An Analysis of How and Why High School Geometry Teachers Implement Dynamic Geometry Software Tasks for Student Engagement

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Abstract

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An Analysis of How and Why High School Geometry Teachers Implement Dynamic Geometry Software Tasks for Student Engagement

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This study examined teachers' use of student tasks involving dynamic geometry software, in which a figure is constructed then altered while maintaining its constructed properties. Although researchers, professional organizations, and policy makers generally have been proponents of dynamic geometry for instruction, there is little research about how and why teachers implement dynamic geometry tasks for student engagement. This study sought to fill this gap.

This investigation explored how and why 12 high school geometry teachers from southwestern Ohio engaged their students with dynamic geometry tasks. In addition, this study examined the teachers' enactment of such tasks in prior years and their plans for future use. Via a naturalistic inquiry design, the researcher interviewed each teacher and studied the printed dynamic geometry tasks that the teachers used with their students. The data analysis applied grounded theory methodology.

The teachers had been using dynamic geometry 1–8 years, were teaching at a wide variety of schools, and were using dynamic geometry tasks in similar ways. The teachers used convergent dynamic geometry tasks (tasks for which students followed the same steps to arrive at the same results) to guide students to discover conjectures or

verify theorems, to provide students with accurate dynamic visual aids, and to have a change of pace from the usual classroom routine. Many teachers experienced restricted access to computers. The teachers were proficient in what they had students do with dynamic geometry but not much beyond that, and they were interested in professional development. Teachers' present use of dynamic geometry built minimally on their past use, and it is likely that their future use will build only slightly on their present use.

The researcher hypothesizes that teachers used convergent tasks due to their limited conception of high school geometry. In particular, there was a conspicuous absence of a connection to proof in the tasks. Consequently, the researcher developed a framework for dynamic geometry tasks with four phases leading to proof: construct, explore, conjecture, and prove. The researcher recommends that future investigations be conducted in conjunction with curriculum development or teacher professional development or both.

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Chapter 1: Introduction

Although there is little consensus concerning how and when to use technology in K–12 mathematics instruction, many researchers (e.g., Ball & Stacey, 2005; Demana & Waits, 1990; Fey, Hollenbeck, & Wray, 2010; Heid, 2005; Knuth & Harmann, 2005; Zbiek & Hollebrands, 2008), professional organizations (e.g., Association of Mathematics Teacher Educators [AMTE], 2006; National Council of Teachers of Mathematics [NCTM], 2000) and policy makers (e.g., Common Core State Standards Initiative [CCSSI], 2010) agree that, when used appropriately, technology can enhance the learning and teaching of K–12 mathematics. Because mathematics teachers can use technology well or poorly in instruction, they need to have knowledge of technological tools and self-confidence in how to implement them effectively into instruction to positively impact student learning (NCTM, 2000). "Effective teachers maximize the potential of technology to develop students' understanding, stimulate their interest, and increase their proficiency in mathematics" (NCTM, 2008, p. 1). Students can use technology to explore and to deepen their understanding of mathematical concepts (CCSSI, 2010). When mathematics teachers effectively use technology in their instruction, students often can engage with mathematics on a deep conceptual level and make connections that would be more difficult or impossible without the use of technology.

Just as many researchers, professional organizations, and policy makers are proponents of the use of technology in K–12 mathematics instruction, Battista (2009), Contreras and Martinez-Cruz (2009), de Villiers (1998), Hollebrands and Smith (2009), Olive (1998), and Sinclair (2008) have encouraged the use of dynamic geometry software (DGS) in K–12 geometry instruction. In a position statement, NCTM (2008) claimed that dynamic geometry software is one type of technological tool that is a "vital component of a high-quality mathematics education." Students can use DGS to discover patterns and formulate conjectures by examining many cases to develop understanding of mathematical relationships (CCSSI, 2010; NCTM, 2000). In Jones's (2002) research summary, he concluded that using DGS "for conceptual exploration leads to conceptual gain" and can facilitate "some types of learning activities, for example, exploration and visualization, and can enhance some others, such as proof and proving;" however, he also noted that it was difficult to tell if the "measurable learning gains" were due to DGS or the "rethought curriculum and pedagogy" (p. 20).

Even though many researchers, professional organizations, and policy makers have supported the implementation of dynamic geometry software in K–12 geometry instruction, teachers are arguably the most important aspect of implementation because they ultimately decide how, when, and why to use DGS in their instruction. There has been little, if any, research regarding how geometry teachers "on their own" enact tasks for student engagement with DGS. As Hollebrands and Smith (2009) stated, even though DGS "has been shown to have a positive impact on students' understanding of geometry and reasoning ability, the implementation of these tools and the nature of the tasks selected by teachers are crucial factors in determining their effects on students' learning" (p. 231).

The remainder of this chapter gives a general overview of the research study. The following paragraph describes dynamic geometry software. The next section explains the problem statement and presents the research questions. The following section discusses each research question. The next section details the educational significance. The final section describes the delimitations and the limitations of the study.

Dynamic geometry software is a computer environment in which geometric objects are created (e.g., points, lines, and circles). Constructions of these geometric objects can be performed such as parallel lines, perpendicular lines, midpoints of line segments, and angle bisectors. If properly constructed, when features of a geometric object are moved with the mouse (called *dragging*), it maintains all of its intended properties and any measurements are updated to reflect the changes. For example, a parallelogram should be constructed so that its opposite sides remain parallel no matter how the parallelogram is altered. The sides and angles of the parallelogram can be measured. When the parallelogram is changed, the measures of the sides and angles automatically update.

Statement of the Problem

In spite of NCTM (2000) stating that the effective use of technology depends on the mathematics teacher, frequently, while conducting their study, researchers have provided both technical and pedagogical support to teachers who were willing to allow researchers access to their students (Kasten & Sinclair, 2009). In the body of research, teachers have rarely made decisions on technology implementation without the influence of the researchers. Even though most would agree that teachers and the instructional decisions they make have a large impact on students and student achievement, Kasten and Sinclair (2009) found that researchers have not focused on how, when, and why teachers choose to use technology in their instruction. Further, there is little research on long-term teaching with regular use of DGS or how teachers choose to implement DGS over the course of a school year (Hollebrands, Laborde, & Straesser, 2008; Kasten & Sinclair, 2009).

It is not enough for a teacher just to use technology; it matters how the technology is being used. "The teacher plays several important roles in a technology-rich classroom, making decisions that affect students' learning in important ways. Initially, the teacher must decide if, when, and how technology will be used" (NCTM, 2000, p. 26). NCTM recommends that teachers need to be able to implement mathematical tasks that use the power and capabilities of technology to enhance students' learning.

Using a collective case study, this research project investigated (a) how geometry teachers are implementing dynamic geometry tasks into their instruction; (b) what factors influenced how geometry teachers implemented dynamic geometry tasks; (c) geometry teachers' past enactment of dynamic geometry tasks for student use; (d) and geometry teachers' future aspirations of implementing dynamic geometry tasks for student use. The corresponding research questions are as follows:

1. In what ways are high school geometry teachers currently enacting dynamic geometry tasks for student engagement?
2. Why are high school geometry teachers currently enacting dynamic geometry tasks for student engagement in the ways that they are?

3. How have high school geometry teachers enacted dynamic geometry tasks for student engagement in the past?
4. How do high school geometry teachers foresee themselves enacting dynamic geometry tasks for student engagement in the future?

Although there are different types of geometry taught in high schools such as informal or honors geometry, the focus of this study is on the geometry taught to the majority of the students in that particular high school. This course frequently is called *geometry* without any modifiers in front of the word "geometry."

Elaboration of the Research Questions

The central question this study investigated is how geometry teachers currently are having students use DGS. This question can be conceptualized using Stein, Smith, Henningsen, and Silver's (2009) mathematical tasks framework. Tasks are seen as having three phases. Teachers initially select a mathematical task from instructional or curricular materials, or teachers may create their own. Next, teachers set up, or launch, the task with students. The third stage is how students work on and engage with the task. All three phases, but especially how students interact with the task, affect student learning (Stein et al., 2009). In alignment with the mathematical tasks framework, the major focus of this study was on what DGS tasks geometry teachers use from the written curriculum, how geometry teachers set up these DGS tasks with students, and geometry teachers' perceptions of how the students interacted with these DGS tasks.

Stein, Remillard, and Smith (2007) stated that "curriculum refers to the substance or content of teaching and learning—the 'what' of teaching and learning" (p. 321).

Aligning with the three phases of mathematical tasks framework are three corresponding curriculum types: the written curriculum, the intended curriculum, and the implemented or the enacted curriculum. The written curriculum is what is actually on the "printed pages" (Stein & Smith, 2010, p. 353). The printed pages included the textbooks, textbook ancillaries, stand-alone activity books, and teacher created activities. Stein and Smith use the term "intended curriculum" differently from most other authors. Stein and Smith's intended curriculum means the teacher's lesson plan—either written or in the teacher's head. In other words, what is intended by the teacher as opposed to what is intended by the author of the written curriculum. They use the term to mean that between the written and intended phases, teacher experiences, attitudes, and beliefs along with norms and values of the classroom and the school district influence the curriculum as teachers alter the written curriculum into what they believe to be workable in the intended curriculum (Stein et al., 2007). Within the enactment phase or enacted curriculum, teachers and students "bring the curriculum to life" by how they interacted with each other and the written curriculum (p. 321).

The second question focused on how teacher knowledge and teacher affect, composed of beliefs and attitudes, influences teachers to use DGS in the ways that they are. Further, question 2 also investigated how experiences inside the school such as the school context and culture and experiences outside the school such as preservice programs and professional development have altered teacher knowledge and affect.

Bennison and Goos (2010) developed a framework to model the factors affecting technology use. Their model is based upon the concept of "zones" and extends

Valsiner's (1997) work, which was based upon Vygotsky's (1978) research. The zone of proximal development (ZPD) consists of various types of teacher knowledge, teacher beliefs, and teacher attitudes. The zone of free movement (ZFM) "structures learners' interactions within the learning environment," in contrast; the zone of promoted action (ZPA) represents "the actions of a more experienced or knowledgeable person to promote specific types of learning" (Bennison and Goos, 2010, p. 33). The ZFM consists of access to resources; support from colleagues; institutional culture; curriculum and assessment requirements; and students. The ZPA consists of a teacher's preservice program, field experience, student teaching experience, and professional development.

Bennision and Goos's model can serve as a lens to conceptualize the second research question, which sought to answer why teachers are using DGS with students in the ways that they are. Teachers' ZPD, consisting of knowledge, attitudes, and beliefs about areas such as geometry, the teaching of geometry, the learning of geometry, and DGS, directly influences how teachers use DGS with students. In contrast, the ZFM and the ZPA do not directly effect how teachers are using DGS with students, but instead affect the ZPD.

Teachers' ZFM is focused on how the constraints and affordances inside the school concerning the use of DGS influence teachers' knowledge, attitudes, and beliefs. Initially, teachers need access to DGS software, computers (or other hardware), teaching materials, and time to use DGS with students. Further, the level of support concerning DGS that teachers receive from colleagues and administrators may affect their use of DGS with students. Curriculum and assessment requirements also may influence the use of DGS with students. For example, teachers may be concerned with using DGS depending upon if and how its use is explicitly stated in the school-approved curriculum.

Teachers' ZPA consists of how the knowledge of "experts" affects teachers' knowledge, attitudes, and beliefs about DGS. Beginning with their own experiences as high school students with mathematics, teachers have numerous opportunities to hear or observe others' knowledge, beliefs, and attitudes about using DGS. While teacher candidates in college, teachers' knowledge, attitudes, and beliefs about DGS are influenced by their professors and cooperating teachers. As practicing teachers, there may be opportunities to attend state or national conferences and commercial workshops to learn about using DGS.

The third and fourth questions acknowledged that change happens along a continuum and sought to answer what teachers have done in the past concerning student use of DGS and how teachers want to further develop and extend their students' use of DGS. From NCTM's (2007) *Mathematics Teaching Today*:

> The education of teachers of mathematics is an ongoing process. Teachers are in a constant state of "becoming." Being a teacher implies dynamic and continuous process of growth that spans a career. Teachers' growth requires commitment to professional development aimed at improving their teaching on the basis of increased experience, new knowledge, and awareness of educational reforms. This growth is deeply embedded in teachers' philosophies of learning, their attitudes and beliefs about learners and mathematics, and their willingness to make changes in how and what they teach. Teachers' growth potential can be

> enhanced or limited by the actions of others, including school administrators, educational policymakers, college and university faculty, parents, and the students themselves. (p. 111)

The factors affecting technology integration and teacher change are numerous, complex, and interconnected:

> Issues of teacher change are central to any discussion of technology integration. In general, when teachers are asked to use technology to facilitate learning, some degree of change is required along any or all of the following dimensions: (a) beliefs, attitudes, or pedagogical ideologies; (b) content knowledge; (c) pedagogical knowledge of instructional practice, strategies, methods, or approaches; and (d) novel or altered instructional resources, technology, or materials…. Furthermore, the context in which teachers work often constrains or limits individual efforts" (Ertmer & Ottenbreit-Leftwich, 2010, p. 258).

The model in Figure 1 is used to summarize the research questions and to see how they are connected. The first research question was focused on how teachers are currently enacting dynamic geometry tasks for student engagement. The second research question is represented by the three rectangles in Figure 1. This research question identified factors influencing how geometry teacher were using student tasks. These factors included teacher knowledge, attitudes, and beliefs; teacher experiences inside the school; and teacher experiences outside the school. The third and fourth research questions pertained to teacher change by looking at past enactment and proposed future enactment of student tasks with dynamic geometry software.

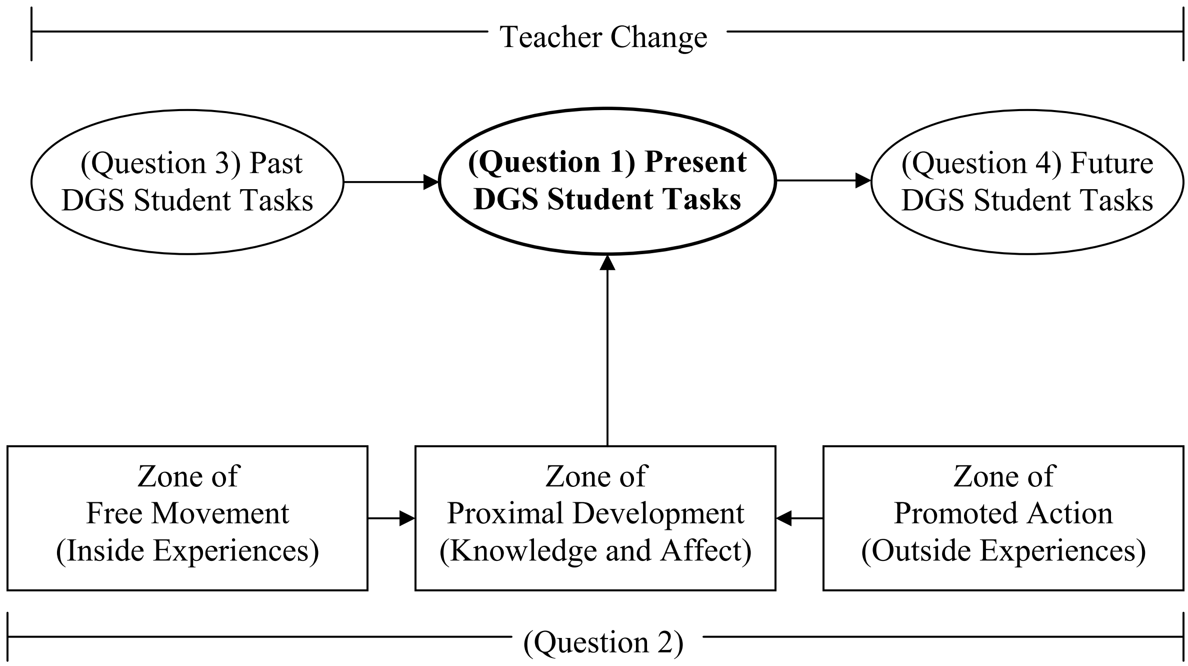

Figure 1. A Visual Model Showing How the Research Questions Are Related.

Educational Significance

This study was focused on how geometry teachers were having students use dynamic geometry software. This study gives a deep analysis of the dynamic geometry tasks teachers implement into their instruction without researcher mediation. The analysis offers insights into how and why geometry teachers use dynamic geometry tasks. It shows whether and to what extent teacher actions align with the recommendations of AMTE, CCSSM, NCTM, teacher candidate programs, and in-service professional development programs for teachers. This study provides information about the development of geometry teachers and what experiences they have had with DGS.

This study helps to fill a void in the current research on DGS. Most of the research has focused on student achievement in settings that were highly influenced by researchers. There are very few studies (e.g., Kasten & Sinclair, 2009; Monaghan, 2004) that focus on how teachers are implementing DGS to engage students with the usual amount of directives and constraints and without mediation from outside the school district.

Delimitations of the Study

There are several delimitations of this study. The use of DGS was only studied in the context of geometry even though DGS has evolved into a technological tool being used throughout the high school mathematics curriculum as well as with elementary and middle school students. The geometry course was chosen because the original focus of DGS was the learning and teaching of high school geometry, so it is an ideal setting to investigate how teachers implement DGS into their instruction because it has had more than 20 years to become accepted as a learning and teaching tool in the geometry curriculum. The study did not take into account geometry teachers who are using DGS in their instruction as a demonstration tool during class discussion or lecture.

Although DGS has been used in high school geometry classrooms with students of varying abilities, this study focused on the geometry taught to the greatest proportion of the student population at each high school. Informal geometry teachers or honors geometry teachers who use dynamic geometry tasks with students were not studied. It may be likely that there are differences in the tasks teachers have students engage in depending on student ability. For example, more closed-ended tasks might be given to low achievers, but more open-ended tasks given to high achievers. Limiting the study to the "standard" geometry course taught was an attempt to decrease the variation in ways that DGS are used due to student ability, and instead focused on describing how dynamic geometry tasks are being implemented into instruction to engage "regular" students.

Limitations of the Study

There are some limitations to this study. In any type of research and especially in qualitative research, it is important for the researcher to maintain "emphatic neutrality" (Patton, 2002, p. 49). As a geometry teacher who uses DGS, I have certain views about teaching and learning with DGS. However, as a researcher, I am not "out to prove a particular perspective or manipulate the data to arrive at predisposed truths" (p. 51). I aspired to be a "neutral investigator" with "no ax to grind, no theory to prove, and no predetermined results to support" (p. 51).

The study was limited to be convenient for the researcher (author) in that the cases are volunteers from southwestern Ohio. As such the results of this study may not generalize to a larger population.

The data collection methods also limited this study. The results were based on two forms of data: analysis of the written tasks and teacher interviews. It is possible that a neutral observer of the enactment of the dynamic geometry tasks might have a different interpretation from that of the teachers. Also, student perspectives might not be the same as the teachers. Last, this study only captures teacher perceptions at a particular point in time because only one interview was conducted. Different results may have arisen with multiple interviews over a school year or multiple school years.

Definitions of Terms

> *Dynamic geometry software (DGS)*:
>
> provides certain primitive objects (e.g., points, lines, circles), basic tools (e.g., perpendicular to line *l* through point *P*) for assembling these into composite objects, and several possible transformations, including, for example, reflection through a point or line. It also allows the user to measure certain parts of the drawing, and typically, to trace the path of points, segments, or circles as dynamic transformations are applied. The term dynamic geometry, originally coined by Nick Jackiw and Steve Rasmussen, has quickly entered the literature as a generic term because of its aptness of characterizing the feature distinguishing dynamic geometry from other geometry software: the continuous real-time transformation often called "dragging." This feature allows users, after a construction is made, to move certain elements of a drawing freely and to observe [how] other elements respond dynamically to the altered conditions. As these elements are moved smoothly over the continuous domain in which they exist, the software maintains all relationships that were specified as essential constraints of the original construction, and all relationships that are mathematical consequences of these (Goldenberg & Cuoco, 1998, p. 351).

Several dynamic geometry software packages are available both commercially and for free such as The Geometer's Sketchpad (GSP), GeoGebra, Cabri II, and Wingeom.

"A *task* in a pedagogical setting consists of activities for the purpose of learning" (Leung, 2011, p. 324).

Open-ended tasks or *divergent tasks* using technology are tasks that:

(a) the instructions require that students identify relationships and make conjectures, (b) students play a role in determining what to investigate and how to investigate it, (c) mathematical reasoning and decision making are required throughout the lesson, and (d) if the lesson is implemented successfully, students will have used a variety of methods and made a variety of conjectures. (McGraw & Grant, 2005, pp. 309–310).

Closed-ended tasks or *convergent tasks* using technology are tasks that:

(a) the instructions require that students identify relationships and make conjectures, (b) the instructions specify both what is to be investigated and how it is to be investigated, (c) students move sequentially through the instruction with little or no decision-making required, and (d) if the lesson is implemented successfully, students will have used similar methods and made similar conjectures (McGraw & Grant, 2005, p. 305).

Chapter 2: Literature Review

The review of the literature is organized into three major areas: (a) theoretical perspectives (b) variables or factors that may influence how teachers choose to use DGS with their students, and (c) research on DGS. Theoretical perspectives or frameworks are broken down into two categories: (a) theoretical models of teacher change and (b) frameworks for classifying DGS tasks. The history of the geometry curriculum in the United States is reviewed including the history and development of DGS. The related research on DGS is divided into two areas (a) theoretical perspectives and (b) studies oriented toward classroom use of DGS. In addition to influences from the geometry curriculum and the software itself, three more variables were identified directly related to teachers that may affect how teachers enact DGS student tasks in geometry instruction: (a) teacher knowledge, (b) teacher affect consisting of attitudes and beliefs, and (c) teacher experiences. Teacher knowledge and affect are reviewed through the lens of teacher experiences including preservice teacher programs, the school culture and context, and professional development for in-service teachers.

Theoretical Models of Teacher Change

Teachers changing their instructional practice is not a one-time event, but a developmental process. “The route towards a new style of mathematics teaching is evolutionary rather than revolutionary” (Beaudin & Bowers, 1997, pp. 133–134). Teachers thinking about technology and using technology in their instruction is also an ongoing process. “Teachers move along a continuum from nontechnology stances to incorporating technology extensively and well in their teaching of mathematics” (Zbiek

& Hollebrands, 2008, p. 288). In the literature, researchers have proposed several theoretical models for understanding how in general change might occur in schools and in particular how teachers might choose or choose not to implement a new technology into their instruction. In this section of the literature review, the models are elaborated, compared, and contrasted. Also, acknowledging that adoption of technology does not occur at the same time for everyone in a school, adopter categories are discussed.

Rogers's IDP Model

Rogers's (2003) initial innovation-decision process (IDP) model was first published in 1962, since then his model has become the seminal work on this subject. "An innovation is an idea, practice, or object that is perceived as new by an individual or other unit of adoption" (p. 12). The model initially was developed to study how agricultural innovations diffused through a community of farmers, but since then has been applied to many different areas of social research. Rogers's innovation-decision process model includes five stages: *knowledge*, *persuasion*, *decision*, *implementation*, and *confirmation*. Only the last two stages of the model are stages of "use."

Niess et al.'s Development Model

In 2009, Niess et al. recast Rogers's framework to encompass how mathematics teachers develop in their use of technology. In AMTE's mathematics teacher development model the stages are *recognizing*, *accepting*, *adapting*, *exploring*, and *advancing*. The model's stages of use and nonuse are slightly different from Rogers's initial model because the model includes three stages of "use." The development model can be applied to a teacher's use of DGS.

- When a teacher is at the recognizing level, the teacher uses DGS for personal use, perhaps to verify an answer or gain insight about a theorem, or uses it to create a diagram for a worksheet or test.
- When a teacher is at the accepting level, the teacher begins to think about how to use DGS with students, and may attend a related conference or workshop or discuss DGS with a colleague.
- When a teacher is at the adapting level, the teacher uses some DGS activities with students in class, but there is not extensive use or a systematic plan for the use of DGS.
- When a teacher is at the exploring level, the teacher is consistently using DGS. The activities have a coherence that the teacher is able to articulate.
- When a teacher is at the advancing level, the teacher uses DGS in a way that challenges what is taught and how it is taught; for example, some of the activities used may be impossible to do without DGS or would not exist without DGS.

The development model is further broken down into the four major themes of (a) curriculum and assessment, (b) learning, (c) teaching, and (d) assessing. Within each of the four themes except for curriculum and assessment are additional subcategories called descriptors. In the learning theme, the additional descriptors are (a) mathematical learning and (b) conception of student thinking. In the teaching theme, the additional descriptors are (a) mathematical learning, (b) instructional, and (c) environment. In the assessing theme, the additional descriptors are (a) usage, (b) barrier, and (c) availability.

Models and frameworks are useful for thinking about particular phenomena, but there is a risk in the model becoming too complicated to be useful. With the elaboration of ten descriptors across four themes, the extended model may be too cumbersome. The beginning development model with the five stages based upon Rogers's initial work has sufficient detail to be a useful lens for viewing a teacher's development of technology use.

Beaudin and Bowers's PURIA Model

Beaudin and Bowers (1997) developed the PURIA (Play, Use, Recommend, Incorporate, and Assess) model for how teachers respond to being introduced to Computer Algebra Systems (CAS), but this model can be applied to any technological tool including DGS. Teachers initially *play* around with DGS, trying out its features. Next, teachers *use* it for their own work. For example, teachers might construct a geometric figure using DGS to check an answer to a test question. Next, teachers might *recommend* DGS to some students to check answers to their homework, but DGS is not directly implemented into instruction until teachers are at the incorporating phase. Teachers *incorporate* DGS into class instruction, but without assessment. Last, teachers begin to *assess* students' use of DGS, perhaps through a project or lab activity. Once teachers have reached the assess phase, the technology is now "firmly established in the teaching and learning process" (p. 134).

Hall and Hord's CBAM

Hall and Hord (1987, 2001) developed and refined the Concerns Based Adoption Model (CBAM) to frame how change takes place in schools based upon over 15 years of research in schools. Within their framework are three diagnostic tools, two of which are discussed here: Stages of Concern (SoC) and Levels of Use (LoU). There is an important conceptual distinction between the two. Stages of Concern “addresses the affective side of change—people’s reactions, feelings, perceptions and attitudes—Levels of Use has to do with behaviors and portrays how people are acting with respect to a specified change” (p. 81).

Stages of Concern (SoC)

Hall and Hord (2001) define a concern as the “composite representation of the feelings, preoccupation, thought, and consideration given to a particular issue or task” (p. 61). The seven stages of concern are *awareness*, *informational*, *personal*, *management*, *consequence*, *collaboration*, and *refocusing*. The stages are grouped into three categories: self, task, and impact. The first two stages relate to self, the third stage relates to task, and the last four stages relate to impact. The stages as related to DGS can be summarized as follows:

- In the awareness stage, a teacher is not concerned with DGS.
- In the informational stage, a teacher would like to know more about DGS.
- In the personal stage, a teacher wonders how using DGS will affect him/her including the role as a teacher and his/her adequacy or inadequacy in using DGS.
- In the management stage, a teacher wrestles with how to manage DGS and use it in instruction.
- In the consequence stage, a teacher begins to question how the use of DGS is affecting students, such as achievement and attitudes.

- In the collaboration stage, a teacher begins to communicate and work with other teachers to implement DGS.
- In the refocusing stage, a teacher begins to reinvent his/her use of DGS moving from the initial implementation scheme to something he/she thinks is even better. For example, this could be a teacher changing from using closed-ended DGS tasks to using open-ended DGS tasks in instruction.

Levels of Use (LoU)

In Hall and Hord's (2001) Levels of Use of the Innovation there are eight levels. The first three levels pertain to the nonuser, and the remaining five levels represent the user. The levels are *nonuse*, *orientation*, *preparation*, *mechanical use*, *routine*, *refinement*, *integration*, and *renewal*. Each level can be related to the use of DGS:

- In the nonuse level, the teacher has little or no knowledge about DGS, does not use it, and is doing nothing to become involved with DGS.
- In the orientation level, a teacher is acquiring information about DGS such as reading about DGS on a website or print material.
- In the preparation level, a teacher has decided to use DGS and is beginning to plan how to use it.
- In the mechanical use level, a teacher is using DGS, but much of the focus is on short-term use and day-to-day use with no long terms plans as he/she evolves in the use of DGS.

- In the routine level, a teacher's use of DGS is "stable". He/she has established classroom routines and procedures for using DGS and does not currently plan to make any changes to how DGS is being used.
- In the refinement level, a teacher is assessing the use of DGS and making changes in an attempt to maximize student benefits.
- In the integration level, a teacher is pooling resources and using DGS activities from other sources or one that have been created collaboratively.
- In the renewal level, a teacher is re-evaluating the use of DGS and making major modifications; for example, a teacher may begin to use DGS as their primary curricular materials instead of a textbook.

Discussion of Teacher Change Models

Even though the frameworks have been presented as linear, they are not necessarily sequential as teachers may move through each previous stage to get to the next one. In fact, Zbiek and Hollebrands (2008) proposed using "modes" instead of "stages" for the PURIA model to emphasize that teachers could be at one of the modes without necessarily passing through the previous ones.

All the models have pro-innovation bias, as it does not discuss points where teachers might decide to not use the technology. Rogers's (2003) original model is an exception as the third stage is the decision stage, explicitly showing that a user makes a decision to use or not to use the innovation. Table 1 shows how the "use" phases of each model are related.

Table 1

Comparison of Use Stages of Change Models

Niess et al.'s Development Model	PURIA	CBAM (SoC)	CBAM (LoU)
Adapting	Incorporate	Management	Mechanical
Exploring	Incorporate	Consequence	Routine Refinement
Advancing	Assess	Collaboration Refocusing	Integration Renewal

At the beginning phase, all of the models focus on the individual teacher's personal knowledge and affect concerning the innovation. First with a teacher's own personal use or "playing around with it," then even when incorporating it into instruction with students the teacher is not yet thinking about how it will impact student achievement until later in the development. In Zbeik and Hollebrands's (2008) review of the literature, they found teachers must work through their personal concerns and management concerns with technology before they focus on student learning.

There has been little or no research applying these models to a teacher's enactment into instruction of student DGS tasks. Wiske and Houde's (1993) research on teacher use of The Geometric Supposer, a precursor to DGS, used Hall and Hord's (1987) Stages of Concern and found that this was a reasonable framework to use to model the teachers' concerns with the new technology.

The Individual in Relation to the System

Each of these models has focused on the individual, but each individual teacher functions within several larger more complex systems. For example, each teacher is a member of a mathematics department of varying sizes, the department functions as part of a high school and the high school is part of the school district. Now more than ever due to state and national educational policies, those school districts are part of the state education system and the state education departments are part of federal guidelines, policies, and mandates for education. Within the whole system, this development does not happen uniformly to each individual. Several of the developers of the change models also have categorized adopters according to when they begin to use the innovation.

In Rogers's (2003) model he identified five adopter categories: *innovators*, *early adopters*, *early majority*, *late majority*, and *laggards*. The innovators are outside of the system; for instance, in the case of DGS the creators of DGS environments are the innovators, and they are not a part of the local system such as the teacher's school or school district. The early adopters help to facilitate the more widespread adoption by sharing advice and information with others and often hold leadership roles in the local system either explicitly or implicitly defined. The early majority interact frequently with others and often "deliberate" for some time before adopting an innovation (p. 284). The late majority is often "skeptical" and may not adopt an innovation until their peers pressure them to do so (p. 284). The laggards are resistant and are nearly isolated in their local system. Though their stance may seem rational to them, the laggards are often viewed as being irrational. Rogers found that the five categories are dispersed along a normal distribution with each category making up 2.5%, 13.5%, 34%, 34%, and 16% respectively.

Nocker and Watkins (1997) identified three categories of CAS adopters: flag wavers, general practitioners, and philosophers. Flag wavers "are those who embrace the new technology wholeheartedly, finding innovative ways of breathing new life into tired old syllabi so that mathematics becomes accessible and interesting to a larger proportion of students" (p. 116). At the opposite end of the flag waver are those who are opposed to any use of technology in the teaching and learning of mathematics, the philosophers. A third group, in between the flag wavers and the philosophers are the general practitioners. The general practitioners are the largest group. These teachers could use a new technology, but they are cautious and need to be convinced of the technological tools' benefits.

Summary of Teacher Change Models

Niess et al.'s development model, the PURIA models, and CBAM Stages of Concern and Level of Use were discussed in the previous section. These models provide a useful starting point for framing the process teachers go through when using DGS. All the models emphasize the teacher's personnel knowledge and affect coming before a teacher thinks about how DGS influences student learning. Even if there is access to DGS, adoption in schools and school districts is not uniform across the individuals in the system. In that respect, various adopter categorizations were discussed.

Frameworks for Classifying DGS Tasks

Several different models have been developed to classify student tasks as they appear in print that a teacher implements into instruction. Four different models can be applied to the classification of DGS tasks. In this section, those four classification systems are discussed along with specific examples related to DGS. Then, the classifications systems are compared and contrasted.

McGraw and Grant's Type 1 and Type 2 Lessons

McGraw and Grant (2005) developed a two-tiered framework for classifying technology tasks. *Type 1 lessons* are lessons where all students follow the same instructions and arrive at the same conclusions, whereas *Type 2 lessons* are lessons where students are given choices or must make their own decisions on how to proceed. In a Type 2 lesson, it is unlikely that all students will have the same results. Type 1 lessons can be renamed closed-ended or divergent lessons, and Type 2 lessons can be renamed open-ended or convergent lessons.

Closed-ended lessons are designed so students proceed through a set of directions where if done correctly, all students will reach the same conclusion. For example, if a teacher wanted students to conjecture that the opposite sides of a parallelogram are congruent and so are its opposite angles, the student first would be given explicit directions to construct the parallelogram. Next, explicit directions for measuring the sides and for measuring the angles would be given. Students would be instructed to drag different vertices and to observe the measure of sides and angles. Lastly, students are asked to write conjectures about the sides and angles. "If the lesson is implemented successfully, students will have used similar methods and made similar conjectures" (McGraw & Grant, p. 305).

On the other hand, in an open-ended lesson, students would be either given choices or must make their own decisions on how to proceed. For example, a lesson on parallelograms might still give students instructions for constructing a parallelogram, but after, students would be asked to find as many properties of parallelograms as they can. Suggestions would be given to measure angles and sides and to construct midpoints, altitudes, and angle bisectors to find conjectures. "If the lesson is implemented successfully, students will have used a variety of methods and made a variety of conjectures" (McGraw & Grant, p. 310).

Stein, Smith, Henningsen, and Silver's Task Analysis Guide

Although the mathematical tasks framework is oriented toward the curriculum prospective, Stein et al. (2009) also developed a task analysis guide (TAG) for judging mathematical tasks as they appear in print either as published materials or teacher-created materials. Their classification system is based upon the level of cognitive demand placed on students as they complete the tasks. Stein et al. defined cognitive demand as "the kind and level of thinking required of students in order to successfully engage with and solve the task" (p. 1).

The TAG differentiates between a low level and a high level of cognitive demand. In the low level category, there are two types of tasks: *memorization* and *procedures without connections to understanding, meaning, or concepts*. *Procedures with* *connections to understanding, meaning or concepts* and *doing mathematics* are two types of tasks that are classified as having a high level of cognitive demand.

The TAG can also be applied to DGS tasks; for example, tasks for the Pythagorean theorem can be designed for each of the four types. In a memorization task, a premade construction of squares built outwardly on each side of a right triangle is given to students. Next, students are told to drag the legs of the right triangle to specified integer lengths, to find the length of the hypotenuse, and then to confirm this length with the DGS measurement tool.

In a procedures without connections task, students are given specific instructions on how to construct a right triangle and how to construct the squares outwardly off of each leg of the right triangle. Students then are told to measure the areas of the squares and to drag to make a conjecture based on the Pythagorean theorem.

In a procedures with connections task, students might still be given steps to arrive at the Pythagorean theorem as in the procedures without connections task, but then students are led to use the capabilities of DGS to develop a way to explain why the Pythagorean theorem is true. One way to do this is to use dissections and transformations where the smaller squares are divided into pieces and moved to show they "cover" the larger square (e.g., see Bennett's (2003) A Dissection in *Pythagoras Plugged In*).

Last, in a doing mathematics task, students are first shown a DGS demonstration of the Pythagorean theorem using squares constructed outwardly on each side of a right triangle. Students then are challenged to investigate several scenarios related to the Pythagorean theorem (a) constructing similar polygons other than squares off the sides of the right triangle, (b) constructing polygons inwardly instead of outwardly, (c) using a triangle other than a right triangle, and (d) using a polygon other than a triangle. Examples of these investigations can be found in Bennett's (2003) Wrong-Way Squares in *Pythagoras Plugged In* and Phelps (2010).

Laborde's Four Tier Classification System

Laborde (2001) originally developed a classification system with four types when using Cabri, but it can be applied to all DGS environments. In the first type, DGS is used to facilitate the material while the mathematical task for the most part remains unchanged. For example, essentially the task of constructing a triangle and measuring its three angles in DGS versus drawing and measuring its three angles with paper, pencil, ruler, and protractor are the same. DGS is used to facilitate the mathematics is a second type of task; for example, it is easier to see the centroid divides the median into a 2:1 ratio with DGS than it is without it because of the accuracy of measuring and the ability to drag in DGS. The third type is when using DGS, the mathematical strategies to complete the task are modified. "Making" a parallelogram with paper and pencil is much different from making the parallelogram on DGS. On paper and pencil, students are drawing the parallelogram, but in DGS if the parallelogram is to maintain all of its properties when it is dragged, it must be constructed. Last, a task may only exist in a DGS environment often called "black box tasks" (Laborde, 1998). Students are asked to reconstruct a diagram that is shown or demonstrated by the teacher.

Ng and Teong's Framework

Ng and Teong (2003) developed a framework of DGS tasks with four different levels along with four instructional purposes. Even though Ng and Teong's framework was intended for the instruction of geometry in elementary grades, it can also be applied to the use of DGS at the secondary level. The four different levels are (a) *teacher demonstrations*, (b) *templates and premade sketches*, (c) *guided exploration and construction tasks*, and (d) *black box tasks*. The levels range from highly structured tasks to "free explorations" (p. 5). The four instructional purposes are teach the concept, consolidate the concept, informal proof, and problem solving. The instructional purposes are not elaborated in this review because the intent is to analyze tasks that are in print. Without a teacher interview, it would be difficult to discern where these tasks came from in the instructional sequence to be able to determine the purpose.

Discussion of Task Frameworks

The main theme of all four frameworks presented is the degree of problem solving flexibility and options available to the students when completing the DGS task. Using McGraw and Grant's classification as the widest lens, Table 2 shows how the tasks in the other frameworks can be classified as either Type 1 or Type 2 tasks though there is some overlap. In the TAG, the low cognitively demanded activities; students are completing the problems with only one path to the correct answer with little room for creativity or exploration. In contrast, tasks that have a high level of cognitive demand, allow students to explore variety of solutions methods.

