INFRASTRUCTURE DESIGN AND COST ALLOCATION IN HUB AND SPOKE AND POINT-TO-POINT NETWORKS

DISSERTATION

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ABSTRACT

This dissertation begins with fundamental questions about hub and spoke and point-to-point networks. The current hub network problem has received particular attention in terms of models and methods. However, current hub models do not meet important requirements for an optimal network design. These requirements include incorporation of flow economies of scale, infrastructure types, and cost allocation. The objective of this research is to investigate and characterize hub and point-to-point networks. This goad is achieved by [1] developing a model that designs an optimal hub network and an optimal point-to-point network with right-sized infrastructure, [2] allocating the network costs of hub networks and point-to-point networks to users (passengers), and [3] comparing cost allocations between hub networks and point-to-point networks. This research contributes to our understanding of fundamental questions about the merits of hub networks versus point-to-point networks.

Keywords: hub network, point-to-point network, infrastructure, cost allocation

DEDICATION

To my Mother in Korea, my Father in Heaven, and my Wife in Columbus who all made this accomplishment possible

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FIELDS OF STUDY

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CHAPTER 1

INTRODUCTION

Airline deregulation in the U.S. has led to intense changes in the arrangement of the network structure. Deregulation has allowed airlines to expand and rearrange their route structure, resulting in hub-and-spoke networks. In one particular form of hub networks, a set of hub nodes is fully connected while other non-hub nodes are connected to those hub nodes. The interaction between nodes is done largely via hubs. From the passenger's point of view, the benefits include more frequent flights at the expense of longer journey times, or inconvenience. For the airline side, airline industries can achieve higher traffic densities than the traditional point-to-point network structure by concentrating passengers in hubs. Studies show that economies of density are the main reason for lowering cost per passenger. However, current hub network models do not represent economies of scale in a realistic way. Such economies of scale are embedded in a hub network, and are interrelated with cooperation of origin-destination flows. For example, economic advantages can be obtained by installing right-sized infrastructure. To establish the optimal infrastructure of hub networks, it is sometimes necessary for the origin-destination pairs (hereafter called O-D pairs) of passengers to cooperate with each other, or for the carriers to coordinate such flows. Moreover, the route strategy for the flows in a hub network should be compared with the point-to-point network provider in order to gauge costs and benefits comparatively.

In theory, a new airline network may be designed optimally by considering some important factors, such as flow economies of scale, optimal infrastructure, and cost allocation. The current hub network design literature has not successfully integrated these factors. Such a research finding can also be applied to other fields. In addition to airline traffic flows in hub networks, there are other major applications for hub-and-spoke networks: telecommunications, fuel transmission, and mail delivery networks.

This dissertation is organized as follows. Previous research is reviewed in chapter 2. In chapter 3, the methodology for the developed models is described, and the various approaches to solving hub, and point-to-point models are characterized algorithmically. Chapters 4, 5, and 6 show numerical results for the developed models. Chapter 7 consists of a discussion of the results and recommendations for future research.

1.1 Problem Statement

This section introduces relevant research issues by describing the problem motivating this research and the purpose of this work. The current hub network problem has received particular attention in terms of models and methods. However, current hub models do not meet important requirements for optimal network design. These requirements include incorporation of flow economies of scale, infrastructure types, and cost allocation. The objectives of this research are [1] to develop a model that designs an optimal hub network and an optimal point-to-point network with right-sized infrastructure, [2] to allocate the network costs of hub networks and point-to-point networks to the users (passengers), and [3] to compare cost allocations between hub networks and point-to-point networks. These objectives contribute to our understanding of fundamental questions about the merits of hub networks versus point-to-point networks.

1.2 Infrastructure Design

The topological infrastructure questions involved in establishing links on both hub networks and point-to-point networks are represented in this section. First, the infrastructure design of hub networks involves an interhub link installation problem with a choice of link types. Infrastructure design in hub networks can be described as follows: given a set of network nodes, [1] select a subset of possible hub nodes, [2] build a set of interhub links where the type of an interhub link can be specified by cost function and flow condition, and [3] assign non-hub nodes to hub nodes so as to minimize the total cost of location (hub), allocation (hub-spoke), and infrastructure (interhub link). An efficient heuristic method for designing the optimal infrastructure of interhub links is developed by solving a multiple hub problem with flow economies of scale. This approach employs piecewise-linear concave cost functions on the interhub links. Second, the infrastructure design in point-to-point networks involves the decision to open any link based on a piecewise linear concave cost function. Within the limited flows generated in networks, the question is which link can be selected to open for achieving flow economies of scale.

1.2.1 Infrastructure of the FDMAP[1] Model

In this section, the issue of optimal interhub infrastructure in hub networks is addressed through an approach which incorporates flow economies of scale. Interhub infrastructure in this research represents the types of linkages, such as the size of airplane, frequency level of airplane service in a passenger network, or the types of backbone link pipelines (trunks) in a telecommunication network. According to the applications of the hub network model, models considering economies of scale represent the real world better than those which use a constant discount. Previous research has pointed out that a constant discount factor does not represent the real volume carried on an interhub link because interhub links may have either lower or higher volume than expected [see O'Kelly (1998) for more details]. This implies that current hub network research[2] does not provide accurate algorithms for addressing optimal interhub infrastructure. The optimal level of the interhub link is a critical factor in network performance. For example, it is expensive to install large volume interhub links if they are unmatched with network flows (called over-provision of infrastructure), or vice versa. Thus, a hub network itself should find the optimal infrastructure types and optimal flows

[1] Flow-based Discount Multiple Allocation Hub Problem (FDMAP).
[2] Bryan (1998) developed the complete enumeration of the interhub links for the smaller size of networks.

simultaneously. The model developed in this study ensures optimality[3] with respect to interhub link types and interhub flows.

1.2.2 Infrastructure of the FDPTP[4] Model

The idea of flow-based discount point-to-point network is based on the observations of previous hub network models and point-to-point models. The results of hub models reveal that some interhub links carry only a small amount of flow while other non-interhub links are busy carrying significant flow. On the other hand, there is not enough flow to maintain all direct links in point-to-point networks. To fill the gap between these conflicting observations, the model allows discounts on heavily used links based on a piecewise-linear concave cost function. This implies that some O-D pairs may have to deviate from their shortest path to build up higher flow levels if necessary.

1.3 Cost Allocation and Game Theory

The cost allocation of a shared link is investigated in this section. Interhub link installed is the shared link in hub networks and any opened link is the shared link in point-to-point networks. Little attention has been paid to the problem of allocating the costs of setting up infrastructure among users (passengers). Allocating the costs properly among users is critical in network design. The model that incorporates a fixed charge for using infrastructure affects the individual routing strategy resulting from flow economies

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[3] Optimality here means proper infrastructure that is based on location-allocation using a heuristic method. Therefore, it may not be an optimal solution strictly speaking.
[4] Flow-based Discount Point-to-Point Problem (FDPTP).

of scale with bundling flows. It is possible for each group of users to cooperate and set up infrastructure satisfying the demand and supply of its own members. The nature of this problem is such that if any disjoint coalitions unite, their total cost will not increase [see Tamir (1992) for more details]. Therefore, with respect to the total cost for all users who join the coalition, there exists an incentive for them to act as a grand coalition. However, a question arises over how to allocate the total network cost (especially the fixed cost of infrastructure) to the users. Fair cost allocation should demonstrate that no group of users will have the motivation to break up the grand coalition and act on its own. In game theory terminology, such a cost allocation is called a core allocation. A suitable cost allocation among cooperating participants is a critical prerequisite to create a coalition.

Using the above argument, the problem of setting cost allocation involves network design. Thus, cost allocation should be reflected in designing both the hub network and the point-to-point network. Mathematical programming for cost allocation featuring a game theory approach is used to set up the hub network efficiently. A common cost allocation method, cores, is incorporated into the hub location model in order to provide fair cost to users. Unlike the hub network, a cost allocation of the point-to-point network can be achieved without resorting to game theory. The main reason is that there is no interhub link in the point-to-point network. In hub networks, O-D pairs can find a cheaper cost by switching their routes due to flow economies of scale on interhub links. However, O-D pairs in point-to-point networks cannot find better routes.

CHAPTER 2

LITERATURE REVIEW

This chapter is a general overview of previous research related to the hub-and-spoke networks and point-to-point networks. Network studies can be reviewed in various ways. This chapter reviews previous literature by focusing on aspects directly relevant to the dissertation. These factors include location-allocation, network design, cost allocation, game theory, competition and so on. Network models can also be formulated with various network environments: continuous (plane) model and discrete (non-plane) model. However, the review section mainly focuses on discrete models.

2.1 Hub Network Design

This section reviews major aspects of hub network design such as location-allocation using heuristic algorithms. Most recent hub problem methods locate the hub nodes given the number of demand nodes, O-D spatial interaction flows, and discounted link costs to minimize the total transportation cost. Since O'Kelly (1986) first formulated the *p*-hub problem, there has been a great deal of analytical research on the hub location problem. Optimizing current air passenger patterns can solve hub location problems in which central facilities acting as switching points for interactions that are chosen as hubs.

The main decision in single hub assignment is to assign one node to another whereas the big concern in the multiple hub problem is how to connect multiple hubs or where to put multiple hubs. The common fundamental interest for both single and multiple hub problems is based on the total minimum cost. One of the most difficult components in hub location problems is the allocation of spoke nodes to hub nodes. In the easiest case, multiple allocation allows each origin-destination node to be routed directly from several hubs based on finding the cheapest routing. In other words, each node can use more than one hub node in a multiple allocation system. A single allocation restriction permits each node to be assigned only to a single hub. This strict restriction makes a single allocation model more difficult to solve than a multiple allocation model. Recent studies focus on the multiple allocation hub problem because it is more realistic. A solution approach of complete enumeration using shortest paths for the multiple hub problem was addressed by Campbell (1994). He stated that a multiple allocation problem gives a good indicator of a lower bound for a single allocation problem (see Campbell (1996) for more details). O'Kelly (1998) presented a multiple hub assignment using the inter-hub discount factor. He stated that the usage of hub linkages depends on the discount pattern.

Heuristic methods are most commonly used in the areas of transportation planning and location modeling. They are useful when optimal algorithms cannot solve the problem or take a very long time to solve because of problem size and complexity. Heuristic methods are analytical solution methods that are based on an intuitive understanding of the model. In general, a good understanding of the relationship between the decision variables and the constraints and the objective function is required to formulate good heuristic procedures. Numerous meta-heuristic methods, in addition to the enumeration-based approaches, have been proposed in recent years. Klincewicz (1991) used exchange heuristics with clustering. He recommended a multi-criteria assignment procedure in allocating spokes to hubs. In his two-stage procedure, both distance from a spoke to a hub and flow between spokes are considered. Later Klincewicz (1992) applied a tabu search and a greedy randomized adaptive search procedure (GRASP) to hub problems. The computational results indicated that good solutions tended to be found in the early stages of tabu search. Skorin-Kapov and Skorin-Kapov (1994) developed a tabu search heuristic to solve single allocation *p*-hub problems. Their tabu search method defined the neighborhood structure for location and allocation separately. Their computational results showed that tabu search is good for solving the single allocation hub problem. At the time of publication, the best-known solutions for the single allocation *p*-HLP (Hub Location Problem) for the CAB[5] data set were achieved by the tabu search heuristic. Later, O'Kelly, Skorin-Kapov and Skorin-Kapov (1995) extended a hub model with a new approach to develop the best bounds obtained for the hub location problem. Their methodology identified good lower bounds for the single allocation hub problem by linearizing a quadratic term.

Some recent research has shown that the important algorithmic benefits of using tight linear programming relaxations. Skorin-Kapov et al. (1996) provided a tight linear programming formulation to find optimal solutions to the problem. Although their solutions were very close to optimality, computation time was extensive. Therefore, the

[5] Civil Aeronautics Board (CAB) data sets of U.S. city airline passenger flows in 1970 are used in this study.

application to the large-scale network is prohibitive. To get over this limitation of size, Sohn and Park (1997) showed that the 2-hub problem could be solved in polynomial time with fixed hub locations based on enumeration of all possible pairs of hubs using a minimum cut algorithm implementing a large network problem. They concentrated on the allocation parts of terminal nodes to two fixed hubs. However, their application is somewhat less realistic because it only deals with 2 hub locations. More recently, Sherali et al. (2000) developed a model for the design of local access transport area networks by applying the reformulation-linearization technique. They designed a hybrid heuristic procedure that combines a limited run of an exact method with a Lagrangean dual procedure. Their problem requires hub nodes to be equipped with multiplexers in order to process the aggregated transmission traffic. The problem decides on which nodes from a potential set should be designated as hubs, what multiplexer capacity should each of these hubs be provided with, and how should the demand of the clients be routed. The cost function in this model involves equipment component costs that exhibit nonlinear economies of scale. They argue that its presence influences the network design, and it specifically needs to be accounted for in developing solution procedures.

Wheeler (1989) has provided “check-lists” of factors in hub location. He discussed how to determine hub location by listing several factors. He stated that the most critical factor is to locate the hub in an area with a large volume of local origin and destination traffic. He demonstrated that hubs should be centrally located near the weighted midpoint of cities served by a hub in order to minimize network circuity and total elapsed travel time. A heuristic for the hub model can utilize this implication. The hub model needs to consider the hub node candidates within the convex hull of all nodes to reduce the search space. The author also maintained that the creation of multiple hub networks allows an airport to serve an increased percentage of the potential O-D markets by better serving existing flows and by entering into markets that were previously served only by circuitous routes.

The proper infrastructure also plays a very important role in designing a hub network. However, there has been little attention to the effect of linkages, cables, or interhub links on the structure of network. Gavish and Neuman (1989) employed Lagrangean relaxation to solve the joint problem of assigning linkage capacity and selecting a route for each interacting node pair to achieve a minimum network cost while satisfying a required performance level. The model included fixed setup and variable traffic linkage costs. This paper demonstrates that there are two costs in network cost: transportation and fixed.

Several algorithms have been proposed for the hub network design. Some of the algorithms are heuristic, and some others are based on more rigorous mathematical programming. As can be observed above, hub models belong to the difficult combinatorial problems thanks to the number of integer variables and constraints involved so the computation of an optimal solution is expected to be a challenging task. The computational requirements for obtaining exact solutions to hub problems lead to many heuristic approaches. Previous heuristic methods show that the use of a collection of linked subproblems is a good modeling approach. However, heuristic methods do not guarantee an exact optimal solution, although they may be fast and deals with large data sets compared to branch-and-bound methods. In this research, both a linearization of a quadratic term and a tabu heuristic algorithm are used to solve the hub problem. Because of the complexity of a hub network design problem and interdependency among different variables, an exact solution for large networks is not yet available. This topic is pursued in Section 3.2 of this dissertation.

2.2 Economies of Scale

This section reviews how cost functions are incorporated in network models to address flow economies of scale. This section also focuses on the effect of economies of scale on network structure. The cost of shipping from node to node often uses a simple linear function. However, the linear cost assumption is often not realistic because it cannot account for economies of scale. If fixed charges, discounts, or economies of scale are involved in the transportation network, concave cost functions make more sense. The question of economies of scale is a major focus in the analysis of concave cost functions. The concavity of the cost function corresponds to economies of scale, which results in decreasing marginal cost and decreasing mean cost with respect to higher volume of flow. Therefore, to adopt economies of scale in a hub model, the concave cost function must be employed.

Zangwill (1968) developed theorems which explicitly characterize the extreme points for certain networks. By exploiting this characterization, he developed algorithms that determine the minimum concave cost solution for networks with a single source and a single destination. His main concern was how to determine minimum cost flows in certain types of concave cost networks. He found that there is an optimal solution to the single-source minimum concave cost network flow problem with at most one positive incoming flow into each node. One important finding is that it is not necessary to search all the extreme flows in order to find the optimal flow. His finding helps solve the uncapacitated concave flow problem because it has many local optima. His results are quite useful in characterizing extreme flows in networks and useful for analyzing the concept of extreme flow. However, this idea of extreme flows is restricted for certain networks. The links of the FDPTP involve multiple sources and destinations so the model may have to split flows into many links to achieve flow economies of scale [see Section 5.2 for more details about flow splitting property]. This topic is pursued in Section 3.2 of this dissertation.

Yaged (1971) proposed the fixed point heuristic to solve the Zangwill' problem. His fast search algorithms are efficient for finding local optima but not a global optimum. He developed techniques for selecting a path through the network for each point-to-point demand for communication channels. He demonstrated that a modification of the iterative algorithm provides acceptable solutions when the link cost displays a fixed charge. This approach is efficient when an exact solution is difficult. He also suggested a method for routing future demands and installing transmission facilities. His model is based on two critical assumptions. First, the transmission systems to be installed on a link must display economies of scale. Second, each link cost is assumed to be a concave function of link size. His study showed that concave link cost functions are realistic because they reflect economies of scale. Magnanti and Wong (1984) reviewed the concave cost network design problem. They discussed the concave cost network flow problem that generalizes the uncapacitated fixed-charge network design model. They also showed that the basic fixed-charge design problem is quite flexible and contains many well-known network optimization problems, including the shortest path and uncapacitated plant location.

Earlier Rech and Barton (1970) proposed a solution method for solving nonconvex transportation problems. They dealt with a minimal cost transportation problem for shipping a single commodity, available at *m* sources, to *n* destinations with known demands. These *n* destinations are connected to supply points by a transportation network, which has features of transshipment points and restrictions on the capacity of a pipeline. They argued that some cost functions can be nonconvex piecewise linear cost functions, with an example of warehouse location under economies of scale. They showed that a particular case of such a cost function is the concave piecewise linear cost function arising whenever economies of scale are present. Hall (1989) showed that the optimal route for each origin-destination pair can be found from the optimal flows on the shared arcs alone. A "shared arc" means it carries flow for more than one origin-destination pair. He argued that the optimal shared arc flows can be found with an exhaustive search over the feasible region for flows. His model procedure for selecting shipment routes accounts for economies of scale.

The following paper represents different aspects of concave cost network flow problem. Balakrishnan and Graves (1989) considered the problem of routing multiple commodities between various origin-destination pairs in a network. They developed a composite algorithm to create both lower bounds and feasible solutions as a mixed integer program. They incorporated economies of scale in arc flow costs using piecewise linear concave total cost functions for each arc. They showed that the cost function can be partitioned into several cost ranges using a piecewise linear assumption. They demonstrated that the composite algorithm is very effective in generating solutions with small gaps between the upper and lower bounds, even for relatively large problems. They stated that the performance of their algorithms substantially improve when the networks have certain special topologies.

The relationships between scale economies and network shapes were mathematically and empirically explored by Gordon (1974). He drew several conclusions. First, fully connected transportation networks are uncommon because of the existence of scale economies for most transportation modes. This contention is supported by the results of the FDMAP and the FDPTP. The models do not build fully connected networks due to flow economies of scale. Second, the greater the scale economies, the less connected the network shape and the more intense the traffic pattern. Third, congestion at nodes should cause a more connected network. Fourth, the network shape, given a fixed cost function, should rely on supply-demand equilibrium.

Explicit flow economies of scale were not exploited in hub models until recent years. O’Kelly and Bryan (1998) formulated a model incorporating a piece-wise linear concave cost function. The cost of utilizing an interhub link in their model depends on the total amount of flow traveling across it with the per unit cost decreasing as flows increase. Their approach approximated a nonlinear cost function to represent more realistic interhub discounts. Later Bryan (1998) presented four extensions of the hub location problem, one being a model that incorporates a flow-dependent cost function for the spokes as well as the interhub links. This extended the basic hub model systematically. She also allowed discounts on all link flows. This idea is very related to point-to-point networks in which any link can get a discount based on threshold. O'Kelly and Bryan (2001) showed that the resulting hub facility locations are interdependent due to the flows between them. Their solution approach favors the assembly of flows into bundles so that the model can penalize fractional facility locations. They discussed non-integer solutions resulting from fractional facilities. They proposed different models depending on the allocations and flow-dependent cost functions. Their approach represents hub-and-spoke networks from a more realistic point of view. Horner and O'Kelly (2001) explored an endogenous hub location problem based on equilibrium traffic assignment. They implemented a non-linear cost function, which allows economies of scale on all network links. They also showed that network flows are re-routed to utilize the cost savings for amalgamation under discounted conditions.

As discussed above, a simple hub model fails to consider economies of scale. When researchers realized that there was little concentration on the interhub links with a simple multiple allocation protocol, studies with discounts shifted to investigate economies of scale. Therefore, the discount factor alpha should be differentiated based on the level of interhub flows. This assertion is also applied to the point-to-point network. Little flow concentration on the links by a simple model motivates the development of a flow discount model of this dissertation.

Over the years the effect of economies of scale has received considerable attention in the transportation network literature. Economies of scale generate tendencies toward concentration. Hub-and-spoke operations of airlines benefit from economies of scale. For example, smaller airplanes bring in passengers from smaller or medium-size cities into a hub, and a larger airplane carries them to another hub that is close to the destinations, and then smaller airplanes carry the passengers to the destinations. Often times, passengers to a common destination are served by a larger airplane directly from the first hub. The benefit of hub networks is the usage of a small number of links and exploitation of economies of scale by concentrating flows. Transportation costs are characterized by significant economies of scale, so that average costs on a connection decline as a number of users increases.

Considerable research has been spent on the cost function in network models, although not all cost function models use the concave cost function. A concave cost function differs from the general cost function in that economies of scale are present in the network. To implement the concave cost function, a piecewise linear concave cost function is employed in a design problem with link type selection. This review supports the practicability of a piecewise-linear concave cost function for the network design to employ flow economies of scale. This topic is pursued in Sections 3.3 and 3.4 of this dissertation.

2.3 Cost Allocation

This section demonstrates how cost allocation is efficient and how it works in a network context by reviewing previous studies. The following research addresses a fundamental question about cost allocation in network design, i.e. how to fairly allocate costs for users. Although there is a much analytical research on hub location models, cost allocation studies do not get much attention. The cost sharing is common in many economic activities. The issues of cost sharing appear both explicitly and implicitly. However, the difficulty arises explicitly whenever a group of individuals jointly use a shared infrastructure. This argument similarly corresponds to the issues of the interhub link costs in hub networks, and the link costs in point-to-point networks.

Heaney and Dickinson (1982) cited four well-known cost apportionment methods to allocate costs among participants in common usage: Separable Cost Remaining Benefit (SCRB) method, Shapley method, Linear Programming method (LP), and Minimum Cost Remaining Saving method (MCRS). They developed the minimum cost remaining saving method to overcome weakness in the Shapley, and LP. They argued that in the presence of a core, all methods listed do not differ significantly. If there is no core, however, the other three methods (Shapley, LP, and SCRB) do not produce a rational allocation of cost while the MCRS method utilizes an additional LP technique that relaxes the core constraints to find a core solution. Their method is based on the axioms of fairness from cooperative *n*-person game theory. Young et al. (1982) also compared different methods for allocating the joint costs. They contrasted the separable cost remaining benefits method (SCRB) with the Shapley value and variants of the core from cooperative game theory. The SCRB method is shown to be flawed in that it is not monotonic[6] in total costs. In other words, an increase in total costs may result in some participants having to pay less than before due to the way in which marginal costs are introduced. They concluded that proportional allocation according to a single numerical criterion may be preferable to the more complicated SCRB method.

There are two comparable works which focus on cost allocation of location models on networks. Tamir (1992) discussed a class of location problems for which there always exists a core allocation. The core itself was characterized by a dual linear program to the location problem. The important concluding remark from his research is that there exists core allocation on tree graphs. Granot (1987) considered cost allocation as a cooperative game in facility locational models, but only in a single facility case. He argued that the location *y* and a corresponding cost allocation *q* should be selected from a least core associated with the locational model. His model assumptions on the utility functions of the users are less restrictive than those in Tamir (1992). He maintained that the allocation of the system's operating cost and the locations of the service stations are strongly related. That is, the farther a user is from the service station, the lower the cost share a user should be willing to pay. However, his argument is questionable in the case of infrastructure cost allocation in hub networks and point-to-point networks because the link usages involve at least two locations.

The game theoretical approach has been used not only to model various cost allocation problems, but also to design algorithms for networks. In a network system, the

[6] See Megiddo (1974) for the monotonicity property. If total costs increase then no participant will be charged less; conversely, if total costs decrease, no participant will be required to pay more.

players need to compete for nodes or arcs, but also need to cooperate in order to efficiently perform the system. Relatively little has been done on the analysis of game theoretic cost allocation for general network problems.

Sharkey (1990) studied a class of cooperative game involving the shared use of fixed cost facilities. He showed that a sufficient and a necessary condition for a nonempty core coincide with optimal values for a pair of integer and linear program. It is shown that shadow prices do not exist to sustain a minimal cost network when there is decentralized ownership of individual links. Combined approaches that incorporate hub network design with cost allocation have been discussed by Skorin-Kapov (1998, 2001). The most relevant game theoretical cost allocation method in hub network design was studied by Skorin-Kapov (1998). He addressed the cost allocation problem associated with the *p*-hub location problem. He described basic hub games, and classified the hub game into 6 different possible games based on players, characteristic function, and network traffic. He defined a hub game as follows. First, it is concerned with the distribution of the cost of flow among users. Second, it takes care of the user's contribution to economies of scale. Thus, a hub game model exploits economies of scale by cooperation of users and by routing traffic via hubs. Using this insight, he approached cooperative game theory to examine the behavior and the associated cost allocation of different users of hub networks. He applied core and nucleolus techniques to solve cost allocation. Thus, his study's main objective is to develop a computationally tractable framework for a fair allocation of the cost of communication services delivered via the hub network among its users. Because a hub network game model is computationally prohibitive, it is very difficult to solve the problem with general practical algorithms. There are very noticeable differences between previous cost allocation approaches and hub network game approaches. While the distribution of the cost of flow among users (network nodes) is the main concern in cost allocation, the division among pairs of nodes is considered in a hub network game. There are some complexities in hub network games. First, it is problematic how to define the set of players for the hub game. Second, how the cost should be determined is an issue. Moreover, most of cost allocation solution concepts require the enormous amount of data to compute and exponential number of constraints. Later, Skorin-Kapov (2001) considered cost allocation of hub-like network design in which each pair of nodes can communicate via a direct link and the origin-destination flow can be delivered through any path in the network. He introduced an incentive to combine flow from different sources. For example, if the total flow through a link exceeds the prescribed threshold, then the cost of this flow is discounted by a factor alpha. This topic is pursued in Sections 3.5 and 3.6 of this dissertation.

2.4 Competition

This section emphasizes competition issues in the networks. There is little literature dealing with competition issues in a hub network. According to previous literature, competing hubs have locational advantages in certain markets. Thus, location benefits impact competitive behavior of existing hubs, and even produce opportunities for the expansion of new hubs. Hansen (1988) suggested three basic principles in a hub competition model. First, an airline's hubs obtain competitive advantages in certain markets. Second, these competitive advantages can lead to additional profits by increasing either load factors or fares. Third, hub competition includes both exploitation of existing hub assets to their best advantage, and the modification of these assets in order to enhance the value of the advantages they can give. He proposed that sound planning of system infrastructure requires an understanding of how hub competition will affect future traffic levels, of how investment decisions may influence the competitive process, and of the potential for management strategies to shift demand in order to make better use of existing facilities. He argued that a relatively small number of hubs make a hub location a critical variable in the competitive process. He pointed out that the model should address service as well as location because different hubs provide different services. Using this insight, he suggested a model of airline hub competition should address four phenomena. First, a model should offer an understanding of the forces leading to hub location. Second, a model must explain how services offered from any given hub are determined, termed as the process of hub configuration. Third, a model should take into account the potential influence of one hub's configuration on that of another, termed as hub interaction. Fourth, a model should estimate how these processes will determine the success of different competitors, termed the competitive outcome. Later Hansen (1990) developed a game theoretical model of airline hub competition using the United States air transportation system. His hub competition model can be depicted as a noncooperative game between a set of airlines seeking to maximize profit with two types of competitors: hub carriers and direct carriers. Although his model had some simplifying assumptions, it represented a considerable advance to fit passenger service preferences, aircraft technology characteristics, and airline economic behavior into a unified model. He concluded that the competition game does not have true equilibrium points in general. However, he found that a state of quasi-equilibrium can be found in the case in which round-to-round strategy adjustments by the airline competitors were small.

An equilibrium model of airline network considering airline competition and passenger routing preference was developed by Ghobrial (1983). The results suggested that network hubbing is efficient, and airlines will probably find it beneficial to hub despite the pricing penalties due to the airport congestion fee imposed on airlines using major hubs. Phillips (1987) presented statistical information regarding air carriers' altered operating and marketing practices in strengthening their hubs and reducing interline operation in hubs. He found that if an airline dominates its hub, it may be in a better position to prevail over actual and potential competitors. There are also negative effects of hub competition. Brueckner and Spiller (1991) examined a negative effect of competition in airline hub-and-spoke networks. They found that competition in an airline hub-and-spoke network may have harmful effects outside the market where it occurs. They expected competition in a single market of a hub-and-spoke network to generate negative externalities. Starr and Stinchcombe (1992) investigated hub networks in which systems are optimal under a variety of cost and demand configurations, and typically demonstrate large, pervasive economies of scale. Moreover, those pervasive scale economies indicate that the industry is a natural monopoly assuming the presence of high costs and low marginal costs. However, they admit that if passenger flows to or from most non-hub destinations exceed several times the economic capacity of a single aircraft, then the scale economy can be fully exploited by each of several firms rather than rely on a monopolistic structure. This statement tells us that there is room in the market for several airlines and several hubs. Marianov et al. (1999) proposed an algorithmic approach to deal with direct competition. Their model enabled an airline to capture customers if the competitor can provide shorter distance for each O-D pair. Their model is based on only cost without considering pricing policy. This topic is pursued in Section 6.4 of this dissertation.

2.5 Summary of Review

A variety of aspects on current hub networks and point-to-point networks were reviewed. Moreover, the topics related with solution approaches were also given. The review included comprehensive views on both conceptual and applied sides. Many previous model formulations and solution approaches are critically examined to overcome the current research gap and to provide a more comprehensive model framework.

CHAPTER 3

METHODOLOGY

This chapter presents methodologies to support design decisions for infrastructure and cost allocation in both hub networks and point-to-point networks. Such networks interconnect geographically dispersed nodes and switching centers. Topological design decisions incorporating these issues are very important in a hub network design. They are important because of the dynamic network adjustment required for interhub links and switching facilities, and the significant impact of network configuration choices on several levels. With increasing competition and service diversity in the airline network industry, understanding network characteristics of both hub and point-to-point networks have become increasingly important.

This chapter develops models to design an optimal hub network and an optimal point-to-point model with respect to infrastructure and shows an approach to solve them. The models are motivated by several observations regarding hub network design practices. From a design perspective, network planners must often accommodate various explicit as well as implicit design considerations that cannot be adequately represented in optimization models. Optimal infrastructure is an explicit decision while fair cost allocation is an implicit consideration. To do this, it is necessary to completely characterize underlying network features of infrastructure and cost allocation in a convenient form for mathematical modeling. From the optimization point of view, since most discrete hub network design problems are NP-hard, optimization-based heuristic procedures that provide feasible solutions are suitable solution approaches. For the point-to-point network, a complete enumeration method, rather than a heuristic approach, is employed.

The network design in this research has two major methodologies: [1] Design the optimal infrastructure on both hub networks and point-to-point networks using an optimization-based model, [2] Allocate the total network cost (especially fixed cost of infrastructure) to the users (O-D pairs) based on cooperative game theory in hub networks based on a player's flow contribution in point-to-point networks. Then, user costs between hub networks and point-to-point networks are compared to characterize different network designs.

On the hub network side, the methodology expands a traditional hub model into a flow-based discount hub model by incorporating flow economies of scale. This modification building on prior research [see Chapter 2 for more details] combines some network design refinements, such as creating alternate routes for origin-destination pairs by incorporating more possible ways to transport flows in a hub network. The hub network design considers major topological decisions of location-allocation, and it also determines the optimal levels of service for installed infrastructure.

On the point-to-point network side, the methodology expands a pure point-to-point model into a flow-based discount point-to-point model by adding flow economies of scale on opened links. The point-to-point network design considers major topological decisions of location-allocation, and it also determines the optimal levels of service for installed infrastructure. To compare the results of the point-to-point network model with those of a hub network model, restrictions on the number of connections between origin and destination are imposed. For each origin-destination pair, for example, the number of links on the interconnecting route is limited by the intermediate hop restrictions to follow hub network structure [see Balakrishnan and Altinkemer (1992) for more details]. In air passenger networks, especially, the number of links is a critical consideration unlike in telecommunication networks because the maximum number of intermediate stops to be tolerated by passengers is at most two. Thus, the model only allows at most two stops on an origin and destination route. It is also expected that the constraints in the number of links affect arc (link) densities and cost structures.

The rest of this chapter is organized as follows. Sections 3.1 and 3.2 define substantial components of the models. In Section 3.3, the infrastructure design of the flow-based discount multiple allocation hub problem (FDMAP) is presented as a mathematical programming formulation with formal descriptions, and the modeling assumptions. A tabu search heuristic is also described as a solution approach to solve the FDMAP. Section 3.4 describes the infrastructure design of the flow-based discount point-to-point problem (FDPTP) as a mathematical program, and details modeling assumptions. Sections 3.5 and 3.6 describe cost allocation schemes for the FDMAP and the FDPTP respectively.

3.1 Network Design Model

For the hub network design problem involves selecting a subset of links (network infrastructure) with optimal service levels on each selected interhub link, and routing all internodal traffic requirements at minimum total investment and operating cost. Thus, the decisions of the topological design of hub nodes and interhub links, and routings of O-D pairs are required. This study develops a hub model that addresses all these problems. The model framework differs from many previous models in two ways. First, fast algorithms are possible for flow-based discounts utilizing a piecewise-linear cost function in the objective function. Second, flows of O-D pairs in the model play an important role in building infrastructure.

The hub network design is defined over a directed network G (N, E) with N nodes and E edges. For every pair of nodes i, j in the network, W_{ij} represents the internodal traffic between i and j. The model also accommodates economies of scale with respect to volume discounts in the form of piecewise-linear concave costs (see Figure 3.1). To establish a piecewise-linear concave cost function on each link of the network, two types of cost, fixed and variable (transport), are introduced. As can be seen in Figure 3.1, the variable costs that represent the cost per unit of flow are dependent on the discount of the link associated with fixed costs. For example, the fixed costs might be investments for building infrastructure while the variable cost component approximates flow-dependent transportation expenses. In Section 3.2.2, how these fixed and variable costs account for routings of the origin-destination pairs is described.

For the point-to-point network, the model employs flow-based discounts on opened links. The mixed-integer programming formulation of the point-to-point problem distinguishes the direction of flow on each of the original undirected network *G*.

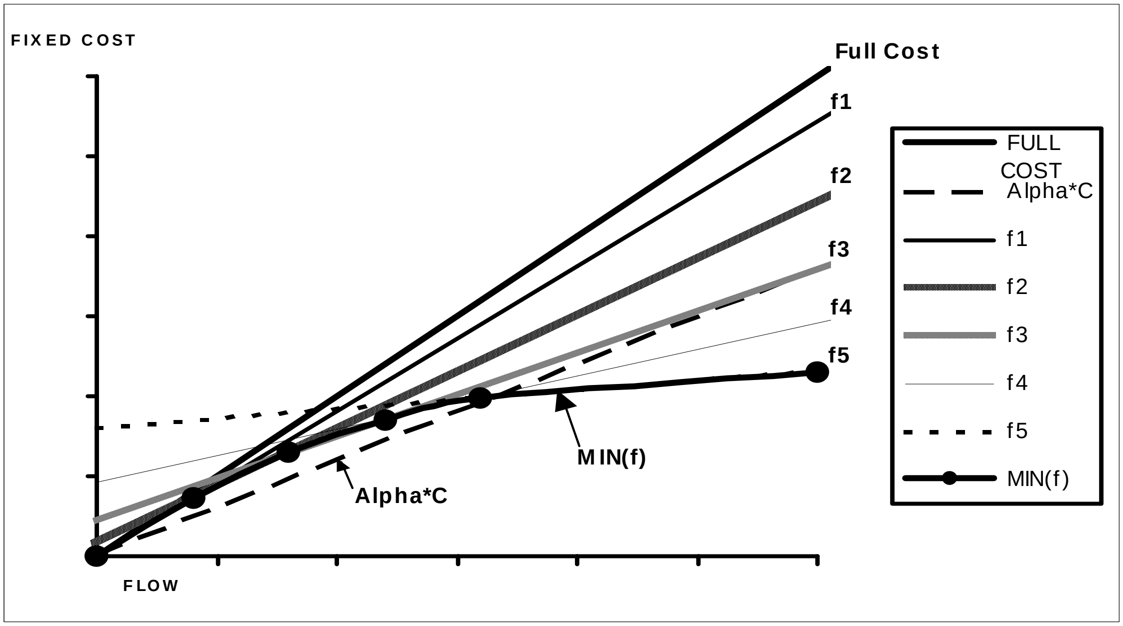

Figure 3.1 Piecewise-Linear Concave Cost Function

3.2 Network Modeling Framework

In this section, the network models are presented with two different sub-models (see Figure 3.2). In the FDMAP model, a hub network design problem considering infrastructure is solved, and infrastructure design variables, such as interhub flows and link cost functions, are incorporated. The infrastructure design of the hub network model seeks to find an optimal interlink under multiple allocation restriction. The FDMAP model is implicitly based on several characteristics, described individually in Sections 3.2.1 - 3.2.4 below. The FDPTP model is described in Section 3.4. The cost allocation of the infrastructure models is then developed in Sections 3.5 - 3.6 where the model assigns the fair cost of infrastructure to the O-D pairs based on game theory in hub networks, and based on player's contribution to flow economies of scale in point-to-point networks.

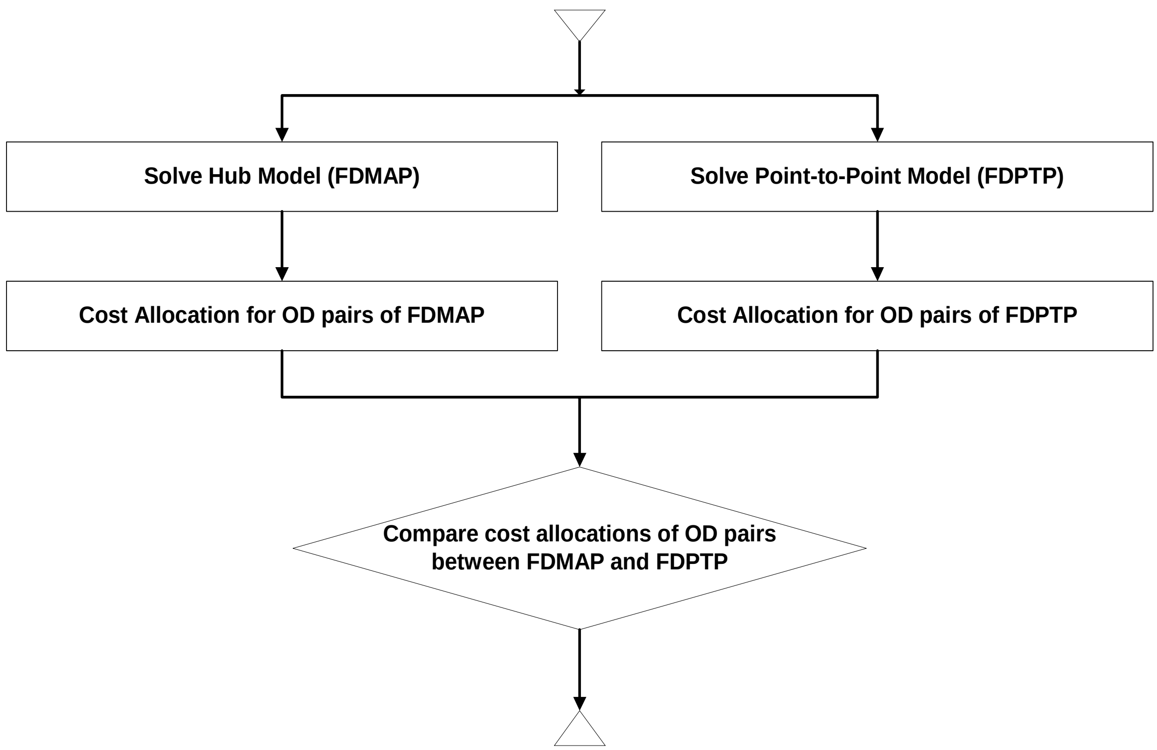

Figure 3.2 Network Design Framework

3.2.1 Tabu Search in Hub Network

The heuristic method for hub models in this study uses the tabu search method. According to Glover et al. (1992), a successful application of the tabu search prevents entrapment in local optima. A suitable neighborhood and fast move evaluation are essential for that application. The basic principle of the tabu search is to define a set of possible solutions from the initial solution in order to locate a better one in its neighborhood by avoiding cycling and trapping into a local optimum. Thus, the tabu search algorithm allows moves which result in a better objective value, and the intermediate solutions obtained recently are considered as tabu to prevent the algorithm from exploring a local minimum more than once. Moreover, it creates more opportunities to reach a global optimum.

Tabu search uses flexible forms of memory to guide search processes dynamically. This enables local search algorithms to escape from local minima and to exploit information that has been gathered during the search. It also relies on a tabu list. The tabu list stores attributes of a move, i.e., the transition mechanism that enables the search algorithm to move from a solution to its neighbor solution. Moves with these attributes are then forbidden during the next iterations. The number of such iterations is called the tabu tenure. Tabu search also allows the tabu status to be revoked if an aspiration criterion is met. These principles define the basic short-term memory, form of tabu search. In addition to the basic algorithm, tabu search uses long-term memory concepts. These include intensification and diversification. Intensification concepts are based on methods that identify promising regions of the search space and then search these regions more thoroughly. Diversification strategies exploit information about the search history and guide the search toward yet unexplored regions. Both intensification and diversification are often based on counting principles. The strength of tabu search is its flexibility with respect to implementation. It contains a wide variety of ingredients and parameters, which have to be properly adjusted if the algorithm is to work well. For more details for tabu search, see Glover (1989, 1990a, 1990b, 1998) and Glover et al. (1993, 1997a, 1997b).

3.2.2 Piecewise-Linear Concave Cost Function

In the real world, infrastructure link can often be built in different sizes or types. The set of possible sizes is often discrete and finite with a reasonable number of links. Each possible size yields a certain fixed cost of the link. One useful property of the piecewise-linear concave cost function is its separable attribute. This allows fixed costs to appear at several levels of link flow, and also allows the linear transportation cost coefficients to vary between different intervals of flow amount. As can be seen in Figure 3.1, there are tradeoffs between fixed costs and discounts depending on flows. To achieve higher discounts, for example, a group of links needs to pay a higher fixed cost. The optimal combination of fixed costs and discounts is decided by the level of flows. The lower envelope called *Min(f)* in Figure 3.1 of the cost function guarantees the lowest transportation cost for volume of flow. The transportation cost of the linear cost function is identical to that of the non-linear cost function at the black circles in Figure 3.1 of the lower envelope cost function. Each segmentation between black circles of the lower envelope cost function provides both discount rate and fixed cost. This lower envelope cost function can also be specified by the predetermined flow range[7].

There are also tradeoffs between the number of linear cost functions and the complexity of the problem. The model can divide the non-linear cost function into as many as possible to mimic the realistic world pattern of rates for flows. However, the large number of piecewise-linear cost function adds difficulty to the problem. In this study, the model approximates the non-linear cost function by five different piecewise-linear cost functions as can be seen in Figure 3.1. For more details about the piecewise-linear approximation, see also Bryan (1997), Holmberg (1994), Runggeratigul (1999), and Vogt and Even (1972).

