STYLISTIC COMPARISONS OF PRIMARY THEMES IN THE STRING QUARTETS OF HAYDN, MOZART AND BEETHOVEN

A Thesis

Presented in Partial Fullfillment of the Requirements for the degree Master of Arts in the Graduate School of the Ohio State University

by

Mark Lewis Lochstampfor, B.M., A.A.

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The Ohio State University

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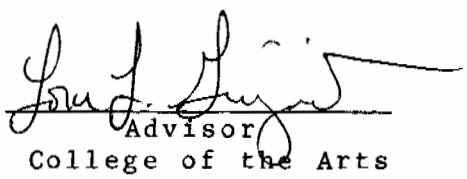
Master's Examination Committee:

Lora L. Gingerich

Marshall H. Barnes

A. Peter Costanza

Approved by



Advisor
College of the Arts

To My Parents

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VITA

January 22, 1956	Born - Portsmouth, Virginia
1975	A.A., Lakeland Community College, Mentor, Ohio
1978	B.M., Ohio University, Athens, Ohio
1978-1980	Entertainer, Joe Gadd and Associates, Inc., Aurora, Ohio
1980-1985	Instructor in Piano and Music Theory, Conservatory of Piano, Columbus, Ohio
1985	Advisor/Instructor of Music Theory, Encore Music Studios, Westerville, Ohio
1980-Present	Music Director, Parkview Presbyterian Church, Reynoldsburg, Ohio
1985-Present	Graduate Teaching Associate, Ohio State University, Columbus, Ohio

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The Quartets and Their Historical Relationships

Introduction

This study explores stylistic features of works by Haydn, Mozart, and Beethoven, by comparing the melodic intervallic content and the harmonic progressions in excerpts from their string quartets. The computer is used as a tool in collating and processing musical data. Information theory is used as a method for comparing the results. Works by the Classic Viennese composers are appropriate subject materials for such a study because of the moderately restricted harmonic vocabulary and use of chromaticism. Works of the same genre, the string quartet, are studied to ensure the most reliable comparison. Therefore, the works I have chosen for this study are: Haydn Op.33, Mozart "Haydn Quartets", KV 387, 421, 428, 458, 464, and 465, and Beethoven Op.18. There are similarities

among these three sets of quartets. Each set comprises six quartets, five in major keys and one in a minor key and each set was composed at a transitional time in the composer's career. These similarities, and others which I recognize in the following paragraphs, should evince my interest in comparing these particular sets of quartets.

Haydn - The Opus 33 Quartets

Haydn's Op.33 quartets were published in 1781, a landmark in his career for many reasons. In each quartet the minuet is quicker than in his previous quartets, thus the title "scherzo" is used to indicate the livelier tempo. During the evolution of this opus, the scherzo movement is transferred from the second movement of the earlier quartets to the more traditional third movement in the later quartets. The finales are lighter rondo and variation forms rather than heavier forms like the fugal finales of Op.20. Donald F. Tovey refers to them as the "lightest in all Haydn's mature comedies."[1] In Op.33, Haydn becomes more economical with his materials, showing a greater ability to fragment thematic materials for developmental purposes. Rosemary Hughes refers to them as "not mere tunes but

1. Donald Francis Tovey, "Haydn," Cobbett's Cyclopedic Survey of Chamber Music, 2v., compiled and edited by Colin Mason, (London: Oxford, 1963), I, 538.

pregnant figures capable of organic growth."[2] Haydn also shows greater concern for regular phrase structure within the early stages of each movement, yet maintains irregular phrases in later stages, often through the use of phrase extension. The reduced use of the sonata form, from two or three movements in a quartet to the first movement only, is another significant new feature. The most salient feature of Op.33 is Haydn's approach to the treatment of the instrumentation. The four instrumental voices are harmonically and melodically equal, creating a texture that unites "style galant" melody with counterpoint.[3]

Mozart - The 'Haydn' Quartets

Mozart's string quartets, KV 387, 421, 428, 458, 464, and 465, were composed between December 1782 and January 1785. They were first published in Vienna by Artaria in 1785 as 'Opera X.' The publication includes a dedication to Haydn on the title page.[4] They represent Mozart's first mature set of string quartets. Mozart's sketches indicate,

2. Rosemary Hughes, Haydn String Quartets, (Seattle: University of Washington Press, 1966), 28.

3. Homer Ulrich, Chamber Music; The Growth and Practice of an Intimate Art, (New York: Columbia University Press, 1948), 223.

4. Stanley Sadie, The New Grove: Mozart, (New York: Norton, 1980), reprint 1985, 95.

and scholars concur, that these quartets are the result of toiling efforts for the composer.[5] In 1782 Haydn's Op.33 were already completed and published, and thus available to Mozart as examples of Haydn's mature quartet form. Consequently, it is not surprising that Mozart's quartets share many features with Haydn's Op.33. Several scholars have cited parallels between these sets of quartets.[6] Stanley Sadie expresses confidence in Mozart's knowledge of Haydn's Op.33 and its resulting influence on his own music.[7] Two significant aspects of this influence are Mozart's use of texture in passing the melody among the instrumental voices, and his approach to quartet form. These quartets also mark the beginning of a "new modern spirit" in Mozart's chamber music which Hermann Abert attributes to a change in Mozart's inspiration for composing, as these quartets are the first chamber pieces Mozart wrote to fulfill his own artistic needs rather than composed in response to an external stimulus such as a commission.[8]

5. Sadie, 94.
Ulrich, 222.

6. Hermann Abert, "Mozart," in Cobbett's Cyclopedic Survey of Chamber Music, 2v., compiled and edited by Colin Mason, (London: Oxford, 1963), II, 155.
Reginald Barrett-Ayres, Joseph Haydn and the String Quartets, (London: Barrie and Jenkins, 1974), 188, 192-3, 196.

7. Sadie, 96.
Charles Rosen, The Classical Style: Haydn, Mozart, Beethoven, (New York: Norton, 1972), 266.

Beethoven - The Opus 18 Quartets

Beethoven composed his first set of string quartets, Op.18, between 1798 and 1800. Parallels have been drawn between this set of quartets and the aforementioned mature quartets by Haydn and Mozart. Ulrich indicates that Op.18 represents Beethoven's first maturity.[9] Kerman however, is quick to remind us of the many weaknesses in these early quartets, but he does so with expressed empathy for the challenge which the composer had undertaken with these works.[10] The Op.18 quartets required the composer to pursue new compositional territory in a genre dominated by the veterans Haydn and Mozart. The debate over the maturity of these quartets is not the purpose of this study, so let it suffice to state that these quartets do represent Beethoven's work at a significant transition in his compositional style. In this way these quartets parallel the other composers' quartets. Several scholars including Kerman, Marliave, Mason, Rosen, Tyson and Ulrich cite similarities in these Beethoven quartets with the works by Haydn and Mozart.[11] Kerman in particular cites the

8. Abert, 155.
Ulrich, 222.

9. Ulrich, 264.

10. Joseph Kerman, The Beethoven Quartets, (New York: Knopf, 1967), 59.

relationship of a borrowed melody in the Adagio of Op.18, No.5 from the Andante of KV 464, which Mozart borrowed from Haydn's quartet Op.20, No.4.[12] Historically, there is little evidence to prove a direct influence by Haydn or Mozart on Beethoven as he wrote these quartets. However, we do know that Beethoven studied composition with Haydn for at least fourteen months during the years preceding these works. We also have strong evidence that Beethoven studied Mozart's compositions. It is recorded in the Koechel Catalogue that Beethoven copied movements from Mozart's quartets, including the finale of KV 464.[13]

11. Vincent D'Indy, "Beethoven," in Cobbett's Cyclopedic Survey of Chamber Music, 2v., compiled and edited by Colin Mason, (London: Oxford, 1963), I, 86-7.
Kerman, 34, 44-5, 58-60.
Joseph De Marliave, Beethoven's Quartets, (New York: Dover, 1957), 12-3, 17-8, 20-1.
Daniel Gregory Mason, The Quartets of Beethoven, (New York: Oxford, 1957), 29, 41-3, 54, 56-69.
Rosen, 380-1, 385.
Ulrich, 265-7.

12. Kerman, 60.

13. Daniel Gregory Mason, 56.

Chapter II

The Computer as a Tool for Analysis

Preparing The Melodic Data

A study such as this, in which a computer is used for data processing, requires specialized preparation of the musical features to be studied. In order to maintain a reasonable length, this study includes only the melodic and harmonic features of the primary theme areas in the first movements of the aforementioned quartets, all of which exhibit sonata form. As a basis for comparison, the first movement of four works from earlier and later periods are also examined including Sammartini, Symphony for Strings in D Major, a Baroque work, Haydn, String Quartet Op.17, No.6 an early Classic work, Beethoven, String Quartet Op.95, a late Classic work, and Schubert, String Quartet No.5, D.36, a Romantic work. A Baroque example in the same genre was not chosen for two reasons. First, the string quartet was

not yet a standard instrumental genre, and second, the sonata form had not yet evolved. The Sammartini work provides a suitable choice for comparison as it is composed for string orchestra in parallel binary form with a tonic theme area followed by a transition to the second theme area in the dominant, thus corresponding adequately to sonata form for the purposes of this study.

The musical excerpts used in this study are chosen because of their common elements; first movements of the same genre and form, with the entire primary theme areas remaining in the tonic key of each quartet. To avoid subjective decisions in ambiguous situations, the primary theme area is determined to close at the last clearly articulated cadence before a transition or modulation. The main melodic voice to be examined is determined by these factors:

1. A melody accompanied by the other instrumental voices.
2. If the melody exchanges voices, the main melodic line will be recognized in the new voice.
3. If the melody exchanges voices in an imitative manner, the main melodic line will be recognized in the new voice. It is important to clarify that none of these theme areas involve a true

fugal exposition, which would greatly complicate this study (see Ex. 5 below).

4. In ambiguous situations, the uppermost instrumental voice will be considered the main melodic line.

The computer is an important tool in this study for processing the data. The main melodic line is translated into Musicode-A, an encoding system developed by Ann K. Blombach , then filed in the computer for processing.[14] This data contains an information field providing important processing data about the clef, key signature, and meter of the encoded music. The encoding of each note provides the information of pitch, octave register, and duration. The duration data accomodates note values from whole notes to sixty-fourth notes, dotted and doubly dotted notes, triplet figures, and tied notes. Bar lines, and double bar lines are encoded in the data, repeat signs however, are excluded for this study because of the brevity of the excerpts. The meter can be processed from the information field and used to calculate the values of the duration symbols. Rests are

14. Ann K. Blombach, "An Introductory Course in Computer-assisted Music Analysis: The Computer and the Bach Chorales," in Journal of Computer-Based Instruction, Feb. 1981, Vol.7, No.3, 70-77.

_____ "A Conceptual Framework for the Use of the Computer in Music Analysis." (Doctoral dissertation, The Ohio State University, 1976). Dissertation Abstracts International, 1976, 37, 4683A.

also included in this data. Musical features such as ornamental figures, phrasing, articulation, and dynamics are not encoded by this system.

Preparing The Harmonic Data

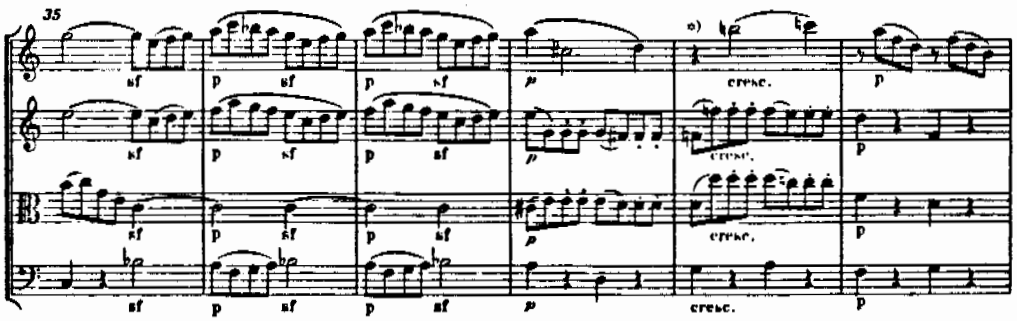
The harmonic features are translated using Harmonic Musicode, a system I developed, based on Musicode-A, then filed in the computer for processing. Harmonic Musicode utilizes the same encoding system as Musicode-A for durations, bar lines, double bar lines and calculating meter. It also encodes the root of the chord, the chord quality, the inversion, the presence of a seventh, a major or minor ninth, and an augmented sixth (see Appendix A). A harmony which lasts over a bar line will only be counted once. This does not imply that the harmony is tied, but ensures accuracy in counting the frequency of harmonic change.

Encoding harmonic data creates inherent difficulties not encountered with melodic data. Musicode-A provides a means of notating the physical characteristics of the music in a musical format. In other words it gives us the means of writing the music as a string of notes divided into appropriate measures which the computer can read. Creating an encoding system and computer programs which can

accurately assimilate harmonic data is not an easy a task. Seldom do composers create their art in a brick-like fashion in which each brick may be easily recognized by any amateur bricklayer, although this can be found in some nineteenth century church hymns. More often composers weave the melodic and harmonic features into a single textural fabric. Individual features may only be recognized by the more experienced musician. Consequently, the harmonic data is a result of subjective decisions and experience, not simple facts like the melodic data. The following paragraphs and musical examples will illustrate these problems, and explain their resolutions.

The use of non-chord tones, including suspensions, appoggiaturas, retardations, escape tones, passing tones and neighbor tones were determined by presence of a clearly defined harmony and by the logic and rules of voice leading and chord progression of the Classic tonal period. These tones are omitted from the harmonic data. Example 1, an excerpt from Mozart's Quartet in C Major, KV 465, shows the harmonic analysis with several non-chord tone types represented.

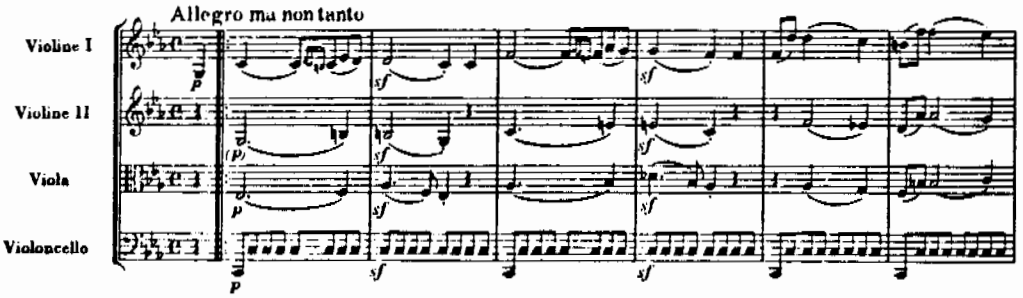
Ex.1 KV 465;mm 35-40



C:I V4/3/IV IV V4/3/IV V/iiV/V V7 vi ii6 V7

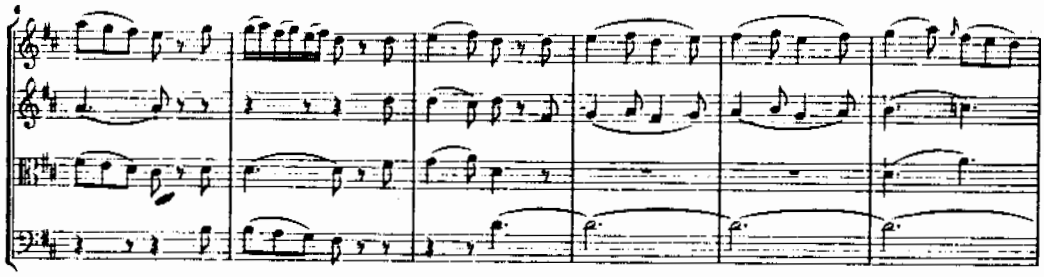
Pedal tones are not treated consistently, but considered harmonic or non-harmonic depending on the musical context. In Example 2 below, the harmonic function is governed by the pedal and its constant driving rhythm. Therefore these six measures contain only tonic function harmony. In Example 3, the pedal is woven into the fabric of the harmony, as it is a chord tone for one moment then a colorful non-chord tone in the next. Here, each harmonic change is recognized and the pedal tone is treated in the same manner as the other non-chord tones.

Ex.2 Beethoven, Op.18, No.4, mm 1-6



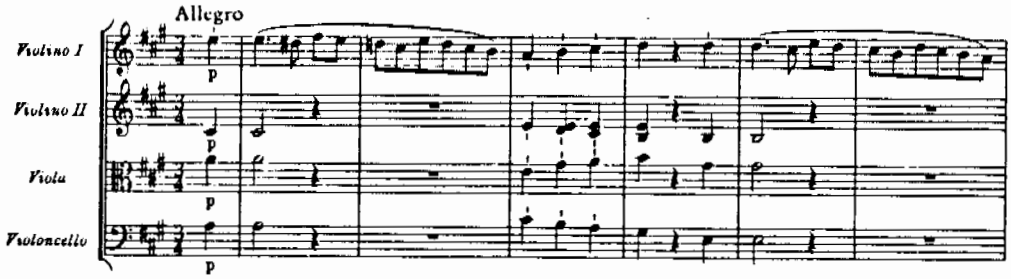
c: i

Ex.3 Haydn, Op.33, No.6, mm 6-11



When rests occur in all instrumental voices as in Example 4, the harmony preceding the rests is considered to continue throughout the rests' duration. The reasoning for this lies in our aural perception of no harmonic change.

