Joint Design of Redundancy and Maintenance for Parallel-Series Continuous-State Systems

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Abstract

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Joint Design of Redundancy and Maintenance for Parallel-Series Continuous-State Systems

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System reliability models are usually developed for binary-state systems and multi-state systems. Indeed, the performance degradation of some systems is a continuous process in consecutive time. So far, however, there have been few studies about reliability modeling and optimization for continuous-state systems. Thus, this study attempts to build a redundancy optimization model, an age-based replacement model, and a joint design of redundancy and maintenance model for continuous-state systems. A key component of multi-state models and continuous-state models is to find the relationship between the states of components and the states of systems. Hence, a structure function is presented to give an expression of the relationship between the continuous state of the system and the continuous states of its components over time with the assumption that the individual component degradation is modeled by a Gamma process, which is widely used to model monotonic degradation paths. The most popular design method for reliability optimization problems is to increase the number of redundant components, which is the so-called redundancy allocation problem. In addition to adding redundant components, maintenance design, which includes a series of activities to restore a system, is another method to prolong the lifespan of the system. However, very little attention has been paid to the joint models of redundancy design and maintenance optimization, especially for

continuous-state systems. To recognize this gap, this study proposes a joint design of redundancy and maintenance for parallel-series continuous-state deteriorating systems. The growing number of components and the increasing frequency of maintenances give a push to the lifecycle cost of the system. Taken this into consideration, this dissertation focuses on building an optimization model that minimizes the system cost while satisfying the constraint of the system reliability on the basis of the degradation level of the system to find the optimal redundancy design. Based on the redundancy design of the continuous-state system, an age-based replacement strategy that minimizes the long-run expected maintenance cost rate to find the optimal maintenance interval is proposed in this study. Both the preventive replacement and the corrective replacement strategies are considered in the age-based replacement maintenance design. Then, a joint model of redundancy and maintenance design for continuous-state parallel-series systems consisting of degrading components is proposed in this study. Finally, this dissertation studies the degradation of the battery pack system which is a continuous–state parallel-series system related to the configuration of the battery pack system and the state-of-health of cells in electric vehicles to illustrate the proposed methodology.

Dedication

This dissertation is dedicated to my advisor, Dr. Yuan who supported me to pursue my doctoral degree and finish my dissertation.

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I have a wonderful time at Ohio University during the last three years. And I have received a lot of support and assistance during my Ph.D. studies. Thanks to all people I met at Ohio University.

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Chapter 1: Introduction

1.1 Objective and Motivation

The objectives of this dissertation are to propose a new redundancy optimization model to optimally design parallel-series continuous-state systems composed of continuously degrading components, an age-based replacement model, and a new joint redundancy and maintenance optimization model for such systems, and to apply those newly proposed models to the battery pack systems for electric vehicle (EV) applications.

Reliability is a critical performance measure when designing and operating a complex system. An important design method to increase system reliability is to add redundant components. One example of redundant design is the battery pack systems for EV applications. Redundant lithium-ion cells are included in the battery pack to improve reliability and safety. Another example is the redundant array of independent disks (RAID) used by some computer systems to prevent data loss due to hard disk failures. Corrective and preventive maintenance actions are widely used to prolong the life of a system. There has been very rich literature related to redundancy optimization and maintenance optimization. But two gaps in existing studies and one practical system have motivated this study.

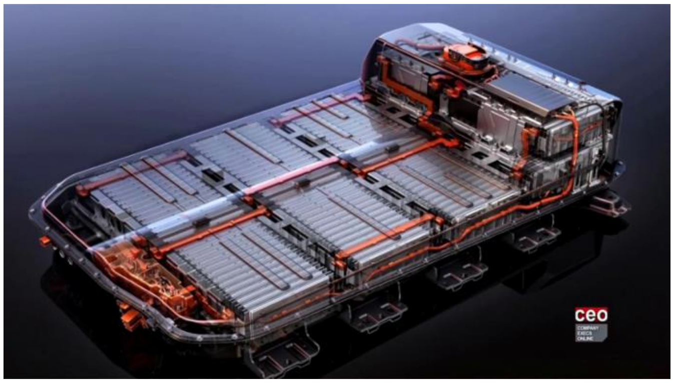
First, existing system reliability models and redundancy optimization methods are usually based on the binary-state and multi-state assumptions. A binary-state system (BSS) and its components have only two possible states, namely, completely functioning and non-functioning. However, the performance of some systems may not be simply characterized by those two states. For instance, the states can be grouped into excellent, average, and poor [1]. Such as computer systems in real-world problems, the component performance is classified by the data processing speed. Hence, the multi-state assumption, which is an extension of the binary-state assumption, with more practical significance and wider applicability was introduced in the 1970s. As a matter of fact, some systems applied to practical life experience continuous degradation over consecutive time, and then continuously perform multiple levels of performance range from full functioning to fatal failure among these complicated systems. Due to their practical importance, continuous-state systems (CSS) composed of degrading components have been increasingly common. However, reliability modeling and optimization for CSS have not been well studied. This study proposes a redundancy optimization model for parallel-series CSS.

Second, redundancy optimization and maintenance design are usually performed separately. The joint design of redundancy and maintenance design for a multi-state system is studied by some researchers [2]. The joint redundancy and maintenance design approach is shown to be more cost effective than the traditional separate designs of redundancy and maintenance. Joint design of redundancy optimization and maintenance for CSS with degrading components is however a still underexplored area. Hence, this dissertation proposes a joint design of redundancy and maintenance for parallel-series CSS with degrading components.

This study is motivated by the battery pack systems for EVs shown in Figure 1. It is a parallel-series CSS with continuously degrading cells. Redundancy is commonly used for system reliability and safety improvement. However, very limited studies have been considered to optimally design the redundancy and maintenance for the battery pack system.

Figure 1

The Battery Pack System in Electrical Vehicles [3]



1.2 Background

In the manufacturing industry, the influence of equipment and system reliability on the lives and property of humans is unallowable to be neglected. The improvement of reliability is important for a wide range of scientific and industrial processes. As the complexity of equipment and system increases, the study of reliability design has been brought to the forefront. It has been applied to both repairable and non-repairable systems. The improvement of reliability based on the redundancy design and maintenance design has been studied extensively in recent years. Some basic concepts and models of reliability, redundancy design, and maintenance design are introduced in this section.

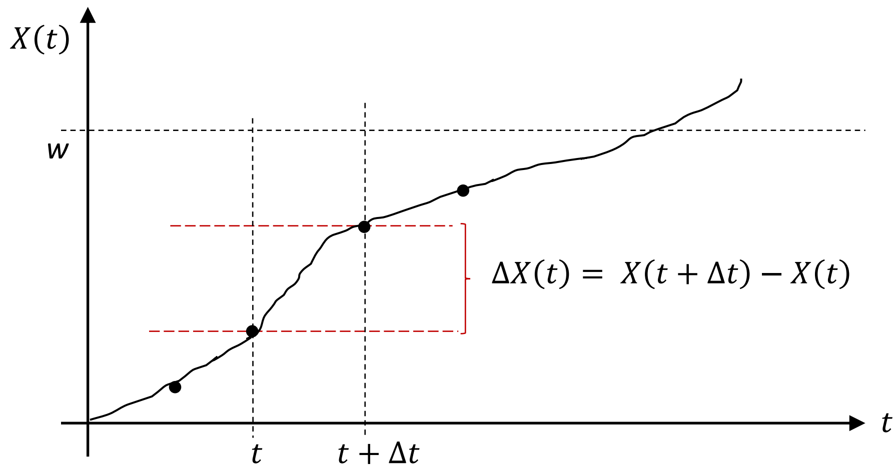
1.2.1 Reliability

Reliability, which is generally denoted by R, is the probability that a product, which may consist of hardware, software, and human resources, performs its intended function adequately under given operating conditions for a stated time interval [4]. It is usually represented by the probability that the item keeps functioning during a stated period. Generally, there are two kinds of models to evaluate the reliability of a system, i.e., failure-time-based models and degradation-based models. In the failure-time-based models, reliability is measured by the decreased probability as time goes on based on some lifetime distributions. The mathematical expression is shown as $R(t) = P(T > t) = \int_t^\infty f(\tau)d\tau$, where $f(t)$ is the probability density function (pdf) of the time to failure random variable T. The lifetime distribution of products is evaluated by the failures in a stated time interval on the basis of a failure-time data set that comes from reliability tests [5], which may be difficult to be applied to highly reliable products due to very few or even no failures occur during the tests. The experiments for these products are expensive and time-consuming for both manufacturers and customers. For example, the costly accelerated outdoor tests for paint and coating products often take time and the results from laboratory tests are also unsuccessful to obtain the prediction of the failure [6]. Hence, the performance of products as a characteristic function of time is proposed to describe the degradation of these products for reliability estimation. The failure of a degradation-based model, which is relevant to the physical state of the item, is defined as the performance of an item exceeds a pre-specified failure threshold. It is also called “soft failure”.

Hence, the reliability of a degradation-based model is denoted by the probability that the performance measure does not reach a pre-defined failure threshold. The mathematical expression is described as $R(t) = P(X(t) > w) = \int_{w}^{\infty} f_X(\tau) d\tau$ if the performance measure is monotonically decreasing or $R(t) = P(X(t) < w) = \int_{-\infty}^{w} f_X(\tau) d\tau$ if the performance measure is monotonically increasing, where $X(t)$ is the degradation level at time t, w is the pre-determined failure threshold, and $f_X(t)$ is the pdf of $X(t)$. Stochasticity is one of the features in a system during operation. The degradation process of a component is generally assumed to be a stochastic process, such as the Winner process, Gamma process, and inverse-Gaussian process. In this dissertation, the Gamma process is applied to describe the performance degradation of a component in a system. The Gamma process shown in Figure 2, which has been widely used to describe the degradation process and data analysis, is a stochastic process with independent, stationary and nonnegative increments $\Delta X(t)$ when no maintenance actions are performed. The Gamma process $\{X(t)_{t \geq 0}\}$ is parameterized by $\alpha(t)$ and β, which can be estimated by historical data/information. The increments $\Delta X(t) = X(t + \Delta t) - X(t)$ follows the Gamma distribution, Gamma $(\alpha(t + \Delta t) - \alpha(t), \beta)$ [7]. The mean of $X(t)$ is $\alpha(t) \cdot \beta$, and its variance is $\alpha(t) \cdot \beta^2$.

Figure 2

Gamma Process Degradation



Two approaches have been used to model and analyze the system reliability. The first approach is to consider a system as one unit, measuring and/or modeling its degradation at the system level. This is the so-called black-box approach for reliability analysis of continuous-state systems [8]. The second approach, i.e., the white-box approach, derives the system reliability via the degradation modeling of its components [8]. Some studies modeled the component-level degradation but assumed the system to be binary [9]–[11]. Their study of the reliability modeling for continuous-state systems consisting of continuous-state components is limited. For such a system with J components, the system's state $X_s(t)$ can be related to its components' states, $Y_j(t), j = 1,2,\ldots,J$,according to a structure function $X_s(t) = \varphi\big(X_1(t)\,, X_2(t)\,,\ldots, X_J(t)\,\big)$, where both $X_s(t)$and $X_j(t)$'s are continuous random variables [12]. An example of such systems is the polymer electrolyte membrane fuel cell (PEMFC) stack [13], which is seen as a continuous-state system with multiple continuously degrading fuel cells connected in series. The voltage degradation of individual cells causes the performance of the whole stack to degrade continuously.

1.2.2 Reliability Optimization

In the industrial field, degradation with age and usage of machines and products are inevitable. And it has a great impact on both the producers and customers. For example, a production line is composed of hundreds of parts and components. The failure of a small part may lead the whole line to be shut down which results in the loss of unpredictable labor and material resources for manufactures. Concerning safety and cost, the machines on the line are required to be highly reliable. For customers, the end result of unreliability is the increased cost of ownership. The safety of people's life and property are seriously affected by unreliable machines and products. Hence, reliability improvement has become an important goal for enterprises to enhance their market competitiveness. To obtain the most reliable machines and products, one possible solution is that the machines and products which can be seen as systems should be designed to slow down the degradation at the utmost by the system designers. Therefore, some researchers focus on reliability optimization problems that evaluate the system reliability by using a degradation-based model.

Obviously, the direct method to enhance the reliability of a system is to improve the reliability level of each component in the system which is generally seen as a reliability allocation problem. However, a lot of labor and material resources are required for manufacturers to improve the reliability of the components. Hence, researchers shift their focus to the entire system instead of a single component. The most popular design method for reliability improvement is to increase the number of redundant components in the system. However, the cost also increases as the growing number of redundant components meet the required system reliability level. Hence, numerous studies focus on redundancy allocation problems (RAP) which has a critical goal to find the tradeoff between the total investment cost of redundant units used in the system and the performance of the system. Redundancy design is widely used in the industrial field. For example, a data center generally maintains multiple power generators which are only used in case of the power grid failure. Severs can be designed with two or three power supplies and one of them reserved as a cold standby. In this study, the redundancy optimization model is applied to the EV battery pack systems.

1.2.3 Maintenance Design

Maintenance design, which includes a series of activities to restore a system, keeps on seeking the improvement of system reliability and the effectiveness of technical actions. Two maintenance strategies, time-based maintenance (TBM) and condition-based maintenance (CBM), have been widely applied in the industrial areas. TBM is a technique that provides maintenance decisions based on the historical failure time data, i.e. lifetime distribution, of the system; while CBM collects the current state information to make maintenance decisions by analyzing the present condition of the system. For example, the model is defined as a TBM if the engine oil changed every three months and the engine oil in CBM is changed when the oil service light is on which means the condition detected by a monitoring sensor is not good. The main goal of CBM is to improve the effectiveness of the operations and reduce the related cost based on the assessment of machines by making maintenance decisions. However, the condition of a system is detected by the condition monitoring sensors. Hence, CBM has a high requirement of monitoring sensors which may be costly. This study focuses on the TBM models which are easier to control the states of the system. The basic purpose of TBM is to reduce the system cost and optimize system reliability based on the mean time to failure (MTTF) or the failure rate of components that comes from failure time analyses by using various statistical tools. Maintenance activities generally include two categories: preventive maintenance (PM) and corrective maintenance (CM). The PM actions are put into force on elapsed time before the system fails, while the CM actions are taken after a sudden failure takes place. The failures include “soft failures”, which is defined as the failure in deteriorating systems that the degradation level reaches the pre-decided failure level, and “hard failures”, which means the component suddenly breaks down due to degradation or traumatic events. The system failure is decided by whichever comes first. And the maintenance actions taken by the engineers for different failure modes may be various due to the complexity and influence during functioning. PM is an effective way to reduce the failure rate of the system and the occurrences of “hard failures”. Furthermore, it helps to prevent some unnecessary costs and protecting human life. In this study, the combination of PM and CM which is an effective way for system performance improvement is implemented.