Table 3.1 illustrates the tradeoffs between fixed costs and transportation costs. Numerical values in Table 3.1 are based on a simple 6-node network (Figure 3.3). The network consists of 6 nodes and 10 flows for all origin-destination pairs. Table 3.1 also shows that the highest interhub discounts (the last row) do not necessarily guarantee the lowest total network cost. As can be seen in Table 3.1, setting the highest discounts for every interhub link does not provide the cheapest total network cost. This illustrates tradeoffs between fixed cost and discount rate. In this particular example, the model also does not install a fully-connected hub network[8] as a result. It rather builds the infrastructure in a selective way. It is worth explaining the behavior of the flow-based discount hub model. There is not enough flow to support a fully-connected hub network.

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[7] This flow range is the deterministic factor for the fixed costs, which changes the total network cost significantly. The numerical results with different flow ranges are shown in Chapter 4.
[8] Traditional hub models install all possible interhub links whereas the FDMAP opens partial intehub links.

In other words, some of the infrastructure is under-utilized so the model fails to achieve economies of scale on every interhub link. Thus, the model decides to close partial interhub links. Notice that the total interhub link flow changes from 240 [discount type: 5, 5, 5] to 280 [discount type: 5, 5, 1] in Table 3.1. Fixed cost for the infrastructure is equal to [distance of interhub link] * [fixed cost of interhub link]. It is also same as *y*-intercept of a piecewise-linear cost function in Figure 3.1.

Level of Discount on Interhub Link*			Fixed Cost			Flow			Transport Cost	Total Cost
H1H2	H1H3	H2H3	H1H2	H1H3	H2H3	H1H2	H1H3	H2H3		
1	1	1	0.00	0.00	0.00	80	80	80	1870.50	1870.50
…	…	…	…	…	…	…	…	…	…	…
…	…	…	…	…	…	…	…	…	…	…
5	5	1	115.38	115.38	0.00	140	140	0	1296.51	1527.27
…	…	…	…	…	…	…	…	…	…	…
…	…	…	…	…	…	…	…	…	…	…
5	5	5	115.38	115.38	128.00	80	80	80	1200.50	1559.26

* There are five kinds of discount rates: 1 is the lowest discount (larger alpha, $\alpha = 1$), and 5 is the highest discount (smaller alpha, $\alpha = 0.2$).

Table 3.1 Tradeoffs between Fixed Costs and Transport Costs (based on Figure 3.3)

Figure 3.3 shows the hub network configuration based on the results of the FDMAP with $p = 3$. Unlike the traditional hub model, interhub links are not fully connected in the hub model. The dotted line between [H2-H3] in Figure 3.3 was feasible in the traditional hub model with a fully interconnected interhub link, but not feasible any more in this hub model. One of the main reasons for this is the number of interhub links between origin and destination allowable in the model. In this research, the hub model relaxes the number of interhub links for O-D pairs to travel. For example, the [C-H2] O-D pair in Figure 3.3 may travel [C→H3→H2] path under the fully interconnected interhub link constraint while its actual path is [C→H3→H1→H2] under the multiple interhub links relaxation [see also Section 3.2.3 for more details]. Another reason for this is that the model finds that this setting provides much cheaper total network cost than the fully connected hub construction.

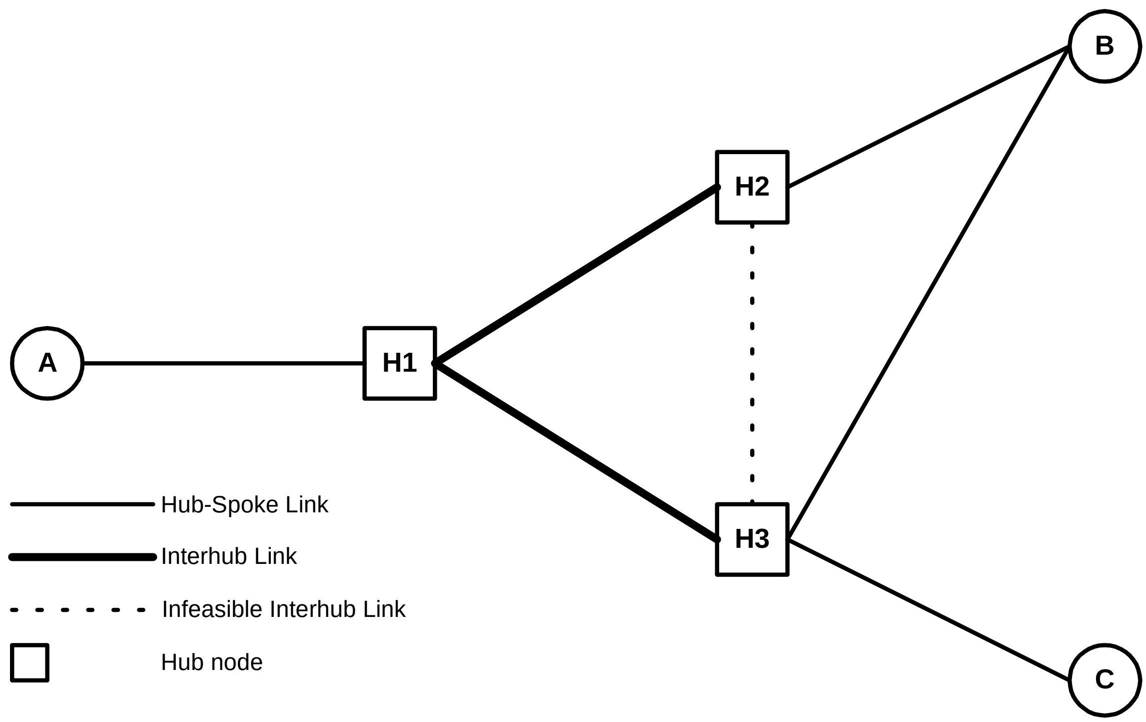

Figure 3.3 Simple Hub Network (6-Node)

3.2.3 Multiple Interhub Links in Hub Network

Because the interhub network is not fully connected, the hub model needs to relax the restrictive assumption that the flows between origin and destination traverse at most one interhub link. Now two interhub links are permitted. The multiple interhub links are introduced in the hub model because flow economies of scale make it desirable in some cases to use two interhub links. In a traditional hub model, only one interhub link was permissible. Now in a special case of two interhub links, it needs three hubs. However, the multiple interhub links are only feasible when a hub node corresponds to either origin or destination node of the path. In other words, flows between origin and destination can utilize two interhub links if the two interhub links are heavily used. The multiple interhub links on the path eventually achieve a cheaper transportation cost than the single interhub link model due to flow economies of scale on the multiple interhub links.

O'Kelly and Miller (1994) classified 8 hub network systems depending on design variables, such as non-hub assignment, internodal connection, and interhub connectivity. The network configuration of the FDMAP corresponds to Protocol F design class with multiple hub allocation, no internodal connection, and partial interhub connectivity. This network type was never studied in the traditional hub models until flow economies of scale are observed in multiple interhub links. This relaxation might break the triangle inequality in some cases. For example, in traditional hub network studies, there is no way for the [H2→H1→H3] path to get the cheaper transport cost than the [H2→H3] path in the triangle of the [H1, H2, H3] in Figure 3.3 because hubs are fully connected. However, it is possible that the [H2→H1→H3] path cost is cheaper than cost of the [H2→H3] direct link in the hub model due to flow economies of scale on multiple interhub links.

3.2.4 Shortest Path Enumeration in Hub Network

The routing policy for the flow-based discount multiple allocation hub problem (FDMAP) is based on the shortest path for each O-D pair. Under the multiple allocation restriction, a common characteristic of flow assignment algorithms is to assign flows along the shortest path. The shortest path computation is based on all link distances (hub-spoke link, and interhub link) for each O-D pair [see Klincewicz (2002) for more details]. Once the hub locations are decided, and the interhub links are installed, flows need to be routed optimally based on the shortest path. For example, once the *p* hubs within the FDMAP are identified, and the model decides values for X_{ijkm} for each origin-destination pair (i, j) using intermediate hubs (k, m). It is also necessary to decide the optimal settings for infrastructure of interhub links to select which piecewise-linear cost function would minimize the total network cost (fixed cost + transport cost) on the interhub link (k, m). That is, each origin-destination pair (i, j) finds the cheapest combination of cost in $W_{ij}(C_{ik} + S_l C_{km} + C_{jm}) + F_l$ [see equation 3.1 for notation]. It is feasible to solve the FDMAP by enumerating all possible combinations of *p* hubs (${}_nC_p$ cases) with different types of infrastructure (interhub links). Each interhub link may have as many as user-specified infrastructure types associated with it. The FDMAP adopts 5 different infrastructure types.

3.3 Infrastructure Design of the Flow-based Discount Multiple Allocation Hub Problem (FDMAP)

The infrastructure design of hub networks involves an interhub link installation problem with a choice of link types. Current algorithms for hub network research do not provide the optimum interhub infrastructure in an efficient way. The infrastructure design of the FDMAP can be described as follows: given a set of network nodes, select a subset of possible hub nodes, build a set of interhub links where the type of an interhub link can be specified by cost function and flow condition, and assign non-hub nodes to hub nodes so as to minimize the total cost.

3.3.1 Model Assumption

Due to the complexity of the hub location problem, several assumptions for the FDMAP are made. First, all origin-destination pairs are routed via the selected hub locations. Under the multiple allocation restriction, each non-hub node can be allocated to any hub. Second, there are no capacities on both the selected hub locations, and a set of interhub links. In other words, they can handle the entire flow volume if necessary. Third, multiple interhub links are possible for each O-D pair. Fourth, flows travel via, at most, three hubs in the 4-length [*i*-*k*-*m*-*j*] path only in the special case where origin or destination is hub itself. Fifth, a homogenous flow is assumed. Sixth, the fixed cost of operating the hub is ignored. Last, neighboring search methods for location-allocation follows tabu search heuristic procedure.

3.3.2 Flow-based Discount Multiple Allocation Hub Problem (FDMAP)

The FDMAP is an improvement over the FLOWLOC (O'Kelly and Bryan, 1998), which has been an inspiration for the present model. Three approaches to find improved solutions are incorporated in this model. First, a tabu search neighboring heuristic is used to explore the location-allocation parts. This enables the model to solve the large network size in an efficient way. Second, a total enumeration method of identifying the shortest path for each O-D pair is applied to finding optimal infrastructures. Third, the model implements flow economies of scale on multiple interhub links applying a piecewise-linear concave cost function rather than a fixed discount function (alpha).

The basic elements of the model are a set of nodes, a set of arcs, and O-D flows. The installation of interhub links (arcs) to be selected in the hub network depends on various factors, such as location of hub nodes, tradeoffs between fixed costs and interhub discount for transport costs, and bundling flows. For example, selecting the interhub link with a lowest per unit flow cost increases fixed costs of building the cheaper operational interhub link (lower alpha). On the other hand, an expensive interhub link with low-priced fixed cost increases the overall per unit flow cost.

The two main decision variables of the FDMAP are X_{ijkm} (O-D path) and S_l (interhub link discount). Interhub link discount variables (S_l) are directly related to fixed costs (F_l). The objective function (3.1) of the FDMAP minimizes total network costs by opening p hubs of n interacting nodes. The model gives flow discounts whenever the interhub link is identified in the routing path. Constraint (3.2) requires that all flow be routed via exactly one path. Constraint (3.3) specifies that p hubs should be open.

Constraints (3.4) and (3.5) require that a hub to be open before a node is assigned to a hub. Constraint (3.6) is necessary in order for the correct fixed cost to be associated with its corresponding interhub discount. Constraints (3.7), (3.8) and (3.9) allow only integer variables.

The Flow-based Discount Multiple Allocation Hub Problem (FDMAP) of the mathematical formulation based on O'Kelly and Bryan (1998) is formalized as follows:

$$Min\sum_{i}^{n}\sum_{j}^{n}\sum_{k}^{n}\sum_{m}^{n}W_{ij}(C_{ik}+C_{mj})X_{ijkm}+\sum_{k}^{n}\sum_{m}^{n}C_{km}\sum_{i}^{n}\sum_{j}^{n}X_{ijkm}W_{ij}\sum_{l}^{5}S_{l}+\sum_{k}^{n}\sum_{m}^{n}\sum_{l}^{5}Y_{lkm}F_{l} \tag{3.1}$$

$s.t.$

$$\sum_{k\in N}X_{ijkm}=1 \tag{3.2}$$

$$\sum_{k\in N}Z_{k}=p \tag{3.3}$$

$$\sum_{m\in N}X_{ijkm}\le Z_{k} \qquad \forall\, i,j,k \tag{3.4}$$

$$\sum_{k\in N}X_{ijkm}\le Z_{m} \qquad \forall\, i,j,m \tag{3.5}$$

$$\sum_{i\in N}\sum_{j\in N}W_{ij}X_{ijkm}\le Y_{lkm}\sum_{i\in N}\sum_{j\in N}W_{ij} \qquad \forall\, k,m \tag{3.6}$$

$$Z_{k},Z_{m}\in\{0,1\} \tag{3.7}$$

$$X_{ijkm}\in\{0,1\} \tag{3.8}$$

$$Y_{lkm}\in\{0,1\} \tag{3.9}$$

where :

$X_{ijkm} = 1$ if flow i to j is routed via hubs in k and m; 0 otherwise

Z_k = node k is selected as a hub; 0 otherwise

N = the number of nodes in a network

p = the number of hubs to open

W_{ij} = the amount of flow traveling between i and j

C_{ij} = the per unit cost of traveling between i and j

S_l = interhub discount factor (slope of the piecewise lines)

F_l = fixed cost (the intercepts of the piecewise lines)

$Y_{lkm} = 1$ if flow on interhub link (k, m) will be charged with F_l ; 0 otherwise.

3.3.3 Tabu FDMAP Algorithm

This section explains the procedures for routing of origin-destination pairs given flows (W_{ij}). The idea underlying this algorithm is based on the shortest path enumeration: given installed infrastructures with fixed p hub locations, the choice of slopes (discounts) and fixed costs are known.

Tabu FDMAP algorithm is formalized as follows:

<u>Begin Loop 1</u>

Step 1. Set initial p nodes by creating a hub candidate list.

→ ${}_n\mathrm{C}_p$

<u>Begin Loop 2</u>

Step 2. Set the initial slope and fixed cost.

→ (# of infrastructure types)$^{{}_pC_2}$

Begin Loop 3

Step 3. Evaluate the objective function. Given W_{ij}, find the best allocation set, which minimizes the total network cost applying flow discount.

→ p^2 (# of possible paths for every i and j)

Step 4. Save the objective function, X_{ijkm}, interhub flows, the fixed cost (F_l) and the slope (S_l) for each interhub link.

End Loop 3

Step 5. Repeat Loop 3 (*Step 3 – 4*) until all (# of infrastructure types) ${}_pC_2$ are evaluated.

Step 6. Save the best solutions (all variables listed in *Step 4*) for each inner Loop (Loop 3), which provide the fixed cost and slope also.

End Loop 2

Step 7. Swap one hub node with non-hub nodes based on tabu search.

Step 8. Repeat Loop 2 (*Step 2*)

Step 9. Save the best solutions (all variables listed in *Step 4*) for each inner loop (Loop 2)

End Loop 1

Step 10. Save the best solutions (all variables listed in *Step 5*) for each inner loop (Loop 1)

Step 11. Check the criteria for tabu search parameters, and terminate the loop if it meets the criteria; otherwise keep Loop 1.

From the initial construction (with largest weights of flow), the tabu FDMAP model obtains only the location part. The allocation part is relaxed to the all hubs rather than a single hub, this part needs p^2 possible paths to find a path between origin and destination. Once hubs are selected, the minimum path between origin and destination is saved. The objective value is equivalent to adding all the paths and fixed costs. The model searches for a new hub location by swapping a hub node with non-hub nodes based on hub frequency, tabu lists, then it updates the best selection at each iteration along with updating the hub frequency, and tabu lists. However, it cannot move tabu nodes unless there is a better solution than the previous best. The algorithm repeats the loop until the maximum iteration for hub exchange is reached. The model saves the best solutions for each LTM[9] by updating the incumbent solution out of LTM sets. While the simple multiple allocation must evaluate p^2 possible paths between origin and destination, the FDMAP requires 5 times more than the traditional one for each interhub link ($5^{{}_pC_2} = 5*5*5$ possible combinations where $p = 3$) in addition to p^2 possible paths to find the optimal slopes and fixed costs for the minimum path between origin and destination. Numerical results are shown in Chapter 4.

3.4 Infrastructure Design of the Flow-based Discount Point-To-Point Problem (FDPTP)

The idea of flow-based discounting on direct links is based on the observations of previous hub models and point-to-point models. The results of hub models reveal that

[9] LTM stands for long-term memory.

some interhub links carry only a small amount of flow while other non-interhub links are busy with carrying a significant flow. On the other hand, there is not enough flow to maintain all direct links in point-to-point networks. This issue can be resolved to give flow-based discounts on any link with a choice of link types. The infrastructure design of the FDPTP can be described as follows: given a set of network nodes, build a set of links where the type of links can be specified by cost function and flow condition, restrict a number of intermediate stops between origin and destination, and transport O-D flows so as to minimize the total network cost.

3.4.1 Model Assumption

Several assumptions for the FDPTP are made due to the complexity of the point-to-point problem. First, all origin-destination pairs are routed by at most two intermediate stops under the hop restriction. Second, there are no capacities on both the selected locations, and a set of opened links. In other words, they can handle the entire flow volume if necessary. Third, a homogenous flow is assumed. Fourth, there is a directed link between every pair of nodes. Last, the fixed cost of operating the selected locations is ignored.

3.4.2 Flow-based Discount Point-To-Point Problem (FDPTP)

Given the above assumptions, the objective is to build links with minimal total network cost. The problem for the Flow-based Discount Point-to-Point Model (FDPTP) is formalized as follows:

$$Min \sum_{i,j,k,m:i\neq j,k\neq m,q} C_{km}(S^q \cdot X^q{}_{ijkm}) + \sum_{k,m:k\neq m,q} F^q{}_{km} \cdot C_{km} \cdot Y^q{}_{km} \tag{3.10}$$

s.t.

$$\sum_{i,j:i\neq j,q} X^q{}_{ijkm} \geq Y^q{}_{km} \cdot T^q \qquad \forall k,m: k\neq m, q \tag{3.11}$$

$$X^q{}_{ijkm} \leq Y^q{}_{km} \cdot W_{ij} \qquad \forall i,j,k,m: i\neq j, k\neq m, q \tag{3.12}$$

$$\sum_{i,j:i\neq j} X^q{}_{ijkm} \leq (1-Y^q{}_{km}) \cdot T^2 \qquad \forall k,m: k\neq m, q=1 \tag{3.13}$$

$$\sum_{m:m\neq i,q} X^q{}_{ijim} = W_{ij} \qquad \forall i,j: i\neq j \tag{3.14}$$

$$\sum_{k:k\neq i,q} X^q{}_{ijki} = 0 \qquad \forall i,j: i\neq j \tag{3.15}$$

$$\sum_{k:k\neq j,q} X^q{}_{ijkj} = W_{ij} \qquad \forall i,j: i\neq j \tag{3.16}$$

$$\sum_{m:m\neq j,q} X^q{}_{ijjm} = 0 \qquad \forall i,j: i\neq j \tag{3.17}$$

$$\sum_{k:k\neq l,q} X^q{}_{ijkl} = \sum_{m:m\neq l,q} X^q{}_{ijlm} \qquad \forall l,i,j: l\neq i,j: i\neq j \tag{3.18}$$

$$\sum_{q} Y^q{}_{km} \leq 1 \qquad \forall k,m: k\neq m \tag{3.19}$$

$$\sum_{q} X^q{}_{ijkm} \leq 3 \cdot W_{ij} \qquad \forall i,j,k,m: i\neq j, k\neq m \tag{3.20}$$

$$X^q{}_{ijkm} \geq 0 \qquad \forall i,j,k,m: i\neq j, k\neq m, q \tag{3.21}$$

$$Y^q{}_{km} \in \{0,1\} \qquad \forall k,m: k\neq m, q \tag{3.22}$$

The complete set of variables used in the FDPTP model is given as:

$$Y^q{}_{km} = \begin{cases} 1 \text{ if link } (k, m) \text{ is discounted on piecewise linear cost } q, \\ 0 \text{ otherwise.} \end{cases}$$

$X^q{}_{ijkm}$ = discounted flow from i to j through link (k, m) on piecewise linear cost q.

T^q = flow range on piecewise linear cost q.

The W_{ij} is exogenously given flow between origin and destination. The number of q of piecewise-linear cost function is five as same as the FDMAP. The Y variables are the arc selection decisions while X variables represent the routing decisions for each origin-destination flow. Objective function (3.10) finds a feasible flow with the minimal total network cost. The fixed costs, $F^q{}_{km}$, are exogenously given by the piecewise-linear cost function. Constraint (3.11) assures that the discounted flow is greater than the given piecewise-linear cost function depending on Y. Constraint (3.12) ensures that if a link (k, m) is non-discounted ($X^1{}_{ijkm}$), the discounted flow ($X^2{}_{ijkm}$, $X^3{}_{ijkm}$, $X^4{}_{ijkm}$, $X^5{}_{ijkm}$) will be zero. This constraint also limits the discounted flow through the link (k, m). Constraint (3.13) guarantees that if there is any non-discounted flow ($X^1{}_{ijkm}$) through a link (k, m), then its value must be less than given threshold T^2. The term $\sum_{i,j:i\neq j} X^q{}_{ijkm} \leq (1 - Y^q{}_{km}) \cdot T^2$ is equivalent to $\sum_{i,j:i\neq j} X^1{}_{ijkm} \leq (1 - Y^1{}_{km}) \cdot T^2$ because the constraint with $q = 1$ is tighter than any other q values, so the other constraints with other q values are unnecessary. Constraint (3.14) implies that all (i, j) flow must originate from i. Constraint (3.15) prevents flow originating from node i to return to its origin. Constraint (3.16) represents that all (i, j) flow must come into j. Constraint (3.17) prevents flow with destination j to leave node j. Constraint (3.18) specifies that the conservation of flow at node l is met. Constraint (3.19) allows the only one discount on the link. Constraint (3.20) prohibits any origin-destination pair that has more than 3 intermediate stops. The purpose of this constraint is to compare the results with the FDMAP because of the maximum number of passenger trips. Without this constraint, many O-D pairs are observed with more than 3 intermediate stops. Constraint (3.21) is non-negative flow variables and constraint (3.22) is binary variable for link (k, m). Variable X^1_{ijkm} captures the flow that goes from node i to node j via link (k, m) whose cost is not discounted. Variables, X^2_{ijkm}, X^3_{ijkm}, X^4_{ijkm}, and X^5_{ijkm} are the flow from node i to node j via link (k, m) whose cost is discounted.

To illustrate the idea of the FDPTP model, a 6-node simple network is studied in detail. Fixed costs and discounts are obtained by cost function 1 of Table 3.3 based the flow range 1 of Table 3.2. As can be seen in Figure 3.4, the FDPTP model optimally opens directed discounted arcs [H1→H2], [H2→H3], and [H3→H1] whose flows are above the flow range of the fifth piecewise-linear cost function. The arcs, [H2→H1], [H3→H2], and [H1→H3], in the network get the discount of the second piecewise-linear cost function. The both directions of [A-H1], [B-H2], [C-H3] arcs get the discount of the fourth piecewise-linear cost function. As can be seen in Figure 3.4, the flow of each discounted link is above the given flow range in Table 3.2. All the links can achieve discounts depending on the level of flows in this particular example.

Interestingly, routings of some O-D pairs are not symmetric. For example, the [H2→C] pair in Figure 3.4 has different routings from [C→H2]. The [H2→C] travels [H2→H3→C] whereas the [C→H2] passes through [C→H3→H1→H2]. Te costs of symmetric routings are different. For example, the [B-C] pair in Figure 3.4 has a symmetric routing but different costs between [B→C] and [C→B]. The [B→C] path enjoys the cheapest discount (S = 0.2) in its path while the [C→B] path uses the less cheaper discount (S = 0.8) in its path. Some O-D pairs without hop constraint travel circuitous routing to contribute their flows on links. For example, the [H3→H1] pair in Figure 3.5 utilizes the one stop path [H3→H2→H1] rather than the direct [H3→H1] path. Solutions of the model also depend on the concave cost function as shown in Table 3.4.

Piece	Flow Range1	Flow Range2	Flow Range3
1	0	0	0
2	15	22.5	30
3	30	45	60
4	45	67.5	90
5	60	90	120

Table 3.2 Ranges of the Interhub Link Flows (6-Node)

Piece		1	2	3	4	5
Cost Function 1	Fixed Cost	0	3	9	18	30
	Slope	1	0.8	0.6	0.4	0.2
Cost Function 2	Fixed Cost	0	4.5	14	27	45
	Slope	1	0.8	0.6	0.4	0.2
Cost Function 3	Fixed Cost	0	6	18	36	60
	Slope	1	0.8	0.6	0.4	0.2

Table 3.3 Slope (Discount) and Fixed Cost for the Piecewise-Linear Cost (6-Node)

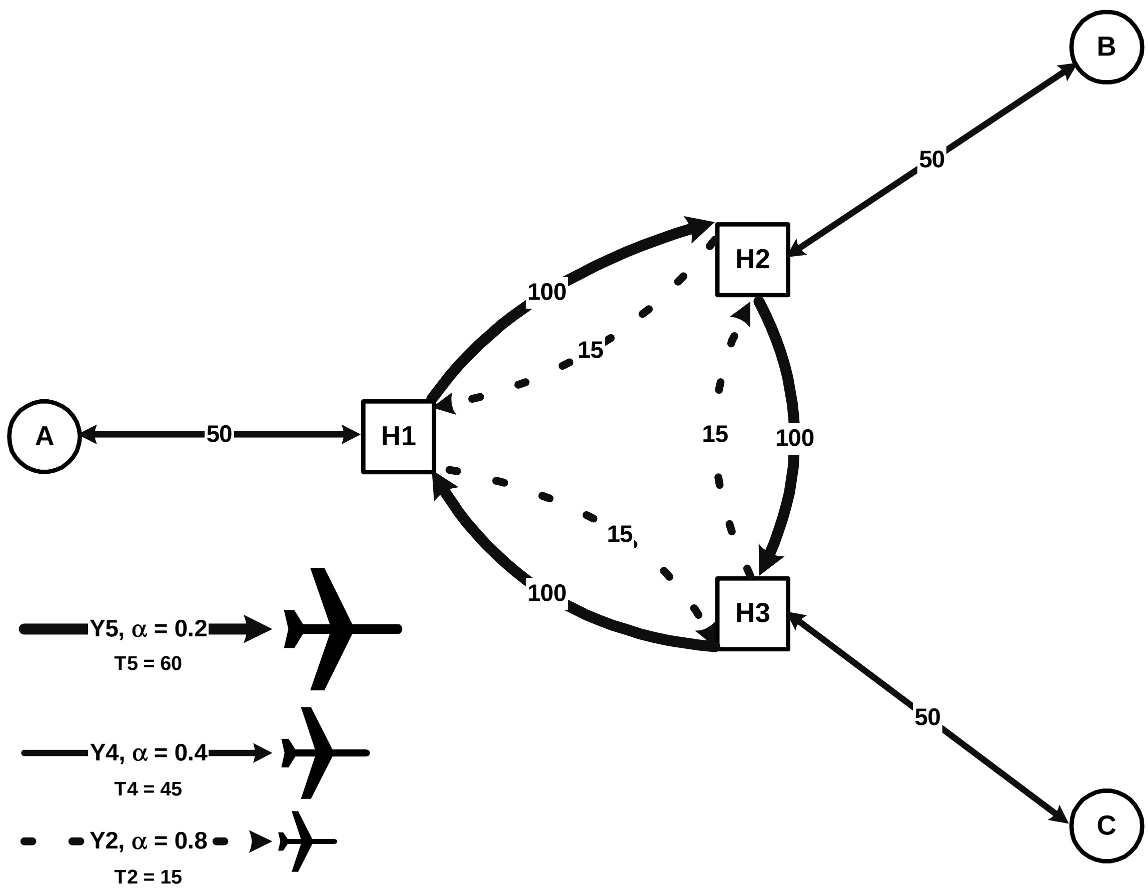

Figure 3.4 Solution (6-Node) of FDPTP with Hop Constraint with Cost Function 1 under Flow Range 1

Figure 3.5 illustrates the role of constraint (3.20) in the model. The FDPTP without the hop constraint (3.20) produces some O-D pairs that take the cheapest link rather than building direct links. For example, the path of [B→C] O-D pair is [B→H2→H1→H3→C] in Figure 3.5 instead of [B→H2→H3→C] path in Figure 3.4. The FDPTP with constraint (3.20) has built Y^2 infrastructure due to the hop constraint as can be shown in Figure 3.4. Total network costs are also increased by the constraint (3.20) as can be seen in Table 3.4. Table 3.5 also shows the decomposition of transport cost for each link. Table 3.4 clearly shows the FDPTP without the hop constraint (3.20) provides the lower total network cost than the FDPTP with the hop constraint. This result is very obvious because adding more constraints always increases the total network cost. However, the hop constraint does not affect the total network cost in the case of the cost function 3 as can be seen in Table 3.4. There are two possible reasons for that: [1] there is no room for O-D pairs to save the costs anymore in the special case of the cost function 3, and [2] there are no O-D pairs more than 2 intermediate stops in the FDPTP solution. Numerical results of CAB dataset are described in Chapter 5.

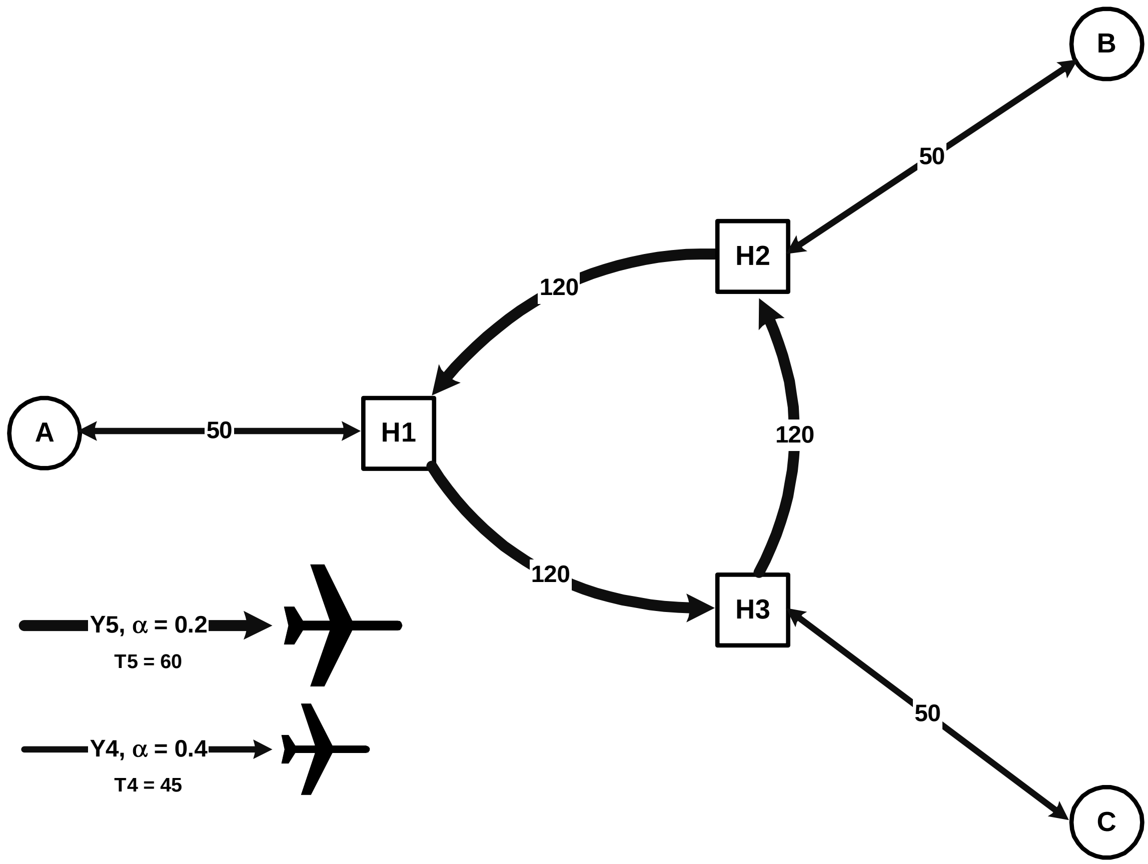

Figure 3.5 Solution (6-Node) of FDPTP without Hop Constraint with Cost Function 1 under Flow Range 1

	Total Network Cost		Fixed Cost		Transport Cost	
	FDPTP	FDPTP-(3.20)	FDPTP	FDPTP-(3.20)	FDPTP	FDPTP-(3.20)
C.F.1	1482.34	1381.44	737.57	703.93	744.78	677.51
C.F.2	1716.99	1672.14	386.81	790.41	1330.18	881.73
C.F.3	1791.47	1791.47	257.07	257.07	1534.40	1534.40

*Cost functions 1(CF1), 2(CF2), 3(CF3) are based on Flow Range 1 (Table 3.4).
See also Table 3.3 for the fixed costs and discount structure.

Table 3.4 Total Network Cost of FDPTP with/without Hop Constraint (6-Node)

	X5		X4		X3		X2		X1		Transport Cost	
	FDPTP	FDPTP-(3.20)	FDPTP	FDPTP-(3.20)	FDPTP	FDPTP-(3.20)	FDPTP	FDPTP-(3.20)	FDPTP	FDPTP-(3.20)	FDPTP	FDPTP-(3.20)
C.F.1	201.80	269.07	408.44	408.44	~	~	134.53	~	~	~	744.78	677.51
C.F.2	~	269.07	~	~	612.67	612.67	717.51	~	~	~	1330.18	881.73
C.F.3	~	~	~	~	~	~	1534.40	1534.40	~	~	1534.40	1534.40

*Cost functions 1(CF1), 2(CF2), 3(CF3) are based on Flow Range 1 (Table 3.4).

Table 3.5 Discounted Link Cost of the FDPTP with/without Hop Constraint (6-Node)

3.5 Cost Allocation in the Hub Network

The complexity of the cost allocation problem has led researchers to conclude that there is no economically justifiable way to allocate joint costs. Nevertheless, the costs must be allocated in some way among beneficiaries. Each O-D pair in the hub network plays an important role in determining the cost and performances of the network link. Effective and economical usage of an interhub link depends on the number of its supporting users. This raises the issue of fair cost associated with installation of shared interhub links. Therefore, a fair[10] and cost-effective configuration is the main concern of cost allocation model. The model employs a cooperative game theory to analyze and investigate the cost allocation problem of shared infrastructure for hub networks. The hub network problem is formulated as a cooperative game, and corresponding cost allocation schemes are chosen from the core of the associated game.

3.5.1 Model Assumption

Players are represented as ordered pairs of nodes (O-D pair). Interhub links have no capacity limits. The cost allocation in the hub network model is classified as transferable utility (TU) game. The coalition of the game would not necessarily use the globally optimal network.

[10] See Savas (1978) for fairness (equity) in cost allocation.

3.5.2 Notations[11], Definitions, and Properties

Following definitions and properties are used to solve the core of a cooperative game in hub networks. Let $N = \{1, 2, \dots, n\}$ be a finite set of nodes, and $P = \{N^*(N\text{-}1)\}$ be players, C: $2^P \rightarrow R$, with $C(0) = 0$ a characteristic function defined over subsets of P referred to as coalitions. If $C(P)$ designates a cost that has to be shared by all the players, then the pair $(P; C)$ is called a cooperative game. For $x \in R^{|P|}$ and $S \subseteq P$, let

$$x(S) = \sum_{j \in S} x_j \tag{3.23}$$

where $x(S)$ is the part of the total network cost paid by the coalition S. A cost allocation vector x in a game $(P; C)$ satisfies $x(P) = C(P)$, and the solution of cooperative games is equivalent to the selection of a reasonable subset of cost allocation vectors. Cost games are stated in the context of hub networks.

Definition 3.1: A cooperative cost game is a set P of players, $\{N^*(N\text{-}1)\}$ together with a characteristic function from 2^P to R, which assigns to each subset of P called a coalition. If the game is a cost game, the valuation is a cost denoted by C. So the game can be written as $(P; C)$.

For the hub network problem, P is set of pairs of nodes. For $S \subset P$, $C(S)$ is the optimal cost that can be achieved by coalition S. Notice that the value of a set is always non-negative for the hub network game, and $C_i > 0$ for singletons. It is desirable for a cooperative game to be zero-normalized; the value that the game attributes to a single

[11] The definition and notation are adapted from the models of Skorin-Kapov (1998) and Solymosi (1984).

player is zero. A cooperative game is zero-normalized if $\forall\, S \subset P$, $C(S) = \sum_{i \in S} C_i$ for a cost game.

Definition 3.2: A cooperative game is monotonic if as coalitions get larger, and their valuations change steadily. If $S \subset T \subset P$, $C(S) + \sum_{i \in T \setminus S} C_i \geq C(T)$ for a cost game. This is equivalent to $C(T) - C(S) \leq \sum_{i \in T \setminus S} C_i$.

Definition 3.3: A cooperative cost game is called subadditive (3.24) if its characteristic function satisfies the properties below.

$\forall\, S, T \subset P$

$$C(S) + C(T) \geq C(S \cup T) \tag{3.24}$$

Definition 3.4: A cooperative cost game is concave (3.25) if its characteristic function satisfies the properties below.

$\forall\, S, T \subset P$

$$C(S) + C(T) > C(S \cup T) + C(S \cap T) \tag{3.25}$$

The characteristic function calculates the cost accrued by any coalition. The coalition needs to distribute the costs to its joining members in a fair way. This is called cost allocation. Let a cost allocation vector function, CA: $P \rightarrow R^n$, assign an amount of costs to each member of P.

Definition 3.5: An allocation of CA is called an imputation if

$$\forall i \in P, CA_i \leq C_i \text{ and } CA(P) = C(P) \tag{3.26}$$

$CA(a)$ is efficient if $CA(P) = C(P)$ where $a \in P$. It allocates all of the costs the grand coalition makes to players. This property is also called as efficiency of the cost allocation vectors. *CA* assigns a nonnegative allocation to each player. That is, $\forall i, CA_i \geq 0$. One always receives a positive cost (non-negativity of the cost allocation vectors). Each player pays less cost by joining the group than by playing alone. That is, $\forall i, CA_i \leq C_i$. This property is called as ***individual rationality*** of the cost allocation vectors. For the cost game, no player should be charged more than the additional charge (the marginal cost) the group incurs by taking the individual player in. That is $\forall i, CA_i \leq C(P) - C(P \setminus \{i\})$. This property is called as ***marginal cost*** of including player *i* in the cost allocation vectors. No coalition *S* should get more cost in total than its valuation. There should be no incentive for the coalition to go it alone. That is, $\forall\, S \subset P,\ CA(S) \leq C(S)$. This property is called as ***standalone cost*** of including player *i* in the cost allocation vectors.

3.5.3 The Core

The core is a simple and intuitive solution concept for *n*-person cooperative games. It contains all feasible payoff vectors in which no subset of the players can break its cooperation with the rest of the players. The core of a game is the set of imputations that satisfies the standalone cost. Mathematically, the core of a cost game (*P*; *c*) consists of all vectors $x \in R^{|P|}$ such that $x(S) \leq c(S)$ for all $S \subseteq N$, and $x(N) = c(N)$. The cost is summarized by a joint cost function *c*(*S*), which is defined for all subsets $S \subset N$ of potential players. The *c*(*S*) represents the least cost of serving the players in *S* by the most efficient means. The cost of serving no one is assumed to be zero; that is, $c(\phi) = 0$. The cost, *c*, is called the characteristic function. A cost allocation method is a function defined for all N and all joint cost functions c on N such that $\varphi(c) = (x_1,..., x_n) \in R^N$ and $\sum x_i = c(N)$, where x_i is the charge assessed to player i.

In a hub game context, the value of $c(S)$ is determined by estimating the least-cost routing of each O-D pair. The $c(S)$ is defined so that it includes the possibility that some or all members of S develop independent ways of cooperation if it is the least-cost alternative of supplying S. Under these circumstances, c will be subadditive (see Definition 3.3): $c(S) + c(T) \geq c(S \cup T)$ for all disjoint S, T. This is based on the simple possibility that the cost of serving two disjoint groups includes the possibility of serving them separately.

There are two major principles of cost allocation necessary to satisfy the core: stand-alone cost, and incremental (marginal) cost. For the stand-alone cost, the following inequality should hold for every subset S:

$$\sum_{i \in S} x_i \leq c(S) \tag{3.27}$$

This condition indicates that no participant be charged more than their stand-alone costs. The incremental cost states that no participant should be charged less than the marginal cost of including itself. In general, the incremental cost of any set S is defined to be $c(N) - c(N\text{-}S)$. This condition requires the allocation $x \in R^N$ to satisfy:

$$\sum_{i \in S} x_i \geq c(N) - c(N - S) \text{ for all } S \subset N \tag{3.28}$$

Whereas (3.27) provides incentives for voluntary cooperation, (3.28) arises from consideration of equity. Therefore, the core of c is the set of all allocations $x \in R^N$ such that (3.27) and (3.28) hold for all $S \subset N$.

A characteristic function game is a purely cooperative game among *n* players who see a fair distribution for a cost that is freely transferable. It is assumed that all players would like as little as possible of the cost, and that one unit of the cost is worth the same to all players. The fairness of a distribution is assumed to depend on the bargaining strengths of the various coalitions that could possibly form among some or all of the players. However, the fundamental assumption of a characteristic function game is that all players are cooperating. In other words, a grand coalition of all *n* players has formed. Thus, coalitions of fewer than *n* players can be used to leverage the strength they would have had without the others (see also Mesterton-Gibbons (2001) for more details). In this research, this assumption is relaxed. Instead of "all players are cooperating", the model only considers the players who use the interhub links.

To illustrate the cost allocation in hub networks, a 6-node simple network is studied in detail. With respect to the hub network, the total network flow for the 6-node network is 300. Concave cost function 1 (Table 3.3) with flow range 1 (Table 3.2) is incorporated in the FDMAP. The total network cost from the FDMAP is 5.76 (1729.18)[12] with transport cost of 4.32 (1296.51) and fixed cost of 1.44 (432.67) as can be seen in Table 3.6. The core allocation is obtained by assigning the fixed cost to the interhub link users proportionally. It does not make sense if the cost allocation model divides the total fixed cost by the number of interhub supporters evenly. Each O-D pair has a different contribution to the interhub link flows. Out of 30 directed O-D pairs, 22 are the interhub link players. As can be seen in Figure 3.3, the FDMAP opens 4 directed interhub links

[12] The total network cost is normalized by total network flow. For example, 5.76 is equal to 1729.18/300 (cost / total network flow).

and closes 2 directed interhub links out of 6 possible directed interhub links. Each interhub link has 7 interhub link users summing up 70 interhub link flows. There are four interhub links summing up 280 total interhub link flows (see Table 3.6). The flows of interhub links enable the interhub link to achieve the cheapest discount (e.g. 5^{th} piece) in piece-wise linear cost functions. As can be seen in Table 3.4, the interhub link should reach at least 60 to get the 5^{th} piecewise-linear cost function. It is shown that every interhub link flow (70) is greater than 60 in this case as can be seen in Table 3.6.

Table 3.7 shows the cost allocation vectors for the 6-node network based on two principles [equations (3.25-3.26)]. Results provide two different cost allocation vectors for each O-D pair. First, cost allocation vectors are achieved under the global hub network. In other words, the coalitions cannot modify the hub network. In this case, the cost allocation is proportional to their contribution to the intherhub link flows. Second, cost allocation vectors are obtained under the coalition optimal hub network. That is, the coalitions can modify alternative routings if they can save their own network cost. Under this situation, the cost allocation is not only proportional to their contribution to the interhub link flows, but also dependent on alternative routing possibilities. For example, the O-D pairs ([3-4], [3-6], [4-3], [4-5], [5-4], [6-3]) with multiple interhub links tend to leave the global optimal hub network to save their network cost. These multiple interhub link O-D pairs try to use single interhub link. To remedy this controversy, the cost allocation model obtains a fair cost allocation by charging cheaper fixed costs for these players so that they can stay in the optimal network. The results support that the grand coalition for each interhub link is the best strategy for the total network cost under the global hub network configuration. In other words, none of the players can achieve a cheaper cost by other coalitions assuming that players cannot change the globally optimal hub network. However, the grand coalition for each interhub link is not necessarily the best strategy to individual players under the coalition optimal hub network configuration. The core allocations are changed if players can modify the globally optimal hub network. This fair cost allocation is only possible if and only if the cost savings by the grand coalition is greater than the opportunity costs by the players who are willing to leave the grand coalition.