Table 2

Comparison of Task Frameworks

McGraw and Grant (2005)	Stein, Smith, Henningsen, and Silver (2009)	Laborde (2001)	Ng and Teong (2003)
Type 1: closed-ended	Memorization Procedures without connections	DGS facilitates the material aspect DGS facilitates the mathematical aspect Tasks changed when given in DGS environment	Level 1: teacher demonstration Level 2: teacher or premade sketches Level 3: guided exploration
Type 2: open-ended	Procedures with connections Doing mathematics	Tasks changed when given in DGS environment Tasks only existing in DGS environment	Level 3: construction tasks Level 4: black box tasks

The first three types of tasks in Ng and Teong's framework would not allowed for much student exploration or investigation. The construction tasks may allow for exploration depending on how the task is enacted by the teacher; for this reason, construction tasks are classified as both Type 1 and Type 2 lessons. Black box tasks that are referred to by Ng and Teong and Laborde are Type 2 lessons because students are shown a sketch or told a scenario and ask to produced a dynamic sketch that behaves in that manner. Students still do not have complete control in the black box tasks unless they have posed their own problems; but nonetheless, there is a high degree of student autonomy.

Laborde's other three types of tasks also fit into McGraw and Grant's framework. Both DGS facilitating the material aspect and DGS facilitating the mathematical aspect would likely be structured as Type 1 lessons. Tasks that are changed when given in the DGS environment could be either Type 1 or Type 2 lessons depending on the amount of direction given to students. In the example of the parallelogram, if students are given explicit instructions on how to construct the parallelogram, then it is a Type 1 lesson. On the other hand, if students are asked to create a parallelogram that cannot be "messed up" in a DGS environment, then it is a Type 2 lesson. The parallelogram task could also be viewed as a black box task because students are asked to create a diagram in DGS.

Summary of Task Frameworks

In this section, four frameworks for analyzing DGS tasks as they appear in print were described: McGraw and Grant's; Stein et al.'s; Laborde's; and Ng and Teong's. McGraw and Grant's framework was used as a lens to view the other three frameworks. The degree of flexibility and independence that students have when working on the task is the main theme of all four frameworks. The more open-ended the task, the higher the cognitive demand for students. The classification of DGS tasks will be revisited in Chapter 3 when discussing the methodology for the document analysis.

The History of the Geometry Curriculum in the United States

This section gives an overview of the history of the geometry curriculum in the United States with special attention to the different catalysts for change in the geometry curriculum. The overview of the history is divided into three time periods: 1844–1929, 1930–1979, and 1980–2000. Even though Masingila (1993) suggests that geometry has been taught the same for the last 100 years, there still has been considerable debate among the mathematics education community concerning the teaching and learning of geometry. For example, in Ludwig's (1997) comprehensive geometry curriculum bibliography of NCTM's monthly journal *Mathematics Teacher*, there were 97 articles published from 1909–1996 about the teaching and learning of geometry. Ludwig's bibliography was used as a starting point for this review and analysis of how the geometry curriculum has changed in the United States.

1844–1929: Beginnings and Committee Reports

Geometry was not taught in high schools in the United States until 1844, when universities began to place geometry on their lists of entrance requirements effectively moving the study of at least some geometry from the universities to the high schools (Masingila, 1993; Peterson, 1973; Sinclair, 2008). Having been translated into English in 1819 by Charles Davies, Legendre's (1794) geometry textbook was used until late in the 1870s (Sinclair, 2008; Slaught et al., 1911a). Exercises were considered a "new innovation" when they began appearing in textbooks in 1885 (Stone, 1930, p. 238). Exercises included theorems, constructions, and numerical problems. Before 1885, there usually were no exercises in geometry textbooks only theorems to memorize and reproduce.

The Committee of Ten was formed in 1893 and recommended that concrete or experimental geometry be taught in grammar school. The purpose of this course would be to "familiarize the pupil with the facts of plane and solid geometry" (Newcomb et al., 1894, p. 106). It was recommended that the course last for 3 years for 1 hr per week beginning at the age of 10.

The committee also suggested that demonstrative geometry should be studied after the first year of algebra, thus reinforcing that geometry should be taught in the 10th grade. Much of the course should be focused on exercises in reproducing demonstrations of theorems in plane geometry. One of the seven considerations set forth by the committee was the extensive use of oral recitation for the demonstration of theorems and to reject “all proofs which are not formally perfect” (Newcomb et al., 1894, p. 114). Once students have shown proficiency in demonstrative geometry, they should begin to create their own constructions and demonstrations.

In 1911, the provisional report of the National Committee of Fifteen on Geometry Syllabus was published in three parts in the journal *School Science and Mathematics*. The major contribution of the report was that it gave a specific geometry syllabus that included the theorems to be studied both informally and formally. Like the Committee of Ten, it recommended that there be preliminary work on geometry in the elementary grades. In high school, there should be some informal proofs of theorems, but also there were approximately 100 theorems that “must receive formal proof in any well-regulated course in geometry” (Slaught, 1911b, p. 441). Plane geometry should be taught for at least a year, but not more than a year and a half. The committee did not recommend applications problems because they felt “the formal side would suffer” (p. 443). It also stated that there should not be different courses for students with varying abilities, instead the syllabus should be followed except for omitting certain theorems and perhaps solid geometry. In contrast to the Committee of Ten, the committee was concerned that too many exercises interrupted the flow of the presentation and learning of the theorems.

After 1910 when junior highs become prevalent in the United States, most of the recommended concrete geometry was taught in the junior high (Sinclair, 2008). Reeve (1930) also advocated the study of geometry earlier to make the transition to formal geometry easier for high school students.

1930–1979: Curricular and Content Debates

The geometry curriculum from 1930–1979 was centered on a couple of debates: solid geometry combined with plane geometry, proving “obvious” theorems, and different approaches to teaching geometry.

Solid geometry stood as a stand-alone semester course in the 11th grade in high schools, but there was much debate about that course in the 1930s. Much of NCTM’s fifth yearbook *The Teaching of Geometry* focused on this debate. Some argued that solid geometry should be combined with the plane geometry course (Reeve, 1930; Stone, 1933; Wilt 1930). Longley (1930) claimed that for many teachers solid geometry was “distasteful” and for some “mathematicians the study of solid geometry in preparatory school appears to be futile” (p. 32). Allen (1930) summarized the views of teachers who had been part of an experiment in teaching geometry in different configurations. Teachers thought both courses should be kept, but instead the material should be divided into elementary and advanced geometry instead of plane and solid geometry. In Austin’s (1931) Report of the Second Committee on Geometry, he summarized the results of a survey of teachers. Twenty-six teachers were in favor of a one year combined course, but

92 teachers were opposed. Even though there was much debate about plane and solid geometry combined into a single course in the 1930s, the two geometries were not made into one course until the 1960s (Usiskin, 1980).

The second debate focused on eliminating proving many theorems that were "obvious," even though this would lead to a decrease in the "rigor" of geometry as a complete axiomatic system (Birkhoff & Beatley, 1930; Nygaard, 1941; Stone, 1931). Nygard (1941) stated that just "because the early Greeks were led by their enthusiasm to prove everything in sight, it does not follow that present-day high school students should let loose their formal deductive logic whenever and wherever there is a chance to use it" (p. 270). Ending in 1899, several mathematicians including Hilbert spent considerable time fixing some of the flaws that were found in Euclid's geometry as a complete axiomatic system (Fehr, 1973; Masingila, 1993; Webb, 1926). These resulted in axioms that were too complicated to be used in high school geometry; instead, Birkhoff's work from 1929 with the order and completeness of real numbers became the basis for the ruler and protractor axioms that still currently are used (Fehr, 1973; Masingila, 1993). Thus, the sequence of teaching geometry in the 1930s was simpler and more usable than Euclid's geometry (Reeve, 1930).

In the 1960s and 1970s, different approaches to teaching geometry were espoused. In the 1973 NCTM yearbook, *Geometry in the Mathematics Curriculum*, seven different approaches to high school geometry were elaborated: synthetic Euclidean geometry, coordinates, transformations, an affine approach, vectors, an integrated program, and an eclectic program.

Although much of the debate focused around rhetoric and not action, the approaches using coordinates and using transformations resulted in notable published materials that teachers used with geometry students. The School Mathematics Study Group (SMSG) published *Geometry with Coordinates* in 1962. In 1971, Coxford and Usiskin took a transformation approach in their geometry textbook. In addition to the debate about different methods for teaching geometry, manipulatives also become readily available beginning in the 1960s (Sinclair, 2008). Various tools and manipulatives were made or commercially available for the experimentation in geometry such as three-dimensional models, Mira mirrors, and geoboards, Dienes blocks, and Cuisinaire rods.

1980–2010: Three Major Catalysts of Change

There are three main forces behind the reform geometry has undergone since 1980: (a) NCTM publications, (b) acknowledgement of the van Hiele theory as a model for student geometric learning, and (c) the development of computer software, particularly dynamic geometry software (DGS).

NCTM Publications

In 1980, NCTM published *An Agenda for Action: Recommendations for School Mathematics of the 1980s*. Its major recommendation was that "problem solving should be the focus of school mathematics in the 1980s" (p. 1). Additional recommendations included student access to computers to be used in "imaginative ways for exploring, discovering, and developing mathematical concepts," use of classroom time should change, and instruction should consist of both discovery and basic skills (p. 9). Even though the *Agenda* did not contain course specific recommendations, it still carried implications for the geometry teacher. If a geometry teacher wished to follow the recommendations set forth by NCTM, the teacher would have moved the geometry course to one where students worked cooperatively to gather empirical evidence to discover and prove conjectures.

In the 1989 *Curriculum and Evaluation Standards for School Mathematics*, NCTM listed items to receive increased attention and decreased attention in the mathematics curriculum. Items to receive decreased attention in geometry included Euclidean geometry as a complete axiomatic system, geometry from a synthetic viewpoint, and two-column proofs. Even though these recommendations helped to move the geometry course away form a purely demonstrative geometry course, many teachers took these recommendations from NCTM to mean eliminating most or all proof from a geometry course (Usiskin, 2007). One of the intentions was to move the teaching and learning of geometry to a classroom where the inductive and deductive perspectives were strengthened through student experiences (NCTM, 1989).

In the 2000 update *Principles and Standards for School Mathematics*, NCTM presents four broad goals for K–12 geometry:

- Analyze characteristics and properties of two- and three- dimensional geometric shapes and develop mathematical arguments about geometric relationships
- Specify locations and describe spatial relationships using coordinate geometry and other representational systems
- Apply transformations and use symmetry to analyze mathematical situations

- Use visualization, spatial reasoning, and geometric modeling to solve problems (p. 41)

The two geometry standards of synthetic and algebraic from 1989 were combined into a single geometry standard in 2000. The combination of these two standards emphasizes the position that algebra and geometry should not exist in isolation. Also, to emphasize the importance of proof, reasoning and proof was identified as one of the five process standards not just for geometry but also for the entire K–12 mathematics curriculum.

Van Hiele Theory

In 1957, Dutch educators Pierre van Hiele and his wife Dina van Hiele-Geldof developed the van Hiele model for the progression of developmental thinking in geometry (Fuys, Geddes, & Tischler, 1988). The van Hiele model did not receive much attention in the United States until it was translated into English in the 1980s. Since then, the model has influenced the teaching and learning of geometry (Masingila, 1993). Showing the developing importance of the van Hiele model, the first chapter in the 1987 NCTM yearbook *Learning and Teaching Geometry, K–12* was about the van Hiele model (Crowley, 1987).

The model consists of five levels (Battista, 2009):

- Level 1: Visual-holistic reasoning. Using a rectangle as an example, at Level 1 students identify an object such as a door as rectangle.
- Level 2: Descriptive-analytic reasoning. At Level 2, students can name properties of rectangles.

- Level 3: Relational-inferential reasoning. At Level 3, students can determine minimum conditions needed to define a rectangle and make deductive arguments.
- Level 4: Formal deductive proof. At Level 4, students work formally within a deductive system understanding the role of definitions, postulates, and theorems.
- Level 5: Rigor. At Level 5, students can work in other axiomatic systems.

The major implication of the van Hiele model is that students need learning opportunities and experiences to help them develop their thinking and reasoning before they are ready to do formal proof. Most high school geometry students begin at Level 1 (Serra 2003). Supporters of the van Hiele research argued that the organization of the geometry curriculum needed to be changed for students to be successful with proof. Most textbooks had proof throughout the book, but the van Hiele model proposed that students needed time to work with definitions of figures, and then to work with properties of geometry figures before they were ready to successfully write proofs.

Dynamic Geometry Software (DGS)

Ross Finney (1973) envisioned what was to come in high school geometry with dynamic geometry software:

> What has not been mentioned yet is a technological force that is going to make available in the future to most children a kind of mathematics education that simply has not been available: the use of computers, both for instruction and for discovery…. More important still, a computer with a television output can portray dynamic aspects of geometry—aspects whose portrayal is more or less impossible with the standard equipment of blackboard, chalk, and hand computation…. After

> all, much of geometry is dynamic, and with a computer that dynamism can be portrayed…. Once a student can see things move, can see geometric objects change continuously, he can develop a fine intuition about the relation—and it is a dynamic one—between elementary algebra, functions, and geometry (pp. 430–431).

In the mid-1980s, Judah Schwartz and Michal Yerushalmy developed The Geometric Supposer, a precursor to the dynamic geometry software environments of Cabri and GSP. The Supposer consisted of three different programs, one for triangles, one for quadrilaterals, and one for circles. A key feature of the Supposer was to create geometrical figures by repeating a construction using different starting conditions (Ruthven, Hennessy, & Deaney, 2008). Even though the Supposer was not a dynamic environment, particular examples of a general case could be generated and geometric objects could be measured. For example, a triangle with the three medians could be made, so students could see that the medians are concurrent. Multiple triangles could be generated with the medians, giving students more empirical evidence that the medians of a triangle are always concurrent.

Cabri and GSP were developed independently of each other and released at about the same time. Cabri, developed by Jean-Marie Laborde, was published in 1989 in France (Laborde & Laborde, 2008). In 1994, Cabri II was released for the Macintosh and in 1998 for Windows. Cabri also has been available on two Texas Instruments handheld calculators including the TI-92 in 1995 and Cabri-Junior in 2002 on the TI-83 Plus.

GSP began at Swarthmore College as part of the Visual Geometry Project (VGP) directed by Eugene Klotz and Doris Schattschneider (Goldenberg, Scher, & Feurzeig, 2008). Nick Jackiw, an undergraduate student at the time, worked on the programming. The initial version of GSP was released in 1991 with new versions in 1993, 1995, 2001, and 2009.

Cabri and DGS are both very similar in how the user interacts with them with one difference. The order of acting on objects is reversed in the two software environments. Cabri is action then selection, but GSP is selection then action (Laborde & Laborde, 2008). For example, in Cabri when constructing a perpendicular line, the command is selected from the menu first then the segment that the line will be perpendicular to and the point the line is to pass through are selected. In GSP, the line and point are selected first then perpendicular line is chosen from the menu.

Even though GSP and Cabri have been the dominant DGS environments used (GSP typically in the U.S. and Cabri typically in Europe), there are two others DGS environments that warrant mention. Wingeom is a DGS environment that is available for free created by Rick Parris, a teacher at Phillips Exeter Academy in Exeter, New Hampshire. In 1985, Parris had two working versions of Geom: one for two dimensions and one for three dimensions (R. Parris, personal communication, January, 2, 2012). Even though a computer mouse was not yet readily available, Parris had worked on dragging capabilities using the arrow keys on the keyboard before ever seeing the Supposer or any DGS (R. Parris, personal communication, January, 17, 2012).

Eventually, the two-dimensional and three-dimensional programs became one software, and with the advent of Windows was renamed Wingeom.

GeoGebra is another free software available for download available from www.geogebra.com. Markus Hohenwarter who serves as project director created GeoGebra. Michael Borcherds is the lead developer. Version 1.0 was released in 2002 with subsequent versions in 2004, 2008, and 2011. Version 4.0 is currently available in 50 languages. Although Wingeom has been available longer, it was first developed before the Internet, which might explain why GeoGebra likely is used more than Wingeom.

Summary of the Geometry Curriculum in the United States

As Sinclair (2008) pointed out in her review of the history of the geometry curriculum, hers is one of the written curriculum because it mainly was conducted using textbooks. This review of the history of the geometry curriculum is similar in that it is based on committee reports, NCTM publications, and largely opinions of the mathematics education community in *Mathematics Teacher*. It is another matter to see what actually was taught and learned inside the classrooms of the past. In part, this is what this study is concerned with—to find out teacher's perceptions of how DGS tasks are enacted in their classrooms.

Even with the shortcomings of this review of the literature, it still traces a timeline from the beginning of geometry taught in high school in the United States in 1844 to the present—where DGS is readily available for student use both at home and at school. The timeline shows how geometry was viewed as a purely demonstrative course to now when it is viewed as a course with many opportunities for students to engage in problem solving.

Theoretical Perspectives of Dynamic Geometry Software (DGS)

This section of the literature review provides an overview of DGS generally oriented toward the theoretical perspective of the "dragging" process and how this altering of geometric figures has influenced what it means to construct, define, and prove when DGS is integrated into the high school geometry curriculum.

Many mathematics teacher educators, researchers, and classroom teachers have been enthusiastic about the potential for DGS in geometry instruction including de Villiers (1996) who said, "the development of dynamic geometry is the most exciting development in geometry since Euclid," and it saved Euclidean geometry from the "trashcan of history" in some countries (p. 25). In Osta's (1998) view, the geometry curriculum has the potential for the most change because of DGS environments. DGS has the ability to visually make explicit what often beforehand were only thoughts (Leung, 2008). The study of geometry traditionally relied heavily on the use of static figures, but because of DGS, figures can be manipulated and changed as the user interacts with them. In fact, Straesser (2001) claimed that geometry for the user of DGS is "(a) lived in differently; (b) broader in scope; (c) has a new, more flexible structure; and (d) offers easy access to certain heuristic strategies" (p. 331).

Dragging and Its Different Forms

One of the most important aspects in the teaching and learning of geometry with DGS is the ability to drag geometric objects and manipulate them dynamically; this makes the DGS environment distinctly different from the paper-and-pencil environment (Goldenberg et al., 2008; Holzl, 1996; Marrades & Gutierrez, 2000; Scher, 2000). In this respect, DGS is a *reorganizer* as opposed to an *amplifier* (Pea, 1985). Even though DGS can serve as an amplifier when the tasks are fundamentally unchanged, the reorganizer theme fits because proponents (e.g., Goldenberg et al., 2008; Healy, 2000; Laborde, 1998) of DGS environments claim that what they are doing is fundamentally different from what was done before in the teaching and learning of geometry (Heid & Blume, 2008).

The different types of dragging can be viewed through the construction of geometric points in DGS (Hollebrands et al., 2008). A point may be a *free point* that can be dragged anywhere on the computer screen. A second type of point is a *point on an object* whose path is limited to the object it is constructed on; for example, a point constructed on a circle can only be dragged along the circumference of the circle. Last, a *constructed point* cannot be dragged in some DGS environments such as Cabri. On the other hand, in GSP a constructed point can be dragged, but the entire geometric figure is dragged without changing the size or the shape of it. Jones (2000) found that it takes students time to realize the "functional dependency" of some objects—especially how some points can be dragged and others cannot (p. 69).

Mariotti (2000) conceptualized dragging as initially an "externally oriented tool" that helps identify the correctness of the construction task, but then develops to become part of the interpersonal activities, typically as a form of mathematical discussion—interwoven with deductive proof (p. 49). Leung (2011) also took a developmental perspective to dragging, creating an evolutionary model of "student's dragging strategy from the primitive to the sophisticated" (p. 330). The model has three distinct phases (a) establishing dragging practices; (b) focusing on drag-invariant features; and (c) establishing DGS discourse. The first phase is "playing around" with the figure, dragging parts of the figure to get an idea of how it behaves. The second phase is looking for patterns and conjectures, and the third phase is taking what is found in the second phase to pursue additional explorations or constructions.

Other researchers also have classified various methods of dragging, but without a developmental perspective (e.g., Baccaglini-Frank & Mariotti, 2010; Holzl, 2001; Olivero & Robutti, 2007; Ruthven, 2006). Baccaglini-Frank & Mariotti (2010) differentiated between dragging modalities and dragging utilization schemes "to separate what might be observed externally" and what is "an internal mental construct of the user" (p. 229). They identified four types of dragging modalities

- *Wandering/Random Dragging* is looking for interesting configurations or regularities of a DGS figure.
- *Maintaining Dragging* is dragging so that the DGS figure maintains a certain property.
- *Dragging with Trace Activated* is using the trace command with a point so the path of the point is traced out (locus) as the object is dragged
- *Dragging Test* is to see if the constructed figure maintains the desired properties. (p. 230)

Measurement in a DGS Environment

"According to conventional wisdom, geometry students ought not to rely on measurements in order to make statements about geometrical objects" (Gonzalez & Herbst, 2009, p. 160). Despite opposition to measuring in traditional paper-and-pencil Euclidean geometry, measuring is an important tool in DGS environments working in close conjunction with dragging. "Distances, lengths, perimeters, area, and angles of constructed figures can be measured and the measurement changes 'continuously'" when dragged (Olivero & Robutti, 2007, p. 137). Even with this importance of measuring, there has not been as much research focused on measuring in DGS as there has been with dragging. This may be because it is assumed that if a figure has been dragged when looking for conjectures appropriate measurements of the figure also have been made. Much like dragging modes, Olivero and Robutti (2007) identified several different measuring modes.

- *Wandering measuring* is analogous to wandering dragging. Students choose some objects to measure and then drag.
- *Guided measuring* is analogous to maintaining dragging.
- *Perceptual measuring* is used to check a student's intuition about a property.
- *Validation measuring* is used after formulating a conjecture.
- *Proof measuring* is used after constructing a proof perhaps to attempt to find some underlying knowledge about the geometric figure to improve upon the proof.

Constructing in a DGS Environment

In a paper-and-pencil environment, students can *sketch*, *draw*, or *construct* a geometric figure. Even though the definitions for sketching and drawing with paper-and-pencil are not universal, these definitions along with the definition for constructing serve as a useful contrast to drawing and constructing in a DGS environment. A sketch of a geometric figure is done "freehand" with no additional tools. A drawing is completed with a ruler and a protractor, and finally a construction is made with a straightedge and a compass. A sketch of a geometric figure is not very useful in determining conjectures by empirical evidence because it is not accurate enough to measure or to see other invariants such as three lines intersecting in a single point. On the other hand, drawing or constructing a figure, with some care, is accurate enough for students to gather convincing empirical evidence for geometric relationships.

In contrast to paper-and-pencil constructions, in DGS they usually can be completed with different types of tools. For example, classic paper-and-pencil constructions can be duplicated in a DGS environment using the basic tools of the compass, straightedge, and point. A geometric figure also can be constructed by using commands from a menu such as midpoints, angle bisectors, parallel lines, and perpendicular lines in conjunction with the basic tools. A final way to construct a geometric figure in DGS is to use commands from a menu involving transformations such as rotations, translations, and reflections along with the basic tools.

In a DGS environment sketching does not exist, only drawing and constructing. A simple view of drawing and constructing in a DGS environment is whether the geometric figure can be *messed up* or not when parts of it are dragged (Hoyles & Noss, 1994). If the figured can be messed up, the figure was drawn; if it cannot be messed up, the figure was constructed. This is also called *appropriately constrained* or a *robust construction* (Finzer & Bennett, 1995; Healy, 2000). According to Hollebrands et al. (2008), researchers have found that beginners have difficulty in constructing geometric figures that *pass the drag test* and cannot be messed up.

Finzer and Bennett (1995) also classified certain DGS constructions as either *under-constrained* or *over-constrained*. An under-constrained figure is somewhere between a drawing and a construction. It has some of the properties of the geometric figure to be constructed, but not all of them. For example, when attempting to construct a right isosceles triangle, if there is a right angle, but the two legs are not congruent the triangle is under-constrained. These are also *soft constructions* if the user purposefully under-constrains the geometric construction to allow for exploration when dragging to gather empirical data (Healy, 2000). An isosceles triangle with the three angles fixed 50°, 50°, and 80° no matter how it is dragged is an example of an over-constrained construction. Only a class of isosceles triangles has been constructed, all mathematically similar.

Defining in a DGS Environment

Usiskin and Griffin (2008) wrote extensively about the debate on defining quadrilaterals. The debate has centered on the use of *inclusive* versus *exclusive* definitions, typically focused on trapezoids and kites. Inclusive and exclusive definitions for the trapezoid are respectively (a) a quadrilateral with *exactly* one pair of parallel sides and (b) a quadrilateral with *at least* one pair of parallel sides. If the inclusive definition is used for a trapezoid, then a parallelogram is also a trapezoid. On the other hand, if the exclusive definition is used, then a parallelogram is not a trapezoid. Inclusive and exclusive definitions for the kite are respectively (a) a quadrilateral with *exactly* two pairs of congruent sides and (b) a quadrilateral with *at least* two pairs of congruent sides. If the inclusive definition is used for a kite, then a rhombus and a square are also kites. On the other hand, if the exclusive definition is used, then a rhombus and a square are not kites.

Although Usiskin and Griffin researched how quadrilaterals were defined in textbooks, there are important implications for defining in DGS environments. As Straesser (2001) remarked, the geometry for the DGS user is "lived in differently" (p. 331). Proponents for the use of DGS often advocate using it as a tool for student-exploration. De Villiers (2007) has suggested teachers give students premade sketches to explore the properties of geometric figures. Two GSP activities books include explorations for students using already constructed quadrilaterals (Bennett, 2002; Wyatt, Lawrence, & Foletta, 2004). In the high school version of the activity, the trapezoid follows the inclusive definition where it can be dragged to from a parallelogram (Bennett, 2002). Likewise, the kite is inclusive because it can be dragged to form a rhombus or a square. Interestingly, the trapezoid also can be dragged, so it becomes a "crossed" quadrilateral, and the kite can be dragged, so it becomes concave (sometimes called a dart). In the middle school version, the trapezoid is constructed as part of a triangle with the top of the triangle hidden. Constructing it in this manner forces the exclusive definition prohibiting more than one pair of parallel sides and also preventing the trapezoid from crossing.

Triangles are another example of geometric figures that are "different" in DGS environments versus paper-and-pencil environments. A scalene triangle is defined as a triangle with no equal sides, but a scalene triangle does not exist in a DGS environment. When an ordinary triangle is constructed with segments and points, it can be dragged to include all three types of triangle classification by sides: scalene, isosceles, and equilateral.

These differences in how constructions "behave" in DGS have the potential to greatly influence novices' conceptions of definitions of geometric figures especially if they are using DGS regularly. In fact, in the high school geometry activity, students are asked to write definitions for the trapezoid and kite based upon their observations. Interestingly in the activity notes, teachers are told that the inclusive definitions "might be more appropriate here," but no mention is given to the crossed trapezoid or the concave kite (p. 237). On the other hand in the isosceles trapezoid student activity sheet, a trapezoid is defined as "quadrilateral with exactly one pair of parallel sides" (p .97). Although most experienced users are aware of the issues related to how constructing figures in DGS at times contradicts the "standard" paper-and-pencil definitions, there appears to be little or no research in this area.

The Role of Proof in a DGS Environment

It has been well documented in the research (e.g., Clements & Battista, 1992; Suydam, 1985; Usiskin, 1980) that students struggle with both seeing the purpose for proof as well as in actually doing proofs. Some teachers in a paper-and-pencil environment have focused on trying to convince students that there is some doubt to what students have been asked to proof, typically out of a textbook, is really true. Students already "know" these are true, or they would not be in the textbook. Teachers have had as much success in trying to convince students that the empirical evidence from DGS could be wrong as they did at convincing students that there was doubt that the proposed theorems in their textbooks were true. The case is the same; in a DGS environment, students are convinced what they see on the screen by dragging combined with measurements is true. De Villiers (1996, 1998, 2003, 2007) along with others has repeatedly advocated that the role of proof needs to change (e.g., Chazan, 1993; Olive, 1998). Even some teacher candidates in undergraduate programs after using DGS have questioned the need for proof (Pandiscio, 2002). The role of proof should not be *verifying*, but be on *explaining* why the conjecture is true.

Along with this role of proof as explaining, it must be part of the problem-solving and exploration process. Many researchers and teachers see DGS as a bridge between construction activities and proof (Battista, 2009). Even though exploring and proving are different activities; they are related and connected serving to "reinforce each other" (Hanna, 2000, p. 14). Justification comes from the need to explain why a construction works (Mariotti, 2000). There are both empirical justifications and deductive justifications, and these need to be linked together thoughtfully (Guven, Cekmez, & Karatas, 2010; Hoyles & Jones, 1998; Marrades & Gutierrez, 2000). Sinclair (2004) believed that some student issues with proof could be mediated by using DGS as an amplifier when working with proofs containing overlapping figures. She created premade sketches that used action buttons to highlight, color-code, or separate the figures in an attempt to help students "see" the path to a proof.

The pedagogical shift is on how teachers present the role of proof. How teachers communicate to students about proof is important for student engagement and for students understanding the need of proof. According to de Villiers (1998):

> Instead of saying the usual, 'We cannot be sure that this result is true for all possible variations and we therefore have to (deductively) prove it to make absolute sure,' students find it much more meaningful if the teacher says: 'We now know this result to be true from our extensive experimental investigation. Let us however now see if we can EXPLAIN WHY it is true in terms of other well-known geometric results, in other words, how it is a logical consequence of these other results.' (p. 388)

In either a paper-and-pencil or DGS environment, there is an opportunity for interpersonal communication that can arise from justifying and proving—classroom discourse (Mariotti, 2000). Proofs need not be done in isolation where students work individually; DGS adds another layer to discourse because students can actively explore dynamic geometric figures. Researchers have discussed how proof can play a social and socio-cultural role when students interact with each other and with the teacher (e.g., Hollebrands et al., 2008; Jones, 2001). Jones (2001) mentioned a benefit from this social setting as students begin "talking the language of geometry even before being introduce to the technical terminology" (p. 53).

Summary

In this section of the literature review, theoretical perspectives of dynamic geometry software were elaborated. DGS fundamentally changes the nature of geometry due to its dynamic capabilities through dragging. Because of the ability to drag geometric objects, the nature of what it means to construct and define in geometry is changed. Also, the nature of proof changes due to the ability to gather convincing empirical evidence. The shift in proof from verification to explanation serves as a venue for student discourse in conjunction with DGS.

Dynamic Geometry Software and the Teacher

This section of the literature review looks at the empirical research on DGS. First, the existing research that was reviewed was classified into several categories. Next, studies that most closely align with this research project are reviewed and discussed more in-depth.

The existing research on DGS has fallen into one or more categories.

- The researcher has either designed the activities, selected the activities, or both either with or without collaboration from the teacher (Choi-Koh, 1999; Gonzalez & Herbst, 2009; Hannafin, Burress, & Little, 2001; Hannafin, Truxaw, Vermillion, & Liu 2008; Healy, 2000; Idris, 2009; Jones, 2000; Kasten & Sinclair; 2009; Ubuz, Ustun, & Erbas, 2009, Uworwabayeho, 2009).
- At times the researchers have not only designed and selected the activities, but also delivered the instruction to students either exclusively or cotaught with the

classroom teacher (Hannafin et al., 2001; Hollebrands, 2003; Shafer, 2004; Ubuz et al., 2009; Uworwabayeho, 2009).

- Much of the existing research has focused on student thinking when using DGS usually in pairs and often not part of the “normal” instructional process (Choi-Koh, 1999; Healy, 2000; Jones, 2000; Mariotti, 2000; Vanicek, 2008).
- The time period for the research studies are often very short ranging from 2 weeks to 10 weeks (Choi-Koh, 1999; Hannafin et al., 2001; Hannafin et al., 2008; Hollebrands, 2003; Idris, 2009; Sinclair, 2004; Ubuz et al., 2009; Yanik & Porter, 2009).
- Studies that have spanned for a year often did not have frequent DGS use and instead had large breaks in between subsequent student-use of the software (Jones, 2000; Uworwabayeho, 2009).
- Students were often using the software for the first time, and therefore had to learn to use some aspects of the software before they could use the software to learn mathematics (Hannafin et al., 2001; Healy, 2000).
- Frequently, research was conducted using the researcher’s own students. Many times students in college who are in teacher candidate programs are the subjects of the research (Guven et al., 2010; Habre, 2009). This type of research is still useful when teacher candidates are studied through the lens of how they are beginning to develop their use of DGS as they transition to practicing teachers.

This section of the literature review looks at the research that is mostly closely related to the first research question of this study: the sort of student DGS tasks teachers are enacting in their instruction. Within this question are three distinct phases, (a) how and what tasks teachers are selecting, (b) the different roles that teachers take on while students are working on the tasks, and (c) after the use of DGS how teachers connect those lessons to classroom instruction.

A useful lens for viewing this part of the literature review is through curriculum types. The teacher takes the *written curriculum*—the curriculum on the printed pages of textbooks, activity books, and worksheets—and decides what they intend to do with those written pages in their instruction. The *implemented or enacted curriculum* is how teachers and students interact with these materials during instruction. During and after the enactment of the lesson, there is the *learned curriculum*—what students have learned and are able to apply from the lesson.

The Written Curriculum

According to Kasten & Sinclair (2009), based upon anecdotal and hearsay evidence when using GSP, teachers tend not to design their own tasks, but instead use activities available from Key Curriculum Press in the form of curriculum modules or from the Math Forum. Even with Kasten and Sinclair's presumption, there has been some research that looked at teacher-created tasks. Teachers who wrote their own materials had an orientation very close to paper-and-pencil geometry and focused on creating observational tasks to facilitate student conjecturing (Labrode 2001; Ruthven et al., 2008). Belfort and Guimaraes's (2004) study focused critically on analyzing and classifying the materials teachers had created.

Teacher Created Materials

Belfort and Guimaraes (2004) analyzed teacher created instructional materials for DGS using three different orientations: software, subject, and instructional. *Software perspective* materials focused on teachers mastering the use of software such as using animation and action buttons. In this type of instructional materials, students are "treated as spectators" where they are just clicking on buttons and the software does the rest (p. 506). At the opposite end, materials are oriented toward the *subject perspective* with minimal use of DGS tools. For example, students are given a triangle with the angle bisector of each interior angle constructed—all that is left for students to do is drag one of the vertices to notice the angle bisectors intersect in the same point. In this perspective, students are also mostly just spectators. The last category is the *instruction perspective* where the interest is in creating "computer assisted learning activities" (p. 507). An example of a teacher-created item with this perspective is one where students were given rectangles and asked to find how many unit squares were needed to "cover" the rectangles. Belfort and Guimaraes found that 10% of the activities created were from the software perspective, 10% toward the subject perspective, and about 50% toward the instructional perspective. The remaining 30% of the activities was labeled "well balanced" because they had all three orientations (p. 508). Teachers who had well-balanced materials created a laboratory lesson where students were engaged in discourse.

How, Why, and What Teachers Chose

A few studies have reported the frequency of the geometry topics teachers used when implementing DGS with students. The most frequent use of GSP was to establish angle properties such as relationships among angle pairs created by parallel lines cut by a transversal (Ruthven, 2006). Leong (2003) found that the most taught topics with GSP for lower secondary teachers were (a) angle properties of a polygons; (b) angle properties of points and lines; (c) properties of triangles; and (d) transformations, and upper secondary teachers used it for (a) angle properties relating to circles; (b) transformations; and (c) locus problems.

Some research has focused on the reasons that teachers chose to implement particular DGS tasks in their instruction. Seventy-one percent of teachers stated they chose activities to enhance student understanding (Kasten & Sinclair, 2009). Teachers also chose activities that were closely integrated with their existing curriculum. Another major theme in Kasten & Sinclair's view was teachers chose activities to "solve current and persistent difficulties they have in delivering that curriculum" (p. 142). Teacher also chose to use DGS for its ability to guide students to discover properties for themselves (Ruthven et al., 2008).

The Implemented Curriculum and Teacher Roles

In a study of European countries, teachers do not appear to be using DGS regularly (Intergeo Consortium, 2008). It is conjectured that similar results probably hold for teachers in the United States. Few studies have reported the number of times that teachers used DGS during the school year, but Coffland and Strickland (2004) found there to be a statistically significant inversely-related relationship between the number of geometry sections taught and teacher technology use. Ruthven et al. (2008) observed that classes over the course of the year usually were not involved in more than a handful of lessons with DGS. Marrades and Gutierrez's (2000) study and Shafer's (2004) study exemplified extended frequent use of DGS with a research-collaborator approach. Monaghan's (2004) study appears to be one of the few that was not highly mediated by the team of researchers, although it did have some training at the start of the project and three half-days during the project. Monaghan was also available to provide both technical assistance and suggestions for activities, but he believed his help was kept to a minimum.

Three Research Studies

Marrades and Gutierrez's (2000) study consisted of 30 Cabri activities over 30 weeks. One of the researchers was also the teacher of the class. They were able to eliminate the software acclimation period for students because students had used the software extensively the previous year. Another strength of Marrades and Guiterrez's study was that it was part of normal class content. The activities students completed focused on three phases: create a figure and explore it, generate conjectures, justify conjectures they had stated for each activity. The results of the research showed that students moved toward more elaborate explanations mainly attributed to carefully structured activities.

Shafer's study is one of the few examples in the research that focused on high school teachers' implementation of DGS into instruction even though it was highly mediated by the researcher. Over the course of 2 months, one teacher implemented 13 GSP labs, and the other teacher implemented 14 GSP labs. One teacher's use was oriented toward using technology to save time and to control while avoiding anything that was unfamiliar to her. On the other hand, the other teacher used GSP as a tool for discovery. Both of these teachers' use of GSP generally was aligned with their overall conception of teaching and learning mathematics. Unfortunately, after the research period during the remainder of the school year, one teacher did one more GSP activity and the other teacher did not use GSP anymore. The teachers cited lack of time and that GSP did not meet state standards.

Monaghan's (2004) study consisted of 13 mathematics teachers who decided they wanted to implement more technology during the school year. The teachers were selected because they were novices at incorporating technology into instruction. They received instruction for a day with different technological tools including spreadsheets, graphical packages, calculators, algebra systems, and DGS. Monaghan found several themes. Activities with technology were found to be more open-ended than activities without technology. Lessons used one form of technology with a worksheet. Tasks moved from a technology-oriented perspective to a mathematically-oriented perspective. Social interactions changed to address groups or pairs of students instead of individuals. Monaghan rejected the notion that using technology moves teachers from teacher-directed instruction to student-centered instruction; instead, he argued that this shift is tied to more than just the technology.

The Role of the Teacher

Meaningful teacher-student interaction is necessary for effective instruction and learning to take place (Olive & Lobato, 2008 Yu, Barrett, & Presmeg, 2009). "A major responsibility for teachers, regardless of the role of technology, is the orchestration of mathematical activities in the classroom" including the amount of assistance and the amount of student control (Heid & Blume, 2008, p. 421). The role the teacher assumes while students are working with DGS is another important factor (Jones, 2001; Mariotti, 2000).

Zbiek and Hollebrands's (2008) review of the literature identified 11 teacher roles for "technology-using mathematics classrooms" (p. 298). There are a three teacher roles that directly relate to what the teacher is doing when the students are working with DGS: *collaborator*, *counselor*, and *technical assistant*. Collaborator and counselor are opposites in that as a collaborator the teacher is not familiar with the problem or the solution, so they are also a learner. In contrast, as a counselor the teacher is familiar with the problem and solution and is able to assist students when they ask for help. As technical assistant, teachers help students with difficulties with the software or the computers—a role Ruthven (2006) also found.

