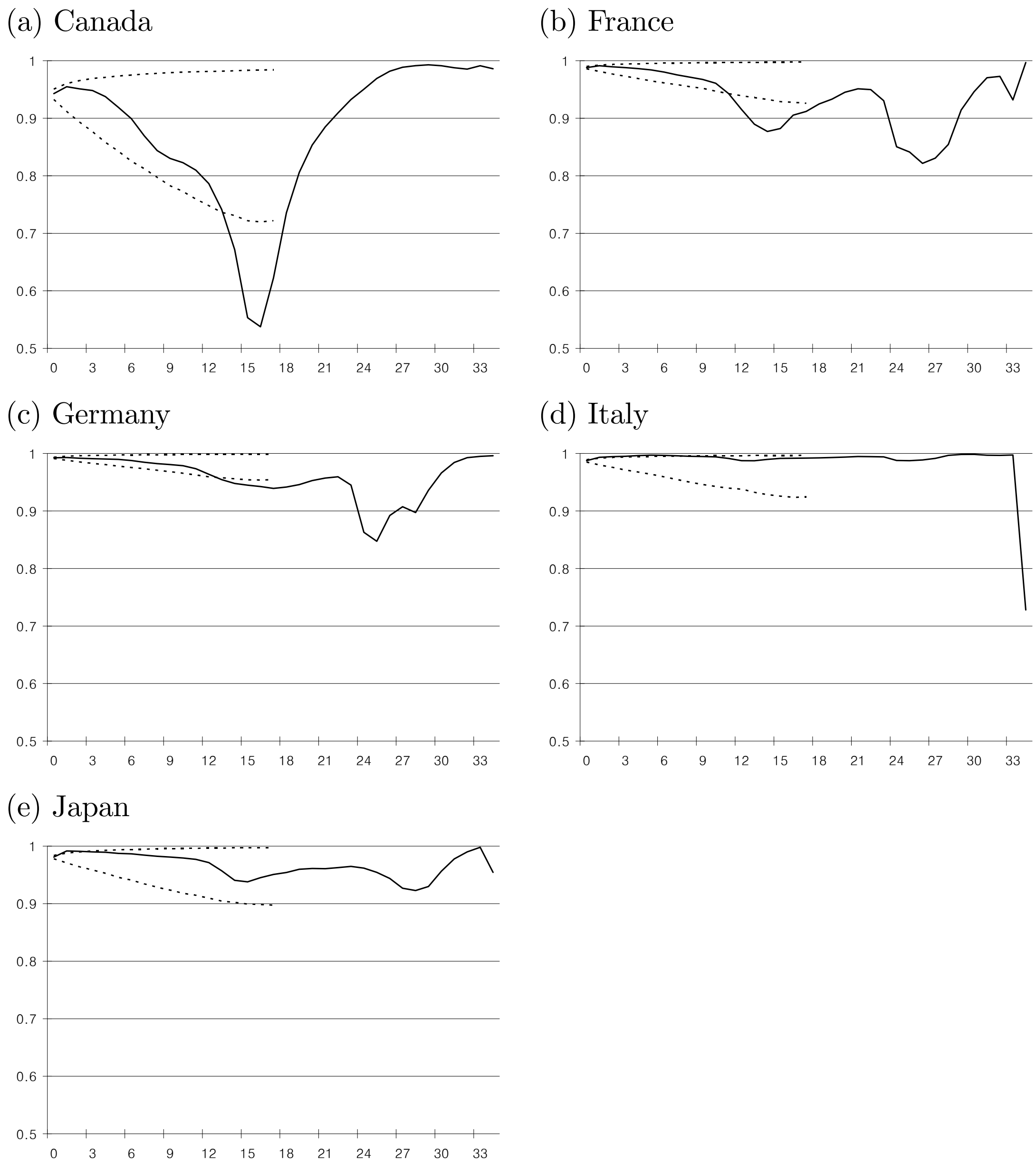


Figure 2.4: Variances of k-differences for x_t of US-Italy pure traded goods



Notes: i) The solid line is the mean, and dotted lines are the 90% confidence intervals.

Figure 2.5: Ratio of V_k for US-Italy pure traded goods



Notes: i) The solid line is the ratio, and dotted lines are the 90% confidence intervals.
ii) The horizontal axis is the time horizon, k (years).

Figure 2.6: Ratio of $V_k's$ for the US RER(1962:01-1995:12)

	$k=1$	k is small and fixed	$k/T \to 0$	$k/T \to b > 0$	$k = T-1$
general case	γ_0	$\sum_{\tau=-k+1}^{k-1} \frac{k-\lvert\tau\rvert}{k}\gamma_\tau$	Ω	Ω	γ_0
random walk	γ_0	γ_0	γ_0	γ_0	γ_0
stationary	γ_0	$\sum_{\tau=-k+1}^{k-1} \frac{k-\lvert\tau\rvert}{k}\gamma_\tau$	0	0	γ_0

Notes: i) γ_τ is the τ-th order autocovariance of Δz_t.
ii) Ω is the long run variance of Δz_t.

Table 2.1: The limit of $V_k(z)$ or the limit of the mean of its distribution

CHAPTER 3

A MONTE CARLO INVESTIGATION ON THE ESTIMATOR OF RATIO OF LONG RUN VARIANCES

3.1 Introduction

To measure the relative importance of traded goods component in accounting for real exchange rate movement, Engel (1999) adopted the ratio of the variance of k-differences in his celebrated empirical work. Engel (1999) computes the ratio of the variance of k-differences of the traded goods component to that of the real exchange rate. With few exceptions, he finds is that the ratio is very close to one even in the long time horizon. He takes the empirical findings as evidence against long run PPP.

The variance of k-differences is, under certain conditions, asymptotically equivalent to the Bartlett kernel estimator, one of heteroskedasticity and autocorrelation consistent (HAC) estimators for the long run variance (LV). Engel's (1999) inference about the long run behavior of real exchange rates in his paper relies on the econometric theories about the HAC estimators. The statistical properties of HAC estimators for LV have been one of the central subjects in time series econometrics literature. However, the statistical properties of the "ratio" of an estimator has not been studies much[29]. The statistical properties of the ratio may not be fully explained by the

[29] One exception is Albuquerque (2005) who studies the properties of the long run correlation of two variables.

statistical properties of the estimator itself because of the nonlinearity between the two.

In this paper, we investigate the statistical properties of the ratio of HAC estimators using a Monte Carlo simulation in order to have a better understanding of the implication of Engel's (1999) empirical findings for long run behavior of real exchange rates. We will consider not only the variance of k-differences used in Engel (1999) but also two other popular estimators; the Bartlett kernel estimator and the Quadratic Spectral kernel estimator. We will focus on the mean of the ratio in the simulation and the power of the tests in our simulation to investigate the accuracy of the ratio as a point estimator and its usefulness in a hypothesis test.

The two main parameters which determine the results in our simulation are the relative volatility and the degree of persistence of the traded goods component in the real exchange rate. Engel's interpretation of his empirical results for the long run movement of real exchange rates focuses on the role of the persistence of the traded goods component. Our simulation results will highlight that the relative volatility of the traded goods is the major determinant of the value of our estimators even at the time horizon as long as twenty or thirty years when the value of the relative volatility is as high as observed in the data Engel (1999) uses. Our results imply that the high level of the estimated ratio found in Engel (1999) is not necessarily a reliable piece of evidence against long run PPP. Our approach is connected to Taylor (2002) who decomposes the real exchange rates into the two factors and shows that the large fluctuation of the real exchange rates in the free floating regime can be attributed to the volatility of the error, not to the persistence of the real exchange rate.

Engel (1999) mainly uses the sum of the demeaned variances of k-differences and the squared drift to measure the movement of a variable instead of the variance of k-differences itself although his inference relies on the properties of the variances of k-differences. We study the sum of the variances of the variances of k-differences and the squared drift when there is no drift in the true data generating process. Interestingly, the mean of the sum at the largest time horizon becomes twice bigger than the true long run variances if the variable has no drift, leading to a huge bias in the estimation. As shown in Chapter 2, demeaning is a source of bias of sample autocovariances. To avoid the bias, we may simply use the uncentered moments to construct our estimators instead of estimating the variances and the drift separately and add the two. That strategy is adopted by Betts and Kehoe (2006). Our simulation results show that the means of the ratio of uncentered moments for the variance of k-differences or the Bartlett kernel estimator are lower than the demeaned counterparts but still far away from zero even the half life of the traded goods component is short. In case of the QS kernel, the mean of the ratio at the long time horizon has significantly lower than the other two estimators, though it is still far away from zero.

One important measure of the usefulness of an estimator is the power of a test based on the estimator. Engel (1999) construct a test using the ratio of the variance of k-differences. Under the null that both the traded goods component and the nontraded component follow a random walk, he constructs confidence intervals of the variance of k-differences by means of a parametric bootstrap method. He finds that there is no evidence against his null, implying that there is no mean reversion in the traded goods component of the real exchange rate. We compute the power of the tests based on the three kernel estimators for each relative volatility and each half life of the traded goods component. Unlike the case of the means of the simulation results, the power of a test is almost invariant to the value of the relative volatility. The half life determines the power of the test. When the half life is one year, the maximum power of a test with centered moments ranges from 45% to 60% depending on the choice of kernels. When the half life is four years, however, the maximum power of a test is 10 to 20% depending on the choice of the kernels and the null of the test. Thus, for given parameter values in the data, the test is not very powerful. Using uncentered moments increases the maximum power of a test by 10 to 20 percentage points.

While the power of the test based on the variance of k-differences decreases quickly after the time horizon with the maximum power, the ratio of uncentered moments of the other two kernel estimators does not decline much. The time horizon at which the test has the maximum power is unknown. Since we do not have a reliable rule to choose the time horizon, an alternative way is to perform a test at the largest time horizon. The ratio for the Bartlett and QS kernel estimators based on uncentered moments gives much higher power than that for the variance of k-differences at the longest time horizon.

The rest of the paper is organized as follows. Section 2 will explain our Monte Carlo simulation. Section 3 presents our simulation results and findings. Section 4 concludes.

3.2 Monte Carlo simulation

In this section, we will explain the Monte Carlo simulation method used in this paper. We will discuss the models of the data generating processes (DGP), our choice of the values of the parameters in the models for the DGPs, and the estimators of the ratio of the long run variances in our Monte Carlo simulation.

In Engel (1999), a real exchange rate consists of the traded goods component, x_t, and the nontraded goods component, y_t. He computes the ratio of the variances of k-differences (V_k) of x_t over the variance of k-differences of the real exchange rate[30]. In our Monte Carlo simulation, we first generate two thousand bootstrap sample of x_t and y_t for given parameter values under the two different models of the DGP for x_t and y_t. Second, we compute the estimator of the ratio of long run variances for each bootstrap sample at each time horizon. Third, we investigate the properties of each estimator by computing the mean and the power of certain tests based on the ratio with the two thousand estimates from bootstrap samples.

3.2.1 Two models for the data generating processes

As for the two models of the DGPs for x_t and y_t, y_t is assumed to be unit root nonstationary in both models since it is affected by preference and technology shocks which we believe have an autoregressive unit root. On the other hand, x_t will be stationary if the law of one price for the traded goods hold in the long run as in Model 2 while x_t may not be stationary when the law of one prices does not hold even in the long run as in Model 1. Thus, the two models are different from each other due to the different assumption on the stationarity of x_t.

For simplicity, we assume that a unit root nonstationary process follows a simple random walk process and that a stationary process follows a AR(1) process. Then Model 1 in the following is the data generating process (DGP) which we will consider

[30] In addition, Engel (1999) assumes that x_t and y_t are indepent. It implies that the variance of k-differences of the real exchang rate equals the sum of the variance of k-differences of x_t and that of y_t.

for nonstationary x_t cases while Model 2 is the DGP for stationary x_t cases. In the simulation, all error terms are assumed to be normally distributed.

$$
\begin{aligned}
\text{Model 1} \quad &: \quad x_t \text{ and } y_t \text{ are random walks without drift.} \\
x_t &= x_{t-1} + \varepsilon_t, \qquad \varepsilon_t \sim iidN(0, \sigma_\varepsilon) \\
y_t &= y_{t-1} + \eta_t, \qquad \eta_t \sim iidN(0, \sigma_\eta)
\end{aligned}
$$

$$
\begin{aligned}
\text{Model 2} \quad &: \quad x_t \text{ is } AR(1) \text{ and } y_t \text{ is a random walk without drift.} \\
x_t &= \rho x_{t-1} + \varepsilon_t, \qquad 0 < \rho < 1, \quad \varepsilon_t \sim iidN(0, \sigma_\varepsilon) \\
y_t &= y_{t-1} + \eta_t, \qquad \eta_t \sim iidN(0, \sigma_\eta)
\end{aligned}
$$

3.2.2 Two main parameters in the models

We have two parameters (σ_ε and σ_η) in Model 1 and three parameters (ρ, σ_ε, and σ_η) in Model 2. Since we are interested not in the level of the long run variances but its ratio, we will set the standard deviation of Δy_t, σ_η to one without loss of generality in our simulation. Let's denote the standard deviation of Δx_t as $stdv(\Delta x_t)$. In Model 1, $stdv(\Delta x_t)$ is σ_ε while it equals $2\sigma_\varepsilon/(1-\rho)$ in Model 2. Our Monte Carlo simulation results depend on the values of the relative size of standard deviation of Δx_t, $stdv(\Delta x_t)$ and AR(1) coefficient, ρ. The larger ρ is, the more time it takes for the effect of a shock to traded goods component to dissipate. Half life is a term for the time it takes for half the effects of a given shock to dissipate. In Model 2, the half life can be computed as $-\ln(2)/\ln(\rho)$. In short, the relative size of standard deviation of Δx_t and the half life for traded goods component are the two parameters in the models of x_t and y_t which will determine our simulation results. We will denote $stdv(\Delta x_t)/stdv(\Delta y_t)$ as RS and half life as HL.

Engel (1999) finds that the ratio of the variances of k-differences are almost always above 90% at any time horizon. He attributes the very high ratio at the long time horizon to the strong persistence of the traded goods component. Under certain values of parameters, Engel's inference may not be valid. We will see that the ratio for a short half life of the traded goods component at fairly long time horizon can be far away from zero if the relative volatility is as big as observed in Engel's data set.

Our approach is comparable with Taylor (2002). He also decomposes the residual variance of his estimated AR model for the real exchange rate into the two factors: the half-life of disturbances and the variance of the (stochastic) error disturbance. He finds that the larger deviation from PPP in the floating exchange rate regime than in the Gold standard or Bretton Woods era is not attributable to significantly greater persistence (longer half-lives) of deviations, but is due to the larger shocks to the real exchange rate process in the period.

3.2.3 Choice of values of parameters

To get a benchmark for parameter values which are relevant to the discussion in this paper, we estimate Model 1 and Model 2 with the first data set in Engel (1999). The data set spans from 1962:01 to 1995:12 for six countries including U.S. So it is a monthly data with 408 observations. For the estimation of Model 2, we include an intercept in the regression for x_t since the price level itself is not available in the data but only an index for price level is. Table 3.1 shows the estimated parameter value for Model 2 from the data set we have.

Estimation results show that the traded good component for some countries in 1962:01-1995:12 data set has much longer half-life (4.1-25.1 years) than we normally expect in this line of literature. Rogoff (1996) mentions the "remarkable consensus" of the half life of 3-5 years for the real exchange rate based on general price index of CPI. According to the Balassa Samuelson theory, the real exchange rate based on the traded goods prices like x_t in Model 2 should have shorter half life than that for the general price index.[31]

One reason for such high value may be due to misspecification error caused by the simplicity of our model. However, allowing longer time lag in our does not significantly reduce our estimated value of half life. Another explanation of failure to get a shorter half life estimate is the existence of the nontradable distribution and retail service in the CPI of tradable goods as emphasized by Burstein, Neves and Rebelo (2003) and Burstein, Eichenbaum and Rebelo (2006).

Since accurate estimation of half life of the traded goods component is not the main interest in this paper, we just set the value of half life of the traded goods component to one, two, and four years (that is, $HL \in \{1, 2, 4\}$) for the traded goods component. We take Kim's (2004) estimate for the half life of the traded goods component as our low bound. We set four years as our upper bound since increasing half life longer than four years does not change our simulation results much.

As for the relative size of the volatility of error, estimates from the data range from 4 to 11. This results imply very high volatility of the traded goods component relative to that of the nontraded goods component during the given sample period. Burstein, Eichenbaum, and Rebelo (2006) shows that the relative size can be as low as one if we use at-the-dock prices for traded goods prices. For this simulation, we

[31] Recent empirical results such as Crucini and Shintani (2002), Kim and Ogaki (2004), and Kim (2005) support for shorter half of traded goods component than real exchange rate itself.

set the estimates from data in Engel (1999) as our upper bound. We will consider $RS \equiv stdv(\Delta x)/stdv(\Delta y) \in \{1, 2, 4, 8\}$.

3.2.4 Two more kernel estimators

For this paper, we will consider two more HAC estimators of the long run variances other than the variance of k-differences (V_k); the Bartlett kernel estimator (BT_k) and the Quadratic Spectral kernel estimator (QS_k). The definitions for BT_k and QS_k, are given in Andrews (1991). k is a bandwidth parameter for each kernel estimator. BT_k is considered because it is asymptotically equivalent to V_k under certain condition and Engel relies on the asymptotic properties of the Bartlett kernel estimator in his inference. QS_k is considered here since Andrews (1991) shows that QS_k has some desirable properties compared with other kernel estimators.

One of our findings in Chapter 2 is that demeaning of first difference is a source of big gap between the true long run variance and the variance of k-differences. Based on the observation, we will also investigate the differences between the method with centered moments and that with uncentered moments in estimating the ratio of long run variances.

