DETERMINING OPTIMAL STAFFING LEVELS FOR THE PICKING AND PACKING OPERATIONS IN A DISTRIBUTION CENTER

A Thesis Presented to

The Faculty of the

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Ohio University

In Partial Fulfillment

of the Requirement for the Degree

Master of Science

by

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November 2002

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1 INTRODUCTION

A supply chain is a network of facilities and distribution systems that are used to procure materials, transform them into finished goods, and distribute the products to customers. The distribution center (DC) plays an integral part in a supply chain by providing a means for storage and distribution of products. They are widely used to store and sort bulk shipments into smaller customized shipments and help create a buffer between the producers and customers. This study is primarily concerned with DCs that receive, store and distribute finished products and does not include DCs that may be part of a manufacturing facility.

Growth in retail distribution has led to a situation where some manufacturers are increasingly delivering their products from their DC directly to the retailers stores, without going through the retailers' DC. Electronic retailing through the World Wide Web also has expanded significantly over the last decade and customers can now buy directly from the manufacturer bypassing the intermediate wholesalers/retailers. All this has led to DCs having to deal with a larger number of orders, which are smaller in size and much varied in composition.

In order to stay ahead of competition, companies need to be able to respond to the diverse customer requirements within a shorter time. Therefore, efficiency in filling customer orders is a source of competitive advantage that can help a company differentiate itself from others. The DC facilitates this process by helping to maintain a stock of items that could be dispatched swiftly to meet the customer requirements.

Automation has helped improve the productivity of many activities in DC operations. However there are still a lot of manual operations involved, particularly in order picking and packing. This can be attributed to the variety in size and shape of items that have to be processed. Effective allocation of personnel to attain work objectives with the minimal work force size is a key to achieving labor productivity in a DC.

1.1 DC Operations

In a DC, the goods are received from the manufacturer or a distributor and stored before they are sent to another distributor or the ultimate customer. The main operations performed in this process are receiving, storage, replenishment, picking, sorting, packing, and shipping. As with all other operations, warehouse management too has benefited from technological advances over the past few years. Automated material handling equipment, bar code readers, and automated packaging equipment have increased the pace at which these activities could be performed. However, manual operations are widely used in many DCs because the volume and variety of items handled makes it difficult to automate. The activities that are studied in this research are picking, sorting, and packing because picking and packing are two operations that are highly labor-intensive and sorting is an intermediate operation between them.

1.1.1 Order Picking

Order picking is the process of retrieving a number of items from their storage locations to satisfy customer orders. Order picking throughput depends on factors such as the location of the items to be picked in the warehouse, the storage policy used to locate items in the DC, the order picking routes followed to retrieve items, the configuration of the warehouse, and the number of pickers employed for the process. Total picking time is a measure of order throughput efficiency in a DC and depends on the factors such as those described.

Though order picking is a relatively simple operation, it is the most costly activity in a typical warehouse. Despite the improvement in technology to handle inventory automatically, order picking has remained the most labor-intensive operation with direct labor accounting for about 60% of the total warehouse costs [25]. This is mainly due to the high variation in the size and shape of items to be handled which makes it difficult to automate.

1.1.2 Sorting

In order to increase the efficiency of the picking operations, several orders are often combined to form a wave. The items for these orders are then picked from the warehouse area simultaneously. Sorting is the process by which these items are separated into individual orders. Order pickers place the items on a staging conveyor, which transports them to the order accumulation/sortation system. The products are circulated and released to the sortation lane to which the order is assigned. Once the items for a wave have been sorted into lanes for packing, sorting of the next wave is begun.

1.1.3 Packing

The next stage in the process is packing together all the items that belong to a single order. Contemporary DCs often have to deal with the ultimate consumer whose orders are much smaller and will consist of a variety of items that differ in shape and size. This makes the use of automated packing equipment more difficult. Therefore packing too remains one of the more labor-intensive tasks in a DC.

The time taken to pack a set of orders depends on factors such as the number of accumulation lanes in the facility, the number and variety of items to be packed, and also the number of packers available for the operations. However, the research on packing time estimation or even the packing operation in general is very limited.

1.2 Problem Background

An important issue in the business environment is to simultaneously reduce the cost and increase the speed of DC operations. Order picking and packing are two stages in the process that consume a large amount of labor and account for a major cost. Therefore any improvements in these activities that help reduce the cost and/or increase throughput will contribute towards enhancing the productivity of DC operations.

The performance of a DC depends on the performance of its component subsystems, including receiving, storage and replenishment, order picking, sorting, packing and shipping. The highest rate at which the DC can perform is restricted by the slowest subsystem or the 'bottleneck'. Thus building capacity in non-bottleneck activities will not help improve the performance of the overall system. Improving performance in a DC requires balancing the capacity throughout the different tasks.

Typically, three major activities need to be carried out in a DC between arrival of customer orders and shipping: order picking, sorting, and packing. The efficiency of these activities determines how quickly a DC can respond to customer requests. Order picking and packing are performed before and after sorting, respectively. These two former operations are highly labor intensive and the number of pickers and packers employed determines how fast the items can be retrieved and packed. However, as the two are consecutive activities in one system, increasing the capacity of the non-bottleneck activity by adding more labor will not improve the system itself. For example, if the bottleneck activity is the picking operation, adding more packers will not help improve the performance of the system. Thus determining the optimal number of pickers and packers required to deliver the throughput within the available time can aid in balancing the capacity throughout the process (picking, sorting and packing) and minimize the labor cost.

1.3 Research Objectives

The objective of this research was to develop a model that can be used to determine the optimal staffing requirements of a DC to pick and pack a given set of customer orders within the specified time using a given set of operating strategies.

The model is based on the time taken to pick, sort, and pack a set of orders, which depends on the number of workers engaged in these operations. Analytical expressions were developed for these times. The objective was to minimize the number of pickers and packers required, while delivering the set of orders within the time available for picking and packing. Thus, the objective will be to minimize the number of pickers and packers employed.

Objective function: $Min\ (W_p + W_{pa})$ (1)

Subject to:

$$WT_P \leq S_p \tag{2}$$

$$WT_{Pa} \leq S_{Pa} \tag{3}$$

$$T_{total} = f(T_P, T_{Pa}, T_S, W) \leq S_{total} = f(S_{pick}, S_{pack}) \tag{4}$$

where,

W_p = number of pickers employed

W_{pa} = number of packers employed

T_p = time taken to pick a single wave of orders (hours)

T_{pa} = time taken to pack a single wave of orders (hours)

S_p = time during which picking shift is in operation (hours)

S_{pa} = time during which packing shift is in operation (hours)

S_{total} = time from start of picking operation to end of packing operation (hours)

T_{total} = the total time to complete all operations (i.e. pick, sort and pack) for all orders

W = the number of waves that are processed to service all the orders

When the orders to be processed are separated into W waves and each wave takes T_p amount of time to pick, the total time taken for picking, WT_p, should be less than the time during which the picking shift will be in operation, S_p. This constraint is given in equation (2) . The similar constraint for packing is given in equation (3). The total time taken from the start of picking to complete packing all the orders should be less than the time designated for these processes in the DC from start of picking shift to end of packing shift. This constraint is given in equation (4).

The focus of this research was a DC with manual order picking and manual packing. Orders were initially formed into waves and each wave was processed separately. It was assumed that the various stock keeping units (SKUs) are distributed throughout the DC in a random manner and that pickers follow a return path to pick items from the storage area. Different wave sizes to group the orders were considered. An automated conveyor sortation system was used to sort the items picked. All items belonging to a single order are directed to one accumulation lane and each lane has the capacity to hold items from several orders. A manual packing system that has different number of chutes/lanes was considered. Once orders are sorted, packers then place the items in boxes and seal them.

2 LITERATURE REVIEW

This section presents a summary of the research that has been done in relation to the operations in a DC that are relevant to this thesis. Initially the various studies that have been carried out in the area of order picking in a manual DC are discussed. This is followed by a review of studies relating to the sorting operation. The order packing operation, which has received little attention in previous work, is discussed in the third section. The fourth section reviews studies that have looked at integrated models in DCs. Finally, studies that consider the importance of human resources planning in a DC are presented.

2.1 Order Picking

Order picking is the process of retrieving items from storage to fulfill customer orders. It has been estimated that the overall logistics cost in the United States is 21% of the Gross National Product and 28% of this cost is attributed to order picking systems [12]. Although it appears a simple process, order picking is the most costly activity in a typical warehouse operation and accounts for around 60% of the operating cost [12], [22]. There are several factors that affect the performance and efficiency of order picking operations. Among these are the assignment policy used to store the items in the warehouse, the routing method used to determine the sequence of the items to be picked, and the manner in which individual orders are clustered to form a wave of orders.

2.1.1 Storage Policies

Several techniques are available to assign items to the various storage locations in a warehouse. In random storage, an item can be placed anywhere in the DC since it does not have any specific place designated. In demand-based storage, items are assigned to storage locations based on the expected demand or popularity of the item. Thus, items that have high expected demand (i.e. fast moving items) are assigned to locations that are closer to the pick-up/drop-off point because these items require the greatest number of trips to its locations.

The class-based storage system places stock in demand-based groups or classes with items being distributed randomly within each group or class. While random storage may require large travel times to traverse the whole warehouse, demand-based storage may cause aisle congestion and unbalanced utilization of the warehouse space. Class-based storage, which is a hybrid of the two former methods, attempts to reduce these disadvantages.

As described by Heskett [11] there are four basic elements in determining the placement of stock in a DC: compatibility, popularity of item, size per unit, and the complementarity of the items. All of the storage assignment methods described above are based on one of the elements identified by Heskett as being important in determining placement of stock. He introduced an improved method for stock storage based on the cube-per-order index (COI). The COI for an item is determined using the following: (1) the space required, in cubic feet, by the item when it is stored in an order selection area, and (2) the average number of times the item appears on orders over a period of time. Items with the lowest COI are assigned to locations that are closer to the pick-up/drop-off location and those with higher COI are stored at locations further away. This minimizes the overall distance that needs to be traveled to pick items for an order.

Malmborg [17] explained dedicated storage as a policy where specific locations in an order picking area are reserved for specific items. This differs from randomized storage policies because it is assumed that storage locations for items do not change over time. Malmborg developed a model to analyze tradeoffs in space requirements and retrieval efficiency associated with dedicated and randomized storage policies using the state probability distribution of aggregate space requirements in a randomized storage system. He demonstrated that the storage space requirement at the start of the picking period is lower for randomized storage than when using dedicated storage.

Petersen [21] defined the placing of items that are required in large volumes (high demand) closer to the pick-up/drop-off point as volume-based storage. This is identical to the demand-based storage policy. He studied the impact of two variations of volume-based storage policy on warehouse efficiency in comparison to randomized storage. In diagonal storage items were stored in the DC in a diagonal pattern with the highest volume items in locations closest to the pick-up/drop-off point and the lowest volume items in the farthest locations from the pick-up/drop-off point. In within-aisle storage the higher volume items were stored in the aisle closest to the pick-up/drop-off point and the lowest volume items stored in the aisles farthest from the pick-up/drop-off point.

Petersen [21] found that higher percentage savings were indicated with the use of volume-based storage compared to random storage. The within-aisle arrangement of items (based on volume) was found to outperform a diagonal arrangement of items (based on volume) with all the routing policies evaluated, the largest percentage savings being when optimal routing was used.

As discussed above, many studies have emphasized the advantage of using methods other than random storage to store SKUs in the active storage area from which order pickers will be retrieving the items given in their pick lists [10], [21]. However, these methods often require consideration of other factors and are more difficult to follow than random storage. Random storage is more frequently used in practice and analytical models are also available to evaluate picking time with random storage. Therefore it was assumed in this research that all the SKUs are randomly stored in the warehouse.

2.1.2 Order Batching

Individual orders are often batched together to form a wave of orders before they are retrieved from the storage locations. Order batching reduces the overall time spent on picking and improves picking efficiency.

Goetschalchx and Ratliff [6] investigated the approach that should be used to cluster items in a wide aisle. They focused on finding the optimal number and location of vehicle stops and determining the specific items that have to be picked at each stop point. Various travel paths from the vehicle stop location to retrieval point were studied to determine the optimal order clustering approach. Of the different methods evaluated, they found that the approaches that use euclidean and rectilinear travel assumptions were superior in minimizing the travel time for picking the order.

Gray, et al. [7] presented a methodology to determine batching for a predetermined number of orders based on the length of all aisle facings, the mean number of items per order and the accumulation lane width at the sortation system assuming that picking and sorting are both performed manually. Choe, et al. [4] compared the single-order-pick (SOP), sort-while-pick (SWP), and pick-and-sort (PAS) strategies for order picking in an aisle-based system with horizontal travel. They found that the PAS strategy can process the highest volume of orders while the SWP method was found to process a relatively high volume without much additional investment compared to SOP systems.

A procedure to determine the order picking strategy depending on the order quantities (EQ) and the number of items in an order (EN) was studied by Lin and Lu [15]. They defined five different classes of orders, based on the lower and upper bounds established for EQ and EN. The various classes of orders were evaluated for two order picking strategies, single order processing (SOP) and batching and zoning (BZ). It was found that the BZ strategy gives shorter picking times and better picker utilization for orders with few quantities. They also found that SOP is a better method when there are a large number of orders.

In this research it was assumed that several orders of different sizes are processed in a wave. Thus the all orders to be processed in a day are divided into several waves. Wave formation may be done at the point of receiving customer orders depending on criteria such as the sequence in which they are received or the geographical location to which they have to be delivered. Only order batching to form waves was important to this study. Details of how orders will be assigned to waves is beyond the scope of this study.

Since this research is primarily concerned with determining optimal staffing levels it was assumed that wave formation would be completed before picking orders are released to the pickers. Further, as recommended by Lin and Lu [15], a zoning strategy was used in this research. Thus the aisles in the storage area are divided into several zones depending on the number of pickers employed and each zone may consist of one aisle, several aisles or part of an aisle.

2.1.3 Routing Policies

A picking tour is a specification of the sequence in which the items in a specific order will be retrieved from their storage locations. Determining the sequence that minimizes the distance traveled is crucial for improving picking efficiency. The various policies used for routing order pickers range from simple heuristics to optimal procedures.

Hall [8] described the various policies that are used for routing. In a traversal strategy, the simplest for routing order pickers in a warehouse, the picker enters any aisle that contains at least one item and exits at the other end, traversing the entire distance of the aisle. A picker following a return policy enters an aisle at one end, picks all items on one side of the aisle and returns in the same aisle while picking items from the opposite side of the aisle.

Different alternatives of these two strategies have also been used to improve picking performance. A split policy is a traversal or return policy from both ends of an aisle. In the midpoint strategy, items in an aisle are picked by dividing it into two halves, entering at the front end to retrieve items from the first half and from the back to retrieve from the latter half. In both halves a return policy is followed. The largest gap strategy is an improvement over the midpoint strategy with the picker still following a return policy from each end of an aisle. Here, instead of entering an aisle only up to its midpoint the picker travels as far as the largest gap to pick all items from the aisle, the largest gap being the portion of the aisle that the picker does not visit [8]. In addition to these, various forms of combined strategies have also been applied to determine picker routing [21], [22], and [24].

Ratliff and Rosenthal [23] presented an algorithm for picking an order in minimum time using the shortest route in a rectangular warehouse without cross aisles. They used the traveling salesman approach and considered all possibilities for traveling between and within aisles. This optimal strategy was obtained assuming picking is done with a vehicle that can pick only one order at a time.

Goetschalckx and Ratliff [5] investigated a situation in which items have to be picked from a wide aisle where the picker cannot reach items on both sides without moving closer to the side of the aisle. They developed an optimal traversal algorithm to determine the within-aisle route, which was found to reduce travel time by up to 30% when compared with the Z-pick strategy. The Z-pick strategy is one in which each slot in the aisle is picked in a fixed sequence, which remains the same for all the orders. They also compared the optimal traversal algorithm with the TSP approach to find the optimal picking path presented by Ratliff and Rosenthal [23]. The approach of Ratliff and Rosenthal was found to be worthwhile compared to Goetschalckx and Ratliff's traversal policy only when the number of aisles is very small or the order density is low.

Hall [8] compared the different heuristic routing policies in a manual warehouse and developed a distance approximation for routing order pickers in a manual warehouse. The routing strategies were evaluated with respect to two warehouse configurations: with negligible aisle width and non-negligible aisle width. For narrow aisles with uniformly distributed pick locations, the largest gap return policy was found to be best with low picks per aisle, whereas the single traversal policy was found best when the number of picks per aisle was higher.

Petersen [21], [22] studied the effects of different routing heuristics and compared them with the optimal policy. Six different routing strategies — traversal, return, midpoint, largest gap, composite (a combination of the best features of traversal and return strategies, which aims to reduce the travel distance between furthest picks in two adjacent aisle) and optimal — were evaluated in a random storage warehouse.

Petersen [21] also found that the picker route length is shorter in a warehouse with fewer-but-longer aisles (a deep warehouse) compared to a wider warehouse with shorter aisles. When within-aisle storage was used, it was found that the composite strategy is the best next to the optimal, while the traversal strategy was almost identical at a higher number of picks [22]. When diagonal storage was used, the composite strategy was found to give routes closest to the optimal. On the other hand, in a random storage environment, the largest gap strategy was found to result in the best routes, closest to the optimal strategy.

Ratliff and Rosenthal's routing strategy was found to give the shortest routing distance for order picking [22]. However this method is more difficult to use than other heuristic methods and is not worthwhile when the number of aisles is very small or the order density is low. On the other hand, the traversal strategy is much simpler to implement and has been found to yield shorter travel distance than the return policy when the pick densities are not very high [5]. Return and traversal strategies are also easier to follow in practice compared to other more complicated methods.

In this research a return strategy was assumed. The manner in which the analytical picking time estimation can be modified if a traversal strategy is used instead is discussed.

2.1.4 Travel Time Models

The time spent by pickers in traveling through the DC to retrieve the items from their storage location constitutes a significant component of the time taken to pick a set of orders. Travel time models are used in a DC to determine the expected route length for order pickers. These models provide one measure through which the performance of an order picking system (with a given storage strategy, routing policy, warehouse configuration and other parameters) can be evaluated.