In Jones's (2002) review of the research, one of his findings was that it matters how DGS is used. Holzl (2001) lamented that DGS often is only used in a verifying manner instead of an active method of knowledge acquisition. Rarely was DGS used to solve or prove; instead it was used only for empirical confirmation. One way to view these criticisms of DGS use is through the variable of control. On one hand, activities may be completely control by the teacher where the teacher is using the software while students watch. On the other hand with student-controlled activities, students are using the software and are able to make choices as they work on the tasks. Leong (2003) researched the modes of GSP that were used. The most widely used mode was teacher-controlled, and the least used mode was student-controlled. Teachers used GSP to draw diagrams for worksheets and test papers, teachers showed animations in front of the class, and teachers clicked and dragged predesigned templates to show geometrical properties were the three highest modes. Providing templates for students to observe and conjecture properties and students exploring hands-on activities freely were the two least selected modes. These results might be attributed to teachers still being at an early stage of technology integration because GSP had only been introduced in Singapore schools in the last 3 years.

Similar to Leong's research, Ruthven (2006) found themes of teacher control. Teachers controlled the access; some did not let students touch the software, but other teachers did let students use the software, albeit with premade sketches. There was a wide range in wanting to let students experience issues with the tool versus not wanting students to be exposed to any issues. Hannafin et al.'s (2001) study found that the teacher believed she had lost control over the students' learning when students used DGS.

Issues with DGS and the Enacted Curriculum

Even when teachers implement DGS into their instruction, de Villiers (2007) listed several pitfalls of DGS that need to be addressed to ensure effective implementation. The *no change* pitfall and *insufficient rethinking and evaluation* pitfall both fit with Pea's (1985) amplifier concept as opposed to moving the use of DGS to a reorganizer. In the no change pitfall, the teaching style does not change and essentially DGS is used as a "glorified blackboard" (p. 47). With the insufficient rethinking and evaluation pitfall, teachers have troubles envisioning what new thinking, learning, and teaching DGS is available for students and teachers.

The *first master software* and c*onstruct dynamic figure*s pitfalls both work together putting teachers in the role of gate-keeping students from using DGS to study mathematics that currently is accessible to them. The first master software pitfall occurs when teachers expect students to be completely proficient with the software before studying appropriate mathematics. In conjunction, the construct dynamic figures pitfall occurs when before allowing students to study properties of geometric figures, students first must complete the more difficult cognitive task of constructing these figures. De Villiers offers the solution of introducing specific features of the software when needed and having students using premade sketches; however, Sinclair (2004) noted that students usually stopped dragging and instead focused on static diagrams when given premade sketches. Sinclair (2004) suggests students need to be directed to make explicit use of dragging to explore and task designers should focus on questions directly related to motion when using premade sketches.

Three more pitfalls also work together: *painless learning* pitfall, *visualization always makes learning easier* pitfall, and *DGS makes practical obsolete* pitfall. Some believe that because DGS is an amplifier that visualization always makes learning easier and painless. Because of the two previous attributes of DGS, there is no need for any concrete learning experiences with physical manipulatives. Teachers need to be aware that the learning probably is more difficult with DGS because the cognitive level of demand is higher and concrete experiences should be used in conjunction with DGS.

The Learned Curriculum

It also is important how teachers connect DGS to the classroom or if the DGS tasks exist as only stand-alone experiences for students. For example, Marrades and Gutierrez (2000) had students work in pairs and write a single explanation agreed upon by both students. At the next class, the teacher gave students a list of different answers to the problem. Several students were selected to present their solutions to the group. The teacher used a guided class discussion for "the solutions presented, the correctness of the conjectures, and the validity of their justifications" (pp. 98–99). With this technique, DGS is directly connected to subsequent classroom discourse and did not exist in isolation.

Even though most of the advantages for using DGS that "have been studied have been observed in small, usually well-supported educational contexts," several of the highlights are summarized (Clements, Sarama, Yelland, & Glass, 2008, p. 144).
In Jones's (2000) study, students moved from mere description to explanation with software to explanations entirely in mathematical context. Hollebrands (2003) found similar results to Jones, but with the study of transformations. Idris (2009) and Ubuz et al. (2009) found a significant difference in student achievement, but in Ubuz et al. the significant difference was no longer present after 25 weeks. Researchers also found that student dispositions improved as they gained some control over their learning (Idris, 2009; Hannafin et al., 2001).

Summary

This section of the literature review summarized the empirical research on DGS. Much of the research has not focused on teachers having students use DGS in the natural setting within the school; instead, a team of researchers have altered and changed the classroom instructional setting. There is still some research that related indirectly to this research project. The three studies that mostly closely align with this study are Marrades and Gutierrez (2000), Shafer (2004), and Monaghan (2004). Some of the issues of why teachers are using DGS were superficially presented; the next section explicitly discusses these variables.

Teacher Knowledge, Affect, and Experiences

This section of the literature review discusses teacher knowledge and teacher affect in the context of experiences mainly related to the use of technology. Teacher experiences including both preservice and in-service experiences influence their knowledge, affect, and beliefs. In turn, teacher knowledge, affect, and beliefs influence how and why teachers choose to do what they do in the classroom. The cycle starts over again as teachers encounter new experiences. This cyclical process often causes teachers to change their practice as they progress through their careers. Some general remarks are made about affect and knowledge, then instead of organizing the review by affect and knowledge it is organized by preservice teaching experiences, experiences within the school culture and school context, and professional development opportunities for inservice teachers.

Researchers seldom study knowledge and affect as static measures. Often, researchers are interested in measuring the changes in these variables with respect to various instructional technologies including DGS:

> Part of the reason for the lack of explanatory power in this research base is that the majority of studies have been devoted to tracing changes in individual teachers' knowledge, beliefs, and instructional practice, while ignoring the fact that teachers' thinking is often influenced by both the social contexts in why they operate and the institutional cultures that profoundly shape the meaning of their work (Windschitl & Sahl, 2002, p. 166).

When discussing knowledge and beliefs, there are several different terms used in the literature that often are not clearly defined (Phillip, 2007). For this review, Philipp's definitions of affect and knowledge are used. Affect is "a disposition or tendency or an emotion or feeling attached to an idea of object" comprised of emotions, attitudes, and beliefs (p. 259). Knowledge is "beliefs held with certainty or justified true belief" (p. 259). Finally, Phillip differentiated the notion of *conception* as depending if it is a belief or knowledge to the particular individual that holds it. It is knowledge if that person could not see the disagreement of that conception as legitimate. On the other hand, the concept is a belief if that person could see another persons' point of view different from theirs even if they do not agree with it.

Teacher Knowledge

Although researchers have conceptualized and studied various forms of teacher knowledge (e.g., Ball, Thames, & Phelps, 2008; Shulman, 1986), this review takes the perspective of three domains of knowledge and their possible intersections. *Content knowledge* (TK), *pedagogical knowledge* (PK), and *technology knowledge* (TK) are three basic forms of teacher knowledge. Each of these domains of knowledge can be paired with the others domains to form three additional domains of *pedagogical content knowledge* (PCK), *technological content knowledge* (TCK), and *technological pedagogical knowledge* (TPK). Finally, all three of the domains can intersect to form the domain of *technological pedagogical content knowledge* (TPACK).

Before Shulman's (1986) address, content knowledge and pedagogical knowledge usually were considered separately. To be an effective geometry teacher, one needs knowledge of geometry and knowledge of general teaching techniques and practices. Shulman proposed that there was a domain of knowledge for effective teaching that involved the interaction of content knowledge and pedagogical knowledge: pedagogical content knowledge (PCK). PCK goes beyond general subject matter knowledge and extends to subject matter knowledge needed for teaching (Shulman, 1986). For example, having knowledge of how to do formal geometric proofs is not enough; a geometry teacher must have knowledge of how to develop student understanding of proof and student capacity to do and write proofs as well as knowledge of what misconceptions or difficulties students might have and how to help students when they encounter these obstacles.

Mishra and Koehler (2006) acknowledged Shulman's framework of pedagogical content knowledge and extended it to include the new domain of technological knowledge. Technological knowledge (TK) was seen much the same way as content knowledge and pedagogical knowledge before Shulman's work, that is, as a separate domain of knowledge for teaching. For example, geometry teachers need to have technological knowledge of how to use DGS. Mishra and Koehler argued that the relationships among content, pedagogy, and technology are complex and interconnected, and therefore identified the new domain of technological pedagogical content knowledge (TPACK). According to Cox (2008):

> Technological pedagogical content knowledge is a knowledge of the dynamic, transactional negotiation among technology, pedagogy, and content and how that negotiation impacts student learning in a classroom context. The essential features are (a) the use of appropriate technology (b) in a particular content area (c) as part of a pedagogical strategy (d) within a given educational context (e) to develop students' knowledge of a particular topic or meet an educational objective or student need (p. 65).

The "TPACK acronym has been identify differently in many publications: Technological Pedagogical Content Knowledge; Technological, Pedagogical, and Content Knowledge; Technology, Pedagogy, and Content Knowledge; Information and Communication Technological Pedagogical Content Knowledge" (Niess, 2011, p. 303). For example, Shafer (2004) found that there were three forms of technological knowledge needed to teach with GSP: *technical knowledge*, *pedagogical knowledge of technology*, and *pedagogical content knowledge of technology*. Technical knowledge is a form of knowledge where teachers know about the technology; for example, knowing how to rescale the axes in GSP. Pedagogical knowledge of technology is knowing how to use GSP to meet the goals of instruction. Shafer defined pedagogical content knowledge of technology as knowing "how students think about and learn mathematics with technology" (p. 175).

Criticisms of TPACK

TPACK is likely somewhat of a misnomer as it also includes attitudes and beliefs and not just knowledge. Archambault and Barnett (2010) appear to have been the only researchers to question the construct of TPACK. They used 595 surveys completed by teachers at online schools to study the existence of the seven mutually exclusive domains of CK, PK, TK, PCK, TPK, TCK, and TPACK. In their results they found that only three factors existed: pedagogical content knowledge, technological-curricular content knowledge, and technological knowledge. "Measuring each of these domains is complicated and convoluted, potentially due to the notion that they are not separate" (Archambault & Barnett, 2010, p. 1661). Koehler and Mishra (2008) also acknowledged that technology integration involved a degree of convolutedness. "Integrating technology in the classroom is a complex and ill-structured problem involving convoluted interaction of multiple factors, with few hard and fast rules that apply across context and cases" (p. 10).

The teams of researchers who usually are mathematics teacher educators with strong beliefs and attitudes about how to use technology have researched the development of TPACK in their preservice teachers. The researchers have taken a student-centered constructivist approach to teaching. Kelly (2010) performed a content analysis of TPACK journal articles from 2006 to 2009. He found that the TPACK framework was used as a conceptual tool. Authors often claimed that there was a development or change in TPACK despite that there does not appear to be accurate methods for measuring TPACK (Kelly, 2010). Even though, this review has discussed the idea that TPACK is a "fuzzy" construct as a form of knowledge, it still is referred to as such when reviewing the related literature, but still acknowledging that TPACK also includes the affective domain and not just the cognitive domain.

Teacher Affect

Attitudes and beliefs are divided into two categories: the area where teachers form attitudes and beliefs about technology and the area where teachers develop self-efficacy. Concerning technology, Phillipp (2007) summarized that "teachers' beliefs about appropriate use of technology for children are constrained by their beliefs about mathematics and their beliefs about teaching and learning mathematics" (p. 294). Also, as other researchers have pointed out, when to use technology—especially after mastery appears to still be a major belief.

Teacher Experiences

Experiences that influence how one teaches begins when that person is a student and continues when they are a preservice teacher at a university. The experiences continue as an in-service teacher within the context and culture of the school and various professional development opportunities and workshops. Because teachers' careers often extend 2 or 3 decades, it is doubtful that their preservice teacher programs are adequate for the duration of the career no matter how good the programs (Niss, 1998):

> A teaching career may span thirty to forty years. The ultimate test of a teacher's ability to adapt to new curriculum developments over such a time span may lie in the extent to which he has developed the inclination and ability for continued self-education in mathematics (Crosswhite, 1973, p. 460).

In Hall and Hord's (2001) own research as well as their review of existing research they found "that most changes in education take three to five years to be implemented at a high level" (p. 5). Each new unit, for example, school, district, or state, takes 3–5 years. A one-time professional development experience in August does not have much effect on change; "change is a process, not an event" (p. 4).

Preservice Teachers

Koh and Divaharan's (2011) study of 74 elementary preservice teachers early in their course work learning to use interactive whiteboards over 10 weeks revealed very little development of TPACK. From student reflections completed at the end of the 10 weeks, only 3.4% of the comments showed evidence of TPACK. An interesting implication of this study is that perhaps the preservice teachers are not yet ready to think about the interaction of technology, pedagogy, and content. They may need to instead first develop knowledge in the separate domains of TK, CK, and PK. Koh and Divahraran suggested long-term sustained intervention is needed.

Ozgun-Koca, Meagher, and Edwards's (2010) study of 20 preservice teachers in a methods class showed signs of shifting from learner to teacher of technology. Even though the preservice teachers were able to move from thinking about technology use as strictly reinforcement of content to using it to develop understanding of content, there still was skepticism about the use of technology in teaching. Students with field placements with a great deal of technology had lessons that were more discovery-based and open-ended. This brings up the notion that the context and the culture of the school play an important role in technology use. This idea is developed more fully later in the review with respect to in-service teachers.

Garofalo, Drier, Harper, Timmerman, and Shockey (2000) discussed the activities incorporating technology that they have created and used with preservice teachers. They believed in using various activities to help preservice teachers think about their roles as future educators. They developed five guidelines for the creation of their activities: "(a) introduce technology in context, (b) address worthwhile mathematics with appropriate pedagogy, (c) take advantage of technology, (d) connect mathematics topics, and (e) incorporate multiple representations" (p. 67). Field-testing, both by the team of researchers as well as others, indicated that they were useful in developing preservice teachers thinking about the integration of technology.

The three studies previously discussed serve to illustrate the representative research reports concerning preservice teachers in the literature. The researchers have also been the preservice teachers' professors. The researchers usually have designed materials for the preservice teachers to use in the researchers' classes. From there, the researchers' aim is to aid the developmental process of the preservice teachers' knowledge and affect in how technology can be used to foster students' conceptual understanding of mathematics. The goal of moving the preservice teachers from having a conception of a teacher-centered direct-instruction classroom to a student-centered constructivist classroom also is implied.

Culture and Context for In-Service Teachers

Many researchers have found inconsistencies in teacher beliefs and teacher practices. Researchers have been able to explain many of these inconsistencies when considering the school culture and school context (Phillip, 2007). School culture and context have been found to be a major factor in the integration of technology (Bennsion & Goos; 2010; Etmer & Pttenbreit-Leftwich, 2010; Goos & Bennsion, 2007; Lampert, 1993; Palak and Walls, 2009; Windschitl and Sahl; 2002). Etmer and Ottenbreit-Leftwich (2010) noted that teachers feel both implicit and explicit pressure to conform. A pressure that they found most teachers give in to regardless if the consequence is considered negative or positive.

Windschitl and Sahl's (2002) 2-year study of three teachers in a private school with laptops found that the social and professional interactions with colleagues were of importance. They differentiated between *learning about* and *learning how* with technology. Teachers learned about technology from institutional voices, such as meetings, but they learned how to use technology from conversational voices such as informal social interactions with fellow teachers. They also found that the teachers did not become more constructivist teachers and that mutual planning time with a colleague who shared the same goals, beliefs, and conceptions influenced the integration of technology.

Palak and Walls's (2009) study of "technology-using teachers in technology-rich schools" consisted of surveys from 113 teachers and a case study of four teachers (p. 419). Attitude was significantly correlated to teacher software use, student software use, and the selection of instructional strategies. The strongest relationship was that 21% of the variation in teacher use could be explained by attitude. Technical and overall support increased the chance that teachers used the software reinforcing the influence of the school culture and context. Palak and Walls (2009) found that what teachers' believed to be good teaching played a large part in how they had students use technology.

There appears to be little research concerning how school culture and context have influenced teacher use of DGS. Lampert (1993) noted in her study of teachers using the Supposer that "the culture of the schools in which they were working had not prepared either the teachers or their students to feel secure that students following their own intellectual roadmaps would learn what they were supposed to know" (p. 153). Coffland and Strickland's (2004) found that teacher attitude towards computers were related to principal attitude towards computers.

Stigler and Hiebert (1999) discussed mathematics teaching in the broad perspective of a cultural activity. "Teaching, like other cultural activities, is learned through informal participation over long periods of time. It is something one learns to do more by growing up in a culture than by studying it formally" (p. 86). They "evolve over long periods of time in ways that are consistent with the stable web of beliefs and assumptions that are part of the culture" (p. 87). "These beliefs, often implicit, serve to maintain the stability of culture systems" such as teaching over time (p. 88). Stigler and

Hiebert gave the example that teachers appear to believe their role is to make sure students do not struggle with instructional material and that teacher should assign many problems that are very similar to what was demonstrated in class.

Aligned with Stigler and Hiebert (1999), Gates (2006) argued, “that mathematics itself is a social construct constituted by social forces and social needs and conventions” (p. 348). Gates cited research that there is a generally accepted mathematics education method firmly entrenched in the culture of teachers presenting new concepts through examples following by practicing those exercises. Gates argued that beliefs were influence by habits, ideas, and discourse. Habits are a part of “the way we have always done it” mentality. Ideas are part of social structures allowing us to engage socially with others. Teacher discourse both formally and informally influenced the individual beliefs.

Professional Development for In-Service Teachers

Researchers have analyzed the effect the duration of professional development had on attitudes and beliefs. The amount of time for professional development with calculators and GSP was found to produce significantly higher changes in attitudes and beliefs when the professional development was a semester-long course versus three workshops (Gnigue, 2003). Other researchers also have found the length of the professional development to have an effect on attitudes and beliefs (Polly 2010; Wozney, Venkatesh, & Abrami, 2006). In addition to the length of the professional development, how frequently teachers used technology in their personal lives was a significant factor in how much they used it in instruction (Wozney et al., 2006)

Levin and Wadmany (2006) concluded that their “results suggest that it is not just technology, but the overall learning environment and its emphasis on non-structured tasks, rich sets of technology-based information resources, and exposing teachers to new visions, that ultimately changes teachers’ belief and practices” (p. 173). Just integrating technology is not enough to ensure meaningful lessons (Levin & Wadmany, 2006).

Polly’s (2010) study consisted of two elementary school teachers’ experiences in a yearlong professional development program. When lessons were planned with the project staff, the tasks had a higher-level of cognitive demand especially focused on problem solving. Instructional methods, however, did not align with those modeled in the professional development program. Results of study illuminated the need for sustained support after the workshops. Polly’s study serves as an example of how formal professional development can also exist inside the school culture and context when there is follow-up to the professional development in the form of a research-collaborator model.

Yanik and Porter (2009) is another example of a study using a teacher-researcher collaboration model to aid in the developmental process of using DGS in elementary classroom instruction over a month. The researchers concluded that due to the collaboration model the teacher was able to develop in the use of DGS. Yanik and Porter stated that their teacher-researcher model could be a viable method to help those teachers, who are willing to incorporate technology. At the end of the study, the teacher held the view that DGS could enhance the teaching and learning of mathematics.

Chval, Abell, Pareja, Musikul, and Ritzka (2008) conducted a survey of 241 mathematics and science teachers concerning their experiences, needs, and expectations for professional development. The teachers taught at the middle school or the high school. Only 50% had accumulated more than 35 hours of professional development the last 3 years with 74% responding they had attended a workshop on mathematics or science teaching during that same time period. The second most cited need from teachers was in using technology to instruct. It is interesting to note the highest reason for needing professional development was in developing critical thinking as most mathematics educators would agree that technology use and higher level thinking skills go hand-in-hand. Teachers expected that the professional development would give them instructional strategies and activities that they could use immediately in their own classrooms.

Reflection and Self-Efficacy

In the research, reflection has been identified an important component for change in beliefs and change in practice (Phillipp, 2007). Teacher self-reflection is often a way for teachers to engage in developing a sense of who they are as a teacher—an identity as a teacher (Sowder, 2007). In-service activities that go across teaching levels and schools seem to be the most worthwhile because it causes teachers to reflect upon their practice and discuss this among other teachers (Niss, 1998).

Teachers' sense of self-efficacy for the use of technology has been identified as key variable in determining if teachers use instructional technology with their students (Etmer & Ottenbreit-Leftwich, 2010; Glazer, 2004; Palak & Walls; 2009 Wozney et al., 2006). Giving teachers time to develop mastery of technology appears to be the best method to help teachers develop a sense of self-efficacy (Etmer & Ottenbreit-Leftwich, 2010). Concerning DGS, Coffland and Strickland (2004) found that 51% of teachers indicated they had a low or medium level of awareness of the capabilities of DGS.

Summary

In this section of the literature review, teacher knowledge and teacher affect were discussed first in isolation, then how they exist in the context of experiences. Experiences of preservice teachers and in-service teachers were reviewed with emphasis on the school culture and context for in-service teachers. Overall, reflection, self-efficacy, and a willingness to incorporate instructional technology have been identified as key factors for change in teacher knowledge and affect.

Literature Review Conclusion and Summary

The literature review first considered theoretical models for teacher change and task analysis. Related to this study, the task analysis models provide a lens to view the first research question. The teacher change frameworks apply to the third and fourth research questions. Next, the history of the geometry curriculum in the United States served to situate DGS within the historical context of teaching and learning of geometry. Then, theoretical and empirical research on DGS was considered. The theoretical research was centered on the notion that dragging in DGS changed the nature of teaching and learning geometry. The empirical research identified three studies that are related to this research project. Finally, teacher knowledge and teacher affect were discussed in the context of preservice and in-service teacher experiences.

Chapter 3: Methodology

This chapter details the data collection and data analysis methods used to investigate the four research questions posed in chapter 1. The subsequent sections of this chapter elaborate on the design choice, the researcher, the subjects, the data collection, the data analysis, and the pilot study.

Design Choice

"Qualitative research is a systematic approach to understanding qualities, or the essential nature of a phenomenon within a particular context" (Brantlinger, Jimenez, Klinger, Pugach, & Richardson, 2005, p. 195). Qualitative designs are naturalistic in that the researcher generally does not manipulate the phenomenon under study (Patton, 2002). In this study, the main focus was on how geometry teachers were implementing dynamic geometry tasks to engage students. As a researcher I had no control over this; the phenomena was not manipulated and unfolded naturally because one of the main goals of the study was to understand how teachers were implementing dynamic geometry tasks without researcher influence.

Often the type of research questions dictates the type of research design. A case study has a "distinct advantage" as a research method "when a 'how' or 'why' question is being asked about a contemporary set of events over which the investigator has little or no control" (Yin, 2009, p. 13). The four research questions mainly focused on "how" and "why," thus making a case study an appropriate methodology.

"A case study is an empirical inquiry that investigates a contemporary phenomenon in depth and within its real-life context, especially when the boundaries between phenomenon and context are not clearly evident" (Yin, 2009, p. 18). The study was in-depth because I collected information on more than just a survey of items relating to the dynamic geometry tasks teachers selected for instruction. I conducted an interview with each teacher that averaged 50 min, and I collected the documents that teachers used with students when enacting dynamic geometry tasks. The study was focused on "real-life context" of teachers' decisions pertaining to DGS. In contrast to many previous studies on student use of DGS, I did not influence or manipulate how teachers chose to enact dynamic geometry tasks with their students.

Brantlinger et al. (2005) define a collective case study as "a study that takes place in multiple sites or includes personalized stories of several similar (or distinctive) individuals" (p. 197). This study took place at 12 different schools with 12 geometry teachers. According to Stake (2006):

> Whether leaning toward standardization or diversity, almost every educational or social service program will be far from uniform across its different situations. To understand complex programs, it is often useful to look carefully at persons and operations at several locations. The multicase project is a research design for closely examining several cases linked together (p. v).

New educational technology and other school innovations are common examples of topics studied with a multiple-case design (Yin, 2009). Yin stated that the purpose of conducting a multiple-case study is replication in much the same way as an experiment is repeated. Each case either predicts similar results or predicts contrasting results but for reasons that can be explained. In this study, geometry teachers were the units of analysis.

A collective case study methodology was appropriate because this investigation addressed how geometry teachers across multiple schools were using DGS, a particular educational technology, with their students.

The Researcher as the Data Collection Instrument

In qualitative research, the researcher is the primary data collection instrument (Nastasi & Schensul, 2005). A potential threat to credibility of qualitative research is that the researcher may shape the findings without being transparent about predispositions and biases (Brantlinger et al., 2005; Patton, 2002). One way to combat this barrier is to attempt to state those predispositions and biases explicitly (Patton, 2002). As the researcher of this study, I was a threat to the credibility due to my personal biases, which might have affected my interpretation of the data. Research reflexivity is a researcher's "attempt to understand and disclose his assumptions, beliefs, values, and biases" (Brantlinger et al., 2005, p. 201). With regard to researcher reflexivity, I have a bias toward the use of DGS. It is my belief that DGS should be an essential part of students learning geometry. Activities should be structured for students to learn geometry through guided discovery or preferably open-ended discovery in which students are actively engaged in creating geometry. Because I researched how DGS was being used instead of whether it should be used, bias is less of an issue in this study.

The Researcher as a Connoisseur

Although researcher bias is a potential pitfall of qualitative research, the researcher also can be a benefit to the study. Because I have participated in many types of activities involving technology, geometry, and dynamic geometry software, I am a *connoisseur* (Patton, 2002). "The researcher as connoisseur or expert uses qualitative methods to study a program or organization, but does so from a particular perspective drawing heavily on his or her own judgments about what constitutes excellence" (p. 172). My professional life has included roles as a teacher, author, department chair, workshop leader, and student. I have taught mathematics at Troy High School in Troy, Ohio, since 1997. During that time, I have taught 12 of the 14 mathematics courses offered, including Algebra 1, Geometry, Algebra 2, Precalculus, AP Statistics, and AP Calculus BC. Every year except for three, I have taught some form of a geometry course. In 2007, I received National Board certification in adolescent to young adult mathematics.

I started using dynamic geometry software with students in 1998, beginning with Cabri II, and then GSP in 2004. Generally, I have used Cabri II and GSP with students between 15–20 times per year, along with many times during class discussions and demonstrations. The activities that I have my students use are mainly ones I have written, field-tested, and revised for more than a decade. I coauthored *Active Investigations With the Geometer's Sketchpad* (Ruland & Nirode, 2001), a stand-alone activity book of over 30 GSP activities and contributed to the writing of an activity ancillary containing over 20 GSP activities for the textbook *Geometry* (Larson, Boswell, & Stiff, 2001).

I have led almost 100 days of professional development workshops for teachers across the United States since 1999 including 20 days of Geometer's Sketchpad workshops. The remaining 80 days have been implementation workshops for Key Curriculum Press's *Discovering Mathematics* textbook series including Murdock, Kamischke, and Kamischke's (2002) *Discovering Algebra* and Serra's (2003) *Discovering Geometry*—textbooks that use an inductive investigative approach.

There are a couple of additional activities that have influenced my professional life. Since 2002, as mathematics department chair at Troy High School, I have established and led 1–2 days per year of professional development often with a focus on technology implementation. I earned a Masters of Science in Education with an emphasis on technology in education in 1999 from the University of Dayton. Since 2007, I have been a doctoral student in the Appalachian Collaborative Center for Learning, Assessment, and Instruction in Mathematics (ACCLAIM) program.

Subjects

Subjects were selected according to purposeful sampling. In this study, the criterion for purposeful sampling was high school geometry teachers whose students used dynamic geometry software. I proposed using maximum variation when selecting participants. Maximum variation sampling is a type of purposeful sampling with the goal of choosing and "describing the central themes that cut across a great deal of variation" (Patton, 2002, pp. 234–235). There are two major strengths of maximum variation sampling according to Patton (2002): the findings will be "(1) high-quality, detailed descriptions of each case, which are useful for documenting uniquenesses, and (2) important shared patterns that cut across cases and derive their significance from having emerged out of heterogeneity" (p. 235).

I planned to use teacher experience and school characteristics as criteria in the maximum variation sampling scheme. I conjectured that years of experience in teaching mathematics, teaching geometry, and implementing student dynamic geometry tasks in geometry instruction influenced how teaches are currently using DGS with their geometry students. In addition, the school can influence the individual. For example, school size was one attribute of the school that might influence how geometry teachers have students use DGS. In some of the smaller schools in the sampling frame, there was only one geometry teacher. On the other hand, some of the larger schools had seven or eight geometry teachers giving the teachers collaborative opportunities. Student demographics likely also played a role in teachers' instructional decisions.

The subjects of this study were from southwestern Ohio. Originally, a 14-county area in Ohio was chosen for close proximity to my location. Due to low participant response, the geographical area was expanded to include most of southwestern Ohio. Southwestern Ohio is a part of the megapolis of Columbus, Dayton, and Cincinnati. Within the megapolis are rural, urban, and suburban school districts with a wide range of high school size; for example, public high school graduation classes can range from 50 to over 800 students. Due to this convenience sample, there is potential for bias because geometry teachers in other geographical areas may be using DGS in other ways.

To locate research participants, I created a list of potential subjects by searching for e-mail addresses on schools' websites and e-mailing those teachers. For the high schools in the original 14-county area, I e-mailed 635 mathematics teachers at 118 high schools on March 28, 2012. Appendix A shows the initial contact e-mail message. From this initial e-mail, six teachers responded and subsequently became participants in this research project. On April 20, 2012, the GeoGebra Institute of Ohio sent an e-mail to its participants about the research project. From this e-mail, two teachers became participants. On April 21, 2012, I e-mailed 420 teachers at 50 high schools that were known to have a site license for Geometer's Sketchpad. These high schools were located throughout southwestern Ohio. From this second round of e-mails, three teachers responded. I contacted one additional teacher because of a suggestion made by a high school teacher who was no longer teaching geometry.

In qualitative research there is no set predetermined ideal sample size. The sampling is concluded once saturation is met, and the data collection methods produce the same, consistent information as previously given. Saturation is defined as when findings become redundant and discrepancies have been resolved (Nastasi & Schensul, 2005). In Guest, Bunce, and Johnson's (2006) analysis, they found that saturation was reached after analyzing 12 interviews. Even though one truly never knows if saturation has been met, using Guest et al.'s analysis as a guide, I stopped recruiting participants after 12 teachers agreed to participate.

As shown in Table 3, there was little variation in the geometry teachers according to years of teaching mathematics, years of teaching geometry, and years of using DGS with geometry students. Even though my original plan was to have participants with a wide range of years of experience, because of low participant response, everyone who responded to my initial contact e-mail became a subject in the study. All of the teachers except for Bill, Leah, and Jill had less than 10 years of teaching experience. Only Bill and Leah had more than 10 years teaching geometry. None of the teachers had used student dynamic geometry tasks for more than 10 years. Nine of the teachers have been using software almost the entire time they have been teaching geometry. For the other three teachers, this gap can be explained. Bill did not begin to use DGS until he moved to his current school 8 years ago; this explains his 10-year gap. Jill's gap can be explain because she taught geometry in the mid 1990s for four years before staying at home with her children for 8 years. Leah's school did not purchase Geometer's Sketchpad until 8 years ago, which is when she started using it with students.

Table 3

Basic Information Regarding Each Geometry Teacher

Name	Years teaching mathematics	Years teaching geometry	Years having students use DGS	Date of interview
Alan	7	7	7	April 3
Bill	18	18	8	April 4
Cara	7	7	5	April 9
Dawn	6	6	5	April 10
Eric	7	7	7	April 11
Faye	2	1	1	April 18
Gina	7	4	4	April 25
Hank	2	2	2	May 1
Iris	6	2	2	May 2
Jill	10	9	4	May 8
Kate	6	3[a]	3	May 16
Leah	21	13	8	May 17

Note. A pseudonym is used for each teacher. All interviews occurred during 2012.
[a]Kate also taught 2 years of honors geometry.

In contrast to the general homogeneity in teacher experience, there was a great deal of variation in the teachers' schools. The graduation classes ranged from 15–550 students. Out of the 12 schools, four schools are private and eight schools are public. Three of the private schools are Catholic schools. One of the Catholic Schools is an all-girls school. For the public schools, the percentage of economically disadvantaged students ranged from 8%–44%. Most of the public schools are predominately white, non-Hispanic with five of the schools over 90%. The most diverse public school's student population was about 60% white, non-Hispanic.

The Ohio Department of Education classifies public school districts by nine categories. As shown in Table 4, six of those categories are present in this study. The three categories that are not accounted for were not a part of the sampling frame; therefore, all the variation in the public school districts that could be a part of this study is represented.

Table 4

Typology of Participating Public School Districts

Typology of Ohio School Districts	Number of Schools
Rural/agricultural—small student population, low poverty, low to moderate median income	3
Rural/small town—moderate to high median income	1
Urban—low median income, high poverty	1
Major urban—very high poverty	1
Urban/suburban—high median income	1
Urban/suburban—very high median income, very low poverty	1

Data Collection

The data collection consisted of an interview and documents in the form of the printed DGS student tasks along with any necessary premade files needed for the tasks. Patton (2002) identified three different types of interviews, two of which are the general interview guide approach and the standardized open-ended interview. "The general interview guide approach involves outlining a set of issues that are to be explored with each respondent" (Patton, p. 342). On the other hand, "the standardized open-interview consists of a set of questions carefully worded and arranged with the intention of taking each respondent through the same sequence and asking each respondent the same questions with essentially the same words" (p. 342). These two interview approaches were combined to strength the overall design and data collected from each case. By combining these two approaches, I asked specific questions, but I was open to ask additional questions depending on the responses of the interviewee. Appendix C lists the initial interview questions.

I collected activity sheets that the teachers had students use with DGS along with any necessary premade files. Yin (2009) explained the most compelling reason to use documents is to verify evidence as well as gain additional evidence from other sources. The documents were used as triangulation with the interview data. The document analysis is described in more detail in the Data Analysis section of this chapter.

Data Analysis

For the analysis of the data collected, I used grounded theory methodology based on Charmaz's interpretation of it (2006). "Grounded theory methods consist of systematic, yet flexible guidelines for collecting and analyzing qualitative data to construct theories 'grounded' in the data themselves" (p. 2). In grounded theory, it is difficult to separate completely the data collection from the data analysis. According to Patton (2002):

> Grounded theory focuses on the process of generating theory rather than a particular theoretical content. It emphasizes steps and procedures for connecting induction and deduction through the constant comparative method, comparing research sites, doing theoretical sampling, and testing emergent concepts with additional fieldwork (p. 125).

The process is more cyclical and iterative than linear.

Figure 2 shows the general outline of the data analysis beginning at the top of the diagram and moving toward the bottom. The horizontal dashed line shows the division between the data analysis for the individual cases and the cross-case analysis for the collective case study. To being the data analysis process, I conducted the first interview and collected the documents. Next, I transcribed the interview, or a professional service transcribed it. Next, I completed the initial coding of the interview and the document analysis. Based upon the initial coding and document analysis, I wrote an early memo to record my thoughts about each case to raise the codes to tentative categories. To refine the tentative categories to conceptual categories, I used focused coding and advanced memo writing. Next, I used the advanced memo to construct a narrative about each case. I conducted a second-level member check by returning a copy of the narrative to each participant. This process was repeated for all 12 cases.

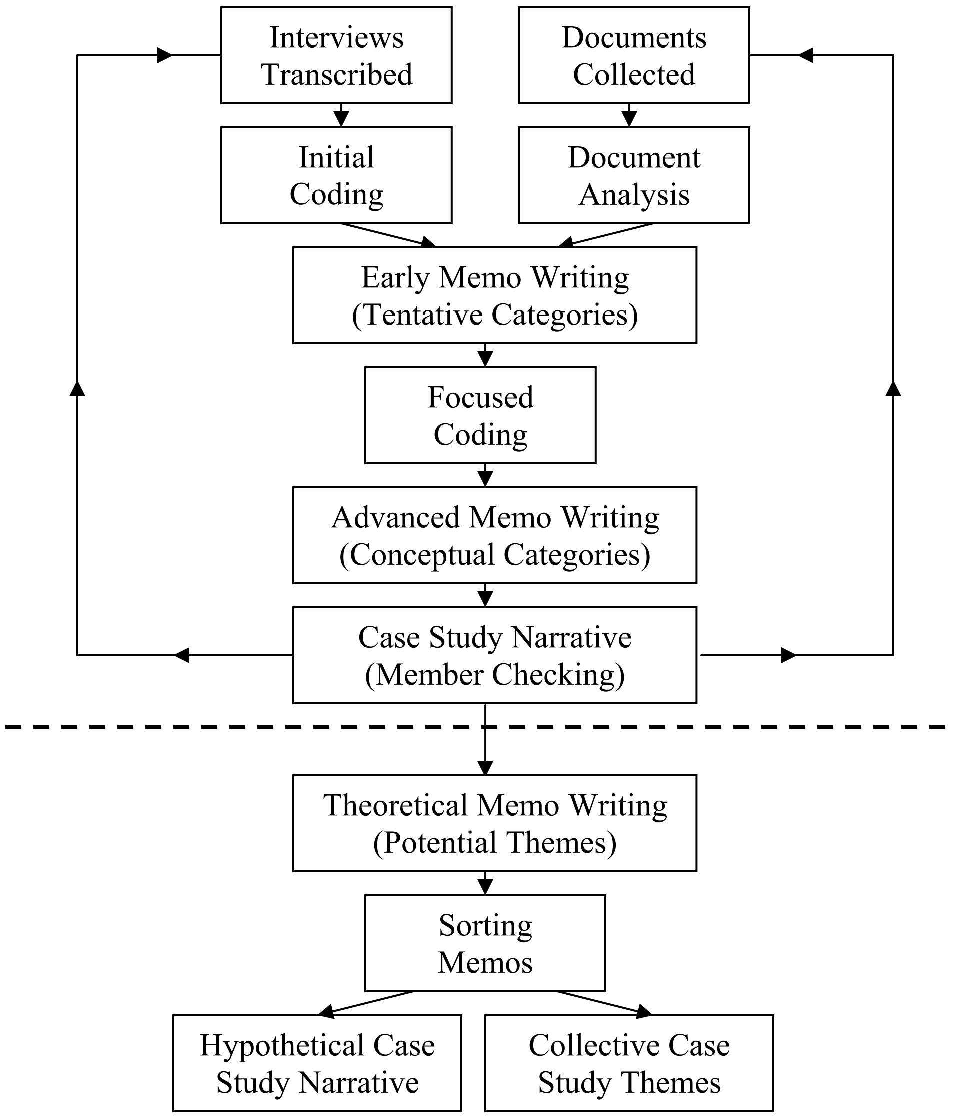

Figure 2. Data Analysis Process (adapted from Charmaz, 2006, p. 11).

At this point, I shifted my focus to the cross-case analysis of the collective case study. After reading the narratives and returning to the interview transcripts and documents when necessary, I wrote theoretical memos. Through the writing of the theoretical memos, some of the conceptual categories from the individual cases became themes. After the memos were sorted, I integrated memos and diagrammed concepts. The integration of the memos formed the basis for the writing of hypothetical case study representative of the 12 individual cases. By diagramming the concepts, the themes for the collective case study were fully developed.

The preceding two paragraphs and Figure 2 briefly explained the data analysis process. The subsequent paragraphs in this section explain this process in more detail. The first paragraph explains how coding and memo writing are central to the data analysis process. The second paragraph describes member checking. The third paragraph outlines the dilemma between the individual cases and the collective case study. The next component details the data analysis processes for each individual case and the collective case study. The last component explains additional credibility techniques used throughout the study.