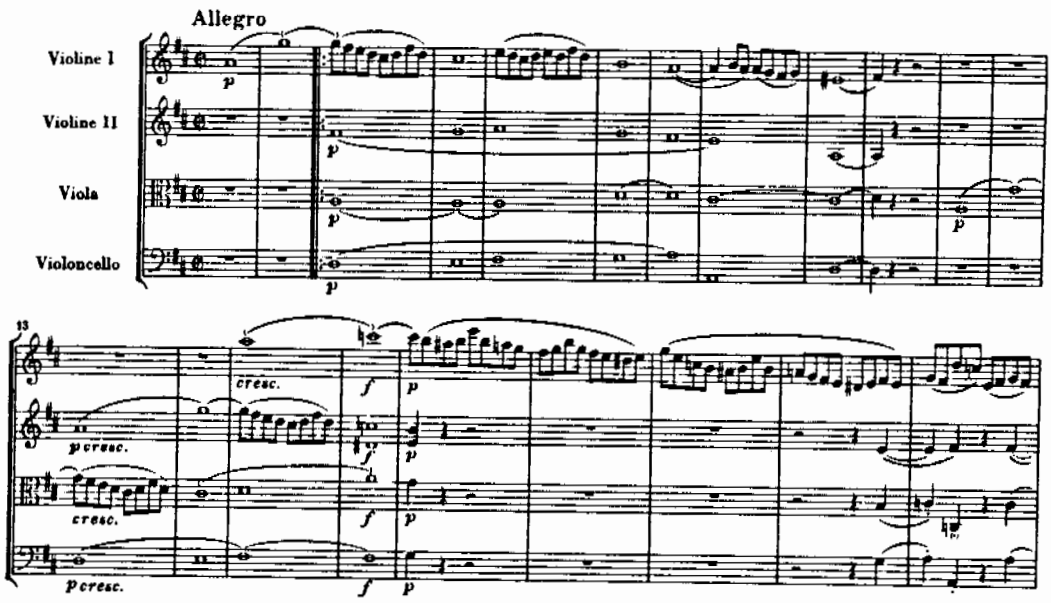
Ex.4 Mozart, KV 464, mm 1-6



A: I _____

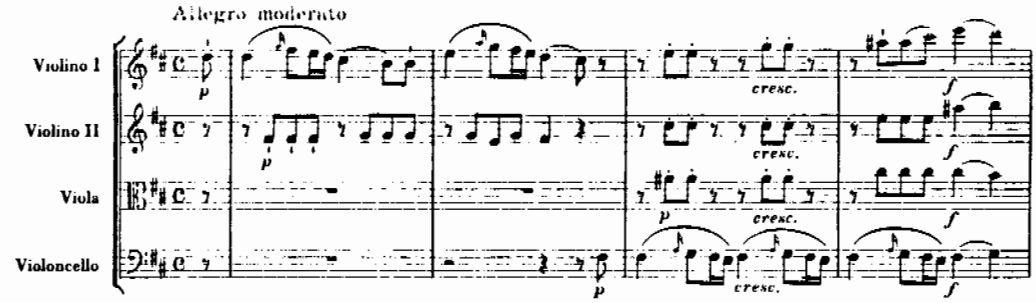
A similar situation is shown in Example 5 where the use of ornate melodic passages in one voice with rests occuring in all other voices. Again the harmony is considered to continue through the duration of the melodic figure unless there is clear definition of harmonic change inferred by the nature of the melodic figure such as chordal arpeggiation.

Ex.5 Beethoven, Op.18, No.3, mm 1-21

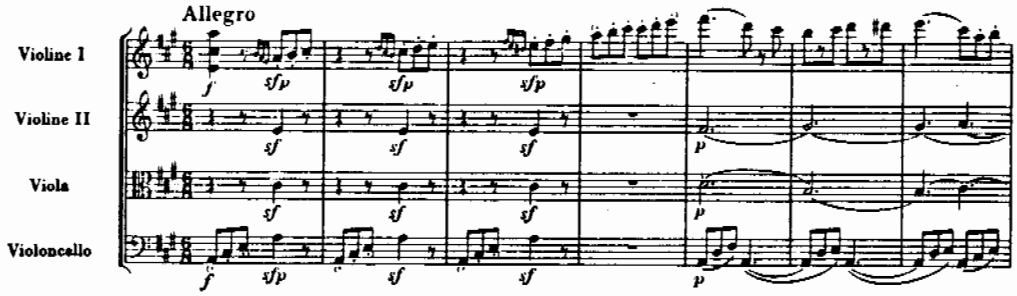


These quartets are among the first to use all four instruments as equal partners both harmonically and melodically. This allows the lowest voice, usually the cello, to carry melodic figures and not be restricted to chordal foundations. The voicings of these harmonies are decided by an emphasized chord tone in the lowest voice. If there is no clear emphasis to a chord tone in the lowest voice, as in Example 6 from Haydn's Op.33 No.1, the voicing is decided according to the lowest voice tone at the arrival of the harmony. If the lowest voice is arpeggiating the harmony, as in Example 7 from Beethoven's Op.18 No.5, the voicing is decided by the lowest voice of the arpeggiation.

Ex.6 Haydn, Op.33 No.1, mm 1-4



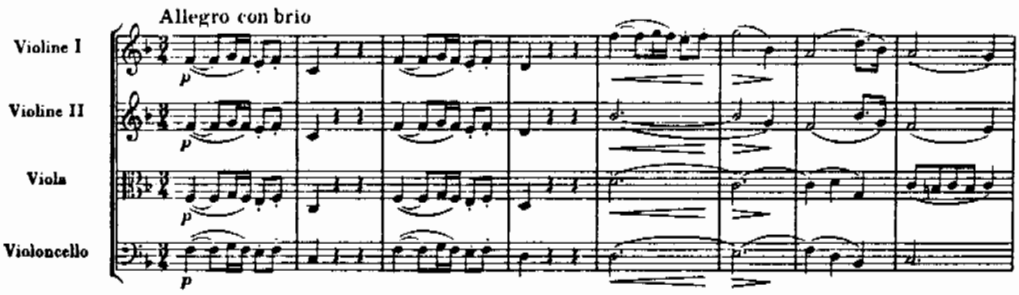
Ex.7 Beethoven, Op.18 No.5, mm 1-7



An anacrusis at the opening of a piece is considered to be a melodic feature in this study and consequently is encoded as an anticipation to the arrival of the tonic. The exception to this is Beethoven's Op.18 No.3, shown in Example 5 above, which opens with a clearly defined dominant seventh in a melodic figure of two full measures.

The harmonic implications of passages in which the instrumental voices are in unisons or octaves such as the passage shown in Example 8, are left to experienced but subjective decision.

Ex.8 Beethoven, Op.18 No.1, mm 1-8



Processing The Data

Processing the melodic data results in a count of all intervals by size and quality, and sorts these results into five tables showing:

1. absolute direction of the intervals,
2. quality of the intervals,
3. size and direction of the intervals,
4. quality, size, and direction of the intervals, and
5. summary analysis.

The first four tables provide frequency and percentage of each interval. The summary analysis assimilates the data from the first four tables into one table showing the frequency of each interval by direction, subtotals by quality, subtotals by direction and the total intervals

counted (see Appendix B).

Processing the harmonic data results in a count of all chords used by frequency of harmonic change and calculation of the durations of each harmony. It sorts these results into four tables showing:

1. frequency of primary chords,
2. subtotal frequencies of primary and secondary chords, and total chords counted,
3. total duration of each primary chord type used,
4. subtotal durations for primary chords combined, secondary chords, and the total durations counted (see Appendix D).

The chords which are considered primary function in this study are: tonic, tonic second inversion when functioning as a cadential six-four, supertonic first inversion, supertonic triad root position, second inversion and supertonic seventh chords, subdominant, dominant, and dominant seventh chords. Tonic function chords are tonic triads in root position and first inversion. Second inversion tonics are only counted as tonic function when they are not followed by a dominant function chord, V or V7. When this occurs, the second inversion tonic is classified and counted as a cadential six-four, one of the primary chord categories in this study. Subdominant triads in all

positions are classified as subdominant chords. Dominant function chords are categorized as dominant triads in all positions, or dominant seventh chords in all positions. This provides a perspective on the possible importance of the seventh's use by each composer. Leading tone triads and seventh chords were not found in the excerpts studied. Two additional classifications are used in this study; the supertonic and the supertonic seventh. Although by a textbook definition these are not primary chords, they occur as frequently as some the primary chords, and thus will be counted individually. The **supertonic 6** is a tally of the supertonic triad in first inversion only, where it often functions as a subdominant substitute, especially in works of the Classic Period. All other supertonic chords, root position and second inversion triads, and seventh chords, all positions, are classified as supertonic.

All other chords are counted as secondary chords. This category is not subdivided into individual chord functions because the programming requires too much time and data storage to be justifiable for the short musical excerpts in this study.

Chapter III

Information Theory and Music

Introduction

Typically, studies in musical style have been laborious processes of analysis involving harmonic and melodic features, phrase, period and formal structures. These standard methods of exploring composers' works are necessary for our understanding of these works, yet many musicians, theorists primarily, are constantly searching for new methods of analysis in hopes of gaining deeper insights into the music. This is apparent by the continued development of musical theories and methods of analysis. Through time the features of musical style, melody, harmony, form, and expression, are mutated in adaptation to composers' ideas and artistic needs. Similarly, our approach to studying music should expand if we are to continue to increase our understanding of musical works individually and as a part of

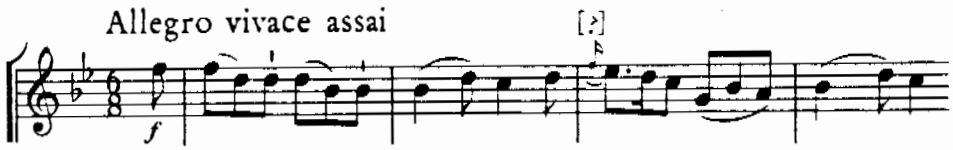
a total artistic body of musical literature.

The concept of information theory as applied to music has existed for roughly three decades, yet it remains a seldom used tool. Information theory provides a measurement for the level of uncertainty or predictability within a reservoir of working materials. For a composer of Western music, one such reservoir is the twelve chromatic tones. When pitches are selected at random from the total collection, the composer has twelve choices for each particular pitch. Our uncertainty is related to that number of choices. If he should choose to use only the seven diatonic scale degrees of a major scale, then there are fewer materials in the reservoir or **low information.** The composer has seven choices for each particular pitch, and the few choices or low information results in low uncertainty or conversely, high predictability.15

Such random selection of pitches, as discussed above, is unrealistic in a musical work. Composers create their musical art with organized efforts which are influenced by their experiences and musical preferences. Let us consider Example 9, the first four measures of melody from Mozart's String Quartet in B flat Major, KV 458.

15. Joseph E. Youngblood, "Style as Information," Journal of Music Theory, 2(1958), 26.

Ex.9. Mozart, KV 458



Twenty notes comprise this excerpt which draws its pitches from the seven diatonic pitch classes of the B flat major scale. A tally of the pitches used is: B flat - 5, C - 3, D - 7, E flat - 1, F - 2, G - 1, and A - 1. It's unlikely that the emphasis on the first and third scale degrees is random, as this choice of pitches accentuates members of the tonic chord pitch. When the melody continues, the listener expects the same basic distribution of pitches, that is, he expects to hear B flat about 5/20 or 25% of the time, C about 3/20 or 15% of the time, D about 7/20 or 35% of the time, F about 2/20 or 10% of the time, E flat, G and A each about 1/20 or 5% of the time.

Knopoff and Hutchinson have explored the effect of the information size on the resulting predictability or entropy. Their work confirms that the larger the information pool, the greater the accuracy in the results.[16] Thus the percentages calculated in connection with Example 9 above would more accurately represent our expectations as the

16. Leon Knopoff and William Hutchinson, "Entropy as a Measure of Style: The Influence of Sample Length," Journal of Music Theory, 27(1983), 85.

melody continues if the first forty notes of the melody were considered rather than just the first twenty.

Most studies such as this, for example the work of Knopoff and Hutchinson, and Youngblood, analyze melodic data by transposition of all music studied to the key of C (major or minor in accordance to the mode of the work) or using a twelve tone system where I is tonic and XII is the major seventh scale degree, and then counting the occurrences of specific scale degrees within a melodic passage as in the excerpt above. The problem with this method, as brought to our attention by John L. Snyder, is that the transpositions are performed for each key modulation and tonal area which detracts from the accuracy of the entropy calculations.[17]

Few theorists have explored the use of this tool in the manner of this study, examining the frequency of specific melodic intervals and harmonic sonorities, and none have published results. Knopoff and Hutchinson mention such possibilities and they have done some research with harmonic data, it is however, limited to a study of consonance verses dissonance.[18]

17. John L. Snyder, "Entropy as a Measure of Musical Style: 'The Influence of a Priori Assumptions'," (Paper delivered at the Ninth Annual Meeting of the Society for Music Theory, Bloomington, Indiana, November 9, 1986).

Probability and Entropy

In the example above, each randomly selected pitch from the diatonic group has a one in seven (1/7) probability of being selected. Each pitch in the chromatic group has a one in twelve (1/12) probability of being selected. Thus, a particular pitch has a greater probability of being selected from the diatonic group than from the chromatic group. A high probability of an event occurring indicates less uncertainty for predicting that event. Probability (P) is calculated by dividing the number of occurrences of an event (x) by the total events counted (n).

Equation 1 - Probability Equation

$$P = x/n.$$

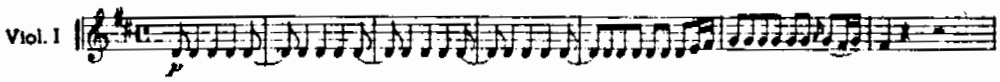
A good example of probability calculations can be illustrated using the excerpt from Mozart's KV 458 shown in Example 9. Using the probability equation I will determine the probabilities for each pitch in the Mozart example.

P(Bb) = 5/20 = .25
P(C) = 3/20 = .15
P(D) = 7/20 = .35
P(Eb) = 1/20 = .05
P(F) = 2/20 = .10
P(G) = 1/20 = .05
P(A) = 1/20 = .05

18. Leon Knopoff and William Hutchinson, "Information Theory for Musica Continua," Journal of Music Theory, 25(1981), 17-44.

A very different example can be observed in the following probability calculations taken from Mozart's Symphony No.38 in D Major shown in Example 10. This excerpt is a motivically derived theme from the first movement.

Ex.10 Mozart, KV 504, Symphony No.38 in D Major, mm 37-43



Examining only the first measure of Example 10 which is the first motive established in this theme, shows that all five pitches are D. The probability of D in this motive is 5/5 or 1.0, hence there is no uncertainty within the first motive. Calculation of probability for this entire sample is as follows:

P(D) = 23/35 = .657
P(E) = 1/35 = .029
P(F#) = 3/35 = .086
P(G) = 8/35 = .229

We know the probability for any one pitch occurring within the sample, but how do we measure the overall uncertainty during the entire sequence? Is there a way to measure the average uncertainty? Claude E. Shannon has devised the concept of entropy to measure the average or overall uncertainty in a string of events.[19] Entropy (H) is a logarithmic function of the probability, P(x), of each

event's occurrence in a sequence of events. It is calculated by multiplying the logarithm of the probability by that probability for each event in the string of events, and taking the negative sum of all the results. The formula for H is shown below using t as the total number of probabilities.

Equation 2 - Entropy Equation

$$H = -[(P_1 \log P_1) + (P_2 \log P_2) + \ldots + (P_t \log P_t)]$$

$$\text{or } H = -\sum_{t=1}^{n} P_t \log P_t$$

Base ten is used to calculate the logarithmic functions in this study. A calculation using log base 10 results in a measurement of uncertainty comparable to the uncertainty experienced when randomly choosing a number between one and ten. If I were to use log base 2, the uncertainty would be relative to the experience of flipping a coin or a fifty-fifty chance. Similarly, log base 6 measures uncertainty relative to the uncertainty of throwing a die.

Applying this concept to a string of musical events, such as intervals in a melodic sample, I can calculate the overall uncertainty experienced during the sample, in relation to the melodic interval content of the sample. To

19. Claude E. Shannon, and Warren Weaver, The Mathematical Theory of Communication, (Urbana: University of Illinois Press, 1949).

demonstrate this application, I will calculate the entropy for the two melodies shown in Examples 9 and 10.