3.2.5 Mean of the simulations and power of the tests

In order to check the usefulness of our estimators, we will compute the mean of the ratio among the two thousand bootstrap samples at each time horizon. We will also compute the size corrected power in the tests based on the estimator of the ratio of long run variances. The mean of the ratio in the simulation shows how accurate an estimator is compared with its true value. In some cases, tests based on inconsistent estimator may show a good performance when it is used in a test. Engel

(1999) constructs a test based on the variance of k-differences. The power of the tests computed from our bootstrap samples will provide information about how useful the estimator of the ratio of long run variances is for testing a hypothesis.

3.3 Results of Monte Carlo simulation and implications

3.3.1 Mean of the ratio in the simulation

Mean of the ratio in Model 1

Chapter 2 shows that the mean of the variance of k-differences equals the variance of the error at any time horizon if the variable follows a random walk as in Model 1. Due to Jansen's inequality, however, the mean of the ratio of long run variance estimators may not equal the ratio of the two means. Figure 3.1 presents the mean of the ratio of long run variance estimators in our simulation for each relative size of the traded goods component error in Model 1. Panel (a), (b), and (c) present the mean of the simulation for V_k, BT_k, and QS_k respectively. The four graphs in each panel are for $RS = 1, 2, 4, 8$ from the bottom to the top.

As a benchmark, we define the ratio of population variances of k-period differences as defined by RPV_k:

$$RPV_k \equiv \frac{V(x_{t+k} - x_t)/k}{V(x_{t+k} - x_t)/k + V(y_{t+k} - y_t)/k}.$$

In Model 1, the ratio of population variances of k-period differences equals $RS^2/(RS^2+1)$ regardless of the time horizon since every shock is permanent and does not disappear even in the long run.

Panel (a) in Figure 3.1 is the mean of the simulation for V_k. When the relative size of the volatility, RS, is one, the graph is not different from the ratio of population variances of k-period differences so that it is a horizontal line. When RS is as big as 8, the graph is very close to a horizontal line. In that case, the gap between the population counterpart and the mean of the ratio of $V_k's$ over different time horizons can not be bigger than three percentage point. However, for the middle range of RS, the graph is decreasing. The gap between the graph and the theoretical value is as big as 8 percentage point for $RS = 2$. The decrease in the mean of the ratio is also observed in the simulation for BT_k and QS_k as in Panels (b) and (c) in Figure 3.1. The graph for BT_k is quite close to that for V_k while the graph for QS_k shows bigger difference from the theoretical value.[32]

Mean of the ratio in Model 2

In Model 2, the long run variance of the traded goods component is zero since the traded goods component is stationary. If an estimator of the long run variance is consistent and the sample size is large enough, the estimator of the ratio of long run variances should be close to zero. Engel finds that the estimated ratios are above 90% at any time horizon from his 34 years monthly data. Engel (1999) takes this empirical results as evidence against long run PPP. For his inference to be valid, the time span of the data should be long enough to apply the asymptotics for given values of parameters. To check whether the condition holds, first we will see what value the ratio of population variances of k-period differences would have for given parameterization.

[32] We also computed the median of the ratio in our simulation under Model 1. In case of the median, it was not different from the ratio of long run variances under our normality assumption for the error terms.

In Model 2,

$$
\begin{aligned}
V(x_{t+k}-x_t)/k &= \frac{2(1-\rho^k)}{1-\rho^2}\sigma_\varepsilon^2 \\
V(y_{t+k}-y_t)/k &= \sigma_\eta^2,
\end{aligned}
$$

and the ratio of population variances of k-period difference for given $RS^2 = V(\Delta x_t)/V(\Delta y_t)$ is

$$
RPV_k = \frac{RS^2(1-\rho^k)}{RS^2(1-\rho^k)+(1-\rho)k}.
$$

RPV_k is decreasing in RS and k while it is increasing in ρ.

Each panel in Figure 3.2 shows the four graphs of RPV_k for given half life with 34 year data. Each line in a panel in Figure 3.2 is different from each other in terms of the value of RS. Each line in a panel is for $RS = 1, 2, 4, 8$ respectively from the bottom to the top. The top line in Panel (a) in Figure 3.2 is the graph for the ratio with $RS = 8$ and one year of half life. In that case, even with the quick mean reversion in the traded component, 90% of the real exchange rate movement in ten years and about three fourths in 34 years are attributed to the movement of the traded goods component. The ratio is far away from the true ratio of long run variances, that is zero at any time horizon. Figure 3.2 shows that higher value of ratio of variances of k-period differences may not reflect the failure of long run PPP since the ratio may be huge even with quick mean reversion of the traded goods component if the volatility of the traded goods component is as high as measure in Engel's (1999) data. The relative volatility of the traded goods component, RS, determines the location of the graph of the mean. The half life of the traded goods component determines the slope of the graph. For given sample size of 34 years, even a short half life can not make the slope of the graph steep enough to push down the mean of our simulation results at the long time horizon to the level close to zero if RS is as big as observed in the data.

Our observation here is comparable with Taylor's (2002) finding that larger deviations from PPP in the floating exchange rate regime can be attributed to larger shocks, not to longer half life. If, in fact, half life of the traded goods component is as long as 4 years like one observed in Engel's (1999) data set, then the graph never goes below 90% even in 34 years as in the top line in Panel (c) in Figure 3.2. Thus, the fact that the ratio is over 90% at all time horizon does not necessarily imply that PPP fails to hold in the traded goods component.

Figure 3.3 shows that the estimator of the ratio with V_k gives even higher value than the ratio of population variances of k-period differences presented in Figure 3.2, especially for longer time horizon. The graphs in Figures 3.2 and 3.3 are very close when the time horizon is very short although the graph for V_k in Figure 3.3 is never lower than the theoretical value in Figure 3.2 at each parameterization. As explained in Chapter 2, the ratio of V_k for Model 2 does not decrease monotonically but shows a U-shape graph. Thus, as the time horizon increases, the ratio of V_k becomes much higher than the population counterpart in Figure 3.2.

The top line in Panel (a) in Figure 3.3 is the ratio of V_k with $RS = 8$ and one year of half life. Under the volatility observed in the data, the ratio of V_k is greater than 90% at each time horizon even when the half life of the traded goods component is as low as one year. For a longer half life as in Panel (c) in Figure 3.3, the graphs are even flatter.

We can compare the mean of the ratio of V_k for Model 2 in Figure 3.3 with that of BT_k and QS_k in Figures 3.4 and 3.5. Different from the U-shaped curves in Figure

3.3, the means in Figures 3.4 and 3.5 settle down to a certain level. It is because the Bartlett and QS kernel estimators are designed to give a smaller weight to a higher order sample autocovariance. For the first half of the sample size, the graph for BT_k is very close to that for V_k while the graph for QS_k is lower than those for V_k and BT_k. While the graph for V_k is always above that for the theoretical value in Figure 3.2, the graph for HQ_k is lower than that for the Figure 3.2 at certain short time horizons when the half life is short and the relative size of volatility, RS, is high.

To sum, the mean of the ratio of variances in our simulation in Model 2 does not decrease much when the relative size of the traded goods component is as high as observed in Engel's (1999) data. It is true even when the half life of the traded goods component is relatively short. It is very different from what an asymptotic theory suggests about the ratio of long run variances in Model 2. According to the asymptotic theory, we may expect our estimators to become close to zero as the time horizon increases especially when the mean reversion of the traded goods component is quick. Under given relative size of volatility, the ratio of V_k, BT_k, or QS_k is a very bad point estimator of long run variances. Considering that the true value of the ratio of long run variances in Model 2 is zero, the ratio of QS_k seems to serve better as an estimator of the ratio of long run variances since the graph of its mean in the simulation is closer to zero than the other two estimators. However, it may be partly due to the downward bias which is also observed in Panel (c) in Figure 3.2. In terms of the distance between the graph from Model 1 and that from Model 2, QS_k may or may not be as good an estimator of the ratio of long run variances. Our simulation in later section on the power of a test based on the ratio of variances will address this point.

3.3.2 Centered and uncentered moments

The mean squared error (MSE)

In his paper, Engel (1999) defines the mean-squared error (MSE) as the sum of the variance of k-differences (V_k) and the squared drift (M_k) as in equation (3.1).

$$MSE_k(z) \equiv V_k(z) + M_k(z), \tag{3.1}$$

$$V_k(z) \equiv \frac{T}{(T-k)(T-k+1)k} \sum_{t=0}^{T-k} [z_{t+k} - z_t - k\overline{\Delta z}]^2, \tag{3.2}$$

$$M_k(z) \equiv [k\overline{\Delta z}]^2/k = k\overline{\Delta z}^2, \quad \text{where } \overline{\Delta z} = \frac{1}{T}(z_T - z_0). \tag{3.3}$$

The drift is measured as a sample mean of the first difference of the variance. Although his inference is based on the asymptotic theory on the variance of k-differences, Engel uses the MSE to measure the movement of a variable instead of the variance of k-differences. He argues that the MSE measures the movement of a variable comprehensively.

The Balassa-Samuelson effect normally implies the existence of a deterministic trend in the nontraded good component. However, it is not clear why deviation from the law of one price in traded good component contains a deterministic trend. Moreover, the empirical evidence for the existence of deterministic trend even in the nontraded good component of real exchange rate between the industrialized countries is at most mixed. To check the relevance of including the drift terms, we test for the null of zero drift by computing the t-value of the sample mean of the first difference of traded good component and that of nontraded good component with Engel's (1999) 1962:01-1995:12 data set. The standard error for the sample mean is computed with the QS kernel heteroskedastic autocorrelation consistent (HAC) estimator of long run variance. For 1962:01-1995:12 data set, only nontraded good components for

US-Canada RER and US-Italy RER reject the null of zero drift. No evidence against the null of zero drift is found for traded goods component. Both economically and empirically, there is not much support for the drift in the traded goods component and even in case of the nontraded goods component, the empirical evidence is not strong.

Nevertheless it seems innocuous to include the drift term. If the drift terms in the two components in the real exchange rate are in fact negligible, we would expect that the estimates of the drifts should be small, too. However, our following results show that to include the drift term may distort our estimation when there is no drift term in the true data generating processes.

Although $\overline{\Delta z}$ is a consistent estimator of the drift of z_t, the estimate of the drift term may have a significant value when the true drift is zero. To see this, first,

$$\begin{aligned} M_k(z) &= k\left[\frac{1}{T}(z_T - z_0)\right]^2 \\ &= k\left(\frac{1}{T}\sum_{t=1}^{T}\Delta z_t\right)^2 \\ &= \frac{k}{T}\left(\frac{1}{\sqrt{T}}\sum_{t=1}^{T}\Delta z_t\right)^2 \end{aligned} \tag{3.4}$$

Equation(3.4) shows that the squared drift term, $M_k(z)$, is a function of a partial sum.

Under *Assumption* 2.1 in Chapter 2, when $k/T \to b > 0$,

$$\lim_{T\to\infty} M_k(z) = \lim_{T\to\infty}\frac{k}{T}\left(\lambda \cdot W(1)\right)^2 = b\Omega W(1)^2 \tag{3.5}$$

where $W(\cdot)$ is standard Brownian motion.

And, by taking expectation of Equation (3.5),

$$E\left(\lim_{T\to\infty} M_k(z)\right) = \lim_{T\to\infty} E\left(M_k(z)\right) = b\Omega. \tag{3.6}$$

Equation (3.6) shows that the mean of the squared drift term is proportional to the long run variance. If a variable is stationary, the drift term is still zero for large k since the long run variance is zero. However, if a variable contains autoregressive unit root, the mean of $M_k(z)$ increases proportional to the increase in k even when there is no drift in the variable. Figure 3.6 shows our simulation results for the MSE_k. In Panel (a) in Figure 3.6, the dotted line is the mean of the MSE_k for a stationary AR(1) with one year of half life and the solid line is for a random walk without a drift. The graph for a stationary AR(1) shows almost the same U shape as that for the variance of k-differences. On the other hand, the graph for a random walk without a drift monotonically increases unlike the horizontal graph for V_k. The value for the mean of the MSE_k becomes almost the double when $k = T - 1$.

Panel (b) in Figure 3.6 shows the mean of the ratio of the MSE_k in the simulation. The dotted line is for Model 1 where the two components in the real exchange follow a random walk without a drift while the solid line is for Model 2 where the traded goods component is a stationary AR(1) with one year of half life. The relative volatility of the traded goods component, RS, is assumed to be one. The graph of the MSE_k for Model 2 does not go all the way back to the original level as the time horizon gets close to the sample size unlike that of V_k while the graph of the MSE_k for Model 1 is similar to that of the V_k. As such, it is not clear which variable the MSE_k is trying to estimate when a variable contains a unit root but no drift. To sum, considering the case of a unit root process with no drift, the MSE_k suggested by Engel (1999) is not a proper measure of the movement of a variable.

The ratio of uncentered moments

Another way to take care of a possible drift in a variable is using uncentered moments. This strategy is adopted in Betts and Kehoe (2006)[33]. We check whether using uncentered moments produces different results. Figure 3.7 shows the mean of the ratio of uncentered moments for each kernel estimator at each relative volatility in Model 1. Like the graphs for centered moments in Figure 3.1, the means are decreasing as the time horizon increases. In case of BT_k and QS_k, the graphs for centered moments and those for uncentered moments are very close. On the other hand, in case of V_k, the graphs for uncentered moments are significantly smaller than the population counterpart in Figure 3.2 at long time horizons for some relative volatility levels.

Figure 3.8 presents the mean of the ratio of uncentered moments for V_k in Model 2. Unlike the case of centered moments in Figure 3.3, it does not show a U-shape but decreases monotonically. While the graphs at short time horizon in Figure 3.8 are not very different from those in Figure 3.3, the graphs in Figure 3.8 at medium to long time horizons are significantly lower than those in Figure 3.3. Those are even smaller than the theoretical value of the ratio in Figure 3.2, especially for the cases of longer half lives and higher relative volatility of the traded goods component. Figure 3.9 shows the graphs for BT_k. Compared with the cases for centered moments in Figures 3.4, the ratios for uncentered moments have lower values for long time horizon by 5 to 15 percentage point. For the ratio for V_k and BT_k, when RS is eight, the mean is

[33] Huizinga (1987) also uses uncentered moments to investigate the long-run behavior of real exchange rates. However, he rationalizes his approach by assuming no drift in the true process instead of viewing it as a comprehensive measure of variances and drifts. He expalins that his approach has the benefit of eliminating a major source of small sample bias in the estimated autocorrelations due to the removal of an unknown mean as in Fuller (1996).

above 80% even at the time horizon of 30 years and with one year half life. Figure 3.10 presents the mean of the ratios in the simulation for QS_k. At the wide range of time horizon, the mean for QS_k has the lowest value among the three kernel estimators considered. Especially when the half life is as short as one year, the mean of the ratio of uncentered moments for QS_k is lower than that for centered moments in Figure 3.5 by 30 percentage points.

The ratio of long run variances is $RS^2/(RS^2+1)$ in Model 1 while the true ratio is zero in Model 2. The mean of the ratio in the simulation is closest to the ratio of long run variances at the shortest time horizon in Model 1. The mean equals the population ratio at $k = 1$ by construction of the simulation. On the other hand, the estimators of the ratio of the long run variances in Model 2 mainly depend on the relative size of the traded goods component. If RS is as large as eight, the mean in the simulation does not get close to the true value of long run variances, that is zero, for given sample size of 34 years even when the traded goods component has a short half life and the time horizon is long. Thus, the estimators are very inaccurate as a point estimator of the ratio of long run variances under Model 2. Using uncentered moments helps estimators have lower value at long time horizon. However, the difference between the ratio with centered moments and that with uncentered moments is not so big as to change our main conclusion in case of V_k and BT_k. In case of the ratio of uncentered moments for QS_k, some significant decrease in the mean of the simulation results is observed when the traded goods component is as short as one year. The mean of the ratio is about 60% for one year of half life and $RS = 8$, which is still not close to zero but very different from 100%. However, we take this fact with caution because the ratio for QS_k in Model 1 also tends to become smaller than the true ratio of long run variances as the time horizon increases. In other words, the ratio of uncentered moments for QS_k has a bigger downward bias than the other two kernel estimators.

3.3.3 Power of tests based on the ratio of variances

Although the previous subsection shows that the ratios of kernel estimators considered in this paper are very poor point estimators of the ratio of long run variances, they may be useful for a hypothesis testing. This subsection considers the possibility. Engel (1999) introduces a test based on the variances of k-differences. He shows that the ratio of the variance of k-differences computed from the data are within two-sided confidence intervals constructed by parametric bootstrapping under Model 1 at all time horizons. He argues that his test result support the hypothesis that the traded goods and the nontraded goods components are independent random walks. In this subsection, we will investigate the power of the tests based on the ratio of variances. The tests in this paper are based on the test performed in Engel (1999), but we make some adjustments in the testing method. Details on the changes are explained in Chapter 2.

We will consider the following two tests in this paper.

Test I	H_0: Model 1
	H_1: Model 2
Test II	H_0: Model 2
	H_1: Model 1

Engel (1999) performs Test I only. On the other hand, we flip the null and think about the power in the opposite case, too.

To compute the power of Test 1 at each time horizon, we pick, as the critical value for 5% size test, the 100th smallest value among the two thousand ratios computed from the bootstrap samples under Model 1. We generate another two thousand bootstrap samples under Model 2 and reject the test when the ratio is smaller than the 95% critical value. The power of the test is the rejection rate in this exercise. One limitation of this approach is that the test is performed at each given time horizon, not over the entire time horizons.

