Backward-Turning: Aristotelian Contradictions, Non-Contradiction, and Dialetheism

A thesis presented to

the faculty of

the College of Arts and Sciences of Ohio University

In partial fulfillment

of the requirements for the degree

Master of Arts

Duke J. Cruz

December 2016

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This thesis titled

Backward-Turning: Aristotelian Contradictions, Non-Contradiction, and Dialetheism

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Abstract

CRUZ, DUKE J., M.A., DECEMBER 2016, PHILOSOPHY

Backward-Turning: Aristotelian Contradictions, Non-Contradiction, and Dialetheism Director of Thesis: Advisor Donald S. Carson

In this thesis the problem to be confronted concerns the principle of noncontradiction (PNC) as understood by Aristotle in his Metaphysics (Γ). For over a hundred years scholars have been questioning Aristotle's defense of the principle, and whether he succeeded in defending or demonstrating the veracity of the principle. One such scholar is Graham Priest (1998), who thinks that Aristotle's arguments are not only unsuccessful at demonstrating PNC, but they also do not provide any arguments against (A) Dialetheism or (B) Trivialism. Priest concludes with a positive thesis (C) that Aristotle's arguments do show that trivialism must be rejected. Thesis (A) is countered by a conception of contradiction (α) that is formed from original Aristotelian texts and Alan Code (1987). Thesis (B) is countered by (α), and three of his own refutations. All of this is done in an effort to show that Aristotle established more than merely thesis (C). Dedication

To my brother, Joshua

Acknowledgements

I would like to first thank Ohio University and the individuals that I had the fortunate opportunity to meet as a teaching assistant, instructor, and graduate student. Once a Bobcat, always a Bobcat. I also would like to thank the Department of Philosophy, which has given me so many unique and rewarding opportunities to grow not only as a philosopher and scholar, but also as a person. In addition to the department as a whole, I would like to thank the members of my thesis committee: Dr. Scott Carson, Dr. James Petrik, and Dr. Yoichi Ishida. I am forever grateful for your guidance, patience, and assistance in this process. I would like to thank my family, friends, and especially my Halimat, for your constant support, encouragement, and love.

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Introduction

The principle of non-contradiction (PNC)¹ has been with Western philosophy

since the ancient Greek Presocratics², specifically Parmenides who says the

following:

What is there to be said and thought must needs be: for it is there for being, but nothing is not. I bid you ponder that, for this is the *first way* of enquiry from which *I hold you back*, but then from that on which mortals wander knowing nothing, two-headed; for helplessness guides the wandering thought in their breasts, and they are carried along, deaf and blind at once, dazed, undiscriminating hordes, *who believe that to be and not to be are the same and not the same*; and the path taken by them all is *backward-turning* (my emphases) (Fr. 6, Simplicius in Phys. 86, 27-28; 117, 4-13 qtd. in Kirk, Raven & Schofield 247)³.

Parmenides wants to hold us back from this backward-turning way of inquiry, which

posits that to be and not to be are the same and not the same. This path is backward

turning, as Parmenides says, because it doesn't go forward, it leads nowhere; an

epistemological abyss, as Nietzsche might say. The Greek philosopher Aristotle

(384-322 BC) carried this idea forward and gave it new life in *Metaphysics*⁴ (Γ),

¹ This principle is called by various names including, but not limited to the following: (1) Law of Non-Contradiction (Tahko 2009); (2) Principle of Contradiction (Łukasiewicz 1971); (3) Law of Contradiction (Barnes 1969). In what follows I will use PNC strictly to refer to the idea, but this information will be useful in what follows.

² Plato apparently held PNC as well: "It is obvious that the same thing will never do or suffer opposites in the same respect in relation to the same thing and at the same time" (*Republic*, 436, b, Trans. Hamilton and Huntington in *Collected Dialogues*).
³ Again: "For never shall this be forcibly maintained, that things that are *not are*" (my emphasis) (Fr. 7, Plato *Sophist* 242A (lines 1 – 2); Sextus *adv. math.* VII, 114 (lines 2 – 6) qtd. in Kirk, Raven & Schofield 248).

⁴ As Paula Gottlieb (2015) notes in her Stanford Encyclopedia of Philosophy (SEP) entry *Aristotle on Non-Contradiction*: "There are also snippets of discussion about the principle of non-contradiction early in the corpus, for example in *De Interpretatione*, and there is the obscure chapter 11 of *Posterior Analytics* I, but none

where he states and defends it. Concerning the principle of non-contradiction, Aristotle says such grand things as it is "the most certain principle of all", it is "the best known", it is "impossible to be mistaken" about it, and finally it is one of the "Ultimate Beliefs" (*Met.* IV.3 1005b 9 – 33)⁵. What does such a grand principle amount to? Although there are many statements in (Γ), the best known and most widely cited version in the literature is the following: "...the same attribute cannot at the same *time* belong and not belong to the same *subject* in the same *respect*; we must presuppose, in face of dialectical objections, any further qualifications which might be added" [my emphases] (Γ .3 1005b 19-20). It is hard to see, even given seemingly tough cases, how anyone could question PNC without risking triviality, and that is largely how Aristotle characterizes it. That is, the principle about which "it is impossible to be mistaken" (IV.3 1005b 11-12). Or again: those who demand demonstration of PNC do so through "...want of education. For it is impossible that there should be demonstration of absolutely everything" (IV.4 1006a 5-9). Nonetheless, Aristotle's PNC which most philosophers, logicians and indeed himself take as the foundation of all knowledge and science, as the unquestioned given, has been heavily scrutinized the last century. It is not difficult to make the connection between the rise of Western logic beginning with Gottlob Frege and Bertrand Russell and many others, to our questioning of this foundational principle of

⁵ Following Ross' translations here.

of these rival Aristotle's treatment of the principle of non-contradiction in Metaphysics IV [my emphases]". *Metaphysics* (Γ) is the part of the corpus, to which I restrict myself in my research.

reasoning. Wherever this scrutiny came from, it leaves us with problems, which we are left to solve and contemplate; problems with which my research is concerned.

It may seem very obvious, but what is a contradiction⁶? Contemporary philosophers understand a contradiction to be a conjunction of some atomic statement Φ and its denial, or $\Phi \land \neg \Phi$. Represented by means of a truth table it is a proposition that is non-contingent and necessarily false:

$$\begin{array}{c|c} \Phi & \Phi & & \neg \Phi \\ \hline T & F & F \\ F & F & F \end{array}$$

Aristotle himself says the following: "...it is plain that every affirmation has an opposite denial, and similarly every denial an opposite affirmation ... We will call such a pair of propositions a *pair* of contradictories [emphases added] (*De Int*.17a 30)⁷. So a contradiction is a *pair* of propositions one member of the pair being an affirmation and the other a denial of this affirmation. What Aristotle thinks is this: I cannot predicate the pair 'man and not-man' of myself at the same time in the same respect, if by man we mean something like (1): "a living human-being with sexchromosomes XY". The four main qualifications of this initial formulation of the principle are very important for Aristotle, and worth getting clear on: Same Attribute (A), Same Time (T), Same Subject (S), and Same Respect (R). Think about the situation where conditions *R*, *S*, and *A* are met and condition *T* is not. Using the definition of "man" from above, (1) "a living male human-being with sexchromosomes XY", we could conceivably predicate a contradiction (1) & ~(1), to my

⁶ Aristotle's conception of this notion will be made more precise in chapter I.

⁷ Another topical passage would be *Posterior Analytics* I 72a 12-14.

present self and my 100,000 year old decayed body. The T condition is a restriction in the temporal scope of predicates. In a second case conditions A, T, and *R* could be met, while *S* is not. Consider the case where Bill is your father's name as well as my father's name. The following contradiction could then be asserted: Bill is my father and Bill is not my father. The same subject or S condition is postulated to prevent ambiguous predication. In a third and final scenario we could have conditions *T*, *A*, and *S* met, but *R*⁸ is not. In this situation we could take "man" to mean: (2) "a member of the human species". We would end up having another contradiction in applying "(1) & \sim (1)" to a single female subject Susie, where \sim (1) = (2). Or it is the case that Susie is a man (by 2) and not a man (by 1). The same respect condition could be seen as a semantic safe guard against equivocation in predication. Finally, keep in mind that Aristotle adds to the preceding four qualifications of PNC a principle (E)⁹ that we should add further extra qualifications where these fail (1005b 20-22). This principle amounts to a sort of pragmatism of PNC. That is, given the option to describe the world in a consistent or inconsistent way, choose the consistent¹⁰.

⁸ Cf. "...it will not be possible for the same thing to be and not to be, except in virtue of an ambiguity..." (*Met.* IV.4 1006b 19-22).

⁹ In what follows, I refer to these qualifications as the complex acronym SARTE when referring to all of them at once.

¹⁰ An interesting philosophical issue I see arising in the context of Dialetheism, and Graham Priest's research generally, and Tuomas Tahko's (2009) research about Priest, is concerning why we should describe the world consistently? Yes, it makes science and philosophy easier, but what other reasons do we have besides pragmatism? Also what is it in us, which wants to correct a contradiction? Tahko (2009) himself thinks that metaphysics could be seen to circumscribe some, if not all, principles of logic—that there might be one "true logic". And any other formal systems, say paraconsistent logic, it is permitted if we have them, but "…we must be

Beginning in 20th century up to present day, the veracity of the principle of non-contradiction as a logical principle, and principle of reasoning in general, has been called into question by several scholars. The polish philosopher and logician Jan Łukasiewicz's (1878-1956) work can be seen as a reinvigoration of thinking about the principle of non contradiction, and his work with multivalent logical systems can be seen as a dismissal of it¹¹. In his "On the Principle of Contradiction in Aristotle" (1910), he dismisses each of Aristotle's arguments for PNC and thinks the principle has no "logical worth", but it has "practical ethical value" (508). To support the latter idea he cites the case of a court room verdicts: "Were we not to recognize this principle and hold joint assertion and denial to be possible, then we could not defend other propositions against false or deceitful propositions. One falsely accused of murder could find no means to prove his innocence before the court" (508). If it was right and not right that x stole z from y, or wrong and not wrong that x murdered y, then morality it seems would collapse into the abyss. If it was right and not right that w fulfills their obligations to v, or right and not right that w lies to *v*, then human cooperation and social communities would be very difficult to

wary of any *metaphysical* implications that one might derive from these frameworks..." (44-5).

¹¹ Though JC Beall thinks that this article (Łukasiewicz 1910) wasn't a strong denial of PNC, since "...[he] concludes that Aristotle was right to preach (as it were) the 'unassailable dogma'..." (Beall 3). Putting ourselves in Aristotle's position, it's clear that there was an issue in his own time with the "writers on nature" (Heraclitus, Democritus, etc.), and he is concerned: "For if those who have seen most of what truth is possible for us (and these are those who seek and love it most)—if these have such opinions and express these views about the truth ["writers on nature"], is it not natural that beginners in philosophy should lose heart? For to seek truth would be to pursue flying game" (1009a 5 – 1009b 38—following Ross). So even if the PNC was 'unassailable dogma', it was not without reason that Aristotle chose to side with it. I think this is Łukasiewicz's point as well (Łukasiewicz 508-509).

maintain. It is clear then that PNC has practical ethical value even if Aristotle didn't say as much in his defense of it, but the question is whether that is all the value it possesses.

There have been many to follow Łukasiewicz and dismiss the principle of non-contradiction. One figure who is central to my research is the logician and philosopher, Graham Priest. Priest¹² is a founder and developer of "Dialetheism¹³" or the view that *some* contradictions are true (Priest 2006, 1). He grounds these ideas in a relatively new branch of logic called "Paraconsistent Logic", which is described by him and others in the following passage:

The contemporary logical orthodoxy has it that, from contradictory premises, anything can be inferred. Let \vDash be a relation of logical consequence, defined either semantically or proof-theoretically. Call \vDash explosive if it validates {A, \neg A} \vDash B for every A and B (ex contradictione quodlibet (ECQ)). Classical logic, and most standard 'non-classical' logics too such as intuitionist logic, are explosive. Inconsistency, according to received wisdom, cannot be coherently reasoned about. Paraconsistent logic challenges this orthodoxy. A logical consequence relation, \vDash , is said to be paraconsistent if it is not explosive. Thus, if \vDash is paraconsistent, then even if we are in certain circumstances where the available information is inconsistent, the inference relation does not explode into triviality. Thus, paraconsistent logic accommodates inconsistency in a sensible manner that treats inconsistent information as informative [my emphases] (Priest, Koji & Weber 2015)¹⁴.

Paraconsistent logic denies the validity of an application of the logical principle *ex*

contradictione quodlibet (ECQ)¹⁵, or the common idea that contradictions imply

¹² In addition to his work in several non-classical logics.

¹³ One point about Dialetheism that should made clear is the following: "...nothing in dialetheism requires the existence of *observable* contradictions—true contradictions that have observable (but inconsistent) consequences (Beall 8).
¹⁴ It should be made clear that one can be a paraconsistent logician and not be a dialetheist (Beall 6).

¹⁵ "The earliest appearance of the principle that I am aware of seems to be in the twelfth-century Paris logician, William of Soissons" (Priest 2004 25). The denial of

anything—that they are "explosive". Thus, "contradiction elimination" and other such rules are invalid in this system. What sorts of contradictions does Priest have in mind when he says that some are true? Take for example the following two familiar logical paradoxes¹⁶: the (1) "Liar's Paradox", and the (2) "Barber's Paradox¹⁷":

(1) This sentence is false.

(2) $\exists x (Man(x) \land \forall y (Man(y) \supset (Shaves(x, y) = \neg Shave(y, y))))$

Consider the Liar's Paradox. If we assume the sentence is true, well then it is true

that it is false (i.e. it is false). And if we assume it is false, then it is false that it is false

(i.e. true). The paradox is that we have the following truth conditions: the sentence

is false *iff* it is true. Or more formally, where x is the sentence above and T = x is

true: $\exists x (Tx \land \neg Tx)^{18}$. A similar problem arises with the Barber's Paradox. Here's the

paradox spelled out: there exists an x such that x is a man and for all y, if y is a man

ECQ here is not wholesale. That is, there may be a paraconsistent logical consequence *x* such that *x* is explosive, but unqualified application of the principle is not valid. Consequently, William of Soissons may have formalized the principle, but Aristotle's second refutation or W2 in *Metaphysics* Γ seems to contain the principle (cf. chapter III).

¹⁶ Cf. Priest (1998), Pg. 94 for his asserting that he believes Russell's set (from which the Barber's Paradox is derived) is a true contradiction. And his thinking that the Liar's Paradox is evidence for Dialetheism (and for more support that the set theoretic paradoxes are evidence) can be found in Priest's *In Contradiction* (1987). Also, Beall (2004) is clear that the Liar's paradox counts as a true contradiction or *dialethia*.

¹⁷ This is a version of Russell's *set-theoretic* paradox, whereas the Liar is a *semantic* paradox.

¹⁸ Crivelli (2004) thinks that Aristotle could be seen to discuss the Liar (and provide a solution) in *Sophistici Elenchi* 180a 34 – 180b 7 (Crivelli 31-34). I will not address this work or Aristotle's discussion of the Liar here, since that is not the aim of my research, and it is difficult to ascertain what Aristotle would say about "extended" or "strengthened" Liars. That is, "(4)—(4) is not true" (Priest 2006b 15-16).

then *x* shaves *y* if and only if *y* doesn't shave *y*. Well the barber is a member of this town, and assuming he is man, he needs a shaving every now and then. If we assume that the barber shaves himself, then he doesn't according to the biconditional in the second conjunct above. If we assume that the barber doesn't shave himself, then he does. The result is similar to the Liar's Paradox: the barber shaves himself *iff* he does not shave himself. Or where S = x shaves himself, we get the following contradiction whether or not the barber shaves himself: $\exists x (Sx \land \neg Sx)$. These true contradictions are called, by Priest, Beall and others, *dialetheias*. It is clear that Priest rejects PNC and ECQ as principles of logic. What is unclear is whether his rejecting PNC as a principle of logic, is at the same time a rejection of Aristotle's formulation of the principle of non-contradiction.

In *To be and not to be – That is the Answer. On Aristotle on the Law of Non-Contradiction* (1998), Priest takes himself to be reawakening a classical philosophical debate about the principle of non-contradiction, and challenges all of Aristotle's arguments for it. As he says: "...developments in contemporary logic itself have made it possible to countenance, *if not the old pre-Socratic views, at least others that endorse the truth of some contradictions.* It therefore becomes crucial to recharge the debate that has lain dormant for over two millennia, and ask: did Aristotle settle the matter?" [my emphasis] (92). Priest is referring to paraconsistent logic in saying that new developments in logic enable us to countenance "other views" (i.e. Dialetheism) outside of the Presocratics' that endorse true contradictions. In his analysis of Aristotle's arguments Priest comes down on the negative side to his question above, and does not think that Aristotle "settled the matter". In fact, he thinks not only did Aristotle not establish PNC, but that he neither provides an argument against dialetheism nor the ridiculous position of trivialism (i.e. everything is true):

[A] They [Aristotle's arguments] do not provide any kind of argument against dialetheism. [B] Neither do they provide any kind of argument against a trivialist. As we noted, nothing can do this... [C] They do show, however, that a rejection of triviality is a precondition for reflective purposive activity, and especially for the institution of communication. This is a lot less than Aristotle advertised, but it is still an interesting and important conclusion [my emphases] (1998 128).

So Priest concludes three main things¹⁹ from his analysis of Aristotle's arguments in

Γ. There are two negative theses (A) and (B), and one positive one (C). The first negative thesis is that (A) Aristotle's arguments don't provide an argument against dialetheism or that some contradictions are true. The second negative thesis is that (B) Aristotle's arguments don't even give an argument against the trivialists. But concluding more positively (C), Priest says that his arguments do give reason to reject trivialism in order to have reflective purposive activity and to be able to communicate effectively. According to Priest then, Aristotle's arguments in Γ established *nothing more than a principle restricting our reflective purposive activity and communication; a principle that impels us to reject trivialism.* Thus, Priest's

¹⁹ Priest does have another conclusion: "Nor do they give any *transcendental* reason for the LNC, or the LNT (as we saw, there are problems concerning the latter enterprise in any case)" [my emphasis] (Priest 128). Since it's not clear that Aristotle meant to give such a reason as Priest and Kirwan define *transcendental reason* (Priest 110, 116; Kirwan 204 – 5), I don't concern myself with this critique here. Whereas in the case of thesis (A) and (B), these are views, with which Aristotle is directly in conflict (i.e. Dialetheism and Trivialism).

analysis concludes with a similar restricted utility of PNC (though not the same) as Łukasiewicz.

Are we to accept Priest's positive thesis (C) that Aristotle is merely giving us a restriction on our reflective purposive activity in *Metaphysics* $(\Gamma)^{20}$? Or can Aristotle be seen to be giving us more than a mere restriction on thought? In my thesis I will argue that Aristotle is not just giving us (C), but that he can be seen to be giving arguments against theses (A) and (B) in *Metaphysics* Γ . Concerning (A), we will see in chapter IV that because of limitations imposed by the conception of contradiction that is sketched in chapter I, we may dismiss thesis (A) since Priest and Aristotle do not share their conceptions of contradiction. In dismissing (A), I do *not* take the further step to say that Dialetheism is false, and Aristotle's conception of contradiction is true. Rather, I make the more modest claim that the difference of the two conceptions of contradiction makes it so Aristotle's conception, or (α) as we understand it in chapter I, would not count the dialetheias that result from the Liar's or Barber's Paradoxes as genuine contradictions. And thus, that Aristotle can't be giving an argument against dialetheism, if they do not share a conception of contradiction. Concerning (B), I use the Aristotelian conception of contradiction that is built in chapter I, the fourth refutation or W4²¹, and Priest's own interpretations of the third (W3) and sixth (W6) refutations to show that thesis (B) and trivialism

²⁰ I am leaving aside Łukasiewicz' limited principle here, which I think is equally shortsighted. The arguments I will offer contra Priest (1998) in what follows could more or less be marshaled against Łukasiewicz' arguments as well. Though, Wedin (2000) does a good job at putting Łukasiewicz (1910) to rest.

²¹ This numbering of the arguments will be explained in what follows, and derive from Wedin (2000, 2003). See Appendix II at the end of the thesis.

are false. In chapter I, I will first go over a conception of Aristotelian contradiction that I will use in what follows using Alan Code's (1987) work. In chapter II, I go over what the Aristotelian scholar Michael V. Wedin (2000, 2004) dubs the Indubitability Proof (IP), which occurs at the end of *Metaphysics* Γ.3. In chapter III, I will go over Aristotle's seven refutations of those who reject PNC or something more explicit, with suitable interpolations from Wedin (2000). Chapters II and III are generally considered to give the reader an idea of the Aristotelian treatment of PNC even though they are not entirely necessary for main argument. Finally, in chapter IV, I will proceed to show the falsity of Priest's twin negative theses (A) and (B).

In this chapter I want to get more specific about the general characterization of the concept of contradiction²² that we gave in the introduction. The justification here is clear: once we understand what Aristotle meant when he talked about contradictions, we will become even more clear about his actual refutations concerning them. In the introduction, the difference between our own notions of contradiction and Aristotle's were intuitive enough. But we must ask ourselves: what does our concept have to do with Aristotle's? Can we simply assume that Aristotle must have meant what most modern logicians generally take to be a contradiction: $\Phi \land \neg \Phi^{23}$? That is, a non-contingent and necessarily false conjunction of any atomic statement and its negation (in the associated truth-functional semantics for conjunction and negation). The answer here is a clear *no*. This chapter begins by delving into Aristotle's own ideas on contradiction. We will then consider work by the renowned Aristotelian scholar, Alan Code on Aristotle's conception of contradiction. Finally, we conclude with a conception (α) of contradiction based on Aristotle's and Code's ideas, and refine this conception in light of Aristotle's conception of PNC²⁴.

²² In line with the aim of this chapter it is interesting to note the 19 formulations of the concept of contradiction, and 6 formulations of PNC that Patrick Grim gives in *What is a Contradiction?* (2004). Even the conception of contradiction is variegated issue.

²³ E.g. Stan Baronett *Logic* 3rd Edition (2015).

²⁴ This conception is original to this thesis, not due to other scholars in the area.

In *Metaphysics and Logic* (1987) by Alan Code, he considers the concept of contradiction for Aristotle and his *dual* usage of it (131-32). That is, between two different works, Aristotle considers the concept of contradiction in two different, but as we will see, intimately related senses²⁵. Let's consider Aristotle's project in *De Interpretatione*²⁶ before introducing the concept of contradiction in the first sense. Aristotle takes himself to be laying a foundation for language and logic in chapters 1 - 5, discussing such topics as the meaning of names, verbs, sentences, and statement-making sentences. Pertinent to our discussion is his concept of a sentence and specifically a "statement-making sentence" (SMS), which is a category of sentences consisting of "...only those in which there is truth or falsity" (17a 1-3). Of these SMS's: "...the first single statement-making sentence is the *affirmation*, next is the *negation*. The others are *single* in virtue of a *connective*²⁷" [my emphases] (17a 8-9). These "single SMS's" are *single* in one of two ways: as "simple statements" or as "compound statements" (17a 20-23). And finally, we are told that single SMS's are sentences that "reveal a single thing" or "reveal more than one thing" (17a 16-18). First, Aristotle tells us that SMS's are in a category of sentences that have the truthvalues of true or false, and that two types of these are affirmations and negations. Affirmations and negations are *single* SMS's in that they consist of one claim (e.g. the affirmation: "I am a guitar player"). And other statements outside of these, presumably "compound statements" that consist of two or more SMS's, are *single* in

²⁵ He himself does not distinguish these senses, or contrast them.

