# DEVELOPMENT OF COST ESTIMATION OF EQUATIONS FOR FORGING

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John C. Rankin

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by

JOHN C. RANKIN

has been approved for

the Department of Mechanical Engineering

and the Russ College of Engineering and Technology by

Bhavin Mehta

Associate Professor of Mechanical Engineering

Dennis Irwin

Dean, Russ College of Engineering and Technology

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DEVELOPMENT OF COST ESTIMATION EQUATIONS FOR FORGING (115 pp.)

Director of Thesis: Bhavin Mehta

Following are the processes and results of the development of a more accurate forging cost estimating equation useful for any forged part of given material and final dimensions. A current forging cost estimating equation standard is used as a benchmark. Error from this equation is calculated at 23 percent. Prototype equations are developed using current methods of metal processing. Models are then tweaked or discarded as testing progresses through varied methods of error trapping. The final equation (below) has an error of 15 percent, a reduction of eight percent over the benchmark.

$$\text{process cost} = K\left(AP_{ave}\right)^{n} EF_{com}$$

Where K and n are constants, A is the cross-sectional area of the forging, $P_{ave}$ is the average pressure needed to produce the forging using the "slab" method of calculation, E is an escalation factor ($102.57 in this study), and $F_{com}$ is a forging shape complexity multiplier.

Approved:

Bhavin Mehta

Associate Professor of Mechanical Engineering

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# HISTORY

## *Introduction*

Forging, by definition, is the process by which the bulk, plastic deformation of a work-piece is carried out via compressive forces on a discrete part in a set of dies ([1]Kalpakjian, 1997). The forging process, by compressively removing large inconsistencies in the forged material's particle lattice, generally improves the strength of a material significantly ([2]Avallone, 1996). There are, or course, many different types of forging, including: open-die, closed-die, orbital, coining, heading, piercing, hubbing, cogging, fullering, and rolling ([3]Kalpakjian, 1997).

Unfortunately, it is very difficult to estimate the approximate cost of these forging techniques before producing one or multiple dies and/or forgings. Even after production begins, many questions remain unanswered concerning the true cost of producing a forged part including: Does it cost more to forge one material over another? Should price increases correspond to larger forging sizes? How should the complexity of a part affect price? Additionally, even if a forging firm is able to accurately price a work piece, how does an outside firm know that it is being charged fairly for work received?

These questions and more, from both a production and purchasing perspective, make the development of forging cost estimating equation highly desirable. Currently little work has been done in this area, instead more research has been focused on machine time equations. These formulas can be readily converted to cost equations using a base cost per unit time rate for operations such as turning, drilling, milling, grinding, etc., but comprehensive cost equations for more complex industrial operations such as welding, casting, and forging have received little cost analysis attention ([4]Abdalla & Shehab, 2001; [5]Leep, Parsaei, Wong & Yang, 1999; [6]Locascio, 2000; [7]Schreve,1999). The developments that follow in this paper attempt to build a discrete equation to adequately describe the cost of forging a given part. A lesser mention using similar analyses will be given to equations in the areas of flash welding and ring rolling.

## *Cost Estimation*

Before going into the details of the development of a forging cost estimation model it is first important cover some basics on cost modeling itself. Cost modeling is a methodology for estimating the costs associated with a project generally to justify a planned capital expenditure, determine likely production costs, or merely bring attention to an area of potentially high cost ([8]TWI World Centre for Materials Joining Technology, 2000).

Though there are other types of cost estimation, the most widely used are methods of parametric modeling. Parametric modeling (sometimes called Algorithmic modeling in more complex modeling situations) employs equations that describe relationships between measurable system attributes affect cost. Parametric techniques use past and current experience to forecast the economics of future activities ([9]International Society of Parametric Analysis, 2004). As most parametric models were first developed in the high technology computer industries most fall into two general categories – those developed to predict hardware costs and those developed predict software costs. The former include such models as PRICE H, SEER H, NAFCOM, and ParaModel; the latter: COCOMO, COCOMO II, PRICE S, and SEER-SEM ([10]Algorithmic Cost Models, n.d.; [11]Department of Defense, 1999). Having said this it is important to denote that though developed for the computer industry, most models have been refined and extended to be useful in all fields of large projects with multiple cost inputs. The following discussion is based around the development of specific use (forging) cost model.

## *Benchmark Forging Equation*

The forging equations presented in this document were developed in order to create a computer program utilizing a system of cost estimation equations to derive the approximate cost of an assembled good containing many complex forgings. The following text will first describe the currently used benchmark cost estimation program. The benchmark equations were used as a standard from which later calculations were built and/or compared. The prototype cost estimation program will later be examined in a similar fashion.

The benchmark system of equations is a sophisticated cost estimation program developed for the specific purpose of estimating the cost of complex assembled products. This cost data is then used in price and contract negotiations with potential customers and suppliers. Figure 1 contains a flow chart depicting the different operations used to obtain a final cost estimate for any given assembly.

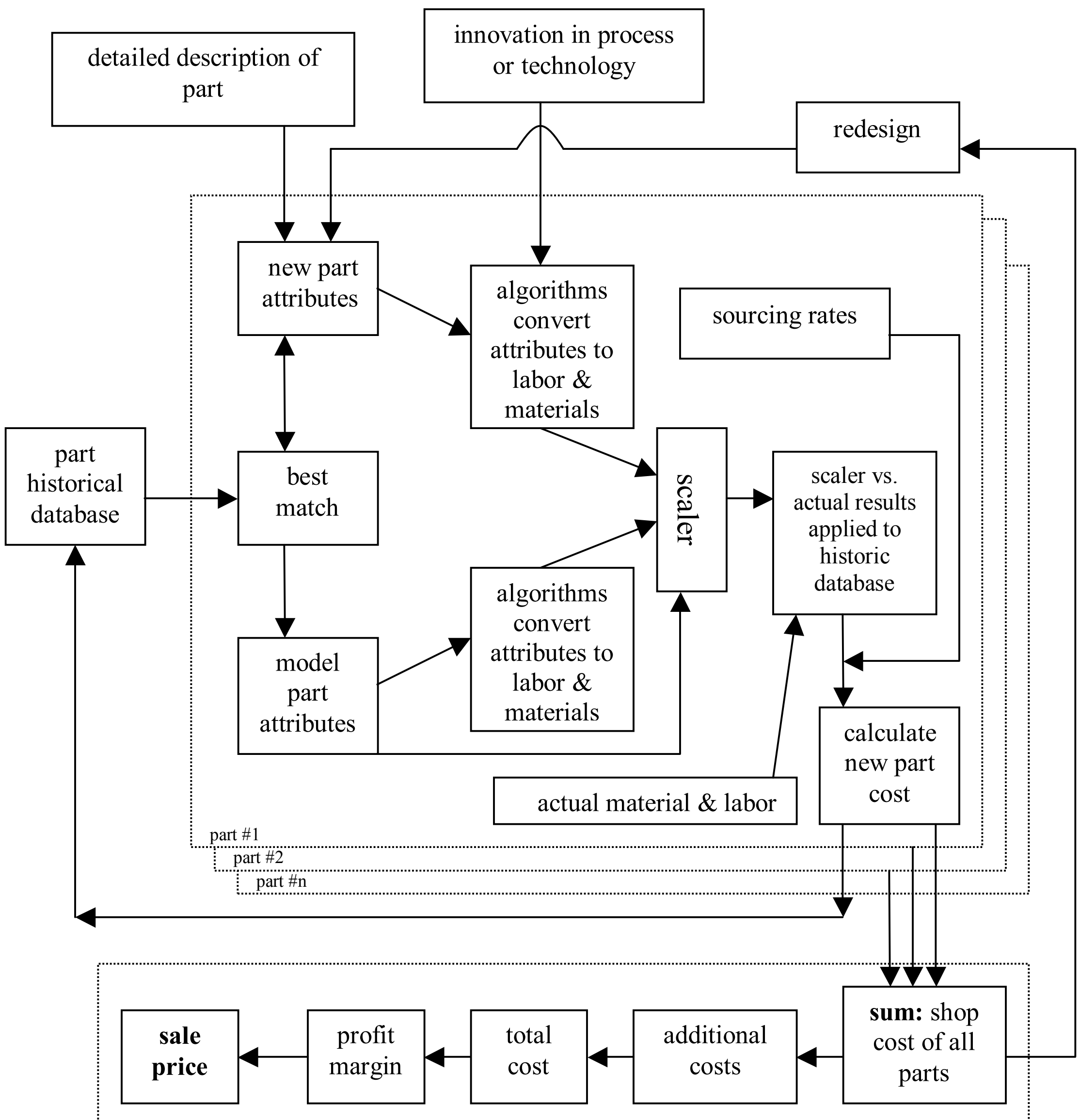


**Figure 1: benchmark program's cost estimation flowchart**

As seen in the above flow chart, the chief inputs into the formulaic system are part descriptions, production techniques and historic database figures. In this system, new parts are compared to a library of "historic" parts in terms of notable part attributes. Through a series of data compiling algorithms, these attributes, along with a host of constants, are eventually fed into a series of cost equations for each part and/or task performed on a part. The resulting costs are then summed, scaled, and added to administrative costs and required profit margins in order to get final part sales price estimates.

