INCORPORATING IMAGE-BASED DATA IN AADT ESTIMATION: METHODOLOGY AND NUMERICAL INVESTIGATION OF INCREASED ACCURACY

DISSERTATION

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By

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ABSTRACT

Annual Average Daily Traffic (AADT) is one of the most fundamental traffic statistics used for highway planning, design, and maintenance. State departments of transportation invest heavily in personnel and equipment to collect traffic counts supporting AADT estimation on all highway segments in their systems on a regular basis.

Vehicles are detectable in air photos, high-resolution satellite images, and LiDAR data of highway segments, which are regularly collected for various purposes. A Bayesian approach is developed to incorporate the traffic data extracted from these images in the existing practice of AADT estimation. The uncertainty in the AADT on a segment is expressed by a probability distribution. In any year of interest, the approach begins with a prior AADT distribution that is updated to a posterior distribution when a traffic count is available. When incorporating the uncertainty in traffic growth, this approach can be applied year by year. Methods are developed to model the prior distribution of the AADT and the probability distribution of short-term traffic counts conditional on the AADT, which are two important components of this approach. Parameters are estimated to make the approach operational.

A numerical study is conducted to simulate AADT estimation during a typical cycle of traffic count collection on the ground. The results show that a small amount of image-based data could be exploited through the Bayesian approach to improve accuracy in AADT estimates while reducing the number of costly and dangerous ground counts. Sensitivity analysis indicates that the Bayesian approach would provide positive benefits for a large range of conditions.

Operational issues are discussed for the Bayesian approach, and it appears that the method could be implemented in state DOTs if the institutional means are developed to extract image-based data and place them in a format that could be easily integrated with data presently used to estimate AADT. Additional areas are suggested for future study.

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CHAPTER 1

INTRODUCTION

Knowledge of vehicular traffic on highway segments is essential for highway planning, design, maintenance, and analysis. Annual average daily traffic (AADT) is a fundamental statistic of this traffic. Basically, AADT is the number of vehicles that would use a highway segment on a typical day of the year. A highway segment for the purpose of AADT estimation is a section of highway through which the traffic volume does not change (FHWA, 2001). In this work, a highway segment will be considered a section of highway between two consecutive interchanges. Because of its importance, Departments of Transportation (DOTs) in the 50 states estimate AADT on all their highway segments. The DOTs are also required to provide AADT estimates to the federal government to be eligible for the federal funding of state projects. Collecting traffic counts to estimate AADT is costly in terms of both equipment and personnel. Data collection involving the installment of traffic sensors on the surface of a busy highway also risks road crews' safety and disrupts traffic. While a large amount of resources are invested in collecting traffic data from the pavement or from the roadway vicinity, vehicles are actually detectable in high-resolution imagery obtained from airborne and space-based platforms. Air photos, high-resolution satellite images, and increasingly, LiDAR (light detection and ranging) data are regularly collected for various purposes. When the images contain highway segments, they can provide additional traffic observations at little marginal cost. Airborne and space-based platforms take images off the road, so such traffic data collection is neither dangerous to road crews nor disruptive to traffic. Airborne and space-based platforms can easily and quickly access segments of interest in remote areas, where it can be costly or difficult to collect traffic data on the ground. In addition, airborne and space-based platforms can cover a large spatial area in a much shorter time period, compared to ground-based data collection. Therefore, traffic data extracted from high-resolution imagery, called image-based data in this work, appear to be appealing for AADT estimation.

However, a given image only provides an observation at an instant in time, which is equivalent to a very short duration observation on the ground. Using such short-duration observations would produce large errors when estimating long time-scale measures such as AADT. The major issue addressed in this work, then, is how to efficiently use the "snapshot" information in the current practice of AADT estimation. A theoretically justified and operational method is required to combine the image-based data with traditional ground-based data in AADT estimation. In addition, estimating the benefits of adding the image-based data, in terms of improved accuracy in AADT estimation, would help determine the desirability of modifying existing practice.

In the research work presented here, we develop a Bayesian approach that allows the combination of data traditionally collected on the ground with the "snapshot" information obtained from airborne and space-based platforms, demonstrate the practicality of the methodology, and quantify the improved accuracy in AADT estimation. Parameters are estimated to make the Bayesian approach operational. A numerical study shows that the image-based data could be exploited to improve accuracy in AADT estimates while reducing the number of costly and dangerous ground counts. Sensitivity analysis indicates that the Bayesian approach would provide positive benefits for a large range of conditions.

In Chapter 2, the use of AADT is briefly discussed. Then, the practice and research related to AADT estimation are reviewed from the early 1900s through the present. The history of traffic data collection from airborne and space-based platforms is also introduced. Finally, the promising features of AADT estimation from the image-based data are discussed.

In Chapter 3, an approach is developed based on Bayesian analysis for AADT estimation. Two important components of this approach are discussed: the prior distribution of AADT and the probability distribution of observing the number of vehicles in an image, conditional on a given segment AADT. Given the prior distribution and the conditional distribution, algorithms are developed for the calculation of the posterior distribution, which is the direct output of the Bayesian approach. In addition, choosing a point estimate based on the posterior distribution is discussed.

In Chapter 4, parameters required for the 3-stage model that characterizes the distribution of image-based counts conditional on the AADT are discussed in detail. Empirical data are used to evaluate the reasonableness of the 3-stage model. The 3-stage model outperforms two other potential models in terms of producing the greatest probability of observing the numbers of vehicles in 22 images of Ohio highway segments. The 3-stage model can also capture the impacts of imaged segment length and traffic volumes on the variability of image-based counts. The possibility of approximating the conditional distribution of image-based counts given the AADT by a lognormal distribution is investigated. It is seen that the approximation is reasonable for implementing the Bayesian approach while eliminating the need for extensive computer simulation.

In Chapter 5, a numerical study is conducted to document the benefits of adding image-based data in AADT estimation by the Bayesian approach. Different quantities of image-based data available for AADT estimation on a segment are considered. It is seen that adding only a small amount of image-based data by the Bayesian approach could decrease the error in AADT estimation while reducing the amount of ground-based traffic data that need to be collected. Sensitivity analysis is also conducted. It is seen that adding image-based data by the Bayesian approach could bring positive benefits for a large range of variations in the parameters required in the approach and even when the parameters are incorrectly quantified.

In Chapter 6, the research is briefly summarized. Operational concerns are discussed for the Bayesian approach. Additional related research is suggested for future study.

CHAPTER 2

LITERATURE REVIEW

In this chapter, an extensive literature review and the background of the research are presented. In Section 2.1, the use of AADT is briefly introduced. In Sections 2.2 and 2.3, the literature reviews the studies and practice in AADT estimation from the early 1900s to present. In Section 2.4, the history of traffic data collection from airborne and space-based platforms is reviewed. In Section 2.5, the motivation of the research, adding image-based data in AADT estimation, is discussed.

2.1 Use of AADT

Annual Average Daily Traffic (AADT) is a summary measure of traffic volumes of special significance to the highway engineer (Wright and Dixon, 2004). AADT is the average daily traffic volume on a given road segment, where the average is taken over a full year (McShane *et al.*, 1998). Assuming 365 days in the year to simplify notation, the AADT on segment *i* in year *y* can mathematically be expressed as:

$$AADT_{i}(y) = \sum_{\delta=1}^{365} V_{i}^{24}(\delta, y) / 365, \qquad (2.1.1)$$

where $V_i^{24}(\delta y)$ is the daily (24-hour) traffic volume on segment *i* on day δ of year *y*. According to Equation (2.1.1), AADT represents a typical daily traffic volume on a road segment for all days of the week, Sunday through Saturday, during the year (AASHTO, 1992). In general, AADT is the volume in both directions, and usually expressed in terms of mixed traffic, *i.e.*, all classes of passenger cars and trucks. Sometimes AADTs on divided highways only consist of one-direction volumes.

A fundamental summary measure of traffic activity, AADT serves as a starting point for many other important traffic statistics. Its applications are wide-ranging in highway planning, design, maintenance, operations and research. AADT values in consecutive years measure and establish trends in traffic, which can be used to predict future traffic (FDOT, 2002). They also are integral components for calculation of another common summary statistic of traffic – Vehicle Distance Traveled (VDT) (or Vehicle Miles Traveled (VMT)). VDT (VMT) is the distance (miles) that vehicles are driven over a highway system during a period of time (*e.g.*, a year). Since AADT on a segment is the number of vehicles that use the segment during an average day of the year, AADT vehicles would travel the distance of the length of the segment on the average day. That is, AADT on a segment multiplied by the length of the segment yields the "VDT" on the specific segment for an average day of the year. Therefore, the VDT of a highway system for a year can be calculated as

$$VDT_I = 365 \sum_{i \in I} AADT_i \times length_i , \qquad (2.1.2)$$

where *I* represents the set of segments in the highway system considered, and *AADT_i* and *length_i* are, respectively, the AADT and length of segment *i*. There are many uses of VDT. For example, VDT is used for the development of highway financing and taxation schedules. Under TEA-21, Federal-aid Highway Funds are apportioned to each state based on its statewide VDT (FHWA, 2000). VDT is also used as the base for accident rates in the evaluation of traffic safety programs. In addition, VDT is used to estimate on-road vehicle fuel consumption by the Energy Information Administration (EIA) of the U.S. government (EIA, 1997).

The design-hour volume (DHV) is an important concept for planning, design and operational purposes (Mannering *et al.*, 2005). DHV is used in the determination of the number of lanes, ramp design, shoulder design, intersection design and other geometric design features. DHV is also used to determine deficiencies in capacity and develop traffic operation programs. In practice, DHV is typically derived from AADT. Specifically, AADT is converted to DHV by a K factor, which is defined as the proportion of AADT occurring in the peak hour (*i.e.*, K = DHV/AADT) (TRB, 2000).

AADT also plays a significant role in designing pavements to produce safe and comfortable roads in a cost-effective manner. The Equivalent Single Axle Load (ESAL) is the fundamental variable considered by AASHTO researchers in predicting pavement damage (Alberta Transportation and Utilities, 1997), and is an input to many current

pavement design procedures. AADT is often needed to determine ESAL (Alberta Transportation and Utilities, 1997).

AADT is used in many other ways, as well (AASHTO, 1992). Private sectors use AADT in determining businesses locations. Agencies use it in estimating aggregate vehicle emissions for air quality analysis. Traffic engineers use it in analyzing rail-crossing safety. AADT is also used in various research studies, for example, Kayhanian *et al.* (2003) studied the impact of AADT on storm water runoff pollutant concentrations generated from California Department of Transportation highway sites. The widespread uses of AADT have led to extensive data collection efforts and systematic methodologies for AADT estimation. In the following two sections, a review of practice and research in AADT estimation will be presented.

2.2. Early Studies in AADT Estimation

When traffic volumes on a segment are collected continuously during the year of interest, its AADT for the year can be calculated by using Equation (2.1.1). Automatic Traffic Recorders (ATRs) are mechanical devices that are able to automatically and continuously collect traffic volumes (Cleveland, 1964). However, because of considerable expenses, ATRs can be installed permanently on only a small portion of highway segments. For example, in Indiana, approximately 90 ATRs are installed permanently for about 11,000 miles of the state highway systems (Fricker and Whitford,

2004). If the typical highway segment length is 2 miles long, about 1 in every 60 segments has a permanent ATR (PATR) installed. For all other segments (greater than 95% of a highway system), AADTs must be estimated on the basis of a sample of traffic volumes. These sample volumes are obtained from coverage counts – traffic counts collected during short periods of time (*i.e.*, limited parts of a year) to guarantee adequate geographic coverage for all segments of interest. Therefore, the accuracy of AADT estimates from a sample of traffic volumes would be a major concern in AADT estimation programs.

Early procedures of estimating AADT from samples fell into two general categories (Shelton, 1936). One was to use a systematic sampling procedure to take a number of traffic counts at different times on the segment considered. For example, the Bureau of Public Roads (BPR) used the coverage of 16 8-hour counts taken 24 days apart through the year to estimate AADT in most of its traffic volume surveys before World War II. The second general early method for estimating AADT was the "short-count method", which was the rudiment of the current AADT estimation method. This method depends largely on the successful establishment of temporal traffic patterns and the selection of the length of the short counting duration.

Collecting traffic counts is the premise of AADT estimation. Traffic count collection efforts date back to the early 1900s when there were no automatic mechanical trafficrecording devices. Traffic counts at that time were taken manually. Observers recorded volume data with tally marks on prepared field data sheets for light volumes or with mechanical hand counters for heavier volumes (Pignataro, 1973). For example, the State Roads Commission of Maryland began taking traffic counts manually over its entire state highway systems in 1917 (Johnson, 1929).

Manual counts result in relatively high costs and are subject to the limitations of human factors, generally precluding 24-hour continuous counts. However, manual counts possess some advantages (*e.g.*, obtaining vehicle classification and turning movement data) over automatic counts in urban areas (Pignataro, 1973). Therefore, before and even after the advent of ATRs, there were a number of research studies focusing on how to reduce the manual counting durations and identify the most cost-effective manual counting hours for obtaining average daily traffic volumes with desired precision. To address this issue, it was first necessary to recognize the temporal traffic patterns over different time scales – hours of the day, days of the week, and months of the year. The temporal traffic patterns of repetitive volumes and variation in these volumes through time were observed at different locations from early manual counts.

Johnson (1929) presented an early analysis of monthly variation of traffic. He analyzed traffic counts that were taken manually one day per month from 6 am to 6 pm over the state highway system in three areas (Baltimore, Frederick, and Salisbury) of Maryland for the years 1917 to 1920, and 1926 to 1928. Average monthly factors of each area for each three-year period were estimated. Johnson claimed that the difference of

monthly traffic patterns between the two three-year periods might be caused by the improvement of vehicles and road surfaces during bad weather.

There were also several unpublished studies on temporal traffic patterns. For example, according to Merrill (1934), Ohio was attempting to obtain traffic factors for both daily and seasonal variations to replace those that were produced in 1925. Ohio researchers had located 72 counting stations on the state highway system in Ohio and classified the stations into six groups based on their seasonal variations. These groups were determined according to land use patterns: industrial primary, industrial secondary, industrial tertiary, agricultural primary, agricultural secondary, and agricultural tertiary. Such groups would be the predecessors of the seasonal factor groups commonly used afterwards.

Shelton (1934), an analyst of BPR, presented preliminary results of a study on the 1933 traffic records of the Holland Tunnel connecting NJ and NYC. A detailed follow-up study was presented two years later (Shelton, 1936). He compared the accuracy of using 1-hour and 8-hour observations in estimating average daily traffic volumes based on the known traffic volumes of the Holland Tunnel and the George Washington Bridge (also connecting NJ and NYC). The variability of the 1-hour and 8-hour volumes was investigated. Since the observations were scattered systematically throughout the year, Shelton claimed that any regular temporal variations (*e.g.*, day-of-week, and month-of-year) might cancel out by averaging over enough number of observations. Detailed

expansion of 1-hour or 8-hour observations to an average daily volume was not mentioned.

Of special interest to the work presented in later chapters, Shelton found that a number of independent observations in short durations could lead to a better average traffic estimate than fewer observations taken over longer durations.

Cherniack (1936) presented methods of estimating traffic volumes that accounted for systematic temporal traffic patterns. Based on the vehicular traffic records of toll crossings scattered over the United States and Europe, he believed that traffic samples taken in certain hours of the day, days of the week, and months of the year could result in an average traffic volume with less dispersion compared to the traffic samples taken at random through the year. Three types of traffic temporal pattern cycles were studied: namely, monthly cycle within a year, daily cycle within a week, and hourly cycle within a day. He made several important observations that can still be considered guidelines for current practice: (1) May and October were found to be the months most representative of average daily traffic volume for a year; (2) weekday traffic volumes did not reflect weekend traffic, and weekend and holiday traffic must be given special consideration when estimating average daily traffic volumes; (3) traffic volumes on Wednesdays were usually a good average of the five weekdays; (4) hourly traffic patterns differed widely between weekdays and Sundays (the Saturday pattern largely depended on whether the Saturday half-holiday or the five-day week was adopted in the community at that time);

(5) hourly traffic patterns of any highway facility differed slightly from season to season, but remained stable year by year.

Subsequently, Shelton investigated the dispersion of highway traffic by time periods for the Holland Tunnel and the George Washington Bridge (Shelton, 1937), two stations in farm areas in Iowa (Shelton, 1938), and nine stations in Michigan (Shelton, 1939). He found that the temporal variations of traffic might vary greatly among different sites. An important contribution of Shelton's work was his approach to ensuring the integrity of data. This data integrity was violated in the following 40 years when data imputation became popular (BPR, 1965). Data imputation was criticized until the early 1990s (Albright, 1991, and AASHTO, 1992).

The predominant studies of the 1930s on estimating AADT focused on averaging the short duration manual counts distributed either randomly or systematically throughout the year considered. The usefulness of the methodologies and findings, however, later diminished because of the widespread implementation of mechanical ATRs.

Around 1940 mechanical ATRs began to replace the intensive manual counts and quickly established their value for economically counting vehicles (Petroff, 1946). The advent of mechanical ATRs responded to the need for large-scale traffic counting operations, especially in rural areas (Petroff and Blensly, 1954). Petroff (1956) estimated that 90 percent of the cost of an ATR traffic count was attributed to the wages of the person who installed and removed ATRs, and the associated travel costs. Correspondingly,

it became cost effective to operate ATRs at the same location for at least 24 hours, or in multiples of 24 hours (*e.g.*, 48 or 72 hours). Since then, it was popular to estimate AADT from short counts taken over periods of 24 hours or multiples thereof.

With the widespread use of ATRs, the research emphasis shifted to estimating AADT from mechanical short duration counts for a large-scale geographic area. Traffic data from segments equipped with PATRs facilitated the research. Petroff (1946) summarized the immediate results from one such study. In this study, 20 nationwide PATR-equipped segments on low-volume (less than 300 vpd) roads were selected to form two groups, northern and southern. Weekday traffic counts of 24-, 48-, and 72-hour duration from 1937 to 1940 collected at these PATR-equipped segments were studied. All volumes on Saturdays, Sundays, and important holidays were omitted. The accuracy of estimating average weekday traffic for each month by daily averaging traffic volumes of the three different durations was measured in terms of coefficients of variation (CVs) – standard deviation divided by mean. The empirical CVs ranged from 8.38% to 32.73%, depending on the length of duration, month, group, and to some extent, volume. It was found that marginal accuracy increased more when increasing the count duration from 24 to 48 hours than when increasing the duration from 48 to 72 hours.

Since there would be some errors in converting the average weekday traffic for each month to an AADT estimate by a corresponding monthly factor, Petroff's CVs given above could be considered lower bounds for the CVs of AADT estimates developed from the weekday traffic volumes of 24-, 48-, and 72-hr durations. Petroff appears to be the first to imply a probability distribution of AADT estimation errors due to spatial and temporal sampling. His interpretation required two assumptions: the data used in the study were representative of the larger population, and the distribution of the errors was normal or approximately normal.

By the end of 1947, the State of Ohio supplemented Petroff's findings and produced a figure showing CVs as a function of annual average weekday traffic volumes for different lengths of counting periods (Figure 1 in Petroff and Blensly, 1954). The figure showed that the relative error of volume estimates from any length of count durations decreases with increasing volume at a much greater rate when the traffic volumes are less than 500 ADT, and then approximately converges to a constant when traffic volumes become much greater than 500 ADT. Although no methodology for producing the data graphed in the figure was found, the figure itself was used for a long time. For example, it appeared in the Traffic Monitoring Guide as recently as 1985 (see page 3-3-3 of FHWA, 1985).

A series of follow-up studies formed the framework of the first published manual for traffic counting procedure by the BPR in 1963. Petroff and Blensly (1954) introduced the first application of the newly improved BPR method in Oregon in 1951 for a rural road system of 8245 miles. The improved BPR method consisted of two steps: first averaging weekday coverage counts to yield an estimate of average weekday traffic volumes for the month; then expanding the averaged weekday estimate to an AADT estimate by an appropriate month-of-year factor that represents the ratio of AADT to the average weekday traffic volume of the month. Petroff and Blensly believed that the sampling error involved in the first step had been quantified in Petroff's previous work (1946). In the second step, Petroff and Blensly described that the segments with coverage counts were first assigned to different factor groups based on geographical locations and/or professional judgments, and then control counts (equally distributed over 18 to 20 locations in one group) were used to develop the appropriate factors for coverage counts in the same group. The control counts were taken several times a year, for periods of time ranging from 24 hours to several weeks. They claimed that the BPR method allowed a determinable and controllable increment of error in the second step, and that the error was additive with the sampling error in the first step. A maximum 10-percent range of relative variations in factors within each month was suggested for each group. They asserted that all estimates except a few would have relative errors within ± 20 percent when using this method, though there was no way of determining the size of the error for the count at a single location. Finally, they recommended PATR installations on segments that exhibited monthly variation patterns most similar to the mean of the control area so that the seasonal factors could be obtained in a more efficient way.

Petroff (1956) presented a framework for using statistical methods to estimate AADT from 24- or 48-hr coverage counts. He showed that investigating PATR-equipped segments in several states resulted in CVs of AADT estimates ranging from 8.3% to 12.6% for 24-hr counts, and from 7.2% to 12.1% for 48-hr counts. He defended the 10-percent range of relative variations in factors within each month for each factor group and found that the most cost-effective control count was approximately 1-week duration, equally spaced 6 times throughout the year. He finally indicated that the AADT estimates on low-volume roads (less than 500 vehicles per day) became rapidly less accurate, and a CV ranging from 20% to 25% might be expected.

The BPR method originally focused on AADT estimation on rural roads. Petroff and Kancler (1958) extended this method to urban roads in Tennessee. Most of their results were comparable to the results of another study conducted by Darrell *et al.* (1958). It is worth mentioning that they used data from more than 30 PATR-equipped segments to simulate manual counts in urban areas. They showed that weekday manual counts of 4-hr duration during most daytime periods were able to produce satisfactory estimates of 24-hr weekday traffic volumes by appropriate expansion factors. They claimed that possible refinements of seasonal factor grouping might decrease the error of AADT estimates based on 24-hr counts, but by only about 1%.

In parallel, the Highway Research Board Committee on Urban Volume Characteristics was establishing urban traffic volume characteristics for the varying needs and interests of cities and states (Adams, 1955). The Committee proposed five-minutecluster sampling for determining urban traffic volumes. In this method, one person consecutively took very short (*e.g.*, 5 minutes) manual counts at four to six close counting stations (within a few minutes walk distance) per hour in CBDs, and repeated every hour for 12 hours. This cluster-count method was believed to be efficient and economical for reliably estimating AADT in urban areas.

Based on Petroff *et al.*'s work, which covered the applications in over 30 states, the BPR issued the first *Guide for Traffic Volume Counting Manual* in 1963, and shortly followed with the 2nd edition in 1965. The procedures provided in the guide (BPR, 1965) were divided into two main categories: one for rural highways, and one for urban roads and streets. The guide proposed that the coverage counting on all roads of interest should be made in a cycle of length ranging from one year up to five years.

For rural highways with AADT greater than 500, the AADT estimation procedure consisted of three major steps: 1) grouping PATR-equipped segments according to similar monthly factors; 2) assigning road segments to the groups (with the aid of seasonal control counts if available); 3) locating and operating short-duration coverage counts. The guide stated that the BPR method should result in AADT estimates with a CV not exceeding 10%. However, the coverage count durations required to obtain this 10% CV boundary were not clearly stated. Since few PATRs would be installed on the rural roads bearing less than 500 vehicles per day, seasonal control and coverage counts formed the traffic counting programs on roads with AADT between 25 and 500. The guide also

mentioned that much longer coverage count durations might be necessary to produce a desirable degree of accuracy in AADT estimation for rural roads with AADT less than 25.

For urban roads and streets, the guide indicated that the normal traffic volumes on weekdays could be considered the same as the AADT without the application of adjustment factors. The guide stated that the resulting CV of AADT estimates from a normal 24-hr weekday coverage count was 10%, which satisfied the majority of the cities. The guide claimed that applying monthly adjustment factors as proposed for rural highways could reduce the CV to about 7%. Manual counts of six-minute duration repeated every hour for eight hours were also proposed for urban roads carrying AADT greater 2000. This type of manual count was believed to result in a CV approximately equal to 12%. The possibility of using origin-destination surveys and traffic assignment was also mentioned for very low volume urban areas, but no details were provided.

There was a full section in the BPR guide concerning editing and smoothing data, especially data collected by ATRs. The guide suggested that all abnormal counts (*e.g.*, counts differing by 30% or more from the record of the same location for the previous year) should be omitted. Regression techniques were also introduced to impute and smooth data based on historical data. The practice of data imputation and smoothing was common, although it violated the concept of "Truth-in-Data". It might sometimes be necessary to edit data because of measurement errors introduced by ATRs, but one was

cautioned to be aware that studies based on edited data would overestimate the accuracy in AADT estimation by the BPR method.

Drusch (1966) evaluated the BPR method in Missouri. He found that PATR-equipped segments could be grouped better by averaging monthly adjustment factors over several consecutive years. He recommended a 7-day coverage count program for the Missouri State Highway Department, where each coverage location would be counted once a year between March and November.

Around the same time, Bodle (1967) specified three sources of error when using the BPR method to estimate AADT: 1) the monthly factor at a coverage-count location would not exactly equal the group mean; 2) the coverage count would differ from the average weekday traffic of the month; and 3) the coverage count may have been assigned to a wrong factor group. He explicitly formulated the CV of an AADT estimate (CV_{aadt}) as a function of the CV of coverage counts (CV_x) and the CV of the monthly adjustment factors (CV_f). Assuming the two variables are uncorrelated, he derived $CV_{aadt} = \sqrt{CV_x^2 + CV_f^2}$. He analyzed the data from 386 PATR-equipped segments in five states. The results strongly indicated that 24-hr counts taken Monday through Friday would not generally produce AADT estimates with CV_{aadt} less than 10%. However, the results showed that 24-hr counts taken Monday through Thursday only would result in CV_x less than 10%. The study also indicated that the common practice of excluding the winter months from coverage counts did not markedly decrease the error in AADT estimates.

In the 1960s and 1970s, the BPR method became common practice among state highway agencies. In 1970 the Bureau of Public Roads was replaced by the Federal Highway Administration (FHWA). FHWA continued many BPR programs and issued the *Guide for Traffic Volume Counting Manual* (FHWA, 1970). In 1975, FHWA published the *Guide to Urban Traffic Volume Counting* or (*Urban Guide*)(FHWA, 1975), which complemented the 1970 manual. The *Urban Guide* summarized most research that had been considered to date and provided the following broad guidelines:

"Urban traffic follows daily and hourly variation patterns which are generally consistent and often predictable. Urban traffic volume patterns exhibit relatively little weekday and seasonal variation. The percent of total traffic occurring in any given period is approximately the same along any route.