²⁶ Following J. L. Ackrill's translation in Barnes (1984).

²⁷ I think it is safe to assume here that Aristotle is thinking of any connective including those outside of conjunction.

that they are composite functional units joined by a connective/s. Finally, single SMS's are said to *reveal*²⁸ *a single thing* in that they are about discrete subjects or objects, and if there is more than one single thing revealed then this would involve the case of multiple subjects and objects in a compound statement (e.g. "Beethoven is a man and Secretariat is a horse").

With these ideas in mind, Aristotle's first concept of contradiction as

described in chapter 6 of *De Interpretatione* is the following:

An *affirmation* is a statement affirming something of something, a *negation* is a statement denying something of something. Now it is possible to state of what does hold that it does not hold, of what does not hold that it does hold, of what does not hold that it does not hold. Similarly for times outside the present. So it must be possible to deny whatever anyone has affirmed, and to affirm whatever anyone has denied. Thus it is clear that for every affirmation there is an opposite negation, and for every negation an opposite affirmation. Let us call an affirmation and a negation which are opposite a *contradiction*. I speak of statements as opposite when they affirm and deny the same thing of the same thing—*not homonymously, together with all other such conditions that we add to counter the troublesome objections of sophists (<i>De Int.*17a 25-37).

Aristotle spells out the function of the two types of SMS's (i.e. affirmations and

negations), and then gives us four possible combinations of these. That is, it is

possible to change your affirmation to a negation and vice versa accounting for each

of the preceding possibilities with respect to present time (and time outside of this).

²⁸ This conception of "revealing a certain thing" is made even more apparent by his discussion in the *Categories*, where the secondary substances are said to reveal the primary substances (2b 29 – 35 and 3b 10-23). This further discussion of 'revealing' could be used here, but I'm not concerned solely with Aristotle's theory of substance and its relation to contradiction. As Wedin (2000) shows, Aristotle's theory of PNC can be generalized while remaining consistent with his set of philosophical ideas, to be applicable to more than just primary substances (man, horse, dog, etc.) (141-148). For more on *substances* for Aristotle see: *Metaphysics* (Λ.3) or *Categories* (Ch. 5).

Once this is in place then he characterizes statements as "opposite" when they affirm and deny the same thing of the same thing, and an affirmation and negation, which are opposite, "a contradiction". Finishing out the chapter Aristotle adds that these contradictions affirm and deny the same thing of the same thing, but not "homonymously," so as to counter the "troublesome objections of sophists". Homonymous definitions: "...have only a *name* in common and the *definition of being* which corresponds to the name is different..." [my emphases] (Cate. 1a 1-2). A homonymous pairing would then be where the name is the same and the definitions are different (where condition *R* (same respect) is *not* met). That is: Sally is a man (1) and not a man (2), where (1) = x is a living human-being with sex-chromosomes XY, and (2) = x is a is a member of the human species. But, says Aristotle, this isn't a contradiction—these *seeming* contradictions aren't what he wants to call a contradiction. That is: those opposite SMS's that affirm and deny the same thing of the same thing are what he calls a contradiction. It is this characterization of contradiction that can never be the case according to *Metaphysics* (Γ). Finally, Aristotle mentions that we should add "other conditions" to counter sophists who presumably think contradictions can be true. These other conditions would most likely be the ones that we listed in the introduction: Same Subject (S), Same Respect (R), Same Attribute (A), Same Time (T), and any extra qualifications that need to be added (E). Once the concept of contradiction is clear, the sophists' apparent or seeming contradictions should disappear. An initial conception of contradiction, then, is this: if something x is a contradiction, then x consists of two single SMS's in a

compound statement, where there is one SMS that affirms and one that denies the same thing of the same thing.

It is this conception of contradiction, which arises from *De Interpretatione* chapter 6 that Alan Code (1987) thinks of as a *pairwise* conception. Code thinks that a contradiction according to Aristotle:

...is not the statement formed by the *conjunction* of the affirmation with the corresponding denial, rather it is a *pair* of statements. To believe a ...[contradiction]... is *not* to believe a single *conjunctive* proposition. It is to have two separate beliefs: one a belief corresponding to the affirmation, the other a belief corresponding to the denial... He is not arguing against the claim that it is impossible to have the conjunctive belief, but instead is considering *two* beliefs²⁹ [my emphases] (132).

Code's conception of contradiction as a pair of SMS's rather than a conjunction of

propositions makes sense of the doxastic commitments that Aristotle thinks are

impossible in *Metaphysics* (Γ .3)³⁰. Given that this is the case, we may still ask here:

how is conceiving a contradiction as a conjunctive compound statement of SMS's

different than conceiving it as a *pair* of SMS's one affirming and one denying the

same thing of the same thing? Code could answer here by saying that these SMS

pairs are different from conjunctive compound statements of SMS's in that beliefs

are involved. But we can give another answer to this question by noting that

contradictions conceived as a conjunctive statements adds our semantics for the

²⁹ The last line of this passage is in reference to *Metaphysics* Γ .3, and Code thinks that Aristotle isn't arguing against conjunctive beliefs therein, but rather two beliefs, which affirm and deny the same thing of the same thing. Code's exegesis of *Metaphysics* Γ beyond his thoughts on Aristotelian contradictions will not enter my main discussion, since I think Wedin (2000, 2003, 2004) provides the best exegesis to date on *Metaphysics* Γ (he himself advances on what Code (1987) and others have thought).

³⁰ This is the case in *Metaphysics* Γ generally.

operator "•" or " \wedge " (= conjunctions are only true if both propositions are true). Further, Aristotle's point in his refutations, as we will see in the following chapter, is less about the *truth-functional* result of a conjunctive contradiction, and more about the impossibility of a pair of SMS's that are contradictorily opposed being true, whether believed or *not*. So conceiving contradiction as pairwise, is conceiving contradictions as pairs of statements, whether believed or not³¹. This appears to give further sense to Code's claim, and it seems clear that conceiving of contradiction as a pair of SMS's sets aside potential anachronistic approaches to interpreting contradictions for Aristotle.

The second passage of Aristotle's in which contradiction is discussed is in *Metaphysics* (I.7)³². Aristotle tells us: "...for contradiction is this—an opposition, one or other side of which must attach to anything whatever, i.e. which has no intermediate" (*Meta.* I.7, 1057a 33-5)³³. Code says here that where the pairwise conception of contradiction above applies 'contradiction' to pairs of statements, the preceding sense of contradiction attaches to "...pairs of items that may be present in a *subject*" [my emphasis] (Code 132). Not mentioned in Code's analysis is that Aristotle is quick to point this out in the book of interest for PNC and the "Law of

³¹ At bottom, I think Aristotle's conception of contradiction could be conceived in the modern truth-functional sense of conjunction. But I think the pairwise conception of contradiction sets aside our modern semantics, even if they were spot on. Further, it makes it clear that contradictions come down to an opposite pair consisting of an affirmation and a negation as Aristotle says in *De Int*.17a 25-37. No matter how many simple statements make up the affirmation or negation, in a contradiction, there is always a pairing of these (i.e. affirmation/s and negation/s). ³² Following W. D. Ross in Barnes (1984).

³³ See also *Metaphysics* (I.4) 1055a 37 – 1055b 17.

Excluded Middle" (LEM), or *Metaphysics* (Γ .7): "...there cannot be an intermediate between contradictories, but of one subject we must either affirm or deny any one predicate" (1011b 23-5)³⁴. Code says that in this sense 'contradiction' is "...an opposition to which the law of excluded middle (LEM) applies" (Code 132). That is, LEM dictates in every contradictory pair of propositions, where one proposition is a negation of the other, one proposition is true and the other false—each pair of propositions is mutually exclusive and jointly exhaustive. In this sense, a contradiction is an opposition, where one or the other of an opposition without an intermediate is true of a given subject. Where predicate "man" means "is a member of the human species", then we must apply one or another side of this opposition (man or not-man) to any given thing. If I pick up rock I find in the Hocking river, then it is either the case that it is a man or not a man: there is no middle term between the pair man and not man—one or the other must be true. It happens to be the case in our world that this rock is not a man, and thus, LEM is met. One final question to ask here is the following: what is the relation between contradiction as a pair of SMS's, and contradiction as a pair of SMS's where LEM applies? Code tells us: "Two statements will be contradictories in the first sense just in case the items ascribed to the subject are contradictories in the second" (132). That is: if two statements are such that LEM³⁵ applies to them, then they are pairs of statements

³⁴ As shown below this idea is again repeated in *Posterior Analytics* 72a 12 – 14. ³⁵ A somewhat important addition to the Aristotelian conception of LEM (and LEM as it relates to (α) below) is noted in *De Interpretatione* Book 9. The addition is that SMS's that are possibly so or possibly not so (e.g. 'a sea battle will happen tomorrow', he says) are not restricted by LEM and hence: "...it is not necessary that of *every* affirmation and opposite negation one should be true and the other false.

that are contradictories or affirm and deny the same thing of the same thing. For example, the following statements are contradictories in the second sense (i.e. LEM applies): (3) "The rock is a man" and (4) "The rock is not a man". One or the other of these must be true where there is no intermediate between being a man, and not being a man. If we know 3 and 4 fit the second sense of 'contradiction', then it also fits the first sense of contradiction as a pair of opposing statements³⁶.

In this last section I want to proffer a conception of contradiction (α) that follows from our discussion above and that we can use in the chapters that follow to delimit our discussion of the principle of non-contradiction (PNC) to a conception of contradiction that is textually motivated.

(α) If something γ is an Aristotelian contradiction, then γ is a compound statement consisting of a pair of statement-making sentences (SMS), the Law of Excluded Middle (LEM) jointly applies to the component SMS's in γ, and γ consists of one SMS that affirms and another SMS that denies the same thing Ψ of the same thing φ.

For what holds for things that *are* does not hold for things that *are not* but may possibly be or not be..." [my emphases] (19b 1 - 4). LEM applies to events described in SMS's that are past and present, but not for future events.

³⁶ It is important to keep in mind Code's (1987) restriction of this duality of senses to *unquantified* statements, since as he mentions Aristotle thinks that in the situation of "universals not taken universally" contradictory statements can be true (132). See *De Interpretatione* chapters 7 and 8, specifically 17b 27-37. Although our intuition is that this supports the basic Dialetheic claim that some contradictions are true, this isn't the case. Here Aristotle is merely saying that "...it is true to say at the same time that *a* man is white and that *a* man is not white..." [my emphasis] (17b 30-1). John may be white, while Ted is not white. These are not the contradictions that Aristotle wants to rule out as impossible—they don't meet the *S* (Same Subject) qualification above. Furthermore, these are better described as sub-contraries as described in the Square of Opposition. Following Code, I restrict my discussion to unquantified SMS's.

This conception of contradiction has three necessary conditions registered in the consequent of the above conditional. The first necessary condition follows from both Aristotle's discussion in *De Interpretatione* and our discussion of Code's exegesis. The second necessary condition is based on LEM³⁷, which, as was discussed above, is critical to Aristotle's conception of contradiction. And finally, the third necessary condition is the formulation of contradiction using *De Interpretatione* (cf. *De Int*.17a 25-37) that was initially introduced above.

Since we want to make sure that this conception of contradiction applies to the Aristotelian formulation of PNC given in the introduction, we need to make a revision. Recall the PNC as given in the Introduction: "For the same *thing* to hold good and not to hold good *simultaneously* of the same *thing* and in the same *respect* is impossible (given any further specifications which might be added against the dialectical difficulties)" [my emphases] (*Meta.* (Γ), 1005b 18 – 21)³⁸. Given this formulation of PNC, we revise (α) as follows:

(α) If something γ is an Aristotelian contradiction, then γ is a compound statement consisting of a pair of statement-making sentences (SMS), the Law of Excluded Middle (LEM) jointly applies to the component SMS's in γ , and γ consists of one SMS that affirms Ψ holds good and

³⁷ It is important to point to out that Aristotle is aware of those who would deny LEM of any given proposition and he recommends the following: "...the starting-point in dealing with all such people is *definition*. Now the definition rests on the necessity of their meaning something; for the formula, of which the word is a sign, becomes its definition" (1012a 12-25). Once we define what we mean by river clearly, then LEM applies and the problem disappears. Or in other cases where LEM fails (the Liar's Paradox and Russell's Paradox), it is possible that we don't have a genuine Aristotelian contradiction.

³⁸ Kirwan glosses over the qualifications same attribute (A), and same subject (S) that are part of SARTE in the translation above. These are more clearly presented in Ross, but I use Kirwan for consistency with Wedin and Priest.

another that denies Ψ holds good, where Ψ is understood to be of the same *attribute* A in the same *respect* R of the same *subject* S at the same *time* T (we may add extra E^{39} qualifications to SART, if needed)⁴⁰.

In the revised conception of contradiction we add the qualifications of *SARTE* that Aristotle himself gives in stating PNC. Another addition is the concept of "holding good", where this makes transparent the meaning of the SMS's. This is not anything too far beyond the initial characterization of an SMS, since we were told above that single SMS's *reveal a single thing* in that they are about discrete subjects or objects, and these SMS's are a category of sentences consisting of two truth-values "True" (*x* holds good), and "False" (*x* does not hold good). But adding the explicit phrase does make clear again, that Aristotle's PNC is an ontological⁴¹ principle of the study of being *qua* being. Thus, PNC says that it is impossible that something that meets the

³⁹ This *E* qualification is ambiguous as it stands, but think of it as covering any qualifications that may arise in given token situations, where the typical four (SART) do not block against true contradictions. We may imagine a situation where each of SART are met, but a true contradiction still arises—here we would add extra qualifications.

⁴⁰ In the formulation above, the affirmation and negation of Ψ are both single SMS's, but an application of this conception could be made more complex, where contradictions would consist of more SMS's than a statement Ψ and its contradictory opposite. And, of course, in this more complex application of (α) the same necessary conditions would apply.

⁴¹ Also see the following sources, who are in accord that Aristotle is defending an ontological version of PNC. Łukasiewicz (1910) himself thinks that logical and ontological formulations of PNC are "logically equivalent" (489, 501, 502). Paul Thom (2010) thinks, "Aristotle certainly wished to maintain the Law of Non-Contradiction as an ontological principle" (231). Charlotta Weigelt (2006) thinks that there is no sharp boundary between logic in metaphysics in Aristotle, as there is in modern philosophy (511). Michael V. Wedin (2000, 115) (2003, 108) thinks that the PNC is Aristotle's "preferred formulation". Even Priest thinks this is right (2006, 2). Cf. also Tahko (2009).

conception of contradiction in (α) is ever actually the case (i.e. PNC is the case *iff* any given compound statement that satisfies (α) is not possibly the case).

Given our discussion in the introduction, the qualifications of SARTE should be clear enough, but I want to further clarify the same time requirement (*T*), since it can cause difficulties in understanding. *T* refers to both (1) the time specified in the semantic content of a belief, and (2) the time at which the act of belief occurs. That is there might be the following scenario: *x* believes "It snowed yesterday at 8 a.m.", and "It did *not* snow yesterday at 8 a.m.". These would be contradictory in sense (1) of the *T* requirement, given the time specified in the semantic content of the belief "It snowed yesterday at 8 a.m.". But it is also possible to violate the same time requirement in sense (2) by adding temporal factors relative to a believer: *x* believes today at 8 a.m. that "It snowed yesterday at 8 a.m.", and *x* does *not* believe today at 8 a.m. that "It did snow yesterday at 8 a.m.". Thus, the same time requirement refers both to (1) and (2)⁴².

As a final and important clarification of (α), we need to think about what it takes for something to be a genuine Aristotelian contradiction. For the present, without taking a positive position, assume that it is possible to have an Aristotelian contradiction without (α) being true. What are the conditions for (α) being true? Since this conception arose from the original Aristotelian passages, we'll assume that he would not recant his assertions, and thus, (α) is *true* just in case both the

⁴² I'm indebted to a comment given by Dr. James Petrik for this critical distinction. Aristotle does not make this distinction of the *T* requirement, but given *E* which permits adding extra qualifications to block contradictions, I don't think he would/could disagree with the distinction of (1) and (2) for *T*, since the distinction further blocks against apparent contradictions.

antecedent and consequent are true⁴³. Now consider an example where one thinks the following case would still yield an Aristotelian contradiction: the second necessary condition of (α) is not true (i.e. concerning LEM), and the other two necessary conditions are true (in addition to the antecedent). This does not seem impossible, say, in other conceptions of contradiction, but it would be difficult to square this example as being a genuine Aristotelian contradiction given the text. Where it is stated in *Metaphysics* (Γ .7) after the Indubitability Proof (IP) and the seven refutations that: "...there cannot be an intermediate between contradictories, but of one subject we must either affirm or deny any one predicate" (1011b 23-5). And further, he strongly repeats this later in the *Metaphysics*, saying that contradictions *are* oppositions that are restricted by LEM⁴⁴: "...for contradiction *is* this—an opposition, one or other side of which must attach to anything whatever, i.e. which has no intermediate" [my emphasis] (Meta. I.7, 1057a 33-5). This idea of LEM's critical role in the conception of contradiction in *Metaphysics* Γ .7 and I.7 is one that Aristotle does not take lightly repeating it in *Posterior Analytics*: "A

⁴³ The semantics for the conditional presented here are clearly different from classical logic's, which has conditionals being true in every case *except* where the antecedent is true and the consequent false. The semantics here are the same as those for classical logic's semantics for conjunction. The consequent of (α) follows standard semantics for conjunction (i.e. the conjunction is true *iff* all conjuncts are true.). The antecedent, I take to have to be trivially true since in order for something *x* to be an Aristotelian contradiction, it needs to actually be an Aristotelian contradiction. If it was false and one claimed that they still had an Aristotelian contradiction, then this is clearly contradictory and one would have to provide grounds for such a claim.

⁴⁴ We learned from the other major passage talking about contradiction what Aristotle means by *opposite* statements: "...I speak of statements as *opposite* when they affirm and deny the same thing of the same thing..." [my emphasis] (*De. Int.* 17a 34 – 35).

contradiction is an opposition of which of itself excludes any intermediate..." (*Post. Anal.*, Book 1, 72a 12 – 14). Finally, without an all out appeal to authority, it also gives one pause to consider that a scholar who specializes in Aristotle's *Metaphysics*, Alan Code, thinks that a 'contradiction' is "...an opposition to which the law of excluded middle (LEM) applies" (Code 132). Given these things then, the claim that the necessary condition involving LEM could be false and we could still have a genuine Aristotelian contradiction, seems unlikely if not clearly false. The other two necessary conditions of (α) seem equally absurd to deny and still have a genuine Aristotelian contradiction. Since, again, one (the first) is at the foundation of the conception of contradictions in his logic, and the other (the third) details precisely what he takes a contradiction to be. But still one might attack the truth conditions for the conditional in (α) as too strict, if not altogether false as an interpretation of the conditional. Again, we would have to imagine Aristotle recanting his views about contradiction, which are the motivation for these truth conditions. Given the research here, there is no evidence to substantiate such a claim that he would recant his views. Thus, given the preceding line of reasoning, we have a genuine Aristotelian contradiction if (α) is true in the sense stated above. In the next two chapters we look to Aristotle's arguments concerning PNC.

Chapter II: The Indubitability Proof

In the next two chapters I will go over Aristotle's arguments for the principle of non-contradiction that appear in *Metaphysics* (Γ). In addition to Aristotle's own statement of his views, this chapter utilizes the English translations of the *Metaphysics* by Christopher Kirwan (1993)⁴⁵. The argument for PNC to be covered in this chapter is the Indubitability Proof. The Indubitability Proof (IP), named by Aristotelian scholar Michael V. Wedin⁴⁶ (2000, 2004⁴⁷), occurs in *Metaphysics* (Γ) Chapter 3 (1005b 11 – 32). This argument for PNC, like the seven refutations to be covered in the next chapter, does not give a direct demonstration⁴⁸ of PNC. Instead it gives us persuasive reasons for accepting it. We begin our discussion of IP here by going over a few general questions about the layout of the arguments in *Metaphysics* (Γ), the philosophers that Aristotle sets himself up against, and his possible purposes for defending PNC. After this we enter the exposition of IP, which can be

⁴⁵ Originally published in 1971. Where appropriate these translations will be complemented with suitable interpolations from, the other major contemporary translator of Aristotle's *Metaphysics*, W. D. Ross (1924). Ross's division of the arguments (R2 – R7 = W2 – W7 (see Appendix II)) will be used here since that is what Wedin and Priest both use. In the initial elenchus or W1 I adhere to Wedin (2000, 2003). In contrast to my use of Kirwan, I explicitly note when Ross's translations are used.

⁴⁶ I follow the ways that Wedin (2000, 2004) presents both IP and W1 – W7 for the most part, but leave out parts of Wedin's thorough defense of both that are not crucial for my purposes here.

⁴⁷ Unless otherwise indicated in this chapter, Wedin (2004) is intended in all citations involving Wedin.

⁴⁸ In the next chapter we will give a characterization of the type of argument that Aristotle thinks he can give for PNC in regard to the seven refutations. This type of argument is called a *negative demonstration* or *elenctic proof*. In IP this method is not intended or at least it is not obvious that Aristotle intended this method for IP, since he introduces the characterization of elenctic proofs in (Γ .4), rather than in (Γ .3) where IP is discussed.

divided roughly into three stages. In the first stage Aristotle defines what he means by a firmest principle, and then gives a defense of this. The second stage claims that PNC is *the* firmest principle, and gives reasons for this identification. The third and final stage is set up to show that it is impossible to believe the negation of PNC—it is impossible to believe in contradictions. The Indubitability Proof is intended to establish PNC as *the* firmest principle, and that it is an indubitable principle of metaphysics.

Before getting into the specifics of the arguments it is necessary to reply to a few questions: (1) What is Aristotle's purpose in introducing and defending the PNC in *Metaphysics I*? (2) Who are the philosophers that he sets himself against and why? (3) What is the layout of the arguments, or how can he be understood to have fit them together into a unified whole? Code's analysis in *Metaphysics and Logic* (1987) gives us a way to answer (1). Code's aim in the essay is to seek a "unified characterization" of Aristotle's *Metaphysics*, and with this aim he gives us four principles, of which the third is that:

...[T]he metaphysician must *both* state the general (propositional) principles that apply to being as such *and* treat of their properties or features. An example would be the principle of non-contradiction (PNC). One of its features is that it is the firmest of all principles, another is that it is a prerequisite for rational thought and discourse" (Code 1987 127).

Given Code's analysis of Aristotle's *Metaphysics*, we can answer (1) by saying, as Code says here: Aristotle introduced and defended PNC, because it is a general principle of being *qua* being, of which the metaphysician must give an account. As Aristotle says, the metaphysician/philosopher who is also to study *substance*: "...must inquire also into the principles of deduction⁴⁹" (Aristotle 1005b 6-7). PNC is the "firmest principle" of deduction, thus, it ought to be studied by the metaphysician. So Aristotle includes PNC in the *Metaphysics* since it is a general principle of deduction that is crucial to metaphysics and the study⁵⁰ of being *qua* being⁵¹.

Now what about an answer to (2)? A few of these figures and the places in Aristotle's argument where they make an appearance are the following: Heraclitus (1005b 24-26; 1012a 25; 1012a 33-34)), Protagoras (1007b 19 – 1008a 8; 1009a 6-7), and Anaxagoras (1007b 25-29; 1009a 25-27; 1009b 27-28; 1012a 26)⁵². From my research, these figures, against whom Aristotle is reacting, are largely advancing metaphysical theses. For example, in the citations involving Heraclitus of Ephesus, he is said by some to think that one can "believe the same thing to be and not to

⁴⁹ Following Ross here. Kirwan translates more neutrally: "...principles of *trains of reasoning*".