Even with a 6-node network, the possibilities of 2^n-1 coalition are very computationally prohibitive as can be seen in Table 3.8. To reduce the complexity of the problem, the model tries to separate all interhub link players (22) into a single interhub link player (7) based on aggregation (see Section 3.5.6). The results show that the core allocations for the all interhub link players (22) are equal to the core allocation for the single interhub link player (7). That is, the cost allocation for the whole network is equal to the cost allocation for the sub-network.

6-Node	Location			Discount			Flow				Cost		
p=3	Hub1	Hub2	Hub3	Interhub 1_2	Interhub 1_3	Interhub 2_3	Interhub 1_2	Interhub 1_3	Interhub 2_3	Total Flow	Obj	Transport Cost	Fixed Cost
C.F.1	4	2	3	5	1	5	140	0	140	280	5.76	4.32	1.44
C.F.2	4	2	3	2	2	2	80	80	80	240	6.13	5.80	0.34
C.F.3	4	2	3	2	1	2	80	0	80	160	6.23	5.94	0.29

* Cost function 1 is based on Flow Range 1, Cost function 2 for Flow Range 2, and Cost function 3 for Flow Range 3.

Table 3.6 Optimal Solutions of FDMAP (6-Node)

I	J	Transport Cost	Cost Allocation under Global Optimum	Cost Allocation under Coalition Optimum
1	2	30.00	30.00	30.00
1	3	37.21	52.66	58.84
1	4	37.21	52.66	55.24
1	5	73.27	88.72	94.90
1	6	73.27	88.72	91.29
2	1	30.00	30.00	30.00
2	3	7.21	22.66	28.84
2	4	7.21	22.66	25.24
2	5	43.27	58.72	64.90
2	6	43.27	58.72	61.29
3	1	37.21	52.66	58.84
3	2	7.21	22.66	28.84
3	4	14.42	45.33	32.45
3	5	36.06	36.06	36.06
3	6	50.48	81.38	68.51
4	1	37.21	52.66	55.24
4	2	7.21	22.66	25.24
4	3	14.42	45.33	32.45
4	5	50.48	81.38	72.11
4	6	36.06	36.06	36.06
5	1	73.27	88.72	94.90
5	2	43.27	58.72	64.90
5	3	36.06	36.06	36.06
5	4	50.48	81.38	72.11
5	6	108.86	108.86	108.86
6	1	73.27	88.72	91.29
6	2	43.27	58.72	61.29
6	3	50.48	81.38	68.51
6	4	36.06	36.06	36.06
6	5	108.86	108.86	108.86

Table 3.7 Cost Allocation of FDMAP with Cost Function 1 under Flow Range 1 (6-Node)

3.5.4 Aggregate Cost Allocation

A cooperative hub game with aggregated players is applied to solve the cost allocation problem efficiently and to reduce the complexity of the problem. In this study, the players are defined as all sets of all node pairs ($N*N-1$). All sets of node pairs are aggregated (grouped) based on interhub link usage due to the complexity of all possible coalitions. For example, Table 3.8 shows the number of possible coalition considerations based on a 6-node simple hub network (see Figure 3.3). More generally, among $n*(n-1)$ players, 2^n-1 coalitions are possible. It is obvious that the CAB dataset is impossible to obtain the cost allocation without the aggregation scheme due to the large number of coalitions. Table 3.9 summarizes the grouping of the sets of node pairs based on the previously supported interhub links. Once the fair cost allocation for the aggregate players is obtained, those costs (fixed costs for infrastructure) are allocated to each individual user. Comparing Table 3.9 with Table 3.8 shows how efficient the aggregation scheme is in the cost allocation.

Combination	S	# of Possibilities
${}_{12}C_{1}$	1	12
${}_{12}C_{2}$	2	66
${}_{12}C_{3}$	3	220
${}_{12}C_{4}$	4	495
${}_{12}C_{5}$	5	792
${}_{12}C_{6}$	6	924
${}_{12}C_{7}$	7	792
${}_{12}C_{8}$	8	495
${}_{12}C_{9}$	9	220
${}_{12}C_{10}$	10	66
${}_{12}C_{11}$	11	12
${}_{12}C_{12}$	12	1
$2^{12}-1$	Sum	4095

Table 3.8 Individual Players (6-Node)

Combination	S	# of Possibilities
${}_{3}C_{1}$	1	3
${}_{3}C_{2}$	2	3
${}_{3}C_{3}$	3	1
$2^{3}-1$	Sum	7

Table 3.9 Aggregate Players (6-Node)

The value of the characteristic function, $c(S)$, for each subset of players S ($S \in P$) should describe the cost associated with the delivery of service to S. For the case when $P = N*(N-1)$, $c(T)$ should be the cost of traffic between pairs of users in T where $T \subseteq P$. For the sets of node pairs, a game $\{N*(N\text{-}1), c\}$ of the characteristic function C is defined as follows:

$C(T) = equation\,(3.1)$, such that $C(\phi) = 0,$ and for $\varphi \neq T \in N*(N-1)$, where X_{ijkm} is an optimal solution subject to (3.2) – (3.9). Notice that there are two optimal situations. First, the traffic between pairs in T is carried over a globally optimal hub network. Second, coalitions optimize the hub network instead of using an optimized hub network. Players are identified as pairs of users, and the cost allocated to each pair could later be equally divided among users in that pair. Table 3.10 shows the model allocates the fixed cost, and the total network cost separately based on a 6-node network.

6-Node	Interhub Link			Total Cost	Total Cost without Fixed Cost	Fixed Cost
	H1H2	H1H3	H2H3			
S	C(S)	C(S)	C(S)	C(S)	C(S)	C(S)
{H1H2, H1H3}	1.84	1.84	0	5.09	4.71	0.38
{H1H2, H2H3}	1.82	0	1.92	5.15	4.34	0.81
{H1H3, H2H3}	0	1.82	1.92	5.15	4.34	0.81
6-Node	Interhub Link			Total Cost	Total Cost without Fixed Cost	Fixed Cost
	H1H2	H1H3	H2H3			
S	C(S)	C(S)	C(S)	C(S)	C(S)	C(S)
{H1H2, H1H3, H2H3}	1.84	1.84	0	5.09	4.71	0.38

Table 3.10 Fixed Costs of Interhub Link for FDMAP Hub Game (6-Node)

Table 3.11 shows the results of the FDMAP hub game with respect to grouping costs, installation built, and total network cost. Table 3.11.a illustrates the payoffs for the 3-player hub game based on their own strategies. Table 3.11.b specifies the infrastructures selected. Table 3.11.c shows that the cooperation of all players (H1H2, H1H3, and H2H3) is the best strategy for reaching a minimum total cost. The role of [H2H3] is not dominant in reducing the total cost. However, the participation of [H2H3] player in this cooperation reduces the infrastructure cost, so that the total cost can be reduced. As can be seen in Figure 3.6, the configuration of the hub network game allows for the selective interhub links rather than a fully connected hub network. Since each pair is restricted to a 4-length stop trip, the node B is connected to both H2 and H3. Although the [B-C] O-D pair has a less expensive 5-length stop way of [B-H2-H1-H3-C] trip, it will still undertake a [B-H3-C] trip. If we relax the maximum trip length, the [BC] O-D pair will select a less expensive route.

a. Group Cost

Group costs	H2H3 JOIN	
H1H2 \ H1H3	Join	Not Join
Join	(160.96, 160.96, 224.23)	(160.96, 218.64, 230.13)
Not Join	(218.64, 160.96, 230.13)	(218.64, 218.64, 240.22)
Group costs	H2H3 NOT JOIN	
H1H2 \ H1H3	Join	Not Join
Join	(200.04, 200.04, 240.22)	(218.64, 218.64, 240.22)
Not Join	(218.64, 218.64, 240.22)	(218.64, 218.64, 240.22)

b. Infrastructure built

Infrastructure	H2H3 JOIN	
H1H2 \ H1H3	Join	Not Join
Join	(5, 5, 1)	(4, alpha = 0.6, 5)
Not Join	(alpha = 0.6, 4, 5)	alpha = 0.6
Infrastructure	H2H3 NOT JOIN	
H1H2 \ H1H3	Join	Not Join
Join	(4, 4, alpha = 0.6)	alpha = 0.6
Not Join	alpha = 0.6	alpha = 0.6

c. Total Network Cost

Total Cost	H2H3 JOIN	
H1H2 \ H1H3	Join	Not Join
Join	5.09	5.17
Not Join	5.17	5.2
Total Cost	H2H3 NOT JOIN	
H1H2 \ H1H3	Join	Not Join
Join	5.14	5.2
Not Join	5.2	5.2

Table 3.11 Three Group Players of FDMAP Hub Game in a Normal Form (6-Node)

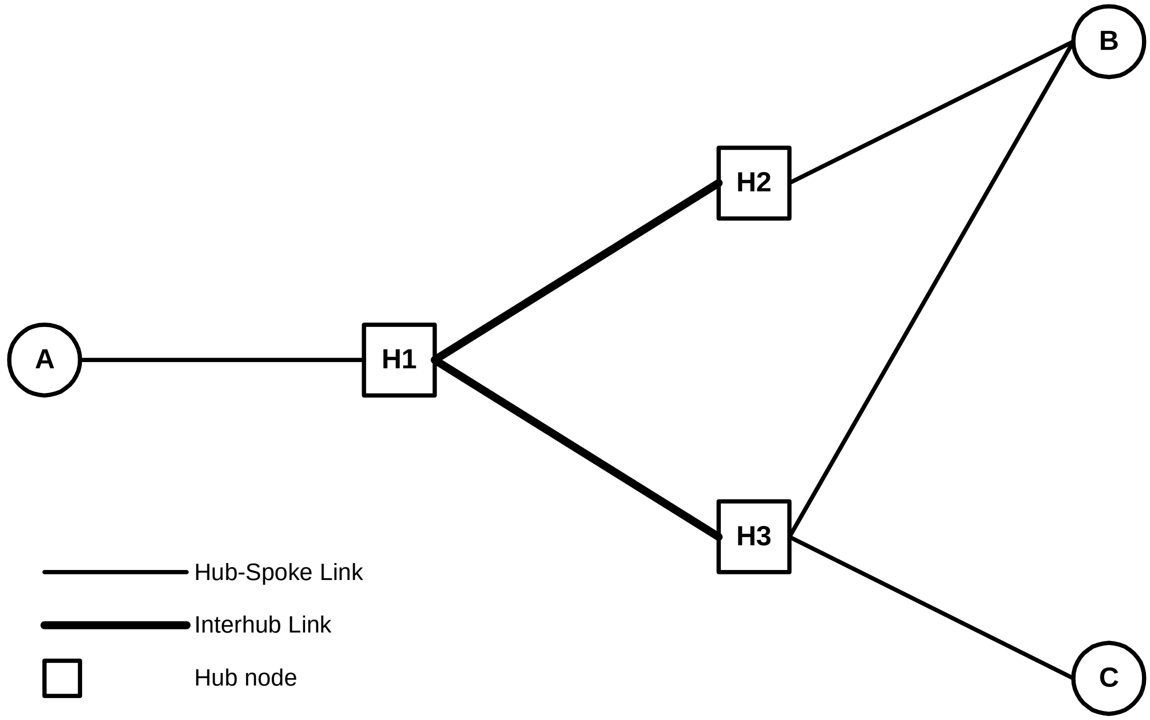

Figure 3.6 The Hub Network with FDMAP Hub Game (6-Node)

3.5.5 Individual Cost Allocation

Once an optimal hub network has been reached as can be seen in Figure 3.2, the cost/mile and the fixed cost for each interhub link can be calculated. Then, those fixed costs to each individual player of the previous interhub link grouping can be fairly allocated. Table 3.13 shows cost allocation of fixed costs for each individual player. By joining the coalition, every individual player in each interhub link group has a lower fixed cost than that of members of the partial coalition. Table 3.13 also demonstrates that joining all players is better than the partial coalition.

a. H1H2, H1H3, H2H3 Coalition

FIXED COST ALLOCATION	H1H2, H1H3, H2H3 COALITION		
Grouping Player	H1H2	H1H3	H2H3
FDMAP Hub Game	0.82	0.82	0.00
Individual Player	H1H2	H1H3	H2H3
FDMAP Hub Game	0.14	0.14	0.14

b. H1H2, H1H3 Coalition

FIXED COST ALLOCATION	H1H2, H1H3 COALITION		
Grouping Player	H1H2	H1H3	H2H3
FDMAP Hub Game	0.82	0.82	1.50
Individual Player	H1H2	H1H3	H2H3
FDMAP Hub Game	0.20	0.20	0.37

Table 3.12 Cost Allocation of Fixed Costs for Individual Player in FDMAP Hub Game (6-Node)

3.6 Cost Allocation in the Point-To-Point Network

In this section, cost allocation schemes of the FDPTP model are presented. The model provides a fair cost allocation among users of discounted links based on their contribution to flow economies of scale. Each pair of nodes transports flows either with discounts, or without discounts. The amount of flow on the links decides the level of discount rates. Many O-D pairs use shared discounted links. Therefore, the cost of discounted links in the point-to-point network should be distributed among its users in a fair way. In other words, the cost allocation should be solved with respect to its own flow because each O-D pair has different flow to transport on shared links.

3.6.1 Model Assumption

Players are represented as ordered pairs of nodes ($N*N-1$). Discounted links in the FDPTP model have no capacity limits. The O-D pairs necessarily use the globally optimal network. Unlike the hub network, the O-D pairs in the point-to-point network cannot reduce their cost by changing their routings.

3.6.2 Proportional Cost Allocation

A proportional cost allocation scheme is developed to solve the cost allocation model. The total network cost is obtained by constraints (3.10) – (3.22). Then, the model separates the total network costs into transportation costs and fixed costs. The goal is to allocate the fixed costs among users of the discounted link users [13] based on their contribution to flow economies of scale. Let PCA_{ijkm} be a proportional cost allocation of the [I-J] pair on [K-M] link. It is possible to divide the fixed costs to user of shared links fairly. Equation (3.29) allocates fixed costs on every opened links to the users with respect to flow contribution on the link.

$$PCA_{ijkm} = F_{km} \cdot [W_{km} / \sum_i \sum_j X_{ijkm}] \qquad \forall\, k, m \qquad (3.29)$$

Tables 3.13-3.14 show the cost allocations of the FDPTP without hop constraint for each O-D pair (players) based on a 6-node network [see Figure 3.5 for the optimal solution of the FDPTP without hop constraint]. Tables 3.15-3.17 show the cost allocations of the FDPTP with hop constraint for each O-D pair based on a 6-node network. Notice that [Difference] columns of Tables 3.13-3.17 have zero, positive or negative numbers. Zero in [Difference] column represents that each user's flow contribution to links is same as others. In other words, cost allocation is not necessary in this particular case. Positive number in [Difference] column shows that O-D pairs pay more fixed costs with respect to flow by proportional cost allocation than those without cost allocation. Negative number in [Difference] column represents that they pay less fixed costs with respect to flow by proportional cost allocation than those without cost allocation. In this particular example, every O-D pair has a unit flow of 10. For example, Table 3.15 provides the cost allocation of O-D pairs that use the discount link (X^2). Their total network cost [Total Network Cost2] represents proportional cost allocations based on their flow contribution

[13] The cost allocation for the non-discounted link users is trivial because there are no flow economies of scale involved in that link.

to the link because each O-D pair contributes different flow to the link. The numerical results of the CAB dataset are shown in Chapter 6.

X	I	J	K	M	Flow	Transport Cost	Total Network Cost 1	Total Network Cost 2	Difference
X4	1	2	1	2	10	12	14.16	14.16	0
X4	1	3	1	2	10	12	14.16	14.16	0
X4	1	4	1	2	10	12	14.16	14.16	0
X4	1	5	1	2	10	12	14.16	14.16	0
X4	1	6	1	2	10	12	14.16	14.16	0
X4	2	1	2	1	10	12	14.16	14.16	0
X4	3	1	2	1	10	12	14.16	14.16	0
X4	4	1	2	1	10	12	14.16	14.16	0
X4	5	1	2	1	10	12	14.16	14.16	0
X4	6	1	2	1	10	12	14.16	14.16	0
X4	1	5	3	5	10	14.42	17.02	17.02	0
X4	2	5	3	5	10	14.42	17.02	17.02	0
X4	3	5	3	5	10	14.42	17.02	17.02	0
X4	4	5	3	5	10	14.42	17.02	17.02	0
X4	6	5	3	5	10	14.42	17.02	17.02	0
X4	1	6	4	6	10	14.42	17.02	17.02	0
X4	2	6	4	6	10	14.42	17.02	17.02	0
X4	3	6	4	6	10	14.42	17.02	17.02	0
X4	4	6	4	6	10	14.42	17.02	17.02	0
X4	5	6	4	6	10	14.42	17.02	17.02	0
X4	5	1	5	3	10	14.42	17.02	17.02	0
X4	5	2	5	3	10	14.42	17.02	17.02	0
X4	5	3	5	3	10	14.42	17.02	17.02	0
X4	5	4	5	3	10	14.42	17.02	17.02	0
X4	5	6	5	3	10	14.42	17.02	17.02	0
X4	6	1	6	4	10	14.42	17.02	17.02	0
X4	6	2	6	4	10	14.42	17.02	17.02	0
X4	6	3	6	4	10	14.42	17.02	17.02	0
X4	6	4	6	4	10	14.42	17.02	17.02	0
X4	6	5	6	4	10	14.42	17.02	17.02	0

Table 3.13 Cost Allocation (FDPTP without Hop Constraint) of X^4 with Cost Function 1 under Flow Range 1 (6-Node)

X	I	J	K	M	Flow	Transport Cost	Total Network Cost 1	Total Network Cost 2	Difference
X5	1	3	4	3	10	4	18	18	0
X5	1	5	4	3	10	4	18	18	0
X5	2	3	4	3	10	4	18	18	0
X5	2	5	4	3	10	4	18	18	0
X5	4	1	4	3	10	4	18	18	0
X5	4	2	4	3	10	4	18	18	0
X5	4	3	4	3	10	4	18	18	0
X5	4	5	4	3	10	4	18	18	0
X5	6	1	4	3	10	4	18	18	0
X5	6	2	4	3	10	4	18	18	0
X5	6	3	4	3	10	4	18	18	0
X5	6	5	4	3	10	4	18	18	0
X5	3	1	3	2	10	3.61	16.22	16.22	0
X5	3	2	3	2	10	3.61	16.22	16.22	0
X5	3	4	3	2	10	3.61	16.22	16.22	0
X5	3	6	3	2	10	3.61	16.22	16.22	0
X5	4	1	3	2	10	3.61	16.22	16.22	0
X5	4	2	3	2	10	3.61	16.22	16.22	0
X5	5	1	3	2	10	3.61	16.22	16.22	0
X5	5	2	3	2	10	3.61	16.22	16.22	0
X5	5	4	3	2	10	3.61	16.22	16.22	0
X5	5	6	3	2	10	3.61	16.22	16.22	0
X5	6	1	3	2	10	3.61	16.22	16.22	0
X5	6	2	3	2	10	3.61	16.22	16.22	0
X5	1	3	2	4	10	3.61	16.22	16.22	0
X5	1	4	2	4	10	3.61	16.22	16.22	0
X5	1	5	2	4	10	3.61	16.22	16.22	0
X5	1	6	2	4	10	3.61	16.22	16.22	0
X5	2	3	2	4	10	3.61	16.22	16.22	0
X5	2	4	2	4	10	3.61	16.22	16.22	0
X5	2	5	2	4	10	3.61	16.22	16.22	0
X5	2	6	2	4	10	3.61	16.22	16.22	0
X5	3	4	2	4	10	3.61	16.22	16.22	0
X5	3	6	2	4	10	3.61	16.22	16.22	0

Table 3.14 Cost Allocation (FDPTP without Hop Constraint) of X^5 with Cost Function 1 under Flow Range 1 (6-Node)

X	I	J	K	M	Flow	Transport Cost	Total Network Cost 1	Total Network Cost 2	Difference
X2	1	6	2	4	10	28.84	36.06	34.25	-1.80
X2	2	6	2	4	5	14.42	18.03	19.83	1.80
X2	3	1	3	2	5	14.42	18.03	19.83	1.80
X2	5	1	3	2	10	28.84	36.06	34.25	-1.80
X2	4	3	4	3	5	16	20	22	2
X2	6	5	4	3	10	32	40	38	-2

Table 3.15 Cost Allocation (FDPTP with Hop Constraint) of X^2 with Cost Function 1 under Flow Range 1 (6-Node)

X	I	J	K	M	Flow	Transport Cost	Total Network Cost 1	Total Network Cost 2	Difference
X4	1	2	1	2	10	12	22.80	22.80	0
X4	1	3	1	2	10	12	22.80	22.80	0
X4	1	4	1	2	10	12	22.80	22.80	0
X4	1	5	1	2	10	12	22.80	22.80	0
X4	1	6	1	2	10	12	22.80	22.80	0
X4	2	1	2	1	10	12	22.80	22.80	0
X4	3	1	2	1	10	12	22.80	22.80	0
X4	4	1	2	1	10	12	22.80	22.80	0
X4	5	1	2	1	10	12	22.80	22.80	0
X4	6	1	2	1	10	12	22.80	22.80	0
X4	1	5	3	5	10	14.42	27.40	27.40	0
X4	2	5	3	5	10	14.42	27.40	27.40	0
X4	3	5	3	5	10	14.42	27.40	27.40	0
X4	4	5	3	5	10	14.42	27.40	27.40	0
X4	6	5	3	5	10	14.42	27.40	27.40	0
X4	1	6	4	6	10	14.42	27.40	27.40	0
X4	2	6	4	6	10	14.42	27.40	27.40	0
X4	3	6	4	6	10	14.42	27.40	27.40	0
X4	4	6	4	6	10	14.42	27.40	27.40	0
X4	5	6	4	6	10	14.42	27.40	27.40	0
X4	5	1	5	3	10	14.42	27.40	27.40	0
X4	5	2	5	3	10	14.42	27.40	27.40	0
X4	5	3	5	3	10	14.42	27.40	27.40	0
X4	5	4	5	3	10	14.42	27.40	27.40	0
X4	5	6	5	3	10	14.42	27.40	27.40	0
X4	6	1	6	4	10	14.42	27.40	27.40	0
X4	6	2	6	4	10	14.42	27.40	27.40	0
X4	6	3	6	4	10	14.42	27.40	27.40	0
X4	6	4	6	4	10	14.42	27.40	27.40	0
X4	6	5	6	4	10	14.42	27.40	27.40	0

Table 3.16 Cost Allocation (FDPTP with Hop Constraint) of X^4 with Cost Function 1 under Flow Range 1 (6-Node)

X	I	J	K	M	Flow	Transport Cost	Total Network Cost 1	Total Network Cost 2	Difference
X5	1	3	2	3	10	7.21	19.23	18.03	-1.20
X5	1	4	2	3	10	7.21	19.23	18.03	-1.20
X5	1	5	2	3	10	7.21	19.23	18.03	-1.20
X5	2	3	2	3	10	7.21	19.23	18.03	-1.20
X5	2	4	2	3	10	7.21	19.23	18.03	-1.20
X5	2	5	2	3	10	7.21	19.23	18.03	-1.20
X5	2	6	2	3	5	3.61	9.61	14.42	4.81
X5	4	3	2	3	5	3.61	9.61	14.42	4.81
X5	4	5	2	3	10	7.21	19.23	18.03	-1.20
X5	6	3	2	3	10	7.21	19.23	18.03	-1.20
X5	1	4	3	4	10	8	21.33	20	-1.33
X5	2	4	3	4	10	8	21.33	20	-1.33
X5	2	6	3	4	5	4	10.67	16	5.33
X5	3	1	3	4	5	4	10.67	16	5.33
X5	3	2	3	4	10	8	21.33	20	-1.33
X5	3	4	3	4	10	8	21.33	20	-1.33
X5	3	6	3	4	10	8	21.33	20	-1.33
X5	5	2	3	4	10	8	21.33	20	-1.33
X5	5	4	3	4	10	8	21.33	20	-1.33
X5	5	6	3	4	10	8	21.33	20	-1.33
X5	3	1	4	2	5	3.61	9.61	14.42	4.81
X5	3	2	4	2	10	7.21	19.23	18.03	-1.20
X5	4	1	4	2	10	7.21	19.23	18.03	-1.20
X5	4	2	4	2	10	7.21	19.23	18.03	-1.20
X5	4	3	4	2	5	3.61	9.61	14.42	4.81
X5	4	5	4	2	10	7.21	19.23	18.03	-1.20
X5	5	2	4	2	10	7.21	19.23	18.03	-1.20
X5	6	1	4	2	10	7.21	19.23	18.03	-1.20
X5	6	2	4	2	10	7.21	19.23	18.03	-1.20
X5	6	3	4	2	10	7.21	19.23	18.03	-1.20

Table 3.17 Cost Allocation (FDPTP with Hop Constraint) of X^5 with Cost Function 1 under Flow Range 1 (6-Node)

CHAPTER 4

NUMERICAL RESULTS OF THE FLOW-BASED DISCOUNT MULTIPLE ALLOCATION HUB PROBLEM (FDMAP)

This chapter presents analytical results of the FDMAP. The computational results from the FDMAP are discussed and compared to the results for previous hub models. The new model particularly focuses on building optimal infrastructure for interhub links with a concave cost function. The numerical results are also compared to other developed models in later chapters.

4.1 Data Description

Civil Aeronautics Board (CAB) data sets are used in this study. Table A.1 in the Appendix A shows 25 U.S. city airline passenger flows in 1970. This is a standard benchmark data used to assess hub model characteristics. Subsets of the data are also generated in order to compare 10*10, 15*15, and 20*20 interactions. The 100 U.S. city airline passenger flows for 1970 in Table A.2 are also used. Figure 4.1 shows the configuration of each data. The data include values for the O-D flows (W_{ij}), and distances (C_{ij}) calculated from latitude and longitude coordinates of each airport. The total network flow for the dataset is given in Table 4.1. All flows are symmetric.

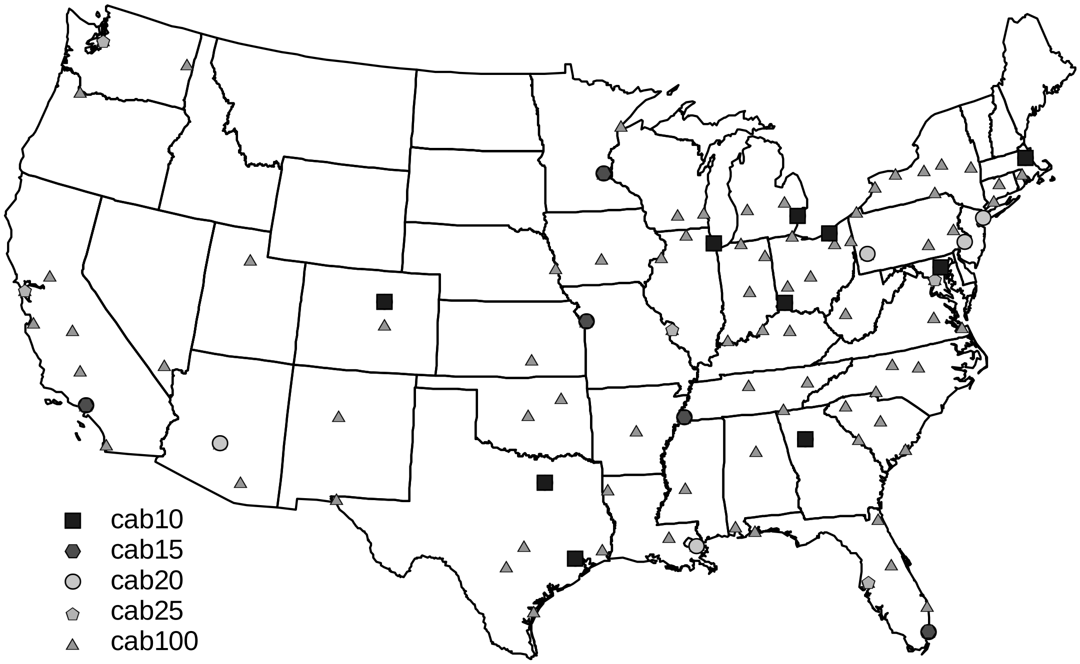

Figure 4.1 CAB Data Set

Dataset	Total Network Flow
10	999,026
15	2,364,942
20	5,754,594
25	8,540,006
100	16,549,732

Table 4.1 Total Network Flow (CAB Data)

4.2 Numerical Results of the FDMAP

To illustrate the efficiency of the new model, a 20 node network, a 25 node network, and a 100 node network are analyzed. The 20 node network is typically used. Unlike the previous FLOWLOC model, the FDMAP solves a 100 node network without fixed hub locations in a reasonable amount of time. The FDMAP determines infrastructure of the interhub link along with optimal hub locations and the allocations of non-hub nodes to hubs for 20, 25, 100 cities. Figure 4.2 illustrates different types of infrastructure in hub network. Total flows from Dallas to Atlanta, for example, are bundled from other city pairs, including Dallas to Atlanta. Then, the model decides a type of plane that provides the minimum total network cost.

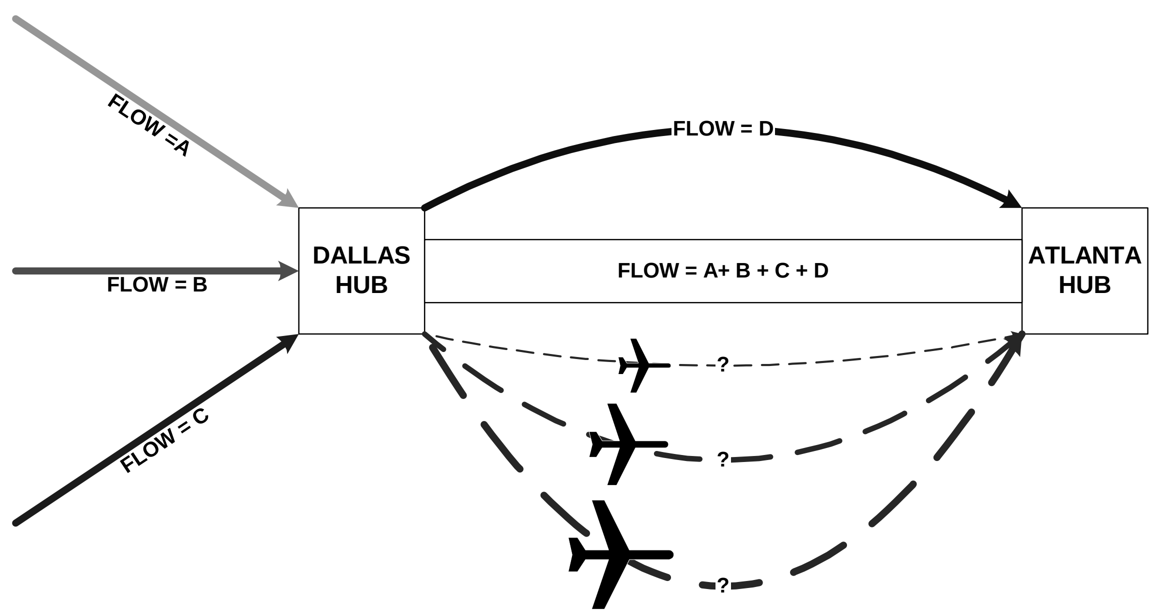

Figure 4.2 Infrastructure of the FDMAP

To exemplify the impact of the flow dependent cost function on interhub flows, a completely interconnected hub network is relaxed. In other words, some interhub links might be permitted to be closed in optimal situations. The number of hubs to open is exogenously fixed at three. Practically, the empirical data on costs and rates would be used to obtain the cost function. To investigate the flow dependency of the new model, costs are approximated by piecewise-linear cost functions. Each nonlinear concave cost function to be approximated is based on equation (4.1). Tables 4.2, 4.7 and 4.10 show the ranges of the interhub link flows for a 20 node network, a 25 node network and a 100 node network respectively. Tables 4.3, 4.8, and 4.11 show slopes and fixed costs for the piecewise-linear functions of the 20 node network, the 25 node network, and the 100 node network.

The FDMAP solves 15 different cost functions for the 20 node network, 4 for the 25 node network and 11 for the 100 node network. The results are compared with the previous hub models. Tables 4.4, 4.9, and 4.12 show the optimal hub locations and the average total network cost per unit flow for CAB20, CAB25 and CAB100 respectively. For the 20 node network, the cost functions 1 to 10 are solved with 1 representing the cost function with the smallest discounts for each piecewise of the cost function and 10 indicating the cost function as the largest discount based on RANGE1 (see Table 4.2). The cost functions 11 to 13 are evaluated to give the lower threshold of flows (RANGE2) as can be seen in Table 4.2. The cost functions 14 to 15 have the same flow range (RANGE1) with different cost functions. For the 25 node network, the cost functions 1 to 2 are solved with 1 representing the cost function with the smallest discounts for each piecewise of the cost function and 2 indicating the cost function as the largest discount based on RANGE1. The cost functions 3 to 4 are evaluated to give the lower threshold of flows (RANGE2) as can be seen in Table 4.7. For the 100 node network, the cost functions 1 to 7 (RANGE1) are solved with 1 representing the cost function with the smallest discounts for each piecewise of the cost function and 7 indicating the cost function as the largest discount. The cost functions 8 to 9 are evaluated to give the lower threshold of flows (RANGE2) as can be seen in Table 4.10. The cost functions 10 to 11 have a same flow range (RANGE1) with a different cost function.

$$\Omega = \sum_q \sum_r W_{qr} \left[1 - \theta \left(\frac{\sum_i \sum_j W_{ij} X_{ijkm}}{\sum_i \sum_i W_{ij}} \right)^{\beta} \right] X_{qrkm} \tag{4.1}$$

W_{qr} = the amount of flow traveling between nodes q and r

$$X_{qrkm} = \begin{cases} 1 \text{ if the flow between nodes } q \text{ and } r \text{ is routed via hub } k \text{ and } m \text{, respectively} \\ 0 \text{ otherwise} \end{cases}$$

θ, β = parameters; $\theta > 0, \beta > 0$

$\sum_i \sum_j W_{ij} X_{ijkm}$ = the total amount of flow traveling across the interhub link (k, m)

$\sum_i \sum_j W_{ij}$ = the total network flow

$$\Phi = \theta \left(\frac{\sum_i \sum_j W_{ij} X_{ijkm}}{\sum_i \sum_i W_{ij}} \right)^{\beta} \tag{4.2}$$

In the three hub model of CAB20, hubs are optimally located at Chicago, New York, and Dallas for all runs as can be seen in Table 4.4. In the multiple allocation hub models, the hub locations vary depending on the discount. Optimal infrastructures in the interhub links are also shown in Table 4.4. Results show that when the available discount increases, it tends to close a partial interhub link to achieve a large interhub discount in other interhub links instead of maintaining all interhub links. More interestingly, the total interhub link flows are distributed differently depending on the range set of interhub link flows. For example, both C.F.10 and C.F.11 have similar total interhub link flows but C.F.11 allocates the flows relatively evenly between two open interhub links while C.F.10 concentrates the flows on one interhub link as can be seen in Table 4.4.

Piece	Flow Range 1	Flow Range 2
1	~	~
2	250,000	125,000
3	500,000	250,000
4	750,000	375,000
5	1,000,000	500,000

Table 4.2 Flow Ranges of the Interhub Link Flows (CAB20)

As can be seen in Table 4.4, the average total network cost (obj) decrease as the available discount increases. This is a similar characteristic of the traditional multiple assignment model. The total network costs are divided into transport costs and fixed costs. As earlier mentioned, fixed costs increase as the available discount increases (see Figure 3.1 for details). There is a positive relationship between the total network cost and the total interhub flow among different cost functions. As the interhub link gets larger flow, the total network cost gets cheaper holding everything else same.

There are several differences between the FDMAP and the traditional hub model. Bryan (1997) argued that the gap between two objective function values (traditional models vs. flow dependent hub models) represents the extra cost paid by the flow dependent hub model for routing some flows via their non least-cost path. The total network cost per unit flow in the traditional hub model is always less than the per unit total network cost in the flow dependent hub model. However this argument neglects a very important network characteristic. It is not appropriate to compare the traditional hub model and the flow dependent hub model directly. There are a couple of reasons why the comparison is not proper. First, it is important to point out that the traditional hub model did not include the fixed costs for the interhub links. Therefore, comparing two models without considering the fixed cost does not make sense. Second, the flow ranges may change the total network cost. For example, the flow range set 2 in Table 4.2 provides much cheaper the total network cost than the flow range set 1. In this case, the flow dependent hub model achieves much cheaper per unit total network cost than the traditional hub model (see Table 4.4 and Table 4.5 for results). Therefore, Bryan's argument that the value of the objective function in the traditional hub model must always be less than or equal to the value in the flow dependent hub model is no longer valid.

There are also noticeable differences between the FLOWLOC and the FDMAP. Moreover, the FDMAP allocates flows to paths that are necessarily their least cost path unlike the FLOWLOC model. This is much clearer once the FDMAP finds the optimal cost allocation vectors for each O-D pair. It is important to note that optimal hub locations between the traditional multiple allocation model and the FDMAP are not always identical. One possible way to compare two models is to fix the hub locations for both models.

For CAB20 data set, Figures 4.3 and 4.4 illustrate the changes in allocation of the FDMAP under different cost functions and flow ranges. There are two reasons why Figure 4.4 has more flows across the interhub links. First, the discount on the interhub link increases in the cost function 11. Second, the flow range in the cost function 11 is lower than the cost function 10. This illustrates a characteristic of the FDMAP that allocates flows optimally in order to minimize the total network cost. One interesting network configuration in the FDMAP different from the FLOWLOC is that the FDMAP does not necessarily open all the interhub links. See also the model assumptions of the FDMAP that allows the multiple interhub links discussed in Chapter 3.

Piece		1	2	3	4	5
Cost Function 1	Fixed Cost	0	21721.78	65165.33	130330.7	500000
	Slope	0.956556	0.869669	0.782782	0.695895	0.326226
Cost Function 2	Fixed Cost	0	26066.13	78198.39	156396.8	500469.7
	Slope	0.947868	0.843603	0.739339	0.635074	0.291001
Cost Function 3	Fixed Cost	0	30410.49	91231.46	182462.9	510896.2
	Slope	0.939179	0.817537	0.695895	0.574253	0.24582
Cost Function 4	Fixed Cost	0	34754.84	104264.5	208529	500469.7
	Slope	0.93049	0.791471	0.652452	0.513432	0.221492
Cost Function 5	Fixed Cost	0	39099.2	117297.6	234595.2	516109.4
	Slope	0.921802	0.765405	0.609008	0.452611	0.171097
Cost Function 6	Fixed Cost	0	43443.55	130330.7	260661.3	521322.6
	Slope	0.913113	0.739339	0.565564	0.39179	0.131129
Cost Function 7	Fixed Cost	0	47787.91	143363.7	286727.4	516109.4
	Slope	0.904424	0.713273	0.522121	0.330969	0.101587
Cost Function 8	Fixed Cost	0	52132.26	156396.8	312793.6	500469.7
	Slope	0.895735	0.687206	0.478677	0.270148	0.082472
Cost Function 9	Fixed Cost	0	56476.62	169429.9	338859.7	496994.2
	Slope	0.887047	0.66114	0.435234	0.209327	0.051193
Cost Function 10	Fixed Cost	0	60820.97	182462.9	364925.8	510896.2
	Slope	0.878358	0.635074	0.39179	0.148506	0.002536
Cost Function 11	Fixed Cost	0	25000	75000	150000	250000
	Slope	1	0.8	0.6	0.4	0.2
Cost Function 12	Fixed Cost	0	12500	62500	137500	237500
	Slope	1	0.9	0.7	0.5	0.3
Cost Function 13	Fixed Cost	0	12500	37500	75000	125000
	Slope	0.9	0.8	0.7	0.6	0.5
Cost Function 14	Fixed Cost	0	25000	75000	150000	250000
	Slope	0.9	0.8	0.7	0.6	0.5
Cost Function 15	Fixed Cost	0	50000	150000	300000	500000
	Slope	1	0.8	0.6	0.4	0.2

*Cost functions 1-10, and 14-15 are based on Flow Range 1 and cost functions 11-13 are based on Flow Range 2.