1.3 Significance and Contribution

To improve the system reliability, this dissertation proposes a redundancy optimization model to determine the optimal design and then an age-based replacement model to find the optimal maintenance interval as a separate design for parallel-series CSS which has limited research. Subsequently, this dissertation introduces a joint design model of redundancy and maintenance for parallel-series CSS. For the redundancy optimization model, this dissertation builds an optimization model with reliability constraints to determine the cost-optimal redundancy design. Based on the system design obtained from the redundancy optimization model, the optimal maintenance interval is determined by solving an expected lifecycle cost rate minimization problem. In addition to the separate design model, a joint design model is introduced later. The objective of the joint design model is to minimize the expected lifecycle cost rate during the system operational time with reliability constraints. The system design and maintenance interval are calculated simultaneously in this joint model. And the lithium-ion battery pack system for electrical vehicles is used as an illustrative example to demonstrate the application of the proposed redundancy optimization model, the age-based replacement model, and the joint design model in this dissertation. The redundancy design, especially, the joint redundancy and maintenance design for CSS, has not been studied previously. This dissertation will contribute to the new models and tools for the reliability design of CSS with degrading components, which are becoming increasingly common.

1.4 Overview

This dissertation is organized into the following four chapters. In Chapter 2, a literature review of redundancy and maintenance is summarized, respectively, to point out the significance of this study. A redundancy optimization model, an age-based replacement model, and a joint model of redundancy design and maintenance design for series-parallel CSS are described in detail, respectively, in Chapter 3. In Chapter 4, the corresponding discussions, models, and results applied to the battery-pack systems for electrical vehicles are presented. Chapter 5 concludes this study first, then points out the contributions and future study of this dissertation.

Chapter 2: Literature Review

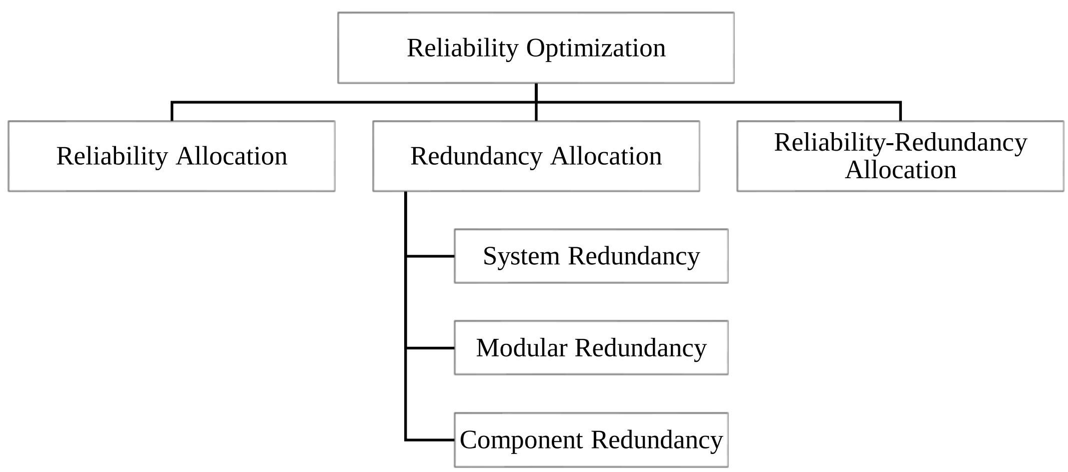
Both redundancy design and maintenance design are popular methods for the improvement of system reliability. They have received considerable attention. The specific objective of these two methods is to find the trade-off between system reliability and system cost. This chapter gives a literature review of the previous studies of redundancy optimization and maintenance for system designs. In addition, this chapter also mentions the issue of EVs which is an increasingly important topic in the automobile industry.

2.1 Redundancy Optimization

The improvement of the reliability of a product is the primary purpose for engineers and managers. The former aims to strengthen safety and the latter aims to enhance profit. Reliability optimization has attracted a great deal of scholarly research to develop various techniques to maximize system reliability. The reliability of the system can be boosted by enhancing the reliability levels of the components in the system, adding redundant components, or the adjustment of the system configuration. In brief, the reliability optimization problems as shown in Figure 3 are divided mainly into three categories: the reliability allocation problems, the redundancy allocation problems (RAPs), and the reliability-redundancy allocation problems (RRAPs). The objective of reliability allocation problems is to improve the reliability level of each component in the system with fixed redundancy levels, namely the system has a fixed structure. Soltani [14] presented a brief literature review on reliability allocation problems for non-repairable systems. A lot of researchers focus on the reliability allocation problems in the early study of reliability optimization problems. In 1977, Ching-Lai et al. [15] proposed the Hooke and Jeeves (H-J) pattern search method which is a sequential search routine for system reliability maximization in combination with a heuristic approach proposed by Aggarwal et al. [16] to solve the reliability allocation problems. Additionally, many other methods have been put forward and implemented in the study of the reliability allocation problems such as GAG2 [17], Fuzzy non-linear programming [18], ECAY (exact) [19], to name a few.

Figure 3

Reliability Optimization Problems [14]

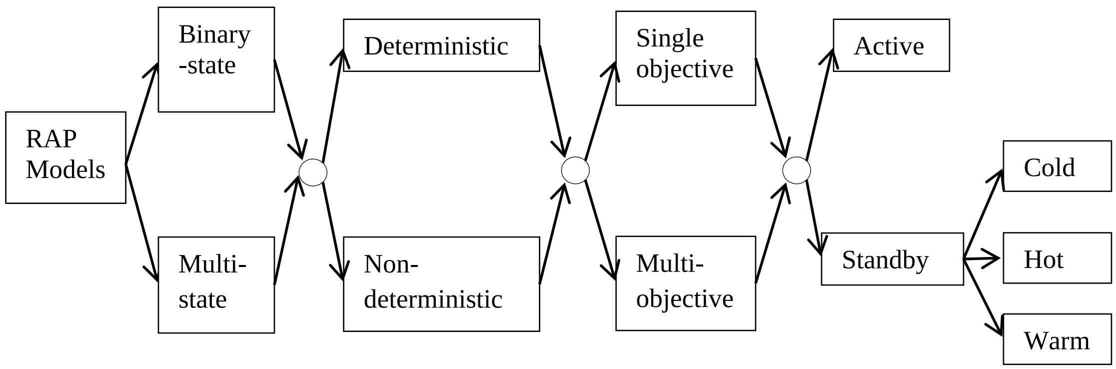


However, a lot of labor and resources are required for manufacturers to improve the reliability of each component in a system. Hence, RAP that improves reliability by adding redundant components in the system is the most popular design method to improve system reliability. The focus of this section is on the review of RAP models. Previous research has shown that redundancies can be assigned at the component level, sub-system level, or system level. In general, it is better to assign the redundant components at the component level than the system level in the case of the usual stochastic order. However, Boland and El-Neweihithe [20] concluded that modular redundancy (subsystem redundancy) is more effective than component redundancy for non-identical spare parts. In general, users have multiple choices of components. Accordingly, RRAP, which is a combination of components selection problem and redundancy level decision problem, is a more general model to improve system reliability. This dissertation mainly focuses on RAP models.

Several researchers have done a meta-analysis of the literature on reliability optimization problems. Tillman et al. [21] provided a state-of-art review of optimization techniques used in small-scale RAPs, and this review firstly grouped the papers by the system configuration and solution method, which are important factors for system optimization. Multiple heuristic methods were proposed during the 1970s. In a 2000 report, a sketch of system reliability optimization problems is summarized by Kuo and Prasad [22]. They classified papers by the system structure and optimization method. Heuristic methods and metaheuristic algorithms for RAPs are summarized in [22]. Then, Kuo and Wan [23] discussed more recent research in RAPs after the publication of [22]. Several classification methods for RAPs are summarized in Figure 4.

Figure 4

Classification of RAP Models [14]



As shown in Figure 4, the RAP models have multiple classification criteria. The most common classification criteria are the number of states. Existing RAPs have been mainly focused on BSS and MSS. In the real world, new products degrade with age and usage and ultimately fail. For the purpose of convenience, a system traditionally only allows two states: totally functioning and completely failed. However, as systems grow more complex, the multi-state systems (MSS) with more practical significance and wider applicability attract more attention in RAP models. The BSS is a special case of the MSS, and many BSS models have been extended to MSS. And the fundamental problem of MSS models is to find the relationship between the states of components and the states of the system.

Generally, the RAPs are grouped into deterministic and non-deterministic models based on the status of parameters. For deterministic models, all parameters in the system are conclusively precisely known. Conversely, the models that are composed of at least one uncertainty parameter are named non-deterministic. The uncertainty can be considered as stochastic uncertainty[24], [25], fuzzy uncertainty[26], chaos uncertainty[27], and so on.

Note that the cost of the system will increase as more and more redundant components are added to improve reliability. The tradeoff between the number of redundant components used in the system, the configuration of the system, and the reliability of the system is the primary goal for designers. On the basis of the number of objective functions, there are two kinds of RAP models: single-objective models and multi-objective models. For single-objective models, scholars generally maximize the reliability of the system under the constraints of the cost or minimize the cost based on the specified system reliability requirement. Although cost minimization and reliability maximization are two competing objectives, from the perspective of system designers, the two objectives mentioned above need to be considered simultaneously. It also has been taken into account that the cost constraints are fluctuant and difficult to determine. In order to solve this kind of multi-objective optimization problem, Kuo and Rajendra Prasad [28] mentioned that interactive decisions can be made according to a bunch of non-dominated feasible solutions.

The redundancy strategies can be separated into two methods, which are active redundancy and standby redundancy. In an active redundancy system, all components start operating simultaneously at time zero although the system could function well with one component at any particular time. Also, all components in the system share the load of the overall system so that the load on each component is reduced. Hence, the performance of the system is generally decided by the component with the best performance. In a standby redundancy system, the original component is replaced by a standby component which can be switched on and operate only when the original one fails. As a result, the performance of the system is the combined performance of the original components and the standby redundant components [29].

Standby can be divided into three forms: cold standby, hot standby, and warm standby. The redundant components are protected from operational stresses to keep full performance until substituted for failed components as parts of the functioning system when the cold standby strategy is applied to the system. The redundant components with hot standby strategy can immediately be active in the system with an increased failure rate when the system control is switched to them. In a system with hot standby strategy, the mathematical models are the same as active redundancy systems [30]. Finally, the warm standby redundant components fall in the middle condition between cold standby and hot standby. Hence, redundant components used for warm standby systems have a lower failure rate than the hot standby redundancies and enter in an active mode faster than cold standby redundancies. More classification rules about standby redundancies are shown in a review by Yearout et al. [31]. Active redundancy strategy and cold standby strategy are the most two popular redundancy design strategies considered so far.

Next, this dissertation summarizes the literature with its characteristics according to the number of system states in the following two subsections.

2.1.1 Binary-State System

For deterministic models, all parameters are determined in the system. Mathematical programming algorithms, which include exact solution methods and approximation solution methods, and some heuristic methods, are introduced to solve deterministic models. Fyffe et al. [32] initially proposed the model for active redundancy optimization with a single objective function that maximizes the reliability of the system subject to the required cost and weight by using the Lagrangian multiplier method, which is a mathematical programming algorithm. Subsequently, Nakagawa and Miyazaki [33] proposed the surrogate constraint approach, which was improved by Onishi et al. [34] for that Lagrangian multiplier may be inefficient sometimes. In addition to Lagrangian multiplier and surrogate constraint, many other exact search and mathematical programming methods such as dynamic programming, branch and bound, integer programming, partial bound enumeration, and lexicographic search method are used to solve optimization problems. Dynamic programming, also known as a multistage decision process, is a well-known method to solve complicated problems by breaking them down into simpler steps adopted by Yalaoui et al. [35]. The branch and bound algorithm proposed by Amari and Dill [36] is a method based on underlying knapsack problems to solve nonlinear integer models related to the design of an optimal series-parallel system.

Conversely, the models that are composed of at least one uncertainty parameter are named non-deterministic models. The uncertainty parameters can be considered as stochastic uncertainty, interval uncertainty, fuzzy uncertainty, robust optimization, and so on. Stochastic uncertainty means the distribution, or the mean value and standard deviation of the system performance are unknown. Coit and Smith [24] introduced a redundancy optimization model with random Weibull scale parameters by using generic algorithm (GA). Several algorithms proposed by Tekiner-Mogulkoc and Coit [25] are used to minimize the coefficient of system reliability variation, which considered both the mean and the standard deviation estimates concerning a minimum system reliability constraint, and some other constraints that related to the system itself. Ravi et al. [26] established an optimization model as a fuzzy multi-objective optimization problem. Besides the system reliability, system cost, weight, and volume are all regarded as fuzzy objectives.

Table 1 and Table 2 summarized the literature of RAPs for binary-state systems with a single objective and multi-objective, respectively. This dissertation analyzed the literature by configuration, the status of parameters, redundancy level, redundancy strategy, number of objectives, and the solution method.

Table 1

Binary-State RAPs with Single-Objective

Reference	Configuration	Parameters	Redundancy level	Redundancy strategy	Objective	Solution method
Amari et al. (2010) [36]	K-out-of-n	Deterministic	Component level	Active and warm-standby	System reliability	Linear programming
Hu et al. (2018) [37]	Parallel-series system	Deterministic	Component level	Warm-standby	System reliability/ lifetime/cost	Simulation
Nahas & Thien-My (2010) [38]	Series–parallel system	Deterministic	Component level	Active	System reliability	Harmony search algorithm (HSA)
Agarwal & Sharma (2010) [39]	Series-parallel; parallel-series; bridge system	Deterministic	Component level	Active	System reliability	Ant colony approach
Yeh (2009) [40]	Series–parallel system	Deterministic	Multiple multi-level	Active	System reliability	Two-stage discrete PSO (2DPSO)
Tavakkoli-Moghaddam et al (2008) [30]	Series–parallel system	Deterministic	Component level	Active and cold-standby	System reliability	GA
Onishi et al. (2007) [34]	Series–parallel system	Deterministic	Component level	Active	System reliability	Improved surrogate constraint method
Young et al. (2006) [41]	Series–parallel system	Deterministic	Multi-level	Active	System reliability	SA
Nahas et al. (2007) [42]	Series–parallel system	Deterministic	Component level	Active	System reliability	A heuristic method based on ant colony meta-heuristic optimization method and the degraded ceiling local search technique
Juang et al. (2008) [43]	Series–parallel system	Deterministic	Component level	Active	System availability	GA
Yeh (2014) [44]	Series-parallel system	Deterministic	Component level	Active	System reliability	Orthogonal simplified swarm optimization scheme (OSSO)
Chambari et al. (2013) [45]	Series-parallel system	Deterministic	Component level	Active and cold-standby	System reliability	simulated annealing algorithm (SA)