⁵⁰ Also referred to as the study of things-that-are *qua* things-that-are in Kirwan and other places.

⁵¹ Aristotle sees himself in (Γ .3) as having a solution to the problem posed in *Metaphysics* Book III (B) 995b 5 – 11, where he was unsure whether the science of being qua being which studies substance will also study PNC and LEM, or the "common axioms". In *Metaphysics* Γ .3, he comes away with a solution to the problem posed in *Metaphysics* B, as Code says: "...the science of substance is the general science of being qua being (1005a 13-18), and this latter is the science that studies what belongs or holds good per se of all things that are (1003a 21-2). Each common axiom holds good of all things that are qua things that are (1005a 22-3). These axioms hold good of all things, and do not have an application merely in one particular kind... apart from the rest of what there is (see also a27-28). Thus the study of the common axioms falls within the scope of metaphysics" (Code 1987 131).

⁵² A few others: Democritus (1009a 25-30; 1009b 13-16), Empedocles (1009b 13-22), Parmenides (1009b 23-25), Homer (1009b 29-31), Cratylus (1010a 10-14), and Epicharmus and Xenophanes (1010a 5-6).

be⁵³" (1005b 24-26). Protagoras⁵⁴ similarly is defending a contentious

metaphysical thesis: "All contradictories are true of the same subject at the same

time" (1007b 19-20). Finally, Anaxagoras is said by Aristotle to be defending the

idea that "all things are mixed together—so that nothing really exists" (1007b 25-

26). So to answer (2), Aristotle is defending himself against these and other

philosophers, since they advance contentious metaphysical theses, with which he

doesn't agree. Further, as we pointed out in chapter I, Aristotle's PNC functions as an

ontological principle.

Finally what are we to think of (3)? That is, what is the layout of the

arguments in *Metaphysics* Γ , or how can he be understood to have fit them together

into a unified whole? Aristotle first gives us the Indubitability Proof (IP) in Γ .3. As

⁵³ Here are a few existent fragments (Fr.), with which Aristotle could be seen to be in disagreement: "(Fr. 61) Sea is the most pure and the most polluted water"; "(Fr. 60) The path up and down is one and the same"; "(Fr. 10) Things taken together are wholes and not wholes" (Heraclitus qtd. in Kirk, Raven, and M. Schofield 188 & 190). As is the case with Aristotle's interpretations of other predecessors (see the following footnote), Kirk et al. think that he unfairly misinterprets Heraclitus and I think this is right. Heraclitus isn't asserting Aristotelian contradictions, which we formalized in chapter 1. In respect of Fr. 10, he goes on to make clear that the sea is pure for one type of being and not for another type, but he does not assert the same thing of the same thing as Aristotelian contradiction requires: "...for fishes it is drinkable and salutary, but for men it is undrinkable and deleterious". Kirk et al. take this to be an anachronistically unfair move by Aristotle, and this is not wrong (186). This doesn't necessarily pose a problem for the current project though, since right or wrong, these are the views that Aristotle is attacking, and against which he poses his refutations. Another project could assess the accuracy of Aristotle's interpretations, and call each of the "writer's on nature" by an individual constant W_1, \dots, W_n to distinguish them from the actual writers' positions. But in order to understand Aristotle's arguments we need to leave these interpretations in place. ⁵⁴ Priest (1998) and others think that this charge against Protagoras is an unfair interpretation by Aristotle. Wedin (2003) shows why this view follows from or is implied by Protagoras' views, even though he and his followers might not have explicitly accepted it.

mentioned above, this proof is meant to show that PNC is the firmest principle, and that a negation of it cannot be *believed*. As such, the conclusion of IP represents a *doxastic* variant of PNC⁵⁵. After IP, Aristotle gives seven different arguments or "refutations" for PNC, which will be discussed in the next chapter. IP and W1 – W7 (the seven refutations) are intended to demonstrate the strength of PNC—the ultimate principle of Aristotle's *Metaphysics*. Finally, in what follows, it is assumed with Wedin (2000) that Aristotle's "main concern": "…in *Metaphysics* Γ … is…the proposal that it is possible to believe the negation of PNC…So the chapter's [Γ] extended defense of the principle itself is arguably motivated by a desire to secure PNC's status as a premiss in the argument that proves its firmness [IP]" (Wedin 2000 113-4). That is, Aristotle is concerned mostly with the negation of the conclusion of IP (i.e. that it is possible to believe a contradiction—the negation of (21a) in what follows), and the seven refutations are plausibly seen as further

⁵⁵ The talk of *versions* of PNC, which Aristotle might have defended/formulated represented some of the early literature on the topic. Including Łukasiewicz (1910) who thought that Aristotle could be seen to formulate PNC in an ontological, logical, and psychological way "...without making explicit in any way the difference among them" (487). His psychological formulation makes up part of what we are calling here the indubitability proof. Wedin (2000, 2003, 2004) doesn't take this approach to his exegesis, and this looks to be an exegetically sound path, since it's not clear that Aristotle differentiated so strongly between logical and ontological matters, as we do now. This contrast seems even more apparent as soon as we remember who Aristotle is defending himself against (i.e. the so-called "writers on nature"). Aristotle defends PNC against these writers, but it is unclear that he meant it as either an ontological principle or logical principle, or both; whereas nowadays for the most part we are used to talking about the concept in purely formal terms (i.e. (assuming classical logic) it is a non-contingent necessarily false proposition). Even though Łukasiewicz' exegesis seems to fall short, I do not defend that here, since he is not central to my research and there are others who have shown this (i.e. Wedin (2000, 2003, 2004)). But it is clear that inasmuch as he fell short in exegesis he succeeded in reviving the scholarship on this foundational issue.

support for PNC's presence in IP (i.e. (6*) or (13) in what follows). The degree of plausibility of the latter idea is high when one remembers that Aristotle thinks that PNC is *indemonstrable*: "But if there are things of which one should not demand demonstration, these persons cannot say what principle they regard as more indemonstrable than the present one [PNC]⁵⁶" (1006a 9-11). Further Aristotle says at the beginning of Γ .4 immediately after giving IP that he *accepted*⁵⁷ PNC, and *showed* on these grounds that PNC is *the* firmest principle (Wedin 2000 114). For these reasons, in what follows this (Wedin 2000) conception of Aristotle's argumentative purposes will be utilized.

Now that some context has been set for the arguments in *Metaphysics* (Γ), let's enter the Indubitability Proof (IP). Before laying down IP, Aristotle gives a "rule to case" argument (1005b 8 – 12), which aims to show that the philosopher, who studies being qua being, can state the firmest principles of everything. It goes like this:

Rule: If someone has the best understanding of a genus, then that person can state the firmest principles of that domain.

⁵⁶ Following Ross.

⁵⁷ Here Ross' translation could complement Kirwan's, since he translates Kirwan's "accepted" as "posited". Where "an immediate deductive principle" is a *posit*: "…if one cannot prove it but it is not necessary for anyone who is to learn anything to grasp it" (*Post. Ana.*, Book I, 72a 15-16 {trans. Jonathan Barnes}). But it could also create problems, since PNC is not a posit it is an *axiom* according to *Metaphysics* Γ .3, 1005a 19 – 20. But is it an axiom as it is strictly conceived by Aristotle in the *Posterior Analytics* (72a 16-19)? That is: "…[0]ne {immediate deductive principle} which it is necessary for anyone who is going to learn anything whatever to grasp, I call an axiom…" (*Post. Ana.* 72a 16 – 19). This also could create problems, since it is not clear that it is necessary to grasp PNC to learn anything. Here Kirwan avoids these problems with "accepted".
Case: If someone has the best understanding of things that are *qua* things that are, then that person can state the firmest principles of *everything*⁵⁸.

The philosopher is the person who has the best understanding of being *qua* being, and thus, can state the firmest principles of everything. It seems odd to consider being qua being a *genus* like say "animal", so why does Aristotle think this? Firstly following Wedin, being qua being is a "ubiquitous genus" in that it consists of "everything that is *qua* thing that is" (226). Most importantly, Aristotle tells us in $\Gamma.2$:

For it falls to one discipline to study not only things called [what they are] *by virtue of one thing*, but also things called [what they are] *with reference to one nature*; indeed in a certain sense the latter too are called [what they are] by virtue of one thing. Plainly, therefore, the things-that-are also fall to be studied by one discipline *qua* things-that-are (1003b 12-15)⁵⁹.

There are two ways something can be a single discipline: when either (a) things are called what they are by virtue of one *thing*, or (b) things are called what they are with reference to one *nature*⁶⁰. An example of (a) would be the science/discipline of "healthy things," Aristotle says, since things in the discipline are all called what they are with respect to one *notion* (i.e. Health)⁶¹. An example of (b) he implies is the discipline of being *qua* being, which studies things that are called what they are by reference to a one *nature* (i.e. being qua being). He also says that the discipline of being qua being can fit into (a) as well, since being qua being could be seen as a single notion with everything it studies called what they are with respect to it.

⁵⁸ Wedin 2004 P. 226.

⁵⁹ Wedin doesn't use this passage to justify the rule to case argument. He uses 1004a 23-25, but this passage doesn't make the above distinction apparent, nor its connection to the study of being *qua* being.

⁶⁰ Aristotle says this earlier in *Metaphysics* Γ .2 as well (1003b 12 – 15).

⁶¹ "Notion" finds expression in Ross's translation, and is useful here.

Aristotle finishes out this passage by saying that things-that-are *qua* things-thatare, given the distinction above, is studied by one discipline⁶². With this we can close out the rule to case argument: being qua being is a genus or single discipline in sense (b), if not also (a). Therefore, the philosopher is the person who has the best understanding of being *qua* being, and thus, can state the firmest principles of everything.

What exactly does a firmest principle amount to—what does it mean to be a *firmest* principle? Aristotle answers: "A principle about which it is impossible to be in error is firmest of all" (1005b 11). A firmest principle is then just a principle about which it is impossible to be in error. Immediately after introducing the concept of the firmest principle, Aristotle goes on to give reasons for it, and so forms what we will now refer to as the following thesis:

(F)⁶³ If (a) error is impossible regarding a principle, P, then (b) P is firmest⁶⁴.

What does Aristotle mean in (F)? He means something along these lines: regardless of what a person may think, if error is impossible with respect to a principle *P*, then *P* is firmest. According to Aristotle, *firmness* is not dependent on the believer—one cannot simply make a principle firm. Firmness, as Wedin says, is an "odd sort of doxastic property" (227). It is *odd* because firmness is "...a property *inherited* by a

 $^{^{62}}$ Aristotle repeats this thesis elsewhere: 1003b 21 – 23; 1003b 31 – 37; 1004a 31 – 33. This thesis is strongly repeated in 1005a 1 – 6, where he repeats the distinction above.

⁶³ N.B.: (F) doesn't require that there be only one firmest principle (Wedin 227).
⁶⁴ Wedin P. 227.

principle" [my emphasis] (227). The firmness is inherited by a principle because

error with respect to it is impossible by believers.

Now that we have an understanding of what Aristotle takes firmest

principles to be, what is the argument that Aristotle gives for (F)? Immediately after

introducing the concept of a firmest principle (F), Aristotle gives reasons for it in the

following passage:

[*i*] A principle about which it is impossible to be in error is firmest of all. For [*ii*] a principle of that kind is necessarily the most intelligible, since [*iii*] everyone makes mistakes on matters about which he does not have understanding; and [*iv*] it is non-hypothetical, since [*v*] what is necessarily part of the equipment of one who apprehends any of the things-that-are is not a hypothesis, and [*vi*] what one necessarily understands who understands anything is necessarily part of the equipment he comes with. It is plain, then, that [*vii*] a principle of that kind is firmest of all [Wedin/my emphases⁶⁵] (1005b 11-18)⁶⁶.

Aristotle starts off by laying out the firmest principle in (i) (= F), and then goes on to

support it. Wedin formalizes the argument⁶⁷ for (F) as follows:

1. If (a) error is impossible regarding a principle, *P*, then (c) *P* is necessarily most intelligible.

2. If (a) error is impossible regarding a principle, *P*, then (d) *P* is non-hypothetical.

3. If (a) error is impossible regarding a principle, *P*, then (c) *P* is necessarily most intelligible, and (d) *P* is non-hypothetical.

4. If (c) *P* is necessarily most intelligible and (d) *P* is non-hypothetical, then (b) *P* is firmest.

: (F) If (a) error is impossible regarding a principle, *P*, then (b) *P* is firmest⁶⁸.

⁶⁵ Wedin (2004) Pgs. 226 – 7. Wedin is responsible for the numbering. I took out the Greek that Wedin included in the parentheses.

⁶⁶ Following Kirwan. Ross puts "certain" in the place of "firmest" in Kirwan's translation.

⁶⁷ I have added an appendix of the entire Indubitability Proof to the end of the thesis.

⁶⁸ Wedin 227-8. In Wedin F = A. I use *F* here to avoid confusion with the 'same attribute' qualification in *SARTE* (*cf.* Introduction, Ch. 1).

In the preceding argument (1) derives from (ii) in the original passage, and (2) derives from (iv). We get (3) *via* conditional proof from (1) and (2). (4) is unstated in the original argument, but as Wedin notes, is needed for the premises to support (F) (228). In order to fully understand the argument for (F), we need to look at the justification for (1) and (2) that Aristotle gives in the passage above.

Starting with the argument for (1), Aristotle says in (ii) that if it is impossible to be in error about a principle, then it is most intelligible since (iii) everyone errs or makes mistakes about that which they do not understand. That is: if one does not understand a principle, then they err with respect to it. This is just plain false though. One could not understand a principle, but not err with respect to it, as Wedin says, due to luck (228). Wedin alters the original text to yield⁶⁹:

1.1: if it is possible that *x* does not understand *P*, then it is possible that *x* errs regarding *P*.

1.1 weakens the strict reading of the text, and adds the modal operators that are implicit in (iii)⁷⁰. 1.1 can account for the lucky person who does not err even though they do not understand *P*, as well as the person who does not understand *P* and errs regarding it. 1.1 is at least plausible⁷¹, and is logically equivalent to 1.1^* , which when combined with 1.2^* implies (1):

⁶⁹ Wedin 228. Wedin's 1.1* = my 1.1.

⁷⁰ As expected, Wedin employs the principle of charity here and other places in his exegesis, but it is probable that this is what Aristotle intended given his explicit use of the modal operators in (i), (ii), (v), and (vi). Wedin does not use the principle without reservation though, as his other work (2003) attests.

⁷¹ One could raise further counterexamples to 1.1, perhaps concerning the meaning of *understands* or *erring*, or again concerning the insertion of the modal operators. Wedin's position on this issue is sided with here that 1.1, if not true, is "...at least plausible" (229).

1.1*: if (a) x cannot err regarding P, then (e) x necessarily understands P⁷².
1.2*: if (e) x necessarily understands P, then (c) P is necessarily most intelligible.

: (1): If (a) error is impossible regarding a principle, P, then (c) P is necessarily most intelligible.

This effectively completes the support argument for (1), but what are we to think of it? 1.2* is not explicit in the passage above, but as in the case of (4) in the argument for (F), it is needed for the support argument that Aristotle offers to imply (1). But even putting this aside there are two issues with 1.2*: what does it mean to say that a proposition is "necessarily intelligible"?; what does it mean to say that a proposition is "most intelligible"? To the first question, we may reply with Wedin that a proposition that is *necessarily intelligible* is one which "...can't fail to be understood whenever entertained" (229). In answer to the second question, Wedin says: "...for *P* to be the *most intelligible* principle would, on this account, entail that it is more intelligible than any other principle and that there is no principle as intelligible as it" [my emphasis] (229). With this understanding of the support argument for (1) in the argument for (F), let's move on to the support argument for (2).

In the argument for (2), Aristotle uses (v) and (vi) in the passage above. (2) connects (a) the impossibility of error regarding a principle *P* to (d) its status as non-hypothetical. (v) supports the connection in (2) by saying that something that anyone, who apprehends the "things-that-are" must necessarily have as part of their "equipment", is not a hypothesis. That is, if error is impossible regarding a principle *P*, then it is non-hypothetical because anyone who apprehends being must

⁷² Wedin 229. Wedin's 1.2* = my 1.1*; his 1.3* = my 1.2*.

necessarily have *P* as part of his or her equipment. (vi) further supports the

connection in (2) by saying that something that is necessarily "part of the

equipment" of anyone is what anyone necessarily understands who understands

anything. That is: if error is impossible regarding a principle P, then it is non-

hypothetical not only because it is needed to understand being (hence, (v)), but to

understand anything at all. The locution "part of the equipment" can be understood

to refer to necessary or stock principles of either an area of study (i.e. "being" in (v))

or any study at all. Following Wedin, we can formalize the support argument for (2):

2.1: if (f) x's understanding anything presupposes x's understanding P, then
(g) x must already have P ["....is part of his equipment...."].
2.2: if (g) x must already have P ["....is part of his equipment...."], then (d) P is non-hypothetical.
2.3: if (f) x's understanding anything presupposes x's understanding P, then
(d) P is non-hypothetical.

2.4: if (a) error is impossible regarding a principle *P*, then (f) *x*'s understanding anything presupposes *x*'s understanding *P*.

: (2): if (a) error is impossible regarding a principle, *P*, then (d) *P* is non-hypothetical⁷³.

The first premise 2.1 is derived from (vi) in the passage above from Aristotle, which

is: "what one necessarily understands who understands anything is necessarily part

of the equipment he comes with". Here the phrase "what one necessarily

understands who understands anything" is plausibly understood in 2.1 to mean "x's

understanding anything⁷⁴ presupposes *x*'s understanding *P*". Premise 2 is derived

from (v) in the original passage, which reads: "what is necessarily part of the

equipment of one who apprehends any of the things-that-are is not a hypothesis".

⁷³ Wedin P. 230. His numbering is retained here.

⁷⁴ Although not fully spelled out here, the "Ultimacy Claim" is present (*Meta.* 1005b 31-3). We will return to this in the concluding comments of IP.

Again (v)'s "what is necessarily part of the equipment of one who apprehends any of the things-that-are" is understood in 2.2 to mean "x must already have P". 2.3 is implied as the logical consequence of 2.1 and 2.2, and connects "x's understanding anything presupposes x's understanding P" to the non-hypothetical status of P. Premise 2.4 like 1.2* in the support argument for (1) is not in the actual Aristotelian text, but is required to derive (2) in the conclusion. With this addition, the argument is complete. As a final note here we can briefly consider what Aristotle takes to be hypothetical, and thus, non-hypothetical. Aristotle in *Posterior Analytics* (76b 23-34) describes "...a hypothesis⁷⁵ as what is provable but accepted without proof" (231). A principle that is *non-hypothetical* then is one, which is *not* provable and accepted without proof—it is either not provable or not accepted without proof. As 2.3 makes clear, it is the principles that are not provable with which it and the argument for (F)is concerned. Further, Aristotle says of PNC, the firmest principle that is the focus of the argument in (F): "... if there are things of which one should not demand demonstration, these persons ["...writers about nature..." (i.e. Heraclitus and others)] cannot say what principle they regard as more indemonstrable than the present one [PNC]" [my emphases] (*Meta*. Γ.4, 1006a 9 – 11)⁷⁶. Aristotle also goes on to give elenctic proofs or negative proofs of PNC (i.e. PNC is not provable, but not accepted without (negative) proof). Thus, this rendering of 'non-hypothetical' is utilized in understanding the nature of PNC (232). With the final support argument

 ⁷⁵ Wedin isn't clear about the translation of *Posterior Analytics* to which he is referring (perhaps he is translating it himself). Jonathan Barnes translates hypothesis as "supposition" and "postulate", which are different types of hypothesis for Aristotle. This isn't too troubling given the clear connection.
 ⁷⁶ I follow Ross' translation here, since it makes the point clearer.

for (2), Aristotle infers (F) from (1) – (4), and the argument for the

characterization of a firmest principle is complete.

Now that we have given the complete argument that Aristotle offers in

support of what a firmest principle is, what is the firmest principle? The principle, as

the reader might have guessed, is none other than the principle of non-

contradiction, and Aristotle states it thus:

For the same thing to hold and not to hold of the same thing at the same time and in the same respect is impossible, given any further specifications added to guard against dialectical objections $(1005b\ 18 - 20)^{77}$.

We may state PNC as a modal proposition following Wedin:

6. It is not possible that there is something, *x*, such that *x* has a property, *F*, and *x* does not have F^{78} .

The proposition is modal because it states that it is not possible for something to

have and not have a property. This principle can be more formally stated as follows:

 $6^*. \neg \langle (\exists x)(Fx \land \neg Fx) \rangle$

As previously stated, Aristotle's principle functions as an ontological principle in

Metaphysics, and thus, "...ranges over things and their properties" (234). So (6) will

be understood to be stating 6^{*79} . Here we can understand PNC as characterized by 6

⁷⁷ Wedin's translation of this key passage varies from Kirwan—it looks to have features of the Ross and Kirwan translations.

⁷⁸ Wedin Pg. 234. Also, notice how this formulation of the principle doesn't quantify over time, which a more formally rigorous approach to Aristotle's PNC might do. Although, the formally rigorous account is not the intention here, it seems like a better formulation of the principle would be: $\neg \langle (\exists x) (\forall \Phi) (\forall r) (\forall t) (\Phi(x, r, t) \land \neg \Phi(x, r, t)) = \Box \neg (\exists x) (\forall \Phi) (\forall r) (\forall t) (\Phi(x, r, t) \land \neg \Phi(x, r, t)), where$ *r*= the respect or $definition of <math>\Phi$, *t* = time at which Φ is asserted.

⁷⁹ This formulation is opposed to a "logical" formulation, which ranges over propositions or atomic statements. Given our previous discussion in chapter I, I take

or 6*, *or* more complexly, the modal negation of (α) (i.e. it is not possible that a compound statement, which consists of a pair of SMS's, fits the specification given in (α) and is the case). That is, 6 or 6* will represent the shorthand or elliptical sense of (α) as it was developed in chapter I.

After Aristotle gives the argument for the characterization of the firmest principle in (F) and states PNC, he begins what Wedin refers to as the Indubitability Proof (IP). The purpose of this proof is to establish that it is not possible to err with regard to PNC, or given (F) above, to establish PNC as meeting the antecedent (a) of the conditional (F)—to establish PNC as *a* firmest principle. Recall (F): if (a) error is impossible regarding a principle, *P*, then (b) *P* is firmest. Aristotle gives IP in the following passage:

[*viii*] This [PNC], then, is the firmest of all principles, for [*ix*] it fits the specification stated. For [*x*] it is impossible for anyone to believe that the same thing is and is not, as some consider Heraclitus said – for it is not necessary that what one says one must also believe. But if [*xi*] it is not possible for contraries to hold good of the same thing simultaneously . . . and if [*xii*] the opinion contrary to an opinion is that of the contradictory, then [*xiii*] obviously it is impossible for the same person to believe simultaneously that the same thing is and is not. For [*xiv*] anyone who made that error would be holding contrary opinions simultaneously [Wedin's/my emphases⁸⁰] (1005b 22-32)⁸¹.