Table 4.3 Slope and Fixed Cost for the Piecewise Linear Cost (CAB20)

CAB20	LOCATION			SLOPE			FLOW				COST		
p=3	Hub1	Hub2	Hub3	Interhub 1_2	Interhub 1_3	Interhub 2_3	Interhub 1_2	Interhub 1_3	Interhub 2_3	Total Flow	Obj	Transport Cost	Fixed Cost
C.F.1	7	4	17	1	1	2	42,846	71,504	865,690	980,040	968.33	962.89	5.44
C.F.2	7	4	17	1	1	2	42,846	71,504	865,690	980,040	966.39	959.87	6.53
C.F.3	7	4	17	1	1	3	127,308	0	1,090,594	1,217,902	963.65	940.81	22.84
C.F.4	7	4	17	1	1	3	127,308	0	1,167,180	1,294,488	960.64	934.53	26.11
C.F.5	7	4	17	1	1	3	142,406	0	1,213,390	1,355,796	957.13	927.76	29.37
C.F.6	7	4	17	1	1	3	142,406	0	1,274,344	1,416,750	953.35	920.71	32.63
C.F.7	7	4	17	1	1	3	142,406	0	1,274,344	1,416,750	949.51	913.61	35.90
C.F.8	7	4	17	1	1	4	246,870	0	1,688,108	1,934,978	945.03	866.71	78.32
C.F.9	7	4	17	1	1	5	246,870	0	2,423,966	2,670,836	936.31	811.87	124.45
C.F.10	7	4	17	1	1	5	270,180	0	2,437,302	2,707,482	924.68	796.76	127.93
C.F.11	7	4	17	5	1	5	1,157,294	0	1,786,434	2,943,728	914.06	782.81	131.25
C.F.12	7	4	17	1	1	5	94,358	0	1,688,108	1,782,466	934.59	875.12	59.47
C.F.13	7	4	17	2	1	5	277,338	0	1,282,100	1,559,438	941.19	906.46	34.73
C.F.14	7	4	17	1	1	3	127,308	0	1,090,594	1,217,902	959.46	940.68	18.78
C.F.15	7	4	17	1	1	3	71,504	0	1,259,246	1,330,750	965.32	927.76	37.56

**Node 7 is Dallas, 4 is Chicago, and 17 is New York.*

**Cost functions 1-10, and 14-15 are based on Flow Range 1 and cost functions 11-13 are based on Flow Range 2.

Table 4.4 Optimal Solutions of the FDMAP (CAB20)

CAB20	LOCATION			FLOW				COST		
p=3	Hub1	Hub2	Hub3	Interhub 1_2	Interhub 1_3	Interhub 2_3	Total Flow	Obj	Transport Cost	Fixed Cost
alpha = 0.1	17	4	12	2,130,552	404,560	506,284	3,041,396	659.661091	659.661091	~
alpha = 0.2	17	4	12	1,486,002	346,816	564,028	2,396,846	712.090342	712.090342	~
alpha = 0.3	17	4	12	1,341,292	346,816	459,432	2,147,540	759.853456	759.853456	~
alpha = 0.4	17	4	12	1,094,846	346,816	459,432	1,901,094	803.810129	803.810129	~
alpha = 0.5	17	4	12	943,680	346,816	437,586	1,728,082	845.39077	845.39077	~
alpha = 0.6	17	4	12	943,680	346,816	400,534	1,691,030	884.636317	884.636317	~
alpha = 0.7	17	4	12	810,090	295,602	272,462	1,378,154	918.928021	918.928021	~
alpha = 0.8	7	4	17	126,820	196,840	985,438	1,309,098	948.41448	948.41448	~
alpha = 0.9	7	4	17	55,804	71,504	865,690	992,998	964.809994	964.809994	~
alpha = 0.95	7	4	17	42,846	71,504	660,210	774,560	970.903212	970.903212	~

**Node 4 is Chicago, 7 is Dallas, 12 is Los Angeles, and 17 is New York.*

Table 4.5 Optimal Solutions of the Multiple Allocation Hub Model (CAB20)

CAB20	FIXED LOCATION			FLOW				COST		
p=3	Hub1	Hub2	Hub3	Interhub 1_2	Interhub 1_3	Interhub 2_3	Total Flow	Obj	Transport Cost	Fixed Cost
alpha = 0.1	7	4	17	673,106	754,266	1,674,852	3,102,224	727.620418	727.620418	~
alpha = 0.2	7	4	17	636,942	670,904	1,161,914	2,469,760	769.110606	769.110606	~
alpha = 0.3	7	4	17	636,942	670,904	1,017,204	2,325,050	806.845276	806.845276	~
alpha = 0.4	7	4	17	537,920	596,098	845,564	1,979,582	841.264865	841.264865	~
alpha = 0.5	7	4	17	537,920	596,098	764,340	1,898,358	872.833211	872.833211	~
alpha = 0.6	7	4	17	486,264	526,156	764,340	1,776,760	902.535237	902.535237	~
alpha = 0.7	7	4	17	126,820	510,206	689,836	1,326,862	927.308084	927.308084	~
alpha = 0.8	7	4	17	126,820	196,840	985,438	1,309,098	948.41448	948.41448	~
alpha = 0.9	7	4	17	55,804	71,504	865,690	992,998	964.809994	964.809994	~
alpha = 0.95	7	4	17	42,846	71,504	660,210	774,560	970.903212	970.903212	~

**Node 7 is Dallas, 4 is Chicago, and 17 is New York.*

Table 4.6 Optimal Solutions of the Multiple Allocation Hub Model with Fixed Hub Location (CAB20)

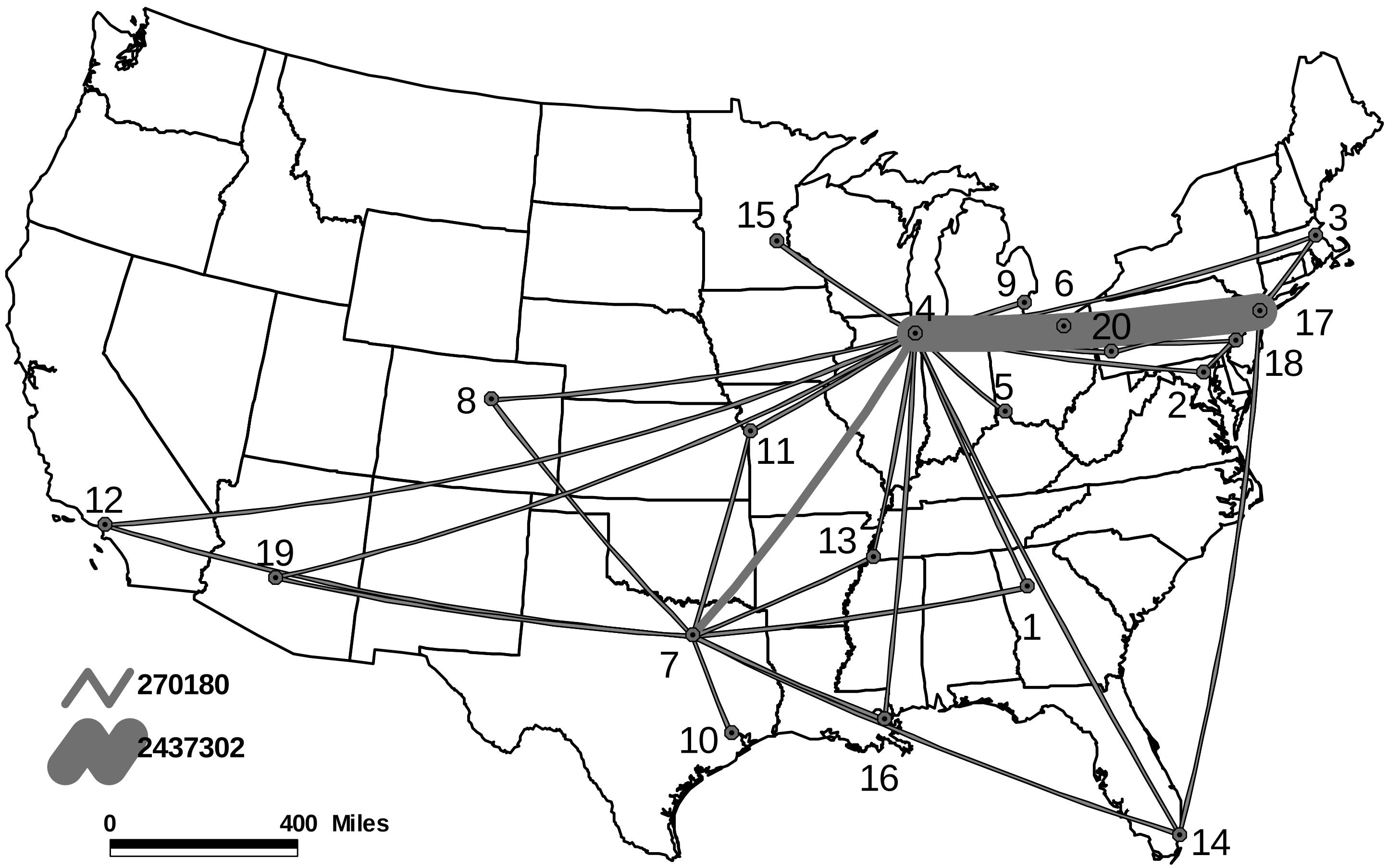

Figure 4.3 Interhub Flow with Cost Function 10 (CAB20)

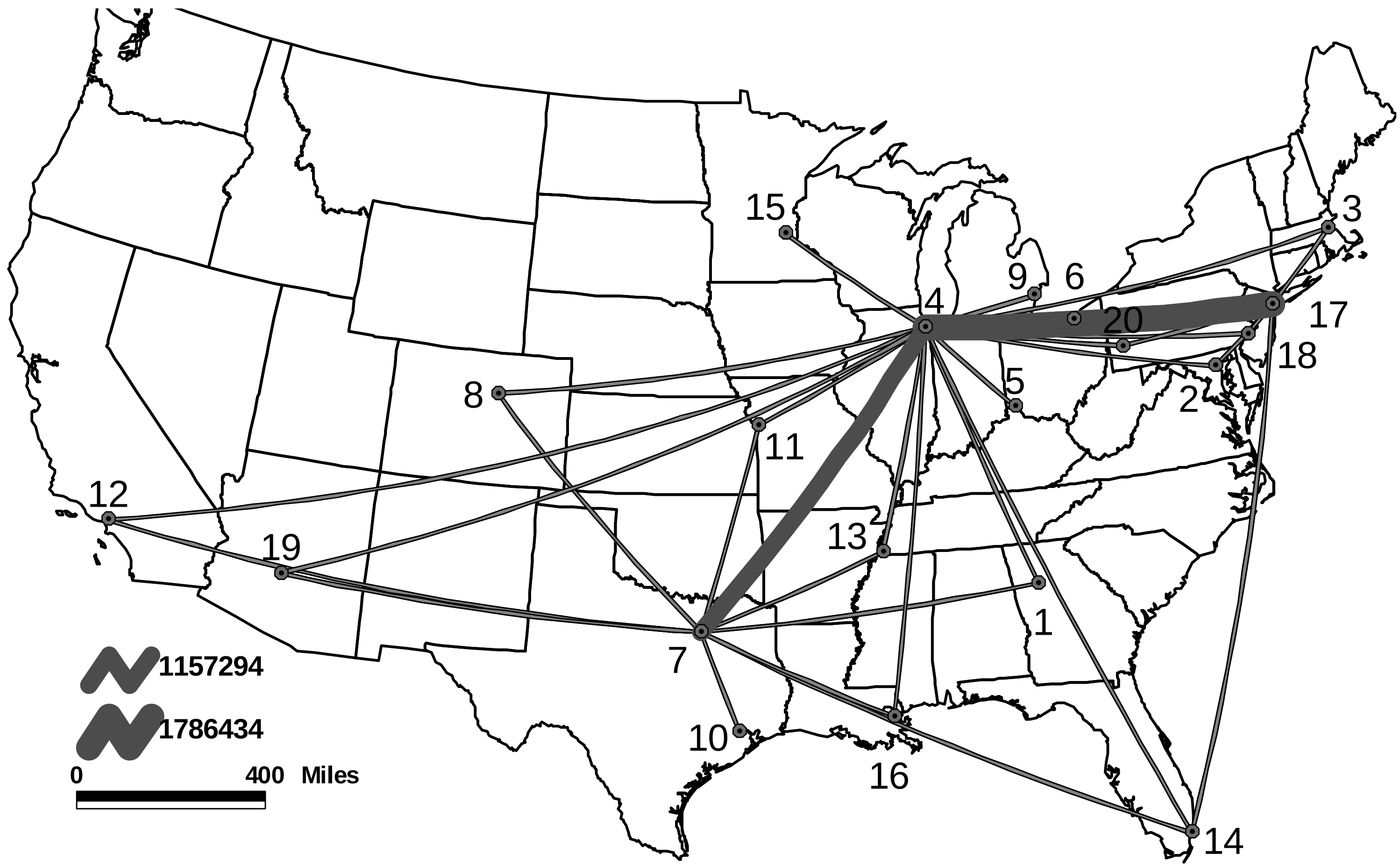

Figure 4.4 Interhub Flow with Cost Function 11 (CAB20)

In the three hub model of CAB25, hubs are optimally found among four different locations depending on the cost functions. Chicago, Los Angeles, and Philadelphia are found as hubs for the cost function 1 as can be seen in Table 4.9. Chicago, Los Angeles, and New York are selected as hubs with cost functions 2-4. Chicago and Los Angeles are strong candidates as hub locations in most cost functions due to the geographical distribution of locations whereas New York, Philadelphia, Washington D.C. are competing each other on east side. In this particular data, the FDMAP does not build a fully-connected hub network under any cost function. Figures 4.5 and 4.6 illustrate the changes in allocation of the FDMAP under different cost functions and flow ranges.

Piece	Flow Range 1	Flow Range 2
1	0	0
2	300,000	150,000
3	600,000	300,000
4	900,000	450,000
5	1,200,000	600,000

Table 4.7 Ranges of the Interhub Link Flows (CAB25)

Piece		1	2	3	4	5
Cost Function 1	Fixed Cost	0	60000	180000	360000	600000
	Slope	1	0.8	0.6	0.4	0.2
Cost Function 2	Fixed Cost	0	30000	90000	180000	300000
	Slope	0.9	0.8	0.7	0.6	0.5
Cost Function 3	Fixed Cost	0	30000	90000	180000	300000
	Slope	1	0.8	0.6	0.4	0.2
Cost Function 4	Fixed Cost	0	15000	45000	90000	150000
	Slope	0.9	0.8	0.7	0.6	0.5

* Cost functions 1-2 are based on Flow Range 1 and cost functions 3-4 are based on Flow Range 2.

Table 4.8 Slope and Fixed Cost for the Piecewise Linear Cost (CAB25)

CAB25	LOCATION			SLOPE			FLOW				COST		
p=3	Hub1	Hub2	Hub3	Interhub 1_2	Interhub 1_3	Interhub 2_3	Interhub 1_2	Interhub 1_3	Interhub 2_3	TOTAL	Obj	Transport Cost	Fixed Cost
C.F.1	18	4	12	4	1	2	1,981,060	0	758,928	2,739,988	1049.09	967.68	81.41
C.F.2	17	4	12	3	1	2	1,427,932	0	955,316	2,383,248	1036.32	1008.90	27.42
C.F.3	17	4	12	5	1	4	2,314,474	0	1,525,088	3,839,562	961.92	837.88	124.05
C.F.4	17	4	12	5	1	5	1,853,632	0	1,371,992	3,225,624	996.33	909.83	86.50

**Node 4 is Chicago, 12 is Los Angeles, 17 is New York, and 18 is Philadelphia.*

** Cost functions 1-2 are based on Flow Range 1 and cost functions 3-4 are based on Flow Range 2.

Table 4.9 Optimal Solutions of the FDMAP (CAB25)

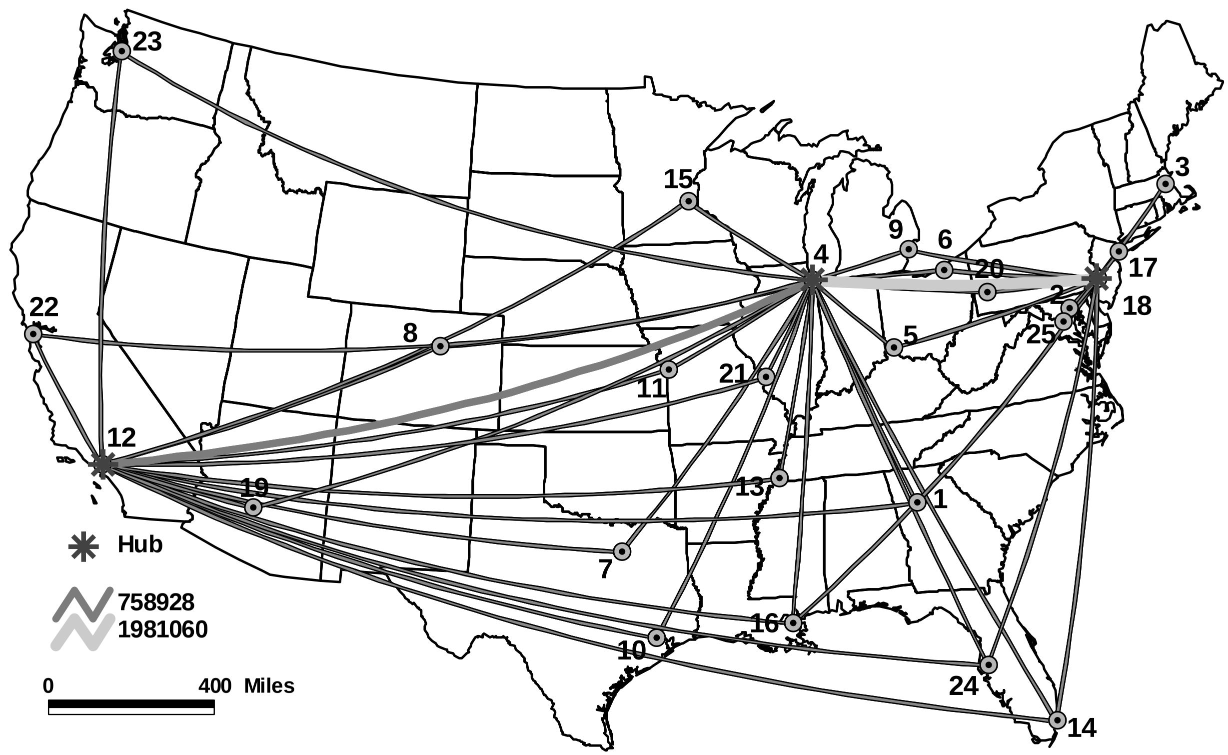

Figure 4.5 Interhub Flow with Cost Function 1 (CAB25)

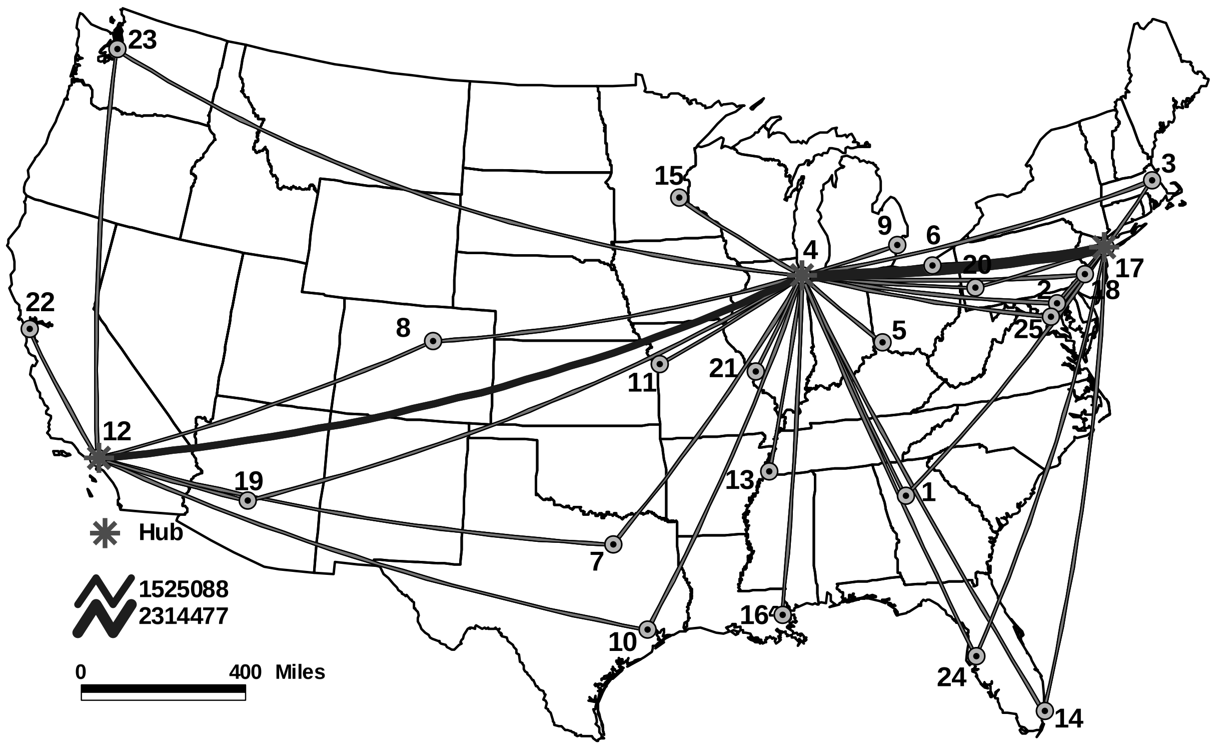

Figure 4.6 Interhub Flow with Cost Function 3 (CAB25)

In the three hub model of CAB100, hubs are optimally found among 8 different locations depending on the cost functions as can be seen in Table 4.12. Figures 4.7 and 4.8 illustrate the changes in allocation of the FDMAP under different cost functions and flow ranges. The results support that there is no relationship between the amount of flow and the length of interhub link. The longer interhub link gets higher volume of flows in Figure 4.7 whereas the shorter interhub link gets higher volume of flows in Figure 4.8.

Piece	Flow Range 1	Flow Range 2
1	~	~
2	250,000	125,000
3	500,000	250,000
4	750,000	375,000
5	2,000,000	1,000,000

Table 4.10 Ranges of the Interhub Link Flows (CAB100)

Piece		1	2	3	4	5
Cost Function 1	Fixed Cost	0	9063.59	27190.8	108763	619225
	Slope	0.981873	0.945618	0.909364	0.800601	0.54537
Cost Function 2	Fixed Cost	0	12084.8	36254.4	145018	825633
	Slope	0.97583	0.927491	0.879152	0.734135	0.393827
Cost Function 3	Fixed Cost	0	15106	45317.9	181272	985465
	Slope	0.969788	0.909364	0.84894	0.667668	0.265572
Cost Function 4	Fixed Cost	0	18127.2	54381.6	217526	1008540
	Slope	0.963746	0.891237	0.818728	0.601202	0.205696
Cost Function 5	Fixed Cost	0	21148.4	63445.1	253780	1024360
	Slope	0.957703	0.87311	0.788516	0.534736	0.149447
Cost Function 6	Fixed Cost	0	24169.6	72508.7	290035	1005450
	Slope	0.951661	0.854983	0.758304	0.468269	0.11056
Cost Function 7	Fixed Cost	0	27190.8	81572.3	326289	1000620
	Slope	0.945618	0.836855	0.728092	0.401803	0.064637
Cost Function 8	Fixed Cost	0	12500	37500	75000	175000
	Slope	0.9	0.8	0.7	0.6	0.5
Cost Function 9	Fixed Cost	0	25000	75000	150000	350000
	Slope	1	0.8	0.6	0.4	0.2
Cost Function 10	Fixed Cost	0	50000	150000	300000	700000
	Slope	1	0.8	0.6	0.4	0.2
Cost Function 11	Fixed Cost	0	25000	75000	150000	350000
	Slope	0.9	0.8	0.7	0.6	0.5

* Cost functions 1-7 and 10-11 are based on Flow Range 1 and cost functions 8-9 are based on Flow Range 2.

Table 4.11 Slope and Fixed Cost for the Piecewise Linear Cost (CAB100)

CAB100	LOCATION			SLOPE			FLOW				COST		
p=3	Hub1	Hub2	Hub3	Interhub 1_2	Interhub 1_3	Interhub 2_3	Interhub 1_2	Interhub 1_3	Interhub 2_3	Total Flow	Obj	Transport Cost	Fixed Cost
C.F.1	65	36	51	1	1	1	141,090	242,082	163,462	546,634	1013.73	1013.73	0.00
C.F.2	51	36	65	4	1	2	1,854,488	0	868,842	2,723,330	1011.41	983.56	27.85
C.F.3	43	51	81	1	1	4	0	294,610	1,953,690	2,248,300	1004.04	973.98	30.06
C.F.4	65	51	46	1	4	4	0	2,296,634	2,177,880	4,474,514	995.17	936.15	59.02
C.F.5	71	51	46	1	4	4	0	2,565,810	2,163,900	4,729,710	990.70	923.72	66.98
C.F.6	71	51	46	1	4	4	0	2,624,364	2,380,418	5,004,782	979.54	903.00	76.55
C.F.7	65	51	46	1	5	4	0	5,208,820	2,553,768	7,762,588	958.47	816.65	141.82
C.F.8	71	51	46	1	5	5	0	2,602,418	2,327,038	4,929,456	959.39	913.21	46.19
C.F.9	65	46	51	5	1	5	4,240,902	0	3,316,108	7,557,010	883.46	788.50	94.96
C.F.10	71	46	51	4	1	4	2,838,700	0	2,575,478	5,414,178	959.50	880.32	79.18
C.F.11	65	51	46	1	4	4	0	2,309,128	2,177,880	4,487,008	976.49	935.79	40.70

** Node 36 is Evansville, 43 is Harrisburg, 46 is Indianapolis, 51 is Las Vegas, 65 is New York, 71 is Philadelphia, 81 is St. Louis.*

** Cost functions 1-7 and 10-11 are based on Flow Range 1 and cost functions 8-9 are based on Flow Range 2.

Table 4.12 Optimal Solutions of the FDMAP (CAB100)

CAB100	LOCATION			FLOW				COST		
p=3	Hub1	Hub2	Hub3	Interhub 1_2	Interhub 1_3	Interhub 2_3	Total Flow	Obj	Transport Cost	Fixed Cost
alpha = 0.1	65	36	54	4,613,042	1,144,008	1,940,114	7,697,164	717.66	717.66	~
alpha = 0.2	65	36	51	3,016,982	1,189,500	2,254,634	6,461,116	777.13	777.13	~
alpha = 0.3	65	36	51	2,835,694	1,072,384	1,714,584	5,622,662	823.54	823.54	~
alpha = 0.4	65	55	51	2,358,216	1,030,886	1,586,438	4,975,540	870.33	870.33	~
alpha = 0.5	4	46	51	2,410,748	999,510	1,387,346	4,797,604	901.90	901.90	~
alpha = 0.6	4	46	51	1,925,546	831,392	1,381,092	4,138,030	934.47	934.47	~
alpha = 0.7	43	46	51	2,065,420	855,168	1,049,080	3,969,668	967.44	967.44	~
alpha = 0.8	43	46	51	1,483,992	665,160	877,058	3,026,210	991.74	991.74	~

**Node 4 is Allentown, 36 is Evansville, 43 is Harrisburg, 46 is Indianapolis, 51 is Las Vegas, 54 is Los Angeles, 55 is Louisville, and 65 is New York.*

Table 4.13 Optimal Solutions of the Multiple Allocation Hub Model (CAB100)

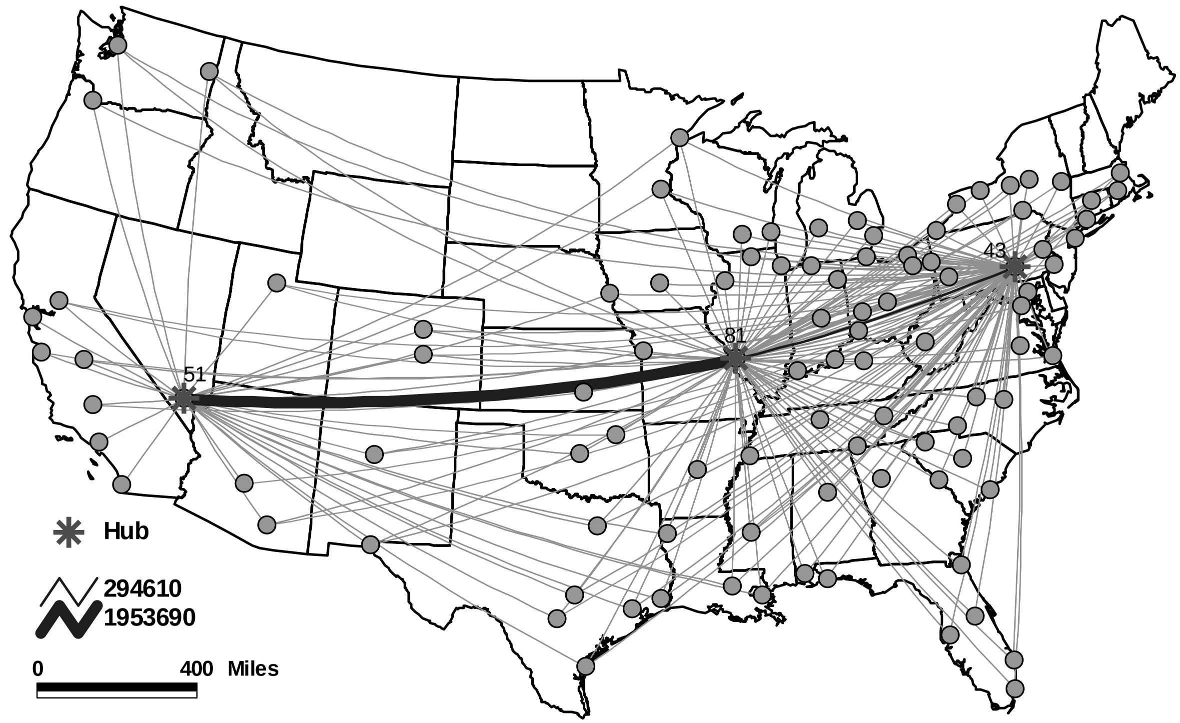

Figure 4.7 Interhub Flow with Cost Function 3 (CAB100)

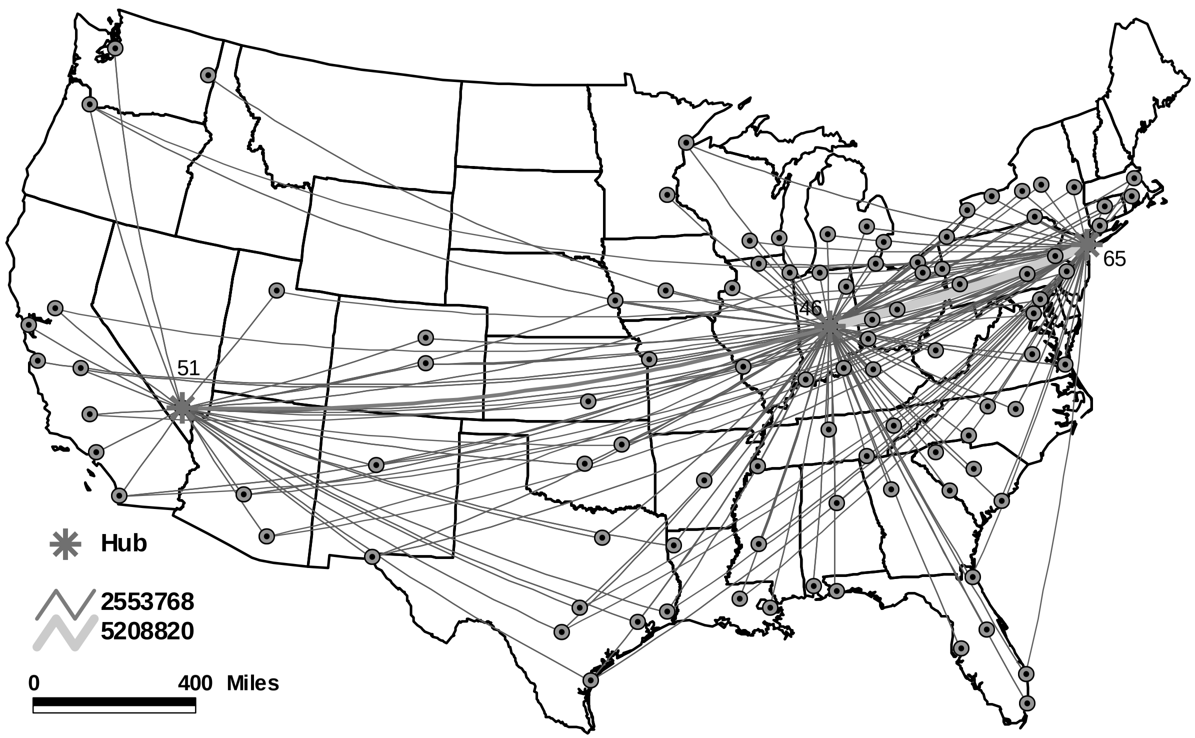

Figure 4.8 Interhub Flow with Cost Function 7 (CAB100)

4.3 Summary of Results

Computational results clearly support the notion that the model can design optimal hub locations and infrastructure. The algorithm solves problems of practical size within acceptable computation times, so it provides a flexible means to model infrastructure in hub networks. Identifying optimal flows is one of the key tasks in building hub network models. This research shows that the study of the relationship between optimal flows and infrastructure is critical to reach an optimal network. It is also necessary to understand the linkage between cost functions and optimal flows. This study investigates the role of flows in building interhub links as a means of routing paths.

This chapter presents empirical results of the FDMAP that incorporates economies of scale utilizing the piecewise-linear cost function. The model focuses on the interhub link installation along with hub locations and allocations of non-hub nodes. The numerical results of CAB data show that the FDMAP is different from the FLOWLOC and the traditional multiple allocation hub model. In terms of modeling aspect, first, the FDMAP allows the multiple interhub links for the O-D pair path. Second, the results of FDMAP show that a completely connected hub network is not necessary to an optimal hub network. In other words, the model does not open fully-connected interhub links to save the total network cost by achieving the flow economies of scale. With respect to speed of computation, the tabu search algorithm solves a relatively large 100 node hub network in reasonable time. The model also points out that the comparison of the total network cost between the flow dependent hub model and the traditional multiple allocation model is not appropriate. On one hand, the fixed cost has not been considered in the traditional multiple allocation model. On the other hand, the range of interhub link flow significantly affects the total network cost.

There are several expected properties of the tabu FDMAP heuristic over previous multiple allocation hub models. First, results of the tabu FDMAP heuristic show similar network structures to the results of the single allocation model. In other words, the usage of interhub links is intensified comparing to the traditional multiple allocation hub problem. The results show that most of the interhub links have significant flows to achieve the discount by bundling flows. Moreover, flows on the interhub links are more reasonable because the discounts are based on the pre-specified threshold. Second, the tabu FDMAP heuristic[14] can be extended to larger networks with a reasonable computation time while the previous FLOWLOC model, which used the LP solution, is limited to small networks with an expensive computation time.

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[14] The tabu FDMAP heuristic is coded in C++.

CHAPTER 5

NUMERICAL RESULTS OF THE FLOW-BASED DISCOUNT POINT-TO-POINT MODEL (FDPTP)

This chapter presents numerical results for the flow-based discount point-to-point model in which every pair of nodes can communicate directly. The flow economies of scale idea in the FDMAP is adopted in this model to compare point-to-point networks with hub networks. A fully-connected pure point-to-point network is infeasible because there is not enough flow to support such a network. The point-to-point network based on the flow-dependent cost function is a reasonable approach to solve the problem. There is an incentive to combine flows from different origins and destinations when the flow through a link surpasses a threshold with various discount factors. Computational results of CAB data are presented to illustrate this idea.

In a hub-and-spoke network, a smaller number of links are chosen to serve a large number of O-D pairs while a large number of links are necessary in a point-to-point network to transport flows. The ideas behind this new model are as follows: [1] the non-interhub links can achieve flow economies of scale, [2] air travelers prefer direct flights and more frequency, and [3] competition increases among direct service providers and hub service providers.

5.1 Numerical Results of the FDPTP

10-node and 15-node versions of CAB data set are performed with different predefined concave cost functions which provide the flow economies of scale. CPLEX, a mixed integer problem solver, is used to solve those problems. Both the 10-node and the 15-node network are solved optimally, but not all instances of the 20-node and the 25-node networks are solved optimally due to increase in memory consumption and running time. For the large network, a number of fractional decision variables increase the size of branch and bound tree significantly.

The behavior of the model depends also on concave cost functions that determine input parameters of discounts and fixed costs along with flow ranges. The numerical results show that discount values and flow ranges are directly related with the complexity of the problem in addition to the problem size. For example, as a discount value gets lower (more discount), the possibility of bundling flows gets bigger. This makes the model complex to solve in addition to the problem size. In the similar context, the higher flow range level makes the model search more O-D pairs to bundle flows than the lower one. Decreasing the discount value alpha (more discount) creates a situation where there is a larger incentive to cooperate. So more re-routing needs to be done at the expense of the level of difficulty. The increase in the flow range level reduces the number of discounted links. The ratio between discounted and non-discounted links illustrates the attractiveness of discounting for a specific cost function.

Figure 5.1 shows the results of the FDPTP (CAB10) with cost function 3 of Table 5.3. In this particular problem, Cleveland (6) and Dallas (7) have the most interacting flows with other nodes. Out of 90 possible directed links, the model opens 20 discounted links which are four Y^5, four Y^4, four Y^3, and eight Y^2 discounted links. As can be seen in Table 5.2 and 5.7, the flows on discount links are corresponding to the flow range each other. Among most discounted links, the discount link between Cleveland (6) and Chicago (4) [6→4] has the largest flows. Notice that the model does not necessarily open the link between Chicago (4) and Cleveland (6) [4→6]. On the other hand, the model opens 4 pairs of two-way links. More interestingly, different discounts are found on those two-way links. For example, the link between Dallas (7) and Houston (10) [7→10] is discounted by Y^2 variable whereas the link between Houston (10) and Dallas (7) [10→7] is discounted by Y^4 variable.

Figure 5.2 shows the results of the FDPTP (CAB10) with cost function 3 without hop constraint (3.20). The model without hop constraint makes the network denser than one with hop constraint. Out of 90 possible directed links, the model opens 15 discounted links which are ten Y^5, four Y^3, and single Y^2 discounted links. This illustrates that most of opened links achieve the large discounts. As can be seen in Table 5.2 and 5.8, the flows on discount links are corresponding to the flow range each other. Table 5.1 shows the total network cost with different discount levels between Figure 5.1 and Figure 5.2. As can be seen in Table 5.1, the model without hop constraint achieves the higher level of the flow economies of scale than the one with hop constraint.

The reason for the dense network result without hop constraint is that the model with hop constraint needs to re-route flows of the geographically separated O-D pairs. For example, the model needs to build extra links to carry the flows from Dallas (7) to

Boston (3) [7→3] which are geographically far from each other. To make this feasible, the model builds the new link from Dallas (7) to Cleveland (6) [7→6] which was not feasible in the previous result (Figure 5.1). On the other hand, the model also removes some links of the previous result to achieve the flow economies of scale. For instance, the links from Boston (3) to Baltimore (2) [3→2], and from Atlanta (1) to Dallas (7) [1→7] are not opened any more. The level of discounts for the links is also changed. For example, the discount level of the link from Baltimore (2) to Cleveland (6) [2→6] changes from Y^2 to Y^5 variable without hop constraint.

Figure 5.3 shows the results of the FDPTP (CAB10) with cost function 1 without hop constraint (3.20). The model with a higher flow range makes the network denser than the one with a lower flow range. The model with a higher flow range cannot achieve the flow economies of scale as much as the one with a lower flow range. The model opens relatively a large number of lower discount links. For example, the proportion of Y^2 is much higher than the one of Y^5. Out of 90 possible directed links, the model opens 16 discounted links which are four Y^5, one Y^3, and eleven Y^2 discounted links. As can be seen in Table 5.2 and 5.9, the flows on discount links are corresponding to the flow range each other.

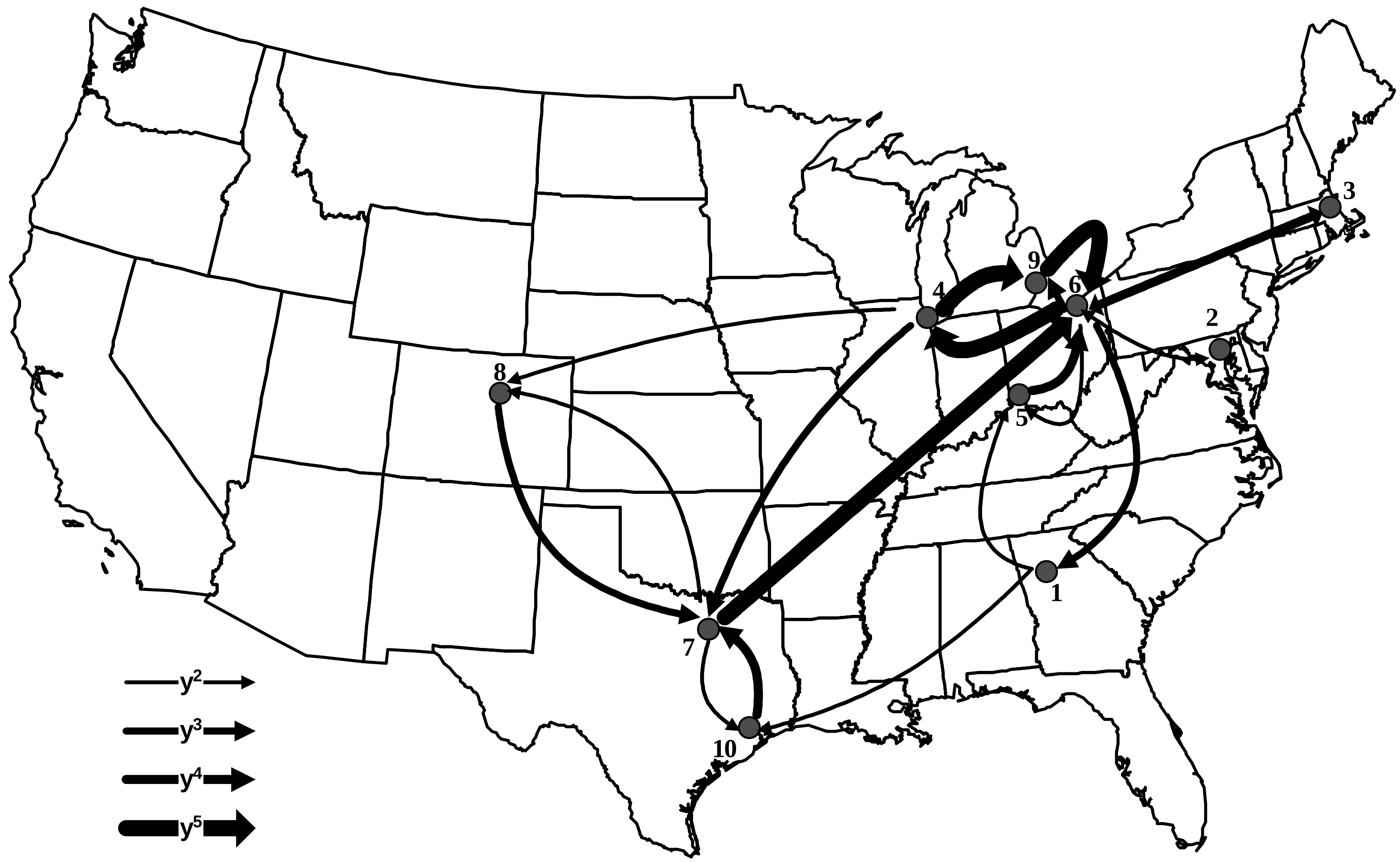

Figure 5.1 Result (CAB10) of the FDPTP with Cost Function 3 under Flow Range 2

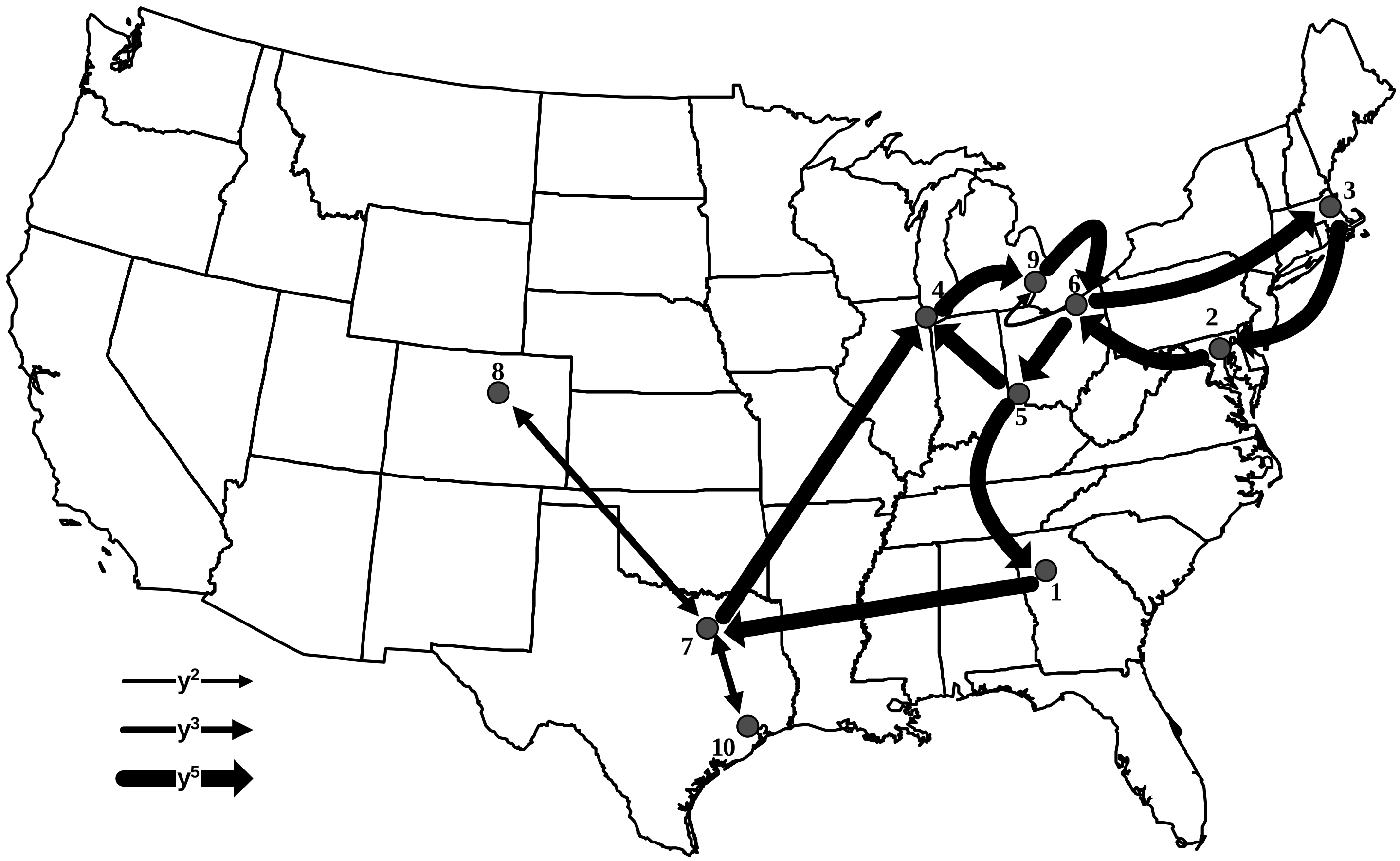

Figure 5.2 Result (CAB10) of the FDPTP without Hop Constraint with Cost Function 3 under Flow Range 2

CAB10	X^5		X^4		X^3		X^2		X^1	
	FDPTP	FDPTP-(3.20)	FDPTP	FDPTP-(3.20)	FDPTP	FDPTP-(3.20)	FDPTP	FDPTP-(3.20)	FDPTP	FDPTP-(3.20)
C.F.3	166.08	403.69	130.67	~	139.10	111.60	157.61	3.25	~	~

*Equation 3.20 is the hop constraint.
**X^1-X^5 is the types of discount flow where X^5 is the largest discount flow and X^1 is no discount flow.