Appendix A details, via flow chart, the process by which the benchmark program calculates the approximate cost of a forging process for individual parts. The resulting generic forging equation is as follows:

$$\text{Total forging cost} = \text{material cost} + \text{forging process cost} + \text{machining cost}$$

Similar flow charts and total cost equations could be constructed for all part tasks such as grinding, turning, etc. These costs, in turn, are formulated from various part-specific inputs as well as historic database information. The following sections will attempt to detail the formulas behind the material, process and machining costs that sum to estimate the overall forging cost.

## Forging Process Cost

Equation 1 is the benchmark forging process cost equation. This equation is intended to represent the monetary cost for the labor, material, and machinery usage for the forging of any given part.

$$\text{process cost} = \mathrm{W}^{0.7}\, CP(F + M)E$$

**(Equation 1)**

Where individual variables are described as follows:

W = billet weight = billet (forge) volume $\times$ material density
C = configuration factor
P = process factor
F = forge factor
M = market factor
E = escalation factor

Taking a closer examination of the process cost input variables; "W" is defined as the billet weight. This means the weight of the proper sized material billet needed to completely fill (without excess) the forging dies of a specified part. Consequently, this billet weight will also be the weight of the corresponding forged part prior to any machining. Thus billet weight can also be referred to as forging weight. Logically, in order to arrive at the billet weight it is necessary to merely multiply the volume of the billet or pre-machining forged part by the density of the material from which it was formed.

The configuration factor, C, and process factor, P, are both derived from the forging database (see Appendix A for information on the forging database and where it fits into the forging process cost). Both multipliers are variables less than or equal to one but greater than zero with the former indicating the complexity of the die and part configurations. The latter variable indicates the complexity of the individual forging. However, the true extent to which these two variables differentiate themselves from one another is unclear and apparently somewhat arbitrary as will later come into play with the development of prototype cost models.

The forging factor, F, is equal to the Battel Forgability Factor for the material from which the part is forged. This factor indicates the ease with which any given material may be plastically deformed.

Finally, the market factor, M, and escalation factor, E, are both general business factors that compensate for any price inflation over a given period of time as well as the current overhead costs for skilled labor, respectively. These factors may be obtained through either market and/or individual forging firm research and are assumed to be constant at M = 0 and E = $102.57 throughout the remainder of the study.

## Material Cost

Equation 2 is the benchmark material cost equation. This equation represents the principle cost of the forging process, or the cost of the bulk material used in the process.

$$\text{material cost} = BW$$

**(Equation 2)**

Where individual variables are described as follows:

B = material cost per pound
W = billet weight = billet volume × material density

Material cost per pound, B, is a material specific variable indicating the current market value of the material of the part to be forged. Again, the billet weight, as discussed above, corresponds to the variable W.

## Machining Cost

Due to the increased levels of strength imparted by forging to a material, many lightweight metals can be effectively used in high performance assemblies. Also due to the high strength and resiliency needed in performance parts, it is important that all forgings be devoid of defects that might weaken any portion of the part, causing it to fail catastrophically. Sonic testing or other methods of internal examination are generally used to detect such interstitions. However, in order to run a complete sonic inspection it is necessary to perform a certain amount of machining. This machining is generally done under the supervision of the forging firm instead of the shop responsible for finish machining due to the necessity to remake a part should it fail testing. It usually consists of a rough turning process designed to quickly create clean, parallel testing surfaces.

Equation 3 is the benchmark system's machining cost equation representing the monetary cost of all machining work necessary to prepare a forged part for sonic or other internal inspection processes.

$$\text{machining cost} = (W - S)^{.57} \times \left[0.1D + \ln\left(\frac{100}{I}\right)\right]E$$

**(Equation 3)**

Where individual variables are described as follows:

W = billet weight
S = sonic weight = sonic volume × material density
D = machining difficulty factor
I = machinability index
E = escalation factor

Billet weight, W, as in the process and material cost equations, represents the weight of the forged part prior to machining. Sonic weight, S, is the weight of the part after it has been machined into a shape suitable for sonic testing. As with billet weight, sonic weight may also be derived through the multiplication of the sonic volume, or the part's volume after sonic machining has been performed, and material density.

The machining difficulty factor, D, is a multiplier of a value greater than one that indicates the difficulty in performing the sonic machining. The more complex the machining processes the greater process time and costs will grow. However, it is unknown what factors constitute the reasoning behind an individual part's difficulty rating.

The machinability index, I, indicates the ease by which a part's forged material may be machined. A lower variable indicates a more machinable material, which, in turn, lowers machining costs. The inverse is true for tougher, more brittle, or harder materials – higher machining times and costs require a higher machinability index rating. However, as with previously discussed variables, the scale upon which these variables rest is unclear.

Finally, as in the process cost equation, the escalation factor, E, is an estimation of current labor rates. Again, it is assumed that skilled labor runs at $102.57 in this project.

# RESEARCH

## *Introduction*

Having examined the benchmark forging cost model, this study seeks to propose a new forging cost model with improved results over the benchmark model. A flow diagram of this new model can be viewed in Appendix B. As can be seen, the proposed new model is set up similarly to the benchmark model with the chief cost constituents stemming from material, processing, machining, and inspection. However, each of these factors has been calculated differently than under the benchmark model. Additionally, part volume calculations have been altered to increase formula accuracy. The following sections will attempt to detail the development of an acceptable new forging cost equation.

## *Forging Cost Equation*

As mentioned above, the new forging cost equation is chiefly made up of the contributing costs of materials, machining, processing, and inspection. Material costs are calculated in the same way as the benchmark program. Additionally, inspection costs are assumed to be nil when compared to other cost constituents. Hence neither one of these sectors of the prototype forging equation will be addressed. The following segment instead seeks to discuss only the process cost of the prototype forging equation as well as differences in machining and volume calculation methodologies.

### Volume Estimation

The following section details some of the supporting equations used in order to utilize the forging cost equation model. Both models discussed supply an adequate level of estimation. However, since development of such volumetric models was not the focus of this study, both volume estimation models are only discussed in brief, by no means covering the true depth of calculation behind each.

## Benchmark Attribute Analysis

One of the most significant ways the new forging model was altered from the benchmark model is in the method used to estimate the volume of parts. The old method used a complex system in which the major features making up a new part were compared to the features contained in a library of parts. Based on the basic three dimensional shapes of a part, such as cylinders and cones, a formula for estimating the volume was compiled from a list of basic formulas corresponding to their appropriately described shapes. These basic formulas were then compiled in a series of additions and subtractions to estimate the overall volume of the part in question. The following figures show some examples of how parts would be sectioned into distinct volumes and summed to estimate the whole part volume.