The more counts, even though of very short duration, the greater the reliability of the resulting estimates. Similarly, the heavier the traffic volume at a particular location, the greater the reliability of a given sample.

The distribution of counts throughout a day is more significant than the total time during which the traffic is counted. The number of separate and independent observations is more important than the number of hours of each observation.

•••

Low-volume roads and streets exhibit a higher day-to-day relative variation in traffic flow than high-volume streets and expressways. Five-day clusters of weekday traffic counts reduce the amount of variation. The reduction, however, is more than offset by the costs of obtaining the counts." (pp. 19, 20, and 30 of FHWA, 1975)

Table 4 in the *Urban Guide* provided suggested CVs for weekday traffic volumes of 1-day, 2-day, 3-day, and 5-day durations, as a function of the daily traffic volume. Therefore, one could use these values to determine the approximate sample size needed for a given precision in AADT estimation. Short counts of duration much less than 24 hours (*e.g.*, 5 or 10 minutes, and six or eight hours) were also discussed in the *Urban Guide*. The Poisson distribution was used as an a-priori approximation to derive the relationship between the variation of minute counts and the magnitude of hourly volume. We will use the Poisson distribution in a similar manner later in this work.

The costs of manual and automatic counting programs were quantified for the first time in the *Urban Guide*. Based on the 1974 experiences in operating and maintaining traffic counters in urban areas, typical costs of automatic counts were listed: a single-location single-day count would cost about \$15.00, a 48-hour count would cost about \$20.00, and a five-day count would cost about \$35.00. The decreasing marginal cost might be because the ATR only needed to be installed and picked up once for any length of counting duration. The figures are all in 1974 dollars. The comprehensive estimated costs for automatic counting programs were given in Table 18 (pp.82) of the *Urban Guide*.

Up to the early 1980s, different statewide traffic counting programs had been established for AADT estimation, since each state had its own needs, priorities, budgets, and geographic and organizational constraints (FHWA, 1985), Specifically, three types of traffic counts – continuous, control, and coverage – were most popular in the practice.

Continuous counts are taken 24 hours a day, 365 days a year on a small portion of segments by PATRs. Continuous counts are the backbone of State traffic counting programs (FHWA, 1985). These counts can lead to a direct calculation of AADT. However, the main purpose of continuous count programs is to provide a cost-effective approach for the development of temporal adjustment factors that are used for converting coverage counts to AADT estimates.

Control counts, also called seasonal counts, are taken several times a year, for periods of time ranging from 24 hours to several weeks. These counts can also be used to estimate AADTs, but their main purpose is to provide a seasonal assignment linkage for coverage counts.

Coverage counts are short-duration counts, for periods of time ranging from several hours to several days. The purpose of these short-duration counts is to guarantee adequate geographic coverage for all roads under the jurisdiction of the state highway authority, providing point-specific AADT information. The coverage counts are converted to AADT estimates by applying factors obtained from either continuous or control counts. This
conversion is based on the premise that similar traffic temporal variation patterns exist for like facilities in a given area.

2.3 Recent Practice and Research in AADT Estimation

Since the 1980s, there has been renewed research interest in AADT estimation. The interest was motivated in part by the counting requirements of the Highway Performance Monitoring System (HPMS), which was originally developed in 1978. Most of the new studies in AADT estimation could be divided into two categories: (1) evaluation and improvement of the traditional methodology, and (2) new methodologies. FHWA also continued its efforts to provide improved methodologies for monitoring the use of America's highways, and periodically updated the *Traffic Monitoring Guide (TMG)*. The first edition of the *TMG* was published in 1985, and the most recent edition was published in 2001.

Several studies investigated the accuracy of AADT estimates from short-duration coverage counts. Sharma (1983) investigated the error of AADT estimates from short-duration counts through the analysis of Alberta's PATR data. He proposed a formula for estimating AADT from a traffic count of duration less than 1 day:

Estimated AADT = short-duration volume count \times H \times D \times S (2.3.1) where H, D, and S represent corresponding hourly, day-of-week and seasonal factors, respectively. He found that short-duration counts with a midpoint at 3 or 4 pm would lead to the most accurate AADT estimates. He identified three main sources of error in estimating AADT by using Equation (2.3.1): error in hourly factor, error in daily factor and error in seasonal factor. He claimed that "the magnitude of error due to the hourly factor is generally expected to be a function of the duration and schedule of a particular short survey... any variation in the duration and schedule of a short-period count will not affect the errors due to the daily (day-of-week) factor and the seasonal factors." He finally concluded that the relative errors in AADT estimation were expected to be less for the roads that carry large volumes of traffic, and the errors were not likely to decrease significantly with further increase in traffic volume beyond a certain critical range.

Albright (1991) provided a historical review of traffic volume estimation. In the review, he challenged data imputation commonly used in traffic counting practice, and cast doubts on the reported ranges of errors of AADT estimates resulting from the traditional method. The concept of "Truth-in-Data" was subsequently introduced in the "AASHTO Guidelines for Traffic Data Programs" (FHWA, 1992).

Sharma *et al.* (1996) investigated the effects of various factors on AADT estimation errors from a short duration coverage count. He studied the data from a large number of PATR-equipped segments in Minnesota and found that the errors were much more sensitive to the factor group assignment than to the duration of counts. Davis (1997) presented a review concerning the accuracy of AADT estimates. He pointed out that using incorrect seasonal or day-of-week factors would produce substantial increases in estimation errors.

Hu *et al.* (1998) presented a study on the accuracy of AADT estimate from a 24-hour count. In the study, AADT was estimated from each available 24-hr count for each PATR-equipped segment by using the corresponding averaged monthly and day-of-week factors for the group the PATR-equipped segment belongs to. The estimated precision in terms of CVs ranged from 5.7% to 15.7%, across the 20 studied PATR-equipped segments.

As noted above, the accuracy of AADT estimation from short-duration counts largely relies on the correct assignment of the coverage counts to factor groups. There are a number of studies concerning seasonal factor group establishment and assignment.

Sharma and Werner (1981) proposed an improved method for grouping PATR-equipped segments. This method consisted of two standard procedures: hierarchical grouping and the so-called Scheffe's S-method of multiple group comparisons. The authors claimed that the proposed method was simple, objective, computer-oriented, and statistically credible, compared to existing grouping methods that were subjective and manual in nature. Sharma and Allipuram (1993) developed an index of assignment effectiveness to evaluate the duration and frequency of control (seasonal) counts. The authors concluded that a 1-week count repeated in 4 different months was much more accurate than a 1-week count repeated twice, but repetition more than 4 times would contribute little additional improvement. Sharma and Leng (1994) investigated the problem of determining the duration and timing of a seasonal count given a specified precision. However, during the last decades, control counts became more and more unpopular in the United States and the newest edition of the *TMG* has left out the suggested use of control counts.

Flaherty (1993) investigated the clustering methods for grouping PATR-equipped segments with similar temporal variation patterns, which was recommended by the *TMG* (FHWA, 1985). Data from 28 PATR-equipped segments throughout Arizona were used in this study. Two distinct groups were finally determined. Flaherty claimed that such grouping provided "best" estimates of AADT.

The *TMG* was updated four times during the last two decades, reflecting various developments in AADT estimation. The newest edition of the *TMG* (FHWA, 2001) only recommended two types of traffic counts, continuous and coverage counts, to combine system and point estimation in an efficient manner without use of control or seasonal count programs.

The *TMG* (FHWA, 2001) recommends that factor groups be created by some combination of three techniques: cluster analysis, geographical/functional assignment of roads to groups, and same road factor application. In the process of cluster analysis, a

least-squares minimum distance algorithm is used to determine which PATR-equipped segments have the "most" similar factors. In the process of geographical/functional assignment of roads to groups, factor groups are initialized based on professional experience with traffic patterns, and finalized on the basis of results of analysis. In the process of same road factor application, the factors from a single PATR-equipped segment are assigned to all segments on the same road as the PATR-equipped segment within the influence of the PATR-equipped segment. The boundary of the influence zone is defined by a road junction that causes the traffic volume to change largely.

Different types of temporal adjustment factors can be adopted according to the conditions of different states. The common types are separate month and day-of-week (19 factors – 12 monthly factor and 7 day-of-week factors), combined month and average weekday (24 factors – one weekday and one weekend factors for each month of the year), separate week and day-of-week (59 factors – 52 weekly factors and 7 day-of-week factors), combined month and day-of-week (sectors), combined month and day-of-week (sectors), combined month of the year).

Since few PATRs function without error in any given year, the *TMG* also recommends an approach using the AASHTO method (AASHTO, 1992) to account for missing data when calculating AADT for continuous count locations:

$$AADT = \frac{1}{7} \sum_{i=1}^{7} \left[\frac{1}{12} \sum_{j=1}^{12} \left(\frac{1}{n_{ij}} \sum_{k=1}^{n_{ij}} V_{jik}^{24} \right) \right]$$
(2.3.2)

where V_{ijk}^{24} is the 24-hour traffic volume for the kth time that a 24-hour volume is available on day-of-week *i* and month *j*; and n_{ij} is the number of times that 24-hour volumes are available on day-of-week *i* and month *j*.

The *TMG* (FHWA, 2001) provides arguments in support of the 48-hour recommendation for coverage counts. As stated, the recommendation of a 48-hour counting period is a compromise to maximize the AADT estimation accuracy subject to cost and equipment limitation constraints, given various alternatives. The marginal decrease of estimation error when increasing counting period from 24 hours to 48 hours is much more than that when increasing counting period from 48 hours to 72 hours. The moveable ATRs will not work reliably over longer periods of time, *e.g.*, pneumatic tubes may not last longer than 48 hours without being reset. Another consideration is that 48-hour volumes provide hourly volumes of two days, a comparison of which would assure the data quality and identify the "unusual" circumstances.

Also the *TMG* (FHWA, 2001) recommends a 3-year cycle for coverage counts on the HPMS, and 6-year cycle for coverage counts on all other segments. The 3-year cycle for HPMS segments is determined to meet adequate accuracy in a cost-effective manner. A research study performed by FHWA showed that relative variability of daily volumes, at a vast majority of locations, ranged from 2 to 25 percent while annual growth ranged from 1 to 4 percent. The advantage of using a 3-year cycle instead of an annual cycle is that the annual coverage counting efforts are reduced by a factor of 3. The main consideration for recommending a 6-year cycle for all other coverage counts is to provide a basic count on a periodic basis. However, more frequent counts are recommended for high growth areas.

The *TMG* (FHWA, 2001) recommends the application of growth factors when using traffic counts collected in previous year to estimate contemporary AADT. The growth factors can be developed from either continuous counts or coverage counts. However, as the *TMG* stated, no "best" mechanism for growth factor estimation has been determined according to available research.

The *TMG* (FHWA, 2001), unfortunately, does not provide discussions in depth about the possible range of error of AADT estimates by the recommended methodologies. It is only mentioned that, "For sites with higher levels of variability, if estimates of annual average daily traffic volumes are desired with better than 10 percent precision, a minimum of 48 hours must be counted. For sites with little traffic variability, a 24-hour count may be sufficient" (pp. 3-12 of FHWA, 2001).

In addition to the traditional estimation methods, several research studies have been conducted that propose new approaches for estimating AADT from short coverage counts.

Phillips and Blake (1980) proposed a pooled estimate of AADT in the case where two short duration counts were available. For a short count of length less than one day, they used a method similar to the traditional one: expanding the short count to a daily volume estimate by a mean daily expansion factor (called *E*-factor) for the group, and then converting the daily estimate to an AADT estimate by a mean monthly factor (called M-factor) for the group. The variances of E- and M-factors were formulated for a particular road type, day type and month. By assuming that the two factors were normally distributed and independent with each other, they derived a formula to calculate the variance in the AADT estimates. They claimed that there were two ways of improving the accuracy in AADT estimates: one was to increase the length of short count; the other was to repeat the count on another day in the year. The latter way raised the issue of combining two short counts. They proposed a combined estimate by assigning weights to the AADT estimates obtained from the two counts. They indicated that the weights should be inversely proportional to the variance of the individual estimate, so that greater weight was given to the more accurate estimate. The weights summed up to 1. In the empirical study, however, the authors only focused on combining two counts collected in the same month. The results indicated that either May or June is the best month to collect counts in the U.K.

Claiming the difficulty in determining the weights used in Phillips and Blake's method, Erhunmwunsee (1991) proposed a multiple regression method for estimating AADT from two or more short-duration counts for periods of less than 24 hours. The regression method had the form

$$\hat{Y}_{i} = \alpha + \sum_{m=1}^{12} \beta_{m} X_{im,j} , \qquad (2.3.3)$$

where \hat{Y}_i was the estimated AADT at a highway segment *i*; α was the constant coefficient; and β_m was the coefficient for a specific short duration count $X_{im,j}$ on day *j* in month *m*. Erhunmwunsee believed that this method could provide a way to combine the short duration counts from different months to estimate AADT.

Data from PATR-equipped segments were used to estimate the regression coefficients. Although the method was established for multiple counts from different count durations, Erhunmwunsee only considered the case where at most two counts were available to estimate the AADT in his numerical study. He claimed that the results showed that regression method worked better than the traditional method where expansion factors were used, but failed to provide the results. Most of his findings were consistent with previous research, such as Sharma (1983) and Phillips and Blake (1980). He failed to interpret the meaning of the regression coefficients in his method. These coefficients should not be considered a combination of the temporal expansion factors and the corresponding weights for different months. Possible high correlations of the coefficients were not discussed. However, he made that point that estimating AADT from short counts would save time and money in the long run, and was useful when limited staff was available.

Aldrin (1998) developed a basic curve method for estimating AADT from short duration counts of length ranging from a few hours to days. The method was based on regression techniques. Hourly volume was the dependent variable, the logarithm of which was expressed as a linear combination of a set of basic curves (*i.e.*, functions). The set of basic curves were developed based on data from PATR-equipped segments and assumed common for all highway segments. The coefficients of the basic curves could be estimated for the location of interest, based on the short duration counts available on the segment. According to the amount of data available, one could select the number of basic curves required in the model. With the estimated model (the chosen basic curves and corresponding estimated coefficients), all unobserved hourly volumes were estimated, and summed up with the observed hourly volumes to get the estimate of AADT. The author conducted a study based on data from 32 PATR-equipped segments. He claimed that the basic curve method reduced the error of AADT estimates to 7.2% compared to the traditional method that produced an error of 9.0%, averaged across the short duration counts of length randomly selected between two hours and two weeks. Unfortunately, the detailed procedure used and comparison results were not provided, and the type of error quantified in the study was not clearly stated. Except for the four basic curves (functions) related to growth trend, seasonal pattern, special days, and day-of-week patterns, all other basic curves appear less interpretable. More-refined basic curves appear to reflect specific

variation patterns on the PATR-equipped segments that would not be necessarily common for all other segments.

Davis and Guan (1996) proposed a Bayesian assignment of coverage count locations to factor groups and the corresponding Bayes estimate of the AADT (called mean daily traffic in their paper). They expressed the daily traffic volumes as a lognormal linear regression model. Since the noise terms appeared autocorrelated in 7 days, a multiplicative autoregressive (MR) model was estimated based on the data from PATR-equipped segments. The authors claimed that all PATR-equipped segments showed similar autocorrelation patterns. The coefficients of MR model were assumed common for segments in the same factor groups. However, these autocorrelation patterns were likely caused by the implicit use of "averaged" day-of-week factors in the model. The authors showed that at least 14 daily volumes were required to reliably assign the coverage counts to the correct factor group. They also proposed a Bayes estimate of AADT by using the posterior probability of group assignment. The estimate performed credibly with 14-day volumes, but the 14-day volumes had to be distributed throughout the year, in an "optimal" pattern that was obtained by a "trail-and-error" method. It would seem very difficult to implement such an "optimal" sample design on coverage count locations.

The authors also claimed that the optimal counts made during March and July were the most informative, as well as those made on Sundays. If one carefully checks the data used in their study, one would find that the reason for the "optimal" sample design was because that the differences between the March and July factors were the greatest across different factor groups they considered, and the differences between Sunday and weekday factors were also the greatest. Such "optimal" sample design for coverage counts could likely lead to reliable group assignment by using simple methods rather than the complicated model proposed. For example, Sharma *et al.* (2001) claimed that a seasonal count consisting of two one-week counts made in different months could provide reliable assignment of a site to a factor group.

Yang and Davis (2002) later extended the Bayesian method for classified AADT estimation. Similar comments can be made for this work. It is noteworthy that the above method used a Bayesian approach. It is similar to that proposed here in that it updates prior beliefs with new information. However, they addressed a problem with additional complexity because of the involved factor group assignment and autocorrelation in daily volumes. Also, our attention is on incorporating very short duration counts in AADT estimation, whereas they needed at least 14-day counts to get a reliable AADT estimate.

Sharma *et al.* (1999) proposed a neural network approach for estimating AADT from 48-hour coverage counts. The neural network did not need the establishment of factor groups and assignment of coverage count sites to the factor group. The authors carried out a detailed comparison between the neural network approach and the traditional method by using data from 63 PATR-equipped segments in Minnesota. The results showed that the traditional method produced better AADT estimates than the neural network approach for a single 48-hour coverage count when it was correctly assigned to a factor group. The error for two 48-hour counts using the neural network approach was comparable to that for only a single 48-hour count using the traditional method. The two 48-hour counts used by the neural network approach were from different months. The authors pointed out that the error could be much higher for coverage count locations assigned incorrectly to factor groups in practice when using the traditional method, and that their method did not depend on group assignment. However, they did not mention the increased possibility of correct group assignment in the traditional method when two 48-hour counts are available. Sharma *et al.* (2001) extended the neural network approach to estimate AADT on low-volume roads. Similarly, two 48-hour counts were necessary to produce reliable AADT estimates by the neural network approach.

There are also several studies on estimating AADT for roads by data other than traffic counts. Most of them adopted a multiple regression method, where the dependent variable was the AADT and the explanatory variables would consist of various factors that contributed to AADT on the road. For example, Mohamad *et al.* (1998) used four explanatory variables to estimate AADT – county population, location type (urban/rural), access to other roads, and total arterial mileage in a county. They achieved an R^2 of 0.75 using data from 89 sites in Indiana. Xie *et al.* (1999) included more variables in the

regression model – function classification, number of lanes, area type, auto ownership, presence of nonstate roads nearby, and service employment. The model had an adjusted R^2 of 0.60 for nonstate roads in Broward County, Florida. Zhao and Chung (2001) continued Xie *et al.*'s study. They extended the study by using a larger data set that includes all state roads in Broward County. The function classification was updated, and land-use and accessibility variables were analyzed more extensively. They presented four models with different combinations of explanatory variables considered, which achieved R^2 of 0.66 to 0.82. As Zhao and Chung pointed out, the method of estimating AADT by not using traffic counts might not be adequate to meet the need of engineering design and planning, but could be used for tasks that do not need a high level of accuracy.

2.4 Traffic Data Collection from Airborne and Space-based Platforms

The previous two sections were concerned with AADT estimation using traffic volumes collected manually or by ATRs on the ground, except for the methods that did not use traffic data at all. A substantial effort is required for State DOTs to collect traffic volumes across statewide highway systems from the ground. Airborne and space-based platforms are available for collecting traffic data by taking photos and imagery over the facilities of interest. They have some advantages over ground-based traffic data collection. For example, these platforms can easily access remote highway segments where it is difficult or costly to send ground crews to collect traffic data. Also, sensors on these

platforms are "off-the-road" so that traffic will not be disrupted and ground crews will not be exposed to danger.

The image-based data collected from these platforms could potentially be used for AADT estimation, and such use will be the main issue of the work presented in the following chapters. Before going into more detail about the use of imagery for AADT estimation, we will briefly review the literature on traffic data collection through air photography, satellite imagery, and other airborne sensors.

Aerial photography has been recognized as a useful tool for collecting traffic data for almost 80 years. One of the earliest applications of aerial photography in traffic studies dates back to an aerial survey of highway traffic conducted by the State Roads Commission of Maryland in 1927 (Johnson, 1928). In this survey, 127 photos were taken approximately 13 seconds apart from an aircraft at an altitude of about 3600 feet between 4:30 and 5:00 P.M.. The photos covered nearly 29 miles of highway between the Baltimore City line and the District of Columbia line. Each photo had approximately 50% overlap with the succeeding one. In addition, traffic counts in both directions were made manually at four stations on the road from 3:00 to 6:00 P.M. on the same day. One of the four stations was located at the last 2 miles of the road near the District of Columbia line, where more traffic was observed due to a nearby junction of a major highway. The traffic volumes observed at the other stations were very uniform during each of the three hours, ranging from 739 to 940 vehicles per hour. Six spot cars were used during the aerial survey to drive with the traffic, and the speeds of cars were recorded by observers in the cars. The recorded speeds ranged from 20 to 33 mph.

From the 127 photos, the number of vehicles was obtained for each quarter mile of the 29 miles. The average number of vehicles per quarter mile was about 7, which led to about 800 vehicles per hour at a speed of 25 to 30 mph. The result of this study showed a satisfactory comparison between ground- and air-based traffic flow data collection. It was noted that the distribution of the traffic was very uneven along the road, from one vehicle in some quarter mile sections to 15 or 20 vehicles in others. The study also investigated the relationship between velocity, spacing, and flow by using the photos.

Among the early proponents of aerial photography in traffic studies, Greenshields proposed the photographic method of studying traffic behavior (Greenshields, 1933), and discussed the potential use of air photos in traffic analysis based on experiments in which air photos were taken from a helicopter and a blimp (Greenshields, 1947). He believed that all implementation difficulties would be overshadowed by the complete and accurate traffic record within the area studied by air photos.

Forbes and Reiss (1952) reported the study of driver behavior based on 35millimeter time-lapse air photos. They concluded that air photos provided a practicable way of collecting information on driver behavior that would be very difficult to collect in other ways. Wohl (1959) presented a collection of traffic volumes and vehicle speeds by Sonne stereo continuous-strip photography. The development of formulas for determining vehicle speeds and volumes were included. He indicated that both volume and speed data could be collected simultaneously over a large area during a short period of time.

Wagner and May (1963) reported a successful study of obtaining time-lapse aerial photographs for studying traffic operation along a considerable length of freeway in Los Angeles. They thought that aerial photography would be useful in observing traffic operation problems that were isolated at locations. They developed a new method of presenting traffic flow data – the time-distance-density contour map, which could be used to determine the origin, duration, and extent of congestion.

Rice (1963) described the use of aerial photography for traffic operations by the Washington D.C. Department of Highways and Traffic. He pointed out that an aircraft was an effective means of reaching trouble spots quickly, while congestion would make it difficult to reach the spots on the ground. He concluded that aerial photography was a powerful tool for research and study of traffic problems not otherwise approachable.

Jordan (1963) described Project Sky Count, which was initiated by the Port of New York Authority. In two of the studies, photographs were taken every 5 seconds from a light airplane at altitudes of 6000 to 10000 feet over the study areas. Vehicle speeds were determined through three sequential photos (*i.e.*, based on vehicle movement during a 10-second period). Traffic speed was calculated by averaging all vehicle speeds. Traffic density was determined by dividing the number of vehicles by the segment length. The product of density and speed led to the traffic volumes. Jordan concluded that aerial photography could serve as a basic medium for highway traffic data collection with great potential.

McCasland (1965) presented a comparison of time-lapse and continuous-strip aerial photography for obtaining traffic data. The study showed that time-lapse photography was more costly for measuring density than continuous-strip photography, but could obtain the acceleration-deceleration data that could not be obtained by continuous-strip photography. However, the continuous-strip photography provided more coverage at the same cost.

Treiterer and Taylor (1966) developed an aerial photogrammetric method appropriate for the test and validation of most of the present traffic flow theories at that time. The method consisted of placing a vehicle in the traffic stream and following it by a helicopter from which air photos were taken at fixed intervals of time. Vehicle spacings and speeds were determined for a platoon of vehicles at short intervals of time, and accurate vehicle trajectories were then obtained. Their analysis showed that the standard error in the velocity determinations was no more than 1.0 mph and the standard error of the spacing determination was no more than 1.0 ft. However, they indicated that the major bottleneck of this technique was the economic feasibility of extracting data from the photos. A thorough description of this study can be found in the dissertation of Taylor (1965). Treiterer (1972) later presented the use of the aerial camera in the studies of intersection operations. He claimed that a camera mounted on a helicopter was the best alternative for studying traffic progression through several signalized intersections.

Buhr *et al.* (1967) discussed gap acceptance in freeway operations. Time-lapse photography was used to investigate various factors affecting freeway merging operations. The resultant time-distance-density contour map effectively illustrated the operation of ramps studied on a continuous basis in both time and space.

Syrakis and Platt (1969) reported the application of color aerial photographs in a parking study in Stark County, Ohio. An aircraft took photographs over the cities of Canton, Massillon, and North Canton every 15 minutes between 10am and 6pm on an average business day. At the same time, data were collected on the ground in an 8-block area in downtown Canton. The comparison indicated that aerial photography achieved a 72% savings in cost and an 85% savings in time.

Cyra (1971) reported a Wisconsin DOT study on traffic data collection through aerial photography. Vehicle accumulation (density) and speed data collected by aerial photographic techniques were compared to manually recorded volumes and speeds during peak periods. The study showed that the aerial method was less costly when collecting vehicle accumulation data but much more costly when collecting speed data than the manual method. Munjal and Hsu (1973) used aerial photographic data to evaluate three lane changing models (linear, nonlinear, and stochastic), and Gazis and Szeto (1974) used aerial photographic data to test a Kalman filtering methodology for the estimation of traffic densities on multilane roadways.

Makigami *et al.* (1985) used aerial photographs to investigate the causes of traffic congestion on an 800-m section of the Hanshin Expressway, Japan. Traffic in the study section was photographed every 5 seconds for one hour with a 35 mm still camera from a helicopter hovering at an altitude of 750 m. To check the accuracy of the data derived from the aerial photography, traffic was recorded by a video camera placed on the roadside. Based on the theory of 3-D representation of traffic flow, 3-minute traffic volume counts were computed from aerial photographs. The difference between these computed volumes and volumes from the video screen was at most \pm 1 vehicle per 3 minutes. The study successfully identified that traffic congestion was caused by merging traffic from another route and small disturbances (such as sudden lane changes) in the stable saturated flow.

With the end of the cold war, high-resolution satellite imagery became available to the civilian community. Merry *et al.* (1995) investigated the feasibility of traffic data collection using satellite imagery. They used 0.4-0.7 μ m resolution aerial photography to simulate the performance of three spatial resolutions, 1.0 m, 2.1 m and 4.2 m. The result indicated that 1.0 m resolution was satisfactory for identifying two types of vehicles –

larger trucks and small vehicles. They also found that a 1-m resolution satellite could cover about 1% of the highways in the continental U.S. per day. Satellites with this resolution were still not available to civilian community when this study was conducted. This changed with the launch of IKONOS in September 1999, a satellite with a near polar, sun-synchronous orbit equipped with a 1 m panchromatic sensor and a 4 m multi-spectral sensor.