This passage begins with identifying PNC not only as *a* firmest principle (remember

we noted that (F) left it open that there could be more than one firmest principle in

it as a given that Aristotle's version of PNC is the ontological version of PNC as understood above. Further, as also stated previously, it is not clear that Aristotle distinguished between the logical and ontological as smoothly as we do nowadays. ⁸⁰ Wedin (2004) Pgs. 235 – 6. I merely clarify the demonstrative pronoun. Wedin provides the numbering, and other emphases.

⁸¹ From here on out, I will reference this passage by the line number/s (i.e. [x], [xi], etc.), rather than the full text citation each time.

footnote 56 (227)), but *the* firmest principle of all principles (firm or not)⁸². PNC

also fits the "specification stated" of a firmest principle (i.e. it is impossible to err

with respect to it). From this point Aristotle gives reasons for thinking PNC is a

firmest principle. We may state the first two premises of IP as follows:

7. If (g) for all x it is impossible that x err with respect to a principle, P, then (a) error is impossible regarding P.

8. For all x, if (h) it is impossible that x believes $\neg P$, then (g)⁸³ it is impossible that x errs with respect to P^{84} .

Starting with premise (7), this is an assumption that Aristotle has working in the

background of IP (Wedin 2004 236-7). That is, error is impossible regarding a

principle *P* if it is impossible that a person *x* errs with respect to *P*, since it is

humans⁸⁵ that err or do not err and thus, make erring impossible or possible with

respect to a principle. Premise (7) makes clear exactly what the "immunity to error"

⁸² As mentioned briefly above, this is the so-called "Ultimacy Claim". This will not be fully expanded upon till the end of IP where it is said to "complete" the argument (Wedin 259). I'm careful in what follows about PNC's identification as *a* firmest principle, since he doesn't get to the Ultimacy claim till the end of Γ .3, and hence, the end of IP.

⁸³ The consequent of premise (8) is labeled (a) by Wedin, but it clearly is the same as the antecedent of (7), so I have labeled it (g), since it is a clear typo. This emendation also serves to more firmly establish the connection between (h) and (a), and ultimately (h) and (b). In personal correspondence through email, Wedin has agreed to the validity this emendation.

⁸⁴ Wedin Pgs. 236 – 237. Wedin's 8* = my 8.

⁸⁵ In response to a comment by Dr. James Petrik, I'm inclined to agree with him that the scope of PNC as a firmest principle should be extended past *actual* doxastic agents, to all actual and *possible* doxastic agents, whether or not the scope included things that are clearly not doxastic agents (i.e. rocks, and mountains). If this scope was not fixed, then as Petrik says, "There might be a principle with respect to which all actual doxastic beings cannot err but that there were logically possible but not actual doxastic beings who might err with respect to the Principle". It's clear that in possible future revisions of his exegesis, Wedin would have to amend this difficulty.

in (a), as Wedin calls it, amounts to. Here IP moves in the direction of what persons or individuals are able to do with respect to a principle, and (7) can be seen as establishing the jumping off point for a new argumentative focus for Aristotle. This new focus in his argument changes from the focus in the argument for (F) which dealt with error being impossible with respect to a firmest *principle*, to the new focus, which concerns "...what individual *agents* can and cannot do..." with respect to a firmest principle in IP [my emphases] (236). Now that the meaning of immunity to error in (a) is clarified, premise (8) derives from (x) in the original passage, where the concern is to support PNC's identification as a firmest principle that was made in (viii) and (ix). Aristotle goes on to give a reason for PNC being a firmest principle in [x] by saying that "...it is impossible for anyone to believe that the same thing is and is not...". That is, PNC is the firmest principle because it is impossible for any person to believe the negation of PNC (i.e. the same thing is and is not some property, Φ), and if that is impossible then it is impossible that anyone errs with respect to PNC. (7) and (8) together imply (9), and this together with (F) implies (10):

9. If (h) it is impossible that *x* believes ¬*P*, then (a) error is impossible regarding *P*.
(F) If (a) error is impossible regarding a principle, *P*, then (b) *P* is firmest.
10. If (h) it is impossible that *x* believes⁸⁶ ¬*P*, then (b) *P* is firmest⁸⁷.

⁸⁶ As Wedin notes in Footnote 6 on Pg. 237, Aristotle does not have a discussion of the relation between *error* and *belief* in Gamma 3. This leaves it up to speculation as to the relation between the two for Aristotle. What is clear is that he thinks that PNC is the firmest principle and the reason he gives in (x) and (8), as we formulated it above, is that "...it is impossible for anyone to believe that the same thing is and is not". In order to derive (10) or PNC's firmness we have to assume he is referencing

Premise (9), which is the logical consequence of (7) and (8), makes it very clear why Wedin thinks that Aristotle "must hold" (7) (236). Without (7), Aristotle could neither draw the consequence in (9), nor the consequence in (10), and both are needed to draw the conclusion that PNC is the firmest principle, which is the statement that Aristotle introduced for defense in (viii) and (ix). The "Ultimacy Claim" that PNC is *the* firmest principle will be addressed at the end of IP, which is fitting since Aristotle saves the reason for this claim till the end of Γ 3. For now Aristotle is concerned to establish the truth of PNC as a firmest principle, and show that PNC is a firmest principle, even though later he intends to give reason why it is *the* firmest of all.

The next phase of the proof⁸⁸ lays out two more premises, which deal with the important question of the *target* of the proof. The claims are as follows:

11⁸⁹. $\neg \langle (\exists x) (\exists z) (x \text{ bel } (Fz \land \neg Fz)) \rangle$

his initial argument for (F). This is warranted since Aristotle says that PNC "...fits the specification stated" of a firmest principle (1005b 23 – 24). ⁸⁷ Premises (9) and (10) are not stated in Wedin's original analysis, but since these logical consequences make the connection between (h) and (a), and (h) and (b) more pronounced, respectively, I go over them here. ⁸⁸ Wedin P. 238 – 239. Wedin's 9 = my 11; Wedin's 10 = my 12. "bel" is the epistemic operator for "belief". In other words: "x bel y" = "x believes y". ⁸⁹ In section 7 of Wedin (2004) P. 253 – 258, he goes over two types of contradiction entailment in order to clear up the kinds of belief in contradiction that Aristotle's IP would not allow. "Extrinsic" belief in contradictions would be where something you believe, given outside information unavailable or unknown to you, entails a contradiction. These Wedin says would not be restricted by IP. An example would be where one believes that the author of *The Castle* is great, but Franz Kafka is not great, where they didn't know that Kafka was the author of the book. Here Wedin says: "...because extrinsic information is not available within the relevant doxastic context, it cannot be used to generate *belief* commitments..." (254). This seems right. If the doxastic context were always open to contradiction, knowledge would be

12. $\neg \langle (\exists x)(x \text{ bel } \langle (\exists z)(Fz \land \neg Fz)) \rangle$

Both (11) and (12) are referred to as potential targets of the indubitability proof. Premise (11) is referred to as the "Instantial" reading of the target of PNC, since it claims that the target of IP is specific instances of PNC (i.e. the water is pure and not pure). Or (11) above reads as: it is not possible that there exists two objects x and zsuch that x believes z is F and not F. Premise (12) is referred to as the "Principled" reading of the target of PNC, since it claims that the target of PNC is the principle itself. Or (12) above reads as: it is not possible that there exists an x such that x believes that it is possible that there exists a z such that z is F and not F. Wedin assumes the target of PNC to be the principle itself or the principled reading for a couple reasons. Firstly, as Wedin notes: "...the entire discussion [of the *Metaphysics*] proceeds from presumptions about the highest principles of any given science and aims to identify the highest principle or principles of the science of things that are" [my emphasis] (237). That is, Aristotle sets forth the argument for (F), including both support arguments for (1) and (2), and then proceeds in identifying PNC as the firmest principle of all principles. It would be strange in light of these developments, if he intended the target of PNC not to be the principled reading, but the instantial

impossible. Or if it were never closed, then our knowledge would be incomplete (?)—this seems less troubling than the latter option. It's hard to know "Intrinsic" belief in contradictions would be where something one believes entails a contradiction through reason—these are restricted by IP. An example of this would be if one believed that a pen was red, but was not colored. The pen being red entails that it is colored, and that they believe a contradiction that would be restricted by IP. Wedin bases the conclusions about these two types of contradiction entailment in what Aristotle says about those philosophers he is against. For example, Protagoras is one such philosopher. Wedin has a nice piece on Aristotle's indictment of Protagoras in *Metaphysics* Γ (cf. Wedin 2003).

reading, about which, say, Heraclitus or Protagoras talks. It would be strange for the target to be the instantial reading precisely because he is defining what a firmest principle is, which is the firmest of all, and why it is. Secondly, in an introductory passage where Aristotle is discussing the concept of a firmest principle (F) in Γ .3, he concludes the passage by saying the principle itself is firm: "It is plain, then, that *a principle* of that kind [i.e. as defined in (1) – (F)] is firmest of all" (1005b 18 – 19). Thus, the target of IP is (12), and by establishing this, each instance that would be covered in (11) would be also be ruled out as impossible.

Now that we have gone over preliminary assumptions and claims, and established the target of the indubitability proof, we can now continue into the proof proper starting with premise (13).

13. $\neg \langle (\exists x)(Fx \land \neg Fx)^{90} \rangle$

Premise (13) should be familiar, since we are just are reiterating (6*) or the ontological version of PNC. Now why does it reappear here? At the beginning of Γ .4 (after giving IP), Aristotle says: "...we have just *accepted* that it is impossible to be and not be simultaneously, and we have *shown* by means of this that it [PNC] is the firmest of all principles" [my emphasis] (1006a 4 – 7). That is, after finishing IP, Aristotle says that he accepted or assumed that PNC was true, and then showed through IP that it is the firmest of all principles. Further, just after giving the

⁹⁰ Wedin Pg. 234. Wedin's 6a = my 6*. Wedin (2004) has an extended discussion in Section 4 about why PNC is included in the proof, if it is not stated therein. The answer is through (13) and (15) we are able to derive (14) or the principle of noncontrariety (to be discussed next). And further, "...Aristotle opens Gamma 4 by announcing that the proof's conclusion was reached "by means of" PNC..." (244). This is sufficient to warrant PNC's use in IP.

characterization of a firmest principle in the argument for (F), Aristotle states PNC in 1005b 18-20. Should we worry about using PNC to prove something *about* it in IP (i.e. it is not possible to believe that the same thing is and is not)? Wedin thinks, "...that there is nothing illegitimate about using PNC to prove something about PNC. For what is proved is not PNC but a different proposition *about* it, namely, that its negation cannot be believed" (239). Surely Aristotle would be guilty of *petitio principii* if he assumed PNC for a demonstration of PNC, but in IP he uses PNC instead as a premise to prove something different than PNC. Thus, the charge of circularity is disarmed⁹¹.

The next two steps of IP deal with what Aristotle says in (xi) in the passage above: "...it is not possible for contraries to hold good of the same thing simultaneously" (1005b 26-7). Wedin takes this to be expressed in (14):

14. $\neg \langle (\exists x) (Fx \land F^*x) \rangle$ 15. $(x) (F^*x \rightarrow \neg Fx)^{92}$

Following the Aristotelian interpretation of PNC, Wedin expresses what may be called the principle of non-contrariety in (14), ontologically, since (xi) is talking about it not being possible that contraries can hold good of the same *thing* at the

⁹¹ Another issue here is that Aristotle doesn't strictly say 13 after talking about (8), but I don't think this is an important discrepancy given that Aristotle tells us after the proof in Γ.4 that he accepted PNC and on the basis of this showed that it was the firmest principle (1006a 4 – 7). Not every argument need have a strict formal layout, especially here, since Aristotle clearly indicates what he did in IP. ⁹² Wedin 239-40. Wedin's 11 = my 13; his 12 = my 14; his 13 = my 15. I think (15) could possibly be said to be present in the text at (xii) as well. Notice how we can't derive the biconditional of (15), if we sought a proof of it. The truth table for (x)(¬Fx → F*x) is invalid if we take (9') (below) as the only premise, and assuming the propositional calculus. This rightly maintains the distance between contraries and contradictories, which are distinct for Aristotle.

same time⁹³. (14) reads as: it is not possible that there is an *x* such that *x* is *F* and *F**, where Φ * indicates the contrary of any property Φ . Wedin takes (14) to follow from (15)⁹⁴, which Aristotle formulates later in Γ .6 (1011b 15-21): "Since it is impossible for a contradiction to be true simultaneously of the same thing, it is obviously impossible too that contraries should simultaneously hold good of the same thing". To make the deduction⁹⁵ of (15) in line with Wedin's formulation above, we'll assume sentential logic:

1'. $\neg \langle (\Phi x \land \neg \Phi x) \rightarrow \neg \langle (\Phi x \land \Phi^* x) \rangle$ 2'. $\Box \neg (\Phi x \land \neg \Phi x) \rightarrow \Box \neg (\Phi x \land \Phi^* x)$ (Modal Square of Opposition, 1') 3'. $\neg (\Phi x \land \neg \Phi x)$ (Theorem) 4'. $\Box \neg (\Phi x \land \neg \Phi x)$ (Rule of Necessitation, 3') 5'. $\Box \neg (\Phi x \land \Phi^* x)$ (MP 2', 4') 6'. $\neg (\Phi x \land \Phi^* x)$ (M axiom: $\Box p \rightarrow p, 5'$) 7'. $\neg (\Phi x \land \Phi^* x) v (\Phi x \land \neg \Phi x)$ (Add 6') 8'. $(\Phi x \land \neg \Phi x) v \neg (\Phi x \land \Phi^* x)$ (Com 7') 9'. $\neg (\Phi x \land \neg \Phi x) \lor \neg (\Phi x \land \Phi^* x)$ (Impl 8') 10'. $(\Phi x \land \neg \Phi x) \lor (\neg \Phi x \lor \neg \Phi^* x)$ (Impl, DM, 9') 11'. $[\Phi x \lor (\neg \Phi x \lor \neg \Phi^* x)] \land [\neg \Phi x \lor (\neg \Phi x \lor \neg \Phi^* x)]$ (Dist 10')

⁹³ The principle of non-contrariety differs from PNC, of course, in that, as is standard of Aristotelian logic, contraries cannot be true at the same time, but they can be false at the same time—contradictories on the other hand are mutually exclusive, *and* jointly exhaustive (i.e. *at most* one must be true, and *at least* one must be true). ⁹⁴ (15) is stated even more clearly in 1011b 18 – 20, but wading through Aristotle's theory of "opposites" in Categories Chapter 10, which I think would be necessary given Kirwan's use of "lack" and "contrary" in the same sentence (further, it's not entirely clear which work came first), would require much more space than is necessary here. I think Ross's translation of the end of Metaphysics Γ .6 is more consistent with the Categories discussion of the opposition of "privation and possession".

⁹⁵ Wedin's explanation of (15) is bare, so I offer this proof of (15), which is not Wedin's. I give many thanks to Ohio U. alumnus Chris Arledge, who helped me with the modal steps here. Unfortunately, Wedin does not indicate which sentence/s he is thinking about in the above text citation, but merely gives the citation. As mentioned in the preceeding footnote, it could be the case that he is talking about the other sentence. In that case though, more would need to be said about the above-cited things. 12'. $\neg \Phi x \lor (\neg \Phi x \lor \neg \Phi^* x)$ (Simp 11') 13'. $\neg \Phi x \lor \neg \Phi^* x$ (Assoc, Taut, 12') 14'. $\neg \Phi^* x \lor \neg \Phi x$ (Com 13') (\therefore) $\Phi^* x \rightarrow \neg \Phi x$ (Impl 14')

Given (15) and PNC in (13), we can derive (14), which Aristotle gives in (xi). That is, (15) is a principle that allows us to relate "contraries and contradictories", and so (14) is not a free-floating claim in IP as it might be interpreted without (15) in place (240). Once (15) is put in place as one of the reasons (14) follows, the argument is not only tighter, but it makes sense of the inclusion of (14) in the argument.

The next steps of the Indubitability Proof involve what Wedin calls "Property Attribution" (PA), as Wedin says: "...[IP] concerns constraints on what one can believe, in particular, constraints *against* the possibility of believing contradictory *states of affairs.* So Aristotle needs a principle that relates belief to *objects* of belief in such a way as to explain why such beliefs are impossible" [my emphases] (240). IP then is an argument that is meant to show exactly one constraint on what one may possibly believe. In order to do this, as Wedin says, Aristotle needs a principle to relate belief to objects of belief, and returning to the passage above, he gives that in (xii): "... [*xii*] the opinion contrary to an opinion is that of the contradictory...". We can represent (xii) as (16):

16. (x)(x believes Fa is contrary to x believes \neg Fa) 16a. (x)(x bel Fa \rightarrow [B:Fa]x) 16b. (x)(x bel \neg Fa \rightarrow [B:Fa]*x)⁹⁶

⁹⁶ Wedin 240-1. Once again, my steps are his +2, and this will be the same in what follows.

In (16), the contrary belief to "*a* is *F*" is the contradictory, or, "*a* is *not*-*F*". As Wedin makes clear, Aristotle needs a way for the *beliefs* of (16) to relate to the contrary *properties* that (14) "proscribes" or prohibits, because then we can talk about which beliefs are similarly impossible (240). In line with (xiv) of the passage above, Aristotle says: "...anyone who made that error [i.e. believing simultaneously that the same thing is and is not"] would be holding contrary opinions simultaneously". In order for "holding contrary beliefs" to be an error, we need some way of relating his principle in (16) about contrary *beliefs*, to (14)'s restriction on contrary *properties*. To do Wedin takes the following route, and he says: "...Aristotle must require that in general believing something involves attribution of a property to the believer, and that such a property is possessed much as any standard property is possessed by a subject" (240). The premises of (16a) and (16b) are extensions of (16), and both involve relating beliefs of a subject to properties possessed by them. " $[B:\Phi\alpha]x$ " is understood as the doxastic property "x believes α is Φ " which could be possessed by any subject *x*, could be about any thing α , and any predicate's Φ . So (16a) says that "*x* believes *Fa*", and since (16) tells us that this is contrary to "x believes $\neg Fa$ " (and *ipso facto* it is contrary to "x believes F^*a "), and assuming Property Attribution (PA), the property possessed by x is [B:Fa]x. Similarly in (16b): if through PA "*x* believes *Fa*" amounts to the doxastic property [B:Fa]x, and by means of (16), the contradictory "x believes $\neg Fa$ " is contrary to "x believes Fa", and given the formal notation that we used to indicate non-doxastic contrary properties (i.e. following (14) above, $Fx \wedge F^*x$), then the doxastic property

possessed by such an x that believes $\neg Fa$ is going to be contrary to the one

possessed by the *x* that believes *Fa* or [*B*:*Fa*]**x*.

After (xii), Aristotle finishes the Indubitability Proof in (xiii) and (xiv).

Aristotle says: "[xiii]...obviously it is impossible for the same person to believe

simultaneously that the same thing is and is not. For [xiv] anyone who made that

error would be holding contrary opinions simultaneously" [Wedin/my emphases]

(1005b 29 – 31). We may provisionally lay out the final steps of IP as follows.

17. (x)(x bel ($p \land q$) \rightarrow (x bel $p \land x$ bel q)) 17 a^{97} . (x)(x bel (Fa $\land \neg Fa$) \rightarrow (x bel Fa $\land x$ bel $\neg Fa$)) 18a. (x)(x bel (Fa $\land \neg Fa$) \rightarrow [B:Fa]x) 18b. (x)(x bel (Fa $\land \neg Fa$) \rightarrow [B:Fa]*x) 19. (x)(x bel (Fa $\land \neg Fa$) \rightarrow ([B:Fa]x \land [B:Fa]*x)) 20. (x)(x bel (Fa $\land \neg Fa$) \rightarrow ([B:Fa]x $\land \neg$ [B:Fa]x)) 21. (x) \neg (x bel (Fa $\land \neg Fa$)) 21a. \neg (**E**x)(x bel (Fa $\land \neg Fa$))⁹⁸

⁹⁷ As regards our discussion in chapter 1, Wedin is careful to point out that the direct target of the argument is the right side of (17a). That is, the target is not the belief in conjunctive propositions (the left side), but *pairs* of SMS's (245). As Wedin says, "The argument against belief in conjunctive propositions... is an extension of the direct argument" (245). But as Wedin also notes and defends, Aristotle wouldn't allow something like: $(x)\langle x \text{ bel } (Fa \land \neg Fa) \land \neg (x \text{ bel } Fa \land x \text{ bel } \neg Fa))$; that is, he wouldn't allow a belief in a conjunctive contradiction, so long as it didn't postulate pairwise contradictions (245). So although the right side of (17a) is the *direct* target of the proof, "It does not shorten the logical reach of the argument. In particular, *it does not compromise the argument's effectiveness against belief in a single conjunctive proposition* [the left side of 17a]" [my emphasis](Wedin 247).

⁹⁸ Wedin 241 – 242. I have added parentheses to 17, 17a, 19, and 20 to preserve the statements as well formed formulas. Also in (21) and (21a) of Wedin's own proof (remember his steps are just mine – 2), he curiously left out the modal operators. Whether this was a typo or not is unclear since he definitely seems aware of the claims that he should be deriving, and yet he did not add them (242). They are added here, since they are clearly stated in the text at (x) and (xiii). He does say that: "...(19a) [(21a) here] can be strengthened to deny the *possibility* of someone's holding such a belief..." (242). So he does indicate that you could add the modal operators. And in fact, in personal correspondence through email, he can't recall

Premise (17), or as Wedin calls it "Doxastic Simplification" (DS)⁹⁹, is an assumption, which is required by Aristotle's argument (241). And DS is the principle if you believe many statements, then you believe each one as well. The belief that IP began with postulating as impossible in (x) is that one could believe that "the same thing is and is not" (1005b 24-5). So the substitution in (17a) is the belief in the negation of PNC (i.e. the belief in question). Premises (18a) and (18b) take this belief in the negation of PNC, and derive the doxastic properties that would follow from such beliefs. (18a) follows from property attribution (16a) and doxastic simplification (17a), and (18b) similarly follows from PA (16b) and DS (17a). Together (18a) and (18b) give us (19). Premise (19) approximates what Aristotle says in (xiv). There he says that if anyone were to make "that error" (i.e. believing in the negation of PNC), then they "would be holding contrary opinions simultaneously". We might sketch the argument here. If someone say, x, believed in the negation of PNC, then x would believe each conjunct of the belief (*Fa* and \neg *Fa*) and *x* would possess the corresponding doxastic properties according to property attribution. This would be the situation expressed in (20). If a person has the contradictory doxastic properties in (20), then from (15) we know they have the contrary properties in (19), which follows from (18a) and (18b). From modus tollens, the principle of non-contrariety

why did not add them, but that one could add them. He also told me that an explicit formulation is useful. So this issue seems to be resolved, if only informally. ⁹⁹ As Wedin says, Jonathan Barnes (1969) also thinks IP requires DS (241).

(14) and the consequent of (19), which is a denial of (14), we get $(21)^{100}$. The conclusion (21) is just the initial target of the proof in (x): for all *x*, it is impossible that *x* believes something *a* has some property *F* and doesn't have it. And through quantifier equivalences we get the claim in (21a), which is reflected in (xiii), that it is impossible that there exists a person *x* that believes something *a* has some property *F* and doesn't have it. Thus, concluding IP, since (*instances*¹⁰¹ of) negations of PNC cannot be believed then the antecedent of (8) is established, which as the reader will recall is:

8. For all *x*, if (h) it is impossible that *x* believes $\neg P$, then (g) it is impossible that *x* errs with respect to *P*.

With the antecedent established in IP, we get the result that PNC is immune to error and with this, the Indubitability Proof is almost complete.