The picking time is also dependent on other factors, some of which were described above. According to Gray, et al. [7] the time to pick a set of orders consists of four components: travel time, retrieval time, stop time and unload time. They formulated a model to estimate the picking cycle time based on picking system parameters for a manual warehouse. Thus the total pick cycle time for a set of orders is given by,

$$PCT = (WT + PT + UL) \times (1 - \beta) \tag{5}$$

where,

$$WT = \frac{2L}{V \cdot Z} \tag{6}$$

$$PT = \left[\frac{R \cdot E}{Z}\right] \times \left[ST + \sum_{k=1}^{TECH} GTPT_k \times RFPT_k + NUPT_k \right] \tag{7}$$

PCT	Picker cycle time
PT	Pick time

WT	Walk time
UL	Constant time to unload the cart, get new boxes and get a new pick list
L	Length of all facing aisles (ft)
V	Speed of a picker (ft/min)
Z	Number of zones
R	Number of customer orders picked simultaneously
E	Number of items per order
ST	Stop time
$TECH$	Number of different storage types in use
$GTPT_k$	Grip time per unit using $TECH_k$ (min)
$RFPT_k$	Relative frequency for picking an item stored using $TECH_k$
$NUPT_k$	Mean number of units packed per item stored in $TECH_k$
β	The imbalance effect due to the variation in zones workload

The walk time depends on the length of all the aisles in the zone assigned to a picker and the speed at which the picker walks through the aisles. This is given in equation (6). The picking time depends on the number of items that each picker has to retrieve from the zone assigned, the number of stops that have to be made to retrieve these items and the time to grip and remove these items, which depends on the storage methods used for different types of items. This relationship is given in equation (7).

Masel and Medeiros [18] modified the expression from Gray, et al. by using a binomial distribution to estimate the expected number of stops that must be made by each picker. The model by Gray, et al. is deterministic since the number of stops are determined based on the average number of SKUs to be retrieved. The model presented by Masel and Medeiros is more representative of the situation that may arise in practice in a DC where the number of items to be retrieved by each picker is not uniform. For this reason the time required to pick a wave of orders was estimated using the model proposed by Masel and Medeiros.

Chew and Tang [3] developed a model to obtain the probability mass function that characterizes the tour of an order picker under general item location, which can be used to analyze the expected travel time for the picking operation. This was done considering the expected number of aisles that were visited and the furthest aisle visited. The model was formulated by considering the analogy of the picking process to the occupancy problem: given that there are n balls and M urns, one seeks the probability of having exactly J ($\leq M$) urns filled with at least one ball. They found that the average turnover time of an order (the total time spent in the system) is a convex function of the batch size, thus giving an optimal batch size to attain the minimum turnover time.

2.2 Sorting

Many warehousing systems use a wave approach to group orders together with each picker being responsible for picking items for all the orders in the wave in his/her zone. This consolidation of orders in picking introduces another stage into the warehousing operations: sorting the items picked in the wave to form the initial order that were combined. This is typically achieved through an order accumulation/sortation system (A/SS). The A/SS is a conveyor system where the items picked are accumulated in a staging area and move as a wave into the sortation system which routes each order to the designated shipping lane. The body of literature available on sorting can be classified into two categories based on the method of analysis: simulation and analytical methods.

2.2.1 Simulation-based Studies

Bozer and Sharp [1] examined the effect of having a recirculation loop in the sortation system. Simulation was used to study the impact of recirculation on systems with various throughput capacities and different numbers of accumulation lanes. It was found that the throughput capacity of a system with recirculation was not sensitive to the number of lanes, whereas the capacity declined for a system with no recirculation as the number of accumulation lanes increased. They also evaluated the effect of two different strategies for deciding the accumulation lane to which a particular order is to be diverted: decision making at the induction point and at the divert point closer to the lanes. The studies revealed that decision making at the divert point improved throughput capacity for a recirculating system with low lane capacity (less than 5), while there was no significant difference when the lane capacity was increased. This was found to be the case irrespective of the number of accumulation lanes considered in the model.

Bozer, et al. [2] used simulation to study how the A/SS throughput was affected by the number of sortation lanes, the wave profile, the assignment of orders to lanes, and the manner in which the waves were released to the sorting system after picking was complete. The research was based on a recirculating sorting conveyor with multiple accumulation lanes. They assumed that no intermixing of the waves was allowed and that items from the subsequent wave are not allowed to enter the loop until all the items of the current wave were processed. The items were referred to as totes, meaning that each was a collection of items. The wave size was restricted to be within 120 to 132 totes.

Incidental lane assignment (where a scanned item is directed to the lane if the order it belongs to has already been assigned one or else a new lane is assigned for that order) when one becomes available was found to yield the highest throughput ratio. They also found that for a given number of lanes and given lane assignment logic with a fixed wave size, the throughput ratio was inversely related to the number of orders. While priority-ranking orders in advance did not improve the throughput ratio, 5%-25% improvement was reported when the second wave was released after 90% of the orders of the first wave were assigned to sortation lanes.

In this study a strategy similar to that was used by Bozer, et al. was assumed to release the waves from the picking area to the sorting conveyor. It was also assumed that accumulation lanes were sufficiently large to hold all orders assigned. Bozer, et al. too considered a similar arrangement. However, the sortation system used in this research was different to that explained by Bozer, et al. because a recirculation loop was not considered here.

2.2.2 Studies using Analytical Methods

Johnson [13] developed an analytical model to determine the sorting time in an A/SS considering a fixed priority rule (FPR) and incorporating the stochastic elements of the system. A fixed priority rule could be any defined scheme for routing the order items to the accumulation lanes such as by order size, where the largest order is sorted first, followed by the second largest order, and so on. The model was limited to the sorting operation at a single lane and assumed that there was no blocking between the shipping lanes and the staging/accumulation area. In a separate study [14], the same analytical model was further developed to consider the next available rule (NAR) to assign incoming items to accumulation lanes. The NAR differs from a FPR because each time an order is completely sorted, the next item to pass the induction point/bar code scanner defines the new order to be sorted. It was found that the NAR outperforms any fixed priority rule, resulting in shorter wave sorting times.

Meller [20] investigated the manner in which orders arriving to the sortation system could be assigned to the accumulation lanes when constrained by the loading sequence. A mathematical model was developed to minimize the maximum recirculation delay over the observed values assuming that order-to-lane assignment must be done prior to the induction point. It is also assumed that items enter the recirculating conveyor in a random sequence. Lane assignments that are capable of reducing the total sorting time up to 50% are obtained by optimizing the mini-max problem.

Masel and Medeiros [19] showed that the expressions developed by Johnson [13], [14] to determine the expected sorting time for a continuous conveyor using the FPR and the NAR can be also applied to a conveyor with discrete carriers such as tilt tray conveyors. They extended the expressions to develop a model to estimate expected sorting time for a partially full conveyor with a single sorting lane. Masel and Medeiros also modified the expression by Johnson to find the expected sorting time when there are multiple sorting lanes. The accuracy of the improved model was found to increase when more orders were sorted at each lane and when the numbers of orders sorted at each lane were not equal.

Most of the research that has been done on sorting considers assigning one order at a time, in a single lane, requiring recirculation. In this research it was assumed that there are several sorting lanes. Therefore items belonging to many orders can be sorted simultaneously without the need for recirculation.

2.3 Packing

Packing is the process of placing the items corresponding to a single order into boxes/cartons or on a pallet. Literature pertaining to the different aspects of the packing operation is minimal compared to the extent of studies that have been carried out with regard to other operations of the DC. Hemminki, et al. [10] looked at on-line packing with boxes of different sizes in automated systems. At each stage, only the layout of the previous boxes on the partially-filled pallet and the size of the box to be placed next are known, but no information is available about forthcoming ones. The research investigated the possibility of using robots instead of traditional manual pallet loading assuming that there is no information about the boxes that are to arrive. The objective of their research was to achieve higher packing densities in on-line packing of boxes.

Heady, et al. [9] evaluated carton-packing rules used by manual pickers for high-volume operations with sets of boxes whose number and sizes vary according to known frequencies. They applied simulation to study sixteen packing rules commonly observed when packing items into small boxes. They demonstrated that the percentage of empty space in cartons is strongly dependent on proper selection of packing rules. They also found that packing larger boxes first gave better results by reducing the carton's empty space by about one-third when compared to a disorganized approach when packing procedures having random components. They did not evaluate the effect of these different rules on packing time, but it is indicated that experienced packers can explicitly follow complicated packing rules without spending any more time than they would otherwise.

2.4 Integrated Models

Most of the research that has been done on DCs to date has investigated the operations and performance of only a single operation in the DC. Gray, et al. [7] carried out one of the most comprehensive studies to deal with DC operations. The study focused on the design and operations of an order consolidation warehouse and covered a variety of issues including equipment and technology selection, picker routing, pick list generation and order batching. The decision variables—facility design and technology selection, item allocation, and operating policy—are first divided into different levels and a hierarchical solution method is applied to each of these levels.

According to Gray, et al., design and technology selection are often dependent on the existing facility constraints. Item allocation refers to the way in which items are allocated to the aisles and zones. The operating policy is concerned with order batch size and number of zones. The parameters corresponding to each of these levels are determined and analytical models are developed to calculate all aspects of concern: total aisle length, batch size for picking, picker utilization, and picker cycle time. The solution was found by iterating between these decision levels to converge upon a solution that minimizes the total operating cost. A simulation model developed with SIMSCRIPT was used to confirm the solution of the analytical models. Thus the model provides an extensive analytical tool to determine the design and operating policy of a warehouse at minimum cost.

Masel and Medeiros [18] investigated the interdependency of the picking and sorting operations in processing waves of orders. They developed analytical models to determine the picking and sorting times in a DC with multiple storage lanes and multiple sortation lanes. Through the application of the models they show that the overall minimum time to process (pick and sort) a given group of orders is accomplished when the time to pick a wave and sort the wave are balanced. The values obtained through the analytical models are verified by similar values obtained from a simulation model, except when the number of items per order is small.

2.5 Human Resources in a DC

Despite the advance of automated equipment and facilities, there are many operations in a DC that cannot be easily automated. Thus labor costs are still a significant portion of DC costs. Therefore, one of the important factors that would contribute to reducing the cost of DC operations is better management of the human resources. However, no research done has concentrated on optimizing staffing requirements in a DC operation.

Luxhoj, et al. [16] conducted a study that considered managing human resources in a DC, though in a different perspective. They focused on the apparel industry where seasonality of products often calls for seasonal changes in labor requirements. The study was aimed at developing a causal regression model for predicting the manpower requirements of the DC, using historic data from 26 months to predict such as the units processed on standard hours, off-standard hours, and covers six direct labor operations.

Their formulation, which was developed to run on a 256K computer and called the Dynamic Manpower Prediction and Allocation Model (DMPAM) was supported by a software package through which it was implemented. The model was tested on a very large apparel distribution center to the satisfaction of its management. However, the main limitation of the DMPAM is that it does not provide an optimization technique. In the present retail distribution environment, product life cycles are much shorter and orders have changed in size and composition compared to 15 years ago, when this study was done. Hence for a contemporary DC the variables that are used for the model will have much different values compared to those used by Luxhoj, et al.

3 METHODOLOGY

The objective of this research is to develop a method for determining the optimal staffing requirements for the picking and packing operations of a manual DC to deliver a given number of customer orders. The principal constraint in doing this is completing all activities involved within the time available. To maintain efficient operations it is also advantageous to ensure that the pickers and packers will not finish processing the orders earlier than the specified time, thus having to pay for idle time. Therefore, in order to find the optimal staffing requirements, it is essential to accurately estimate the time taken for the different operations involved.

The optimization model that was developed utilizes stochastic analytical models to estimate the expected picking, sorting, and packing times. The preliminary stage of developing the model involved identifying time estimation models to estimate the time needed to pick, sort and pack a wave of orders. The models were then tested to verify their accuracy.

The analytical models were developed for a particular set of operating policies for a DC. If the operating conditions were different, the time estimation models would need to be adjusted to reflect the new conditions. Ways in which the models used in this research can be modified to accommodate different situations is discussed where this would be possible.

3.1 System Description

The picking and packing operations of a DC were the focus of this study. The picking and packing operations were assumed to be manual because this is typical when dealing with broken case picking, processing of soft goods, and items that are varied in shape and size. The schematic representation of the DC operations considered in this study is shown in Figure 3-1.

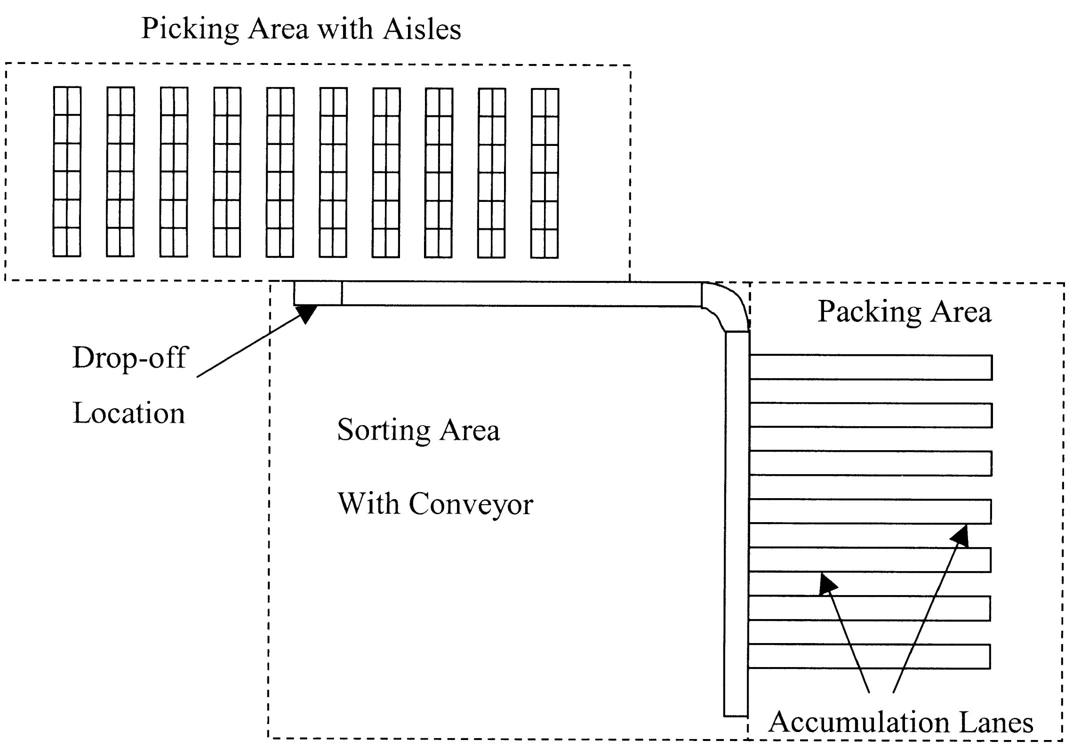

Figure 3-1: Schematic Representation of Picking, Sorting and Packing Areas

The picking area consists of several zones with one picker per zone. Each picker retrieves the items specified in the pick list from the zone assigned to him/her. Pickers are informed how many of each SKU should be retrieved, where the SKUs are located within the zone, and the sequence in which he/she should travel from one SKU location to another to retrieve the items in the pick list. All the items retrieved are given a label for identification and are collected into a cart.

The picked items are then unloaded onto a non-recirculating sorting conveyor. It was assumed that all pickers have to travel approximately the same distance from their respective zones to the point at which they drop off the load in the cart on to the sorting conveyor.

After being picked and unloaded, the items are then transported to the packing area via a conveyor where there are multiple accumulation lanes. A sorting system with no recirculation was assumed, so the accumulation lanes were assumed to be sufficiently large to hold all orders assigned without having to be recirculated. All items that enter the conveyor at the induction point have a lane assigned and all items belonging to a single order are directed to the same lane.

Once sorting of all the orders belonging to a wave is complete, packers begin to place the items in boxes and seal them. Every packer gets a list of orders and the items belonging to each order that he/she has to pack. Multiple orders are assigned to a single accumulation lane and the packer sorts out the individual orders. Boxes of different sizes are available to the packers and a box that can hold all the items corresponding to a particular order is chosen. It was also assumed that one packer is assigned to each accumulation lane and that a packer has to pack items only in that particular lane. If more than one lane is assigned to each packer the additional time necessary to travel between lanes too will have to be accounted for in the packing time estimation model.

The total time taken to pick, sort and pack a given set of orders and the minimum number of workers necessary to accomplish these tasks within the given time are limited by three constraints. First, the above activities have to be performed for each wave in the given sequence because orders cannot be packed until the items belonging to them are retrieved from storage and grouped to form orders.

Second, packing cannot begin before the picking and sorting has been completed for a particular wave. This is because all the items for individual orders may not have been picked and the packers can be idle until those items reach the correct sorting lanes. Also, the same operation cannot be performed on two waves simultaneously. Thus, picking items for the second wave cannot begin until picking has been completed on the first wave. This is necessary to ensure that items corresponding to each wave remain separate. However, picking items for subsequent waves continues while packing is going on. Finally, the time to complete processing of all the orders depends on the total time taken for the three operations for the individual waves.

These constraints have to be satisfied when finding the optimal staffing requirements. Thus an optimization model was required to determine the number of pickers and packers necessary to process the given set of orders subject to the constraints defined. The optimization model was then tested to find the number of pickers and packers necessary to process a given number of orders for different scenarios. Finally the data generated from the optimization model were used to develop a regression model that can be used to predict the number of pickers and packers.

3.2 Time Estimation Models

The initial step in formulating a model to determine the optimum staffing levels in a DC involved developing analytical models to estimate the time required to complete each of the relevant operations. Picking and packing operations were selected for the study as they are the two most labor intensive operations in a DC. The sorting operation is the intermediary stage that connects picking and packing. Thus, while picking and packing were the focal operations used to determine optimal staffing levels, sorting was included to account for the time taken from the start of picking to end of packing. This section describes the approach used to formulate time estimation models for each of these operations.

3.2.1 Picking time estimation

The storage area contains several aisles, which are classified into zones. Each picker is assigned to a zone, which consists of one or more aisles or part of an aisle depending on the number of pickers employed.

According to Gray, et al. [7] the time taken to pick a wave of orders consists of the travel time (time required to walk through the pick area), retrieval time (time to retrieve one item from storage), stop time (associated with the number of points at which the picker has to stop), and the unload time (time to unload the cart at the sorting conveyor and get a new pick list). Based on this they formulated an expression to determine the expected picking time for a set of orders. Their model is based on the assumption that the stop time incurred for each SKU can be estimated using the average number of units to be retrieved from each SKU. However, in practice, data on the number of items of each SKU that have to be retrieved may not be readily available.