Agarwal et al. (2010) [46]	**Complex system**	**Deterministic**	**Component level**	**Active**	**System reliability**	**The proposed algorithm in this research searches for a possibly improved solution in the k-neighborhood of the current best feasible solution**
Yue et al. (2015) [47]	Smart grid communication network	Deterministic	Component level	Active	System cost	Improved GA
Yun et al. (2007) [48]	Series system	Deterministic	Multi-level	Active	System reliability	GA
Ziaee (2013)[49]	Hierarchical series-parallel system	Deterministic	Component level	Active	System reliability	Mixed-integer programming
Zangeneh et al. (2015) [50]	Bridge system	Deterministic	Component level	Active and cold-standby	System reliability	Improved GA
Najafi et al. (2013) [51]	Series-parallel system	Deterministic	Component level	Active and cold-standby	Mean time to failure	Simulated annealing (SA) and Genetic algorithm (GA)
Sharifia & Yaghoubizadeh (2015) [52]	Series-parallel system	Deterministic	Component level	Active and cold-standby	System reliability	SA and GA
Ramezani & Poutdarvish (2016) [53]	Bridge network	Deterministic	Component level	Hot standby and cold standby	System reliability	Hierarchical memetic algorithm (HMA)
Peiravi et al. (2019) [54]	Series-parallel system	Deterministic	Component level	Active and standby	System reliability	GA
Han et al. (2015) [55]	Multi-level system	Deterministic	Multi-level	Active	System cost	Simulation-based optimization procedure
Chang & Kuo (2018) [56]	Generalized (typically complex) network	Deterministic	Component level	Active	System reliability	Partitioning-based simulation optimization method for reliability optimization (PSORO)
Guilani et al. (2016) [57]	Series-parallel system	Deterministic	Component level	Active and cold-standby	System reliability	Simulation and GA
Kong et al. (2015) [58]	Series-parallel system	Deterministic	Component level	Active and cold-standby	System reliability	Simplified particle swarm

Soltani et al. (2013) [59]	Series-parallel system	Deterministic	Component level	Active	System reliability	Heuristic and meta-heuristic
Sadjadi & Soltani (2009) [60]	Series-parallel system	Deterministic	Component level	Active	System reliability	Hybrid genetic algorithm
Sahoo (2010) [61]	Series-parallel; parallel-series; complex system	Interval reliability	Component level	Active	System reliability	GA
Chen et al. (2018) [62]	Series–parallel system	Fuzzy lifetime	Component level	Standby	System lifetime	Hybrid particle swarm optimization algorithm with local search
Zhao & Liu (2005) [63]	Series-parallel system	Fuzzy lifetime	Component level	Standby	Expected system lifetime/ system reliability	Fuzzy simulation, neural network, and GA
Chatwattanasiri et al. (2016) [64]	Series–parallel system	Uncertain stress	Component level	Active	Minimization of the maximum regret	Nonlinear programming. Neighborhood search heuristic method
Feizollahi & Modarres (2012) [65]	Series–parallel system	Interval reliability	Component level	Active	System reliability	Min-Max regret approach
Gupta et al. (2009) [66]	Series–parallel system	Interval reliability	Component level	Active	System reliability	GA based penalty function technique
Soltani & Sadjadi (2014)[67]	Series–parallel system	Fuzzy	Component level	Active	Expected value of system reliability	Robust
Mousavi et al. (2016) [68]	Series-parallel system	Fuzzy	Component level	Active	System reliability	Improved fruit fly optimization algorithm (IFOA)
Wang et al. (2016) [69]	Series-parallel system	Interval uncertainty	Component level	Cold-standby	System reliability	GA-based searching approach
Feizollahi et al. (2015) [70]	Series-parallel system	Budgeted uncertainty	Component level	Cold-standby	System reliability	Robust optimization approach. MIP model iteratively in a Benders' decomposition framework, and single binary linear model
Sadjadi & Soltani (2015) [71]	Series–parallel system	Uncertainty	Component level	Active and cold standby	System reliability	Min-max regret method
Tekiner & Coit (2011) [25]	Series–parallel system	Stochastic reliability	Component level	Active	The coefficient of variance	Linear programming

Table 2

Binary-State RAPs with Multi-Objective

Reference	Configuration	Parameters	Redundancy level	Redundancy strategy	Objective	Solution method
Khalili-Damghani et al. (2013) [72]	Series-parallel system	Deterministic	Component level	Active	System reliability and system cost	DSAMOPSO
Chambari et al. (2012) [73]	Series-parallel system	Deterministic	Component level	Active and cold-standby	System reliability and system cost	Non-dominated sorting genetic algorithms (NSGA-II)
Azizmohammadi et al. (2013) [74]	Series-parallel system	Deterministic	Component level	Active and standby	System reliability and cost and volume minimization	Hybrid multi-objective imperialist competition algorithm (HMOICA), based on imperialist competitive algorithm (ICA) and genetic algorithm (GA)
Salazar et al. (2006) [75]	Series-parallel system; complex system	Deterministic	Component level	Active	System reliability and system cost	NSGA-II
Coit & Konak (2006) [76]	Series-parallel system	Deterministic	Component level	Active	Subsystem reliability	MWO2 heuristic
Zhao et al. (2007) [77]	Series-parallel system; gearbox	Deterministic	Component level	Active	System weight and system cost	Multi-objective ACS algorithm
Zoulfaghari et al. (2014) [78]	Series-parallel system	Deterministic	Component level	Active	System reliability and system cost	GA
Zhang et al. (2014) [79]	Series-parallel system	Deterministic	Component level	Active	System reliability, system weight, and system cost	Barebones multi-objective particle swarm optimization algorithm (BB-MOPSO)
Dolatshahi & Damghani (2015) [80]	SCADA system	Deterministic	Component level	Cold-standby and hot-standby	System reliability and system cost	Multi-objective particle swarm optimization (MOPSO)
Sooktip et al. (2012) [81]	K-out-of-n	Deterministic	Component level	Active	System reliability and system cost	Genetic Algorithm Approach with Penalty Function
Panda & Jagadev	Series-parallel	Deterministic	Component	Active	System reliability	Multi-objective evolutionary

(2016) [82]	system		level		and system cost	algorithms (MOEA)
Hemmati et al. (2018) [83]	K-out-of-n system containing s independent subsystems	Deterministic	Component level	Active	MTTF and system cost	Multi-objective harmonic search (MOHS)
Govindan et al. (2016) [84]	Series-parallel system	Deterministic	Component level	Cold-standby	System reliability and system cost	NSGA-II and APBSA
Ardakan et al. (2015) [85]	Series-parallel system	Deterministic	Component level	Active and standby	System reliability and system cost	Non-dominated sorting genetic algorithms (NSGA-II)
Sadjadi et al. (2014) [86]	Series-parallel system	Deterministic	Component level	Cold-standby	System reliability, system weight, and system cost	Compromise programming approach
Soltani et al. (2015) [87]	Series-parallel system	Deterministic	Component level	Active and cold-standby	System reliability, entropy, entropy weight, and system cost	Compromise programming approach
Chen & Liu (2011) [88]	Series-parallel system	Type-2 fuzzy lifetimes	Component level	Standby	System lifetime and cost	Approximation-based PSO algorithm

2.1.2 Multi-State System

With the in-depth study, the performance of the system or the component cannot be simply classified as perfect operating and complete failure. Many industrial systems are operating under multiple states of degradation. For instance, the performance of the components in a power generation station has multiple de-rated capacity states [89]. Therefore, MSS was introduced into redundancy optimization problems [90], and several binary-state configurations were extended to MSS [91]. Gürler et al. [92] introduced a system reliability maximization model, which divides the states of components into good states, doubtful states, PM due states, and failed states, with a constraint on the cost for series-parallel systems. Tian et al. [93] proposed a method to obtain the optimal design which includes the number of redundancy and a bunch of technical actions for a multi-state series-parallel RRAP at the subsystem level. The objective function is to minimize the consumption of resources under restrictions on system availability. Li et al. [94] studied a redundancy optimization model, which minimizes the system cost subject to common cause failures, for a multi-state series-parallel system. The mixture of components with variety of types in this model is analyzed by using universal generating function (UGF). And then genetic algorithm (GA) is applied to this model to obtain the optimal solutions. Similar to the methods used for the BSS, the methods for the MSS include mathematical programming, heuristics, meta-heuristic, and so on as summarized in [91].

Many authors have contributed to evaluating the reliability of MSS models. The UGF based on meta-heuristics is the most popular method for reliability optimization of MSS [91]. [95] published by Lisnianski and Levitin is the earliest paper that using UGF to analyze the reliability of power systems. Levitin et al. [96] proposed an algorithm using UGF to evaluate the performance of complex series-parallel MSS with different kinds of failures and imperfect protections. GA, which was inspired by the biological phenomenon of evolution that selects the best gens for offspring, is one of the most widely used metaheuristics for MSS [97]. Monte-Carlo (MC) simulation methodology is a general method to evaluate the system reliability for almost all MSS in practice. Billinton and Wenyuan [98] proposed a hybrid approach using MC simulation and an enumeration technique to evaluate the reliability of large-scale composite generation-transmission systems. Zio and Podofillini [99] presented an MC simulation approach to evaluate the system performance with some constraints that originated from its multi-state elements. In addition, numerous studies have adopted many other metaheuristic algorithms such as variable neighborhood search [100], ant colony optimization [101], and particle swarm optimization [102].

Table 3 and Table 4 summarized the literature of RAPs for binary-state systems with a single objective and multi-objective, respectively. The same summary method is used in multi-state systems. Based on the literature reviewed above, the study of continuous-state systems for RAPs has not been well studied. The continuous-state system is a new area for RAPs. Hence, this dissertation considers constructing reliability models for continuous-state systems.

Table 3

Multi-State RAPs with a Single Objective

Reference	Configuration	Parameters	Redundancy level	Redundancy strategy	Objective	Solution method
Li et al. (2010) [94]	Series–parallel system	Deterministic	Component level	Active	System cost	Analyzed by UGF and solved by GA
Nahas & Thien-My [38]	Series–parallel	Deterministic	Component level	Active	System reliability	Harmony search algorithm (HSA)
Ouzineb et al (2011) [103]	Series–parallel	Deterministic	Component level	Active	System cost	A combination of space partitioning,GA, and tabu search (TS)
Liu et al. (2013) [2]	Three-stage coal transportation system	Deterministic	Component level	Active	System reliability	GA
Ebrahimipour et al. (2010) [104]	Series–Parallel;k-out-of-n System	Deterministic	Component level	Hot-standby	System cost	UGF and GA
Ouzineb et al. (2008) [105]	Series–parallel	Deterministic	Component level	Active	System cost	TS

Table 4

Multi-State RAPs with Multi-Objective

Reference	Configuration	Parameters	Redundancy level	Redundancy strategy	Objective	Solution method
Taboada et al. (2008) [106]	Series-parallel system	Deterministic	Component level	Active	Maximization of the system availability, and the minimization of system cost and weight	UGF; multi-objective multi-state genetic algorithm (MOMS-GA)
Attar et al. (2017) [107]	Series-parallel system	Deterministic	Component level and subsystem level	Hot standby and cold standby	System cost and system availability	Simulation based optimization (SBO) and GA
Mousavi et al. (2015) [108]	Series-parallel system	Fuzzy	Component level	Active	System cost and system availability	Controlled elitism non-dominated ranked genetic algorithm (CE-NRGA)

2.1.3 Continuous-State System

As a matter of fact, the degradation of some system performance is a continuous process in consecutive time. It may degrade continuously so that it can exhibit various performance levels range from fully functioning to complete failure. Hence, some scholars extended the discrete-state system to the most general case of CSS, such as the polymer electrolyte membrane fuel cell (PEMFC) stack presented by Bae et al [109]. The CSS first introduced by Baxter [110] is a special kind of system in which the states of the system and its components degrade continuously, changes between two extreme cases of perfect functioning, and total failure over time [111]. To evaluate the system performance, Gámiz and Miranda [112] built a structure function for a CSS by using multivariate nonparametric regression techniques. The structure function gives an expression of the relationship between the continuous state of the system and the state of the components over time, varied according to the distribution of the performance.

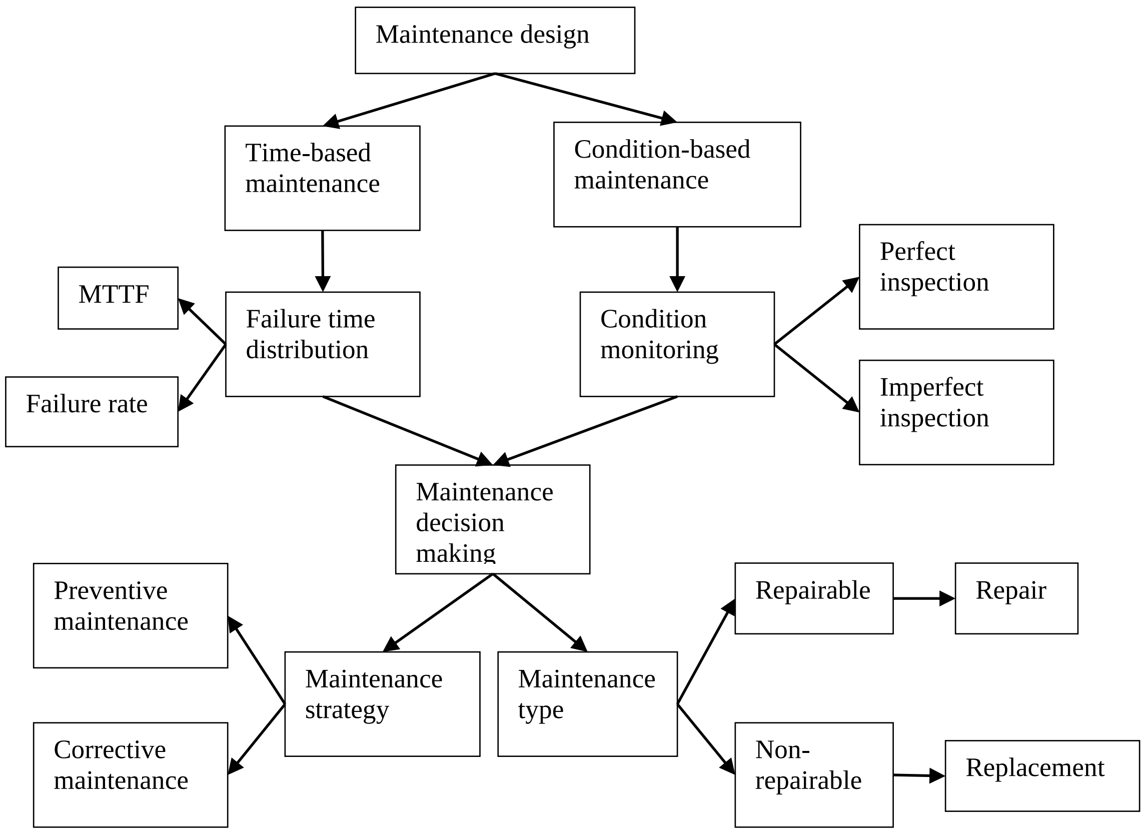
Together, these studies indicate that redundancy design has been widely used to solve various reliability optimization problems. However, such studies remain narrow in focus dealing only with the BSS and the MSS. Hence, the CSS is introduced in this study to solve RAPs.

2.2 Maintenance Design

The past decade has seen the rapid development of maintenance functions in industrial scenarios with the advancement of technology, such as the ongoing automation of production processes [113]. A comprehensive review of TBM models and CBM models in industrial applications is presented by Ahmad and Kamaruddin [114]. A brief summarize of maintenance is shown in Figure 5. This section focuses on an overview of TBM.