¹⁰⁰ There are two ways to derive the conclusion in (21), and the second way is from PNC (13), and the consequent of (20). Wedin prefers the approach given above, since it is more kosher with Aristotle not explicitly using PNC in IP. ¹⁰¹ In the proof as presented here I only go over the "instantial reading" ((11) above) of the target of IP (cf. P. 43). Viewing the target of IP as that of the "principled reading" ((12) above) has three different exegetical interpretations as offered by Wedin (2004) P. 249 – 250. The second and third (Wedin's interpretation) of which are more plausible than the first, but the second and third seem to be almost equally plausible. Either we assume that Aristotle would accept Wedin's "C-based beliefs" (i.e knowledge assuring—sort of Cartesian in nature—this is Wedin's approach) in the third interpretation, or we think that Aristotle just accepted that PNC is the firmest principle and showed that it is the firmest by showing that it is impossible to believe an instance of the negation of PNC (the second approach) (250). Either interpretation here seems almost equally plausible, but the main point is that it is extremely unlikely that Aristotle would think that the negation of (12) or $(\exists x)(x be)$ $(\exists z)(Fz \land \neg Fz)$ would be true. So I think that the demonstration of (12) is intended in *Metaphysics* Γ .3 and IP. So, I would agree with, Wedin that Aristotle intends something like the third approach, whether that is in the form of the C-based believes or not.

Recall that Aristotle claimed the following above: "...it [PNC] is the firmest

of all principles" [my emphases] (1005b 22-23). IP established that no

contradictions can be believed, and thus derived that PNC is immune to error or (g). And premise (9) was a logical consequence of (7) and (8), and finally, premise (10)

was a consequence of (9) and the concept of a firmest principle (F).

9. If (h) it is impossible that *x* believes ¬*P*, then (a) error is impossible regarding *P*.
(F) If (a) error is impossible regarding a principle, *P*, then (b) *P* is firmest.
10. If (h) it is impossible that *x* believes ¬*P*, then (b) *P* is firmest.

IP then technically derives that PNC is *a* firmest principle, but not *the* firmest principle as Aristotle wishes to establish. Thus, Wedin thinks that Aristotle completes IP with what he calls the "Ultimacy Claim," which is just the claim that PNC is *the* firmest principle. In order to do this, Wedin gives a defensible reading¹⁰² of 1005b 22-23, which has received critique from scholars, to which we now turn.

At the end of the Indubitability Proof, Aristotle says that due to the conclusions of IP: "...all those who demonstrate go back to this [PNC] opinion in the end: it is, in the nature of things, the principle of all the other axioms also" (1005b 33-36). So PNC is *a* firmest principle because it is impossible to believe negations of PNC, but PNC is *the* firmest principle because everyone who demonstrates "goes back" to PNC, and every principle goes back to PNC as well. There are two worries¹⁰³ here: (a) it is unclear that PNC is in every demonstration, which we might think if

¹⁰² My account of his defense is altered, but the basic and fundamentally important elements are present.

¹⁰³ This discussion is in Section 6 of Wedin (2004) or Pgs. 258 – 262. Wedin (2000) also is another source on these worries, but I keep my discussion to Wedin (2004).

every demonstration and principle "goes back" to PNC; (b) there seem to be other fundamental logical (firmest) principles, and it is unclear why PNC is *the* firmest. Addressing (a) first, consider the following two deductive¹⁰⁴ principles:

$$(MP) \quad \phi \to \psi$$
$$\phi$$
$$\therefore \psi$$
$$(ID) \quad \phi \to \phi$$

Modus Ponens (MP) does not have PNC present in the demonstration. So why think that, as Aristotle says: "...all those who demonstrate go back to this [PNC] opinion in the end"? Wedin thinks that:

...PNC is the doctrine that everyone who demonstrates goes back to in the end – not as the principle *from which* all deductions start, in which case it would be used in all deductions, but rather as a *presupposition* of the *validity* of the principles that are used in such deductions, namely, the principles of deductive reasoning [my emphasis] (261).

According to Wedin, Aristotle's claim makes sense if we think of PNC as the

"presupposition of the validity" of the principles of demonstration. That is, PNC isn't

present in MP as a premise in the deduction of ψ , but it is presupposed by the

principle. That is, if we assume the opposite of the conclusion or $\neg \psi$, then we can

deduce " $\psi \land \neg \psi$ " and only through PNC can we discharge the initial assumption and

¹⁰⁴ Wedin 260 – 261. Wedin assumes that PNC is solely a deductive principle, but I think one could argue with this very claim—anachronism? It could be an inductive principle (or both as it seems), not unlike the theorem $Pr(\Phi \& \sim \Phi) = 0$, which is derivable from Kolmogorov's (1933) original three axioms of probability theory (Ellery Eells (1991) Pg. 400 has a brief and useful account of the axioms of probability theory). This sort of idea would be kosher with Aristotle's valuing PNC as an ontological principle of things that are. I leave this aside here since this is not the central concern.

establish the conclusion¹⁰⁵. Similarly for the Law of Identity (ID): if we assume the opposite of ID or " $\neg(\phi \rightarrow \phi)$ ", then we end up with another contradiction " $\phi \land \neg \phi$ ", which can only be eliminated through PNC. So PNC can be seen here to be the presupposition of these principles' validity, and this gives meaning to (a). One who expressed worry (b) then would state here: well I see that PNC is presupposed by the principles of reasoning, but why think that it is *the* firmest principle, since there are equally firm principles out there (i.e. ID, law of excluded middle, etc.)? Wedin defends Aristotle by saying:

...the firmness attaching to PNC is *inherited* by all principles whose denials flout the principle of non-contradiction. Because these principles inherit their firmness *from* PNC and because PNC establishes its *own* firmness, he declares that it is the principle of all other principles. Thanks to its role in *explaining* the firmness of other principles, PNC can be declared *the* firmest principle of all (261).

So because all principles inherit their firmness from PNC since it explains their

firmness, and PNC establishes its own firmness, PNC is *the* firmest principle of all¹⁰⁶.

¹⁰⁵ Wedin uses Quine's conception of inconsistency in his exposition: "One schema implies another if and only if the one in conjunction with the other's negation is inconsistent" (Quine (1966, Pg. 100) qtd. in Wedin 2004, 260). I use Indirect Proof (IP or \perp elimination) here, since it looks like Wedin is using some combination of IP and Quine's inconsistency method. As far as this method is concerned, it doesn't seem to give Wedin what he wants with a transparent contradiction, at least if Quine is assuming some sort of truth table method.

¹⁰⁶ Assume with Wedin (2004) that the ultimacy claim "...effectively *completes* Aristotle's argument" [my emphases](259). If this is the case, then it appears that Aristotle completes IP at the end of Γ .6 where he says: "It has now been fully enough stated that the opinion that opposite assertions are not simultaneously true is the firmest of all, and what are the consequences for those who make this statement, and why they make it" (1011b 13 – 15, following Kirwan). In personal correspondence through email, Wedin disagrees with this change of the end of IP, but given the above, it is hard to see how he could. The result either way is not crucial to my main argument here and it is not my main purpose here to definitively

With this final addition, the Indubitability Proof is complete, and we can now move on to consider arguments that further support both PNC's placement in IP.

establish such a result, so I leave the reader to ponder the result for themselves, while I remain neutral in the text above and what follows concerning such a result.

Chapter III: The Seven Refutations

In Chapter II, which dealt with the "Indubitability Proof" that Aristotle gives in Γ .3, we were given a defense of the idea that contradictions cannot be believed. In this chapter we are given seven arguments, or as they are called by Aristotle, "refutations", which further support PNC's presence in IP. This appears to be a more fitting title because, as we will see, Aristotle is not giving demonstrations or arguments proper, but rather, is providing reasons that refute those who think that PNC is false. In chapter II, Aristotle's argumentative purposes in *Metaphysics* Γ were briefly covered. You'll recall that Aristotle is mostly concerned with the claim that it is possible to believe a contradiction (Wedin 2000, 113 – 4)¹⁰⁷. So even though IP centrally deals with this doxastic claim in Γ .3, the whole of his defense in the rest of *Metaphysics* (Γ) is mostly concerned with this proposition.

The first of the seven refutations is W1, and is the longest of the seven. W2-W7 are referred to as "shriek arguments" by Wedin, and are intended to derive an absurd or "shriek-worthy" consequence from the postulating of contradictions. These refutations together are meant to further demonstrate PNC's status as *the* firmest principle. This survey of the refutations will rely on the individuation, numbering and exegesis of the arguments in Michael V. Wedin (2000, 2003)¹⁰⁸. Although many translators and interpreters of Aristotle vary on how they

¹⁰⁷ In this chapter, Wedin (2000) will be assumed in all quotations of him, unless otherwise indicated.

¹⁰⁸ I make this explicit since there are alternative numberings of the arguments given by Łukasiewicz (1910), Ross (1924), and Kirwan (1993). See Appendix II at the end of the thesis for a complete listing of all individuations and numberings. Although the numbering and exegesis will be limited to Wedin (2000, 2003), I will use others' exegetical understanding where necessary.

individuate the arguments, most agree on the *number* of arguments he gives (with the exception of Łukasiewicz (1910), who counts five), which is counted at seven according to Ross (1924), Kirwan (1971), and Wedin (2000, 2003).

Before getting into the refutations, what kind of argument does Aristotle think that he can give against those who would deny the principle of noncontradiction? First, it is important to keep in mind that Aristotle clearly doesn't think that a proper demonstration¹⁰⁹ of it can be given in the sense of his *Posterior Analytics*: "Some, owing to a lack of training, actually ask that it [PNC] be demonstrated: for it lack of training not to recognize of which things demonstration ought to be sought, and of which not (1006a 5 – 7). Granting that he thinks he

...can, however, *demonstrate negatively* even that this view is impossible ["...it is possible for the same thing to be and not to be..." (1005b 34 - 1006a 2)] if our opponent will only *say something*; and if he says nothing, it is absurd to attempt to reason with one who will not reason about anything, in so far as he refuses to reason. *For such a man, as such, is seen already to be no better than a mere plant*¹¹⁰ [my emphases] (1006a 12-15).

Aristotle immediately goes on to explain that a "negative¹¹¹ demonstration", or as it

is also known as *elenctic* demonstration, is different than a proper demonstration,

¹⁰⁹ There is a side issue with a few scholars in this field of research of whether the *elenctic* demonstrations of PNC count as "knowledge". And whether Aristotle shifts his theory of knowledge from the Analytics to the Metaphysics. See T.H. Irwin (1977), Alan Code's (1986) response, and S. Marc Cohen's (1986) response to both. This issue is not dealt with here, and the result either way seems to be innocuous for what I have to say.

¹¹⁰ Christopher Kirwan's (1993) translation of this reads more dramatically substituting "vegetable" for W.D. Ross's (1924) "plant".

¹¹¹ "When Socrates uses the elenchus, he gets his opponent to refute himself out of his own mouth" (Gottlieb 2015). Aristotle is clearly trying to do much the same thing here.

because he thinks if you were to demonstrate PNC you could be accused of begging the question. So as long as someone else does the assuming, or as he says above "says something", we have a negative demonstration or proof, and a defense of PNC is given¹¹². The person who would *say* or *assume* nothing about PNC would be, via *ad hominem*, no better than a mere "plant" or "vegetable" (cf. Kirwan) in that they would be denying PNC, but not open to reasoning about it, or at least not through the only method Aristotle thought was available (i.e. elenctic proof.).

Given this preliminary discussion of negative proofs, one might still wonder about the difference between proofs proper and elenctic proofs, and any issues that may arise. Wedin further defines the important *pragmatic* difference between these:

...[A]n elenctic proof purports to establish something directly, say *q*, and it does so by finding something else, *p*, such that *p* is the case and *p* entails *q*. But the proof is aimed at an opponent of *q*, and *so it is critical that the opponent accept the truth of p*. Aristotle puts this by requiring that the opponent accept the truth of *p*. Otherwise, he says, one might be open to the charge of begging the question (my emphases) (127).

As we learned above, as long as the opponent of PNC says something (i.e. accepts the truth of *p*), then we can get an elenctic demonstration of PNC. But why do we have an elenctic demonstration when the opponent (i.e. the person who "says something") commits *petitio principii* by asserting *p*, but not when the demonstrator does so? How is there any difference here? Łukasiewicz (1910) noted this problem as well: "Whoever wants to demonstrate the law of contradiction [=PNC] commits the fallacy of *petitio principii* and the demonstration is false. If, however, another is

¹¹² Going back to Code (1986) he thinks that with each refutation of PNC, Aristotle establishes (what Priest calls a *perinomic* fact) something *about* PNC (i.e. "...it must be believed, or that it is a necessary condition for thought" (Priest 100-101)).

guilty of making this mistake, then an elenchus is possible—and everything is in order. I cannot grasp what is being said here" [my emphasis] (495). So the issue is how is begging the question on the part of the opponent different from on the part of the demonstrator? Why do we get a successful demonstration with the one but not the other?

In order to get an answer to these questions Wedin takes the fallacy of *petitio principii* or "…begging the question…[to be] relative to beliefs and believers" (129). He uses David Sanford's understanding¹¹³ of the fallacy to get across this idea: "On his [Sanford's] view the fallacy occurs whenever one must believe that the conclusion of an argument is true, in order to believe of a premiss, or a conjunct of a premiss, that it is true" (128). Let's use the following argument¹¹⁴ as an example:

All the members of the club attended the University of Texas.

Twardowski is a member of the club.

(\therefore) Twardowski attended the University of Texas.

So a person *x* begs the question in Sanford's sense, if *x* were to believe the first premise is true just because he believes the conclusion is true, and because *x* knows the club's six members and that they attended the University of Texas (129). If premise one were believed by *x* through an outside source or evidence, then *x* would not be begging the question. It is in this sense that Wedin thinks that Aristotle might have this understanding of begging the question in mind in *Metaphysics* Γ , since begging the question here is relative to "beliefs and believers". Begging the

¹¹³ David H. Sanford's *Begging the Question as Involving Actual Belief and Inconceivable Without It* (1988) Pgs. 32 – 7.

¹¹⁴ Wedin 127 – 128. Reproduced here with just a name change in the explanation afterwards.

question's relativity to beliefs is clear from above: *x*'s belief that does beg the question reasons from the conclusion to the truth of premise one. Begging the question's relativity to believers is understood by thinking about who is responsible for a belief: demonstrator or bystander/opponent. Someone who was listening to *x* would clearly not be responsible for the *petitio principii*. So to return to the initial issue noted above: just as *x* doesn't commit *petitio principii* by believing the first premise because of some outside source or evidence, the demonstrator in elenctic proofs doesn't beg the question by having the opponent assert and "say something significant". The opponent is *responsible* for the priof and thus, the *petitio principii* is relative to him (1006a 25 – 27). And the belief is held by him after saying something *significant* and thus, is relative to him (1006a 19 – 24). This conception, or something like it, seems to be what Aristotle is after with the idea that there are no questions begged, if an opponent, not the demonstrator, is responsible for the assumption.

W1. The 1st Refutation: Signification and Substance

The first refutation (W1) in *Metaphysics* Γ , spans from 1006a 28 – 1007b 18, and has three parts according to Wedin (2000). In the following introductory passage, Aristotle lays out the two cases that he will deal with in the first elenctic proof:

(1)¹¹⁵ If (a) a name signifies being or not being something, then (b) it could not be the case that something was and was not so and so¹¹⁶.

¹¹⁵ My premise or claim numbers, in what follows, are not those of Wedin's.
¹¹⁶ Wedin (2000), P. 133; Wedin's Greek is taken out. As Wedin (2000) indicates, in
1006a 28 – 31 Aristotle actually uses the universal quantifier 'everything' rather
than the particular or existential quantifier 'something'. But as Wedin points out, the

In the first part (W1.I: 1006a 31 – 1006b 34), Aristotle considers a case where a name signifies being. In the second part (W1.II: 1006b 34 – 1007a 20), he considers an opposite case where a name signifies not being. And finally, in the third part (W1.III: 1007a 20 – 1007b 18), which isn't mentioned in this passage, he offers support for the results of these two cases with an argument concerning substance.

W1.I is dealt with in three stages according to Wedin. In the first stage the goal is to show that "a name signifies one thing" (Wedin 134). Aristotle says the following:

...if 'man' signifies one thing, let it be *two-footed animal*. By signifying one thing I mean this: if that thing (that is signified) is man, then if anything is a man, then that thing (that is signified) will be what it is to be a man (1006a 31 - 34)¹¹⁷.

Wedin takes the preceding passage to claim the following:

(2) 'M' signifies one thing, T = (x) (x is $M \rightarrow T$ is what it is to be x)¹¹⁸.

So 'man' signifies one thing, 'two-footed animal', if and only if for all *x*, if *x* is a 'man'

then 'two-footed animal' is what it is to be *x*. Wedin takes signification here to be

"modally laden" (135). That is, whatever is a man has the essence of being two-

footed. Aristotle continues in order to defend (2):

But it makes no difference even if someone were to assert that it [man] signified more than one thing, provided that these were definite; for a different name could be assigned to each formula. (I mean, for instance, if someone were to assert that 'man' signified not one but several things, of one

elenctic proof is set to refute the weak denial of PNC (i.e. *something* is so and not so) rather than the strong denial of PNC (i.e. *everything* is so and not so) in 1006b 33 – 4 (133, footnote 37). Wedin offers further textual evidence for this exegetical move (134, footnote 40). This quote is also a rough translation from Kirwan's original other than the quantifier.

¹¹⁷ This is another Wedin translation.

¹¹⁸ Wedin (2000), Pg. 135.

of which the formula was 'two-footed animal', but there was more than one other as well, but a definite number; for a distinct name could be assigned in respect of each of the formulae.) But if, instead of so assigning, he were to assert that it signified infinitely many things, it is obvious that there would be no statement. For not to signify one thing is to signify nothing, and if names do not signify, discussion is eliminated with others; and, in truth, even with oneself, since it is not possible even to conceive if one is not conceiving one thing and, if it is possible, one name could be assigned to that actual thing. Let the name, then, as was said originally, signify something and signify one thing (1006a 34 – 1006b 13).

Aristotle begins then by giving us an understanding of what he means by "signifying one thing". He then tells us that, even if someone were to say that man means more than one thing, we can assign a name to each definition or *formula*, if there are a definite number of meanings. If someone did not take this route and asserted that 'man' (or any other term) has an infinite number of meanings, then, he tells us, there is "no statement". Further, he tells us that if one does not signify one thing, then they signify nothing, and discussion is eliminated with others and oneself. Being careful he tells us that if it is possible to conceive while conceiving more than one thing, then initial idea that a name signifies one thing¹¹⁹.

How can we make sense of the claim that there will be "no statement" in the preceding scenario? Assuming Aristotle is referring to his conception of statement making sentences (SMS) that we went over in chapter 1 (cf. P. 14), then there is no statement when one says 'man' has an infinite number of meanings or formulae for

¹¹⁹ The Ross translation makes Aristotle's idea clearer in my view: "Let it be *assumed* then, as was said at the beginning, that the name has a meaning and has one meaning..." [my emphasis] (1006b 11 – 12). In other words, let's assume that a name signifies one thing, and see what follows. He doesn't think he's giving a knock down argument for the idea that names can only signify one thing in order for there to be meaning.

a few reasons. Since we know SMS's are statements that are bivalent (are true or false), then it is hard to see how there could be a statement, if one did not assign one meaning to 'man', but instead insisted that it had several without restriction. Further, we know that SMS's are either affirmations or negations; it is difficult to understand how it could be one or the other of these, if the meanings were unlimited¹²⁰.

After establishing that a name signifies one thing in stage one of W1.I, in

stage two Aristotle seeks to show that a name and its contradictory opposite cannot

signify the same thing (e.g. horse and not horse cannot signify the same thing) (134).

He continues:

Then it is not possible that 'what it is to be a man' signifies precisely what is signified by 'what it is not to be a man', if 'man' signifies not only about one thing but also signifies one thing (1006b 13 - 15).

Wedin represents this passage as claiming the following:

(3) (e) 'N' signifies one thing \rightarrow (f) $\neg \Diamond$ ('what it is to be N' and 'what it is not to be N' signify the same thing)¹²¹

Aristotle thinks that (3) is a consequence of stage one of W1.I, where claim (2) sets

out what it means to signify one thing (138). Aristotle continues:

And it will not be [possible] to be and not to be the same thing unless homonymously, as if others were to term not-man what we term man¹²². But

¹²⁰ Stage one of W1.I harkens back to Aristotle's elenctic proof strategy. That is, Aristotle says that the opponent of PNC should "say something" and this something should "signify something both to himself and to someone else" (1006a 11 – 23). It is clear that he takes signification, even if his arguments don't make it certain or necessary, to mean signifying one thing. This assumption is right in line with the respect or *R* qualification in SARTE, which reaffirms the idea that Aristotle is defending a specialized and specific conception of PNC. ¹²¹ Wedin (2000), Pg. 137.

what is found perplexing is not whether it is possible that the same thing should simultaneously be and not be a man in name, but in *actual fact* (1006b 19 - 23).

Aristotle thinks that it is impossible for anything to be and not be at the same time,

unless it is by means of a homonymous relation (i.e. same name, but different

definition—where condition R in SARTE is not met; cf. Pg. 15 – 16 here). Also,

Aristotle harkens back to the ontological PNC, with which he is concerned. That is, it

is *perplexing* that *x* could be a man and not a man simultaneously in the world (i.e.

both could signify the same thing simultaneously).

Arguing against this idea that man and not-man can signify the same thing

simultaneously, Aristotle continues:

But if 'man' and 'not-man' do not signify something different, it is plain that [neither] does 'not to be a man' from 'to be a man', so that to be a man will be to be a not-man; for they will be one thing... But if they are one thing, 'to be a man' and 'not-man' signify one thing. But it had been shown that they signify something different (1006b 22 - 29).

Wedin frames this passage in the following premises, which include places in the text, from which they are taken:

- (4) (g) 'N' and 'not-N' signify the same \rightarrow (i) 'what it is to be N' and 'what it is not to be N' will have the same definition (1006b 24 5).
- (5) (i) 'what it is to be N' and 'what it is not to be N' will have the same definition \rightarrow (j) *what it is to be N* and *what it is to be not-N* will be the same thing (1006b 25).
- (6) (j) what it is to be N and what it is to be not-N will be the same thing →
 (h) 'what it is to be N' and 'what it is not to be N' signify the same thing (1006b27 8).
- (.:.) (g) 'N' and 'not-N' signify the same \rightarrow (h) 'what it is to be N' and 'what it is not to be N' signify the same thing (1006b 22 5)¹²³.

¹²² Ross provides a good contrast in translation of 1006b 19 – 21: "And it will not be possible for the same thing to be and not to be, *except in virtue of an ambiguity*, just as one whom we call 'man,' others might call 'not-man'..." [my emphasis].
¹²³ Wedin (2000), Pg. 137.

Premises (4), (5), and (6) give us the logical consequence in the conclusion. In the very last sentence in the passage above, Aristotle tells us: "...it had been shown that they ['to be a man' and 'to be a not-man'] signify something different". Wedin takes Aristotle to mean that (h) has been shown to be false through (3), which followed from stage one of W1.I (138). That is, (3) tells us: (e) 'N' signifies one thing \rightarrow (f) $\neg \diamond$ ('what it is to be N' and 'what it is not to be N' signify the same thing). In the case where 'N' signifies one thing, we are told that it is impossible that 'what it is to be N' and 'what it is not to be N' and (3) only establishes (h) in an "attenuated" sense, since (j) in (6) involves the case of two things (i.e. what it is to be N and what it is to be not-N) and (g) also involves the case of two things (i.e. 'N' and 'not-N') (138). But (3) does give us strong reason to reject (h), and thereby reject (g)—N and not-N do not signify the same thing.