Coding and memo writing were two major components of the data analysis process. Each of these was completed multiple times at different stages of the analysis. To begin to build theory, the first step is to code the data. Coding is an ongoing and "emergent process" (Charmaz, 2006, p. 59). Coding "categorizes segments of data with a short name that simultaneously summarizes and accounts for each piece of data," showing how the researcher analytically selects, separates, and sorts the data (p. 43). After coding is completed, the next step is memo writing. "Memo-witting constitutes a crucial method in grounded theory because it prompts you to analyze your data and codes early in the research process" (p. 72). Memo writing is used to begin to elevate codes and data to theoretical categories and is used throughout the research process. Memo writing is also a form of reflexive journaling. “Reflexive journaling refers to the personal notes of researchers that document thinking, impressions, and potential biases during the research design and data collective, analysis, and interpretation” (Nastasi & Schensul, 2005, p. 185). The use of a reflexive journal makes the researcher explicitly aware of personal thoughts and impressions during the analysis of the data.

Member checking is another major part of the data analysis process, and according to Stake (2006) it is a vital component of field research. Member checks allow participants in a study to review interview transcripts (Brantlinger et al., 2005; Nastasi & Schensul, 2005). Patton (2002) classifies member checks as a form of analytical triangulation where multiple analysts are used to review findings. Member checks at both the first and second level were used. At the first level, participants were able to read the transcripts of the interviews before the analysis and interpretation of the data. At the second level, participants were able to read the researcher’s analyses and interpretations of the data. By using both levels of member checks, the participants confirm the accuracy of the data collected, validating the researcher’s analysis and interpretation of the data.

In any collective case study, the “case-quintain dilemma” exists (Stake, 2006, p. 1). The quintain is the target of the research, or as Stake describes it, the quintain is the phenomenon being studied. In this investigation, the quintain is how and why geometry teachers were having their students engage in dynamic geometry tasks. Before analyzing the quintain, the researcher first needs to develop an analysis of each case, and then describe these cases for the reader (Patton 2002; Stake, 2006). The cross-case analysis is shorter in length than the sum of the cases, but it is expected to describe the most important findings from each case along with the major themes about the quintain (Stake, 2006). The next paragraphs describe the data analysis process for each case and then how the cross-case analysis was used to gain insight in the ultimate target of the research.

After each interview, I either transcribed it or a professional service completed the transcription, and then I performed a first-level member check by returning the transcript to the interviewee to verify the accuracy of the transcribed interview. All of the study participants returned the first-level member check.

After the initial coding of the interview transcript, I analyzed the student activity sheets. Although the interview protocol contained questions pertaining to how geometry teachers were having students use DGS, the document analysis of the student tasks was a source of data triangulation for how geometry teachers were having students engage in dynamic geometry tasks. "Triangulation refers to the use of multiple sources, methods, theories, and investigators" (Nastasi & Schensul, 2005, p. 185). The purpose of triangulation is to test for consistency—meaning that different data sources give essentially the same results (Patton, 2002). The data from the document analysis were compared to the interview to test for consistency. I analyzed the documents for geometry topic, type of task (open-ended or closed-ended), perspective of the tasks (described in the next sentence), what students were asked to write, whether students explicitly were told to drag objects, source of the written activity (teacher-created or other source), and use of premade DGS files. Tasks have a *geometry perspective* if the steps on the sheet tell students only what to do geometrically, for example, construct the angle bisectors of the triangle. On the other hand, tasks have a software perspective if the steps tell students how to perform the geometry tasks with the software. For example, to construct the angle bisector, select the three points of the angle with the vertex in the middle, and then from the construct menu choose angle bisector.

After I completed the initial coding and the document analysis, I wrote an early memo to describe my impressions and informal thoughts about the data pertaining to each teacher to begin to raise initial codes to tentative categories. Next, I performed focused coding to revise the initial coding by eliminating redundant codes and strengthening the tentative categories. I used focused coding and advanced memo writing to compare and analyze the data across the case. Focused coding is making decisions about which initial codes are the most significant and frequent to categorize the data (Charmaz, 2006). From the focused codes, I developed the conceptual categories. Categories are higher-level concepts under which a researcher groups initial codes. Codes are raised to categories that best assess what is happening in the data (Charmaz, 2006). I wrote an advanced memo after the focused coding to revise and expand the initial summary about that particular case as well as to begin comparing that teacher to previous cases in order to develop the tentative categories into conceptual categories.

After the advanced memo writing, I constructed a narrative describing each geometry teacher. After an introduction of each geometry teacher, the narrative was divided into three parts: present student use of dynamic geometry, factors affecting present student use of dynamic geometry, and past and future student use of dynamic geometry. After writing the narrative about a particular geometry teacher, I e-mailed a copy of it to the teacher to conduct a second-level member check. According to Lincoln and Guba (1985), member checks are the most "crucial technique for establishing credibility" where "data, analytical categories, interpretations, and conclusions are tested" with those whom the data was originally collected (p. 314). I repeated this data analysis process with all 12 cases.

At this point, the analysis shifted to comparing the cases to obtain an overall picture through the cross-case analysis. As suggested by Stake (2006) to begin the cross-case analysis, I read the 12 individual narratives and identified potential themes. For each potential theme I drafted a theoretical memo that contained a table that consisted of the evidence from the cases that supported that potential theme. The case narratives, original interview transcripts, and the documents all were used to build evidence for the potential themes.

Throughout the theoretical memo writing process, I used negative case analysis. "Negative case analysis may be regarded as a process of revising hypotheses with hindsight" (Lincoln & Guba, 1985, p. 309). Negative case analysis is when a researcher looks for evidence that is inconsistent with categories and conceptual themes (Brantlinger et al., 2005). "Where patterns and trends have been identified, our understanding of those patterns and trends is increased by considering the instances and cases that do not fit within the pattern" (Patton, 2002, p. 554). When searching for negatives cases, there are three possible results. No negative cases may be found implying that the emerging themes are correct. Negative cases are identified and can be easily explained. Lastly, the negative cases are found, but cannot be explained. This suggests a limitation to the study.

By carefully reviewing any negatives cases, the emerging themes and conclusions of the data analysis are revamped and made stronger. The results of the negative cases analysis are included with the results of the cross-case analysis in Chapter 4.

After the theoretical memo writing, I sorted the memos. Grounded theorists use sorting memos to serve the initial theoretical development of their analysis (Charmaz, 2006). Sorting memos "gives you a logic for organizing your analysis and a way of creating and refining theoretical links that prompts you to make comparisons between categories" (p. 115). I sorted the memos into groups to assist in identifying the most prevalent themes that cut across all 12 cases. After identifying themes, I grouped these according to each research question. I divided the cross-case analysis report into two parts. The first part consisted of hypothetical case narrative who was representative in a modal sense of the 12 teachers in the study. The second part consisted of a discussion of each theme along with supporting evidence and the results of the negative case analysis.

Throughout the analysis of the individual cases and the cross-case analysis of the collective case study, I employed some additional credibility techniques including: a reflexive journal, thick descriptions, and an audit trail. I kept a reflexive journal throughout the process to document additional thoughts and impressions about each geometry teacher individually and the geometry teachers as one collective group. Thick, detailed descriptions are those reported with "sufficient quotes and field note descriptions to provide evidence for researchers' interpretations and conclusions" (Brantlinger et al., 2005, p. 201). The final report of this study includes an abundance of quotations. An audit trail is the "systematic documentation and record keeping of all the procedures and data relevant to the study" (Nastasi & Schensul, 2005, p. 185). Throughout the study, I kept an audit trail containing a log that included information about all interviews including where, when, and how long they were. I also categorized and logged the documents.

Pilot Study

I conducted a pilot study of three cases before the main study. According to Yin (2009), the pilot study report "should be explicit about the lessons learned from both the research design and field procedures" (p. 94). The pilot study served two primary objectives. The first of these objectives was to determine whether the interview protocol would provide the necessary data to address the two purposes of the research: (a) to determine how and why high school geometry teachers are implementing DGS tasks for student engagement and (b) to describe how they have used such tasks for student engagement in the past as well as their intentions for the future. The pilot study's second objective was to gain familiarity with the data analysis process.

In the pilot study, in keeping with the first objective, I conducted individual interviews with three geometry teachers who I knew well. At the end of each interview, I read the research questions to each teacher, and then I asked each teacher if there were other questions that I should ask to help answer the research questions. I also asked each teacher whether there were anything else that he or she would want to know about other geometry teachers using DGS with students. Two of the three teachers did not have any suggestions for additional questions. The second interviewee said she would like to know about how other teachers assessed student learning related to the DGS activities, so

I added this question to the interview protocol. Furthermore, I divided several of the questions into shorter questions when I realized that the pilot participants were only answering part of the question because I was asking multiple items in a single question. This added to the overall list of questions in the interview protocol, but generally did not increase the length of the interviews.

The pilot study's second objective was to test and refine the data analysis techniques. After each interview, I transcribed and coded the recording; however, several times I started over coding the transcript to become more adept at coding interview transcripts and to refine the codes. The primary issue was too many codes generated, especially codes that were redundant. After a few attempts at coding the transcriptions, a much smoother process emerged which generally eliminated the redundant codes. I also performed the document analysis a couple of times with each teacher's activities to become more familiar and more thorough with this part of the analysis.

Summary

This chapter described the research design of the qualitative collective case study that investigated how geometry teachers were implementing dynamic geometry tasks for student engagement, how teachers' prior experiences, attitudes, and beliefs influenced that implementation, and teachers' future aspirations with DGS. The qualitative collective case study was an appropriate design because a natural phenomenon was studied across multiple sites that were not manipulated. Researcher biases were disclosed as well as how the researcher as a connoisseur was beneficial to the design. A description of how potential participants were contacted and subsequently become subjects was given. Data collection consisted of an interview and documents. Grounded theory was used as the basis for the data analysis to construct theory “grounded” in the data. An integral part of the data analysis process was multiple iterations of coding and memo writing in conjunction with reflexive journaling and member checking. Additional credibility techniques that were implemented were explained.

Chapter 4: Results

This chapter presents the results from the data analysis of the individual case studies and the results from the cross-case analysis of the collective case study. The results are based upon the analysis of the teacher interviews and the document analysis of the printed student tasks. The section on the data analysis of the individual case studies is divided into 12 parts—one for each case. The section on the cross-case analysis of the collective case study is divided into two parts. The first part describes Zoey, a hypothetical high school geometry teacher, who is representative in a modal sense of the 12 teachers in this study using dynamic geometry software with students. Zoey's narrative was constructed based on the data analysis of the 12 teachers in the study. The second part presents and explains the 11 themes derived from the cross-case analysis.

The Individual Cases

This section presents a narrative about each research participant and their use of DGS as a means of instruction. A pseudonym is used for each teacher. After a general description of the geometry teacher and the school in which he or she teaches, the narrative is divided into three parts. The first part describes how the teacher is currently enacting dynamic geometry tasks for student engagement. The second part details factors affecting how the teacher is currently enacting dynamic geometry tasks for student engagement. The third part explains each teacher's past experiences and future aspirations concerning DGS. In this section, any text in quotation marks is that particular teacher's words unless otherwise noted.

Alan

For 7 years, Alan has been teaching mathematics. His entire teaching career has been spent in a "very rural" high school with about 50 students in each graduation class. Alan describes the school district as having "a decent amount of poverty" and some "low income housing." The K–12 school district is all in one building. There are three mathematics teachers for Grades 7–12. One teaches exclusively high school, and Alan and the other mathematics teacher split the junior high and the remaining high school classes between them.

Throughout his career, Alan has taught geometry. He teaches all of the geometry classes at his school on an 18-week semester block. Both geometry classes are second semester and constitute two of the blocks out of the four 90-min blocks Alan teaches. His two geometry classes have 10 and 15 students. His geometry classes consist of all sophomores, but the students can have different prior mathematics courses. Some of the students only had Algebra 1 as freshmen; on the other hand, some had taken Algebra 1 as eighth graders and Algebra 2 as freshmen. Alan is comfortable with the geometry content that he teaches, but he tries "to learn something new every year" and is "never going to know it as well" as he possibly could.

Alan's geometry classes follow the same format each day. He tries "to have a warm-up problem on the board when they walk in and try to activate their prior knowledge and get them ready for the day's lesson." A lecture usually lasts about 30 min where Alan asks the students many questions. "I have note cards with all the kids' names on it, and I randomly ask questions to different kids." Students then have time to work homework and practice problems, while Alan walks around "table to table, asking questions making sure they understand it." In the 90-min block period, there is still usually 20 min remaining for the next lesson, and then students have homework on that section. Occasionally, Alan uses Geometer's Sketchpad with the interactive whiteboard as "motivation to spark their interest and get them ready for the day's lesson." Alan has tried to design some lessons to cover the whole 90 min where they have the last 20 min to work on homework citing that sometimes he does not like the monotony of "do homework, go over it, and do another lesson, go over homework again."

Present Student Use of Dynamic Geometry

Alan's students have been engaging in dynamic geometry tasks by using the Geometer's Sketchpad his whole career. Alan's students use GSP about 10 times to complete tasks that allow for some exploration but have a focused goal. He describes the activities as "guided discovery," and he is "a big believer in a big part of learning is struggling through it a little bit." Students engaged in dynamic geometry tasks on such topics as transformations, parallel lines, triangle properties, triangle congruency, quadrilaterals, circles, and trigonometry. His general goal for his students when they use GSP is to give them "an opportunity to explore things and see how things are related." Alan thinks that a positive aspect of GSP is that it gives students the ability to visualize and to manipulate different geometric objects:

> I just think for the pure fact that they can actually visualize it that much better. And, as a teacher you can explain things the best way you think you possibly could, and you're going to have a lot of kids with a lot of different pictures in

> their heads. So, I think the fact that they can manipulate the different objects and different figures in Sketchpad and see exactly how everything works. I think that's the best way. A lot of kids are very visual learners. To give them a chance to see that I think it's very worthwhile.

In the computer lab, students either have a teacher-created activity sheet that gives them a specific task to complete, or Alan gives students a task verbally to complete. The printed sheet does not have an introduction, and there are no diagrams. The activity sheet is oriented toward the geometry perspective and not the software perspective. That is there are no explicit instructions on how to do something with GSP. Because the directions are not oriented toward the software, students are not instructed to drag points on the sheet. To introduce an activity, Alan gives his students goals to accomplish. Alan gives students a task to try to figure out how to construct a geometric figure in GSP using a new file; for example, students are asked to construct a parallelogram. In some of the tasks, students create a static figure. For example, when exploring triangles, students are told to construct a triangle given specific lengths for the three sides. "But, overall I give the goal that we want to accomplish and then give them some time to try to figure it out on their own." Although each student has his or her own computer, students often discuss ideas with each other:

> They'll bounce ideas off each other, and a lot of times we'll have kids do it different ways. Kids will be really curious to go over and see how the other kids are approaching it. So, they'll learn new methods that way, too.

While students are working, Alan says he does “a lot of observation and a lot of questioning:”

> And, if there’s always a few kids in class that struggle with it a little bit more, so walk around and pay closer attention to them and make sure that they’re understanding it. But, at the same time the kids that get it real quick have questions ready for them to try to challenge them a little bit.

There is also monitoring software on the teacher’s computer that Alan can use to see what all students’ are doing on their computers:

> I make sure I let them know that I can see what they’re doing. If you have a kid that forgets that, and they’re doing something on the Internet you can real easy without signaling them out you can say you know “I don’t want you on the Internet right now. Let’s make sure we’re working on Sketchpad.” And, usually right away they’ll do it and if not, you can lock down their computer, so they can’t do anything from your home computer…. And, you can even, if they’re struggling with something, send them a message or a quick hint on how to do things.

Alan sets up the activity sheets for students to record and organize their notes about specific examples and to write conjectures about geometric properties. After students have had time to construct the figure and explore some of its properties, Alan facilitates a class discussion to ensure that students have reached specific conjectures. Sometimes, Alan uses the teacher monitoring software to help facilitate the discussion:

> So, I can see people using different methods to make a parallelogram and with a Smartboard and projector in that classroom, I can take one particular student's computer and make that my entire computer screen and then put that up on the Smartboard for all the kids to see. So, they can explain one of their methods that might be different that somebody else's method on how to do it.

Alan assesses student learning from GSP in two different ways. One way, students turn in their GSP files electronically to Alan. A GSP assignment is worth 2–3 homework assignments. Alan also includes a GSP task on quizzes and tests 8–10 times per year. Students are called up alphabetically to his desk to complete a GSP task as part of a quiz or test. For example, on a quiz students constructed the circumscribed circle or inscribed circle for a triangle. This task is about 15% of the quiz grade.

Alan has one dynamic geometry task that is unique when compared with all the others tasks. Students complete a take home project on transformations using GSP. They create an artistic design that has to meet specific requirements. For example, some of the project requirements are to have at least two transformations of each type, parallel lines, perpendicular lines, and different colors.

Factors Affecting Present Student Use of Dynamic Geometry

Alan feels supported by his mathematics department colleagues and administration in having his students use GSP. "The guy that teaches strictly high school loves it. He loves the technology aspect of it. He loves seeing me go in there. The other teacher does as well." The principal has also been very supportive. "My principal's told me he wishes he would have had Sketchpad back when he was learning geometry." The administration has also been very supportive when it comes to purchasing materials for GSP. “The first time I went in there to ask them to buy the software, there’s no hesitation whatsoever. So, they bought the software and the license for all the computers. Bought the extra teacher’s edition and all the extra activities.” Alan cites his principal and his students’ enjoyment as the biggest influences on why he has continued to use GSP with his students. “He sees the benefits of it really helps out a lot. Also, just watching the kids enjoy it. It makes them enjoy math a little bit more.”

Alan’s personal experiences with GSP have been mostly as a student. He remembers having a teacher in junior high who had students use it and credits her with being the biggest influence in getting him started using GSP with students. “I still remember her bringing us down to the computer lab, and I think, the first thing we did was—it was wintertime, so we made a snowman.” In college, Alan used it some in a geometry class for tessellations, but he did not use it during student teaching. As a teacher, Alan has never participated in any formal professional development, but has learned GSP on his own:

> And, once I got it as a teacher, played around with it a lot more, learned a lot of the different things you can use with it. Like, I’ve just grown to love it more by just teaching with it than I did really when I was just using it as a student.

Alan states his proficiency to be an 8.5 or 9 on a 10-point scale:

> I mean there’s a lot of things I still have to learn about it, but every once in awhile I’ll have a kid ask a question I don’t know, but for the most part I feel like I always—I know it well enough that I can answers the kids questions.

Past and Future Student Use of Dynamic Geometry

Alan's current use of GSP is different from what it was in the past due to his access to computers.

> The first 6 years I taught here, I used laptops probably 2 times a year because there was always a computer class in the computer lab preventing us from using the computers. However, during those years, I used Sketchpad as a demonstration tool in our class lectures/discussions probably once or twice a week.

Now that the computer lab is open during the blocks that he teaches geometry he is able to have students use GSP more frequently. "It would have been nice if I could have taught like this for the past 6 years."

Given his current constraints, Alan sees using GSP in additional ways. He would like to have students complete some long-term projects using GSP at home. Even though he is not completely sure what those would look like, a possibility is something combining art with geometry and mathematics. Alan also would like to have class in the computer lab everyday, so the computers would be available whenever students wanted to use them. As a teacher, Alan tries "to learn about new things we can do." He would be interested in meeting with a group of teachers who use GSP to compare notes:

> I'd love just to sit around with 8–10 teachers and talk about specifically different activities that we use and compare notes on how you can make it better and what teachers have learned from the kids. That probably would be one of the best things for me. I'd rather have discussions like that then just sit there listening to somebody talk about it the whole time.

Bill

Bill has been teaching mathematics for 18 years. He currently teaches in a rural high school with 75 students in each graduation class. Bill describes the school district as “very wealthy” with “state of the art” facilities, “very high academics,” and “a lot of pride in the small community.” The high school mathematics department has three mathematics teachers; one teacher divides his time in half between teaching mathematics and directing athletics.

Throughout his career, Bill has taught geometry. He teaches all the geometry classes at his school consisting of three periods out of the seven 42-min periods he teaches. His geometry classes consist of 20 students each made up of freshmen and sophomores. Bill describes his comfort level with the content as very high because “I’ve been at three schools, and it’s been the same textbook every time. My comfort level with this book is very, very high.”

Bill uses a variety of instructional activities in his geometry classroom. Sometimes, he gives “short lectures with kids at the board in between.” Other times, he uses the computer that is connected to the projector to show short demonstrations of geometric concepts with Geometer’s Sketchpad. “We’ll get some of those things in there, and then go back up to the book, maybe, and say ‘Ok, now open your book to this page. And, look at the three theorems. Is this what we just talked about?’” Students also occasionally complete a worksheet in groups. In summary, Bill says, “That would be—try not to stand up front and lecture. I try not to, but obviously in geometry, I think there are days you have to do that. You know, but I try not to make that an everyday thing.”

Present Student Use of Dynamic Geometry

Over the last 8 years, Bill's geometry students have been completing dynamic geometry tasks with Geometer's Sketchpad. He describes the activities as a "reinforcer-by-doing," "interactive," and "self learning." Students use GSP six times to complete mostly closed-ended tasks. Student activities include angle relationships, parallel lines, transformations, triangle properties, and triangle congruence. His general goal for his students when they use GSP is "just to deepen the understanding:"

> Where I like it the most is after we've covered something. What I try to do is—certain activities, I kind of know going in this is something that's harder. The kids aren't going to get it the first time. So, I like to take them into the lab or get the laptops whichever is easiest…. Some of those are 30–40 steps and some are 8 or 10 steps where they have to kind of discover it on their own.

Bill believes that the greatest benefits to having students use GSP is that it is a form of a technology and "kids like technology:"

> When you say, "We're going to the lab," automatically it's like, "Ah." I mean you can see the difference in their faces. It's, "Oh, this is a fun day. We're not going to sit and listen to you talk to us for any amount of time." And, they're getting just as much out of it, but it's an attitude. It's a whole different attitude, mindset in their minds, so their focus is different.

Students use GSP by opening a new file and following the step-by-step directions on a single-paged, teacher-made activity sheet. The printed sheet does not have an introduction, and there are no diagrams. Bill has students use the computers in the lab, but sometimes they also use the laptops in the classroom. Bill does not give the students much direction when they are beginning an activity. "I might go around and answer questions, but we're not going through it step by step. It's here it is. It's on the paper. Follow the steps." The activity sheet gives specific steps on what to do, but they do not tell students explicitly how to do the step on GSP; for example, a step on a sheet might be to "construct the angle bisectors from each vertex." Although each student has his or her own computer, students often help each other:

> But, I've seen them getting done and instead of getting their other work done, helping other kids. And, then you see that progress the next time you go in and do another activity that that kid that was helped last time, now has a much better idea of how to navigate the…program.

While students are working, Bill walks around answering students' questions, or if there are many student questions about the same step, he shows all the students using the projector. If Bill's students are using the computer lab, he can take control of any student's computer by using the teacher computer to show students how to do a particular task or to redirect them if they are off task:

> Then, I can actually click on Johnny's computer and take control of it or just observe it and see what he's doing. So, if they ask a question, I can just click on that real quick and say, "You need to go to your text tool. You're on your pointer tool that's why it keeps highlighting that. Or you have two things highlighted because you measured something then it stayed highlighted." So, it's a very nice program also, or if someone else decided that they're bored and now they're

> surfing the Internet, I can click on it and take control and make it a Word document quick and say get back to work. Never have to say anything out loud, but they know I'm watching. So, it's a very nice tool.

Bill ties the GSP tasks back into classroom instruction when they come up during discussions. "Or say, 'You know the other day we were in the lab we did this on Sketchpad, what did it tell you?' And, right away for the ones that got to that next level, it's like, 'Oh, yeah, that's right. That was that activity.'"

Students hand in work on a printout of GSP including answers to any questions asked on the activity sheet. The questions on the activity sheets usually ask about particular relationships that the students have found with GSP and then ask students why these are true. Rarely are students instructed to drag a point on the activity sheet. When answering the "why" question, Bill wants students to restate the correct theorem that they have discussed previously in class:

> And with each step or sequence, I'm making them go back and find the theorem or postulate that related to that step or sequence and then explain why that worked on Sketchpad. Hoping that they almost restate that theorem again because I'm just trying to have students make that connection.

Bill puts the point value for each step on the student sheet and uses it for formative assessment when he returns the paper to a student to fix errors:

> And, we've kind of gone to formative assessment here. And, that's the other nice thing, if it's not acceptable, I'll give it right back to them and say, "You need to

clean up part 5, 9, 12, and 13." So, they know what they need to do then. You know, get back on it.

When Bill's DGS tasks are compared, students complete a unique geometry research project. The project is part of school-wide requirement where each teacher is obligated to have a writing project in each course. Initially, the project was not intended to use GSP, but many students have started to use GSP to complete the geometry aspect of the project. Students research three locations in the world that they would like to visit. They use those three locations as vertices of triangle. Next, students classify the triangles by their sides and angles and also find the circumcenters.

Factors Affecting Present Student Use of Dynamic Geometry

Inside his school, Bill feels supported in having his students use GSP with minimal constraints. Bill describes the school staff as supportive of his use of GSP and technology. The school has a lab facilitator who was interested in learning how to help study hall students with GSP. His colleagues in the mathematics department are "very interested in the fact that I use it and what it does." The administration is supportive because the principal "loves the fact that we do different things," and it has "never been an issue" when Bill has wanted to upgrade existing technology or purchase new technology.

Bill did not have any experience with DGS until he came to his current school 8 years ago. At that time, he "kind of started playing with it." When Bill became the geometry teacher, he received all of the materials from the former geometry teacher:

> He was actually doing about what I'm doing, and then he at the time went to the change over of being athletic director, so I got all his stuff, and saw he had a few activities in there. He wasn't using it quite as much as I am now; just saw that they were using it. And, that's kind of what got me started playing with it.

Bill has continued to use GSP because the district monetarily supports technology and "the students enjoying it and seeing progress in it and seeing it evolve into something that they kind of actually look forward to." Bill has gone to a few OCTM and NCTM conferences where he attended some sessions on GSP. Bill describes his level of proficiency with GSP on a 10-point scale as a "7, 6–7. Like I said because I don't use it for everything yet, there's probably parts of it if I realized we could use, I'd make it better."

Past and Future Student Use of Dynamic Geometry

Bill's current use of GSP is not much different from when he first started having students use it. Some changes were made to make the activities fit within the class period. "The biggest thing is time to make it—I had one that was way too long. Nobody got it done in a period, so I had to cut some parts out of that. Others were way too short." Other changes Bill made were to make sure the number of points that the activity was worth was "a realistic reflection of what they did, and for their ability to learn it and do it right and to actually do it:"

> I don't want for them to go in there with a C, and come out with an A because I made it worth too many points, and vice versa. I don't want them to go in with an

A, and come out with an F because they didn't get it and couldn't figure out how to navigate Geometer's Sketchpad.

Bill does not foresee himself doing anything differently in the future with GSP. "But, as far as using it more or differently here, no, and I think that's just because I have the ability to use it in so many different ways." Although he does not see using it differently, he still looks for conference sessions about GSP to attend. "That's usually really the first thing that I look at when I look through the OCTM, NCTM. You know, some of the speakers are great. Some are boring, so you always pick up something."

Cara

Cara has been teaching mathematics for her 7-year career at a large high school. The district is "excellent with distinction" and has a "very diverse" population with 20% of the students being minorities. The graduation class is approximately 500 students. In the high school, there are 17 teachers in the mathematics department, and Cara describes them as working "very well together" and "very collaboratively."

Cara has taught geometry since she began teaching. Seven members of the mathematics department teach 15 sections of geometry. Cara currently teaches three sections of geometry out of the five 50-min periods that she teaches. There are two different tracks for mathematics courses: a lower Math 1, Math 2, and Math 3 sequence and the more traditional Algebra, Geometry, Algebra 2 sequence. The school does not have an honors mathematics program, but instead students are accelerated where it is possible for a seventh or eighth grader to take geometry in the junior high. Cara can have freshmen through seniors in her geometry classes, but typically students are freshmen and sophomores. This year she has more freshmen than any other grade. Cara is “very” comfortable with the geometry content she teaches because “this is my 7th year teaching and my eighth time teaching geometry out of this book because I student taught here and student taught geometry as well.”

Cara’s geometry classes usually follow the same basic structure each day. “We start off the day with a warm-up problem, so there’s a warm-up problem on the board when they come in.” A student or pair of students works the problem on the interactive whiteboard after everyone is finished with it. Cara then goes “over the homework and those answers are on my Smartboard.” Next, Cara teaches the lesson for the day. This might consist of “a guided notes lesson,” “an activity,” or “working with a partner for something.” The activities often consist of something hands-on such as cutting out geometric figures and folding to verify geometric properties. After the lesson, students “have some time to work on the homework.”

Present Student Use of Dynamic Geometry

For 5 years, Cara’s geometry students have engaged in dynamic geometry tasks using Geometer’s Sketchpad. She describes the activities at the beginning of the year as “more learning how to use the software,” then the farther they get along in the year students should know how to construct certain geometric figures without directions. Her hope is that students begin to use that knowledge to “talk about more in-depth thinking that we’ve been doing in class.” The activities are either precursors or follow-ups to the classroom instruction depending on when Cara can reserve the computer lab. There is one GSP activity per chapter except there are not activities for the chapters about congruent triangles and three-dimensional shapes. Students complete a total of eight closed-ended tasks. Her general goal is "to kind of take what we've been doing in class and show how that really works:"

> So for me, it's to draw that picture on the board for them. I don't think they—they're like, "You just fudged that. You made it look like that." And, I feel like they can really see that with Sketchpad…. I think that my goal is not only that they are seeing it mathematically in class how the numbers work out, but physically how it shows up on Sketchpad as well.

Cara thinks the greatest benefit to having student use GSP is that it is "just a day out of the routine of math:"

> It's not me sitting at the board showing them how to do something. They're running it. And, I think as silly as it is; they get excited. "We're going to the IMC today? We get to play on the computers?" You know what I mean? I think they very much get excited about that, and it's something different where I don't think math always lends itself to something different.

Students use GSP by completing a teacher-created activity sheet in one of two available computer labs. The printed sheets do not have an introduction, and only one of the activities includes diagrams. Cara gives the students few verbal directions at the beginning; instead expecting them to begin with a new file and to follow the written directions on the sheet. The activities are generally explicit about the steps to use on GSP to construct the figure. For example, if students are to measure the length of the radius, on the sheet it says, "Highlight the circle and go to the *Measure* menu then click *Radius* to find the radius of the circle." As the students are going through the steps, "lots of students like to cross off the tasks as they go just to make sure that they're doing everything in order." The activity sheets are three or four pages with multiple parts. Most of the time each part is titled. Within each part, there are multiple steps for the students to follow. Most activities have between 40–50 steps for students to complete; some of the steps require students to write something. Frequently, but not always, students are asked to drag a point before making an observation. This is either to record a measurement or to make a conjecture based upon those measurements. During an activity, Cara walks around monitoring students "making sure that they're on task and trying to answer questions." Cara leaves it open to the students to decide how they interact with each other:

> I don't say one way or the other. If Johnny and Sally are sitting next to each other and Johnny sees and turns to Sally and says, "Hey, how did you get that to do that?" And she tells him, then that's fine with me.

Cara assesses student learning from GSP by creating a rubric for each activity. The rubric explicitly states how many points student get for each part. Students turn in their paper copies of the completed activity sheet and GSP files electronically by e-mail. "But, they're e-mailing me their sketches which I love because I can see and drag those points and see really how these are connected." The GSP tasks are worth 15% of a student's quarter grade. When Cara returns the graded papers, they take time to go over them:

> When I hand them back to them, then we go over those Sketchpads. So, I would have those sketches up to makes sure they're okay. That's what we did, and try to make sure that that equates. So, my thought would be if we did that sometime in the chapter, they would get their Sketchpad back in that chapter before the test, so we would have gone over that in class before the test.

Factors Affecting Present Student Use of Dynamic Geometry

Cara's four colleagues who teach geometry and her administration support her use of GSP. "Our group of four, I think works very well together. I would say that they—just the fact that they are like, 'Ok, we have a Sketchpad coming up; and we're still committed to doing that." This group of geometry teachers plans GSP activities together:

> We'll sit down at the end of the chapter and plan the next chapter. And, we just do that on our own time, and then if we see if there's something that needs to be fixed. "Well, you fix the review worksheet. You make Sketchpad work with Google now, and you do this." So, we kind of divvy up the tasks.

The administration is supportive of Cara's use of GSP because she is using technology. "The buzzword is technology, so I feel like they will support me because it's technology, and I'm using technology with my students."

Cara does have some issues with access to computers, which can dictate if she does the lab before or after discussing the material in the classroom:

> We have 60 for 2000 kids. So, like I said when we're definitely planning the next chapters, it's like, "Okay, where looking three weeks out." And, we're booking the computers labs…. So, if the lab's not open, I might be doing it before or I

might be doing it at the end of the chapter as opposed to wherever I wanted to necessarily really do it.

Cara has struggled a bit with if students are getting out of GSP what they are supposed to be learning. Sometimes, she feels “they just follow the steps, and they don’t think about how that worked.” Cara’s dilemma is she devotes a lot of time to GSP and although students think GPS is fun, the students do not think it is aiding their learning of geometry:

> I devote almost a week and a half by the time the year is over to going to the lab to do Sketchpad. I feel like that’s a decent chunk when you’re only looking at four nine-week quarters. I’ve asked them at the end of the year, “Do you like that?” And, they like doing it, but when I ask them the question, “Do you feel it benefits your geometry?” And, they do not. So, then I struggle with if it’s not benefiting you, then maybe we need to do it in a different way, or we need to do something else so that it is benefiting you. Yeah, it can be fun, but if you’re not getting anything out of it, then what’s the point?

On the other hand, Cara continues to use GSP with students because of department support and students enjoying it:

> I would say just our department because if everyone else is still doing it, I feel, “Okay, then I will keep doing this.” And, I think just the fact that my kids enjoy it. Maybe they might not get the most out of it. They’re still doing it, and I think that they enjoy going to the computer lab.

Cara has had various personal experiences with GSP. She used GSP some in her own high school geometry class. "I remember we had Sketchpad days in geometry." In college, she had a class where she extensively used GSP especially for the basic constructions. She also had to "create a lesson using Sketchpad that wasn't like anything we had done." When her school first purchased GSP, "somebody from the company came out and did a little one-day professional development thing about how to use it, but that was about it." Since then, she has participated in some online webinars:

> I have done those through the company. They'll have a little webinar at 8 o'clock at night, and they'll show something. So, I'll log on and watch them. They have the newer—we don't have whatever the newest one is. We don't have that version of the software, so there were a couple of things that I wasn't able to do because we don't have the new version. But, there were still some cool ideas.

Currently, Cara would say that she is "pretty proficient with Sketchpad", but "if you wanted me to construct something random, I might not be as comfortable with it if it might not be something that I've done before."

Past and Future Student Use of Dynamic Geometry

Cara's current method of having students use GSP is not that much different from when she started 5 years ago. "The content is still pretty similar." When she first started using GSP, it was when students returned from winter break:

> When we first started using it, we got it like half way through the year, so I didn't—couldn't start off. So, I felt like I want you to construct a parallelogram, but you don't even know how to make a point. So, I felt like we had to digress

and go back to chapter one, and I remember doing it in the middle of the year when we got it. We started in the middle of the year, which was silly, but we did.

Cara has also revised the point values on the rubrics, and students previously did not turn in the files to her electronically.

In the future, Cara envisions using GSP in a couple of different ways. Instead of computers, she "would love to see an app" on the iPad. She would "also like to see just more of them investigating the stuff than me telling them how to do it." One way that Cara thinks she could help accomplish this shift is by using premade files that the students would just manipulate. Initially, her problem with doing this was how to get the files to the students, but she could do it now with e-mail or her website. "And, so now I feel like I could very easily get those files to them, and maybe they would spend less time creating and more time manipulating to get to answers which I think is good."

Cara is interested in more professional development in the future with GSP. She would be interested in more webinars because "those things are easy. They're in the evenings. I can sit on the coach in my pajamas. I don't have to spend the whole day learning about Sketchpad." She would also be interested in talking about GSP with other teachers in her geographical area. "Even if it was just something in the area, where a half-day professional development or even just a 3 to 5 o'clock thing where in the area those who use Sketchpad could bring their ideas."

Dawn

Dawn has been teaching mathematics at the same rural school throughout her 6-year career. The district comprises several small towns. The graduation class sizes are a little less than 150 students. The high school population is about 95% white, non-Hispanic with 40% of the overall student population economically disadvantaged. There are four teachers in the mathematics department in the high school.

The only course Dawn has taught during her career is geometry. She teaches all the geometry classes each year. She teaches six geometry sections in 50-min class periods. The class sizes are 20 students with half the students as freshmen and the other half coming from the other three grades. The school has an integrated course sequence that has been stereotyped as for lower students. Dawn feels comfortable with the geometry content because she has taught it so many times. “I feel like I have mastered it pretty well.”

Dawn’s geometry classes follow a similar plan each class period. When students walk into the classroom, there is a problem on the board for them to work out. Next, they go over the homework from the previous night. After going over the homework, Dawn gives them notes. During the last 20 min of class, students have time to work on homework and, “they should get it done in class and the next day do that again. That’s kind of a standard day…. Most of the time it’s pretty much up there at the board with notes, that’s a pretty typical day.”

Present Student Use of Dynamic Geometry

For 5 years, Dawn’s geometry students have been completing dynamic geometry tasks using Geometer’s Sketchpad. Students complete seven core activities and then they also can work on a few other ones for extra credit. She describes the activities as “pretty self-explanatory. They pretty much tell the kids exactly what to do.” Topics include triangle properties, quadrilaterals, transformations, circles, and parallel lines. Dawn's general goal is for students to see how the geometric properties hold in general:

> Ideally, I would love to have them figure out these mathematical concepts on their own, seeing it's dynamic and how it works. For instance, how it works for all circles and not just the one that they drew. That's the ultimate goal. I'm not sure if they get there every time, but that's the plan.

Dawn cites that the greatest benefit to having students use GSP is that the software is dynamic.

> Like I said before, the dynamic part of it; the fact that they can see. You know, I can put up 100 examples, and I don't think they grasp the concept of proving something for all cases. And, I think that helps them. If they're doing a triangle and they can drag it, and they see the measurements changing on the angles or sides, but the relationship is holding true no matter what they do to it. I like that aspect of if, that they can see things as a whole no matter what kind of shape they try to change it into.

In the school library, students use GSP by following the directions on a teacher-created activity sheet. The activity sheets do not have an introduction, but usually include a couple of diagrams. Students start with a new file to complete the closed-ended task. Dawn launches the activity with a classroom introduction before they go to the library. "I pull it up here, and I show them, you know, what they should look for when they get down to the computer lab. And, I open it up and show them what it looks like, and after that I give them the paper, and they do it own their own." Most activities contain at least 20 steps for students to complete, and some due to multiple parts may have as many as 90 steps. The steps are oriented toward the software perspective and tell students exactly where to find the correct commands.

The setting in the library can be a little chaotic. “It’s the only computer lab in the school, so kids are coming in to see if there are open computers. And, if you don’t get down there fast enough, sometimes you have to kick kids off. It’s usually a headache.” Students each have his or her computer, but they help each other. “A lot of times they would be helping each other and making sure if someone is stuck. Usually the kid next to them is like, ‘Did you read the directions?’” Dawn spends most of her time “running from computer to computer answering questions…. But, most of the time it’s going from one person to the other, basically pointing out to them the step that they skipped because that is usually the questions that they have.”