Power of Test I

Our simulation results show that different from the mean of the ratio in the simulation, the power of the test is almost invariant to the value of the relative volatility of the traded goods component. Due to the invariance, we report the simulation results for $RS = 8$ only. Figure 3.11 shows the power of Test I for each kernel estimator. The solid line represents V_k, the normal dotted line is for BT_k, the bold dotted line is for QS_k. In Test I, the null is that both x_t and y_t follow a random walk without a drift while the alternative is that x_t follows a stationary AR(1) and y_t follows a random walk without a drift. The half life of x_t in the alternative is 1, 2, and 4 years from the top panel to the bottom.

First of all, the power for each kernel estimator is very low when the half life of x_t in the alternative is 4 years as observed in Engel's (1999) data set. Even at a time horizon where the test gives its maximum power, the power is less than 15%. As the half life decreases, the power of the test increases, especially in the middle range of time horizon. Depending on the kernel, the test has a maximum power of 45 to 60% at a certain time horizon.

In case of V_k, the power increases at the short and medium time horizon. However, as k gets close to the sample size, the distribution under the null and the distribution under the alternative become similar to each other as shown in Chapter 2. Thus, the test based on the ratio of V_k barely has any power at the time horizons near the sample size. On the other hand, the power of the tests based on BT_k or QS_k reaches higher maximum and does not decrease rapidly.

Figure 3.12 presents the power of Test I based on uncentered moments. The maximum power of the test with V_k is 5 to 15% lower than tests with the other two kernel estimators. Unlike the case of centered moments, the maximum power for V_k is 5 to 10 % lower than the other two even when the half life is 4 years. The decrease of power in the test with V_k is much faster than the other two kernel estimator although the power does not go all the way down to the size of the test.

To sum, in Test I, the power of the test for the half life of x_t in the alternative observed in Engel's (1999) data set (4 years) is as low as 15% even at the time horizon where the power reaches the maximum. Among the kernel estimators considered in this paper, using the Bartlett kernel estimator (BT_k)or the Quadratic Spectral kernel estimator (QS_k) gives higher maximum power of the test than the variance of k-differences (V_k).

Power of Test II

Figure 3.13 presents the power of Test II for each kernel estimator at each time horizon. In Test II, the null is that x_t follows a stationary AR(1) and y_t follows a random walk without a drift and the alternative is that both follow a random walk without a drift. The power of Test II is 5 to 10% higher than Test I at the maximum. However, the maximum power is still less than 20% when the half life of x_t in the null is 4 years as observed in Engel's (1999) data set. The power of Test II with centered BT_k at the largest time horizon in Figure 3.13 is much greater than that with QS_k while the two does not show much difference in Figure 3.11.

As in Figure 3.14, Test II based on uncentered moments has 10 to 18% higher maximum power than that for centered moments. As the time horizon approaches the sample size, the power does not decrease much in case of BT_k and QS_k while the power for V_k declines faster than the other two. The gap between the power at the largest time horizon and the maximum power is within 5 percentage points in case of BT_k and QS_k for a given half life.

In sum, unlike the mean of the ratio of variances considered in the previous section, Test I and Test II considered in this paper are almost invariant to the relative volatility of the traded goods component. The tests do not have a high power enough to give a definite answer for given sample size and persistence of the traded goods component observed in the data.

In practice, we do not know a priori the time horizon at which the test gives the maximum power. Literature on the estimation of long run variances has studied much about a sample-dependent bandwidth choice rule which gives the smallest mean squared errors of the estimation in the large sample. However, the efforts do not seem to be successful so far at the small sample available in a usual empirical study. Moreover, the efficient bandwidth for the estimation of long run variances is not necessarily efficient for the tests based on the ratio of long run variances. Kiefer and Vogelsang (2002b) show that valid tests (asymptotically pivotal) for regression parameters can be constructed with a kernel estimator whose truncation lag is equal to sample size. Given the fact that we do not have a reliable rule to choose the time horizon for the test, using the ratio of BT_k or QS_k at the longest time horizon can be considered as an alternative method. To follow the approach, it would be desirable if a test which has a flat power graph at longer time horizons after it reaches its maximum power. BT_k and QS_k are designed to give smaller weight to higher order sample autocovariances which helps the test based on those kernels has such property.

3.4 Conclusion

Using a Monte Carlo simulation methods, we have investigated the properties of the ratio of k-period variances as an estimator of the ratio of long run variances of two variables. Engel (1999) measures the importance of the traded goods component in the movement of the real exchange rate with the ratio. He finds that the traded goods component determines almost all the movement of the real exchange rate at any time horizon and that there is no statical evidence that the importance of the traded goods component decreases as the time horizon gets longer.

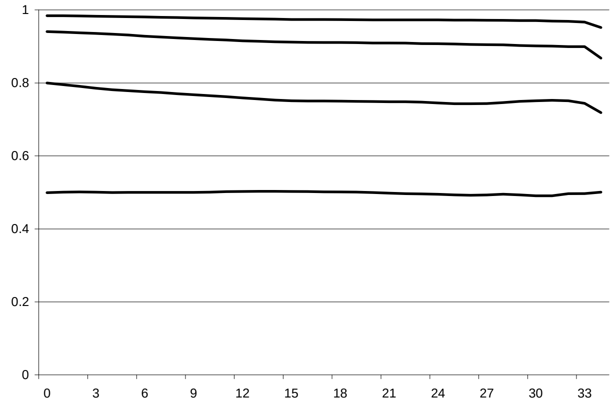
Our simulation results suggest that the ratio of the variance of k-differences or other HAC estimators do not provide a reasonable point estimate for the ratio of the long run variances of the traded goods component and the real exchange rate. Under realistic parameterization, the large value of the ratio should not be attributed to high persistence of the traded goods component but to its great relative volatility.

The ratio can be used to construct a hypothesis test. Unlike the estimation problem, the power of the test does not depend on the relative volatility of the traded goods component but on its persistence. Under four years of half life which is observed in the data, the maximum power of the test is about 10 to 20%. Thus, the test does not have a high power. Among the three estimators considered in this paper, the power of the test based on the variance of k-differences is the lowest.

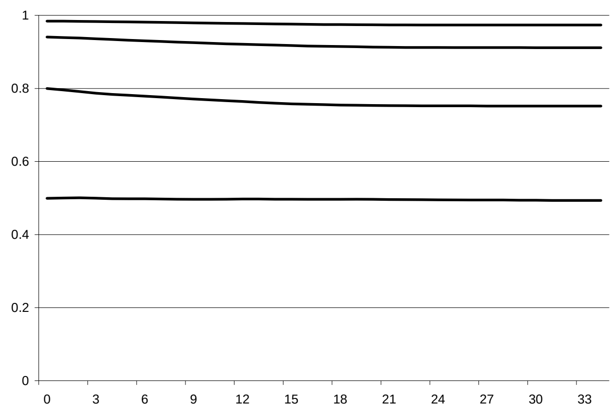
Engel mainly uses the sum of the variance of k-differences and the squared drift as the measure of the movement. We show that the sum may have a serious bias when the true data generating process of the variable is a random walk without a drift. Instead, we investigate the properties of the ratio based on uncentered moments which is adopted in Betts and Kehoe (2006). The estimation is a little biased downward but not very serious under the parameterization observed in the data. In general, the mean of the ratio of uncentered moments in the simulation under Model 2 is lower than that for centered moments. The test based on uncentered moments has 10 to 15 percentage points higher maximum power than its counterpart based on centered moments.

In conclusion, our simulation results indicate that the ratio of variances considered in this paper is a very poor point estimator of the ratio of long run variances and the test based on the ratio has low power under the parameterization observed in the data. Our findings raise a doubt about the conclusion in Engel (1999) based on the ratio we have studies in this paper.

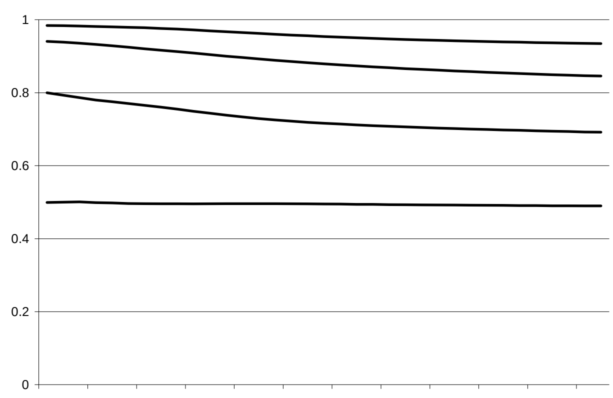
(a) The variance of k-differences



(b) The Bartlett kernel estimators



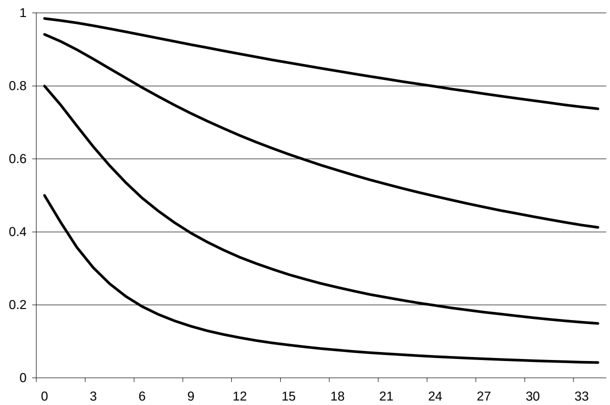
(c) The QS kernel estimators



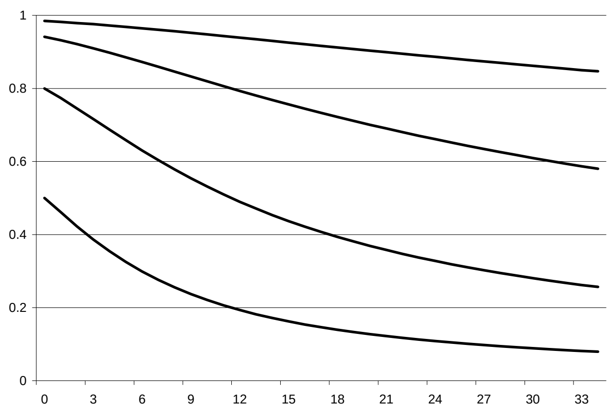
Notes: i) Each line represents the case for $RS = 1, 2, 4, 8$ from bottom to top.
ii) The true value of the ratio of LV's is $RS^2/(RS^2+1)$ in Model 1.

Figure 3.1: The mean of the ratio of variances under Model 1

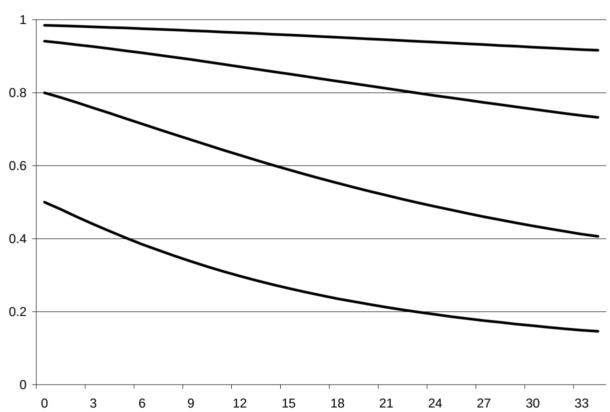
(a) Half life = 1yr



(b) Half life = 2 yrs



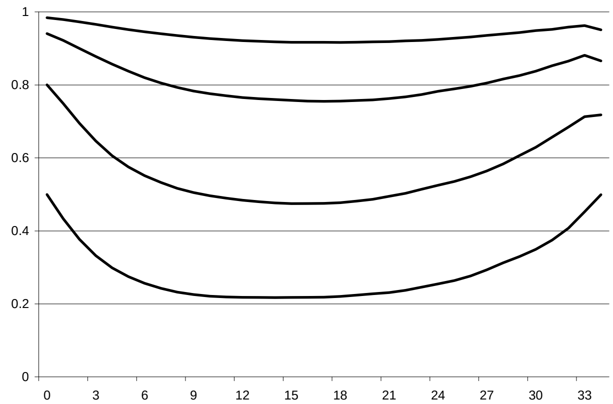
(c) Half life = 4 yrs



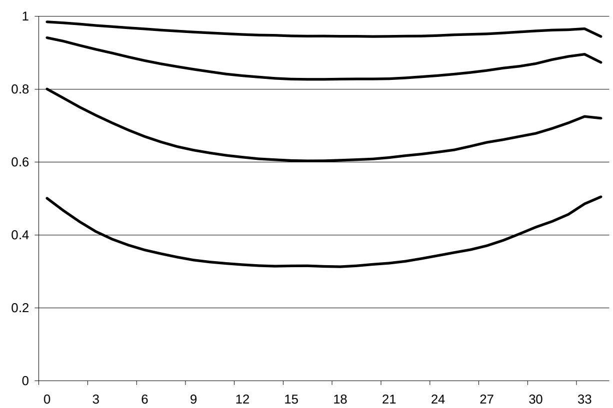
Notes: i) The value is computed by $RS^2(1-\rho^k)/[RS^2(1-\rho^k)+(1-\rho)k]$.
ii) Each line represents the case for $RS = 1, 2, 4, 8$ from bottom to top.

Figure 3.2: The theoretic ratio of k-period variances under Model 2

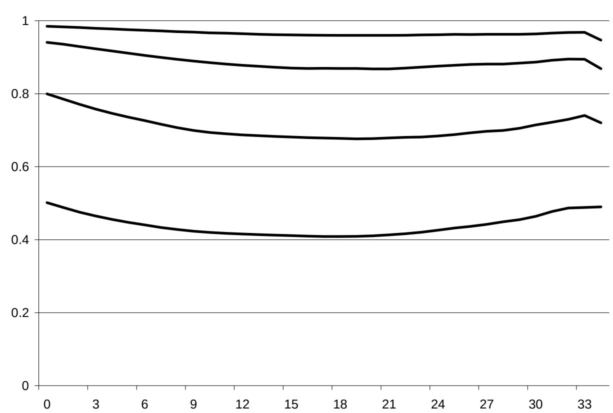
(a) Half life = 1yr



(b) Half life = 2 yrs

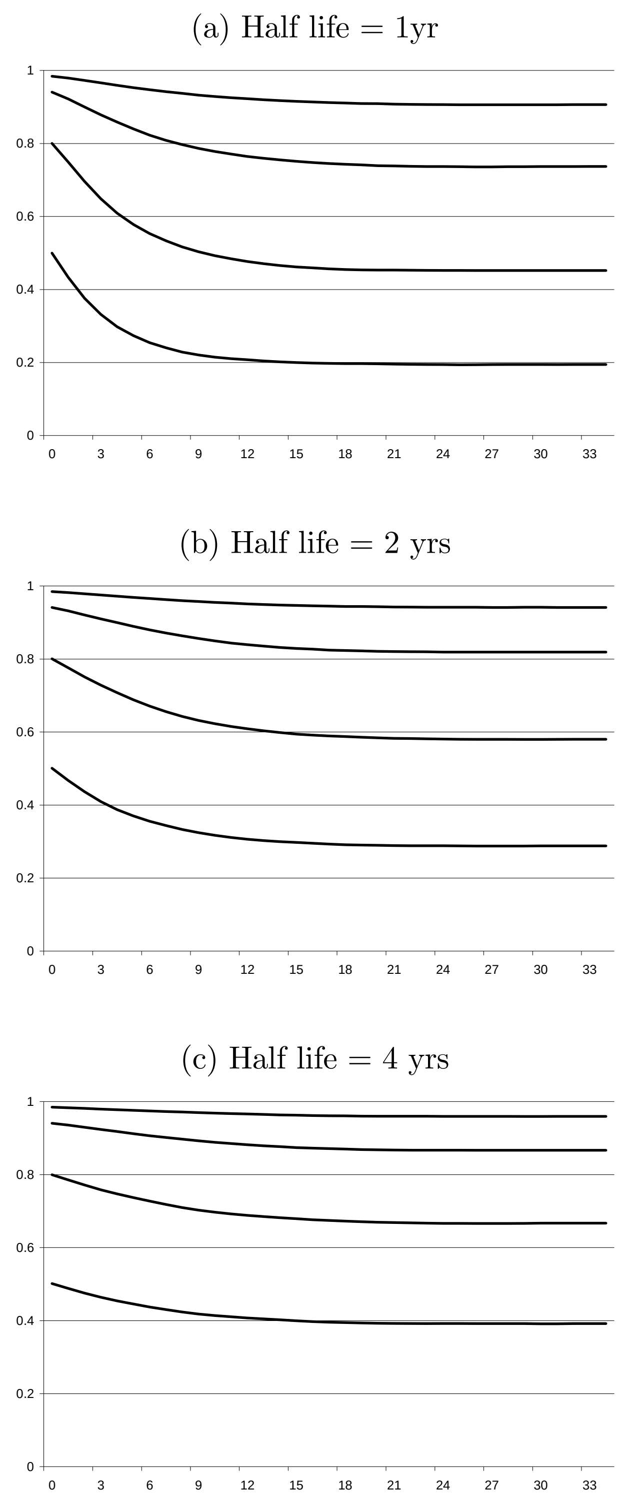


(c) Half life = 4 yrs



Notes: i) Each line represents the case for $RS = 1, 2, 4, 8$ from bottom to top.