Masel and Medeiros [18] modified the expression from Gray, et al. by using a binomial distribution to estimate the expected number of stops that must be made by each picker. This expected picking time estimation model, using the same notations used by Masel and Medeiros is given below in equation (8). The model by Gray, et al. is deterministic since the number of stops is determined based on the average number of SKUs to be retrieved. On the other hand the modified expression by Masel and Medeiros is more representative of the situation that may arise in practice in a DC where the number of items to be retrieved by each picker is not uniform. For this reason the time required to pick a wave of orders was estimated using the model proposed by Masel and Medeiros. The travel time, number of stops and number of retrievals per picker as given by Masel and Medeiros are shown in equations (9), (10), and (11) respectively.

$$E(T_p) = \frac{1}{W_p}\left[2a\left(\frac{L}{V} + T_e\right) + nm\left(Z_\alpha\sqrt{\frac{W_p - 1}{nm}} + 1\right)T_r + Ka\left(1 - \left(1 - \frac{W_p}{Ka}\right)^{\frac{nm}{W_p}\left(Z_\alpha\sqrt{\frac{W_p-1}{nm}}+1\right)}\right)T_I\right] + T_U \quad (8)$$

$$\text{Travel time per picker} = \frac{a}{W_P}\left[2\left(\frac{L}{V} + T_e\right)\right] \tag{9}$$

$$\text{Number of stops per picker} = \frac{Ka}{W_P}\left[1-\left(1-\frac{W_P}{Ka}\right)^{\frac{nm}{W_P}\left(Z_\alpha\sqrt{\frac{W_P-1}{nm}}+1\right)}\right] \tag{10}$$

$$\text{Number of retrievals per picker} = \frac{nm}{W_P}\left[Z_\alpha\sqrt{\frac{W_p-1}{nm}}+1\right] \tag{11}$$

where,

m	Number of orders in the wave
n	Number of items per order in the wave
W_p	Number of pickers
a	Number of aisles in the storage area
L	Length of each aisle in the storage area
V	Picker walking speed
T_e	Time to cross over from one side of the aisle to the other
Z_α	Normal distribution Z-value that corresponds to a probability of α
T_r	Time to retrieve one item from storage
K	Number of different SKUs in an aisle
T_I	Time to stop and identify the storage location of the next item to be retrieved
T_u	Time to unload (if necessary) and prepare for picking the next wave
T_p	Expected picking time for the wave

For this study it was assumed that the SKUs are stored in a random manner. Random assignment was assumed because this generally will create uniform activity throughout and balance picking load. Without a random layout it is very difficult to formulate analytical models because the location of picks is highly dependent on the arrangement of SKUs and the particular demand distribution.

There was no constraint on cart capacity and it was assumed that the cart can hold all items retrieved by a picker for one wave. Therefore every picker has to unload the items onto the sorting conveyor only once for each wave. This is accounted for by T_U in the picking time estimation model in equation (8). An approach that can be used to deal with cart capacity constraints is presented subsequently.

It was assumed that the pickers follow a return strategy, where they enter the aisle at one end, pick all required items, cross to the other side and return from the same side that was entered. With a return path all pickers have identical travel distances, which is equal to twice the length of all the aisles assigned to each picker. The travel time per picker based on this approach is given in equation (9).

If a traversal strategy was assumed, the distance traveled by the pickers would approximately be reduced by half and the travel time estimation could be modified accordingly if the aisle width is assumed to be zero. However, if a strategy other than these was used to route order pickers, all pickers will not follow the same path to retrieve the items from storage and therefore the travel time will depend on the exact location of items or the route taken. Thus, the picking time estimation model would have to be modified to take account of change in travel distance.

It was assumed that all items required to deliver a given set of customer orders are available in storage before picking begins. The analytical model for the number of retrievals per picker is given in equation (11). Here it was assumed that the retrieval time for every item is approximately equal irrespective of shape, size or weight.

3.2.2 Sorting time estimation

Sorting is the process of separating the items picked as a wave into individual orders. With respect to sorting, the time that was important in this research was the time taken for the last item (of the particular wave) placed on the sorting conveyor to reach the designated accumulation lane. This time is referred to as the sorting time in this study.

During the preliminary stages of evaluation of the sorting time estimation model, it was assumed that the sortation system directs the items to a single accumulation lane. Thus it takes the same amount of time to sort every item that is placed on the conveyor. Here it was assumed that it takes the same amount of time for all pickers to travel from their respective zones to the unloading point. This may be achieved in practice for example by arranging the aisles on both sides of the conveyor or having multiple parallel conveyors. A simple expression, assuming a single accumulation lane was initially used to estimate the sorting time for a single wave. When the length of the conveyor between the loading point and the accumulation lane is D (ft) and the conveyor travels at a speed of U (ft/min), the time taken to sort all the items can be expressed as follows:

$$T_s = \frac{D}{U} \tag{12}$$

However, if there were more than one accumulation lane, the time taken to sort a particular item depends on the lane it is assigned to and therefore the distance it has to travel to reach that particular lane. The sorting time estimation model was later modified to take account of multiple accumulation lanes based on the results obtained from simulation.

The model given in equation (12) was used as the basic expression to determine sorting time. After experimentation to verify its accuracy, the model was later expanded to consider the effect of multiple accumulation lanes.

3.2.3 Packing time estimation

The packing activity involves placing each item of an order into the box then sealing the box. The time to pack a wave is a function of the time taken to put a single item in the box (T_C) and the time taken to seal a box (T_B). In this study an average number of items per order were considered in the analytical models. However, in reality the number of items per order can fluctuate. This can be handled through an intelligent assignment of orders to packers such that every packer is allotted orders with a large number of items as well as those with lower number of items per order to produce an equal number of items per packer. It was assumed that boxes of different sizes are available and that each packer selects a box that will hold all items corresponding to a particular order. Thus for each order a T_B amount of time is spent for sealing the box. It was also assumed that T_B is same for all boxes and T_C is the same for all items.

In formulating the packing time estimation model it was also assumed that the time taken by packers to locate items belonging to an order and place them in a box is not significantly affected by the number of orders to be packed or the average number of items per order.

Assuming that items and orders are uniformly distributed among packers, when there are m orders with average n items in each order, the number of items and the number of orders assigned to a packer can be denoted by nm/W_{Pa} and m/W_{Paa}, respectively. Thus the packing time for a wave (T_{pa}) can be expressed as shown in equation (13). This simple model assumes that all packers get an identical number of orders and items.

$$T_{pa} = \frac{nm}{W_{Pa}} T_C + \frac{m}{W_{pa}} T_B \qquad (13)$$

This expression was modified to reflect the packing time if all packers do not get an identical number of items. The approach used in the picking time estimation model by Masel and Medeiros [18] to determine the number of retrievals per picker was modified and used here to determine the number of items assigned to each packer. When the variation of number of items per packer is assumed to follow a binomial distribution, the expected packing time can be expressed as shown in equation (14).

$$T_{pa} = \frac{nm}{W_{Pa}}\left[Z_{\alpha}\sqrt{\frac{W_{Pa}-1}{nm}}+1\right]T_C + \frac{m}{W_{pa}}T_B \tag{14}$$

The equal distribution of orders among packers can be managed by intelligent allocation of orders. Therefore the binomial distribution approximation was used only to account for the imbalance in the number of items assigned to packers.

3.3 Simulation Testing of Time Estimation Models

This section describes the experimentation carried out to compare the results obtained by using the analytical models with those obtained from simulated waves generated in Microsoft Excel.

3.3.1 Analysis of Picking Time Estimation Model

The model that was used to estimate the picking time was given in equation (8) previously. The picking time found by this model was compared with the same results found through simulated waves using Microsoft Excel. The values used to describe the system are shown in Table 3-1.

The expected picking time to retrieve a single wave of 750 items (150 orders of 5 items each) was calculated using the analytical model when different numbers of pickers were employed. To determine the actual order picking time, 20 waves were randomly generated in Excel for a given number of pickers. This was done by first randomly determining the aisle and the location within that aisle in which each item would be located and then assigning that item to a picker.

Table 3-1: Values used for Different Parameters in Picking Time Estimation

Parameter	Notation	Value
Aisle length	L	150 ft
Picker walking speed	V	60 ft/min.
Number of aisles in storage area	a	100
Time to retrieve one item	T_r	0.2 min.
Time to stop and identify next location	T_I	0.2 min.
Number of SKUs in an aisle	K	20
Time to crossover from one side of the aisle to the other	T_e	0.5 min.
Time to unload and prepare for picking the next wave	T_u	1.0 min.

Each picker was assigned an equal number of aisles. For example if there were 100 aisles and 25 pickers, aisles 1- 4 were assigned to the first picker, aisles 5 - 8 were assigned to the second picker, and so on. Once the aisle an item was located in was determined it was then assigned to the picker for that aisle. After the number of retrievals for each picker was calculated the number of stops for each picker was found. For each item, the location of the item in the storage area was compared with all the previous locations assigned to the picker who was to pick the particular item. If the picker was not assigned an item at that location previously, the number of stops for the picker was increased by one. However, if the particular picker had retrieved an item from the same location previously, the number of stops was not changed.

The total number of retrievals and stops for each picker was calculated to determine the maximum retrievals and stops for a picker in that wave. The time taken to completely pick the wave of orders depends on the time taken by the picker who has the highest number of retrievals and stops. Therefore, for different number of pickers, 20 trials of the above testing were carried out to determine the maximum retrievals and stops per picker. During each trial, the composition of the items in the wave was changed. Thus the aisles in which the items were located and the location of items within the aisle were changed.

The analytical picking time estimation model used in this research assumes that the number of SKUs (number of stops) assigned to each picker follows a binomial distribution [18]. When the sample size, which is equal to the total number of items to be processed, is increased, the binomial distribution could be approximated with a normal distribution [27]. Accordingly, the number of SKUs assigned to a picker depends on the number of pickers employed, the number of orders to be processed, the number of items per order, and the Z_α value that is used. Z_α is the normal distribution Z-value that corresponds to a probability of α.

In order to determine the appropriate Z-value that should be used, the analytical model was tested by varying the Z-value used in the picking time estimation model to determine a value that will give approximately the same picking time as from the simulated results. The simulated results were compared against those obtained from the analytical model with Z-values corresponding to different probabilities. A comparison of the percentage difference between the simulated picking time and those obtained from the analytical model with different α values is shown in Figure 3-2. The picking times found through simulation and from the analytical models is given in Appendix I.

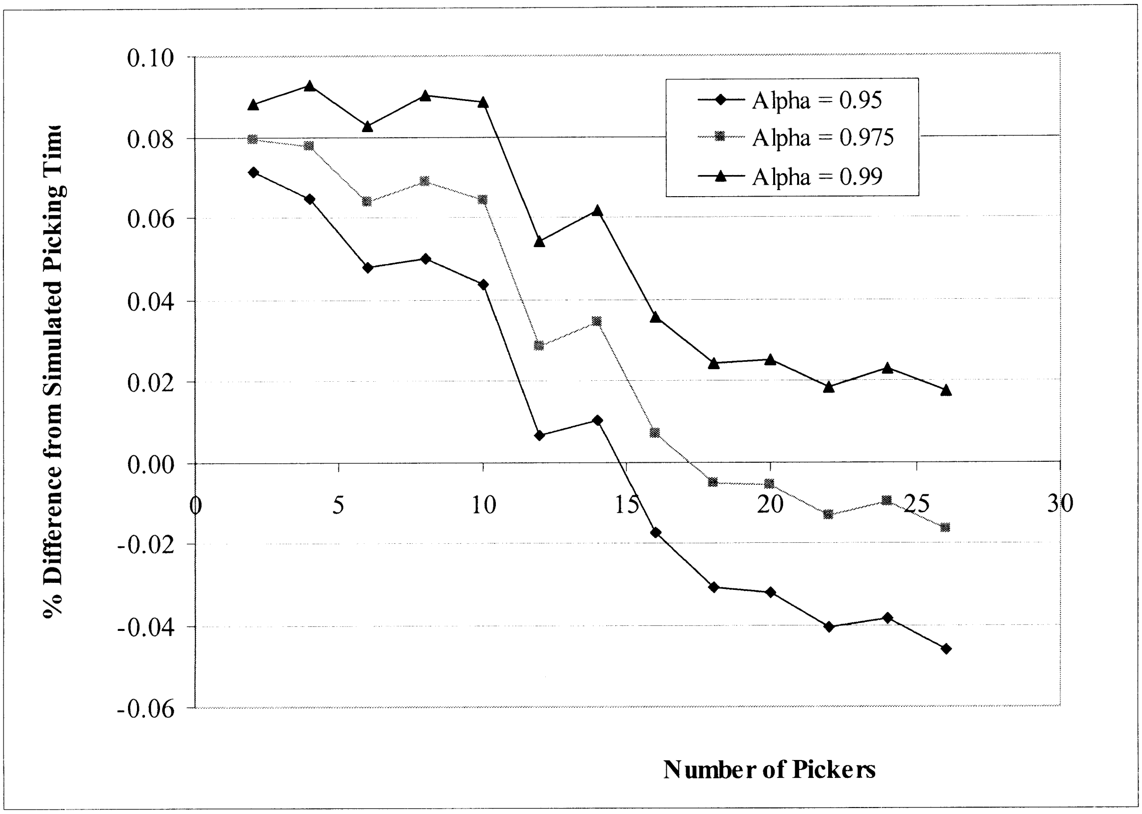

Figure 3-2: Percentage Difference in Picking Time with Different α Values

As can be seen from Figure 3-2, when the number of pickers is very low the percentage difference between the simulated picking time and analytical picking time are not very different with either α value. However as the number of pickers is increased, the % difference between the simulated picking time and analytical picking time approaches zero when α = 0.975. In order to ascertain whether this α value produces comparable results with even higher numbers of pickers, the simulation was carried out with 50 pickers. The time to pick the wave of orders was 54.35 minutes from simulation while the time estimated from the analytical model was 53.44 minutes. Thus the results from the two methods are most comparable when $\alpha = 0.975$. Therefore the Z-value corresponding to a probability of 0.975 was chosen for all the subsequent experiments.

The number of retrievals and the corresponding number of stops were obtained using the simulated waves (with $\alpha = 0.975$) as described above and these results were used to calculate the travel time, retrieval time and stop time and the picking time per picker for the wave. The average picking time obtained for 20 runs for a given number of pickers was compared with the picking time estimated using the analytical model for the same number of pickers. The picking time as obtained through the analytical model and by simulation is shown in Figure 3-3.

As can be observed from Figure 3-3, the simulated time to pick the wave of orders compares favorably with the picking time estimated from the analytical model. Thus it was concluded that the picking time estimation model is accurate in determining the time taken to pick a wave of orders when the picker follows a return path to retrieve items that are similar in shape, size and weight that are randomly distributed in the storage area.

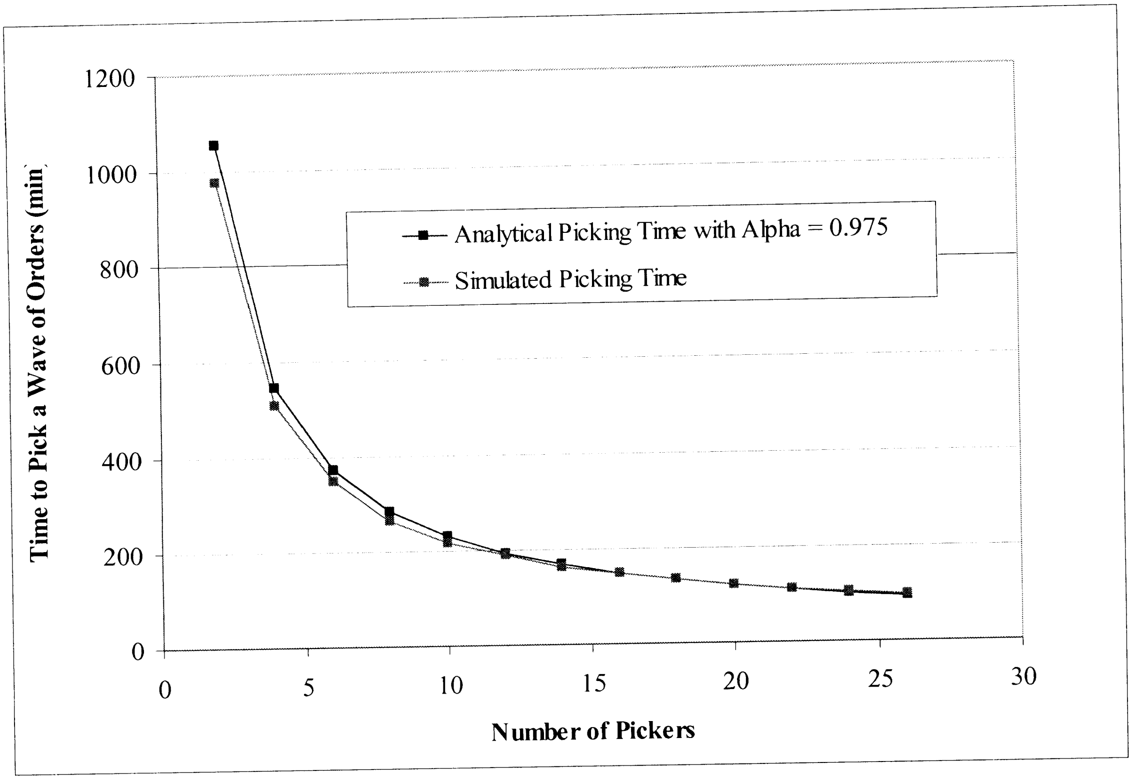

Figure 3-3: Picking Time per Wave as a Function of Number of Pickers

A Z-value corresponding to a probability of 0.975 also resulted in the smallest percentage difference between the number of stops and number of retrievals given by the simulated process and the picking time estimation model. This means that the values estimated by the picking time estimation model are most comparable to simulated results when α equals 0.975. A comparison of the number of retrievals and stops obtained from the two methods when $\alpha = 0.975$ was used for the picking time estimation model is shown in Figure 3-4. A table with the values for number of retrievals and number of stops obtained using the two methods using 0.975 probability are shown in Appendix II.