If students finish the activity early, they are to work on a new activity such as creating a Ferris wheel or kaleidoscope. “Overall it seems like I get a group of kids who really excel and then I try to pull things for them to do extra because they finish their stuff really quickly and they like it.” These activities are worth extra points for students and serve to keep them busy. “Some of those activities are like bonus activities. Once we get down in the library, some kids are faster than others and I don’t like them to have a lot of free time in there. So, I try to keep bonus activities on hand that they can work on.”

Students either turn in their answers on the activity sheet or print out the GSP file with their answers in a textbox. Students are instructed to drag certain parts of their construction before they answer the questions on the sheet. The questions ask students to make conjectures about geometric relationships and theorems. Dawn grades the students' answers to the questions on the activity sheet as well as what the file looks like if she had students print it out. "I try not to have them print it off to save paper if they can, but sometimes we print it off." A GSP assignment is worth 15–20 points, but overall "it's not a big part of their grade." To tie the activity back into the classroom, Dawn discusses it the next day:

> The next day we go over it together, and then we do the lesson that is related to what they did. I will say, "Remember, when you did this on Sketchpad that is tying into this concept or idea?" On my board I will pull up Sketchpad and show them what they should have been doing on a certain step and how it relates to what we are learning. We usually have to spend a day or at least half a class period going over it.

Factors Affecting Present Student Use of Dynamic Geometry

Dawn has felt indifference from her mathematics department colleagues and her administration concerning her use of GSP. Her mathematics colleagues do not use it and are generally unaware of it:

> It's pretty much just me. I'm the only one who uses it. I think they're just not familiar with it, and I've tried to show them how to use it, especially for day-to-day classroom stuff because I have used it in my classroom all the time to show things. I think they just don't know how to use it, so they haven't tried…. I think they just think it's for geometry, and it's unrelated to their classes at all.

Dawn feels similarly toward her administration. "I wrote the grant to get it and got it. I think that I needed like $100, so the school topped it off. Otherwise, it has been sort of my thing; no one else acknowledges it, I guess." On the other hand, the administration is supportive of technology provided the faculty can secure external funding for it. "The problem is we're a little school and don't usually have the money for things. They're really supportive when we write grants and can get it ourselves."

Even though Dawn has had some frustration with using GSP with students, she continues to use it with them. Access to computers is one source of irritation for Dawn. "We only have one computer lab so usually you have to reserve it about a month in advance and it's really difficult to know exactly what I'm doing. I try to get it about once a month." She sometimes has to rearrange the order that she was planning to teach the material:

> Sometimes, if I am doing the chapter on circles, there's a general format I want to do, but if I can move one concept to the end of the chapter and maybe teach it the next week I will try to do that, so I can get into the library. Sometimes I will just change the typed out paper to accommodate what they know or don't know or I will bump things out if I have to.

Dawn often is concerned about if all her students are learning material when using GSP:

> I've been a little bit frustrated because I feel I have to simplify the instructions a lot more than I'd like to. Sometimes I think that they don't get the insights that maybe I want them to get on their own. I really have to push them to it, a lot of

> them just kind of say they don't understand how to do it right off the bat even though I know they could figure it out.

Some students think that GSP is difficult to use. "I think their attitudes sometimes towards it and the fact that they think that they can't use it so they don't try as hard as they could even though it's not that hard." Sometimes a few students do not work on GSP and instead become classroom management problems. "I get a handful of kids just like anything else that just don't do it and it's frustrating. So then those kids will try to distract others." Dawn believes that many of her problems could be reduced if she was able to use GSP in her classroom and use it more frequently. "Just being in a different setting outside of my classroom, they get crazy sometimes so that's difficult. If I had it in here and they were using it often, it would be different." Despite these difficulties, students seem to like using GSP because it is a day out of class. "The kids generally when I say we're going to be in Sketchpad tomorrow they say, 'Oh good we're out of class.'" Overall, Dawn thinks the majority of students are learning material using GSP. "It's usually about, I'd say two thirds of them understood it, had it finished and the other third, maybe half of them just didn't have enough time to finish or the other half were messing around." Despite these issues, "it's definitely worthwhile to use."

Dawn has had some personal experiences with GSP both before and after writing the grant to get it at her school. She first had experience with GSP in a geometry course for middle school teachers that she took during her undergraduate work. "That is the first time I had ever seen any sort of geometry software, so I thought it was pretty cool." During student teaching, she had some experience with a Ferris wheel animation with

GSP in a precalculus class. "Then, I saw it at the school I student taught, and that was about it so, when I came here, I knew I wanted it." She has been to some presentations at OCTM conferences, though "it's been awhile. Our funding was cut for those, so it's been four years since we have been to one." She describes her current level of proficiency at "about 50%. I feel really good with basic constructions, but nothing too much beyond that. I can figure it out following directions, but I wouldn't be able to come up with it myself."

Past and Future Student Use of Dynamic Geometry

Dawn has changed some from how she was first having students use GSP to how she is having them use it now. Students use GSP more frequently now. "I think I just keep adding more, I try to add more as we go. I keep the old ones and try to add more to it. The old topics have not changed a lot I guess." Initially, Dawn was using materials she did not create, but found those difficult for her students:

> I bought the book that goes with the software for geometry, and some of them are adapted from there. When I just copy them out of the book, the kids a lot of times have a hard time following the directions so I have to alter them.

Another change that Dawn has made is she has added questions throughout the activity:

> Now I change a lot of, you know, my own directions, trying to incorporate questions throughout and type in the questions instead of just doing the activity and talking about it the next day. I try to make them answer questions. That's the biggest change from when I first started it.

Next year, Dawn envisions some changes with using GSP because her school is implementing new curriculum:

> With our curriculum planning, as we're going through the topics that are going to be in geometry next year, I have several that I am like, "Okay, I am going to make a Sketchpad activity for this, Sketchpad activity for this."

One particular area where Dawn sees using Sketchpad more is transformations. "There is so much more transformations involved in the new curriculum coming up, so I think I will be able to use Sketchpad for that quite a bit." Dawn also would be interested in professional development opportunities. "If there was training I would definitely be interested in doing it. The more complex with animations; I think the basic stuff I wouldn't need so much but more advanced training would be great."

Eric

Eric has been teaching mathematics for 7 years. His whole teaching career has been spent in a "rural school, small" with graduation classes of a little over 100 students. Eric describes the community as "a lot of low income." Farming is an important part of the community. "It's a big farming community; a lot of people live on farms and work on the farms. We even have a bring your tractor to school day each year." There are three teachers in the high school mathematics department.

Eric has taught geometry throughout his career. He teaches all five sections of geometry at his school with 54-min periods. His geometry classes consist of all grades, but mostly sophomores and 25% are freshmen. Eric's geometry class sizes are usually 20 students. Eric is "comfortable" with the geometry content that he teaches now that he has taught it for many years. "It took me a couple of years to get used to everything."

Eric's instruction for geometry tends to be focused on his use of an interactive whiteboard. According to Eric, "[I] put all my notes up on the whiteboard, and then I take those notes—I have a free website where I put the notes on everyday, so if a student misses a day." Eric also has his students use different manipulatives or he "will pull up Geometer's Sketchpad and try to explain something to them there." Eric does several examples for students. "I will work a lot of problems."

Present Student Use of Dynamic Geometry

As long as he has been teaching, Eric's geometry students have been completing dynamic geometry tasks with Geometer's Sketchpad. His students complete 10–12 GSP assignments with many spanning 2 days. The tasks are closed-ended and focus on creating a geometric construction to verify particular theorems. Geometry topics include polygons, triangle properties, geometric mean, transformations, and circles. The GSP activities are follow-up assignments to classroom instruction. "We usually go over the topic in class, and then I will write up something and tell them, 'Here's your directions. Here is what I want you to do on Geometer's Sketchpad.'" Eric thinks that GSP "really helps them understand the subject matter we're trying to go over in class. It gives them another way to look at it rather than looking at their books, if they do look at their books." Using GSP also gives students access to technology that they might not otherwise have. "It brings technology into the classroom for them. If we didn't have that, the only technology is a little handheld calculator for them."

Students use GSP either in the computer lab or in the classroom with laptops. Students have a teacher-created activity sheet that gives them specific tasks to complete. The activity sheet does not have an introduction or any diagrams. The activity sheet is mostly oriented toward the geometry perspective and not the software perspective in that it does not usually give specific steps to do with GSP. To introduce an activity, Eric usually "show[s] them [students] certain things on how to do it on Geometer's Sketchpad because they're not used to doing it yet." Students always begin with a new file. On the written activity, students are not instructed to drag any points. Each student has his or her own computer, but students get help from Eric and from each other:

> In the beginning of the year, I have a really hard time because there is only one of me and 25 students, and I can't get around to everybody at the same time. They're always asking questions. I tell them, "Ask your neighbor. Ask your neighbor." I tell them, "There is only one of me, and I can't get to everybody all at once." I have that problem at the beginning of the year and as the year goes on the students know there are students who are really good at it, and they can go ask them how you do something.

If necessary, Eric also demonstrates aspects of GSP to students using the projector in the computer lab or the interactive whiteboard in his classroom. When students are working with GSP, students sign a sheet with their name and computer that they are using in case there is inappropriate use of the computer. "We had an instance this year where a student got into someone else's and changed their password to an inappropriate saying, so we tracked that back." Throughout the year, Eric has students use GSP to work on a geometry poster that uses different designs. "They do that throughout the year. I use that as a fill-in. If they're done with GSP number three, and the rest of the class isn't, they can go back and start working on the shapes project."

Eric assesses student learning from GSP by having students turn in a paper copy of the file. Students seldom have questions to answer on the activity sheet; if they do, they answer those questions on GSP using a textbox:

> They print it off when they're done, and I grade their printed project when they turn it in. I base that on points. For instance, most of my GSPs are worth 200–250 points and are graded accordingly depending if they did all seven steps in that assignment or if they only did half of them.

Even with the large point values, the GSP grades are a part of homework, which is 35% of a student's grade. Homework can be worth 1,000 points in a quarter. "Then again it's not something if you don't do, it's not affecting your grade that much."

Factors Affecting Present Student Use of Dynamic Geometry

Eric is isolated in his use of GSP in his building. "I'm really the only one that uses it. The other teachers don't use it. I don't know if they know how to use it." Even though the other mathematics teachers do not use it in Eric's building, they do not appear to be opposed to Eric using it. "I mean, they have no quarrels about me using it." The administration is supportive, but only because "they, of course, think it's a great thing because it's bringing technology into the classroom."

Eric thinks that his students enjoy using GSP, but he also makes sure to keep them busy for the whole period. "I think most of them would tell you they like it. They like going down there because they haven't seen anything like that before because of the environment that they live in." Eric believes that students like using GSP because it is a break from his lectures. "Well, it's different than me sitting up there lecturing in front of the class. It's a different avenue of learning for them, and they like that." Eric wants to make sure students are working on GSP:

> I try to keep them busy. I try to do things where I know it is going to take them in the beginning more than one class to do. I want them to sit down there and work on the subject. If I only give them something to do that they can do in 10 min, most of the class will just do it in 10 min and goof off the rest of the time.

Because Eric teaches five sections of geometry, computer access can sometimes be an issue:

> I try to keep every class the same everyday what I cover. It's kind of hard to schedule when the Spanish 2 class goes up there and takes up two periods a day. The others are open, but I want to take all five classes up there at once.

Even with these access issues, Eric has been able to do all the activities that he wants to do:

> If something like that happens, I will take four classes today, and the other class I will do tomorrow's assignment. And, the next day it is flipped. I will take them down to the lab and the other four will be in the classroom.

Eric would like to use GSP more, but he has been limited due to access issues. "Ten to twelve times a year does not sound like much, but it's about three times a quarter. When you're fighting for computer time, it's not really that bad."

Eric's first experience with geometry software was right before he started teaching when he was taking his course work. "There was a professor who—the requirements of the class were we had to purchase the bundle that had Geometer's Sketchpad in it." Eric also saw a presentation of GSP at a local OCTM affiliate. During his teaching career, Eric has learned about GSP by "going to conventions or conferences" and "a few webinars on the Internet." At an NCTM conference, Eric was able to meet and talk with the person who led one of the webinars that Eric participated. Eric would rate his GSP proficiency on a 100-point scale as a 75.

Past and Future Student Use of Dynamic Geometry

Eric's current use of GSP is "somewhat" different from what it was in the past. Eric "know[s] how to do things different now than [he] did in the beginning." When asked what he meant by "differently," he replied:

> I don't know. I can't really pinpoint one thing. Like the students themselves, they will come up and show me how to do something a certain way and get an answer and draw it correctly, which I hadn't done it that way before.

He previously used some published GSP activities, but "I have kind of gotten away from that because I think they're a little confusing sometimes for the students, so I rewrite my own instructions based on what I know and what I know about my students."

For the future, Eric mainly is concerned about his students' abilities and attitudes toward the use of the software, but he is also interested in learning more about GSP:

> I would like to see them catch on quicker. It just seems like, I don't know if it's our school, but the students and their ability to pick things up and do things on their own, just kind of really has gone down through the years, I think.

Eric attributes this issue to students wanting instant results, instead of putting in the time and effort to learn the software. "They kind of want to push a button and be done. You hear many complaints, 'This is going to take forever to do.'" Eric is open to learning more about using GSP. "I am open to anything really, anything that I don't know how to do right now. I know there's a lot more than can be done with Sketchpad."

Faye

For 2 years, Faye has been teaching mathematics. She teaches in "a low-income area" with over 40% of the students in the high school are classified as economically disadvantaged. "There are some students that really have some tough backgrounds." About 250 students graduate each year at Faye's high school. The high school mathematics department has six full-time teachers and two additional teachers who split time between mathematics and another academic department.

This is Faye's first year teaching geometry. Her school has three different geometry tracks. About 50% of the students are in the middle track in which Faye teaches all five sections with average class sizes of 25 students. The class consists mostly of sophomores and a few juniors. Rarely are there any freshmen in Faye's geometry classes because they are in the honors track. The school day is made up of seven 50-min periods. Faye feels a little uncomfortable with the material that she teaches given that this is her first year of teaching geometry.

> Am I as comfortable as I would like to be? No, because I haven't taken geometry since eighth grade…. Do I feel uneasy some days? Not because I don't feel that I don't know the material, but I just don't feel as comfortable as algebra. I've been teaching algebra for two years…. A scale of 1–10? I'd say I'm a 7, I guess. There's room for improvement. That's for sure.

During her first year of teaching geometry, Faye has been using some different instructional approaches, but due to the constraints of a novice teacher she has not been able to incorporate as many activities as she would like. Students frequently have a warm-up on the board to complete when they come into class. "There are a couple of days that are just notes using the projector, or I'll give them guided notes and have them use the textbook." During classroom instruction, Faye does not use Geometer's Sketchpad much. "I pretty much will just draw a picture up or something like that. It's just a lot of hassle to boot up Sketchpad and Notebook and switch back and forth." Occasionally, students work in a group of three or four students. "They all have to work through it. They all have to initial that they agree with their group's answer because they have to compare them. And, then I shuffle them behind my back and staple it and grade the top one." Faye has students complete some activities and projects:

> I wish I had more activities because I think that geometry has a lot of opportunities for manipulatives. Being my first year teaching it, I have not incorporated as much as I would like to. I still have incorporated some projects and some activities, and I think the kids have really done well with it and really excelled with it.

Present Student Use of Dynamic Geometry

Because this is Faye's first year teaching geometry, this is also her students' first year engaging in dynamic geometry tasks with Geometer's Sketchpad. Students use GSP six times to complete mostly closed-ended tasks.

> I have about six labs here, but I would say a lot of these labs end up being like a two-day lab because they're a little bit longer or asking them to do different things. I would say I've spent a good 10–12 school days in the lab.

Student DGS activities included basic geometric relationships, parallel lines, types of triangles, points of concurrency, and optical art. Faye's general goals for her students when they use GSP are to be hands-on and to introduce geometric concepts that connect back to the classroom lessons:

> I think my general goal is to be hands-on, first of all. It's nice that they are physically sitting there working on the computers. Also, it's to lead right into what we're going to be going over in class the next day. I want them to see it a little bit.

Faye believes that the greatest benefits to having student use GSP is that students "can drag points anywhere, everywhere to explore what's going on:"

> I really like that they can measure the angles, put them over in the corner, drag the vertices wherever they want, or add segments. Here's the length. Here's the length. Here's the length. What do you observe? Here's the angle. Here's the angle. It's very interactive as far as finding out your own theorems or whatever we're trying to go for.

Students use GSP by opening a new file and following the directions on an activity sheet in the computer lab. The activity sheets frequently have an introduction and some diagrams. Sometimes to begin an activity, Faye demonstrates some ideas on the projector before students begin:

> It varies depending on what we've talked about and what we haven't, and also the difficulty that I think it is. If I think that the instructions maybe have used something that we aren't familiar with on Sketchpad or something, I'll have my projector ready to go with Sketchpad and I'll say, "Let's review how did we find the midpoint of a segment and how did we do this?"

Faye has students use GSP in a couple of different ways:

> I've done it where I've kind of used it as almost a guided note type thing. They have a sheet in front of them to follow steps, and I kind of do it with them or, "Here, try this," and then check in with them after step five, check in with them after step ten. I've done other things where I make it purely exploration. "Here's your worksheet. Go." I just sit in the back and roam and observe what's going on.

The steps on the activity sheet vary in that sometimes students are told explicitly how to construct geometric objects with GSP, and other times they are not. Most of the activities are published activities. Even though each student has his or her own computer, there is still student collaboration:

> There's a lot of partner work. It ends up being, “Hey, how did you make that segment?” And, then back on their own computer. There's definitely some collaboration between their neighbors. I kind of encourage that.

Faye walks around answering questions and helping students. If necessary she stops the class to show them something on the projector:

> I circulate and answer questions and sometimes, there’s so many questions that I am running around with my head chopped off. I just sit in the back of the classroom so that I can see all their monitors, which is to see what they’re struggling with, what they’re grasping and what they’re not. Like I said, sometimes I’ll just interrupt them in the middle of their lab and go up to the front on their projector and say, “Let’s recap. How do we do this?” I'll go over some things.

Students turn in their answers to questions from the activity sheet on paper. The questions consist of students defining geometric objects or stating conjectures that they have found. Students are frequently told to drag points in the written directions before answering the questions. Faye usually does not grade students’ work, but she takes a completion grade if students are “just goofing off” in the computer lab:

> Often times, I won’t even grade it. If they start to take advantage of that, then I'll just do a completion grade because I don’t want to know exactly what they missed and what they got correct. I want to know what they explored.

Faye connects the GSP activities back to classroom instruction by using them as a guide for the class notes the next day:

> It might not click what they're actually exploring or figuring out or doing on Sketchpad, but I think my goal is to connect my notes the next day to say, "Okay, remember when we did this on Sketchpad. What did you figure out?" That's when I'll kind of ask, "What did you put for question four?" And, I'll have them literally refer back to whatever they put. Some of them will come up with some crazy answers, but usually we'll get around to the correct answer. It just leads us right through our notes, so it's a good bridge to get them thinking before I just point blank say, "Okay, here's the theorems."

Factors Affecting Present Student Use of Dynamic Geometry

Inside her school, Faye is generally met with apathy concerning how to use GSP except for a fellow geometry teacher who is "very technology proficient." Faye's colleague has been extremely supportive. "She's answered any question that I have. She seems very familiar with it as well, and like I said, has been teaching a lot longer, so she knows these labs forward and backwards." The rest of her colleagues in the mathematics department, Faye describes as "indifferent:"

> If that's what I want to use, I think that they're like, "Go for it." There's nothing wrong with that, but at the same time, I don't think they incorporate it because when it says Geometer's Sketchpad, they associate it with only geometry.

Faye has not "really talked to the administration about it [GSP] at all."

Because there is only one computer lab available for teacher use, Faye has experienced access issues:

> It's kind of a fight. The English teachers do a lot of research projects, so they sign up for that thing all the time. I have to plan well in advance that, “Okay, I want to go to the lab this day.” And, then I never know if it’s going to bleed over because it’s very hard for me to judge if it’s going to be a one-day lab or a two-day lab. On paper, it’s the exact same length. But if the kids will fly through one, the next one, they're not even halfway through, and I have to sign up for the next day. I think that's a big struggle for me because I don’t plan far enough in advance being my first year teaching geometry.

Even though Faye, at times, has been flustered concerning her students’ lack of proficiency and work ethic when using GSP, she feels that students still enjoy it because it is something different from the classroom. Most of Faye’s frustrations with her students have been because of their reading and comprehension:

> Some of the other students don’t read very well, so the other geometry teacher and I have conversations, we just joke around, they just don’t want to read…. There is some frustration. Lab days are frustrating for the students, for me because I literally just go, “Here; it says it. Read.”

Faye also had some classroom management issues where some students were off task or did not do anything with GSP:

> I just had an issue with one or two of my classes. The class clowns trying to be class clowns and nothing on the computer screen, or they figure out all sorts of stuff on Sketchpad. When that happened, I literally did not let them go down to the lab for a couple labs, and it was an eye-opener for them. I did end up having

to take a completion grade. That was solely classroom management type stuff and not because I thought it was necessary.

Despite these issues, Faye still thinks that students enjoy using GSP because it is a novelty:

I think they like it. I think it's a change of pace that they like to just get out of the classroom. They consider it kind of like a fieldtrip. I don't know. I think if I used it too much, it would take away from the excitement.

Faye had used GSP a great deal in a college geometry course and considers that course a big influence on her using GSP as a teacher. "I had used it so much in college and thought that it was a great way to get them on technology." Besides the college course, Faye has not had any additional experience or professional development with GSP. Faye says her level of proficiency is "about an 8 or a 9" on a 10-point scale. "Like I said, I took a course in college. My geometry course was 110% on Sketchpad…. I would say I'm pretty comfortable with it."

Past and Future Student Use of Dynamic Geometry

Because this is Faye's first year of using GSP with students, Faye has no past use, but concerning future use Faye is still feeling her way through it due to how early she is in her teaching career. Faye would like to work on addressing some of her students' reading and comprehension issues when using GSP:

It's tempting for me to just take out all the pictures and that way, they don't have any choice but to read. But at the same time, there's some things that maybe it's worth it just enough that they need a picture to figure out what the words mean.

> That's where it gets tricky because if I give them something without words on it and they're trying to follow and they don't have something that looks correct. But at the same time, they're reading. It might be something that I'm willing to trial and error as far as just take the pictures out, and then they're forced to read all of it.

As far as professional development opportunities, Faye is not sure what is out there, and this year she was focused on professional development to learn how to use the clickers the school had purchased:

> I think that the focus this year was that I got these clickers that I had no idea how to use. I couldn't even use them if I wanted to. So, I took two professional developments on that. Those I didn't go find. The school district contacted all of us that had the clickers, and said, "This is what we're doing and let us know if you're interested."

Concerning professional development with GSP, Faye has not had time to look into what is available. "Honestly, I don't even know what's available. I've never looked into it."

Gina

For 7 years, Gina has been teaching mathematics. She teaches in a Catholic, all-girls school in a suburban area with 180 students in each graduation class. Gina describes the school as "fairly conservative," but "not as conservative as you would think for Catholic." There are eight teachers in the mathematics department. Her school uses a mod schedule consisting of 23 min each, so teachers do not see the same kids everyday or at the same time. The schedule repeats itself every 6 days:

> Our schedule is very weird. We're in a mod schedule, so I don't see the same kids every day or at the same time so, tomorrow I'll see half of my geometry kids and then next day I'll see the other half. Some days I might see all of them but that just kind of depends. It's an A through F schedule.

Gina has taught geometry for 4 years. Her school has two tracks: honors and geometry. Gina teaches two geometry classes with 20 students in each section. Her classes generally consist of an even split between freshmen and sophomores. Gina sees her students for either two or three mods at a time. Gina is comfortable with the geometry content that she teaches giving herself “an 8 or 9” on a scale from 1–10. Gina really enjoys teaching geometry. “I love teaching geometry because there’s a lot you can do with it.”

Gina uses many activities and group work in conjunction with notes in her geometry instruction. “You know, generally, we’ll take some sort of notes at the beginning.” Gina gives her students packets for the chapters that include figures and diagrams for the notes. “I don’t want them to try to worry about that more than figuring out what’s on the page.” After the notes, Gina’s students “do some sort of exploration with it and come up with conjectures, and then they’ll fill in the theorems or the postulates.” The explorations consist of patty paper or ruler, compass, and protractor. Sometimes, Gina uses the projector and the interactive whiteboard to show short Geometer’s Sketchpad demonstrations.

Present Student Use of Dynamic Geometry

Every year that she has taught geometry, Gina's students have been completing tasks with the Geometer's Sketchpad. Due to constraints in scheduling the laptop carts, Gina's geometry students use GSP "every 6 days or so" to complete mostly closed-ended tasks. Gina describes the activities as "explorative dynamic…. And just, I guess, I would say fun for them. They enjoy it." Dynamic geometry topics included angles, parallel lines, quadrilaterals, exterior angles in polygons, points of concurrency, and circles. Gina's general goals for her students when they use GSP depend on if she is using it to introduce a geometric topic or to follow it up:

> General goals, if it's something they're exploring I haven't talked about, like, learning those new concepts, coming up with definitions on their own, which I think is really good and putting math into words. Sometimes it's hard for them to do that. And, yeah, just generally—and then if it's a follow-up, just making sure that they really understand the topic where they can relate it to a big word problem or whatever it is that they're doing so just making sure that they fully understand the topic.

Gina thinks that the greatest benefits to having student use GSP is that they can complete it independently and they are learning to communicate mathematically. "I think it's something that they can do on their own, and they can become confident in their learning of geometry…. And, I like that they are answering in sentences and developing that vocab."

Students use GSP by opening a new file and following the directions on a published activity sheet using a laptop in the classroom. The activity sheet usually has an introduction and some diagrams. “They’re usually—we turn off the lights and they like—they’re girls, so whatever. Set the mood.” Gina launches an activity by “explain[ing] to them what they’re doing” and briefly assessing prior knowledge. “So I do a little bit of a review or a little bit of what they're going to do, but mostly it's just them getting started with Sketchpad.” The steps on the activity sheet usually do not tell students explicitly how to do the steps on GSP, but if it is something new, there is an explanation in the margin for that particular step. Although each student has her own computer, students work with each other:

> They're girls so they'll probably get together in groups of two or three, and there will be a lot of conversation about the different questions. And, I mean, they really hate to be wrong, so I guess they make sure that they're on the right track.

While students are working, Gina walks around answering question, assists with technical issues, or grades papers:

> I'll be walking around making sure they're on task and answering questions. Sometimes we have technical issues, so I have to send tickets to our tech people, which takes some time. So, it's just a variety of things, or maybe I'm just sitting there grading.

Students turn everything into Gina electronically including answers to any questions. “They turn it in electronically, and I usually have a little rubric. And, I’ll just make sure they answered all the questions correctly.” She sometimes gives points for what they do on GSP and sometimes the points are only for answering the questions. “So I'll give them credit for if it was something kind of involved with this making up whatever it is. I'll give them points for that sometimes or sometimes it might just be answering the questions.” The questions on the activity sheets focus on defining geometric terms and occasionally stating conjectures about geometric relationships. Before answering the questions, students are told on the sheet to drag a specific point.

Gina frequently ties the GSP activity back into classroom instructions in the next few days:

> Normally, it's the next day, and we'll talk about what kind of things they came up with. Normally, I don't have it due until the following cycle, so they have time because they don't have it on their home computers for the most part so it kind of just depends. If it was something that they all finished in class that day, I'll probably bring it up the next day and we'll go over things. If it's something that was longer, I might wait a couple days, so it just depends on the length of it.

Gina has one GSP activity that is atypical. Students complete a fall picture that is required to have two pairs of vertical angles, two pairs of adjacent angles, two pairs of linear pairs, and two different polygons. Students print the project and then color it.

Factors Affecting Present Student Use of Dynamic Geometry

Inside her school, Gina feels supported by her mathematics colleagues and her principal in her use of GSP. When she learned there was new version of GSP available, she talked to her colleagues in the mathematics department. “I think it was really expensive and they were all about it, and our principal ordered it for us and, so they’re really supportive." When Gina was first hired at her school, she felt that it was an expectation to use GSP frequently:

> Well, when I started teaching there 7 years ago, that was kind of just an expectation that we would do that. So, I felt like I didn't really have a choice but to do that because that's what I fell into. And, it was my first year teaching, and I was just going to do what I was told pretty much. So, it was more of an expectation.

Gina has continued to use GSP with students because of collaboration within the mathematics department. "I guess, just other geometry teachers that I teach with, just collaborating with them and making sure what we're doing is useful to the students."

The scheduling of one of the four available laptop carts in the school affects Gina's use of GSP. "I usually do Sketchpad on F days because the computer situation is a little weird at our school. We have to get a cart, so we have to schedule the cart. So, I'll do it probably maybe every 6 days or so." Because of scheduling the laptop cart on a specific day, the GSP lesson might change from year to year:

> Like I said, with the whole cart situation if I'm at a certain point that's when I can—they can use the computers, so the topics are generally the same. But, I'll probably choose a different section of the chapter, but similar topics.

The scheduling also influences if the activity is an introduction or a follow-up activity. The woman in charge of the computer carts requires teachers to sign-up far enough in advance of the day they plan to use it:

> You have to reserve the cart and our cart lady is kind of anal. If we don't have it scheduled ahead of time, we can't just go up to her the day before and say, "Hey, I want the cart." So, I just schedule at the beginning of the year every F day.

Gina thinks that overall her students enjoy using GSP because it is something different from the normal classroom routine. “I think, in general, the girls enjoy doing them. It’s something different to do.” Gina believes her students “love to use technology so I think they like that, too:”

> They like it. So I think if they were always like, "Oh, gosh," you know, I would probably think twice about what I'm giving them or possibly not do it. But, they enjoy it, so I think that does affect why we continue.

When asked what her students would say about using GSP, Gina responded:

> I think they would say that they like it; something different. And, I feel like they would say that they do learn a lot from it, and it's a good follow up to what they've already learned in mastering the material. I think it would be positive

Gina had little or no experience with DGS until she started teaching. She may have had some experience with GeoGebra in her undergraduate program, but she is not sure. “I’m wondering if we maybe did something with GeoGebra because it sounds familiar.” Gina took an online course over the summer a couple of years ago:

> So, I did take a class on how to teach with Geometer's Sketchpad. It's through Key Curriculum Press. It was a couple years ago, so I don't really remember all that but it was all about creating dynamic lessons and things like that. And, they really stressed that.

Gina rates her current proficiency at using GSP as “probably like a 7” on a scale of 1–10.

Past and Future Student Use of Dynamic Geometry

Gina’s current use of GSP is not much different from when she first started having students use it. “I think I just know which activities are better than others so, I mean, not significantly. I think just my choice in activities have changed.” She also is using GSP “probably a little bit more” with students now than in the past.

In the future, Gina sees a couple of different ways of having students use GSP. If students had their own computers, she would be able to use GSP whenever she wanted:

> If we do go to where the students each have their own computer, which I don't know if we're going to do that. That'd be nice if we could do it whenever we want.

Gina also would like to incorporate more partner or group activities instead of individual activities. “So, I think it would be cool if it was more of a group or partner type activity. I don't know how I would do that, but that would be cool.”

Gina would be interested in learning how other teachers are using GSP with students:

> I would be interested in how other schools use it if they had something where somebody presented, not an online situation. I feel like you learn more in those situations. So, I definitely would.

Hank

Hank has been teaching mathematics for 2 years. Hank teaches at a small private school “specifically for students who have ADHD and dyslexia, like that’s the student clientele we usually go for.” The school has existed for about 30 years, but the high school has been around for 10–15 years. The graduation classes are about 14 students. Because of the small size of the school, Hank is the only high school mathematics teacher.

Hank has taught geometry since he began his teaching career. Hank's geometry class is 55 min and has 16 students in it. The majority of the class consists of sophomores along with some eighth graders and freshmen. It is rare for Hank to have three different grades of students in his geometry class. Hank is very comfortable with the geometry content that he teaches. "I mean, I feel 10 out of 10 most times, but, I have no problem hopping into a topic and being able to talk about it, know what I'm talking about."

Hank uses many activities in his geometry instruction. In fact, activities are an integral part of the school philosophy. "Our motto at the school is just that we teach to bright students who learn differently and what that usually translates into is we do a lot more in-class involved activities, lots of projects, things like that." Almost daily, Hank has an activity that is followed by a discussion:

> Almost every single day starts out with some sort of activity that ranges from anywhere from 10 min to 30 min, and then after that usually we have a period of talking about the activity. What was the activity trying to achieve?

Hank sometimes uses PowerPoint-type software with his interactive whiteboard, but prefers to "do problems with the kids to kind of introduce new concepts." Hank seldom uses lectures in his instruction. "But, usually I don't devote a ton of time to the lecture–note-taking format."

Present Student Use of Dynamic Geometry

Hank's students have been completing tasks with GeoGebra both years that he has taught geometry. The school has a laptop program where each student has his or her own laptop. Individually, students use GeoGebra in the classroom almost weekly. "Well, typically, they're pretty familiar enough with me now that all I need to do is say, 'Open up your laptops.' And, so that means open up GeoGebra." Hank's general goal for his students when they are using GeoGebra is for them to be able to complete the given task in any manner:

> I mean, the ultimate goal is for them to be able to produce what I wanted them to produce and sometimes it's a more step by step process that I lay out for them, and sometimes it's like, 'Here's what I want you to make. Go for it.' So if they can create that on their own, or find a new way to get to that, that's always really awesome in my opinion. If they can come up with multiple ways to do it, that's more fun for me because I can show each one up here and show like how this person did it. But, then at the same time this person achieved the same thing doing it a different way. I try to promote like different ways of doing the same, like getting the same answer so that they can see that there's more than one way to get there.

Hank thinks that the greatest benefits to having student use GeoGebra are that there is instantaneous feedback for the students:

> It's just like more instantaneous like in terms of you make something and then you immediately see what it can do so like when you're trying to understand what

are the important properties of certain figures, you know what happens…. It's so quick for them to understand the idea whereas like if I didn't have that I'd have to draw something up there and then it's not moveable so I have to introduce more examples so they can start to see the differences in each example and see kind of what's the important thing there. Whereas in GeoGebra, I have something I can move immediately, and they can see hundreds of examples instantaneously. So, I think, it's just the instant understanding of the object. It gets rid of all the need for some of the physical explanations that teachers without software would have used…. It's just instantaneous for them.

Students use GeoGebra is a couple of different ways. Frequently, Hank has his laptop projected and students have their laptops as they work together to create a construction:

> Sometimes it could be as simple as we're building it all together. They have their laptops open and I have mine open, and we're all kind of piecing it together as a group. So, they might be watching as I like make suggestions up here to their own drawings, and they'll be raising their hands asking me questions like, 'I'm having trouble doing this. Can you explain it to me?' So I might say, 'Did you try doing this?' They're okay now I got it working, and then we'll keep moving forward.

Sometimes Hank gives his students a printed sheet from an activity book from his textbook that was written generically for dynamic geometry:

> Other times I will have particular assignments, from like the McDougal books here, there's a lot of particular assignments in here that involve using geometry software so whatever software they had originally planned for, in place of that I just use GeoGebra instead. I'll have the students, I'll open up the activity and I'll have the students work on it for maybe somewhere in the neighborhood of 20 minutes in the classroom and then we'll come back together and talk about what they created and what they found throughout the entire activity. What kind of important things did they draw from the activity?

Hank also has written his own activities for his students to work on in class:

> Sometimes I'll have my own written out activities for them to work on where I give them a goal and they have to try to create this thing by any means necessary…. I'll tell them you have to create for me a parallelogram, for example, or I had them do one time I had them build a golf course where they had to like build the reflection so it hit off the wall, and try to make it show it would show all the different possible ways in which you could hit the balls off the wall so it went into the hole. They would have to create something like that, and they would have to piece it all together on their own. Just making the box, the course itself, was a challenge for some of them because they had to start thinking about parallel lines and actually making right angles so that the box didn't, like when you moved the box, like slump over or something like that.

Once in a while, Hank gives students a GeoGebra task for homework on the spur of the moment:

> I try to avoid that with GeoGebra as a homework assignment because when I do that more often than not the homework assignment is like an inspiration I have in class and I'm like, "Oh, this would be such a great thing for them to do for homework." So I'll give it to them for homework, so it's usually on the fly. It's hard to follow up with them the next day sometimes with a particular homework assignment that was done in GeoGebra because half of them say they sent it to me but then they didn't send it to me, or somehow the file might like magically get lost…. It doesn't work out perfect or how you envisioned it. I'm also a young teacher so like I'm still kind of doing a lot of things by like oh this is such a great deal. I'll try it and then I'll realize it was a bad idea, or maybe it was a really great idea but I didn't execute it very well.

Depending upon the type of task, Hank assumes a couple of different roles:

> Well, I mean if I have them creating something where I want them to work individually without my guidance, I'm just a monitor. But if we're doing something together as a group, I will typically have my laptop up here. I'll have mine being displayed on the board, and I'll be kind of walking around asking questions, having them give me suggestions, and then I'll do some of the suggestions up there. Sometimes their suggestions aren't really going in the right direction but they'll see that when I do it up there, that that's not what they intended for me to do…. I mean a lot of kids will ask me questions like when we're creating something, and they're all trying to keep up with the creation and they have a hiccup somewhere along the way, they'll ask me and I'll stop and I'll

> take a look at the computer and tell them what they need to do to fix it or make a suggestion as to what might be going on, then they figure it out on their own. So I'm usually pretty mobile in the classroom when that's going on.

Because GeoGebra is such an integral part of Hank's geometry classroom, he tends to assess student learning in a more formative than summative manner. To close an activity, Hank has a class discussion. "Like what can you tell me? And, then I kind of work with them on that, and hopefully they do have a really good response to that. If not, I try to guide them to what was the real intention here." There is a participation grade for the in-class GeoGebra work "so as long as you're actively trying to do this and working on it, you get credit for the assignment." Hank grades the homework assignments "for accuracy. I try to see if they accomplished the goal…. Usually it's out of five points, something simple."

Factors Affecting Present Student Use of Dynamic Geometry

Even though Hank's school is very focused on integrating technology into instruction, the size of the school limits the support available to him. The administration is supportive because Hank's school "is very technology driven, so anything involving computer based activities, they want us to use a lot." Hank often goes to conferences or to visit other schools for support:

> When I'm really looking for support, though, or for confirmation or new ideas, I try to make an effort to go to as many conferences as possible or to go observe other math classrooms at other schools. I use my professional development days and actually go observe somewhere else.

Hank attended a GeoGebra conference last year. "Last year, I felt like I learned a lot of really cool new tricks that I didn't think were even there." Also, Hank looks online for resources. "I'm pretty good at finding things that are done by GeoGebra online."

This year's geometry class has influence how Hank has used it because of the behavior and maturity:

> I can easily say my Geometry class was one of my most challenging classes this year because it's a larger group. That particular group of kids is a little bit more, like the older kids in that class are a little bit more in terms of higher needs, weaker math students, and behaviorally immature. So, I mean at the beginning of the year I was more timid with things I let them do. I tried to really restrict them in terms of what they could use in class. I did a lot more me presenting while their laptops are shut in class. But now as we're getting close to the end of the year they've gotten a lot better about that. I've opened them up a lot more. I'm not as concerned about them being on task as I was before. So it can have a big effect on how much I use it in class.