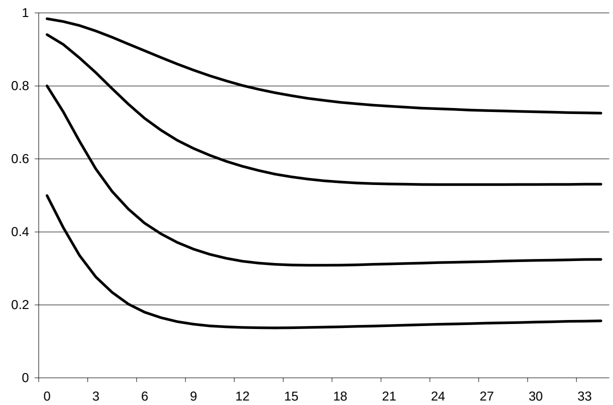
Figure 3.3: The ratio of V_k under Model 2



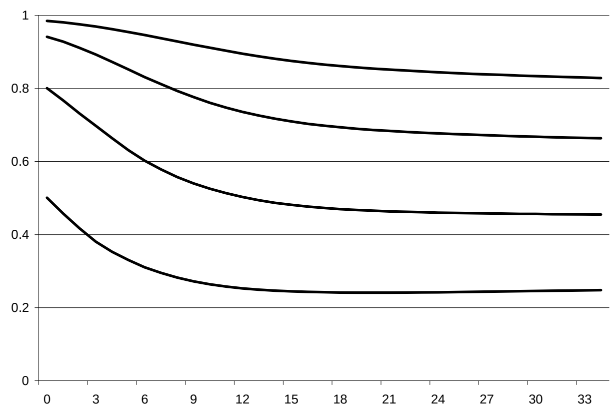
Notes: i) Each line represents the case for $RS = 1, 2, 4, 8$ from bottom to top.

Figure 3.4: The ratio of BT_k under Model 2

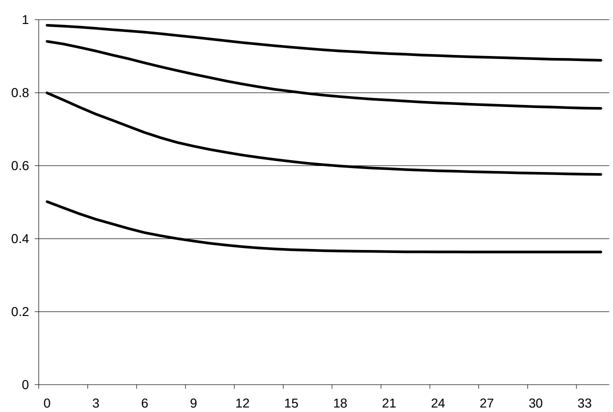
(a) Half life = 1yr



(b) Half life = 2 yrs



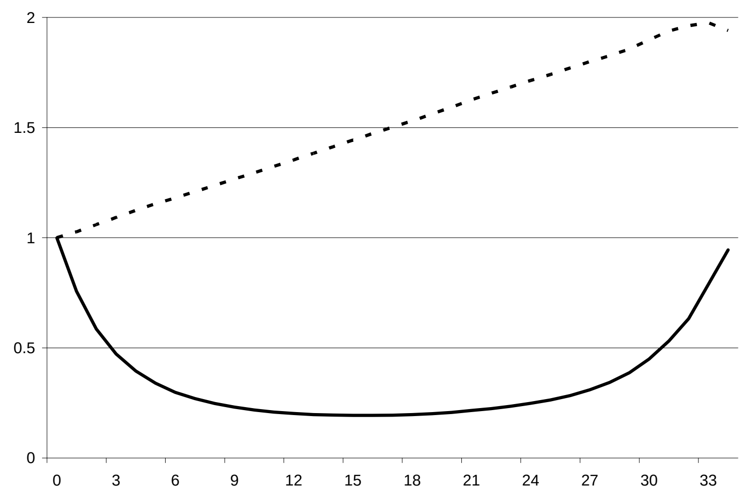
(c) Half life = 4 yrs



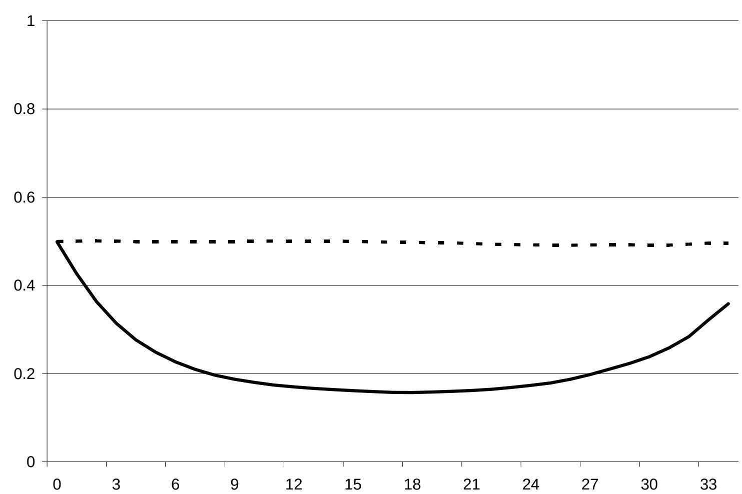
Notes: i) Each line represents the case for $RS = 1, 2, 4, 8$ from bottom to top.

Figure 3.5: The ratio of QS_k under Model 2

(a) The mean of the MSE_k



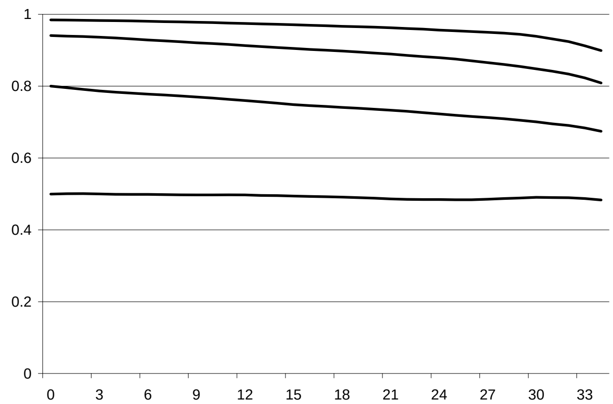
(b) The mean of the ratio of the MSE_k



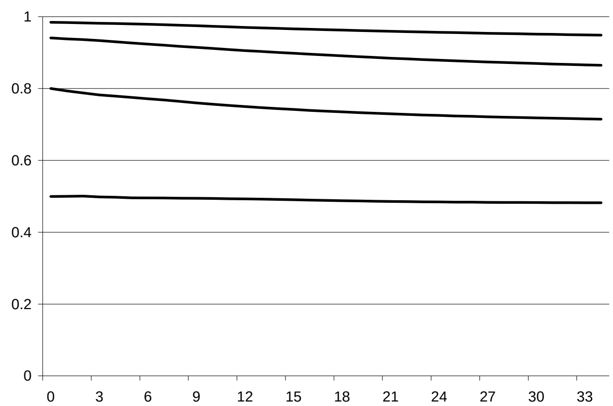
Notes: i) The solid line is AR(1) and the dotted line is for random walk in (a).
ii) The solid line is for Model 2 and the dotted line is for Model 1 in (b).

Figure 3.6: Simulation results for the MSE_k

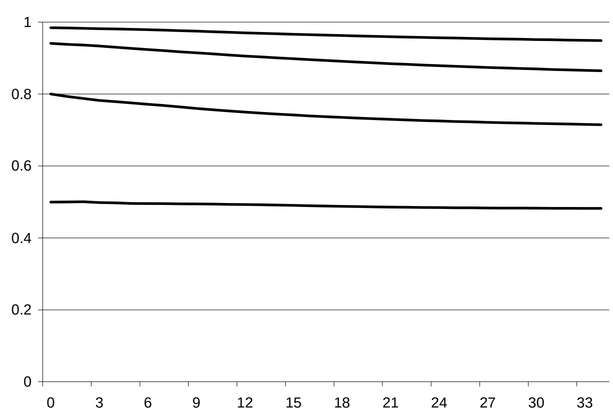
(a) The variance of k-differences



(b) The Bartlett kernel estimators



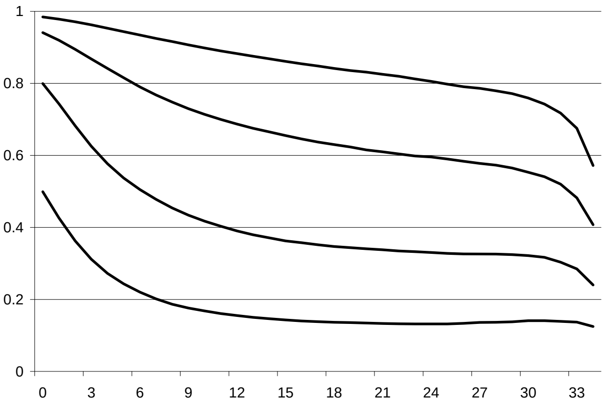
(c) The QS kernel estimators



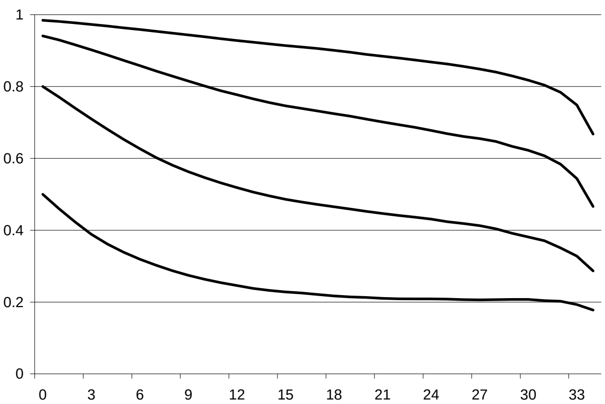
Notes: i) Each line represents the case for $RS = 1, 2, 4, 8$ from bottom to top.
ii) The true value of the ratio of LV's is $RS^2/(RS^2+1)$ in Model 1.

Figure 3.7: The mean of the ratio of uncentered moments under Model 1

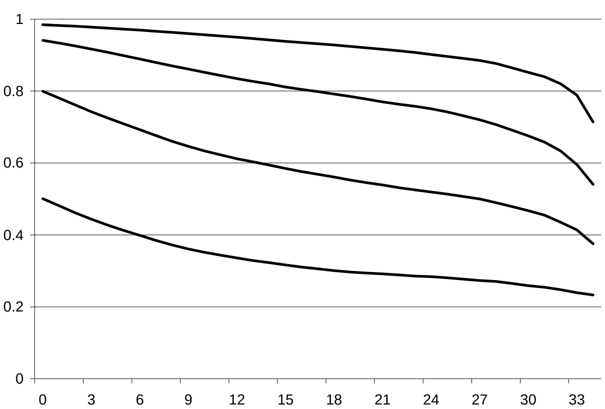
(a) Half life = 1yr



(b) Half life = 2 yrs

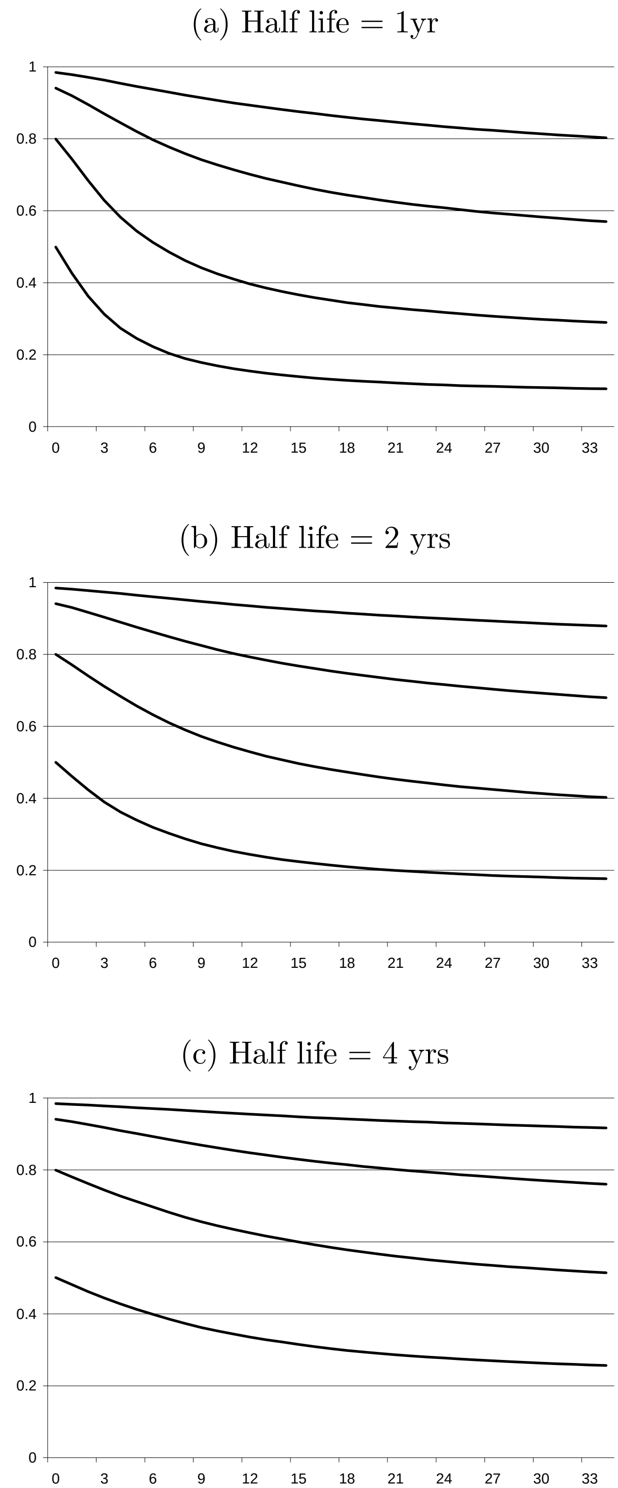


(c) Half life = 4 yrs



Notes: i) Each line represents the case for $RS = 1, 2, 4, 8$ from bottom to top.

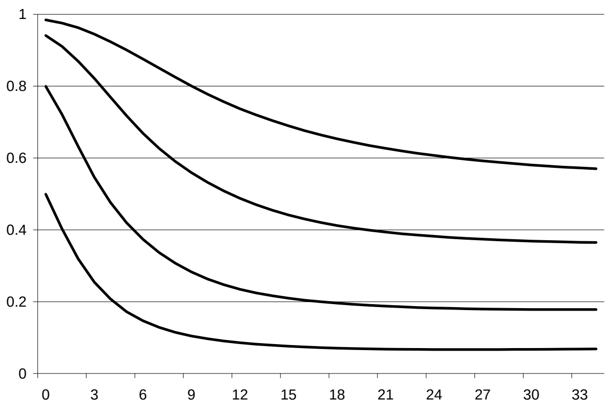
Figure 3.8: The ratio of uncentered moments for V_k under Model 2



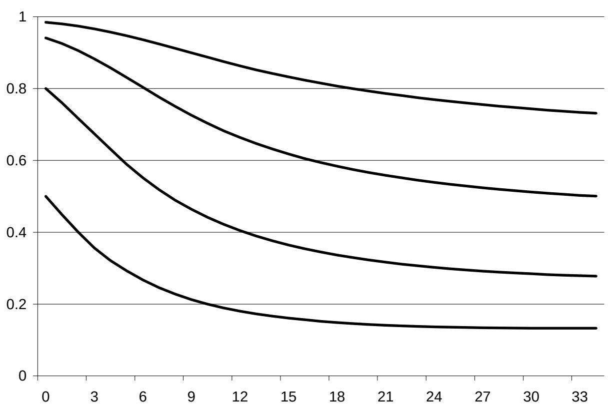
Notes: i) Each line represents the case for $RS = 1, 2, 4, 8$ from bottom to top.

Figure 3.9: The ratio of uncentered moments for BT_k under Model 2

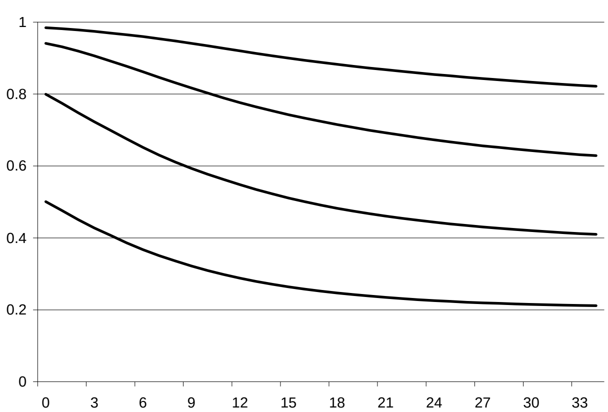
(a) Half life = 1yr



(b) Half life = 2 yrs



(c) Half life = 4 yrs

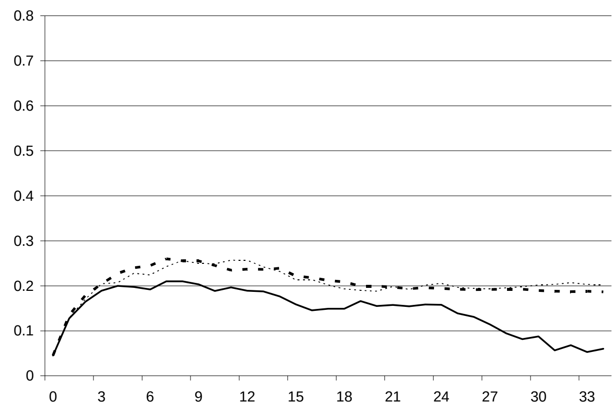


Notes: i) Each line represents the case for $RS = 1, 2, 4, 8$ from bottom to top.