Table 5.1 FDPTP (CAB10) with Cost Function 3 under Flow Range 2

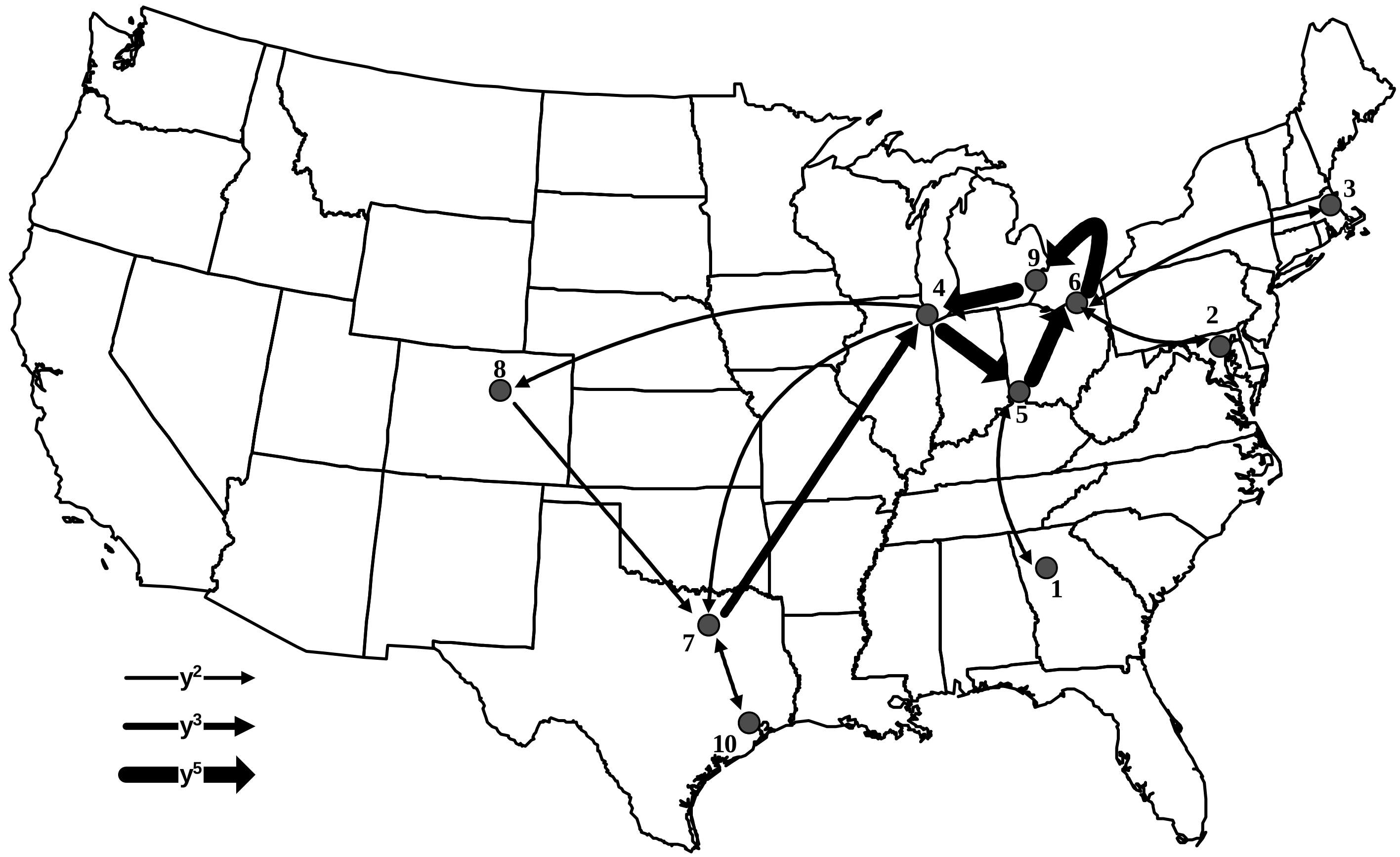

Figure 5.3 Result (CAB10) of the FDPTP without Hop Constraint with Cost Function 1 under Flow Range 1

Piece	Flow Range 1	Flow Range 2
1	0	0
2	62,500	31,250
3	125,000	62,500
4	187,500	93,750
5	250,000	125,000

Table 5.2 Ranges of Link Flows (CAB10)

Piece		1	2	3	4	5
Cost Function 1	Fixed Cost	0	12,500	37,500	75,000	125,000
	Slope	1	0.8	0.6	0.4	0.2
Cost Function 2	Fixed Cost	0	6,250	18,750	37,500	62,500
	Slope	0.9	0.8	0.7	0.6	0.5
Cost Function 3	Fixed Cost	0	7,500	20,000	38,750	63,750
	Slope	1	0.8	0.6	0.4	0.2

*Cost functions 1-2 are based on flow range 1, and cost function 3 is based on flow range 2.

Table 5.3 Slope and Fixed Cost for the Piecewise-Linear Cost Function (CAB10)

Comparing Table 5.4 and Table 5.5 shows how efficient the FDPTP model is with respect to the total network cost. Given the same input parameters of discounts and fixed costs with flow range, the FDPTP provides the cheaper total network cost than the FDMAP in cost function 3 whereas the FDMAP finds the cheaper total network cost in cost functions 1 and 2. Even the FDPTP with hop constraint in cost function 3 (Table 5.6) which behaves like hub network due to the number of stops achieves the cheaper total network cost than the FDMAP. More interestingly, the decomposition of total network costs into transport costs and fixed costs provides the clear idea of network structure.

Unlike the other results, the proportion of fixed costs in the FDPTP with cost function 3 as can be seen in Table 5.4 is larger than the transport cost. Although the FDPTP obtains the cheaper transport costs than the FDMAP, fixed costs are very high due to the large number of links to be opened. The number of links built by the FDPTP cost relatively higher fixed costs than the FDMAP solution. Therefore, the optimal network configuration between hub network and point-to-point network depends on the input parameters.

CAB10	Total Network Cost	Transport Cost	Fixed Cost
C.F.1	682.87	485.40	197.48
C.F.2	643.40	585.39	58.01
C.F.3	531.69	245.15	286.53

*Cost functions 1-2 are based on flow range 1, and cost function 3 is based on flow range 2.

Table 5.4 Cost of the FDPTP Solution without Hop Constraint (CAB10)

CAB10	Total Network Cost	Transport Cost	Fixed Cost
C.F.1	651.25	643.46	7.79
C.F.2	641.59	637.70	3.90
C.F.3	624.08	552.71	71.37

*Cost functions 1-2 are based on flow range 1, and cost function 3 is based on flow range 2.

Table 5.5 Cost of the FDMAP Solution (CAB10)

CAB10	Total Network Cost	Transport Cost	Fixed Cost
C.F.3	588.91	352.91	236.01

*Cost function 3 is based on flow range 2.

Table 5.6 Cost of the FDPTP Solution with Hop Constraint (CAB10)

K	M	Flow	Discount
6	4	286643	5
9	6	240692	5
4	9	174417	5
7	6	148210	5
6	3	108379	4
3	6	108379	4
5	6	103292	4
10	7	96709	4
6	1	89859	3
4	7	75580	3
8	7	67896	3
6	9	66275	3
7	10	60725	2
6	2	60035	2
2	6	60035	2
1	5	53875	2
6	5	49417	2
4	8	36646	2
1	10	35984	2
7	8	31250	2

Table 5.7 Discount Levels in the FDPTP (CAB10) with Cost Function 3 under Flow Range 2

K	M	Flow	Discount
9	6	413904	5
6	5	380208	5
4	9	380208	5
5	1	202085	5
7	4	202085	5
1	7	202085	5
5	4	178123	5
3	2	155415	5
6	3	155415	5
2	6	155415	5
10	7	82778	3
7	10	82778	3
8	7	67896	3
7	8	67896	3
6	9	33696	2

Table 5.8 Discount Levels in the FDPTP (CAB10) without Hop Constraint with Cost Function 3 under Flow Range 2

K	M	Flow	Discount
4	5	344,623.00	5
5	6	344,623.00	5
6	9	344,623.00	5
9	4	344,623.00	5
7	4	166,862.00	3
3	6	108,379.00	2
6	3	108,379.00	2
4	7	98,966.00	2
7	10	82,778.00	2
10	7	82,778.00	2
1	5	75,054.00	2
5	1	75,054.00	2
4	8	67,896.00	2
8	7	67,896.00	2
2	6	62,500.00	2
6	2	62,500.00	2

Table 5.9 Discount Levels in the FDPTP (CAB10) without Hop Constraint with Cost Function 1 under Flow Range 1

Figure 5.4 shows the results of the FDPTP (CAB15) with cost function 1 in Table 5.10. In this particular problem, Chicago (4), Cincinnati (5) and Kansas City (11) have the most interacting flows with other nodes. Out of 210 possible directed links, the model opens 26 discounted links which are twelve Y^5, five Y^4, four Y^3, and five Y^2 discounted links. As can be seen in Table 5.10 and 5.12, the flows on discount links are corresponding to the flow range each other. Among most discounted links, the discount link between Chicago (4) and Cincinnati (5) [4→5] has the largest flows. There are 9 pairs of two-way links. Eight pairs of those two-way links have the same level of discount on both ways. Only one pair has a different level of discount. For example, the link between Baltimore (2) and Cleveland (6) [2→6] is discounted by Y^4 variable whereas the link between Cleveland (6) and Baltimore (2) [6→2] is discounted by Y^2 variable. Not only the level of discount but also the amount of flow on two-way links are different. As can be seen in Table 5.12, the flows between Chicago (4) and Kansas City (11) [4→11] and Kansas City (11) to Chicago (4) and [11→4] are different [see also model property in section 5.2 for more details].

Piece	Flow Range 1	Flow Range 2
1	0	0
2	125,000	62,500
3	250,000	125,000
4	375,000	187,500
5	500,000	250,000

Table 5.10 Ranges of Link Flows (CAB15)

Piece		1	2	3	4	5
Cost Function 1	Fixed Cost	0	25,000	75,000	15,0000	250,000
	Slope	1	0.8	0.6	0.4	0.2
Cost Function 2	Fixed Cost	0	12,500	62,500	137,500	237,500
	Slope	1	0.9	0.7	0.5	0.3
Cost Function 3	Fixed Cost	0	12,500	37,500	75,000	125,000
	Slope	0.9	0.8	0.7	0.6	0.5
Cost Function 4	Fixed Cost	0	12,500	37,500	75,000	125,000
	Slope	1	0.8	0.6	0.4	0.2
Cost Function 5	Fixed Cost	0	6,250	31,250	68,750	118,750
	Slope	1	0.9	0.7	0.5	0.3
Cost Function 6	Fixed Cost	0	6,250	18,750	37,500	62,500
	Slope	0.9	0.8	0.7	0.6	0.5

*Cost functions 1-3 are based on flow range 1, and cost functions 4-6 are based on flow range 2.

Table 5.11 Slope and Fixed Cost for the Piecewise Linear Cost Function (CAB15)

K	M	Flow	Discount
4	5	704,756	5
5	6	642,256	5
6	9	642,256	5
9	4	642,256	5
11	4	485,092	5
4	11	422,592	5
8	11	338,512	5
11	8	338,512	5
5	1	318,805	5
8	12	276,108	5
12	8	276,108	5
1	5	256,305	5
2	6	233,277	4
7	11	211,591	4
11	7	211,591	4
1	14	193,323	4
14	1	193,323	4
3	2	166,021	3
6	3	166,021	3
4	15	128,003	3
15	4	128,003	3
7	10	119,699	2
10	7	119,699	2
6	2	67,256	2
1	13	62,500	2
13	11	62,500	2

Table 5.12 Discount Levels in the FDPTP (CAB15) without Hop Constraint with Cost Function 1 under Flow Range 1

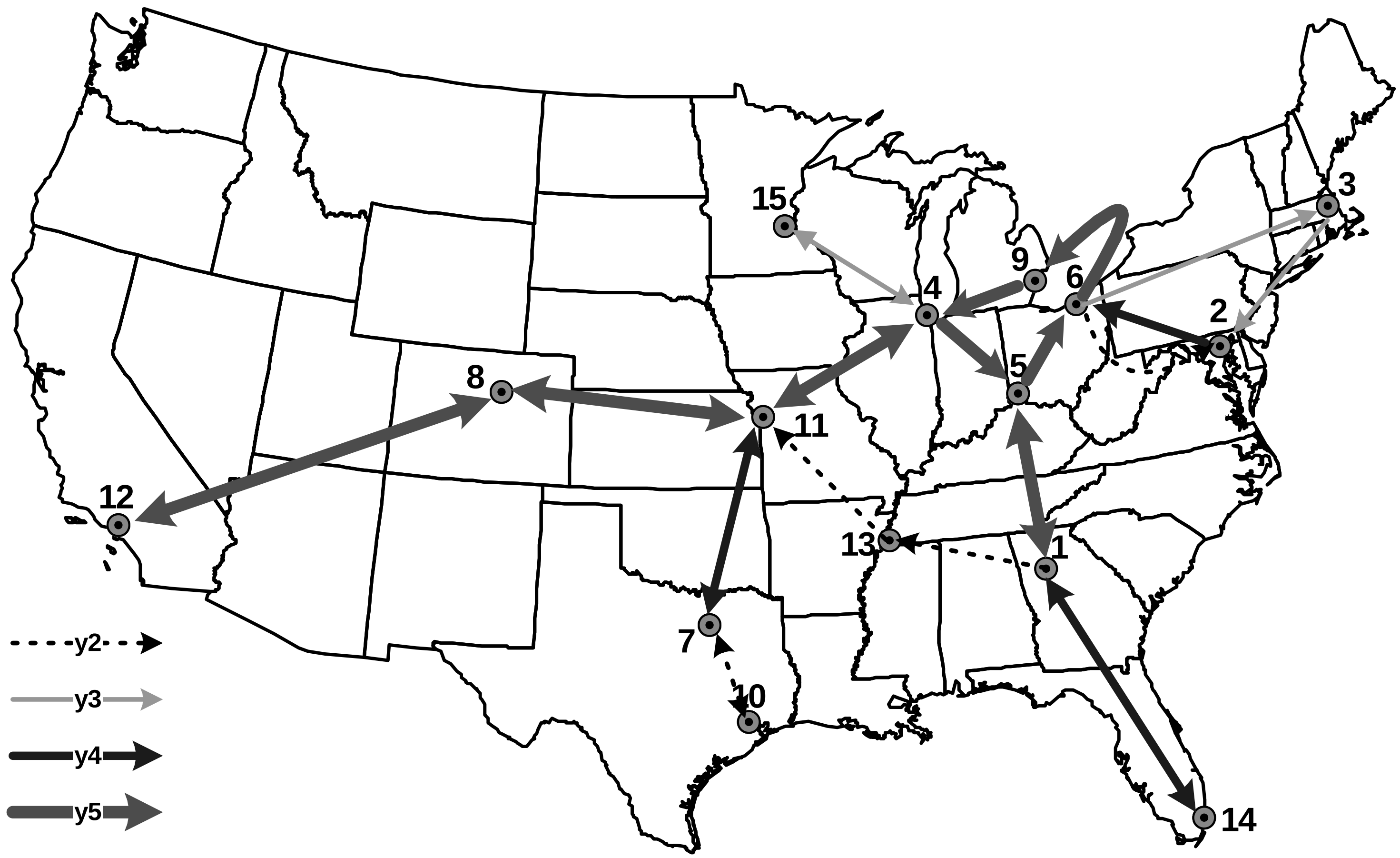

Figure 5.4 Result (CAB15) of the FDPTP without Hop Constraint with Cost Function 1 under Flow Range 1

5.2 Properties of the FDPTP

There are several model properties from the results of the FDPTP model. First, individual routings are not symmetric unlike the traditional hub model as can be seen in Figure 5.5. The model strategically allocates the flows based on the flow range (threshold level) to achieve the optimal network. This leads to produce the paths (*i*, *j*) and (*j*, *i*) differently. For example, the *i*-*k*-*j* is the indirect path for (*i*, *j*) while the *j*-*i* is the direct path for (*j*, *i*).

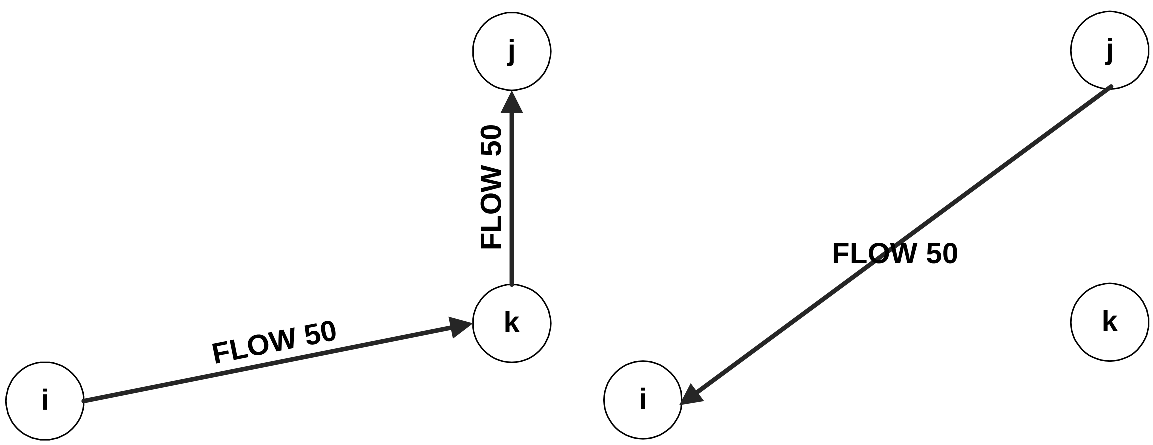

Figure 5.5 Asymmetric Individual Routing

Second, the FDPTP model splits the flow of O-D pair selectively based on the flow range (threshold level) as can be seen in Figure 5.6. For example, the flow of O-D pair (*i*, *j*) is split two ways. The flow of 50 is kept in path (*i*, *k*) and it is split into two arcs *k*-*j* and *k*-*m*-*j*.

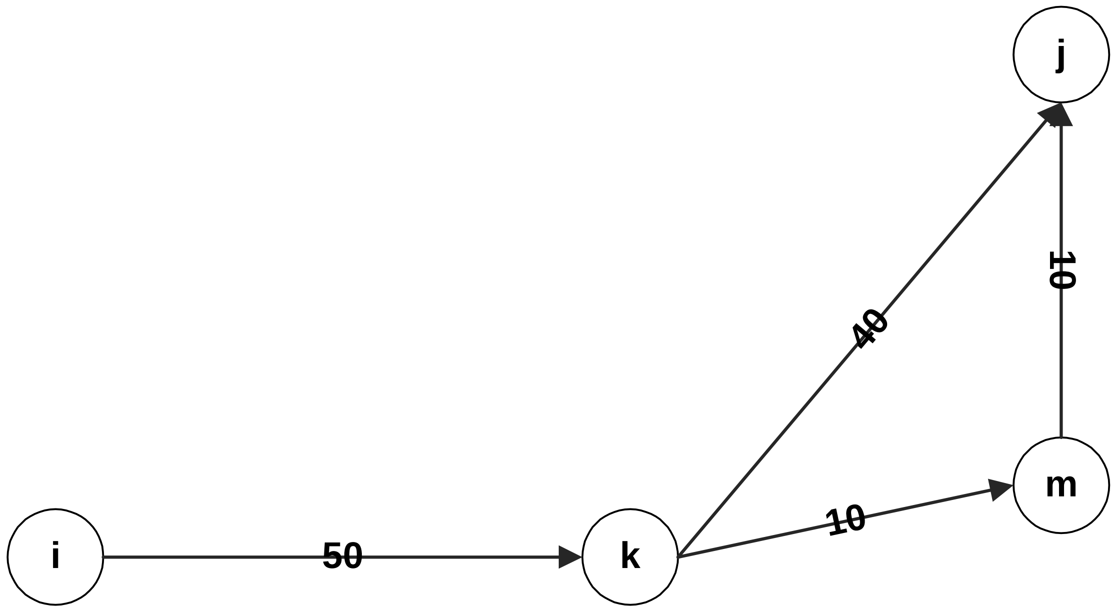

Figure 5.6 Splitting Flows of O-D pair

Third, there is a mixture of discount and non-discount link for the O-D pair as can be seen in Figure 5.7. This model property is closely related to second model property. Based on the flow range, the model amalgamates the flows on a specific arc while once a specific arc reached the flow range, the model deviates the surplus flow into other arcs.

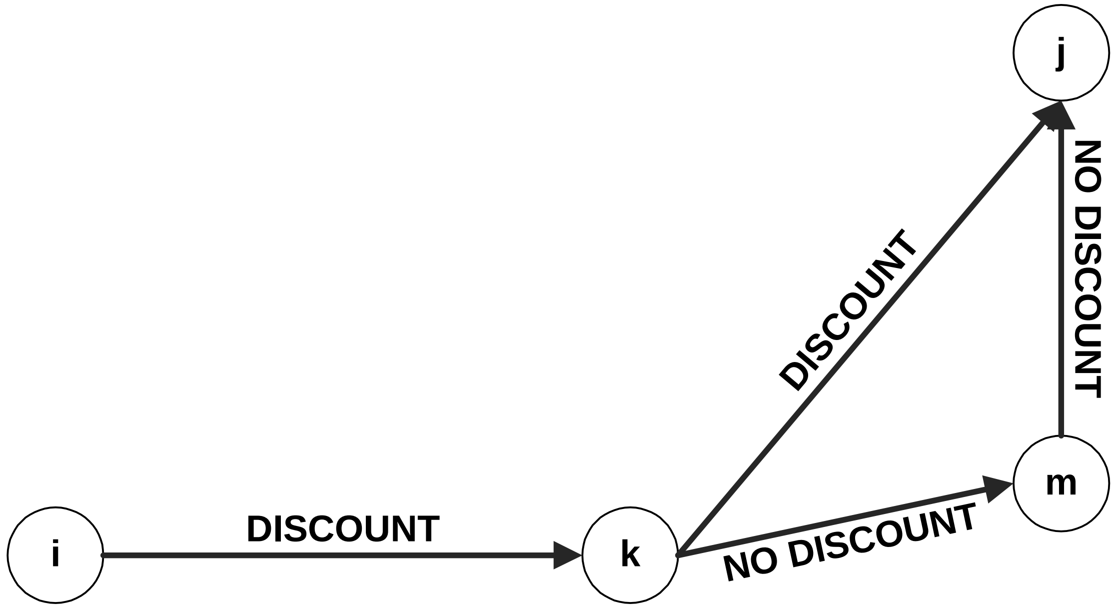

Figure 5.7 Mixture of Discount and Non-discount Link

Numerical results also show other interesting characteristics of the FDPTP model. Results show that discounting is not guaranteed even if flow of O-D pair is greater than the flow range. It is not necessarily guaranteed due to the model properties above, especially the property of splitting the flows. In this case, the fairness issue is involved which will be discussed in cost allocation chapter. In the FDPTP model, the distance between the origin and the destination does not affect the routing strategy at all. In other words, the short distance of O-D pair does not necessarily favors a direct connection.

5.3 Summary of Results

The numerical results show the effectiveness of flow-based discount point-to-point network model. For smaller networks, it is possible to obtain optimal solutions in a reasonable time. However, heuristic solution techniques for larger problems are necessary because the model formulation is too memory intensive for the largest network evaluated.

The behavior of the model depends also on concave cost functions that determine input parameters of discounts and fixed costs along with flow ranges. The numerical results show that discount values and flow ranges are directly related to the complexity of the problem, in addition to the problem size. The FDPTP model, with a higher flow range, cannot achieve flow economies of scale as much as the one with a lower flow range. It also displays a less dense network structure than the lower one. Given discounting incentives based on the amount of flow on network links, O-D pairs are motivated to amalgamate their flows. It turns out that a high level of flow range makes the model difficult to solve because the model needs to bundle flows in many different possible ways. This property is more sensitive to the model with hop constraint. The result of network structure is also affected by the hop constraint. The reason for the less dense network result without hop constraint is that the model with hop constraint needs to re-route flows of the geographically separated O-D pairs.

CHAPTER 6

NUMERICAL RESULTS OF COST ALLOCATION

This chapter presents the results of the cost allocations in both hub network games and point-to-point network games. The cost allocation problems associated with the hub network design and the point-to-point network design are addressed. The hub network is efficient with a small number of links under the flow economies of scale. This network efficiency utilizing the flow economies of scale is accomplished by the cooperation of users (O-D pairs). Thus, it is essential to allocate the cost of delivering services through interhub link infrastructure among its users in a fair way. Fair cost allocation not only provides the efficient hub network but also compares the hub network with the point-to-point network. In this manner, no O-D pair should want to secede if they receive a fair cost from the hub network. In other words, users (O-D pairs) of the hub network with unfair cost allocation may seek point-to-point network services. Such a network with unfair cost allocations results in a higher total network cost due to failure in acheiving flow economies of scale. The total network cost, $c(P)$, in hub networks obtained by the FDMAP is separable into fixed costs of infrastructure and transportation costs. In the cost allocation model, the objective is to allocate the fixed costs of infrastructure to interhub link users in the hub network. The results of cost allocations between users in the hub network and users in the point-to-point network users are also compared to show the cost difference of the links. Figure 6.1 illustrates how infrastructure costs in the hub networks that involve the users. For example, the Dallas hub collects flow (users) from other spoke nodes and sends those flows, including its own, into the Atlanta hub in order to achieve flow economies of scale along the interhub link. This involves cost allocation issues about how much each flow contributes to the aggregated flows. The cost allocations associated with the FDPTP model are also tackled with empirical data. The FDPTP game is characterized to provide a fair cost allocation among users of discounted links based on their contribution to flow economies of scale.

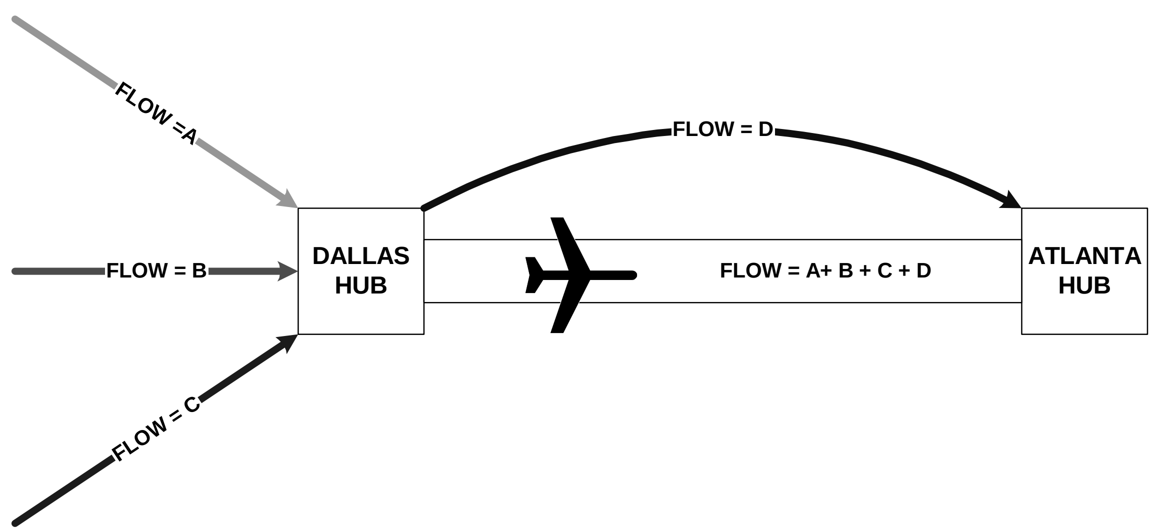

Figure 6.1 Cost Allocation of Infrastructure (Interhub Link)

6.1 Numerical Results of Cost Allocation in the FDMAP

The numerical analysis of cost allocation in the FDMAP is based on CAB15 data. Total network flow for CAB15 data is 2,364,942. Table 6.1 shows flow ranges for each piece-wise linear cost function. Concave cost functions 1-3 of Table 6.2 are based on the flow range 1 in Table 6.1, while concave cost functions 4-6 of Table 6.2 are based on the flow range 2 in Table 6.1. The FLOW RANGE2 (third column of Table 6.1) gives more discounts than the FLOW RANGE1 (second column of Table 6.1) for the same amount of flow. For instance, to achieve the second piece of the concave cost function in Table 6.1, there is a flow difference to reach depending on flow ranges. The model allows 125,000 units of flow to get the second piece of the concave cost function under Flow Range1 while 62,500 units of flow are only necessary under Flow Range2. Cost allocation solutions of the FDMAP (CAB15) are shown in Table 6.3. Out of 6 different simulations tested, 4 are presented to illustrate the empirical results.

Piece	Flow Range 1	Flow Range 2
1	0	0
2	125,000	62,500
3	250,000	125,000
4	375,000	187,500
5	500,000	250,000

Table 6.1 Ranges of the Interhub Link Flows (CAB15)

Piece		1	2	3	4	5
Cost Function 1	Fixed Cost	0	25000	75000	150000	250000
	Slope	1	0.8	0.6	0.4	0.2
Cost Function 2	Fixed Cost	0	12500	62500	137500	237500
	Slope	1	0.9	0.7	0.5	0.3
Cost Function 3	Fixed Cost	0	12500	37500	75000	125000
	Slope	0.9	0.8	0.7	0.6	0.5
Cost Function 4	Fixed Cost	0	12500	37500	75000	125000
	Slope	1	0.8	0.6	0.4	0.2
Cost Function 5	Fixed Cost	0	6250	31250	68750	118750
	Slope	1	0.9	0.7	0.5	0.3
Cost Function 6	Fixed Cost	0	6250	18750	37500	62500
	Slope	0.9	0.8	0.7	0.6	0.5

* Cost functions 1-3 are based on Flow Range 1 and cost functions 4-6 are based on Flow Range 2.

Table 6.2 Slope and Fixed Cost for the Piecewise Linear Cost (CAB15)

Simulation Set 1: Cost Function 1 under Flow Range 1

Table 6.3 shows the cost allocations for the interhub link users with the cost function 1 under the flow range 1. As can be seen in Table 6.4, Chicago (4), Dallas (7), and Los Angeles (12) are selected as hubs to open. However, the FDMAP only opens 2 directed interhub links (1 undirected interhub link) out of 6 directed ones due to the high flow volume requirement. The model does not agglomerate enough flows to reach the high flow range for the cheaper discount, and the concave cost function is too expensive to motivate players to cooperate each other. Combination of a high flow range and an expensive concave cost function does not produce any multiple interhub link[15] users at all in this particular simulation. This absence of the multiple interhub link users makes the cost allocation model less complex to solve. Table 6.3 provides the fair cost allocation for the interhub link users. Players (O-D pairs) get the fair cost allocation based on their contribution to the flow economies of scale. Total network costs consist of transportation costs and fixed costs. Transportation costs do not change while fixed costs are allocated differently depending on the player's role in the FDMAP. The core allocation provides the proportional fixed cost to the player instead of evenly divided fixed costs. The cost allocation considers two different network optimal situations: [1] global optimal network, and [2] coalition optimal network. Under the global optimal network, the players cannot alter the interhub link with given hub locations. In other words, no player can achieve the cheaper transportation cost than the FDMAP solution itself. The solution of cost allocations guarantees the cheapest individual cost for the player. In other words, no player has a motivation to leave the grand coalition, which is the solution of the FDMAP. if cost allocation vectors of Table 6.3 are obtained. Under the coalition optimal network, however, the players can modify (alter) the interhub link with given hub locations.

[15] There are 3 hubs involved in the multiple interhub link. This multiple interhub link occurs only when either origin or destination is hub itself.

I	K	M	J	Transport Cost	Flow	Fixed Cost	Total Network Cost
2	4	12	12	16,002,131.30	7975	2,508,529.85	18,510,661.15
3	4	12	12	50,112,203.46	22254	6,999,977.84	57,112,181.30
4	4	12	12	91,116,669.42	65387	20,567,428.38	111,684,097.80
5	4	12	12	9,810,393.34	5951	1,871,882.27	11,682,275.62
6	4	12	12	24,569,654.56	14412	4,533,283.03	29,102,937.59
9	4	12	12	36,627,461.11	22463	7,065,718.62	43,693,179.74
12	12	4	2	16,002,131.30	7975	2,508,529.85	18,510,661.15
12	12	4	3	50,112,203.46	22254	6,999,977.84	57,112,181.30
12	12	4	4	91,116,669.42	65387	20,567,428.38	111,684,097.80
12	12	4	5	9,810,393.34	5951	1,871,882.27	11,682,275.62
12	12	4	6	24,569,654.56	14412	4,533,283.03	29,102,937.59
12	12	4	9	36,627,461.11	22463	7,065,718.62	43,693,179.74

**Hub node 4 is Chicago, and 12 is Los Angeles.*
*** Index I and J stands for origin and destination, and K and M for hubs.*

Table 6.3 Cost Allocation (FDMAP) with Cost Function 1 under Flow Range 1 (CAB15)

CAB15	LOCATION			SLOPE			FLOW				NETWORK COST		
p=3	Hub1	Hub2	Hub3	Interhub 1_2	Interhub 1_3	Interhub 2_3	Interhub 1_2	Interhub 1_3	Interhub 2_3	TOTAL	Obj	Transport Cost	Fixed Cost
C.F.1	4	12	7	2	1	1	276,884	0	0	276,884	1068.37	1031.54	36.83
C.F.2	4	12	7	2	1	1	276,884	0	0	276,884	1070.35	1051.93	18.41
C.F.3	4	12	7	2	1	1	276,884	55,804	148,606	481,294	1042.44	1024.02	18.41
C.F.4	4	12	7	1	5	4	0	599,030	491,492	1,090,522	1006.51	843.72	162.79
C.F.5	4	12	7	1	5	5	0	961,768	552,216	1,513,984	947.15	742.29	204.86
C.F.6	4	12	7	1	4	4	0	451,806	456,064	907,870	1028.93	964.24	64.69

**Node 4 is Chicago, and 7 is Dallas, and 12 is Los Angeles.*

*** C.F.1-C.F.3 are based on Flow Range 1 and C.F.4-C.F.6 are based on Flow Range 2*

**** Obj is the total network cost.*

Table 6.4 Solutions of the FDMAP (CAB15)

Simulation Set 2: Cost Function 2 under Flow Range 1

Table 6.5 shows the cost allocations for the interhub link users with the cost function 2 under the flow range 1. The difference from the simulation set 1 is the cost function. The cost function 2 is slightly cheaper than the cost function 1, so the FDMAP does not bundle more flows than the cost function 1. The cost allocation distributes the fixed costs to the players proportionally depending on their contribution to the flow economies of scale. The results of this simulation set can be interpreted as similar as the simulation set 1.

I	K	M	J	Transport Cost	Flow	Fixed Cost	Total Network Cost
2	4	12	12	17,391,274.86	7975	2,508,529.85	19,899,804.71
3	4	12	12	53,988,567.19	22254	6,999,977.84	60,988,545.03
4	4	12	12	102,506,253.10	65387	20,567,428.38	123,073,681.47
5	4	12	12	10,846,981.84	5951	1,871,882.27	12,718,864.12
6	4	12	12	27,080,041.64	14412	4,533,283.03	31,613,324.67
9	4	12	12	40,540,229.98	22463	7,065,718.62	47,605,948.61
12	12	4	2	17,391,274.86	7975	2,508,529.85	19,899,804.71
12	12	4	3	53,988,567.19	22254	6,999,977.84	60,988,545.03
12	12	4	4	102,506,253.10	65387	20,567,428.38	123,073,681.47
12	12	4	5	10,846,981.84	5951	1,871,882.27	12,718,864.12
12	12	4	6	27,080,041.64	14412	4,533,283.03	31,613,324.67
12	12	4	9	40,540,229.98	22463	7,065,718.62	47,605,948.61

**Hub node 4 is Chicago, and 12 is Los Angeles.*
*** Index I and J stands for origin and destination, and K and M for hubs.*

Table 6.5 Cost Allocation (FDMAP) with Cost Function 2 under Flow Range 1 (CAB15)

Simulation Set 3: Cost Function 3 under Flow Range 1

Table 6.6 shows the cost allocations for the interhub link users with the cost function 3 under the flow range 1. As can be seen in Table 6.4, Chicago (4), Dallas (7), and Los Angeles (12) are selected as hubs to open. In this simulation, the FDMAP opens all 6 directed (3 undirected) interhub links. The cost function 3 gives a discount of the first piece of cost function on Chicago-Dallas, and Los Angeles-Dallas interhub links unlike the cost functions 1, and 2. In this case, the interhub link users of Chicago- Dallas (Dallas-Chicago), and Dallas-Los Angeles (Log Angeles-Dallas) do not pay fixed costs but enjoy the discount rate because there is no fixed charge involved. These fully opened interhub links are possible due to the low flow range requirement. Individual cost allocations of O-D pairs are provided in Table 6.6.

I	K	M	J	Transport Cost	Flow	Fixed Cost	Total Network Cost
2	4	12	12	16,002,131.30	7975	1,254,264.93	17,256,396.22
3	4	12	12	50,112,203.46	22254	3,499,988.92	53,612,192.38
4	4	12	12	91,116,669.42	65387	10,283,714.19	101,400,383.61
5	4	12	12	9,810,393.34	5951	935,941.14	10,746,334.48
6	4	12	12	24,569,654.56	14412	2,266,641.52	26,836,296.08
9	4	12	12	36,627,461.11	22463	3,532,859.31	40,160,320.43
12	12	4	2	17,391,274.86	7975	1,254,264.93	17,256,396.22
12	12	4	3	53,988,567.19	22254	3,499,988.92	53,612,192.38
12	12	4	4	102,506,253.10	65387	10,283,714.19	101,400,383.61
12	12	4	5	10,846,981.84	5951	935,941.14	10,746,334.48
12	12	4	6	27,080,041.64	14412	2,266,641.52	26,836,296.08
12	12	4	9	40,540,229.98	22463	3,532,859.31	40,160,320.43

**Hub node 4 is Chicago, and 12 is Los Angeles.*
*** Index I and J stands for origin and destination, and K and M for hubs.*

Table 6.6 Cost Allocation (FDMAP) with Cost Function 3 under Flow Range 1 (CAB15)

Simulation Set 4: Cost Function 4 under Flow Range 2

Tables 6.7-6.10 show the cost allocations for the interhub link users with the cost function 4 under the flow range 2. As can be seen in Table 6.4, Chicago (4), Dallas (7), and Los Angeles (12) are selected as hubs to open. The FDMAP opens 4 directed (2 undirected) interhub links out of 6, and closes Chicago-Los Angeles (Chicago-Los Angeles) interhub link. The FDMAP obtains the optimal hub network by closing 2 interhub links instead of providing the fully connected hub network. Combination of a low flow range and a steep (cheaper) cost function produces many multiple interhub link users. This presence of the multiple interhub link users adds more complexity to the problem. The reason for the complicated cost allocation is that the O-D pairs involving the multiple interhub links interact each other.

As discussed in Chapter 3, the total infrastructure cost allocation is equivalent to the sum of the individual infrastructure cost allocation. Tables 6.7-6.10 provide cost allocation vectors for each interhub link both under the global hub network configuration and under the coalition hub network configuration respectively. Each table includes different fixed costs both under the global hub network (Fixed Cost 1 column) and under the coalition hub network (Fixed Cost 2 column). Notice that the transport cost (Transport Cost column) is same under both conditions. Each of Tables 6.7 and 6.8 provides the fair cost allocation for the interhub link of Chicago-Dallas, and Dallas-Chicago respectively. In terms of multiple interhub links, there are 7 multiple interhub links out of 28 interhub link players. Total Cost 1 and Fixed Cost 1 are obtained under the global hub network while Total Cost 2 and Fixed Cost 2 are gained under the coalition optimal hub network. Notice that Total Cost 2 with multiple interhub links is decreased whereas Total Cost 2 of non-multiple interhub links is increased. Under the coalition hub network configuration, the players can modify their interhub links to save their individual costs by altering their paths. This is called as a coalition optimal network. This coalition optimal situation makes the total cost much higher than that of the global optimal network. Thus, it is necessary to find the core solution to make those willing-to-leave-players stay in the network while at the same time to maintain the remainder-players still happy.

Each of Tables 6.9 and 6.10 shows the fair cost allocation for the interhub links of Los Angeles-Dallas, and Dallas-Los Angeles respectively. For example, the player of [2-12] O-D pair in Table 6.9 has multiple interhub links in its paths. The path, [2-4-7-12], involves [4-7] interhub link and [7-12] interhub link at the same time. The multiple interhub link players would make their own coalition to save their costs if the other players are not willing to offsets their overly charged fixed costs. Results in Tables 6.7-6.10 show the offsets among players. For example, individual costs for the single interhub link players are increased under the coalition optimal network restriction compared to the global optimal network while individual costs for the multiple interhub link players are decreased under the coalition optimal network configuration. The solutions of cost allocation guarantee the cheapest individual cost for the players under both network configurations. Players (O-D pairs) get the fair cost allocations based on their contribution to the flow economies of scale. Results[16] of Simulation Set 5 (Flow

[16] These results are not presented.

Range 2, Cost Function 5) and Simulation Set 6 (Flow Range 2, Cost Function 6) could be obtained in the similar context of simulation set 4.

The results support the grand coalition for each interhub link is the best strategy for the total network cost under the global hub network configuration. In other words, none of the players can achieve the cheaper cost by other coalitions assuming players cannot change the globally optimal hub network. On the other hand, the grand coalition for each interhub link is not necessarily the best strategy to individual players under the coalition optimal hub network configuration. The core allocations are changed if players can modify the globally optimal hub network. This fair cost allocation is possible if and only if the cost savings by the grand coalition is greater than the opportunity costs by the players who are willing to leave the grand coalition.

Multiple interhub link players pay more fixed costs than the single interhub link players under the global hub network with respect to their contribution to flow economies of scale. The reason is that they have to pay fixed costs twice whereas the single interhub link users pay only once. However, the multiple interhub link players pay less fixed costs than the single interhub link players under the coalition optimal hub network configuration with respect to their contribution to flow economies of scale. The coalition optimal network is only feasible if and only if there are the multiple interhub link players in the solutions of the FDMAP.