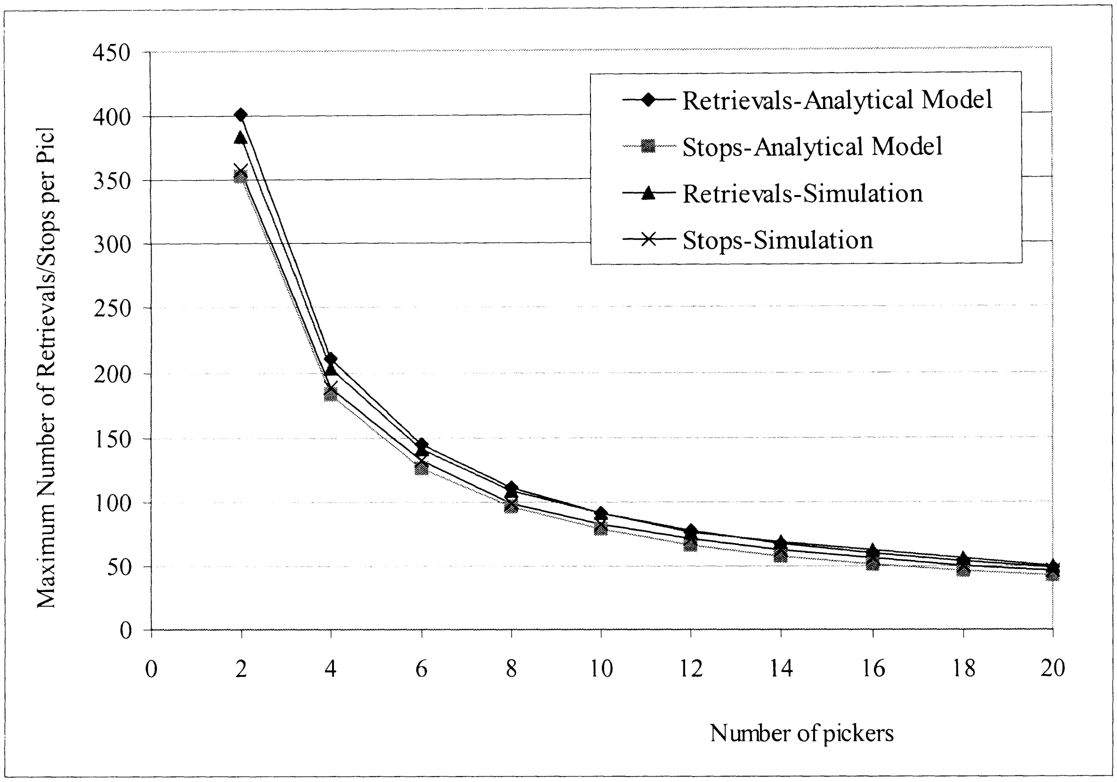

Figure 3-4: Maximum Number of Stops and Retrievals per Picker with α = 0.975

3.3.2 Analysis of Sorting Time Estimation Model

The expression given in equation (12) previously was used to calculate the sorting time. These results were compared with the experimental results generated through Microsoft Excel. In order to simulate the sorting operation, the last item in a wave was assigned a random number between 1 and W_{pa}, which was used to determine the lane it is assigned. Once the destination of the item was known, the time taken to travel to that location was calculated based on the distribution.

Fifty replications of this were conducted and the sorting time was taken as the average of the time taken by the last item in each of the replications. However, when there is only one accumulation lane, the last item in every wave has the same destination and therefore will take the same time to be sorted. With only one accumulation lane, the sorting time estimated by the simulation model was identical to that given by the analytical model. For a 20 ft (D) long conveyor that is traveling at 75 ft/min (U), the time taken to sort is 0.267 minutes (D/U) using the analytical model and the same result was obtained from simulation. However when more than one accumulation lane was used the sorting time obtained from the simulation model was lower than that given by the analytical model in equation (12). This was because the number of orders and items assigned to each lane were lower. The sorting time estimation model was then modified when there are multiple lanes, as shown below.

$$T_S = \frac{D + \frac{D_l}{2}}{U} \qquad (15)$$

In equation (15), D is the distance from the drop off point in the picking area to the first sorting lane and D_l is the length of the sorting conveyor between the first and the last sorting lanes (number of lanes is equal to l). U denotes the speed at which the conveyor is traveling. The simulation model was again used to determine the sorting time with different numbers of accumulation lanes and the results were compared with the time obtained using the model given in equation (15). This is shown in Figure 3-5.

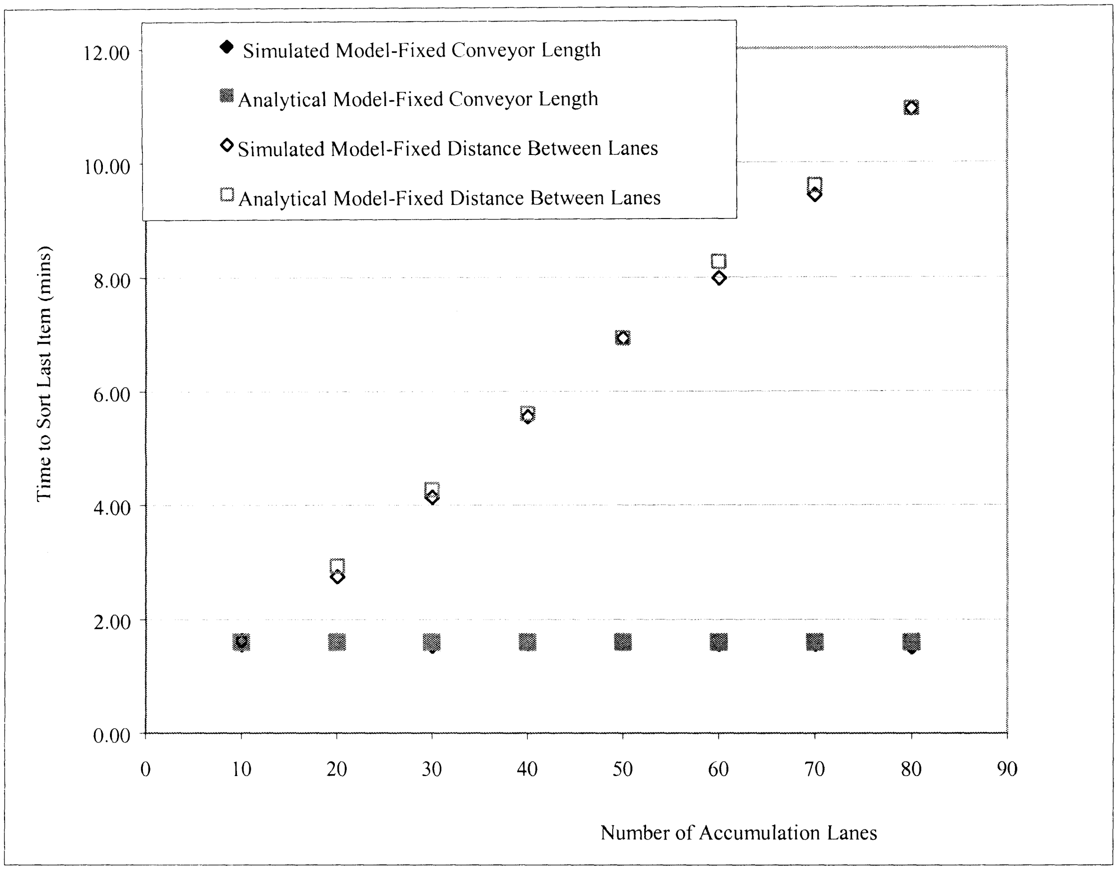

Figure 3-5: Comparison of Time Taken to Sort Last Item in a Wave

The sorting times shown in Figure 3-5 were obtained assuming a conveyor traveling at 75 ft/min (*U*), having a distance of 20 ft (*D*) from the drop-off point in the picking area to the first accumulation lane. Sorting times were determined considering a fixed conveyor length of 200 ft (D_l) and also when the distance between adjacent lanes was fixed at 20 ft (D_l =20 × number of lanes).

Based on these parameters the analytical model gives a sorting time of 1.6 min. when the conveyor length is fixed. As could be observed this result is comparable with that given by simulation for the same situation and the percentage mean square error between the two values, within the range evaluated, was 0.11%.

When a fixed distance between the lanes was assumed the time taken to sort the last item shows a linear increase and the analytical results are comparable with the simulated values. The percentage mean square error between the two values using this approach was 2.06%. Therefore it was concluded that the modified sorting time estimation model in equation (15) reasonably reflected the time taken to sort a wave of items.

3.3.3 Analysis of Packing Time Estimation Model

The following approach was used to evaluate the appropriateness of the packing time estimation model. A predetermined number of orders with a specific average number of items per order were generated randomly through simulation. Two approaches were used to assign these orders to packers.

In the intelligent assignment approach the orders were assigned such that the largest order goes to the first packer; the second largest order goes to the second packer and so on. The process was continued until each packer was assigned an order in this sequence. If all orders had not been assigned the cycle was repeated by reversing the sequence in each cycle until all orders were assigned such that all packers receive approximately the same number of items. Once the process was completed, the number of orders and the number of items assigned to each packer was determined. In the random assignment approach, orders were randomly assigned to packers by selecting a packer at random and assigning one order to each packer until all packers were assigned an order. If all orders were not assigned the cycle was repeated.

For example, consider 200 orders, with an average of 8 items per order to be assigned to 20 packers where the orders were generated randomly. The two assignment approaches described above were used to assign orders and items to packers. The average (from 5 trials) of the maximum number of items and the maximum number of orders assigned to a packer are shown in Table 3-2. For the given number of orders (m = 200) with average of 8 items per order (n = 8), the maximum number of items and orders assigned to a packer were also found using the simple model (equation (13)) and the probabilistic model (equation (14)).

Table 3-2: Results from Packing Time Estimation Models

	Intelligent Assignment	Simple Packing Model	Random Assignment	Probabilistic Packing Model
Number of items/packer	81	79.8	103.6	96.8
Number of orders/packer	10	10	10	10

The number of items found using intelligent assignment is comparable with the value obtained from the simple model. Though slightly different from each other, the maximum number of items found using the random approach compares well with that found using the probabilistic model. The difference in the two values is equal to approximately 6 items. If it takes 0.5 minutes to place an item in a box these extra items will take 3 minutes more to process. On the other hand, it will take 54.8 minutes to pack 96.8 items belonging to 10 orders as given by the probabilistic model. Thus a difference of 6 items will not have a significant impact on the packing time. The number of orders assigned to each packer was 10 with both assignment strategies and is equal to the value given by the analytical models.

Based on these findings it was concluded that the simple packing model might be more appropriate for use with a simple picking time model whereas the probabilistic packing time model may be more suitable for use when a similar approach is used to evaluate the picking process. The picking time estimation model used in this research is based on a probabilistic model. Therefore the same approach, the probabilistic packing model, was used for modeling the packing process.

3.4 Optimization Model

The minimum number of pickers and packers necessary to process the specified number of orders was determined by developing an optimization model. The model aims to minimize the total number of employees subject to certain conditions that define the operating schedules of the DC.

3.4.1 Development of the Model

The objective of the optimization model was to find the minimum number of pickers and packers required to process the orders. Where W_p denotes the number of pickers and W_{pa} denotes the number of packers, the objective function could be expressed as shown in equation (16):

$$\text{Objective function:} \quad Min\,(W_p + W_{pa}) \tag{16}$$

The DC considered to formulate this model was assumed to be working on 8-hour shifts in performing the picking and packing activities. The reason for considering an 8-hour shift was to take account of a normal 40-hour work week. The total orders to be processed in a day are divided into W waves. The time to pick one wave is denoted by T_p and T_{pa} denotes the time taken to pack a single wave. Thus, in order to complete picking and packing all the orders (divided into W equally-sized waves), the following constraints must be satisfied:

$$W\,T_p \leq 8 \text{ hours} \tag{17}$$

$$W\,T_{pa} \leq 8 \text{ hours} \tag{18}$$

In addition to other operating parameters T_p and T_{pa} in the above equations depend on W_p and W_{pa}, respectively. In order to ensure non-negative values and integers for W_p and W_{pa} following constraints are also necessary.

$$W_p \geq 1 \,\&\, \text{int} \tag{19}$$

$$W_{pa} \geq 1 \,\&\, \text{int} \tag{20}$$

Order processing has to be performed sequentially. First, by picking the items that belong to one wave from the storage aisles, and sorting them into individual orders and then packing them into cartons or boxes. Packing cannot begin until picking and sorting have been completed. Therefore if the packers start the work shift at the same time as pickers, there will be some idle time before they could begin work. The idle time depends on the time taken to pick and sort the first wave of orders, which in turn would vary based on the configuration of the orders that have to be processed on the given day.

Having to pay packers for non-productive time can be avoided by staggering the start of the packing operation. However, the size of the delay may vary depending on factors such as the number of orders to be processed, and items per order. But, a work schedule where packers have to begin work at different times everyday is not easy to implement under practical circumstances. Therefore to avoid paying packers for non-productive time, the packing shift was assumed to commence one hour after the picking operation has begun. This is a variable condition and the time lag after which the packing shift will begin can be changed as deemed necessary. Hence in this study the total time available for all the three activities, given that picking and packing activities are carried out on 8-hour shifts and that packing begins one hour after picking has begun, will be 9 hours. The arrangement of the waves in the three processes studied is shown graphically in Figure 3-6. It is assumed that the orders are processed in 5 waves (merely for representation).

According to Figure 3-6, $T_p + T_s$ is spent to pick and sort the first wave and T_{pa} is the time spent to pack the last wave. In between these two tasks picking, sorting, and packing have to be completed for the remaining W-1 waves. The time taken for this will depend on which of the processes takes longer to complete. Therefore the total processing time can be expressed, in terms of the times involved and the total number of waves to be processed on a given day, as shown in equation (21).

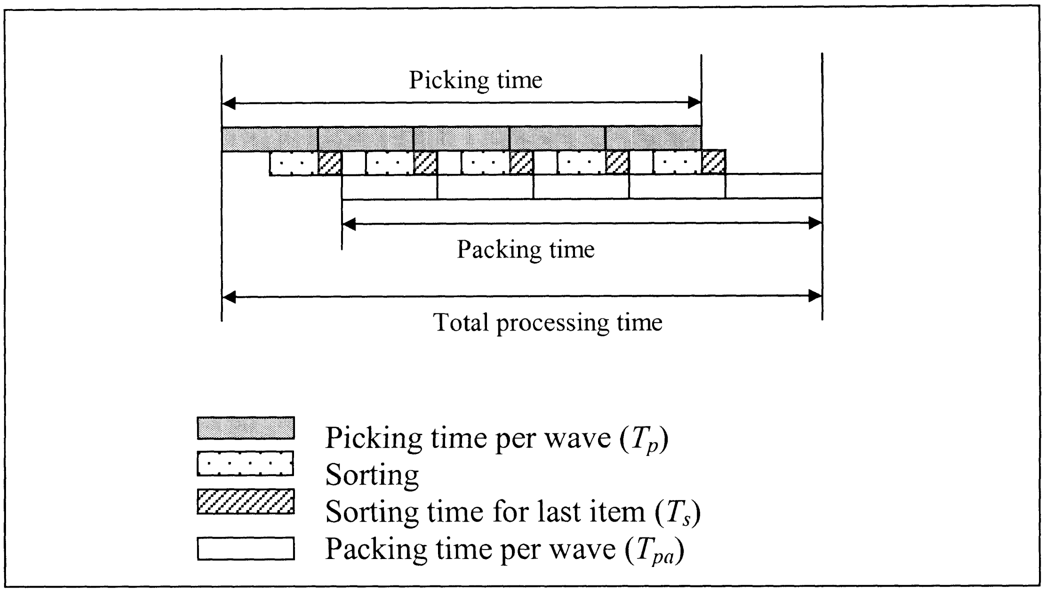

Figure 3-6: Arrangement of Picking, Sorting and Packing Processes

$$T_{Total} = [(T_p + T_s) + T_{pa} + (W\text{-}1)\ \{\max\ (T_p + T_s,\ T_{pa})\}] \leq 9 \text{ hours} \tag{21}$$

3.4.2 Testing the Optimization Model

The optimization representation formulated to find the minimum number of pickers and packers was then modeled in Microsoft Excel and the different operating parameters were set at the values specified in Table 3-3.

The model was run to find the optimum number of workers and the wave configuration with 3, 6, and 12 average items per order. At each level the Excel Solver was used to determine the minimum number of workers (pickers and packers) and the number of waves that should be formed to achieve this minimum worker level by varying the numbers of orders, from 1000 to 40,000 in increments of 2000 orders.

Table 3-3: Systems Parameters used to Evaluate Time Estimation Models

Picking operation		Sorting Operation		Packing Operation	
Parameter	**Value**	**Parameter**	**Value**	**Parameter**	**Value**
a	100	D	20 ft	T_C	0.25 min
L	150 ft	l	100	T_B	1.00 min
V	60 ft/min	D_l	150 ft		
T_e	0.5 min	U	80 ft/min		
T_r	0.2 min				
T_l	0.2 min				
T_u	1.0 min				
K	50				
Z	1.96				

Results generated by the Excel Solver are exact only in situations when one uses scenarios that could be formulated as linear programming/integer programming models. When the constraints are non-linear, the results generated cannot be taken as conclusive because the Excel Solver does not have the capability to handle non-linear constraints. The model used to find the optimal number of pickers and packers uses the time estimation models that were developed in section 3.2 and some of the constraints used are non-linear. Therefore, it is likely that the solutions provided by the Solver are not the optimal but sub optimal.

In order to overcome the limitations of solving a non-linear model using the Microsoft Excel Solver, more versatile optimization software was used to formulate the models. LINGO, a comprehensive tool designed to model and solve linear, nonlinear and integer optimization models [26] was used to formulate and solve the optimization model. This model used the same parameters as were used for the Excel model. The LINGO model was then executed to find the optimal number of workers and the wave configuration with 3, 6 & 12 average items per order by varying order size from 1,000 to 40,000.

3.5 Regression Model

One objective of this research is to formulate a model that could be used to estimate the optimal number of pickers and packers required to process a given number of orders during a working day. Hitherto in this research, time estimation models have been developed and tested for the different activities. An optimization model was formulated to depict the operations of interest in the DC subject to the constraints established. In order to be able to apply the results of the optimization model to formulate a human resources management strategy for a DC that can be implemented, it is necessary to devise a tool that could be used easily.

A regression model allows one to express the relationship between two or more variables algebraically and indicates the nature of the relationship between the variables. In particular, it indicates the extent to which a variable's value could be predicted by knowing other values. Thus, in this research a regression model was developed to predict the number of pickers, number of packers and the number of waves necessary as a function of number of orders and the total number of items when the operating parameters of the DC are specified.

The data gathered by using the optimization model were used to generate a regression model to predict the variables of interest. Three separate regression models were formulated to determine the number of pickers, number of packers and the number of waves. The number of orders and the total number of items were used as independent variables in formulating the regression models to estimate the number of pickers, number of packers and the number of waves.

The regression model was then used to predict the number of pickers, number of packers and the corresponding number of waves when there are an average of 9 items per order. Data were collected by varying the number of orders from 1000 to 40,000 in increments of 1,000 orders up to 4,000 and in increments of 2,000 orders thereafter. These values were then compared against the optimal number of pickers, packers and waves generated by the optimization model under the same operating parameters to test the effectiveness of the regression model.

4 RESULTS

The findings of the experimentation carried out with different optimization models to determine the optimal staffing requirements of a DC are presented in this section. The regression model developed based on the results of the above models too is presented.

4.1 Optimization Models using Microsoft Excel and LINGO

The same model was formulated using Microsoft Excel and LINGO to determine the optimal number of pickers and packers and the number of waves to be processed for different numbers of orders per day. The number of orders was varied from 1,000 to 40,000 in steps of 2,000 for an average 3, 6 and 12 items per order.