Forging Volume Cylinder = $V_1 - V_2 - V_3 - V_4$

$$V_1 = \frac{\pi D_1^2 L_1}{4} \qquad V_3 = \frac{\pi D_2^2 L_2}{4}$$

$$V_2 = \frac{\pi D_2^2 L_1}{4}$$

$V_4 = 0$ if shaft ID $\leq 6$

$$V_4 = \frac{\pi D_3^2 L_2}{4}$$

Where:
$D_1$ = shaft OD +2+2A
$D_2$ = shaft OD + 2A
$D_3$ = shaft OD - 2 - 2A
$L_1$ = 2A + 0.175
$L_2$ = shaft length + 2A

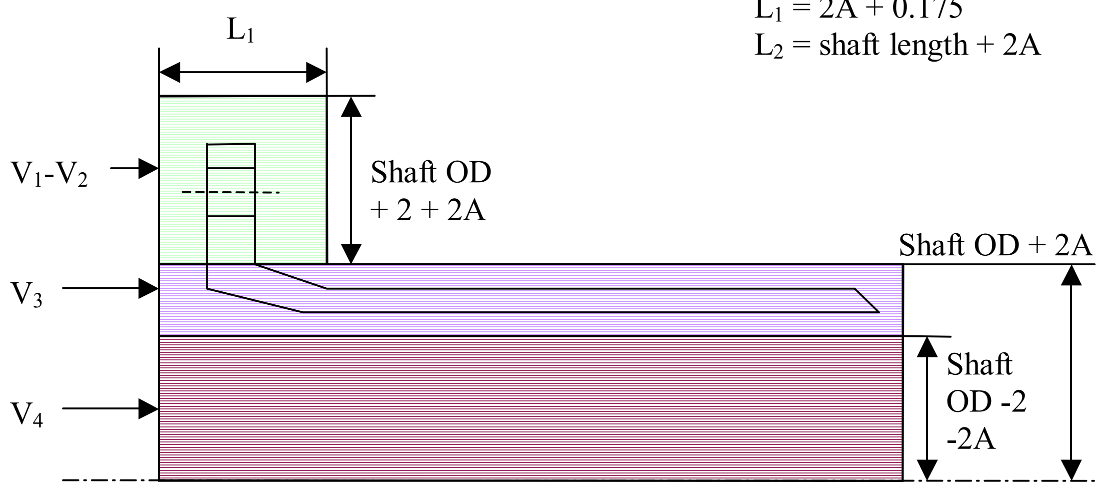


**Figure 2: benchmark system's volume analysis of a shaft consisting of a cylinder and OD flange**

Figure 2 shows a shaft with a flange on the outside diameter. As can be seen by the figure, the program has divided the part into three distinct volume regions, V1, V2, V3, and V4, based on similar parts and features in the part library. Using these simplified "block" volumes, it is estimated that the volume of the part is V1 – V2 + V3 – V4. This volume calculation, though effective for a host of parts and features due to the extensive depth of the part library, fails to take into consideration the most efficient shape needed to forge a part given shape limitations of inspection techniques and the desire to minimize material losses to machining processes. Such considerations are crucial to forging shops in order to reduce costs and, thus are also important to consider when estimating forged part shape and volume. Thus, the attribute method can only be effective as long as the part library behind it contains any and all exceptions to the general shape rule. Without these exceptions the system would soon break down as parts increased in forging complexity.

For instance, in the above part such material excesses may be noted through inconsistent material thickness on the ID and OD of the part length. Additional material inconsistencies may be noted on the flange length. Granted, such material cost additions may be considered insignificant on this part and may be attributed to sketching errors. However, such excesses become magnified when examining a more complex part such as the one shown in Figure 3:

Volume = $V_3$ - $V_4$ + $V_5$

$$V_3 = \frac{\pi D_1^2 L_1}{4}$$

$$V_4 = \frac{\pi D_2^2 L_2}{4}$$

$$V_5 = \frac{\pi\left(D_3^2 + D_4^2 + D_3 D_4\right)L_3}{12}$$

Where:
$D_1$ = cone $OD_{max}$ + 2A
$D_2$ = shaft OD + 2A
$D_3$ = cone $OD_{max}$
$D_4$ = cone $OD_{max}$
$L_1$ = flange thickness + 2A
$L_2$ = cone length + 2A
$L_3$ = cone length - flange thickness

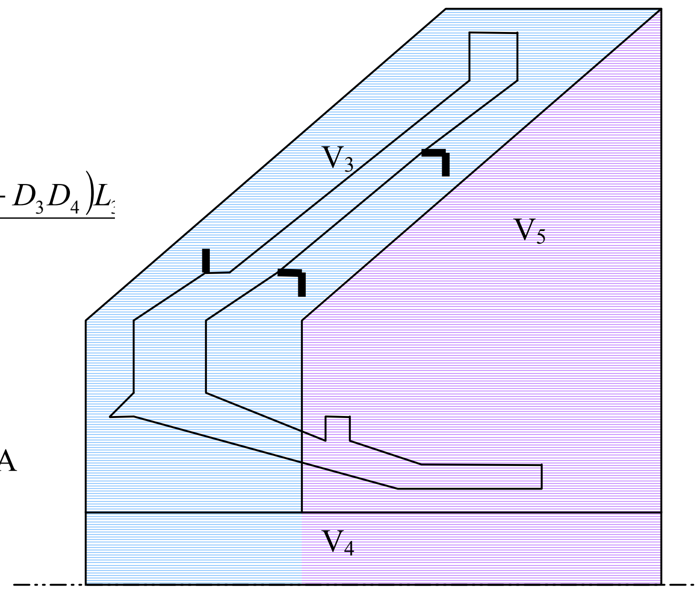


**Figure 3: benchmark system's volume analysis of a cone with an inside appendage**

Figure 3, as with more simple parts, shows that the part has been divided into several distinct volume blocks based appendage location for further analytical purposes based on part shape, V3, V4, and V5. The part forging volume is then calculated using the equation: V3 – V4 + V5. As can be clearly seen in the figure, this part is a far greater example of potential material excesses assumed through the benchmark system's method of volume estimation. Please refer to Appendix C for additional information and equations behind attribute based volume estimation.

## "Stick" Analysis

The principle behind the stick method is an assumption that even though the finished shape of a part is complex; the forged shape will be near net shape while still maintaining simplicity sufficient to allow removal from the forging dies. This simplicity allows one to assume the forged shape of a part to be the summation of a limited number of simple shapes. In the case of this experiment shapes are rotations of varied complexity around a central axis, it can be assumed that forged parts can be simplified even further than the traditional three dimensional geometries to a set of two dimensional shapes projected around an axis. These two dimensional shapes can be simplified even further by assuming that all two dimensional shapes can be reduced to as set of one dimensional lines that can be projected across space into two dimensional shapes. As mentioned, these shapes can then be rotated to produce three dimensional shapes whose volume can be calculated in order to estimate the forged volume of a part.

Certainly the above one dimensional process would work for very simple shapes but what about more complex shapes with protrusions, webs, and flanges? In order to project the proposed model for volume estimation from simple shapes to more complex it is first necessary to understand a few forging basics. Using a simple set of dies it is impossible to forge complex details on a larger part due to the costly difficulties they would present in removing the part from the dies once forged. Instead, all details are forged as outcroppings surrounded by fill material; as shown in Figure 4, where the lines are the shape to be rotated around a central axis and the shading represents valleys that will be filled with additional forging material. Additional examples and explanations of actual forged parts can be found in Appendix D.

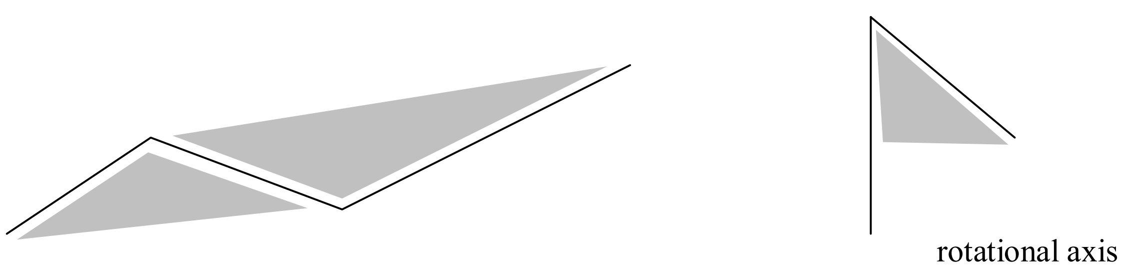


**Figure 4: forged part models with filled valleys**

Now that the methodology of the stick method has been briefly explained, the following is a mathematical explanation of the model. The two principle shape volumes needed to estimate the volume of a forging are shells and disks. An explanation of the two principle shapes follows – this includes the volumetric estimation of a cone which is built upon to arrive at the volume of a shell. Additionally, while calculating volumes, the surface area of individual shapes should also be calculated for later use. An explanation of surface areas is explored after the volumetric equations.