I	K	M	J	Transport Cost	Flow	Fixed Cost 1	Total Cost 1	Fixed Cost 2	Total Cost 2
2	4	7	7	2,990,181.77	3878	1,278,771.68	4,268,953.45	2,504,783.31	5,494,965.09
2	4	7	8	4,594,674.87	3202	1,055,860.48	5,650,535.35	2,281,872.11	6,876,546.98
2	4	7	10	4,166,451.44	4198	1,384,291.78	5,550,743.22	2,610,303.41	6,776,754.85
2	***4***	***7***	***12***	10,135,969.32	7975	2,629,758.68	12,765,728.00	1,314,879.34	11,450,848.66
3	4	7	7	6,048,328.96	5951	1,962,344.06	8,010,673.03	3,188,355.70	9,236,684.66
3	4	7	8	9,691,573.91	5768	1,901,999.76	11,593,573.66	3,128,011.39	12,819,585.30
3	4	7	10	5,250,650.29	4242	1,398,800.79	6,649,451.08	2,624,812.43	7,875,462.71
3	***4***	***7***	***12***	33,742,853.16	22254	7,338,263.28	41,081,116.44	3,669,131.64	37,411,984.80
4	4	7	7	3,385,353.72	21423	7,064,240.78	10,449,594.50	8,290,252.41	11,675,606.13
4	4	7	8	22,472,402.38	27342	9,016,032.83	31,488,435.21	10,242,044.46	32,714,446.84
4	4	7	10	6,005,116.51	15826	5,218,628.32	11,223,744.83	6,444,639.95	12,449,756.47
4	***4***	***7***	***12***	43,020,025.75	65387	21,561,383.17	64,581,408.93	10,780,691.59	53,800,717.34
5	4	7	7	1,281,295.25	3102	1,022,885.45	2,304,180.69	2,248,897.08	3,530,192.32
5	4	7	8	1,682,165.85	1562	515,069.98	2,197,235.82	1,741,081.61	3,423,247.46
5	4	7	10	1,216,291.57	1917	632,131.33	1,848,422.90	1,858,142.97	3,074,434.53
5	***4***	***7***	***12***	5,433,022.82	5951	1,962,344.06	7,395,366.88	981,172.03	6,414,194.85
6	4	7	7	2,357,451.42	5023	1,656,335.78	4,013,787.20	2,882,347.41	5,239,798.83
6	4	7	8	3,979,824.95	3512	1,158,083.07	5,137,908.02	2,384,094.70	6,363,919.65
6	4	7	10	2,447,339.15	3543	1,168,305.33	3,615,644.48	2,394,316.96	4,841,656.12
6	***4***	***7***	***12***	13,968,635.57	14412	4,752,361.39	18,720,996.96	2,376,180.70	16,344,816.27
9	4	7	10	2,742,265.66	4448	1,466,729.36	4,208,995.01	2,692,740.99	5,435,006.65
9	***4***	***7***	***12***	20,104,376.91	22463	7,407,181.09	27,511,558.01	3,703,590.55	23,807,967.46
9	4	7	7	2,559,817.65	6479	2,136,452.22	4,696,269.88	3,362,463.86	5,922,281.51
9	4	7	8	5,946,120.82	5615	1,851,547.96	7,797,668.78	3,077,559.59	9,023,680.41
15	4	7	7	2,357,235.12	4678	1,542,571.93	3,899,807.05	2,768,583.56	5,125,818.68
15	4	7	8	10,389,687.59	8897	2,933,788.46	13,323,476.05	4,159,800.09	14,549,487.68
15	4	7	10	1,996,806.13	2753	907,802.59	2,904,608.71	2,133,814.22	4,130,620.35
15	***4***	***7***	***12***	17,781,368.82	17714	5,841,196.90	23,622,565.72	2,920,598.45	20,701,967.27

**Fixed Cost1 & Total Cost1 are under global optimal whereas Fixed Cost2 & Total Cost2 are under coalition optimal.*
*** Index I and J stands for origin and destination, and K and M for hubs.*
**** I-J pairs with italic and bold numbers are multiple interhub links.*

Table 6.7 Cost Allocation (FDMAP) with Cost Function 4 under Flow Range 2 on Interhub Link 4-7 (CAB15)

I	K	M	J	Transport Cost	Flow	Fixed Cost 1	Total Cost 1	Fixed Cost 2	Total Cost 2
7	7	4	9	2,559,817.65	6479	2,136,452.22	4,696,269.88	3,362,463.86	5,922,281.51
7	7	4	15	2,357,235.12	4678	1,542,571.93	3,899,807.05	2,768,583.56	5,125,818.68
7	7	4	2	2,990,181.77	3878	1,278,771.68	4,268,953.45	2,504,783.31	5,494,965.09
7	7	4	3	6,048,328.96	5951	1,962,344.06	8,010,673.03	3,188,355.70	9,236,684.66
7	7	4	4	3,385,353.72	21423	7,064,240.78	10,449,594.50	8,290,252.41	11,675,606.13
7	7	4	5	1,281,295.25	3102	1,022,885.45	2,304,180.69	2,248,897.08	3,530,192.32
7	7	4	6	2,357,451.42	5023	1,656,335.78	4,013,787.20	2,882,347.41	5,239,798.83
8	7	4	9	5,946,120.82	5615	1,851,547.96	7,797,668.78	3,077,559.59	9,023,680.41
8	7	4	15	10,389,687.59	8897	2,933,788.46	13,323,476.05	4,159,800.09	14,549,487.68
8	7	4	2	4,594,674.87	3202	1,055,860.48	5,650,535.35	2,281,872.11	6,876,546.98
8	7	4	3	9,691,573.91	5768	1,901,999.76	11,593,573.66	3,128,011.39	12,819,585.30
8	7	4	4	22,472,402.38	27342	9,016,032.83	31,488,435.21	10,242,044.46	32,714,446.84
8	7	4	5	1,682,165.85	1562	515,069.98	2,197,235.82	1,741,081.61	3,423,247.46
8	7	4	6	3,979,824.95	3512	1,158,083.07	5,137,908.02	2,384,094.70	6,363,919.65
10	7	4	15	1,996,806.13	2753	907,802.59	2,904,608.71	2,133,814.22	4,130,620.35
10	7	4	2	4,166,451.44	4198	1,384,291.78	5,550,743.22	2,610,303.41	6,776,754.85
10	7	4	3	5,250,650.29	4242	1,398,800.79	6,649,451.08	2,624,812.43	7,875,462.71
10	7	4	4	6,005,116.51	15826	5,218,628.32	11,223,744.83	6,444,639.95	12,449,756.47
10	7	4	5	1,216,291.57	1917	632,131.33	1,848,422.90	1,858,142.97	3,074,434.53
10	7	4	6	2,447,339.15	3543	1,168,305.33	3,615,644.48	2,394,316.96	4,841,656.12
10	7	4	9	2,742,265.66	4448	1,466,729.36	4,208,995.01	2,692,740.99	5,435,006.65
12	***7***	***4***	***15***	17,781,368.82	17714	5,841,196.90	23,622,565.72	2,920,598.45	20,701,967.27
12	***7***	***4***	***2***	10,135,969.32	7975	2,629,758.68	12,765,728.00	1,314,879.34	11,450,848.66
12	***7***	***4***	***3***	33,742,853.16	22254	7,338,263.28	41,081,116.44	3,669,131.64	37,411,984.80
12	***7***	***4***	***4***	43,020,025.75	65387	21,561,383.17	64,581,408.93	10,780,691.59	53,800,717.34
12	***7***	***4***	***5***	5,433,022.82	5951	1,962,344.06	7,395,366.88	981,172.03	6,414,194.85
12	***7***	***4***	***6***	13,968,635.57	14412	4,752,361.39	18,720,996.96	2,376,180.70	16,344,816.27
12	***7***	***4***	***9***	20,104,376.91	22463	7,407,181.09	27,511,558.01	3,703,590.55	23,807,967.46

**Fixed Cost1 & Total Cost1 are under global optimal whereas Fixed Cost2 & Total Cost2 are under coalition optimal.*
*** Index I and J stands for origin and destination, and K and M for hubs.*
**** I-J pairs with italic and bold numbers are multiple interhub links.*

Table 6.8 Cost Allocation (FDMAP) with Cost Function 4 under Flow Range 2 on Interhub Link 7-4 (CAB15)

I	K	M	J	Transport Cost	Flow	Fixed Cost 1	Total Cost 1	Fixed Cost 2	Total Cost 2
1	7	12	12	11,147,511.99	9221	3,517,064.93	14,664,576.93	8,480,470.85	19,627,982.85
7	7	12	12	13,672,403.94	27350	10,431,810.64	24,104,214.58	15,395,216.56	29,067,620.50
10	7	12	12	12,455,154.69	17267	6,585,962.50	19,041,117.19	11,549,368.42	24,004,523.11
11	7	12	12	14,487,634.82	15287	5,830,752.81	20,318,387.63	10,794,158.73	25,281,793.55
13	7	12	12	4,968,694.24	5454	2,080,259.42	7,048,953.67	7,043,665.34	12,012,359.59
14	7	12	12	23,980,271.85	15011	5,725,481.15	29,705,753.00	10,688,887.07	34,669,158.92
2	***4***	***7***	***12***	10,135,969.32	7975	3,041,816.81	13,177,786.14	1,520,908.41	11,656,877.73
3	***4***	***7***	***12***	33,742,853.16	22254	8,488,099.23	42,230,952.39	4,244,049.62	37,986,902.77
4	***4***	***7***	***12***	43,020,025.75	65387	24,939,846.53	67,959,872.28	12,469,923.26	55,489,949.02
5	***4***	***7***	***12***	5,433,022.82	5951	2,269,824.69	7,702,847.50	1,134,912.34	6,567,935.16
6	***4***	***7***	***12***	13,968,635.57	14412	5,497,011.15	19,465,646.72	2,748,505.58	16,717,141.15
9	***4***	***7***	***12***	20,104,376.91	22463	8,567,815.81	28,672,192.73	4,283,907.91	24,388,284.82
15	***4***	***7***	***12***	17,781,368.82	17714	6,756,456.81	24,537,825.63	3,378,228.40	21,159,597.23

**Fixed Cost1 & Total Cost1 are under global optimal whereas Fixed Cost2 & Total Cost2 are under coalition optimal.*
*** Index I and J stands for origin and destination, and K and M for hubs*
**** I-J pairs with italic and bold numbers are multiple interhub links.*

Table 6.9 Cost Allocation (FDMAP) with Cost Function 4 under Flow Range 2 on Interhub Link 7-12 (CAB15)

I	K	M	J	Transport Cost	Flow	Fixed Cost 1	Total Cost 1	Fixed Cost 2	Total Cost 2
12	12	7	13	4,968,694.24	5454	2,080,259.42	7,048,953.67	7,043,665.34	12,012,359.59
12	12	7	14	23,980,271.85	15011	5,725,481.15	29,705,753.00	10,688,887.07	34,669,158.92
12	12	7	1	11,147,511.99	9221	3,517,064.93	14,664,576.93	8,480,470.85	19,627,982.85
12	12	7	7	13,672,403.94	27350	10,431,810.64	24,104,214.58	15,395,216.56	29,067,620.50
12	12	7	10	12,455,154.69	17267	6,585,962.50	19,041,117.19	11,549,368.42	24,004,523.11
12	12	7	11	14,487,634.82	15287	5,830,752.81	20,318,387.63	10,794,158.73	25,281,793.55
12	***7***	***4***	***15***	17,781,368.82	17714	6,756,456.81	24,537,825.63	3,378,228.40	21,159,597.23
12	***7***	***4***	***2***	10,135,969.32	7975	3,041,816.81	13,177,786.14	1,520,908.41	11,656,877.73
12	***7***	***4***	***3***	33,742,853.16	22254	8,488,099.23	42,230,952.39	4,244,049.62	37,986,902.77
12	***7***	***4***	***4***	43,020,025.75	65387	24,939,846.53	67,959,872.28	12,469,923.26	55,489,949.02
12	***7***	***4***	***5***	5,433,022.82	5951	2,269,824.69	7,702,847.50	1,134,912.34	6,567,935.16
12	***7***	***4***	***6***	13,968,635.57	14412	5,497,011.15	19,465,646.72	2,748,505.58	16,717,141.15
12	***7***	***4***	***9***	20,104,376.91	22463	8,567,815.81	28,672,192.73	4,283,907.91	24,388,284.82

**Fixed Cost1 & Total Cost1 are under global optimal whereas Fixed Cost2 & Total Cost2 are under coalition optimal.*
*** Index I and J stands for origin and destination, and K and M for hubs*
**** I-J pairs with italic and bold numbers are multiple interhub links.*

Table 6.10 Cost Allocation (FDMAP) with Cost Function 4 under Flow Range 2 on Interhub Link 12-7 (CAB15)

6.2 Numerical Results of Cost Allocation in the FDPTP

The numerical analysis of cost allocation in the FDPTP is based on CAB10 data. Total network flow for CAB10 data is 999,026. Table 6.11 shows flow ranges for each piece-wise liner cost function. Concave cost functions 1-2 of Table 6.12 are based on the flow range 1 of Table 6.11 while concave cost function 3 of Table 6.12 are based on the flow range 2 of Table 6.11. The FLOW RANGE2 (third column of Table 6.11) gives more discounts than the FLOW RANGE1 (second column of Table 6.11) for the same amount of flow. To achieve the second piece of concave cost function in Table 6.12, there is a flow difference to reach depending on flow ranges. For example, the model allows 62,500 flows to get the second piece of concave cost function under the FLOW RANGE1 while 31,250 flows are only necessary under the FLOW RANGE2 setting. Out of 3 different simulation sets tested, 2 simulation sets are presented to illustrate the empirical results.

Piece	Flow Range 1	Flow Range 2
1	0	0
2	62,500	31,250
3	125,000	62,500
4	187,500	93,750
5	250,000	125,000

Table 6.11 Ranges of Link Flows (CAB10)

Piece		1	2	3	4	5
Cost Function	Fixed Cost	0	12500	37500	75000	125000
1	Slope	1	0.8	0.6	0.4	0.2
Cost Function	Fixed Cost	0	6250	18750	37500	62500
2	Slope	0.9	0.8	0.7	0.6	0.5
Cost Function	Fixed Cost	0	7500	20000	38750	63750
3	Slope	1	0.8	0.6	0.4	0.2

* Cost functions 1-2 are based on Flow Range 1 and cost function 3 is based on Flow Range 2.

Table 6.12 Slope and Fixed Cost for the Piecewise Linear Cost (CAB10)

Simulation Set 1: Cost Function 3 under Flow Range 2 (Without Hop Constraint)

The simulation set 1 is based on cost function 3 under flow range 2 of the FDPTP without hop constraint. Cost allocation solutions of the FDPTP (CAB10) are shown in Tables 6.13-27. Each table provides the cost allocations of the O-D pairs for the opened links. As can be seen in Figure 5.2, the FDPTP (CAB10) of simulation set 1 opens 15 links out of 90, which are ten y^5, four y^3, and one y^2 discounted links.

Table 6.13 provides the fair cost allocation of the O-D pairs for [6-9] link. Players (O-D pairs) get the fair cost allocation based on their contribution to the flow economies of scale. Total network costs consist of transportation costs and fixed costs in the table. Transportation costs do not change while fixed costs are allocated differently depending on the player’s role in the FDPTP. The core allocations are affected mainly by the proportional fixed costs to the player instead of evenly divided fixed costs. As can be seen in Table 6.13, Total Network Cost 1 and Total Network Cost 2 are different for each

O-D pair which uses [6-9] link. Total Network Cost 2 is obtained with cost allocation solutions whereas Total Network Cost 1 is calculated without cost allocations. Difference column in Table 6.13 shows difference between evenly-distributed fixed costs and proportionally-distributed fixed costs. Positive sign in Difference column shows that the O-D pair of cost allocation pays more fixed costs than the one of without cost allocation solutions. The main reason is that the O-D pairs of positive sign have high volume of flows compared to the ones of negative sign. One the other hand, the O-D pairs of negative sign pay less fixed costs in cost allocation solutions due to their low volume of flows compared to the ones of positive sign. In other words, the O-D pairs in negative sign will pay unnecessary high fixed costs without cost allocation solutions. Notice that the O-D pairs of higher volume of flows pay more fixed costs so their total network costs are increased. Tables 6-14-6.27 of different opened links can be explained same as Table 6.13. Table 6.13 shows the cost allocations of the O-D pairs for y^2 discounted link, Tables 6.14-6.17 show the cost allocations of the O-D pairs for y^3 discounted link, and Tables 6.18-6.27 show the cost allocations of the O-D pairs for y^5 discounted link.

X	I	J	K	M	Flow	Transport Cost	Total Network Cost 1	Total Network Cost 2	Difference
X2	2	9	6	9	6699	505,151.76	740,798.76	645,696.57	-
X2	3	9	6	9	16578	1,250,097.91	1,485,744.91	1,597,903.82	+
X2	6	9	6	9	10419	785,665.95	1,021,312.95	1,004,256.24	-

Note: *X: a type of discount, I: origin, J: destination, and K-M: opened link.*

Table 6.13 Cost Allocation (FDPTP) of 6-9 Link with Cost Function 3 under Flow Range 2 (CAB10)

X	I	J	K	M	Flow	Transport Cost	Total Network Cost 1	Total Network Cost 2	Difference
X3	1	8	7	8	2243	893,444.59	2,368,725.03	1,332,078.48	-
X3	2	8	7	8	3202	1,275,438.96	2,750,719.40	1,901,611.81	-
X3	3	8	7	8	5768	2,297,542.75	3,772,823.20	3,425,514.33	-
X3	4	8	7	8	27342	10,891,021.84	12,366,302.28	16,237,935.66	+
X3	5	8	7	8	1562	622,184.77	2,097,465.22	927,644.48	-
X3	6	8	7	8	3512	1,398,919.93	2,874,200.37	2,085,715.38	-
X3	7	8	7	8	11557	4,603,450.35	6,078,730.79	6,863,500.20	+
X3	9	8	7	8	5615	2,236,598.92	3,711,879.36	3,334,650.31	-
X3	10	8	7	8	7095	2,826,120.98	4,301,401.43	4,213,596.43	-

Table 6.14 Cost Allocation (FDPTP) of 7-8 Link with Cost Function 3 under Flow Range 2 (CAB10)

X	I	J	K	M	Flow	Transport Cost	Total Network Cost 1	Total Network Cost 2	Difference
X3	1	10	7	10	7248	962,919.99	1,454,968.88	1,350,671.97	-
X3	2	10	7	10	4198	557,717.73	1,049,766.62	782,301.45	-
X3	3	10	7	10	4242	563,563.27	1,055,612.16	790,500.90	-
X3	4	10	7	10	15826	2,102,534.74	2,594,583.63	2,949,190.76	+
X3	5	10	7	10	1917	254,679.58	746,728.47	357,234.85	-
X3	6	10	7	10	3543	470,698.89	962,747.78	660,241.55	-
X3	7	10	7	10	34261	4,551,683.49	5,043,732.37	6,384,571.24	+
X3	8	10	7	10	7095	942,593.45	1,434,642.34	1,322,160.27	-
X3	9	10	7	10	4448	590,931.03	1,082,979.92	828,889.20	-

Table 6.15 Cost Allocation (FDPTP) of 7-10 Link with Cost Function 3 under Flow Range 2 (CAB10)

X	I	J	K	M	Flow	Transport Cost	Total Network Cost 1	Total Network Cost 2	Difference
X3	8	1	8	7	2243	893,444.59	2,368,725.03	1,332,078.48	-
X3	8	2	8	7	3202	1,275,438.96	2,750,719.40	1,901,611.81	-
X3	8	3	8	7	5768	2,297,542.75	3,772,823.20	3,425,514.33	-
X3	8	4	8	7	27342	10,891,021.84	12,366,302.28	16,237,935.66	+
X3	8	5	8	7	1562	622,184.77	2,097,465.22	927,644.48	-
X3	8	6	8	7	3512	1,398,919.93	2,874,200.37	2,085,715.38	-
X3	8	7	8	7	11557	4,603,450.35	6,078,730.79	6,863,500.20	+
X3	8	9	8	7	5615	2,236,598.92	3,711,879.36	3,334,650.31	-
X3	8	10	8	7	7095	2,826,120.98	4,301,401.43	4,213,596.43	-

Table 6.16 Cost Allocation (FDPTP) of 8-7 Link with Cost Function 3 under Flow Range 2 (CAB10)

X	I	J	K	M	Flow	Transport Cost	Total Network Cost 1	Total Network Cost 2	Difference
X3	10	1	10	7	7248	962,919.99	1,454,968.88	1,350,671.97	-
X3	10	2	10	7	4198	557,717.73	1,049,766.62	782,301.45	-
X3	10	3	10	7	4242	563,563.27	1,055,612.16	790,500.90	-
X3	10	4	10	7	15826	2,102,534.74	2,594,583.63	2,949,190.76	+
X3	10	5	10	7	1917	254,679.58	746,728.47	357,234.85	-
X3	10	6	10	7	3543	470,698.89	962,747.78	660,241.55	-
X3	10	7	10	7	34261	4,551,683.49	5,043,732.37	6,384,571.24	+
X3	10	8	10	7	7095	942,593.45	1,434,642.34	1,322,160.27	-
X3	10	9	10	7	4448	590,931.03	1,082,979.92	828,889.20	-

Table 6.17 Cost Allocation (FDPTP) of 10-7 Link with Cost Function 3 under Flow Range 2 (CAB10)

X	I	J	K	M	Flow	Transport Cost	Total Network Cost 1	Total Network Cost 2	Difference
X5	1	2	1	7	6469	917,332.02	2,591,410.56	2,364,245.84	-
X5	1	3	1	7	7629	1,081,825.00	2,755,903.55	2,788,194.70	+
X5	1	4	1	7	20036	2,841,190.95	4,515,269.50	7,322,620.14	+
X5	1	5	1	7	4690	665,062.17	2,339,140.71	1,714,069.10	-
X5	1	6	1	7	6194	878,335.83	2,552,414.38	2,263,740.72	-
X5	1	7	1	7	11688	1,657,408.66	3,331,487.20	4,271,650.24	+
X5	1	8	1	7	2243	318,067.04	1,992,145.59	819,756.29	-
X5	1	9	1	7	8857	1,255,960.69	2,930,039.23	3,236,995.74	+
X5	1	10	1	7	7248	1,027,797.57	2,701,876.11	2,648,949.43	-
X5	2	7	1	7	3878	549,917.08	2,223,995.62	1,417,304.90	-
X5	2	8	1	7	3202	454,057.37	2,128,135.91	1,170,245.04	-
X5	2	10	1	7	4198	595,294.45	2,269,372.99	1,534,256.31	-
X5	3	7	1	7	5951	843,877.39	2,517,955.93	2,174,930.75	-
X5	3	8	1	7	5768	817,927.20	2,492,005.74	2,108,049.16	-
X5	3	10	1	7	4242	601,533.84	2,275,612.38	1,550,337.12	-
X5	4	7	1	7	21423	3,037,873.52	4,711,952.06	7,829,531.41	+
X5	4	8	1	7	27342	3,877,213.17	5,551,291.71	9,992,767.01	+
X5	4	10	1	7	15826	2,244,194.85	3,918,273.39	5,783,978.16	+
X5	5	7	1	7	3102	439,876.94	2,113,955.48	1,133,697.73	-
X5	5	8	1	7	1562	221,498.32	1,895,576.86	570,869.07	-
X5	5	10	1	7	1917	271,838.84	1,945,917.38	700,612.04	-
X5	6	7	1	7	5023	712,283.00	2,386,361.54	1,835,771.66	-
X5	6	8	1	7	3512	498,016.70	2,172,095.24	1,283,541.72	-
X5	6	10	1	7	3543	502,412.63	2,176,491.18	1,294,871.39	-
X5	9	7	1	7	6479	918,750.06	2,592,828.60	2,367,900.57	-
X5	9	8	1	7	5615	796,231.14	2,470,309.69	2,052,131.77	-
X5	9	10	1	7	4448	630,745.53	2,304,824.07	1,625,624.59	-

Table 6.18 Cost Allocation (FDPTP) of 1-7 Link with Cost Function 3 under Flow Range 2 (CAB10)

X	I	J	K	M	Flow	Transport Cost	Total Network Cost 1	Total Network Cost 2	Difference
X5	2	1	2	6	6469	404,808.15	1,578,119.78	1,235,053.62	-
X5	2	3	2	6	12999	813,433.48	1,986,745.11	2,481,753.29	+
X5	2	4	2	6	13692	856,799.08	2,030,110.71	2,614,060.01	+
X5	2	5	2	6	3322	207,879.53	1,381,191.16	634,232.20	-
X5	2	6	2	6	5576	348,927.23	1,522,238.86	1,064,563.15	-
X5	2	7	2	6	3878	242,672.13	1,415,983.76	740,383.05	-
X5	2	8	2	6	3202	200,370.34	1,373,681.96	611,321.95	-
X5	2	9	2	6	6699	419,200.78	1,592,512.40	1,278,964.94	-
X5	2	10	2	6	4198	262,696.65	1,436,008.28	801,477.06	-
X5	3	1	2	6	7629	477,397.03	1,650,708.66	1,456,519.41	-
X5	3	4	2	6	35135	2,198,629.54	3,371,941.17	6,707,931.52	+
X5	3	5	2	6	5956	372,706.35	1,546,017.97	1,137,112.29	-
X5	3	6	2	6	14121	883,644.45	2,056,956.08	2,695,964.17	+
X5	3	7	2	6	5951	372,393.47	1,545,705.09	1,136,157.69	-
X5	3	8	2	6	5768	360,941.94	1,534,253.57	1,101,219.55	-
X5	3	9	2	6	16578	1,037,395.21	2,210,706.83	3,165,051.62	+
X5	3	10	2	6	4242	265,450.02	1,438,761.65	809,877.49	-

Table 6.19 Cost Allocation (FDPTP) of 2-6 Link with Cost Function 3 under Flow Range 2 (CAB10)

X	I	J	K	M	Flow	Transport Cost	Total Network Cost 1	Total Network Cost 2	Difference
X5	1	2	3	2	6469	478,101.41	1,863,849.03	1,458,668.43	-
X5	3	1	3	2	7629	563,832.99	1,949,580.62	1,720,232.10	-
X5	3	2	3	2	12999	960,711.11	2,346,458.74	2,931,091.50	+
X5	3	4	3	2	35135	2,596,706.28	3,982,453.91	7,922,447.86	+
X5	3	5	3	2	5956	440,187.35	1,825,934.98	1,342,994.15	-
X5	3	6	3	2	14121	1,043,634.25	2,429,381.88	3,184,086.70	+
X5	3	7	3	2	5951	439,817.82	1,825,565.44	1,341,866.72	-
X5	3	8	3	2	5768	426,292.92	1,812,040.55	1,300,602.80	-
X5	3	9	3	2	16578	1,225,222.62	2,610,970.25	3,738,105.61	+
X5	3	10	3	2	4242	313,511.54	1,699,259.17	956,511.28	-
X5	4	2	3	2	13692	1,011,928.35	2,397,675.97	3,087,353.24	+
X5	5	2	3	2	3322	245,517.53	1,631,265.15	749,064.23	-
X5	6	2	3	2	5576	412,102.87	1,797,850.49	1,257,309.50	-
X5	7	2	3	2	3878	286,609.56	1,672,357.19	874,434.40	-
X5	8	2	3	2	3202	236,648.74	1,622,396.37	722,005.92	-
X5	9	2	3	2	6699	495,099.91	1,880,847.54	1,510,530.19	-
X5	10	2	3	2	4198	310,259.65	1,696,007.28	946,589.90	-

Table 6.20 Cost Allocation (FDPTP) of 3-2 Link with Cost Function 3 under Flow Range 2 (CAB10)

X	I	J	K	M	Flow	Transport Cost	Total Network Cost 1	Total Network Cost 2	Difference
X5	1	2	4	9	6469	306,721.55	726,533.54	563,863.69	-
X5	1	3	4	9	7629	361,721.86	781,533.85	664,973.88	-
X5	1	5	4	9	4690	222,371.94	642,183.93	408,798.99	-
X5	1	6	4	9	6194	293,682.69	713,494.68	539,893.60	-
X5	1	9	4	9	8857	419,946.33	839,758.32	772,011.23	-
X5	4	1	4	9	20036	949,988.11	1,369,800.10	1,746,417.19	+
X5	4	2	4	9	13692	649,193.31	1,069,005.30	1,193,449.00	+
X5	4	3	4	9	35135	1,665,893.00	2,085,704.99	3,062,505.89	+
X5	4	5	4	9	19094	905,324.06	1,325,136.05	1,664,308.73	+
X5	4	6	4	9	35119	1,665,134.37	2,084,946.36	3,061,111.26	+
X5	4	7	4	9	21423	1,015,751.41	1,435,563.40	1,867,313.61	+
X5	4	8	4	9	27342	1,296,395.23	1,716,207.22	2,383,237.11	+
X5	4	9	4	9	51341	2,434,285.25	2,854,097.24	4,475,085.09	+
X5	4	10	4	9	15826	750,374.91	1,170,186.90	1,379,456.90	+
X5	5	2	4	9	3322	157,509.51	577,321.50	289,558.69	-
X5	5	3	4	9	5956	282,398.14	702,210.13	519,148.57	-
X5	5	6	4	9	7284	345,364.01	765,176.00	634,902.32	-
X5	5	9	4	9	7180	340,432.95	760,244.94	625,837.26	-
X5	7	1	4	9	11688	554,175.53	973,987.52	1,018,772.42	+
X5	7	2	4	9	3878	183,871.72	603,683.71	338,021.85	-
X5	7	3	4	9	5951	282,161.07	701,973.06	518,712.75	-
X5	7	5	4	9	3102	147,078.41	566,890.40	270,382.62	-
X5	7	6	4	9	5023	238,160.82	657,972.81	437,824.59	-
X5	7	9	4	9	6479	307,195.69	727,007.68	564,735.32	-
X5	8	1	4	9	2243	106,349.74	526,161.73	195,508.77	-
X5	8	2	4	9	3202	151,819.82	571,631.81	279,099.01	-
X5	8	3	4	9	5768	273,484.30	693,296.29	502,761.75	-
X5	8	5	4	9	1562	74,060.76	493,872.75	136,150.11	-
X5	8	6	4	9	3512	166,518.18	586,330.17	306,119.84	-
X5	8	9	4	9	5615	266,229.95	686,041.94	489,425.66	-
X5	10	1	4	9	7248	343,657.11	763,469.10	631,764.41	-
X5	10	2	4	9	4198	199,044.22	618,856.21	365,914.32	-
X5	10	3	4	9	4242	201,130.44	620,942.43	369,749.54	-
X5	10	5	4	9	1917	90,892.75	510,704.74	167,093.32	-
X5	10	6	4	9	3543	167,988.01	587,800.00	308,821.93	-
X5	10	9	4	9	4448	210,897.74	630,709.73	387,705.31	-

Table 6.21 Cost Allocation (FDPTP) of 4-9 Link with Cost Function 3 under Flow Range 2 (CAB10)

X	I	J	K	M	Flow	Transport Cost	Total Network Cost 1	Total Network Cost 2	Difference
X5	2	1	5	1	6469	483,638.87	1,366,252.19	1,246,485.64	-
X5	2	7	5	1	3878	289,929.13	1,172,542.45	747,236.25	-
X5	2	8	5	1	3202	239,389.65	1,122,002.97	616,980.53	-
X5	2	10	5	1	4198	313,853.14	1,196,466.46	808,895.77	-
X5	3	1	5	1	7629	570,363.42	1,452,976.74	1,470,001.39	+
X5	3	7	5	1	5951	444,911.88	1,327,525.19	1,146,674.30	-
X5	3	8	5	1	5768	431,230.33	1,313,843.65	1,111,412.77	-
X5	3	10	5	1	4242	317,142.69	1,199,756.01	817,373.95	-
X5	4	1	5	1	20036	1,497,942.25	2,380,555.57	3,860,656.42	+
X5	4	7	5	1	21423	1,601,637.89	2,484,251.21	4,127,911.88	+
X5	4	8	5	1	27342	2,044,157.37	2,926,770.69	5,268,420.23	+
X5	4	10	5	1	15826	1,183,191.96	2,065,805.28	3,049,448.42	+
X5	5	1	5	1	4690	350,636.31	1,233,249.63	903,697.28	-
X5	5	7	5	1	3102	231,913.40	1,114,526.72	597,711.93	-
X5	5	8	5	1	1562	116,779.09	999,392.41	300,975.51	-
X5	5	10	5	1	1917	143,319.79	1,025,933.11	369,379.04	-
X5	6	1	5	1	6194	463,079.17	1,345,692.49	1,193,497.00	-
X5	6	7	5	1	5023	375,532.24	1,258,145.56	967,861.71	-
X5	6	8	5	1	3512	262,566.04	1,145,179.36	676,713.18	-
X5	6	10	5	1	3543	264,883.68	1,147,497.00	682,686.45	-
X5	7	1	5	1	11688	873,824.57	1,756,437.89	2,252,113.81	+
X5	8	1	5	1	2243	167,692.38	1,050,305.70	432,194.67	-
X5	9	1	5	1	8857	662,171.82	1,544,785.14	1,706,619.78	+
X5	9	7	5	1	6479	484,386.50	1,366,999.82	1,248,412.50	-
X5	9	8	5	1	5615	419,791.66	1,302,404.98	1,081,931.81	-
X5	9	10	5	1	4448	332,543.78	1,215,157.10	857,067.27	-
X5	10	1	5	1	7248	541,878.89	1,424,492.21	1,396,588.03	-

Table 6.22 Cost Allocation (FDPTP) of 5-1 Link with Cost Function 3 under Flow Range 2 (CAB10)

X	I	J	K	M	Flow	Transport Cost	Total Network Cost 1	Total Network Cost 2	Difference
X5	2	4	5	4	13692	698,374.97	2,504,839.60	1,948,112.64	-
X5	3	4	5	4	35135	1,792,097.92	3,598,562.54	4,999,045.99	+
X5	5	2	5	4	3322	169,442.13	1,975,906.76	472,657.77	-
X5	5	3	5	4	5956	303,792.09	2,110,256.72	847,426.15	-
X5	5	4	5	4	19094	973,909.71	2,780,374.33	2,716,715.07	-
X5	5	6	5	4	7284	371,528.14	2,177,992.77	1,036,375.44	-
X5	5	9	5	4	7180	366,223.51	2,172,688.14	1,021,578.20	-
X5	6	4	5	4	35119	1,791,281.82	3,597,746.45	4,996,769.49	+
X5	9	4	5	4	51341	2,618,702.13	4,425,166.75	7,304,853.28	+

Table 6.23 Cost Allocation (FDPTP) of 5-4 Link with Cost Function 3 under Flow Range 2 (CAB10)

X	I	J	K	M	Flow	Transport Cost	Total Network Cost 1	Total Network Cost 2	Difference
X5	1	2	6	3	6469	719,444.14	2,804,708.89	2,194,995.54	-
X5	1	3	6	3	7629	848,452.52	2,933,717.27	2,588,594.99	-
X5	2	3	6	3	12999	1,445,672.35	3,530,937.10	4,410,688.98	+
X5	4	2	6	3	13692	1,522,743.73	3,608,008.48	4,645,830.72	+
X5	4	3	6	3	35135	3,907,508.11	5,992,772.86	11,921,652.23	+
X5	5	2	6	3	3322	369,453.31	2,454,718.06	1,127,187.38	-
X5	5	3	6	3	5956	662,391.30	2,747,656.05	2,020,929.58	-
X5	6	2	6	3	5576	620,129.93	2,705,394.68	1,891,991.83	-
X5	6	3	6	3	14121	1,570,454.59	3,655,719.34	4,791,394.65	+
X5	7	2	6	3	3878	431,288.36	2,516,553.11	1,315,843.67	-
X5	7	3	6	3	5951	661,835.23	2,747,099.98	2,019,233.03	-
X5	8	2	6	3	3202	356,107.61	2,441,372.36	1,086,470.20	-
X5	8	3	6	3	5768	641,483.04	2,726,747.79	1,957,139.32	-
X5	9	2	6	3	6699	745,023.39	2,830,288.14	2,273,036.81	-
X5	9	3	6	3	16578	1,843,707.68	3,928,972.43	5,625,079.00	+
X5	10	2	6	3	4198	466,876.88	2,552,141.63	1,424,422.83	-
X5	10	3	6	3	4242	471,770.30	2,557,035.05	1,439,352.46	-

Table 6.24 Cost Allocation (FDPTP) of 6-3 Link with Cost Function 3 under Flow Range 2 (CAB10)

X	I	J	K	M	Flow	Transport Cost	Total Network Cost 1	Total Network Cost 2	Difference
X5	1	5	6	5	4690	211,889.89	611,912.99	389,529.23	-
X5	2	1	6	5	6469	292,263.47	692,286.57	537,284.56	-
X5	2	4	6	5	13692	618,591.96	1,018,615.07	1,137,192.80	+
X5	2	5	6	5	3322	150,084.90	550,108.01	275,909.62	-
X5	2	7	6	5	3878	175,204.47	575,227.58	322,088.35	-
X5	2	8	6	5	3202	144,663.41	544,686.52	265,942.99	-
X5	2	10	6	5	4198	189,661.78	589,684.88	348,666.04	-
X5	3	1	6	5	7629	344,671.20	744,694.31	633,628.68	-
X5	3	4	6	5	35135	1,587,366.98	1,987,390.08	2,918,147.03	+
X5	3	5	6	5	5956	269,086.60	669,109.70	494,677.21	-
X5	3	7	6	5	5951	268,860.71	668,883.81	494,261.93	-
X5	3	8	6	5	5768	260,592.93	660,616.04	479,062.82	-
X5	3	10	6	5	4242	191,649.66	591,672.76	352,320.47	-
X5	4	1	6	5	20036	905,208.05	1,305,231.15	1,664,095.46	+
X5	4	5	6	5	19094	862,649.35	1,262,672.46	1,585,857.39	+
X5	4	7	6	5	21423	967,871.43	1,367,894.54	1,779,293.12	+
X5	4	8	6	5	27342	1,235,286.41	1,635,309.51	2,270,897.29	+
X5	4	10	6	5	15826	715,004.12	1,115,027.22	1,314,432.76	+
X5	6	1	6	5	6194	279,839.22	679,862.33	514,444.36	-
X5	6	4	6	5	35119	1,586,644.11	1,986,667.21	2,916,818.15	+
X5	6	5	6	5	7284	329,084.42	729,107.52	604,974.61	-
X5	6	7	6	5	5023	226,934.52	626,957.62	417,186.64	-
X5	6	8	6	5	3512	158,668.93	558,692.03	291,690.12	-
X5	6	10	6	5	3543	160,069.48	560,092.58	294,264.83	-
X5	7	1	6	5	11688	528,053.09	928,076.19	970,750.04	+
X5	7	5	6	5	3102	140,145.51	540,168.61	257,637.46	-
X5	8	1	6	5	2243	101,336.68	501,359.78	186,292.98	-
X5	8	5	6	5	1562	70,569.72	470,592.83	129,732.34	-
X5	9	1	6	5	8857	400,151.11	800,174.22	735,620.56	-
X5	9	4	6	5	51341	2,319,539.15	2,719,562.25	4,264,140.79	+
X5	9	5	6	5	7180	324,385.79	724,408.90	596,336.86	-
X5	9	7	6	5	6479	292,715.26	692,738.36	538,115.12	-
X5	9	8	6	5	5615	253,680.53	653,703.64	466,355.36	-
X5	9	10	6	5	4448	200,956.55	600,979.65	369,429.86	-
X5	10	1	6	5	7248	327,457.97	727,481.08	601,984.62	-
X5	10	5	6	5	1917	86,608.30	486,631.40	159,216.96	-

Table 6.25 Cost Allocation (FDPTP) of 6-5 Link with Cost Function 3 under Flow Range 2 (CAB10)

X	I	J	K	M	Flow	Transport Cost	Total Network Cost 1	Total Network Cost 2	Difference
X5	1	2	7	4	6469	1,022,258.94	2,887,823.12	2,634,674.69	-
X5	1	3	7	4	7629	1,205,567.08	3,071,131.26	3,107,115.97	+
X5	1	4	7	4	20036	3,166,174.07	5,031,738.25	8,160,201.27	+
X5	1	5	7	4	4690	741,133.78	2,606,697.96	1,910,128.97	-
X5	1	6	7	4	6194	978,802.27	2,844,366.45	2,522,673.52	-
X5	1	9	7	4	8857	1,399,620.87	3,265,185.05	3,607,252.08	+
X5	7	1	7	4	11688	1,846,987.55	3,712,551.73	4,760,253.17	+
X5	7	2	7	4	3878	612,818.08	2,478,382.26	1,579,420.07	-
X5	7	3	7	4	5951	940,402.37	2,805,966.55	2,423,705.22	-
X5	7	4	7	4	21423	3,385,353.72	5,250,917.90	8,725,094.42	+
X5	7	5	7	4	3102	490,191.25	2,355,755.44	1,263,373.15	-
X5	7	6	7	4	5023	793,755.86	2,659,320.04	2,045,752.19	-
X5	7	9	7	4	6479	1,023,839.18	2,889,403.36	2,638,747.46	-
X5	8	1	7	4	2243	354,448.42	2,220,012.60	913,522.23	-
X5	8	2	7	4	3202	505,993.68	2,371,557.86	1,304,100.84	-
X5	8	3	7	4	5768	911,483.93	2,777,048.11	2,349,173.53	-
X5	8	4	7	4	27342	4,320,699.32	6,186,263.50	11,135,766.78	+
X5	8	5	7	4	1562	246,833.89	2,112,398.07	636,166.62	-
X5	8	6	7	4	3512	554,981.20	2,420,545.38	1,430,356.70	-
X5	8	9	7	4	5615	887,306.22	2,752,870.40	2,286,860.16	-
X5	10	1	7	4	7248	1,145,359.84	3,010,924.02	2,951,943.44	-
X5	10	2	7	4	4198	663,385.84	2,528,950.02	1,709,748.70	-
X5	10	3	7	4	4242	670,338.91	2,535,903.09	1,727,668.89	-
X5	10	4	7	4	15826	2,500,891.94	4,366,456.12	6,445,565.25	+
X5	10	5	7	4	1917	302,932.51	2,168,496.69	780,749.94	-
X5	10	6	7	4	3543	559,879.95	2,425,444.13	1,442,982.29	-
X5	10	9	7	4	4448	702,891.91	2,568,456.09	1,811,567.94	-

Table 6.26 Cost Allocation (FDPTP) of 7-4 Link with Cost Function 3 under Flow Range 2 (CAB10)

X	I	J	K	M	Flow	Transport Cost	Total Network Cost 1	Total Network Cost 2	Difference
X5	1	2	9	6	6469	121,952.04	276,028.92	215,868.04	-
X5	1	3	9	6	7629	143,820.08	297,896.96	254,576.80	-
X5	1	5	9	6	4690	88,414.75	242,491.64	156,503.50	-
X5	1	6	9	6	6194	116,767.80	270,844.69	206,691.40	-
X5	4	1	9	6	20036	377,713.86	531,790.75	668,593.62	+
X5	4	2	9	6	13692	258,118.30	412,195.18	456,896.78	+
X5	4	3	9	6	35135	662,356.59	816,433.47	1,172,441.44	+
X5	4	5	9	6	19094	359,955.51	514,032.39	637,159.44	+
X5	4	6	9	6	35119	662,054.96	816,131.84	1,171,907.53	+
X5	4	7	9	6	21423	403,861.25	557,938.14	714,877.27	+
X5	4	8	9	6	27342	515,444.82	669,521.71	912,392.03	+
X5	4	10	9	6	15826	298,347.95	452,424.84	528,107.54	+
X5	5	2	9	6	3322	62,625.55	216,702.43	110,853.86	-
X5	5	3	9	6	5956	112,281.08	266,357.97	198,749.43	-
X5	5	6	9	6	7284	137,316.22	291,393.10	243,064.28	-
X5	7	1	9	6	11688	220,339.37	374,416.26	390,024.07	+
X5	7	2	9	6	3878	73,107.13	227,184.01	129,407.37	-
X5	7	3	9	6	5951	112,186.82	266,263.71	198,582.58	-
X5	7	5	9	6	3102	58,478.16	212,555.04	103,512.55	-
X5	7	6	9	6	5023	94,692.39	248,769.28	167,615.58	-
X5	8	1	9	6	2243	42,284.50	196,361.38	74,848.05	-
X5	8	2	9	6	3202	60,363.34	214,440.22	106,849.51	-
X5	8	3	9	6	5768	108,736.95	262,813.84	192,475.94	-
X5	8	5	9	6	1562	29,446.45	183,523.33	52,123.34	-
X5	8	6	9	6	3512	66,207.38	220,284.27	117,194.09	-
X5	9	1	9	6	8857	166,970.04	321,046.92	295,554.69	-
X5	9	2	9	6	6699	126,287.94	280,364.82	223,543.05	-
X5	9	3	9	6	16578	312,524.48	466,601.36	553,201.49	+
X5	9	4	9	6	51341	967,868.21	1,121,945.09	1,713,229.43	+
X5	9	5	9	6	7180	135,355.64	289,432.52	239,593.84	-
X5	9	6	9	6	10419	196,416.49	350,493.37	347,678.02	-
X5	9	7	9	6	6479	122,140.55	276,217.44	216,201.74	-
X5	9	8	9	6	5615	105,852.63	259,929.52	187,370.39	-
X5	9	10	9	6	4448	83,852.63	237,929.51	148,428.05	-
X5	10	1	9	6	7248	136,637.56	290,714.44	241,862.97	-
X5	10	2	9	6	4198	79,139.69	233,216.57	140,085.65	-
X5	10	3	9	6	4242	79,969.17	234,046.05	141,553.91	-
X5	10	5	9	6	1917	36,138.82	190,215.71	63,969.55	-
X5	10	6	9	6	3543	66,791.79	220,868.67	118,228.55	-

Table 6.27 Cost Allocation (FDPTP) of 9-6 Link with Cost Function 3 under Flow Range 2 (CAB10)

Simulation Set 2: Cost Function 3 under Flow Range 2 (With Hop Constraint)

The simulation set 2 is based on cost function 3 under flow range 2 of FDPTP with hop constraint. The hop constraint restricts the number of stops between origin and destination to at most 2 in the FDPTP. Cost allocation solutions of the FDPTP (CAB10) are shown in Tables 6.28-47. Each table provides the cost allocations for the link model opened. As can be seen in Figure 5.1, the FDPTP (CAB10) of simulation set 1 opens 20 links out of 90, which are four y^5, four y^4, four y^3, and eight y^2 discounted links.