Figure 5

Maintenance Design Process



2.2.1 Time-Based Maintenance Design

There is a growing body of literature that recognizes the importance of maintenance design. TBM models, as a traditional maintenance technique, have been studied by a considerable amount of literature. The basic assumption of TBM models is the failure state is predictable based on the hazard rate function of the components. In the earlier studies of TBM models, the researchers generally assumed the system failure is caused by hard failures which mean the system suddenly breaks down. The maintenance time T which generally is the decision variable is decided by the failure time data [115].

Generally, there are two processes, failure data analysis, and maintenance decision making, of TBM models shown in Figure 5. The failure characteristics, such as mean time to failure (MTTF) and failure rate based on the bathtub curve process [116], are obtained by analyzing gathered failure data through statistical models. The most popular statistical model used for data analysis is the Weibull distribution due to the various aging classes of life distribution [117]. The Weibull distribution with scale parameter θ and shape parameter β which represents the lifetime characteristics is widely modeled in TBM models. The cumulative distribution of Weibull distribution is defined as,

$$F(t) = 1 - \exp\left\{-\left(\frac{t}{\theta}\right)^{\beta}\right\}, \ \ t > 0$$

where $\alpha > 0$ and $\beta > 0$. And the hazard rate function is

$$h(t) = \frac{\beta}{\theta}\left(\frac{t}{\theta}\right)^{\beta-1}, \ \ t > 0$$

However, only the case when $\beta > 1$ with an increasing failure rate is meaningful for the process to make maintenance decisions in TBM models. The MTTF is calculated as,

$$MTTF = \theta\Gamma\left(1 + \frac{1}{\beta}\right)$$

where $\Gamma(x)$ is the gamma function.

Subsequently, the cost assessment and the mechanism assessment are carried out to make decisions. The maintenance action, as we all know, is costly due to the expensive labor cost and many unpredictable costs. Hence, the aim of cost assessment is to keep the system costs down as best as we can. And the PM which is an effective way to reduce the breakdown cost is introduced in the TBM models. Hence, the system cost includes the PM cost (c_{pm}) and the CM cost (c_{cm}), which is much higher than preventive cost due to the unpredicted environment. The mechanism assessment groups the structures of components into repairable and non-repairable. For non-repairable components, the only maintenance policy is replacement. The most popular maintenance strategy of TBM models is the age-replacement strategy which minimizes the maintenance cost to find the optimal PM interval T. The general method of age-replacement models to find the optimal interval is to minimize the cost function per unit time developed by [118]. The cost function is modeled by

$$C(T) = \frac{C_{cm}F(T) + C_{pm}R(T)}{\int_0^T R(T)\,dt}, \quad T > 0$$

where $C(T)$ is the system cost per unit time, $F(T)$ is the cumulative distribution function (CDF) of the system and $R(T)$ is the reliability function of the system.

Ideally, perfect maintenance, which means the state of the system will back to the initial state after maintenance, is assumed in many existing CBM models. That is the so-called replacement. However, the state of the system may be restored to as-bad-as-old after PM for repairable components. Hence, both perfect maintenance and imperfect maintenance have been considered in the maintenance design for repairable components. The state of the system will be as-good-as-new when the perfect maintenance is implemented, while the state of the system falls between as-good-as-new and as-bad-as-old after imperfect maintenance. Huynh et al. [119] considered minimal repairs, which improves the state of the system back to the previous state for degrading systems. Furthermore, Wu et al. [120] proposed a degradation-based maintenance optimization model with imperfect repair which helps to reduce the degradation of the system. Then, Le and Tan [121] proposed an optimization model with an extension that assumes the state of the system can be improved to a better state with probability p instead of a random amount. The improvement factor is another point for researchers to consider for maintenance optimization models. Zhang et al. [122] proposed an imperfect maintenance decision model, which improves the degradation rate of the system after an imperfect maintenance.

In recent years, the stochastic process is considered in the TBM models. And the soft failures are also introduced to TBM models instead of hard failures. Abdel-Hameed [123] proposed an age-replacement policy that minimizes the average replacement cost subject to a gamma wear process. It is assumed that the maintenance action is either a corrective replacement when the system fails or a preventive replacement when the gamma wear process reaches a prespecified threshold, whichever occurs first. The cost of replacement is a function of component degradation level. And both the continuous version and discrete version are analyzed in [123]. The special case which is a discrete age replacement model in [123] is applied to a cylinder on a swing bridge by Van Noortwijk [124]. Numerous researchers focus on modeling the TBM models based on the degradation process[124], [125]. However, it is impractical to assume that the system suffers the degradation process only. In addition to the degradation of components over operational time, the sudden failures caused by traumatic events and shocks also exist in practice. Both hard failures and soft failures can be considered simultaneously during system operation.

The TBM models for dynamic systems that fail when the degradation state exceeds the critical threshold or a shock occurs before the degradation reaches the threshold are the so-called Degradation-Threshold-Shock (DTS) models, which were firstly analyzed by Lemoine and Wenocur [126]. Singpurwalla [127] reviewed the literature about stochastic-process-based reliability which includes DTS models published before this paper. Lehmann [128] proposed the system survival function and the system failure rate of DTS models. The DTS models are not only applied to TBM models but also CBM models as described by Deloux et al. [129]. The maintenance policy is optimized by using the combination of Statistical Process Control (SPC) and CBM models. Van Noortwijk et al. [125] presented a new method which is a combination of two stochastic processes for DTS models to obtain structural system reliability. Instead of independent failure modes of DTS models analyzed in the literature above, Huynh et al.[119] proposed several age-based maintenance strategies for DTS models with the assumption that the degradation level of the system and the occurrence of shocks are dependent. And minimal repairs for repairable components are considered in [119] based on the time-based decision, which depends on the system operational age, and degradation-based decision, which depends on the system degradation state.

In this dissertation, an age-based replacement model with an assumption that all failure modes are independent is added to the redundancy optimization problems to build a joint model for redundancy and maintenance optimization. And the maintenance actions are assumed to be perfect maintenance, which is the so-called replacement.

2.2.2 Condition-Based Maintenance Design

Due to the rapid development of industrial systems based on condition monitoring, the CBM technique has also been widely discussed in the literature. The main goal of CBM is to enhance the effectiveness of the operations and reduce the related cost based on the assessment of machines by making maintenance decisions. Maintenance decisions for CBM strategies are made by engineers to optimize system performance defined in the mathematical model based on specific criteria. The objective of CBM models includes the PM threshold and the inspection schedule [130]. Hence, this section focuses on the review of inspection quality and the optimization criteria to make maintenance decisions.

Generally, researchers assumed the inspections are perfect, which means the actual condition of the system is detected by the monitoring sensors accurately, without any error. However, it is more realistic to assume for the model that imperfect inspection occurs during the system in service. A great number of studies have been published under different assumptions of inspection quality [131]–[133]. Lam [132] proposed a maintenance policy that only consists of imperfect inspections for a deterioration system with an increasing failure rate. It is assumed that the inspection is related to the probability of a wrong alarm occur. Berrade et al. [134]constructed a CBM model with the consideration of the false positives, which are actually false alarms and false negatives, are allowed. The probability of inspection errors for imperfect inspections is generally assumed to be constant for convenience. In reality, the inspection errors are influenced by the degrading parameters in a deteriorating system. Vast literature with imperfect inspections under Gamma process assumption for degrading systems is reviewed by Van Noortwijk [135]. Recently, Tang et al. [136] and Ye et al. [137] proposed similar models under different stochastic processes with the imperfect inspection assumption.

The primary purpose of the maintenance design problem is to minimize the cost of the system during working. Hence, the cost minimization model is widely used in maintenance design problems. The cost parameters generally include the PM cost C_{pm}, the CM cost C_{cm}, the inspection cost C_i, and an additional cost that the system functions under failure state at a cost rate C_d. Grall et al. [138] introduced a new predictive-maintenance policy for degrading systems composed of a single component with continuous-state. The purpose of their article is to make the decision of the optimal schedule of inspection based on the system state and the threshold of PM to minimize the total cost caused by maintenance actions and system failure. The system state under stochastic degradation is analyzed by using regenerative and semi-regenerative process theory. Then, the long-run expected cost of the system is obtained based on the steady-state derived in the previous process. The cumulative cost of this system is defined as [138],

$$C(t) \equiv C_i * N_i(t) + C_{pm} * N_p(t) + C_{cm} * N_c(t) + C_d * d(t)$$

where $N_i(t)$, $N_p(t)$, $N_c(t)$ represent the random number of inspections, PM times, and CM times in $[0, t]$. And $d(t)$ is the time function elapsed during the system performance falls in the failure state. Then the long-run cost can be deducted by using elementary renewal theory [138],

$$EC_\infty = \lim_{t\to\infty}\left[\frac{E_0[C(t)]}{t}\right]$$

$$= C_i * \lim_{t\to\infty}\left[\frac{E_0[N_i(t)]}{t}\right] + C_{pm} * \lim_{t\to\infty}\left[\frac{E_0\left[N_p(t)\right]}{t}\right] + C_{cm} * \lim_{t\to\infty}\left[\frac{E_0[N_c(t)]}{t}\right]$$

$$+ C_d * \lim_{t\to\infty}\left[\frac{E_0[d(t)]}{t}\right]$$

The same maintenance policy was also applied in [139] for deteriorating systems. Huynh et al. [140] introduced traumatic events in the system to find the optimal maintenance design that minimizes the expected long-run cost.

2.3 Electric Vehicles

Electric vehicles (EVs) first introduced in the mid-19th century are the trend of the development future because of the environmental problem of exhausting fossil fuel and the emission of air-borne pollution. Renewable clean energy and power provided by the battery pack used in EVs are important for the sustainable development of our planet.

To drive the vehicle with enough power, hundreds of cells are needed in parallel or series connections assembled in the battery pack. The lithium-ion batteries have advantages in energy density, power density, and service life [141] in comparison with other batteries. It is also the so-called environmentally friendly energy without any polluting gases are released. Based on these advantages, it is widely used in EVs. Most research about EVs focused on the high cost of the battery pack and the limited driving range [142]. The high cost of the battery actually is the problem related to the replacement based on the life of the battery. In order to reduce the cost of replacement, the reliability of the battery pack is an important performance index to improve. Hence, the degradation of battery pack capacity under extended cycling is the main problem for the operation of EVs. In the traditional reliability method, the batteries in the system are considered as identical components in the system. Actually, the battery pack of EVs is a combination of more than 100 lithium-ion batteries with an individual variation that can never be eliminated. Hence, the difference among the batteries that influence the performance of the system should also be taken into account. When the capacity of one cell in the battery pack faded, the other cells need to share the load of the aged cell, which is an accelerated factor of the degradation of the other cells [143]. On the other hand, it means that the life of the battery can be extended by adding batteries in the battery pack system to share the load. And the entire battery pack needs to be replaced and maintained when a few cells degraded to the level of safety use which causes prohibitive cost. Under the consideration of the cost, redundant cells are necessary to prolong the life span of the battery pack and reduce the replacing frequency. Also, maintenance and repair are important for vehicles. As we all know, vehicle maintenance needs to be performed regularly. In electrical vehicles, the replacement of the battery pack system has a great influence on the life of vehicles and drivers. Hence, maintenance design for EVs is an important issue to be studied.

In summary, both reliability optimization problems and maintenance design for stochastically deteriorating systems are reviewed in this section. As the most popular method for reliability optimization problems, RAP model is studied in this dissertation. Because the continuous-state systems have not been well studied in RAP models, this dissertation proposes a continuous-state optimization model for redundancy design. For a stochastically deteriorating system, maintenance reviewed in this section is necessary for further improvement of the reliability during the continuous degradation process under consecutive time. Both the PM and CM are considered in this dissertation to minimize the lifecycle cost rate of the system under the assumption of the Gamma process to describe the system degradation.

Chapter 3: Proposed Methodology

A general model for redundancy and maintenance design is proposed to solve the problem described in Section 3.1. Section 3.1 also lists the basic assumptions of the models presented in this study. Section 3.2 gives a detail of the degradation process of the components in the system and the system itself. Section 3.3 and Section 3.4 introduce a redundancy optimization model and a maintenance design model, respectively, for CSS parallel-series systems. The joint model of redundancy and maintenance design is derived in Section 3.5.

3.1 Problem Description

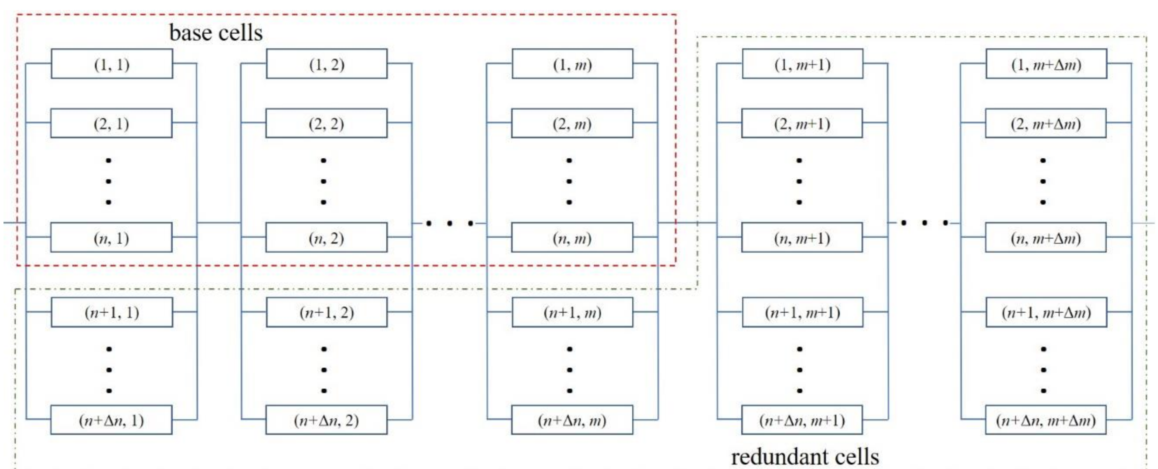
The structure of the continuous-state parallel-series system is shown in Fig. 3. The original system consists of m subsystems in series connections, and each subsystem includes n components in parallel connection. In order to slow the degradation rate and improve the system performance, a number of $(m + \Delta m)(n + \Delta n) - mn$ redundant components are supposed to be added to the system. The configuration of the system determines the relationship between the performance of the components and the performance of the system. And the performance of the whole system can be expressed by the performance of the subsystems in several ways. Hence, the configuration of the system is an important decision to be decided to improve system reliability.

In addition to improving the system reliability by adding active redundant components, prolonging the lifespan of the system which is another aim for engineers to achieve should also be considered in the process of system design. The most effective way is to carry out maintenance actions. In this study, both the PM and CM are implemented in the maintenance design. To construct a joint model of redundancy design and maintenance design for parallel-series continuous-state systems, the following assumptions are necessary:

- The components are independent.
- All components come from a homogeneous population and the initial degradation levels of all components are zero.
- All redundant components are active in the system, which means that the redundant components and the components in the basic structure degrade simultaneously.
- The system fails when the system-level degradation measure exceeds a pre-specified failure threshold for the first time.
- The maintenance time interval is negligible.