Volume of a cone:

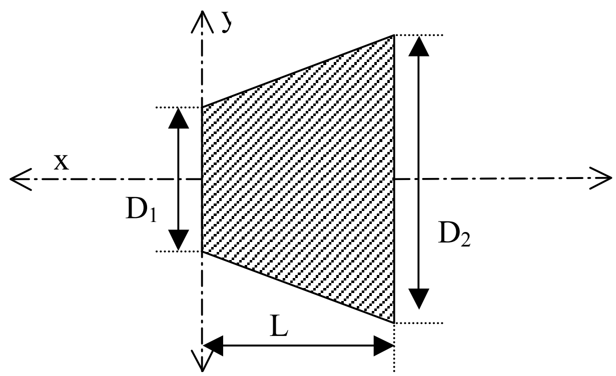


**Figure 5: cutaway of a right circular cone**

Where:

L = length measured axially
$D_1$, $D_2$ = diameter at each end
$r_1$, $r_2$ = radius at each end

By geometric convention the volume of a frustum of a right circular cone is as follows:

$$V = \frac{\pi L}{3}\left(r_1^2 + r_1 r_2 + r_2^2\right)$$

**(Equation 4)**

By substituting diameters for radii in (Equation 4):

$$V = \frac{L}{12}\pi\left(D_1^2 + D_1 D_2 + D_2^2\right)$$

**(Equation 5)**

Using (Equation 5) the volume of a conical shell may be calculated.

Volume of a shell:

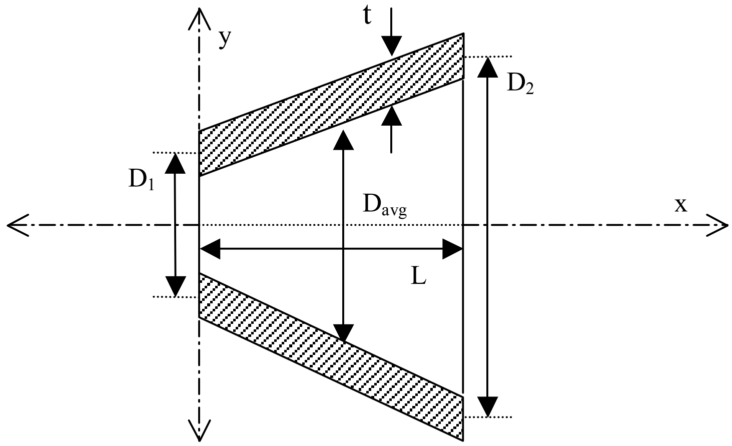


**Figure 6: cutaway of a right circular cone shell**

Where:

L = length measured axially
t = thickness measured radially
$D_1$, $D_2$ = diameter at each end measured to the midpoint

If:

$V_O$ = volume of outer cone
$V_I$ = volume of inner cone

Then the volume of a right circular cone shell may be calculated by:

$$V_{Shell} = V_O - V_I$$

**(Equation 6)**

By substituting into (Equation 5) based on dimensions from Figure 6:

$$V_O = \frac{L}{12}\pi\left[(D_1 + t)^2 + (D_1 + t)(D_2 + t) + (D_2 + t)^2\right]$$

**(Equation 7)**

$$V_I = \frac{L}{12}\pi\left[(D_1 - t)^2 + (D_1 - t)(D_2 - t) + (D_2 - t)^2\right]$$

**(Equation 8)**

Substituting (Equation 7) and (Equation 8) into (Equation 6):

$$V_{Shell} = \frac{L}{12}\pi\left(4D_1t + 2D_1t + 2D_2t + 4D_2t\right)$$

$$V_{Shell} = \frac{L}{12}\pi\left[6t\left(D_1 + D_2\right)\right]$$

$$V_{Shell} = \pi Lt\left(\frac{D_1 + D_2}{2}\right)$$

**(Equation 9)**

Where the average diameter:

$$D_{avg} = \frac{D_1 + D_2}{2}$$

**(Equation 10)**

Therefore, by substituting (Equation 10) into (Equation 9), the volume of a conical shell may be expressed:

$$V_{Shell} = \pi LtD_{avg}$$

Volume of a disk:

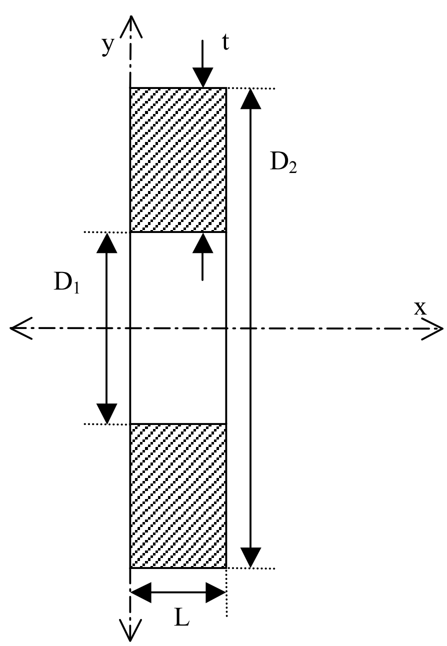


**Figure 7: cutaway of a hollow cylinder**

By geometric convention, (Equation 11) is equal to the volume of a cylinder.

$$V_{cylinder} = \frac{\pi}{4} L D^2$$

**(Equation 11)**

Altering (Equation 11) for a hollow cylinder using the notation from Figure 7 results in the following:

$$V_{disk} = \frac{\pi}{4} L \left( D_2^2 - D_1^2 \right)$$

**(Equation 12)**

$$V_{disk} = \frac{\pi}{4} L \left( \frac{D_2 + D_1}{2} \right) \left( \frac{D_2 + D_1}{2} \right)$$

And, since the wall thickness of the cylinder can be expressed:

$$t = \frac{D_2 - D_1}{2}$$

**(Equation 13)**

Then the volume of a hollow cylinder can be expressed as:

$$V_{disk} = \pi L t D_{avg}$$

As $V_{shell} = V_{disk}$ both disk and shell volumes can be calculated using the same formula.

$$V = \pi L t D_{avg}$$

**(Equation 14)**

However, bear in mind that though the simplified equation forms of $V_{shell}$ and $V_{disk}$ are similar, the basis from which each figure is dimensioned is very different, as noted in Figure 6 and Figure 7. Dimensions for a shell are measured from the midpoint while the corresponding distances on a disk are measured from the more standard endpoints. As will be seen, this difference in distancing becomes inconsequential during the measurement of actual parts.

Surface area of a cone:

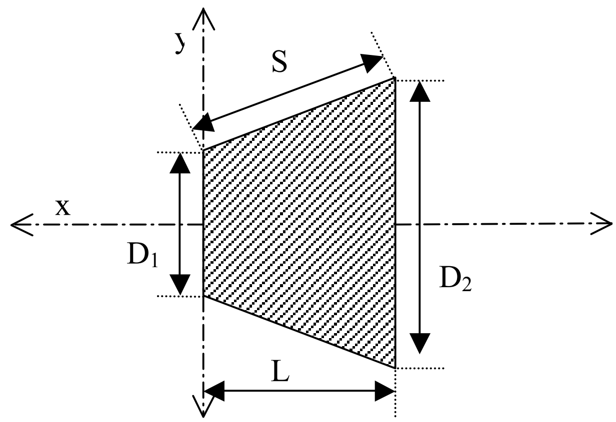


**Figure 8: surface area of a right circular cone**

By convention the surface area (excluding the ends) of a right circular frustum is as follows:

$$A = \pi S\left(\frac{D_2}{2} + \frac{D_1}{2}\right)$$

**(Equation 15)**

or

$$A_{side} = \frac{\pi}{2}(D_1 + D_2)\sqrt{\frac{1}{4}(D_2 - D_1)^2 + L^2}$$

Where:

$$S = \sqrt{\frac{1}{4}(D_2 - D_1)^2 + L^2}$$

**(Equation 16A)**

Therefore by substitution:

$$A_{side} = \pi D_{avg} S$$

**(Equation 17)**

Surface area of a shell:

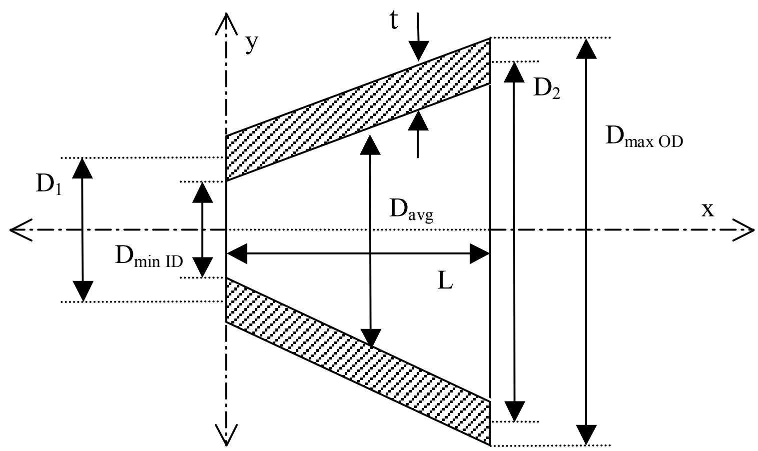


**Figure 9: surface area of a right circular cone shell**

Based on Figure 4 the equation for the area of a shell is as follows:

$$A_{Shell} = A_{OD} + A_{ID} + A_1 + A_2$$

Where:

$A_{OD}$ = surface area of the outside "side" surface
$A_{ID}$ = surface area of the inside "side" surface
$A_1$ = surface area of the small end of the shell
$A_2$ = surface area of the large end of the shell

Speaking first of the "side" surface areas only, based on (Equation 17), the surface areas of the ID and OD surfaces of a shell are as follows:

$$A_{OD} = \pi\left(D_{avg} + t\right)S$$
$$A_{ID} = \pi\left(D_{avg} - t\right)S$$

Based on (Equation 10) and the dimensions shown in Figure 8 the following is true:

$$D_{\min ID} = D_1 - t$$

**(Equation 18A)**

$$D_{\max OD} = D_2 + t$$

**(Equation 18B)**

Therefore:

$$D_{avg} = \frac{D_1 + D_2}{2} = \frac{D_{\min ID} + D_{\max OD}}{2}$$

**(Equation 19)**

Simplifying:

$$D_2 - D_1 = \left(D_{\max OD} + t\right) - \left(D_{\min ID} - t\right)$$
$$D_2 - D_1 = \left(D_{\max OD} - D_{\min ID}\right) - 2t$$

If:

$$R_{diff} = \frac{\left(D_{\max OD} - D_{\min ID}\right)}{2}$$

Then by substitution:

$$D_2 - D_1 = R_{diff} - t$$

Therefore based on (Equation 16A):

$$S = \sqrt{\left(R_{diff} - t\right)^2 + L^2}$$

**(Equation 16B)**

So:

$$A_{ID} + A_{OD} = 2\pi D_{avg} S$$

**(Equation 20)**

$$A_{ID} + A_{OD} = 2\pi D_{avg} \sqrt{\left(R_{diff} - t\right)^2 + L^2}$$

Finally, the surface areas of the fore and aft ends of the shell are based on the area of a conventional circle and (Equation 18A) and (Equation 18A) as follows:

$$A_1 = \frac{\pi}{4}\left[\left(D_1 + t\right)^2 - \left(D_1 - t\right)^2\right] = \pi D_1 t$$
$$A_2 = \frac{\pi}{4}\left[\left(D_2 + t\right)^2 - \left(D_2 - t\right)^2\right] = \pi D_2 t$$

Added together to get the total end area of the shell:

$$A_1 + A_2 = 2\pi\left(\frac{D_1 + D_2}{2}\right)t = 2\pi D_{avg} t$$

$$A_1 + A_2 = 2\pi D_{avg} t$$

**(Equation 21)**

Surface area of a disk:

Using Figure 7 of a cutaway of a hollow cylinder, the areas of the ID and OD surfaces are as follows:

$$A_{OD} = \pi D_2 L$$
$$A_{ID} = \pi D_1 L$$
$$A_{ID} + A_{OD} = 2\pi L D_{avg}$$

If in the case of a hollow cylinder L = S, therefore:

$$A_{ID} + A_{OD} = 2\pi D_{avg}\sqrt{(R_{diff} - t)^2 + L^2} = 2\pi D_{avg} L$$

The areas of the fore and aft ends of the disk are similarly simple:

$$A_1 = A_2 = \frac{\pi}{4}D_2^2 - \frac{\pi}{4}D_1^2$$
$$= \frac{\pi}{4}\left(D_2^2 - D_1^2\right)$$
$$= \pi\left(\frac{D_2 + D_1}{2}\right)\left(\frac{D_2 - D_1}{2}\right)$$
$$= \pi D_{avg} R_{diff}$$
$$= \pi D_{avg} t$$
$$A_1 + A_2 = 2\pi D_{avg} t$$

Summary:

Finally, as may have been realized in the calculations above, there are only three shape cases that should be treated the same in terms of volume and area calculation.

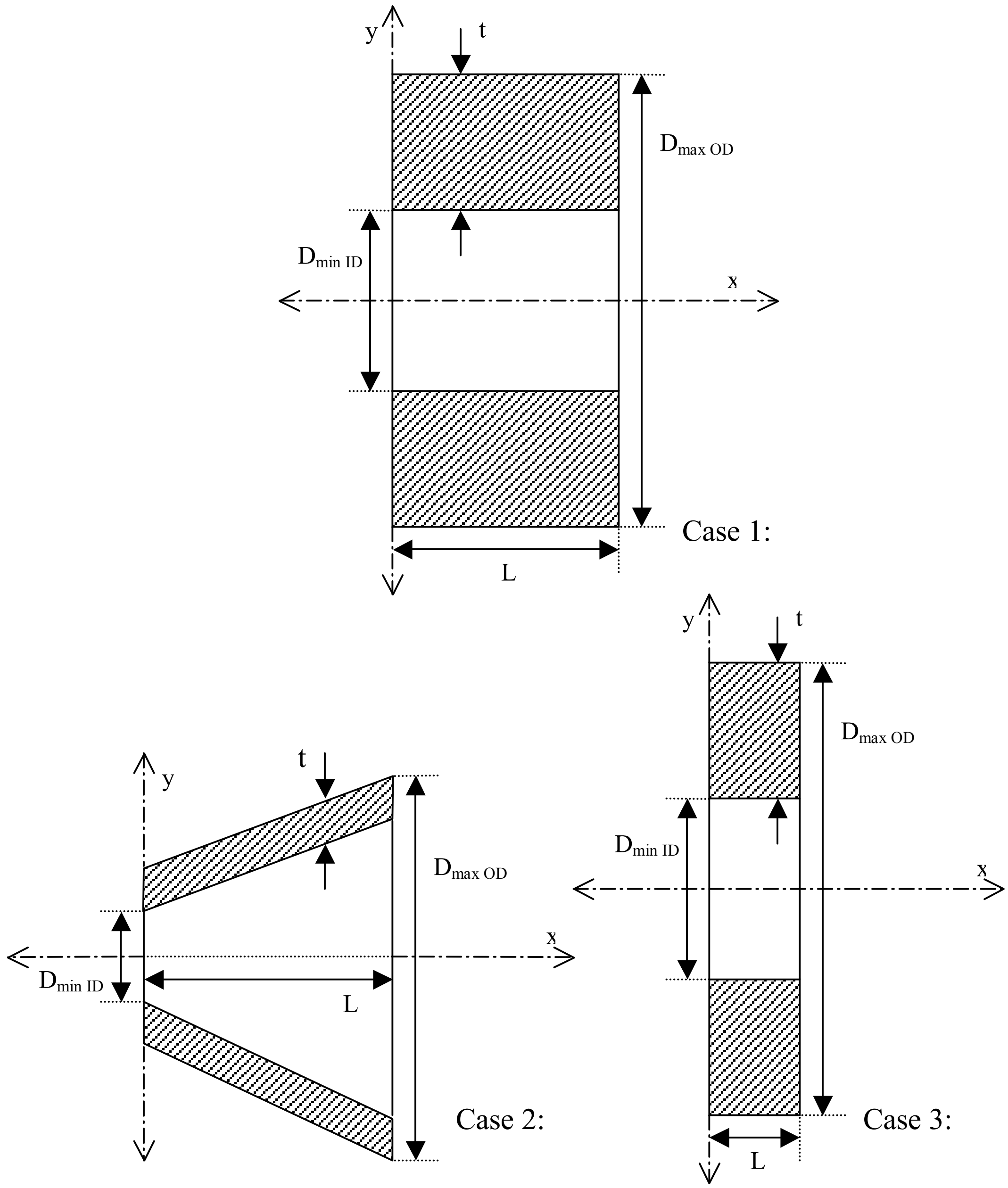


**Figure 10: volume and area case summary (cases 1 – 3)**

Where $D_{avg}$ and $R_{diff}$ can be defined as:

$$D_{avg} = \frac{D_{\max OD} + D_{\min ID}}{2}$$

**(Equation 22)**

$$R_{diff} = \frac{D_{\max OD} - D_{\min IN}}{2}$$

**(Equation 23)**

Note that in cases 1 and 3: $t = R_{diff}$

Thus:

$$V = \pi L t D_{avg}$$

**(Equation 24)**

$$A_{TB} = 2\pi D_{avg}\sqrt{\left(R_{diff} - t\right)^2 + L^2}$$

**(Equation 25)**

$$A_{FA} = 2\pi D_{avg} t$$

**(Equation 26)**

Where:

V = volume
$A_{TB}$ = surface area of the top and bottom or "side" surfaces
$A_{FA}$ = surface area of the fore and aft surfaces

The above mathematics represents the basis of the stick method (additional mathematics explaining the stick method can be found in Appendix E). Please note that there is no such thing as a vertical stick, instead, as shown in case 3 of Figure 10, vertical rises are modeled using a stick of short length and vast thickness. Individual stick cases can be connected together at will to model more complex forged shapes. The corresponding volume and area of more complex shapes can be calculated through simple summation of a small set of formulas.