A comparison of the optimal results obtained using the two models with 12 items per order are shown in Figure 4-1. The LINGO model used is shown in Appendix III. The numerical results obtained are shown in Appendix IV.

As can be observed from the Figure 4-1, though Excel results are comparable to those found using LINGO, the latter consistently generates better or as good solutions. The total number of workers employed to process a particular number of orders is less according to LINGO. From the perspective of cost management in a DC, lower staffing levels are more favorable.

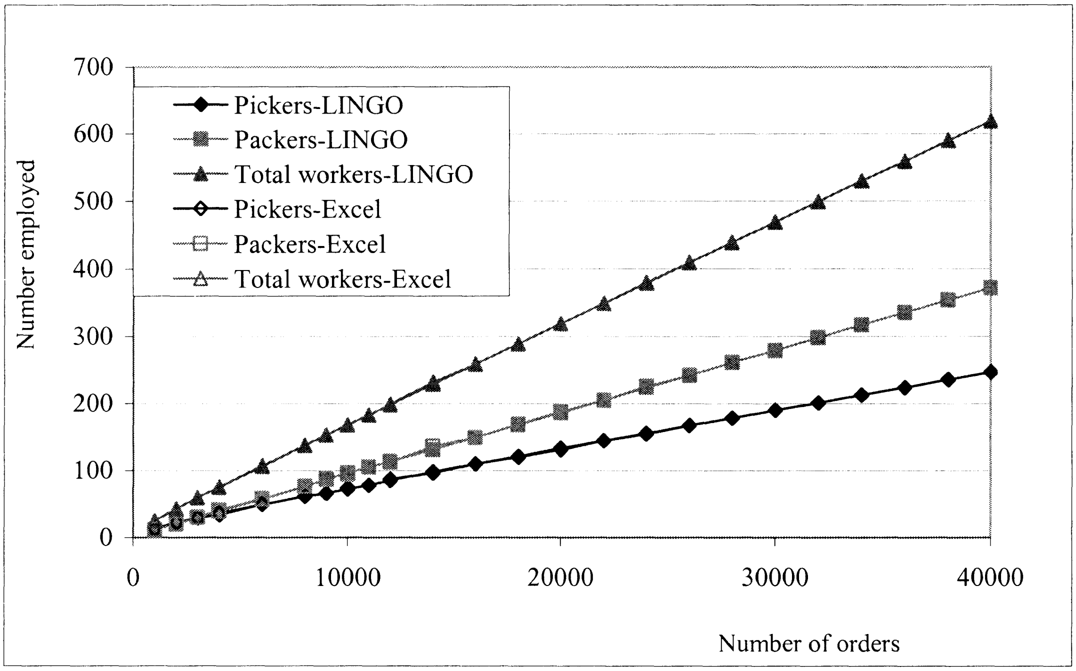

Figure 4-1: Comparison of LINGO and Excel Solver Results with 12 Items per Order

Therefore, since LINGO is better equipped to handle non-linear and integer constraints and has proven to generate optimal solutions more frequently, it was used for all subsequent analyses.

4.2 Workers Employed Based on Number of Orders and Number of Items

The number of pickers and the number of packers as a function of number of orders and number of items processed are shown below in Figure 4-2 and Figure 4-3, respectively. The figures show the combined results from three different experiments with average 3, 6, and 12 items per order.

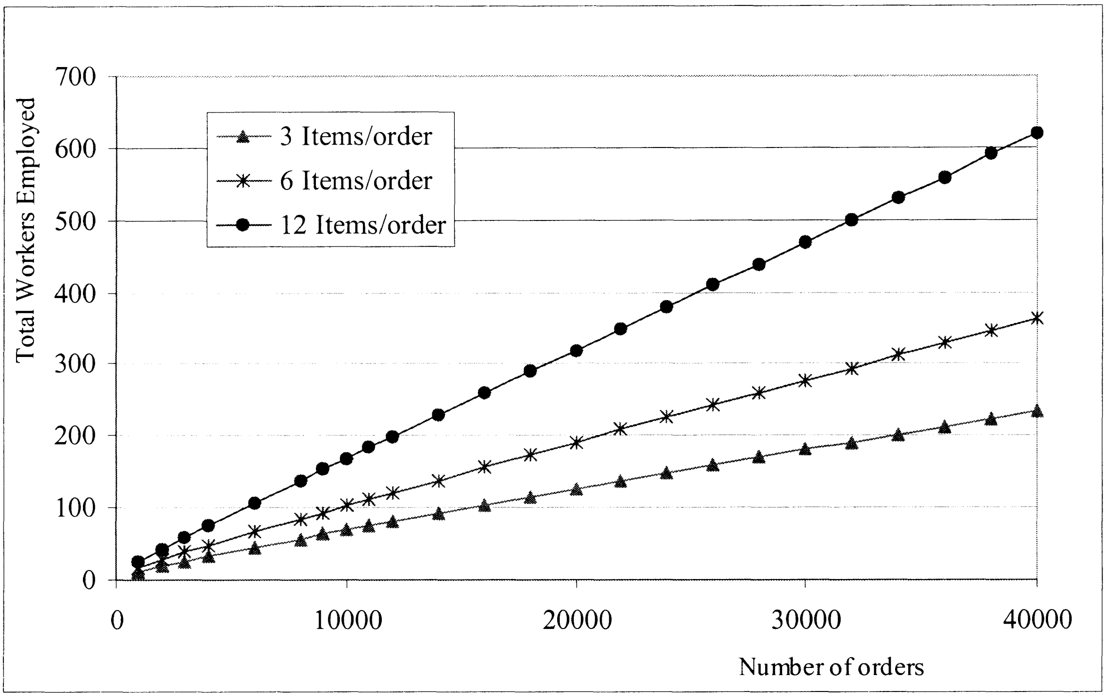

Figure 4-2: Total Employed as a Function of Number of Orders per Day

As can be observed from Figure 4-2, the total number of workers employed varies linearly with the number of orders per day. However, the size of workforce does not double when the number of orders per day is doubled. This may be due to savings in time as pickers will be able to retrieve more items without a significant increase in the number of stops or traveling required.

The absence of a cart capacity constraint in picking too may have contributed to the linearity of results over the range of orders studied. If a cart capacity constraint is introduced it may have an impact on the amount of traveling that needs to be done by pickers and thus the picking time. This is because the amount of traveling that needs to be done will depend on how many times the picker has to unload, unless the amount of work assigned to a picker is limited to the cart capacity defined.

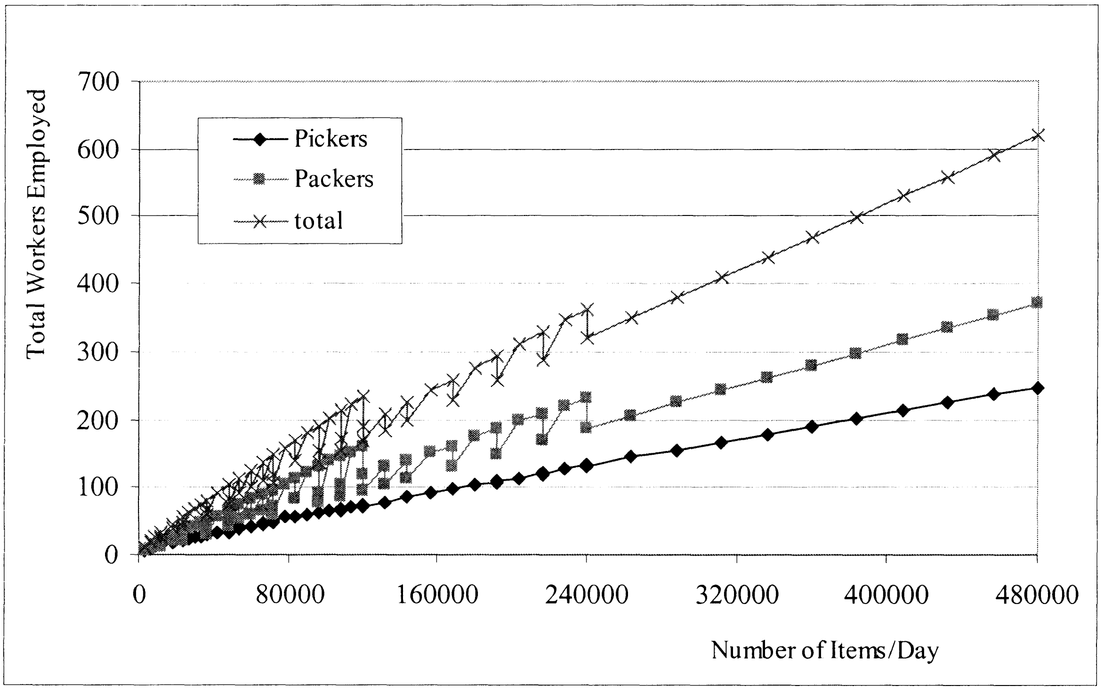

Figure 4-3: Number of Pickers, Packers and Total Employed as a Function of Total Number of Items per Day

According to Figure 4-3, the number of pickers employed is directly related to the total number of items to be processed and does not depend on the number of items per order. This is because orders are not separated out in the picking area and therefore the time taken to pick a set of orders depends only on the total number of items in the orders. However, the time taken to pack and seal the cartons depends on the number of orders per day and number of items and this explains the variation of the number of packers required between neighboring numbers of items in Figure 4-3, up to 240,000 items; after this point, the results correspond to only 12 items per order. Hence the number of packers shows a linear variation with the number of items because the number of items per order is constant above this limit.

Further analysis was conducted to determine the variation of the number of items to be picked by each picker and the number of items to be packed by each packer per wave when the number of orders per day increases from 1,000 to 40,000. The results are shown in Figure 4-4 and Figure 4-5.

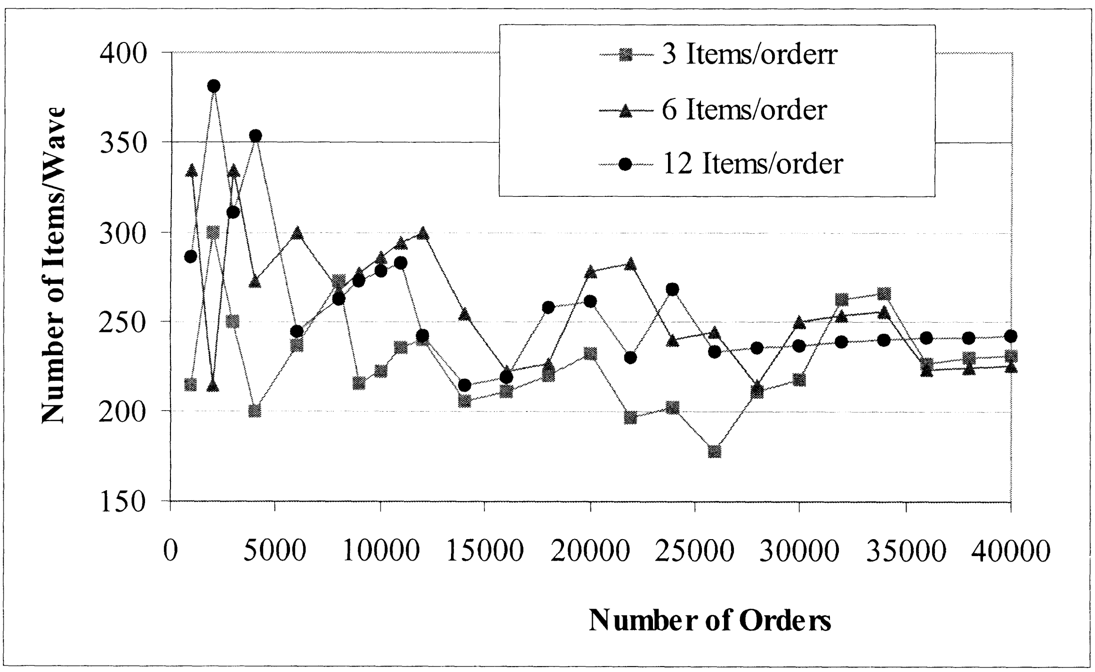

Figure 4-4: Variation of Number of Items per Wave per Picker

According to the figures, when the number of orders to be processed is low (approximately $\leq$ 6,000) the number of items per picker and the number of items per packer are comparatively higher and are fluctuating.

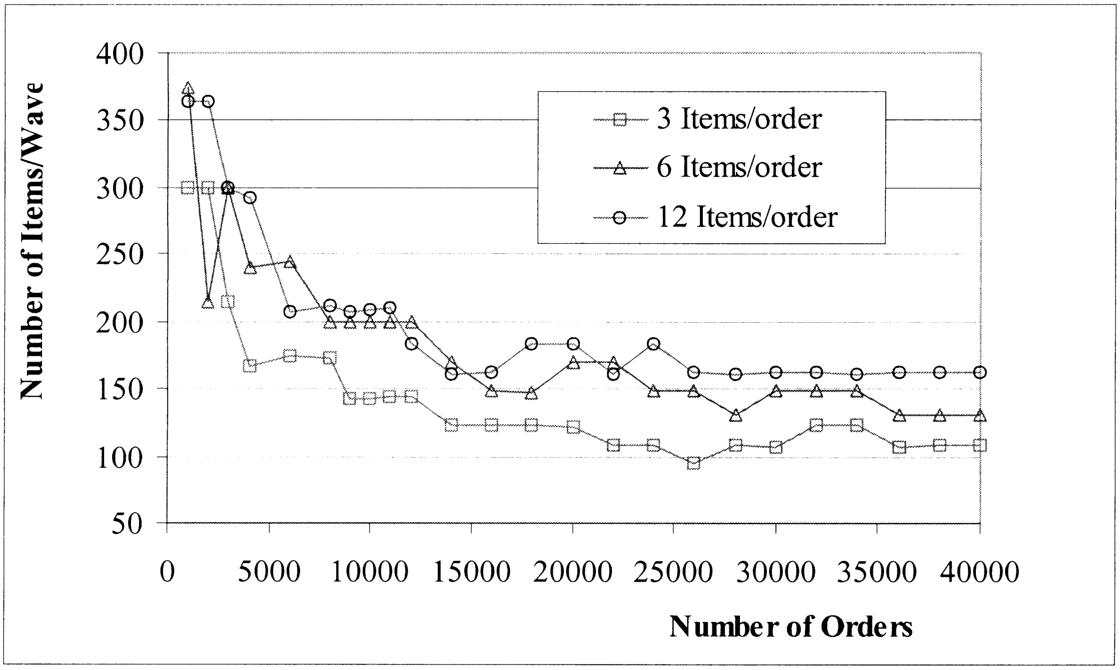

Figure 4-5: Variation of Number of Items per Wave per Packer

However, as the number of orders to be processed is increased, the number of items per worker decreases and remains almost constant after a certain limit. Thus considering the range of orders above 6,000, the number of items per picker per wave varies in the range 200 – 300 and the number of items per packer per wave varies between 100 and 200. Therefore, it can be concluded that the level or work per employee is not varying significantly in the range 6,000 to 40,000 orders per day. Hence the time estimation models accurately reflect the time involved for the different activities in this range because the work can be performed in the same way throughout this range.

One of the main causes for the high and fluctuating workload with lower number of orders per day can be attributed to the wave configuration. The results from the optimization model indicate that the number of waves is especially low when the number of orders to be processed is low (Appendix IV). This gives rise to a larger workload per worker per wave. This may require additional travel than what was accounted for in the picking time estimation model for pickers and more time to sort and pack items for packers. Having two additional constraints in the optimization model can help control the workload per worker in this range. For example, if the number of items per picker per wave is to be limited to 250 (this could be considered as the cart capacity) and the number of items per packer per wave is to be limited to 150 (lane capacity), the following constraints can be included in the optimization model.

$$\frac{n \cdot m}{W \cdot W_P} \leq 250 \tag{22}$$

$$\frac{n \cdot m}{W \cdot W_{Pa}} \leq 150 \tag{23}$$

In order to determine how the workload per worker changes with these two additional constraints the optimization model was executed with 2,000 orders of average 3 items each. The findings are shown in Table 4-1.

Table 4-1: Number of Items/Wave for Pickers and Packers with 2,000 orders of average 3 Items

Optimization Model	W_p	W_{pa}	W	Total Workers	Items per wave	
					Pickers	Packers
No additional constraints	10	10	2	20	300	300
With additional constraints	12	10	4	22	125	150

When the additional constraints are included, the optimal number of workers required to complete the same number of orders increases slightly. However, the workload per worker decreases significantly since the number of waves is doubled. This approach can be used in the optimization model to control the workload assigned to each worker and also to take account of cart capacity during picking.

4.3 Probabilistic vs. Simple Optimization Model in LINGO

The time taken to process a set of orders is a function of the time taken for the three primary operations considered in this study: picking, sorting, and packing. Two variations of the optimization model were formulated by adopting different approaches to estimate the time involved.

In practice the orders received by a DC could have varying number of items in each order. For processing them within the DC, the orders are commonly distributed among the pickers based on other criteria such as limiting the maximum number of aisles assigned to a picker or limiting the maximum number of pickers assigned to a particular aisle. Such methods often lead to unbalanced loads among different workers. Thus the probabilistic model best models the routine adopted in practice. Items are not evenly distributed among the workers, so the picking time and packing time models use a binomial distribution to estimate the expected number of different items that must be retrieved by each picker and packed by each packer. Hence different workers will have a different number of items to pick or pack and to estimate times requires identifying who has the most work.

For the simple model, it is assumed that the orders to be processed on a particular day and items corresponding to those orders are evenly distributed among the workers. This is how a manager may make scheduling decisions typically because it can be calculated quickly. This model formulated in LINGO is shown in Appendix V.

The simple and probabilistic models were solved using LINGO to determine the optimal staffing levels with average 3, 6, and 12 items per order and varying the number of orders per day from 1,000 to 40,000 in steps of 2,000. The variation of the total number of workers employed as a function of number of orders is shown in Figure 4-6. The complete results are given in Appendix VI.

As can be observed from Figure 4-6, the optimal results generated by the simple model are consistently lower that those generated by the probabilistic model.

Effective labor management in a DC calls for a schedule that requires the minimum number of employees to process a given number of orders during a working day. The results shown indicate that a simple distribution of items to workers, where each picker or packer gets the same number of items, requires a smaller work force.