Finally, after showing why a name signifies one thing in stage one, and that a name and its contradictory opposite do not signify the same thing in stage two, in stage three Aristotle seeks to prove the principle of non-contradiction. To conclude the first case of the elenctic proof in W1.I, Aristotle says:

It is accordingly necessary, (α) if it is true of anything to say that it is a man, that it be a two-footed animal (for that was what 'man' signified); and (β) if that is necessary, it is not possible that the same thing should not be at that time, a two-footed animal . . . Consequently, (γ) it is not possible that it should be simultaneously true to say that the same thing is a man and is not a man (1006b 28 – 34).

As Wedin says, this argument proves PNC for the instance of 'man', but it is "meant to be general in effect" applying to more than just 'man' (139). We may put the claims in this passage as follows¹²⁴:

(7) 'M' signifies $T \rightarrow (x)(Mx \rightarrow Tx)$ (8) $(x)(Mx \rightarrow Tx) \rightarrow \neg (\exists x)(Mx \land \Diamond \neg Tx)$ (9) $\neg (\exists x)(Mx \land \Diamond \neg Tx) \rightarrow \neg (\exists x)(Mx \land \Diamond \neg Mx)$ (10) $\neg (\exists x)(Mx \land \Diamond \neg Mx) \rightarrow \neg \Diamond (\exists x)(Mx \land \neg Mx)$ (11) $\neg (\exists x)(Mx \land \Diamond \neg Tx) \rightarrow \neg \Diamond (\exists x)(Mx \land \neg Mx)$ (12) $\neg \Diamond (\exists x)(Mx \land \neg Mx)$

The first claim (7) is drawn from (α) in the original passage above. The second claim is drawn from (β) in the passage above. Claims (9) and (10) are additional enthymemes that are needed to deduce (11), and (12), which is derived from (γ). The conclusion to W1.I begins with (7), which tells us that if man signifies one thing (recall stage one of W1.I), two-footed animal, then for all *x*, if *x* is a man, then necessarily *x* is a two footed animal. Claim (8) continues the argument by adding that if the consequent of (7) is true, then it is not possible that "the same thing" (i.e. man) is not a two-footed animal. Granting that the interlocutor initially assumes that *M* signifies *T*, we get the consequent of (7), which allows us to deduce the

¹²⁴ This is the "narrow scope" reading of the end of this proof, but there is the "wide scope" reading, which won't be discussed here (cf Pg. 139 in Wedin (2000)). Kirwan thinks that the wide scope reading succeeds in retaining the generality of the principle that looks to be restricted to essential predication in W1. As Wedin (2000) notes, the wide scope version has the same difficulty as the narrow scope version (141). Wedin also defends an extended version of PNC as present in W1, which covers "colored individuals" (141 – 148). I think Wedin (2000) is right to insist that Aristotle didn't maintain a restricted PNC, and the textual evidence shows that to be true whether one looks, as he says, at Gamma 5's arguments or any other part of *Metaphysics* Gamma (142). Priest (1998) spends some time arguing for this, and he rightly does so. Aristotle doesn't think that PNC just applies to substances, but any ontological entity, as anyone who carefully studies *Metaphysics* Gamma can see for themselves.
consequent of (11) and thus, an instance of PNC in (12). This concludes the first case in the first elenctic proof or W1.I.

We now turn to the second case¹²⁵ of the first elenctic proof. In the first case of the first elenctic proof or W1.I, Aristotle deals with the case where a name signifies being something. In the second case of the first elenctic proof or W1.II, Aristotle discusses the case where a name signifies *not* being something. He tells us:

The same argument applies also in the case of not being a man. For 'to be a man' and 'to be a not-man' signify something different, if even being pale and being a man are different. For the former is much more strongly opposed, so that it signifies something different. But if [the disputant] asserts that 'pale' signifies one and the same thing too, we shall repeat just what was stated before also, that everything, and not only opposites¹²⁶, will be one (1006b 34 – 1007a 7).

First, Aristotle says that the same argument from W1.I and the case of where a name signifies being something applies here in the case where a name signifies not being something (i.e. man). By this he could mean that substituting not-man in the place of 'man' in W1.I will give us the same result in (12) above. Next, he tells us that contradictories such as 'man' and 'not-man' are more strongly opposed and signify something different than the pair 'being pale' and 'being a man'. In the latter case a man could be pale, which would mean that the predicates signify the same thing, but in the former case, being a man and being a not-man each signify something

¹²⁵ Wedin (2000) doesn't go over parts II, and III of W1 in his analysis. Thus, interpretations offered here do not complement his or add to his. Similarly, only in the cases of W4 and W5 of the six shriek arguments do I utilize Wedin's specific exegeses. His understanding of shriek arguments is also utilized.

¹²⁶ The four Aristotelian classes of opposites (i.e. relatives, contraries, privation and possession, and affirmation and negation) are to be found in *Categories* Chapters 10 & 11 or 11b 15 – 14a 25.

different. But if, as Aristotle says, someone were to say that 'pale' signifies one and the same thing as 'man', then everything will be *one*.

It is useful to recall an earlier claim at this point: (3) (e) 'N' signifies one thing \rightarrow (f) \neg ('what it is to be N' and 'what it is not to be N' signify the same thing). A reason why W1.II is important is because it offers insights on a variant of this claim, where *N* is not the only name involved and the other name is not just the negation of the original one—variants and cases, which are neither covered under (3), nor are *opposites*. The reason why Aristotle adds "not only opposites" to his final claim in the passage above is because the opposition of "affirmation and negation" was the focus of W1.I, and now he is concerned with names and relations outside of those.

The case under consideration is where a disputant claims that *pale* and *man* signify one and the same thing. In this case it won't help to revert back to Aristotle's meaning of signifying one thing, and substituting *P* (i.e. 'pale) for *M* in the original formulation: (2) 'P' signifies one thing, T = (x) (x is $P \rightarrow T$ is what it is to be x). Assuming that *P* in fact signifies *T*, then the claim is a true biconditional. But clearly *pale* does not signify the same thing that *man* does. And this seems to be the point for Aristotle. Assuming that it does, then it becomes unclear how we are to accept any intuitive or objective method for signification, and Aristotle claims that all things will be *one*. It seems Aristotle means that if signification is not principled, then opposites, and any other relations and names (e.g. 'man' and 'pale') signify the same thing (i.e. are one) or could be in truth equally applied to anything. If, even at the fundamental level, *man* and *pale* do not signify something different, then it

seems that we cannot say that any member of an opposition or relation signifies some one thing. If this is impossible, then Aristotle tells us that:

...[W]hat we have stated follows, if he will answer the question asked [Is it a man?]. But if, asked the question baldly, he appends the denials also, he is not answering the question asked. For nothing prevents the same thing being both a man and pale and a thousand other things; nevertheless, if one is asked whether it is true to say that this thing is a man or not, the answer ought to signify one thing, not append that it is also pale and tall. For it is certainly impossible to go right through the coincidentals of a thing, which are infinite; so let him go through either all or none. So equally, even if the same thing is a thousand times a man and not a man, one ought not to append, to one's answer to the question whether it is a man, that it is simultaneously not a man also; unless one is to append all the other things too which coincide in it, the things that it is or is not. But if one does that, there is no discussion [my emphasis] (1007a 7 – 1007a20).

If the opponent answers the question "Is it a man?", then the result that it is impossible that the same thing is a man and not a man at the same time. Answering the question amounts to the opponent signifying one thing in the sense of (2), and not adding other coincidentals. Further, if the thing is a thousand times man and not man, then Aristotle tells us, one ought not add to one's answer that it is also not a man simultaneously, unless we are to add all the other things which are coincidental properties of it. Aristotle concludes that if one does this, then there is no discussion. There are two basic routes the opponent may take here. *Route 1*: answer (in the sense given above) the question "Is it a man?" by saying, "It is not a man" after one has already answered. If one does this, then one has to add the coincidental properties and there is no discussion to be had. *Route 2*: answer the question "Is it a man?" by answering and not adding what the opponent does in route 1. There is no discussion to be had if the opponent takes route 1, because they will be listing an infinite number of coincidental properties, and more importantly, because it is

difficult, if not downright impossible, to have a serious discussion with someone

who wants to assert that man and not man are the same thing. For these reasons,

Aristotle concludes for case two of W1.I that it is impossible that the same thing is a

man and not a man at the same time.

Finally, W1.III, Wedin takes Aristotle to be giving an argument for W1.I and

W1.II, which concerns the ineliminability of essence. Aristotle says the following in

the first line of argumentation:

Those who say this [1007a 17 - 20] entirely eliminate substance and what it is to be. For it is necessary for them to maintain that all things are coincidences and that there is no such thing as just what to be a man or to be an animal [is]. For if anything is just what to be a man [is], that will not be to be a not-man or not to be a man: yet those are its denials. For what it signified was one thing, and that was something's substance, and to signify a thing's substance is to signify that, for it, to *be* is nothing else. But if, for it, just what to be a man [is] should be either just what to be a not-man [is] or just what not to be a man [is], it *will* be something else; so that it is necessary for them to say that this kind of formula applies to nothing, and that all things are coincidentally (1007a 20 – 31).

Continuing from the thoughts of W1.II Aristotle tells us that those who take "route

1" above "entirely eliminate substance and what it is to be". The commitments of this route entail that *all* things are coincidences and that substances do not exist (cf. Pg. 15, footnote 24). 'What it is to be a man' is not the same thing as the denials of *this* (i.e. to be a not-man or not to be a man). If for any *x*, 'to be a man' is the same as 'to be a not-man' or 'to not be a man', then Aristotle tells us, *x* will be something

else—something coincidental. He continues:

But if everything is said coincidentally, there will not be anything which things are initially about, if 'coincidental' always signifies a predication about a certain subject. Consequently it will be necessary to go on to infinity. But that is impossible, for not even more than two combine; for the coincidental is not coincidental in the coincidental, unless because both coincide in the same thing—I mean for instance that the pale may be artistic and the latter pale because both coincide in a man. But Socrates is not artistic in that way—that both coincide in some other thing. Accordingly, since some things are called coincidental in the latter way, some in the former, those so called in the latter, as the pale is in Socrates, cannot be an infinite upward series, e.g. some other thing coincidental in Socrates the pale; for not everything makes up some one thing. Nor indeed will there be any other thing coincidental in the pale, as for instance the artistic; for the latter no more coincides in the former than the former in the latter, and at the same time there is a distinction between things that coincide in this way and things that coincide as the artistic in Socrates. In none of the latter cases does the coincidental coincide in something coincidental, but it does in all the former cases; so that not everything will be said coincidentally. Consequently, there will be something signifying substance even in such a case. And if that is so, it has been shown that it is impossible to predicate contradictories simultaneously. (1007a 33 - 1007b 18).

Assuming that "everything is said coincidentally" and something that is coincidental signifies a "predication *about a subject*", then there will *not* be a subject about which these coincidental predications apply and one will have to list coincidental predications infinitely. This "infinite series of predication" results due to the conception of "coincidental," which is about a *subject*. One has to list these predications infinitely since there is no subject that they are about, but they, so to speak, *need* a subject. But this is impossible, Aristotle says, because "not even more than two combine". Aristotle seems to mean that in coincidental predications, there doesn't seem to be a way in which coincidental predications above two could be true at the same time, if they are not about a subject. That is: musical is coincidental in talented, because both are coincidental of Robert. But musical could not be coincidental in talented, without a given subject.

There are two ways to be coincidental, one of which is where (C₁) both properties coincide in the same thing, and one coincidental property is coincidental

in something else coincidental. Aristotle gives the example of the pale and the artistic both coinciding in the same man. The pale could be artistic, and vice versa, because both inhere in the same man. The second way to be coincidental (C_2) is where a property/s do not coincide in some other thing, and the coincidental is not coincidental to something else coincidental. For example, Josh is sad. Both of these do not coincide in some other subject. One (sadness) coincides in the other (Josh). Those things coincidental in sense (C_2) , Aristotle says, do not require one to infinitely list coincidental predications, because they have a subject in which to coincide, so that the coincidental is not coincidental to something else coincidental. Given this, Aristotle says: "...not everything will be said coincidentally". Even in the (C₁) first sense of coincidental, there is something signifying substance, Aristotle tells us. To get this result we must assume his definition of coincidental above (i.e. "predication about a subject"). If these things are true then Aristotle concludes: "..it has been shown that it is impossible to predicate contradictories simultaneously" (1007b 17–18). And with this conclusion, W1 is finished.

W2. The 2nd Refutation¹²⁷: Everything Will be One¹²⁸

The first refutation is the longest of the seven, but the next six refutations proceed somewhat differently and Wedin calls them "shriek" arguments (157). They are called shriek arguments because they are meant to establish an impossible or shriek-worthy consequence, which results from the assertion of a contradiction by

¹²⁷ Ross (1924) interprets the 2nd refutation much the same as I do here.
¹²⁸ The titles of W2 – W7 are drawn from Wedin (2000) Pg. 159.

an opponent. Wedin takes the final six arguments (W2 – W7) to take the following form¹²⁹:

$$(1) \neg PNC \supset q$$

(2) !q
(..) !¬PNC

The opponent or person that 'says something significant' denies PNC (1), and through doing so, encounters an impossible or shriek-worthy result (*q*). This impossible consequence leads them to think *q* is shriek-worthy, impossible or absurd (!q)¹³⁰, and this impossibility and absurdity *transfers* also to the proposition that entails it ($!\neg$ *PNC*) (158). The transfer of awkwardness or absurdity

...leaves *truth values* unaffected; in particular, it does not give us the negation of \neg PNC. In effect, Aristotle says to his opponent, perhaps more for the benefit of his friendly auditors, 'See what embarrassments you have brought upon your house by this brazen denial of the principle of principles!' Aristotle may hope that this has persuasive effect but he does *not* suggest that the embarrassments are grounds for denying the negation of PNC [my emphasis] (Wedin 158 – 159).

This is an important point to keep in mind as we go through the next six refutations. Aristotle sets up these arguments with the goal of showing the absurdity of denying the principle of non-contradiction. But in demonstrating this awkwardness or absurdity Aristotle knows he is not giving definitive proof of PNC, as we learned at the beginning of the chapter when Aristotle says that PNC is indemonstrable. In transfer of awkwardness from what is entailed to what entailed it, "truth value is unaffected", as Wedin tells us above. An awkward or seemingly impossible

¹²⁹ Wedin (2000), Pg. 157.

¹³⁰ Wedin takes "!" to be the shriek sign.

consequence is not the same thing as a false consequence. That is the difference

between a demonstration proper, and a refutation.

We may move on to the second refutation in W2¹³¹. Aristotle tells us:

Again, if contradictories are all simultaneously true of the same thing, it is plain that everything will be one. For the same thing will be both a warship and a wall and a man, if it is possible either to affirm or to deny something of everything, as is necessary for those who state the thesis of Protagoras. For if a man is thought by someone not to be a warship, it is plain that he is not a warship; so that he also *is*, if the contradiction is really true. Indeed we also get the doctrine of Anaxagoras, that 'every article is mixed together'; so that nothing is truly one. These people seem, therefore, to be stating something indefinite; and while they consider that they are stating that which *is*, their statement is actually concerning that which is *not* (for the indefinite is what *is* potentially and *not* in complete reality). (1007b 18 – 29).

The thesis of interest for Aristotle is a consequence he thinks follows from

Protagoras' views: (P) It is possible either to affirm or to deny something of

everything. Aristotle grants this consequence and shows the absurdities that follow.

For example: (a) the same thing can be a warship, wall, and man, and (b) the same

thing can be a warship and not a warship simultaneously. Given *P*, if it is possible to

affirm or deny something of everything, then clearly one can affirm 'warship', 'wall',

and 'man' of the same thing. This thesis does not limit possible applications of it to

ones we would call valid (i.e. "this is a man, because it has all the features of such a

being"). Without such a limitation of the principle, then contradictories can be true

of the same thing. That is, everything would be "one" because there is nothing to

conceptually differentiate anything from anything else, if all contradictories are true

¹³¹ For a more extensive look at this refutation, see Wedin's *A Curious Turn in Metaphysics Gamma: Protagoras and Strong Denial of the Principle of Non-Contradiction* (2003), which is listed in the bibliography here. There he strongly shows that Protagoras' views entail the sorts of claims that Aristotle attributes to him in *Metaphysics* Gamma.

of something. As a final addition to this argument, Aristotle brings in the idea of Anaxagoras that "every article is mixed together", which implies that "nothing is truly one". Aristotle counters this attack by saying that people who follow Protagoras and Anaxagoras are speaking of the indefinite or that which is *not* or that which is only *potentially*¹³². A human being can potentially be a man or a woman, but not *actually*. He continues:

On the other hand their statements, *at least,* must affirm or deny everything of everything; for it would be absurd if the denial of *itself* held good of each thing, but the denial of some other thing, which does not hold good of it, did *not* hold good of it. I mean for instance that if it is true to say of a man that he is not a man, plainly he is also either a warship or not a warship. So if the affirmation holds good of him, necessarily its denial does too. But if the affirmation does not hold good, at least *its* denial will hold good of him more readily than his own. So if even the latter does hold good, that of warship will too; and if it does, its affirmation will too. This is one consequence, then, for those who state this [Protagorean] thesis... [my emphasis] (1007b 29 – 1008a 2).

Aristotle first considered the weaker thesis P, which he attributed to Protagoras and company. Now he thinks that their views entail *at least* the following: (P^*) *It is possible to affirm or deny everything of everything.* This follows, for Aristotle, because if we can get the result that the affirmation and denial of Φ holds good of x, then why not the affirmation and denial of any other thing, especially when they definitely do not hold good of x? If at the basic level even the predicate of man can be contradictorily applied to a man, then why not everything else? Without a principled mode of signification that excludes contradictions as understood in our first chapter

¹³² For more on Aristotle views on Anaxagoras see his *Physics*, Book I, Ch. 5. This is also a good place to learn about his views on the *potential* and *actuality* (cf. Books I – III).

and W1, then it is hard to see how on such a view as *P**, one can escape the charge that all contradictions are true of everything¹³³. To return to the initial claim, everything will be one because if all contradictions hold good of something or everything, then everything will be conceptually indistinguishable¹³⁴. With such a shriek-worthy and absurd consequence of views like Protagoras and others, Aristotle's interlocutors (i.e. those who "say something") are likely then to think absurd that which implied the consequence (i.e. the negation of PNC).

W3. The 3rd Refutation: That the Law of Excluded (LEM) Middle Will Fail

The third refutation concerns the law of excluded middle and how it will fail

given that one denies PNC. Aristotle tells us:

...Those, then, who maintain this view...are driven...to the further conclusion that it is not necessary either to assert or deny. For if it is true that a thing is a man and not-man, evidently also it will be neither man nor not-man. For to the two assertions there answer two negations. And if the former is treated as a single proposition compounded out of two, the latter also is a single proposition opposite to the former (1008a 2 – 7)¹³⁵.

So we are told if the proposition that 'x is a man and x not a man is true', then so is

the proposition 'x is neither a man nor not a man'. We are then told that the former

proposition is compounded out of two statement making sentences (SMS), and the

¹³³ This seems to be the rough formulation of *ex contradictione quodlibet* (ECQ), as discussed in the introduction, which is often associated with arguments for PNC, and has nowadays come to be understood the rule of inference Contradiction Elimination (\perp Elim). See Pg. 9.

 $^{^{134}}$ This claim strangely enough gets clarified in W4, when Aristotle reasserts it (1008a 23 – 8).

 $^{^{135}}$ I prefer Ross's translation. Here is Kirwan's for the contrast: ...[A]nother [consequence of denying PNC] is that it is not necessary either to assert or deny. For if it is true that he is a man and not a man, plainly also he will be neither a man nor not a man; for the two have two denials, and, if both make up the one former, there must also be the one latter opposed to it (1008a 2 – 7)

latter is an opposite assertion to the former. Aristotle then concludes that it is not necessary to assert or deny. Recall the thesis *P** which Aristotle attributes to Protagoras: *it is possible to affirm or deny everything of everything*. Well, if this is true, then something like 'x is a man and x is not a man' can result for all x. And the opposite proposition to this is 'x is neither a man nor not a man', which can be asserted given *P**. Well if for all x, we can either affirm that it is a contradiction, which tells us nothing about what it is, or we can affirm its opposite, which tells us nothing as well, then it seems altogether pointless or non-necessary to assert or deny anything. And thus, the law of excluded middle (LEM), which is that for any proposition, it or its negation is true, fails because it is not necessary to affirm or deny that either of the above propositions are true. Through a strong denial of PNC we reach the shriek-worthy result that asserting and denying are pointless, and as in the previous refutation, this shriek-worthiness and absurdity transfers to that which entails the absurd result (i.e. the negation of PNC).

W4. The 4th Refutation: That It Will Be Impossible to Truly Assert Anything

The fourth refutation continues with the theme of assertion and denial from W3, and says that it will be impossible to truly assert anything. Aristotle tells us the following:

Again either $[\Delta]$ this [presumably ~PNC] is so in every case, i.e. a thing is both pale and not pale, both a thing-that-is and not a thing-that-is, and in a similar way for all other assertions and denials; or $[\Delta_2]$ it is so in some cases but not in others. If it is not so in all cases, these would be agreed [upon]. But if it is so in all, then in turn either [a] anything asserted may also be denied and anything denied also asserted; or [b] the things asserted may also be denied but not everything denied also asserted. But if the latter, something would be securely *not* a thing-that-is, and that opinion would be firm; and if not to be is something firm and certain, the opposite assertion would be still more certain. But if anything denied may equally be asserted too, necessarily it is either true to state separately, for instance, that a thing is pale and again that it is not pale, or not. If it is not true to state separately, then not only does he not state these things but nothing whatever is—and how can things-that-are-not walk and talk? Also, everything would be one, as we said before [1007b 20 {i.e. W2}], and a man and a god and a warship and their contradictories will be the same thing; for if it applies equally to each thing, nothing will differ from anything else, since if it did differ, *that* would be true and distinctive [my emphases] (1008a 7 – 28).

Aristotle begins by sketching two cases: (Δ) either ~PNC is true in all cases; or (Δ_2) ~PNC is true in some cases but not in others. Beginning with the second case (Δ_2), if PNC not true in all cases then "the exceptions will be agreed upon"¹³⁶. If (Δ_2) is the case then we reach the shriek-worthy conclusion that there are exceptions to the PNC. The first case (Δ) takes up more space and has two cases within it. If it is so in all, he tells us, then either (a) everything asserted may also be denied and vice versa, or (b) everything asserted may also be denied and *not* everything denied is asserted. If (b) is the case within the first case (Δ), then Aristotle tells us that we will have something that is "not a thing-that-is". And if we know something is definitively not, then we would surely know when that something is. But if (a) is the case within (Δ), then Aristotle tells us that it is either true or untrue of any given compound statement Ψ to state each statement making sentence (SMS) separately. If it is not true to state the SMS's separately, then the opponent is not really stating them when he asserts both parts of a contradiction separately, and further, nothing is (i.e. exists). That is, not only is the opponent wrong in stating the parts separately, but also assuming that their mode of analysis (~PNC is true in all cases) is what we have

¹³⁶ This phrase comes from the Ross translation of 1008a 11.

to understand the world, then we can't say that anything exists—we can't truly assert anything, if our every assertion is coupled with a denial of that assertion. Lastly, everything is *one*, since if everything can be asserted and denied of everything, then everything is conceptually one. In this scenario there will be nothing to distinguish anything from anything else, so everything will be conceptually one. He continues:

Equally, even if it is possible to have the truth in stating things separately, the result we have stated follows; and in addition it follows that everyone would have the truth and everyone would be in error, and [the disputant] himself is in error by his own admission. At the same time it is obvious that in response to this person there is nothing for an investigation to deal with; for he says nothing. For he says neither that it is so-and-so nor that it is not so-and-so; and again he also *denies* both these, saying that it is neither so-and-so nor not so-and-so. For if he did not, something would already be *definite*. [my emphases] (1008a 28 – 34).