Mozart, KV 458

$$H = -[(.25\log.25) + (.15\log.15) + (.35\log.35) + (.05\log.05) + (.10\log.10) + (.05\log.05) + (.05\log.05)]$$

$$= -[(-.15051) + (-.12359) + (-.15958) + 3(-.06505) + (-.10000)]$$

$$= .72883$$

Mozart, KV 504

$$H = -[(.657\log.657) + (.029\log.029) + (.086\log.086) + (.229\log.229)]$$

$$= -[(-.11986) + (-.04459) + (-.09163) + (-.14660)]$$

$$= .40268$$

The entropy calculated for the Mozart KV 458 example, .72883, means that we are approximately 7/10 times as uncertain of the next occurring pitch as when we randomly choose a number between one and ten. The other example, Mozart KV 504, with a .40268 entropy value means that we are approximately 4/10 times as uncertain as when we randomly choose a number between one and ten. Notice that the closer the value is to zero, the less uncertain we are of the next event which will occur.

Entropy and Musical Style

Style in music can be briefly defined as characteristic tendencies of organizing musical features. Tendencies affect the predictability of expected events. Expectation gives rise to musical meaning for both the listener and the performer. Leonard B. Meyer defines two types of expectations we experience, latent expectation and active expectation.[20] Latent expectations are those expectations ingrained in our subconscious, things which are routine, common or daily occurrences. They become active or conscious expectations when they are delayed or disrupted. For example, we usually expect to hear a ii7 I6/4 V7 progression followed by a root position tonic triad at the end of a phrase in traditional tonal music. We hear this occur frequently and give little conscious thought to what we have heard. We become actively aware of our expectations when we hear the progression move to a different harmony such as the submediant. These expectations are affected by harmonic and melodic ambiguities, dissonance, consonance, and other musical features. Measuring entropy may be a way to quantify some aspects of musical style, and help explain our perception and reactions to music. Meyer has devoted a

20. Leonard B. Meyer, Music, The Arts, and Ideas: Patterns and Predictions in Twentieth-Century Culture, (Chicago: University of Chicago Press, 1967), 9.

complete study to the subject of emotion and meaning in music.[21] His work reveals the usefulness of entropy as measuring device for musical style and our intellectual-emotional perceptions of music. It must be remembered however, that entropy is only one of many tools we may use in analyzing musical style. Entropy holds no mystical secrets, nor does it provide all the answers to our questions about musical style. However, it gives us another way to examine music in conjunction with other more conventional analytic tools and provides an enriched perspective of musical style.

21. Leonard B. Meyer, Emotion and Meaning in Music, (Chicago: University of Chicago Press, 1956).

Chapter IV

Results of Processing the Data

Absolute Direction of the Melodic Intervals

Examining the absolute direction of the intervals regardless of their size, as recorded in Appendix E, shows interesting parallels in the compositional styles of Haydn, Mozart and Beethoven. Descending intervals are more frequent than ascending intervals in most of the quartets written by all three composers. In each quartet set, descending intervals dominate at least three of the six quartets, but examining the actual difference between the frequency of descending and ascending intervals shows that Haydn maintains a more equal use of ascending and descending intervals compared to Mozart and Beethoven, who demonstrate larger differences between the frequency of ascending and descending intervals. Beethoven, having the largest range, is most likely to favor either ascending or descending

intervals. Table 1 below shows the range of difference for each composer. These differences are determined by subtracting the smaller value (s), either ascending or descending interval count, from the greater value (g), qnd then converting it to a percentage of the whole interval count: [(g-s) X 100] ÷ (g+s).

TABLE 1

Range Extremes of Ascending and Descending Intervals

COMPOSER/QUARTET	ASCENDING	DESCENDING	DIFFERENCE(%)
Haydn, Op.33, No.1	18	18	0 %
Op.33, No.3	22	34	21.42 %
Mozart, KV 465	48	41	7.87 %
KV 421	24	11	45.47 %
Beethoven, Op.18, No.2	35	35	0 %
Op.18. No.5	25	10	42.85 %

It is important to note a peculiar occurrence in the range extremes exhibited by Mozart and Beethoven as shown in Table 1. The quartets in which they exhibit the greatest contrasts of frequency in interval direction, KV 421 and Op.18, No.5, show significantly greater frequencies of ascending intervals than descending intervals which contrasts with their propensities for descending intervals in these quartet sets.

Another interesting contrast to note is that Haydn uses an equal balance of descending and ascending intervals in his minor key quartet (Op.33, No.1), whereas Mozart produces his largest range of deviation in the minor key (KV 421). It is this same quartet by Mozart in which the use of stationary intervals intervenes in greater proportion to ascending intervals. This parallels the c minor quartet by Beethoven (Op.18, No.4) which also utilizes greater use of stationary intervals than ascending intervals. This feature is unique to these two quartets.

Studying the interval direction also shows that the use of stationary intervals or perfect primes is an extreme and significant difference in the melodic styles among the three composers. Repeated pitches show a significant decline in use from Haydn to Mozart and Beethoven respectively. Haydn uses primes extensively in both major and minor keys as can be seen in Table 2 below and Appendix E. Mozart and Beethoven use only half as many primes as Haydn. As mentioned earlier, an unusually large number of stationary intervals are heard in the minor quartets of Mozart and Beethoven, thus examining the major and minor key quartets separately will allow a more accurate comparison of the composers' styles. Table 2 shows that the frequency of perfect primes increases from the major key quartets to the

minor key quartets in the themes by all three composers, especially for Beethoven whose use of primes shows a substantial difference from 5.08% to 31.91%, or a 26.83% difference. Mozart shows a lesser difference from 6.68% to 25.00%, a 18.32% difference, and Haydn shows the least difference, 16.80% to 21.74%, a 4.94% difference. Table 2 below presents the mean percentage of absolute deviation for each set of quartets. Dividing the major key quartets from the whole set shows the varied use of stationary intervals in the minor key by Mozart and Beethoven.

TABLE 2

Percent Means for Absolute Direction

COMPOSER	DESCENDING	ASCENDING	STATIONARY
Haydn, Op.33			
major keys only	44.45 %	38.75 %	16.80 %
minor key only	39.13 %	39.13 %	21.74 %
all quartets	43.56 %	38.81 %	17.63 %
Mozart			
major keys only	49.45 %	43.67 %	6.68 %
minor key only	54.55 %	20.45 %	25.00 %
all quartets	50.30 %	39.80 %	9.90 %
Beethoven, Op.18			
major keys only	46.70 %	48.22 %	5.08 %
minor key only	40.43 %	27.66 %	31.91 %
all quartets	45.66 %	44.79 %	9.55 %

Melodic Interval Frequency and Melody Length

The frequency any interval is heard within a melody is related to the length of the melody, but interval frequency and melody length are not interchangable stylistic elements. Melodic length may be measured by the number of intervals or notes it contains, but this information does not present a total representation of the melodic length. The meter and the total number of measures influence an accurate assessment of a given melody's length. Independent of these parameters, interval frequency can only reveal the tendencies of a composer to be either melodically rich, utilizing a great variety of intervals, or melodically sparse utilizing just a few different intervals. By considering the interval frequency in a musical excerpt in the context of its total length in number of measures, it can be determined if the composer is rhythmically vigorous, by crowding interval information into a few measures, or rhythmically terse, using very little interval information in each measure. Table 3 below shows the total melodic interval count and total length in measures for each thematic excerpt. The length in measures is determined by the total number of beats in each excerpts as entered in the computer data file, divided by the number of beats in each measure as indicated by the meter. This calculation process results in partial measures for some of the pieces because

many melodies incorporate partial measures such as anacruses. The information is listed by composer with each quartet in publication order from top to bottom. Table 4 shows the calculated mean density of intervals per measure and standard deviation for the excerpts.

Table 3

Total Melodic Intervals

COMPOSER QUARTET	Haydn	Mozart	Beethoven
1	46	65	62
2	187	44	76
3	66	40	113
4	88	115	47
5	98	56	40
6	96	100	120
MAJOR	107.0	75.2	82.2
ALL	96.83	70.0	76.33

Total Number of Measures

COMPOSER	Haydn	Mozart	Beethoven
1	10.38	9.38	20.0
2	12.0	8.0	20.0
3	8.13	11.5	27.5
4	9.0	18.0	12.25
5	31.5	15.0	10.33
6	17.5	21.75	28.5
Major	15.65	15.13	21.26
All	14.75	14.02	19.76

Table 4

Mean Density of Intervals Per Measure

COMPOSER	Haydn	Mozart	Beethoven
1	4.43	6.93	3.10
2	15.58	5.50	3.80
3	8.12	3.48	4.11
4	9.78	6.39	3.84
5	3.11	3.73	3.87
6	5.49	4.60	4.21
Major (mean)	8.42	5.03	3.82
All (mean)	7.75	5.11	3.82
Standard Deviation	4.15	1.29	0.35

Several observations can be made after examining these tables. Haydn uses the most intervals as evidenced by an overview of the quartets' melodic interval frequency and supported by the calculated means of interval frequency. Mozart and Beethoven not only use fewer intervals, but they are generally less consistent than Haydn in the number of melodic intervals used within the first theme areas. Conversely, Beethoven is the most consistent in his treatment of rhythmic activity as measured by the density of intervals per measure. This consistency carries into his writing in the minor key, unlike Haydn and Mozart. The minor key quartets (Haydn's Op.33, No.1, Mozart's KV 421, and Beethoven's Op.18, No.4) demonstrate close similarities in their melodic interval frequencies.

It is interesting to note that although the themes differ in length by their total number of measures, the average theme length for each quartet set is similar for Haydn and Mozart. Beethoven's primary themes are on the average substantially longer.

Melodic Interval Quality

Examination of the qualities of intervals used in the string quartets is divided into two areas: (1) use of the major, minor and perfect intervals, and (2) use of augmented and diminished intervals. No doubly augmented or doubly diminished intervals were found in these excerpts. Appendix F lists the count of interval qualities for each quartet theme.

Each composer uses major and minor intervals in a unique way. Haydn uses 6.32% more major intervals than minor intervals in the excerpts studied. Three of the five major key quartets by Haydn are dominated by use of major intervals, and one of the remaining major quartets employs a reasonably equal use of major and minor intervals. When Haydn utilizes fewer major than minor intervals, the difference is usually small. Conversely, the difference is significantly larger when major intervals dominate. Perfect intervals account for 25.37% of the total number of

intervals in the Op.33 quartets. This results from Haydn's more extensive use of perfect primes than either Mozart or Beethoven. This characteristic has been previously examined in regards to interval direction.

Mozart displays a predisposition towards minor intervals, but only by a small percentage, 2.31%, in the major key excerpts. This near equilibrium is however, a misrepresentation of Mozart's approach to the use of interval quality in the individual quartets. Major intervals dominate three of the five major key quartets as was characteristic in Haydn's quartets. In contrast to Haydn's style, Mozart makes larger differentiations between the use of major and minor intervals in the individual quartets, thus his theme areas show large predominances of either major or minor intervals. Mozart uses slightly fewer perfect intervals than Haydn, and his choice of perfect intervals differs from Haydn's. Mozart uses far fewer perfect primes and more perfect fourths and fifths.

Beethoven shows a preference for minor intervals. Unlike Mozart, this general tendency is a reasonable representation of all the Op.18 quartets. Minor intervals dominate four of the six quartets. One of the remaining two quartets exhibits an approximate balance in use of major and minor intervals. Only the A Major quartet (Op.18, No.5),

shows a distinct inclination towards major intervals. The reader should recall that it is this quartet which has been historically connected with Mozart's KV 464 quartet. The patterns of interval quality used in these two quartets resemble one another. Since it has been confirmed that Beethoven studied the KV 464 quartet and imitated some of Mozart's style in the writing of his own A Major quartet, it is reasonable to speculate the domination of major intervals in Op.18, No.5 is a Mozartian influence and not Beethoven's propensity. His use of perfect intervals resembles the other two composers, but like Mozart, fewer primes and more fourths and fifths are ascribed to this classification of interval quality compared to Haydn.

Investigating the use of augmented and diminished intervals can not be done without recognizing the exact size of the intervals if creditable information is to be acquired. The following table collates the intervals extracted from the musical excerpts.

Table 5

Augmented and Diminished Intervals

COMPOSER	AUGMENTED	DIMINISHED
Haydn	7 total	4 total
	5 aug primes	1 dim 4th
	2 aug 2nds	3 dim fifths
Mozart	8 total	4 total
	4 aug primes	1 dim 3rd
	2 aug 2nds	2 dim 5ths
	2 aug 4ths	1 dim 8ve
Beethoven	2 total	5 total
	2 aug primes	2 dim 3rds
		1 dim 5th
	(NO aug 4ths)	1 dim 6th
		1 dim 7th

The majority of augmented intervals used by Haydn and Mozart, and all those used by Beethoven, are augmented primes. Examination of Appendix C shows that all can be associated with ascending direction. Examination of the music confirms the obvious, that augmented primes serve as vehicles for melodic chromaticism. It is only in the minor quartets of Haydn and Mozart, that augmented seconds are heard. Each occurrence is, as we might suspect, the minor sixth scale degree ascending the harmonic minor scale through the major seventh to the tonic scale degree. Mozart is the only composer of the three to use the tritone spelled as an augmented fourth. All three composers use the tritone

spelled as a diminished fifth. The tritone occurs in Beethoven's themes just once, in the minor key quartet. Haydn and Beethoven restrict their use of the tritone to its natural occurrence within the diatonic scale. Mozart proves to be bolder, introducing half of the tritones used via secondary leading tones. Mozart's use of the diminished third and diminished octave are also results of secondary leading tones. This indicates that secondary leading tones may play a significant role in Mozart's chromatic, melodic style in these quartets. Beethoven's use of the diminished seventh is similar to that of the diminished fifth; it occurs only in the minor key and within the context of the harmonic minor scale. He uses the diminished third and diminished sixth only in the D Major quartet (Op.18, No.3). Beethoven is the only composer of the three to use these intervals, and he uses them in just one of his quartet themes, where they arise from the use of secondary leading tones within the thematic areas of his Op.18 quartets.

Melodic Interval Quality, Size and Direction

Composers from closely related eras and geographical regions inevitably share some features in their compositional styles while retaining their individuality. Examining all parameters of the melodic intervals, quality,

size and direction, shows important common and contrasting features of melodic style. Appendix G provides a list and count of the intervals used in each quartet. The same intervals predominate in the quartet themes in the music of Haydn, Mozart and Beethoven. These intervals are the primes, major and minor seconds, both ascending and descending, and minor thirds, descending for Haydn and Beethoven and ascending for Mozart. It is in the order of the composers' preference among these intervals which provides the characteristics of the individual melodic styles in these quartets. Table 6 below shows the order of the first seven most frequently used intervals in each set of quartets. This order represents the mean for each set and not the individual quartets. The standard deviation is given with each of the intervals in each set. Seven intervals are compared because of the similarities among the three composers. The standard deviation provides a perspective on the accuracy of this order in representing the individual quartets. Low deviation indicates high consistency of use, and conversely high deviation indicates less consistency.

TABLE 6

Order of Frequently Used Melodic Intervals

Haydn

Interval	Mean Percent	Deviation
-MAJ2	18.08	4.1411
* PP	17.63	3.4752
+MAJ2	11.86	9.5612
-MIN2	9.12	1.1721
+MIN3	7.63	6.0560
+MIN2	7.58	3.6695
-MIN3	6.88	5.0400

Mozart

Interval	Mean Percent	Deviation
-MAJ2	21.73	8.0820
-MIN2	15.66	6.5808
+MIN2	13.57	5.2703
* PP	9.90	8.0082
+MAJ2	8.92	8.6660
+MIN3	5.40	2.2696
-MIN3	4.55	5.4379

Beethoven

Interval	Mean Percent	Deviation
+MIN2	14.51	4.0613
+MAJ2	13.81	9.9923
-MAJ2	12.71	9.1783
-MIN2	10.07	5.7052
-MIN3	5.30	1.9357
* PP	5.08	10.8251
+PER4	4.32	2.9405

There are two methods of counting the intervals in the quartet sets for the purposes of comparing the most frequently used intervals by each composer. The first method is to count each interval type in all the quartet themes collectively within a set as shown in Appendix H.

For example, descending major seconds tally 80 of from a total of 420 intervals, or 20.48%, counted in all of Mozart's quartet themes. The second method requires counting the intervals in each quartet, then calculating the mean of all six quartets in each set. Using the same example of descending major seconds in Mozart's quartet set, the mean is 21.73%. The two methods show similar results but the second method provides a more accurate representation of the frequency of intervals which characterize the style of each quartet set. Thus, using the means of interval frequency does not void the creditability of this study, because the importance lies in the general patterns exhibited by the composers and not the quartet themes themselves.

Haydn's generally low deviation figures in the above table indicates that he is the most consistent in his use of these intervals, with the exception of the ascending major second. Table 6 shows evidence that Mozart and Beethoven exhibit a relatively high degree of consistency in using minor seconds but less consistency in using major seconds and perfect primes. Haydn shows faithful use of the smaller intervals, where as Mozart and Beethoven explore increased uses of larger intervals. Both Mozart and Beethoven make greater use of perfect fourths, especially in ascending

direction. Study of Appendix G shows that Beethoven incorporates much greater use of perfect fifths, major and minor sixths and sevenths, and as discussed previously, a few infrequent intervals such as diminished thirds, sixths, and sevenths. In general an expanded vocabulary of melodic intervals is observed from the older composer Haydn, to the younger composer Beethoven. Comparing the three quartet sets with earlier pieces by Sammartini and Haydn, and later pieces by Beethoven and Schubert using Appendix G, supports that this expansion is a developmental pattern evolving through the music of the Classic Period composers.