## Forging and Sonic Volumes

In the cases of both the benchmark and stick methods of volume estimation discussed above there is little difference between calculating forged and sonic volumes. Both assume that the excess material (when compared to the final part) needed to produce a forged part is equal to 0.175 inches on all sides of the part; sonic parts have in excess of 0.1 inches of material. Appendix F shows an example of volume calculation using both the benchmark and stick methods for both forging and sonic volume estimation. Please see this appendix for more practical detail in the step-by-step volume estimation using an actual part.

## Machining Cost Equations

As mentioned previously, in addition to volume calculations, the other major alteration to the modeling process used to test versions of forging process cost equations is the process used to determine machining time cost. Both the benchmark and prototype methods of calculating machining costs will presently be contrasted. As seen in (Equation 1), the benchmark model is as shown in (Equation 3):

$$\text{machining cost} = (W - S)^{.57} \times \left[ 0.1D + \ln\left(\frac{100}{I}\right) \right] E$$

Where:

W = billet weight
S = sonic weight = sonic volume*material density
D = machining difficulty factor
I = machinability index
E = escalation factor

Unlike the benchmark volume calculations, the machining cost equation, shown below as (Equation 27), is straightforward in that it remains constant for all part shapes. However, in the equation variables lay irreconcilable difficulties. Both the machining difficulty factor and the machinability index are not standard, measurable materials values. Instead they are the deeply imbedded combination of material factors hidden inside the benchmark part library. Hence, it is unknown what physical principles this machining equation may or may not be based upon. Without this knowledge it is impossible to use, much less judge the logical or effective value of the benchmark machine cost equation. Hence, it must be discarded in favor of other cost analysis methods.

The alternate model used consists of the conventional set of equations used to calculate the time needed to turn a work piece given a set of ideal material feeds, speeds, and cutting depths ([12]Green, Horton, Jones, McCauley, Oberg, & Ryffel, 2000). Having calculated the time needed to turn a given volume of material off the outside of a part it is simple to deduce the cost of the operation using an escalation factor. Note that this machining cost model is the same used for both rough and finish turning. The only difference is that in the rough turning performed on a forged part before sonic inspection assumes the maximum conventional cutting depth for the material. In this way the part can be cleaned up to inspection standards at a minimum cost.

The following example illustrates the involved equations and arithmetic for a turning process.

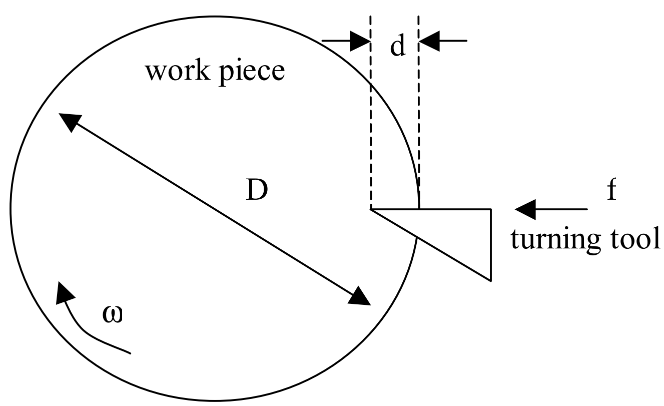


**Figure 11: example of a rough turned work piece**

The basic machining cost for any machining process is equal to time × labor costs:

$$\text{machining cost} = T \cdot E$$

**(Equation 27)**

Where:

$$T = \frac{V}{R_R}$$

**(Equation 28)**

And:

$$R_R = \omega \cdot \pi \cdot D \cdot f \cdot d = 12 \cdot S \cdot f \cdot D$$

**(Equation 29)**

And:

$$S = \frac{\omega \cdot \pi \cdot D}{12}$$

**(Equation 30)**

Where (Figure 11):

T = machining time
E = escalation factor
V = volume to be removed or the difference between forged and sonic volumes
$R_R$ = material removal rate
S = turning speed
f = feed rate
ω = rotational velocity in RPM
D = part diameter
d = depth of cut

## Process Cost Equation

The most critical changes made to the forging model pertained to the actual forging process equation. This equation seeks to explain the cost of the actual forging process, which can then be combined with other value-added process cost estimations to arrive at the total forging cost estimation. The following (Equation 31) presents an early version of the forging process cost equation. Further equation formats will later be detailed in the data analysis section (an overview record of all prototype process cost equations can be seen in Appendix G).

$$\text{process cost}_{\text{initial}} = AP_{ave}CK(F+M)EF_{com}$$

**(Equation 31)**

Where individual variables are described as follows:

C = configuration factor
F = forge factor
M = market factor
E = escalation factor or labor costs per hour
A = work-piece area in contact with die
$P_{ave}$ = average die pressure
K = error constant
$F_{com}$ = shape complexity factor

(Equation 31) should be compared to the benchmark process cost equation (Equation 1) shown below.

$$\text{process cost} = \mathrm{W}^{0.7}\, CP(F + M)E$$

As can be seen, there are obvious similarities between the two process equations including configuration factor (C), forge factor (F), market factor (M), and escalation factor (E). This is due to first attempts to only alter (Equation 1) to better describe the physical factors of forging that intuitively effect costs while still leaving as much (Equation 1) intact as possible. Such physical factors (A, $P_{ave}$, K, $F_{com}$) will be discussed presently and expanded upon during further alterations to (Equation 31) during data analysis. Information on the remaining variables (C, F, M, E) can be defined using the same parameters as discussed in the process cost equation discussion of the benchmark system.

## Work-Piece Area (A)

Work-piece area in contact with the die, A, is an indication of the size (in terms of surface area) of the part to be forged. Usually, the larger the work-piece area the larger will be the corresponding part. Such larger parts are more difficult and, consequently, more expensive to forge.

The estimated area in contact with the forging die varies from part to part and even depends on which part axis the forging pressure is utilized upon. During initial testing of forging process equation it was assumed that since all parts being forged are, basically, cylindrical in shape that the area in contact with a die would simply be the average circular footprint of any given part.

$$A = \pi \cdot R_{ave}^{2}$$

**(Equation 32)**

Where $R_{ave}^{2}$ is equal to the average radius of the part in question.

It was realized in later models that most parts' optimal forging positions would not present this circular footprint as an area in contact with the die. For example, a lengthy shaft would not be forged with pressure along the long axis. Instead it would be forged while setting horizontally. Hence, the formula for area changed to:

$$A = length \cdot 2 \cdot R_{ave}$$

**(Equation 33)**

Later versions of the process cost model, including the final version, used this formula to calculate the area of a part in contact with the die.

## Average Die Pressure ($P_{ave}$)

Similar to work-piece area, as the average die pressure, $P_{ave}$, on a part increases so will forging costs. The equation used to describe this die pressure is as follows.

$$P_{ave} = Y + \frac{Y \cdot D}{6h}$$

**(Equation 34)**

Where Y is equal to the yield stress of the material, D is the diameter of the work-piece, and h is the height of the work-piece. This equation is derived using a "slab" forging model analysis. The complete derivation of which can be seen in Appendix H ([13]Avitzur, 1968; [14]Caddel and Hosford, 1993).