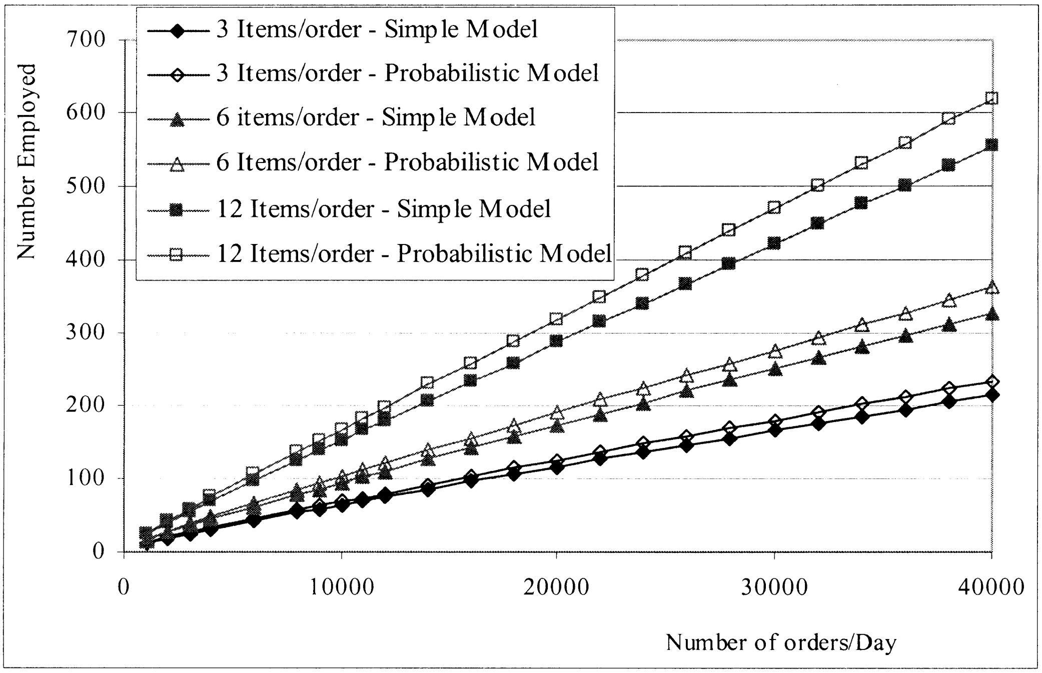

Figure 4-6: Comparison of Results for the Simple and Probabilistic Models using LINGO

The simple model requires fewer workers, but assumes a level of work that is lower than actually occurs. Therefore if a manager used the simple assumption he/she would not be able to finish processing the orders in time. The average percentage difference in the total number of workers employed between the two models is shown in Table 4-2.

Table 4-2: Average Percentage Difference in Total Number of Workers Employed by the Two Models

Number of Items per Order	Average Percent Difference
3	8.54
6	15.83
12	29.75

The average additional time necessary if the optimal results obtained from the simple model were used in the probabilistic model was approximately 44 minutes. This is 8.09% higher than the total time available from the start of the picking operation to completion of packing (9 hours). Thus because the simple model assumes a level of work that is lower than that actually occurs, the time constraints will not be satisfied if these number of pickers and packers are used.

Though the simple model results in lower labor requirements for the picking and packing operations in a DC, the probabilistic model is more representative of how the workload is distributed. For this reason the probabilistic model was chosen to analyze the optimization model further and formulate an approach to determine optimal staffing requirements given the operating parameters for a DC.

4.4 Wave Formation in the Probabilistic Optimization Model

The variation of number of waves required to process all the orders during a particular workday as a function of number of orders and number of items to be processed is shown below in Figure 4-7 and Figure 4-8, respectively. The figures show the combined results from three different experiments with average 3, 6, and 12 items per order. The logarithmic regression models for the number of waves as a function of number of orders per day and the total number of items per day are shown in equations (17) and (18) respectively.

$$W = 1.5852 \text{ Ln } (m) - 9.024 \tag{17}$$

$$W = 1.3967 \text{ Ln } (m) - 9.7383 \quad (18)$$

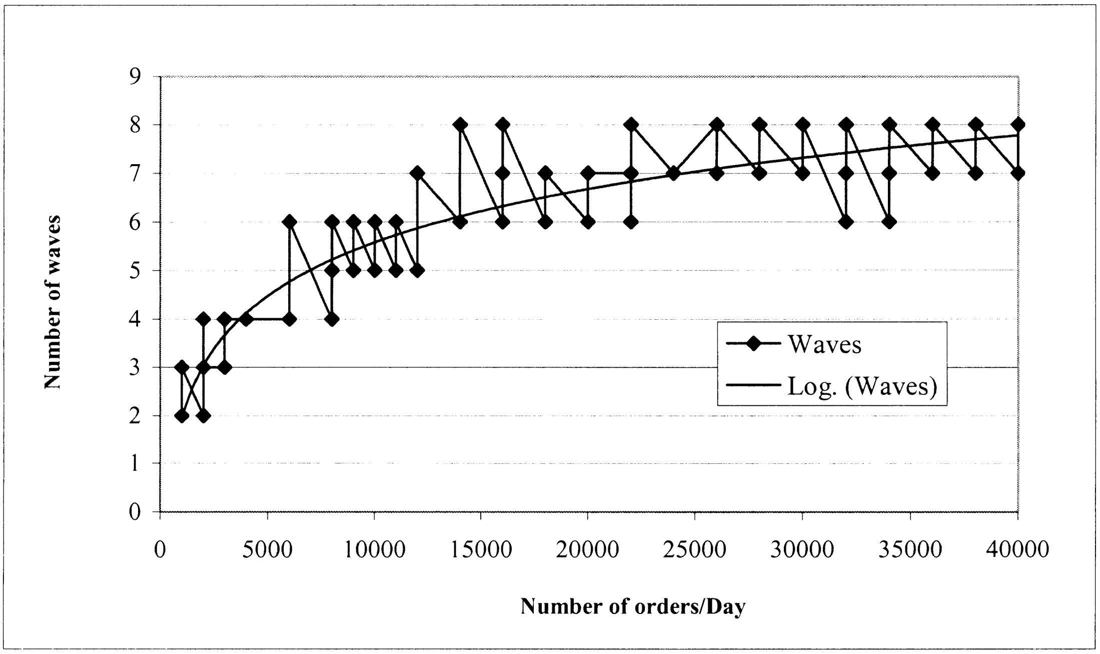

Figure 4-7: Number of Waves as a Function of Number of Orders/Day

As can be observed from the figures, the number of waves required to process the orders increases rapidly at first. However, as more and more orders are processed, the number of additional waves required increases at a slower rate. With the parameters that were used for the experimentation, the maximum number of waves necessary to process all the items levels off to 8. This means that any wave will not take longer than an hour to either pick and sort or pack.

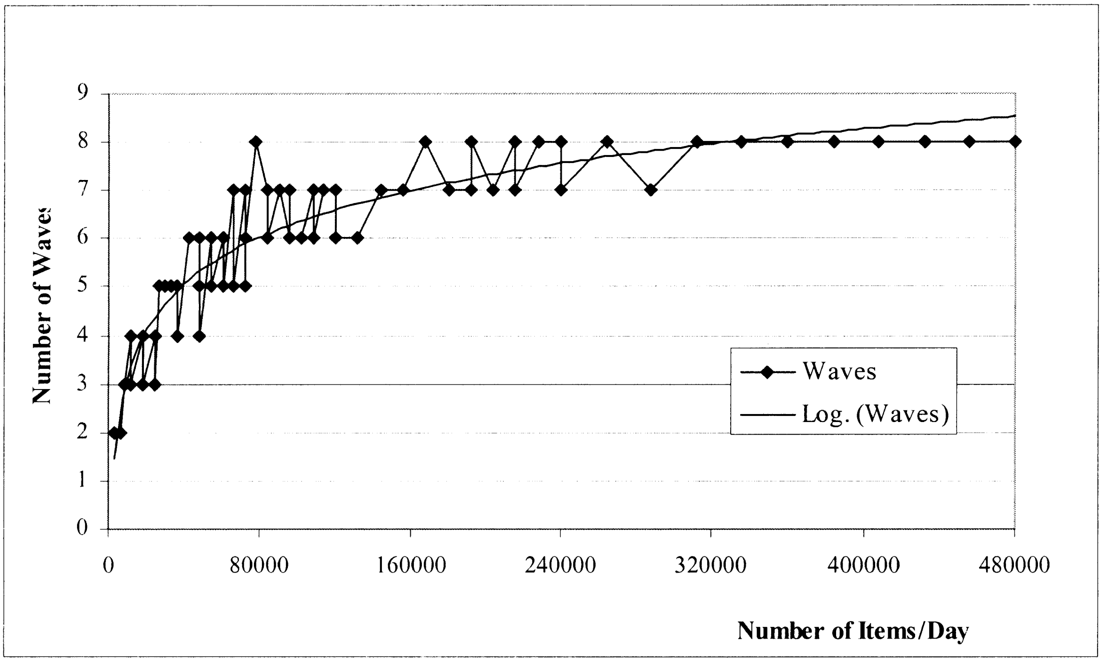

Figure 4-8: Number of Waves as a Function of Number of Items/Day

The upper bound of only 8 waves to process all orders (based on the values used for the variables in the model) means that each wave takes not more than an hour to process. In order to determine the reason for the upper limit of 8 waves and to determine the relationship to the time lag, if any, further experimentation was conducted.

This maximum time per wave is equal to the time lag defined between the picking and sorting operations and the packing operation during the formulation of the model. The numbers of waves required to process up to 500,000 orders with 6 items per order was found using the optimization model. Such a large number of orders per day was considered merely to appreciate how the staffing levels change with the time lag. The experimentation was repeated by setting the time lag between the operations to 2 hours, 1 hour, 45 minutes, and 30 minutes. The variation of the number of waves is shown in Figure 4-9.

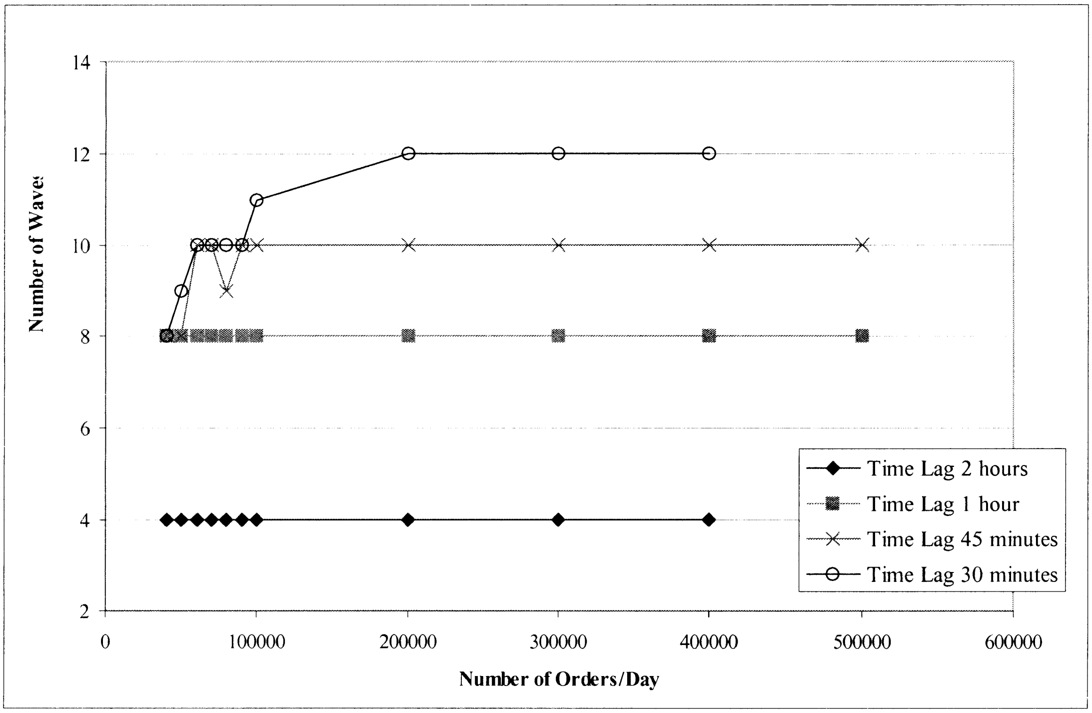

Figure 4-9: Variation of Number of Waves for Different Time Lags

When the time lag between the picking and packing shifts is 1 hour (so total time available is 9 hours) the number of waves required to process all the orders remains at 8 even when 500,000 orders per day are considered. This means that the time per wave is equal to the time lag between work shifts. (8 waves = 8-hour shift/ 1-hour time lag).

Based on this approach, when the time lag is 45 minutes, the maximum number of waves should be 10 (10.67 waves = 8-hour shift/45 minute time lag). As could be observed from Figure 4-9, the maximum number of waves required with a 45-minute time lag is equal to

10. Similarly if the time lag is 30 minutes, the maximum number of waves required should not exceed 16 (16 waves = 8-hour shift/ 30 minute time lag). Within the range of orders per day considered in the figure above, the maximum number of waves reaches a maximum of 12. However, following the pattern observed with a 1-hour and 45-minute time lag, this number should level off to 16 waves if a large enough number of orders were considered. LINGO was not able to find the optimal solution when the number of orders was set to higher than 500,000 so it was not possible to verify this postulation. When the time lag was increased to 2 hours, all items were processed using 4 waves as expected (8 hours/2hours). Here again it was not possible to determine the result when 500,000 orders were considered.

These observations indicate that the maximum number of waves a DC should use to process all orders depends on the time lag between the end of the picking and sorting operations and start of the packing operation for a particular wave. On the other hand, one of the variables that influences that total number of workers required for all the operations is the number of waves to be formed. Therefore the time lag chosen will have an impact on the number of workers employed.

In order to ascertain how the time lag between the operations and the resulting maximum number of waves influences the total number of workers required, further experimentation was conducted using LINGO. The total number of workers required to process 40,000 to 500,000 orders of 6 items each (in steps of 10,000 from 40,000 to

100,000 and in steps of 100,000 thereafter) were found for different amounts of time lag. The results obtained are shown in Figure 4-10 below.

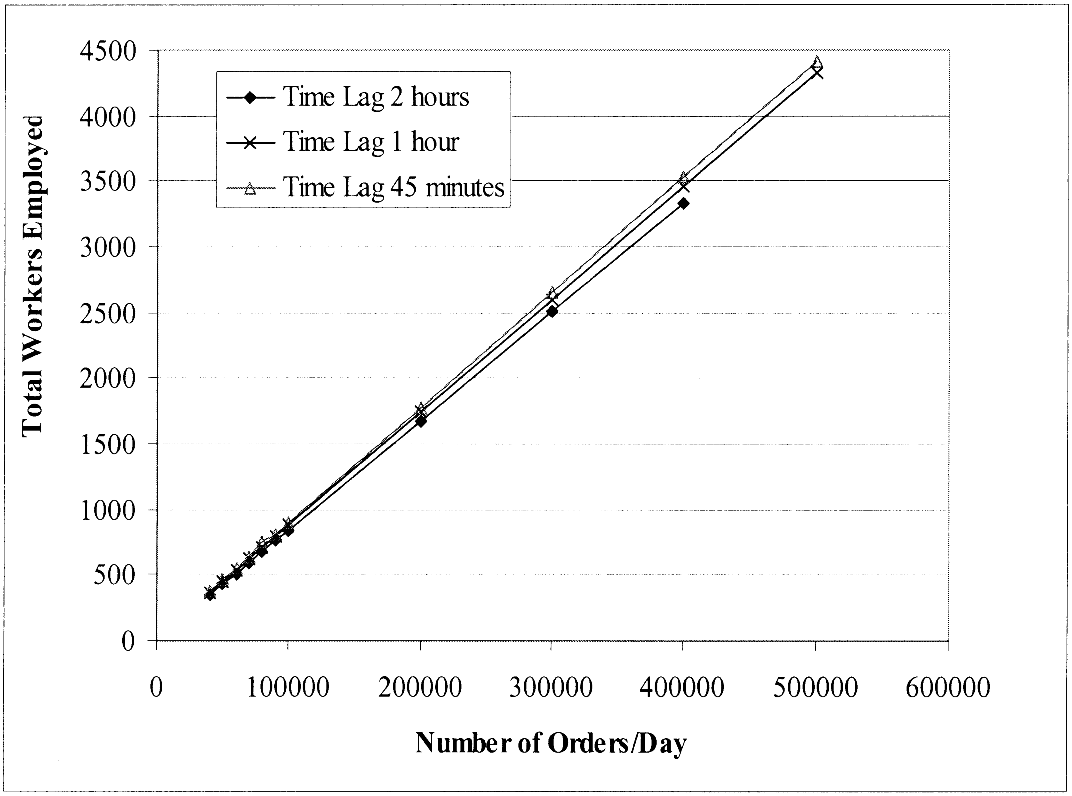

Figure 4-10: Total Number of Workers Required with Different Time Lags

A time lag of two hours between the operations requires the smallest workforce to process all the orders, as indicated by the figures above. With a 2-hour time lag, all orders were processed in 4 waves, irrespective of the total number of orders per day considered. This is because a picker can retrieve more items in a wave when the number of waves is less. For example, with an 8-hour time lag, there will be one wave to pick and pickers will have to make only one trip through the particular zone.

In order to further understand how the time lag affects the number of waves and the number of workers, the optimization model was run with different time lags to process 8,000 orders with average 6 items per order. The time lags tested were 0.5 hours, 0.75 hours, and 1 hour and thereafter increased in steps of 1 hour up to 8 hours. The results are shown in Figure 4-11. The details are given in Table 4-3.

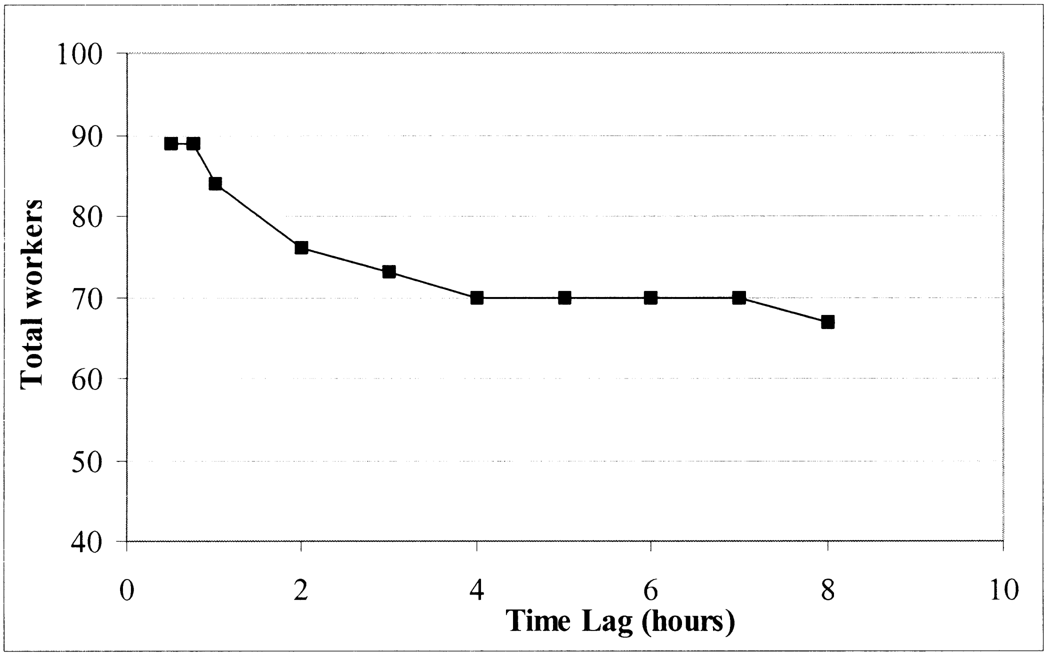

Figure 4-11: Total Workers to Process 8,000 Orders of Average 6 Items/Order with Different Time Lags

The results indicate that even lower staffing levels can be achieved by changing the time lag between the picking and sorting operations and the packing operation. However, according to the results in Table 4-3, the number of items per worker increases considerably when the time lag is increased and the number of waves to be processed decrease.