Continuing from above, even if stating the SMS's separately gives us truth, we are still left with the problem that nothing exists since each affirmation is canceled by a negation. Further, assuming the first case (Δ) is true, then "everyone would have the truth and everyone would be in error". Also, the initial requirement of elenctic proofs is for the opponent to "say something significant". But if the opponent says that the negation of PNC is true in all cases, hence (Δ), then it seems difficult to understand how they would say something meaningful or significant—or as Aristotle says, they "say nothing". Once again we are left with the result that it is impossible to truly assert anything (i.e. cue the shriek operator "!", and transfer of absurdity to the negation of PNC).

W5. The 5th Refutation: That the Argument May Be Self-Defeating

The fifth refutation picks up where W4 left off, and makes the case that the argument that one is able to simultaneously affirm and deny the same thing is self-defeating. Aristotle says the following:

Again, if whenever an assertion is true its denial is false and when the latter is true its affirmation is false, there can be no such thing as simultaneously asserting and denying the same thing truly. However, they would doubtless assert that this is the question originally posed (1008a 34 – 1008b 2).

So if when an assertion is true, we know its denial is false, and vice versa, then the claim that there can be such a thing as asserting and denying the same thing truly is self-defeating. Just by understanding what assertions and denials are, it is seemingly impossible for an assertion and denial to be simultaneously true. Notice how Aristotle is careful to back away from a potential *petitio principii* at the end of the passage. Wedin notes that there is the thought that Aristotle takes the preceding argument in the following way¹³⁷:

(1) (p)(p is true = $\neg p$ is false) (2) (p)(p is true = $\neg (p \text{ is not true}))$ (3) $p \rightarrow \neg (\neg p)$ (\therefore) $\neg \langle (\exists p)(p \land \neg p)$

But in the final line of the passage above, Aristotle refrains from the inference that is registered in the conclusion (\therefore) and in the penultimate line. So he is aware that he cannot simply say that it is impossible to truly and simultaneously affirm and deny the same thing of the same thing. It seems plausible that he is offering up this idea as another shriek-worthy consequence of denying PNC, which would again transfer to

¹³⁷ Wedin (2000), Pg. 160 – 161.

the original denial of PNC, but he is not asserting that therefore PNC is true, so no

charge of *petitio principii* can be brought against W5.

W6. The 6th Refutation: Denier's Beliefs & Actions Contravene His Denial

In the sixth refutation, Aristotle is interested in the practical value of PNC,

and its ramifications for those who negate PNC. Aristotle says the following:

Again, are we to say that he who believes that things are in a certain state, or are not, is in error, while he who believes both has the truth? For if he has the truth, what can be meant by saying that the nature of things-that-are is of that kind? If he does not have the truth, but has more truth than the one who believes the former way, then the things-that-are would already be in some state, and that would be true and not simultaneously also not true. But if everyone equally both is in error and states the truth, there will be nothing for such a person to speak or say; for he simultaneously says this and not this. And if a man believes nothing, but considers it equally so and not so, how would his state be different from a vegetable's¹³⁸? (1008b 2 - 30).

Aristotle begins by bringing the question in the background of his refutations to the surface: who is right in the debate concerning PNC? And if the PNC denier is right, then it is unclear what is meant by asserting: *the nature of things-that-are is contradictory*. Also, if the denier of PNC has *more* truth than the PNC affirmer, then, Aristotle says, things-that-are would be in a *definite* state, which cannot possibly be contradictory. Additionally, there is nothing for the denier to speak or say, if everyone speaks truth and is in error, if everyone asserts the truth of contradictions; he might as well be a Pyrrhonian and seek quietism. Finally, such a denier's beliefs would contravene his denial of PNC, since he cannot possibly think everything so and not so; he cannot, via *ad hominem*, be a vegetable. Not only do the denier's beliefs contravene his denial of PNC, but also his actions. Aristotle continues:

 $^{^{138}}$ Ross reads: "...what difference will there be between him and the plants?" (1008b 10 – 12).

From which it is also quite obvious that *nobody actually is in that condition*. neither those who state this thesis nor anybody else. For why does anyone walk to Megara rather than stay where he is, when he considers that he should walk there? Why does he not proceed one morning straight into a well or over a precipice, if there is one about: instead of evidently taking care to avoid doing so, as one who does *not* consider that falling in is equally a good thing and not a good thing? It is consequently plain that he believes that one thing is better, another not better. And if so, he must also believe that one thing is a man and another not a man, one thing sweet and another not sweet. For he neither seeks nor believes everything indifferently when, considering that it is better to drink water and see a man, he thereupon seeks to do so; and yet he ought to, if the same thing were equally a man and not a man. But just as we said, there is nobody who does not evidently take care to avoid some things and not others; so that it seems that everyone holds some beliefs *baldly*, if not about everything then about what is better and worse. And if this is not knowledge but opinion *one would have to be all the more anxious about the truth*, as a sick man is more anxious about his health than one who is healthy. For indeed a man who holds an opinion is in an unhealthy condition with regard to the truth, compared with one who has knowledge [my emphases] (1008b 12 - 30).

Aristotle builds on the ideas of the last passage by extending the practical value of PNC not only to beliefs, but also to actions. Aristotle tells us that neither those who deny PNC, nor anybody else is "actually in *that* condition" (i.e. a vegetable's— considers everything equally so and not so, and believes nothing). As he says, one decides to walk to Megara and not stay where they are. Others decide to avoid falling in a well or off a precipice. These people implicitly think one thing preferable to the other, and do not think that both options for action in each case are desirable. From here Aristotle thinks that such people must also believe that one thing is a man, and another piece of food is sweet, but not both contradictories in each case simultaneously. And further, even if one did think that *x* is a man and not a man when going to see him, they go see him regardless, so it seems like the belief is also uneconomical. Additionally, Aristotle says, one "avoids some things and not others";

everyone at least, holds the belief in PNC baldly in that they hold it, at least, in

cases involving things that are "better or worse". So again, the denier's actions

contravene their denial of PNC. Therefore, we can apply the shriek operator to these

consequences of the denial of PNC, and to the negation of PNC itself.

W7. The 7th Refutation: Nothing is Nearer to or Further from the Truth, or More or Less the Case

The final refutation concerns the nature of truth. Aristotle finishes the seven

refutations by telling us:

Again, however much everything is so-and-so and not so-and-so, at least the more and the less are present in the nature of things-that-are. For we would not assert that two and three are *equally* even, or that one who considered that four things were five and one who considered that they were a thousand were *equally* in error. So if they are not equally, it is plain that one of them is less, so that he has more truth. So if what is more is nearer, there must be something true which the more true view is nearer. And even if that is not so, at least there is already something more firm and more truthlike, and we should be rid of the unadulterated¹³⁹ thesis which would prevent us from having anything definite in our thinking (1008b 31 – 1009a 5).

Leaving behind the truth of PNC for a second, Aristotle tells us that there is at least

"the more and the less" in existing things. He gives the example¹⁴⁰ of the numbers 2 and 3 not being *equally even*. And another example: someone who thought that four things were five things, and another who thought that 4 things were a thousand, would not be *equally in error*. If, in these examples, evenness and error are not equal, then there must be an extreme of true that is more closely approximated by the truer claim. And if there are not such extremes of truth and falsity, Aristotle tells

 ¹³⁹ Ross' translation on this word seems more appropriate (i.e. "...unqualified...").
 ¹⁴⁰ As an interesting side note, Aristotle is not a mathematical Platonist: http://plato.stanford.edu/entries/aristotle-mathematics/.

us, then *at least* there is something more true and less true, *at least* we would have something more *definite* than unrestricted contradictions in the unqualified doctrine¹⁴¹. For a final time, we reach a shriek-worthy consequence with the denial of PNC that it is seemingly impossible to have anything determinate and definite in our thinking and theorizing. As with the other refutations, this shriek-worthy and absurd consequence transfers to the denial of PNC, which implied the absurdities under investigation.

The seven refutations have been critiqued for being several ways to commit *petitio principii*¹⁴². But as Wedin's exegetical work has shown, whether in the first elenchus (W1), or the seven shriek arguments or refutations that follow (W2 – W7), no questions are begged. As you'll recall from the beginning of this chapter and in the previous one, Aristotle is for the most part concerned with the assertion that it is possible to believe a contradiction (Wedin 2000, 113 – 4). The Indubitability Proof (IP) was mainly a rail against those who would purport that it is possible to believe contradictions, and the seven refutations now offer further support for that doxastic claim. Additionally, the seven refutations also provide support for PNC's presence in IP ((6*) and (13)). Now that we have completed the exegetical work necessary to understand PNC and the arguments that Aristotle offers for it, we now turn to the critique of Aristotle's ideas in *Metaphysics* (Γ) given by Graham Priest (1998).

¹⁴¹ I take Aristotle's thoughts in this last refutation to reflect his intellectual humility with regards to his refutations concerning PNC.

¹⁴² Cf. Łukasiewicz (1910), Priest (1998), etc.

Chapter IV: Critique of Priest Contra Aristotle

Now that we have thoroughly examined Aristotle's arguments concerning the principle of non-contradiction, in this final chapter we will evaluate whether the critique leveled by Graham Priest (1998) succeeds. In the introduction we went over Priest's two negative theses *contra* Aristotle's arguments concerning PNC, and one positive thesis¹⁴³:

- *Thesis* (A): Aristotle's arguments do not provide any kind of argument against Dialetheism.
- *Thesis* (B): Aristotle's arguments do not provide any kind of argument against a Trivialist.
- *Thesis* (C): Aristotle's arguments show that a rejection of triviality is a precondition for reflective purposive activity, and especially for the institution of communication.

In this chapter we will examine both of the negative theses (A) and (B) to see if they

can withstand criticism, and whether Aristotle's analysis gives us more than merely

a restriction on reflective purposive activity and communication (i.e. (C)). As a final

note before heading into the critiques, here's a passage given by Priest in his

discussion of a specific issue in what he calls the desperation of the exegetical

analyses of Aristotle's *Metaphysics* (Γ) and PNC:

I doubt that there is any single answer to this question¹⁴⁴, or even that Aristotle has any clearly thought-out aims. He is just shooting with everything he can think of (Priest¹⁴⁵ 101).

¹⁴³ Priest (1998) Pg. 128.

¹⁴⁴ Concerning the relation of the negative proof method to the six shriekarguments.

¹⁴⁵ Unless otherwise indicated, whenever Priest is cited in this chapter it is Priest (1998).

In what has preceded and what follows in this chapter, it is clear that in his analysis and arguments, Aristotle is not shooting in the dark with lack of argumentative foresight or organization. It is this opinion and the accompanying theses that we now seek to raze.

Aristotle *Contra* Thesis (A)

The first critique that Priest¹⁴⁶ launches at Aristotle concerns Dialetheism, and it is the thesis, which is arguably the most crucial to establish given Priest's purposes (i.e. he is concerned to defend his dialetheic commitments). Priest is also eager to critique Aristotle because: "There is hardly a defense of the Law [PNC] since Aristotle's worth mentioning" (91). The first negative thesis given above is (A): Aristotle's arguments do not provide any kind of argument against Dialetheism. Before countering this thesis, it is useful to ask the following: *what were Aristotle's arguments clearly intended to be arguments against*? Aristotle was seeking to counter positions like those of Heraclitus, Protagoras, and others who held a sort of *metaphysical*¹⁴⁷ or *ontological* dialetheism (cf. Γ .5 and Γ .6). That is, that there could be existent true contradictions in reality, or as he would say, concerning the *thingsthat-are.* But as he says: "…what can they mean by saying that the nature of existing things is of this kind?" (1008b 4 – 5). For example, he mentions the metaphysical

¹⁴⁶ For comparison, Priest's (1998) layout of his essay is just by an argument-byargument critique, and these theses are ones he thinks that he has established by the end of his critique. Whereas I start from his theses, and try to make sense of them in light of everything he says, and everything we have said here concerning the arguments.

¹⁴⁷ Not in Edwin Mares (2004) sense of the phrase.

and physical views of Democritus and Anaxagoras¹⁴⁸ in Γ .5 after finishing the seven refutations and IP, in order to explain how people come to believe the negation of PNC: "If, then, that which is not cannot come to be, the thing must have existed before as both contraries alike, as Anaxagoras says all is mixed in all, and Democritus too; for *he* says the void and the full exist alike in every part, and yet one of these is being, and the other non-being" (1009a 25 – 30). We can now answer our initial question: Aristotle's arguments were clearly intended to confront the metaphysical, ontological, and epistemological theses of various contemporaries and past philosophers.

And what specific views did he attack under this heading? In chapter II we went over arguments that were part of the Indubitability Proof (IP), where Aristotle attacked the idea that it is possible to believe contradictions¹⁴⁹. In chapter III we discussed the seven refutations. The long elenchus W1 and its three stages (W1.I – W1.III), given an understanding of 'signification', showed in both cases W1.I and W1.II that the same thing could not be a man and not a man simultaneously. W1.III supported both W1.I and W1.II by showing absurdities that follow for Aristotle's theory of substance when PNC is denied. W2 attacked the thesis that "...all contradictories are true of the same subject at the same time..." (1007b 19 – 20), which led to the absurdity that all things are one. W3 confronted the previous thesis as well and showed how it led to a denial of the law of excluded middle. W4 was

¹⁴⁸ There are many more people who Aristotle sees himself in opposition with. For a more complete list see the Introduction.

¹⁴⁹ As was said in previous chapters in discussions of his argumentative purposes, this is his aim generally in *Metaphysics* (Γ) and his treatment of PNC.

concerned with the thesis that all contradictories are true, which led to the absurd result that it is impossible to assert anything with truth. W5 showed that given an understanding of assertion and denial, the argument that it is possible to do both simultaneously is self-defeating, but backs away from a possible *petitio principii*. W6 showed the practical necessity of PNC for thought and action. And finally W7 showed that a denial of PNC leads to the absurd result that we cannot have anything *definite* in our thinking. One last thing to get clear on is that Aristotle thought the principle of non-contradiction was indemonstrable (1006a 1 – 11), and he is not giving a demonstration *proper* for PNC, according to his *Analytics* conception of demonstrations (*cf.* Chapter III, 55 – 6 (above)). He is giving an *elenctic* or *negative* demonstration of PNC, wherein an opponent "says something significant" and takes responsibility for the *petitio principii*.

Returning to thesis (A), a very basic question could be the following: can Priest fairly critique Aristotle and *Metaphysics* (Γ) for not having a response to a late-twentieth century advancement in logic? What would we think in other cases? Conveniently, we will use the example of *geocentrism* from Aristotle's astronomy (a theory Ptolemy followed some 500 years later). In *On the Heavens*, Aristotle tells us: "Earth then has to exist; for it is earth which is at rest¹⁵⁰ at the *centre*" [my emphasis] (*De Caelo* Ch. 3, 286a 20 – 21). When Copernicus came up with the theory of *heliocentrism* a little over 1300 years later (setting aside the additional theory of Tycho, *geo-heliocentrism*, which came after), we expect Aristotle's and Ptolemy's geocentrism to be able to respond to the issue of whether the Earth is at the center ¹⁵⁰ Only the geocentric problem is dealt with here. of the universe, since it is a hypothesis that claims the earth *is* at the center. In this case, Copernicus' heliocentrism is a *direct* critique of the theory of geocentrism (i.e. both disagree about the position of earth in the solar system). In the present case, Aristotle would clearly disagree with the basic dialetheic claim that *some* contradictions are true, but the *formal* contradictions (e.g. Liar's Paradox and Russell's Paradox) that Priest is dealing with are clearly beyond the scope of Aristotle's critique. Even so, it is clear that there are direct areas of disagreement: Priest thinks that *some* contradictions are true *per* the basic dialetheic claim (Priest (1998) 94 – 5; Priest (2006) & (2006b)) and that one can believe them (Ibid. and Priest (1998), Pg. 94), he gives a few examples of true contradictions (Priest (2006b)) and does not think PNC is indemonstrable (Priest (1998) 98). Whereas, Aristotle thinks that it is neither the case that any contradictions are true (1011b 15 - 18) nor that one can believe them (IP), he gives negative demonstrations contra the claim that it is possible to believe contradictions and *contra* the negation PNC itself, and finally, thinks PNC is indemonstrable (1006a 6 – 13). Thus, even if one were to say that Aristotle is mostly concerned with establishing that one cannot believe contradictions (Wedin (2000) $113 - 4^{151}$) rather than giving a demonstration proper for PNC, there are still various points of genuine disagreement between the two. To answer our initial question, then, it is fair of Priest to critique Aristotle and *Metaphysics* (Γ) for these and other points of disagreement. So one cannot simply escape thesis (A) by saying that Priest is dealing with a new problem, for which Aristotle didn't intend his arguments.

¹⁵¹ A position, with which I am in accord.

What we can do is both affirm and dismiss (not deny) thesis (A). We have reason to *dismiss* thesis (A), because Priest and Aristotle do not share their conception of contradiction. And because of this, we have reason to *affirm* thesis (A), since Aristotle could not have any arguments against dialetheism, if it does in fact have another conception of contradiction from which it is arguing. To support this dismissal of (A), recall the Liar's Paradox (semantic paradox) and the Barber's Paradox (set-theoretic paradox) that we went over in the introduction:

(1) This sentence is false.

(2)
$$\exists x (Man(x) \land \forall y (Man(y) \supset (Shaves(x, y) = \neg Shave(y, y))))$$

In the Liar's Paradox we have the following truth conditions: the sentence is false *iff* it is true. So that we have the following true contradiction or *dialetheia*: $\exists x (Tx \land \neg Tx)$. The result is similar in the Barber's Paradox¹⁵²: the barber shaves himself *iff* he does not shave himself. Where S = x shaves himself, we get the following *dialetheia*, whether or not the barber shaves himself: $\exists x (Sx \land \neg Sx)$. These two paradoxes are pieces of evidence for Dialetheism for Priest, JC Beall, and others¹⁵³. The conception of contradiction here is a conjunction of some property and its

¹⁵² I use the Barber's Paradox here, since it is derived from Russell's original settheoretic paradox by someone that Russell doesn't name in his original text: see <u>http://plato.stanford.edu/entries/russell-paradox/</u>, and *The Philosophy of Logical Atomism* (2010), Pg. 101. We leave aside the fact that the Barber's Paradox is able to be solved, Russell's original paradox is not as amenable to a solution. ¹⁵³ Cf. Priest (1998), Pg. 94 for his asserting that he believes Russell's set (from which the Barber's Paradox is derived) is a dialetheia. And his thinking that the Liar's Paradox is evidence for Dialetheism (and for more support that the set theoretic paradoxes are evidence) can be found in Priest's *In Contradiction* (1987). Also, Beall (2004) is clear that the Liar's paradox counts as a dialethia.

negation, and can be non-explosively (*contra* ECQ) true in dialetheic conceptions of paraconsistent logic.

Now recall the conception of contradiction (α) that we built up from textual evidence in chapter I:

(α) If something γ is an Aristotelian contradiction, then γ is a compound statement consisting of a pair of statement-making sentences (SMS), the Law of Excluded Middle (LEM) jointly applies to the component SMS's in γ , and γ consists of one SMS that affirms Ψ holds good and another that denies Ψ holds good, where Ψ is understood to be of the same *attribute* A in the same *respect* R of the same *subject* S at the same *time* T (we may add extra E^{154} qualifications to SART, if needed)¹⁵⁵.

In our discussion of (α) in chapter I, we said that this conditional is true *iff* both the antecedent and consequent are true. We arrived at these truth conditions for (α) through considering the claim that one of the necessary conditions (i.e. LEM) in the consequent could be false, while (α) is still true (i.e. an Aristotelian contradiction is present). The absurdity of such a claim became apparent after considering the overwhelming amount of textual support for such a condition, and also support from Code's analysis. Thus, we were left with the above truth conditions for (α).

¹⁵⁴ This *E* qualification is ambiguous as it stands, but think of it as covering any qualifications that may arise in given token situations, where the typical four (SART) do not block against true contradictions. We may imagine a situation where each of SART are met, but a true contradiction still arises—here we would add extra qualifications.

¹⁵⁵ In the formulation above, the affirmation and negation of Ψ are both single SMS's, but an application of this conception could be made more complex, where contradictions would consist of more SMS's than a statement Ψ and its contradictory opposite. And, of course, in this more complex application of (α) the same necessary conditions would apply.

In the Aristotelian conception of contradiction, we can see that the law of excluded middle (LEM) is a necessary condition. LEM is the principle that for any pair of contradictory propositions, one or the other is true—the propositions are mutually exclusive *and* jointly exhaustive. In the conception of contradiction that Priest is utilizing, LEM is not necessary to it, or at very least, it may be violated. That is, if a conjunction of some property and its negation can be true in cases like the Liar's or Barber's Paradoxes, then that shows that LEM, which requires that one or the other of a pair of contradictories is true, is not absolutely necessary to the conception of contradiction that is at work. And as a result, the Liar and other similar cases would fail to be classified as Aristotelian contradictions.

We could also point out that other than this major difference, Priest assumes that modern formulations of contradiction using "classical logic¹⁵⁶" (i.e. $\Phi \land \neg \Phi$) will capture Aristotle's ideas of contradiction (Priest 93). But contradictions are made up of statement making sentences (SMS) for Aristotle, which are statements with two truth-values (i.e. true and false) and they come in two types (i.e. affirmations and negations). One can also have single SMS's that are *simple* (i.e. consisting of a single SMS) or *compound* (i.e. consisting of multiple SMS's). And further, as we learned from Alan Code in chapter I, a contradiction for Aristotle:

...is not the statement formed by the *conjunction* of the affirmation with the corresponding denial, rather it is a *pair* of statements. To believe a ...[contradiction]... is not to believe a single conjunctive proposition. It is to have two separate beliefs: one a belief corresponding to the *affirmation*, the other a belief corresponding to the *denial*... He is not arguing against the

¹⁵⁶ A somewhat misapplied phrase in this context.

claim that it is impossible to have the conjunctive belief, but instead is considering two beliefs [my emphases] (Code (1987) 132).

So Aristotle is not arguing against a conjunctive belief, but instead, is arguing against the claim that it is possible to have a *pair* of beliefs one of which is an *affirmation* and the other is a *denial*, where both the affirmation and denial may consist of more than one SMS. Further, the conception of contradiction that Priest is assuming is not obviously concerned with qualifying contradictions with respect to time, hence *T*, and he is certainly not concerned with adding extra qualifications *E* when *SART* fails to both exclude true contradictions and secure PNC.