In June 1998, U.S. Congress passed the Transportation Efficiency Act for the 21st Century (TEA-21), which called for research in remote sensing and spatial information technologies. This led to new interest in collecting and analyzing traffic data from airborne and space-based platforms. For example, Angel and Hickman (2002) presented a method for measuring freeway level of service from airborne imagery. Toth *et al.* (2003) developed a method for using airborne LiDAR (light detection and ranging), which uses the same principle as RADAR, data to identify and classify vehicles into three categories: passenger cars, multi-purpose vehicles, and trucks. Toth and Brzezinska (2004) discussed technical aspects of using airborne photography, high-resolution satellite imagery and LiDAR data to support traffic flow monitoring and management. They believed that the great amount of LiDAR data and imagery collected for routine aerial mapping over highways provides an opportunity for obtaining traffic flow data at practically no extra effort.

There is also an increasing international interest in this area. Ernst *et al.* (2003) described the LUMOS project conducted in Germany. This project conceptualized an airborne wide area traffic monitoring system. To validate the complete concept and its implementation, a flight was performed in Berlin in May 2003. Preliminary results of the comparison between the airborne and conventional traffic measurements were presented, and areas of future work were suggested.

Schreuder *et al.* (2003) presented a traffic data collection from aerial imagery in the Netherlands. Sequences of digital aerial images were obtained from a helicopter over different motorway sites. They developed software to automatically detect and track vehicles from the digital images, and determine individual vehicle trajectories. They achieved a 98% success rate when weather conditions were reasonable. Stilla *et al.* (2004) described the possibilities of vehicle extraction by three different airborne sensors: visual, thermal infrared, and active synthetic aperture radar.

In this section, we focused primarily on still imagery, which provides discrete time coverage rather than continuous time coverage. There are also several research studies involved with aerial videos. The reader can find a review of such studies in Angel (2002).

2.5 Imagery and AADT

In this section, we introduce the use of traffic data collected by sensors on airborne or space-based platforms for estimating AADT. For simplicity, we will refer to air photos, satellite images, and LiDAR data as imagery, and the traffic data extracted from them will be called image-based data.

In most of the previous studies, the image-based data were considered to represent traffic flow data during relatively short periods of time. In the work presented here, the image-based data will be used for estimating traffic volumes over a long-term period (*e.g.*, a year).

Imagery covers a segment at an instant in time, which can be considered a snapshot, while traditional ground-based traffic data (*e.g.*, coverage counts) are collected at a specific point during a relatively much longer period. Compared to ground-based ATRs, airborne or space-based platforms can access a segment of interest more quickly and easily. Also, keeping in mind that much imagery taken for other purposes might likely cover highway segments, traffic information in imagery will already exist in archives or databases in many cases. Many research studies discussed in Section 2.2 indicated that more observations during short periods at different time points would provide better AADT estimation than fewer observations taken over longer period. Therefore, image-based data offer the potential to complement traditional ground-based data in AADT estimation with additional, useful information.

McCord *et al.* (2003b) estimated AADTs from several air photos and satellite images for several highway segments in Ohio. A sequential approach of five steps was proposed to produce the AADT estimate from a single image: 1) obtain the vehicle density from the image; 2) covert the density to a volume, called "t-minute volume", that would be obtained on the ground during a short period ("t minutes"); 3) expand the t-minute volume to an hourly volume; 4) expand the hourly volume to a daily volume; 5) de-seasonalize the daily volume to produce an image-based annual average daily volume. Mathematically, the sequential approach was expressed as

$$AADT^{img} = N^{img} / L \times Us[N^{img} / L] \times F^{H}(h; f) \times 24 \times F^{MD}(m, d; f), \qquad (2.5.1)$$

where $AADT^{img}$ is the image-based AADT estimate; N^{img} is the number of vehicles appearing on the image of the highway segment considered; L is the length of the highway segment covered by the image; $Us[N^{img}/L]$ is the space-mean speed on the segment considered when the traffic density is N^{img}/L ; $F^{H}(h;f)$ is the corresponding hourly adjustment factor and $F^{MD}(m,d;f)$ is the corresponding monthly and day-of-week factor for segments in highway functional class f to which the segment considered belongs. Equation (2.5.1) is slightly simpler than Equation (4b) in McCord *et al.* (2003b), which divided the vehicles into trucks and passenger cars.

The authors indicated that one satellite image or air photo provides information equivalent to traffic counts of very short duration. They, therefore, raised the possibility that the information would be too noisy to be of use in estimating AADT. They used the equivalent of Equation (2.5.1) to produce AADT estimates from air photos or satellite images of 14 different Ohio Interstate segments, where vehicles were observed under free-flow conditions. The errors in the 14 image-based AADT estimates were quantified by calculating the differences between the image-based AADT estimates and the corresponding AADT estimates developed from traditional, ground-based data. (The authors noted that the ground-based AADT estimates were also subject to errors.)

An earlier simulation study (McCord *et al.*, 2002b) showed the overall error level in AADT estimation for segments across a highway network could be decreased when using imagery while also decreasing the ground-based sampling efforts. The simulation assumed error levels in image-based AADT estimates that were much larger than the error level found in the above study (McCord *et al.*, 2003b). Therefore, the results of McCord *et al.* (2003b) indicated that the quantified errors in the 14 image-based AADT estimates seemed small enough that imagery could be considered as a useful resource of information for AADT estimation. The factors affecting the quality of image-based estimates were also discussed.

McCord *et al.* (2003b) did not provide a way to integrate the image-based data with traditional data collected on the ground in the process of AADT estimation for a specific segment. This issue will be the primary topic of the work presented in later chapters.

McCord *et al.* (2003b) suggested that obtaining air photos or satellite images only for the purpose of AADT estimation might not be cost-effective at present. However, they believed that these images could be used for AADT estimation at only marginal cost if a sufficiently large and reliable market could be established for the data. This belief is consistent with Paine and Kiser's claim (2003) that it was increasingly important for all agencies, whether county, state, federal, or private, to maximize the use of aerial photography and related imagery.

The market for images from airborne and space-based platforms is actually much broader than the area of traffic data collection. For example (Paine and Kiser, 2003), topographic mapping was one of the earliest applications of aerial photography. Other uses of these images include land-use planning, area and corridor studies, and highway planning and design. In addition, aerial photography and satellite imagery are used in the fields of astronomy, architecture, archaeology, geomorphology, oceanography, hydrology and water resources, conservation, ecology, mineralogy, and national defense. Geographic information systems also have a large connection with the use of aerial photography and satellite imagery.

Therefore, any image containing highway segments, originally obtained for other purposes, might be used in AADT estimation. Below, we will briefly introduce possible sources of the imagery.

Existing aerial photography could cover nearly all the United States and Canada (Wolf and Dewitt, 2000). The Earth Resources Observation System (EROS) Data Center in Sioux Falls, South Dakota, has archived millions of air photos and satellite images (Wolf and Dewitt, 2000). Their archived coverage includes photos taken through the National Aerial Photography Program (NAPP) and the National High Altitude Photography program (NHAP). The EROS Data Center also archives photos taken by the

U.S. Geological Survey (for its topographic mapping projects) as well as other federal agencies such as the National Aeronautics and Space Administration (NASA), the Bureau of Reclamation, the Environmental Protection Agency (EPA), and the U.S. Army Corps of Engineers. The U.S. Department of Agriculture is another resource for aerial photography.

The Departments of Transportation (DOT) of most states also obtain air photo coverage for use in highway planning and design. Thus their coverage typically follows state and federal highways. For example, the Aerial Engineering Office of Ohio DOT flies aerial survey missions about 150 times per year, and takes about 2000~3000 air photos containing highway segments each year (private communication with John Ray, Director of the Aerial Engineering Office of Ohio DOT).

New imagery coverage is obtained routinely or upon demand. Many countries also have repeated periodic coverage. In addition, there are a number of survey companies that possess aircraft and have the ability to obtain imagery (Angel, 2002). Two companies – PAR Government Systems and Skycomp – have been engaged in aerial surveying of traffic flow (Murray, 2002).

In summary, imagery is a potentially useful source for traffic flow data collection, and has been used in traffic flow studies, traffic operations, and many other areas in transportation. The imagery appears to have potential for AADT estimation, especially if imagery originally obtained for other purposes is used. However, a theoretically justified and operational means of combining the image-based data with traditional ground-based data in AADT estimation is lacking. Also, the "benefits" in terms of improved accuracy in AADT estimation would need to be documented. This study proposes a methodology to address the above issues. In the next chapter, the proposed approach for AADT estimation will be developed.

CHAPTER 3

ESTIMATING AADT BASED ON BAYESIAN ANALYSIS

It is impracticable to collect traffic volumes 365 days a year on every segment of a highway system. AADT values for most segments are estimated based on traffic volume samples obtained from counts taken within some period much shorter than a year. Uncertainty in the AADT values based on this sampling procedure is therefore unavoidable. The profession has recognized the need to address the uncertainty in AADTs. The American Association of State Highway and Transportation Officials (*AASHTO*) suggests that the "precision and bias" of AADT estimates be assessed when providing them to users (*AASHTO*, 1992). Investigation of the accuracy of AADT estimates from traffic volume samples can be traced back many decades, as discussed in Chapter 2.

One of the best languages to handle the uncertainty is probability (Berger, 1985). This chapter develops an approach for AADT estimation based on Bayesian analysis. The approach addresses the uncertainty in AADT by producing a probability distribution of the AADT on a given segment instead of a point estimate.

Because of the limitation on funds, equipment and manpower, state DOTs usually take several years to cover all roadway segments with short-duration traffic counts (*i.e.*,

coverage counts) in their jurisdiction. For example, the *Traffic Monitoring Guide* or *TMG* (FHWA, 2001) recommends a 6-year sampling cycle for covering the entire roadway system in a state. That is, a segment would actually be updated with new count data once every six years. The counts from previous years could provide useful information in determining the AADT for the contemporary year. However, there would be more uncertainty in AADT when the estimation is based on older counts than when it is based on more recent counts. To estimate AADT in the contemporary year (the year when AADT is to be estimated), the *TMG* recommends inflating count data collected in the most recent year by appropriate growth factors for segments without count data collected in the contemporary year. The *TMG* does not mention the use of previous data when count data are available in the contemporary year.

No studies have been found that investigate the potential combination of previous and contemporary data. An approach based on Bayesian analysis provides an efficient way to combine prior information with newly observed data. The previous data form prior information about the AADT in the contemporary year, and the Bayesian approach provides a mean of updating the prior information based on the recently collected traffic counts.

Detailed discussion on general Bayesian analysis can be found elsewhere (*e.g.*, Berger, 1985). In the following sections, the Bayesian approach is developed for AADT estimation, and the components of the approach are discussed. The algorithms used to

implement the approach are then presented. Finally, the choice of point estimates based on the distribution is discussed.

3.1 Framework of Bayesian AADT Estimation

In general, the AADT on a segment of interest can be considered a population parameter (mean) of the daily traffic volumes on the segment in a year. For ease in exposition it will be assumed that there are 365 days in the year. Any traffic count taken within a time period shorter than a year would provide a traffic volume sample taken from this population. Probabilistic models can be used to characterize this traffic volume sample conditional on AADT. Let $f(V_i^T|AADT_i)$ denote such a probabilistic model, *i.e.*, the probability distribution of a traffic volume V_i^T on segment *i* during a time period *T*, conditional on the segment's true AADT being $AADT_i$. When observing a sample described by a parameter, probabilistic models allow the deduction of an inference about the parameter from these observations (Berger, 1985).

A prior distribution π for $AADT_i$ can be established based on prior information. The prior information might consist of traffic volume samples collected several years previously and volumes on other segments that allow one to estimate growth in traffic. After collecting new traffic counts, the Bayesian approach allows one to update the prior AADT distribution for a highway segment to a posterior distribution, conditional on newly observed traffic counts. This update is realized through Bayes's Therom (Berger, 1985). Mathematically, it can be written as

$$\pi(AADT_{i}(y)|V_{i}^{T}(y)) = \frac{f(V_{i}^{T}(y)|AADT_{i}(y))\pi(AADT_{i}(y))}{m(V_{i}^{T}(y))},$$
(3.1.1a)

where $\pi(AADT_i(y)|V_i^T(y))$ is the posterior distribution of the AADT in year *y* on segment *i* - *i.e.*, $AADT_i(y)$ – after the traffic volume $V_i^T(y)$ is obtained from a count taken during time duration *T* in year *y* on segment *i*; $f(V_i^T(y)|AADT_i(y))$ is defined above; $\pi(AADT_i(y))$ is the prior distribution of $AADT_i(y)$ before any traffic data are collected in year *y*; and $m(V_i^T(y))$ is the marginal distribution of $V_i^T(y)$. The marginal distribution can be obtained by the following integration:

$$m(V_i^T(y)) = \int_{AADT_i(y)} f(V_i^T(y) | AADT_i(y)) \pi(AADT_i(y)) dAADT_i(y).$$
(3.1.1b)

There could be more than one traffic count taken for the same segment during different periods in a year. In such a case, the update can be implemented in two ways. One way is to consider a joint distribution of all the traffic volumes obtained from the counts conditional on AADT (or reduce the situation through a sufficient statistic). Equation (3.1.1a) can then be expressed as

$$\pi(AADT_i(y) | \underline{V}_i^T(y)) = \frac{f(\underline{V}_i^T(y) | AADT_i(y))\pi(AADT_i(y))}{m(\underline{V}_i^T(y))}, \qquad (3.1.2a)$$

where $\underline{V}_{i}^{T}(y)$ represents the vector of *n* traffic volumes, $\{V_{i}^{TI}(y)_{1}, V_{i}^{T2}(y)_{2}, ..., V_{i}^{Tn}(y)_{n}\}$, obtained from counts in year *y* on the segment *i*, each of which might cover a different duration $T_{j}, j = 1, ...n; f(\underline{V}_{i}^{T}(y)|AADT_{i}(y))$ is the joint distribution of $\underline{V}_{i}^{T}(y)$ conditional on $AADT_{i}(y)$; and $m(\underline{V}_{i}^{T}(y))$ is the marginal joint distribution of $\underline{V}_{i}^{T}(y)$, obtained by

$$m(\underline{V}_i^T(y)) = \int_{AADT_i(y)} f(\underline{V}_i^T(y) \mid AADT_i(y)) \pi(AADT_i(y)) dAADT_i(y).$$
(3.1.2b)

A sequential analysis can also be adopted when more than one traffic volume is available. Using sequential analysis, the prior distribution of AADT can be updated to a "interim-posterior" distribution based on the first traffic volume collected. Then, when another new traffic volume is obtained, the interim-posterior distribution can be updated to another interim-posterior distribution. That is, the update becomes a dynamic procedure and is implemented at any moment when a new traffic volume is obtained. The posterior distribution is eventually obtained at the time when the last traffic volume is obtained in the year of interest.

If all traffic volumes collected during different short-term periods in a year can be assumed mutually independent, updating through a joint distribution or sequential analysis would result in the same posterior distribution (Berger, 1985). Most of the temporal variations in the traffic volumes during different time periods can be explained by the temporal adjustment factors used in practice. No other factors would be expected to introduce correlation between traffic volumes collected far apart in the year, so this independence assumption appears good in such a case. However, traffic volumes collected in consecutive time periods, such as the consecutive days on which the two daily volumes are often collected in present practice, might have some correlation, even after the adjustment of the temporal factors. Such correlation would be difficult to model unless enough data were collected on the segment considered. As a model approximation, the independence assumption will be considered appropriate in this work, and there will therefore be no difference in using either the sequential or joint distribution approach when updating the prior distribution with contemporary traffic counts. The sequential approach is adopted in this work, since it has the advantage of providing an updated AADT estimate as soon as new volume data become available and does not need to address the joint distribution.

In summary, the Bayesian approach for updating the prior distribution based on contemporary traffic counts contains two components: a conditional probability distribution of traffic counts given the AADT $f(\underline{V}_i^T(y)|AADT_i(y)))$, and a prior distribution on the AADT $\pi(AADT_i(y))$. These two components will be discussed in Sections 3.2 and 3.3. The algorithms for implementing the approach will be presented in Section 3.4. Finally, choosing a point estimate based on the posterior distribution will be discussed in Section 3.5.

3.2 Probabilistic Model for Traffic Volumes conditional on AADT

The Bayesian approach is intended to be used when updating a prior AADT distribution for a highway segment to a posterior distribution, conditional on newly observed traffic volumes. (Below we will use count and volume interchangeably except when needed for clarity.) The probability of observing a traffic volume conditional on the AADT plays a key role in this updating process, as shown in Equations (3.1.1).

Various traffic volume samples can be collected by different methods within periods of different lengths. This work only considers the traffic volumes collected within a period much shorter than one year. Specifically, this work will primarily concentrate on the probabilistic models for two types of volumes — 48-hour ground-based traffic volumes and image-based volumes. In current practice, a short-duration traffic volume is collected on the ground usually during a consecutive 48-hour period (see more in Chapter 2). The motivation for this work is the incorporation of traffic information contained in images in AADT estimation. The so-called "image-based" volume for segment i is derived from the number of vehicles on the segment appearing in the image, denoted as N^{img} . The variable N^{img} can be considered equivalent to a traffic volume collected on the ground during a period on the order of minutes (McCord et al., 2003b). Because of the characteristics of the probabilistic models proposed here, the concepts are mainly developed for modeling image-based volumes. The reason will be clarified at the end of this section.

Assuming there is a sample of n image-based counts on a segment i in year y, the joint distribution density function of the n image-based counts conditional on the AADT in year y is denoted as

$$f(\underline{N}_i^{img}(\mathbf{y})|AADT_i(\mathbf{y})), \tag{3.2.1}$$

where $\underline{N}_i^{img}(y)$ is the vector of *n* image-based volumes, $\{N_i^{img}, N_i^{img}, ..., N_i^{img}, ...$

the segment will be omitted below unless required for clarity. Below an image-based volume will be denoted as $N^{img}(l,h,\delta)$, where *l* represents the length of the segment covered in the image, and *h* and δ represent the hour and the day the image is taken.

If the *n* image-based volumes can be assumed to be independent, their joint density would become the product of their individual marginal densities. Since images are often taken for a specific segment far apart during a year, the independence assumption seems appropriate in most cases for this work. Therefore, a method is developed to model the distribution of a single image-based volume conditional on AADT – $f(N^{img}(l,h,\delta)|AADT)$.

The following three additional distributions are introduced to model the distribution $f(N^{img}(l,h,\delta)|AADT)$, which is called "3-stage model" in this work:

- 1) $f(V^{24}(\delta)|AADT)$: the density function of 24-hour daily volume $V^{24}(\delta)$ on day δ when the image is taken, conditional on the AADT of the segment;
- 2) $f(V^{H}(h, \delta) | V^{24}(\delta), AADT)$: the density function of hourly volume $V^{H}(h, \delta)$ in hour *h* of day δ when the image is taken, conditional on the daily volume $V^{24}(\delta)$ on day δ and the AADT on the segment;
- 3) $f(N^{img}(l,h,\delta)|V^{H}(h,\delta), V^{24}(\delta), AADT)$: the density function of image-based volume $N^{img}(l,h,\delta)$, conditional on the hourly volume $V^{H}(h,\delta)$ in hour *h* of day δ , the daily volume $V^{24}(\delta)$ on day δ and the AADT on the segment.

The joint distribution of $\{N^{img}(l,h,\delta), V^{H}(h,\delta), V^{24}(\delta)\}$ conditional on AADT is the
product of the above three distributions. Therefore, the distribution $f(N^{img}(h, \delta)|AADT)$ can be obtained by integrating this joint distribution over $V^{H}(h, \delta)$ and $V^{24}(\delta)$

$$f(N^{img}(l,h,\delta)|AADT) = \iint_{V^{24}(\delta),V^{H}(h,\delta)} f(N^{img}(l,h,\delta),V^{H}(h,\delta),V^{24}(\delta)) | AADT)d[V^{24}(\delta)]d[V^{H}(h,\delta)^{D}]$$

$$= \iint_{V^{24}(\delta),V^{H}(h,\delta)} f(N^{img}(l,h,\delta)|V^{H}(h,\delta),V^{24}(\delta),AADT)f(V^{H}(h,\delta)|V^{24}(\delta),AADT)f(V^{24}(\delta)|AADT)d[V^{H}(h,\delta)]d[V^{24}(\delta)].$$
(3.2.2)

Modeling the three conditional distributions will be discussed next.

The distribution $f(V^{24}(\delta)|AADT)$ reflects the day-to-day variability of 24-hour traffic volumes around the AADT. As seen in Chapter 2, 24-hour daily traffic volumes show systematic monthly and day-of-week variations from the AADT, which can be captured by monthly and day-of-week adjustment factors. However, traffic flows are caused by human travel behavior that cannot be completely captured by monthly and day-of-week adjustment factors, so traffic volumes still show some additional "unexplained" variations. These unexplained variations would contribute most to the error in AADT estimates from short-term traffic counts when the systematic variations are accounted for, by monthly and day-of-week factors, for example. The magnitude of the "unexplained" variation would tend to increase with increased traffic volumes (BPR, 1965). Relative error is a popular measure for comparing the accuracy of the AADT estimates across segments with large differences in AADT values (Petroff, 1956). Relative error also has a

simple relationship with multiplicative error (Jiang *et al.*, 2004). A multiplicative error term is, therefore, introduced to represent the "unexplained" part of the variation in a daily volume from the AADT. This error term will be denoted Noise(D), and defined as the ratio of the de-seasonalized daily volume to the true AADT:

$$Noise(D(\delta)) = V^{24}(\delta) \times F^{M}(m(\delta)) \times F^{D}(d(\delta)) / AADT, \qquad (3.2.3a)$$

where $F^{M}(m(\delta))$ and $F^{D}(d(\delta))$ are the monthly and day-of-week factors, respectively, for the month $m(\delta)$ and day-of-week $d(\delta)$ that are used to de-seasonalize the observed daily volume on the day. Given the monthly and day-of-week factors, a daily volume $V^{24}(\delta)$ can therefore be modeled as a function of *AADT*:

$$V^{24}(\delta) = AADT \times (1/F^{M}(m(\delta))) \times (1/F^{D}(d(\delta))) \times Noise(D(\delta)).$$
(3.2.3b)

In practice, highway segments are grouped into categories or groups considered homogeneous in terms of similar temporal variations (FHWA, 2001). A set of common adjustment factors is applied to all segments in the group. The common adjustment factors are estimated by averaging the factors developed from the PATR-equipped segments in that group. This work proposes to model one distribution of Noise(D) for one homogeneous group through the analysis of the continuous daily volumes collected on PATR-equipped segments in the group. Details are given in Chapter 4. Systematic temporal variations would not be identical for all segments in a homogeneous group. Therefore, the Noise(D) obtained by applying one set of common adjustment factors would result from two types of "unexplained" variations: one representing the "unexplained" temporal variation on a specific segment; the other representing the "unexplained" spatial variation in systematic temporal variation across the segments in the group. After modeling the distribution of Noise(D), the distribution $f(V^{24}(\delta)|AADT)$ can ultimately be modeled for a given segment through Equation (3.2.3b).

The distribution $f(V^{H}(h,\delta)|V^{24}(\delta), AADT)$ reflects the variability of the hourly volume $V^{H}(h,\delta)$ around the average hourly volume in the day. The average hourly volume in day δ is defined as $[V^{24}(\delta)/24]$. On a typical day, knowing $V^{24}(\delta)$ provides enough information to model $V^{H}(h,\delta)$, and knowing the AADT does not provide much more information. Therefore, after excluding the holidays and days with special events or bad weather from the population, it appears reasonable to assume that $V^{H}(h,\delta)$ conditional on $V^{24}(\delta)$ is independent of *AADT*. That is

$$f(V^{H}(h,\delta) | V^{24}(\delta), AADT) = f(V^{H}(h,\delta) | V^{24}(\delta)).$$
(3.2.4)

Similar to what was done with Noise(D), a multiplicative error term is introduced to represent the "unexplained" part of variation in the hourly volume in hour *h* of day δ . Denoted Noise(H), this error term is defined as the ratio of the adjusted hourly volume by the hourly factor to the average hourly volume in the day:

$$Noise(H(h,\delta)) = V^{H}(h,\delta) \times F^{H}(h) / (V^{24}(\delta)/24), \qquad (3.2.5a)$$

where $F^{H}(h)$ is the hourly factor for hour *h*. $F^{H}(h)$ represents the average ratio of the average hourly volume in a day to the hourly volume expected in hour *h* of the day.

Therefore, given the independence in Relation (3.2.4), the hourly volume in hour h can be modeled as

$$V^{H}(h,\delta) = (V^{24}(\delta)/24) \times (1/F^{H}(h)) \times Noise(H(h,\delta)), \qquad (3.2.5b)$$

The hourly factors would also be assumed common across all segments in the same homogeneous group. Again, this work proposes to model *Noise*(*H*) from the analysis of continuous hourly volumes on PATR-equipped segments. Details are given in Chapter 4. After modeling the distribution of *Noise*(*H*), the distribution $f(V^H(h, \delta) | V^{24}(\delta))$ can ultimately be modeled through Equation (3.2.5b).

Since an image-based volume is actually a density measurement of traffic, rather than traffic flow passing a point on the segment, $f(N^{img}(l,h,\delta)|V^H(h,\delta),V^{24}(\delta),AADT)$ is modeled differently than above. Consider an image-based volume $N^{img}(l,h,\delta)$ taken on an imaged segment of length l in hour h of day δ . To develop the distribution of $f(N^{img}(l,h,\delta)|V^H(h,\delta),V^{24}(\delta),AADT)$, we consider the $V^H(h,\delta)$ vehicles using the segment during hour h to be scattered along a hypothetical, long segment at the instant time twhen the image was taken. If traffic in hour h is not increasing towards or decreasing from a more congested state (*i.e.*, if the traffic is non-transitioning) and operating under free-flow conditions (*i.e.*, if the movement of a given vehicle is not generally affected by the movements of other vehicles), the vehicles can be considered to be scattered at random so that equal sections of the hypothetical segment would be equally likely to contain the same number of vehicles, *i.e.*, to be uniformly distributed along the hypothetical segment. Since traffic flow rate can be expressed as a product of traffic density and space-mean speed (FHWA, 2005), we assume, as a modeling approximation, that the length of the hypothetical segment containing the $V^{H}(h,\delta)$ vehicles (more strictly, the same number of vehicles) is equal to the space-mean speed *Us* times one hour. Here, the space-mean speed *Us* is based on the conventional definition (Daganzo, 1997), namely, the average of the $V^{H}(h,\delta)$ vehicles' speeds along the hypothetical long segment at the instant time *t*:

$$Us = \frac{1}{V^{H}(h,\delta)} \sum_{i=1}^{V^{H}(h,\delta)} u_{i}, \qquad (3.2.6)$$

where u_i is the speed of vehicle *i* among the $V^H(h, \delta)$ vehicles along the hypothetical long segment at the instant time *t*.