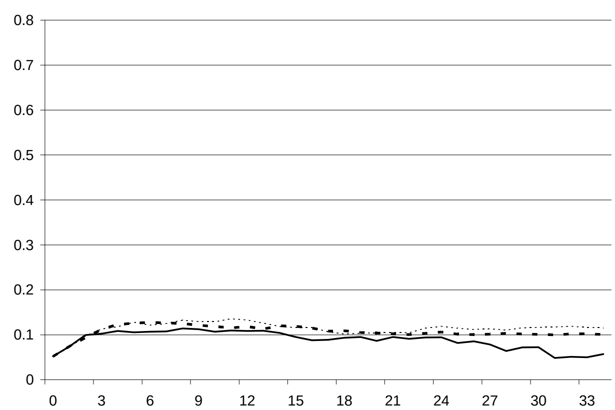
Figure 3.10: The ratio of uncentered moments for QS_k under Model 2

(a) The half life of x_t in the alternative = 1 yr

(b) The half life of x_t in the alternative = 2 yrs



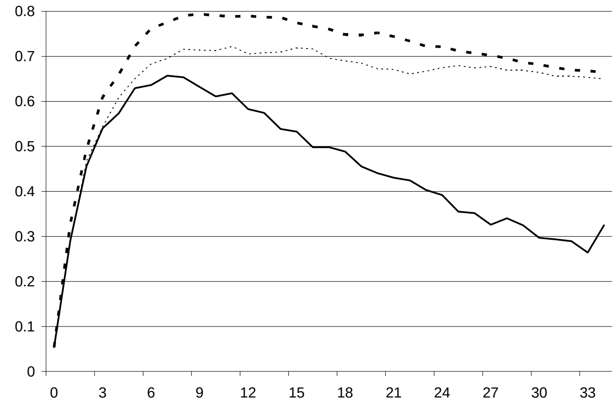
(c) The half life of x_t in the alternative = 4 yrs



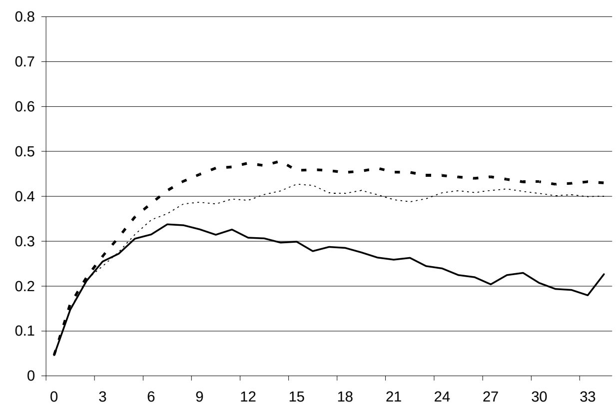
Notes: i) H_0 : x_t is random walk. ii) $stdv(\Delta x)/stdv(\Delta y) = 8$.
iii) Solid lines: V_k, Normal dotted lines: BT_k, Bold dotted lines: QS_k.

Figure 3.11: Power of Test I with the ratio of centered moments

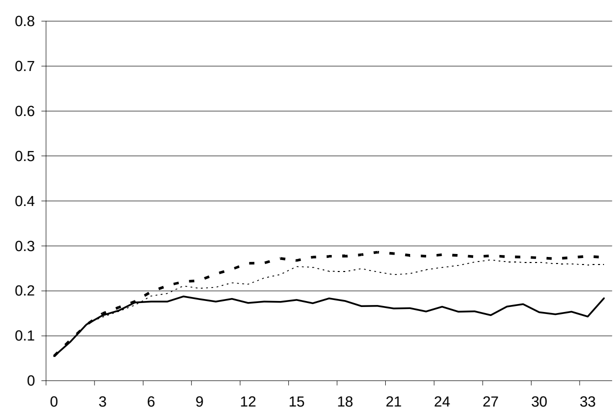
(a) The half life of x_t in the alternative = 1 yr



(b) The half life of x_t in the alternative = 2 yrs



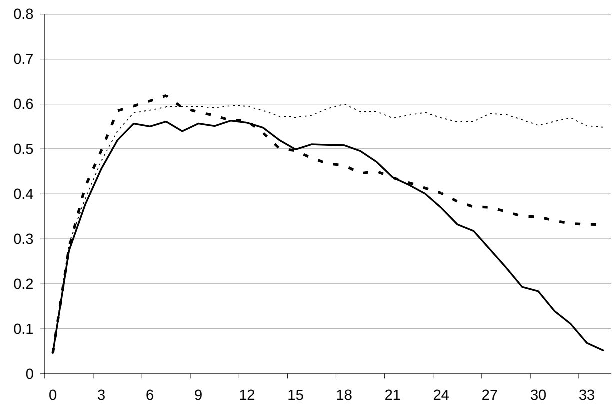
(c) The half life of x_t in the alternative = 4 yrs



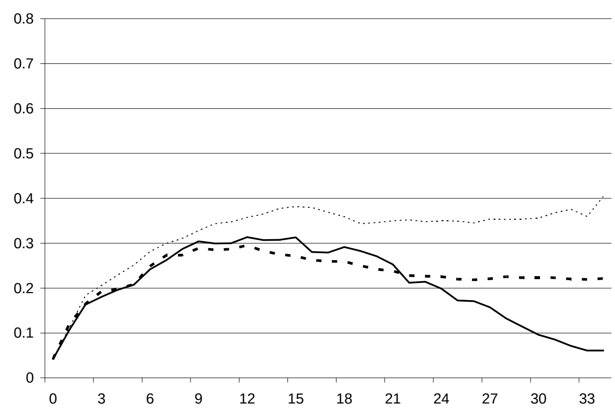
Notes: i) H_0 : x_t is random walk. ii) $stdv(\Delta x)/stdv(\Delta y) = 8$.
iii) Solid line: V_k, Normal dotted line: BT_k, Bold dotted line: QS_k.

Figure 3.12: Power of Test I with the ratio of uncentered moments

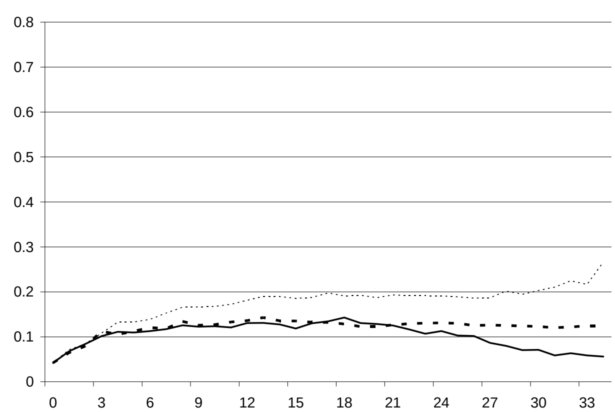
(a) The half life of x_t in the null = 1 yr



(b) The half life of x_t in the null = 2 yrs



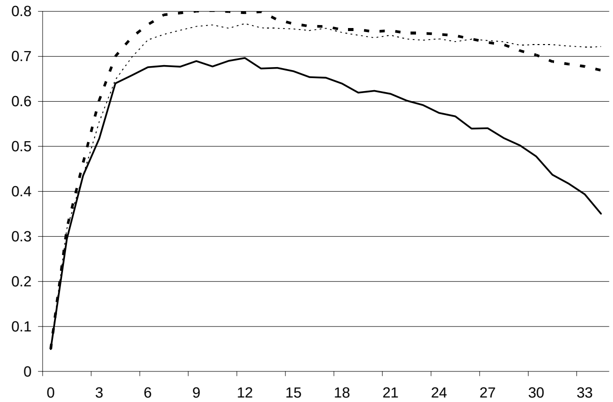
(c) The half life of x_t in the null = 4 yrs



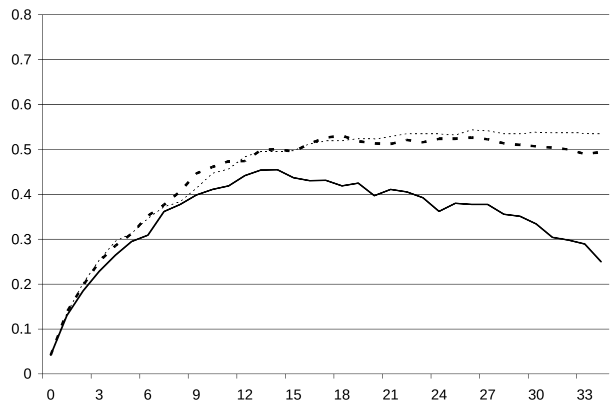
Notes: i) H_0 : x_t is stationary AR(1). ii) $stdv(\Delta x)/stdv(\Delta y) = 8$.
iii) Solid lines: V_k, Normal dotted lines: BT_k, Bold dotted lines: QS_k.

Figure 3.13: Power of Test II with the ratio of centered moments

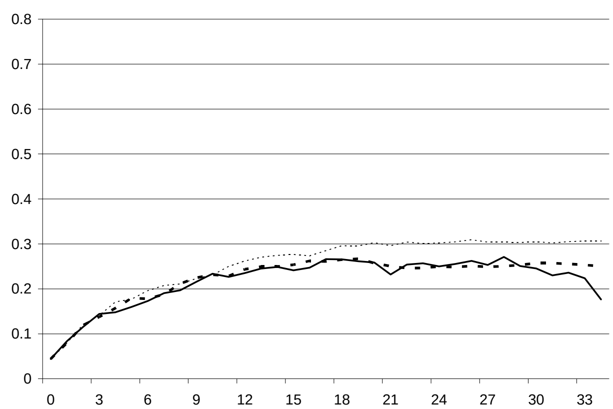
(a) The half life of x_t in the null = 1 yr



(b) The half life of x_t in the null = 2 yrs



(c) The half life of x_t in the null = 4 yrs



Notes: i) H_0 : x_t is stationary AR(1). ii) $stdv(\Delta x)/stdv(\Delta y) = 8$.
iii) Solid lines: V_k, Normal dotted lines: BT_k, Bold dotted lines: QS_k.

Figure 3.14: Power of Test II with the ratio of uncentered moments

Parameters	Canada	France	Germany	Italy	Japan
$\widehat{\sigma}_\eta$	.00259	.00258	.00215	.00251	.00360
$\widehat{\alpha}^*$	.00382	.01443	.00259	.08745	.00904
	(.00237)	(.01360)	(.00420)	(.05243)	(.01690)
$\widehat{\rho}^*$	.98611	.99185	.99408	.98803	.99771
	(.00878)	(.00712)	(.00586)	(.00710)	(.00316)
half life (yrs)	<4.13>	<7.06>	<9.73>	<4.80>	<25.19>
$\widehat{\sigma}_\varepsilon$	.01045	.02360	.02460	.02212	.02593
stdv(Δx)	.01049	.02365	.02464	.02219	.02594
stdv(Δx)/stdv(Δy)	4.05	9.17	11.46	8.84	7.21

Notes: i) Items with * have the standard error in the parenthesis below.

Table 3.1: Estimation of Model 2 from 1962:01-1995:12 data

CHAPTER 4

HIGHER POWER TESTS FOR THE FAILURE OF LONG RUN PURCHASING POWER PARITY

4.1 Introduction

The long run version of Purchasing Power Parity(PPP) can be tested in a unit root test context. The standard augmented Dickey-Fuller (ADF) test, though, rarely rejects the null of unit root for industrialized countries with the US dollar as the numeraire currency, showing no evidence for the long run PPP. In a globalized world like today, it is hard to imagine that a large price discrepancy for a tradable good between two industrialized economies can last for a long time without arbitrage. Due to the strong plausibility of the long run PPP, literature in this context has attributed the failure of the rejection of the unit root test to the low power of the ADF test[34].

Frankel (1986), Lothian and Taylor (1996) and others try to solve the low power problem of the unit root test by using longer horizon data and succeed in finding more favorable results for the long run PPP. On the other hand, another way to increase the power is to pool bilateral exchange rates across countries by applying panel unit root tests. Abuaf and Jorion (1990), Higgins and Zakrajsek (1999) and others provide

[34] For instance, according to table II in Elliott, Rothenberg, Stock (1996), the asymptotic power of a simple AR(1) DF test with 5% size is only 12% with 3.4 years of half life (AR(1) coefficient is .95) and 25 years of quarterly data.

evidence for the long run PPP with a panel unit root test. However, century-long data includes not only a flexible exchange rate period but also a fixed exchange rate regime. A simple ADF test may be applicable to the long horizon data only under the assumption that the behavior of real exchange movement in the two regimes are similar. The assumption may not be true, as emphasized by Mussa (1986). In the case of panel unit root tests, rejection of the unit root null does not necessarily imply that all individual currencies satisfy long run PPP. It is possible that only part of bilateral real exchange rates in the panel are stationary and reject the null.

Another way to improve the power of the unit root test is to apply a more powerful univariate test than the standard ADF test. Cheung and Lai (2000) perform both the ADF test and more powerful DF-GLS test of Elliott, Rothenberg, and Stock (1996) with 1973.04-1994.12 monthly data for 94 currencies. In only 17 out of the 94 cases, the ADF test can reject the null at the 10% size. Out of 77 countries left, the DF-GLS gives additional 13 rejections. Their success is at its most modest since the majority of currencies still fail to reject the unit root null. Other more powerful univariate unit root tests include the feasible point optimal test (PT) of Elliott, Rothenberg, and Stock (1996) and the modified feasible point optimal test (MPT) by Ng and Perron (2001).

More recently, Elliott and Pesavento (2006) and Amara and Papell (2006) simultaneously find much stronger evidence for long run PPP by applying a covariate augmented feasible point optimal test (CPT) developed by Elliott and Jansson (2003). Elliott and Pesavento (2006) can reject the null 14 out of 15 bilateral real exchange rates (RERs) with 5% size and Amara and Papell (2006) can reject 12 out of 20 cases.

In this paper, we look at two different aspects of the CPT test based on the two previous empirical works. First, both of the papers use CPI general price indexes to construct the real exchange rates. If the Balassa-Samuelson theory is important to explain the long run movement of the real exchange rates, it is possible that long run PPP holds for traded goods price RER but not for CPI based RER. We compare unit root test results for CPI based RERs with those for traded goods price RERs. As a proxy for traded goods prices, PPI and Export/Import prices will be used.

We are not the first who use tradable goods price for testing PPP. Using a long run data set, Kim (1990) and Ito (1997) find that WPI/PPI based real exchange rates are more likely to reject the unit root test than CPI based real exchange rates. Coakley, Kellard, and Snaith (2005) find that panel unit root tests with a floating period data (1973-1998) are more likely to reject the null for PPI based RERs than CPI based RERs. Our paper uses tradable goods prices for PPP test without relying on long run data or a panel unit root test.

The second aspect is related to the choice of a covariate. The two previous papers perform the CPT test using 6 or 7 different covariates. In fact, as explained in Elliott and Pesavento (2006), the CPT test leaves us quite free in choosing the covariates to use. The only two conditions for a variable to be a covariate are (i) economic relevance and (ii) stationarity of covariates. One possible downside of the freedom of the choice of covariates is that it is hard to compare one set of the CPT results with another since each test may use different set of covariates. In some cases, availability of covariates may differ from one country to another. There is also a risk of overrejection by adding a large set of covariates.

Elliott, Jansson, and Pesavento (2005) show that the CPT test developed by Elliott and Jansson (2003) can be directly applied to testing for nonstationarity of potential cointegrating relationship with known parameters. Elliott, Jansson, and Pesavento's method (2005) efficiently exploits the information which is already used to construct the real exchange rates without adding an additional data set as covariates. In this paper, we will see if adopting the CPT test itself with the same set of data can give more rejections than performing univariate tests by following Elliott, Jansson, and Pesavento (2005). We use, as a covariate, the inflation difference of the two countries.

What we find in the paper is that adopting the CPT test produces a huge increase in rejection rate of the test when we use PPI or Export/Import price indexes. The CPT test gives approximately 50% rejection rate while that for any powerful univariate test does not exceed 20%. On the other hand, with CPI data, the CPT test is not applicable in many cases. It is because the covariate in our test does not satisfy the assumption of the test, the stationarity of covariates. The rejection rate of the CPT test with CPI data for available countries is about 30%.

The rest of the paper is organized as follows. Section 2 will review unit root tests adopted in this paper. Section 3 presents empirical results. Section 4 concludes.

4.2 univariate and covariate unit root tests

4.2.1 univariate unit root tests

We perform the same four univariate tests used in Elliott and Pesavento (2006) and compare with the result from a covariate test in this paper[35]. The first unit

[35] Haldrup and Jansson (2005) provide an excellent insight of the developement in unit root test literature during the last decade. Amara and Papell (2006) and Elliott, Pesavento (2006) give a

root test used in this paper is the augmented Dickey-Fuller (ADF) t-test[36]. The test statistic, $t_{\widehat{\rho}}$, is the OLS t-value for the hypothesis that $\rho = 1$ in the following regression

$$q_t = \mu + \rho q_{t-1} + \sum_{j=1}^{k} \gamma_j \Delta q_{t-j} + \upsilon_t. \tag{4.1}$$

q_t is the logarithm of the real exchange rate. We reject the null if $t_{\widehat{\rho}}$ is smaller than the critical value, -2.89. The critical value comes from Case 2 with the sample size of 100 in Table B.6 of Hamilton (1994). The improper choice of k may result in a severe size distortion of the test. Following the suggestion by Haldrup and Jansson (2005), we use a modified form of the Akaike Information Criterion (MAIC) developed by Ng and Perron (2001) for all the univariate unit root tests in this paper. In MAIC, the penalty function against increasing k is data dependent unlike other popular methods such as AIC, Bayesian Information Criterion (BIC), or general-to specific rule in Hall (1994) and Ng and Perron (1995).[37]

Equation (4.1) has a deterministic term, μ. Elliott, Rothenberg, and Stock (1996) show that the asymptotic power curve of the Dickey-Fuller t-test virtually equals the bound when power is one-half and is never very far below in the case where there is no deterministic component. However, in the case where a deterministic mean or trend is present, power can be improved considerably over the standard Dickey-Fuller test by modifying the method employed to estimate the parameters characterizing

brief summary of the CPT test we use. Parts of exposition on unit root tests in this section are borrowed from those papers.

[36] Said and Dickey (1984) show conditions under which the ADF test has the same asymptotic distribution as that of the original Dickey-Fuller (1979) unit root test even when k is unknown. According to the condition, k should be increased at a certain rate as the sample size increases, $o(T^{1/3})$. Chang and Park (2002) generalize Said and Dickey's (1984) condition. For more explanation, see Haldrup and Jansson (2005).