Table 4-3: Results of Processing 8,000 Orders with Average 6 Items/Order

Time Lag (hrs)	Pickers	Packers	Waves	Total Workers	Items per Picker	Items per Packer
0.5	38	51	5	89	253	189
0.75	37	52	5	89	260	185
1	36	48	5	84	267	200
2	31	45	4	76	388	267
3	28	45	3	73	572	356
4	26	44	2	70	924	546
5	26	44	2	70	924	546
6	26	44	2	70	924	546
7	26	44	2	70	924	546
8	23	44	1	67	2087	1091

Thus, reaching lower staffing levels and maintaining a practical workload among packers cannot be achieved at the same time. Therefore a trade-off has to be made at some point as to the extent to which employee workload can be increased while trying to lower staffing levels. As the time lag between the operations is increased, the number of orders per wave increases due to the decrease in the number of waves. Thus it also important to consider whether the different functions of the DC have the capacity to handle waves consisting of a very large number of orders.

These findings can have significant implications on managerial decision making with respect to DC operations. According to these findings the time lag defined during the design of the optimization model has a direct impact on the maximum number of waves that will be processed. This means that the time lag between the picking and sorting operations and the packing operation can be used to maneuver the number of waves and thereby achieve the minimum number of workers required. However, it is also important to complete processing all orders in time for delivery. Therefore using a time lag of, for example, 8 hours may not be practical though it will result in lower labor requirements.

4.5 Regression Model based on LINGO Results

The optimal staffing levels established using the probabilistic optimization model by means of LINGO were used to formulate regression models for the number of pickers, the number of packers, and the number of waves required to process a given number of orders consisting of different items. The complete models formulated using Minitab is shown in Appendix VII and the formulas are given in equations 19, 20, and 21.

$$W_p = 10.7 + 1.95\text{x}10^{-4}\, m + 4.82\text{x}10^{-4}\, nm \quad (19)$$

$$W_{pa} = 2.5 + 2.16\text{x}10^{-3}\, m + 5.9\text{x}10^{-4}\, nm \quad (20)$$

$$W = 1.5852\ \text{Ln}\,(m) - 9.024 \quad (21)$$

The regression models were then used to estimate the number of pickers and number of packers required and the number of waves that should be used to process 1,000 to 40,000 orders, in steps of 2,000 with 9 items per order. A comparison of the results obtained from the regression model and those found using LINGO, for the same number of orders and items per order are shown in Figure 4-12 and Figure 4-13. Comprehensive results obtained using LINGO and the regression models are shown in Appendix VIII.

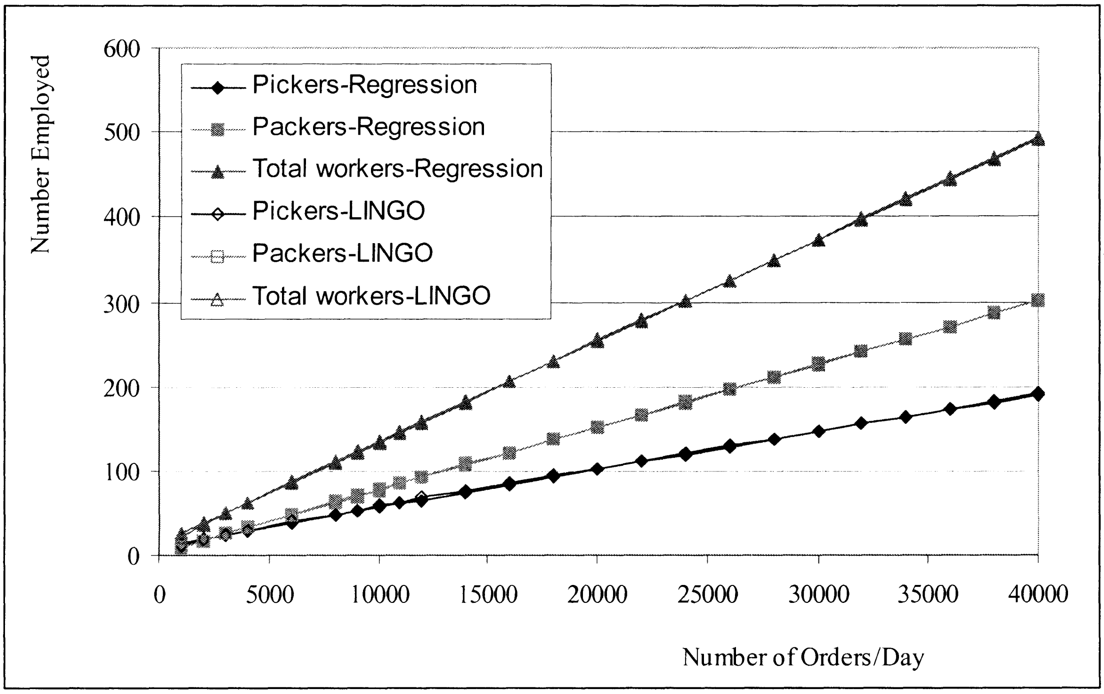

Figure 4-12: Number of Pickers, Packers, and Total Workers from Regression and LINGO for Average 9 items per order

As can be observed, the regression model overestimates the number of workers required when the number orders to be processed are lower. When 1000 orders are considered, the regression model overestimates the number of pickers by 27%, the number of packers by 11% and the total workers by 43%. However, as the number of orders increase, the results generated by regression are comparable to those found using LINGO. Thus when there are 28,000 to 32,000 orders, regression results are comparable to LINGO values. Overall the regression model generated solutions that are very much comparable to the optimal solutions found using the LINGO model.

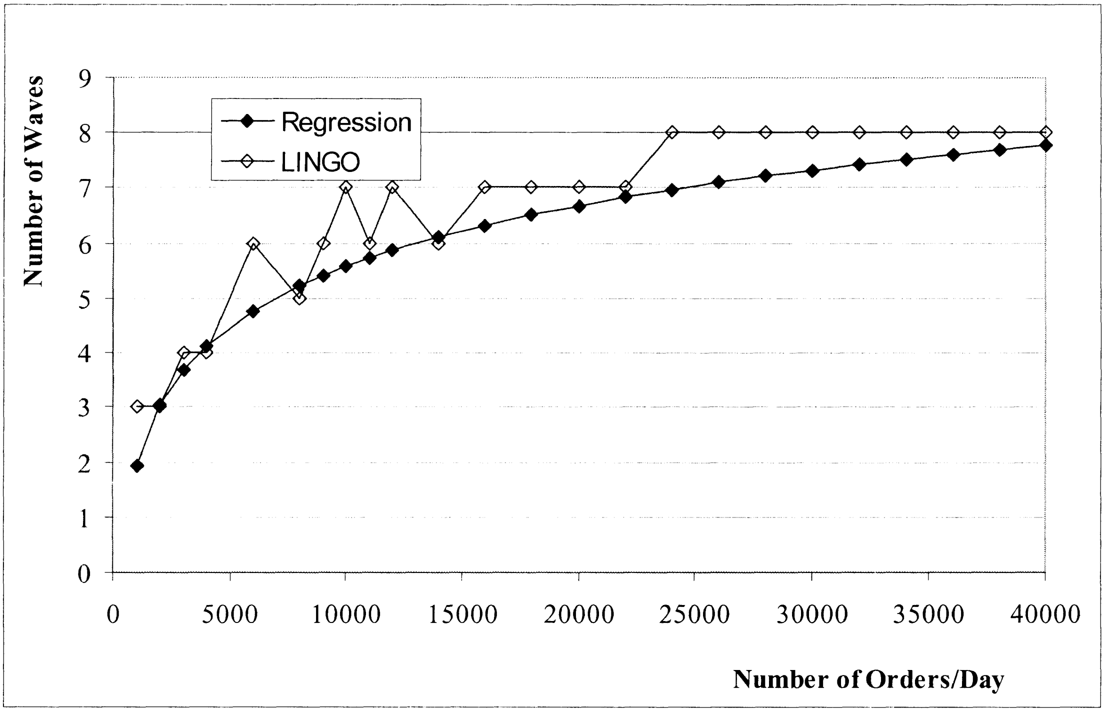

Figure 4-13: Number of Waves Required from LINGO and Regression for Average 9 Items/Order

The regression model developed for the number of waves to be processed is logarithmic and increases with the number of orders. Within the range of the number of orders per day considered in the LINGO model, the number of waves levels off to 8 as the number of orders increased. This is represented quite accurately by the results from the regression model in the range of orders considered. With 40,000 orders, the number of waves required according to the logarithmic regression model is 8.

Further study was carried out by introducing the regression results to the model formulated in Microsoft Excel. The purpose of this was to ascertain if the given number of pickers and packers could process all the orders with the specified number of waves subject to the time constraints established. The time taken to pick and pack all the orders and, the total time taken for the process with the number of pickers and packers found from regression is shown in Figure 4-14.

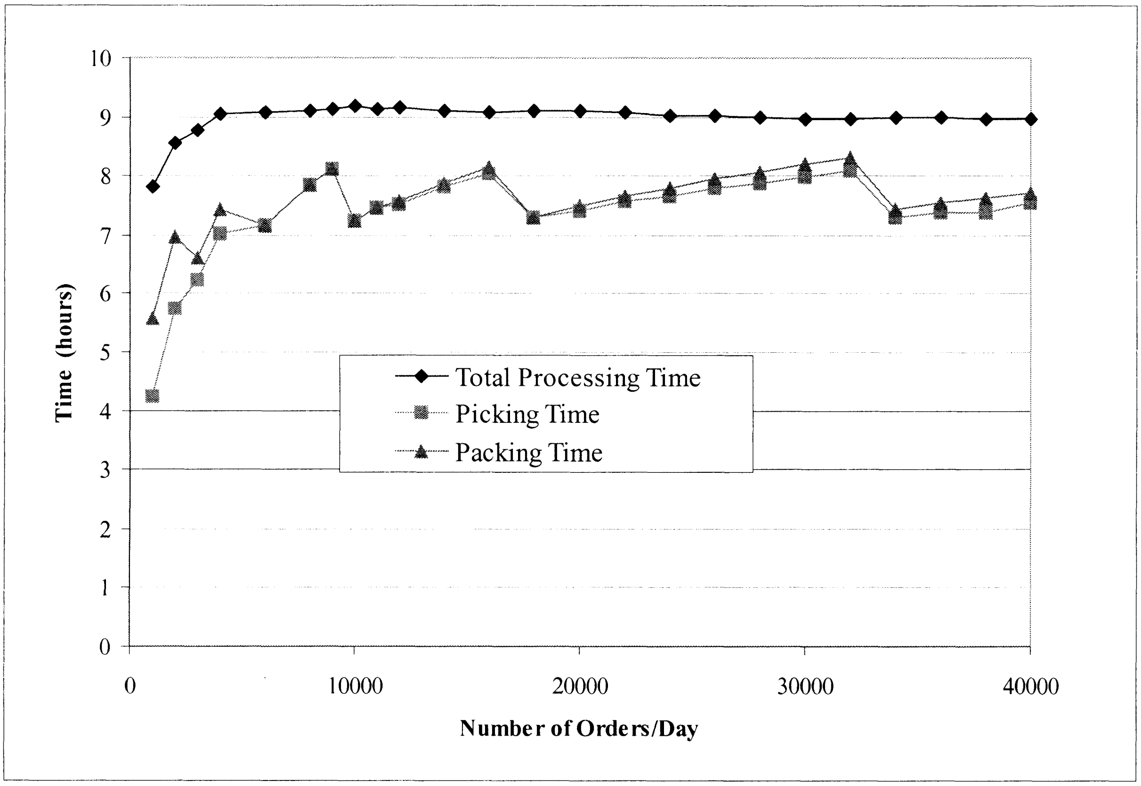

Figure 4-14: Picking Time, Packing Time and Total Time to Process Orders

For the models tested in this research the total time available to process all orders was 9 hours. Thus arrangements that are capable of processing the orders within this time are satisfactory. According to the processing times found from the Excel model, the regression results were found to satisfy the constraints in most instances while they violated them in a few instances. However, the maximum overshoot in the total time taken to process all the orders was less than 11 minutes.

5 CONCLUSIONS

This research was aimed at developing a method to determine the optimal staffing requirements for a DC with manual picking and packing operations for a given set of strategies.

5.1 Summary of Results

Initially time estimation models were formulated to determine the time to pick, sort and pack a given number of orders. Experimentation was conducted to test these models and verify their accuracy. The picking and packing operations were assumed to be operated in 8-hour shifts and all orders due on a particular workday had to be processed during this time. In addition it was assumed that the packing of a wave of orders does not begin until picking and sorting of it were completed. In order to avoid idle time by pickers in waiting for the items, the packing operation was staggered. Therefore all the operations had to be completed during the total time available. And for the purpose of this research the total time was taken as 9 hours. An optimization model was then formulated to find the optimal number of pickers and packers and the number of waves required subject to the constraints established. The model was designed using LINGO and experimentation was conducted to evaluate the model.

5.1.1 Optimization Model

Two alternative forms of the optimization model were tested with LINGO. In the first model, the probabilistic approach, it was assumed that the number of retrievals by each picker and the number of items packed by each packer follow a binomial distribution. For the second model, the simple approach, it was assumed that all pickers have an equal number of retrievals and that all packers have an equal number of items to pack. Optimal results found using LINGO from the two models revealed that the optimal staffing levels found by the simple approach were lower than those from the probabilistic approach. Considering all the different situations experimented, the average percentage error in the total number of workers required with the simple model is 8.09%.

However, in assigning items to pickers and packers in a DC during the normal course of operations, the number of retrievals for various pickers and the number of items packed by different packers are not equal. The probabilistic model reflects this situation better than the simple approach. Therefore though the former resulted in lower optimal staffing requirements the probabilistic approach was used for all the subsequent analyses. Using the simple model will delay processing the orders.

The probabilistic optimization model was also formulated and tested using the Solver tool in Microsoft Excel. These results were comparable to those found using LINGO. However, Excel has limited capabilities in dealing with non-linear optimization models. Since the optimization model used in this research contained non-linear constraints, the results obtained using LINGO were used to formulate the next stage of the methodology that could be used to find optimal staffing levels.

5.1.2 Regression Model

A method was required to estimate the number of pickers, number of packers and the number of waves required to process a given set of orders during a workday based on the results obtained from the optimization model. For this purpose, three regression models were formulated using Minitab. The regression models were based the number of orders per day and the total number of items to be processed as the independent variables.

The results obtained from the LINGO optimization model and the regression models were comparable particularly when the number of orders to be processed was higher. The regression model was found to overestimate the number of workers required when the number of orders per day was low. However as the number of orders was increased the results were similar to those found through optimization. The number of waves generated by the regression models was continuously increasing whereas those required according to LINGO leveled off to a particular value. However, the time taken to process all the orders with the total number of workers and the number of waves found from the regression model was only 10 minutes more than the time available even in the worst situation.

Therefore it is concluded that the approach developed in this research can be used by any DC that has manual picking and packing operations to estimate the optimal staffing levels. However it is important to emphasize that the time estimation models used in this formulation can be used as they are only by a DC that follow similar strategies with respect picking, sorting and packing. If the operating policies are significantly different and if the assumptions made in this research are very different to the actual situation, the user will have to develop own time estimation models that can accurately reflect the time taken for the different processes. However the approach to formulating the optimization and regression models will still be accurate.

Another interesting discovery from this research was the correlation of the time lag between the operations and the maximum number of waves that was generated. It was found that the maximum number of waves formed was not greater than the number of time lag durations that can be accommodated in a work shift. The total number of workers required to process the throughput is dependent on the number of waves formed. Therefore the time lag between the operations can be adjusted until the optimal number of waves that gives the minimum number of workers is found. However, it was also discovered that with higher time lags, the maximum number of waves that can be processed is reduced. This leads to higher work loads per worker and may results in situations that cannot be dealt with existing facilities and operating policies. Thus a trade-off has to be made on how much more workload can be allocated to a worker in trying to achieve lower staffing levels. These finding that can contribute significantly to reduce the DC labor cost by determining the appropriate workforce.

5.2 Future Work

Further research is necessary to determine if the relationship between the time lag between operations and the maximum number of waves that can be processed is valid with all possible time lag durations and irrespective of the number of orders to be processed. If a DC does not have any constraint on how much time lag can be allowed between the operations, the optimization model can be further developed to determine the most suitable time lad and the corresponding optimal staffing levels.

No cart capacity was considered in this research. It will be useful to know how the optimal staffing levels are influenced when an additional constraint for cart capacity is included in the optimization model. When the time lag between the operations was increased, it was seen that the optimal staffing levels were reduced. However, it also resulted in higher workload per worker. A more comprehensive model may be developed to achieve the most appropriate staffing levels by finding the best time lag and also complying with workload constraints. Further experimentation is required to achieve this.

In this study, each picker was assigned to a zone, which can be one aisle, several aisles of part of an aisle. When the number of pickers is low, a zone may consist of several aisles. However as the number of pickers is increased the zone size decreases and may lead to a picker being assigned past of an aisle. This may cause congestion in the aisles and slightly different travel times. Therefore it will be useful to evaluate how optimal staffing levels change if the zone size is restricted to at least one aisle.

The regression model that was formulated to determine the number of waves required to process the throughput was logarithmic and gradually levels off as the number of orders is increased. Further experimentation is required to determine if the maximum number of waves generated by the regression model will be equal to the maximum number of waves that can be processed according to the relationship that was found when the number of orders is much higher.

In order to ascertain the appropriateness of the optimization and regression models, it is also important to implement and test them in an actual situation.