Figure 6

The Configuration of the Continuous-State Parallel-Series Systems



3.2 Degradation Process

This dissertation evaluates the system reliability according to the system degradation level. The increasing length of operation time leads to the continuous degradation of the performances of components. A general degradation- model uses a linear or nonlinear regression to describe an observed degradation path and the failure time is usually defined as the time when the mean degradation path crosses a pre-defined failure threshold. An observed degradation path can also be modeled and analyzed by a stochastic process, e.g., the Wiener process, Gamma process, and inverse Gaussian process. When a stochastic process is used, the failure occurs when the degradation path exceeds a prespecified failure threshold for the first time. This dissertation assumes the degradation path model of the component is modeled by a gamma process which is a continuous-time process with independent, stationary, and nonnegative increments when no maintenance actions are performed. A degradation-based failure occurs when the degradation path reaches the failure threshold for the first time. Other degradation models can certainly be used. The general performance function, which is a degradation path for an individual component (i, j), is considered to be a Gamma process $\{Y_{ij}(t), t \geq 0\}$ where $Y_{ij}(t)$ represents the degradation level at time t with identical scale parameter β and shape parameter $\alpha(t)$. The Gamma process has the following properties [135]:

1) $Y_{ij}(0) = 0$ and $\alpha(0) = 0$ with probability one.
2) Increments in Gamma process, $\Delta Y_{ij}(t) \equiv Y_{ij}(t + \Delta t) - Y_{ij}(t) \sim G(\Delta\alpha(t), \beta)$, are independent and non-negative.

3) $\alpha(t)$ as the shape parameter of Gamma process is a monotone increasing function.

The probability density function of Gamma process with parameters $\Delta\alpha(t) \equiv \alpha(t+\Delta t) - \alpha(t)$ and β is then given by,

$$f\left(\Delta y_{ij}; \Delta\alpha(t), \beta\right) = \frac{1}{\Gamma\left(\Delta\alpha(t)\right) \cdot \beta^{\Delta\alpha(t)}} \cdot \Delta {y_{ij}}^{\Delta\alpha(t)-1} \cdot \exp\left(-\frac{\Delta y_{ij}}{\beta}\right) \cdot I_{\{\Delta y_{ij} \geq 0\}}$$

where $I_{\{\Delta y_{ij} \geq 0\}} = \begin{cases} 1, & if \;\; \Delta y_{ij} \geq 0 \\ 0, & if \;\; \Delta y_{ij} < 0 \end{cases}$ and $\Gamma\left(\Delta\alpha(t)\right) = \int_0^\infty z^{\Delta\alpha(t)-1}\, e^{-z} dz$ is the gamma function. Various degradation processes are modeled by the value of parameters β and $\Delta\alpha(t)$. $E(\Delta Y_{ij}(t)) = \beta \cdot \Delta\alpha(t)$, and $VAR(\Delta Y_{ij}(t)) = \beta^2 \cdot \Delta\alpha(t)$.

The failure of the components in a deteriorating system occurred when the degradation level exceeds the prespecified failure threshold w. Define the failure time $\sigma_w = \inf\left\{t \geq 0, Y_{ij}(t) \geq w\right\}$ and the cumulative distribution $F_{\sigma_{w_{ij}}}(t)$ of the performance of an individual component is defined as,

$$\begin{aligned} F_{\sigma_{w_{ij}}}(t) &= Pr\left(\sigma_{w_{ij}} \leq t\right) \\ &= Pr\left(Y_{ij}(t) \geq w\right) \\ &= \int_w^\infty f\left(y_{ij}; \alpha(t), \beta\right) dy_{ij} \\ &= \frac{\Gamma\left(\alpha(t), \frac{w}{\beta}\right)}{\Gamma\left(\alpha(t)\right)}, t \geq 0 \end{aligned} \tag{1}$$

where

$$\Gamma(\alpha, x) = \int_x^\infty z^{\alpha-1}\, e^{-z} dz, \quad\quad x \geq 0 \quad \alpha \geq 0$$

is the incomplete gamma function. And the probability density function of the first hitting time is denoted by [135],

$$f_{\sigma_{w_{ij}}}(t) = \frac{\partial F_{\sigma_{w_{ij}}}(t)}{\partial t} = \frac{\alpha}{\Gamma\big(\alpha(t)\big)} \int_{\frac{w}{\beta}}^{\infty} \Big\{\log(z) - \psi\big(\alpha(t)\big)\Big\} z^{\alpha(t)-1}\, e^{-z} dz, \;\; t \geq 0 \qquad (2)$$

where the digamma function $\psi(x)$ is defined as,

$$\psi(x) = \frac{\Gamma'(x)}{\Gamma(x)} = \frac{\partial log\Gamma(x)}{\partial x}.$$

When the specific performance of the system cannot be fulfilled, the system is deemed to be failed. Indeed, the degradation levels of the components in the system differing from each other. The performance of the system may be still reliable even though the performances of some components are lower than the mean required performance in the traditional reliability method. Hence, the consideration of system-level reliability has an advantage in cost reduction of replacement. Upon the expression of performance for a single component and specific rules, the performance of the entire system can be described by a deterministic structure function given by

$$Y_s(t) = \Phi\left(Y_{11}(t), Y_{12}(t), \dots, Y_{ij}(t), \dots, Y_{nm}(t)\right)$$

The structure function $\Phi(\cdot)$ indicates the relationship between the states of the components and the state of the system itself, varying according to different rules decided by the property of the components and the configuration of the system. It also means that the continuous state of the system stems from the states of different parts of the system through the structure function. Note that the degradation path of a component may be affected by the redundancy design because adding active redundancy can reduce the load of each cell and slow the degradation of the cell. Examples of structure functions include $sum()$, $min()$, $max()$, etc. For example, when modeling and analyzing the voltage degradation of a fuel cell stack system consisting of multiple fuel cells in a serial configuration, Yuan et al. [13] defined two system-level degradation measures: one is the sum of voltages of all components and the other is the minimum voltage among all components. Those two system-level degradation measures are related to the components' degradation measures via the $sum()$ and $min()$ structure functions, respectively. In addition, some empirical techniques, e.g., regression, may be used to find approximations for complex structure functions [112].

Then, the lifetime distribution of the system is described by using structure function based on the distribution of an individual component. The system cumulative density function for the degradation process is defined as,

$$\begin{aligned} F_s(t) &= Pr\left(\sigma_{w_s} \leq t\right) = Pr(Y_s(t) \geq w_s) \\ &= Pr\left(\Phi\left(Y_{11}(t), Y_{12}(t), \dots, Y_{ij}(t), \dots, Y_{nm}(t)\right) \geq w_s\right), \quad t \geq 0 \end{aligned} \tag{3}$$

And the probability density function of the degraded system is

$$f_s(t) = \frac{\partial F_s(t)}{\partial t} = \frac{\partial\, Pr\left(\Phi\left(y_{11}(t), y_{12}(t), \dots, y_{ij}(t), \dots, y_{nm}(t)\right) \geq w_s\right)}{\partial t} \tag{4}$$

3.3 Redundancy Optimization Model

First of all, this dissertation builds a redundancy optimization model. Minimizing the entire cost of redundant components for a parallel-series continuous-state system is the main objective of this model with the requirement of the reliability of the system. In this section, the model itself and GA used to solve the problem are described in detail.

3.3.1 Model Construction

Starting from a given base design consisting of $n \times m$ components, the optimization model for parallel-series continuous-state systems is formulated to determine the cost-optimal redundancy design, satisfying some system reliability and design constraints.

The system cost $C_s(\boldsymbol{X})$ is analyzed in two parts, the intrinsic cost c_r of the redundant components related to production and the installation cost C_{act} associated with technical actions applied to the installation of components. The technical actions include the installation process of series and the installation process of parallel. c_p is the cost that redundant components organized in the parallel system, and c_s is used to represent the expenses that the subsystem in series with the original system. Hence, we can get the expression for cost as below,

$$C_s = C_{com} + C_{act}$$
$$C_{com} = c_c mn + c_r\big((m + \Delta m)(n + \Delta n) - mn\big) \tag{5}$$
$$C_{act} = c_p(m + \Delta m)\Delta n + c_P(n - 1)\Delta m + c_s\Delta m$$

where c_c is the unit cost of components in the original system, c_r is the unit cost of redundant components in the new system, c_p is the unit cost of components installed in parallel, c_s is the unit cost of the subsystem installed in series with the other subsystems. Based on the above description and discussion, the optimization model for the parallel-series continuous-state system can be defined as below,

$$\begin{aligned} \min \quad & C_s(\boldsymbol{X}) \\ \text{s. t.} \quad & R_s(t_m; \boldsymbol{X}) \geq R_{req}(t_m) \\ & \mathbf{D}(\boldsymbol{X}) \leq \boldsymbol{d} \end{aligned} \tag{6}$$

where t_m is the mission time and $R_{req}(t_m)$ is the system reliability requirement at t_m. $R_s(t_m; \boldsymbol{X})$=1-$F(t_m; \boldsymbol{X})$ =1-$F_s(\boldsymbol{X}; t)$ is the reliability function of the system. $\mathbf{D}(\boldsymbol{X}) \leq \boldsymbol{d}$ represents a set of possible design constraints. For example, there may be an up limit on the total number of components assembled in the system or a size/weight limit. The design variable vector $\boldsymbol{X}$ in this parallel-series system, which includes the redundant number of components in each subsystem and the number of redundant subsystems, is described as $\boldsymbol{X} = (\Delta n, \Delta m)$ where Δn and Δm are nonnegative integers. GA is implemented to solve this nonlinear integer programming model.

3.3.2 Genetic Algorithm

Apart from the GA described in this section, many other algorithms can be used in this study. For example, particle swarm optimization (PSO) can also be used to solve the optimization problem. However, GA algorithm is sufficient to solve the optimization problem described in this study with different kinds of systems, especially for the illustration problem in section 4. And it has great advantages in solving integer problems. Hence, GA has a detailed explanation in this section.

The basic idea of GA is to produce offspring of the next generation by cross-genes of parents which have the best fitness. Due to the limitations of searching algorithms, the solution computed by GA may be a locally optimal solution if the initial parameters are unreasonable. Hence, this study compares the GA solutions with the results computed by the enumeration method to determine the parameters in this problem.

The GA starts with an initial randomly generated population of solutions. A fitness score that is used to evaluate the chromosome is computed for each solution and a set of solutions are selected as parents to produce a new generation of solutions (offspring) via the crossover and mutation operators. And then repeat this procedure for a given number of generations. The top solutions in the last generation are examined to find the best solution(s). Detailed implementation of GA is described below.

The population which is formed with a set of individuals is a subset of solutions to the proposed problem in the current generation. Also known as chromosomes, individuals are strings combined with several genes. Binary encoding, which translates the non-negative integer decision variables $(\Delta n, \Delta m)$ into binary values, i.e. 0s and 1s, is used in this study. The initial population can be populated with completely random solutions or using a heuristic function. In this dissertation, the initial population of n_{pop} solutions are randomly generated in the search space.

The fitness function is used to evaluate each solution and is defined as,

$$Fitness(\Delta n, \Delta m) = -\left[C_s(\boldsymbol{X}; t) + max\left(0,\ R_{req}(t_m) - \ R_s(t_m; \boldsymbol{X})\right) \cdot \mathcal{M}\right]$$

where the second term is a penalty term when a solution does not satisfy the reliability constraint and $\mathcal{M} = 10^7$ is a big number. According to the fitness scores of all solutions, the chromosomes are selected as parents of the next generation for reproduction. If the solution exceeds the constraints, the fitness score for this solution will decrease which means it has a smaller chance to be selected.

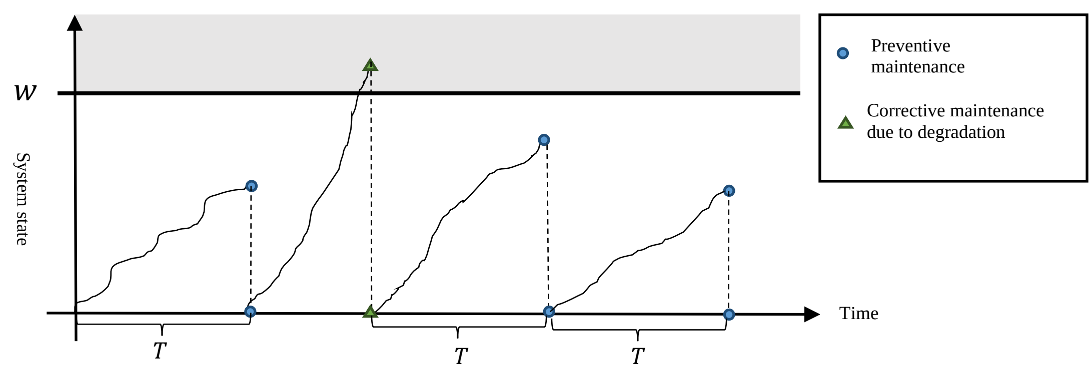
Parent selection is a process that selects the fittest chromosome from the population and the selected parents will pass their genes to the next generation. Chromosomes with higher fitness scores have more success to be selected. This study implements the tournament selection method [88]. This method randomly selects k_{sel} chromosomes from the population and then choose the one with the highest fitness among the k_{sel} selected chromosomes. This procedure is repeated n_{pop} times. Crossover is then applied to generate offspring from the n_{pop} parents. The one-point crossover method is used in this study. A crossover point is randomly picked and the parts of two parents after the crossover point are swapped to generated two new offspring. The mutation operator is applied to increase the diversity of the offspring. Each gene of a chromosome has a low mutation probability p_{mut} to be mutated. The algorithm is terminated after n_{gen} generations. The unique solutions in the last generation are ranked according to their fitness scores in order to find the optimal solution(s).

3.4 Age-Based Replacement Model

Once the system configuration is determined, maintenance strategies are applied to the system with a deterministic configuration shown in Figure 7. Since the CBM models have a high requirement on the sensors which is costly, this dissertation focuses on TBM models. The components in the system are assumed to be “as good as new” after any maintenance actions, which is the so-called replacement. The age-based replacement model has been widely used in the industrial field. For example, the engine drive chain on old Volvos is supposed to be replaced every 80,000 km. The oil and oil filters are generally replaced every 3 months. The reliability of the components in a plane has a great impact on passengers’ life. It is necessary to implement a regular component replacement. Both preventive maintenance and corrective maintenance as maintenance strategies are introduced to the age-based maintenance model in this study. And it is decided by whichever occurs first. The preventive replacement is carried out at regular intervals T which is a planned downtime from the last replacement time with a cost C_{pm}. The corrective replacement actions are taken when the system fails which means the degradation level reaches the prespecified threshold w. The unplanned downtime caused by item failures, internal or external events may lead to additional cost C_d. Hence, the cost of corrective replacement is defined as $C_{cm} = C_{pm} + C_d$. The only design variable of this maintenance model is the regular preventive replacement time interval T. And the additional cost C_d may have a great impact on the optimal solutions.

Figure 7

Schematic Evolution of an Age-Based Replacement System



The analysis of system cost is a generally used measure to evaluate the maintenance policy. In this study, the objective function of this model is to minimize the lifecycle cost rate which includes two parts, system design cost, and maintenance cost. The system design cost C_{design} is the total cost of the system redundancy design cost defined in Eq. (5) because all the components in the system are replaced by new components. And the cost is a constant number since the system configuration has been determined in the redundancy optimization model.