Harmonic Data Results

The results from the harmonic data are listed in Appendix I, and reveal several interesting characteristics. All three composers use the dominant seventh chord more often than the dominant triad by amounts ranging from 36% in Mozart's music to 52% in Beethoven's music. Frequency of the first inversion supertonic triad as a subdominant function is almost equal to the frequency of the subdominant triad in Haydn's quartets. In fact, it is used in all five of the major key quartets, while the subdominant triad is heard in only three of them. The first inversion supertonic triad occurs less frequently than the subdominant in

Mozart's and Beethoven's quartets. It is interesting to note that with the exception of the String Quartet in C Major, KV 465, Mozart uses little to no subdominant function harmony in his quartet themes. In contrast to Haydn, Beethoven never uses the first inversion supertonic triad without the presence of the subdominant triad in the theme area. The first inversion supertonic does not appear in any minor key quartet themes among these three sets. The possible reason for this is that the diminished quality of the supertonic triad was less pleasing as a subdominant substitute in the minor mode than the minor supertonic in the major mode.

The other supertonic chords, the supertonic seventh and the triad in root position and second inversion, occur approximately 60% less in Beethoven's quartets than either Haydn's or Mozart's quartets. In Haydn's and Beethoven's music, the cadential six-four appears in only three quartets each. Mozart, however, uses it in five quartets to establish strong, final thematic cadences.

Primary functioning chords, the II6, and all positions of I, IV, V, and V7, account for the majority of the harmonic vocabulary, while secondary chords occur an average of only approximately sixteen percent in these theme areas. The accuracy of this is indicated from two measurements,

frequency and duration of the harmonies. Comparison of these measurements shows close correlation between frequency and duration. Table 7 summarizes the percentages of primary chords used as measured by frequency and duration. The mean and standard deviation are given for each set of quartets.

TABLE 7

Summary of Primary Chord Usage

COMPOSER/ QUARTET	TOTAL NO. OF CHORDS	PERCENT PRIMARY	TOTAL DURATION BY BEAT	PERCENT PRIMARY
Haydn				
Op.33, No.1	25	56.00	41.5	63.86
No.2	30	93.33	48.0	91.67
No.3	25	68.00	32.5	75.38
No.4	31	97.77	31.0	98.61
No.5	45	82.22	45.0	81.75
No.6	57	89.47	105.0	88.57
MEAN		80.97		83.31
STAN. DEV.		14.54		11.37
Mozart				
KV 387	29	65.52	37.5	64.00
KV 421	19	68.42	32.0	71.88
KV 428	14	100.00	46.0	100.00
KV 458	31	100.00	106.0	100.00
KV 464	20	100.00	45.0	100.00
KV 465	40	75.00	87.0	81.61
MEAN		84.82		86.25
STAN. DEV.		15.43		14.66
Beethoven				
Op. 18, No.1	25	80.00	60.0	80.83
No.2	28	89.29	40.0	94.69
No.3	34	91.18	55.0	91.82
No.4	24	83.33	49.0	91.84
No.5	14	92.86	62.0	95.16
No.6	37	78.38	57.0	87.72
MEAN		85.84		90.34
STAN. DEV.		5.56		4.90

A parallel relationship between frequency and duration is evident in this table. Two quartets from each set show relatively larger differences between the measurements of freqeuncy and duration. Among these are the minor key quartets, Haydn Op.33, No.1, Mozart KV 428, and Beethoven Op.18, No.4, all which exhibit larger measurements by duration than frequency. This indicates a similar tendency toward expanded durations of primary chords in minor mode by Haydn, Mozart and Beethoven.

Mozart's music demonstrates the widest contrast in the presence of secondary chords. He is the only composer to omit secondary chords entirely from primary theme areas. This characteristic occurs in three of his quartet themes, KV 428, KV 458, and KV 464. This feature may allude to important characteristics of Mozart's harmony in thematic areas, but an expanded study Mozart's themes is necessary to form any creditable conclusion.

Mozart and Beethoven exhibit an expanded harmonic vocabulary not heard in Haydn's quartet themes. Both Mozart and Beethoven introduce the use of the German augmented sixth chord in primary theme areas. In his C minor quartet, Beethoven expands the dominant through use of a dominant ninth chord and an altered dominant seventh with an augmented fifth. Such expansion within primary themes

indicates an expansion in harmonic vocabulary from the early Classic composers to the later Classic composers.

More evidence of the expansion of the harmonic vocabulary can be found by examining the earlier work by Sammartini and later works by Beethoven and Schubert. Sammartini's work maintains a very small harmonic vocabulary, utilizing tonic and dominant triads only (see Appendix H). In comparison to Sammartini's work, the later works by Beethoven and Schubert use much larger harmonic vocabularies including bII, VI, +6, $vii^{o}7$, secondary dominants and leading tone chords.

Finally, comparing the standard deviations on Table 7, reveals that Beethoven shows markedly lower deviations than either Haydn or Mozart. Thus, although Beethoven does not use more secondary chords as indicated in the mean percentage, his use of secondary chords is a significantly consistent characteristic in his harmonic style.

Chapter V

Entropy and the Quartet Themes

Introduction

The function of entropy as a measurement of stylistic uncertainty is discussed in Chapter 3. This chapter discusses the results of calculating the values of entropy for the musical excerpts in this study. The melodic interval entropies are graphically displayed in Figure 1 with the corresponding values listed in Table 9. The harmonic entropies are displayed in Figure 2 with the corresponding values listed in Table 10.

An example of an entropy calculation relative to this study will be beneficial to the reader's understanding of the results discussed in the following pages. The melodic entropy for Beethoven's String Quartet, Op.18, No.5, is calculated below. Table 8 shows the melodic interval count for this example.

TABLE 8

Beethoven, Op.18, No.5

INT.	UP	DOWN	SAME	TOTAL
PER1			5	5
AUG1	2			2
MIN2	8	2		10
MAJ2	13	1		14
MIN3		1		1
MAJ3		1		1
PER5		1		1
PER8		1		1
MAJ9	2	1		3
MAJ10		1		1
TOTAL	25	10	5	40

The values for each interval counted (x_1, x_2, ... x_t) and the total number of intervals counted (n) can now be introduced into the entropy equation. Remember that the logarithmic base is ten, therefore a value of 1.00 means that our uncertainty is equal to the uncertainty experienced when randomly choosing a number between one and ten. Lower values indicate less uncertainty, and conversely higher numbers indicate greater uncertainty than is experienced when randomly choosing a number between one and ten.

The reader should recall that probability is calculated as $P = x/n$. Therefore $P_1 = x_1/n$, $P2 = x_2/n$, ... $P_t = x_t/n$. $H = -[(P_1 \log P_1) + (P_2 \log P_2) + \ldots + (P_t \log P_t)]$. Introducing the values from Table 8 we arrive at the following calculation of entropy for Beethoven's Op.18, No.5.

$H = -[((5/40)\log(5/40)) + ((2/40)\log(2/40)) +$

$$((8/40)\log(8/40)) + ((2/40)\log(2/40)) + ((13/40)\log(13/40))$$
$$+ ((1/40)\log(1/40)) + ((1/40)\log(1/40)) + ((1/40)\log(1/40))$$
$$+ ((1/40)\log(1/40)) + ((1/40)\log(1/40)) + ((2/40)\log(2/40))$$
$$+ ((1/40)\log(1/40)) + ((1/40)\log(1/40))]$$
$$= -[.125(-.90309) + 3(.05(-1.30103)) + .2(-.69897)$$
$$+ .325(-.48812) + 7(.025(-1.60206))]$$
$$= -[-.11289 - .19515 - .13979 - .15864 - .28036]$$
$$= .88683$$

TABLE 9

Melodic Entropies

COMPOSER/ PIECE	ENTROPY
Sammartini	
Symphony for Strings	.79551
Haydn	
Op.17, No.6	.97430
Op.33, No.1	1.01254
Op.33, No.2	1.16395
Op.33, No.3	1.00655
Op.33, No.4	.97350
Op.33, No.5	.92700
Op.33, No.6	1.00623
MEAN (ALL)	1.01496
STAN. DEV.	.07281
RANGE	.92700 to 1.16395
MEAN (MAJOR ONLY)	1.01545
STAN. DEV.	.07975
RANGE	.92700 to 1.16395

Mozart		
KV 387		1.03320
KV 421		.88075
KV 428		.95853
KV 458		.92776
KV 464		.81877
KV 465		1.01376
MEAN (ALL)		.93880
STAN. DEV.		.07396
RANGE	.81877 to	1.03320
MEAN (MAJOR ONLY)		.95040
STAN. DEV.		.07586
RANGE	.81877 to	1.03320
Beethoven		
Op.18, No.1		.93638
Op.18, No.2		1.16184
Op.18, No.3		1.02311
Op.18, No.4		.88514
Op.18, No.5		.88683
Op.18, No.6		1.14878
MEAN (ALL)		1.00701
STAN. DEV.		.11466
RANGE	.88514 to	1.16184
MEAN (MAJOR ONLY)		1.03139
STAN. DEV.		.11026
RANGE	.88683 to	1.16184
Op. 95		1.04090
Schubert		
String Quartet No.5		1.08810

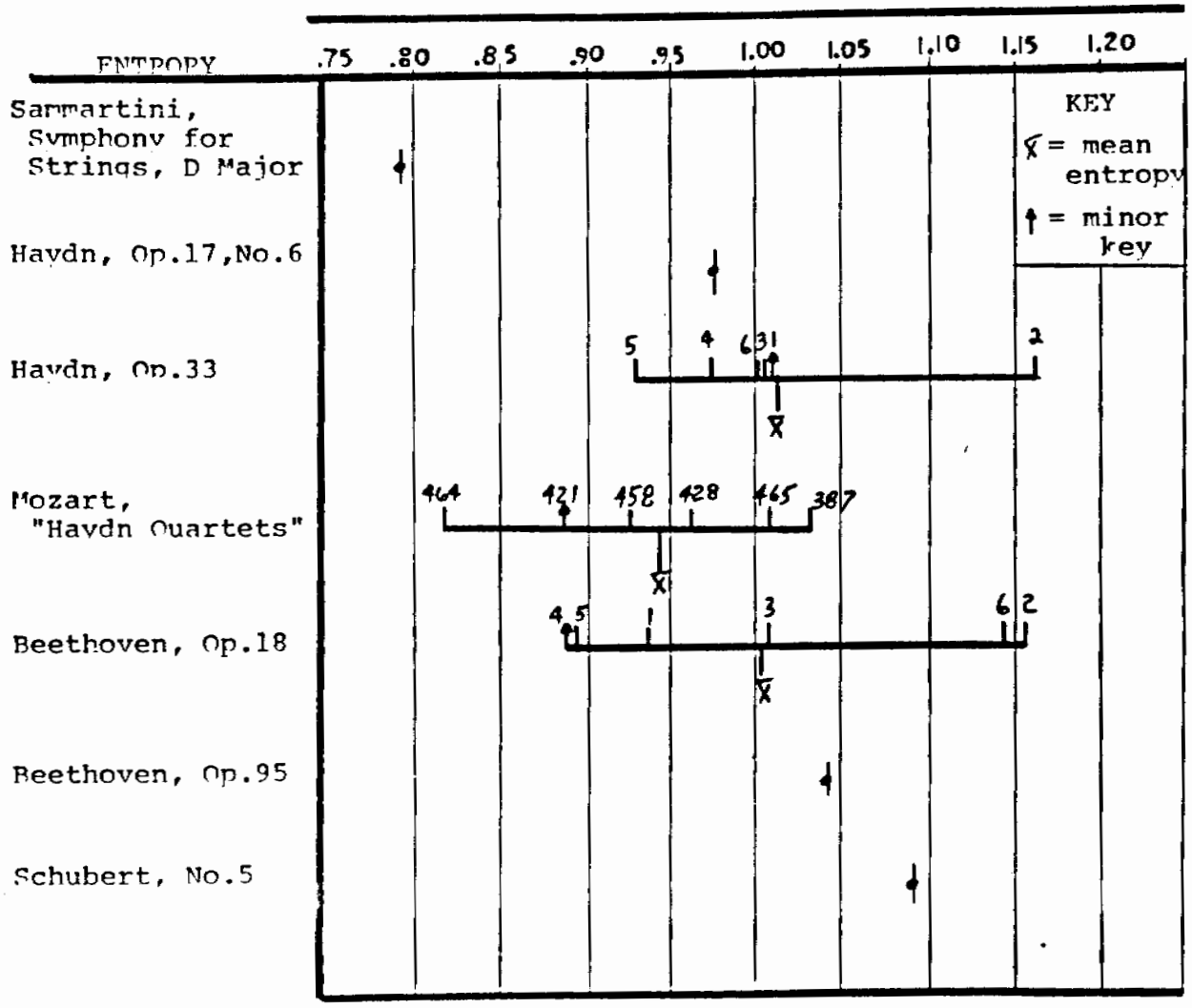


Figure 1 - Melodic Entropies Graph

Entropy: The Melodic Intervals

Haydn and Beethoven exhibit similar melodic styles as reflected by the entropy values for these quartet themes. Three methods of measurement support this statement due to the closely related values between the two sets of quartets. These measurements are the mean entropy, the standard deviation, and the range of entropy values. When compared to Haydn's and Beethoven's music, Mozart's themes reflect relatively lower values in all three measurements. Beethoven's quartet themes exhibit the greatest standard deviation, .11466, and span the largest range, .2767, and the range of entropy values sits between the outer boundaries established by Haydn and Mozart. Mozart's quartet themes generally exhibit the lowest entropies, smallest range, and least deviation. This denotes high predictability and consistency in Mozart's use of melodic intervals within these primary theme areas. Haydn's music incurs the highest entropy of the three composers, but Beethoven's themes produce the greatest deviation. This might indicate that Haydn generally evokes the greatest uncertainty and Beethoven demonstrates the least consistency.

Comparing the entropies of the major key quartet themes with the complete quartet sets, reveals that all three composers exhibit higher entropy means in their major key quartet themes; Haydn, 1.01496 to 1.01545, Mozart, .93880 to .95040, and Beethoven, 1.00701 to 1.03139. This indicates that all three composers generally exhibit less uncertainty in minor key themes than in major key themes. Haydn and Mozart also show higher standard deviations in their major key themes, .07975 and .07586 respectively, when compared to the the complete quartet sets. Conversely, Beethoven shows a lower standard deviation, .11026. This is expected because Beethoven's lowest entropy occurs in his minor key quartet theme (Op.18, No.4). Thus, omitting this entropy from the complete quartet set causes a decrease in the value of the standard deviation.

When compared, the highest and lowest entropy values for the major key quartets of Haydn and Beethoven reveal similar melodic treatments by the two composers. Their lowest major key entropies, Haydn, .92700 and Beethoven, .88683, reflect the relatively small difference of .04017 and their highest major key entropies are nearly the same value, Haydn, 1.16395 and Beethoven, 1.16184, a minute difference of .00211. However it is interesting to note that in respect to the size of entropy range, Haydn and

Mozart show the closer similarity, .07975 and .07586 respectively. It is the minor key themes where Mozart and Beethoven exhibit closely related entropies, .88075 and .88514 respectively, and consequently exhibit similar melodic styles.

When the entropies are examined for other possible relationships such as specific keys, flat keys versus sharp keys, or the chronological order of composition, no other significant relationships are found. When these three sets of quartet themes are compared to earlier and later works, a pattern of increasing entropy is evident. Sammartini's Symphony for Strings in D Major exhibits a low entropy that is only surpassed by the unusually low entropy of Mozart's String Quartet in A Major, KV 464. The entropy for Haydn's String Quartet in D Major, Op.17, No.6, corresponds closely with the mean entropy of the major key themes in Op.33. Beethoven's String Quartet in F Minor, Op.95, exhibits a considerably higher entropy than his C minor quartet in Op.18. If Beethoven maintains the entropy relationship between major and minor keys evident in Op.18, then his later works should reflect significantly higher entropies compared to his earlier works such as Op.18. That conclusion however, is not the purpose here and therefore is left for a separate study of Beethoven's music. Schubert's

String Quartet No.5 in B flat Major shows a relatively higher entropy compared to the entropies exhibited by 79% of the music examined in this study. This entropy is not as high as might be anticipated, but String Quartet No.5 is not Schubert's most adventuresome quartet and thus may reflect a lower than normal entropy for his work. Nevertheless, the pattern of increasing entropy should be evident.

The entropy values for the quartet themes in Haydn's Op.33, Mozart's 'Haydn Quartets', and Beethoven's Op.18 are so close that it is not possible to observe any pattern of increasing entropy values among these works. However, when compared to earlier and later works such as those mentioned above the pattern is clear, especially when compared to the earlier works.