Hank did not have any experience with DGS, until he had a couple of professors who used GeoGebra during his undergraduate work. He credits a particular professor with being his biggest influence in using GeoGebra with students:

> Because the way in which he used it with us is the way I wanted to use it with my students, which was he'd give us a task and then just say go for it, and when he first did that to us we'd never even used GeoGebra before…. So I supposed that's kind of what I—he was the one that got me into wanting to use GeoGebra because

> I realized how good of a teaching tool it was just by forcing myself to figure out all these things. I learned a lot in the process.

Hank believes that GeoGebra works really well to help him teach his curriculum the best that he knows how. “But in terms of GeoGebra working with my curriculum, it works fine because it’s just as good as any other—I mean it’s better than any other tool. That’s why I used it.” Hank rates his current proficiency at using GeoGebra as an “8 or 9” on a scale of 1–10. “I mean, I’m really good at what I know, and I’m quick to learn new ideas.”

Past and Future Student Use of Dynamic Geometry

Hank’s use of GeoGebra last year is different from how he is using it this year:

> Well, I was a little bit more timid last year using it. I would only use it for very select things, and then there’d be periods of time where I wouldn’t use it hardly at all. But, I think this year for my geometry class, I made a more conscious effort to try and use it more because I realized after my first year how much better some of the lessons could have been just in the explanation part of it, like showing kids ideas and walking them through it. It’s like a more active systems of notes than just like me sitting down and having them write down all these rules and properties of particular objects. I can actually talk to them and actively move things and have a discussion with them as a class…. I feel like I jump back and forth between a more traditional and a very nontraditional style of teaching and I think that’s kind of our philosophy here in this particular school, too. We want

> hands on. We want more activities, and GeoGebra lets us do that in my math classes.

Hank has gotten "some cool ideas" from some other schools that he would like to try in the future. He might try having groups of students teach a lesson to the class.

> I like the idea of even the kids just presenting the material, so they actually go through the—I assigned them particular things, and they are the ones that are going to teach the class. So, they have to be the ones who go and learn it and then get ready to teach it.

Hank may also have students design an activity. "And, I might try to have them design an activity for the class to do on this particular topic, so I might encourage them to try and use GeoGebra as a way of making an activity."   Hank is also interested in an app becoming available for GeoGebra on the iPad. "I think our school's probably going to start having more iPads in the school. We've already got one cart of them, and we're probably going to end up getting a couple of more for next year."

Hank is planning on going to a GeoGebra conference this summer. He prefers to go to professional development that is more oriented toward users of the software sharing ideas. "I like people who are GeoGebra users coming in and throwing out their ideas to everybody because it's a little bit more realistic in terms of what we can use, and also I can get new ideas."

Iris

Iris has been teaching mathematics for 6 years. She teaches in a suburban high school with graduation classes of a 100 students. The school is "98% Caucasian, middle to upper class, and 1–3% of the students are on free and reduced lunch. So, we are super college prep, like APs out the wazoo. Parents are super involved, which is a good thing and bad thing." The high school mathematics department has five teachers.

Iris has taught geometry for two years. Her school has four different mathematics tracks: basic, college prep, advanced college prep, and honors or AP, though the basic track is only available for freshman and sophomore year. The basic track has 5% of the students, and the honors or AP track has 25%. The middle 70% of the students is divided almost evenly among the college prep and advanced college prep tracks. Iris teaches two sections of the advanced college prep geometry with average class sizes of 20 students. The school day consists of seven 48-min periods. Iris is comfortable with the geometry content that she teaches. "Like if I measure myself against the high school content, I would say an 8" on a 10-point scale.

Iris uses student explorations and activities in her instruction to lead students to learn new material:

> I try to get them in groups. I try and give them worksheets that lead them where they should go, because who likes to listen to somebody talk? So, I would say maybe 60% of the time they're in groups or doing something. The rest of that time, we're either going over homework, or I'm giving them a short little lecture time. But, I try not to do that.

Present Student Use of Dynamic Geometry

Both years that she has taught geometry, Iris's students have been completing dynamic geometry tasks using GeoGebra. When students are learning the material in the chapter of the textbook about geometric constructions and points of concurrency, they use GeoGebra eight times. Students complete the given tasks using three different tools: compass, patty paper, and GeoGebra. "We'll first do it compass because that's the worst one; they hate that the most. Then we do patty paper, and then we do the GeoGebra." Iris's general goal when using GeoGebra for her students is for them to learn how to use a specific technology. "I also think they need to learn a 21st century skill where you've got to be able to learn a technology. You've got to be able to learn a software or program and become fluent in it." Iris believes that the greatest benefit of using GeoGebra is that it makes it easier for students to focus on the mathematical relationships of the points of concurrency:

> My first year I did it on compass and protractor, and I was just like—it took a full day to do one point of concurrency. Then, I'm like, 'What did they learn? How will this benefit them? It's not.' They're just frustrated that the line didn't go in the right place. There's no mathematics behind that. So, that's when I started using GeoGebra because they see the point, plus you can investigate what does it look like in an acute triangle? What does it look like in a right triangle? What does it look like in an obtuse triangle? I feel like it just gives you so much more math rather than being caught up in the specifics of where do I fold it? You can see the math happen which I think is fantastic about GeoGebra.

In the classroom, students have individual laptops. They use an investigation from the textbook to guide them, but the textbook does not give directions on how to complete the investigation using GeoGebra or any dynamic geometry software:

> Well, they have like in our book—we follow along with the investigation. But, as far as how to do the GeoGebra, I don't give them a step-by-step kind of a worksheet. At first, they're frustrated because they just want me to tell them how to do it, and usually it's my kids who don't do as well that are better at GeoGebra. Because honestly, what kind of a kid does well in school? A kid who sits there and does what you say. What kind of kids are good at technology? Kids who don't listen to you and figure it out on their own. So, it's kind of nice for those kids who are not waiting for me to tell them what to do because then there are friends who sit there and do everything right, which in general is successful. They're like, "Yeah I know how to do this." And, the other kid is like, "But, she didn't tell me." It kind of levels the playing field a little bit.

Iris leads students through the first couple of lessons step-by-step on GeoGebra, but after that she prefers to give students time to figure out how to complete the task:

> After the first two or three—like once we're used to how the menus work, we're used to how to make a point, we're used to how to make a segment. At that point when they realize that most of those tools are up there, you just got to find where it is, then I go, "What do you think it is?" I'll perform it up here, but I don't perform it until they give me an idea. So, "Hey, look, I found this. It's the third one over. Scroll down at the second one." So, only one person in the class really has to come up with the answer, but I'm trying to give them as much investigation in a structured environment as I can. I don't want the kid who wants me to tell him to be frustrated for 45 min. I want them to be frustrated for like 90 s.

In class, Iris assesses learning by having students show her the construction on GeoGebra. "And, I would say, 'Okay, I'm going to come around and you're going to show it to me.' And, I just have a list of my kids, and I put a checkmark." Iris also has students use Geogebra on homework, quizzes, and tests:

> Then a lot of their homework, like it'll specifically say here number 11, technology. Use geometry software. So I require, if there's geometry software, they have to get on their laptop at home or in the computer lab, and they have to save it and e-mail it to me. Or, they have to print it off and give it to me. Same thing for a test or a quiz. There's stuff that has them use GeoGebra, or they can choose to use GeoGebra. And, in that case they have to either e-mail it to me or they have to print it off.

Students complete Geogebra tasks on quizzes and tests such as constructing an angle bisector or constructing the inscribed circle for a triangle. During the remainder of the school year, students "maybe once a month" have a homework exercise from the textbook to complete using GeoGebra.

Factors Affecting Current Student Use of Dynamic Geometry

Iris feels very supported by one particular mathematics colleague as well as her entire department and administration:

> Well, I got geometry from him. He used to teach it. So, when we were talking about what to do—I mean, he's never been pushy at all. He just says what he does, and if I go in I'm like, "What do you think about this?" He just says it, and it usually involved GeoGebra…. When we would be in math meetings, he'd be

> like, "Dude, look at this!" So, I think it was just like a very organic sharing. I mean there's only five of us. We're like a family. Like if something broke in my house, I would probably call them if my dad wasn't available or my fiancé. We have a Memorial Day picnic. I mean I would—we just share things.

Iris describes the support from her administration as "superb" because they have provided approval to seek funding for conferences and professional development. "They approve us to get PTA funding for whatever we want and they pay for us to go to the T-cubed every year."

Iris has open access to the laptops to use with her students:

> And, I sign up every day at the beginning of school and then if somebody needs it—next year we'll have four, this year we had three carts, and the computer lab. So if somebody needed them, I was like, "I just don't want them sitting in the room." Like it's not saying that I have to have them every day, but if they're not going anywhere else they better be coming to me. I mean, why let technology sit in a room with no kids?

Iris's students influence her somewhat in how she uses Geogebra with them depending on their frustration:

> Sometimes if I've got a class where they have no idea, then the first person who says it, I'll go up and be like, "You're exactly right. This is how you do it." But, if I have kids who I know I can let them kind of hang a little bit, I'll wait until 10 people say it, and then I'll go up. So, I just try and read the class because I mean I do investigations with all my classes, and I really think it depends on the kids. If I

> let one kid stay frustrated for 3 or 4 min, they're fine. But if I let a different kid stay frustrated for that long, they've shut down, and I don't get them back. So, I think it's knowing who your kids are, who you can let go, or I walk by and say, "I'm just going to show you in a minute so just calm down. I'm not going to let you leave frustrated. You can trust me."

Iris did not have any experience with dynamic geometry software until she started teaching. She went to the GeoGebra conference last year and has gone to T-cubed three times. She also has participated in some workshops given by her colleague on Saturdays. Iris's main method of professional development is to use her own time during the summer:

> If I want to learn anything, I do it on my own time in the summer. I pick up maybe one or two things here and there during the year. That's why professional development frustrates me because all of a sudden they're presenting on this topic, and they're like you should really use it. And I want to be like, "So you're telling me this in October and you want me to implement it next week?" But, if I'm a planner and I've done my job correctly, you missed the wagon.

Iris says that she is comfortable with what she does in class with GeoGebra. "For what I use, very. But as far as understanding all the capabilities, not much."

Past and Future Student Use of Dynamic Geometry

Iris's current student use of GeoGebra is not any different from how she had students use it last year. "I haven't changed how I have them use it. I've changed how I do other stuff, so I don't do points of concurrency any other way now because the first year it was just the worst thing."

In the future, Iris does not foresee using GeoGebra in any other ways with students, but she is going to investigate the possibility at this year's GeoGebra conference:

> I'm going to talk to others about it at the GeoGebra conference and just try and see if there are other things in geometry I could do. Right now, I don't see anything. But, it's not that I don't want to. It's just that I personally don't see it, so maybe getting somebody else's perspective would let me. But for me to use it, I have to feel like it's useful. So unless it's useful, then no because I don't believe in just using technology to say I'm using it.

Jill

For 10 years, Jill has been teaching mathematics. She began her teaching career, at a private high school, and then she stayed at home with her children for the next 8 years. The last 5 years, she has been teaching at an independent K–12 Catholic school. Because the school is independent, it has "a lot of leeway, probably, compared to a lot of other schools." Located in a suburban area, the school has graduation classes of 100 students. The high school mathematics department has five teachers.

Jill has taught geometry for 9 years with 5 of those years at her current high school. She is one of two teachers who teach geometry. Jill teaches one section of geometry with 15 sophomores. There are not any freshmen in Jill's geometry class because they are in the honors course. The school is on "modified block scheduled," so each class meets "three times a week for 70 min." Jill really enjoys teaching geometry. "I love it…. It's one of my most favorite things to teach just because there's—everything is not black and white, and there's room in between."

Jill describes her instructional approach as "laid back" with a great deal of dialogue between her and the students:

> My class is very—kids aren't raising their hands. It's very interactive in terms of, it's more of a dialogue back and forth in terms of, I'll pose a question…. I'm not, like, inquiry-based but I like to ask questions. It's constant question, question. What do you know? And, when they can answer the short questions, I think it gives them confidence in getting to the end because they've been answering these along the way correctly.

To help facilitate student learning, Jill gives students the notes prior to class. "So, I always type out my notes ahead of time with—you can see, so they kind of—they'll be typed out. And, they'll have blanks they have to fill in on their own."

Present Student Use of Dynamic Geometry

For 4 out of the 5 years that she has taught geometry at her current school, Jill's students have been completing dynamic geometry tasks using Geometer's Sketchpad. Students complete 15–20 activities on topics including constructions, parallel lines, quadrilaterals, polygons, and similar triangles. The activities are usually closed-ended. Jill's general goal when having her students use GSP is "just to make a connection between the diagrams and the theorems:"

> It's to give them a concrete understanding of why the theorems are, why the definitions work, why when you bisect the chords they go through the center of the circle. You know, why does that work? So, just to reiterate, so they're not just simply memorizing something when they can kind of go back to a picture in their head about it if they forget it.

Jill believes that the greatest benefits of using GSP is making connections, using technology, and breaking from the routine:

> Just to make the connection and just, like I said, any time you can—with computers, you know, kids are constantly on their phones. They're constantly texting on their phones…. And, that's what kind of floors me that some kids kind of draw back from it a little bit and aren't as excited about it, because you'd think that in this technological age everybody would want to do it. So, it amazes me that it does still scare some of them, but I do think it's a break from just your normal everyday just me up there talking and lecturing. And, it just kind of gives them a little bit of a break and be just—puts it hands-on.

In the classroom, students use laptops individually. The GSP activity sheet is already a part of the teacher-created notes handout that Jill has given them for the chapter. The sheet does not have an introduction, and it does not have any diagrams. The sheet does not explicitly tell students how to do the construction, but it tells them what to measure. For example, students are instructed to construct a parallelogram, and then measure the four angles and the four sides of the parallelogram. Next, students construct the diagonals and measure the four segments created by the intersection point of the diagonals. Frequently, students have blanks to fill in to complete a theorem. The activity sheets do not explicitly say anything about dragging points. Jill describes the activities as:

> Anything where we're trying to introduce a theorem we usually construct it and kind of see, okay, the reason why the isosceles triangle theorem worked is, okay, we're going to construct this triangle and you're going to measure the angles. And, so, I'll give them the sheet, and they have to come up with the theorem based on what they saw on the construction on the Sketchpad. It's pretty much, I think, the main goal when I use it is for them to kind of understand why the theorems work.

When students are first using GSP, they are doing constructions like they would with a compass and straightedge. They do not use the commands out of the menu such as perpendicular line or parallel line; instead, they use the compass and straightedge tools:

> I like that there's built-in things on Sketchpad, but you don't have to go through all that rigmarole to find the perpendicular, to find the parallel, but when you're first learning it that you have to go through all of that. And then when they finally kind of get it all then you can show them the shortcuts.

Jill does little to introduce the activity: "And usually when we do Sketchpad…it's in their notes for the chapter at the time. And then I'll just say, "Okay, fire it up and get started on it." Jill gives students some time to work individually and with each other before demonstrating the construction to students using the projector:

> I give them probably about 10—depending on what it is I'll give them 5–10 min on their own to come up with or collaborate with somebody next to them and walk around and see what they're doing. And, then I'll hook mine up and put it on the screen, so they can if they weren't getting the construction right. Here's what you can do. And, they can do it while I'm doing it.

The GSP activity is tied back into classroom instruction through the notes handout for the chapter:

> Because they have in their—in terms of the notes, it has the Sketchpad activity, and then it usually has the theorem. And, then it has the examples, so they can see, "Okay, I went from this to this to this, so I can see how I can apply what I did here to the theorem to my examples."

Jill does not grade the in-class GSP tasks, but students also have a few GSP assignments for homework that are "maybe 5–10%" of their quarter grade.

> It's more probably over the course of the year they've probably had maybe five Geometer's Sketchpad assignments they've taken home, and they've been pretty big ones in terms of what we went through, you know, the hierarchy of the different types of quadrilaterals. That was a big assignment; construct a rectangle. Construct and show me that it is what it is. And, then they did the regular polygons. But, most of it's class work just to kind of help reinforce the theorems, so not much of it was probably part of their overall grade. It was just more part of their class work.

Factors Affecting Current Student Use of Dynamic Geometry

Jill feels very supported by her colleagues in the mathematics department:

> And, there's a lot of collaboration within our math department because we're all—there's five of us and we're very—we work well together. Outside of work we're all very good friends. So, if there was ever a need that I had, I know I could go to any of them.

On the other hand, Jill says that her administration "probably does not even know we use it [GSP]."

Jill's students are at times frustrated with GSP and struggle with it:

> And, some kids take to it like ducks, and some kids just struggle with it and they're, like, "Why is this pulling my grade down? I just can't do this…" Some kids, though, have really struggled with Sketchpad and I'll say, "Okay, let's—we're getting the computers out." They'll say, "Oh, no, not this again." I'm, like, "Really? This is a treat. This is a break in me just talking."

In class, if Jill senses that the student frustration level is too high, she stops the activity to demonstrate:

> My goal is to give it to them, and then me never have to put my thing up on the board, but then sometimes it's…just, like, okay, we're getting nowhere because they're getting frustrated. So, at that point in time I was plugging my laptop in and I was saying, "Okay, construct a line segment," and I'm doing it as they're doing it. I don't like doing it that way.

For the out of class assignments, Jill has had some conversations with parents who are concerned about their child's frustration with using GSP:

> And, I've had conversations with parents…. But that frustration level, "My kid was home all weekend trying to do this, and I can't help him" because they know nothing about the software. And, I've gotten some of those e-mails and calls. And, that's what I tell them. "This is not meant to frustrate them. This is meant for something fun to do that they can just kind of take it outside." Some of them buy into that, and some of them think it's crap. It all depends on the parent.

Jill thinks that her class has mixed feelings about using GSP. Some students like it, but others do not like it:

> You'd get a split poll. I mean, I think some of them would say they really like it. Some of them would say, "It's so frustrating. It's so hard, and I don't get why we're doing it." So, I think you'd get a split. I mean, I think some kids, like I said, like it just because it's something different, and so they welcome that. But, I mean, I would say probably of my 15 kids six of them would say, "I hate it…." So, I say they'd say, "I hate it," meaning, "I'm frustrated with it."

Because of students' frustration and struggle, Jill tends to help them quite a bit:

> So, I probably tend to walk them through a little bit more than a normal, but I'm a step by step person, so that's how I thrive. I probably walk them through a little bit too much, handhold a little bit too much, but I know what I'm dealing with, so I've learned walking them through is a little bit better.

Some of the students' frustration and struggle according to Jill is because students do not practice using GSP at home:

> I said, "Some of this is you taking this home and practicing it, and you becoming comfortable with it. It's like anything else. It's just your comfort level so that when you come back that you feel a little bit better acquainted with the software." Some kids probably did it, and maybe that's why those are the ones who really took to it. Some kids just thought, "Nah, one more thing to do, and I'm not going to do it. And, how is she going to know if I don't do it?" I think then that shows later when they just are still frustrated with it.

Jill has "total access" to computers. "Here every department has their own carts. And, then we do have two computer labs, but I've never had to take my kids down to computer lab because I've always just gotten them off the cart." Because Jill has laptops readily available, she can be spontaneous and use GSP on a whim:

> And, because some days, I'm not saying I don't know what I'm doing on a day-to-day basis. I do. But, some days if I think that even if it's something that I wasn't planning on doing I'll say, "Hey, let's go get the computers." And, we'll just walk down the hall, grab the computers, and it's just that easy.

Jill's colleagues who were already using GSP and her taking an online GSP course were the biggest influences to get Jill started in using it with students. Jill did not know anything about DGS until she returned to teaching after staying at home with her children:

> I taught Geometry that first year, and I saw all these other teachers using it. I was like, "I didn't know anything about it." And, they said, "Oh, you need to take this class so you know how to use it." I was so excited about taking it and then I started using it, like, at Christmas time next year.

Jill has continued to use GSP because so are her colleagues, and she likes it and believes in its value as a learning tool:

> If I didn't like the software and I didn't believe in what it did, if I didn't buy into it, I think it would be harder for me to use, but since I enjoy it then it's not, like, "Oh, my God. I have to get Geometer's Sketchpad out."

Jill says that she is really "confident" with what she does in class with GSP:

> Like I said, now, could I do more with it? Probably absolutely, and I really haven't take the time to maybe see the dynamic stuff…but, what we do with it I'm very confident with it, but I'm sure there's a lot—I know, there's a lot to the program that I'm not using.

Past and Future Student Use of Dynamic Geometry

Jill's current student use of GSP is not very different from how students used it in prior years, but she has added more activities:

> Some of them I've probably tweaked, but most of them I'd say probably the steps that we've used to get to the end result has been the same. I mean, some of them might have been tweaked. And, some of them I probably made up last year, because, like anything, the more comfortable I feel with it the more I know what

to do with it. The more I've taught it I see, "Oh, maybe this would lend itself to doing it on Sketchpad."

In the future, Jill would like to "do some kind of more fun, creative things with Sketchpad" for student enjoyment. If time permits, Jill would like to use GSP for tessellations:

> Maybe do some of the tessellations and show them how Sketchpad can take the points and change them and then dynamically move them all to create an Escher. I think that would be neat for them to see.

Jill would also be interested in participating in a GSP webinar:

> I've done webinars. I've done, I think, two or three of them this year. None of them have been solely based on Sketchpad. And I get e-mails, you know, about webinars sometimes. I've not seen one that's come across that interests me in Geometer's Sketchpad, but if there ever is one I definitely would want to watch it, participate in it, go to a conference on it because I just would like to learn more.

Kate

Kate has been teaching mathematics for 6 years at a Catholic high school. Her school graduates 175 students each year:

> It's pretty diverse as far as racial makeup and socio-economic kind of things. The neighborhood that the school is located in is fairly poor. And so kids—there are a number of kids that will come from the neighborhood. And then there are also—the majority—I'd say 80% of our kids are from Catholic feeder schools. There

> are six feeder schools. And it depends, some are affluent; some are not. But, it's a fairly broad mix of kids and abilities.

The high school mathematics department has five fulltime teachers and two part-time teachers.

For 3 years, Kate has taught geometry. She teaches two geometry classes of 24 and 28 students. The students in her geometry classes are mostly sophomores and a few freshmen. The majority of freshmen take honors geometry. The school day has seven 50-min periods. Kate describes her comfort level with the content as a 10 on a scale from 1–10. "I guess, I just feel having gone through the curriculum four or five times by now that I feel that I pretty much know it."

Kate spends about half of her class time on whole-class instruction. "I'd say about half the time would probably be lecture kind of stuff or discussion kind of stuff as a class." The other half of class time is spent on practice, activities, and formal assessments:

> I'd say at least one day a week we usually spend on some sort of practice kind of stuff just in class, like doing problems, either with partners or small groups or on white boards or whatever. I'd say probably about once a week we do some sort of exploratory sort of activity, whether it's on Sketchpad or just a worksheet or with protractors and compasses, whatever, some sort of hands-on kind of a thing. And then, some time for quizzes, test kind of stuff.

Present Student Use of Dynamic Geometry

For all 3 years that she has been teaching geometry, Kate's students have engaged in tasks with Geometer's Sketchpad. Students use GSP eight times to complete mostly closed-ended tasks. Topics include properties of parallel lines, triangle congruence, triangle inequalities, quadrilaterals, and circles. Kate describes the activities as "fairly guided." With a little bit of "discovering concepts:"

> I don't do a lot of just draw something and explore. A little bit at the beginning, but they typically have instructions and then they have to do whatever it is that I'm asking them to do, and them make observations based on that. So, I think I use it as a tool for discovering concepts rather than just saying this is the relationship between these angles that you have to kind of see it and experience it so that, hopefully, you'll remember it better.

Kate's general goals are to give her students something fun and interactive to do while problem solving and thinking for themselves to discover geometric concepts:

> I think to make them problem solve a little bit, think for themselves, make them more interested in what we're learning rather than just having to regurgitate something that they can kind of discover something for themselves and make it a little more meaningful to them, make it more fun and interactive. But, I think the independent thinking and the critical thinking kind of stuff and rather than just being told something, having to figure that out is one of the biggest things that I'm kind of going for.

Kate thinks that the greatest benefit to having students use GSP is "kind of the same things as the goals," that is "to make it interactive and a little more interesting" and "independent thinking kind of stuff, to make them think critically."

In the classroom, students use GSP on laptops with a partner because there are not enough laptops for each student, but Kate thinks "in a lot of ways, works better that way, just to make them kind of talk about, and you're not stuck by yourself if you don't really understand what's going on." Students are partnered up with whomever they are sitting next to in class, but Kate switches students' seats every few weeks. Kate does not initially give the students much direction when beginning an activity. "I kind of just let them start and then sort of see how it goes." Students begin by opening a new file and following the directions on a published activity sheet. The activity sheets have an introduction and a few diagrams. When students are working in pairs, one student uses the laptop, while the other one takes care of the paperwork:

> One person's in charge of filling out the questions or answering the questions, and one's kind of doing the computer. And, then part way through, I'll ask them to switch so that one person isn't always doing one job or the other, just to kind of give everybody a chance to play with the computer.

While students are working, Kate "usually walk[s] around" monitoring student progress to "make sure everyone is on task." A couple of times per activity, Kate stops the class, and she or a student demonstrates something on the interactive whiteboard:

> And then, if I see somebody with something interesting then I might stop everybody and let them go up and tell everyone what they've done…. Usually,

> maybe partway through to sort of make sure everybody's kind of getting the idea, and then at the end, as a wrap up, we would usually have some sort of discussion or take some notes or something.

Kate does not grade, but only collects the students' sheets and usually reads over them. "Sometimes I'll collect their worksheets and kind of see their responses and that would sort of tend to have to be a day after kind of thing." Students are instructed to drag one or more points before answering a question. The questions are usually based on making a conjecture. Kate uses what she has found out about student learning to have a class discussion:

> If I look and kind of see is there something people struggled on or really didn't get, then we'll discuss that. And, then I would say 75% of the time, then there's theorems or notes or something that we would then take and do problems and stuff like that on the material.

Kate does not grade the student work. "I don't usually grade it. I'll collect it and make sure they actually did it. I tell them that I'm going to read it, and so that sort of motivates doing it."

Factors Affecting Present Student Use of Dynamic Geometry

Inside her school, Kate does not have much support for using GSP besides open access to the laptops. Her mathematics department colleagues do not use it much:

> I've tried to get some of them using it more, just because we have it and we spent the money on it. But, I think it would be good if some—one of my—so there are three people right now that are teaching geometry. I'd say I use it far more than

anyone else, but one other teacher uses it occasionally. And, then one of them just has never used dynamic geometry software ever before. And, so she's not terribly comfortable with it. But, I was actually talking to her today, and she's going to start doing transformations next week. So, I gave her an activity that I've done before to kind of introduce that. And, so she's going come talk to me about it tomorrow and kind of go through it, so that she can start doing a little bit.

Kate does not think her administration is aware of her use of GSP. "Our technology person is the one that provided the funds to buy it, and he's very supportive of it and likes it. He's a physics guy. But, my principal, I don't know that they would even know what it is." Kate has open access to laptops whenever she wants to use them:

We have a laptop cart that is for one wing of the building, so that's probably 12–15 teachers can share this cart. And so, typically, whenever I want to use it, it's not—a lot of the teachers I don't think use it. If I want to use it, most of the time I'll be able to. And I might have to rearrange a day or two, depending on if other people are using it or not. But for the most part, I can kind of have it when I want it.

Kate has continued using GSP because students are able to connect what they do with GSP to material they are learning in class lectures:

Every, not every time, but often, after you use it, then somebody will bring something up in class later like, "Hey, isn't that just like whatever?" Or, you can see that they've kind of made some connections and are retaining something from that experience. And just the ability to sort of move things around, I think helps

> them think in a different way at times, just sort of, instead of it just being static, that they can kind of manipulate figures in their mind a little bit better, which I think is kind of a good skill to have.

Kate's first experience with DGS and also her biggest influence to get her started using software was from a professor in her master's degree program:

> I was a teaching assistant, and saw it for the first time there. And, so I was teaching a class for—a geometry class for future elementary education people. The professor in charge of that was a math ed person…. And then, I was a teaching assistant and liked it a lot, and so that was the first time that I saw it.

Kate also has taken some professional development. "I took two online courses on it, the geometry course and the algebra course offered by Key Curriculum. And, so I think during those that I got pretty comfortable with it." Kate also participated in the Park City Math Institute the last couple of summers:

> There are math researchers there and graduate students, and undergraduate students. But there's this program for teachers, also. So its about 70 teachers—secondary school teachers and middle school teachers—are there every year. And you just kind of spend the morning doing interesting math problems that they write. And so, Sketchpad has been kind of a big part of that experience, as well. And so I think I learn—because there are a lot of very tech savvy people at this, and so they can do cool stuff. I think I've learned some stuff there, too.

Because of all the professional development, Kate considers herself proficient with GSP.

Past and Future Student Use of Dynamic Geometry

Over the 3 years that Kate's students have been using GSP, she has changed some of the activities that she uses:

> I think I've learned that you have to be fairly specific with kids about expectations and what you want them to do. And, so I try to provide that kind of stuff for them. And, I think different activities that I've done before like there are some in here that I've tried before and just don't go well. And, so then I might try something else the next year, and other ones that I feel have gone well. And, so I do those year after year. So, I don't know that I've necessarily changed a ton like what I have them do, but just kind of maybe when or what activities that I choose for them to do.

Some of the changes resulted from students struggling with the activity, and Kate's thinking that students did not get much out of it:

> Just if they're really struggling to just do it, and I feel like they spent 40 min doing something and they haven't really gotten anywhere, or they just aren't answering the questions or whatever, they're answering them in a way that makes me think that they don't really know what's going on at all, or what they're looking at.

Some of the choices from year to year are due to where Kate is in teaching the material in relation to the school year:

> But it also, I think, depends on just timing of where you are in a specific point in time if it's a Friday before a long weekend or something, and you feel like they're

> really not going to listen to what I have to say, maybe if I put them in a situation where they have to do something, that they'll be more engaged. And, so I think it depends a little bit on that, too.

Kate does not have plans for the future for using GSP because she is not teaching geometry next year.

> We have a teacher that our department head really wanted to only have to focus on one course, just because this is only her second year, and so she doesn't want to have to have her teach three different things. So, she's going to do geometry and honors geometry.

Currently, Kate is not interested in any professional development with dynamic geometry software because she does not know what is available. "None that I know of right now. But if there were more I'd be interested, I think. But I don't really know of any right now."

Leah

Leah has been teaching mathematics for 21 years. She teaches at an urban magnet school for Grades 7–12 with 400 students in each graduation class. The school offers "all AP courses that can be taught." The school also has a "top ranked" orchestra and band along with a "very strong theater program." It also has "almost every single sport you can think of." Leah describes it as "neat place to work." The school has a lot of diversity:

> There is an equal distribution of poor, middle class, and wealthy. We have parents who are on welfare as well as part of the working poor. We have parents

> who are professors, health providers, or have careers in the trades. We also have parents who are doctors, lawyers, and people in business, and some CEOs….We also get a lot of students from outside the district. These students previously attended top private schools, Catholic schools, Montessori, the neighborhood schools, or they were home schooled.

There are 13 mathematics teachers for Grades 7–12.

Leah has been teaching geometry for 13 years. Her school has two mathematics tracks. One-third of the students are in the honors track and two-thirds are in the normal track. Leah teaches two geometry classes in the honors track and three geometry classes in the normal track. Her classes each have 25–30 students. The school day consists of seven 50-min periods. Leah is "very comfortable" with the geometry content that she teaches and describes herself as "a Van Hiele believer."

Leah's instructional techniques are focused on students working together in groups. "My biggest teaching method is they sit in a group, and they work together." Early in the geometry course, there is a focus on constructions:

> At the beginning of the year, there's a lot of construction, so the kids are helping each other with constructions. There is lecture when needed, but mostly it's activity based. And then, you know, Sketchpad or constructions or tracing paper—whatever method is most appropriate for the activity.

Leah's textbook is investigation-based, so that plays a big part in what students are doing:

> Some days it might be, "Let's do these investigations, and then here's some sample problems." And then, I try to do two or three days of activities or then go

> over homework in a lump sum. I mean unless the kids really, "I got to know this." I don't—I just kind of like to keep the ideas rolling, and then go into it.

Present Student Use of Dynamic Geometry

Over the last 8 years, Leah's students have been engaging in dynamic geometry tasks with Geometer's Sketchpad. This year, students have also used the geometry software on the TI-*n*spire. Leah's students use GSP about four times and the TI-*n*spire about 10 times to complete closed-ended tasks. Student DGS activities included points of concurrency, right triangle trigonometry, tessellations, and quadrilaterals. Leah's general goal for her students is "for them to have that opportunity to see whatever picture they need to have to confirm it. So, it's to really build those visual connections between, 'That's that definition. That's that picture. Here are the results.' That's the big thing." The greatest benefit to the software is that it makes use of the "Van Hiele [model of learning], getting them to see all the possibilities:"

> Then also, when kids have—and this has happened once or twice—but, they'll have a funky question, and that's the nice part about the *n*spire is, "Well, let's just see." You know? And, they can move it around. But, just to be able to see all the possibilities is the best part.

When using GSP, students open a teacher-created GSP file and follow the directions on the screen. The files already have the figures constructed along with a table of measurements. Students are asked to drag vertices and make conjectures based upon what they notice in the table. Although each student has his or her own computer or calculator, students often help each other:

> It's really neat to see the kids who get the technology and go through it really fast, sit and help the other kid who doesn't. They're not lost in the manipulation of the ruler and the compass and—that all those little steps. They can actually back up and see their, you know, like this, "Oh, there's the patterns. Can't you see this pattern?" Or, you know, they'll go and want to ask another question a different way.

While students are working, Leah walks "around the room, correcting errors." Leah also encourages students to help each other. "A lot of times I'll be doing, 'You know, so-and-so is having problems with the computer, can you go over and, you know, help him through that problem, or help her through that?'" She also has the ability to monitor students' computer usage from a central teacher station.

To tie it back into the classroom instruction, Leah has students complete a table or have a class discussion about what they did. For assessment, "usually it's participation. Yes, you did it; no, you didn't." Students submit the file, though Leah might not look at it sometimes. "But, I do ask them always to turn it in, always submit it, even if I never look at it." The points of concurrency activity is an exception; that activity is graded.

Factors Affecting Present Student Use of Dynamic Geometry

Leah has some support from her mathematics colleagues and administration in using GSP. In the mathematics department, it is evenly split between those who use software and those who do not. "A lot of them are like, 'No, I had pencil and paper, and that's all I need and software is not necessary.'" Despite the division in technology use, "all of the geometry teachers, except for one of them, have agreed to do the concurrency" activity with GSP. Her department chair and administrator “do believe that software is important, and that you’re developing something beyond stuff.”

Because there are only “two computer labs for 120 teachers,” computer access is “very limited.” The TI-*n*spire should help with this issue. “The *n*spire is great. Now, anytime something comes up, you can just go with it. But, it’s more important for the kids to do it than me do it.”

Leah’s students do not influence how she uses it, but they are why she keeps using it. “I mean obviously since I’ve done it for so long, that I ignore the ones that don’t want to do it.” Most of her students like it though she is not sure why. “All I can say is that I do know that they do like it. For whatever reason, they do.” Her students are the biggest reason that she has continued to use GSP because they “enjoy it” and they work well with each other:

> They enjoy going up in the lab and working at their own pace. That’s the biggest thing that—and I completely forgot about that, is that—especially in the CP class, where you’re dealing with discipline, or you’re dealing with so-and-so can’t remember, they can get up there and they can go at their own pace. And, watching the kids communicate and talk to each other. So, it really is the students.

Leah did not have experience using dynamic geometry before her school purchased GSP. “Somehow they decided to buy it at our school. I couldn’t even tell you who. They just said, ‘Hey, there’s this software.’ And, I just kind of got hooked.” Leah did have a little bit of professional development. “It was just a little kind of thing. But that was just for teachers getting around it." With GSP, Leah's level of proficiency "on a 5 [point scale], I would say 3 because with the geometry side of it, I'm real good with the shapes. But, I don't know hot make the tools, like that kind of stuff." Because she is a novice user of the TI-*n*spire, Leah's rate her proficiency as "a pretty confident 1" on a 5-point scale.

Past and Future Student Use of Dynamic Geometry

Leah's current use of dynamic geometry is not any different from her past use though she is using the TI-*n*spire some this year. In the future, Leah would like to use DGS in more unstructured and spontaneous ways. She thinks that the TI-*n*spire might help her do that:

> Because I would really like for them to be able to just build it and go and not have it be so constrained. And hopefully, with the *n*spires, maybe I can get that to, "Oh, you posed that. You don't understand. Let's pull it out and actually look at it and make it and see what happens." So, yeah, I would like it to be more that way…. So, yeah, I would like to get it more unstructured. And, I would like to get it to be more so getting it from this data and when I could get it in to a graph form to make that connection from—to the graphic representation and not just keep it in table form.

Leah is participating in professional development led by a high school teacher this summer with the TI-*n*spire:

> He's doing the *n*spire on how to use it to set up the questions in the classroom and the whole bit. I guess for a lot of it is building the software to the animation

> where the kids could develop more of a booklet or a—you know, like, "I started here, and now I've gone to that."

Cross-Case Analysis

This section presents the results of the cross-case analysis for the collective case study. In the first part, the results are personified using a hypothetical teacher named "Zoey." Zoey's attributes are representative in a modal sense of the geometry teachers in this study. After Zoey's narrative, the second part describes the most prevalent themes for the collective case study. This part can be thought of as a more elaborate version of Zoey's narrative with explanations and evidence to support each theme.

Zoey, the Typical Geometry Teacher in This Study

Zoey has been teaching mathematics for 7 years. She teaches in a public school near a large urban area with 130 students in each graduation class. The student population is 90% white, non-Hispanic, and 25% of students are identified as economically disadvantaged. The high school mathematics department has five teachers.

Zoey has taught geometry every year of her career except for 1 year. She teaches two or three sections of geometry each year. Her class periods are 50 min in length, and her average class size is 20 students consisting of a mix of freshmen and sophomores. Zoey is comfortable with the geometry content that she teaches.

Zoey uses a variety of instructional activities in her geometry classroom. She often uses her interactive whiteboard to facilitate the lesson by displaying notes, figures, and occasionally DGS demonstrations. At the start of class, Zoey sometimes reviews the homework or has a warm-up activity for students to complete. She occasionally lectures, but usually for no more than half of the class period. The lectures can be interactive consisting of some discussion with her students, or Zoey might provide guided notes with a general outline and blanks for students to fill in. Students often work in groups. Many times while in groups, students are working on hands-on activities using such tools as a compass, a ruler, a protractor, and patty paper.

Present Student Use of Dynamic Geometry

Over the last 6 years, Zoey's geometry students have been completing dynamic geometry tasks with Geometer's Sketchpad. The activities are either guided discovery or used to reinforce material that Zoey has previously taught. Students use GSP nine times to complete mostly closed-ended tasks. Student activities include points of concurrency, parallel lines, quadrilaterals, transformations, and triangle congruence, and circles. Zoey's general goal is for students to explore geometry in a hands-on way without direct instruction from her. Zoey believes the greatest benefit to students using GSP is that it is dynamic and gives students access to hundreds of visual examples by dragging a point in the construction. Other benefits include that students are doing something they enjoy and they do not have to listen to Zoey lecture.