The maintenance part is the long-run expected cost, which can be obtained by the renewal reward theory [144], of maintenance actions between two replacements. It represents the maintenance cost per unit time. The expected long-run unit cost is defined as

$$cost\ rate\ = \frac{expected\ maintenance\ cost\ in\ a\ maintenance\ interval}{expected\ length\ of\ a\ maintenance\ interval}$$

Based on this equation, the expected long-run cost per unit time in this dissertation is defined as

$$C_{\infty}(T) = \lim_{T\to\infty} \frac{E[C(T)]}{T} = \frac{E[C(T)]}{E[T]} \tag{7}$$

where $C(T)$ is the accumulated maintenance cost at time T. Then, the long-run unit cost can be changed into a ratio of the expected maintenance cost in a maintenance interval to the expected length of the maintenance interval. And the expected maintenance cost $E[C(T)]$ is expressed as,

$$E[C(T)] = C_{cm}F_s(T) + C_{pm}(1 - F_s(T))$$

where $(1 - F_s(T))$ is the probability of a PM in a replacement interval and $F_s(T)$ defined in Eq. (3) is the probability that a CM that implemented in a replacement interval. Either a PM is implemented in a regular maintenance interval, or a CM will be done. And the expected length of the maintenance interval is integral to the reliability function according to renewal reward theory [144].

$$E[T] = \int_0^T (1 - F_s(t))\, dt$$

Assuming the system has a limited lifetime l. In order to add the system design cost to the expected long-run unit cost function, the system design cost should have the same unit used with the expected long-run maintenance cost. Hence, the system design cost is divided into l equal parts which are described as the system design cost per unit time. Then, the analytical formula of cost minimization objective function is described as,

$$C_s(T) = C_\infty(T) + ({}^{A}/_{P}, I, l) * C_{design}, \quad T > 0 \tag{8}$$

where $({}^{A}/_{P}, I, l) = \frac{I(1+I)^l}{((1+I)^l - 1)}$ is the capital recovery factor which divides the system design cost into $l - period$ equivalent parts. I represents the interest rate per period and l is the expected lifetime of a system in the number of periods. Then, both two parts are the costs per unit time. The capital recovery factor is a ratio used to obtain the present value, which represents the equivalent periodical cost, of a set of future costs. It has been widely used in the economic engineering area. The optimization variable T is restricted to minimize the lifecycle cost rate $C_\infty(T)$ given in Eq. (7).

In the CBM models, each of the components in the system should be equipped with a monitoring sensor to obtain its real-time status, and the monitoring sensors are required to be sensitive at any time. Hence, the condition-based maintenance model has a high requirement of monitoring sensors which may be costly. Then, this dissertation focuses on age-based maintenance. In the battery pack system which is discussed as an application in chapter 4, mixed cells with different degradation levels in a system may cause accelerated degradation to individual cells [143]. Hence, the whole system is considered to be replaced which is an age-based replacement policy in this dissertation to avoid the increased degradation rate.

3.5 Joint Design Model of Redundancy and Maintenance Optimization

Previous research focuses on the redundancy design and maintenance design respectively. In this section, a joint optimization model of redundancy design and maintenance design is proposed for parallel-series continuous-state systems. The aim of this joint model is to minimize the lifecycle cost rate of the system and the reliability requirement should be satisfied simultaneously. Following this goal, the joint model is to minimize the expected lifecycle cost rate during the system operational time with the constraint of reliability or other design constraints.

Since the maintenance policy in this study is the age-based replacement, the cost of each preventive replacement is the cost of redundancy design which is the construction of the parallel-series system defined in Eq. (5). The design cost C_{design} is varied based on the variation of the system configuration during the searching of solutions. And the corrective replacement cost is the summation of the preventive replacement and an additional unpredictable cost C_d which is caused by system failure. C_d, as an unexpected cost, maybe a large value since the unpredictable failure has a great impact on both safety and cost. Hence, the unit maintenance costs are given by,

$$C_{pm} = C_{design} = C_{\mathrm{s}} = C_{com} + C_{act}$$

$$C_{cm} = C_{pm} + C_d = C_{design} + C_d$$

Then the cost rate minimization model is summarized as,

$$\min C_{\mathrm{s}}(T)$$

$$\text{s. t. } R_s(t_m; \boldsymbol{X}) \geq R_{req}(t_m)$$

$$\mathbf{D}(\boldsymbol{X}) \leq \boldsymbol{d}$$

where $C_s(T)$ is defined in Eq. (8). $R_s(t_m;\boldsymbol{X})$=1-$F(t_m;\boldsymbol{X})$ =1-$F_s(\boldsymbol{X};t)$ is the reliability function of the system. $F_s(\boldsymbol{X};t)$ is defined in Eq. (3). The design variable vector $\boldsymbol{X}$ in this minimization model, includes the redundant number of components in the subsystem, the number of redundant subsystems, and the preventive replacement interval, can be shown as $\boldsymbol{X} = (\Delta n, \Delta m, T)$ where Δn, Δm and T are nonnegative integers.

Since the continuous-state systems have not been well studied for reliability problems, this dissertation firstly proposed a redundancy optimization model and an age-based replacement model for parallel-series continuous-state systems, respectively. The objective of the redundancy optimization model is to minimize the system design cost satisfy the reliability requirements. And this dissertation considers the design cost includes the cost of components themselves and the cost of technical actions for connection. Then, an age-based replacement model is formulated for a determined configuration which is decided by the redundancy optimization model. The objective is to minimize the lifecycle cost rate. It includes two parts, the expected long-run maintenance cost and the design cost which is evenly distributed per unit time. Furthermore, this dissertation introduced a joint model of redundancy and maintenance for parallel-series continuous-state systems. The joint model determines the optimal system design and the optimal maintenance interval simultaneously.

Chapter 4: A Case Study on the Battery Pack System

According to the redundancy optimization model, maintenance design model, and the joint design model proposed in chapter 3, this chapter presents the models and results applied to the battery pack system in EVs. Section 4.1 defines the performance of the battery pack system. Section 4.2, 4.3, and 4.4 build the redundancy optimization model, age-based replacement model, and joint design model, respectively, then analyze and compare the results from different models.

4.1 Overview of the Battery Pack System Performance

This section uses the battery pack system for EVs as an example to illustrate the proposed methodology. This study assumes that all cells in a pack are independent, the initial degradation levels of all cells are zero, and the cells degrade consistently. Mixing cells with different degradation levels may introduce interactions among the cells and accelerate the degradation processes. Gong et al.[143] conducted an experimental study on the cell inconsistency problem for parallel-connected lithium-ion battery cells for EVs. Cells with different degradation levels were connected in parallel. Experimental results and analysis indicated that cell inconsistency may severely reduce the reliability of a battery pack.

4.1.1 Performance of the Battery

The state-of-health (SOH) of the cells defined as a variable in EVs shows the general health condition of a single cell. It is evaluated by the percentage of maximum releasable capacity of an aged battery relative to the maximum capacity of a new battery. The SOH can be expressed as [145],

$$\text{SOH} = \frac{Q_{max}(\text{aged})}{Q_{max}(\text{new})} \times 100\%$$

$$Q_{max}(\text{aged}) = Q_{max}(\text{new}) - Q_{max}(\text{fade})$$

where $Q_{max}(\text{aged})$ is the current maximal releasable capacity of an aged battery that has operated period, while $Q_{max}(\text{new})$ represents the maximum amount of a newly used battery with initial capacity. $Q_{max}(\text{fade})$ is the faded capacity of the aged battery as a result of the cycle number, temperature, and discharge rate. Based on the relationship among $Q_{max}(\text{aged})$, $Q_{max}(\text{new})$, and $Q_{max}(\text{fade})$, the SOH is defined as

$$\text{SOH} = \frac{Q_{max}(\text{new}) - Q_{max}(\text{fade})}{Q_{max}(\text{new})} \times 100\% = \left(1 - \frac{Q_{max}(\text{fade})}{Q_{max}(\text{new})}\right) \times 100\%$$

Hence, the performance of the batteries is reflected by the degradation of the cells which can be defined as,

$$Y(t) = 1 - \text{SOH} = \frac{Q_{max}(\text{fade})}{Q_{max}(\text{new})} \times 100\%$$

Under the extended cycling number, Ramadass et al. [146] summarized that the main factors responsible for the capacity fade of lithium-ion batteries can be separated into three parts to analyze. The first part deals with the loss related to the consistent growth of resistance at both the positive electrode and the negative electrode. Part two indicates that the loss of lithiation at both two electrodes has an impact on the capacity fade. The last part said that the loss of active material Li+ is responsible for the capacity fade. According to the analysis in [146], $Q_{max}(\text{fade})$ of Sony 18650 cells is determined by three parameters: the rate capability, secondary active material and, primary active material losses. The expression of $Q_{max}(\text{fade})$ can be shown as,

$$Q_{max}(\text{fade}) = Q_{lost}(i) + Q_{\text{I}} + Q_{\text{II}}$$

where $Q_{lost}(i)$ is the loss of discharge capacity, which is related to its discharging rate, i. Q_{I} and Q_{II} represent the capacity loss due to primary and secondary active material, respectively. Under very low discharge rate, the semi-empirical capacity fading model constructed by Ramadass et al. [146] shows that Q_{I} and Q_{II} result from the state-of-charge (SOC) of the limiting electrode. The changing rate of SOC of electrode material is defined as

$$\frac{d\mathrm{SOC}_{lost}}{dN} = k_1 N + k_2$$

which is related to the temperature and the charge/discharge cycle number. The parameter k_1 is responsible for the accelerated loss of capacity under unfavorable conditions such as high temperature, while k_2 accounts for the capacity loss under normal conditions of the charge/discharge cycles. And the loss in the SOC was calculated by

$$\mathrm{SOC}_{lost} = \frac{Q_{lost}(T, N)}{Q_{max}(\mathrm{new})}$$

where $Q_{lost}(T, N)$ is influenced by the temperature and the number of charge/discharge cycles. Hence, the capacity loss due to primary and secondary active material can be shown as

$$Q_{lost}(T, N) = Q_{\mathrm{I}} + Q_{\mathrm{II}} = \mathrm{SOC}_{lost} \times Q_{max}(\mathrm{new}) = \frac{1}{2} k_1 N^2 + k_2 N$$

For higher discharge rates, the discharge rate should be considered into the loss of capacity which is responsible for rate capability losses. As shown in [145], the discharge capacity loss is a liner with discharge rate,

$$Q_{lost}(i) = k_3 i$$

$$i = \frac{I}{Q_{max}(\mathrm{new})}$$

where i represents the discharging rate, I is the individual discharging current and k_3 is a parameter that changed based on the cycle numbers.

This study defines the degradation measure of individual cells as $Y(N) = 1 - \mathrm{SOH}$ so that the degradation measure $Y(N)$ increases with N. To model $Y(N)$ by the Gamma process with the scale parameter β and shape function $\alpha(N)$, we assume

$$E[Y(N)] = \beta \cdot \alpha(N) = \left(\frac{1}{2}k_1N^2 + k_2N\right) + \frac{k_3}{Q_{max}(\mathrm{new})}i \tag{9}$$

and then

$$\alpha(N) = \frac{1}{\beta}\left(\frac{1}{2}k_1N^2 + k_2N + \frac{k_3}{Q_{max}(\mathrm{new})}i\right)$$

Because $VAR[Y(N)] = \beta^2 \cdot \alpha(N) = \beta \cdot E[Y(N)]$, increasing β will increases the variance of $Y(N)$.

4.1.2 Performance of the Battery Pack System

Liu et al. [145] constructed a capacity fade system model by using UGF technique based on parallel-series MSS to find the optimal number of redundant cells. And the batteries in the system are assumed to be independent and identical. This paper extends the MSS of the battery pack system to CSS and introduces Gamma process in the parallel-series system.

Next, the system-level degradation measure $Y_s(N)$ is derived. For the subsystem with multiple cells connected in parallel, the battery management system (BMS) generally does not monitor the SOH of single cells due to it does not measure the cell current, instead, BMS tracks the subsystem SOH by an equivalent single value for the whole subsystem, e.g., the average SOH [147]. For multiple subsystems connected in series, the system performance is determined by the worst subsystem [143]. Therefore, the degradation measure of the parallel-series battery pack can be formulated as

$$Y_{\mathrm{s}}(N|\Delta n, \Delta m) = \max_{k=1,\dots,m+\Delta m} \left\{ \frac{1}{n+\Delta n} \sum_{j=1}^{n+\Delta n} Y_{jk}(N|\Delta n, \Delta m) \right\} \quad (10)$$

Herein the cell degradation $Y_{jk}(N|\Delta n, \Delta m)$ is modeled by the Gamma process with the scale parameter β and shape parameter function $\alpha(N|\Delta n, \Delta m) = \frac{E[Y_{\mathrm{s}}(N|\Delta n,\Delta m)]}{\beta}$, where $E[Y_{\mathrm{s}}(N|\Delta n, \Delta m)]$ is given by Eq. (9).

Generally, the battery pack should be replaced when the capacity is degraded to 80% of the initial capacity. It means that the reliability constraint of the system is w_s = 0.20 [148]. Then the system reliability function is defined as

$$\begin{aligned} R_s(N|\Delta n, \Delta m) &= Pr(Y_{\mathrm{s}}(N|\Delta n, \Delta m) \leq w_s) \\ &= \prod_{k=1}^{m+\Delta m} Pr\left(\frac{1}{n+\Delta n} \sum_{j=1}^{n+\Delta n} Y_{jk}(N) \leq w_s \right) \\ &= \prod_{k=1}^{m+\Delta m} Pr\left(\sum_{j=1}^{n+\Delta n} Y_{jk}(N|\Delta n, \Delta m) \leq (n+\Delta n) w_s \right) \\ &= \left(1 - \frac{\Gamma\left[(n+\Delta n)\alpha(N|\Delta n, \Delta m),\ \frac{(n+\Delta n)w_s}{\beta}\right]}{\Gamma[(n+\Delta n)\alpha(N|\Delta n, \Delta m)]} \right)^{m+\Delta m} \end{aligned} \quad (11)$$

Note that if $Y_{jk}(N|\Delta n, \Delta m)$, for $j = 1, \dots, n+\Delta n$, follow the $iid\ Gamma(\alpha(N|\Delta n, \Delta m), \beta)$ distribution, $\sum_{j=1}^{n+\Delta n} Y_{jk}(N|\Delta n, \Delta m)$ is a $Gamma\big((n + \Delta n)\alpha(N|\Delta n, \Delta m), \beta\big)$ random variable.

Liu et al. [145] and Xia et al. [148]assumed a different structure function of the form

$$Y_s'(N|\mathit{\Delta} n, \mathit{\Delta} m) = \max_{k=1,\ldots,m+\mathit{\Delta} m} \left\{ \min_{j=1,\ldots,n+\mathit{\Delta} n} Y_{jk}(N|\mathit{\Delta} n, \mathit{\Delta} m) \right\} \quad (12)$$

that is, the performance of a subsystem with cells connected in parallel is determined by the best cell in the subsystem. Under this assumption, the reliability function of the battery

pack system becomes

$$R_s'(N|\mathit{\Delta} n, \mathit{\Delta} m) = \left(1 - \left(\frac{\Gamma\left[\alpha(N|\mathit{\Delta} n, \mathit{\Delta} m),\ \frac{w_s}{\beta}\right]}{\Gamma[\alpha(N|\mathit{\Delta} n, \mathit{\Delta} m)]} \right)^{n+\mathit{\Delta} n} \right)^{m+\mathit{\Delta} m}$$

Under the same configuration, $R_s'(N|\mathit{\Delta} n, \mathit{\Delta} m)$ is expected to be higher than

$R_s(N|\mathit{\Delta} n, \mathit{\Delta} m)$.