[37] The maximum k is chosen by the following rule: $k_{\max} = int(12(T/100)^{1/4})$ as in Ng and Perron (2001).

the deterministic term. Elliott, Rothenberg, and Stock (1996) propose local-to-unity GLS detrending of the data. GLS detrending is designed to make the test invariant to unknown parameters in the deterministic term[38]. For any series $\{y_t\}_{t=1}^{T}$, of any constant α, define

$$y^{\alpha} = (y_1, y_2 - \alpha y_1, \ldots, y_T - \alpha y_{T-1})'. \tag{4.2}$$

The GLS detrended series $\left\{y_t^{GLS,\alpha}\right\}_{t=1}^{T}$ is given by

$$\begin{aligned} y_t^{GLS,\alpha} &= y_t - d_t' \beta^{GLS,\alpha}, \\ \beta^{GLS,\alpha} &= \arg\min_{\beta} S(\alpha, \beta), \\ S(\alpha, \beta) &\equiv (y^{\alpha} - d^{\alpha\prime}\beta)'(y^{\alpha} - d^{\alpha\prime}\beta), \end{aligned} \tag{4.3}$$

where $d_t = 1$ for a deterministic mean and $d_t = (1, t)$ for a deterministic trend.

As pointed out in Haldrup and Jansson (2005), the unit root testing problem does not admit a uniformly most powerful (UMP) test because the functional form of the optimal test against any specific alternative $\rho < 1$ depends on ρ. Instead, a local-to-unity approach constructs an efficient test against a specific point alternative with the hope that the test will have a good power against a wide range of possible alternatives. The alternative, $\overline{\alpha}$, is a Pitman drift: $\overline{\alpha} = 1 + \overline{c}/T$. Recommended value for $\overline{c}$ is -7 for a deterministic mean and -13.5 for a deterministic trend[39].

One way to apply local-to-unity GLS detrending approach is the modified Dickey-Fuller t-test (DF-GLS) proposed by Elliott, Rothenberg, and Stock (1996). It performs a standard ADF test with the GLS detrended series with $\overline{\alpha} = 1 + \overline{c}/T$. The test

[38] Invariance under the GLS transformation may not be justified when the initial observation is not constant. In this case, there are unknown nuisance parameters in the deterministic term as explained in p820 of Elliott, Rothenberg, and Stock (1996). For more discussion on the initial condition, see Muller and Elliott (2003).

[39] The value for $\overline{c}$ is chosen to make the power of the test 50% when $\alpha = \overline{\alpha}$.

statistic is the OLS t-value for the hypothesis that $\rho = 1$ in the following regression

$$\widetilde{q}_t = \rho \widetilde{q}_{t-1} + \sum_{j=1}^{k} \gamma_j \Delta \widetilde{q}_{t-j} + \upsilon_t, \text{ where } \widetilde{q}_t \equiv q_t^{GLS,\overline{\alpha}}. \tag{4.4}$$

Elliott, Rothenberg, and Stock (1996) also suggest a point optimal unit root test (PT). The PT test is defined as

$$P_T = [S(\overline{\alpha}) - \overline{\alpha} S(1)]/\widehat{\omega}^2, \tag{4.5}$$

where $S(\alpha) \equiv \min_{\beta} S(\alpha, \beta)$ and the long run variance estimate of υ_t, $\widehat{\omega}^2$, is computed by $[\widehat{\sigma}_{vk}/(1 - \sum_{i=1}^{k} \widehat{\gamma}_i)^2]$ where $\widehat{\sigma}_{vk} = T^{-1} \sum_{t=k+1}^{T} \widehat{\upsilon}_{tk}^2$ with $\widehat{\gamma}_i$ and $\widehat{\upsilon}_{tk}$ obtained from the regression in equation (4.4). The PT test is a natural by-product of constructing the asymptotic power envelope which gives an attainable upper bound on local asymptotic power for a class of unit root tests. Local-to-unity approach constructs a test which attains the power envelope at a specific point alternative. According to Neyman-Pearson lemma, the power envelope is related to the likelihood ratio. However, when the variance-covariance matrix of υ_t is not an identity matrix, a simple likelihood ratio statistic like $[S(\overline{\alpha}) - S(1)]$ does not produce a test of correct size. As Elliott, Rothenberg, and Stock (1996) show, though, P_T in equation (4.5) produces a valid large-sample test. Of the two tests suggested in their paper, Elliott, Rothenberg, and Stock (1996) conclude from their simulation results that the DF-GLS test shows a better overall performance than the PT test.

Ng and Perron (2001) suggest a modified version of the PT test (MPT). The MPT test statistic is defined as

$$\begin{aligned} MP_T &= [\overline{c}^2 T^{-2} \sum_{t=k+1}^{T} \widetilde{q}_{t-1}^2 - \overline{c} T^{-1} \widetilde{q}_T^2]/\widehat{\omega}^2 \quad \text{for a deterministic mean;} \qquad (4.6) \\ &= [\overline{c}^2 T^{-2} \sum_{t=k+1}^{T} \widetilde{q}_{t-1}^2 + (1 - \overline{c}) T^{-1} \widetilde{q}_T^2]/\widehat{\omega}^2 \quad \text{for a deterministic trend.} \end{aligned}$$

Ng and Perron (2001) state that the motivation of designing this modified test is to provide functionals of sample moments that have the same asymptotic distributions as well known unit root tests. In fact, the MPT test shares the same limiting distribution with the PT test by Elliott, Rothenberg, and Stock (1996), but has better size and power properties. To sum up, the DF-GLS and MPT tests seem to provide the best overall size and power properties among existing univariate unit root tests.[40]

4.2.2 covariate unit root tests

To fix the idea about the CPT test used in this paper, let's consider a system of equations in the following. Long run PPP implies that the nominal exchange rate and the price difference between the home and foreign countries are cointegrated with the coinegrating vector of (1,1). If we assume, additionally, that the price difference is I(1), the cointegration relation is a part of the following bivariate system with nominal exchange rate (s_t) and the price difference ($x_t \equiv p_t^d - p_t^f$) when $\gamma = 1$:

$$s_t = \mu_s + \gamma x_t + u_{s,t} \tag{4.7}$$

$$x_t = \mu_x + \tau_x t + u_{x,t} \tag{4.8}$$

and

$$A(L)\begin{pmatrix}(1-\rho L)u_{s,t} \\ \Delta u_{x,t}\end{pmatrix} = \varepsilon_t, \tag{4.9}$$

where $A(L)$ is a finite polynomial of order k in the lag operator L which introduces stationary dynamics to the model. If there is no cointegration, then $\rho = 1$. If there is a cointegration relationship, $\rho < 1$.

[40] For univariate tests in this paper, we use the GAUSS code used for Ng and Perron (2001). The code is provided by Perron at http://people.bu.edu/perron/.

Under the system of equations (4.7) to (4.9), any univariate test for a unit root in $q_t = s_t - \gamma x_t = s_t + p_t^f - p_t^d$ amounts to examining (4.7) ignoring information in the remaining equations in the model. The main idea of a covariate unit root test is that we can improve our test results by exploiting correlations between the error terms in such a multivariate system. Hansen (1995) first developed a covariate unit root test, which uses the ADF test and adds covariates to the right hand sides of the regression in order to increase power of the test. Although Hansen's (1995) test delivers large power gain, Elliott and Jansson (2003) point out that it does not make optimal use of all available information in the system. The covariate point optimal test (CPT) is developed by Elliott and Jansson (2003). The CPT builds on the point optimality of the PT test and seeks for additional power gain by extending Hansen's covariate method. When there is no additional information we can exploit from covariates, the univariate PT test and the CPT test become equivalent.

The choice of covariates included in the CPT test does not have much limitations. Any variable which satisfies the two conditions can be a candidate for a covariate. The two conditions are (i) stationarity of the covariate and (ii) economic relevance of the variable. Previous empirical works like Elliott and Pesavento (2006) and Amara and Papell (2006) include various covariates such as money differentials, income differentials, current account deficit, interest rates differentials. A power gain of a covariate test over a univariate test can be attributed either to a larger data set or to a more efficient use of the same data set. To pick up the power gain from the latter, we adopt the test suggested by Elliott, Jansson, and Pesavento (2005), who show how to apply Elliott and Pesavento (2003) to a no-cointegration test. They restrict covariates in the test to the first difference of the right hand side variables in the cointegrating regression.[41] The covariate in their method amounts to the inflation difference between foreign and home countries or Δx_t in our system of equation (4.7) to (4.9). With the variable in the cointegrating regression as a covariate, we can use the same data set as in the univariate tests in the paper. The downside of our approach is that we may lose important information outside our bivariate system which can be included in the CPT test. The rejection rates in their papers are higher than ours. However, their method may have a risk of overrejection by adding irrelevant covariates.

The CPT test is defined as

$$CP_T(1, \overline{\alpha}) = T(tr[\widetilde{\Sigma}(1)^{-1}\widetilde{\Sigma}(\overline{\alpha})] - (m + \overline{\alpha})), \tag{4.10}$$

where m is the number of covariates used. The variance covariance matrices of the residuals ($\widetilde{\Sigma}(r)$ where $r = 1, \overline{\alpha}$) are constructed by GLS-detrending of the data under the null and alternative. In the process of GLS-detrending, estimated R^2 and long run variance of the VAR system under the null are used to exploit the information captured from covariates. In the VAR estimation, the lag length is determined by BIC following Elliott and Jansson (2003). The R^2 value represents the contribution of the covariate in explaining the movement of the nominal exchange rate. Like the PT test, the CPT test is a modified version of a likelihood ratio test. The critical value of the test depends on the model specification about deterministics and the value of R^2.

[41] The only practical difference from the usual CPT test is that Elliott, Jansson, and Pesavento (2005) drop the first observation in estimating R^2 and long run variance matrix in Step (a) in p81 of Elliott and Jansson (2003). For more discussion, see p38 of Elliott, Jansson, and Pesavento (2005).

The test in this paper amounts to Case 3 in Table 1 of Elliott and Jansson (2003). If the test statistic is smaller than the critical value, the test rejects the null.[42]

4.3 Empirical Results

4.3.1 Data

All data are collected from June 2006 International Financial Statistics CD-ROM with a few exceptions. We use quarterly data. Our baseline tests cover 1973-1998 to avoid the effect of a possible structural break due to the end of the Bretton Woods system and the introduction of the Euro. We will compare our baseline test results with test results for longer period. The data used in this paper spans from 1957 to 2005. Twenty eight countries including US[43] are chosen among high-income and middle-income countries based on data availability. Our data set includes nominal exchange rate against US dollar, CPI, PPI, Export/Import price index for each country. We take the logarithm of all data we collect. PPI and Export/Import data are not available for all the 28 countries for the sample period.

For a nominal exchange rate, national currency per US$ period average (RF.ZF) is used. For European countries which had adopted the Euro (Austria, Belgium, Finland, France, Germany, Ireland, Italy, Netherlands, Portugal, and Spain from 1999Q1

[42] For detailed steps of the CPT test, see Elliott and Jansson (2003). Matlab code for the CPT test is available at http://www.econ.berkeley.edu/~mjansson/. We use the program after converting it into GAUSS code.

Following table is the critical value used in this paper. It is taken from Case 3 in table 1 of Elliott and Jansson (2003). For values of R^2 between the ones given in the table, interpolation is used as recommented in Elliott and Jansson (2003).

R^2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Critical value	3.34	3.41	3.54	3.70	3.96	4.41	5.12	6.37	9.17	17.99

[43] Those countries include Australia, Austria, Belgium, Canada, Chile, Denmark, Finland, Germany, Greece, Ireland, Italy, Japan, Korea, Malaysia, mexico, Netherlands, New Zealand, Norway, Portugal, Singapore, South Africa, Spain, Sweden, Switzerland, Thailand, UK, and US.

and Greece from 2001Q1), the series is connected by the Euro exchange rate with the same code[44]. For consumer prices (64...ZF) and producer prices (63...ZF), data for Germany up to 1991Q4 is for West Germany before the unification. PPI/WPI for Ireland of year 2005 comes from the Datastream. For export/import prices, export price index (76...ZF) and import price index (76.X.ZF) are used whenever available (Australia, Finland, Germany, Greece, Japan, Korea, New Zealand, Portugal, Sweden, UK, and US). When export/import price indexes are not available, export unit value index (74...ZF) and import unit value index (75...ZF) are used (Canada, Denmark, Ireland, Italy, Netherlands, Norway, South Africa, Spain, and Thailand).

4.3.2 Testing for a unit root on the CPI based real exchange rates

Table 4.1 shows our test results for the five different unit root tests (ADF, DF-GLS, PT, MPT, and CPT) on 1973-1998 quarterly data for 27 bilateral real exchange rates when U.S. is treated as a base country. For all unit root test results in this paper, the test is rejected if the test statistic is smaller than the critical value. The critical value for a univariate unit root test is at the bottom of the table. As mentioned before, the critical value of the CPT test depends on both the model specification and the R^2 between the variable and the covariates. Like other previous empirical results, the ADF test fails to reject the null for every bilateral real exchange rate. Even in case of more powerful univariate tests, the results are, at most, modest. PT and MPT reject the null for 4 bilateral RERs (France, Germany, Italy, and New Zealand) while DF-GLS rejects one more bilateral RER (Mexico). It is consistent with Cheung and

[44] To connect the individual currency exchange rate data with the Euro exchange rate data, we need the exchange rate of the Euro of 1998Q4. We use the average of daily rates of the ECU which can be obtained from the following FRB website: http://www.federalreserve.gov/releases/H10/hist/.

Lai (2000) who adopt DF-GLS test and succeed in getting more rejections but still cannot get rejections for majority cases.

At first look, results of the CPT test in table 1 seems to have much more rejections than univariate test results. We have five more rejections (Greece, Ireland, Portugal, South Africa, and Thailand) in addition to 5 rejections from univariate tests. However, one big assumption in a covariate unit root test is that covariates used in the test are stationary. Thus, we should not give much credit to the results if there is evidence against the stationarity of covariates. The covariate we use in the test is the difference of inflation rates of the two countries. To check the stationarity of our covariate, we apply Park's (1990) $G(p, q)$ test which exploits the spurious regression result that time polynomials tend to mimic a stochastic trend. The $G(p, q)$ test takes stationarity as the null hypothesis with the alternative hypothesis of unit root nonstationarity. Its asymptotic distribution is chi-squared with $q - p$ degrees of freedom. p denotes the order of the time trend maintained under the null hypothesis, whereas q denotes the number of superfluous time polynomial terms added. In our test, $p = 0$, which indicates that the variable is stationary without any time trend under the null hypothesis. We use $q = 1$. QS kernel is used in estimating the long-run variance of the estimated residual in the regression. Following Kahn and Ogaki (1992), no prewhitening method is applied. Automatic bandwidth is used but $\sqrt{T}$ is used as a bandwidth if the computed automatic bandwidth is greater than $\sqrt{T}$. The last column in Table 4.2 provides the the G-test results. Since the G-test statistic follows $x^2(1)$ under the null of the stationarity, we reject the null if the test statistic is greater than the critical value, 3.84.

It turns out that the inflation differences for 12 countries reject the null of stationarity. After eliminating those 12 countries, the covariate test now rejects the null of unit root for only 5 countries (Greece, Mexico, New Zealand, South Africa, and Thailand) out of 15 countries. The rejection rate is still "modest" in the sense that the majority of the bilateral RERs considered fail to reject the null. None of the five are the industrialized Western European countries which have been the main interest of the PPP literature. In that sense, the covariate test suggested by Elliott, Jansson, and Pesavento (2005) does not help solving the low power problem of a univariate unit root test much when CPI general price index is used. It is because the CPI for many countries seem to be more highly integrated than I(1). Thus in those cases, the basic assumption of the covariate unit root test of Elliott, Jansson, and Pesavento (2005) is not satisfied.

This problem has been noticed in the previous works. Amara and Papell (2006) have to use the growth rate of the inflation rate instead of the inflation rate itself as a covariate to avoid the possibility of nonstationarity of the covariate they use. In case of Elliott and Pesavento (2006), only US inflation is used as a covariate. They find some evidence in favor of stationarity of US inflation rate for their sample period. In that case, though, we lose the flavor of the covariate unit root test as a test for nocointegration when the cointegrating vector is known as considered in Elliott, Jansson, and Pesavento (2005). In this paper, we will check whether we can apply the cointegration test idea in Elliott, Jansson, and Pesavento (2005) for different price indexes.

4.3.3 Testing for a unit root on the traded goods price based real exchange rates

The Balassa-Samuelson theory implies that the real exchange rate based on general price indexes may have a unit root even when the law of one price holds for the trade goods because the nontraded goods component in the real exchange rate may contain a unit root. Thus, according to the Balassa Samuelson theory, it is more desirable to use the traded goods prices rather than the general price index of CPI in the test of the law of one price.

In this subsection, we test for a unit root in the traded goods price based real exchange rates and see if we can find stronger evidence for the law of one price. One practically important issue to deal with the traded goods prices is which price series we should use as the traded goods prices. Betts and Kehoe (2006) show that the choice of price series used for traded goods price may change the empirical results on the real exchange movements. In this paper, we will consider three different price data as the traded goods prices. First data set in this paper is taken from Engel (1999). Engel uses a weighted average of some subindexes of CPI as traded goods price. Engel uses monthly OECD data from January 1962 to December 1995 for Canada, France, Germany, Italy, Japan, and the United States.

CPI is affected by the change in the cost of distribution and retail services which are not tradable. Thus, it is possible that a good of our interest is a tradable but has a different consumer price in each country. For this reason, prices measured at the production site like PPI may be more proper to test for the law of one price since those prices are not directly affected by the distribution cost or retail services. Our second data set in this subsection is PPI/WPI.