TABLE 10

Harmonic Entropies

COMPOSER/ PIECE		ENTROPY
Sammartini		
Symphony for Strings		.29834
Haydn		
Op.17, No.6		.33410
Op.33, No.1		.69451
Op.33, No.2		.59578
Op.33, No.3		.65119
Op.33, No.4		.62114
Op.33, No.5		.72248
Op.33, No.6		.80024
MEAN (ALL)		.68089
STAN. DEV.		.06814
RANGE	.59578 to	.80024
MEAN (MAJOR ONLY)		.67817
STAN. DEV.		.07434
RANGE	.59578 to	.80024
Mozart		
KV 387		.72295
KV 421		.65829
KV 428		.72663
KV 458		.49439
KV 464		.50525
KV 465		.79554
MEAN (ALL)		.65051
STAN. DEV.		.11373
RANGE	.49439 to	.79554
MEAN (MAJOR ONLY)		.64895
STAN. DEV.		.12453
RANGE	.49439 to	.79554

Beethoven		
Op.18, No.1		.80648
Op.18, No.2		.69416
Op.18, No.3		.75134
Op.18, No.4		.69168
Op.18, No.5		.50856
Op.18, No.6		.59987
MEAN (ALL)		.67535
STAN. DEV.		.09756
RANGE	.50856 to	.80648
MEAN (MAJOR ONLY)		.67208
STAN. DEV.		.10657
RANGE	.50856 to	.80648
Op. 95		.64232
Schubert		
String Quartet No.5		.72086

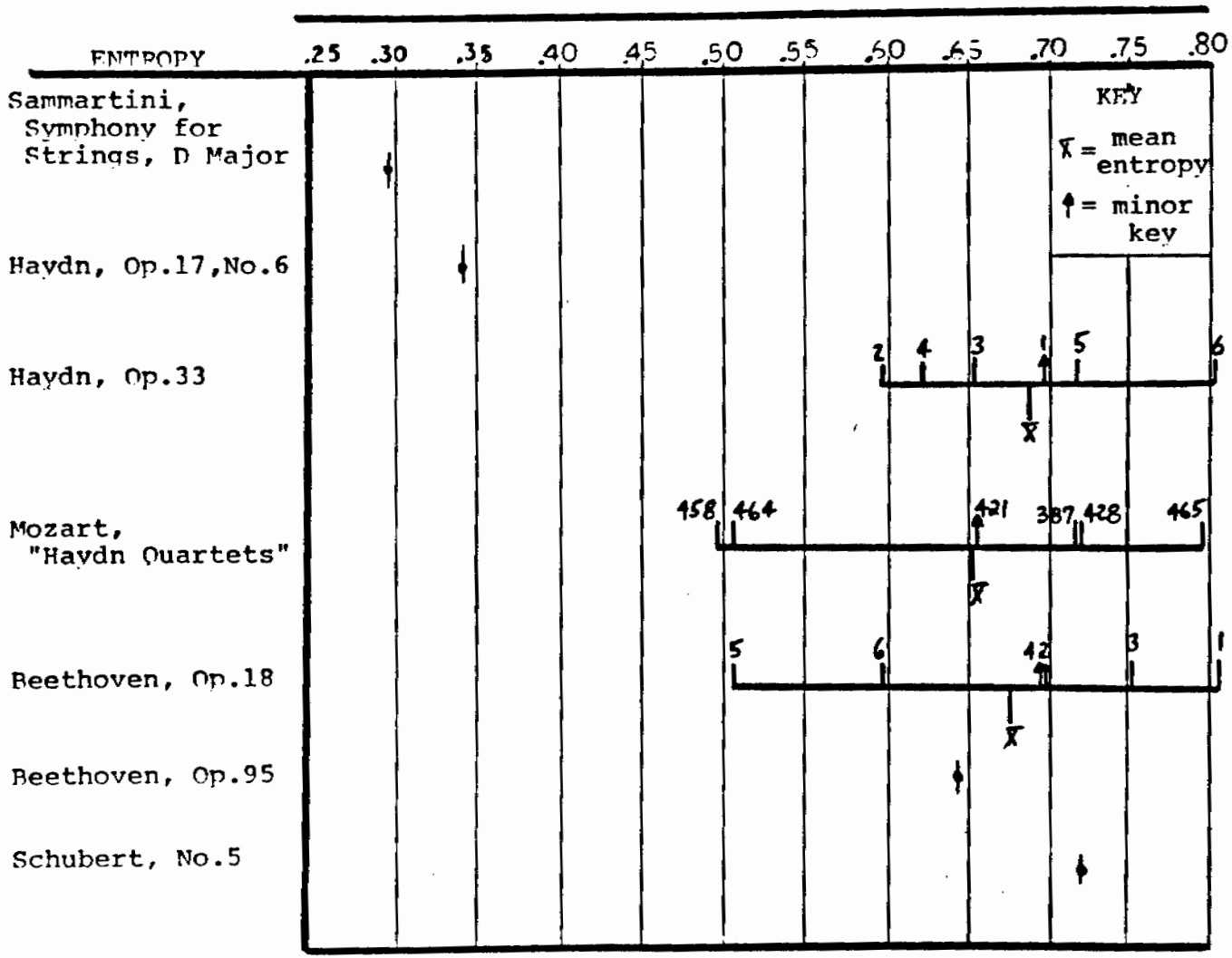


Figure 2 - Harmonic Entropies Graph

Entropy: The Harmonic Intervals

The results of the harmonic data were discussed in Chapter IV. The discussion included explanations of two methods of measuring harmony, the first by frequency, and the second by duration. The calculations of harmonic entropies discussed in this chapter are based on measurements of frequency and not duration. This decision is founded upon the purpose of entropy as a measurement of uncertainty of an event's occurrence, not how long it is endures.

Entropies for the harmonic data are lower than those for the melodic interval data for several important reasons. First, the number of harmonies used during this practice period are substantially fewer than the number of possible melodic intervals used, especially within a primary theme area as opposed to a transitional or developmental section. Second, harmonies change with less frequency than melodic intervals resulting in a smaller data sample. Recall that the size of the data sample has a direct influence on the accuracy of calculation of entropy (see Chapter III). Third, as discussed earlier in this essay, this study does not allow separate classifications for each secondary chord because the programming for such a detailed classification system demands too much time and data storage to be

justifiable for these short musical excerpts.

The harmonic entropies indicate a similarity in the harmonic styles of Haydn, Mozart and Beethoven. These entropies show closer similarities than the melodic entropies. The highest entropies for a quartet within each set, are closely related as shown in Figure 2 and Table 10, Haydn, .80024, Mozart, .79554 and Beethoven, .80648. Mozart's and Beethoven's music also share close values for the lowest entropies, Mozart, .49439 and Beethoven, .50856. The mean entropies, Haydn, .68089, Mozart, .65051, and Beethoven, .67535, are also very close. However, like the melodic entropies, Haydn and Beethoven show the closest mean entropies, reflecting a difference of .01355. It is Mozart and Beethoven who reflect similar ranges differing by only .00323. In the melodic data Mozart and Beethoven proved to exhibit similar styles in the minor key quartet themes, but in the harmonic data it is Haydn and Beethoven who show close similarities in the minor quartet themes which is reflected by their entropies, .69451 and .69168 respectively, a minute difference of .00373.

Examination of the standard deviations for these sets of quartets reveals a contrasting pattern to that observed with the melodic entropies. Mozart exhibits the most deviation, .11373, in his harmonic treatment whereas he

showed the least deviation in his melodic interval treatment. Haydn and Beethoven exhibit less deviation, .06814 and .09756 respectively, in their harmonic treatment.

Comparison of the major key quartets of each set to the entire quartet set including the minor key quartet reveals interesting changes in the entropies of each quartet set. The value for Haydn's mean entropy rises due to the narrow entropy range and to the occurrence of three entropies lower than the overall mean entropy for the entire quartet set and lower than the minor key quartet. Mozart's and Beethoven's music exhibit contrasting patterns compared to the pattern exhibited by Haydn's music. Three quartets in each of their quartet sets exhibit higher entropies compared to their minor key quartets and to their overall mean entropy. Thus, the examination of their major key quartets reveals decreasing mean entropy values when compared to their complete quartet sets. Examination of major key quartets only, does not affect the entropy ranges of these quartet sets, but all three sets do experience increased deviation values. This indicates that the harmonic styles of all three composers exhibit less uncertainty in the minor key themes.

Comparison of the melodic entropies of these quartet sets as a group with earlier and later works revealed a pattern of increasing entropy. This pattern is again revealed with the harmonic entropies. The harmonic entropy values reflect a more marked increase than do the melodic entropies. The earliest works examined in this study by Sammartini and Haydn's Op.17, utilize a very small harmonic vocabulary resulting in extremely low entropies of .29834 and .33410 respectively. The later work by Schubert utilizes a larger variety of harmonies resulting in less predictability or higher entropy of .72086. This entropy is higher than the mean for each quartet set for Haydn, Mozart and Beethoven, but within the range of each quartet set. Other quartets in this study by Haydn, Mozart and Beethoven also reflect significantly higher entropies compared to the Sammartini work and Haydn's Op.17.

The harmonic entropies for Haydn's Op.33, Mozart's 'Haydn Quartets', and Beethoven's Op.18, show even closer values than the melodic entropies, thus a pattern of increasing entropy is not revealed among them. Nevertheless, the evidence found in this study and by the work of Knopoff and Hutchinson and by Youngblood, suggests to me that the entropies would continue to increase through the 19th century composers.22

Chapter VI

Conclusion

Examination of the quartet themes by Haydn, Mozart and Beethoven clearly shows many similarities in their melodic and harmonic styles. These similarities include the composers' use of interval qualities, interval direction, and frequency of primary chord sonorities as compared to secondary chords sonorities. This examination also reveals some individual characteristics of each composer's style, such as each composer's most frequent choices of intervals and chords sonorities.

This study does not evince, nor was it the purpose of this study to do so, any direct influence of Haydn or Mozart upon Beethoven's writing of the Opus 18 String Quartets. Though the scope of comparative examination is narrow in

22. Knopoff and Hutchinson, "Musica Continua." Youngblood, "Style Information."

this study, the documentation indicates valid similarities among these three sets of quartets. It can be concluded from the evidence, that the similarities in the musical styles of Haydn, Mozart and Beethoven, which are revealed through traditional methods of analysis, are also revealed and supported by more recently innovated methods of melodic and harmonic analysis.

The study of the melodic intervals and sonorities used by Haydn in Op.33, Mozart in his 'Haydn Quartets' and Beethoven in Op.18, shows the subtle differences in the in the compositional styles of the three composers. Each of these composers reveals his preference for certain intervals distinguished by size, quality, and direction, and certain sonorities distinguished by function, even though they use the same melodic and harmonic vocabularies. Each composer shows his individuality through his unique combinations and choices. Haydn maintains a more equal use of ascending and descending intervals than either Mozart or Beethoven. Mozart's propensity is to use ascending intervals. However, both Mozart and Beethoven tend to stress either ascending or descending intervals in a particular quartet theme, Beethoven being the least consistant of the two composers.

Haydn exhibits a predominant use of the perfect prime, which accounts for approximately one fifth of all the intervals used in these quartet themes. Mozart uses major intervals more than any other interval quality. He also uses a greater number of perfect fourths than Haydn. Beethoven tends toward more use of minor intervals than other interval qualities. He also shows increased use of larger intervals, such as perfect fifths, major and minor sixths and sevenths, and introduces use of some infrequent intervals during his era such as diminished thirds, sixths and sevenths.

The use of subdominant functioning harmony is an important difference in the harmonic styles of these three composers. Mozart uses very little subdominant harmony in his quartet themes. Haydn uses both the subdominant triad and the first inversion supertonic triad as subdominant functioning harmony freely and independently in these quartet themes. Beethoven also uses both the subdominant triad and the first inversion supertonic triad as subdominant functioning harmony. However, Beethoven does not use the first inversion supertonic triad in a quartet theme without the subdominant triad also present within the harmonic progression of the quartet theme.

Mozart and Beethoven employ the use of German augmented sixth chords in these quartet themes. This sonority is not found in Haydn's quartet themes. Mozart also makes use of secondary functioning areas or tonicizations. These do not occur in the themes by Haydn and Beethoven.

The melodic intervals used by these composers, and the calculated entropies for both the melodic and the harmonic data also show how similar these composers are, in both their melodic and harmonic styles. The mean harmonic entropies for all three composers are close. Resemblances in the melodic and harmonic styles of Haydn and Beethoven are reflected by their similar melodic entropy means and the harmonic entropies of their minor key quartet themes. Examination of the harmonic entropy ranges and the melodic entropies of the minor key quartet themes by Mozart and Beethoven show similarities in their styles.

The statistical evidence documented in this study reflects many parallels among the three quartet sets. All three composers frequently utilize the same intervals, perfect primes, major and minor seconds and descending minor thirds. Descending intervals occur more frequently in the quartet themes by all three composers, and each composer tends to expand the duration of sonorities in minor key themes when compared to their major key themes.

These parallels strengthen our suspicions that the earlier compositions influenced Beethoven as he wrote the Opus 18 quartets. To make convincing conclusions about this theory requires a thorough investigation of the quartets, not just the primary themes of the first movements. Additional historical evidence, which may or may not exist, would be helpful to support such a conclusion. Further studies comparing Haydn's, Mozart's and Beethovens's music might reveal that the subtle similarities and differences among the quartet themes found in this study are common in most of their music.

The new methods of stylistic analysis presented in this essay are beneficial in providing us with other ways of studying musical styles. When used as additional analytical tools, they can enhance our understanding of composers' musical styles. Therefore, the results of this study are only a beginning, perhaps a provocation for a larger study.

APPENDIX A

Melodic Data Encoded in Musicode-A:

```
*C 530 LUDWIG VAN BEETHOVEN
*T 530 STRING QUARTET OP.18, NO.1
*E 530 EDITION: 'BEETHOVEN WERKE:STREICHQUARTETTE I.'
** _ ABTEILUNG VI, BAND 3, ED. PAUL MIES. MUNCHEN-
** _ DUISBERG: G. VERLAG, 1982.
*D 530 DEDICATION: CARL AMENDA
*E 530 ENCODER: MARK LOCHSTAMPFOR, 1986
*S 530 SECTION: MOVEMENT I; ALLEGRO CON BRIO
*X 530 EXCERPT: A THEME, MEASURES 1-20
*P 530 PHRASE: 0:9/1, 20/1
53001001$VIOLIN.I,TREBLE,B-,3-4$
53001002 FN4Q FN4E GN4S FN4S EN4E FN4E/ CN4Q RRRQ RRRQ/ FN4Q
53001003 FN4E GN4S FN4S EN4E FN4E/ DN4Q RRRQ RRRQ/ FN5Q FN5E
53001004 GN5S FN5S EN5E FN5E/ GN5H B-4Q AN4H DN5E. B-4S/ AN4H
53001005 GN4Q/ FN4Q FN4E GN4S FN4S EN4E FN4E/ CN4Q RRRQ RRRQ/
53001006 FN4Q FN4E GN4S FN4S EN4E FN4E/
53001007 DN4Q RRRQ RRRQ/ FN5Q FN5E GN5S FN5S EN5E FN5E/ AN5H
53001008 GN5Q/ GN5Q GN5E AN5S GN5S F 5E GN5E/ B-5H AN5Q/ AN5Q
53001009 AN5E B-5S AN5S GN5E AN5E/ CN6Q. B-5E AN5E GN5E/ FN5H
53001010 AN5E GN5E/ FN5H.//
```

Harmonic Data Encoded in Harmonic Musicode:

```
*C 530 LUDWIG VAN BEETHOVEN
*T 530 STRING QUARTET OP.18, NO.1
*E 530 EDITION: 'BEETHOVEN WERKE:STREICHQUARTETTE I.'
** _ ABTEILUNG VI, BAND 3, ED. PAUL MIES. MUNCHEN-
** _ DUISBERG: G. VERLAG, 1982.
*D 530 DEDICATION: CARL AMENDA
*E 530 ENCODER: MARK LOCHSTAMPFOR, 1986
*T 530 TONALITY: TONIC KEY, F MAJOR
*S 530 SECTION: MOVEMENT I; ALLEGRO CON BRIO
*X 530 EXCERPT: A THEME, MEASURES 1-20
*P 530 PHRASE: 0:9/1, 20/1
*H 530 HARMONIC CADENCES: PAC, PAC
53001001$A.THEME,FNM,B-,3-4$
53001002 FNM53H./ CNM53H./ FNM53H./ B-M63H./ FNM53H./ CNV65H./
53001003 FNM53Q DNN53Q B-M53Q/ FNM64H CNM53Q/ FNM3H./ CNM53H./
53001004 FNM53H. B-M63H./ FNM53H./ C D07H./ GNN53H./ F D07H./
53001005 ANN53H./ CNH42Q. GNN63Q./ FNM64H CNV07Q/ FNM53H.//
```

APPENDIX B

Sample Results from Processing the Melodic Data:

Beethoven String Quartet in F major, Op.18, No.1.

* INDICATES THAT THE PITCH STAYS THE SAME
\+ INDICATES UPWARD MOVEMENT
\- INDICATES DOWNWARD MOVEMENT

INTERVALS EXPRESSED BY QUANTITATIVE NUMERALS (NUM)
1 REPRESENTS A PRIME, 2 REPRESENTS SECOND, ETC.