Table 6.28 provides the fair cost allocation of the O-D pairs for the [1-5] link. Players (O-D pairs) get the fair cost allocation based on their contribution to the flow economies of scale. As can be seen in Table 6.28, Total Network Cost 1 and Total Network Cost 2 are different for each O-D pair which uses [1-5] link. Total Network Cost 2 is obtained with cost allocations whereas Total Network Cost 1 is calculated without cost allocations. Difference column in Table 6.28 shows difference between evenly-distributed fixed costs and proportionally-distributed fixed costs. Positive sign in Difference column shows that the O-D pair of cost allocation pays more fixed costs than the one of without cost allocation solutions. Tables 6-29-6.47 can be explained same as Table 6.28. Tables 6.28-6.35 show the cost allocations of the O-D pairs for y^2 discounted link, Tables 6.36-6.39 show the cost allocations of the O-D pairs for y^3 discounted link, Tables 6.40-6.43 show the cost allocations of the O-D pairs for y^4 discounted link, and Tables 6.44-6.47 show the cost allocations of the O-D pairs for y^5 discounted link.

X	I	J	K	M	Flow	Transport Cost	Total Network Cost 1	Total Network Cost 2	Difference
X2	1	2	1	5	6469	1,934,555.49	2,401,821.36	2,271,195.07	-
X2	1	3	1	5	7629	2,281,453.67	2,748,719.55	2,678,458.37	-
X2	1	4	1	5	20036	5,991,769.01	6,459,034.88	7,034,420.22	+
X2	1	5	1	5	4690	1,402,545.25	1,869,811.13	1,646,607.65	-
X2	1	6	1	5	6194	1,852,316.69	2,319,582.57	2,174,645.58	-
X2	1	9	1	5	8857	2,648,687.27	3,115,953.14	3,109,595.72	-

Table 6.28 Cost Allocation (FDPTP) of 1-5 Link with Cost Function 3 under Flow Range 2 (CAB10)

X	I	J	K	M	Flow	Transport Cost	Total Network Cost 1	Total Network Cost 2	Difference
X2	1	7	1	10	11688	6,500,472.88	7,245,338.60	8,194,057.07	+
X2	1	8	1	10	2243	1,247,481.24	1,992,346.95	1,572,490.59	-
X2	1	10	1	10	7248	4,031,094.07	4,775,959.78	5,081,324.92	+
X2	2	10	1	10	4198	2,334,786.55	3,079,652.26	2,943,074.23	-
X2	3	10	1	10	4242	2,359,257.87	3,104,123.58	2,973,921.12	-
X2	5	10	1	10	1917	1,066,170.99	1,811,036.70	1,343,943.14	-
X2	9	10	1	10	4448	2,473,828.15	3,218,693.86	3,118,340.68	-

Table 6.29 Cost Allocation (FDPTP) of 1-10 Link with Cost Function 3 under Flow Range 2 (CAB10)

X	I	J	K	M	Flow	Transport Cost	Total Network Cost 1	Total Network Cost 2	Difference
X2	2	1	2	6	6469	1,619,232.62	1,879,968.54	1,872,090.22	-
X2	2	3	2	6	12999	3,253,733.93	3,514,469.85	3,761,833.47	+
X2	2	4	2	6	13692	3,427,196.32	3,687,932.24	3,962,383.56	+
X2	2	5	2	6	3322	831,518.13	1,092,254.04	961,367.09	-
X2	2	6	2	6	5576	1,395,708.93	1,656,444.85	1,613,661.31	-
X2	2	7	2	6	3878	970,688.53	1,231,424.45	1,122,270.19	-
X2	2	8	2	6	3202	801,481.35	1,062,217.27	926,639.80	-
X2	2	9	2	6	6699	1,676,803.11	1,937,539.03	1,938,650.85	+
X2	2	10	2	6	4198	1,050,786.60	1,311,522.52	1,214,876.29	-

Table 6.30 Cost Allocation (FDPTP) of 2-6 Link with Cost Function 3 under Flow Range 2 (CAB10)

X	I	J	K	M	Flow	Transport Cost	Total Network Cost 1	Total Network Cost 2	Difference
X2	2	8	4	8	3202	2,324,480.63	3,458,772.00	2,919,143.24	-
X2	3	8	4	8	5768	4,187,259.30	5,321,550.67	5,258,469.14	-
X2	4	8	4	8	16987	12,331,652.86	13,465,944.23	15,486,410.42	+
X2	5	8	4	8	1562	1,133,928.40	2,268,219.78	1,424,016.78	-
X2	6	8	4	8	3512	2,549,524.04	3,683,815.41	3,201,758.60	-
X2	9	8	4	8	5615	4,076,189.49	5,210,480.86	5,118,984.78	-

Table 6.31 Cost Allocation (FDPTP) of 4-8 Link with Cost Function 3 under Flow Range 2 (CAB10)

X	I	J	K	M	Flow	Transport Cost	Total Network Cost 1	Total Network Cost 2	Difference
X2	1	2	6	2	6469	1,619,232.62	1,879,968.54	1,872,090.22	-
X2	3	2	6	2	12999	3,253,733.93	3,514,469.85	3,761,833.47	+
X2	4	2	6	2	13692	3,427,196.32	3,687,932.24	3,962,383.56	+
X2	5	2	6	2	3322	831,518.13	1,092,254.04	961,367.09	-
X2	6	2	6	2	5576	1,395,708.93	1,656,444.85	1,613,661.31	-
X2	7	2	6	2	3878	970,688.53	1,231,424.45	1,122,270.19	-
X2	8	2	6	2	3202	801,481.35	1,062,217.27	926,639.80	-
X2	9	2	6	2	6699	1,676,803.11	1,937,539.03	1,938,650.85	+
X2	10	2	6	2	4198	1,050,786.60	1,311,522.52	1,214,876.29	-

Table 6.32 Cost Allocation (FDPTP) of 6-2 Link with Cost Function 3 under Flow Range 2 (CAB10)

X	I	J	K	M	Flow	Transport Cost	Total Network Cost 1	Total Network Cost 2	Difference
X2	2	5	6	5	3322	600,339.62	812,116.55	714,231.27	-
X2	3	5	6	5	5956	1,076,346.40	1,288,123.34	1,280,542.28	-
X2	4	5	6	5	19094	3,450,597.41	3,662,374.35	4,105,217.30	+
X2	6	5	6	5	7284	1,316,337.67	1,528,114.61	1,566,062.78	+
X2	7	5	6	5	3102	560,582.02	772,358.96	666,931.19	-
X2	8	5	6	5	1562	282,278.89	494,055.83	335,830.60	-
X2	9	5	6	5	7180	1,297,543.18	1,509,320.12	1,543,702.74	+
X2	10	5	6	5	1917	346,433.19	558,210.12	412,155.73	-

Table 6.33 Cost Allocation (FDPTP) of 6-5 Link with Cost Function 3 under Flow Range 2 (CAB10)

X	I	J	K	M	Flow	Transport Cost	Total Network Cost 1	Total Network Cost 2	Difference
X2	1	8	7	8	2243	1,191,259.45	2,436,027.33	1,548,637.29	-
X2	4	8	7	8	10355	5,499,550.44	6,744,318.32	7,149,415.57	+
X2	7	8	7	8	11557	6,137,933.79	7,382,701.67	7,979,313.93	+
X2	10	8	7	8	7095	3,768,161.31	5,012,929.19	4,898,609.70	-

Table 6.34 Cost Allocation (FDPTP) of 7-8 Link with Cost Function 3 under Flow Range 2 (CAB10)

X	I	J	K	M	Flow	Transport Cost	Total Network Cost 1	Total Network Cost 2	Difference
X2	4	10	7	10	15826	2,803,379.66	3,218,545.91	3,236,178.08	+
X2	6	10	7	10	3543	627,598.52	1,042,764.77	724,490.01	-
X2	7	10	7	10	34261	6,068,911.31	6,484,077.56	7,005,857.28	+
X2	8	10	7	10	7095	1,256,791.27	1,671,957.52	1,450,820.39	-

Table 6.35 Cost Allocation (FDPTP) of 7-10 Link with Cost Function 3 under Flow Range 2 (CAB10)

X	I	J	K	M	Flow	Transport Cost	Total Network Cost 1	Total Network Cost 2	Difference
X3	2	7	4	7	3878	1838454.241	3594279.352	2649274.67	-
X3	3	7	4	7	5951	2821207.114	4577032.225	4065454.76	-
X3	4	7	4	7	21423	10156061.17	11911886.28	14635227.24	+
X3	4	8	4	7	10355	4909023.637	6664848.748	7074068.903	+
X3	4	10	4	7	15826	7502675.816	9258500.927	10811609.32	+
X3	5	7	4	7	3102	1470573.764	3226398.875	2119146.474	-
X3	6	7	4	7	5023	2381267.574	4137092.685	3431487.021	-
X3	6	10	4	7	3543	1679639.86	3435464.971	2420417.781	-
X3	9	7	4	7	6479	3071517.542	4827342.653	4426160.543	-

Table 6.36 Cost Allocation (FDPTP) of 4-7 Link with Cost Function 3 under Flow Range 2 (CAB10)

X	I	J	K	M	Flow	Transport Cost	Total Network Cost 1	Total Network Cost 2	Difference
X3	2	1	6	1	6469	2172680.798	3033861.26	2978640.06	-
X3	2	10	6	1	4198	1409941.875	2271122.337	1932961.968	-
X3	3	1	6	1	7629	2562278.839	3423459.301	3512760.089	+
X3	3	10	6	1	4242	1424719.732	2285900.193	1953221.693	-
X3	4	1	6	1	20036	6729298.574	7590479.035	9225542.16	+
X3	5	1	6	1	4690	1575185.182	2436365.644	2159502.532	-
X3	5	10	6	1	1917	643844.3485	1505024.81	882679.3932	-
X3	6	1	6	1	6194	2080319.194	2941499.655	2852016.777	-
X3	7	1	6	1	11688	3925536.121	4786716.583	5381719.743	+
X3	8	1	6	1	2243	753334.8323	1614515.294	1032785.539	-
X3	9	1	6	1	8857	2974715.386	3835895.847	4078190.603	+
X3	9	10	6	1	4448	1493906.97	2355087.432	2048074.043	-
X3	10	1	6	1	7248	2434316.034	3295496.496	3337329.286	+

Table 6.37 Cost Allocation (FDPTP) of 6-1 Link with Cost Function 3 under Flow Range 2 (CAB10)

X	I	J	K	M	Flow	Transport Cost	Total Network Cost 1	Total Network Cost 2	Difference
X3	1	9	6	9	8857	500910.115	736557.115	752845.2916	+
X3	2	9	6	9	6699	378863.8207	614510.8207	569415.2206	-
X3	3	9	6	9	16578	937573.4318	1173220.432	1409130.546	+
X3	5	9	6	9	7180	406066.9104	641713.9104	610300.2364	-
X3	6	9	6	9	10419	589249.4623	824896.4623	885615.343	+
X3	7	9	6	9	6479	366421.6591	602068.6591	550715.2133	-
X3	8	9	6	9	5615	317557.8972	553204.8972	477275.1849	-
X3	10	9	6	9	4448	251557.8854	487204.8854	378080.1464	-

Table 6.38 Cost Allocation (FDPTP) of 6-9 Link with Cost Function 3 under Flow Range 2 (CAB10)

X	I	J	K	M	Flow	Transport Cost	Total Network Cost 1	Total Network Cost 2	Difference
X3	8	1	8	7	2243	893444.59	2368725.034	1332078.476	-
X3	8	2	8	7	3202	1275438.955	2750719.4	1901611.806	-
X3	8	3	8	7	5768	2297542.753	3772823.197	3425514.333	-
X3	8	4	8	7	27342	10891021.84	12366302.28	16237935.66	+
X3	8	5	8	7	1562	622184.7746	2097465.219	927644.4849	-
X3	8	6	8	7	3512	1398919.929	2874200.373	2085715.385	-
X3	8	7	8	7	11557	4603450.346	6078730.79	6863500.2	+
X3	8	9	8	7	5615	2236598.918	3711879.362	3334650.309	-
X3	8	10	8	7	7095	2826120.983	4301401.428	4213596.428	-

Table 6.39 Cost Allocation (FDPTP) of 8-7 Link with Cost Function 3 under Flow Range 2 (CAB10)

X	I	J	K	M	Flow	Transport Cost	Total Network Cost 1	Total Network Cost 2	Difference
X4	3	1	3	6	7629	1,696,905.04	4,091,097.90	3,213,690.36	-
X4	3	2	3	6	12999	2,891,344.69	5,285,537.55	5,475,784.64	+
X4	3	4	3	6	35135	7,815,016.21	10,209,209.07	14,800,499.52	+
X4	3	5	3	6	5956	1,324,782.60	3,718,975.46	2,508,944.79	-
X4	3	6	3	6	14121	3,140,909.18	5,535,102.04	5,948,423.33	+
X4	3	7	3	6	5951	1,323,670.46	3,717,863.32	2,506,838.56	-
X4	3	8	3	6	5768	1,282,966.09	3,677,158.95	2,429,750.43	-
X4	3	9	3	6	16578	3,687,415.36	6,081,608.22	6,983,426.24	+
X4	3	10	3	6	4242	943,540.59	3,337,733.46	1,786,928.11	-

Table 6.40 Cost Allocation (FDPTP) of 3-6 Link with Cost Function 3 under Flow Range 2 (CAB10)

X	I	J	K	M	Flow	Transport Cost	Total Network Cost 1	Total Network Cost 2	Difference
X4	1	2	5	6	6469	584,526.94	1,209,773.13	1,132,740.23	-
X4	1	3	5	6	7629	689,342.40	1,314,588.60	1,335,859.51	+
X4	1	4	5	6	20036	1,810,416.09	2,435,662.29	3,508,360.36	+
X4	1	6	5	6	6194	559,678.44	1,184,924.64	1,084,586.95	-
X4	1	9	5	6	8857	800,302.22	1,425,548.42	1,550,885.79	+
X4	5	1	5	6	4690	423,779.77	1,049,025.97	821,232.29	-
X4	5	2	5	6	3322	300,169.81	925,416.00	581,691.61	-
X4	5	3	5	6	5956	538,173.20	1,163,419.40	1,042,912.47	-
X4	5	4	5	6	19094	1,725,298.71	2,350,544.90	3,343,413.49	+
X4	5	6	5	6	7284	658,168.84	1,283,415.03	1,275,449.03	-
X4	5	7	5	6	3102	280,291.01	905,537.21	543,168.99	-
X4	5	8	5	6	1562	141,139.45	766,385.64	273,510.62	-
X4	5	9	5	6	7180	648,771.59	1,274,017.79	1,257,238.34	-
X4	5	10	5	6	1917	173,216.59	798,462.79	335,672.13	-

Table 6.41 Cost Allocation (FDPTP) of 5-6 Link with Cost Function 3 under Flow Range 2 (CAB10)

X	I	J	K	M	Flow	Transport Cost	Total Network Cost 1	Total Network Cost 2	Difference
X4	1	3	6	3	7629	1,696,905.04	4,091,097.90	3,213,690.36	-
X4	2	3	6	3	12999	2,891,344.69	5,285,537.55	5,475,784.64	+
X4	4	3	6	3	35135	7,815,016.21	10,209,209.07	14,800,499.52	+
X4	5	3	6	3	5956	1,324,782.60	3,718,975.46	2,508,944.79	-
X4	6	3	6	3	14121	3,140,909.18	5,535,102.04	5,948,423.33	+
X4	7	3	6	3	5951	1,323,670.46	3,717,863.32	2,506,838.56	-
X4	8	3	6	3	5768	1,282,966.09	3,677,158.95	2,429,750.43	-
X4	9	3	6	3	16578	3,687,415.36	6,081,608.22	6,983,426.24	+
X4	10	3	6	3	4242	943,540.59	3,337,733.46	1,786,928.11	-

Table 6.42 Cost Allocation (FDPTP) of 6-3 Link with Cost Function 3 under Flow Range 2 (CAB10)

X	I	J	K	M	Flow	Transport Cost	Total Network Cost 1	Total Network Cost 2	Difference
X4	1	7	10	7	11688	1,035,192.13	1,815,201.45	2,072,161.17	+
X4	1	8	10	7	2243	198,659.82	978,669.14	397,660.63	-
X4	10	1	10	7	7248	641,946.66	1,421,955.98	1,284,995.22	-
X4	10	2	10	7	4198	371,811.82	1,151,821.14	744,261.86	-
X4	10	3	10	7	4242	375,708.85	1,155,718.17	752,062.60	-
X4	10	4	10	7	15826	1,401,689.83	2,181,699.15	2,805,785.64	+
X4	10	5	10	7	1917	169,786.39	949,795.71	339,864.22	-
X4	10	6	10	7	3543	313,799.26	1,093,808.58	628,137.15	-
X4	10	7	10	7	34261	3,034,455.66	3,814,464.97	6,074,119.93	+
X4	10	8	10	7	7095	628,395.64	1,408,404.95	1,257,869.91	-
X4	10	9	10	7	4448	393,954.02	1,173,963.34	788,584.26	-

Table 6.43 Cost Allocation (FDPTP) of 10-7 Link with Cost Function 3 under Flow Range 2 (CAB10)

X	I	J	K	M	Flow	Transport Cost	Total Network Cost 1	Total Network Cost 2	Difference
X5	4	1	4	9	20036	949,988.11	3,468,860.04	2,686,107.34	-
X5	4	2	4	9	13692	649,193.31	3,168,065.25	1,835,605.00	-
X5	4	3	4	9	35135	1,665,893.00	4,184,764.94	4,710,340.46	+
X5	4	5	4	9	19094	905,324.06	3,424,196.00	2,559,819.01	-
X5	4	6	4	9	35119	1,665,134.37	4,184,006.31	4,708,195.44	+
X5	4	9	4	9	51341	2,434,285.25	4,953,157.19	6,882,982.49	+

Table 6.44 Cost Allocation (FDPTP) of 4-9 Link with Cost Function 3 under Flow Range 2 (CAB10)

X	I	J	K	M	Flow	Transport Cost	Total Network Cost 1	Total Network Cost 2	Difference
X5	1	4	6	4	20036	1,247,469.81	2,239,761.19	2,634,669.23	+
X5	2	4	6	4	13692	852,483.36	1,844,774.74	1,800,453.74	-
X5	3	4	6	4	35135	2,187,554.99	3,179,846.37	4,620,138.92	+
X5	5	4	6	4	19094	1,188,819.55	2,181,110.93	2,510,799.27	+
X5	6	4	6	4	35119	2,186,558.81	3,178,850.19	4,618,034.97	+
X5	7	4	6	4	21423	1,333,826.40	2,326,117.78	2,817,055.24	+
X5	8	4	6	4	27342	1,702,351.75	2,694,643.13	3,595,384.61	+
X5	9	4	6	4	51341	3,196,563.56	4,188,854.95	6,751,175.52	+
X5	10	4	6	4	15826	985,349.23	1,977,640.61	2,081,067.84	+
X5	2	7	6	4	3878	241,449.79	1,233,741.17	509,944.46	-
X5	3	7	6	4	5951	370,517.71	1,362,809.09	782,537.26	-
X5	5	7	6	4	3102	193,134.92	1,185,426.31	407,902.97	-
X5	6	7	6	4	5023	312,739.11	1,305,030.49	660,508.26	-
X5	9	7	6	4	6479	403,391.74	1,395,683.12	851,967.55	-
X5	2	8	6	4	3202	199,361.07	1,191,652.45	421,052.65	-
X5	3	8	6	4	5768	359,123.87	1,351,415.25	758,473.35	-
X5	5	8	6	4	1562	97,252.34	1,089,543.72	205,397.95	-
X5	6	8	6	4	3512	218,662.11	1,210,953.49	461,816.65	-
X5	9	8	6	4	5615	349,597.87	1,341,889.25	738,354.35	-
X5	6	10	6	4	3543	220,592.21	1,212,883.59	465,893.05	-

Table 6.45 Cost Allocation (FDPTP) of 6-4 Link with Cost Function 3 under Flow Range 2 (CAB10)

X	I	J	K	M	Flow	Transport Cost	Total Network Cost 1	Total Network Cost 2	Difference
X5	7	1	7	6	11688	2,360,249.47	5,425,377.40	7,436,354.46	+
X5	8	1	7	6	2243	452,946.58	3,518,074.50	1,427,082.74	-
X5	10	1	7	6	7248	1,463,645.46	4,528,773.39	4,611,455.95	+
X5	7	2	7	6	3878	783,114.94	3,848,242.87	2,467,332.53	-
X5	8	2	7	6	3202	646,604.96	3,711,732.89	2,037,235.37	-
X5	10	2	7	6	4198	847,735.05	3,912,862.98	2,670,928.82	-
X5	7	3	7	6	5951	1,201,732.09	4,266,860.01	3,786,254.74	-
X5	8	3	7	6	5768	1,164,777.46	4,229,905.39	3,669,823.11	-
X5	10	3	7	6	4242	856,620.32	3,921,748.25	2,698,923.31	-
X5	7	4	7	6	21423	4,326,114.35	7,391,242.27	13,630,135.32	+
X5	8	4	7	6	27342	5,521,384.42	8,586,512.35	17,396,030.43	+
X5	10	4	7	6	15826	3,195,868.26	6,260,996.18	10,069,108.97	+
X5	7	5	7	6	3102	626,411.18	3,691,539.11	1,973,611.53	-
X5	8	5	7	6	1562	315,426.91	3,380,554.83	993,804.39	-
X5	10	5	7	6	1917	387,114.84	3,452,242.77	1,219,669.02	-
X5	7	6	7	6	5023	1,014,333.77	4,079,461.70	3,195,825.50	-
X5	8	6	7	6	3512	709,205.69	3,774,333.62	2,234,469.27	-
X5	10	6	7	6	3543	715,465.77	3,780,593.70	2,254,192.66	-
X5	7	9	7	6	6479	1,308,355.27	4,373,483.19	4,122,188.62	-
X5	8	9	7	6	5615	1,133,880.97	4,199,008.90	3,572,478.64	-
X5	10	9	7	6	4448	898,219.51	3,963,347.44	2,829,988.42	-

Table 6.46 Cost Allocation (FDPTP) of 7-6 Link with Cost Function 3 under Flow Range 2 (CAB10)

X	I	J	K	M	Flow	Transport Cost	Total Network Cost 1	Total Network Cost 2	Difference
X5	4	1	9	6	20036	377,713.86	806,928.04	877,922.82	+
X5	9	1	9	6	8857	166,970.04	596,184.22	388,089.56	-
X5	4	2	9	6	13692	258,118.30	687,332.48	599,946.06	-
X5	9	2	9	6	6699	126,287.94	555,502.12	293,531.89	-
X5	4	3	9	6	35135	662,356.59	1,091,570.77	1,539,519.78	+
X5	9	3	9	6	16578	312,524.48	741,738.66	726,402.70	-
X5	9	4	9	6	51341	967,868.21	1,397,082.39	2,249,622.45	+
X5	4	5	9	6	19094	359,955.51	789,169.68	836,646.95	+
X5	9	5	9	6	7180	135,355.64	564,569.82	314,608.00	-
X5	4	6	9	6	35119	662,054.96	1,091,269.14	1,538,818.70	+
X5	9	6	9	6	10419	196,416.49	625,630.67	456,532.13	-
X5	9	7	9	6	6479	122,140.55	551,354.73	283,892.09	-
X5	9	8	9	6	5615	105,852.63	535,066.81	246,033.97	-
X5	9	10	9	6	4448	83,852.63	513,066.81	194,899.22	-

Table 6.47 Cost Allocation (FDPTP) of 9-6 Link with Cost Function 3 under Flow Range 2 (CAB10)

6.3 Numerical Results of Cost Allocation Comparison

This section compares the cost allocations between hub networks and point-to-point networks. It is shown that some O-D pairs prefer a certain network structure. This contributes to understanding a fundamental question about the merits of hub networks versus point-to-point networks. The computational results from the FDMAP and the FDPTP are used to compare the networks. The numerical analysis of cost comparison is based on CAB10 data.

As can be seen in Figure 6.2, there are many possible ways for each O-D pair to travel depending on network configuration. Under the hub network structure, the [A-B] pair's route depends on how interhub links are connected each other. In the case of 3-hub problem, there are four possible interhub connections available as can be seen in Figure 6.3. Each O-D pair finds its own cheapest route out of all possible ways achieving flow economies of scale. Under the point-to-point network structure, the [A-B] pair's route is not constrained by hub nodes because there are no hubs chosen. Its best route is basically found by flow economies of scale on opened links.

Table 6.48 shows the results of the FDMAP (CAB10) with cost function 3 (see Table 5.3). Three hubs are open, but hubs are not fully connected each other. To achieve flow economies of scale, the model closes a partial interhub link as can be seen in Figure 6.4. Under the same cost function, it is shown that the flow-based discount point-to-point model provides the cheaper network cost than the flow-based discount multiple hub location model. Due to the large number of opened links, the point-to-point network pays higher fixed costs for opened links than the hub network by offsetting the cheaper transportation costs. Table 6.49 shows the results of the FDPTP (CAB10) with cost function 3. Table 6.50 presents cost differences of O-D pairs between hub network and point-to-point network. Out of 90 directed O-D pairs, 67 O-D pairs can save their costs by point-to-point network as can be seen in Table 6.51 whereas 23 O-D pairs maintain their cheapest network costs by hub network as can be seen in Table 6.52. Figure 6.5 illustrates O-D pairs that save the their costs by a hub network configuration. Cost allocation makes it possible to capture the market share of each network provider with respect to the cost of each O-D pair. The FDMAP captures 28% of market shares with respect to passenger flows as can be seen in Figure 6.5. Out of 90 O-D pairs, the FDMAP provides the cheaper costs to 23 O-D pairs that the FDPTP does.

It is shown that O-D pairs with multiple interhub links always favor a point-to-point network. Those pairs includes as [2→7], [3→7], [6→7], [6→10], and [7→9] O-D pairs. There are some O-D pairs with single interhub link that favors a hub network. For example, O-D pairs such as [2→10], [4→6], [5→10], and [8→9] always prefer hub network to point-to-point network. O-D pairs such as [7→6], [7→4], and [4→9] pays relatively high amount of fixed costs. Their fixed cost percentage is more than 65% of their total network costs. The most cost savings by point-to-point network. On the other hand, O-D pairs such as [1→5], [2→5], [2→6], [6→2], and [7→10] pay relatively low amount of fixed costs. Their fixed cost percentage is less than 15% of their total network costs.

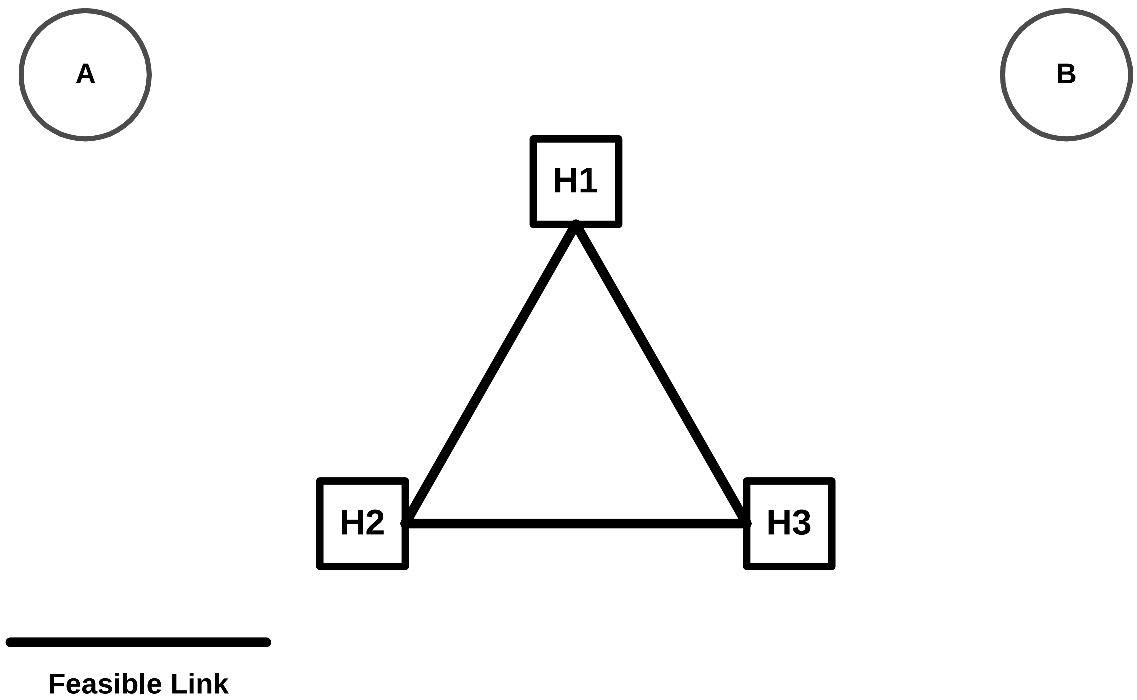

Figure 6.2 Competitive Route in Hub Network

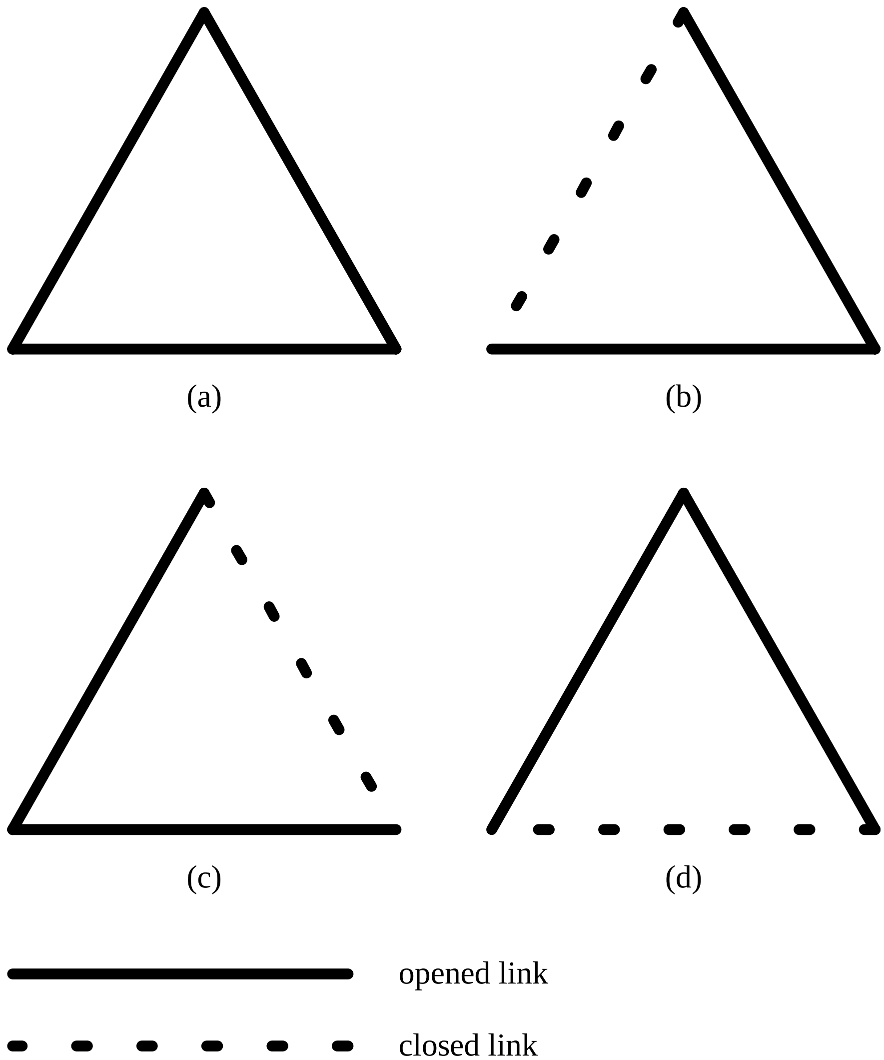

Figure 6.3 Interhub Link Connection

CAB10	LOCATION			DISCOUNT			FLOW				COST		
p=3	Hub1	Hub2	Hub3	Interhub 1_2	Interhub 1_3	Interhub 2_3	Interhub 1_2	Interhub 1_3	Interhub 2_3	Total Flow	Obj	Transport Cost	Fixed Cost
C.F.3	7	6	4	1	3	5	0	143,180	373,396	516,576	624.08	552.71	71.37

**Node 4 is Chicago, 6 is Cleveland, and 7 is Dallas.*
*** C.F.3 is based on Flow Range 2.*
**** Obj is the normalized total network cost, which is equivalent to 623,470,754.44[624.08*999,026] where 999,026 is total network flow.*

Table 6.48 Solution of the FDMAP (CAB10) with Cost function 3 under Flow Range 2

CAB10	COST		
C.F.3	Obj	Transport Cost	Fixed Cost
FDPTP	588.91	352.91	236.01

** C.F.3 is based on Flow Range 2.*
*** Obj is the normalized total network cost.*

Table 6.49 Solution of the FDPTP (CAB10) with Cost function 3 under Flow Range 2

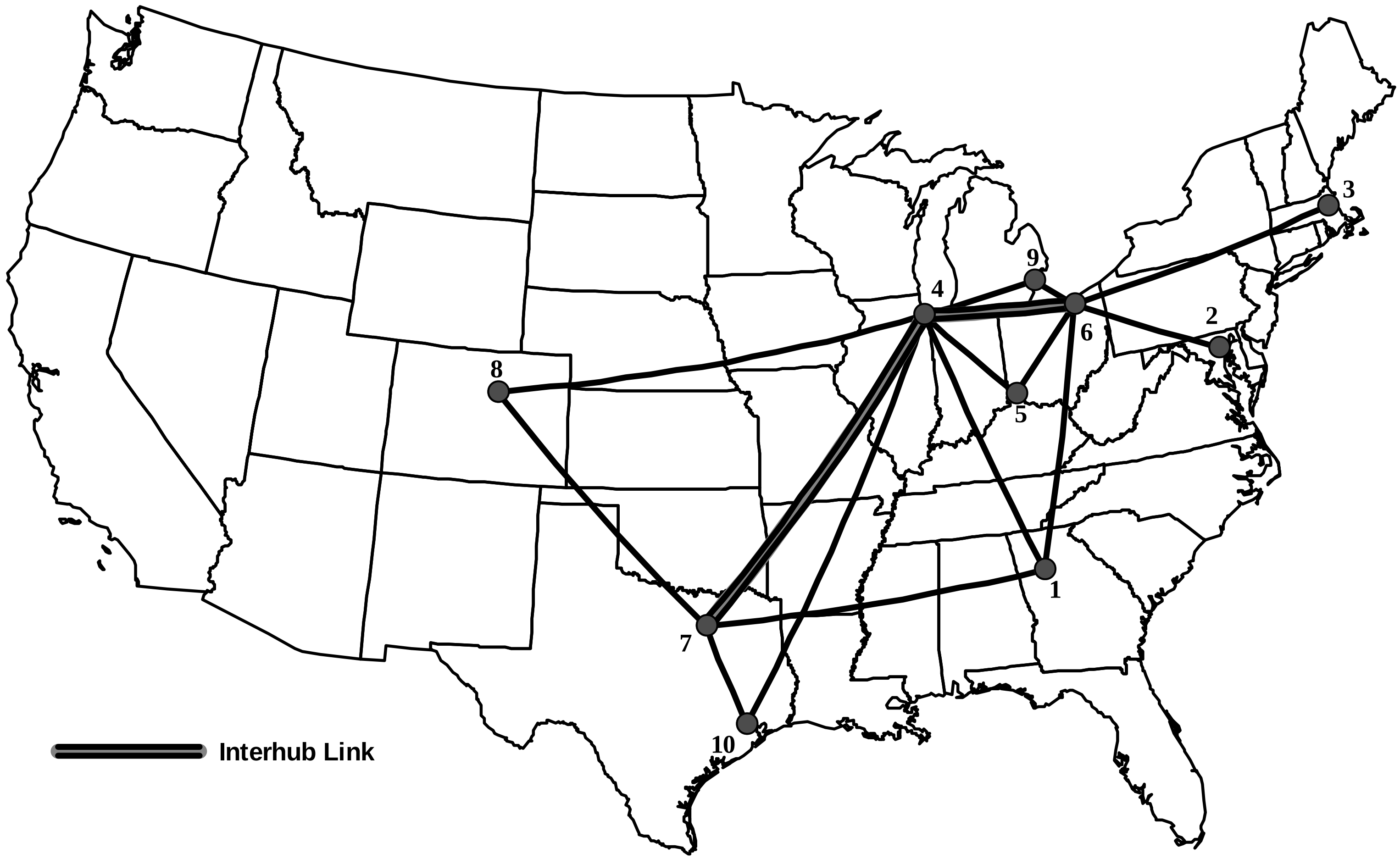

Figure 6.4 Result (CAB10) of the FDMAP with Cost function 3 under Flow Range 2

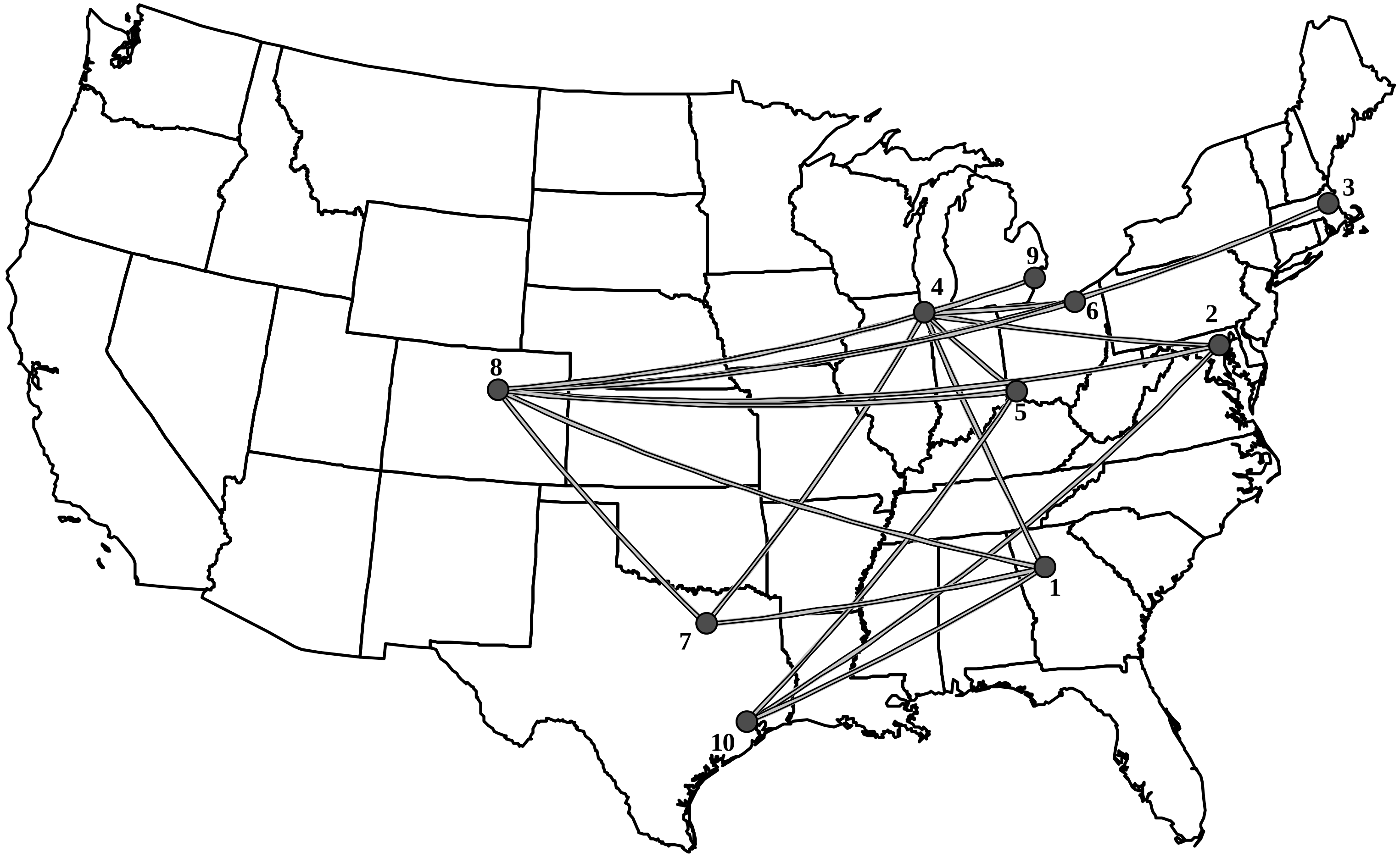

Figure 6.5 Cost Allocation (CAB10) of Favorable O-D Pairs by the FDMAP with Cost function 3 under Flow Range 2