Note that the hourly volume $V^{H}(h,\delta)$ is observed at a specific point on the segment considered in the hour, while the $V^{H}(h,\delta)$ vehicles distributed along the *Us*-mile-long segment correspond to an observation over space at an instant in time. We do not intend to argue that the vehicles observed at the specific point on the segment considered in the hour are exactly the vehicles distributed along the *Us*-mile-long segment at the instant time *t*. However, the number of vehicles obtained from these two different observations should be similar. Since the traffic is non-transitioning and operating under free-flow conditions, the traffic would be steady, and the image-based volume $N^{img}(l,h,\delta)$ would be independent of the instant time *t*. The image-based volume $N^{img}(l,h,\delta)$ can, then, be considered to be the number of vehicles along an *l*-mile segment taken randomly from the *Us*-mile long segment along which the $V^{H}(h,\delta)$ vehicles are distributed uniformly. Therefore, given $V^{H}(h,\delta)$, *Us* and length *l*, the image-based volume $N^{img}(l,h,\delta)$ would have a binomial distribution:

$$N^{img}(l,h,\delta) | V^{H}(h,\delta) \sim \text{Binomial}(n = V^{H}(h,\delta), p = l / Us).$$
(3.2.7)

In the above analysis, $N^{img}(l,h,\delta)$ results from the hourly volume $V^{H}(h,\delta)$, and there is no mention of daily volume $V^{24}(\delta)$ or *AADT*. Therefore, conditional on $V^{H}(h,\delta)$, the image-based volume can be considered independent of both $V^{24}(\delta)$ and *AADT*. That is

$$f(N^{img}(h,\delta)|V^{H}(h,\delta), V^{24}(\delta), AADT) = f(N^{img}(h,\delta)|V^{H}(h,\delta)).$$
(3.2.8)

The length l of the segment in the image of Distribution (3.2.7) can be determined directly from the image. To incorporate the uncertainty in the space-mean speed Us into Distribution (3.2.7), a normal distribution is introduced to model Us as follows. The normal distribution is commonly used for modeling the distribution of vehicle speeds (McShane *et al.*, 1998). So we model the speed of vehicle *i* in Equation (3.2.6) as

$$u_i \sim N(\bar{u}, \sigma_u^2), \tag{3.2.9}$$

where \bar{u} is the distribution mean and $\sigma_{\rm u}$ is the distribution standard deviation. As a first

approximation, a plausible estimate of \bar{u} could be the speed limit for vehicles in urban areas, and the speed limit plus 5 mph for vehicles in rural areas.

Under free-flow conditions, the speeds of vehicles can be considered mutually independent. Therefore, Us given in Equation (3.2.6) would be a sum of independent normal variables and also follow a normal distribution

$$Us \sim N(\bar{u}, \sigma_{\rm u}^2/V^{\rm H}(h,\delta)). \tag{3.2.10}$$

Several states post different speed limits for passenger cars and trucks on the same highway in their jurisdictions. For example, in Ohio, the speed limit on interstate highways is 55 mph for trucks, and 65 mph for cars. In this case, vehicles can be considered as being generated from two subpopulations – cars and trucks. Let P_k denote the truck proportion in the $V^H(h,\delta)$ vehicles, and \bar{u}_k and \bar{u}_c denote the speed distribution means for trucks and cars, respectively. If it is assumed that the standard deviation of speed distribution remains the same for trucks and cars, the distribution of (3.2.10) would become

$$Us \sim N(P_k \bar{u}_k + (1 - P_k) \bar{u}_c, \, \sigma_u^2 / V^H(h, \delta)).$$
(3.2.11)

Distributions (3.2.10) and (3.2.11) model the variability in the space-mean speed, conditional on the hourly volume $V^{H}(h, \delta)$. Note from Distribution (3.2.11) that the truck proportion P_k is required for this model when considering different speed limits for cars and trucks. Various factors affect truck proportion on a segment, such as time-of-day,

functional class, geographic location (FHWA, 2001). Statistical distributions will be introduced to model the truck percentage in numerical studies presented later.

Given the general distributions of (3.2.3) and (3.2.5), and the binomial distribution of (3.2.7), the distribution $f(N^{img}(l,h,\delta)|AADT)$ cannot be derived analytically through the integration of (3.2.2). A numerical technique, such as Monte Carlo simulation, can be used to derive $f(N^{img}(l,h,\delta)|AADT)$ numerically. The algorithm for the simulation procedure used in this work can be summarized as follows:

Algorithm (3.1)

- 1. Given AADT, $F^{M}(m(\delta))$, $F^{D}(d(\delta))$, $F^{H}(h)$, the length *l* of imaged segment, and distribution models for *Noise*(*D*), *Noise*(*H*), *P_k* and *Us*;
- 2. (a) Generate a *Noise*(*D*) value from the *Noise*(*D*) distribution; (b) use this generated *Noise*(*D*) value in Equation (3.2.3) to produce a daily volume $V^{24}(\delta)$;
- (a) Generate a Noise(H) value from the Noise(H) distribution; (b) use this generated Noise(H) value and the V²⁴(δ) value generated in Step 2 in Equation (3.2.5) to produce an hourly volume V^H(h,δ);
- 4. Generate a truck proportion value from the P_k distribution;
- 5. Generate a space-mean speed value from the Us distribution;
- 6. Use the given length l of the imaged segment from Step 1, the $V^{H}(h, \delta)$ value

generated in Step 3 and the *Us* value generated in Step 4 to generate an image-based volume $N^{img}(l,h,\delta)$ from the binomial distribution of (3.2.7).

7. Repeat Steps 2-6 *M* times to produce *M* values of $N^{img}(l,h,\delta)$.

The set of image-based volumes obtained by the above algorithm represents a numerical approximation of the distribution $f(N^{img}(l,h,\delta)|AADT)$. Also, Steps (2)-(6) of the algorithm can be used to generate random samples of image-based volumes under specific circumstances given in Step (1), which will be used in the data generation part of the numerical study presented later.

The approach described above can also be used to establish probabilistic models of hourly or 24-hour volumes, condition on AADT. For example, the probabilistic model of 24-hour volumes conditional on AADT is the distribution $f(V^{24}(\delta)|AADT)$ introduced above. The probabilistic model of hourly volumes conditional on the AADT, $f(V^{H}(h,\delta)|AADT)$, can be obtained by integrating the product of distributions $f(V^{24}(\delta)|AADT)$ and $f(V^{H}(h,\delta)|V^{24}(\delta))$ over all possible values of $V^{24}(\delta)$. Alternatively, the two noise terms Noise(D) and Noise(H) can be aggregated into one noise term – $Noise(DH) = Noise(D) \times Noise(H)$. Then distributions $f(V^{24}(\delta)|AADT)$ and $f(V^{H}(h,\delta)|V^{24}(\delta))$ can be reduced directly to one distribution $f(V^{H}(h,\delta)|AADT)$, the probabilistic model of hourly volumes conditional on the AADT, by modeling the distribution of the noise term *Noise(DH)*.

A similar approach can also be used to produce the probabilistic model of volumes in intervals of various durations. For example, if one regards a 48-hour traffic volume as two independent 24-hour traffic volumes, the probability of observing the 48-hour traffic volume (*i.e.*, two independent 24-hour traffic volumes) is just the product of probabilities of observing each of the two 24-hour traffic volumes. Otherwise, one might develop a temporal adjustment factor corresponding to the period of the 48-hour volume, and model the distribution of the 48-hour volumes conditional on the AADT. A probabilistic model of volumes in duration of several hours conditional on AADT can be modeled in a similar manner.

3.3 Prior Distribution of AADT

A prior distribution is based on prior information. In this section, two types of prior distributions are developed for a specific segment. The two distributions correspond to two types of prior information – one where there is "no prior information"; the other where the prior information is based on the posterior distribution in the previous year and the corresponding growth factors developed from other segments.

This work considers consecutive application of the Bayesian approach in AADT estimation year by year. For a given segment, it is assumed that there is a beginning year when the AADT estimation was of interest, and a traffic sample was collected for the first time on that segment. (Any year can be considered the "beginning" year when this approach is to be used for AADT estimation, even if traffic samples had been collected before that year and previous AADT estimates produced.) A non-informative prior distribution can be used for the prior distribution of the AADT in this "beginning" year. Non-informative priors are often used when insufficient prior information is available to implement a Bayesian analysis. It is assumed that there would not be available information for this "beginning" year. Various methods could be used to choose a non-informative prior. In this work, a uniform distribution will be used as the non-informative prior for the AADT on a segment of interest. The uniform distribution is one of the earliest adopted non-informative priors in Bayesian analysis (Robert, 1994). It is still commonly used because of its simplicity and reasonableness. The uniform distribution can be denoted

$$\pi(AADT) \sim \text{uniform}(B^L, B^U), \qquad (3.3.1)$$

where B^L and B^U are the distribution lower and upper bounds, respectively. The selection of the bounds will be discussed in the numerical studies later. More thorough analysis of choosing a non-informative prior distribution of AADT could be a subject of future study.

Based on the traffic volumes collected in the beginning year, the uniform prior of (3.3.1) can be updated to a posterior distribution by the Bayesian approach. The posterior distribution in the beginning year could be converted to a prior of the AADT in the

following year by incorporating the uncertainty in traffic growth. Then, this prior can be updated when traffic volumes are collected in this following year or remain the same until the end of this following year if no volumes are collected. That is, except for the beginning year, the prior distribution of the AADT in the following years would be obtained from the posterior distribution in the previous year. Therefore, the second prior distribution considered in this work is based on two types of information: one is the posterior distribution of AADT obtained in the previous year; the other is the distribution of growth factors on other PATR-equipped segments with AADT growth assumed to be similar to the growth of the segment considered.

Consider a specific segment *i*. When the posterior distribution of AADT on segment *i* in year *y* is obtained, the prior distribution of AADT in following year y+1 would be derived from the year *y* posterior distribution and the growth in traffic on segment *i* from year *y* to year y+1. A growth factor *GF* is commonly used to represent the growth, and can be defined as

$$GF_i(y, y+1) = AADT_i(y+1)/AADT_i(y).$$
 (3.3.2a)

Rearranging Equation (3.3.2a) yields

$$AADT_i(y+1) = GF_i(y, y+1) \times AADT_i(y).$$
(3.3.2b)

Given the distributions of $GF_i(y, y+1)$ and $AADT_i(y)$, a distribution of $AADT_i(y+1)$ can be derived from Equation (3.3.2b) numerically.

Knowledge about growth factor $GF_i(y, y+1)$ can be obtained from growth factors of PATR-equipped segments whose growth in traffic is considered representative of AADT growth on segment *i*. Specifically, the growth factor $GF_i(y, y+1)$ for segment *i* could be considered a random variable following the distribution of growth factors across the segments with similar traffic growth patterns. The growth factors obtained from those PATR-equipped segments would produce an empirical distribution for $GF_i(y, y+1)$ and could be used to model this distribution. Therefore, a distribution of $AADT_i(y+1)$ can be produced from the distributions of $GF_i(y, y+1)$ and $AADT_i(y)$. This distribution is considered the prior distribution of AADT in year y+1.

3.4 Calculation of Posterior Distributions

In general, there is no closed-form representation for the probability model of image-based volumes conditional on the AADT. Therefore, a numerical technique, such as Monte Carlo simulation, is used to obtain the numerical posterior distribution.

According to Equation (3.1.1a), a plausible straightforward way to produce the numerical posterior distribution is described as follows: first generate an AADT value from the prior distribution; then, conditional on the generated AADT value, generate a short-term traffic volume from the probabilistic model. After repeating the two steps a large number of times, a numerical joint distribution of AADT and short-term traffic volume is produced. The numerical marginal distribution of AADT, conditional on the

specific observed short-term traffic volume, can be produced from the joint distribution. However, this simple method is not computationally efficient, since most of the generated data are not used in the specific marginal distribution that targets the observed short-term traffic volume. Therefore, a more efficient numerical method is introduced.

According to Equation (3.1.1a), the posterior distribution $\pi(AADT|V^T)$ is proportional to the product of the prior distribution and the conditional distribution, since the denominator of the right side in Equation (3.1.1a) does not depend on the AADT. This relation can be mathematically written as

$$\pi(AADT|V^{T}) \propto \pi(AADT) \times f(V^{T}|AADT).$$
(3.4.1)

The right side of Relation (3.4.1), $\pi(AADT) \times f(V^T|AADT)$, can be regarded as a *pseudo* density function for the posterior distribution $\pi(AADT|V^T)$, where $f(V^T|AADT)$ is just a weight for $\pi(AADT)$. To take advantage of this observation, a large number of AADT values are first generated based on the prior distribution $\pi(AADT)$; then, given the observed volume V^T , the weight for each generated AADT value is evaluated based on the probabilistic function $f(V^T|AADT)$; finally these AADT values are re-sampled with replacement according to their weights. The AADT values resulting from the re-sampling process would be a numerical approximation of the posterior distribution $\pi(AADT|V^T)$ (Robert, 1994). The above is the basic idea of the so-called Sampling-Importance -Resampling method (Robert, 1994), which will be adopted in the work presented here.

volumes and image-based volumes. Therefore, we present below algorithms for "calculating" posterior distributions conditional on a 24-hour volume and an image-based volume, respectively.

Algorithm (3.2) – posterior distribution of AADT conditional on a 24-hour volume

- 1. Given the prior distribution $\pi(AADT)$, the observed 24-hour volume $V^{24}(\delta)$, the corresponding monthly factor $F^{M}(m(\delta))$ and day-of-week factor $F^{D}(d(\delta))$, and the *Noise(D)* distribution.
- 2. Generate *N* values of AADT from the prior $\pi(AADT)$, $AADT_n$, n = 1, 2, ...N.
- 3. Evaluate the weight $\omega(n)$ for each $AADT_n$ as follows
 - (i) Use $V^{24}(\delta)$ and $AADT_n$ in Equation (3.2.3a) to calculate the "observed" Noise(D) value, $Noise(D(\delta))_n$;
 - (ii) Calculate the probability distribution $f(Noise(D(\delta))_n)$ based on the *Noise(D)* distribution;
 - (iii) $\omega(n) = f(Noise(D(\delta))_n).$
- 4. Normalize the weights obtained in Step 3: $\omega'(n) = \alpha(n)/\Sigma_{i=1,..N} \alpha(i), n = 1, 2, ...N$.
- 5. Resample the *N* values of AADT in Step 2 with replacement based on the weights in Step 4 to obtain *N* "new" AADT values, which is the numerical approximation of the posterior distribution $\pi(AADT|V^{24}(\delta))$.

Algorithm (3.3) - posterior distribution of AADT conditional on an image-based volume

- 1. Given the prior distribution $\pi(AADT)$, the observed image-based volume $N^{img}(l,h,\delta)$, the corresponding monthly factor $F^{M}(m(\delta))$, day-of-week factor $F^{D}(d(\delta))$, hourly factor $F^{H}(h)$, the length of imaged segment *l*, and the distribution models for *Noise*(*D*), *Noise*(*H*), P_k and *Us*.
- 2. Generate *N* values of AADT from the prior π (*AADT*), *AADT_n*, *n* = 1, 2, ...N.
- 3. Evaluate the weight $\omega(n)$ for each $AADT_n$ as follows
 - (i) Generate a *Noise*(*D*) values from the *Noise*(*D*) distribution, and then use the generated *Noise*(*D*) values in Equation (3.2.3) with the value $AADT_n$ to produce a daily volumes $V^{24}(\delta)$;
 - (ii) Generate a *Noise*(*H*) value from the *Noise*(*H*) distribution, and then use this generated *Noise*(*H*) value and the $V^{24}(\delta)$ value generated in Step 3(i) in Equation (3.2.5) to produce an hourly volume $V^{H}(h, \delta)$;
 - (iii) Generate a truck proportion value P_k from the P_k distribution;
 - (iv) Generate a space-mean speed value Us from the Us distribution;
 - (v) Calculate $f(N^{img}(l,h,\delta)|V^{H}(h,\delta))$, the probability of observing the image-based volume $N^{img}(l,h,\delta)$ conditional on the generated hourly volume $V^{H}(h,\delta)$ in Step 3(ii) based on the binomial distribution of (3.2.7), with the length 1 of the imaged segment, and the generated Us in Step 3(iv);
 - (vi) Repeat Steps 3(i)-(v) nn times to produce nn probability values for $AADT_n$,

$$f(N^{img}(l,h,\delta)|V^{H}(h,\delta))_{n,i}, i = 1, 2, ...,nn;$$

(vii)
$$\omega(n) = \sum_{i=1,2...nn} (f(N^{img}(l,h,\delta)|V^{H}(h,\delta))_{n,i}) /nn;$$

- 4. Normalize the weights obtained in Step 3: $\omega'(n) = \omega(n)/\Sigma_{i=1,..N} \omega(i), n = 1, 2, ...N$.
- 5. Resample the *N* values of AADT in Step 2 with replacement based on the weights in Step 4 to obtain *N* "new" AADT values, which is the numerical approximation of the posterior distribution $\pi(AADT | f(N^{img}(l,h,\delta)))$.

The above algorithms appear more complicated than the "straightforward" method mentioned at the beginning of this section. However, the generation part of the algorithms targets directly the observed traffic counts, and no wasteful data are generated. The size of the approximated distribution (*i.e.*, N) is more controllable, compared to the "straightforward" method. Therefore, the proposed algorithms are more computationally efficient. One common problem for the numerical approximation of the posterior distribution is that the approximated distribution may degenerate to a few points after many updating procedures of the proposed approach (Doucet *et al.*, 2001). This degeneration is unlikely to happen for AADT estimation, unless hundreds of traffic counts are available for a few years, which can rarely happen. In case that the algorithm of calculating the posterior distribution. One can refer to Doucet *et al.* (2001) for details.

3.5 Point Estimates of AADT based on the Posterior Distribution

When the prior distribution $\pi(AADT)$ is available, the posterior distribution $\pi(AADT|V^T)$ can be produced based on the traffic count V^T with distribution $f(V^T|AADT)$ using the procedure presented in Section 3.4. This posterior distribution is the direct result of the Bayesian approach for AADT estimation, integrating simultaneously prior information and information brought by the observed traffic count V^T . The posterior distribution $\pi(AADT|V^T)$ can be used as a probability distribution to describe the properties of the uncertainty in the AADT. For example, the variance of the posterior distribution would reflect a measure of the variability in the AADT.

In practice, a point AADT estimate would likely be required for many applications. In such cases, a point estimate of AADT could be selected based on $\pi(AADT|V^T)$. For example, the mean, the median, or the mode of the posterior distribution could be chosen as the point estimate. An explicit approach for choosing a point estimate is based on decision analysis. A loss function (or disutility function) is introduced as a criterion. Let $L(AADT, AADT^{Est})$ denote the loss function, where $AADT^{Est}$ denotes the chosen point estimate. Given the posterior distribution $\pi(AADT|V^T)$, the expected loss of choosing $AADT^{Est}$ can be calculated as

$$E[L(AADT, AADT^{Est})] = \int_{AADT} L(AADT, AADT^{Est}) \pi(AADT | V^{T}) dAADT.$$
(3.5.1)

In decision analysis, one would choose an estimate that minimizes the posterior expected loss shown in Equation (3.5.1). Therefore, the best point estimate would depend

Loss Function	Chosen Estimates			
$ AADT - AADT^{Est} $	Median of $\pi(AADT V)$			
$(AADT - AADT^{Est})^2$	Mean of $\pi(AADT V)$			
$\begin{cases} K_0(AADT - AADT^{Est}), & if \ AADT - AADT^{Est} \ge 0\\ K_1(AADT^{Est} - AADT), & if \ AADT - AADT^{Est} < 0 \end{cases}$	$K_0/(K_0 + K_1)$ -fractile of $\pi(AADT V)$			
$\omega(AADT)(AADT - AADT^{Est})^2$	$\frac{E^{\pi(AADT V)}[AADT\omega(AADT)]}{E^{\pi(AADT V)}[\omega(AADT)]}$			

Note: K_0 and K_1 are two constants reflecting the seriousness of underestimating and overestimating the AADT, respectively; $\omega(AADT)$ is the weight on the squared errors as a function of the AADT; and $E^{\pi(*)}[.]$ represents the expectation of [.] based on the distribution $\pi(*)$.

Table 3.1: Choice of Estimates associated with Common Loss Functions.

on the choice of the loss function. Different loss functions result in different estimates. In Table 1, some common loss functions are listed associated with the corresponding "best" estimates in terms of minimizing the loss. One can refer to Berger (1985) for an extensive discussion of loss functions.

In summary, an approach for AADT estimation has been developed based on Bayesian analysis. This approach produces a posterior distribution of AADT rather than a point estimate only, which is produced by the traditional approach. If a point estimate is desired, it can be obtained from the posterior distribution. Therefore, the Bayesian approach can be compared to the traditional approach in terms of "which leads to a better point estimate". The numerical study of such a comparison will be presented in the following chapters.

CHAPTER 4

EVALUATION OF 3-STAGE MODEL

In Chapter 3, a Bayesian approach for AADT estimation was proposed. As noted, the probability distribution of traffic counts conditional on the AADT is one of the two primary components in this approach. This conditional distribution is the means for taking advantage of information brought by newly collected traffic counts. A 3-stage model was developed to establish the probability distribution of image-based counts conditional on the AADT. In this chapter, parameters describing the 3-stage model are discussed. Also, the reasonableness of the 3-stage model is evaluated by comparing against other potential models using 22 empirical image-based counts. Finally, the distribution of image-based counts conditional on the AADT resulting from the 3-stage model is investigated as a function of traffic volume and imaged segment length and simplified to a lognormal distribution. Such a simplification would make it easier to implement the Bayesian approach and save computing time.

4.1 Parameters in the 3-stage Model

The conditional distribution of short-term traffic count given the AADT of the segment considered can mathematically be expressed as the probability mass function (pmf) of an observed short-term traffic count conditional on the AADT. This section will focus on the pmf of image-based counts conditional on the AADT. Recall that the proposed conditional distribution is modeled through the 3-stage model, which introduces three additional probability distributions. The 3-stage model describes the "unexplained" (random) variation in traffic volumes at three stages: daily, hourly, and one-image short-term variations. As discussed in Chapter 3, distributions of Noise(D) and Noise(H)account for the "random" variations at the first two stages. At the third stage, a binomial distribution is used to capture the randomness of the image-based counts given the hourly volume. In addition to the hourly volume V^{H} , the binomial distribution incorporates two more variables: space-mean speed Us, and truck proportion P_k in the hour the image is taken. If the length of segment included in the image is fixed, all systematic temporal variation patterns are assumed known, and the AADT is given, the distribution of image-based counts on a given segment during a specific time period would depend on Noise(D), Noise(H), Us, and P_k . Therefore, the distribution characteristics of the four variables would directly specify the pmf of image-based counts conditional on the segment's AADT value. In this section, the distributions of the four variables are developed from either empirical data or assumptions.

4.1.1 *Noise*(*D*)

Noise(D) represents the "unexplained" variation of a daily volume from the AADT, after the adjustment of monthly and day-of-week factors. Equation (3.2.3a) is repeated as Equation (4.1.1) for convenience

$$Noise(D(\delta)) = [V^{24}(\delta) \times F^{M}(m(\delta)) \times F^{D}(d(\delta))] / AADT.$$
(4.1.1)

Traffic data collected on PATR-equipped segments can be analyzed to investigate the distribution of Noise(D). A large number of daily volumes collected on the PATR-equipped segments could lead to very good AADT estimates for these segments. In a homogeneous group, traffic data collected on PATR-equipped segments could provide a good understanding of the systematic temporal variation pattern in traffic, so monthly and day-of-week (as well as hourly factors) can be estimated for each PATR-equipped segment. Averaging these factors across these segments would yield the estimates of "common" temporal adjustment factors for all segments in the homogeneous group. Given the AADT and temporal adjustment factors, each daily volume on a PATR-equipped segment leads to an "observed" Noise(D) value by using Equation (4.1.1). Pooling all available "observed" Noise(D) values from a homogeneous group would produce an empirical distribution of Noise(D) for that group.

Analysis of traffic data collected on 24 PATR-equipped segments in Ohio indicates that Noise(D) for segments in the same group can be reasonably modeled by a lognormal

distribution with mean equal to one (Appendix A). The resulting mathematical expression of the lognormal distribution of Noise(D) is

$$\text{Ln}[Noise(D)] \sim N(-\sigma_D^2/2, \sigma_D^2),$$
 (4.1.2)

where Ln[Noise(D)] is the natural logarithm of Noise(D), which according to (4.1.2) follows a normal distribution, and σ_D is the standard deviation of the normal distribution; In (4.1.2), the use of $-\sigma_D^2/2$ as the mean of the normal distribution ensures that the distribution mean of Noise(D) is equal to one (McCord *et al.*, 2000). Therefore, σ_D is the only parameter specifying the distribution of Noise(D). Analysis indicates that σ_D might vary with different groups of homogeneous segments and with different years. It would be straightforward to estimate σ_D for a homogeneous group with the traffic data collected on PATR-equipped segments every year.

Based on the data from 24 Ohio PATR-equipped segments across four functional classes (01, 02, 11, and 12) (Appendix A), a default σ_D will be set to 0.12 for Section 4.2 and the work presented in Chapter 5. Sensitivity analysis to σ_D will also be conducted in Chapter 5.

4.1.2 *Noise*(*H*)

Noise(H) represents the "unexplained" variation of an hourly volume from the average hourly volume in the same day, after the adjustment of hourly factors. Equation (3.2.5a) is repeated as Equation (4.1.3) for convenience

$$Noise(H(h,\delta)) = [V^{H}(h,\delta) \times F^{H}(h)] / [V^{24}(\delta)/24].$$

$$(4.1.3)$$

Traffic data collected on PATR-equipped segments are usually stored as consecutive hourly volumes, which can be used for the investigation of Noise(H). Specifically, these hourly volumes can be used to produce daily volumes for days in which all hourly volumes are available. Similar to the monthly and day-of-week factors, a set of "common" hourly factors can be estimated for a homogeneous group (Appendix A). When the hourly factors are given, Equation (4.1.3) can be used to produce "observed" Noise(H) values for each hour-of-day from each available pair of hourly and daily volumes (*i.e.*, $V^{H}(h, \delta)$ and $V^{24}(\delta)$). A relatively large difference in the hourly traffic patterns between weekdays and weekends was noted in the empirical study conducted on 24 Ohio segments (Appendix A). Therefore, it is proposed that at least two sets of hourly factors be used when implementing this method – one set for weekdays and another for weekends. Since images would be taken during the daytime, analysis in this work will be limited to daytime hours. In addition, the images obtained in McCord et al. (2002a), which will be used to evaluate the reasonableness of the proposed distribution of image-based counts conditional on the AADT, were all taken between 10:00am and 1:00pm (which will be called "mid-day" hours). Therefore, this work only focuses on Noise(H) corresponding to mid-day hours. Determining the empirical results for images obtained in other than mid-day hours is a topic for future study.