FREQ	NUM	PERCENT	CUM%
2	*1	3.23%	3.23%
17	+2	27.42%	30.65%
28	-2	45.16%	75.81%
4	+3	6.45%	82.26%
3	-3	4.84%	87.10%
3	+4	4.84%	91.94%
2	-4	3.23%	95.16%
1	-6	1.61%	96.77%
2	+10	3.23%	100.00%

INTERVALS EXPRESSED BY QUALITATIVE NAMES

FREQ	QUALITY	PERCENT	CUM%
29	MAJ	46.77%	46.77%
26	MIN	41.94%	88.71%
7	PER	11.29%	100.00%

INTERVALS EXPRESSED BY ABSOLUTE DIRECTION

FREQ	DIRECTION	PERCENT	CUM%
34	DESCENDING	54.84%	54.84%
26	ASCENDING	41.94%	96.77%
2	STATIONARY	3.23%	100.00%
62	**TOTAL**		

INTERVALS EXPRESSED BY QUALITATIVE AND QUANTITATIVE NAMES

FREQ	INTERVAL	PERCENT	CUM%
2	*PER1	3.23%	3.23%
8	+MIN2	12.90%	16.13%
12	-MIN2	19.35%	35.48%
9	+MAJ2	14.52%	50.00%
16	-MAJ2	25.81%	75.81%
2	+MIN3	3.23%	79.03%
2	-MIN3	3.23%	82.26%
2	+MAJ3	3.23%	85.48%
1	-MAJ3	1.61%	87.10%
3	+PER4	4.84%	91.94%
2	-PER4	3.23%	95.16%
1	-MAJ6	1.61%	96.77%
2	+MIN10	3.23%	100.00%

SUMMARY ANALYSIS

INT.	UP	DOWN	SAME	TOTAL
PER1			2	2
AUG1				0
MIN2	8	12		20
MAJ2	9	16		25
MIN3	2	2		4
MAJ3	2	1		3
PER4	3	2		5
AUG4				0
DIM5				0
PER5				0
MIN6				0
MAJ6		1		1
MIN7				0
MAJ7				0
PER8				0
MIN9				0
MAJ9				0
MIN10	2			2
MAJ10				0
TOTAL	28	34	2	62

APPENDIX C

Melodic Data Results: Summary Analysis

Sammartini, Symphony for Strings in D Major

INT.	UP	DOWN	SAME	TOTAL
PER1			1	1
MIN2		1		1
MAJ2		1		1
MIN3		6		6
MAJ3		6		6
PER4		6		6
PER5		1		1
PER8	4	1		5
TOTAL	6	22	1	9

Haydn, Op.17, No.6

INT.	UP	DOWN	SAME	TOTAL
PER1			18	18
MIN2	5	8		13
MAJ2	8	32		40
MIN3		9		9
MAJ3		6		6
PER4	6			6
PER5	3	5		8
MAJ6	8			8
MIN7		2		2
MAJ9	3			3
TOTAL	34	63	18	115

Haydn, Op.33, No.1

INT.	UP	DOWN	SAME	TOTAL
PER1			10	10
MIN2	1	5		6
MAJ2	1	8		9
AUG2	2			2
MIN3	9	1		10
MAJ3	1	1		2
PER4	1	1		2
DIM5	1			1
PER5	1			1
MAJ7	1			1
MIN9		1		1
MAJ9		1		1
TOTAL	18	18	10	46

Haydn, Op.33, No.2

INT.	UP	DOWN	SAME	TOTAL
PER1			38	38
AUG1	3			3
MIN2	12	14		26
MAJ2	5	28		33
MIN3	13	13		26
MAJ3	12	5		17
DIM4		1		1
PER4	9	6		15
PER5	6	1		7
MIN6		1		1
MAJ6	2	5		7
MIN7	4	3		7
PER8	3			3
MAJ9	1			1
MAJ10		1		1
PER11		1		1
TOTAL	70	79	38	187

Haydn, Op.33, No.3

INT.	UP	DOWN	SAME	TOTAL
PER1			14	14
MIN2	6	6		12
MAJ2	2	9		11
MIN3	6	12		18
MAJ3	5	4		9
PER4		2		2
PER5	1			1
MAJ6		1		1
MIN7	1			1
MIN14		1		1
TOTAL	22	34	14	70

Haydn, Op.33, No.4

INT.	UP	DOWN	SAME	TOTAL
PER1			14	14
MIN2	5	9		14
MAJ2	16	23		39
MIN3	3	5		8
MAJ3	2	2		4
PER4	1	1		2
PER5		2		2
MIN6	1			1
MAJ6		1		1
MIN7	1			1
MIN10	1			1
PER11	1			1
TOTAL	31	43	14	88

Haydn, Op.33, No.5

INT.	UP	DOWN	SAME	TOTAL
PER1			16	16
AUG1	2			2
MIN2	14	8		22
MAJ2	25	18		43
MIN3		2		2
MAJ3	1	1		2
PER4		4		4
PER5	1			4
MIN7		2		2
MAJ7		1		1
PER8	1			1
MAJ10	1			1
PER12	1			1
TOTAL	46	36	16	98

Haydn, Op.33, No.6

INT.	UP	DOWN	SAME	TOTAL
PER1			11	11
MIN2	8	9		17
MAJ2	19	18		37
MIN3	7	7		14
MAJ3	2	5		7
PER4	4	1		5
DIM5		2		2
PER5	1	2		3
TOTAL	41	44	11	96

Mozart, KV 387

INT.	UP	DOWN	SAME	TOTAL
PER1			2	2
AUG1	2			2
MIN2	8	9		17
MAJ2	5	19		24
DIM3		1		1
MIN3	6	1		7
MAJ3	2			2
PER4	1	1		2
AUG4	2			2
PER5	1			1
MAJ6	1	1		2
DIM8		1		1
PER8	1	1		2
TOTAL	29	35	2	65

Mozart, KV 421

INT.	UP	DOWN	SAME	TOTAL
PER1			11	11
MIN2	3	12		15
MAJ2		7		7
AUG2		2		2
MIN3	1			1
PER4	1			1
DIM5	1			1
MIN6		1		1
PER8		2		2
MIN10	2			2
PER11	1			1
TOTAL	9	24	11	44

Mozart, KV 428

INT.	UP	DOWN	SAME	TOTAL
AUG1	1			1
MIN2	8	6		14
MAJ2		8		8
MIN3	2	4		6
PER4	4	1		5
DIM5		1		1
PER5	2			2
MIN6		2		2
PER8	1			1
TOTAL	18	22	0	40

Mozart, KV 458

INT.	UP	DOWN	SAME	TOTAL
PER1			9	9
MIN2	22	14		36
MAJ2	27	19		46
MIN3	2	2		4
MAJ3	6	3		9
PER4	1	6		7
MAJ6	2			2
PER8		2		2
TOTAL	60	46	9	115

Mozart, KV 464

INT.	UP	DOWN	SAME	TOTAL
PER1			7	7
MIN2	4	11		15
MAJ2	3	20		23
MIN3	4			4
MAJ3	4	1		5
MIN7		1		1
MIN13	1			1
TOTAL	16	33	7	56

Mozart, KV 465

INT.	UP	DOWN	SAME	TOTAL
PER1			11	11
AUG1	1			1
MIN2	16	6		22
MAJ2	17	13		30
MIN3	7	14		21
MAJ3	3	3		6
PER4	2	1		3
MIN6		4		4
MAJ6	1			1
PER8	1			1
TOTAL	48	41	11	100

Beethoven, Op.18, No.1

INT.	UP	DOWN	SAME	TOTAL
PER1			2	2
MIN2	8	12		20
MAJ2	9	16		25
MIN3	2	2		4
MAJ3	2	1		3
PER4	3	2		5
MAJ6		1		1
MIN10	2			2
TOTAL	26	34	2	62

Beethoven, Op.18, No.2

INT.	UP	DOWN	SAME	TOTAL
PER1			6	6
MIN2	10	8		18
MAJ2	7	10		17
MIN3	2	6		8
MAJ3	4	2		6
PER4	7	3		10
PER5		1		1
MIN6	2			2
MAJ6	1			1
MIN7		1		1
PER8	1	2		3
MAJ9		1		1
MAJ10	1			1
PER11	1			1
TOTAL	35	35	6	76

Beethoven, Op.18, No.3

INT.	UP	DOWN	SAME	TOTAL
PER1			2	2
MIN2	13	25		38
MAJ2	6	24		30
DIM3	1	1		2
MIN3	13	7		20
MAJ3	3	5		8
PER4	2	2		4
DIM6		1		1
MIN6	2	1		3
MAJ6	1	1		2
MIN7	3			3
TOTAL	44	67	2	113

Beethoven, Op.18, No.4

INT.	UP	DOWN	SAME	TOTAL
PER1			15	15
MIN2	3	8		11
MAJ2	1	8		9
MIN3	3	3		6
PER4	2			2
DIM5	1			1
DIM7	1			1
MAJ7	1			1
PER8	1			1
TOTAL	13	19	15	47

Beethoven, Op.18, No.5

INT.	UP	DOWN	SAME	TOTAL
PER1			5	5
AUG1	2			2
MIN2	8	2		10
MAJ2	13	1		14
MIN3		1		1
MAJ3		1		1
PER5		1		1
PER8		1		1
MAJ9	2	1		3
MAJ10		1		1
TOTAL	25	10	5	40

Beethoven, Op.18, No.6

INT.	UP	DOWN	SAME	TOTAL
MIN2	18	16		34
MAJ2	9	1		10
MIN3		8		8
MAJ3		6		6
PER4	7	7		14
PER5	10	13		23
MIN6	6			6
MAJ6	7			7
MIN7	1	1		2
PER8	1	2		3
PER22	1	2		3
PER25		2		2
PER26	2			2
TOTAL	62	58	0	120

Beethoven, Op.95

INT.	UP	DOWN	SAME	TOTAL
PER1			7	7
MIN2	3	7		10
MAJ2	4	7		11
MIN3		2		2
MAJ3		1		1
PER4	3			3
DIM5		1		1
PER5	1	1		2
PER8	4	5		9
MIN9	2			2
TOTAL	18	24	7	49

Schubert, String Quartet No.5

INT.	UP	DOWN	SAME	TOTAL
PER1			4	4
MIN2	5	1		6
MAJ2	4	4		8
MIN3	1	4		5
MAJ3	2	3		5
PER4	1			1
DIM5		2		2
PER5	7	5		12
MIN7	2	1		3
TOTAL	23	20	4	47

APPENDIX D

Sample Results from Processing the Harmonic Data:

Beethoven String Quartet in F major, Op.18, No.1.

PRIMARY CHORD FREQUENCY

TONIC	8
S.DOMINANT	3
S.TONIC6	1
S.TONIC	1
DOMINANT	3
DOMINANT7	2
CADENTIAL.64	2

PRIMARY CHORD DURATIONS BY TOTAL NUMBER OF BEATS

TONIC	22.0
S.DOMINANT	7.0
S.TONIC6	1.5
S.TONIC	3.0
DOMINANT	7.0
DOMINANT7	4.0
CADENTIAL.64	2.0

CHORD FREQUENCY

PRIMARY CHORDS	20
SECONDARY CHORDS	5
TOTAL NO. OF CHORDS	25

CHORD DURATIONS BY TOTAL NUMBER OF BEATS

PRIMARY DURATIONS	48.5
SECONDARY DURATIONS	11.5
TOTAL DURATIONS	60.0

APPENDIX E

Intervals Expressed by Absolute Direction

Haydn, Op.33

Op.33, No.1

FREQ	DIRECTION	PERCENT	CUM%
18	ASCENDING	39.13%	39.13%
18	DESCENDING	39.13%	78.26%
10	STATIONARY	21.74%	100.00%
46	TOTAL		

Op.33, No.2

FREQ	DIRECTION	PERCENT	CUM%
70	ASCENDING	37.48%	37.48%
79	DESCENDING	42.25%	79.68%
38	STATIONARY	20.32%	100.00%
187	TOTAL		

Op.33, No.3

FREQ	DIRECTION	PERCENT	CUM%
22	ASCENDING	31.43%	31.43%
34	DESCENDING	48.57%	80.00%
14	STATIONARY	20.00%	100.00%
70	TOTAL		

Op.33, No.4

FREQ	DIRECTION	PERCENT	CUM%
31	ASCENDING	35.23%	35.23%
43	DESCENDING	48.86%	84.09%
14	STATIONARY	15.91%	100.00%
88	TOTAL		

Op.33, No.5

FREQ	DIRECTION	PERCENT	CUM%
46	ASCENDING	46.94%	46.94%
36	DESCENDING	36.73%	83.67%
16	STATIONARY	16.33%	100.00%
98	TOTAL		

Op.33, No.6

FREQ	DIRECTION	PERCENT	CUM%
41	ASCENDING	42.71%	42.71%
44	DESCENDING	45.83%	88.54%
11	STATIONARY	11.46%	100.00%
96	TOTAL		

Mozart, 'Haydn' Quartets

KV 387

FREQ	DIRECTION	PERCENT	CUM%
29	ASCENDING	44.62%	44.62%
34	DESCENDING	52.31%	96.92%
2	STATIONARY	3.08%	100.00%
65	TOTAL		

KV 421

FREQ	DIRECTION	PERCENT	CUM%
9	ASCENDING	20.45%	20.45%
24	DESCENDING	54.55%	75.00%
11	STATIONARY	25.00%	100.00%
44	TOTAL		

KV 428

FREQ	DIRECTION	PERCENT	CUM%
18	ASCENDING	45.00%	45.00%
22	DESCENDING	55.00%	100.00%
0	STATIONARY	0.00%	100.00%
40	TOTAL		

KV 458

FREQ	DIRECTION	PERCENT	CUM%
60	ASCENDING	52.17%	52.17%
46	DESCENDING	40.00%	92.17%
9	STATIONARY	7.83%	100.00%
115	TOTAL		

KV 464

FREQ	DIRECTION	PERCENT	CUM%
16	ASCENDING	28.57%	28.57%
33	DESCENDING	58.93%	87.50%
7	STATIONARY	12.50%	100.00%
56	TOTAL		

KV 465

FREQ	DIRECTION	PERCENT	CUM%
48	ASCENDING	48.00%	48.00%
41	DESCENDING	41.00%	41.00%
11	STATIONARY	11.00%	100.00%
100	TOTAL		

Beethoven, Op.18

Op.18, No.1

FREQ	DIRECTION	PERCENT	CUM%
26	ASCENDING	41.94%	41.94%
34	DESCENDING	54.84%	96.77%
2	STATIONARY	3.23%	100.00%
62	TOTAL		

Op.18, No.2

FREQ	DIRECTION	PERCENT	CUM%
35	ASCENDING	46.05%	46.05%
35	DESCENDING	46.05%	92.11%
6	STATIONARY	7.89%	100.00%
76	TOTAL		

Op.18, No.3

FREQ	DIRECTION	PERCENT	CUM%
44	ASCENDING	38.94%	38.94%
67	DESCENDING	59.29%	98.23%
2	STATIONARY	1.77%	100.00%
113	TOTAL		

Op.18, No.4

FREQ	DIRECTION	PERCENT	CUM%
13	ASCENDING	27.66%	27.66%
19	DESCENDING	40.43%	68.09%
15	STATIONARY	31.91%	100.00%
47	TOTAL		

Op.18, No.5

FREQ	DIRECTION	PERCENT	CUM%
25	ASCENDING	62.50%	62.50%
10	DESCENDING	25.00%	87.50%
5	STATIONARY	12.50%	100.00%
40	TOTAL		

Op.18, No.6

FREQ	DIRECTION	PERCENT	CUM%
62	ASCENDING	51.67%	51.67%
58	DESCENDING	48.33%	100.00%
0	STATIONARY	0.00%	100.00%
120	TOTAL		

APPENDIX F

Intervals Expressed by Qualitative Names

Haydn, Op.33

Op.33, No.1

FREQ	QUALITY	PERCENT	CUM%
13	MAJ	28.26%	28.26%
17	MIN	36.96%	65.22%
13	PER	28.26%	93.48%
1	DIM	2.17%	95.65%
1	AUG	4.35%	100.00%

Op.33, No.2

FREQ	QUALITY	PERCENT	CUM%
59	MAJ	31.55%	31.55%
60	MIN	32.09%	63.64%
64	PER	34.22%	97.86%
1	DIM	.53%	98.40%
3	AUG	1.60%	100.00%

Op.33, No.3

FREQ	QUALITY	PERCENT	CUM%
21	MAJ	30.00%	30.00%
32	MIN	45.71%	75.71%
17	PER	24.29%	100.00%

Op.33, No.4

FREQ	QUALITY	PERCENT	CUM%
44	MAJ	50.00%	50.00%
25	MIN	28.41%	78.41%
19	PER	21.59%	100.00%

Op.33, No.5

FREQ	QUALITY	PERCENT	CUM%
47	MAJ	47.96%	47.96%
26	MIN	26.53%	74.49%
23	PER	23.47%	97.96%
2	AUG	2.04%	100.00%

Op.33, No.6

FREQ	QUALITY	PERCENT	CUM%
44	MAJ	45.83%	45.83%
31	MIN	32.29%	78.12%
19	PER	19.79%	97.92%
2	DIM	2.08%	100.00%

Mozart, 'Haydn' Quartets

KV 387

FREQ	QUALITY	PERCENT	CUM%
28	MAJ	43.08%	43.08%
24	MIN	36.92%	80.00%
7	PER	10.77%	90.77%
2	DIM	3.08%	93.85%
4	AUG	6.15%	100.00%