Well, could Priest escape from the former critique by saying that it is not crucial or necessary that LEM is part of Aristotle's conception of contradiction? Well this would be rather strange and clearly false thing to say, since Aristotle not only mentions LEM's relation to contradictories in the same book as PNC (*Metaphysics* Γ .7 1011b 23 – 24), but also later in the same work says the following: "...for contradiction *is* this—an opposition, one or other side of which must attach to anything whatever, i.e. which has no intermediate" [my emphasis] (I.7, 1057a 33-5). Further, in another work he remains consistent with this idea, saying in *Posterior Analytics*: "A contradiction is an opposition of which of itself excludes any intermediate..." (*Post. Anal.*, Book I, 72a 12 – 14)¹⁵⁷. Finally, as was said in Chapter I, it is surely not insignificant that a scholar who specializes in Aristotle's *Metaphysics*, Alan Code, thinks that a 'contradiction' is "...an opposition to which the law of excluded middle (LEM) applies" ((1987) 132). So LEM is not just added to (α)

¹⁵⁷ Translated by Jonathan Barnes (1984).

without reason. Contra Priest: LEM *is* crucial and necessary to what Aristotle takes a contradiction to be.

As further support for my preceding conclusion, it is not just as the first and third necessary conditions would have it, that if something γ is an Aristotelian contradiction, then γ is a compound statement consisting of a pair of SMS's, and γ consists of one SMS that affirms Ψ holds good and another SMS that denies Ψ holds good, where Ψ is understood to be of the same *attribute* A in the same *respect* R of the same *subject S* at the same *time T* (we may add extra *E* qualifications to SART, if needed). But it is also the point that what is important with such a possible situation according to Aristotle, is that either (but not both or neither) the affirmation or the denial must be true as LEM would have it. So again, the LEM qualification is not only crucial, but is necessary for Aristotelian contradictions. Therefore, Priest and Aristotle are utilizing different conceptions of contradiction and we may dismiss thesis (A) as a misleading and equivocating critique, since Aristotle's arguments could not be used against Dialetheism, which utilizes another conception of contradiction. Perhaps, stated provocatively: Priest's contradictions are not genuine contradictions at all, or maybe they are. This is beyond the reach of this project. Aristotle Contra Thesis (B)

Priest's second thesis (B) is that Aristotle's arguments do not provide any kind of argument against a Trivialist. Trivialism is the view that *everything is true*, and so a trivialist, though it is safe to say nobody of such a description exists, is one that thinks that everything is true. Connected to this view is the Law of NonTriviality (LNT), which is just the view that *not* everything is true. Trivialism seems so obviously false so as to not even need an argument from Aristotle, let alone anybody else, but Aristotle does just that. So Priest is correct in thinking that Aristotle in the refutations and elsewhere often attacks trivialism, especially in regards to Protagoras¹⁵⁸(Priest 96). Aristotle in this regard seems to be clearing the *impossibilities* of the area of investigation under consideration in *Metaphysics* (Γ) just as he does in other works. As he tells us in *Meteorology*:

We consider a satisfactory explanation of phenomena inaccessible to observation to have been given when our account of them is free from *impossibilities* [my emphasis] (*Meteorology*, Ch. 7, 344a 5 – 7)¹⁵⁹.

In the case under consideration with PNC, the objects of study are accessible to the mind through reason and experience, but as in the case of weather phenomena and other areas of study, one needs to clear the impossibilities. One impossibility with respect to the principle of non-contradiction is where *all* contradictions are true, and *a fortiori*, everything is true (i.e. trivialism). So Aristotle wants to rule out the possibility of the strong denial of PNC (and thus, trivialism) explicitly in his refutations W2, W4, and W6¹⁶⁰. Since the claim in thesis (B) includes all of Aristotle's arguments in *Metaphysics* (Γ), in what follows in my arguments *contra*

¹⁵⁹ I am indebited to Mariska Leunissen's *Explanation and Teleology in Aristotle's Science of Nature* (2010) for this passage (Leunissen 159).

¹⁵⁸ Or at least Aristotle's characterization of his views. Wedin (2003) creates a compelling argument showing just that—that Protagoras' views entail a "strong denial" of PNC (i.e. *a fortiori,* Trivialism).

¹⁶⁰ As is clear from Chapter III, I prefer Wedin's exegetical approach and division of the refutations, but both Wedin and Priest use Ross' division of the refutations, and Kirwan's translations. I do not discuss W2 here, since it is not, in my mind, the strongest of the ones that could be marshaled against (B).

(B), I do not restrict myself solely to the arguments, in which Aristotle explicitly discusses trivialism or to the arguments he explicitly discusses.

The first defense that one could launch against thesis (B) is that whether or not the Indubitability Proof (IP), as presented in chapter II *via* Wedin (2004) (*cf.* Appendix I at the end), is sound is clearly debatable. Recall the conclusion that it reaches in (21a): $\neg \diamond (\exists x)(x \text{ bel } (Fa \land \neg Fa))$. Or as Aristotle says in the text at the close of Γ .3: "...it is impossible for the same person to believe simultaneously that the same thing is and is not [something, Ψ]..." (1005b 26 – 33). What *is* questionable is that it is possible to believe the negation of PNC using the Aristotelian conception of contradiction (α) as sketched in chapter I. This is where we now turn.

Consider an example of a contradiction as defined in (α). The compound statement γ consists of a pair of SMS's, one SMS affirms that "James is a man" holds good and another SMS that denies that *this* holds good, where "James is a man" is understood to be of the same *attribute A* (i.e. "is a man") in the same *respect R* (i.e. two-footed animal) of the same *subject S* (i.e. James) at the same *time T* (July 1st, 2016 @ 5pm) (we may add extra *E* qualifications to SART, if needed). Assume we know that James is a man in the respect defined above, then it could neither be the case nor could anyone believe it to be the case that James is a man and not a man. Thus, LEM jointly applies to the SMS and its contradictory opposite. If this follows then we know something whose affirmation is true and not simultaneously false (i.e. James is a man) *via* the conception of contradiction (α). Therefore, trivialism and thesis (B) are both false, since the negation of the claim above is false (i.e. at least one thing is false), and further, LNT would be true. This example is essentially the same as the first case of the first refutation or W1.I (1006a 31 – 1006b 34).

Consider another more complex example of a contradiction as defined in (α) that goes beyond the Aristotelian arguments themselves. A compound statement γ consists of a pair of SMS's, and γ consists of one SMS that affirms that "The core of the sun is greater than or equal to 27 million degrees" holds good and another SMS that denies that *this* holds good, where "The core of the sun is greater than or equal to 27 million degrees" is understood to be of the same *attribute A* (i.e. "The core of the sun") in the same *respect R* (temperature) of the same *subject S* (e.g. the sun in the Milky Way galaxy in our solar system) at the same *time T* (Monday July 1st, 2016) @ 5pm) (we may add extra *E* qualifications to SART, if needed). Remember that LEM is a necessary condition for Aristotelian contradiction in (α) , so in order for this to be a case that is covered under (α), the case must be one to which LEM applies. We now know that: "At the core of the sun, gravitational attraction produces immense pressure and temperature, which can reach more than 27 million degrees F (15 million degrees C)" ("How Hot is the Sun?"). Since this is the case, LEM applies in this example, and it could not *not* be the case nor could one *believe* that the SMS "The core of the sun greater than or equal to 27 million degrees" holds good and does not hold good. And if this is the case, then we know something is true and not simultaneously false *via* the conception of contradiction. Thus, thesis (B) that Aristotle does not provide any arguments or ideas *contra* trivialism is false, LNT is

true, and trivialism is false, since we know at least one thing is false (i.e. the

contradictory opposite of the affirmative SMS).

A second line of defense against thesis (B) comes from the fourth refutation

(W4), which doesn't get a lot of attention in the literature. The reader will recall that

the fourth refutation tells us that it will be impossible to truly assert anything.

Aristotle says:

Again either $[\Delta]$ this [presumably ~PNC] is so in every case, i.e. a thing is both pale and not pale, both a thing-that-is and not a thing-that-is, and in a similar way for all other assertions and denials; or $[\Delta_2]$ it is so in some cases but not in others. If it is not so in all cases, these would be agreed [upon]. But if it is so in all, then in turn either [a] anything asserted may also be denied and anything denied also asserted; or [b] the things asserted may also be denied but not everything denied also asserted. But if the latter, something would be securely not a thing-that-is, and that opinion would be firm; and if not to be is something firm and certain, the opposite assertion would be still more certain. But if anything denied may equally be asserted too, necessarily it is either [c] true to state separately, for instance, that a thing is pale and again that it is not pale, or [d] not. If it is not true to state separately, then not only does he not state these things but nothing whatever is-and how can things-that-are-not walk and talk? Also, everything would be one, as we said before [1007b 20 {i.e. W2}], and a man and a god and a warship and their contradictories will be the same thing; for if it applies equally to each thing, nothing will differ from anything else, since if it did differ, *that* would be true and distinctive [my emphases] (1008a 7 – 28).

As I spelled out in chapter III, Aristotle gives us two cases: (Δ) either ~PNC is true in all cases; or (Δ_2) ~PNC is true in some cases but not in others. The second case (Δ_2) gives the unsatisfactory result that there are exceptions to PNC, and these will be agreed upon; a clearly shriek-worthy consequence of accepting (Δ_2). On the other hand, in the first case (Δ), the strong denial of PNC in (Δ) implies trivialism. Within (Δ) there are two cases: (a) everything asserted may also be denied and vice versa, or (b) everything asserted may also be denied and *not* everything denied is asserted. If (b) is the case, then something would *not* be a thing-that-is. And if we know this, then we clearly know when something *is* a thing-that-is, he tells us. In a second argument¹⁶¹ here Aristotle tells us that if (a) is the case, then it is either (c) true or (d) untrue to state contradictorily opposed SMS's separately. And if it is untrue to state them separately, then not only do they not state anything in asserting contradictions, but they cannot say that anything is the case. So, at least conceptually, nothing *exists*. Further (following W2), everything is one. This is implied for the following reason that if everything can be asserted and denied of everything, then everything is conceptually one—there will be nothing to distinguish one thing from another. So, if one saw a dog, it would also not be a dog, and it would also be a cat, horse, and rock...*ad infinitum*. Aristotle continues:

Equally, even if it is possible to have the truth in stating things separately, the result we have stated follows; and in addition it follows that everyone would have the truth and everyone would be in error, and [the disputant] himself is in error by his own admission. At the same time it is obvious that in response to this person there is nothing for an investigation to deal with; for he says nothing. For he says neither that it is so-and-so nor that it is not so-and-so; and again he also *denies* both these, saying that it is neither so-and-so nor not so-and-so. For if he did not, something would already be *definite*. [my emphases] (1008a 28 – 34).

Thus, suppose (c) stating the contradictorily opposed SMS's separately gives us truth, we are still left with the problems that nothing exists and that "everyone would have the truth and everyone would be in error". The initial requirement of negative proofs is for the opponent to "say something significant," but if the opponent asserts (Δ), then not only would they not say something significant, they

¹⁶¹ Like other refutations, the fourth refutation seems to be a few arguments that are in close proximity, but not necessarily one or logically the same. It is clear that there are more than seven refutations.

"say nothing", as Aristotle says above. And if they say nothing, then the elenctic proof cannot get off the ground. We are left with the result that it is impossible to truly assert anything.

To conclude with W4 then, although Aristotle seems to be making outlandish claims, he does show some possible consequences for such an equally outlandish position as trivialism. The first absurd result that is drawn is that nothing *exists*. If it is not true to state the predicates separately, then not only is nothing said when a trivialist does this, but we can't say anything is the case or exists. If everything is true, then we cannot even make the basic claim that something is the case or exists. This is not only absurd, but untrue. I am sitting here writing. It is the case that I am sitting here writing, and the negation of this proposition is false. The second absurd consequence that is drawn in W4 (but also present in W2) is that everything is *one*. As Aristotle clarifies in the passage above, he means by this that everything becomes indistinguishable once every contradiction can be predicated of everything as (Δ) would have it. Since we know that things are distinguishable in some sense, this claim is clearly false. And even if things were *not* totally distinguishable, say in some relational ontology¹⁶², we would still be better with this than the position that every contradiction is true of everything—everything is true. The last result that Aristotle

¹⁶² In *An Introduction to Relational Ontology* (2006) by Wesley Wildman, he describes "relational ontology": "The basic contention of a relational ontology is simply that the relations between entities are ontologically more fundamental than the entities themselves. This contrasts with substantivist ontology in which entities are ontologically primary and relations ontologically derivative". See: http://www.wesleywildman.com/wordpress/wp-content/uploads/docs/2010-Wildman-Introduction-to-Relational-Ontology-final-author-version-Polkinghorne-ed.pdf

talks about is that we cannot truly assert anything. Everyone both has truth and does not ("is in error") all the time. This is an absurdity that must be admitted by one who asserts that every contradiction is true of everything, and clearly this is false. The doctors diagnosed that Willy has a traumatic brain injury. They are not in error and not in error simultaneously. The judges convicted Jimmy of arson after video proof was provided. The judges are not in error and not in error simultaneously, the arguments in W4 are not conclusive and were not meant by Aristotle to be as such since it is a negative demonstration as he defines it, but they do give us reason to reject trivialism, and thus, consider thesis (B) to be untenable, if not clearly false.

One final defense we can give for Aristotle against thesis (B) is concerning what Priest says in regards to the third and sixth refutations. The reader will recall that the third refutation (W3) was concerning the law of excluded middle (LEM), and the sixth refutation (W6) dealt with the point that the PNC denier's beliefs and actions override his denial of PNC. Offering his own take on the third refutation Priest tells us:

For a trivialist, assertion, *as an act of communication*, is pointless. In fact, all communicative activity (commanding, questioning, etc.) is pointless. For the point of communicative activity is, in the first instance, to induce certain mental states in the hearer. But the trivialist already believes the hearer to be in that state. Similarly, there is no point in a trivialist even listening to the communicative attempts of another. For whatever information they might hope to gain from the communication, they are already in possession of it: whatever the beliefs, desires, etc., of the other, the trivialist already believes the other integral to the rationale of communication, and hence the possibility of social life (121).

Additionally, he offers his conclusions on the latter part of W6 (1008b 12 – 31):

We have, at any rate, a firm and general conclusion concerning trivialism. A belief in trivialism is incompatible with reflective, purposeful action; and, in particular, with communication. This does not show that trivialism *is* false, or that no one cannot suppose it to be true. But its rejection is a precondition of this central feature of what it is to be a person (and not a vegetable). We can therefore interpret Aristotle's argument as perinomic, establishing this important fact (127).

If a "general rejection of trivialism" is necessary for communication and the possibility of social life, as Priest says in his take on the third refutation, then it is unclear how this would not be *prima facie* evidence for thesis (B) being false. And in the case of W6, if believing trivialism is "incompatible with reflective, purposeful action" and "communication", and its rejection is a precondition of being a person, then it is, again, unclear why this could not be seen as evidence for thesis (B) being false. Once again these arguments (W3, W4, & W6) are not meant to be demonstrations proper, but here we see that even Priest himself is wont to reject trivialism based on the third and sixth refutations. Thus, given W4 and Priest's understanding of W3 and W6, it is clear that Aristotle's arguments must imply a rejection of trivialism, and thus, the truth of (C). More importantly though, given the two cases considered involving the Aristotelian conception of contradiction (α), in addition to W4, and Priest's claims regarding W3 and W6, we can conclude that thesis (B) is false. In conclusion, in this chapter we have shown the falsity of thesis (A), since it mistakenly assumes that Aristotle and Priest share conceptions of contradiction. That is, the falsity is not in terms of the actual thesis (A), but in terms of attributing to Aristotle a conception of contradiction, which is not his own. For this reason, as was said above, we can dismiss thesis (A). We have also shown the falsity of thesis (B), given Aristotle's conception of contradiction (α), W4, and
Priest's own take on W3 & W6. And finally, Aristotle has been shown in his analysis to be giving not only a restriction on reflective purposive activity as thesis (C) would have it, but also a rigorous conception of contradiction which allows us to dismiss thesis (A), several reasons in both IP and W1 – W7 to doubt the truth of the negation of PNC, and finally, a series of reasons to not only reject trivialism, but show it to be false.

Conclusion

Through the research that has been presented we have learned a lot about contradictions and the principle that bears the name, and Aristotle's ideas with respect to both. In chapter I we learned about Aristotelian logic and concepts therein, and formulated a conception of contradiction (α) based on these and Code (1987). In chapter II, we went over the Indubitability Proof (IP) that occurs at the end of Γ .3. In chapter III, we then explored the seven refutations. And finally in chapter IV, we found Priest's theses (A) and (B) were false. So not only have we challenged a modern critique of Aristotle's PNC, we have also learned a lot about the origins of such a central concept in modern logic, essential views on this very concept from of the founder of Western logic, and modern developments with respect to logic and this concept.

The following questions come to mind that are outside of this critique: (a) Are the true contradictions or *dialetheias*, which Priest and other Dialetheists (*cf.* JC Beall) talk about and research, *genuine* contradictions; and for that matter: (b) Which conception of contradiction ought one to prefer (i.e. one where LEM reigns supreme, or one where LEM is not as central, or some other one entirely); (c) Is there any non-arbitrary way to choose a conception of contradiction? These questions are clearly beyond the reach of my research, but present interesting possibilities and questions for the foundations of logic. Since contradiction is so central to the logical enterprise and philosophy, it is clear that we should take the time to get these things right.

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1.1*: if (a) *x* cannot err regarding *P*, then (e) *x* necessarily understands *P*.

1.2*: if (e) x necessarily understands P, then (c) P is necessarily most intelligible

1. If (a) error is impossible regarding a principle, *P*, then (c) *P* is necessarily most intelligible.

2.1: if (f) x's understanding anything presupposes x's understanding P, then (g) x must already have P ["....is part of his equipment...."].

2.2: if (g) x must already have P ["....is part of his equipment...."], then (d) P is non-hypothetical.

2.3: if (f) *x*'s understanding anything presupposes *x*'s understanding *P*, then (d) *P* is non-hypothetical.

2.4: if (a) error is impossible regarding a principle *P*, then (f) *x*'s understanding anything presupposes *x*'s understanding *P*.

2. If (a) error is impossible regarding a principle, *P*, then (d) *P* is non-hypothetical. 3. If (a) error is impossible regarding a principle, *P*, then (c) *P* is necessarily most intelligible, and (d) *P* is non-hypothetical.

4. If (c) *P* is necessarily most intelligible and (d) *P* is non-hypothetical, then (b) *P* is firmest.

(F) If (a) error is impossible regarding a principle, *P*, then (b) *P* is firmest.

6. It is not possible that there is something, *x*, such that *x* has a property, *F*, and *x* does not have *F*.

 6^* . $\neg \Diamond (\exists x) (Fx \land \neg Fx)$

7. If (g) for all x it is impossible that x err with respect to a principle, P, then (a) error is impossible regarding P.

8. For all *x*, if (h) it is impossible that *x* believes $\neg P$, then (g) it is impossible that *x* errs with respect to *P*

9. If (h) it is impossible that x believes $\neg P$, then (a) error is impossible regarding P. 10. If (h) it is impossible that x believes $\neg P$, then (b) P is firmest

11. $\neg \langle (\exists x) (\exists z) (x \text{ bel } (Fz \land \neg Fz)) \rangle$

12. $\neg \langle (\exists x)(x \text{ bel } \langle (\exists z)(Fz \land \neg Fz)) \rangle$	
13. $\neg \langle (\exists x)(Fx \land \neg Fx) \rangle$	(Premise)
14. $\neg \langle (\exists x)(Fx \land F^*x)$	(13, 15) ¹⁶³
15. $(x)(F^*x \rightarrow \neg Fx)$	(Premise)
16. (x)(x believes Fa is contrary to x believes \neg Fa)	(Premise)
16a. (x)(x bel $Fa \rightarrow [B:Fa]x$)	(16, PA ¹⁶⁴)
16b. (x)(x bel $\neg Fa \rightarrow [B:Fa]^*x$)	(16, PA)
17. (x)(x bel ($p \land q$) \rightarrow (x bel $p \land x$ bel q))	(Premise)

¹⁶³ Premise (14) is justified by (13) and (15), a premise that occurs after (14). I maintain Wedin's ordering of the premises here for consistency.

¹⁶⁴ Property Attribution (PA). These justifications for the premises are those given by Wedin (2004) in his own appendix of the proof. The other steps' justification are in the respective parts where they occur.

$17a^{165}$. (x)(x bel (Fa ∧ ¬Fa) → (x bel Fa ∧ x bel ¬Fa))	(17)
18a. (x)(x bel (Fa $\land \neg$ Fa) \rightarrow [B:Fa]x)	(16a, 17a)
18b. (x)(x bel (Fa $\land \neg$ Fa) \rightarrow [B:Fa]*x)	(16b, 17a)
19. (x)(x bel (Fa $\land \neg$ Fa) \rightarrow ([B:Fa]x \land [B:Fa]*x))	(18a, 18b)
20. (x)(x bel (Fa $\land \neg$ Fa) \rightarrow ([B:Fa]x $\land \neg$ [B:Fa]x))	(13, 17)
21. (x) \neg (x bel (Fa $\land \neg$ Fa))	(11, 18/12, 17)
21a. $\neg \langle (\exists x)(x \text{ bel } (Fa \land \neg Fa)) \rangle$	(21)

¹⁶⁵ Wedin thinks that this claim can be strengthened to a biconditional (247).

	D (100.1)		
Łukasiewicz	Ross (1924)	Kırwan	Wedin (2000, 2003)
(1910) ¹⁶⁶		(1971)	
L1: 1006b28	R1 = K1 & K2	K1: 1006a 28-	W1:
- 34		31	
L2: 1006b11	R2: 1007b18	K2: (Parts I –	i: Elenctic Proof of PNC: 1006a28
- 22	- 1008a2	III)	- 1007b18
L3: 1007b18	R3: 1008a2 -	I: 1006a31 –	ii: Outline of Proof (2 Cases):
- 21	7	b34	1006a28 - 31
L4: 1008a28	R4 = K4	II: 1006b34 -	I: Case One: Where a Name
- 30		1007a20	Signifies Being: 1006a31 – b34
L5: 1008a12	R5 = K5	III: 1007a20 -	Stage 1: That a Name Signifies
_ 19		h18	One Thing: $1006a31 - h13$
17	R6 - K6	K3.1007b18	Stage 2: That 'Man' and 'Not-man'
	NO - NO	1002-7	Cannot Signify the Same Thing
		- 1000a7	
	D7 - V7	V4. 1000a7	$\frac{1000013 - 20}{\text{Stage 2: The Dreef of DNC}}$
	K = K	K4: 1008a7 -	Stage 3: The Proof of PNC:
		34	1006b28 - 34
		K5: 1008a 34	II: Case Two: Where a Name
		– b2	Signifies Not Being: 1006b34 –
			1007a20
		K6: 1008b2 –	III: Support Argument: The
		31	Ineliminability of Essence:
			1007a20 – b18
		K7: 1008b31	W2: That Everything Will Be One:
		– 1009a5	1007b18 - 1008a2
			W3: That the Law of Excluded
			Middle Will Fail: 1008a2 – 7
			W4: That It Will Be Impossible to
			Truly Assert Anything: 1008a7 –
			34
			W5. That the Argument May Be
			Solf-Defecting: $1008_234 - h^2$
			W6. That the Denier's Poliofs and
			Actions Contractor a Uis Deniel
			Actions Contravene His Denial:
			100802 - 31
			W7: That Nothing is Nearer to or
			Further from the Truth:
			1008b31 – 1009a5

Appendix II: Alternative Individuations of the Seven Refutations

¹⁶⁶ Here's a key for the lettering: L = Łukasiewicz (1910), R = Ross (1924), K = Kirwan (1971), W = Wedin (2000, 2003). This appears a little different form in the appendices of Wedin (2000, 2003), but the content is wholly the same.



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