4.1.3 Computational Method

The power from the number of complete charge/discharge cycles of the battery pack for well-functioning EVs is a constant P no matter how many cells in the battery pack. The power supplied by 1 cycle of charge/discharge will increase as the number of cells in the system rise. Hence, the cycle number of the system will change with the addition of redundant cells. Assuming the cycle number of the system without redundancies is N. In order to satisfy the same power P, the new cycle number of the new system will decrease, becomes [145]

$$N_{new} = \frac{mnN}{(m + \Delta m)(n + \Delta n)}$$

To supply the same power P, if $(m + \Delta m)(n + \Delta n) - mn$ redundant cells are assigned on the new battery pack, the discharging rate are recalculated as [145],

$$i = \frac{mnI}{(m + \Delta m)(n + \Delta n)}$$

where I is the individual discharging current in the original system with $m \times n$ cells. Hence, the mean degradation path of individual cells in the battery pack system with $(m + \Delta m)(n + \Delta n) - mn$ redundant cells becomes

$$E[Y(N|\Delta n, \Delta m)] = \left\{\frac{1}{2} k_1 \left(\frac{mnN}{(m + \Delta m)(n + \Delta n)}\right)^2 + k_2 \frac{mnN}{(m + \Delta m)(n + \Delta n)}\right\} + \frac{k_3}{Q_{max}(\text{new})} \times \frac{mnI}{(m + \Delta m)(n + \Delta n)} \tag{13}$$

As shown in Eq. (13), adding active redundant cells decreases the power output of individual cells and hence slows down the cell degradation as this degradation is proportional to the using conditions [149].

4.2 Redundancy Optimization Model for the Battery Pack System

Based on the optimization model constructed in Eq. (5), this study tries to find the optimal number of redundant cells to minimize the total cost of the system to satisfy the reliability requirements of the battery pack. Sony 18650 with a capacity of 1.75Ah is studied in this dissertation [145]. The objective considered here is the total cost of battery cells in the system, which is proportional to the total number of cells in the system. Then, the optimization model defined in section 3 is formulated as follow:

$$\begin{aligned} &\min C_s(\Delta n, \Delta m) = c_c (m + \Delta m)(n + \Delta n) \\ &\text{s. t. } R_s(t_m|\Delta n, \Delta m) = \left(1 - \frac{\Gamma\left[(n+\Delta n)\alpha(N|\Delta n,\Delta m), \frac{(n+\Delta n)w_s}{\beta}\right]}{\Gamma[(n+\Delta n)\alpha(N|\Delta n,\Delta m)]}\right)^{m+\Delta m} \geq R_{req}(t_m) \end{aligned} \tag{14}$$

where c_c is the unit cost of the cells, which is assumed to be \$5 in the numerical studies. In this dissertation, we only consider the cost of components. Note that other cost components may be added to the cost objective function, e.g., costs for connecting cells to form modules and packs as we described in section 3.

For illustrative, assuming that the base design is $(n \times m) = (2 \times 5)$. This study investigated respectively the battery pack functioning at 25℃ charge/discharge $t_m = 800$ cycles and 50℃ charge/discharge $t_m = 500$ cycles that are sensitive to cell redundancy to find the relationship among reliability, number of redundancies, configuration, and the cost of cells. Values of the parameters in the semi-empirical cell degradation model at different temperatures are listed in Table 5, and Table 6 summarizes the parameters used in GA with details in section 3.3.2. And this study assumes the user demand for reliability is no less than $R_{req}(t_m) = 0.99$ or $R_{req}(t_m) = 0.9999$. The discharge rate for both conditions is 1C.

Table 5

Values of Parameters at Different Temperatures [145], [147], [150]

Cycling temperature (℃)	k_1 (cycle^{-2})	k_2 (cycle^{-1})	k_3 (A^{-1})	
			a (A^{-1})	b (A^{-1}cycle^{-1})
25	8.5×10^{-8}	2.5×10^{-4}	0	9×10^{-5}
50	1.6×10^{-6}	2.9×10^{-4}	2.77×10^{-2}	8.1×10^{-5}

Table 6

Parameters Used in GA

Population size	n_{pop}	500
Number of generations	n_{gen}	500
Tournament size	k_{sel}	3
Mutation probability	p_{mut}	0.001
Constraint penalty	$\mathcal{M}$	10^7

Table 7 lists the optimal solutions found by GA, assuming charge/discharge $t_m = 800$ cycles at 25℃ and $t_m = 500$ cycles at 50℃, respectively. The scale parameter β used in the numerical study ranges from 0.01 to 0.03. Since the number of batteries assumed in this example is small, an exhaustive search method is used to verify the optimal solutions found by GA. The GA and exhaustive search produce identical solutions for all β and $R_{req}(t_m)$ combinations at different temperatures. Table 8 and Table 9, for example, enumerate a set of possible solutions to identify the optimal solution(s) when $\beta = 0.01$. The optimal solutions for $R_{req}(t_m) = 0.99$ and 0.9999 are marked with * and **, respectively. When $\beta = 0.01$, the cost-optimal solution that satisfies the 0.99 system reliability requirement is $(\Delta n,\ \Delta m) = (2, 0)$, that is, adding two parallel cells to each subsystem. The total number of cells is $(n + \Delta n)(m + \Delta m) = (2 + 2)(5 + 0) = 20$, corresponding to a total cost of 100. If the system reliability requirement increases to 0.9999, two optimal solutions $(\Delta n,\ \Delta m) = (1, 3)$ and $(2, 1)$ with a total cost 120, which is obviously higher than lower reliability requirement, are found. Both solutions use 24 cells to form the battery pack, with the 3×8 and 4×6 configurations, respectively. When we compare the two optimal solutions, the system reliability of the $(2, 1)$ design is higher than that of the $(1, 3)$ design. Hence, one may prefer adding redundant cells in parallel than in serial.

Table 7

Optimal Redundancy Allocation when $t_m = 800$ Cycles at 25℃ and $t_m = 500$ Cycles at 50℃

Temperature (℃)	β	$R_{req}(t_m) = 0.99$				$R_{req}(t_m) = 0.9999$			
		Δn^*	Δm^*	C_s^*	$R_s^*(800)$	Δn^*	Δm^*	C_s^*	$R_s^*(800)$
25	0.01	2	0	100	0.99937	1	3	120	0.99994
						2	1		1.00000
	0.02	1	3	120	0.99217	3	0	125	0.99992
		2	1		0.99885				
	0.03	3	0	125	0.99825	4	0	150	0.99998
50	0.01	2	1	120	0.99994	2	1	120	0.99994
						1	3		0.99912
	0.02	2	1	120	0.99288	3	1	150	1.00000
	0.03	2	2	140	0.99518	3	2	175	0.99997

Table 8

Enumeration of $(\Delta n \times \Delta m)$ *when* $\beta = 0.01$ *and* $t_m = 800$ *Cycles at* 25℃

Δn		Δm					
		0	1	2	3	4	5
	0	$C_s = 50$	60	70	80	90	100
		$R_s(t_m) = 0.00000$	0.00001	0.02899	0.40341	0.81533	0.95699
	1	75	90	105	120**	135	150
		0.38360	0.95579	0.99842	0.99994	1.00000	1.00000
	2	100*	120**	140	160	180	200
		0.99937	1.00000	1.00000	1.00000	1.00000	1.00000
	3	125	150	175	200	225	250
		1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
	4	150	180	210	240	270	300
		1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
	5	175	210	245	280	315	350
		1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

Table 9

Enumeration of $(\Delta n \times \Delta m)$ *when* $\beta = 0.01$ *and* $t_m = 500$ *Cycles at* 50℃

Δn		Δm					
		0	1	2	3	4	5
	0	50	60	70	80	90	100
		0.00000	0.00000	0.00000	0.00374	0.22492	0.71469
	1	75	90	105	120*	135	150
		0.00044	0.50036	0.97207	0.99912	0.99997	1.00000
	2	100	120**	140	160	180	200
		0.97387	0.99994	1.00000	1.00000	1.00000	1.00000
	3	125	150	175	200	225	250
		1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
	4	150	180	210	240	270	300
		1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
	5	175	210	245	280	315	350
		1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

As shown in Table 7, the optimal number of redundant components increases, which means the system reliability decreases, when the β value increases from 0.01 to 0.03. To explain this trend, we examine the effect of β on the component reliability. Table 10 and Figure 8 show the component reliability at $t_m = 800$ cycles for various $(\Delta n,\ \Delta m)$ configurations. With the (0,0) or (0,1) redundancy design, the cell reliability improves as the β value increases from 0.01 to 0.05. The dependence of the cell reliability on β under the (0,2) configuration does not show a monotonic trend. The cell reliability decreases at first and then increases as the value of β decreases. However, the system reliability cannot satisfy the system reliability requirement with the first three configurations. On the other hand, the component reliability decreases as β increases for any $(\Delta n,\ \Delta m)$ when the number of redundant cells is at least 5, and hence more redundant cells are needed when β increases from 0.01 to 0.05. In addition, it can been seen from Table 10 and Figure 8, adding redundant cells improves the reliability of individual cells for a given β value due to the load reduction on individual cells.

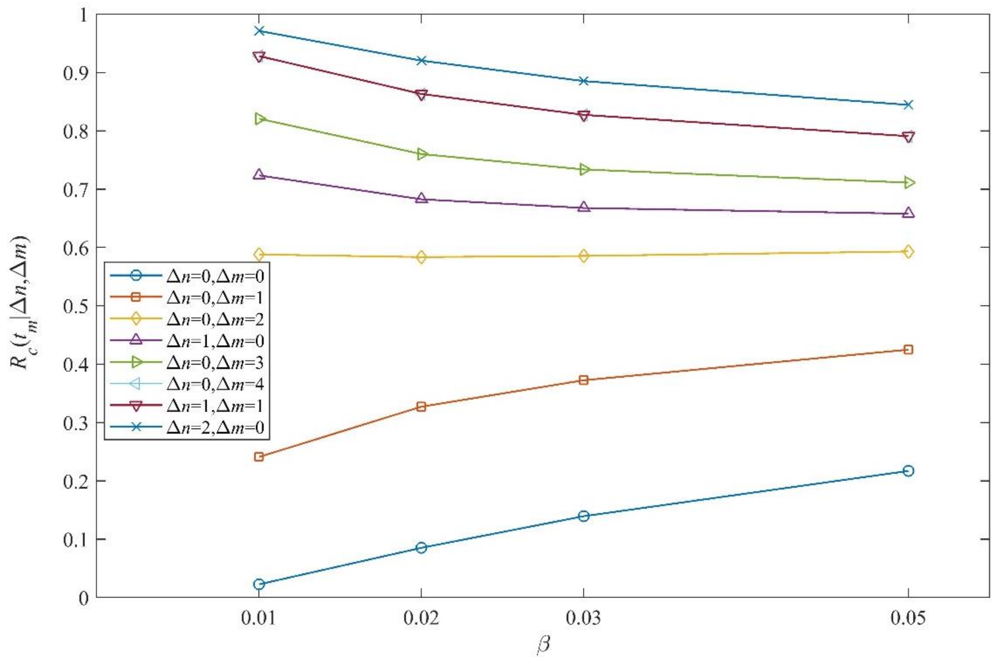
Table 10

Cell Reliability at 800 Cycles for Different β Values and $(\Delta n \times \Delta m)$ Configurations

$(\Delta n \times \Delta m)$		(0,0)	(0,1)	(0,2)	(1,0)	(0,3)	(0,4)	(1,1)	(2,0)
# of redundancy cells		0	2	4	5	6	8	8	10
$R_c(800)$	$\beta = 0.01$	0.0226	0.2409	0.5880	0.7235	0.8207	0.9281	0.9281	0.9712
	$\beta = 0.02$	0.0852	0.3217	0.5835	0.6826	0.7601	0.8631	0.8631	0.9202
	$\beta = 0.03$	0.1394	0.3724	0.5855	0.6677	0.7337	0.8273	0.8273	0.8852
	$\beta = 0.05$	0.2170	0.4249	0.5931	0.6578	0.7111	0.7907	0.7907	0.8444

Figure 8

The Effect of β on the Cell Reliability at 25℃



The structure function used to describe the relationship between system performance and system performance has multiple definitions. Liu et al [145] assumed that the performance of a parallel subsystem is determined by the best cell in the subsystem and used the structure function given by Eq. (12). Table 11 compares the optimal solutions assuming the two different structure functions given by Eq. (10) and Eq. (12), respectively. For a given β value, the structure function given by Eq. (12) yields an optimal solution with less redundant components than that of the structure function given by Eq. (10) because $R_s(N|\Delta n, \Delta m)$ derived from the structure function Eq. (12) is higher than $R_s'(N|\Delta n, \Delta m)$ under a given system configuration.

Table 11

Comparison of Optimal Solutions Assuming Two Different Structure Functions

$R_{req}(t_m)$	β	Y_s in Eq. (20)			Y_s' in Eq. (22)		
		Δn^*	Δm^*	C_s^*	Δn^*	Δm^*	C_s^*
0.99	0.01	2	0	100	1	1	90
	0.02	1 2	3 1	120	2	0	100
	0.03	3	0	125	2	0	100
	0.05	3 4	1 0	150	2	0	100
0.9999	0.01	1 2	3 1	120	2	0	100
	0.02	3	0	125	2	1	120
	0.03	4	0	150	2	1	120
	0.05	5	0	175	3	0	125

4.3 Age-Based Replacement Model for Battery Pack System

Following the configuration decided by the redundancy design in the last subsection, the maintenance design is carried out on the battery pack system to prolong the life of EVs and ensure the safety of drivers and passengers. Regular maintenance and repair are very important in the use of a car. A battery pack system is an important part to support the driving energy for electric vehicles. And since the difficulty in the battery pack system repair, the maintenance strategy generally used for the battery pack system is the replacement. Hence, this subsection focuses on the construction of an age-based replacement model for battery pack systems in EVs. The system reliability function has already been defined in Eq. (11). The age-based replacement model is given in Eq. (8). The decision variable in this model only includes the maintenance interval which is the number of charge/discharge cycles between two maintenance. Finally, the optimization model is summarized as,

$$C_s(T) = C_\infty(T) + \left({}^{A}/_{P}, I, l\right) * C_{design}$$

$$= \frac{C_{cm}F_s(T) + C_{pm}(1 - F_s(T))}{\int_0^T (1 - F_s(t))\, dt} + \frac{I(1+I)^l}{((1+I)^l - 1)} * C_{design}$$

$$C_{pm} = C_{design} = c_c * (m + \Delta m)(n + \Delta n)$$

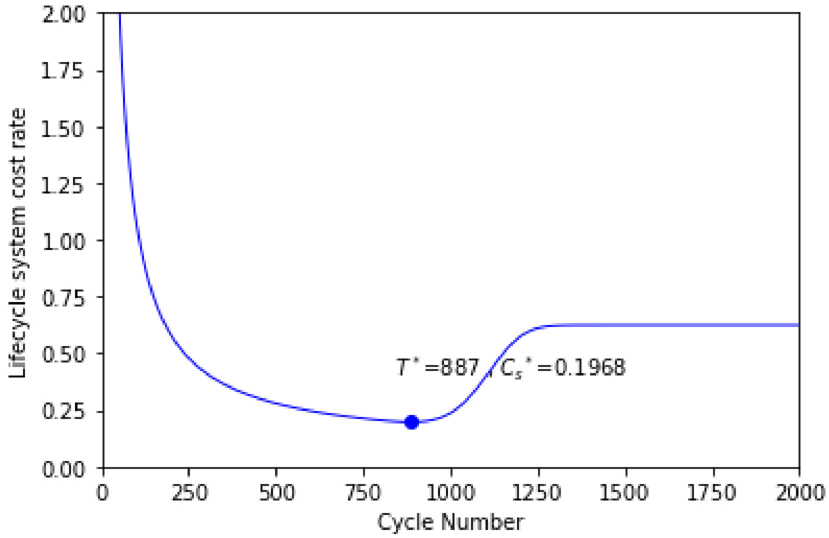
$$C_{cm} = C_{pm} + C_d$$

The average upper bound for most EV batteries to replace is 10 years. The batteries generally fully charge/discharge 200 cycles per year. Hence, this dissertation assumes that the total lifetime of the battery pack system is $l = 200 \times 10 = 2000\ cycles$. Assuming the interest rate is $1\%\ per\ year$ which means the interest rate $I = 1\%/200 = 5 \times 10^{-5}\ per\ cycle$, a 2×5 battery pack system is studied at 25℃.