I	J	Flow	Cost Allocation (FDPTP)	Cost Allocation (FDMAP)	Difference
1	2	6469	5,276,025.51	5,645,175.44	369,149.93
1	3	7629	7,228,008.24	8,512,727.34	1,284,719.10
1	4	20036	13,177,449.81	11,973,457.50	(1,203,992.31)
1	5	4690	1,646,607.65	3,684,758.06	2,038,150.41
1	6	6194	3,259,232.53	3,467,198.66	207,966.13
1	7	11688	10,266,218.23	8,287,043.29	(1,979,174.94)
1	8	2243	3,518,788.51	3,079,409.54	(439,378.97)
1	9	8857	5,413,326.81	5,792,709.17	379,382.36
1	10	7248	5,081,324.92	6,743,854.49	1,662,529.57
2	1	6469	4,850,730.28	5,645,175.44	794,445.16
2	3	12999	9,237,618.11	11,295,529.15	2,057,911.04
2	4	13692	5,762,837.29	6,295,392.18	532,554.89
2	5	3322	1,675,598.36	1,789,822.18	114,223.82
2	6	5576	1,613,661.31	1,744,636.17	130,974.85
2	7	3878	4,281,489.32	4,696,545.07	415,055.75
2	8	3202	4,266,835.69	4,377,836.09	111,000.40
2	9	6699	2,508,066.07	2,727,443.59	219,377.52
2	10	4198	6,090,912.49	5,843,630.55	(247,281.94)
3	1	7629	6,726,450.45	8,512,727.34	1,786,276.89
3	2	12999	9,237,618.11	11,295,529.15	2,057,911.04
3	4	35135	19,420,638.44	24,698,979.75	5,278,341.31
3	5	5956	3,789,487.06	4,657,389.50	867,902.43
3	6	14121	5,948,423.33	7,852,272.94	1,903,849.61
3	7	5951	7,354,830.58	8,654,310.34	1,299,479.77
3	8	5768	8,446,692.92	9,288,826.23	842,133.31
3	9	16578	8,392,556.79	10,781,160.79	2,388,604.00
3	10	4242	6,714,070.92	6,936,480.08	222,409.16
4	1	20036	12,789,572.32	11,973,457.50	(816,114.82)
4	2	13692	6,397,934.61	6,295,392.18	(102,542.43)
4	3	35135	21,050,359.76	24,698,979.75	3,648,619.99
4	5	19094	7,501,683.25	4,869,548.55	(2,632,134.71)
4	6	35119	6,247,014.14	5,159,088.77	(1,087,925.37)
4	7	21423	14,635,227.24	16,094,839.84	1,459,612.59
4	8	27342	29,709,894.89	24,811,035.82	(4,898,859.07)
4	9	51341	6,882,982.49	12,381,491.36	5,498,508.87
4	10	15826	14,047,787.40	15,394,106.25	1,346,318.86
5	1	4690	2,980,734.82	3,684,758.06	704,023.24
5	2	3322	1,543,058.70	1,789,822.18	246,763.48
5	3	5956	3,551,857.26	4,657,389.50	1,105,532.23
5	4	19094	5,854,212.76	4,869,548.55	(984,664.22)
5	6	7284	1,275,449.03	1,645,422.09	369,973.06
5	7	3102	3,070,218.43	3,121,598.93	51,380.50
5	8	1562	1,902,925.36	1,815,767.83	(87,157.53)
5	9	7180	1,867,538.58	2,298,707.16	431,168.58
5	10	1917	2,562,294.66	2,353,577.89	(208,716.77)

I	J	Flow	Cost Allocation (FDPTP)	Cost Allocation (FDMAP)	Difference
6	1	6194	2,852,016.78	3,467,198.66	615,181.88
6	2	5576	1,613,661.31	1,744,636.17	130,974.85
6	3	14121	5,948,423.33	7,852,272.94	1,903,849.61
6	4	35119	4,618,034.97	5,159,088.77	541,053.80
6	5	7284	1,566,062.78	1,645,422.09	79,359.31
6	7	5023	4,091,995.28	4,511,613.02	419,617.74
6	8	3512	3,663,575.25	3,702,828.61	39,253.37
6	9	10419	885,615.34	982,082.44	96,467.09
6	10	3543	3,610,800.84	3,966,788.60	355,987.76
7	1	11688	12,818,074.20	8,287,043.29	(4,531,030.91)
7	2	3878	3,589,602.72	4,696,545.07	1,106,942.36
7	3	5951	6,293,093.29	8,654,310.34	2,361,217.05
7	4	21423	16,447,190.56	16,094,839.84	(352,350.72)
7	5	3102	2,640,542.71	3,121,598.93	481,056.22
7	6	5023	3,195,825.50	4,511,613.02	1,315,787.52
7	8	11557	7,979,313.93	7,672,417.24	(306,896.69)
7	9	6479	4,672,903.83	6,430,081.78	1,757,177.95
7	10	34261	7,005,857.28	7,586,139.14	580,281.87
8	1	2243	3,791,946.75	3,079,409.54	(712,537.21)
8	2	3202	4,865,486.97	4,377,836.09	(487,650.88)
8	3	5768	9,525,087.87	9,288,826.23	(236,261.65)
8	4	27342	37,229,350.70	24,811,035.82	(12,418,314.88)
8	5	1562	2,257,279.47	1,815,767.83	(441,511.64)
8	6	3512	4,320,184.66	3,702,828.61	(617,356.04)
8	7	11557	6,863,500.20	7,672,417.24	808,917.04
8	9	5615	7,384,404.13	6,449,360.73	(935,043.40)
8	10	7095	5,664,416.82	6,281,190.73	616,773.91
9	1	8857	4,466,280.16	5,792,709.17	1,326,429.01
9	2	6699	2,232,182.74	2,727,443.59	495,260.85
9	3	16578	7,709,828.94	10,781,160.79	3,071,331.85
9	4	51341	9,000,797.98	12,381,491.36	3,380,693.38
9	5	7180	1,858,310.74	2,298,707.16	440,396.41
9	6	10419	456,532.13	982,082.44	525,550.30
9	7	6479	5,562,020.19	6,430,081.78	868,061.59
9	8	5615	6,103,373.10	6,449,360.73	345,987.63
9	10	4448	5,361,313.94	5,381,102.15	19,788.22
10	1	7248	9,233,780.45	6,743,854.49	(2,489,925.97)
10	2	4198	4,630,066.96	5,843,630.55	1,213,563.59
10	3	4242	5,237,914.01	6,936,480.08	1,698,566.07
10	4	15826	14,955,962.45	15,394,106.25	438,143.80
10	5	1917	1,971,688.97	2,353,577.89	381,888.92
10	6	3543	2,882,329.81	3,966,788.60	1,084,458.79
10	7	34261	6,074,119.93	7,586,139.14	1,512,019.22
10	8	7095	6,156,479.61	6,281,190.73	124,711.12
10	9	4448	3,996,652.83	5,381,102.15	1,384,449.33

Table 6.50 Cost Allocation (CAB10) of O-D Pairs with Cost function 3 under Flow Range 2

I	J	Flow	Cost Allocation (FDPTP)	Cost Allocation (FDMAP)	Difference
1	2	6469	5,276,025.51	5,645,175.44	369,149.93
1	3	7629	7,228,008.24	8,512,727.34	1,284,719.10
1	5	4690	1,646,607.65	3,684,758.06	2,038,150.41
1	6	6194	3,259,232.53	3,467,198.66	207,966.13
1	9	8857	5,413,326.81	5,792,709.17	379,382.36
1	10	7248	5,081,324.92	6,743,854.49	1,662,529.57
2	1	6469	4,850,730.28	5,645,175.44	794,445.16
2	3	12999	9,237,618.11	11,295,529.15	2,057,911.04
2	4	13692	5,762,837.29	6,295,392.18	532,554.89
2	5	3322	1,675,598.36	1,789,822.18	114,223.82
2	6	5576	1,613,661.31	1,744,636.17	130,974.85
2	7	3878	4,281,489.32	4,696,545.07	415,055.75
2	8	3202	4,266,835.69	4,377,836.09	111,000.40
2	9	6699	2,508,066.07	2,727,443.59	219,377.52
3	1	7629	6,726,450.45	8,512,727.34	1,786,276.89
3	2	12999	9,237,618.11	11,295,529.15	2,057,911.04
3	4	35135	19,420,638.44	24,698,979.75	5,278,341.31
3	5	5956	3,789,487.06	4,657,389.50	867,902.43
3	6	14121	5,948,423.33	7,852,272.94	1,903,849.61
3	7	5951	7,354,830.58	8,654,310.34	1,299,479.77
3	8	5768	8,446,692.92	9,288,826.23	842,133.31
3	9	16578	8,392,556.79	10,781,160.79	2,388,604.00
3	10	4242	6,714,070.92	6,936,480.08	222,409.16
4	3	35135	21,050,359.76	24,698,979.75	3,648,619.99
4	7	21423	14,635,227.24	16,094,839.84	1,459,612.59
4	9	51341	6,882,982.49	12,381,491.36	5,498,508.87
4	10	15826	14,047,787.40	15,394,106.25	1,346,318.86
5	1	4690	2,980,734.82	3,684,758.06	704,023.24
5	2	3322	1,543,058.70	1,789,822.18	246,763.48
5	3	5956	3,551,857.26	4,657,389.50	1,105,532.23
5	6	7284	1,275,449.03	1,645,422.09	369,973.06
5	7	3102	3,070,218.43	3,121,598.93	51,380.50
5	9	7180	1,867,538.58	2,298,707.16	431,168.58

I	J	Flow	Cost Allocation (FDPTP)	Cost Allocation (FDMAP)	Difference
6	1	6194	2,852,016.78	3,467,198.66	615,181.88
6	2	5576	1,613,661.31	1,744,636.17	130,974.85
6	3	14121	5,948,423.33	7,852,272.94	1,903,849.61
6	4	35119	4,618,034.97	5,159,088.77	541,053.80
6	5	7284	1,566,062.78	1,645,422.09	79,359.31
6	7	5023	4,091,995.28	4,511,613.02	419,617.74
6	8	3512	3,663,575.25	3,702,828.61	39,253.37
6	9	10419	885,615.34	982,082.44	96,467.09
6	10	3543	3,610,800.84	3,966,788.60	355,987.76
7	2	3878	3,589,602.72	4,696,545.07	1,106,942.36
7	3	5951	6,293,093.29	8,654,310.34	2,361,217.05
7	5	3102	2,640,542.71	3,121,598.93	481,056.22
7	6	5023	3,195,825.50	4,511,613.02	1,315,787.52
7	9	6479	4,672,903.83	6,430,081.78	1,757,177.95
7	10	34261	7,005,857.28	7,586,139.14	580,281.87
8	7	11557	6,863,500.20	7,672,417.24	808,917.04
8	10	7095	5,664,416.82	6,281,190.73	616,773.91
9	1	8857	4,466,280.16	5,792,709.17	1,326,429.01
9	2	6699	2,232,182.74	2,727,443.59	495,260.85
9	3	16578	7,709,828.94	10,781,160.79	3,071,331.85
9	4	51341	9,000,797.98	12,381,491.36	3,380,693.38
9	5	7180	1,858,310.74	2,298,707.16	440,396.41
9	6	10419	456,532.13	982,082.44	525,550.30
9	7	6479	5,562,020.19	6,430,081.78	868,061.59
9	8	5615	6,103,373.10	6,449,360.73	345,987.63
9	10	4448	5,361,313.94	5,381,102.15	19,788.22
10	2	4198	4,630,066.96	5,843,630.55	1,213,563.59
10	3	4242	5,237,914.01	6,936,480.08	1,698,566.07
10	4	15826	14,955,962.45	15,394,106.25	438,143.80
10	5	1917	1,971,688.97	2,353,577.89	381,888.92
10	6	3543	2,882,329.81	3,966,788.60	1,084,458.79
10	7	34261	6,074,119.93	7,586,139.14	1,512,019.22
10	8	7095	6,156,479.61	6,281,190.73	124,711.12
10	9	4448	3,996,652.83	5,381,102.15	1,384,449.33

Table 6.51 Cost Allocation (CAB10) of Favorable O-D Pairs by Point-To-Point Network with Cost function 3 under Flow Range 2

I	J	Flow	Cost Allocation (FDPTP)	Cost Allocation (FDMAP)	Difference
1	4	20036	13,177,449.81	11,973,457.50	(1,203,992.31)
1	7	11688	10,266,218.23	8,287,043.29	(1,979,174.94)
1	8	2243	3,518,788.51	3,079,409.54	(439,378.97)
2	10	4198	6,090,912.49	5,843,630.55	(247,281.94)
4	1	20036	12,789,572.32	11,973,457.50	(816,114.82)
4	2	13692	6,397,934.61	6,295,392.18	(102,542.43)
4	5	19094	7,501,683.25	4,869,548.55	(2,632,134.71)
4	6	35119	6,247,014.14	5,159,088.77	(1,087,925.37)
4	8	27342	29,709,894.89	24,811,035.82	(4,898,859.07)
5	4	19094	5,854,212.76	4,869,548.55	(984,664.22)
5	8	1562	1,902,925.36	1,815,767.83	(87,157.53)
5	10	1917	2,562,294.66	2,353,577.89	(208,716.77)
7	1	11688	12,818,074.20	8,287,043.29	(4,531,030.91)
7	4	21423	16,447,190.56	16,094,839.84	(352,350.72)
7	8	11557	7,979,313.93	7,672,417.24	(306,896.69)
8	1	2243	3,791,946.75	3,079,409.54	(712,537.21)
8	2	3202	4,865,486.97	4,377,836.09	(487,650.88)
8	3	5768	9,525,087.87	9,288,826.23	(236,261.65)
8	4	27342	37,229,350.70	24,811,035.82	(12,418,314.88)
8	5	1562	2,257,279.47	1,815,767.83	(441,511.64)
8	6	3512	4,320,184.66	3,702,828.61	(617,356.04)
8	9	5615	7,384,404.13	6,449,360.73	(935,043.40)
10	1	7248	9,233,780.45	6,743,854.49	(2,489,925.97)

Table 6.52 Cost Allocation (CAB10) of Favorable O-D Pairs by Hub Network with Cost function 3 under Flow Range 2

In this particular example, it is difficult to generalize the O-D pair's behaviors for its favored network. However, there are some patterns of cost allocations that can be generalized. The O-D pairs with multiple interhub links always prefer the FDPTP to the FDMAP. Examples include [2→7], [3→7], [6→7], [6→10], and [7→9] O-D pairs. The FDPTP finds the exact same routes for [6→4→7] multiple interhub link O-D pairs as does the FDMAP. However, the FDPTP produces more interhub link flows than the

FDMAP, so the sharing fixed cost by the FDPTP is much lower than the one by the FDMAP. Notice that the O-D pair pays fixed costs proportional to player's flow contribution to the link.

A noticeable pattern of cost allocation is that the [6→4] interhub link users prefer the FDPTP to the FDMAP with an exception of the [2→10] O-D pair. The main reason for this is that the FDPTP has built the [6→4] link with the same discount as the FDMAP, so the hub network may lose its merit of interhub link usage. Another clear pattern is that all the O-D pairs originating from node 3 prefer the FDPTP to the FDMAP. Notice that node 3 is only connected to hub node 6, and its average distance to 3 opened hub locations is longer than any other nodes. So these pairs already had a disadvantage under the hub network in this particular example. The similar but weak pattern can be found with node 2. It is only connected to a single hub location, but its average distance to 3 opened hub nodes is much shorter than node 3. On the other hand, the O-D pairs originating from node 8 prefer the FDMAP to the FDPTP with exceptions of [8→7], and [8→10] pairs. Its average distance to 3 opened hub locations is also very long. However, it is connected multiple hub locations, so it can take advantages of the interhub link discounts.

Results of the FDMAP and the FDPTP show the similar network structure as can be seen in Table 6.53. For example, the degree of node shows a similar pattern between two network solutions although the ranks of node 6 and node 6 are interchanged each other. The hub nodes of the FDMAP still maintain their top ranks in the FDPTP solution.

ID	Average Distance to Hub Locations	Rank of Average Distance to Hub Locations	Degree of Node	
			FDMAP	FDPTP
1	1866.386	6	6	3
2	2122.4107	7	2	2
3	2955.6746	10	2	2
4*	1101.4284	1	14	4
5	1275.0983	2	4	3
6*	1320.9963	4	12	11
7*	1799.8105	5	8	6
8	2788.1775	9	4	3
9	1314.0669	3	4	3
10	2258.2133	8	4	3

** Hub node.*

Table 6.53 Degree of Node (CAB10) of the FDMAP and the FDPTP with Cost function 3 under Flow Range 2

To generalize characteristics of O-D pairs, 100 sets of simulation are generated. Each simulation set is randomly selected from CAB25 data set. Two of them are chosen to see the geographical effect of spatial arrangement on data set because they are geographically extensive as can be seen in Table 6.54. Notice that the CAB10 data set is geographically intensive in East region.

City	State	Id	SIMULTATION7	SIMULATION8
Atlanta	Georgia	1	1	0
Baltimore	Maryland	2	0	1
Boston	Massachusetts	3	1	0
Chicago	Illinois	4	0	1
Cincinnati	Ohio	5	0	0
Cleveland	Ohio	6	0	0
Dallas	Texas	7	0	1
Denver	Colorado	8	1	1
Detroit	Michigan	9	0	0
Houston	Texas	10	0	1
Kansas City	Missouri	11	0	1
Los Angeles	California	12	0	1
Memphis	Tennessee	13	0	0
Miami	Florida	14	0	1
Minneapolis	Minnesota	15	1	1
New Orleans	Louisiana	16	0	0
New York	New York	17	1	0
Philadelphia	Pennsylvania	18	1	0
Phoenix	Arizona	19	0	1
Pittsburgh	Pennsylvania	20	1	0
St. Louis	Missouri	21	0	0
San Francisco	California	22	1	0
Seattle	Washington	23	0	0
Tampa	Florida	24	1	0
Washington	District of Columbia	25	1	0

** 1 is selected for simulation set.*

Table 6.54 Selected Nodes for Simulation Set 7 and 8

Simulation Set 7: Randomly Selected Subset (10-Node) of CAB25 with Cost Function 3 under Flow Range 2

Table 6.55 shows that the FDPTP obtains a cheaper total network cost than the FDMAP. Proportion of fixed cost to the total network cost in the FDPTP is much higher than the one in the FDMAP. As can be seen in Figures 6.6 and 6.7, the number of links built as a result is different each other. The FDPTP tends to build more links to transport O-D flows achieving flow economies of scale than does the FDMAP. As stated in Chapter 4, the FDMAP does not provide a fully connected hub network. It closes the longest interhub link to achieve flow economies of scale. One interesting observation from the results is that a node 25 (Washington D.C.) has significantly changed its role in the network. For example, it acts as spoke node in the FDMAP whereas it turns into a node with the largest flow with other nodes in the FDPTP.

Simulation Set 7 (10-Node)	Cost		
CF3	Obj	Transport Cost	Fixed Cost
FDMAP	642.81	483.69	159.12
FDPTP	474.57	226.45	248.12

* *Total network flow is 2,089,852.*
** *Cost function 3 is based on flow range 2.*

Table 6.55 Solutions for Simulation Set 7

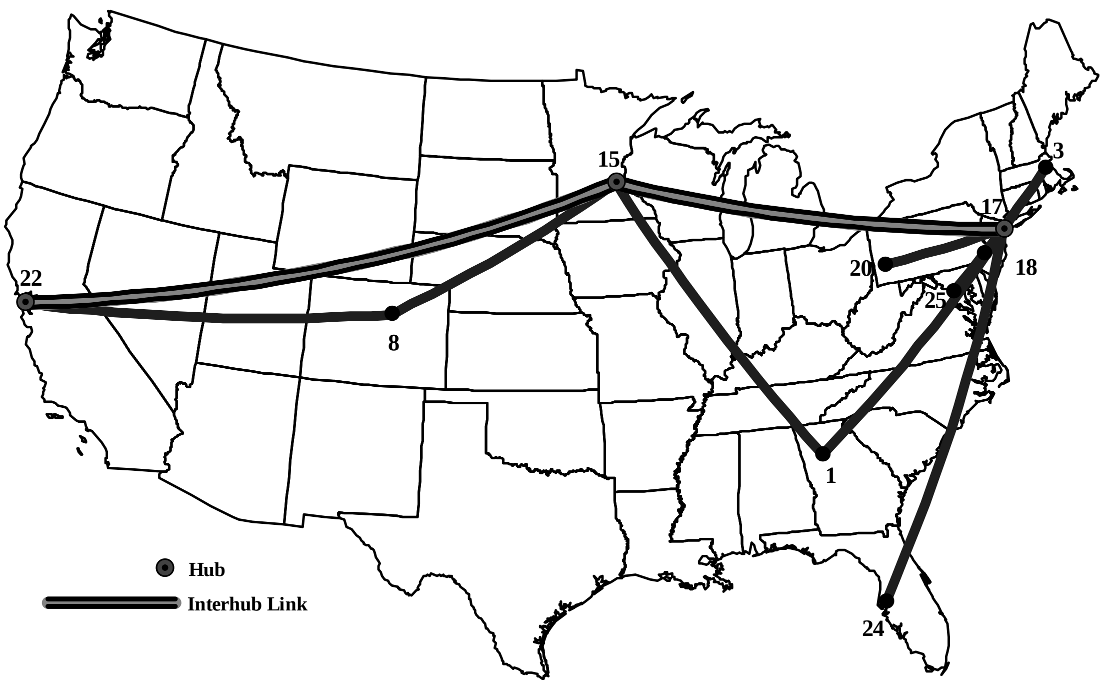

Figure 6.6 Result (Simulation 7) of the FDMAP with Cost function 3 under Flow Range 2

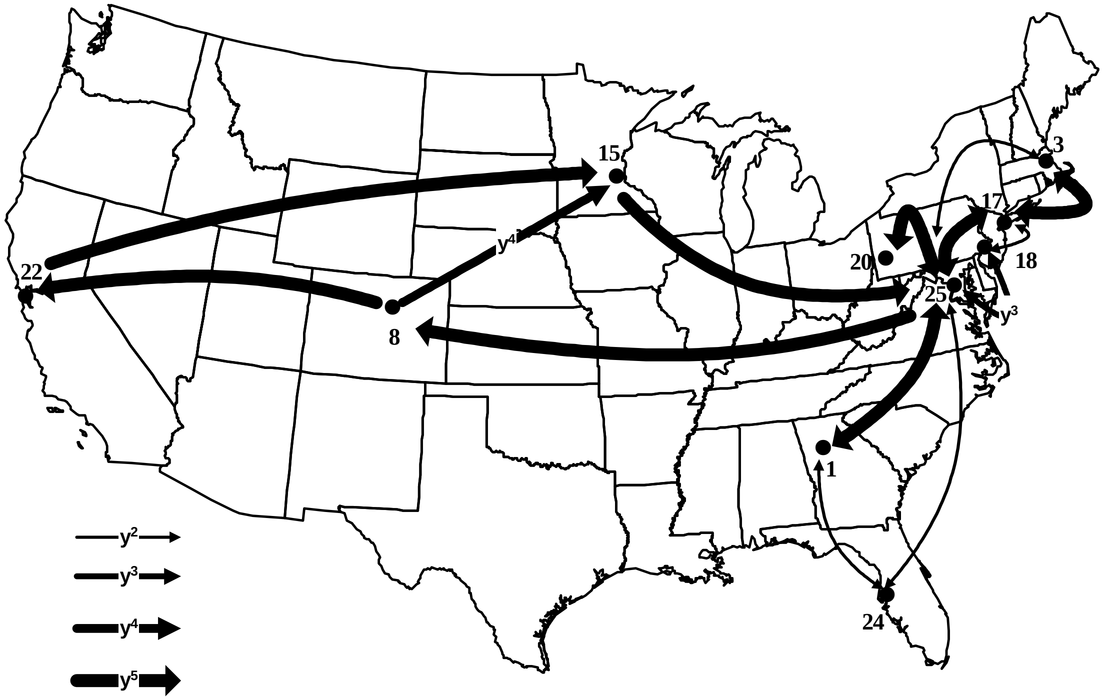

Figure 6.7 Result (Simulation 7) of the FDPTP with Cost function 3 under Flow Range 2

Simulation Set 8: Randomly Selected Subset (10-Node) of CAB25 with Cost Function 3 under Flow Range 2

A similar result with simulation set 7 is obtained in this simulation. Table 6.56 shows that the FDPTP obtains a cheaper total network cost than the FDMAP. As can be seen in Figures 6.8 and 6.9, the structure of infrastructure is different each other. The FDMAP does not provide a fully connected hub network. It closes the longest interhub link to achieve flow economies of scale. One interesting observation from the results is that a node 11 (Kansas City.) has significantly changed its role in the network. It increases interactions with other nodes in the FDPTP. On the other hand, a node 4 acting as a hub in the FDMAP drastically decreases interactions with other nodes in the FDPTP. Unlike the previous results of the FDPTP, the opened links of the FDPTP are all symmetric as can be seen in Figure 6.9.

Simulation Set 8 (10-Node)	Cost		
CF3	Obj	Transport Cost	Fixed Cost
FDMAP	886.15	678.66	207.49
FDPTP	763.27	397.26	366.01

** Total network flow is 1,253,528.*
*** Cost function 3 is based on flow range 2.*

Table 6.56 Solutions for Simulation Set 8

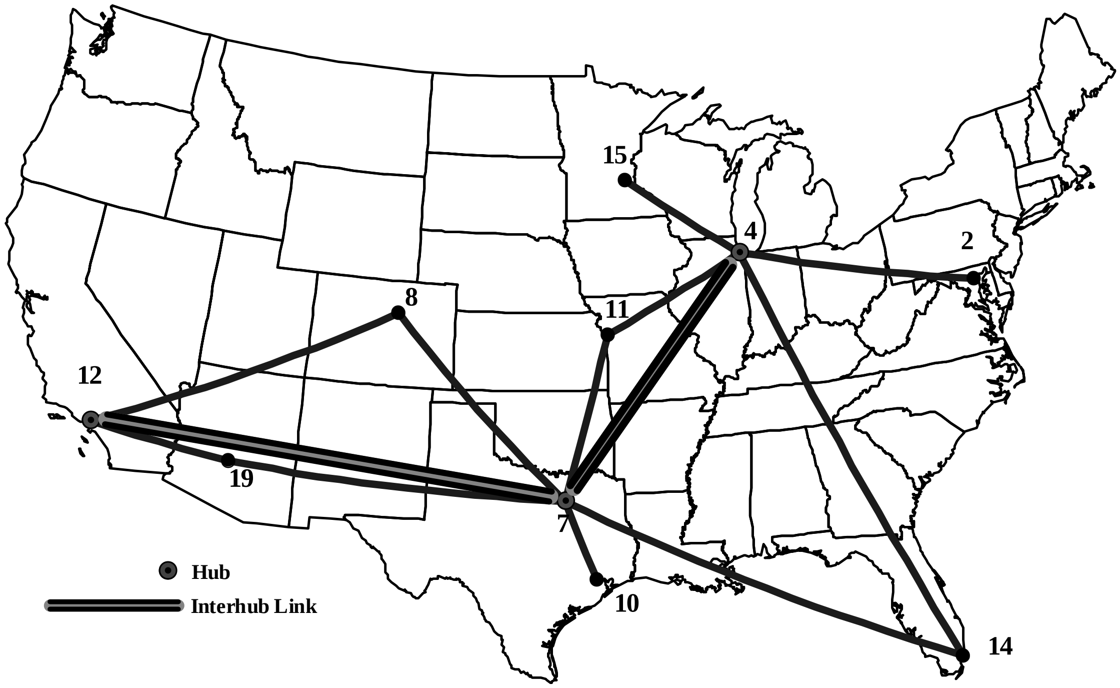

Figure 6.8 Result (Simulation 8) of the FDMAP with Cost function 3 under Flow Range 2

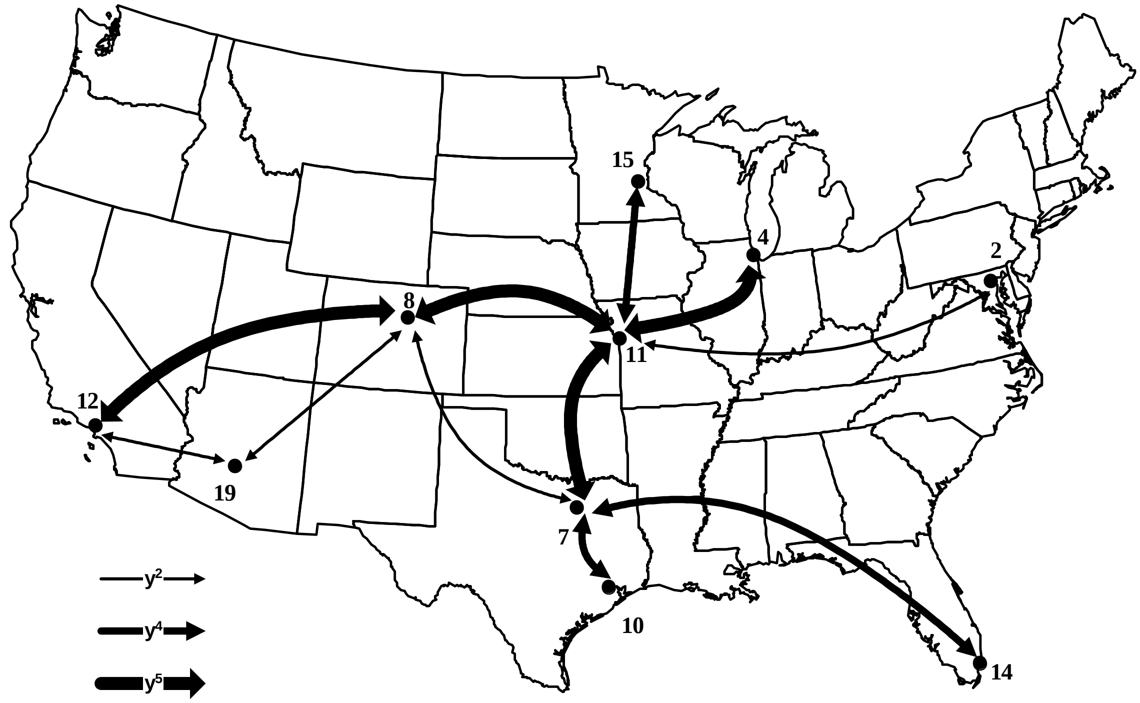

Figure 6.9 Result (Simulation 8) of the FDPTP with Cost function 3 under Flow Range 2

The models tend to produce results that the FDPTP finds the lower total network cost than the one of the FDMAP. This makes sense because the FDPTP could always build same infrastructure as the FDMAP, plus it has merits of other links discounts. With relatively small number of simulations, the geographical effect on spatial arrangement can be ignored on these models. The node's role in the network can significantly change between two models.

6.4 Summary of Results

This chapter expands the scope of the traditional research on hub network and point-to-point network. In network theory, one tried to determine the minimum network cost achievable in a situation modeled by a concave cost function. In this chapter, the question of how to share the cost among users is considered.

In the FDMAP, this work starts with the minimization of the objective function, and analyzes how the saving achieved by minimizing cost is to be divided among the participants in the cooperative system. The reason to employ a cooperative game theory is that the O-D pairs (players) may change the hub network structure to save their own network costs. To represent this possibility, two types of configuration are assumed: [1] global optimal hub network, and [2] coalition optimum network.

Cost allocations are also shown to be different under different characteristic functions. Characteristic functions under the global optimal network and under the coalition optimum network affect the cost allocation vectors. Under the assumption of the global optimal hub network, players who use the multiple interhub links are often charged more than the single interhub link players. One of the reasons is that they have to pay multiple fixed costs in their paths. Another reason is that those multiple interhub link users help the hub network achieve the cheaper total network cost by contributing their flows to the indirect paths. The model finds cost allocations in the sense that no coalition will find it more profitable to form their own localized sub-network system. The core is shown to be nonempty, so that the cores can be used to allocate cost. If the cost savings by the grand coalition is greater than the opportunity costs by the players who are willing to leave the grand coalition, the core of the game is not empty.

Two approaches are applied to reduce the complexity of cost allocation. The numerical results prove the total interhub link cost is the sum of the individual interhub link cost. This proof enables the model to solve the each interhub link game separately. It is also shown that the cost allocation for the total network is equal to the cost allocation for the sub-network. The model also reduces the complexity by aggregating individual players into group players. This way makes it possible to consider the number of player as only the number of interhub links.

In the FDPTP, this work starts with the minimization of the objective function, and analyzes how the saving achieved by minimizing cost is to be divided among the participants with respect to their contributions to flow economies of scale. Their contribution is directly related to their own flows. Therefore, the cost allocations can be easily obtained by proportional payment for the fixed costs with respect to their own flows. Since there is no possibility for the O-D pairs to change the network structure, the individual cost allocation is proportional to its flow for the fixed costs for the opened link.

Cost allocations of O-D pairs between hub network and point-to-point network are compared. It is shown that O-D pairs with multiple interhub links are always favorable with point-to-point network. There are some O-D pairs with single interhub link that favors a hub network. This comparison study of cost allocation shows that there is a certain network structure to give advantages to certain O-D pairs. However, it is rather affected by flow economies of scale on network links. This comparison study also contributes to understand a fundamental question about the merits of hub networks versus point-to-point networks.

CHAPTER 7

CONCLUSION

This section offers conclusions about the study, and points to new research directions. This study investigated the following. First, infrastructure design in hub network and point-to-point network was developed. A mathematical programming model identified the optimal infrastructure that minimizes total network cost in both a hub network and a point-to-point network. A relationship between flow and infrastructure was examined. Second, cost allocation was adopted in network design of both the hub network and the point-to-point network. Mathematical programming for cost allocation featuring a game theory approach is used to set up the hub network efficiently. A common cost allocation method, cores, is incorporated into the hub location model in order to provide fair cost to users. Unlike the hub network, a cost allocation of the point-to-point network can be achieved without resorting to game theory. The main reason is that there is no interhub link in the point-to-point network. In hub networks, O-D pairs can find a cheaper cost by switching their routes due to flow economies of scale on interhub links.

An analysis using mathematical models involves two parts: modeling and solving. In some cases, the hub network problem may be solved optimally by using efficient algorithms. In other cases, it may have to be solved by using some approximate procedures. In any case, the solution to a mathematical model is optimum or near-optimum only for the model, not for the real problem since a model is a only an abstract representation of reality. Generally speaking, the effectiveness of a solution procedure is evaluated by speed and accuracy. Speed refers to the time it takes to solve a model on a computer, while accuracy refers to the closeness of the solution to optimality. The FDMAP model developed in this study solves a relatively complex hub problem efficiently, and it also provides a new finding of multiple interhub links. This saves a total network cost by enabling flow economies of scale on multiple interhub links.

7.1 Summary of Research

On the hub network side, first of all, the main modification adapted by the model is that the optimal network should consider multiple interhub links, which may break triangle inequality. Secondly, O-D pairs do not necessarily use the shortest path with respect to distance because the benefits of the longer path may be determined by the number of O-D pairs patronizing the links. Thus, flow economies of scale in constructing and operating the infrastructures are necessary for cooperation to occur. One of the important aspects of the hub network infrastructure design is the different geographical distribution of network traffic. This may become an important consideration when interhub links are interconnected. The tabu FDMAP heuristic is efficient to solve the flow-based discount multiple hub location problem. Results of the infrastructure design of the FDMAP exactly obey the minimum cost envelope (lower envelope), which configures theoretical relationships between flows and corresponding slopes and fixed costs. The optimal interhub flows match the proper infrastructure and fixed costs. With optimal infrastructure, the best solution satisfies the optimal flow condition. In other words, flows in the interhub link are required to reach large enough flow thresholds to obtain a certain discount or to acquire a high level of infrastructure. For example, if an interhub link flow is large enough, then it receives a high discount and pays a correspondingly higher fixed cost. However, if an interhub link flow is not large enough, it cannot support expensive fixed costs. In other words, each interhub flow user-cost for a specific infrastructure cannot be optimal if it does not reach the appropriate thresholds because it would be better off selecting a low level of infrastructure to accommodate smaller flows. The model supports the optimality condition showing that the high level of infrastructure for all interhub links are not always the best choice for the hub network system. As a concluding remark on the infrastructure design model, the computation time depends mainly on the number of times that the shortest paths must be computed and the enumeration of all possible infrastructures.

On the point-to-point network side, the model solves flow-based discount point-to-point problem. The results of the FDPTP are compared to hub network with respect to O-D pair’s route cost. There are many interesting characteristics of the FDPTP model. Discounting is not guaranteed even if flow of O-D pair is greater than the flow range. It is not necessarily guaranteed due to the model property of splitting the flows. In the FDPTP model, the distance between the origin and the destination does not affect the routing strategy at all. In other words, the short distance of O-D pair does not necessarily favor a direct connection. The behavior of the model depends on concave cost functions that determine input parameters of discounts and fixed costs along with flow ranges. It is shown that discount values and flow ranges are directly related with the difficulty of the problem in addition to the problem size. The FDPTP model with a higher flow range could not achieve flow economies of scale as much as the one with a lower flow range. It displayed a less dense network structure than the lower one. Given discounting incentives based on the amount of flow on network links, O-D pairs are motivated to amalgamate their flows. It turns out that a high level of flow range makes the model difficult to solve because the model needs to bundle flows in many different possible ways. The result of network structure is also affected by the hop constraint. The FDPTP without hop constraint shows the less dense network than the FDPTP because it needs to re-route flows of the geographically separated O-D pairs.

On the cost allocation in hub network, a cooperative game theory has been incorporated into the hub network design to obtain cost allocation. To overcome the difficulty of cost allocation, groupings for the players help to characterize core conditions. Using two-step cost allocation: an aggregate hub game and an individual hub game, costs of total shared network cost for the subset of users are allocated so that no one has an incentive not to cooperate. A necessary and sufficient condition for hub network structure to dominate direct route structure is declining average cost on each route theoretically. Cost allocation is complicated by the multiple interhub link observation (see Section 3.2.3). Multiple interhub link players pay more fixed costs per flow than the single interhub link players under the global hub network with respect to their flows because they have to pay fixed costs for the multiple interhub link. However, the multiple interhub link players pay less fixed costs per flow than the single interhub link players under the coalition optimal hub network configuration with respect to their flows. In other words, if the cost savings by the grand coalition is greater than the opportunity costs by the players who are willing to leave the grand coalition, the core of the game is not empty. On the other hand, a game theory is not necessary to obtain cost allocation in point-to-point network. The main reason for that is that point-to-point networks do not have interhub links that may affect individual routing costs. A proportional cost allocation is rather adopted to obtain cost allocation with respect to player's flow contribution.

The study also compares the cost allocations of O-D pairs between hub network and point-to-point network to see the merits of each network. It is shown that O-D pairs with multiple interhub links are always favorable with point-to-point network. There are some O-D pairs with single interhub link that favors a hub network. The comparison study of cost allocation shows that there is a certain network structure to give advantages to certain O-D pairs. However, it is rather affected by flow economies of scale on network links. The comparison study also contributes to understand a fundamental question about the merits of hub networks vs. point-to-point networks.

7.2 Further Research

Although the present algorithms are adequate for hub network design, there is a need for better approaches and exact hub network design solutions to measure and compare the optimality of solution. More work should be devoted to the analysis of algorithm to form a better concept of the applicability and limits of the various techniques. For example, the infrastructure design problem should be expanded into large networks such as telecommunication, computer network, and post delivery system. Tabu search is effective in hub network. However, it always runs the risk of being restricted to a local minimum, especially for a large data set. To overcome this limitation, a new model will incorporate other approaches, such as VNS (variable neighboring search), Partitioning, and Clustering techniques for the large data set.

The FDPTP was effective in modeling flow-based discount point-to-point network. For a smaller network size, it is possible to obtain the optimal solutions in a reasonable time. However, heuristic solution techniques for larger problems are necessary because the model formulation is too memory consuming for the large network.
Another area of future research is the introduction of interhub link capacities. This is appropriate because passenger inconvenience is a very critical factor in air passenger networks. The optimal assignment of link capacities to a hub network with minimum cost routes is interesting, since routing plays a determining role in the optimization of network performance. This study utilized flow economies of scale using piecewise-linear concave cost functions for the interhub links only, but further research may include economies of

scale on nodes as well. Finally, it is hoped that this study stimulates further investigation in this field.

APPENDIX A

ID	City Name	State Name
1	Atlanta	Georgia
2	Baltimore	Maryland
3	Boston	Massachusetts
4	Chicago	Illinois
5	Cincinnati	Ohio
6	Cleveland	Ohio
7	Dallas	Texas
8	Denver	Colorado
9	Detroit	Michigan
10	Houston	Texas
11	Kansas City	Missouri
12	Los Angeles	California
13	Memphis	Tennessee
14	Miami	Florida
15	Minneapolis	Minnesota
16	New Orleans	Louisiana
17	New York	New York
18	Philadelphia	Pennsylvania
19	Phoenix	Arizona
20	Pittsburgh	Pennsylvania
21	St. Louis	Missouri
22	San Francisco	California
23	Seattle	Washington
24	Tampa	Florida
25	Washington	District of Columbia

Table A.1 25-Node CAB Data Set

ID	City Name	State Name	ID	City Name	State Name	ID	City Name	State Name
1	Akron	Ohio	41	Greensboro	North Carolina	81	St. Louis	Missouri
2	Albany	New York	42	Greenville	South Carolina	82	Salinas	California
3	Albuquerque	New Mexico	43	Harrisburg	Pennsylvania	83	Salt Lake City	Utah
4	Allentown	Pennsylvania	44	Hartford	Connecticut	84	San Antonio	Texas
5	Atlanta	Georgia	45	Houston	Texas	85	San Diego	California
6	Augusta	Georgia	46	Indianapolis	Indiana	86	San Francisco	California
7	Austin	Texas	47	Jackson	Mississippi	87	Seattle	Washington
8	Bakersfield	California	48	Jacksonville	Florida	88	Shreveport	Louisiana
9	Baltimore	Maryland	49	Kansas City	Missouri	89	South Bend	Indiana
10	Baton Rouge	Louisiana	50	Knoxville	Tennessee	90	Spokane	Washington
11	Beaumont	Texas	51	Las Vegas	Nevada	91	Syracuse	New York
12	Binghamton	New York	52	Lexington-Fayette	Kentucky	92	Tampa	Florida
13	Birmingham	Alabama	53	Little Rock	Arkansas	93	Toledo	Ohio
14	Boston	Massachusetts	54	Los Angeles	California	94	Tucson	Arizona
15	Bridgeport	Connecticut	55	Louisville	Kentucky	95	Tulsa	Oklahoma
16	Buffalo	New York	56	Madison	Wisconsin	96	Utica	New York
17	Charleston	West Virginia	57	Memphis	Tennessee	97	Washington	District of Columbia
18	Charleston	South Carolina	58	Miami	Florida	98	West Palm Beach	Florida
19	Charlotte	North Carolina	59	Milwaukee	Wisconsin	99	Wichita	Kansas
20	Chattanooga	Tennessee	60	Minneapolis	Minnesota	100	Youngstown	Ohio
21	Chicago	Illinois	61	Mobile	Alabama			
22	Cincinnati	Ohio	62	Moline	Illinois			
23	Cleveland	Ohio	63	Nashville-Davidson	Tennessee			
24	Colorado Springs	Colorado	64	New Orleans	Louisiana			
25	Columbia	South Carolina	65	New York	New York			
26	Columbus	Ohio	66	Norfolk	Virginia			
27	Corpus Christi	Texas	67	Oklahoma City	Oklahoma			
28	Dallas	Texas	68	Omaha	Nebraska			
29	Dayton	Ohio	69	Orlando	Florida			
30	Denver	Colorado	70	Pensacola	Florida			
31	Des Moines	Iowa	71	Philadelphia	Pennsylvania			
32	Detroit	Michigan	72	Phoenix	Arizona			
33	Duluth	Minnesota	73	Pittsburgh	Pennsylvania			
34	El Paso	Texas	74	Portland	Oregon			
35	Erie	Pennsylvania	75	Providence	Rhode Island			
36	Evansville	Indiana	76	Raleigh	North Carolina			
37	Flint	Michigan	77	Richmond	Virginia			
38	Fort Wayne	Indiana	78	Rochester	New York			
39	Fresno	California	79	Rockford	Illinois			
40	Grand Rapids	Michigan	80	Sacramento	California			

Table A.2 100-Node CAB Data Set

D_{ij}	A	H1	H2	H3	B	C
A	0	3	6.32	6.32	9.43	9.43
H1	3	0	3.61	3.61	7.07	7.07
H2	6.32	3.61	0	4	3.61	7.28
H3	6.32	3.61	4	0	7.28	3.61
B	9.43	7.07	3.61	7.28	0	10
C	9.43	7.07	7.28	3.61	10	0

Table A.3 Distance Matrix of 6-Node Network

F_{ij}	A	H1	H2	H3	B	C
A	0	10	10	10	10	10
H1	10	0	10	10	10	10
H2	10	10	0	10	10	10
H3	10	10	10	0	10	10
B	10	10	10	10	0	10
C	10	10	10	10	10	0

Table A.4 Flow Matrix of 6-Node Network

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