The analysis of hourly volumes obtained on the Ohio PATR-equipped segment indicates that Noise(H) can also be reasonably modeled by a lognormal distribution with mean equal to one (Appendix A). Similar to Noise(D), the mathematical expression of the lognormal distribution of Noise(H) is

$$Ln[Noise(H)] \sim N(-\sigma_H^2/2, \sigma_H^2),$$
 (4.1.4)

Once again the use of $-\sigma_H^2/2$ as the mean of the normal distribution ensures that the mean of *Noise*(*H*) is equal to one. It is proposed that σ_H be estimated every year for each homogeneous group of segments with the traffic data collected on PATR-equipped segments.

Based on the data from 24 Ohio PATR-equipped segments across four functional classes (01, 02, 11, and 12) (Appendix A), a default σ_H will be set to 0.10 for Section 4.2 and the work presented in Chapter 5. Sensitivity analysis to σ_H will also be conducted in Chapter 5.

4.1.3 Space-mean speed distribution

As noted in Chapter 3, space-mean speed Us is an input to the 3-stage model. The distribution of Us was modeled in Chapter 3 by the average of the independent normally distributed speeds of the $V^{H}(h, \delta)$ vehicles passing the segment considered during the hour the image-based count was obtained. Since all images used in the following evaluation were taken in Ohio, where trucks and cars have different speed limits on the same

highway, distribution (3.2.11) is applied to model the space-mean speed *Us*. This distribution is rewritten here for convenience

$$Us \sim N(P_k \bar{u}_k + (1 - P_k) \bar{u}_c, \ \sigma_u^2 / V^H(h, \delta)).$$
(4.1.5)

In this work, the mean truck and car speeds, \bar{u}_k and \bar{u}_c , will be set to truck and car speed limits on the segment for urban areas, and truck and car speed limits on the segment plus 5 mph for rural areas. A large value of σ_u means high variation in the space-mean speed, which would lead to large variability in image-based counts. Intuitively, the benefit when adding more "noisy" data would not be greater than that when adding less "noisy" data. As a conservative estimate, therefore, a default value of σ_u is set to 10 mph, which is higher than all values presented in McShane *et al.* (1998).

4.1.4 Truck proportion distribution

When considering different speed limits for cars and trucks on the same segment, P_k becomes an additional input to the 3-stage model. Ohio DOT provides a published truck proportion for each hour of the day by functional classification (*i.e.*, a functional class as a homogeneous group) (Ohio DOT's website, accessed July 2005). These proportions are calculated using short-term vehicle classification counts taken statewide.

In this work, three different assumptions on truck proportion P_k distribution will be considered:

- 1) P_k equals the published truck proportion p during the hour considered with probability one;
- 2) P_k is normally distributed with mean p and standard deviation σ_k , truncated at 0 and 1. For the numerical study, the standard deviation σ_k is set to 0.1 as a first approximation;
- 3) P_k is uniformly distributed with mean p. The range of the distribution depends on the value of p. If p is less than 0.5, the range will be set as (0, 2p). If p is greater than 0.5, the range will be set as (2p-1, 1).

Although we propose these three assumptions, we will see later that the distribution of image-based counts conditional on the AADT resulting from the 3-stage model does not appear to be sensitive to the distribution of P_k .

4.2 Evaluation of the 3-stage Model using 22 Image-based Counts

In this section, the probability of observing the 22 image-based counts given in McCord *et al.* (2002a) will be simulated to investigate the reasonableness of the 3-stage model compared to two other models. As noted in the previous chapter, the 3-stage model cannot provide a closed form for the distribution of image-based counts conditional on the AADT. However, the distribution of image-based counts conditional on the AADT can be simulated numerically by Algorithm (3.1) described in Chapter 3. That is, the pmf of image-based counts conditional on AADT can be approximated by simulation. Based

on the simulation, one possible way to determine the pmf value of an observed image-based count n^{img} given the AADT is to use

$$f(N^{img} = n^{img} | AADT) = \frac{\text{Number of simulated } N^{img} \text{ values} = n^{img}}{M}$$
(4.2.1)

where *M* is, as given in Algorithm (3.1), the total number of simulated N^{img} values. The pmf value obtained by Equation (4.2.1) is subject to sampling error in the simulation. However, such error would be negligible when the size of the simulated distribution (*i.e.*, *M*) is sufficiently large compared to the possible range of N^{img} for a given AADT. An alternative is to use appropriately weighted average of the pmf values obtained by Equation (4.2.1) to obtain a smooth approximation to the pmf value (Hastie *et al.*, 2001). In the following work, the former method of calculating the pmf value (*i.e.*, Equation (4.2.1) will be adopted for simplicity.

The characteristics of the 22 image-based counts used in the study are presented in Table 4.1. All 22 image-based counts were taken on Ohio highway segments. The first column of Table 4.1 gives the observation number of the 22 image-based counts. The imaged segment length and the functional class to which each imaged segment belongs are given in the second and third columns, respectively. The image-based count, *i.e.*, the number of vehicles appearing in the image, is presented in the fourth columns. The combined monthly/day-of-week factor (which is denoted as $F^{m}*F^{d}$) published by the Ohio DOT (Ohio DOT's website, Accessed July 2005a), corresponding to the day each image-based count was taken, is listed in the fifth column. The published hourly factor (Ohio DOT's website, Accessed July 2005), corresponding to the hour each image-based count was taken, is listed in sixth column. Table 4.1 also includes the truck proportion corresponding to the imaged hour, which is published by the Ohio DOT (Ohio DOT's website, Accessed July 2005), and the assumed distribution means of speed for trucks and cars, which are determined using the approach described above. They are listed in the seventh, eighth and ninth columns, respectively. The last column gives the "ground-based" AADT in the year the image-based count was taken. These ground-based AADT values are obtained from the traffic data collected on the ground. When PATR data were available in the same year the image was taken, the AADT values were directly determined from the data. When sufficient PATR data were not available, the AADT values were obtained from the AADT values published on the Ohio DOT's website (Accessed July 2005b) for the year in which the image was taken. If a published value was not available in the year the image was taken, the values were determined from a combination of published AADT values in previous years and corresponding published growth factors (Ohio DOT's website, Accessed July 2005c). In this empirical study, these ground-based AADT values will be assumed to be the true AADT on the segment in the year the image was taken.

Segment		n ^{img}	Published F ^m *F ^d	Published F ^h	Published <i>P</i> _k	ū _k	ū _c	Ground- based AADT	
No.	Length (mi)	FC*							
1	7.47	11	186	1	0.8681	0.1679	55	65	30178
2	3.07	11	176	1	0.8681	0.1679	55	65	77497
3	4.75	1	123	1.01	0.8013	0.32	60	70	45955
4	13.01	11	325	0.95	0.8681	0.1679	55	65	30112
5	3.82	11	244	0.95	0.8681	0.1679	55	65	78970
6	10.76	1	336	1.04	0.8013	0.32	60	70	47931
7	1.43	11	58	0.93	0.8333	0.1575	55	65	51604
8	2.85	11	134	0.93	0.8333	0.1575	55	65	47852
9	3.74	11	182	0.93	0.8333	0.1575	55	65	45288
10	1.80	11	132	0.94	0.8333	0.1575	55	65	67592
11	2.10	1	91	0.87	0.7440	0.3188	60	70	41920
12	3.45	1	147	0.87	0.7440	0.3188	60	70	42210
13	0.63	11	63	1.29	0.8333	0.1575	55	65	139460
14	2.24	11	182	1.29	0.8333	0.1575	55	65	145120
15	2.19	11	171	1.29	0.8333	0.1575	55	65	134020
16	1.46	11	63	1.29	0.8333	0.1575	55	65	91130
17	0.56	11	24	1.29	0.8333	0.1575	55	65	93490
18	0.63	11	29	1.29	0.8333	0.1575	55	65	102710
19	0.63	11	55	1.29	0.8333	0.1575	55	65	117810
20	1.87	11	87	0.97	0.8333	0.1575	55	65	60942
21	2.17	11	129	0.97	0.8333	0.1575	55	65	70722
22	4.33	11	305	0.97	0.8333	0.157	55	65	84844

Table 4.1: Empirical Image-based Counts and Corresponding Information used in the Study.

Given the information in Table 4.1, the probability of observing each image-based count can be simulated based on the proposed 3-stage model. The simulation steps have been presented in Algorithm (3.1). Specifically, under the same condition that each image-based count was taken, the simulation generates a large number of "possible" image-based counts (*i.e.*, *M* image-based counts) conditional on the "true" AADT (*i.e.*, ground-based AADT listed in Table 4.1). The probability of observing the image-based count given in Table 1 can then be approximated by using Equation (4.2.1).

The simulated probability of observing each of the 22 image-based counts is given in Table 4.2. In this study, M is set to 50,000. A simple simulation study indicates that M= 50,000 would result in a CV of simulated pmf values calculated by Equation (4.2.1) no more than 0.04 for the 22 image-based counts. As mentioned in the previous section, three assumptions of truck proportion variability are considered, so three different probabilities of observing each image-based count are obtained.

As shown in Table 4.2, the three different assumptions of truck proportion variability result in small differences in the simulated probabilities of observing the image-based counts conditional on the "true" AADT. The products of the 22 simulated probabilities under different assumptions are given at the bottom of Table 4.2. The product will be used later as a criterion for the evaluation of the 3-stage model. The three products resulted from different assumptions on the distribution of P_k differ little, and are in the same order of magnitude. It appears that the distribution of image-based counts

				(1)	(2)	(3)
Segment				Fixed at	Normal	Uniform
~~			n ^{img}	Published	P_k	P_k
				P_k		
No.	Length (mi)	FC*				
1	7.47	11	186	0.0107	0.0105	0.0101
2	3.07	11	176	0.0132	0.0127	0.0131
3	4.75	1	123	0.0041	0.0042	0.0051
4	13.01	11	325	0.0071	0.0070	0.0067
5	3.82	11	244	0.0096	0.0095	0.0101
6	10.76	1	336	0.0051	0.0055	0.0060
7	1.43	11	58	0.0330	0.0315	0.0324
8	2.85	11	134	0.0113	0.0112	0.0106
9	3.74	11	182	0.0045	0.0045	0.0039
10	1.80	11	132	0.0057	0.0055	0.0056
11	2.10	1	91	0.0211	0.0217	0.0196
12	3.45	1	147	0.0146	0.0144	0.0142
13	0.63	11	63	0.0238	0.0231	0.0225
14	2.24	11	182	0.0115	0.0118	0.0110
15	2.19	11	171	0.0131	0.0123	0.0135
16	1.46	11	63	0.0148	0.0147	0.0159
17	0.56	11	24	0.0337	0.0331	0.0353
18	0.63	11	29	0.0254	0.0245	0.0267
19	0.63	11	55	0.0221	0.0222	0.0209
20	1.87	11	87	0.0229	0.0239	0.0237
21	2.17	11	129	0.0163	0.0166	0.0164
22	4.33	11	305	0.0076	0.0079	0.0074

Product of the 22 probabilites 1.69E-42 1.53E-42 1.67E-42

Table 4.2: "Probabilities" of Observing the 22 Image-based Counts under Three Different Assumptions on P_k .

conditional on the AADT resulting from the 3-stage model is not sensitive to the distribution of P_k .

The probabilities presented in Table 4.2, by themselves, are not useful in drawing any conclusion about the reasonableness of the 3-stage model. Therefore, two additional models for the distribution of image-based counts conditional on the AADT are introduced as comparison basis for the evaluation of the 3-stage model.

The first model adopts the concept of "maximum density". Maximum density is defined as the maximum number of passenger cars per mile per lane on a given segment at a given level of service (TRB, 2000). Level of service (LOS) is a qualitative measure that describes operational conditions of the traffic stream on a segment. As evident from the observed densities, all 22 image-based counts were taken at LOS A or B, which represents operational conditions at low traffic densities with no or slight restrictions of driver freedom. According to the Highway Capacity Manual (TRB, 2000), the density range for LOS A and B is 0-18 pc/mi/ln. Given the number of lanes and length of the imaged segment, the maximum number of possible passenger cars on the segment that could be present and still produce LOS A/B can be easily calculated. To convert the number of passenger cars to the number of vehicles, the heavy vehicle adjustment factor f_{HV} is used to account for the effects of trucks in the traffic. In the 22 images used here, only trucks are identified as the heavy vehicles. Therefore, we simplify the equation used in the *HCM* for the calculation of f_{HV} to

$$f_{HV} = \frac{1}{1 + P_T (E_T - 1))},\tag{4.2.2}$$

where P_T is the proportion of trucks in the traffic stream, and E_T is the passenger car equivalent for trucks. In this study, the truck proportion in each image is obtained and shown in the fifth column of Table 4.3. E_T for all 22 images is assumed to be 1.5, a value used for level terrains (TRB, 2000). Correspondingly, the factor f_{HV} is calculated using Equation (4.2.2) and given in the sixth column. The seventh column gives the number of lanes of the imaged segment. Therefore, the maximum number of vehicles on the imaged segment can be determined by

$$N^{mg}_{max} = 18 \times [\text{number of lanes}] \times [\text{segment length}] \times f_{HV}.$$
(4.2.3)

When knowing only that the segment considered is operating at LOS A or B, any integer value between zero and N^{img}_{max} would be equally likely to be the number of vehicles observed on the imaged segment. Therefore, the probability of observing any image-based count n^{img} would be

$$f_1(N^{img} = n^{img}) = 1/[N^{img}_{max} + 1].$$
(4.2.4)

By using Equation (4.2.4), the probability of observing each of the 22 image-based counts by the "maximum-density" model is calculated and shown in the last column of Table 4.3.

Segment		n^{img} Observed P_k		Heavy vehicle Number adjustment of factor f mu		N ^{img} _{max}	$f_1(N^{img}=n^{img})$	
No.	Length (mi)	FC*			5 110			
1	7.47	11	186	0.4731	0.8087	5	544	0.0018
2	3.07	11	176	0.1875	0.9143	4	202	0.0049
3	4.75	1	123	0.4146	0.8283	6	425	0.0023
4	13.01	11	325	0.2831	0.8760	6	1230	0.0008
5	3.82	11	244	0.1025	0.9513	4	261	0.0038
6	10.76	1	336	0.3869	0.8379	6	974	0.0010
7	1.43	11	58	0.1724	0.9206	6	142	0.0070
8	2.85	11	134	0.1940	0.9116	6	281	0.0036
9	3.74	11	182	0.2473	0.8900	6	359	0.0028
10	1.80	11	132	0.1364	0.9362	5	152	0.0065
11	2.10	1	91	0.0879	0.9579	5	181	0.0055
12	3.45	1	147	0.0544	0.9735	4	242	0.0041
13	0.63	11	63	0.0476	0.9767	6	67	0.0147
14	2.24	11	182	0.0110	0.9945	7	280	0.0036
15	2.19	11	171	0.0000	1.0000	8	316	0.0032
16	1.46	11	63	0.0476	0.9767	4	103	0.0097
17	0.56	11	24	0.1250	0.9412	6	57	0.0172
18	0.63	11	29	0.0000	1.0000	6	68	0.0145
19	0.63	11	55	0.0545	0.9735	8	88	0.0112
20	1.87	11	87	0.0460	0.9775	4	132	0.0075
21	2.17	11	129	0.0155	0.9923	4	155	0.0064
22	4.33	11	305	0.0295	0.9855	4	307	0.0032

Table 4.3: "Probabilities" of Observing the 22 Image-based Counts by Using the "Maximum Density" Model.

The second model adopts a lognormal distribution for the distribution of image-based counts conditional on the AADT, which is based on the work in McCord *et al.* (2002a). It is assumed that an image-based count can, given the AADT, be modeled as

$$N^{img}|AADT = \left\lceil AADT / \left[F^{M}(m(\delta)) \times F^{D}(d(\delta)) \right] / \left[24 \times F^{H}(h) \right] \times \left[l/Us \right] \times Noise(img) \right\rceil,$$

$$(4.2.5a)$$

where $\lceil \bullet \rceil$ represents the closest integer to \bullet ; and *Noise(img)* follows a lognormal distribution

Ln[Noise(img)] ~
$$N(-\sigma_{img}^2/2, \sigma_{img}^2)$$
. (4.2.5b)

The standard deviation was set at $\sigma_{img} = 0.17$, based on the 22 image-based counts shown in Table 4.1. Details on determining $\sigma_{img} = 0.17$ can be found elsewhere (Jiang *et al.*, 2004). The space-mean speed *Us* used in Equation (4.2.5a) is calculated using the truck proportion observed on the imaged segment

$$Us = [\text{observed } P_k] \times \bar{u}_k + [1 \text{- observed } P_k] \times \bar{u}_c, \qquad (4.2.5c)$$

where \bar{u}_k and \bar{u}_c are defined before.

By using the lognormal model (4.2.5), it is straightforward to simulate a large number of observed image-based counts (*i.e.*, M image-based counts) conditional on the "true" AADT given in Table 4.1, under the conditions the image-based counts were taken. Then Equation (4.2.1) can be used to approximate the probability of observing each of the 22 image-based counts conditional on the AADT. Table 4.4 provides these simulated
Segment		n ^{img}	Ground- based AADT	Published $F^m * F^d$	Published F^h	Observed P_k	\bar{u}_k	ū _c	$lognormal-based$ $f_1(N^{img} = n^{img})$	
No.	Length (mi)	FC*								
1	7.47	11	186	30178	1	0.8681	0.4731	55	65	0.0133
2	3.07	11	176	77497	1	0.8681	0.1875	55	65	0.0132
3	4.75	1	123	45955	1.01	0.8013	0.4146	60	70	0.0033
4	13.01	11	325	30112	0.95	0.8681	0.2831	55	65	0.0074
5	3.82	11	244	78970	0.95	0.8681	0.1025	55	65	0.0091
6	10.76	1	336	47931	1.04	0.8013	0.3869	60	70	0.0051
7	1.43	11	58	51604	0.93	0.8333	0.1724	55	65	0.0378
8	2.85	11	134	47852	0.93	0.8333	0.1940	55	65	0.0120
9	3.74	11	182	45288	0.93	0.8333	0.2473	55	65	0.0050
10	1.80	11	132	67592	0.94	0.8333	0.1364	55	65	0.0054
11	2.10	1	91	41920	0.87	0.7440	0.0879	60	70	0.0211
12	3.45	1	147	42210	0.87	0.7440	0.0544	60	70	0.0129
13	0.63	11	63	139460	1.29	0.8333	0.0476	55	65	0.0215
14	2.24	11	182	145120	1.29	0.8333	0.0110	55	65	0.0128
15	2.19	11	171	134020	1.29	0.8333	0.0000	55	65	0.0135
16	1.46	11	63	91130	1.29	0.8333	0.0476	55	65	0.0168
17	0.56	11	24	93490	1.29	0.8333	0.1250	55	65	0.0267
18	0.63	11	29	102710	1.29	0.8333	0.0000	55	65	0.0215
19	0.63	11	55	117810	1.29	0.8333	0.0545	55	65	0.0182
20	1.87	11	87	60942	0.97	0.8333	0.0460	55	65	0.0264
21	2.17	11	129	70722	0.97	0.8333	0.0155	55	65	0.0167
22	4.33	11	305	84844	0.97	0.8333	0.0295	55	65	0.0078

Table 4.4: "Probabilities" of Observing the 22 Image-based Counts by Using the "Lognormal" Model.

"lognormal" probabilities in the last column. Each simulated probability is based on 50,000 simulated possible observed image-based counts (*i.e.*, M = 50,000).

To compare the "maximum-density" and lognormal models to the 3-stage model, the product of probabilities of observing the 22 image-based counts conditional on the "true" AADTs is calculated for each of the three models. The results are given in Table 4.5. For the 3-stage model, the smallest product using the three different distributions of P_K (*i.e.*, normally distributed P_K) is used in Table 4.5 for conservative purpose of comparison. Assuming independence among the 22 image-based counts, the products can be regarded as the simulated joint probability of observing the 22 image-based counts. As seen in Table 4.5, the 3-stage model produces a much larger joint probability than that produced by the "maximum-density" model, and slightly larger than that produced by the "lognormal" model. In terms of producing the maximum likelihood of observing the 22 image-based counts conditional on the "true" AADT, the 3-stage model would outperform the other two models.

	Max-Density Model	Lognormal Model	3-Stage Model	
Probability	4.75E-52	1.47E-42	1.53E-42	

Table 4.5: Comparison of "Probability" of Observing 22 Image-based Counts using Three Different Models.

The 3-stage model produces a higher probability than the lognormal model but the difference is not great. The performance of the lognormal model, however, exploited some information directly from the 22 image-based counts, compared to the performance of the 3-stage model. The parameter σ_{img} of the lognormal model was directly estimated from the same 22 image-based counts used to determine the probability. In addition, the truck proportions observed in the imaged segments were directly used in the lognormal model. On the other hand, no information in the 22 image-based counts was used to estimate the parameters (*e.g.*, σ_D and σ_H) of the 3-stage model.

The 3-stage model can also capture the impacts of imaged segment length and traffic volumes on the variability of image-based counts, while the $\sigma_{img} = 0.17$ in the lognormal model is not necessarily transferable to a new image-based count. When the imaged segment length and AADT change drastically from the condition of the 22 image-based counts, the 3-stage model would show advantages over the lognormal model. In the next section, more details about this dependence will be discussed.

4.3 Lognormal Approximation to the Distribution of Image-based Counts conditional on AADT

The 3-stage model was able to capture the impacts of imaged segment length and traffic volume on the distribution of image-based counts conditional on the AADT. In section 4.1, the contribution of four variables (Noise(D), Noise(H), Us, and P_k) to the

distribution of image-based counts conditional on the AADT was represented by four statistical distributions. Given these distributions and corresponding temporal adjustment factors, the distribution of image-based counts conditional on the AADT would depend only on the imaged segment length *l* and the hourly volume through the Binomial distribution of (3.2.7), *i.e.*, the third stage of the 3-stage model. In this section, the distribution of image-based counts conditional on the AADT resulting from the 3-stage model will be approximated by a lognormal distribution with parameters as a function of the imaged segment length and traffic volume. Such an approximation will make it much simple to implement the Bayesian approach, and a large amount of computing time could be saved.

Recalling that no closed form is available for the proposed distribution of image-based counts conditional on the AADT resulting from the 3-stage model, the mean of the image-based counts conditional on the AADT ($E[N^{img}|AADT]$), and the standard deviation of the natural logarithms of the image-based counts conditional on the AADT ($sd_{img}|AADT$), will be used to describe the probability distribution of image-based counts conditional on the AADT] and $sd_{img}|AADT$ are two parameters required in the lognormal approximation to the distribution. Since Noise(D) and Noise(H) were argued to be mutually independent and assumed not to depend on the traffic volumes, the expected hourly volume in the hour the image conditional on the AADT is taken can be used to represent the impact of traffic volume on the probability distribution of the

image-based count conditional on the AADT, rather than the AADT itself. The expected hourly volume conditional on the AADT can be shown to be

$$E[V^{H}(h,\delta)|AADT] = AADT/[F^{M}(m(\delta)) \times F^{D}(d(\delta))]/[24 \times F^{H}(h)].$$
(4.3.1)

Equation (4.3.1) shows that the expected hourly volume in the hour the image is taken, conditional on the AADT, is the AADT adjusted by the corresponding temporal factors. All temporal factors used in Equation (4.3.1) are assumed known, so AADT and the expected hourly volume have a one-to-one relationship.

This section investigates $E[N^{img}|AADT]$ and $sd_{img}|AADT$ as a function of the imaged segment length *l* and the expected hourly volume in the hour the image is taken. Based on the law of total expectation (Rice, 1995), the mean of image-based counts conditional on the AADT can be approximated by

$$E[N^{img}|AADT] \approx$$

$$AADT/[F^{M}(m(\delta)) \times F^{D}(d(\delta))]/[24 \times F^{H}(h)] \times l / (E[P_{k}] \times \bar{u}_{k} + (1 - E[P_{k}]) \times \bar{u}_{c}),$$

$$(4.3.2a)$$

where $E[\bullet]$ represents the mean of \bullet . This approximation involves an assumption that the expectation of the reciprocal of *Us* is equal to the reciprocal of the expectation of *Us*, which would be reasonable when the variance of the normally distributed *Us* is not very large, compared to the mean of *Us*. Substituting Equation (4.3.1) into Equation (4.3.2a) yields

$$\mathbb{E}[N^{img}|AADT] \approx \mathbb{E}[V^{H}(h,\delta)|AADT] \times l / (\mathbb{E}[P_{k}] \times \bar{u}_{k} + (1 - \mathbb{E}[P_{k}]) \times \bar{u}_{c}), \qquad (4.3.2b)$$

That is, the mean of the image-based counts conditional on the AADT can be expressed as the hourly volume $E[V^{H}(h, \delta)|AADT]$ multiplied by the fraction of the time that would correspond to an "average" duration during which all N^{img} vehicles passing the *l*-mile-long segment.

Because of the complexity, it is difficult to derive analytically an approximation for the standard deviation of the natural logarithms of the image-based counts conditional on the AADT, $sd_{img}|AADT$. Simulation is used to investigate $sd_{img}|AADT$ as a function of land $E[V^{H}(h, \delta)|AADT]$.

Let us consider an example first. Assume that the AADT on a 1.5-mile highway segment of interest is 10,000. A numerical distribution of the image-based counts on the segment will be simulated for an hour of a day, when the monthly and day-of-week factors are equal to one, and the hourly factor is 0.8333 (0.8333 means that the hourly volume constitutes 5% of the daily volume). Truck and car speed limits are 55 and 65 mph, respectively. The distributions of *Noise(D)* and *Noise(H)* would be the same as those described in Section 4.1. To investigate the impacts of *Us* and *P_k* distributions on the distribution of image-based counts conditional on the AADT, we set the σ_u for the normally distributed *Us* at three levels (0, 10, and 20), and consider three types of *P_k* distribution described in Section 4.1 assuming *p* equals 0.25.