Two approaches to model the time taken to pick and pack a given number of items were used in this research. In the simple approach it was assumed that the number of items to be picked and packed was evenly distributed among the pickers and packers respectively. In the probabilistic approach it was assumed that the maximum number of items assigned to a picker or a packer follows a binomial distribution and the time estimation models were modified accordingly. However, no experimentation was conducted to analyze the results if the number of items assigned to a picker is not uniform (probabilistic) but all packers are given an equal number of items to pack by intelligently assigning orders to packers. Further experimentation is required to determine how the optimal results and the regression model will be influenced by this approach.

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APPENDIX I

Table A1: Simulated Total Picking Time and Analytical Total Picking Time using Different α Values

No Of Pickers	Total Picking Time (mins)				Percentage Squared Error (compared to simulated)		
	Simulated Results	Analytical Results					
		$\alpha = 0.95$	$\alpha = 0.975$	$\alpha = 0.99$	$\alpha = 0.95$	$\alpha = 0.975$	$\alpha = 0.99$
2	1957.60	2094.67	2059.52	2096.13	-7.0%	-5.2%	-7.1%
4	2041.30	2153.02	2124.18	2186.90	-5.5%	-4.1%	-7.1%
6	2109.20	2193.11	2168.46	2248.85	-4.0%	-2.8%	-6.6%
8	2121.80	2225.65	2204.34	2298.88	-4.9%	-3.9%	-8.3%
10	2192.00	2253.75	2235.26	2341.92	-2.8%	-2.0%	-6.8%
12	2239.70	2278.84	2262.84	2380.22	-1.7%	-1.0%	-6.3%
14	2301.45	2301.72	2287.95	2415.03	0.0%	0.6%	-4.9%
16	2351.20	2322.87	2311.14	2447.14	1.2%	1.7%	-4.1%
18	2372.90	2342.65	2332.80	2477.07	1.3%	1.7%	-4.4%
20	2394.00	2361.28	2353.18	2505.19	1.4%	1.7%	-4.6%
22	2480.00	2395.78	2467.03	2643.38	3.4%	0.5%	-6.6%
24	2504.00	2411.88	2486.07	2669.62	3.7%	0.7%	-6.6%
Mean Percentage Error in Total Picking Time					-1.3%	-1.0%	-6.1%

APPENDIX II

TABLE A2: Number of Retrievals and Number of Stops from Analytical Model and Simulated Results using $\alpha = 0.975$

No of Pickers	Analytical Results		Simulated Results		% Difference	
	Retrievals	Stops	Retrievals	Stops	Retrievals	Stops
2	400.74	351.77	384.4	357.4	4.25%	-1.57%
4	209.79	183.12	203.25	188.8	3.22%	-3.01%
6	144.18	125.36	140.75	131.55	2.44%	-4.70%
8	110.77	96.02	108.05	98.8	2.52%	-2.81%
10	90.44	78.19	90.5	82.3	-0.06%	-4.99%
12	76.73	66.18	76.4	70.85	0.43%	-6.59%
14	66.83	57.52	68.55	62.6	-2.51%	-8.11%
16	59.33	50.98	61.8	56.2	-3.99%	-9.29%
18	53.46	45.85	55.75	50.45	-4.11%	-9.12%
20	48.72	41.72	49.8	46.2	-2.17%	-9.71%

APPENDIX III

Probabilistic Optimization Model Formulated using LINGO

```
[OBJECTIVE] MIN = WP+WPA;

! n=items per order and m = Number of orders/day;
! W = Number of waves;
! WP = Number of pickers;
! WPA = Number of packers;

@GIN (WP);     ! Constraints to ensure integer variables;
@GIN (WPA);
@GIN (W);

W >=1;         ! Constraints to ensure non-zero variables;
WP >= 1;
WPA >= 1;

m = 24000;   ! Changed according to number of orders to be processed in
a day;
n=12;          !Changed according to number of items/orders;


! Components of the picking time equation;
a1 = 2*100*(150/60 + 0.5);
a2 = n*m/w*(1.96*(((WP-1)/(n*m/w))^0.5)+1)*0.2;
a3 = 20*100*( 1-(1-WP/(20*100))^(n*(m/w)/WP*(1.96*(((WP-
1)/(n*(m/w)))^0.5)+1)))*0.2;
a4 = 1.0;

! Picking time equation;
x = (1/60)*(1/WP)*(a1 + a2 + a3 + a4 );

! Picking Time Constraint;
X*W <= 8;
```

```
! Number of orders per packer per wave, rounded;
L = m/W;
Q = L/WPA;

! Number of items per packer per wave;

Itemspacker= (1/WPA)*n*m/w*(1.96*(((WPA-1)/(n*m/w))^0.5)+1);

! Packing time equation;
y = (1/60)*(m/w/WPa*1.00 + (Itemspacker)*0.25);

! Packing Time Constraint;
Y*W <= 8;

! Sorting time constraint;
Z = 1.1875/60;
! Based on the parameters equals 1.1875;

! Total time constraints;

(X + Z)*W + Y <= 9;
W*Y+ (X+Z) <= 9;
```

APPENDIX IV

Optimal Results Obtained from Microsoft Excel and LINGO

Table A3: Average 3 Items per Order

Total orders	Results from LINGO				Results from Excel			
	Pickers	Packers	Waves	Total	Pickers	Packers	Waves	Total
1000	7	5	2	12	7	5	2	12
2000	10	10	2	20	9	11	2	20
3000	12	14	3	26	12	14	3	26
4000	15	18	4	33	16	17	4	33
6000	19	26	4	45	19	26	4	45
8000	22	35	4	57	23	34	4	57
9000	25	38	5	63				
10000	27	42	5	69	26	43	4	69
11000	28	46	5	74				
12000	30	50	5	80	30	50	5	80
14000	34	57	6	91	34	57	6	91
16000	38	65	6	103	37	66	5	103
18000	41	73	6	114	41	73	6	114
20000	43	82	6	125	44	81	6	125
22000	48	88	7	136	48	88	7	136
24000	51	96	7	147	49	99	6	148
26000	55	103	8	158	52	106	6	158
28000	57	112	7	169	57	112	7	169
30000	59	121	7	180	59	121	7	180
32000	61	130	6	191	63	128	7	191
34000	64	138	6	202	64	138	6	202
36000	68	145	7	213	70	143	8	213
38000	71	152	7	223	72	152	7	224
40000	74	160	7	234	75	160	7	235

Table A4: Average 6 Items per Order

Total orders	Results from LINGO				Results from Excel			
	Pickers	Packers	Waves	Total	Pickers	Packers	Waves	Total
1000	9	8	2	17	9	8	2	17
2000	14	14	4	28	14	14	3	28
3000	18	20	3	38	19	19	4	38
4000	22	25	4	47	20	29	3	49
6000	30	37	4	67	28	38	4	66
8000	36	48	5	84	36	48	5	84
9000	39	54	5	93				
10000	42	60	5	102	43	59	6	102
11000	45	66	5	111				
12000	48	72	5	120	48	72	5	120
14000	55	83	6	138	54	84	5	138
16000	62	93	7	155	64	92	8	156
18000	68	105	7	173	67	106	6	173
20000	72	118	6	190	74	116	7	190
22000	78	130	6	208	78	132	5	210
24000	86	139	7	225	85	140	7	225
26000	91	151	7	242	91	151	7	242
28000	98	161	8	259	98	161	8	259
30000	103	174	7	277	104	173	8	277
32000	108	186	7	294	110	184	8	294
34000	114	198	7	312	114	198	7	312
36000	121	207	8	328	121	207	8	328
38000	127	219	8	346	127	218	8	345
40000	133	230	8	363	133	230	8	363

Table A5: Average 12 Items per Order

Total Orders	Results from LINGO				Results from Excel			
	Pickers	Packers	Waves	Total	Pickers	Packers	Waves	Total
1000	14	11	3	25	13	12	2	25
2000	21	22	3	43	23	20	4	43
3000	29	30	4	59	29	30	4	59
4000	34	41	4	75	36	39	5	75
6000	49	58	6	107	49	57	6	106
8000	61	76	6	137	61	76	5	137
9000	66	87	6	153				
10000	72	96	6	168	74	94	7	168
11000	78	105	6	183				
12000	85	113	7	198	87	112	8	199
14000	98	131	8	229	96	136	7	232
16000	110	149	8	259	110	149	8	259
18000	120	169	7	289	121	168	8	289
20000	131	188	7	319	133	186	8	319
22000	144	205	8	349	144	205	8	349
24000	154	226	7	380	155	224	8	379
26000	167	242	8	409	167	242	8	409
28000	178	261	8	439	178	261	8	439
30000	190	279	8	469	190	279	8	469
32000	201	298	8	499	201	298	8	499
34000	213	317	8	530	213	317	8	530
36000	224	335	8	559	224	335	8	559
38000	236	354	8	590	236	354	8	590
40000	247	372	8	619	247	372	8	619

APPENDIX V

Simple Optimization Model Formulated using LINGO

```
[OBJECTIVE] MIN = WP+WPA;

! n=items per order and m = Number of orders per day;
! W = Number of waves;
! WP = Number of pickers;
! WPA = Number of packers;

@GIN (WP);   ! Constraints to ensure integer variables;
@GIN (WPA);
@GIN (W);

W >=1;       ! Constraints to ensure non-zero variables;
WP >= 1;
WPA >= 1;

m = 10000;          ! Changed according to number of orders to be
                    processed in a day;
n=12;               !Changed according to number of items/orders;

! Components of the picking time equation;
a1 = 2*100*(150/60 + 0.5);
a2 = n*m/w*0.2;
a3 = 20*100*( 1-(1-WP/(20*100))^(n*(m/w)/WP))*0.2;
a4 = 1.0;

! Picking time equation;
x = (1/60)*(1/WP)*(a1 + a2 + a3 + a4 );

! Picking Time Constraint;
X*W <= 8;

! Packing time equation;
Y = (1/60)*(1/WPA)*(0.25*n*(m/w)+1.0*(m/w));
```

```
! Packing Time Constraint;
Y*W <= 8;

! Sorting time constraint;
Z = 1.1875/60;
! Based on the parameters equals 1.1875;

! Total time constraints;

(X + Z)*W + Y <= 9;
W*Y+ (X+Z) <= 9;
```

APPENDIX VI

Optimal Results obtained from Simple and Probabilistic Models using LINGO

Table A6: 3 Items per Order

Total orders	Simple Model				Probabilistic Model			
	Pickers	Packers	Waves	Total workers	Pickers	Packers	Waves	Total workers
1000	7	5	2	12	7	5	2	12
2000	9	10	2	19	10	10	2	20
3000	12	13	3	25	12	14	3	26
4000	14	17	3	31	15	18	4	33
6000	18	25	4	43	19	26	4	45
8000	21	33	4	54	22	35	4	57
9000	22	37	4	59	25	38	5	63
10000	25	39	5	64	27	42	5	69
11000	26	43	5	69	28	46	5	74
12000	28	47	5	75	30	50	5	80
14000	31	55	5	86	34	57	6	91
16000	35	61	6	96	38	65	6	103
18000	38	68	6	106	41	73	6	114
20000	41	75	7	116	43	82	6	125
22000	44	82	7	126	48	88	7	136
24000	48	88	8	136	51	96	7	147
26000	51	95	8	146	55	103	8	158
28000	53	103	8	156	57	112	7	169
30000	56	110	8	166	59	121	7	180
32000	58	117	8	175	61	130	6	191
34000	61	124	8	185	64	138	6	202
36000	63	132	8	195	68	145	7	213
38000	66	139	8	205	71	152	7	223
40000	69	146	8	215	74	160	7	234

Table A7: 6 Items per Order

Total orders	Simple Model				Probabilistic Model			
	Pickers	Packers	Waves	Total workers	Pickers	Packers	Waves	Total workers
1000	9	7	2	16	9	8	2	17
2000	13	13	3	26	14	14	4	28
3000	16	20	3	36	18	20	3	38
4000	21	23	4	44	22	25	4	47
6000	27	35	4	62	30	37	4	67
8000	33	45	5	78	36	48	5	84
9000	37	49	6	86	39	54	5	93
10000	40	54	6	94	42	60	5	102
11000	41	62	5	103	45	66	5	111
12000	45	65	6	110	48	72	5	120
14000	52	74	7	126	55	83	6	138
16000	57	85	7	142	62	93	7	155
18000	63	94	8	157	68	105	7	173
20000	68	105	8	173	72	118	6	190
22000	74	115	8	189	78	130	6	208
24000	79	125	8	204	86	139	7	225
26000	84	136	8	220	91	151	7	242
28000	89	146	8	235	98	161	8	259
30000	94	157	8	251	103	174	7	277
32000	99	167	8	266	108	186	7	294
34000	104	178	8	282	114	198	7	312
36000	109	188	8	297	121	207	8	328
38000	114	198	8	312	127	219	8	346
40000	119	209	8	328	133	230	8	363

Table A8: 12 Items per Orders

Total orders	Simple Model				Probabilistic Model			
	Pickers	Packers	Waves	Total workers	Pickers	Packers	Waves	Total workers
1000	13	11	3	24	14	11	3	25
2000	20	20	3	40	21	22	3	43
3000	28	27	5	55	29	30	4	59
4000	33	36	5	69	34	41	4	75
6000	45	52	6	97	49	58	6	107
8000	57	68	7	125	61	76	6	137
9000	63	75	8	138	66	87	6	153
10000	68	84	8	152	72	96	6	168
11000	73	93	8	166	78	105	6	183
12000	79	100	8	179	85	113	7	198
14000	89	117	8	206	98	131	8	229
16000	99	134	8	233	110	149	8	259
18000	109	150	8	259	120	169	7	289
20000	119	168	8	287	131	188	7	319
22000	130	184	8	314	144	205	8	349
24000	140	200	8	340	154	226	7	380
26000	150	217	8	367	167	242	8	409
28000	160	234	8	394	178	261	8	439
30000	171	250	8	421	190	279	8	469
32000	181	267	8	448	201	298	8	499
34000	191	284	8	475	213	317	8	530
36000	201	300	8	501	224	335	8	559
38000	211	317	8	528	236	354	8	590
40000	221	334	8	555	247	372	8	619

APPENDIX VII

Regression Models Formulated using Minitab

Regression Analysis: pickers versus orders, items

The regression equation is
pickers = 10.7 +0.000195 orders +0.000482 items

Predictor	Coef	SE Coef	T	P
Constant	10.6987	0.4862	22.00	0.000
orders	0.00019519	0.00003172	6.15	0.000
items	0.00048217	0.00000332	145.29	0.000

S = 2.226 R-Sq = 99.9% R-Sq(adj) = 99.9%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	2	230839	115419	23288.92	0.000
Residual Error	65	322	5		
Total	67	231161			

Source	DF	Seq SS
orders	1	126216
items	1	104623

Unusual Observations

Obs	orders	pickers	Fit	SE Fit	Residual	St Resid
1	1000	7.000	12.340	0.468	-5.340	-2.45R
24	1000	9.000	13.787	0.468	-4.787	-2.20R
51	6000	52.000	46.586	0.397	5.414	2.47R
67	38000	236.000	237.985	0.822	-1.985	-0.96 X
68	40000	247.000	249.948	0.872	-2.948	-1.44 X

R denotes an observation with a large standardized residual
X denotes an observation whose X value gives it large influence.

Regression Analysis: packers versus orders, items

The regression equation is

packers = 2.50 + 0.00216 orders +0.000590 items

Predictor	Coef	SE Coef	T	P
Constant	2.5045	0.2325	10.77	0.000
orders	0.00215903	0.00001517	142.32	0.000
items	0.00058968	0.00000159	371.55	0.000

S = 1.065 R-Sq = 100.0% R-Sq(adj) = 100.0%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	2	551708	275854	243368.54	0.000
Residual Error	65	74	1		
Total	67	551782			

Source	DF	Seq SS
orders	1	395228
items	1	156480

Unusual Observations

Obs	orders	packers	Fit	SE Fit	Residual	St Resid
8	10000	44.000	41.785	0.168	2.215	2.11R
20	34000	134.000	136.059	0.296	-2.059	-2.01R
22	38000	154.000	151.771	0.338	2.229	2.21R
37	22000	130.000	127.841	0.138	2.159	2.05R
67	38000	354.000	353.442	0.393	0.558	0.56 X
68	40000	372.000	371.912	0.417	0.088	0.09 X

R denotes an observation with a large standardized residual

X denotes an observation whose X value gives it large influence.

Regression Analysis: waves versus orders

The regression equation is
Waves = - 9.024 + 1.5852 Ln(orders)

Predictor	Coef	SE Coef	T	P
Constant	-9.0240	0.8337	-10.82	0.000
Ln(order	1.58517	0.08743	18.13	0.000

S = 0.7205   R-Sq = 82.4%   R-Sq(adj) = 82.2%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	1	170.65	170.65	328.73	0.000
Residual Error	70	36.34	0.52		
Total	71	206.99			

Unusual Observations

Obs	Ln(order	Waves	Fit	SE Fit	Residual	St Resid
1	6.9	2.0000	1.9260	0.2409	0.0740	0.11 X
3	6.9	2.0000	1.9260	0.2409	0.0740	0.11 X
7	6.9	3.0000	1.9260	0.2409	1.0740	1.58 X
39	10.4	6.0000	7.5159	0.1186	-1.5159	-2.13R
53	9.5	8.0000	6.1093	0.0851	1.8907	2.64R
56	9.7	8.0000	6.3210	0.0866	1.6790	2.35R

R denotes an observation with a large standardized residual
X denotes an observation whose X value gives it large influence.

APPENDIX VIII

Table A9: Optimal Results from the Regression Model and using LINGO

Total Orders	Regression Results				LINGO Results			
	Pickers	Packers	Waves	Total workers	Pickers	Packers	Waves	Total Workers
1000	15	10	4	25	12	9	3	21
2000	20	17	4	37	18	17	3	35
3000	24	25	5	49	23	26	4	49
4000	29	32	5	61	28	33	4	61
6000	38	47	5	85	40	47	6	87
8000	47	62	5	109	48	63	5	111
9000	51	70	5	121	53	70	6	123
10000	56	77	5	133	59	76	7	135
11000	61	85	5	145	62	85	6	147
12000	65	92	6	157	68	92	7	160
14000	74	107	6	181	75	108	6	183
16000	83	122	6	205	85	122	7	207
18000	92	137	6	229	94	137	7	231
20000	101	152	7	253	103	152	7	255
22000	110	167	7	277	112	167	7	279
24000	119	182	7	301	121	181	8	302
26000	129	197	7	325	130	196	8	326
28000	138	212	8	349	138	211	8	349
30000	147	227	8	373	147	226	8	373
32000	156	242	8	397	156	241	8	397
34000	165	256	8	421	164	256	8	420
36000	174	271	9	445	173	271	8	444
38000	183	286	9	469	181	286	8	467
40000	192	301	9	493	190	301	8	491