Figure 9 presents the optimal maintenance time interval $T^* = 887$ with cost $C_s^* = 0.1968$ of the age-based replacement model when $\beta = 0.01$ and $C_d = 500$ at 25℃. The lifecycle system cost rate decreases first and then increases as the cycle number increases. Finally, the cost rate becomes horizontal gradually. In beginning, the increasing cycle number means the reduction times of PM, and the cost is decreasing accordingly. But the probability of CM occurrence is increasing as the PM time interval increases. Then the system cost will increase as the cycle number increases after a specific cycle number. The optimal solutions are the trade-off that we want to find between these two problems.

Figure 9

Optimization of the Age-Based Maintenance Model when $\beta = 0.01$ *and* $C_d = 500$ *at 25 ℃*



Then this dissertation considers the effect of the value of β. The optimal solutions under the different values of β are shown in Table 12. For example, when $\beta = 0.01$ and $C_d = 5$, the optimal design is $(\Delta n,\ \Delta m) = (2, 0)$ and the optimal maintenance interval is $T^* = 1179$ with cost $C_s^* = 0.1742$. The optimal designs are obtained from the redundancy optimization model shown in Table 7. Then the optimal maintenance interval is found based on the specific configuration. What can be clearly seen is that the lifecycle system cost rate increases as β increase. Since the mean value of battery performance is a constant value in this model, the variance among the batteries in the battery pack system increases as β increases. As mentioned in [143], the degradation of batteries performance is accelerated for the system which is composed of several batteries with different degradation levels. The capacity of batteries degraded faster for the batteries with higher variance. Hence, the system reliability decreases as β increases which leads to the lifecycle system cost rate increases. And as the additional cost C_d increases, the C_{cm} increases, and then the lifecycle system cost rate also increases.

Table 12

Optimal Solutions for Different Additional Cost Values C_d and Different β Values at 25 ℃ with $t_m = 800$ Cycles

β	C_d	$R_{req}(t_m) = 0.99$				$R_{req}(t_m) = 0.9999$			
		Δn^*	Δm^*	T^*	C_s^*	Δn^*	Δm^*	T^*	C_s^*
0.01	5	2	0	1179	0.1742	2	1	1446	0.1884
	50	2	0	995	0.1853	2	1	1220	0.1988
	500	2	0	887	0.1968	2	1	1089	0.2100
0.02	5	2	1	1513	0.1938	3	0	1603	0.1937
	50	2	1	1147	0.2095	3	0	1247	0.2074
	500	2	1	955	0.2298	3	0	1062	0.2240
0.03	5	3	0	1691	0.1971	4	0	2000	0.2129
	50	3	0	1208	0.2141	4	0	1523	0.2272
	500	3	0	977	0.2379	4	0	1259	0.2468
0.05	5	4	0	2000	0.2177	5	0	2000	0.2359
	50	4	0	1477	0.2361	5	0	1795	0.2498
	500	4	0	1125	0.2673	5	0	1400	0.2761

Table 13 shows the optimal maintenance interval for a maintenance model without redundant cells added to the system. It also means there is no reliability constraint when charge/discharge $t_m = 800$ cycles. When $C_d = 5$ and $\beta = 0.01$, the optimal cost in Table 12 is $C_s^* = 0.1742$ with $R_{req}(800) = 0.99$. While the optimal cost in Table 13 is $C_s^* = 0.1576$ without reliability constraint at $t_m = 800$ cycles. Although it is lower than the optimal cost in Table 12, we should note that the system reliability is not guaranteed when charge/discharge $t_m = 800$ cycles. The system without redundant cells is more likely to fail before 800 cycles and the drivers and passengers' lives are threatened by the accidents. Hence, the additional cost C_d caused by unpredictable accidents may be very high. Then if the additional cost C_d increases to 500, the optimal cost in Table 13 is higher than the optimal cost in Table 12. Hence, people generally prefer the system with redundant cells and reliability constraints at $t_m = 800$ cycles in Table 12 with the consideration of safety.

Table 13

Optimal Maintenance Time Interval for Different Additional Cost Values C_d and Different β Values at 25 ℃ without Redundant Cells

β	0.01			0.02			0.03			0.05		
C_d	5	50	500	5	50	500	5	50	500	5	50	500
T^*	492	388	326	516	351	264	554	332	225	652	315	181
C_s^*	0.1576	0.1796	0.2042	0.167	0.2036	0.2544	0.1742	0.2234	0.3054	0.1858	0.2561	0.4143

4.4 Joint Model of Redundancy and Maintenance Design for Battery Pack System

This section focuses on the construction of a joint model of redundancy and maintenance design for battery pack systems in EVs. The objective function of this model is to minimize the lifecycle system cost rate as described in Eq. (8). With the consideration of business, this study builds three kinds of models with different reliability constraints. In other words, the manufacturers have three choices to pick. Each one of them has its advantages and disadvantages.

1) No constraint on reliability at any time.
2) The system reliability no less than the requirement R_{req} when $t = t_m$ which is a pre-specified warranty, that is

$$\text{s. t.} \quad R_s(t_m|\Delta n, \Delta m) \ \geq \ R_{req}.$$

3) The system reliability no less than the requirement R_{req} when $t = T$ which is the maintenance interval, that is

$$\text{s. t.} \quad R_s(T|\Delta n, \Delta m) \ \geq \ R_{req}.$$

Table 14 shows the optimal solutions with different design methods. Table 15 shows the optimal solutions with different additional costs. Table 16 shows the optimal solutions with different β values. According to the optimal solutions of joint design model shown in Table 14, Table 15, and Table 16, the lifecycle system cost rate in the first condition which is a system without reliability constraint is lower than the other two systems. The reason is that the system reliability is not guaranteed at any time during the lifetime of the system. The system has a high probability to be failed before the warranty. Hence, the security of this kind of design is the lowest of the three designs. The system with reliability constraint $R(800) \geq R_{req}$ and the system with reliability constraint $R(T^*) \geq R_{req}$ have higher system reliability requirement, and therefore have higher cost. Hence, manufactures can choose each one of them based on their own consideration. If one prefers lower cost, the model with no constraint is a better choice, while if one prefers more reliable products, the other two models have a greater chance to be selected.

And the system cost for the other two systems are decided by unit cost, β and R_{req}. The joint model with the third constraint has the highest cost when $R_{req} = 0.9999$ which has the stringent reliability requirements. And it also produces the most reliable product among these three models. Then people can choose the system with a lower cost at specific values of parameters in the model.

Since the redundancy design in the separate design system has a constraint on reliability at 800 cycles, it is reasonable to compare the minimal costs in the separate design system with the joint design model with $R(800) \geq R_{req}$. As shown in Table 14, the system cost of a separate design system is $C_s^* = 0.1913$ when $R_{req} = 0.9999$, while the cost of a joint design system when $R(800) \geq R_{req}$ is $C_s^* = 0.1901$. Then this study concludes that the joint design of redundancy and maintenance is more effective than the separate design to reduce the system cost.

Table 14

Optimal Solutions under Different Designs when $\beta = 0.01$ and $C_d = 100$ at 25 ℃

Design models		$R_{req} = 0.99$				$R_{req} = 0.9999$			
		Δn^*	Δm^*	T^*	C_s^*	Δn^*	Δm^*	T^*	C_s^*
Separate design		2	0	958	0.1797	2	1	1175	0.1913
Joint design	No constraint	1	0	654	0.1742	1	0	654	0.1742
	$R(800) \geq R_{req}$	2	0	958	0.1797	3	0	1272	0.1901
	$R(T^*) \geq R_{req}$	1	0	603	0.1786	2	0	748	0.2037

Then this study considers the systems with different additional cost values C_d and different β values shown in Table 15 and Table 16, respectively. The system prefers more redundant cells or more frequent PM when C_d increases which means C_{cm} increases. Because the system tries to avoid using CM if the CM is very expensive. And it has the same trend when β increases from 0.01 to 0.05 due to the component reliability decreases as β increases which is proved in Table 10 and Figure 8. And one can also compare the results in Table 16 with the system without redundant cells shown in Table 13. The cost of a maintenance system with redundant cells has a lower cost than a maintenance system without redundant cells. Hence, both the separate design model and the joint design model have better solutions than the maintenance system without redundant cells. Adding redundant cells is an effective way to minimize system cost.

Table 15

Optimal Solutions for Different Additional Cost Values C_d in Corrective Maintenance and Different Reliability Constraints when $\beta = 0.01$ at 25 ℃

C_d	Reliability constraints	$R_{req} = 0.99$				$R_{req} = 0.9999$			
		Δn^*	Δm^*	T^*	C_s^*	Δn^*	Δm^*	T^*	C_s^*
5	No constraint	0	0	492	0.1530	0	0	492	0.1530
	$R(800) \geq R_{req}$	2	0	1179	0.1651	2	1	1446	0.1774
	$R(T^*) \geq R_{req}$	1	0	603	0.1771	2	0	748	0.2037
50	No constraint	1	0	683	0.1696	1	0	683	0.1696
	$R(800) \geq R_{req}$	2	0	995	0.1762	3	0	1315	0.1873
	$R(T^*) \geq R_{req}$	1	0	603	0.1778	2	0	748	0.2038
500	No constraint	1	0	597	0.1850	1	0	597	0.1850
	$R(800) \geq R_{req}$	2	0	887	0.1877	3	0	1189	0.1967
	$R(T^*) \geq R_{req}$	1	0	597	0.1850	2	0	748	0.2038

Table 16

Optimal Solutions for Different β Values and Different Reliability Constraints when $C_d = 500$ at 25 ℃

β	Reliability constraints	$R_{req} = 0.99$				$R_{req} = 0.9999$			
		Δn^*	Δm^*	T^*	C_s^*	Δn^*	Δm^*	T^*	C_s^*
0.01	No constraint	1	0	597	0.1918	1	0	597	0.1918
	$R(800) \geq R_{req}$	2	0	887	0.1968	3	0	1189	0.2080
	$R(T^*) \geq R_{req}$	1	0	597	0.1918	2	0	748	0.2129
0.02	No constraint	2	0	778	0.2171	1	0	778	0.2171
	$R(800) \geq R_{req}$	3	0	1062	0.2240	3	0	1062	0.2240
	$R(T^*) \geq R_{req}$	2	0	758	0.2177	3	0	810	0.2533
0.03	No constraint	2	0	706	0.2355	2	0	706	0.2355
	$R(800) \geq R_{req}$	3	0	977	0.2379	4	0	1259	0.2468
	$R(T^*) \geq R_{req}$	2	0	660	0.2382	4	0	890	0.2873
0.05	No constraint	3	0	860	0.2636	3	0	860	0.2636
	$R(800) \geq R_{req}$	4	0	1125	0.2673	5	0	1400	0.2761
	$R(T^*) \geq R_{req}$	3	0	749	0.2726	5	0	845	0.3456

In this joint design model, there are three constraints given to select in this dissertation. And the joint design model is more effective than the separate design model to reduce the system cost rate. Then this dissertation provides some business recommendations for manufactures. If the manufacture prefers minimal cost to reliable products, the first constraint is more suitable. But the product may fail before the warranty. If the manufacture has a higher requirement on the system reliability, the last two constraints may be more attractive.

Chapter 5: Conclusion

During the usage of machines in the industrial field, it seems reasonable that the degradation of machines is continuous in the actual application as time elapses. Based on the study of multi-state systems, this study constructs optimization models regarding the parallel-series continuous-state systems based on the concept of structure function. The relationship between the performances of the components and the system is expressed by the structure function.

The main contributions of this dissertation are the construction of three models. The models built in this study include:

1) A nonlinear integer programming model was formulated to find the cost-optimal redundancy design for CSS considering a mission reliability constraint. Both the enumeration method and GA are used to solve the optimization model.
2) An age-based maintenance model is introduced in this study to find the optimal maintenance interval with a minimal period cost rate for CSS.
3) A joint design of redundancy and maintenance model considering different kinds of reliability constraints is formulated to find the optimal redundancy design and optimal maintenance interval for CSS simultaneously.

Another contribution is that the battery pack systems in EVs which is a typical parallel-series continuous-state system are analyzed in this study. To calculate the optimal solutions of parallel-series continuous system, GA and enumeration method are adopted in the optimization models. In the battery pack system of EVs, the following conclusions are summarized:

1) According to the result of the redundancy optimization model, the reliability of the battery pack system could be improved with the addition of redundant cells.

2) Adding redundant cells in parallel connections is better than series connections to improve system reliability.

3) There are three kinds of systems with different reliability constraints built for the battery pack system. Each one has its own characteristics. The manufactures can choose each one of them with their own consideration.

4) The joint design of redundancy and maintenance is more effective than the separate design to reduce the system cost.

5) Both the scale parameter value β and the additional cost C_d have a great impact on the optimal solutions.

For further study, more realistic assumptions can be added in this study. The following aspects are possible future research directions:

1) The current study assumed that components are functionally identical in a system. A possible future extension will consider systems with non-identical components. The continuous-state reliability-redundancy allocation problem, which is a combination of component selection and redundancy allocation, will be explored.

2) The system with multiple failure modes is a more realistic assumption for maintenance models. It is necessary to add shocks which subject to Poisson process in both the maintenance model and the joint design model. And it is possible to assume that the failure modes are not independent.

3) Not only the TBM model, but this study can also consider the construction of CBM models for the maintenance design model and the joint design model.

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