When using GSP, students use computers in a lab setting or laptops in the classroom. Students begin by opening a new file and following the step-by-step directions on a teacher-made activity sheet. To introduce the activity, Zoey sometimes shows students some of what they will be doing using the projector and interactive whiteboard; other times, she does not give any introduction before students begin working. The activity sheet gives specific steps on what to do, but they usually do not tell students explicitly how to do the step on GSP; for example, a step on a sheet might be to “construct the angle bisectors from each vertex.” Although each student has his or her own computer, students often discuss ideas and help each other. While students are working, Zoey walks around answering questions and making sure students are on task. The questions on the activities sheets usually ask students to state a conjecture or theorem about a particular relationship that they have verified with GSP. Students either turn in the assignment electronically or on paper to Zoey. Zoey grades the assignments, but it is not a large part of a student’s grade. When Zoey introduces new material, she uses the GSP assignment as a springboard for a discussion and guided notes for students.

Factors Affecting Present Student Use of Dynamic Geometry

There are some factors affecting Zoey’s present use of GSP with students. According to Zoey, her students enjoy using GSP because it is a change of pace. Her administration is supportive because she is using technology, but they probably are unaware of anything specific about GSP. In college, Zoey had some experience with DGS. In fact, she credits a college professor with being her biggest influence to get her started in using GSP. She also has participated in some professional development on GSP online and by attending some conference sessions. Zoey says that she is fairly proficient with GSP, and she is especially comfortable with what she has her students do. Although she knows there is much more than GSP can do, she is not very knowledgeable about much beyond what she has her students do.

Past and Future Student Use of Dynamic Geometry

Zoey's current use of dynamic geometry tasks is not that much different from when she started. She has increased the number of activities that students are doing when compared to the past. Zoey also has refined some of the current activities to make the directions clearer and easier for students to follow. Activities that she felt were confusing for students or ineffective, she has discontinued using.

In the future, Zoey envisions making some small changes in student use of GSP. She hopes in the future students might have access to dynamic geometry software all the time either through an open lab or on the iPad. This open access would allow the use to be more organic, spontaneous, and unstructured. Zoey would also like to incorporate some more partner or group activities. A student project is possible where groups of students could present their work to the class. Last, she may continue to try to add some topics, particularly transformations and tessellations.

Zoey is open to participating in professional development to learn more about using DGS. She would like to be able to meet with a group of teachers who are using the software to share ideas. She is also interested in attending conferences such as OCTM or NCTM and going to sessions about DGS.

The 11 Themes Derived From the Collective Case Study

This section describes the 11 themes from the collective case study that emerged from the cross-case analysis. These themes are named and briefly described in Table 5. The first four themes pertain to how students are currently engaging in dynamic geometry tasks. These themes are (a) teacher use convergent tasks, (b) students collaborate, (c) teachers circulate, and (d) teachers facilitate whole class follow-up. The next four themes relate to factors affecting implementation of dynamic geometry tasks. These themes are (a) DGS provides accurate dynamic visual aids, (b) DG tasks are a change of pace, (c) teachers experience restricted access to computers, and (d) teachers are proficient with the basics of DGS. The last three themes connect the present DGS usage to the past and the future use of DGS. These themes are (a) present use builds minimally on past use, (b) future use will build minimally on present use, and (c) teachers are open to professional development. Within this section, a separate part elaborates on each theme. The elaboration of each theme contains three components. The first component describes how and which teachers exemplify the theme. The second component contains the results of the negative case analysis. That is, reasons why particular teachers do not fit that theme. The third component displays a table with supporting evidence. The table for the convergent tasks theme contains numerical data. The table for each of the remaining themes displays supporting quotations from the teacher interviews.

Table 5

The 11 Themes Derived From the Cross-Case Analysis

Theme	Description of Theme
Teachers use convergent tasks.	• Specific written steps usually are given. • Objective is to discover conjectures or to verify theorems.
Students collaborate.	• Students discuss ideas. • Students help each other with the software.
Teachers circulate.	• Teachers answer student questions. • Teachers make sure students are on task.
Teachers facilitate whole class follow-up.	• Teachers tie tasks back to classroom instruction. • Teachers lead class discussion or lecture.
DGS provides accurate dynamic visual aids.	• DG figures are more accurate than hand-drawn figures. • DG figures can be dragged to see geometric relationships.
DG tasks are a change of pace.	• DG tasks differ from lectures or taking notes. • Students enjoy using DGS.
Teachers experience restricted access to computers.	• Limited computers are shared among all teachers in the building. • Computer time must be scheduled far in advance.
Teachers are proficient with the basics of DGS	• Teachers know how to construct circles, triangles, and quadrilaterals and how to measure angles and segments. • Teachers are not very comfortable with anything beyond the simple construction and measurement tasks that they have their students do.
Present use builds minimally on past use.	• Teachers have edited some past tasks for clarity. • Teachers have added more tasks.
Future use will build minimally on present use.	• Teachers may edit some of the present tasks for clarity. • Teaches may add some new topics.
Teachers are open to professional development.	• Teachers are open to learning more about DGS. • Teachers are open to discussion with other DGS users.

Teachers Use Convergent Tasks

The first theme is that students are completing convergent tasks with dynamic geometry software. For the document analysis, 12 teachers gave the researcher a total of 121 student activities. Almost all of these tasks were convergent task. When engaging in a convergent task, students do not have much choice on what or how to explore. If students do these tasks correctly, they have been guided to discover the same conjectures or to verify the same theorems. The typical dynamic geometry task asks students to write down a conjecture or to restate a theorem using their construction. Although almost all of the student tasks are convergent, teachers enacted these tasks a little differently.

Alan, Hank, Jill, Eric, and Iris have some tasks that have elements of open-ended exploration. The open-ended element of the tasks is in the construction aspect of the activity. Students are not given specific instructions on how to construct the geometric figures. Alan asks students to construct a figure, for example, a parallelogram, and "then give them some time to try to figure it out on their own." His activity sheets are unique because there is nothing to indicate that they are a dynamic geometry task. Hank uses some tasks similar to Alan. Hank gives students "a goal and they have to try to create this thing by any means necessary." Jill's tasks also ask students to construct a geometric object, and she gives students "5–10 min on their own to kind of come up with or collaborate with somebody next to them." Eric has a few tasks that ask students to create a construction with proper measurements to verify a theorem out of the textbook. Iris uses the investigations in the *Discovering Geometry* (Serra, 2003) textbook to facilitate the student activities. Because these activities are written for compass and straightedge or patty paper, students have some opportunities to explore; however, these opportunities to explore are limited because students only have to find the correct command in the menu. "And, then it comes like hide and seek. They're like, 'Oh, I found it. I found it first.'"

Bill, Cara, Dawn, Eric, Faye, Gina, Hank, and Kate all have tasks that are representative of the "standard" dynamic geometry student task found in this study. This task has explicit steps on an activity sheet for the students to guide them to a particular conjecture or theorem. Among the different teachers, there is some variation in the orientation of the steps on the activity sheet. Bill and Hank do not give students explicit instructions on how to construct a geometric object with the software. On Bill's activity sheets, a step says to "construct the three medians of a triangle." When Hank uses an activity sheet, he uses one from the teaching ancillaries in his textbook (Larson et al., 2001). Because these activities were written in a generic form for any DGS, they do not have specific steps for how to construct a geometric object with Geogebra. Faye, Gina, and Kate use activities from a published GSP activity book (Bennett, 2002). These activities only have specific steps with GSP when students have something unfamiliar to construct. These instructions are listed in the margin next to that particular step. Cara and Dawn have teacher created activities that have very explicit steps on how to construct a geometric figure with GSP. For example, to construct a perpendicular line it says "Highlight AB and point C. Go to Construct and click on Perpendicular Line." Some of Eric's activities give specific GSP steps, but other activities only instruct students to construct a geometric object.

When compared to the other teachers' tasks, Leah's student tasks are unique because students open a premade GSP file. The geometric figure is already constructed, and the student directions are on the screen when students open the file. Most of the files have a table already set up with measurements. Students drag to add new measurements to the table, and then they form a conjecture based on the data in the table.

Teachers have students engage in a variety of tasks across the geometry curriculum, but there are some topics that are more frequent than others. Table 6 shows the topic for the student tasks that each teacher gave the researcher for the document analysis. One way to look at the table is to use the "Total" column to see that the five most frequent geometry topics are points of concurrency, quadrilaterals, circles, and constructions; however, this is misleading because some of the teachers have multiple tasks for the same topics. For example, Iris has six tasks with constructions. Instead of looking at the most frequent topics, it is more representative to look for the topics that the majority of the teachers used. Eleven teachers use a task about points of concurrency, nine teachers use a task about parallel and perpendicular lines, eight teachers use a task about quadrilaterals, and six teachers use tasks about circles; transformations and tessellations; and triangle congruence.

A teacher's students engage in DGS activities nine times per year. For the 121 activities the researcher was given, the mean is 10.08 activities per teacher. Hank uses Geogebra weekly; however, he is an outlier because he is the only teacher whose students have unlimited access with a one-to-one laptop program. If Hank's activities are removed, the mean is 8.36. Jill said she used it 20 times but only gave the researcher 8 activities because she did not have all of them anymore. If Jill's 12 additional activities are included, the mean is 9.45 activities per teacher. For the data in Table 6, all of the teachers except for Hank are within three activities of nine; therefore, it is reasonable that the typical number of activities for the teachers in this sample is nine.

The typical dynamic geometry task used Geometer's Sketchpad with a new file and a teacher-made activity sheet. Hank and Iris used Geogebra, but nine of the teachers used GSP exclusively. Leah used mostly GSP, but she is beginning to use the TI-*n*spire some. Students begin with a new file in 101 out of 121 (93.4%) of the activities. Gina and Kate had one activity each that used a premade file. All but one of Leah's activities started with a premade file. The teachers created 66.9% of the activities sheets. Faye, Gina, and Kate almost always used a published activity sheet (Bennett, 2002). Iris used the investigations out of her textbook (Serra, 2003). Some of Hank's activities are written and some are verbal. His written tasks are divided between activities that he wrote and activities from his textbook (Larson et al., 2001).

Table 6

Dynamic Geometry Topics

Topic	Alan	Bill	Cara	Dawn	Eric	Faye	Gina	Hank	Iris	Jill	Kate	Leah	Total
Angle relationships		0.5		1				1					2.5
Circles	1			2	2		2	4			1		12
Constructions			1			1		1	6	2			11
Parallel and perpendicular lines	1	1	0.5			1	1	2		1	1	1	9.5
Perimeter and area				1				1					2
Points of concurrency	1	2	1	1	3	2	1	2	2		1	1	17
Polygons			0.5				1	1				2	4.5
Pythagorean theorem							1	2					3
Quadrilaterals	1		1	2			1	4		3	3	2	17
Segments and angles		0.5	1			1	1	1					4.5
Similarity and dilations			0.5		3		1	3		2			9.5
Transformations and tessellations	2	1	1.5	1	1			3					9.5
Triangle properties	1							1			1		3
Triangle congruence	1	1	1				1	2			1		7
Triangle trigonometry	1							1				1	3
Other				2	1	1	2						6
Total	9	6	8	10	10	6	12	29	8	8	8	7	121

Note. A fraction is used if a task covered more than one geometry topic. Depending on whether the teachér gave the researcher all of the student activities, the number of activities may not be same as what is reported in the narrative.

Students Collaborate

Students collaborate with each other when working on a dynamic geometry activity. As shown in Table 7, all but one of the teachers talked about student collaboration. In most of the teacher quotations, student collaboration is oriented toward the software aspect of the task. Students discuss and share ideas about how to construct geometric objects with the software. There is one exception; Gina's students collaborate more about the "different questions" on the activity sheet. Eric, Faye, and Hank are explicit about students collaborating. Eric tells students to "ask your neighbor," and Faye "encourages" the collaboration between students. Hank assigns pairs or small groups for students to work. On the other hand, Cara neither encourages nor discourages student collaboration. Kate's student collaboration is unique because her students work in pairs sharing a laptop. Her students take turns where one student reads the written directions and the other student uses the software.

Because Iris is using DGS differently from the other teachers, her collaboration is usually as a whole class. Iris only is using Geogebra for the chapter on constructions and points of concurrency. After giving students a chance to find a particular command in the menu individually, she then demonstrates it when a student has discovered how to do it. "So, only one person in the class really has to come up with the answer, but I'm trying to give them as much investigation in a structured environment as I can."

Table 7

Students Collaborate

Teacher	Supporting Quotation From the Interview
Alan	They'll bounce ideas off each other, and a lot of times we'll have kids do it [the dynamic geometry task] different ways. Kids will be really curious to go over and see how the other kids are approaching it. So, they'll learn new methods that way, too.
Bill	But, I've seen them getting done, and instead of getting their other work done, helping other kids. And, then you see that progress the next time you go in and do another activity that that kid that was helped last time, now has a much better idea of how to navigate the…program.
Cara	I don't say one way or the other. If Johnny and Sally are sitting next to each other and Johnny sees and turns to Sally and says, "Hey, how did you get that to do that?" And she tells him, then that's fine with me.
Dawn	A lot of times they would be helping each other and making sure if someone is stuck. Usually the kid next to them is like, 'Did you read the directions?'
Eric	I tell them, "Ask your neighbor. Ask your neighbor…." And, as the year goes on the students know there are students who are really good at it [GSP], and they can go ask them how you do something.
Faye	There's a lot of partner work. It ends up being, "Hey, how did you make that segment?" And, then back on their own computer. There's definitely some collaboration between their neighbors. I kind of encourage that.
Gina	They're girls so they'll probably get together in groups of two or three, and there will be a lot of conversation about the different questions.
Hank	I might have two people paired up, and so they'll be working together trying to build something, or accomplish a task that I've given them on GeoGebra.
Jill	I give them probably about 10—depending on what it is I'll give them 5–10 min on their own to come up with or collaborate with somebody next to them and walk around and see what they're doing.
Kate	One person's in charge of filling out the questions or answering the questions, and one's doing the computer. And, then part way through, I'll ask them to switch so that one person isn't always doing one job or the other, just to give everybody a chance to play with the computer
Leah	It's really neat to see the kids who get the technology and go through it really fast, sit and help the other kid who doesn't.

Teachers Circulate

As shown in Table 8, while students are working on a dynamic geometry activity, all 12 teachers circulate around the room. When circulating, teachers answer questions, ask questions, make sure students are on task, observe students working, and demonstrate on a teacher computer. Cara, Dawn, Faye, and Gina explicitly say they answer student questions when they are walking around the room. Dawn, Eric, and Faye expressed some frustration with students because so many of them have questions that it is difficult to get around the room quickly enough to help all students who need it. Even though some teachers answer students' questions, other teachers such as Alan and Hank ask students questions when they walk around the room. Cara, Dawn, Gina, and Kate make sure students are on task. Bill and Jill sometimes just observe what students are doing. Hank, Iris, and Jill give students a construction task to complete, and then provide students time to figure out how to do it on their own. After students have tried the construction, the three teachers solicit student input and use a teacher computer projected on an interactive whiteboard to complete the construction. Although all the teachers circulate around the room when students engage in a dynamic geometry task, there are several roles the teachers undertake when students are working.

Table 8

Teachers Circulate

Teacher	Supporting Quote From the Interview
Alan	I'll walk around and make sure that they're actually constructing it instead of just drawing it. And, if there's always a few kids in class that struggle with it a little but more, so walk around and pay closer attention to them and make sure that they're understanding it. But, at the same time the kids that get it real quick have questions ready for them to try to challenge them a little bit.
Bill	But, the rows are not real far apart, so I'm going aback and forth in and out of these rows and there's no way around the one end, so they're pushed up against the wall. So, I just kind of walk back and forth and watch.
Cara	And then you would see me walking around, (1) making sure that they're on task and (2) trying to answer questions if they had questions.
Dawn	I am usually running from computer to computer answering questions. I'm usually circulating and just making sure that they are on task, but most of the time it's going from one person to the other, basically pointing out to them the step that they skipped because that is usually the questions that they have.
Eric	In the beginning of the year, I have a really hard time because there is only one of me and 25 students, and I can't get around to everybody at the same time. They're always asking questions.
Faye	I circulate and answer questions and sometimes, there's so many questions that I am running around with my head chopped off.
Gina	I'll be walking around making sure they're on task and answering questions.
Hank	I'll have mine being displayed on the board, and I'll be kind of walking around asking questions, having them give me suggestions, and then I'll do some of the suggestions up there.
Iris	I model it on the projector, and then they each have a laptop. And, then I try and walk around and make sure they can do it.
Jill	I give them probably about…5–10 min on their own to come up with or collaborate with somebody next to them and walk around and see what they're doing. And, then I'll hook mine [my laptop] up and put it on the screen, so they can see if they weren't getting the construction right. And, they can do it while I'm doing it.
Kate	Usually walk around, talk to partners, make sure everyone's on task.
Leah	I'm walking around the room, correcting errors.

Teachers Facilitate Whole Class Follow-up

As shown in Table 9, to bring closure to a dynamic geometry activity, all but one of the teachers has a whole class follow-up following an activity. For the teachers that are using the activities to introduce new material, the whole class follow-up is the basis for class notes or class discussion. Dawn, Faye, Jill, Kate, and Leah use the activities to facilitate student note-taking. Faye has her students explicitly refer back to the questions on the activity sheet. Because Jill's activities are in the guided notes that she gives students at the beginning of each chapter, it is easy for her and her students to connect what they did with GSP and the classroom instruction. After an activity, Leah has students complete and table or chart for their notes. Alan, Gina, Hank, and Kate usually explain or discuss the activities to follow-up with students. When Cara and Dawn review the activities, they have the completed GSP file displayed on an interactive whiteboard. Because Bill and Eric use DGS to follow-up classroom instruction, they only briefly reiterate the objectives of the activities with students.

Iris is the only teacher who does not follow up an activity with a whole-class discussion. She is using GeoGebra only for constructions and points of concurrency. Gina has students complete the construct three ways: (a) straightedge and compass, (b) patty paper, and (c) GeoGebra. Because one of her goals is for students "to be able to learn a technology," Iris brings closure to a task by verifying whether students can correctly perform the construction on GeoGebra. "I would just pick the hardest point of congruency to make…and I would say, 'Okay, I'm going to come around, and you're going to show it to me.' And, I just have a list of my kids, and I put a checkmark."

Table 9

Teachers Facilitate Whole Class Follow-up

Teacher	Supporting Quotation From the Interview
Alan	But, a lot of it, I'll give them a goal of things we want to accomplish, and then give them time to try to discover on their own, but then I'll always wrap it up where I explain an example of how to do it to give some of the kids that struggle a little bit a chance to see it, too.
Bill	I think you always can use it because that next day when you're passing them back, you can kind of review one last time. Or say, "You know the other day we were in the lab we did this on Sketchpad, what did it tell you?" And right away for the ones that got to that next level, it's like, "Oh, yeah, that's right. That was that activity." And so, it keeps bringing it back together.
Cara	When I hand them back to them, then we go over those Sketchpads. So, I would have those sketches up to makes sure they're okay. That's what we did, and try to make sure that that equates. So, my thought would be if we did that sometime in the chapter, they would get their Sketchpad back in that chapter before the test, so we would have gone over that in class before the test.
Dawn	The next day we go over it together, and then we do the lesson that is related to what they did. I will say, "Remember, when you did this on Sketchpad that is tying into this concept or idea?" On my board I will pull up Sketchpad and show them what they should have been doing on a certain step and how it relates to what we are learning. We usually have to spend a day or at least half a class period going over it.
Eric	Like I said after we go over it in class is when I take them to do Geometer's Sketchpad. From there it may be just as simple as me telling them now you should be able to see what's happening, what we studied in class and why it happens that way.
Faye	It might not click what they're actually exploring or figuring out or doing on Sketchpad, but I think my goal is to connect my notes the next day to say, "Okay, remember when we did this on Sketchpad. What did you figure out?" That's when I'll kind of ask, "What did you put for question four?" And, I'll have them literally refer back to whatever they put. Some of them will come up with some crazy answers, but usually we'll get around to the correct answer. It just leads us right through our notes, so it's a good bridge to get them thinking before I just point blank say, "Okay, here's the theorems."

Continued

Table 9 (continued)

Teachers Facilitate Whole Class Follow-up

Teacher	Supporting Quotation From the Interview
Gina	Normally, it's the next day, and we'll talk about what kind of things they came up with. Normally, I don't have it due until the following cycle, so they have time because they don't have it on their home computers for the most part so it kind of just depends. If it was something that they all finished in class that day, I'll probably bring it up the next day and we'll go over things. If it's something that was longer, I might wait a couple days, so it just depends on the length of it.
Hank	Like what can you tell me? And, then I kind of work with them on that, and hopefully they do have a really good response to that. If not, I try to guide them to what was the real intention here.
Jill	Because they have in their—in terms of the notes, it has the Sketchpad activity, and then it usually has the theorem. And, then it has the examples, so they can see, “Okay, I went from this to this to this, so I can see how I can apply what I did here to the theorem to my examples.”
Kate	I would say usually through class discussions…. If I look and kind of see is there something people struggled or really didn’t get, then we’ll discuss that. And, then I would say 75% of the time, then there’s theorems or notes or something that we would then take and do problems and stuff like that on the material.
Leah	We would definitely for quadrilaterals—there’s the ultimate chart at the end with everything across the top and everything down the side. So, we fill those in. We will also then, for the quadrilaterals, we’ll do all the proofs, say, “Okay, so you saw this really happened. How would we go about proving it so we have that tie back in that way?”

DGS Provides Accurate Dynamic Visual Aids

As shown in Table 10, all 12 teachers credit dynamic geometry software with providing students with particular affordances. The three main affordances are accuracy, dynamic figures, and visual aids. Though the accuracy of DGS in constructing geometric figures is implied in many of the teacher quotations, Cara explicitly discusses when drawing a geometric figure on the board, students may not accept it as true because it may not be drawn accurately. Alan, Dawn, Faye, Gina, Hank, and Iris describe the dynamic aspect of the software that allows students to drag, move, or manipulate the geometric construction to observe what changes and what remains invariant. Nine of the teachers explain that DGS helps students to be able "to see" geometric relationships. Jill states that students are able to make connections between diagrams and theorems, and Leah explains that students can connect diagrams and definitions. Taking the three affordances together, teachers think that a benefit to having students use dynamic geometry software is that it provides accurate dynamic visual aids for students to make connections.

Table 10

DGS Provides Accurate Dynamic Visual Aids

Teacher	Supporting Quotation From the Interview
Alan	I just think for the pure fact that they can actually visualize it that much better…. So, I think the fact that they can manipulate the different objects and different figures in Sketchpad…. A lot of kids are very visual learners. To give them a chance to see that I think it's very worthwhile.
Bill	Interactive. It pulls them in. It [GSP] gives them something to look at.
Cara	So for me, it's to draw that picture on the board for them I don't think they—they're like, "You just fudged that. You made it look like that." And, I feel like they can really see that with Sketchpad.
Dawn	Ideally I would love to have them figure out these mathematical concepts on their own, seeing it's dynamic and how it works.... It's wonderful to have something I can show movement with and show how numbers will change.
Eric	It gives them another way to look at things. It gives them something to experiment with, what can be done with different shapes, how they're made.
Faye	I really like that they can measure the angles…drag the vertices wherever they want, or add segments…. What do you observe?… It's very interactive as far as you can drag and figure out what's going on.
Gina	Well, I think the dynamic-ness is the best part about it [GSP]…. They [Students] can only do so much with a compass and a protractor and a ruler.
Hank	Whereas in GeoGebra, I have something I can move immediately, and they can see hundreds of examples instantaneously.
Iris	GeoGebra shows the math better because you're not caught up in all these other things. You can click and drag a point and see how it works.
Jill	Just to make a connection between the diagrams and the theorems.
Kate	I love it [GSP] as an exploration tool for independent thinking and discovery. I especially like teaching geometry because there's so much chance to explore and discover…. And, I think Sketchpad…helps the kids do that.
Leah	For them to have that opportunity to see whatever picture they need to have to confirm it. So, it's to really build those visual connections between, "That's that definition. That's that picture. Here are the results…." You can manipulate it [GSP] and show it versus just drawing it on the board because that's not enough.

DG Tasks Are a Change of Pace

Teachers think students enjoy engaging in dynamic geometry activities because it is a change of pace from the normal classroom instruction. Table 11 displays quotations from ten of the teachers supporting this theme. Bill, Cara, Eric, and Jill discuss how using DGS is a change of pace because students are not passively listening to the teacher lecture. Dawn states that it is good to sometimes "get out of the classroom," and Faye talks about how students consider it "like a fieldtrip" to go to the computer lab. It is interesting that Bill and Faye talk about how they do not want to use DGS too much because then the novelty would wear off with the students. On the other hand, Iris thinks her students would say that they "wished they could use it more."

Alan and Hank do not exemplify the change of pace theme. Although Alan did not talk about DGS as a change of pace for students, he did say that if his students were asked what they thought of DGS "hopefully, they would say they enjoyed it." It can be speculated that perhaps what Alan's students enjoy about using dynamic geometry software is what other teachers' students enjoy about it. That is, it is a change of pace from the normal classroom routine. Because of a one-to-one laptop program, for Hank's students, using GeoGebra is a normal part of classroom instruction; therefore, Hank would not exemplify this theme. It is also interesting to note that in the interview, Hank did not discuss whether students either liked or disliked using GeoGebra.

Table 11

DG Tasks Are a Change of Pace

Teacher	Supporting Quotation From the Interview
Bill	When you say, “We’re going to the lab,” automatically it’s like, “Ah.” I mean you can see the difference in their faces. It’s, “Oh, this is a fun day. We’re not going to sit and listen to you talk to us for any amount of time….” Probably also why I don’t do it a little more because I want them to look forward to it.
Cara	I think to be honest just a day out of the routine of math…. It’s not me sitting at the board showing them how to do something. They’re running it. And, I think as silly as it is; they get excited. “We’re going to the IMC today? We get to play on the computers?” You know what I mean? I think they very much get excited about that, and it’s something different where I don’t think math always lends itself to something different.
Dawn	But, overall the kids do like it. I feel it’s good to have at least something every once in a while to get out of the classroom, just different
Eric	Well, it’s different than me sitting up there lecturing in front of the class. It’s a different avenue of learning for them, and they like that.
Faye	I think they like it. I think it’s a change of pace that they like to just get out of the classroom. They consider it kind of like a fieldtrip. I don’t know. I think if I used it too much, it would take away from the excitement.
Gina	I think, in general, the girls enjoy doing them. It's something different to do.
Iris	I think they’d say they like it. They’d probably say the wished they could use it more.
Jill	But, I think it’s a break from just your normal everyday just me up there talking and lecturing.
Kate	It’s a change from the routine, so they like that.
Leah	“We get to go play in the lab. We get to goof off a little bit.” All I can say is that I do know that they do like it. For whatever reason, they do.

Teachers Experience Restricted Access to Computers

As shown in Table 12, eight of the teachers experienced restricted access to computers for student use. Even though all the schools had computers available for student use, the teachers often had to share the computers with all the teachers in the building. Bill, Cara, and Dawn explained that they had to sign up for the computer lab almost a month in advance to get computer time for their students. Dawn also pointed out that due to the restricted access, she is not having students use DGS as much as she would like them to use it. Because teachers had to schedule computer time so far in advance, this causes the dynamic geometry task to no longer be in the proper order of the originally planned instructional sequence. Gina's solution to this issue was to sign up for the laptop cart on a fixed schedule at the beginning of the year.

Although the majority of the teachers experienced restricted access to computers, some experienced unrestricted access. Even though Alan had very restricted use last year, this year he has not "had any problems getting in there when I want to" because there is not another class scheduled in the computer lab when he is teaching geometry. Hank has unlimited access because his students all have laptops. Because Iris only uses GeoGebra for one chapter, she does not have scheduling difficulties. Also, Iris signs "up everyday at the beginning of the school year" for one of the laptop carts, and then if another teacher needs them, they can use them. Jill has "total access" because every department in her school has a laptop cart. Kate has a laptop cart available for her wing of the building, and other teachers do not use it very much.

Table 12

Teachers Experience Restricted Access to Computers

Teacher	Supporting Quotation From the Interview
Alan	This year's been the best year so far. I wish I would have been able to teach like this for the last 6 years because this is the first year that I've been able to get into the computer lab. There's always been web design or accounting class in the computer lab during the time that I was teaching geometry.
Bill	To be honest with you, our English department is our laptop Nazis. You have to put in about a month ahead to get a laptop around here.
Cara	We have 2000 kids and we have 60 computers. It is very hard to get in the computer lab…. So, when we're planning the next chapters, it's like okay, we're looking three weeks out and we're booking the computers labs.
Dawn	It's difficult to get to…. We're not using it as much as maybe I would like to be. We only have one computer lab so usually you have to reserve it about a month in advance and it's really difficult to know exactly what I'm doing. I try to get it about once a month…. If you don't get down there fast enough, sometimes you have to kick kids off. It's usually a headache.
Eric	It's kind of hard when I teach five classes. I try to keep every class the same every day what I cover. It's kind of hard to schedule when the Spanish III class goes up there and takes up two periods a day. The other periods are open, but I want to take all five classes up there at once.
Faye	It's kind of a fight. The English teachers do a lot of research projects, so they sign up for that thing all the time. I have to plan well in advance that, "Okay, I want to go to the lab this day." And, then I never know if it's going to bleed over because it's very hard for me to judge if it's going to be a one-day lab or a two-day lab…. There's one [computer lab] available for all the teachers and then there are three laptop carts with about ten computers in each. I did do that one time that the lab was full…. That ended up being a headache….It takes 15 minutes just for them to get logged on.
Gina	You have to reserve the cart, and our cart lady is kind of anal. If we don't have it scheduled ahead of time, we can't just go up to her the day before and say, "Hey, I want the cart." So, I just schedule at the beginning of year every F day.
Leah	We have two computer labs, so it's hard to get in there…. We have two computer labs for 120 teachers. And one of the—we have an engineering course, which books the one lab. So, that even restricts it even more. So, the computer lab—it's very limited to get in there

Teachers Are Proficient With the Basics of DGS

As shown in Table 13, nine of the teachers expressed being comfortable with the basics of using dynamic geometry software. All the teachers were cognizant that they had sufficient knowledge with DGS to assist students when needed, but they also realized that there were many capabilities of the software that they did not know how to use. Although Alan was aware of his deficiencies with GSP, he seemed much more confident in his ability than the other eight teachers. In fact, he spoke about using GSP to work on some of his own problems. "There are certain problems that I'll work on that I don't have the kids work on. I think it's one good way to show the kids that you're still working on something new, trying to figure something out."

The three remaining teachers appear confident in their abilities. This level of confidence likely seemed due to past experiences these teachers had with DGS. Faye had a geometry course in college that made intensive use of Geometer's Sketchpad. Hank was very confident in his ability to figure things out on GeoGebra, and he learned a great deal from last year's GeoGebra conference. As a mathematics graduate student, Kate was a teaching assistant for a professor in mathematics education who used GSP extensively.

Table 13

Teachers Are Proficient With the Basics of DGS

Teacher	Supporting Quotation From the Interview
Alan	I mean there's a lot of things I still have to learn about it, but every once in awhile I'll have a kid ask a question I don't know, but for the most part I feel like I always – I know it well enough that I can answers the kids questions.
Bill	Like I said because I don't use it for everything yet, there's probably parts of it if I realized we could use, I'd make it better…. So, I'm sure there's a lot more out there that I could learn on that, and would probably be beneficial.
Cara	I would say pretty proficient with going through these type of sketches if you wanted me to construct something random, I might not be as comfortable with it if it might not be something that I've done before.
Dawn	I feel really good with the basic constructions, but nothing too much beyond that. I can figure it out following directions, but I wouldn't be able to come up with myself.
Eric	I know there are some things I do not know how to do on there yet or have not found how to do yet.
Gina	It's hard because I want to make up more of my own lessons, but I feel like sometimes that's difficult to do.
Iris	For what I use, very. But as far as understanding all the capabilities, not much.
Jill	But, what we do with it I'm very confident with it but I'm sure there's a lot—I know there's a lot to the program that I'm not using.
Leah	On a 5, I would say 3 because with the geometry side of it, I'm real good with the shapes. But I don't know how to make the tools.

Present Use Builds Minimally on Past Use

As shown in Table 14, 11 of the teachers have not made significant changes to the student dynamic geometry tasks. Faye is not included in this analysis because it is her first year teaching geometry and using DGS. Teachers have made minor changes to the activities, and they are using DGS with students more frequently. In the beginning, Dawn and Eric were both using activities from the book *Exploring Geometry with the Geometer's Sketchpad* (Bennett, 2002), but have since rewritten these activities to meet the needs of their students. Bill, Cara, Gina, Jill, and Kate have made some minor changes to the activities that they have student do. Iris and Leah have not changed how they are having students use DGS, but Leah would like to make some changes. Alan, Dawn, Gina, Hank, and Jill have continued to add more activities.

Table 14

Past Use is Similar to Present Use

Teacher	Supporting Quotation from the Interview
Alan	Researcher: So, is it mainly the change due to the lack of access to… Alan: Correct. Correct.
Bill	Researcher: So, really I guess just kind of editing these. Bill: Yes, editing those and making them fit to my students in this case.
Cara	Researcher: So, would you say that there's been big major changes? Cara: I would say not huge. The content is still pretty similar.

Continued

Table 14 (continued)

Present Use Builds Minimally on Past Use

Teacher	Supporting Quotation from the Interview
Dawn	Dawn: When I first used it I was taking the book and copying activities and saying here you go. Now I change a lot of, you know my own directions, trying to incorporate questions throughout. Researcher: Is there differences in topics? Dawn: I think I just keep adding more, I try to add more as we go. I keep the old ones and try to add more to it.
Eric	I think a couple of them I would copy out of the one book I have. I have kind of gotten away from that because I think they're a little confusing sometimes for the students, so I rewrite my own instructions based on what I know and what I know about my students.
Gina	Gina: I think I just know which activities are better than others so, I mean, not significantly. I think just my choice in activities have changed. Researcher: Is the frequency still the same? The number of times a year? Gina: Probably a little bit more.
Hank	Well, I was a little bit more timid last year using it. I would only use it for very select things and then there'd be periods of time where I wouldn't use it hardly at all. But, I think this year for my geometry class, I made a more conscious effort to try and use it more because I realized after my first year how much better some of the lessons could have been just in the explanation part of it, like showing kids ideas and walking them through it.
Iris	I haven't changed how I have them use it.
Jill	Some of them I've probably tweaked, but most of them I'd say probably the steps that we've used to get to the end result has been the same…. And, some of them I probably made up last year, because, like anything, the more comfortable I feel with it the more I know what to do with it.
Kate	So I don't know that I've necessarily changed a ton like what I have them do, but just kind of maybe when or what activities that I choose for them to do.
Leah	Researcher: Have you changed how you have them use it? Leah: No. But I would like to.

Future Use Will Build Minimally on Present Use

Given teachers current constraints, their future implementation of student tasks will only be minimally different from present use. Table 15 shows quotations from 11 teachers to support this theme. Kate does not have data to support this theme because she is not teaching geometry next year. Teachers' ideas for making changes in student use of DGS include: (a) adding a few activities, (b) assigning student projects or presentations, (c) making some adjustments to existing activities to make them more student-friendly, and (d) not making any changes at all. Dawn is planning to add more activities on transformations because the Common Core State Standards include more emphasis on transformations. Jill would like to have students create an Escher-like tessellation. Alan is thinking about assigning a month-long art project, and Hank would like to have students use GeoGebra to present new material. Cara, Eric, and Faye are interested in addressing student learning issues. Cara thinks students would spend more time investigating geometric concepts if she used premade files in the activities. Eric would like for his students to learn how to use the software more quickly, and Faye would like for her students to improve their reading comprehension to be able to follow the steps better on the activity sheets. Bill and Iris do not have any plans to use it any differently from how they currently are using it, but Iris thinks that getting a different perspective from someone else may change her opinion. Although teachers do not plan to make major changes to how their students are using dynamic geometry software, nine teachers are thinking about making some minor modifications.

Table 15

Future Use Will Build Minimally on Present Use

Teacher	Supporting Quotation from the Interview
Alan	I'm not exactly sure what those are going to be, but maybe like those art projects and things like that…. So, hopefully I'll get into those more long-term projects where they are three or four weeks to work on something and try to kind of let them go with it in the direction that they want.
Bill	But, as far as using it more or differently here, no, and I think that's just because I have the ability to use it in so many different ways.
Cara	I would also like to see just more of them investigating the stuff than me telling them how to do it. I like the—lots of the ones in the book had premade sketches that they would just manipulate, and I like that.
Dawn	There is so much more transformations involved in the new curriculum coming up, so I think I will be able to use Sketchpad for that quite a bit.
Eric	I would like to see them catch on quicker. It just seems like, I don't know if it's our school, but the students and their ability to pick things up and do things on their own, just kind of really gone down through the years I think
Faye	It's tempting for me to just take out all the pictures and that way, they don't have any choice but to read…. It might be something that I'm willing to trial and error as far as just take the pictures out, and then they're forced to read all of it.
Gina	So, I think it would be cool if it was more of a group or partner type activity. I don't know how I would do that but that would be cool.
Hank	I like the idea of even the kids just presenting the material…. I assigned them particular things, and they are the ones that are going to teach the class.
Iris	Right now, I don't see anything. But, it's not that I don't want to. It's just that I personally don't see it, so maybe getting somebody else's perspective would let me.
Jill	Maybe do some of the tessellations and show them how Sketchpad can take the points and change them and then dynamically move them all to create an Escher. I think that would be neat for them to see
Leah	Because I would really like for them to be able to just build it and go and not have it be so constrained. And hopefully, with the *n*spires, maybe I can get that to, "Oh, you posed that. You don't understand? Let's pull it out and actually look at it and make it and see what happens."

Teachers Are Open to Professional Development

Table 16 shows that nine of the teachers are open to participating in professional development opportunities with dynamic geometry software. Alan, Cara, Gina, and Hank were interested in exchanging ideas with fellow teachers using dynamic geometry software. Bill and Jill were interested in finding DGS sessions at state and national conferences. As the only two users of GeoGebra, Hank and Iris were interested in attending the GeoGebra conference this summer.

Even though Faye, Kate, and Leah do not having supporting data in the table, they likely are open to participating in professional development. When asked about their interest in professional development, Faye and Kate appeared to have interpreted the question as asking about specific professional development. Faye gave the following response when asked about professional development, “Honestly, I don’t even know what’s available. I've never looked into it.” This response is most likely due to Faye still being a little overwhelmed in her second year of teaching and her first year teaching at a new school after she moved back to Ohio. Kate responded, “None that I know of right now. But, if there were more I’d be interested, I think. But, I don’t really know of any right now.” Although Leah did not specifically talk about professional development with DGS, she did talk about participating in professional development this summer with the TI-*n*spire, but not with the geometry aspect of it.