Totally there are $3 \times 3 = 9$ different combinations of *Us* and *P_k* distributions. We consider the combination of $\sigma_u = 10$ and (truncated) normally distributed *P_k* as the base

Simulated Distribution of Image-based Counts for Base Case



Figure 4.1: Simulated distribution of image-based counts for base case.

case. By using Algorithm (3.1), 5000 image-based counts are simulated for this base case. The histogram of these counts (*i.e.*, a numerical distribution) is plotted in Figure 4.1.

For the base case, the mean of the 5000 simulated image-based counts is 12.0026, which is almost exactly equal to 12, the number calculated by the approximation in Equation (4.3.2a). The $sd_{img}|AADT$ of the 5000 simulated image-based counts is 0.3576. For comparison purposes, simulations are conducted for two other combinations of Us and P_k distributions: the "zero-variance" case, where there is no variation in Us and P_k (*i.e.*, $\sigma_u = 0$ and $P_k = 0.25$ with probability one), and the "large-variance" case of $\sigma_u = 20$ and uniformly distributed P_k . For the "zero-variance" case, the mean and $sd_{img}|AADT$ 103

based on 5000 simulated image-based counts are 12.0028 and 0.3471, respectively. For the "large-variance" case, the mean and $sd_{img}|AADT$ based on 5000 simulated image-based counts are 12.0682 and 0.3797, respectively. It can be seen that different assumptions on Us and P_k distributions lead to almost the same mean but slightly different $sd_{img}|AADT$. However, the relative difference in the $sd_{img}|AADT$ is not greater than 9%. Therefore, the distribution of image-based counts conditional on the AADT does not appear sensitive to the variation in Us and P_k . In the following section, we will only consider the base case, where σ_u equals 10 and P_k is (truncated) normally distributed.



Figure 4.2: Normal Q-Q plot for the natural logarithms of the 5000 simulated image-based counts.

The natural logarithms of the 5000 simulated image-based counts shown in Figure 4.1 are checked in a Q-Q normal plot, as show in Figure 4.2. Except for the two tails and the step pattern that results because all simulated values are integers, the 5000 simulated values almost fall on a straight line. Therefore, we use the following closed-form function to approximate the non-closed-form distribution of image-based counts conditional on the AADT:

$$N^{img}|AADT = \begin{bmatrix} E[N^{img}|AADT] \times \mathcal{E}_{img} \end{bmatrix},$$
(4.3.3a)

and
$$\log(\varepsilon_{img}) \sim N(-(sd_{img}|AADT)^2/2, (sd_{img}|AADT)^2),$$
 (4.3.3b)

where $\lceil \bullet \rceil$, $E[N^{img}|AADT]$ and $sd_{img}|AADT$ are as defined above. Given $E[N^{img}|AADT]$ and $sd_{img}|AADT$, the pmf value for an image-based count conditional on the AADT can be easily "calculated" by using (4.3.3). As mentioned above, the $E[N^{img}|AADT]$ can be approximated by Equations (4.3.2). The $sd_{img}|AADT$ is a function of l and $E[V^{H}(h, \delta)|AADT]$, and can be simulated for different combinations of l and $E[V^{H}(h, \delta)|AADT]$ to represent different scenarios. For each combination, a large number of image-based counts are simulated so that $sd_{img}|AADT$ can be estimated. The contour lines of $sd_{img}|AADT$ can then be plotted on the plane of l vs. $E[V^{H}(h, \delta)|AADT]$. Figure 4.3 plots the contour lines of $sd_{img}|AADT$ over a portion of the plane $l \times$ $E[V^{H}(h, \delta)|AADT]$.



Figure 4.3: Contour lines of $sd_{img}|AADT$ over the plane $l \times E[V^{H}(h, \delta)|AADT]$.

Figure 4.3 shows an obviously decreasing pattern of $sd_{img}|AADT$ with increasing land $E[V^{H}(h,\delta)|AADT]$. When $E[V^{H}(h,\delta)|AADT]$ is greater than 2000, most levels of $sd_{img}|AADT$ will be below 0.25. When l is greater than 2 miles, most levels of $sd_{img}|AADT$ will be below 0.25. For any length l greater than 2 miles, the level of $sd_{img}|AADT$ decreases rapidly with increasing $E[V^{H}(h,\delta)|AADT]$, and goes into the large flat area, where $sd_{img}|AADT$ is around 0.20, no matter how $E[V^{H}(h,\delta)|AADT]$ increases. Also, for any $E[V^{H}(h,\delta)|AADT]$ greater than 1000, the level of $sd_{img}|AADT$ decreases 106 rapidly with increasing length l, and eventually enters the large flat area no matter how the length l increases.

Such contour lines can be considered to produce "default" values of sd_{img} |AADT. If one does not want to conduct the entire algorithm (3.1) to simulate the distribution of image-based counts conditional on the AADT, this conditional distribution could be simplified by a lognormal distribution given in (4.3.3), with the value of sd_{img} |AADT "read" from a contour-line plot like Figure 4.3. The calculation of the posterior distribution of the AADT conditional on an image-based count would become much simpler, compared to the Algorithm (3.3) proposed in Chapter 3. Below is the corresponding simplified version of Algorithm (3.3) by using the lognormal approximation proposed in (4.3.3) to the distribution of image-based counts conditional on the AADT.

Algorithm (4.1) – posterior distribution of AADT conditional on an image-based volume

- 1. Given the prior distribution $\pi(AADT)$, the observed image-based volume $N^{img}(l,h,\delta)$, the corresponding monthly factor $F^{M}(m(\delta))$, day-of-week factor $F^{D}(d(\delta))$, hourly factor $F^{H}(h)$, the length of imaged segment *l*, and the Distribution (4.3.3).
- 2. Generate *N* values of AADT from the prior π (*AADT*), *AADT_n*, *n* = 1, 2, ...*N*.
- 3. Evaluate the weight $\omega(n)$ for each $AADT_n$ as follows
 - (i) Use $AADT_n$ in Equation (4.3.1) to calculate $E[V^H(h, \delta)|AADT]$;

- (ii) "Read" a value of sd_{img} [AADT from a contour line plot like Figure 4.3, based on l and the E[$V^{H}(h, \delta)$ [AADT] calculated in Step 3(i);
- (iii) Use $N^{img}(l,h,\delta)$ and $E[N^{img}|AADT]$ in Equation (4.3.3a) to calculate the "observed" noise term ε_{img} ;
- (iv) Calculate the value of $f(\varepsilon_{img})$ based on the distribution of (4.3.3b);
- (v) $\omega(n) = f(\varepsilon_{img})$ obtained in Step 3(iv).
- 4. Normalize the weights obtained in Step 3: $\omega'(n) = \alpha(n)/\Sigma_{i=1,..N} \alpha(i), n = 1, 2, ...N$.
- 5. Resample the *N* values of AADT in Step 2 with replacement based on the weights in Step 4 to obtain *N* "new" AADT values, which is the numerical approximation of the posterior distribution $\pi(AADT | N^{img}(l,h,\delta))$.

We will come back to the use of Algorithm (4.1) in Chapter 5.

CHAPTER 5

NUMERICAL STUDY

In this chapter, the value of adding image-based counts in AADT estimation will be evaluated by a numerical study. Chapter 3 developed a Bayesian approach to incorporate different types of short-duration traffic counts (including image-based counts) in AADT estimation. As noted, the approach introduces a dynamic process of updating the probability distribution of the AADT on a given segment. In the year of interest, the approach begins with a prior distribution that is updated when a traffic count is available. The final updated (or posterior) distribution of the AADT is used to derive the prior distribution of AADT in the following year after incorporating the uncertainty in traffic growth.

Incorporating image-based counts with the Bayesian approach will be compared to the traditional ground-based AADT estimation through numerical examples under various assumptions. The sensitivity of the results will be investigated. Before conducting the numerical evaluation, several simple examples will be presented to illustrate the implementation of the Bayesian approach.

Non-informative prior	Uniform between 2000 and 200000		
F ^{md} for the image day	1		
F ^h for the image hour	0.8333		
$\sigma_{ m D}$	0.12		
ĝ ^{y/y-1}	1.05		
$\sigma_{ m f}$	0.05		
$\sigma_{ m H}$	0.10		
P _k	Uniform between 0 and 0.5		
$ar{\mathrm{u}}_\mathrm{k}$	55		
$\bar{\mathrm{u}}_{\mathrm{c}}$	65		
σ_{u}	10		

Table 5.1: Summary of Parameters/Distributions used in the Examples.

5.1 Illustrative Examples

In this section, three examples will be presented to illustrate the basic implementation of the Bayesian approach. A summary of parameters used in these examples is given in Table 5.1.

5.1.1 Observing two ground-based daily volumes in the year of interest

In this example, two ground-based daily volumes, $V^{24}(\delta_1)$ and $V^{24}(\delta_2)$, are collected on the segment considered in the year when the AADT estimate is produced. Assume that the two observed daily volumes are

$$V^{24}(\delta_l) = 48,395,$$

$$V^{24}(\delta_2) = 46,980,$$

and, without loss of generality, that the monthly and day-of-week factors corresponding to the two days all equal one.

Assume that no information was available about the AADT on the segment before the two daily volumes are collected. As discussed in Chapter 3, a uniform distribution can be selected for the non-informative prior of the AADT. Given the range of the uniform distribution, any value in the range is equally likely to be the AADT. Here, the lower bound of the uniform distribution is set as 2000 vpd, and upper bound as 200,000 vpd. These two bounding values are selected based on the analysis of data on the CD of ODOT GIS-file 2002 (Ohio DOT, 2002). According to the CD, average daily traffic volumes on Ohio interstate highways ranged from 2910 to 169,640 vpd. Therefore, the uniform prior selected here covers the range of values on the CD.

The Bayesian approach begins with this uniform prior distribution. According to Algorithm (3.2) described in Chapter 3, a large number of AADT values must be generated from the prior distribution to represent a numerical approximation of the prior. Since the prior distribution is uniform, sampling each integer once from 2000 to 200,000 would faithfully represent the uniform prior. However, sampling every possible value once would lead to very large sample that appears not necessary. In this example, a sequence from 2000 to 200,000 with a step length of 10 is used as the sample from the



Figure 5.1: Numerical approximation of prior distribution of AADT – uniform between 2000 and 200,000 with a step length of 10.

prior uniform distribution. Figure 5.1 plots the histogram of the 19,801 (= (200000-2000)/10+1) AADT values.

After observing the first daily volume $V^{24}(\delta_1) = 48,395$, the prior shown in Figure 5.1 can be updated as described in Chapter 3. The weight needs to be evaluated for each AADT value in Figure 5.1. That weight is proportional to the probability of observing $V^{24}(\delta_1) = 48,395$ on that day conditional on the AADT value. For example, in order to evaluate the weight for AADT = 100,000, the probability of observing $V^{24}(\delta_1) = 48,395$

on that day conditional on AADT = 100,000 needs to be obtained. According to Equation (3.2.3), observing $V^{24}(\delta_1) = 48,395$ conditional on AADT = 100,000 is equivalent to observing a *Noise*($D(\delta_1)$) = 48395 / 100000 = 0.48395. As suggested in (4.1.2), *Noise*(D) is modeled by a lognormal distribution with the parameter $\sigma_D = 0.12$. The probability density of *Noise*($D(\delta_1)$) = 0.48395 is 1.1235e-007 (obtained by using software MATLAB). Therefore, a "rough" weight for AADT=100000 can be estimated as 1.1235e-007. After obtaining the weights for all 19,801 AADT values in Figure 5.1 in this way, these "rough" weights are normalized so that the sum of the resulting weights equals to one.

Based on the weights obtained above, the 19,801 AADT values are then re-sampled with replacement. The re-sampled 19,801 AADT values plotted in Figure 5.2, form a numerical approximation of the posterior distribution of AADT after observing the first traffic volume $V^{24}(\delta_1)$. The mean and standard deviation of the numerical posterior distribution are calculated as

Mean of the posterior after observing
$$V^{24}(\delta_1) = 50,510,$$
 (5.1.1a)

Standard deviation of the posterior after observing $V^{24}(\delta_1) = 6,090.$ (5.1.1b)



Figure 5.2: Numerical approximation of the posterior distribution after observing the first daily volume $V^{2^4}(\delta_1)$ when using numerical prior distribution of Figure 5.1.

As seen in Figure 5.2, the AADT values are clustered roughly around the observed daily traffic volume, *i.e.*, 48,395. The variability of AADT is largely reduced from that of the prior distribution in Figure 5.1. That is, conditional on the first daily volume $V^{24}(\delta_l)$, the uncertainty in AADT is significantly reduced (σ = 6,090 for the posterior versus σ = 57,162 for the prior).

After observing the second daily volume $V^{24}(\delta_2) = 46,980$, the distribution of AADT shown in Figure 5.2 can be updated in a similar manner. Weights are evaluated for all



Figure 5.3: Numerical approximation of posterior distribution after observing the second daily volume $V^{2^4}(\delta_2)$ by updating the distribution given in Figure 5.2.

AADT values in Figure 5.2 by estimating the probability of observing the second daily volume $V^{24}(\delta_2) = 46980$ conditional on each corresponding AADT value, and then these AADT values are re-sampled with replacement according to the weights. These newly re-sampled AADT values plotted in Figure 5.3, form a numerical approximation of the posterior distribution of AADT after observing the second traffic volume $V^{24}(\delta_2)$. The mean and standard deviation of the numerical posterior distribution are calculated as

Mean of the posterior after observing $V^{24}(\delta_2) = 49,207,$ (5.1.2a)

Standard deviation of the posterior after observing $V^{24}(\delta_2) = 4,211$. (5.1.2b) Comparing (5.1.2b) with (5.1.1b), the uncertainty in AADT is reduced again after observing the second daily volume (σ is reduced from 6,090 to 4,211).

This example illustrates the implementation of the Bayesian approach when two daily volumes are collected in the year of interest. The resultant AADT point estimate on the segment would be 49,207, if we choose the mean of the (final) posterior distribution as our default point estimate.

In traditional practice, the two daily volumes would be first de-seasonalized and then averaged to get an annual average. Since the corresponding monthly and day-of-week factors all equal to one, the traditional AADT estimate would be (48395 + 46980) / 2 = 47688. The estimate obtained by the Bayesian approach is 49207, a little (~3%) larger than the traditional estimate. The difference is mainly caused by the prior information considered in the estimation with the Bayesian approach. The estimate is shifted a little to the prior mean (101,000). However, one can see that the difference is small. That is, when a non-informative prior is used in the estimation, the Bayesian approach appears to produce a similar point estimate as the traditional method does. Also, the non-informative prior appears to have little impact on the estimation, in other words, the estimation does not appear sensitive to the choice of non-informative prior. The benefit of the Bayesian approach does not show up in this example. In a following example, we will see that the benefit of the Bayeisian approach is to incorporate the "useful" prior information into the estimation.

5.1.2 Turning year y-1 posterior into year y prior

Figure 5.3 shows the numerical approximation of posterior distribution of the AADT after the observing two daily volumes in the year of interest (say, year *y*-1). In this example, this posterior distribution will be converted to the prior distribution of AADT for the following year (year *y*) by incorporating the uncertainty in traffic growth.

Analysis of PATR-based AADT values in Florida (McCord *et al.*, 2003a) shows that the growth factors appear to follow a lognormal distribution for segments in an area that is believed to have similar growth pattern in traffic everywhere. Mathematically, the growth factor $g_i^{y/y-1}$ of AADT on segment *i* from year *y*-1 to year *y* can therefore be modeled as

$$\log[g_i^{y/y-1}] \sim N(\ln[\hat{g}^{y/y-1}] - \sigma_f^2/2, \sigma_f^2),$$
(5.1.3)

where $\hat{g}^{y/y-1}$ is the distribution mean of growth factors across the area from year *y* to year *y*-1, and σ_f represents the standard deviation of the logarithms of growth factors. Those values can be estimated from the distribution of paired AADT values in the two years *y*-1 and *y* on the PATR-equipped segments across the group of similar segments.



Figure 5.4: Numerical approximation of prior distribution of the year following the year discussed in previous example.

In this example, the distribution mean $\hat{g}^{y/y-1}$ is assumed to be 1.05, and σ_f is assumed to be 0.05. One growth factor is randomly generated from the lognormal distribution shown in Distribution (5.1.3) for each of the 19801 AADT values given in Figure 5.3. As described in Equation (3.3.2b), the product of these growth factors and the AADT values shown in Figure 5.3 would lead to a numerical approximation of prior distribution of the AADT in the following year (year y), plotted in Figure 5.4. The mean and standard deviation of the numerical prior distribution in year y are calculated as

Mean of the year
$$y \text{ prior} = 51660,$$
 (5.1.4a)

Standard deviation of the year y prior = 5124. (5.1.4b)

Compared to Figure 5.3, the prior distribution shifted to the right, reflecting the belief in traffic growth during the two years. The standard deviation increases (from 4211 to 5124) because of the added uncertainty of the traffic growth from year *y*-1 to year *y*.

5.1.3 Observing an image-based count in the year of interest

This example continues with the previous example. The numerical approximation of the prior distribution given in Figure 5.4 will be considered the prior distribution for the year (year y) when an image-based count is obtained. It is now assumed that the image-based count is obtained in year y from an image covering a 2-mile long section of the segment, with the number of imaged vehicles $N^{img} = 84$. The monthly and day-of-week factors corresponding to the image day are assumed to equal one. The hourly factor corresponding to the image hour is assumed to be 0.8333.

The AADT on the segment is estimated for year *y* when the image-based count is taken. With the Bayesian approach, the image-based count is used to update the prior distribution, a numerical approximation of which is shown in Figure 5.4. The weight for each AADT value in Figure 5.4 is evaluated based on Algorithm (3.3) described in Chapter 3. All parameters describing the distributions of *Noise*(*D*), *Noise*(*H*), truck percentage P_k , and space-mean speed *Us* are specified, as given in Table 5.1.

According to Algorithm (3.3), the number *nn*, which is the total simulation runs for calculating an "image-based" weight (as defined in Chapter 3), must be determined. The number *nn* must be sufficiently large to keep sampling errors in the simulation in an ngeligble level. However, it is impractical to select a very large *nn* to evaluate the "image-based" weight for each of the 19801 AADT values in Figure 5.4. This example sets *nn* at two different levels – 50 and 5000. Figures 5.5a and 5.5b plot the resultant numerical approximations of AADT posterior distributions for the two levels of *nn*=50 and *nn*=5000, respectively. The means and standard deviations of the posterior distributions obtained at the two levels of *nn* are calculated as

Mean at
$$[nn = 50] = 51886$$
, $(5.1.5a)$

Standard deviation at
$$[nn = 50] = 4579.$$
 (5.1.5b)

Mean at
$$[nn = 5000] = 51920,$$
 (5.1.5c)

Standard deviation at
$$[nn = 5000] = 4546.$$
 (5.1.5d)

Based on Figures 5.5 and Equations (5.1.5), the two posterior distributions obtained at the two levels of nn are almost the same. That is, the result appears not sensitive to the level of nn. Therefore, nn=50 appears to be sufficiently large for the purpose of approximating the posterior distribution.



Figure 5.5: Numerical approximations of posterior distributions after observing the image-based count: (a) nn = 50; and (b) nn = 5000.

Compared to the prior distribution, the standard deviation of the AADT distribution is reduced from 5124 to 4579. The uncertainty in the AADT has decreased because of the added information provided by the image-based count.

5.2 Numerical Evaluation of Adding Image-Based Counts in AADT Estimation

Monte Carlo simulation is used to investigate the benefit of adding image-based counts through the Bayesian approach, compared to two current practices in AADT estimation. The simulation considers AADT estimation on a specific segment not equipped with a permanent ATR over a 6-year cycle of ground-based sampling efforts. A 6-year cycle means that the segment is sampled with ground-based counts once every six years, where the sample typically consists of 24-hour volumes taken on two consecutive days. Without loss of generality, the segment is assumed to be sampled on two consecutive days in year one. Then, no ground-based counts are taken for the next five years, that is, until the end of the 6-year cycle. Imagery is considered as a secondary source of sample data; that is, the segment can be imaged by air - or space-based sensors during the six years.

At the beginning of each simulation run, a random segment is generated. An AADT value is randomly selected as the first-year true AADT on this segment. The true AADT values is generated from a uniform distribution

$$AADT(y=1) \sim \text{Uniform}(3000, 160000).$$
 (5.2.1)

Distribution (5.2.1) is selected to roughly reflect the fact that the average daily traffic volumes on Ohio interstate highways ranged from 2910 to 169640 vpd (Ohio DOT, 2002).

Then, five true growth factors ($g_t^{y/y-1}$'s) are generated from Distribution (5.1.3) for the following five years (y = 2, 3, ... and 6). Given AADT(y=1) and the five generated true growth factors, Equation (3.3.2b) is used to recursively calculate the true AADT values in the following five years, AADT(y), y = 2, 3, ... and 6. This study assumes that the parameter σ_f in Distribution (5.1.3) does not vary over the years but remains $\sigma_f = 0.05$ (McCord *et al.*, 2003a). However, the distribution mean of growth factors, $g^{y/y-1}$, does vary over the years and is generated through the following distribution

$$\hat{g}^{y/y-1} \sim \text{Uniform}(1, 1.05), y = 2, 3, \dots \text{ and } 6.$$
 (5.2.2)

Distribution (5.2.2) is selected because the analysis of 2003 Florida AADT data (FDOT, 2003) showed that the distribution means of growth factors in a district was rarely ourside the range of 1 to 1.05.

Next, the length l of this segment covered by the image is simulated. In this study, the length l is assumed to be exactly equal to the length of the segment, so all image-based counts for a given segment have the same coverage length of the segment. The length l is generated from a uniform distribution

$$l \sim \text{Uniform}(0.5 \text{ mi}, 10 \text{ mi}).$$
 (5.2.3)

Distribution (5.2.3) reflects a rough guess of the possible distribution of the lengths of segments in a highway system.

Finally, samples of traffic counts available for the AADT estimation on the segment must be generated. Two ground-based daily volumes in year one (y=1) are generated using Equation (3.2.3). Image-based counts are generated using the algorithm given in Chapter 3 for any year y when image-based counts are collected. The monthly, day-of-week and hourly factors are required for the generation of sampled traffic counts. For simplicity, both monthly and seasonal factors are assumed to equal one; and the hourly factor is 0.8333 (corresponding to the mid-day hours). All parameters required for generating ground- and image-based traffic counts are given in Table 5.2.

F^{md}	1
F^{h}	0.8333
$\sigma_{\rm D}$	0.12
$\sigma_{\rm H}$	0.10
P_k	Uniform between 0 and 0.5
$ar{\mathbf{u}}_k$	55
\bar{u}_c	65
σ_{u}	10

Table 5.2: Summary of Parameters used in the Traffic Count Generation.

After generating all ground- and image-based traffic counts during the cycle considered, three different approaches are simulated for the AADT estimation in each year of the 6-year cycle for comparison purpose. The descriptions of the three approaches are given below

• *The Bayesian approach developed in the work*: The prior distribution for year one is assumed non-informative and uniform as Distribution (3.3.1), with $B^L = 2000$ and $B^U = 200000$, as in the previous example. In a given year *y*, its prior distribution is updated when any type of traffic count is available; otherwise the prior distribution turns into the posterior distribution directly. The final posterior in a year would be converted into the prior in the following year by incorporating the uncertainty in traffic growth. This approach produces a distribution of the AADT. As described in Chapter 3, the point estimate can be determined from the distribution based on the loss function selected. In this numerical study, Squared Relative Error (SRE) is used as a criterion of comparison between different approaches. Therefore, the point estimate from the posterior distribution is derived as shown in the last row of Table 3.1

$$\hat{AADT}^{p} = \frac{E^{\pi(AADT|V)}[1/AADT]}{E^{\pi(AADT|V)}[1/AADT^{2}]}.$$
(5.2.4)

• *Ground counts only without growth factors*: This approach first de-seasonalizes and then averages the two daily volumes collected in year one to estimate the AADT values in any year of the 6-year cycle. That is, the traffic growth on the segment during

the 6-year cycle is ignored in the AADT estimation. The obtained estimate is denoted as $A\hat{A}DT^{g}$.

• *Ground counts only with growth factors*: This approach first de-seasonalizes and then averages the two daily volumes collected in year one to estimate the AADT value in year one. Then the year one estimate is inflated by the appropriate growth factors to estimate AADT values in the following years. In our study, the distribution mean of growth factors, $\hat{g}^{y/y-1}$, is considered to be the growth factors used for the inflation, since true growth factors on the segment considered are not available. The obtained estimate is denoted as $A\hat{A}DT^{gf}$.

In each run of the simulation, the three AADT estimates are produced for each year of the 6-year cycle by using these three approaches. (Note that the estimate will be the same in every year when using the approach that considers "ground count only without growth factors".) For comparison purposes, Squared Relative Error (SRE) is calculated for the three estimates, respectively:

$$SRE^{M}_{r}(y) = \left(\frac{A\hat{A}DT^{M}_{r}(y) - AADT_{r}(y)}{AADT_{r}(y)}\right)^{2}, M = p, g, gf, \text{ and } y = 1, \dots 6$$
(5.2.5)

where $A\hat{A}DT^{M}{}_{r}(y)$ is the AADT estimate produced by approach *M* in year *y* for run *r*; $AADT_{r}(y)$ is the true AADT in year *y* for run *r*; and $SRE^{M}{}_{r}(y)$ is the SRE of $A\hat{A}DT^{M}{}_{r}(y)$. After a large number of simulation runs (say, R runs), the SRE's are averaged across all runs to obtain the mean SRE (MSRE) of AADT estimates obtained by each approach for each year of the cycle

$$MSRE^{M}(y) = \frac{1}{R} \sum_{r=1}^{R} SRE^{M}_{r}(y), M = p, g, gf, y = 1, \dots 6$$
(5.2.6)

The MSRE would reflect the accuracy of the AADT estimates produced by the three approaches.

The MSREs of the three estimates across 5000 runs versus each year of the cycle are plotted in Figure 5.6. The figures differ in the quantity of image-based data assumed to be available for AADT estimation. In Figure 5.6(a), the segment has exactly one image-based count in the year of AADT estimation. That is, no more image-based counts are available in either the estimation year or the previous years. In Figure 5.6(b), the segment has one image-based count every year of the cycle. That is, for any given year, one image-based count every year, back to year one, is available for AADT estimation. In Figure 5.6(c), the segment has 50% probability to have one image-based count every year. In Figure 5.6(d), the segment has 33% probability to have one image-based count every year. The latter two quantities are intended to simulate the scenario that the image-based data come from sources that are not completely scheduled and deterministic, for example, satellite images scheduled for other purposes that could also be affected by cloud cover



Figure 5.6: Comparison of MSRE in AADT estimates by three approaches during a 6-year cycle: (a) one image-based count in the estimation year; (b) one image-based count every year; (c) 50% probability of one image-based count every year; (d) 33% probability of one image-based count every year.