KV 421

FREQ	QUALITY	PERCENT	CUM%
7	MAJ	15.91%	15.91%
19	MIN	43.18%	59.09%
15	PER	34.09%	93.18%
1	DIM	2.27%	95.45%
2	AUG	4.55%	100.00%

KV 428

FREQ	QUALITY	PERCENT	CUM%
8	MAJ	20.00%	20.00%
22	MIN	55.00%	75.00%
8	PER	20.00%	95.00%
1	DIM	2.50%	97.50%
1	AUG	2.50%	100.00%

KV 458

FREQ	QUALITY	PERCENT	CUM%
57	MAJ	49.57%	49.57%
40	MIN	34.78%	84.35%
18	PER	15.65%	100.00%

KV 464

FREQ	QUALITY	PERCENT	CUM%
28	MAJ	50.00%	50.00%
21	MIN	37.50%	87.50%
7	PER	12.50%	100.00%

KV 465

FREQ	QUALITY	PERCENT	CUM%
37	MAJ	37.00%	37.00%
47	MIN	47.00%	84.00%
15	PER	15.00%	99.00%
1	AUG	1.00%	100.00%

Beethoven, Op.18

Op.18, No.1

FREQ	QUALITY	PERCENT	CUM%
29	MAJ	46.77%	46.77%
26	MIN	41.94%	88.71%
7	PER	11.29%	100.00%

Op.18, No.2

FREQ	QUALITY	PERCENT	CUM%
26	MAJ	34.21%	34.21%
29	MIN	38.16%	72.37%
21	PER	27.63%	100.00%

Op.18, No.3

FREQ	QUALITY	PERCENT	CUM%
40	MAJ	35.40%	35.40%
64	MIN	56.64%	92.04%
6	PER	5.31%	97.35%
3	DIM	2.65%	100.00%

Op.18, No.4

FREQ	QUALITY	PERCENT	CUM%
10	MAJ	21.28%	21.28%
17	MIN	36.17%	57.45%
18	PER	38.30%	95.74%
2	DIM	4.26%	100.00%

Op.18, No.5

FREQ	QUALITY	PERCENT	CUM%
20	MAJ	50.00%	50.00%
11	MIN	27.50%	77.50%
7	PER	17.50%	95.00%
2	AUG	5.00%	100.00%

Op.18, No.6

FREQ	QUALITY	PERCENT	CUM%
23	MAJ	19.17%	19.17%
50	MIN	41.67%	60.83%
47	PER	39.17%	100.00%

APPENDIX G

Intervals Expressed by Qualitative and Quantitative Names

Haydn, Op.33

Op.33, No.1

FREQ	INTERVAL	PERCENT	CUM%
10	*PER1	21.74%	21.74%
1	+MIN2	2.17%	23.91%
5	-MIN2	10.87%	34.78%
1	+MAJ2	2.17%	36.96%
8	-MAJ2	17.39%	54.35%
2	+AUG2	4.35%	58.70%
9	+MIN3	19.57%	78.27%
1	-MIN3	2.17%	80.45%
1	+MAJ3	2.17%	82.62%
1	-MAJ3	2.17%	84.79%
1	+PER4	2.17%	86.96%
1	-PER4	2.17%	89.14%
1	+DIM5	2.17%	91.31%
1	+PER5	2.17%	93.48%
1	+MAJ7	2.17%	95.65%
1	-MIN9	2.17%	97.88%
1	-MAJ9	2.17%	100.00%

Op.33, No.2

FREQ	INTERVAL	PERCENT	CUM%
38	*PER1	20.32%	20.32%
3	+AUG1	1.60%	21.93%
12	+MIN2	6.42%	28.34%
14	-MIN2	7.49%	35.83%
5	+MAJ2	2.67%	38.50%
28	-MAJ2	14.97%	53.48%
13	+MIN3	6.95%	60.43%
13	-MIN3	6.95%	67.38%
12	+MAJ3	6.42%	73.80%
5	-MAJ3	2.67%	76.47%
1	-DIM4	.53%	77.01%
9	+PER4	4.81%	81.82%
6	-PER4	3.21%	85.03%
6	+PER5	3.21%	88.24%
1	-PER5	.53%	88.77%
1	-MIN6	.53%	89.30%
2	+MAJ6	1.07%	90.37%
5	-MAJ6	2.67%	93.05%
4	+MIN7	2.14%	95.19%
3	-MIN7	1.60%	96.79%
3	+PER8	1.60%	98.40%
1	+MAJ9	.53%	98.93%
1	-MAJ10	.53%	99.46%
1	-PER11	.53%	100.00%

Op.33, No.3

FREQ	INTERVAL	PERCENT	CUM%
14	*PER1	20.00%	20.00%
6	+MIN2	8.57%	28.57%
6	-MIN2	8.57%	37.14%
2	+MAJ2	2.86%	40.00%
9	-MAJ2	12.86%	52.86%
6	+MIN3	8.57%	61.43%
12	-MIN3	17.14%	78.57%
5	+MAJ3	7.14%	85.71%
4	-MAJ3	5.71%	91.43%
2	-PER4	2.86%	94.29%
1	+PER5	1.43%	95.71%
1	-MAJ6	1.43%	97.14%
1	+MIN7	1.43%	98.57%
1	+MIN14	1.43%	100.00%

Op.33, No.4

FREQ	INTERVAL	PERCENT	CUM%
14	*PER1	15.91%	15.91%
5	+MIN2	5.68%	21.59%
9	-MIN2	10.23%	31.82%
16	+MAJ2	18.18%	50.00%
23	-MAJ2	26.14%	76.14%
3	+MIN3	3.41%	79.55%
5	-MIN3	5.68%	85.23%
2	+MAJ3	2.27%	87.50%
2	-MAJ3	2.27%	89.77%
1	+PER4	1.14%	90.91%
1	-PER4	1.14%	92.05%
2	-PER5	2.27%	94.32%
1	-MIN6	1.14%	95.45%
1	-MAJ6	1.14%	96.59%
1	+MIN7	1.14%	97.73%
1	+MIN10	1.14%	98.86%
1	+PER12	1.14%	100.00%

Op.33, No.5

FREQ	INTERVAL	PERCENT	CUM%
16	*PER1	16.33%	16.33%
2	+AUG1	2.04%	18.37%
14	+MIN2	14.29%	32.65%
8	-MIN2	8.16%	40.82%
25	+MAJ2	25.51%	66.33%
18	-MAJ2	18.37%	84.69%
2	-MIN3	2.04%	86.73%
1	+MAJ3	1.02%	87.76%
1	-MAJ3	1.02%	88.78%
4	-PER4	4.08%	92.86%
1	+PER5	1.02%	93.88%
2	-MIN7	2.04%	95.92%
1	-MAJ7	1.02%	96.94%
1	+PER8	1.02%	97.96%
1	+MAJ10	1.02%	98.98%
1	+PER12	1.02%	100.00%

Op.33, No.6

FREQ	INTERVAL	PERCENT	CUM%
11	*PER1	11.46%	11.46%
8	+MIN2	8.33%	19.79%
9	-MIN2	9.38%	29.17%
19	+MAJ2	19.79%	48.96%
18	-MAJ2	18.75%	67.71%
7	+MIN3	7.29%	75.00%
7	-MIN3	7.29%	82.29%
2	+MAJ3	2.08%	84.37%
5	-MAJ3	5.21%	89.58%
4	+PER4	4.17%	93.75%
1	-PER4	1.04%	94.79%
2	-DIM5	2.08%	96.87%
1	+PER5	1.04%	97.92%
2	-PER5	2.08%	100.00%

Mozart

KV 387

FREQ	INTERVAL	PERCENT	CUM%
2	*PER1	3.08%	3.08%
2	+AUG1	3.08%	6.15%
8	+MIN2	12.31%	18.46%
9	-MIN2	13.85%	32.31%
5	+MAJ2	7.69%	40.00%
19	-MAJ2	29.23%	69.23%
1	-DIM3	1.54%	70.77%
6	+MIN3	9.23%	80.00%
1	-MIN3	1.54%	81.54%
2	+MAJ3	3.08%	84.62%
1	+PER4	1.54%	86.16%
1	-PER4	1.54%	87.70%
2	+AUG4	3.08%	90.78%
1	+PER5	1.54%	92.32%
1	+MAJ6	1.54%	93.86%
1	-MAJ6	1.54%	95.40%
1	-DIM8	1.54%	96.94%
1	+PER8	1.54%	98.48%
1	-PER8	1.54%	100.00%

KV 421

FREQ	INTERVAL	PERCENT	CUM%
11	*PER1	25.00%	25.00%
3	+MIN2	6.82%	31.82%
12	-MIN2	27.27%	59.09%
7	-MAJ2	15.91%	75.00%
2	-AUG2	2.27%	77.27%
1	+MIN3	2.27%	79.27%
1	+PER4	2.27%	81.81%
1	+DIM5	2.27%	84.08%
1	-MIN6	4.55%	88.63%
2	-PER8	4.55%	93.18%
2	+MIN10	4.55%	97.73%
1	+PER11	2.27%	100.00%

KV 428

FREQ	INTERVAL	PERCENT	CUM%
1	+AUG1	2.50%	2.50%
8	+MIN2	20.00%	22.50%
6	-MIN2	15.00%	37.50%
8	-MAJ2	20.00%	57.50%
2	+MIN3	5.00%	62.50%
4	-MIN3	10.00%	72.50%
4	+PER4	10.00%	82.50%
1	-PER4	2.50%	85.00%
1	-DIM5	2.50%	87.50%
2	+PER5	5.00%	92.50%
2	-MIN6	5.00%	97.50%
1	+PER8	2.50%	100.00%

KV 458

FREQ	INTERVAL	PERCENT	CUM%
9	*PER1	7.83%	7.83%
22	+MIN2	19.13%	26.96%
14	-MIN2	12.17%	39.13%
27	+MAJ2	23.48%	62.61%
19	-MAJ2	16.52%	79.13%
2	+MIN3	1.74%	80.87%
2	-MIN3	1.74%	82.61%
6	+MAJ3	5.22%	87.83%
3	-MAJ3	2.61%	90.43%
1	+PER4	.87%	91.30%
6	-PER4	5.22%	96.52%
2	+MAJ6	1.74%	98.26%
2	-PER8	1.74%	100.00%

KV 464

FREQ	INTERVAL	PERCENT	CUM%
7	*PER1	12.50%	12.50%
4	+MIN2	7.14%	19.64%
11	-MIN2	19.64%	39.29%
3	+MAJ2	5.36%	44.64%
20	-MAJ2	35.71%	80.36%
4	+MIN3	7.14%	87.50%
4	+MAJ3	7.14%	94.64%
1	-MAJ3	1.79%	96.43%
1	-MIN7	1.79%	98.21%
1	+MIN13	1.79%	100.00%

KV 465

FREQ	INTERVAL	PERCENT	CUM%
11	*PER1	11.00%	11.00%
1	+AUG1	1.00%	12.00%
16	+MIN2	16.00%	28.00%
6	-MIN2	6.00%	34.00%
17	+MAJ2	17.00%	51.00%
13	-MAJ2	13.00%	64.00%
7	+MIN3	7.00%	71.00%
14	-MIN3	14.00%	85.00%
3	+MAJ3	3.00%	88.00%
3	-MAJ3	3.00%	91.00%
2	+PER4	2.00%	93.00%
1	-PER4	1.00%	94.00%
4	-MIN6	4.00%	98.00%
1	+MAJ6	1.00%	99.00%
1	+PER8	1.00%	100.00%

Beethoven

Op.18, No.1

FREQ	INTERVAL	PERCENT	CUM%
2	*PER1	3.23%	3.23%
8	+MIN2	12.90%	16.13%
12	-MIN2	19.35%	35.48%
9	+MAJ2	14.52%	50.00%
16	-MAJ2	25.81%	75.81%
2	+MIN3	3.23%	79.03%
2	-MIN3	3.23%	82.26%
2	+MAJ3	3.23%	85.48%
1	-MAJ3	1.61%	87.10%
3	+PER4	4.84%	91.94%
2	-PER4	3.23%	95.16%
1	-MAJ6	1.61%	96.77%
2	+MIN10	3.23%	100.00%

Op.18, No.2

FREQ	INTERVAL	PERCENT	CUM%
6	*PER1	7.89%	7.89%
10	+MIN2	13.16%	21.05%
8	-MIN2	10.53%	31.58%
7	+MAJ2	9.21%	40.79%
10	-MAJ2	13.16%	53.95%
2	+MIN3	2.63%	56.58%
6	-MIN3	7.89%	64.47%
4	+MAJ3	5.26%	69.74%
2	-MAJ3	2.63%	72.37%
7	+PER4	9.21%	81.58%
3	-PER4	3.95%	85.53%
1	-PER5	1.32%	86.84%
2	+MIN6	2.63%	89.47%
1	+MAJ6	1.32%	90.79%
1	-MIN7	1.32%	92.11%
1	+PER8	1.32%	93.42%
2	-PER8	2.63%	96.05%
1	-MAJ9	1.32%	97.37%
1	+MAJ10	1.32%	98.68%
1	-PER11	1.32%	100.00%

Op.18, No.3

FREQ	INTERVAL	PERCENT	CUM%
2	*PER1	1.77%	1.77%
13	+MIN2	11.50%	13.27%
25	-MIN2	22.12%	35.40%
6	+MAJ2	5.31%	40.71%
24	-MAJ2	21.24%	61.95%
1	+DIM3	.88%	62.83%
1	-DIM3	.88%	63.71%
13	+MIN3	11.50%	75.21%
7	-MIN3	6.19%	81.40%
3	+MAJ3	2.65%	84.05%
5	-MAJ3	4.42%	88.47%
2	+PER4	1.77%	90.24%
2	-PER4	1.77%	92.01%
1	-DIM6	.88%	92.90%
2	+MIN6	1.77%	94.67%
1	-MIN6	.88%	95.56%
1	+MAJ6	.88%	96.45%
1	-MAJ6	.88%	97.34%
3	+MIN7	2.65%	100.00%

Op.18, No.4

FREQ	INTERVAL	PERCENT	CUM%
15	*PER1	31.91%	31.91%
3	+MIN2	6.38%	38.30%
8	-MIN2	17.02%	55.32%
1	+MAJ2	2.13%	57.45%
8	-MAJ2	17.02%	74.47%
3	+MIN3	6.38%	80.85%
3	-MIN3	6.38%	87.23%
2	+PER4	4.26%	91.49%
1	+DIM5	2.13%	93.62%
1	+DIM7	2.13%	95.74%
1	+MAJ7	2.13%	97.87%
1	+PER8	2.13%	100.00%

Op.18, No.5

FREQ	INTERVAL	PERCENT	CUM%
5	*PER1	12.50%	12.50%
2	+AUG1	5.00%	17.50%
8	+MIN2	20.00%	37.50%
2	-MIN2	5.00%	42.50%
13	+MAJ2	32.50%	75.00%
1	-MAJ2	2.50%	77.50%
1	-MIN3	2.50%	80.00%
2	-MAJ3	5.00%	85.00%
1	-PER5	2.50%	87.50%
1	-PER8	2.50%	90.00%
2	+MAJ9	5.00%	95.00%
1	-MAJ9	2.50%	97.50%
1	-MAJ10	2.50%	100.00%

Op.18, No.6

FREQ	INTERVAL	PERCENT	CUM%
18	+MIN2	15.00%	15.00%
16	-MIN2	13.33%	28.33%
9	+MAJ2	7.50%	35.83%
1	-MAJ2	.83%	36.67%
8	-MIN3	6.67%	43.33%
6	-MAJ3	5.00%	48.33%
7	+PER4	5.83%	54.17%
7	-PER4	5.83%	60.00%
10	+PER5	8.33%	68.33%
13	-PER5	10.83%	79.17%
6	+MIN6	5.00%	84.17%
7	+MAJ6	5.83%	90.00%
1	+MIN7	.83%	90.83%
1	-MIN7	.83%	91.67%
1	+PER8	.83%	92.50%
2	-PER8	1.67%	94.17%
1	+PER22	.83%	95.84%
2	-PER22	1.67%	96.67%
2	-PER25	1.67%	98.34%
2	+PER26	1.67%	100.00%