## Shape Complexity Factor ($F_{com}$)

As forging shapes get more complex it generally takes more time, intermittent forging steps, and money to make a single part. Hence, it is logical to include some factor indicating the complexity of a part in the process cost equation – shape complexity factor, $F_{com}$. If one assumes that simplest forging shape is a cylinder where complexity, $F_{com}$, would equal one then for other shapes:

$$F_{com} = 0.3 \frac{S^{\frac{1}{2}}}{V^{\frac{1}{3}}}$$

**(Equation 35)**

Where S is the surface area of the part in contact with the die and V is the volume of the work-piece. Appendix I shows the complete derivation for how the shape complexity equation was calculated.

## Constant (K)

As mentioned above in the forging process equation, (Equation 31), K is a constant utilized to absorb a portion of any error inherent in the equation. As can be seen, it was included in early versions of the process cost equation in conjunction with a host of other constant factors remaining from the benchmark model such as the configuration, forge, and market factors. However, since all these factors are also constants, they were later factored into K and dropped from the formal written equation. Thus (Equation 31) changes to the following,

$$\text{process cost} = A \cdot P_{ave} \cdot K \cdot E \cdot F_{com}$$

**(Equation 36)**

Initially, since K was a constant relating the process equation to the actual forging process cost, K was solved using the following equation:

$$K(C,F,M) = \frac{\sum_{n}^{i=1} \left[ \frac{(\text{actual forgingcost})_i}{(\text{projected forgingcost})_i} \right]}{n}$$

**(Equation 37)**

Where the projected forging cost is the calculated cost of forging without using an error factor and the actual cost of forging is the true dollar cost to produce a forging.

Other versions of constants were used in later versions of (Equation 36) in order to better compensate for error between the proposed process cost equation and actual forging costs. Some of the later versions of the process cost equation are shown below. A more detailed history can be seen in Appendix G. Additional information on when each equation was used and its corresponding degree of effectiveness will be thoroughly discussed in the analysis section of this document.

$$\text{process cost} = A\left(K_1 + K_2 P_{ave}\right) EF_{com}$$

**(Equation 38)**

$$\text{process cost} = KAP_{ave}^{n} EF_{com}$$

**(Equation 39)**

$$\text{process cost} = \left(K_1 + K_2 AP_{ave}\right) EF_{com}$$

**(Equation 40)**

$$\text{process cost} = \left[K\left(AP_{ave}\right)^{n}\right] EF_{com}$$

**(Equation 41)**

All necessary calculations deriving the inputs for each process cost equation can be seen in Appendix J.

# ANALYSIS

## *Introduction*

The following is a detailed analysis of the research steps and data gathered in order to reach the conclusion of whether or not a better forging process equation could be found. First it is necessary to describe the base data from benchmark data trials, then trial forging process equation iteration will be briefly discussed in turn. Since, for reasons that will be explained presently, it is difficult to analyze the cost of the forging process in isolation, total cost is calculated for each cost equation version, using the following (Equation 42), in addition to the basic forging cost.

$$\text{Total Cost} = (\text{material} + \text{forging} + \text{machining}) \cdot \text{vendor markup} \cdot \text{admin costs}$$

**(Equation 42)**

## *Benchmark Data*

Though only one benchmark forging process trial was performed, several different versions or variations of benchmark data were used as bases when looking at total costs. The lone benchmark forging trial as well as the first total cost trial used data resulting from a strict calculation using benchmark equations – that is the benchmark equations exactly as discussed previously summed as in the above (Equation 42).

This methodology works well for the forging process (hence the single benchmark) trial. However, there are two problems with using a strict benchmark system when comparing total costs. The first is that all prototype trials use a completely different system of equations to estimate the cost of machining before sonic analysis. The differences in equations, since they have not been compared independently, potentially bring an unknown degree of error to the total cost that would pass undetected when comparing the standard benchmark methodology to prototype equations, only to later be attributed to the forging process equation. Clearly attributing such hidden error to the forging equation would be a mistake. Thus it is necessary to replace the standard benchmark method of calculating machining costs with the prototype method discussed above and shown in (Equation 27) through (Equation 30).

The second problem, as discussed above and to be shown presently, is that all prototype forging equations use error constants as a scale factor between equation results and actual data. The benchmark forging equation does not do this in any recognizable form (one may recall that it was unknown what factors contributed to several variables). Instead a similar method of scaling takes place when calculating the final cost of individual parts (see the "scaler" in Figure 1). Since the forging step is only one of many processes in a finished part, scaling will not affect the results being analyzed as either forging costs or total costs. Hence it was necessary to replicate the benchmark scaling method for a single processes data as opposed to a complete part. This scaling was done by comparing actual and benchmarked data to calculate multipliers for each of material, forging, and machining costs. These modified costs can then be added per (Equation 42). Refer to Appendix K for detail on the benchmark method of scaling.

### *Version 1*

As shown in Appendix G, Version 1 of the prototype forging Equation is as follows.

$$\text{process cost} = A \cdot C \cdot P_{ave}(F + M)E \cdot F_{com}$$

**(Equation 43)**

This equation is very similar to that used in benchmark method with the addition of shape and pressure factors. Unlike versions 2, 3, and 4, there is no error factor. The error factor used in one form or another in the later prototype versions was added after some brief experimentation with version 1. Typical forging cost results using this equation were in the hundred millions of dollars. Such outrageous results prompted the use of an error factor. All further use of Version 1 in terms of useful data was scrapped from that point on, thus Version 1 data does not appear on any data sheets with the more reasonable data from later cost equations.

### *Version 2*

Prototype equation version 2 is very similar to version 1 with the addition of a single error constant (K) as in the following equation.

$$\text{process cost} = A \cdot C \cdot P_{ave} K(F + M) E \cdot F_{com}$$

**(Equation 44)**

The trials performed using version 2 attempted to determine the relevance of equation variables left over from the benchmark model equation. Trial 2.1a through 2.1e walk through (Equation 44) dropping unknown variables from each consecutive equation in order to test their validity. If such variables are simple constants (presumably error constants in their own right) the effect on the cost should be minimal as the error should be taken up by the K variable. The following chart shows a summary of what was done in each version 2.1 trial:

| trial: | removed variable: | process cost equ.: |
|---|---|---|
| 2.1a | none | $ACP_{ave}K(F+M)EF_{com}$ |
| 2.1b | configuration factor (C) | $AP_{ave}K(F+M)EF_{com}$ |
| 2.1c | forging factor (F), market factor (M) | $ACP_{ave}KEF_{com}$ |
| 2.1d | shape complexity factor ($F_{COM}$) | $ACP_{ave}K(F+M)E$ |
| 2.1e | (C), (F), (M) | $AP_{ave}KEF_{com}$ |
| 2.1f | (C), (F), (M), ($F_{COM}$) | $AP_{ave}KE$ |

**Table 1: Variable changes made throughout prototype equation version 2.1 trials**

After attempting to weed out useless variables from the cost equation in the version 2.1 trials, the version 2.2 trials and continuing throughout the following version 3 and 4 trials the nature of the error constant, K, is developed through experimentation. Version 2.2 uses the methodology used to find K discussed in (Equation 36) and experiments with the averaging shown. Specifically, trial 2.2a averages K based on the forged part's material makeup while trial 2.2b averages K based on the type of part being forged or part family.

## *Version 3*

Continuing the experimentation with the scope of the error constant K, an additional error constant was added to version 3 of the prototype forging cost equation.

In this case two trials were run with each of the following equations – each with two error constants $K_1$ and $K_2$.

$$\text{process cost} = (K_1 + K_2 AP_{ave})EF_{com}$$

**(Equation 45)**

$$\text{process cost} = A(K_1 + K_2 P_{ave})EF_{com}$$

**(Equation 46)**

The two trial equations, differing only in their placement of parenthesis, can be explained by examining the variables of their makeup. The variable E, escalation factor, is a constant that does not depend at all on part dynamics. It is an outside constant added to the equation such that the results equal dollars. The variable $F_{com}$, shape complexity factor, is a multiplier starting at 1 for the simplest of forgings and increasing with shape complexity. When compared to the other two variables A, surface area, and $P_{ave}$, average die pressure, it is clear that the latter two variables make up the most important part of the equation in terms of value and physical importance. Hence, the experimenting done in version 3 attempting to manipulate the equation favorably by shifting K values concerned only the variables A and $P_{ave}$. Had the results compared more favorably, as shown later in the results section, perhaps similar additional experiments would have been performed.