On the other hand, Burstein, Eichenbaum, and Rebelo (2006) show that export/import prices measured at the dock can be a good candidate for the traded goods prices in their empirical analysis for short and medium run movements of the real exchange rates. Our third data set for the traded goods prices is export/import prices. We use the geometric average of the export price and the import price as the traded goods price following Burstein, Eichenbaum, and Rebelo (2006). For the second and third data set, our data source is the same quarterly IFS CD-ROM data that we use in the previous subsection. One big shortcoming of the departure from using CPI general price indexes is limited data availability. While the CPI general price indexes are readily accessible for most countries for reasonably long period, the subindexes of CPI, PPI or Export/Import price indexes for many countries are difficult to collect or the time series available is too short for statistical analysis. Accordingly, comparing with the tests with CPI general price indexes, our data set is small. In this paper, we do not try to find a new data source but use readily available data set only.

Table 4.2 is our results from Engel's CPI data set. For comparison, we first do the unit root tests using the general price index of CPI as in part (a) of Table 4.2. There is no rejection from any univariate test. The G-test rejects the stationarity of the inflation difference for France and Japan. For the other three countries (Canada, Germany, and Italy), the CPT test does not reject the null in the CPT test. With the traded goods prices, more powerful univariate tests (DF-GLS, PT, and MPT) reject the null for Canada but not for other countries. The CPT test is applicable for 4 countries except for Japan. The CPT test does not reject for any bilateral RER based on CPI traded goods prices. Over all, there is no convincing evidence for the law of one price for any bilateral RER based on the CPI traded goods price.

Table 4.3 is our test results from our second data set for PPI/WPI. 18 PPI/WPI series are available among 27 countries we consider in this paper. The ADF test still does not reject the null in any cases. The PT and MPT tests reject the null for Finland, South Africa, and Sweden. The DF-GLS test gives one more rejection (Ireland) in addition to the three countries. It is comparable to the "modest" success of Cheung and Lai (2000) when they apply the DF-GLS test to 90 bilateral RERs: more rejections than the ADF test but not the majority. Since the G-test rejects the stationarity null of the inflation difference for three countries (Finland, Germany, and Switzerland), we consider 15 bilateral RERs for the CPT test[45]. In addition to the rejections from the univariate tests, the CPT test rejects five more cases including Denmark, Netherlands, New Zealand, Spain, and Thailand. As a result, the CPT test rejects the null for 8 out of 15 countries. Compared with the result from the CPI general price indexes in Table 4.1, the rejection rate from the PPI data is higher (a little more than a half) and includes some developed Western European countries.[46]

Table 4.4 shows the test results from our third data set for Export/Import prices. In this case, the ADF test rejects one case, New Zealand. However, the rejection disappears as we apply more powerful tests to the country. The PT and MPT tests reject the null for two countries (South Africa and Thailand) out of 18 countries. The DF-GLS rejects two more countries (Germany and Netherlands) in addition to the

[45] We still report the CPT test results for the three countries (Finland, Germany, and Switzerland) for completion, although our analysis of the results and our conclusion are not based on the results.

[46] The CPT test with the PPI data is applicable to Canada and Japan among the five countries in our first data set from Engel (1999) using the CPI traded goods prices. None of the two reject the null of the law of one price

rejections in the PT and MPT test. In case of Export/Import prices, the G-test does not reject the null of stationarity of inflation difference for any country considered in this paper. In addition to the 4 rejections from the DF-GLS test, the CPT test rejects the null for Denmark, Finland, Ireland, Italy, and Sweden. Thus, the CPT test rejects the null for 9 out of 18 countries. Again, Compared with the result from the CPI general price indexes in Table 4.1, the rejection rate from the PPI data is higher.[47] R^2 for Export/Import price data is 24% on average. It is much higher than that for CPI data (8%) or that for PPI data (7%).

To sum up, the CPT test using PPI or Export/Import data rejects the null for about half of the bilateral RERs, which is much higher than the rejection rate of a univariate test using CPI general price indexes. On the other hand, the CPT test with CPI based traded goods prices does not give any rejection for the five industrialized countries considered. Our test results are consistent with Betts and Kehoe (2006) who find that choice of price index is important in accounting for the movement of the RERs.

4.3.4 Testing for a unit root with longer time span data

In general, using a longer time span data is believed to give higher power to a unit root test. We perform unit root tests for the period of 1973-2005 which includes the introduction of the Euro. Tables 4.5-4.7 are our test results for CPI, PPI, and Export/Import data, respectively. In terms of the rejection rate of the CPT test, it does not make a significant difference to extend the sample period up to year 2005. The CPT test rejects the null for 3 out of 11 available CPI bilateral RERs, 6 out of 11

[47] The CPT test with the Export/Import data is applicable to 4 countries among the five countries in our first data set. In case of Germany and Italy, the CPT test can reject the null.

available PPI bilateral RERs, and 8 out of 16 Export/Import prices bilateral RERs respectively. The rejection rate for the period of 1973-1998 is also about one third, a half, and a half for each price index.

On the other hand, the results from univariate tests are quite different. For this longer period, the rejection rates of univariate tests for CPI and PPI bilateral RERs are as high as that of the CPT test. The DF-GLS, PT, and MPT tests reject the null for 9 to 11 out of 27 CPI bilateral RERs. Those tests reject the null for 7 to 10 out of 18 PPI bilateral RERs.

Therefore, it helps increasing rejection rate of univariate tests for CPI or PPI bilateral RERs to extend the sample period up to year 2005 while there is not much difference in the test results of the CPT test. Such increase in rejection rate for univariate tests does not occur to the tests for Export/Import price bilateral RERs. The test results are similar to those for our baseline period (1973-1998) whether the test is a univariate or covariate test.

We also do the CPT test with a longer time span of 1957-1998 which includes the fixed exchange rate period. Because of data availability, we only have smaller number of bilateral RERs. Table 4.8 shows our results. Although we can construct 22 CPI bilateral RERs for the period, the G-test rejects the stationarity of our covariate for 7 countries. Then, the CPI test reject the null only for New Zealand among 15 bilateral RERs based on the CPI general price indexes. Meanwhile, the DF-GLS and PT test reject the null for three more countries (Finland, France, and Greece)[48] to which the CPT test is not applicable because of the G-test results. The MPT test rejects for Australia in addition to the rejections by the PT test. To sum up, it does not help

[48] The univariate test results for longer time period are not reported in this paer to save space. The results are available upon request.

increasing rejections to include fixed exchange rate data (1957-1972) into the sample. The rejection rate from the CPT test using longer time span CPI data is lower than that for the shorter period (1973-1998). It is also lower than the rejection rate from univariate tests for the same period.

We also do the CPT test with PPI data of 1957-1998. As in the test with CPI data, the CPT test rejects the null for only one country (Thailand) out of 11 countries to which the CPT test is applicable. The DF-GLS, PT, and MPT tests reject the null for two countries (Australia and Finland) out of 13 available countries. In case of Export/Import prices, the CPT test rejects 4 countries (Denmark, Greece, Netherlands, and Sweden) out of 11 available countries, a little higher rejection rate than CPI or PPI cases. The DF-GLS, PT, and MPT tests reject the null for 2, 3, and 3 countries out of 11 countries. Over all, our data sets do not show any convincing evidence that extending the sample period to earlier years would help increasing rejection rate of a univariate test or a covariate test.

4.3.5 Testing for a unit root with a vector of covariates

Instead of taking the difference between US inflation and foreign inflation in order to get a covariate for the CPT test, it is possible to use a vector of (US inflation, foreign inflation) as our covariate vector. This approach may allow our covariate vector to explain better the short run movement of the RERs with a richer dynamics.

However, to perform the CPT test, we should first check whether inflation series is stationary. G-test using each of the three data sets we consider in the paper rejects the null of stationarity of inflation for most of the countries. Especially, the US inflation show no evidence of stationarity. Therefore, the CPT test using US inflation and foreign inflation as a set of covariates is not applicable since the data does not satisfy the assumption of the test. Just for the completion, we report the CPT test results with a set of covariates in Table 4.9. We got more rejections than the test with the difference of inflation rates as a covariate in general. However, it may reflect a spurious regression between the covariates and the RERs.

4.3.6 Testing for a unit root with Germany as the base country

We run the unit root tests with Germany as the base country. Papell and Theodoridis (2001) show that a unit root test with Germany as the base country gives more rejections. Univariate tests with CPI data seem to be consistent with Papell and Theodoridis (2001). Even the ADF test gives 2 rejections. The DF-GLS, PT, and MPT tests reject the null for 6, 8, and 8 out of 27 countries respectively, which are greater than the rejections from the tests with US as the base country.

On the other hand, the number of rejections from univariate tests with PPI or Export/Import data is not very different whether the base country is US or Germany. In case of the covariate test, the G-test rejects the stationarity of the covariate for many countries. As a result, we can apply the CPT test to only small subset of countries we consider. The CPT test rejects the null for 2 out of 6 countries with CPI data, 6 out of 9 countries with PPI data, 5 out of 10 countries with Export/Import data. One interesting fact for this data is that R^2 with the covariate is higher than the data with US as the base country. It is 31% for CPI data, 27% for PPI data, and 44% for Export/Import data.[49]

[49] The entire test results with Germany as the base country are not reported in this paper to save space. Those are avialable upon request.

4.4 Conclusion

Because of the strong plausibility of long run PPP, literature has been searching for a more powerful testing method to correct the very low rejection rate of the standard ADF test for a unit root in the real exchange rate. Using longer run data or adopting panel method help increasing the power but both have a risk of violating the assumption of the test. More powerful univariate tests like the DF-GLS test give more rejections as in Cheung and Lai (2000) but their success was at most modest.

Compared with previous empirical results, the unit root test results presented by Elliott and Pesavento (2006) and Amara and Papell (2006) are quite impressive. They simultaneously apply the augmented feasible point optimal test (CPT) developed by Elliott and Jansson (2003) to the CPI based real exchange rates of industrialized countries during post Bretton Woods period. They succeed in rejecting the null for the majority of the real exchange rates considered. The CPT test is known to be more powerful than univariate tests when the quasi-difference of the real exchange rate has correlation with covariates.

Two aspects of their tests raise some doubt on their success. First, according to the econometric theory, a covariate in the CPT test should be (i) stationary and (ii) economically relevant. Basically any stationary variable can be freely used as a covariate if an economic theory show the relevance between the covariate and exchange rate movements. The two previous works pick 6 or 7 stationary covariates based on various economic theories on the exchange rates. They try one of those covariates for the CPT test one by one and they treat the test result as rejection if a test with one of those covariates rejects the null. The test strategy has a risk of overrejection by including irrelevant covariates in the test if there does exist a unit root for some real exchange rate.

Second, the two previous works use CPI general price indexes to construct real exchange rates. According to the Balassa-Samuelson theory, the real exchange rate based on CPI general price indexes may have a unit root because of nontradable goods. Even the CPI subindexes of the tradable goods may include nontradable components since the cost for distribution and retail services is a big part of the consumer prices.

The long run PPP can be interpreted as a cointegrating regression of a nominal exchange rate on the difference of domestic and foreign prices with known cointegrating vector. Elliott, Jansson, and Pesavento (2005) show that one can directly apply the CPT test developed by Elliott and Jansson (2003) to testing for nonstationarity of potential cointegrating relationship with known parameters. This test constructs a covariate as the first difference of the right hand side variable in the cointegrating regression, which is the difference of domestic and foreign inflation in the context of the long run PPP. We apply the CPT test suggested by Elliott, Jansson, and Pesavento (2005) in this paper. The cointegrating regression is not the only guide to choose a covariate but it is an important one. By relying on this relationship, we are free from the overrejection risk. In addition, this approach uses the same data set that we use in univariate unit root tests while the two previous papers use bigger data sets by including various covariates. Therefore, we can evaluate the effect of switching the testing method from univariate tests to covariate tests with the same data set. We construct the real exchange rates using not only CPI but also PPI and Export/Import price indexes and compare the test results.

We consider 27 bilateral RERs with US as a base country for the post Bretton Woods period of 1973-1998, although the numbers of real exchange rates for PPI or Export/Import price indexes are smaller due to data availability. Our CPT test results with PPI or Export/Import price indexes give approximately 50% rejection rate, which is much higher than univariate tests performed in this paper. Thus, we can find much stronger evidence in favor of the long run PPP than that in the literature, although it is lower than that found by Elliott and Pesavento (2006) or Amara and Papell (2006). We have only weaker evidence from the CPT test with CPI based real exchange rates. In many cases, the CPT test is not applicable because the CPI inflation difference between domestic and foreign countries seems to be too persistent to be stationary. The rejection rate with CPI data for applicable cases is lower than that with PPI or Export/Import price indexes.

To sum, we find stronger evidence in favor of long run PPP from the CPT test than univariate tests reflecting the efficiency of the covariate test. The choice of price index significantly affects our test results, which is consistent with the Balassa-Samuelson theory.

Country	ADF	DF-GLS	PT	MPT	CPT	(R^2)	<G-test>
Australia	-1.05	-0.74	10.05	9.17	6.34	(0.00)	<4.00*>
Austria	-2.08	-1.40	6.80	5.51	5.28	(0.00)	<2.77>
Belgium	-1.95	-1.91	3.42	3.40	3.35	(0.01)	<0.18>
Canada	-0.84	0.09	12.01	10.04	18.37	(0.13)	<2.72>
Chile	-2.54	-0.16	70.51	47.63	46.37	(0.59)	<5.82*>
Denmark	-2.01	-1.65	4.92	4.30	10.54	(0.03)	<8.89*>
Finland	-2.34	-1.63	5.50	4.53	5.40	(0.00)	<5.54*>
France	-2.36	-2.33*	2.35*	2.34*	-0.10*	(0.25)	<4.76*>
Germany	-2.07	-2.03*	3.15*	3.10*	2.90*	(0.04)	<4.91*>
Greece	-1.90	-1.75	4.14	3.77	0.58*	(0.06)	<1.05>
Ireland	-2.19	-1.74	4.93	4.30	1.47*	(0.04)	<9.32*>
Italy	-2.33	-2.33*	2.22*	2.26*	0.78*	(0.03)	<7.65*>
Japan	-1.72	-0.74	14.68	11.27	11.37	(0.03)	<2.49>
Korea	-1.72	-1.55	4.65	4.50	3.72	(0.00)	<4.56*>
Malaysia	-1.15	-0.76	10.88	9.93	6.18	(0.02)	<1.60>
Mexico	-2.46	-2.01*	3.81	3.34	1.23*	(0.02)	<0.04>
Netherlands	-2.17	-1.94	3.54	3.26	3.74	(0.00)	<0.46>
New Zealand	-2.33	-2.34*	2.31*	2.35*	3.08*	(0.17)	<3.40>
Norway	-2.30	-1.92	3.91	3.44	4.39	(0.01)	<0.43>
Portugal	-1.73	-1.52	4.49	4.15	1.40*	(0.22)	<7.88*>
Singapore	-1.65	-1.37	7.59	6.86	4.81	(0.50)	<0.01>
South Africa	-1.75	-1.21	7.38	6.58	2.82*	(0.02)	<2.50>
Spain	-1.98	-1.38	7.09	5.84	6.63	(0.00)	<7.71*>
Sweden	-1.78	-1.68	4.37	4.26	3.57	(0.04)	<1.46>
Switzerland	-2.34	-1.22	8.68	6.57	8.58	(0.01)	<3.00>
Thailand	-1.13	-1.23	7.40	7.43	3.19*	(0.00)	<0.08>
UK	-1.98	-1.47	6.32	5.43	7.65	(0.05)	<8.31*>
5% critical value	-2.89	-1.98	3.17	3.17			<3.84>

Notes: i) * denotes rejections at 5%.

Table 4.1: Unit root tests with 1973-1998 CPI based RER

(a) Unit root tests with CPI general price based RER (1962-1995)

Country	ADF	DF-GLS	PT	MPT	CPT	(R^2)	<G-test>
Canada	-1.72	-1.50	3.81	3.73	14.34	(0.17)	<0.70>
France	-2.24	-1.41	5.49	5.17	5.40	(0.01)	<7.65*>
Germany	-1.66	-0.55	13.60	12.61	12.66	(0.01)	<0.51>
Italy	-2.45	-1.32	6.67	6.16	5.90	(0.00)	<0.04>
Japan	-1.08	0.81	68.63	62.72	55.48	(0.05)	<19.56*>

(b) Unit root tests with CPI Traded goods price based RER (1962-1995)

Country	ADF	DF-GLS	PT	MPT	CPT	(R^2)	<G-test>
Canada	-2.34	-2.33*	1.73*	1.74*	5.93	(0.11)	<0.02>
France	-1.86	-0.97	7.96	7.47	7.35	(0.02)	<1.13>
Germany	-1.50	-0.34	16.55	15.33	14.70	(0.00)	<0.18>
Italy	-2.24	-0.91	10.37	9.53	9.22	(0.00)	<0.08>
Japan	-0.73	1.12	92.68	84.88	58.17	(0.07)	<10.68*>
5% critical value	-2.89	-1.98	3.17	3.17			

Notes: i) * denotes rejections at 5%.