(Merry *et al.*, 1995). Whether a segment was assumed to be imaged in any given year was determined by random generation with probability 0.5 and 0.33, respectively.

In each sub-plot of Figure 5.6, the x-axis represents the year of the cycle, and the y-axis represents the MSRE in AADT estimates. The bold ("grd w/o growth") curve represents the MSRE in AADT estimates produced by using ground counts only without growth factors. The dashed ("grd w. growth") curve represents the MSRE in AADT

estimates produced by using ground counts only with growth factors. Since these two curves are not affected by the quantity of image-based data, the two curves remain the same in the four sub-plots, which only differ in the quantity of image-based data assumed to be available for AADT estimation. The dotted ("Bayesian approach") curves in the four sub-plots represent the MSRE in AADT estimate produced by incorporating image-based counts with the Bayesian approach; each plot corresponds to a different quantity of available image-based data, as mentioned above.

To illustrate how to read the graphs, consider first the AADT estimation in the fourth year of the six-year cycle (Year 4 on the x-axis), three years after the ground counts were taken (recall that the 48-hour coverage count was taken in Year 1). In Figure 5.6(a), when using the 48-hour coverage count to estimate the AADT in Year 4 without applying a growth factor, the "grd w/o growth" curve shows that the MSRE of AADT estimation in Year 4 would be about 0.0181; when applying an appropriate growth factor between Year 1 and Year 4 to the above AADT estimate, the "grd w. growth" curve shows that the MSRE of AADT estimation in Year 4 would be estimation in Year 4 would decrease to 0.0158. If a single image-based count obtained in Year 4 was incorporated in the AADT estimation with the Bayesian approach developed here (specifically, a prior distribution of AADT in Year 4 is developed from the 48-hour coverage count of Year 1 and growth factor distribution from Year 1 to Year 4, and then is updated to a posterior distribution of AADT in Year 4 based on the image-based count), the "Bayesian approach" curve shows that the MSRE in Year

4 would decrease to 0.0110. (In this case, there would have been no image-based counts available in Years 1, 2, or 3, and the MSREs of AADT estimation for these years would be read from the "grd w. growth" or "grd w. growth" curves, depending on whether growth factors were or were not applied.) Similarly, when using the 48-hour coverage count to estimate the AADT in Year 5 without applying a growth factor, the "grd w/o growth" curve shows that the MSRE in Year 5 would be 0.0233; when applying an appropriate growth factor between Year 1 and Year 5 to the above AADT estimate, the "grd w. growth" curve shows the MSRE in Year 5 would decrease to 0.0192. If a single image-based count obtained in Year 5 was used in the AADT estimation with the Bayesian approach developed here, the "Bayesian approach" curve shows that the MSRE in the Year 5 would decrease to 0.0122. Again, the assumption is that an image would be available in Year 5, but no images would have been available in Years 1, 2, 3, or 4, and so the MSRE would be read from the corresponding "ground-based" curves for these years.

As mentioned above, the "grd w/o growth" and "grd w growth" curves of Figures 5.6(b)-5.6(d) are identical to those of Figure 5.6(a). The differences in the "Bayesian approach" curves result from the different assumptions in the supply of image-based data. In Figure 5.6(b), the assumption is that an image-based count is available every year up to and including the estimation year. Specifically, an image-based count obtained in Year 1 is combined with the 48-hour coverage count of Year 1 by the Bayesian approach to develop the posterior distribution of AADT in Year 1. The AADT point estimate (given in

Equation (5.2.4)) from this posterior distribution would result in an MSRE of 0.0068, as seen from the "Bayesian approach" curve at Year 1. The growth factor distribution from Year 1 to Year 2 would then be incorporated into the Year 1 posterior distribution to develop the prior distribution of AADT in Year 2. An additional image based-count obtained in Year 2 is used to update this prior distribution to the Year 2 posterior distribution. The AADT point estimate from this posterior distribution would result in an MSRE of 0.0076, as seen from the "Bayesian approach" curve at Year 2. The Year 2 posterior distribution is combined with Year 2-to-Year 3 growth factor distribution to develop the Year 3 prior distribution. Yet another image-based count obtained in Year 3 is used to update the Year 3 prior distribution to the Year 3 posterior distribution, and so on through the six years of the cycle.

In Figure 5.6(c) and Figure 5.6(d), the "Bayesian approach" curves are produced by randomly generating the available image-based count for each year, so that the segment would have an image-based count every year with probability 50% and 33%, respectively. For example, in Figure 5.6(c), an image-based count would be available with probability 50% in Year 1. If an image-based count is obtained in Year 1, it will be combined with the 48-hour coverage count to develop the posterior distribution of AADT in Year 1; if an image-based count is not obtained in Year 1, only the 48-hour coverage count is used to develop the posterior distribution of either case could produce an AADT point estimate in Year 1, and the "Bayesian approach"

curve at Year 1 shows an MSRE of 0.0073, which is an average performance across the two cases. The appropriate Year 1 posterior distribution will be used to develop the Year 2 prior distribution with the combination of Year 1-to-Year 2 growth factor distribution. Again, an image-based count would be available with probability 50% in Year 2. If an image-based count is obtained in Year 2, the Year 2 prior distribution will correspondingly be updated to the posterior distribution of AADT in Year 2; if an image-based count is not obtained in Year 2, the Year 2 prior distribution will be regarded as the Year 2 posterior distribution without any updating. The "Bayesian approach" curve at Year 2 shows an MSRE of 0.0087, which is an average performance across the two cases in Year 2. Figure 5.6(d) can be read similarly; and the only difference is that the segment is assumded to have an image-based count every year with probability 33%.

From Figure 5.6, it can be seen that using ground-based data without growth factors would lead to a "greater-than-linear" increase in the MSRE of AADT estimates error when the ground-based counts become older. Applying growth factors would reduce the deterioration rate, but the MSREs of the AADT estimates still increase. However, when adding image-based data through the Bayesian approach, even a small amount – *e.g.*, one image-based count only in the estimation year (Fig. 5.6(a)) or an average of one image-based count every three years (Fig. 5.6(d)) – would significantly decrease the deterioration rate. That is, adding image-based counts collected more recently would provide useful information for AADT estimation and improve the estimation accuracy. If
one image-based count every year is available (Figure 5.6(b)), the MSRE of AADT estimates would almost remain constant during the 6-year cycle.

These figures also show that the marginal improvement of adding image-based data (even a small amount) through the Bayesian approach is greater than the marginal improvement of incorporating growth factors when using only ground-based data. That is, the additional accuracy, compared to the recommended practice of using growth factors, gained from adding a small amount of image-based data through the Bayesian approach is greater than the additional accuracy gained from following the recommended practice of using growth factors, compared to not using growth factors. For example, the Year 4 MSREs of Figure 5.6(a) (when only one image-based count was available, and it was available in the year that the AADT was to be estimated) were seen to be 0.0181, 0.0158, and 0.0110. Using growth factors with only the ground-based count would reduce the MSRE by 0.0023 (from the "grd w/o growth" value of 0.0181 to the "grd w. growth" value of 0.0158). The incorporation of an image in Year 4 would reduce the MSRE from the better "grd w. growth" case by an even greater amount, namely, 0.0048 (from the "grd w. growth" value of 0.0158 to the "Bayesian approach" value of 0.0110).

These figures can be interpreted in another way. For example, as shown in Figure 5.6(b), when adding one image-based count every year through the Bayesian approach, the MSRE of the AADT estimate in year five of the 6-year cycle is 0.00760. This MSRE is almost as low as the MSRE produced by the ground-based estimate in the year when

the count is taken (*i.e.*, the MSRE in year one), 0.00755. Therefore, if one image-based count were available every year for a segment, the need to conduct the ground-based count for that segment could be largely reduced, and the resources for ground-based counts collection could be used where the urgency is higher.

Another finding can be seen in these figures. Using ground-based data without growth factors performs almost as well as using ground-based data with growth in the first two years of the cycle. This finding is consistent with the recommendation for using growth factors made in the *TMG* (FHWA, 2001). There, it was mentioned that when the ground-based data is not more than 3 years old, applying growth factors does not show much benefit compared to not applying growth factors.

This section exhibits that image-based counts could decrease the error in AADT estimation while reducing the amount of ground-based data that needs to be collected. Since the results are based on 5000 simulation runs, each of which simulated a different segment with different AADT and imaged segment length, the results would reflect an "average" improvement across a large range of variation in AADT and imaged segment length.

5.3 Sensitivity of Results to the Parameters used in the Bayesian Approach

The simulation results shown in Section 5.2 indicated that AADT estimation errors could be largely reduced by incorporating image-based counts in the estimation. The

results, of course, depend on probability distributions of short-term traffic counts conditional on the AADT and prior distributions of the AADT used in the Bayesian approach. Specifically, the simulation involved two types of the conditional distributions: one for 24-hour volumes and one for image-based counts. Since the estimation was seen not to be sensitive to the non-informative prior, here we focus on the distribution of growth factors across a homogeneous group. In this section, we investigate the sensitivity of results to the parameters describing the two conditional distributions and the distribution of growth factors. Namely, these parameters are σ_D of the *Noise(D)* distribution, σ_H of the *Noise(H)* distribution, σ_U of the *Us* distribution, the standard deviation (denoted as σ_p) of the (truncated) normal P_k distribution, and σ_f of the growth factor distribution. In the sensitivity analyses presented below, one image-based count is considered available only in the AADT estimation year, *i.e.*, the new simulation results obtained from new parameters would be compared to the results shown in Figure 5.6(a).

5.3.1 Sensitivity to the σ_D of the *Noise*(*D*) distribution

According to the 3-stage model, when the σ_D increases, the variability of both 24-hour volumes and image-based counts will increase; when the σ_D decreases, the variability of both 24-hour volumes and image-based counts will decrease. In this subsection, we will investigate the impact of changing σ_D on the results given in Section 5.2. Besides the base level of σ_D ($\sigma_D = 0.12$), two other levels are considered: one where



Figure 5.7: Sensitivity of results to σ_D : (a) $\sigma_D = 0.06$; (b) $\sigma_D = 0.12$; and (c) $\sigma_D = 0.24$.

the level of σ_D is halved ($\sigma_D = 0.06$), and one where the level of σ_D is doubled ($\sigma_D = 0.24$). All other parameters remain the same as those leading to the results in Figure 5.6(a).

Figures 5.7(a) and (c) plot the simulation results for the two levels of σ_D , $\sigma_D = 0.06$ and $\sigma_D = 0.12$, respectively. For comparison purpose, Figure 5.6(a) is re-plotted at the same scale of y-axis as Figures 5.7(a) and (c), and presented in Figure 5.7(b). As it would be expected, the level of AADT estimation errors decreases in Figure 5.7(a), since the variability of both ground- and image-based counts is reduced. Also, the level of AADT estimation errors increases in Figure 5.7(c), since the variability of both ground- and 136 image-based counts becomes larger. Nevertheless, it is still noticeable that the AADT estimation errors are largely reduced by incorporating image-based counts in the estimation for either of the two cases.

One interesting finding is that the curves of ground-based AADT estimation error with and without the use of growth factors cross when the level of σ_D is doubled. A possible reason could be that the ground-based data were so noisy that the use of growth factor might bring some but not significant improvement, which was disguised by the randomness in the simulation.

5.3.2 Sensitivity to the σ_H of the *Noise*(*H*) distribution

According to the 3-stage model, the *Noise*(*H*) distribution only affects the distribution of image-based counts. When the σ_H increases, the variability of image-based counts will increase; when the σ_H decreases, the variability of image-based counts will decrease. Intuitively, if the variability of image-based count becomes smaller while the variability of ground-based count remains the same level, the improvement resulting by adding image-based counts would tend to increase. Therefore, in this subsection, we will only investigate the sensitivity of results to the increase of the level of σ_H . Two additional levels of σ_H are considered: one where the level of σ_H is doubled ($\sigma_H = 0.20$), and one where the level of σ_H is tripled ($\sigma_H = 0.30$). All other parameters remain the same as those leading to the results shown in Figure 5.6(a).

Sensitivity to Sigma_H



Figure 5.8: Sensitivity of results to σ_{H} .

Since the increase of σ_H does not affect the ground-based estimation, the curves of ground-based AADT estimation errors remain the same for different levels of σ_H . Therefore, the simulation results for different levels of σ_H are plotted in one figure (Figure 5.8) to facilitate the comparison. The AADT estimation errors resulting by adding one image-based count in the estimation year are plotted for the two increasing levels of σ_H , as well as the base level ($\sigma_H = 0.10$). It can be seen that the AADT estimation error resulting by adding one image-based count in the estimation year would increase slightly when increasing the level of σ_H . This increase is expected intuitively. However, even 138 tripling the level of σ_H ($\sigma_H = 0.30$, rather than the base case where $\sigma_H = 0.10$) can still improve the AADT estimation accuracy greatly when the ground-based data is more than one year old. In fact, as long as the level of σ_H is estimated correctly, adding the image-based counts by the Bayesian approach would not make the ground-based AADT estimation worse because of the intrinsic feature of Bayesian analysis.

5.3.3 Sensitivity to the parameters describing the P_k and Us distributions

Previously, we have shown that the Bayesian approach does not appear sensitive to the distributions of both P_k and Us. Therefore, it would be expected that the performance of results of Section 5.2 is not sensitive to them, either.

In this subsection, we investigate the sensitivity of results to the parameters describing the P_k distribution, used in the 3-stage model. In the base case, the P_k is (truncated) normally distributed with the mean p equal 0.25 and standard deviation σ_p equal 0.1. Here, we consider two additional levels of p with fixed level of σ_p equal 0.1: p = 0.50 and p = 0.75; and two additional levels of σ_p at the fixed level of p equal 0.25: $\sigma_p = 0.2$ and $\sigma_p = 0.3$. The corresponding simulation results are plotted in Figures 5.9(a) and (b).





(b) Sensitivity to Sigma_p



Figure 5.9: Sensitivity of results to (a) p and (b) σ_p of the P_k distribution.

In Figure 5.9(a), little difference can be found among the three curves of estimation errors resulting by adding one image-based count in the estimation year, each of which reflects a different truck proportion mean p. In Figure 5.9(b), little difference can be found among the three curves of estimation errors resulting by adding one image-based count in the estimation year, each of which reflects a different σ_p .

Using the base case P_k distribution, we also investigate the sensitivity of results to the parameters describing the *Us* distribution. In the base case, the σ_u of the distribution *Us* is 10, and the car and truck speed limits (u_c and u_k) are 65 mph and 55, respectively. Two increasing levels of σ_u are considered when keeping the car and truck speed limits at the base case values: $\sigma_u = 20$ and $\sigma_u = 30$. Two additional sets of the car and truck speed limits are considered when keeping the level of σ_u as the base case value: one where $u_c =$ 60 and $u_k = 50$, and one where $u_c = 70$ and $u_k = 60$. The corresponding simulation results are plotted in Figures 5.10(a) and (b). As in Figure 5.9, little difference can be found either for different levels of σ_u , or for different sets of u_c and u_k .

Therefore, we have shown again that both P_k and Us distributions seem to have little impact on the simulated results demonstrating the improved accuracy when incorporating image-based counts in the AADT estimation. In other words, adding image-based counts by the Bayesian approach appears to perform well for a large range of variation in P_k and Us values.





(b) Sensitivity to uc and uk



Figure 5.10: Sensitivity of results to the Us distribution: (a) σ_u ; (b) u_c and u_k .

5.3.4 Sensitivity to the σ_f of the growth factor distribution

When there is less variability in growth factors across a group, the growth factor on the segment considered can be estimated from the PATR-equipped segments in the same group with more accuracy. Old ground-based data can, therefore, produce better estimates by applying the growth factors. Consequently, the image-based counts collected would not be as valuable for AADT estimation in the contemporary year.

In this subsection, we investigate the sensitivity of the accuracy results to the decrease of the variability in growth factors across a group. According to the growth factor distribution of (5.1.3), the parameter σ_f reflects the variability in the growth factors. Besides the base case where $\sigma_f = 0.05$, two lower levels – $\sigma_f = 0.03$ and $\sigma_f = 0.01$ – are investigated. All other parameters remain the same as those leading to the results shown in Figure 5.6(a). The simulation results for the two lower levels of σ_f , are plotted in Figures 5.11(b) and (c), respectively. For comparison purpose, Figure 5.6(a) is re-plotted and presented in Figure 5.7(a).

As shown in Figures 5.11, adding one image-based count in the estimation year by the Bayesian approach can still decrease the AADT estimation error, although the variability in the growth factors is reduced. Also, such decreases appear to become larger when the ground-based counts get older.

However, as the variability of the growth factors decreases, the marginal improvement of adding image-based counts through the Bayesian approach would not be

greater than the marginal improvement of incorporating growth factors when only using ground-based data. That is because that the growth factor on a specific segment can be estimated more accurately from any other segment in the group if there is less variability in the growth factors. Inflating old ground-based data by the more accurately estimated growth factor would be almost equivalent to the contemporary data. That is why the MSRE curve of applying growth factors to ground-based counts is almost flat in Figure 5.11(c) where σ_f is only 0.01, which is almost negligible.



Figure 5.11: Sensitivity of results to the level of σ_f : (a) $\sigma_f = 0.05$; (b) $\sigma_f = 0.03$; (c) $\sigma_f = 0.01$.

In this section, we saw that the AADT estimation accuracy was sensitive to the distributions of Noise(D), Noise(H) and growth factors. However, in most cases, the improvement by adding image-based counts was still remarkable. Therefore, we believe that the Bayesian approach is able to work well in a large range of variation in the parameters, which represent different scenarios.

5.4 Sensitivity of Results to Incorrect Quantification of Parameters

In the simulations of previous sections, we assumed that the parameters required in the Bayesian approach are well known. Therefore, the parameters used for the data generation are the same as those used for AADT estimation process. These parameters were obtained from either analysis of empirical data or assumptions. If they are poorly representative of the true ones, the accuracy results given in Section 5.2 could be questionable. That is, adding image-based counts might not be able to improve the AADT estimation accuracy as much as shown, and even probably make the AADT estimation worse. In this section, the sensitivity of results to incorrectly quantifying those parameters is investigated.

Section 5.3 showed that the accuracy results are not sensitive to the distributions of P_k and Us. Thus it would be expected that incorrect quantification of the parameters describing the distributions of P_k and Us has little impact on the simulation results. Also, the growth factor distribution can be easily estimated from the data collected on

PATR-equipped segments in a homogeneous group. Assuming there are a sufficiently large number of PATR-equipped segments in a group, the estimation of the growth factor distribution would be fairly good.

There are two major distributions left for the sensitivity analysis, Noise(D) and Noise(H). The distributions of Noise(D) and Noise(H) reflect the "quality" of the traffic counts. The Bayesian approach is established based on Bayesian analysis, which takes advantage of different information sources based on the beliefs on their quality. If the belief on quality is incorrect, one would inappropriately use the information contained in the traffic counts; this could be thought of as putting wrong "weights" on the different traffic counts in the estimation. The accuracy results of the Bayesian approach would largely depend on the relative quality of one type of traffic count to another. As mentioned before, the change of Noise(D) distribution would affect both ground- and image-based traffic counts simultaneously, which would impair the insights from the analysis results. Therefore, we focus here on analyzing the sensitivity of the results to incorrectly quantifying the *Noise(H)* distribution, namely, the level of σ_{H} .

In this analysis, we use different values of σ_H for data generation and estimation in the simulation. Still using σ_H to denote the true value of this parameter (*i.e.* the value used in data generation), we use s_H to denote the "estimated" value that is used in the estimation part of the simulation. Two scenarios are considered in the simulation: one where $\sigma_H = 0.1$ and $s_H = 0.3$, and one where $\sigma_H = 0.3$ and $s_H = 0.1$. In the former scenario, the value of σ_H is overestimated. In the latter scenario, the value of σ_H is underestimated. All other parameters remain the same as those leading to the results shown in Figure 5.6(a).

The AADT estimation errors resulting from the simulation are plotted in Figure 5.12, for the two combinations of σ_H and s_H : ($\sigma_H = 0.1$, $s_H = 0.3$) and ($\sigma_H = 0.3$, $s_H = 0.1$). In Figure 5.12, the AADT estimation errors for the correctly σ_H estimation cases (*i.e.*, ($\sigma_H = 0.1$, $s_H = 0.1$) and ($\sigma_H = 0.3$, $s_H = 0.3$)) are also plotted to serve as a reference. The errors resulting from using only ground-based data are also plotted to serve as a reference.



Figure 5.12: Sensitivity of results to incorrectly quantification of σ_{H} .

As can be seen, a large overestimation of σ_H ($\sigma_H = 0.1$, $s_H = 0.3$) reduces the AADT estimation accuracy improvement that would have been achieved by adding image-based counts. However, the improvement is still noticeable, which is almost at the same level of the improvement that could be achieved when the true value of σ_H equals to what we estimate (*i.e.*, $\sigma_H = 0.3$, $s_H = 0.3$). On the other hand, when underestimating the value of σ_H ($\sigma_H = 0.3$, $s_H = 0.1$), adding image-based count would not improve the AADT estimation accuracy but worsen it, except for the last two years of the cycle compared to using ground-based count without the growth factors. That is, the benefits seem more sensitive to underestimation of σ_H than to overestimation.

5.5 Lognormal Approximation

All results in the previous sections are obtained by using the "entire" 3-stage model for the distribution of image-based counts conditional on the AADT. At the end of Chapter 4, a simple way of calculating the posterior distribution of AADT was introduced as Algorithm (4.1) by using a lognormal approximation to the distribution of image-based counts conditional on the AADT. Correspondingly, rather than going through the simulation of entire 3-stage model, the weights can be calculated easily from a close-formed lognormal distribution. It would be interesting to investigate the sensitivity of the accuracy results to replacing Algorithm (3.3) by Algorithm (4.1). A simulation is conducted to generate exactly the same data as those leading to Figure 5.6(a), but use Algorithm (4.1) instead of Algorithm (3.3) in the estimation part. The results are plotted in Figure 5.13.

As can be seen, using the lognormal approximation to the distribution of image-based counts conditional on the AADT in the calculation of posterior distribution slightly increase the estimation error. However, the error increase appears small, while the approximation saves a large amount of computing time and makes the Bayesian approach more easily implementable.



Figure 5.13: Results using lognormal approximation.

CHAPTER 6

SUMMARY AND CONCLUSIONS

In this work, a Bayesian approach was developed for AADT estimation. This approach allows merging image-based data with ground-based data for AADT estimation on a given segment. Specifically, the uncertainty in the AADT is expressed by a probability distribution. In any year of interest, the approach begins with a prior distribution, which is then updated according to Bayes' theorem when a traffic count is available. Sequential updating is conducted when a series of traffic counts are collected within different time periods or from different collection platforms in the year. The posterior distribution at the end of the year reflects the updated belief on the uncertainty in the segment AADT, conditional on all traffic counts collected in the year. If required, point estimates can be derived from the posterior distribution based on assumed or specified loss functions.

The Bayesian approach contains two important components, the prior distribution of the AADT and the probability distribution of short-duration traffic counts conditional on the AADT. Methods were developed to model the two components, and data parameters required for the two components were estimated based on analysis of extensive empirical data.

It was shown that a non-informative prior distribution could be used for a given year when it is assumed (or believed) that no "prior" information is available on the AADT of interest. Another type of the prior distribution was also developed from a combination of the previous-year posterior distribution and the growth factors estimated from other segments. The latter prior distribution provides a sufficient way to combine the information from traffic counts collected in different years.

The probability distribution of short-duration counts conditional on the AADT plays a critical role in the Bayesian approach. This conditional distribution is the means for the Bayesian approach to take advantage of information brought by the short-duration counts. A 3-stage model was developed for modeling the probability distribution of image-based counts conditional on the AADT. This model is general and could be applied to any short-duration traffic counts, but the focus in this work was on incorporating image-based traffic data into AADT estimation. The 3-stage model is capable of capturing the impacts of the length of segment covered by the image and the magnitude of the traffic volume on the distribution of image-based counts conditional on the AADT. The 3-stage model was seen to perform well on 22 image-based counts, compared to two other potential models. Because of its complexity, the probability distribution of image-based counts conditional on the AADT was also approximated by a lognormal distribution. The two parameters specifying the lognormal distribution can be obtained by approximate calculation (using Equation (4.3.2)) and from a look-up plot (as shown in Figure 4.3). Use of the lognormal approximation was seen to have little effect on the solution quality. Although AADT would be estimated off-line, the reduced need for extensive computer simulation and the greater transparency of the closed-form lognormal function could make the lognormal approximation more attractive for practical implementation.

A numerical study was conducted, which showed that adding image-based data with the Bayesian approach could improve AADT estimation accuracy in current practice. Specifically, segments were assumed to be sampled by a 48-hour coverage count once every 6 years. The study simulated the scenarios of different quantities of image-based data available on the segment. The results showed that adding image-based data extracted from even a single image of the segment in the AADT estimation year could greatly improve the accuracy. The improvement increases as the most recent 48-hour coverage count becomes older. Adding image-based counts could lengthen the cycle of coverage count collection while maintaining the present accuracy level of AADT estimation. Sensitivity analysis showed that the Bayesian approach could bring positive benefits for a large range of conditions.

Default values for the data parameters required for the approach have been provided in this dissertation. Otherwise, one can use data regularly collected by the state DOTs to estimate many of the parameters by using the methods introduced in this work. The algorithms for implementing the Bayesian approach have been presented, and the corresponding programs are straightforward in any coding language. The Bayesian approach does not require any additional hardware investment or additional data collection. Therefore, the Bayesian approach appears implementable by the state DOTs.

In this work, the Bayesian approach primarily showed advantages in AADT estimation because of its capability to add image-based counts to ground-based counts previously collected. The major operational concern would be to obtain the images from various sources in a form that can be easily interfaced with traffic monitoring units that estimate AADT in the state DOTs. The traffic information in the images must be extracted and put in a form (*e.g.*, number of vehicles in the image, the length of the segment covered by the image, date and time the image is taken) that could be used for AADT estimation. Designing the databases, procedures, and institutional responsibilities that would allow the image-based data to be incorporated in AADT estimation on an ongoing basis is a topic for future study.

When such a system is successfully established, existing practice could change correspondingly. The Bayesian approach produces a posterior distribution of the AADT on a given segment. If we use the coefficient of variation (CV) of the distribution to represent the uncertainty in the AADT, the value of CV could serve as a criterion for determining whether new traffic counts need to be collected for a given segment, either on the ground or from imagery. For example, if recently collected image-based data produce a CV less than 0.1 (an assumed CV threshold value for data collection) for a segment, a new cycle of ground coverage counts for that segment might be skipped and the saved resources could be used for collecting traffic counts on other segments in the system with higher CVs. By using the system, the traditional approach of collecting coverage counts on an "n-year" cycle might be converted to a new strategy of coverage count collection that is based on the uncertainty in AADT estimates. In addition, the traffic monitoring unit could request the aerial engineering division of the DOT to obtain images as they fly over highway segments when flying other missions, especially over segments with highly uncertain AADT estimates. If the aircraft is already being used for other missions, the marginal cost of the data collection would be low.