APPENDIX H

Melodic Data for Each Quartet Set Collectively

Haydn, Opus 33

FREQ	INTERVAL	PERCENT	CUM%
103	PER1	17.61%	17.61%
5	+AUG1	0.85%	18.46%
46	+MIN2	7.86%	26.32%
51	-MIN2	8.72%	35.04%
68	+MAJ2	11.62%	46.67%
104	-MAJ2	17.78%	64.44%
2	+AUG2	0.34%	64.79%
38	+MIN3	6.50%	71.28%
40	-MIN3	6.84%	78.12%
23	+MAJ3	3.93%	82.05%
18	-MAJ3	3.08%	85.13%
1	-DIM4	0.17%	85.30%
15	+PER4	2.56%	87.86%
15	-PER4	2.56%	90.42%
1	+DIM5	0.17%	90.59%
1	-DIM5	0.17%	90.76%
10	+PER5	1.71%	92.47%
5	-PER5	0.85%	93.32%
2	-MIN6	0.34%	93.66%
2	+MAJ6	0.34%	94.00%
7	-MAJ6	1.20%	95.20%
6	+MIN7	1.03%	96.23%
5	-MIN7	0.85%	97.08%
1	+MAJ7	0.17%	97.25%
1	-MAJ7	0.17%	97.43%
4	+PER8	0.68%	98.11%
1	-MIN9	0.17%	98.29%
1	+MAJ9	0.17%	98.46%
1	-MAJ9	0.17%	98.63%
1	+MIN10	0.17%	98.80%
1	+MAJ10	0.17%	98.98%
2	-MAJ10	0.34%	99.31%
1	-PER11	0.17%	99.49%
2	+PER12	0.34%	99.83%
1	+MIN14	0.17%	100.00%

Mozart, KV 387-465

FREQ	INTERVAL	PERCENT	CUM%
40	PER1	9.52%	9.52%
4	+AUG1	0.95%	10.47%
61	+MIN2	14.52%	24.99%
58	-MIN2	13.81%	38.80%
52	+MAJ2	12.38%	51.18%
86	-MAJ2	20.48%	71.66%
2	-AUG2	0.48%	72.14%
1	-DIM3	0.24%	72.38%
22	+MIN3	5.24%	77.62%
21	-MIN3	5.00%	82.62%
15	+MAJ3	3.57%	86.19%
7	-MAJ3	1.67%	87.86%
9	+PER4	2.14%	90.00%
9	-PER4	2.14%	92.14%
2	+AUG4	0.48%	92.62%
1	+DIM5	0.24%	92.86%
1	-DIM5	0.24%	93.10%
3	+PER5	0.71%	93.81%
7	-MIN6	1.67%	95.48%
4	+MAJ6	0.95%	96.43%
1	-MAJ6	0.24%	96.67%
1	-MIN7	0.24%	96.91%
1	-DIM8	0.24%	97.15%
3	+PER8	0.71%	97.86%
5	-PER8	1.19%	99.05%
2	+MIN10	0.48%	99.53%
1	+PER11	0.24%	99.77%
1	+MIN13	0.24%	100.00%

Beethoven, Opus 18

FREQ	INTERVAL	PERCENT	CUM%
30	PER1	6.55%	6.55%
2	+AUG1	0.44%	6.99%
60	+MIN2	13.10%	20.09%
71	-MIN2	15.50%	35.59%
45	+MAJ2	9.83%	45.42%
60	-MAJ2	13.10%	58.52%
1	-DIM3	0.22%	58.74%
1	+DIM3	0.22%	58.96%
20	+MIN3	4.37%	63.33%
27	-MIN3	5.90%	69.23%
9	+MAJ3	1.97%	71.20%
16	-MAJ3	3.49%	74.69%
21	+PER4	4.59%	79.28%
14	-PER4	3.06%	82.34%
1	+DIM5	0.22%	82.56%
10	+PER5	2.18%	84.74%
15	-PER5	3.28%	88.02%
1	-DIM6	0.22%	88.24%
10	+MIN6	2.18%	90.42%
1	-MIN6	0.22%	90.62%
9	+MAJ6	1.97%	92.59%
2	-MAJ6	0.44%	93.03%
1	+DIM7	0.22%	93.25%
4	+MIN7	0.87%	94.12%
2	-MIN7	0.44%	94.56%
1	+MAJ7	0.22%	94.78%
3	+PER8	0.66%	95.44%
5	-PER8	1.09%	96.53%
2	+MAJ9	0.44%	96.97%
2	-MAJ9	0.44%	97.41%
2	+MIN10	0.44%	97.85%
1	+MAJ10	0.22%	98.07%
1	-MAJ10	0.22%	98.29%
1	-PER11	0.22%	98.49%
2	+PER22	0.44%	98.93%
1	-PER22	0.22%	99.13%
2	-PER25	0.44%	99.57%
2	+PER26	0.44%	100.00%

APPENDIX I

Harmonic Data

Haydn

Op.33. No.1

	PRIMARY CHORD FREQUENCY	PRIMARY CHORD DURATIONS BY TOTAL NUMBER OF BEATS
TONIC	1	1.0
S.DOMINANT	1	2.0
S.TONIC6	0	0
S.TONIC	3	4.5
DOMINANT	2	2.0
DOMINANT7	5	15.0
CADENTIAL.64	2	2.0

	CHORD FREQUENCY	CHORD DURATIONS BY TOTAL NUMBER OF BEATS
PRIMARY CHORDS	14	26.5
SECONDARY CHORDS	11	15.0
TOTAL	25	41.5

Op.33. No.2

	PRIMARY CHORD FREQUENCY	PRIMARY CHORD DURATIONS BY TOTAL NUMBER OF BEATS
TONIC	14	22.0
S.DOMINANT	2	4.0
S.TONIC6	1	2.0
S.TONIC	0	0
DOMINANT	2	3.0
DOMINANT7	9	13.0
CADENTIAL.64	0	0

	CHORD FREQUENCY	CHORD DURATIONS BY TOTAL NUMBER OF BEATS
PRIMARY CHORDS	28	44
SECONDARY CHORDS	2	4
TOTAL	30	48

Op.33. No.3

	PRIMARY CHORD FREQUENCY	PRIMARY CHORD DURATIONS BY TOTAL NUMBER OF BEATS
TONIC	7	10.5
S.DOMINANT	0	0
S.TONIC6	5	7.5
S.TONIC	3	4.5
DOMINANT	0	0
DOMINANT7	2	2.0
CADENTIAL.64	0	0

	CHORD FREQUENCY	CHORD DURATIONS BY TOTAL NUMBER OF BEATS
PRIMARY CHORDS	17	24.5
SECONDARY CHORDS	8	8.0
TOTAL	25	32.5

Op.33. No.4

	PRIMARY CHORD FREQUENCY	PRIMARY CHORD DURATIONS BY TOTAL NUMBER OF BEATS
TONIC	13	17.5
S.DOMINANT	1	2.0
S.TONIC6	2	2.0
S.TONIC	0	0
DOMINANT	6	6.0
DOMINANT7	8	8.0
CADENTIAL.64	0	0

	CHORD FREQUENCY	CHORD DURATIONS BY TOTAL NUMBER OF BEATS
PRIMARY CHORDS	30	35.5
SECONDARY CHORDS	1	.5
TOTAL	31	36.0

Op.33. No.5

	PRIMARY CHORD FREQUENCY	PRIMARY CHORD DURATIONS BY TOTAL NUMBER OF BEATS
TONIC	15	20.5
S.DOMINANT	0	0
S.TONIC6	2	4.0
S.TONIC	2	1.0
DOMINANT	8	10.0
DOMINANT7	9	15.5
CADENTIAL.64	1	.5

	CHORD FREQUENCY	CHORD DURATIONS BY TOTAL NUMBER OF BEATS
PRIMARY CHORDS	37	51.5
SECONDARY CHORDS	8	11.5
TOTAL	45	63.0

Op.33. No.6

	PRIMARY CHORD FREQUENCY	PRIMARY CHORD DURATIONS BY TOTAL NUMBER OF BEATS
TONIC	19	35.0
S.DOMINANT	10	16.0
S.TONIC6	2	4.0
S.TONIC	4	6.0
DOMINANT	6	10.0
DOMINANT7	8	14.0
CADENTIAL.64	2	8.0

	CHORD FREQUENCY	CHORD DURATIONS BY TOTAL NUMBER OF BEATS
PRIMARY CHORDS	51	93.0
SECONDARY CHORDS	6	12.0
TOTAL	57	105.0

Mozart

KV 387

	PRIMARY CHORD FREQUENCY	PRIMARY CHORD DURATIONS BY TOTAL NUMBER OF BEATS
TONIC	7	8.0
S.DOMINANT	0	0
S.TONIC6	1	1.0
S.TONIC	5	8.0
DOMINANT	2	3.0
DOMINANT7	3	3.0
CADENTIAL.64	1	1.0

	CHORD FREQUENCY	CHORD DURATIONS BY TOTAL NUMBER OF BEATS
PRIMARY CHORDS	19	24.0
SECONDARY CHORDS	10	13.5
TOTAL	29	37.5

KV 421

	PRIMARY CHORD FREQUENCY	PRIMARY CHORD DURATIONS BY TOTAL NUMBER OF BEATS
TONIC	7	15.0
S.DOMINANT	2	3.0
S.TONIC6	0	0
S.TONIC	0	0
DOMINANT	2	3.0
DOMINANT7	1	1.0
CADENTIAL.64	1	1.0

	CHORD FREQUENCY	CHORD DURATIONS BY TOTAL NUMBER OF BEATS
PRIMARY CHORDS	13	23.0
SECONDARY CHORDS	6	9.0
TOTAL	19	32.0

KV 428

	PRIMARY CHORD FREQUENCY	PRIMARY CHORD DURATIONS BY TOTAL NUMBER OF BEATS
TONIC	4	15.0
S.DOMINANT	1	2.0
S.TONIC6	0	0
S.TONIC	3	21.0
DOMINANT	1	2.0
DOMINANT7	3	4.0
CADENTIAL.64	2	2.0

	CHORD FREQUENCY	CHORD DURATIONS BY TOTAL NUMBER OF BEATS
PRIMARY CHORDS	14	46.0
SECONDARY CHORDS	0	0
TOTAL	14	46.0

KV 458

	PRIMARY CHORD FREQUENCY	PRIMARY CHORD DURATIONS BY TOTAL NUMBER OF BEATS
TONIC	18	70.0
S.DOMINANT	0	0
S.TONIC6	0	0
S.TONIC	4	10.0
DOMINANT	4	7.0
DOMINANT7	5	19.0
CADENTIAL.64	0	0

	CHORD FREQUENCY	CHORD DURATIONS BY TOTAL NUMBER OF BEATS
PRIMARY CHORDS	31	106.0
SECONDARY CHORDS	0	0
TOTAL	31	106.0

KV 464

	PRIMARY CHORD FREQUENCY	PRIMARY CHORD DURATIONS BY TOTAL NUMBER OF BEATS
TONIC	10	23.0
S.DOMINANT	1	3.0
S.TONIC6	0	0
S.TONIC	0	0
DOMINANT	1	3.0
DOMINANT7	7	14.0
CADENTIAL.64	1	2.0

	CHORD FREQUENCY	CHORD DURATIONS BY TOTAL NUMBER OF BEATS
PRIMARY CHORDS	20	45.0
SECONDARY CHORDS	0	0
TOTAL	20	45.0

KV 465

	PRIMARY CHORD FREQUENCY	PRIMARY CHORD DURATIONS BY TOTAL NUMBER OF BEATS
TONIC	10	29.0
S.DOMINANT	5	13.0
S.TONIC6	2	4.0
S.TONIC	1	1.0
DOMINANT	5	9.0
DOMINANT7	6	13.0
CADENTIAL.64	1	2.0

	CHORD FREQUENCY	CHORD DURATIONS BY TOTAL NUMBER OF BEATS
PRIMARY CHORDS	30	71
SECONDARY CHORDS	10	16
TOTAL	40	87

Beethoven

Op.18, No.1

	PRIMARY CHORD FREQUENCY	PRIMARY CHORD DURATIONS BY TOTAL NUMBER OF BEATS
TONIC	8	22.0
S.DOMINANT	3	7.0
S.TONIC6	1	1.5
S.TONIC	1	3.0
DOMINANT	3	7.0
DOMINANT7	2	4.0
CADENTIAL.64	2	4.0

	CHORD FREQUENCY	CHORD DURATIONS BY TOTAL NUMBER OF BEATS
PRIMARY CHORDS	20	48.5
SECONDARY CHORDS	5	11.5
TOTAL	25	60.0

Op.18, No.2

	PRIMARY CHORD FREQUENCY	PRIMARY CHORD DURATIONS BY TOTAL NUMBER OF BEATS
TONIC	12	18.875
S.DOMINANT	1	2.0
S.TONIC6	1	.5
S.TONIC	2	1.875
DOMINANT	3	4.125
DOMINANT7	6	10.5
CADENTIAL.64	0	0

	CHORD FREQUENCY	CHORD DURATIONS BY TOTAL NUMBER OF BEATS
PRIMARY CHORDS	25	37.875
SECONDARY CHORDS	3	2.125
TOTAL	28	40.000

Op.18, No.3

	PRIMARY CHORD FREQUENCY	PRIMARY CHORD DURATIONS BY TOTAL NUMBER OF BEATS
TONIC	9	15.0
S.DOMINANT	2	2.5
S.TONIC6	5	9.0
S.TONIC	0	0
DOMINANT	4	5.5
DOMINANT7	10	16.5
CADENTIAL.64	1	2.0

	CHORD FREQUENCY	CHORD DURATIONS BY TOTAL NUMBER OF BEATS
PRIMARY CHORDS	31	50.5
SECONDARY CHORDS	3	4.5
TOTAL	34	55.0

Op.18, No.4

	PRIMARY CHORD FREQUENCY	PRIMARY CHORD DURATIONS BY TOTAL NUMBER OF BEATS
TONIC	9	30.5
S.DOMINANT	2	2.0
S.TONIC6	0	0
S.TONIC	1	3.0
DOMINANT	0	0
DOMINANT7	5	8.5
CADENTIAL.64	3	1.0

	CHORD FREQUENCY	CHORD DURATIONS BY TOTAL NUMBER OF BEATS
PRIMARY CHORDS	20	45.0
SECONDARY CHORDS	4	4.0
TOTAL	24	49.0

Op.18, No.5

	PRIMARY CHORD FREQUENCY	PRIMARY CHORD DURATIONS BY TOTAL NUMBER OF BEATS
TONIC	7	32.0
S.DOMINANT	2	9.0
S.TONIC6	0	0
S.TONIC	0	0
DOMINANT	0	0
DOMINANT7	4	18.0
CADENTIAL.64	0	0

	CHORD FREQUENCY	CHORD DURATIONS BY TOTAL NUMBER OF BEATS
PRIMARY CHORDS	13	59.0
SECONDARY CHORDS	1	3.0
TOTAL	14	62.0

Op.18, No.6

	PRIMARY CHORD FREQUENCY	PRIMARY CHORD DURATIONS BY TOTAL NUMBER OF BEATS
TONIC	16	31.0
S.DOMINANT	0	0
S.TONIC6	0	0
S.TONIC	1	1.0
DOMINANT	6	6.0
DOMINANT7	6	12.0
CADENTIAL.64	0	0

	CHORD FREQUENCY	CHORD DURATIONS BY TOTAL NUMBER OF BEATS
PRIMARY CHORDS	29	50.0
SECONDARY CHORDS	8	7.0
TOTAL	37	57.0

Sammartini, Symphony for Strings in D Major

	PRIMARY CHORD FREQUENCY	PRIMARY CHORD DURATIONS BY TOTAL NUMBER OF BEATS
TONIC	5	8.0
S.DOMINANT	0	0
S.TONIC6	0	0
S.TONIC	0	0
DOMINANT	4	8.0
DOMINANT7	0	0
CADENTIAL.64	0	0

	CHORD FREQUENCY	CHORD DURATIONS BY TOTAL NUMBER OF BEATS
PRIMARY CHORDS	9	16.0
SECONDARY CHORDS	0	0
TOTAL	9	16.0

Haydn, String Quartet, Op.17, No.6

	PRIMARY CHORD FREQUENCY	PRIMARY CHORD DURATIONS BY TOTAL NUMBER OF BEATS
TONIC	19	23.0
S.DOMINANT	0	0
S.TONIC6	0	0
S.TONIC	1	3.0
DOMINANT	0	0
DOMINANT7	2	6.0
CADENTIAL.64	0	0

	CHORD FREQUENCY	CHORD DURATIONS BY TOTAL NUMBER OF BEATS
PRIMARY CHORDS	22	32.0
SECONDARY CHORDS	49	103.0
TOTAL	71	135.0

Beethoven, String Quartet, Op.95

	PRIMARY CHORD FREQUENCY	PRIMARY CHORD DURATIONS BY TOTAL NUMBER OF BEATS
TONIC	7	16.5
S.DOMINANT	0	0
S.TONIC6	0	0
S.TONIC	0	0
DOMINANT	5	10.0
DOMINANT7	6	22.0
CADENTIAL.64	1	2.0

	CHORD FREQUENCY	CHORD DURATIONS BY TOTAL NUMBER OF BEATS
PRIMARY CHORDS	19	50.5
SECONDARY CHORDS	8	21.5
TOTAL	27	72.0

Schubert, String Quartet No.5

	PRIMARY CHORD FREQUENCY	PRIMARY CHORD DURATIONS BY TOTAL NUMBER OF BEATS
TONIC	14	24.0
S.DOMINANT	0	0
S.TONIC6	7	10.0
S.TONIC	7	10.0
DOMINANT	12	21.0
DOMINANT7	2	3.0
CADENTIAL.64	0	0

	CHORD FREQUENCY	CHORD DURATIONS BY TOTAL NUMBER OF BEATS
PRIMARY CHORDS	42	68.0
SECONDARY CHORDS	6	12.0
TOTAL	48	80.0

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