Table 4.2: Unit root tests with 1973-1998 CPI based RER

Country	ADF	DF-GLS	PT	MPT	CPT	(R^2)	<G-test>
Australia	-1.95	-1.53	5.59	4.92	7.37	(0.02)	<2.07>
Austria	-2.20	-1.79	4.32	3.75	7.42	(0.01)	<0.00>
Belgium							
Canada	-2.02	-1.29	5.18	4.45	4.53	(0.01)	<0.00>
Chile							
Denmark	-1.83	-1.57	5.19	4.66	0.54*	(0.40)	<0.65>
Finland	-2.52	-1.99*	3.57*	3.10*	3.15*	(0.05)	<5.36*>
France							
Germany	-1.94	-1.77	4.10	3.83	3.43	(0.05)	<4.08*>
Greece							
Ireland	-1.99	-2.00*	3.27	3.33	0.98*	(0.06)	<3.06>
Italy							
Japan	-1.78	-1.36	6.27	5.46	7.16	(0.00)	<0.00>
Korea	-1.50	-1.22	7.14	6.60	5.09	(0.04)	<3.11>
Malaysia							
Mexico	-1.62	-0.69	21.67	17.41	17.99	(0.04)	<3.11>
Netherlands	-1.86	-1.84	3.51	3.53	2.67*	(0.05)	<3.38>
New Zealand	-1.81	-1.74	3.44	3.41	2.59*	(0.02)	<1.44>
Norway							
Portugal							
Singapore							
South Africa	-2.21	-2.23*	2.03*	2.06*	1.88*	(0.03)	<0.82>
Spain	-1.98	-1.98	3.16	3.20	-2.77*	(0.21)	<3.19>
Sweden	-2.12	-2.04*	3.02*	2.96*	0.00*	0.18	<0.31>
Switzerland	-2.22	-1.70	4.71	3.99	4.71	(0.01)	<6.56*>
Thailand	-1.81	-1.87	4.38	4.44	2.21*	(0.00)	<0.90>
UK	-1.36	-1.37	5.76	5.79	4.76	(0.00)	<0.12>
5% critical value	-2.89	-1.98	3.17	3.17			

Notes: i) * denotes rejections at 5%.

Table 4.3: Unit root tests with 1973-1998 PPI based RER

Country	ADF	DF-GLS	PT	MPT	CPT	(R^2)	<G-test>
Australia	-2.51	-0.07	37.51	26.54	20.38	(0.04)	<0.06>
Austria							
Belgium							
Canada	-2.29	-1.31	8.45	6.56	16.97	(0.58)	<0.66>
Chile							
Denmark	-1.81	-1.55	5.86	5.38	-2.13*	(0.32)	<0.18>
Finland	-1.93	-1.91	3.58	3.61	1.88*	(0.32)	<0.00>
France							
Germany	-2.17	-1.98*	3.45	3.33	1.09*	(0.16)	<3.72>
Greece	-1.77	-1.49	6.46	5.85	3.96	(0.00)	<0.05>
Ireland	-2.55	-1.62	6.26	5.00	1.95*	(0.19)	<0.41>
Italy	-2.00	-1.94	3.63	3.56	2.61*	(0.33)	<2.25>
Japan	-2.53	-1.15	9.29	7.32	7.18	(0.15)	<0.07>
Korea	-2.68	-0.86	11.41	8.28	6.22	(0.02)	<0.15>
Malaysia							
Mexico							
Netherlands	-2.13	-2.05*	3.25	3.19	0.58*	(0.38)	<0.51>
New Zealand	-3.15*	-0.32	31.77	22.03	18.41	(0.03)	<0.02>
Norway	-1.02	-0.35	13.35	11.77	10.51	(0.06)	<0.00>
Portugal							
Singapore							
South Africa	-2.25	-2.25*	2.76*	2.80*	1.75*	(0.03)	<0.78>
Spain	-1.95	-0.82	16.14	12.41	9.20	(0.22)	<0.13>
Sweden	-2.09	-1.83	4.08	3.82	-1.94*	(0.34)	<0.43>
Switzerland							
Thailand	-2.72	-2.39*	2.67*	2.44*	6.61*	(0.80)	<0.13>
UK	-1.97	-1.48	6.16	5.32	5.00	(0.21)	<2.10>
5% critical value	-2.89	-1.98	3.17	3.17			

Notes: i) * denotes rejections at 5%.

Table 4.4: Unit root tests with 1973-1998 Export/Import price index based RER

Country	ADF	DF-GLS	PT	MPT	CPT	(R^2)	<G-test>
Australia	-1.89	-1.47	6.49	5.91	5.68	(0.01)	<5.15*>
Austria	-2.36	-1.65	5.28	4.43	3.54	(0.01)	<3.08>
Belgium	-2.11	-2.11*	2.79*	2.82*	2.43*	(0.02)	<0.11>
Canada	-2.05	-1.35	7.48	6.31	11.97	(0.04)	<3.21>
Chile	-2.42	-0.14	76.85	55.39	49.38	(0.58)	<6.52*>
Denmark	-2.29	-1.93	3.78	3.39	8.28	(0.02)	<11.65*>
Finland	-2.07	-1.76	4.34	3.96	4.50	(0.00)	<7.37*>
France	-2.45	-2.45*	2.10*	2.13*	1.45*	(0.09)	<6.00*>
Germany	-2.27	-2.27*	2.50*	2.52*	1.69*	(0.07)	<5.15*>
Greece	-1.79	-2.47*	2.01*	1.84*	0.78*	(0.07)	<14.38*>
Ireland	-2.17	-1.72	4.76	4.25	-0.34*	(0.25)	<8.57*>
Italy	-2.50	-2.48*	1.98*	2.01*	1.71*	(0.01)	<9.21*>
Japan	-2.10	-1.18	10.02	7.91	5.61	(0.01)	<4.45*>
Korea	-2.12	-1.97	3.18	3.10*	3.07*	(0.01)	<7.76*>
Malaysia	-1.03	-0.28	29.74	18.39	11.02	(0.03)	<1.04>
Mexico	-2.49	-1.89	3.48	3.12*	-0.48*	(0.24)	<1.16>
Netherlands	-2.36	-2.19*	2.80*	2.66*	2.77*	(0.00)	<1.39>
New Zealand	-2.31	-2.30*	2.40*	2.43*	2.33*	(0.06)	5.12*>
Norway	-2.42	-2.13*	3.12*	2.85*	3.75	(0.01)	<1.98>
Portugal	-2.24	-1.96	3.20	2.93*	1.81*	(0.13)	<11.00*>
Singapore	-2.26	-2.58*	5.36	5.36	6.28	(0.49)	<0.02>
South Africa	-1.73	-1.08	8.36	7.40	5.55	(0.00)	<0.19>
Spain	-2.17	-1.49	6.10	5.14	3.83	(0.00)	<10.96*>
Sweden	-1.73	-1.43	5.70	5.31	5.14	(0.01)	<5.56*>
Switzerland	-2.67	-1.38	7.59	5.95	3.94	(0.07)	<3.07>
Thailand	-0.91	-0.57	16.20	15.45	7.55	(0.00)	<0.36>
UK	-2.14	-1.82	4.31	3.67	8.34	(0.05)	<11.62*>
5% critical value	-2.86	-1.98	3.17	3.17			

Notes: i) * denotes rejections at 5%.

Table 4.5: Unit root tests with 1973-2005 CPI RERs

Country	ADF	DF-GLS	PT	MPT	CPT	(R^2)	<G-test>
Australia	-2.08	-1.81	3.93	3.51	6.03	(0.01)	<6.71*>
Austria	-2.39	-2.12*	2.98*	2.77*	5.71	(0.01)	<0.54>
Belgium							
Canada	-2.51	-1.67	4.78	4.15	3.94	(0.05)	<4.70*>
Chile							
Denmark	-2.26	-2.02*	3.42	3.15*	0.43*	(0.22)	<2.80>
Finland	-2.13	-2.01*	3.11*	3.03*	3.55	(0.02)	<9.98*>
France							
Germany	-2.30	-2.17*	2.77*	2.67*	2.26*	(0.01)	<1.41>
Greece							
Ireland	-2.15	-2.13*	2.96*	2.96*	1.43*	(0.03)	<6.21*>
Italy							
Japan	-2.14	-1.82	3.68	3.33	5.31	(0.01)	<1.05>
Korea	-1.22	-1.32	6.87	6.81	6.16	(0.05)	<7.73*>
Malaysia							
Mexico	-2.22	-0.94	14.62	11.82	17.80	(0.02)	<0.59>
Netherlands	-2.27	-2.26*	2.42*	2.44*	1.70*	(0.01)	<1.36>
New Zealand	-1.81	-1.68	4.29	4.11	2.67*	(0.04)	<3.79>
Norway							
Portugal							
Singapore							
South Africa	-1.89	-1.78	3.19	3.14*	2.73*	(0.05)	<0.88>
Spain	-2.16	-2.16*	2.66*	2.70*	-0.05*	(0.13)	<7.25*>
Sweden	-2.28	-2.28*	2.37*	2.40*	1.42*	(0.12)	<3.54>
Switzerland	-2.51	-2.05*	3.29	2.95*	3.74	(0.00)	<1.53>
Thailand	-1.24	-0.99	8.99	8.66	4.59	(0.01)	<0.01>
UK	-1.98	-1.94	3.32	3.31	3.90	(0.01)	<4.87*>
5% critical value	-2.89	-1.98	3.17	3.17			

Notes: i) * denotes rejections at 5%.

Table 4.6: Unit root tests with 1973-2005 PPI RERs

Country	ADF	DF-GLS	PT	MPT	CPT	(R^2)	<G-test>
Australia	-2.26	-0.81	21.14	15.99	19.39	(0.12)	<0.11>
Austria							
Belgium							
Canada	-1.54	-1.33	6.59	6.11	7.76	(0.13)	<0.27>
Chile							
Denmark	-2.02	-1.68	4.98	4.56	-2.94*	(0.32)	<0.08>
Finland	-1.92	-2.28*	2.77*	2.69*	3.56*	(0.32)	<0.45>
France							
Germany	-2.49	-2.20*	2.80*	2.68*	0.09*	(0.15)	<3.50>
Greece	-1.61	-1.48	5.81	5.56	2.73*	(0.00)	<1.88>
Ireland	-2.94*	-1.58	6.17	4.91	2.00*	(0.17)	<1.72>
Italy	-1.07	-0.86	7.95	7.51	5.31	(0.35)	<2.91>
Japan	-2.33	-1.09	8.90	7.17	7.77	(0.17)	<0.66>
Korea	-2.49	-0.52	18.52	14.18	8.62	(0.03)	<0.47>
Malaysia							
Mexico							
Netherlands	-2.51	-2.33*	2.57*	2.47*	-1.13*	(0.32)	<1.22>
New Zealand	-3.42*	-0.65	29.93	22.06	17.07	(0.08)	<0.73>
Norway	-1.98	-1.44	5.66	5.17	7.90	(0.06)	<1.15>
Portugal							
Singapore							
South Africa							
Spain	-2.07	-0.84	16.84	13.36	10.78	(0.22)	<0.63>
Sweden	-2.40	-2.01*	3.38*	3.13*	-2.40*	(0.38)	<0.01>
Switzerland							
Thailand	-2.78	-1.67	3.89	3.42	6.79*	(0.78)	<0.17>
UK	-2.27	-1.61	5.53	4.75	3.48*	(0.19)	<4.23*>
5% critical value	-2.89	-1.98	3.17	3.17			

Notes: i) * denotes rejections at 5%.

Table 4.7: Unit root tests with 1973-2005 Export/Import prices RERs

Country	CPI <G-test>		PPI <G-test>		E/I Price indexes <G-test>	
Australia	3.96	<0.03>	4.51	<1.16>	7.56	<1.39>
Austria	10.28	<2.60>	16.71	<0.89>		
Belgium	9.09	<0.27>				
Canada	20.95	<0.47>	9.61	<2.09>	17.83	<0.91>
Chile						
Denmark			8.24	<0.12>	-1.77*	<0.12>
Finland	2.70*	<3.90*>	1.08*	<6.43*>		
France	0.41*	<7.26*>				
Germany	8.87	<0.87>	7.16	<0.00>		
Greece	2.52*	<8.86*>				
Ireland	6.73	<1.79>	5.29	<1.05>	-0.22*	<0.05>
Italy	4.40	<0.16>			5.94	<2.85>
Japan	31.92	<7.90*>			18.03	<0.12>
Korea			16.27	<3.10>		
Malaysia	13.35	<2.46>				
Mexico	0.08*	<5.73*>	8.22	<4.84*>		
Netherlands	17.97	<6.78*>	5.30	<0.00>	0.52*	<0.03>
New Zealand	3.00*	<0.01>	48.60	<2.24>	11.97	<0.13>
Norway	18.64	<1.00>			11.81	<0.22>
Portugal	11.42	<1.55>				
Singapore						
South Africa	1.83*	<7.75*>				
Spain	6.80	<2.01>			37.45	<0.59>
Sweden	6.80	<0.84>			-0.87*	<3.30>
Switzerland	18.79	<1.06>				
Thailand			1.77*	<0.91>		
UK	5.27	<0.21>	8.13	<0.48>		

Notes: i) * denotes rejections at 5%.

Table 4.8: CPT test with a time span including fixed exchange rate regime (1957-1998)

Country	CPI (R^2)		PPI (R^2)		E/I Price indexes (R^2)	
Australia	-0.48*	(0.13)	-0.46*	(0.15)	8.55	(0.08)
Austria	5.17	(0.08)	-1.24*	(0.42)		
Belgium	2.06*	(0.00)				
Canada	8.81	(0.21)	2.99*	(0.03)	9.22	(0.36)
Chile	29.22	(0.68)				
Denmark	5.34	(0.11)	1.69*	(0.43)	0.81*	(0.52)
Finland	5.14	(0.01)	2.14*	(0.12)	1.10*	(0.33)
France	-0.69*	(0.38)				
Germany	5.24	(0.25)	2.94*	(0.28)	4.42*	(0.61)
Greece	2.34*	(0.06)			2.59*	(0.02)
Ireland	0.82*	(0.05)	1.29*	(0.14)	5.03	(0.39)
Italy	0.86*	(0.05)			1.21*	(0.39)
Japan	4.22	(0.28)	6.34	(0.08)	2.20*	(0.21)
Korea	3.58	(0.06)	3.53*	(0.27)	12.08	(0.12)
Malaysia	-0.24*	(0.10)				
Mexico	-1.11*	(0.13)	14.18	(0.05)		
Netherlands	4.09	(0.07)	2.81*	(0.36)	0.07*	(0.38)
New Zealand	2.39*	(0.08)	3.20*	(0.06)	19.79	(0.33)
Norway	4.74	(0.01)			2.54*	(0.11)
Portugal	3.28*	(0.04)				
Singapore	3.65*	(0.52)				
South Africa	1.95*	(0.41)	0.35*	(0.14)	0.98*	(0.17)
Spain	7.05	(0.00)	-2.95*	(0.25)	4.45	(0.22)
Sweden	2.32*	(0.04)	-0.19*	(0.16)	-0.71*	(0.36)
Switzerland	8.59	(0.03)	2.96*	(0.19)		
Thailand	0.22*	(0.09)	-0.84*	(0.07)	4.44*	(0.67)
UK	3.35*	(0.05)	4.11	(0.01)	4.47	(0.23)

Notes: i) * denotes rejections at 5%.

Table 4.9: Covariate unit root tests with a vector of inflations as covariates (1973-1998))

CHAPTER 5

CONCLUDING REMARKS

The Balassa-Samuelson theory in the long run says that the deviation from the law of one price of the traded goods prices is not a dominant factor which accounts for the long run movement of real exchange rates. In statistical terms, the theory implies that the traded goods component is stationary while the real exchange rate itself may contain an autoregressive unit root if the nontraded goods component in the real exchange rate is nonstationary. As such, the ratio of the long run variance for the traded goods component in the real exchange rate to that for the real exchange rate should be zero. Engel (1999) computes the ratio of the variances of k-differences. As far as the long run is concerned, his ratio amounts to an estimate of the ratio of long run variances between the traded goods component and the real exchange rate. He finds that (i) his ratio, or the importance of the traded goods component in the movement of the real exchange rate, is over 90% at every time horizon, (ii) the ratio is even increasing at the time horizons near the sample size for some countries, (iii) and no statistical evidence is found in favor of the decrease in the importance of the traded goods component in the long run in the test based on his measure.

Chapter 2 is about the second and third parts of Engel's findings. I first show that the increase in the ratio at the time horizons near the sample size found in Engel

(1999) is attributable to the statistical properties of the variance of k-differences, not to the increase in the importance. The variance of k-differences tends to go back to the initial value as the time horizon, k, approaches the sample size whether the variable is stationary or unit root nonstationary on average. I also show that some statistical evidence for the decrease in the importance of the traded goods component in the long run can be found after some appropriate modification of the test used in Engel (1999).

In Chapter 3, by means of a Monte Carlo simulation, I investigate whether the relative volatility of the traded goods component or its persistence is the dominant factor that leads to Engel's first and third findings. Contrary to Engel's (1999) inference, the mean of the ratio in my Monte Carlo simulation mainly depends on the relative volatility of the traded goods component under the parameterization observed in the data. It implies that the high ratio in the long time horizon does not necessarily mean that the traded goods component is highly persistent. On the other hand, the power of the test used in Engel (1999) is mainly determined by the persistence of the traded goods component. For given parameter values, the power is less than 20%. Using different kernel estimators or uncentered moments help increasing the power of the test. However, the power is still very low.

In Chapter 4, I adopt a state-of-the-art covariate unit root test as in Elliott and Jansson (2003) which has a better power than a univariate test. Unlike previous papers which use several covariates, I use the inter-country difference of inflation as the only covariate in order to see if using a more efficient testing method with the same data set can produce a different result. The Balassa-Samuelson theory in the long run implies that the unit root test is more likely to reject the null for the real exchange rate based on production site prices like PPI than for that based on the general price indexes since the production site price is closer to the traded goods price. The rejection rate in the covariate test for the real exchange rate based on production site prices is approximately 50%, which is higher than that from a univariate test result as well as that from the covariate test for the real exchange rate based on CPI. This new evidence is supportive of the Balassa-Samuelson in the long run.

My findings in this dissertation are consistent with the direction which this line of literature is heading for. Taylor and Taylor (2004) state that the Harrod-Balassa-Samuelson model of equilibrium real exchange rates is attracting renewed interest as a desirable modification [of PPP theory] after languishing for some years in relative obscurity.

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