Table 16

Teachers Are Open to Professional Development

Teacher	Supporting Quotation From the Interview
Alan	I'd love just to sit around with 8–10 teachers and talk about specifically different activities that we use and compare notes on how you can make it better and what teachers have learned from the kids…. I'd rather have discussions like that then just sit there listening to somebody talk about it the whole time.
Bill	That's usually really the first thing that I look at when I look through the OCTM, NCTM…. So, you always pick up something…. But, that's the first thing I always go down and kind of look is there anything software wise….So, I'm always looking for some ways with that.
Cara	Like I said, those webinars. I would be interested in more of those kinds of things just because I think those things are easy…. And just different ideas, like you just said, teachers don't talk enough. Why don't I know what other schools are doing? Have that as well. Even if it was just something in the area, where a half-day professional development or even just a 3 to 5 o'clock thing where in the area those who use Sketchpad could bring their ideas.
Dawn	If there was training, I would definitely be interested in doing it. The more complex with animations; I think the basic stuff I wouldn't need so much but more advanced training would be great.
Eric	I am open up to anything really, anything that I don't know how to do right now. I know there's a lot more that can be done with Sketchpad.
Gina	I would be interested in how other schools use it if they had something where somebody presented, not an online situation. I feel like you learn more in those situations. So, I definitely would.
Hank	I mean the conferences that have been held here in Ohio. I've tried to go to anyone that I become aware of…. If there's ones like that where it's just a bunch of GeoGebra users coming and meeting up, I love going to those ones….I like workshops. I like people who are GeoGebra users coming in and throwing out their ideas to everybody because it's a little bit more realistic in terms of what we can use, and also I can get new ideas.
Iris	The GeoGebra conference probably.
Jill	And, I'm anxious to see if I go to an NCTM conference, I would really like to go to a session that was based just on that program because I'm sure there's so much more you could do with it that I'm not doing.

Summary

This chapter presented the results of the research. The results were split into two sections. The first section discussed the results from the analysis of each individual case. These results were divided into four parts: (a) a general overview of each teacher, (b) present student use of dynamic geometry, (c) factors affecting present student use of dynamic geometry, and (d) past and future student use of dynamic geometry. The next section discussed the results from the cross-case analysis of the collective case study. These results were divided into two parts. The first part described a hypothetical case that was representative of the 12 cases in the actual study. The second part stated and explained the 11 themes with supporting evidence from the cross-case analysis. This part also included identifying teachers who did not fit a particular theme and offering an explanation as to why they did not fit that theme.

Chapter 5: Discussion, Recommendations, and Conclusion

This chapter is divided into three main sections. The first section is a discussion in light of the results presented in chapter 4. The discussion includes answers to the research questions, the absence of a connection to proof in the tasks, how the results informed the related theoretical frameworks, and researcher reflexivity. The second section of this chapter presents recommendations for practice and for research. The third section provides an overall conclusion to the research project.

Answers to the Research Questions

This study posed four research questions. Two of the questions focused on current student use of dynamic geometry tasks, and the other two questions focused on past and future student use. This section provides an answer to each research question and where applicable an accompanying discussion in light of the 11 themes derived from the cross-case analysis of the collective case study.

In what ways are high school geometry teachers currently enacting dynamic geometry tasks for student engagement?

Four themes related to the first research question: (a) teachers use convergent tasks, (b) students collaborate when working on a task, (c) teachers circulate when students work on a task, and (d) teachers facilitate a whole-class follow-up discussion to bring closure to the task. Teachers use Geometer's Sketchpad to engage students in convergent tasks approximately nine times per year. The typical tasks have the following characteristics: (a) the geometry content for the tasks is found in a standard high school geometry textbook, (b) students begin with a new file, (c) students follow directions from a teacher-created sheet, and (d) students answer questions related to discovering conjectures or verifying theorems. The most frequent geometry topics include: points of concurrency, parallel lines, quadrilaterals, transformations, triangle congruence, and circles. When students work on dynamic geometry tasks, students help each other with software aspects of the tasks, and teachers walk around helping students with software aspects of the tasks as well as making sure they are on task. To tie the dynamic geometry tasks back into instruction, teachers usually review the activity sheet with students, and many also use it as a basis for class notes.

Why are high school geometry teachers currently enacting dynamic geometry tasks for student engagement in the ways that they are?

Four themes connected to the second research question: (a) DGS provides accurate, dynamic visual aids, (b) DGS tasks are a change of pace, (c) teachers have restricted access to computers, and (d) teachers are proficient with the basics of DGS. One reason for the current enactment of dynamic geometry tasks is because it is an accurate, dynamic visual aid for students "to see" empirical evidence that particular geometric theorems are true. According to Pea (1985), "computers are commonly believed to change how effectively we do traditional tasks, amplifying or extending our capabilities, with the assumption that these tasks stay fundamentally the same" (p. 168). In the context of this study, the "traditional tasks" are student learning of geometric theorems. This learning can take place either by teachers telling students information or teachers facilitating a discovery-based lesson. Historically, teachers told the students the theorems. On the other hand using a guided-discovery approach, students make conjectures based on a paper-and-pencil construction or drawing that students create using geometric tools (e.g., compass, straightedge, ruler, protractor, and patty paper). Instead of telling students theorems or guiding students to discover them with paper and pencil, teachers use dynamic geometry tasks to lead students to discover these theorems. It is “easier” for students to use DGS than paper and pencil because the figures in a DGS environment are accurate when compared to the paper-and-pencil environment. DGS diagrams also allow students to view a vast amount of empirical evidence by dragging instead of a single static paper-and-pencil diagram, thus providing affordances for conjecturing.

The change of pace theme is an important aspect affecting teachers’ current use of DGS. A school year is a long period of time, and even the most innovative teaching techniques can become monotonous to the teacher and to the students. In fact, a couple of teachers in the study mentioned that they do not want to use DGS too often with students because the novelty might dissipate. Although the main goal is students learning, variety in education is important. Students likely are more engaged when teachers use a variety of instructional approaches and techniques. It also is possible that a teacher using a variety of instructional approaches produces larger learning gains as opposed to a teacher who use the same instructional approaches every class period for the entire year.

Even though the restricted computer access theme is prevalent, it is hard to discern how much it affects the usage or if there might be other more important underlying factors affecting usage. There are some teachers with open access, but they generally are using DGS the same as those teachers with limited access. Hank is the only teacher with open access who is using DGS differently from the others, but only Hank has truly unlimited access—being able to use it whenever he wants to use it. For example, Hank uses it frequently as "a more active systems of notes than just me sitting down and having them write down all these rules and properties of particular objects."

The important attribute underlying access is likely *spontaneity*—the ability to make adjustments "on the fly" and to capitalize on unplanned teachable moments. Teachers make unplanned adjustments frequently during instruction, changing methods based upon both verbal and non-verbal feedback from students. Having all teaching tools at their immediate disposal, teachers can be flexible and make adjustments quickly; however, due to the limited access of DGS, this is one less tool that teachers have available to use spontaneously.

Some of the best teaching and learning opportunities in high schools often result from unplanned situations when a student question or response steers the lesson somewhere the teacher originally did not envision. With limited access, students are unable to explore the new learning opportunity. Because most of the teachers had classrooms with interactive whiteboards, the best that can be done is for the teacher to explore the opportunity, while students passively watch.

Teachers likely need access to DGS at all times in their classroom for it to truly affect how they are using it. This is something that Leah hopes she can begin to have students do with the TI-*n*spire—investigate student-posed questions. "Oh, you posed that. You don't understand. Let's pull it out and actually look at it. And, make it, and see what happens." Even those teachers who had relative ease in securing computer time are not able to make instantaneous adjustments to the lesson or to take advantage of spontaneous learning opportunities because they must schedule computer time, and thus their usage is not very different from those with restricted access.

The fourth theme affecting dynamic geometry usage is that teachers are comfortable with the basics of the DGS, but not much beyond that. Teacher conception of high school geometry is likely an underlying influence in teachers only knowing the basics of DGS. Although teachers were not asked about his or her conception of geometry in the interview, they were asked how comfortable they were with the geometry content that they taught. All of them except for Faye, who was teaching geometry for the first time, were very comfortable with the geometry content, but this appeared to be because they had taught from the same textbook long enough that they were very familiar with that material—that is, their conception of the high school geometry content was essentially their textbook. As Stigler and Hiebert (1999) found in their research, the mathematics teacher's script is deeply entrenched in our culture. This phenomenon exists for the geometry teacher, too. The standard script is for teacher to tell or to guide students to discover particular theorems—usually those found in their textbook. To complete the script, students work on problems and proofs involving these theorems and then take a test over this material.

Although there are many factors influencing why teachers use dynamic geometry software in the ways that they do, in this study there appears to be two main factors. The most important factor is teacher conception of high school geometry. This view that there is a specific set of theorems for students helps explain the four themes. The dynamic geometry tasks are used to amplify the students' guided discovery of these particular theorems. Except for having unlimited access, the current level of computer access helps to reinforce that the dynamic geometry tasks are limited to students discovering conjectures or verifying theorems. For their conception of geometry, teachers only need to be comfortable with the basics of DGS. The other important factor is that using DGS is a change of pace. The use of DGS helps to break up the monotony of classroom instruction. Because students generally like using DGS, this enjoyment may help students to learn these theorems.

How have high school geometry teachers enacted dynamic geometry tasks for student engagement in the past?

One theme related to the third research question: teacher's present use of dynamic geometry tasks builds minimally on past use of dynamic geometry tasks. Teachers have made minor changes to the activities, and they are using DGS with students more frequently. This minimal amount of change can be attributed to the same factor affecting the answer to the second research question: teachers' conceptions of the high school geometry curriculum. If teachers have a static view of the geometry curriculum, there is little need for change beyond some minor editing and perhaps adding some new activities from the existing curriculum.

How do high school geometry teachers foresee themselves enacting dynamic geometry tasks for student engagement in the future?

Two themes pertained to the fourth research question: (a) teacher's future use of dynamic geometry tasks will build minimally on present use of dynamic geometry tasks and (b) teachers are open to participating in professional development opportunities. Teachers plan to use DGS more frequently by adding some additional geometry topics, and they also may make some modifications to existing activities to address some concerns they have about student learning. The first theme can be explained by the same explanation that was used for why past dynamic geometry tasks are not much different than the current tasks. Teachers have a relatively set conception of the high school geometry curriculum, so there is not much incentive to do anything dramatically different from what they are currently doing.

In contrast to the planned future enactment of dynamic geometry tasks, teachers are open to participating in professional development opportunities with dynamic geometry software. Even though teachers currently have a set conception of high school geometry, they are open to learning about new ways of using dynamic geometry software. Depending on the type of professional development opportunities in which teachers participate in the future, this could alter how they use dynamic geometry tasks in the future. Perhaps with professional development focused on developing pedagogy and content as well as additional skills with DGS, teachers might move away from the standard convergent conjecturing tasks that they currently are using.

The Absence of a Connection to Proof in the Tasks

When I first began this study, I had not given much thought to how dynamic geometry tasks should connect to proof. As this study progressed, I began to think deeply about the role of proof in the high school geometry curriculum and this connection between dynamic geometry tasks and proof should be made. In light of this, at the conclusion of the data analysis process, I found it interesting that proof was largely absent from this study about geometry teachers engaging their students in dynamic geometry tasks. Proof rarely was present in the dynamic geometry tasks or the teacher interviews. The majority of activity sheets teachers used did not make a connection between generating conjectures and proving those conjectures. Alan had a single activity that made this connection. Alan's activity on angles formed when two parallel lines are cut by a transversal asked students to prove several conjectures about the angles pairs after they used GSP. Although not really connecting proofs to the dynamic geometry tasks, Eric has some activities where students constructed dynamic diagrams for specific theorems from the textbook.

To keep this research project a manageable size, I purposefully avoided asking about proof in the interviews, but teachers rarely mentioned it in conjunction with dynamic geometry software. In their interviews, Faye, Gina, Hank, and Iris never said "proof," "proving," or "prove." Bill, Cara, and Kate brought up proof in the context of their textbook or doing it in class, but they did not connect proof with the dynamic geometry tasks.

Dawn and Jill talked about proof and dynamic geometry software. Dawn thought that DGS helps students understand the concept of a proof. "I can put up 100 examples, and I don't think they grasp the concept of proving something for all cases. And, I think that [the dynamic aspect of the software] helps them." Jill said that she considers proof to be "an integral part of geometry," and even though she does not connect DGS and proof, she wondered whether other teachers had found a way to effectively connect them:

> The thing I didn't use Sketchpad with a lot was the proofs….We used Sketchpad to show why side-angle-side works and side-side-side works….But, then when it came to actually doing the proofs, we put Sketchpad on hiatus for a little while. We didn't really use it. And, I'd be curious to see if people have used it on proofs instead of just kind of—what am I trying to say? Like, this activity, this activity, but to kind of bring it all together with proofs.

Out of the 12 teachers, Leah was the only one to discuss explicitly the integration of proof and the dynamic geometry tasks. When asked how she provided closure for a task, she said proof was one way for that closure:

> What closure is, is like something like this, we would definitely—for quadrilaterals, there's the ultimate chart at the end, with everything across the top and everything down the side. So, we fill those in. We will also then, for the quadrilaterals, we'll do all the proofs, say, "Okay, so you saw this really happen. How would we go about proving it?" So, we have that tie back in that way.

Leah also expressed thinking about what the order of proof and the dynamic geometry tasks should be—if one should come before the other:

> It has made me question, "Do I do the proofs first, and then let Sketchpad back me up?" I've learned that sometimes you think the technology is going to give them the pattern, because it won't be magic anymore. But, with the circles, it's still a magical formula behind it. You know? So, especially in that one, I drive circles more to the similar triangles. I let the proofs push, and then let them see that's what the computer's doing behind it.

Teachers apparently do not connect dynamic geometry tasks to proof. It is unknown how much proof there is in teachers' geometry classes, but the two activities mostly are kept separate. This could be due to computer access, change of pace, or teachers' beliefs about how proof and dynamic geometry tasks should be integrated. Earlier it was argued that teachers' conception of high school geometry goes a long way in explaining "the why" for dynamic geometry tasks. The material in the textbook likely also influences this conception. In textbooks, DGS activities usually appear in supplemental materials instead of the main textbook, and often the purpose of these activities is unclear (Oner, 2008). Textbooks often do not have a connection between proof and dynamic geometry tasks further exacerbating this disconnect in the classroom instruction. In alignment with Stigler and Hiebert's (1999) argument that the teaching and learning of mathematics is deeply entrenched in culture, Herbst, Chen, Weiss, and Gonzalez (2009) found there were norms for "doing proof." Teachers or the textbook decided when students should do a proof, and they provided the diagram, the statement about what was a given, and the statement about what was to be proved. Because teachers have definite norms about how proof takes place in instruction, this conception may be another reason that proof and dynamic geometry are not integrated.

How the Results Informed the Related Theoretical Frameworks

This section discusses how the results informed the related theoretical frameworks in the literature review. The first component discusses the task frameworks and offers a new framework to be used with dynamic geometry tasks. The second component describes the usefulness of Bennison and Goos' (2010) framework in understanding why teachers are implementing dynamic geometry tasks in the ways that they are. The third component critiques the teacher change frameworks as applied to past and future practices of teachers' enactment of dynamic geometry tasks for student engagement in this study.

Task Frameworks

Drawing on the comparison of four different task frameworks (McGraw & Grant, 2005; Stein et al., 2009; Laborde, 2001; Ng & Teong, 2003) that were presented and discussed in Chapter 2, a framework is proposed for dynamic geometry tasks in high school geometry. As shown in Table 17, the framework for dynamic geometry tasks has four phases leading to proof: *construct*, *explore*, *conjecture*, and *prove*. Within in each phase, there are different options of how that phase is enacted. This framework should not be taken as a recommendation that student tasks with DGS must be completely open-ended or that proof must always be a part of the task. A variety of tasks are important. In addition, as this study tries to make clear, each teacher's context is different, and any researcher investigating technology integration also needs to understand the school context and culture.

Table 17

Framework for Dynamic Geometry Tasks

Phase	Options
Construct	Teacher provides students premade file Teacher provides specific software-oriented steps Teacher provides general content-oriented steps Teacher provides task Teacher provides choices Students generate
Explore	Teacher tell students what to measure, drag, and observe Students decide what to measure, drag, and observe
Conjecture	Teacher hint at what to write for a conjecture Students generate written conjectures
Prove	Task is connected to proof Task is not connected to proof

The construct phase focuses on the different ways the geometric figure is made with DGS. These different ways constitute a continuum where at one extreme the teacher has control of the construction to the other extreme where the students have control of the construction. For example, the task of constructing the quadrilateral formed by connecting successive midpoints of a parallelogram can be enacted in different ways. A teacher may give students a premade file with the quadrilateral and parallelogram already constructed. Instead, a teacher may provide written steps for the construction. These steps may be software-oriented with specific directions on where to find particular commands in the software menus, or these steps may be geometry-oriented. To illustrate the difference between the two types of steps, the following example is used. Click on segment *AB* and point *C*, and then from the construct menu, choose parallel line (software-oriented step). Construct a line parallel to segment *AB* through point *C* (content-oriented step). The next three approaches are more open-ended than the first three approaches. A teacher may give students the construction task without guidance of how to do it. The teacher may give students options, for example to construct the quadrilateral formed by connecting consecutive midpoints of a parallelogram or to construct the quadrilateral formed by connecting successive angle bisectors of a parallelogram. Last, the students may control the construction phase where they make decisions on what quadrilateral to construct from a parallelogram.

The explore phase and the conjecture phase are based on the work of McGraw and Grant (2005). In the explore phase, a teacher might tell students explicitly what to explore, or students can choose what to explore. In the conjecture phase, a teacher might hint at what students are to conjecture, or students may be left on their own to generate conjectures. A teacher hint at a conjecture is usually a question on the activity sheet. In the quadrilateral formed by connecting consecutive midpoints of a parallelogram example, students might be asked what they notice about the opposite sides of the quadrilateral. The question also might be very specific, such as asking what kind of quadrilateral is formed.

The prove phase is divided into two options. The overall task can either included or exclude proof of the conjecture. The language of the "task is connected to proof" and the “task is not connected to proof” is based on the task analysis framework language of “procedures with connections” and “procedures without connections” (Stein et al., 2009).

This section presented a new framework for analyzing dynamic geometry tasks. A task is seen as having four phases leading to proof: construct, explore, conjecture, and prove. Although this framework is based in part on the document analysis of the tasks in this study, this framework may need additional refinement as more tasks are scrutinized. For example, a range of possibilities likely exists for the three phases of explore, conjecture, and prove as opposed to the proposed dichotomous nature of those phases.

Bennison and Goos’ Framework

Throughout the data collection and the data analysis of this research study, Bennison and Goos’ (2010) framework captured the intricacies of technology integration. The framework acknowledges that technology integration is complex, and it recognizes that context is an important factor in technology integration. The complexity is conceptualized through the use of three zones: a zone internal to the teacher, a zone consisting of the school culture, and a zone of professional influences outside the school. The zone of proximal development consists of teachers’ beliefs, attitudes, and knowledge. The zone of promoted actions consists of factors outside of the school such as course work, student teaching, and professional development. The zone of free movement consists of factors in side the school such as access to materials, student behavior and knowledge, and colleague and administrator support. Through the zone of free movement, Bennison and Goos’ framework emphasizes that the school context and culture is important to understand why teachers are integrating technology in the ways that they are. The zone framework provides sufficient detail and scaffolding for it to have been a very useful lens in this study about teachers integrating student dynamic geometry tasks into their instruction.

Teacher Change Frameworks

This section applies the three teacher-change models (Niess et al., 2009; Beaudin & Bower, 1997; Hall & Hord, 2001) that were discussed in the literature review to the findings of this research project. Niess et al. (2009) created a technological pedagogical content knowledge development model based upon Roger's (2003) initial work in 1962. This five-stage model included three stages that pertain to teacher's implementing technology tasks into their instruction. These three stages are *adapting*, *exploring*, and *advancing*. All of the teachers in this study currently are at the exploring level because they are consistently using DGS and the activities had coherence to them. Teachers were not at the lower level of adapting because there was regular and systematic use of the activities, and they were not yet at the advancing level because they were not challenging what is taught in geometry and how it is taught.

Beaudin and Bower's (1997) used their PURIA model to assess how teachers were integrating computer algebra systems into instruction. The PURIA model had two stages where teachers were implementing tasks for student engagement. At the *incorporate* stage, teachers were not yet assessing students. In the final *assess* phase, teachers were assessing student work. In the assess phase of Beaudin and Bower's model, there is some ambiguity because it is unknown if this is meant as strictly summative assessment or if it includes formative and summative assessment. In this research study, all of the teachers were assessing student learning with dynamic geometry, but not all of them were using summative assessment methods to do this. Another drawback to the PURIA model is that is does not take into account if the nature of the tasks are extending or challenging the current curriculum.

Although neither of Hall and Hord's (2001) models was specifically developed for teachers integrating technology into the classroom, they are the most useful of the teacher change models reviewed. One benefit is Hall and Hord explicitly separate their model into two different parts—one focused on teacher feelings and beliefs and the other focused on integration. The *stages of concern* framework focuses on a teacher's feelings, perceptions and attitudes about dynamic geometry tasks. Four of the seven stages related to when teachers are having students complete tasks: *management*, *consequence*, *collaboration*, and *refocusing*. The first three phases represented various teachers from this study. Some of the teachers were in the management stage because they were struggling with issues with DGS such as scheduling computer time. Teachers were also at the consequence stage because they were concerned with whether dynamic geometry tasks were enhancing student learning. A couple of teachers were at the collaboration stage because they worked with other geometry teachers on creating and editing the student tasks. The collaboration stage also might be external to the schools, for example, using the online community of teachers to discuss ideas. This also may manifest itself in professional development because some of the teachers expressed that they would like to talk to other teachers using dynamic geometry tasks with students. Teachers did not appear to be at the refocusing stage because they had yet to dramatically alter their implementation of the student tasks.

The other half of Hall and Hord's model, *levels of use*, focused on how teachers actually were using dynamic geometry tasks with students. Five levels related to using DGS: *mechanical*, *routine*, *refinement*, *integration*, and *renewal*. Teachers were past the mechanical level because they were not using DGS in the short-term, and they had a collection of activities that they had students complete. Most of the teachers in the study were at the routine level or the refinement level because either the use was stable, or they were making minor changes. Few teachers were at the integration level because they were not using activities that had been created with other teachers. It does appear that most teachers may enter this level due to their openness toward professional development. Last, teachers were not at the renewal level because they were not using activities that were a major change from their initial use.

Hall and Hord's model is the most useful of the three teacher-change models because it explicitly acknowledges that although connected there is a distinction between teachers' concerns and teachers' use. By having a separate model for teacher concerns, Hall and Hord's model fits well with one of the overall conceptions of this research project: context matters, and it is important for researchers to understand the context of each teacher's experiences both inside and outside of the school.

Researcher Reflexivity

To be reflexive "is to undertake an ongoing examination of what I know and how I know it" (Patton, 2002, p. 64). Further, "these questions challenge the researcher to also be a learner" (p. 495). Professionally, I live in two worlds that are often very different: I am a researcher and I am a practitioner. As a doctoral student, I live in a sometimes very theoretical world, on the other hand, as a teacher I live in the practical world of a vibrant high school community consisting of teenagers, colleagues, administrators, and parents.

Although I had an open mind as qualitative researcher, at the beginning of this study, I had a rather set way as a teacher of using dynamic geometry tasks with my students. My use of dynamic geometry tasks for the most part matched Zoey, the hypothetical teacher who is representative of the teachers in this study. I used it as guided discovery for student to make conjectures based on what they saw in their dynamic constructions, and that was mainly how I used it during the 14 years that I have taught one of the three geometries in the different mathematics tracks available at my school.

Also, like many of the teachers in this study, I was using dynamic geometry tasks because it was a change of pace to the classroom routine. Students enjoyed it, they got excited about doing it, and they looked forward to Sketchpad days for much the same reason as Bill's student looked forward to those days: "Oh, this is a fun day. We're not going to sit and listen to you talk to us for any amount of time."

When I began reading and planning for this study, I came across research that forced me to seriously reconsider what I, as a teacher, was having students do with DGS. McGraw and Grant (2005) described Type 1 and Type 2 lesson structures for technology usage. Type 1 lessons are closed-ended or convergent tasks, and type 2 lessons are open-ended or divergent tasks. Throughout my career, I have always prided myself that I was a teacher who makes students think and discover mathematical concepts, but I had to admit that my supposed discovery tasks were highly orchestrated and very structured. In fact, I was using what Harper and Edwards (2011) call cookbook lessons—lessons "that lead students through a series of procedural steps in a recipe-like fashion" (p. 180). Worse yet, the cognitive level of demand that the dynamic geometry tasks placed on my students was very low. The level of thinking students needed to successfully complete the tasks was minimal, and these tasks could be characterized as tasks that were procedures without connections (Stein et al., 2009).

My use of these convergent, cookbook, low cognitively demanding tasks was also largely due to my conception of geometry. My conception of high school geometry was that there were specific theorems that students should learn. If students were able to use these theorems successfully in problems and proofs, then I had done my job as a teacher well.

Because of reading the existing research on dynamic geometry software, I have tried some different dynamic geometry tasks with my honors geometry students the last two years with mixed results. I have moved past the paradigm of a specific set of theorems for students to know in geometry, instead my view is there is an important process for students to experience in geometry. This process is focused on students deciding what geometric figures to construct and explore, generating their own conjectures, and then seeing those conjectures elevated to theorems through the development of student proofs. Students have struggled with determining what to explore and then proving their conjectures. I have struggled in the process trying to find the right balance in terms of time and assessment. I concede that sometimes what I do as a teacher is influenced by how difficult it is for me to assess student learning. For example, it is much easier to assess and give feedback to 25 students when they are all writing a proof on the same theorem instead of several groups generating proofs for different theorems.

I do not know where this leaves me, but like the teachers in this study, I am open to professional development for integrating DGS in the high school geometry curriculum. I have some thoughts about what I would like to have students do this year, but I know it would be beneficial to have a group of teachers to discuss these ideas with who are currently using dynamic geometry tasks with students. As a teacher, this research project has been one big professional development project for me. One final word about researcher reflexivity: because of this research project I am not the same as when I began—as a researcher or as a teacher.

Recommendations for Practical Purposes

This section details the practical recommendations in light of the results of this research project. The first part recommends moving to unlimited access of dynamic geometry software much like many schools now have with graphing calculators. The second part proposes developing curriculum materials that challenge the present viewpoint of the geometry curriculum. The third part encourages the professional development of geometry teachers as whole and not just limited to technology but inclusive of pedagogical knowledge and content knowledge.

Unlimited Access

As discussed in the themes that related to the second research question, it is recommended that there be unlimited access to dynamic geometry software. None of the schools in this study except for Hank's school had this kind of access yet, but working to eliminate the access issues probably would affect how and the frequency dynamic geometry software is used. Even though many schools are struggling with finances, perhaps the solution to providing unlimited access is going to come about as it has for some schools with graphing calculators. Some schools have classroom sets of graphing calculators for students to use, or they require students to have their own graphing calculators. Tablets such as the iPad may provide a solution once dynamic geometry software is available on them. Either classroom sets of tablets or students having their own tablets may become a realistic future option.

Unlimited access has been tried in the past, but with limited success. Handheld access has been available since 1995 when the TI-92 calculator came equipped with Cabri II. The cost was likely prohibitive as the TI-92 calculator cost twice as much as the TI-83, and they were not permitted on standardized tests because of the QWERTY keyboard. In 2003, Cabri Jr. became available on the TI-83 family of graphing calculators. One reason for the limited success of DGS on graphing calculators is due to the poor resolution of the screen and the cumbersome nature of dragging with cursor keys.

When a classroom is "flipped," instead of students receiving instruction at school via lectures and note taking and then practicing individually at home, students receive information at home and work on mathematics in the classroom. For example, one method of flipping the classroom consists of students watching video lectures at home and then working collaboratively on problems at school. The usual script of teachers lecturing in the classroom and then giving students problems to work on at home has been "flipped."

Perhaps flipping the classroom can alleviate geometry teachers' struggles with access to computers. Home access to dynamic geometry software is a readily available. GeoGebra and Wingeom are both free, and Key Curriculum offers a student home edition for Geometer's Sketchpad for $10 per year. Students could work on a dynamic geometry task at home, and then in the classroom the teacher could facilitate a class discussion on the completed task. Students could share their work by saving their file on a flashdrive or e-mailing the file to the teacher. Flipping the classroom would eliminate two themes from this study: student collaboration and teachers circulating. It is unknown if this would be an advantage or disadvantage. Flipping the classroom would have the distinct disadvantage that it would not provide for adjustments or spontaneous teachable moments using DGS.

Curriculum Materials

The development of curriculum materials that provide cognitively demanding conjecturing tasks are needed. Much of what is currently available as published materials or what teachers are writing are of the "cookbook" variety. Although most teachers in this study used their own tasks, providing teachers with examples of cognitively demanding tasks that thoughtfully integrate dynamic geometry and proof would be a start. Making sure these tasks are available in a file format to make them easy for teachers to edit is a necessity.

Professional Development

Teachers need access to professional development opportunities connecting dynamic geometry software to pedagogy, curriculum, and content knowledge in geometry. It is difficult for teachers to make pedagogical and curricular changes when they do not know what possibilities for change exist. They need a starting point from which to work. Professional development must consist of more than a single day of learning how to use the basics of the software. It must include the integration of pedagogy and content with technology. In addition, this professional development to some extent needs to be teacher-driven and not just some "expert" handing down advice on "how it should be done." Opportunities must be made for meaningful dialogue among teachers. It is important for the professional development to be sustained and not just a one-time event because "change is a process not an event" (Hall & Hord, 2001, p. 4).

Recommendations for Further Research

This section makes recommendations for areas in the teaching and learning of high school geometry that need additional research. The recommendations emphasize using research methodology similar to this project: naturalistic inquiry and qualitative methods. It is important to understand current teacher practices situated in context before attempting to alter or modify their practices. This change must begin with teacher beliefs and attitudes because regardless of proposed changes made by researchers, policy makers, professional organizations, and school administrators, it still must start with teachers willfully deciding to change their instructional practices.

Research in Conjunction with Practical Recommendations

Practical recommendations and research recommendations need to be connected; therefore, all of the practical recommendations also must include research detailing how they have influenced and affected teacher implementation of dynamic geometry tasks for student engagement. Research in the areas of teaching with unlimited access to DGS, developing curriculum materials, and providing professional development is needed.

Observations and Student Perspectives

Research that has a similar inquiry as the existing study but incorporates observations of students using the dynamic tasks is needed. One limitation of this study is that it only had two sources of data: the teacher interview and the student activities sheets. Also, research should be completed on student perspectives through student interviews.

Longitudinal Studies

Longitudinal studies with teachers who are using dynamic geometry tasks with students are needed. All of the teachers in this study have been using DGS for under 10 years. One of the main foci of this study was past and future practice in relation to the present use. Research that follows-up with teachers on a yearly basis is needed to formulate a clear conception of how teachers change their usage as they advance in their careers from early-career teachers to mid-career teachers to late-career teachers.

Teacher Conception of High School Geometry

Teacher conception of high school geometry was hypothesized to be a major factor affecting why teachers were using DGS in the ways that they were. Research is needed in this area to see if this hypothesis is accurate. Although dynamic geometry was the focus of this study, teachers' attitudes and beliefs about the purpose and goals of high school geometry may help explain teachers' instructional and assessment practices.

Proof and Dynamic Geometry Software

Research is needed that explicitly addresses the role of proof in instruction for teachers who are using dynamic geometry tasks for student engagement. A previous section in this chapter discussed the peculiarity of the absence of a connection to proof in the tasks in this study. How and why teachers are connecting proof to dynamic geometry tasks especially is needed.

Nonusers of DGS

Research in teachers' decision process to use or not to use DGS is needed. Some of the teachers in this study stated that there were other geometry teachers in their school that chose not to use the software. The second research question was an important question to ask because "why" is just as important as "how." An equally important question to ask is "why not." Why are geometry teachers who have access to dynamic geometry software not using it with students? Rogers (2003) pointed out that there is often an inherent bias in research on how innovations diffuse through a system. Typically, a team of researchers is studying the innovation because they believe that it should be adopted quickly and by all members. Rarely is nonuse of an innovation researched. In one example of this kind of research, Stols and Kriek (2011) found that DGS was not implemented when teachers believed that DGS was not compatible with their teaching style.

Conclusion

This study began as an attempt to fill a void in the existing research on dynamic geometry software. Researchers rarely had studied how teachers were using DGS without researcher mediation or intervention. An important feature of this research project was that it acknowledged that teachers integrating technology is a complex process based on many factors that are interrelated. Understanding what teachers are currently doing and why they are doing it is paramount to any discussion on teacher change.

The results of this research provided detailed accounts of how 12 geometry teachers are currently using dynamic geometry tasks for student engagement, why they are enacting tasks in the ways they are, how teachers' past DGS use is different from current use, and what their future use of DGS might look like. Through the cross-case analysis of the collective case study, 11 themes were identified. Generally, teachers were using convergent tasks with DGS for students to discover conjectures or verify theorems. Students and the teacher collaborated with each other when working on these tasks. The themes related to ways in which teachers were currently using DGS were explained by teachers' conception of the high school geometry curriculum. Providing students with a change of pace also explained current usage. Generally, teachers had not changed their

DGS tasks, nor did they plan to make major changes in the future, but they were open to participating in professional development opportunities.

This study also provided practical and research recommendations for dynamic geometry tasks. For any research that involves teachers and students, connecting research to practice is important. The results need to be disseminated so they will reach teachers. In addition to mathematics teacher educators conducting and reporting the results of their research, they must undertake outreach to community schools to establish professional development and build communities of learners to affect teacher change. It is emphasized again that any educational change must start with the teachers. Change should not be meted down to teachers from policy makers, researchers, or administrators. Professional development must be an integral part of any research project and its accompanying recommendations. Although this research study is finished, it is not the end. It is a beginning—the first step in developing meaningful, sustained professional development for geometry teachers using dynamic geometry tasks for student engagement.

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Appendix A: Initial Contact E-mail

I'm currently in my 15th year as a mathematics teacher at Troy High School. I'm also a graduate student at Ohio University researching how geometry teachers have their students use geometry software. If you are willing to participate in the study and meet the qualifications asked below, you will receive $75 to compensate you for your time. Your participation in this study is strictly confidential. Your name and school will NOT be used in any written material or disclosed to anyone.

1. Do you teach geometry? ("Geometry" refers to the standard geometry course that the majority of students take at your school, not an honors, informal, or basic geometry course.)
2. If you answered yes to the previous question, do you have your geometry students use Dynamic Geometry Software (DGS) such as The Geometer's Sketchpad (GSP), GeoGebra, Cabri II, Wingeom, etc.?
3. Approximately, how many different Dynamic Geometry Software activities do you have your geometry students complete during a school year? (This question refers to where individual students or groups of students are actively using the software and "driving" the mouse. This question does not refer to when you as the teacher are using the software as part of a class discussion or demonstration.)
4. If you answered yes to both of the first two questions, are you willing to participate in a research study consisting of an interview and a collection of your instructional activities that you give your students when they are using geometry software? If so, please reply to this e-mail.

If you have any questions before deciding whether to participate, feel free to ask.

Thanks for your consideration,
Wayne Nirode

Appendix B: Informed Consent Form

Ohio University Consent Form

Title of Research: How Geometry Teaching are having Students use Dynamic Geometry Software

Researcher: Wayne Nirode

You are being asked to participate in research. For you to be able to decide whether you want to participate in this project, you should understand what the project is about, as well as the possible risks and benefits in order to make an informed decision. This process is known as informed consent. This form describes the purpose, procedures, possible benefits, and risks. It also explains how your personal information will be used and protected. Once you have read this form and your questions about the study are answered, you will be asked to sign it. This will allow your participation in this study. You should receive a copy of this document to take with you.

Explanation of Study

This study is being done because there is little existing research on how geometry teachers use geometry software with students.

If you agree to participate, you will be asked to participate in an interview and to give your activity sheets along with any pre-made files that are needed that you have students complete to the researcher.

Your participation in the study will last approximately two hours along with the collection of your student activity sheets and any accompanying pre-made files.

Risks and Discomforts

No risks or discomforts are anticipated

Benefits

This study is important to mathematics education because many in the mathematics education community believe that teachers using geometry software can improve student achievement and understanding, and to date there is little research investigating how geometry teachers are using geometry software with students.

Individually, you may benefit by receiving a copy of the research report.

Confidentiality and Records

Your study information will be kept confidential by not having your name or school directly tied to any of your data. Your actual name will be replaced with a pseudonym name. There will be a separate computer file where your name is linked to your pseudonym. All data will be kept either in a locked filing cabinet or on a password protected computer.

Additionally, while every effort will be made to keep your study-related information confidential, there may be circumstances where this information must be shared with:

* Representatives of Ohio University (OU), including the Institutional Review Board, a committee that oversees the research at OU

Compensation

As compensation for your time/effort, you will receive $75. Because University funds are being used to compensate participants, your name and address will need to be provided to the Finance Office at OU.

Contact Information

If you have any questions regarding this study, please contact Wayne Nirode (Nirode-w@troy.k12.oh.us or 937-332-6710) or Greg Foley (foleyg@ohio.edu or 740-593-4430).

If you have any questions regarding your rights as a research participant, please contact Jo Ellen Sherow, Director of Research Compliance, Ohio University, (740)593-0664.

By signing below, you are agreeing that:

- you have read this consent form (or it has been read to you) and have been given the opportunity to ask questions and have them answered
- you have been informed of potential risks and they have been explained to your satisfaction.
- you understand Ohio University has no funds set aside for any injuries you might receive as a result of participating in this study
- you are 18 years of age or older
- your participation in this research is completely voluntary
- you may leave the study at any time. If you decide to stop participating in the study, there will be no penalty to you and you will not lose any benefits to which you are otherwise entitled.

Signature____ Date_________

Printed Name____

Version Date: 01/18/12

Appendix C: Interview Questions

1. Describe your school: department size, students, courses offered, community, school culture.
2. What type of dynamic geometry software (DGS) do you have your students use?
3. Tell me about using DGS with your geometry students.
4. What geometry topics do you use with DGS?
5. Describe the types of activities you have your students do with DGS.
6. How do you launch the DGS task or activity?
7. Describe the setting in which your students use DGS (computer lab, classroom, home).
8. Describe how students work with DGS.
9. Describe how students work with each other (individual, partners, groups) when using DGS.
10. Describe your involvement with students when they are using DGS.
11. What general goals do you have for your students when using DGS?
12. How do you bring closure to the task or tied it back to the classroom instruction?
13. How do you assess your students when using DGS?
14. What aspects of the software do you really like to have students use? (commands/actions from the menus)
15. What aspects of the software do you not have students use?
16. What do you think are the greatest benefits to having students use DGS?
17. What do you think are the biggest drawbacks to having students use DGS?
18. Describe your level of comfort with the geometry content you teach.
19. Describe the different teaching techniques you use to teach geometry in your classroom. (with and without software)
20. Describe your level of proficiency with the software.
21. What or who was the biggest influence to get you started in using DGS with your geometry students?
22. What or who has been the biggest influence to sustain this use with students?
23. Describe the level of support from your colleagues concerning DGS.
24. Describe the level of support from your administrators concerning DGS.
25. How do curriculum requirements (board adopted) influence your use of DGS?
26. How do assessment requirements (OGT) influence your use of DGS?
27. Describe your access to computers for using DGS.
28. Describe your access to teaching materials for using DGS.
29. Describe your access to time for planning and implementing DGS.
30. Do your students influence how you use DGS with them, for example, their behavior, abilities, attitudes, and motivation?
31. Describe your own experiences with DGS. (high school, teach ed program, student teaching)
32. Describe any professional development with DGS.
33. Since you started using DGS with students, have you changed how you have students use it?

34. Suppose there was no curriculum and no OGT and you could whatever you wanted with DGS, what would that look like?
35. Describe any additional ways that you would like to have your students use DGS than what you already do.
36. Describe any professional development or other learning opportunities that you are interested in undertaking with DGS.
37. What do you think your students would say if I asked them what they thought of using DGS?
38. What advice would you have to new geometry teachers starting out with DGS?
39. How would you summarize teaching geometry with DGS?