To implement the Bayesian approach, the Monte Carlo representation of the posterior distribution in each year needs to be stored. Otherwise, all previous traffic counts need to be stored. Although there are continuing developments in data storage devices, posterior distributions for all segments in a system would still be a large burden for data storage. An alternative might be to use a theoretical distribution to approximate the resultant posterior distribution. Addressing the storage issues is a left as a topic for future study.

A major reason for the storage of posterior distributions is to provide prior information for the AADT estimation in the following year. One should be careful when using this type of prior information when the highway network or the trip pattern distribution (origin-destination matrix) changes significantly during the period considered. When such changes occur, traffic volumes on a highway segment of interest might change greatly, and prior information obtained from the previous application of the method would not be appropriate for estimating AADT in the contemporary year. A non-informative prior might be an appropriate choice for this case. A more thorough analysis of this problem would be a topic for future study, but it is noted that a similar problem would exist with current practice when using old ground counts to estimate AADT on a segment when there have been recent changes to the network or origin-destination matrix.

In this work, AADT was considered to be the total volume aggregated across all types of vehicles. Increasing attention is being given in practice to classifying traffic volumes according to vehicle type. In Appendix B, a Bayesian approach is sketched out for classified AADT estimation. For the sake of illustration, vehicles are grouped into two classes – large vehicles (called trucks) and small vehicles (called cars). The problem is two-dimensional, which makes the approach more complicated in terms of modeling the correlation between truck and car volumes and calculating the two-dimensional posterior distribution. Therefore, an alternative Bayesian approach is also sketched out in Appendix B that may be capable of turning the two-dimensional problem into two independent one-dimensional problems. Some issues related to the modeling and implemention of the approach are discussed. However, a thorough development requires more intensive

analysis of data collected from segments equipped with permanent classification traffic recorders, and such analysis is left for future study.

This work only considered image-based data obtained during periods when traffic is non-transitioning and operating under free-flow conditions. Such traffic conditions facilitated the modeling of the probability distribution of image-based counts conditional on the hourly volume, the third stage of the 3-stage model for establishing the probabilistic model of image-based counts conditional on the AADT. When image-based data are obtained under congested or transitioning traffic conditions, the framework of the 3-stage model might be adopted, but the third stage (*i.e.*, the probability distribution of image-based counts conditional on the hourly volume) would likely need to be modified. Under congested conditions, the movement of a given vehicle would generally be affected by the movements of other vehicles. When traffic is transitioning, it is not reasonable to assume that equal sections of a given segment are equally likely to contain the same number of vehicles. Thus, the binomial distribution would appear inappropriate for the image-based data taken during transitioning or congested traffic conditions.

For a given segment, there appears to be a maximum traffic volume that could be handled during one hour. When the hourly volume is close to this point, the traffic is more likely to be congested. Thus, whether traffic conditions are considered to be congested would mainly depend on the simulated hourly volume, the "output" of the second stage of the 3-stage model. Whether traffic conditions are transitioning would mainly rely on the hour of the day; for example, traffic density of the segment tends to increase as the peak hour approaches and decrease after the peak hour. It would be tricky to model the third stage for periods that are typically either congested (*e.g.*, peak hours) or transitioning (*e.g.*, hours before and after peak hours). Accomplishing this task would rely on a solid understanding of transitioning and congested traffic phenomena that are justified by empirical data. Again, this is a topic for future study. At present, if image-based data are intended to be incorporated into the AADT estimation through the Bayesian approach, one might consider discarding the image-based data collected when congested or transitioning conditions are suspected.

In summary, the Bayesian approach presented in this work appears ready for pilot tests of incorporating image-based data obtained under uncongested and non-transitioning traffic conditions when estimating unclassified AADT. More refined estimation of the data parameters could be achieved during the tests. The ultimate goal would be a well-established system for AADT estimation, in which all types of traffic counts available for every segment are stored and used in a systematic and theoretically acceptable procedure.

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APPENDIX A

ANALYSIS OF DATA FROM 24 PATR-EQUIPPED HIGHWAY SEGMENTS IN OHIO

In this appendix, we investigated *Noise(D)* and *Noise(H)* defined in Chapter 3 by analyzing one year's worth of traffic data collected from 24 Ohio PATR-equipped highway segments across 4 functional classes (01-Rural Interstate, 02-Rural Other Principle Arterial, 11-Urban Interstate, 12-Urban Other Freeways and Expressways). The information of the 24 segments (their functional classes (FC), PATR numbers and AADT values) is given in Table A.1. The locations of the 24 PATRs are shown in Figure A.1.

FC	PATR	AADT	FC	PATR	AADT	FC	PATR	AADT	FC	PATR	AADT
01	507	19984	02	058	4376	11	154	36523	12	704	35380
01	553	27398	02	509	4595	11	737	49295	12	727	40769
01	156	27723	02	021	6415	11	140	59520	12	554	41421
01	531	42824	02	532	9920	11	535	59521	12	557	45507
01	508	46641	02	136	12322	11	536	62642	12	105	68964
01	506	48328	02	501	15802	11	157	110349			
			02	160	19725						

Note: AADT values are calculated by the AASHTO method, based on traffic data collected from March 2003 to February 2004

Table A.1: Information of 24 PATR-equipped Segments


Figure A.1: Locations of 24 PATRs.

A.1 *Noise*(*D*) across 4 functional classes

Noise(D) represents the "unexplained" variation of a daily volume from the AADT after the adjustment of monthly and day-of-week factors. Equation (3.2.3a) is repeated as Equation (A.1) for convenience:

$$Noise(D(\delta)) = [V^{24}(\delta) \times F^{M}(m(\delta); f) \times F^{D}(d(\delta); f)] / AADT.$$
(A.1)

The notation for monthly and day-of-week factors in Equation (A.1) differs slightly from the notation in Equation (3.2.3a) since Noise(D) is investigated by function class in this appendix. Here, "f" represents the functional class, and each functional class is considered a homogeneous group where the segments "share" a common set of monthly and day-of-week factors. According to Equation (A.1), Noise(D) can be regarded as a ratio of the estimated AADT to the true AADT on the segment, where the estimated AADT is developed from the daily volume by de-seasonalizing this daily volume with monthly and day-of-week factors.

We determined the month-of-year and day-of-week factors for each PATR-equipped segment *i* as (McCord *et al.*, 2002a):

$$F_i^M(m) = AADT_i / \langle V_i^{24}(\delta) \rangle_{m(\delta)=m}, \quad m = 1, 2, ..., 12$$
 (A.2a)

$$F_i^D(d) = AADT_i / \langle V_i^{24}(\delta) \rangle_{d(\delta)=d}, \quad d = 1, 2, ..., 7$$
 (A.2b)

and the month-of-year and day-of-week factors for each functional class f as the harmonic mean of the individual factors:

$$F^{M}(m;f) = [F_{i}^{M}(m)]_{i \in I(f)}, m = 1, 2, ..., 12,$$
(A.3a)

$$F^{D}(d;f) = [F_{i}^{D}(d)]_{i \in I(f)}, \quad d = 1, 2, ...7.$$
 (A.3b)

where $\langle . \rangle_{m(\delta)=m}$ and $\langle . \rangle_{d(\delta)=d}$ represent the arithmetic average over all days-of-the-year δ that are, respectively, in month *m* and on day-of-the-week *d*; [.]_{*i* \in *I*(*f*)} represents the harmonic average over the segments that belong to functional class *f*; and *AADT*_{*i*} is the AADT on the PATR-equipped segment *i* calculated by the AASHTO method.

f	Number of "observed" <i>Noise</i> (<i>D</i>)'s	Mean of <i>Noise</i> (<i>D</i>)	Std. Dev. of Ln[Noise(D)]
01	2070	1.0012	0.1165
02	2361	1.0049	0.1287
11	2015	0.9972	0.1166
12	1721	1.0007	0.1278

Table A.2: Statistical Characteristics of the "Observed" Noise(D)'s by Functional Class

For each functional class, we use Equation (A.1) to produce the "observed" Noise(D) values according to the available daily volumes on all segments, with the "average" monthly and day-of-week factors given in Equation (A.3). Some statistical characteristics of these Noise(D) values are presented in Table A.2.

The natural logarithms of these Noise(D) values are checked in a Q-Q normal plot by functional class, as shown in Figure A.2. We see that the distribution falls mostly along the 45° straight line, with some deviation in the tails. (It is possible that anomalies on holidays, which were not excluded from the analysis, account for some of the deviations from normality in the tails.) Although the deviation in the lower tail is marked, the proportion of observations in the portions of the tails that deviate from the 45° straight line is small. Therefore, as a modeling approximation, it appears that Noise(D) can be modeled by a lognormal distribution.



Figure A.2: Normal Q-Q plot for the natural logarithms of the "observed" Noise(D) values by functional class.

It would be a desirable property that the expectation of Noise(D) is equal to 1, since a de-seasonalized daily volume is expected to be the AADT except for some "random" variation in that day. Also the sample mean of the "observed" Noise(D) values given in Table A.2 does not show a large deviation from one for each functional class. Therefore, the distribution of Noise(D) is mathematically expressed as

$$\operatorname{Ln}[\operatorname{Noise}(D)] \sim N(-\sigma_D^2/2, \sigma_D^2), \tag{A.4}$$

where Ln[Noise(D)] is the natural logarithm of Noise(D), which according to (A.4) follows a normal distribution, and σ_D is the standard deviation (Std. Dev.) of the normal distribution. In (A.4), the use of $-\sigma_D^2/2$ as the mean of the normal distribution ensures that the distribution mean of Noise(D) is equal to one (McCord *et al.*, 2000). Based on the σ_D values of the four function classes presented in Table A.2, a default σ_D value is set at 0.12 for the numerical studies described in the work.

A.2 *Noise*(*H*) across 4 functional classes

Noise(H) represents the "unexplained" variation of an hourly volume from the average hourly volume in the same day, after the adjustment of hourly factors. Equation (3.2.5a) is repeated as Equation (A.5) for convenience

$$Noise(H(h,\delta)) = [V^{H}(h,\delta) \times F^{H}(h,d(\delta);f)) / [V^{24}(\delta)/24].$$
(A.5)

The notation for the hourly factor in Equation (A.5) also differs slightly from the notation in Equation (3.2.3a). Here, "f" represents the functional class, and each functional class is considered a homogeneous group where the segments "share" a common set of hourly factors. We noticed a fairly large amount of variability in the hourly patterns among different days-of-the-week. We, therefore, applied seven different sets of hourly factors, one set for each day of the week, as denoted by $F^{H}(h,d(\delta);f)$. According to Equation (A.5), Noise(H) can be regarded as a ratio of the estimated daily volume to the true daily volume, where the estimated daily volume is developed from the

hourly volume expanded to account for all 24 hours in the day and the hourly variability as represented by the hourly factor.

We determined the hourly factors for each PATR-equipped segment *i* as (McCord *et al.*, 2002a):

$$F^{H}_{i}(h,d) = (\langle V_{i}^{24}(\delta) \rangle_{d(\delta)=d}/24)/\langle V_{i}^{H}(h,\delta) \rangle_{d(\delta)=d}, \quad d = 1,2, \dots,7; \ h = 1,2,\dots,24;$$
(A.6)

and the hourly factors for each functional class f as the harmonic mean of these individual factors:

$$F^{H}(h,d;f) = [F^{H}_{i}(h,d)]_{i \in I(f)}, \quad d = 1,2,...,7; h = 1,2,...24$$
(A.7)

where $\langle . \rangle_{d(\delta)=d}$ represents the arithmetic average over all days-of-the-year δ that are on day-of-the-week *d*, and [.]_{*i* \in *I*(*f*)} represents the harmonic average over the segments that belong to functional class *f*.

Since images would be taken during the daytime, analysis should be limited to daytime hours. In addition, the images obtained in (McCord *et al.*, 2002a), which were used to evaluate the 3-stage model, were all taken between 10:00am and 1:00pm. Therefore, we only considered the "observed" *Noise*(*H*) values from 10:00am and 1:00pm. Specifically, for each functional class, we use Equation (A.5) to produce the *Noise*(*H*) values "observed" from 10:00am and 1:00pm, with the "average" hourly factors given in Equation (A.7). Some statistical characteristics of these *Noise*(*H*) values are presented in Table A.3.

f	Number of "observed" <i>Noise</i> (<i>H</i>)'s	Mean of <i>Noise</i> (<i>H</i>)	Std. Dev. of Ln[Noise(H)]
01	6207	0.9998	0.0925
02	7077	1.0017	0.1059
11	6042	1.0017	0.0847
12	5160	1.0020	0.0877

Table A.3: Statistical Characteristics of the Noise(H) Values "Observed" from 10:00am to 1:00pm by Functional Class

The natural logarithms of these Noise(H) values are checked in a Q-Q normal plot by functional class, as shown in Figure A.3. We see that the distribution falls mostly along the 45° straight line, with some deviation in the tails. (It is possible that anomalies on holidays, which were not excluded from the analysis, account for some of the deviations from normality in the tails.) Similar to the Ln[*Noise(D)*] values in Figure A.2, the proportion of observations in the tails is small. Therefore, as a modeling approximation, it appears that *Noise(H)* can be modeled by a lognormal distribution.

Similar to Noise(D), it would be a desirable property that the expectation of Noise(H) is equal to 1. In addition, the sample mean of the "observed" Noise(H) values given in Table A.3 does not show a large deviation from one for each functional class. Therefore, the distribution of Noise(H) is mathematically expressed as

$$\operatorname{Ln}[\operatorname{Noise}(H)] \sim N(-\sigma_{H}^{2}/2, \sigma_{H}^{2}), \qquad (A.8)$$

where Ln[Noise(H)] is the natural logarithm of Noise(H), which according to (A.8)



Figure A.3: Normal Q-Q plot for the natural logarithms of the Noise(H) values "observed" from 10:00am to 1:00pm by functional class.

follows a normal distribution, and σ_H is the standard deviation (Std. Dev.) of the normal distribution. In (A.8), the use of $-\sigma_H^2/2$ as the mean of the normal distribution ensures that the distribution mean of *Noise*(*H*) is equal to one (McCord *et al.*, 2000). Based on the σ_H values of the four function classes presented in Table A.3, a default σ_H value is set at 0.10 for the numerical studies described in the work.

APPENDIX B

CLASSIFIED (TRUCK AND CAR) AADT ESTIMATION

Increasing attention is being given in practice to classifying traffic volumes according to vehicle type. The *TMG* (FHWA, 2001) suggests a nationwide conversion from traditional total volume-emphasized data collection programs to such classification-based programs. In this appendix, an approach based on the Bayesian analysis is sketched out for classified AADT estimation, parallel to the one proposed in Chapter 3. For the sake of illustration, vehicles are grouped into two classes – large vehicles (called trucks) and small vehicles (called cars). Correspondingly, there are two classified AADTs – truck and car AADTs, denoted as *_KAADT* and *_CAADT*, respectively.

This appendix considers the case that both truck and car volumes are collected during a short-term time period (including an equivalent short-term period corresponding to an image-based count) for the truck and car AADT estimation. As in the case of estimating total AADT, the problem here is to estimate the yearlong traffic average based on short-term observations. Specifically, ${}_{K}AADT$ and ${}_{C}AADT$ will be estimated based on short-term truck and car traffic volumes, respectively. It is assumed that the short-term truck and car volumes are observed during the same time period. To facilitate the 177

following analysis, we denote the observed short-term truck and car volumes by a vector $\tilde{V} = [_k V^T, _c V^T]$, where the subscripts *k* and *c* represent truck and car, respectively; the superscript *T* stands for the length of the short-term period (or the length of imaged segment for image-based counts). In the same manner, we introduce the AADT vector $\tilde{A} = [_{\kappa}AADT, _{c}AADT]$. This is now a 2-dimensional (bi-variate) problem. First, we mathematically express the approach as

$$\pi(\tilde{A} | \tilde{V}) = \frac{f(\tilde{V} | \tilde{A})\pi(\tilde{A})}{m(\tilde{V})},$$
(B.1)

where $\pi(\tilde{A}|\tilde{V})$ is the posterior distribution of the paired truck and car AADTs \tilde{A} conditional on the observed short-term paired truck and car volumes \tilde{V} ; $f(\tilde{V}|\tilde{A})$ is the probability distribution of the short-term truck and car volumes conditional on the truck and car AADTs; $\pi(\tilde{A})$ is the prior distribution of the truck and car AADTs; $m(\tilde{V})$ is the marginal distribution of the short-term truck and car volumes, which can be obtained by integrating the numerator of the right side of Equation (B.1) over the space of \tilde{A} . Equation (B.1) is similar to Equation (3.1.1a). It differs in that it represents bi-variate updating.

The key issue involved with implementing Equation (B.1) is to model the conditional distribution $f(\tilde{V} | \tilde{A})$ and prior distribution $\pi(\tilde{A})$. Recalling the complexity of the conditional distribution in the one-dimensional problem, modeling the two-dimensional distribution $f(\tilde{V} | \tilde{A})$ would not be less complicated.

If it is assumed that the truck and car traffic volumes are mutually independent, one could estimate $_{\kappa}AADT$ and $_{c}AADT$ with two independent procedures. No information from the car AADT estimation is used for the truck AADT estimation, and vice-versa. The prior distribution of $_{\kappa}AADT$ would be updated to a posterior distribution, conditional on the short-term truck volume $_{k}V^{T}$, without the use of any information on the short-term car volume $_{c}V^{T}$ and the prior distribution of $_{c}AADT$; similarly, the prior distribution of $_{c}AADT$ is updated to a posterior distribution, conditional on the short-term car volume $_{c}V^{T}$, without the use of any information on the short-term car volume $_{c}V^{T}$, without the use of any information on the short-term car volume distribution of $_{c}AADT$; similarly, the prior distribution of $_{c}AADT$ is updated to a posterior distribution, conditional on the short-term car volume $_{c}V^{T}$, without the use of any information on the short-term truck volume $_{k}V^{T}$ and the prior distribution of $_{c}AADT$. Mathematically, the conditional distribution $f(\tilde{V} | \tilde{A})$ will become

$$f(\widetilde{V} | \widetilde{A}) = f(_{k}V^{T}|_{K}AADT) \times f(_{c}V^{T}|_{C}AADT), \qquad (B.2)$$

where $f(_{k}V^{T}|_{K}AADT)$ is the distribution of the short-term truck volume $_{k}V^{T}$ conditional on the truck AADT, and $f(_{c}V^{T}|_{C}AADT)$ is the distribution of the short-term car volume $_{c}V^{T}$ conditional on the car AADT. In the same way, the prior $\pi(\tilde{A})$ will become

$$\pi(\tilde{A}) = \pi({}_{K}AADT) \times \pi({}_{C}AADT), \qquad (B.3)$$

where $\pi({}_{\kappa}AADT)$ and $\pi({}_{c}AADT)$ are the priors of truck and car AADTs, respectively. Therefore, the two-dimensional problem given in Equation (B.1) would turn into two independent one-dimensional problems:

Truck
$$\pi({}_{K}AADT|_{k}V^{T}) = \frac{f({}_{k}V^{T}|_{K}AADT)\pi({}_{K}AADT)}{m({}_{k}V^{T})},$$
 (B.4a)

Car
$$\pi(_{c}AADT|_{c}V^{T}) = \frac{f(_{c}V^{T}|_{c}AADT)\pi(_{c}AADT)}{m(_{c}V^{T})},$$
 (B.4b)

where $\pi({}_{\kappa}AADT|_{k}V^{T})$ and $\pi({}_{c}AADT|_{c}V^{T})$ are the posterior distributions for truck and car AADTs after the short-term observations ${}_{k}V^{T}$ and ${}_{c}V^{T}$, respectively; $m({}_{k}V^{T})$ and $m({}_{c}V^{T})$ are the marginal distributions for ${}_{k}V^{T}$ and ${}_{c}V^{T}$, respectively, and can be obtained by integrating each corresponding numerator over the related space. The conditional distributions and prior distributions in Equations (B.4) can be modeled in the same manner as proposed in Chapter 3. The systematic temporal patterns for truck and car volumes are critical for the modeling of the conditional distributions.

The assumption above assumed independence in truck and car volumes. However, truck and car traffic volumes on the same roadway would likely have some dependence. For example, it is not likely that the one-day truck volume exceeds the one-day car volume for most highway segments. Therefore, knowledge of the car AADT could provide some useful information for the estimation of truck AADT. The difficulty in implementing Equation (B.1) lies largely in addressing the existence of inter-variable correlations. More data and assumptions would be required to model this correlation. How to do so is left for the future study.

Since the sum of *KAADT* and *CAADT* is the total AADT, an alternative method might be considered for modeling truck and car AADT estimation. The truck and car AADTs can be uniquely determined from the total AADT and the truck proportion of the AADT

(TPA). Similarly, the short-term truck and car volumes can be replaced by a short-term total vehicle volume and a corresponding truck proportion. That is, the problem becomes one of estimating total AADT and TPA from observed short-term total volume and truck proportion. Similarly, one can think of two vectors: $\tilde{V}_p = [V^T, _k p^T]$, and $\tilde{A}_p = [AADT, _{K}P]$, where $V^T = _k V^T + _c V^T$ and $_k p^T = _k V^T / V^T$; *AADT* and $_K P$ denote the AADT and the TPA. Then, the Bayesian approach can be expressed as

$$\pi(\tilde{A}_{P} | \tilde{V}_{p}) = \frac{f(\tilde{V}_{p} | \tilde{A}_{P})\pi(\tilde{A}_{P})}{m(\tilde{V}_{p})}$$
(B.5)

where like Equation (B.1), $\pi(\tilde{A}_p | \tilde{V}_p)$ is the posterior distribution, $f(\tilde{V}_p | \tilde{A}_p)$ is the conditional distribution, $\pi(\tilde{A}_p)$ is the prior distribution, and $m(\tilde{V}_p)$ is the marginal distribution. Rather than using car and truck volumes as in Equation (B.1), total volume and truck proportion are used in Equation (B.5).

Transforming the truck and car volumes to a total volume associated with the corresponding truck proportion increases the possibility of the independence between the two variables of interest. Intuitively, given the functional class of the highway of interest, knowledge of total volume seems to provide little additional information on the truck proportion; similarly, knowledge of truck proportion seems to provide little additional information on the total volume. The observed truck proportion on the image might have some effects on the estimation of total volume, but such an effect is expected to be very small, as we saw in Chapter 4 and 5. Therefore, it seems reasonable to treat the estimation

of the AADT and the TPA as two independent procedures. The reasonableness of the independence and empirical grounds would be a topic for the future study. Corresponding to the conversion of Equation (B.1) to Equations (B.4), we convert Equation (B.5) to two independent equations:

$$\pi(AADT | V^{T}) = \frac{f(V^{T} | AADT)\pi(AADT)}{m(V^{T})}, \qquad (B.6a)$$

$$\pi({}_{K}P|_{k}p^{T}) = \frac{f({}_{k}p^{T}|_{K}P)\pi({}_{K}P)}{m({}_{k}p^{T})},$$
(B.6b)

Equation (B.6a) is equivalent to Equation (3.3.1a), except for the slight difference in the notation. In Equation (B.6b), $\pi({}_{\kappa}P|_{k}p^{T})$ is the posterior distribution of the ${}_{\kappa}P$ conditional on the observed short-term truck percentage ${}_{k}p^{T}$; $f({}_{k}p^{T}|_{\kappa}P)$ is the distribution of the short-term truck percentage ${}_{k}p^{T}$ conditional on the ${}_{\kappa}P$; $\pi({}_{\kappa}P)$ is the prior distribution of the ${}_{\kappa}P$; $m({}_{k}p^{T})$ is the marginal distribution of the short-term truck percentage ${}_{k}p^{T}$ conditional on the ${}_{\kappa}P$; $\pi({}_{\kappa}P)$ is the prior distribution of the ${}_{\kappa}P$; $m({}_{k}p^{T})$ is the marginal distribution of the short-term truck percentage ${}_{k}p^{T}$, which can be obtained by integrating the numerator of the right side of Equation (B.6b) over the space of ${}_{\kappa}P$.

Most of the work presented here has dealt with the total AADT estimation from short-term volumes based on the Bayesian approach. This work would extend directly to Equation (B.6a). To estimate the $_{\kappa}P$ from the short-term truck proportion, we need to model the prior distribution $\pi(_{\kappa}P)$ and the conditional distribution $f(_{k}p^{T}|_{\kappa}P)$. Again the modeling could follow the approach proposed in Chapter 3. For example, the non-informative prior distribution $\pi(_{\kappa}P)$ can be modeled as a uniform distribution between 0 and 1. The prior can also be empirically modeled based on the distribution of TPA's across a set of highway segments equipped with permanent classification traffic recorders (PCTRs), which are believed to have a similar pattern as the segment of interest in terms of truck proportion. The Bayesian approach would again be implemented year after year, so the prior distribution in year y could be derived from the posterior distribution of year y-1 adjusted by the yearly change in truck proportion observed at the PCTR locations believed to exhibit the similar patterns as the segment of interest. Note we use the term of "yearly change in pattern," rather than "growth pattern" because we do not want to confine changing patterns in truck proportion to increasing only. The conditional distribution $f({}_{k}p^{T}|_{K}P)$ represents the probability of observing the ${}_{k}p^{T}$ conditional on the $_{K}P$. The variations in the truck proportion for different time scales could be modeled through the investigation of traffic data collected at PCTR locations. The conditional distribution could then be developed in a manner similar to that used in Chapter 3.

It would be interesting to compare the approaches proposed in Equations (B.1) and (B.5). If the approaches are modeled and implemented correctly, both approaches should produce the same result for the classified AADT estimation.

Before concluding this appendix, a brief discussion is presented on the computation of the posterior distribution. If the independence assumption does not hold, the problem of the truck and car AADT estimation will be two-dimensional. Consequently, the computation of the posterior distribution would be more difficult than that for one-dimensional problems. Fortunately, a large number of references are available for this subject. During the last decade, Markov Chain Monte Carlo (MCMC) methods have been used in the area of Bayesian inference, especially for high-dimensional problems (Martinez and Martinez, 2002). The MCMC method would be a good point to start when considering a way to numerically estimate the posterior distribution for truck and car AADT estimation.

In this appendix, we sketched two alternative methods for the truck and car AADT estimation. The independence assumption would simplify the problem, but it might not be reasonable. Whatever the approach, the successful implementation of an approach for estimating truck and car AADT would largely depend on a good understanding of the patterns of truck and car traffic volumes across the highway system. Such an understanding would begin with analysis of the data at PCTR locations.