In addition to performing trials the differing (Equation 45) and (Equation 46), similar to the version 2.2 trials, trial "a" assumes a single error constant for all forged parts with no differences due to either material makeup or part family. The error constants for trials "b" are material dependant but not part family dependant.

## *Version 4*

After experimenting with simple equations and error constants, version 4 uses more complex error constants to create a more complex equation. In theory this should, in turn, better explain predictable fluctuations in cost in the forging operation. As can be seen in the following equations, similar math experiments as performed in version 3 were also done in version 4.

$$\text{process cost} = KAP_{ave}^{n}EF_{com}$$

**(Equation 47)**

$$\text{process cost} = K\left(AP_{ave}\right)^{n}EF_{com}$$

**(Equation 48)**

Furthermore, identical to version 3, trials “a” assume a single error constant across the board and trials “b” limit error constants to a single material.

# RESULTS

## *Introduction*

The following section will discuss the results derived from the above analysis. Unlike the analysis section, results from the prototype cost equations can be divided into two segments. First, as seen in equation Version 2.1, is the development of which variables contribute to cost results. Second is the form and complexity of the equations error factor as developed in prototype Versions 2.2 through 4. Each segment of analysis will be discussed presently in more detail.

## *Variable Analysis*

As mentioned, the variable analysis takes place completely within prototype cost equation version 2.1. Table 2 (in addition to data presented in Table 1) shows the differences between different v2.1 trials with the error of each result when compared to the actual total forging cost.

| trial: | removed variable: | process cost equ.: | total error: |
|---|---|---|---|
| 2.1a | none | $ACP_{ave}K(F+M)EF_{com}$ | 20.33% |
| 2.1b | configuration factor (C) | $AP_{ave}K(F+M)EF_{com}$ | 20.33% |
| 2.1c | forging factor (F), market factor (M) | $ACP_{ave}KEF_{com}$ | 17.17% |
| 2.1d | shape complexity factor ($F_{COM}$) | $ACP_{ave}K(F+M)E$ | 20.31% |
| 2.1e | (C), (F), (M) | $AP_{ave}KEF_{com}$ | 17.17% |
| 2.1f | (C), (F), (M), ($F_{COM}$) | $AP_{ave}KE$ | 38.43% |

**Table 2: Variable changes made throughout prototype equation version 2.1 trials with compared results**

The error shown by Table 2 shows several variable developments. First, there is no difference between the results of version 2.1a of the prototype equation and 2.1b. Thus, the configuration factor, C, plays no role that cannot be absorbed by the error factor, K. Second, as seen in the error level of version 2.1c when compared to 2.1a, forging and market factors can also be combined favorably into the error factor, K. In fact, when all three of these variables are removed from the equation its error is reduced as seen in version 2.1e. Finally, using a similar comparison, version 2.1d suggests that similar improved results can be derived by eliminated shape complexity factor as well. However, the results shown in version 2.1f wherein configuration, forging, market and shape complexity factors are all removed from the equation show increased levels of error from 2.1e. Hence the version 2.1e or the following (Equation 49) is used for further analysis in later versions of the forging cost equation.

$$\text{process cost} = A \cdot P_{ave} KE \cdot F_{com}$$

**(Equation 49)**

### *Scope of K Values*

The nature of experimentation of versions 2.2 through 4 follow two different lines of parallel thought: First, how should the error factor be applied to individual parts, and second how should the error factor be expressed in the cost equation. Hence, equation versions 2.2 through 4 all contain an equation “a” and “b”. As the versions progressed the placement and/or complexity of the error constant was modified. Simultaneously, the results were applied to individual parts either by part shape (as indicated by a larger family shape) or part material. For example, in version 2.2 the error constant K was derived using an average comparison to the actual cost. In 2.2a the results were averaged across part families while in 2.2b results were averaged across part material.

However, as the shape factor is designed to describe the complexity of forging any individual part, it was assumed inappropriate to continue to use part family as a viable scope for the error factor as in 2.2b. Instead, versions 3 and 4 trial “a” use the same value for K for all parts across the board while “b” utilizes material specific K values. Table 3 shows the prototype equation alterations between both versions and trials.

| trial: | K scope: | process cost equ.: | total error: |
|---|---|---|---|
| 2.2a | material | $AP_{ave}KEF_{com}$ | 27.08% |
| 2.2b | part family | $AP_{ave}KEF_{com}$ | 50.64% |
| 3.1a | all the same K value | $A(K_1+K_2P_{ave})EF_{com}$ | 30.37% |
| 3.1b | material | $A(K_1+K_2P_{ave})EF_{com}$ | 25.37% |
| 3.2a | all the same K value | $(K_1+K_2AP_{ave})EF_{com}$ | 32.07% |
| 3.2b | material | $(K_1+K_2AP_{ave})EF_{com}$ | 20.15% |
| 4.1a | all the same K value | $AK(P_{ave}{}^{n})EF_{com}$ | 46.64% |
| 4.1b | material | $AK(P_{ave}{}^{n})EF_{com}$ | 19.34% |
| 4.2a | all the same K value | $K(AP_{ave})^{n}EF_{com}$ | 21.50% |
| 4.2b | material | $K(AP_{ave})^{n}EF_{com}$ | 15.09% |

**Table 3: Variable changes made throughout prototype equation version 2.2 – 4.2 trials with compared results**

It is easy to notice that the total error in version 2.2 indeed casts doubt on the idea of using a single K value across part families. Additionally note the difference between the total error presented in 2.1e in Table 2. Even though the recorded process cost equations are identical the total error is different than that of 2.2a or 2.2b. This is because scope of K was altered from 2.1 to 2.2. In version 2.1 the error scope was limited to parts of the same material within part family. When the idea occurred that it might be improper to make the error constant dependant on part family the two were broken apart in version 2.2 in order to note the differences in error. As was mentioned previously, part family error was then discarded.

Later trials focused specifically on how changing the mathematical format of the error value, K, would affect cost results. For this reason each consecutive version tends to use a more complex error factor or factors. As expected the more complex the equation the better resulting projected cost tended to mimic actual cost. Additionally, error factors seemed to perform better when made dependant upon material instead of trying to use one factor for all parts.

Please note that more complex the cost equation grew the more difficult it becomes to extract a valid error factor from the decreasing part per material pool. In this study some material categories had to be discarded from the results due to too few participants to effectively calculate a material error factor.

As can be seen in Table 3 the prototype equation with the lowest total error when compared to actual costs was version 4.2b with 15.09 percent error.

$$\text{process cost} = K\left(AP_{ave}\right)^{n} EF_{com}$$

**(Equation 50)**

## *Final Results*

| | Total Cost | | | |
|---|---|---|---|---|
| | **Shaft** | **Disk** | **Seal** | **Total** |
| actual | * | * | * | * |
| bench | 23.91% | 98.56% | 151.75% | 83.87% |
| $\text{bench}_{MACH}$ | 20.72% | 36.87% | 24.47% | 27.71% |
| scaled | 26.79% | 19.85% | 25.97% | 23.98% |
| $\text{scaled}_{MACH}$ | 23.02% | 20.65% | 24.71% | 22.55% |
| 2.1a | 17.07% | 9.68% | 41.19% | 20.33% |
| 2.1b | 17.07% | 9.68% | 41.19% | 20.33% |
| 2.1c | 8.61% | 9.73% | 41.19% | 17.17% |
| 2.1d | 17.55% | 12.04% | 36.86% | 20.31% |
| 2.1e | 8.61% | 9.73% | 41.19% | 17.17% |
| 2.1f | 38.73% | 28.23% | 53.29% | 38.43% |
| 2.2a | 13.71% | 22.46% | 54.08% | 27.08% |
| 2.2b | 75.82% | 31.76% | 41.19% | 50.64% |
| 3.1a | 26.27% | 9.44% | 67.93% | 30.37% |
| 3.1b | 31.89% | 18.37% | 26.09% | 25.37% |
| 3.2a | 25.47% | 10.48% | 74.36% | 32.07% |
| 3.2b | 15.22% | 12.86% | 37.26% | 20.15% |
| 4.1a | 29.97% | 50.97% | 65.17% | 46.64% |
| 4.1b | 13.24% | 12.22% | 37.63% | 19.34% |
| 4.2a | 5.34% | 20.34% | 47.49% | 21.50% |
| 4.2b | 11.64% | 11.25% | 25.17% | 15.09% |

**Table 4: Comparison of error between calculated and actual costs for versions of the benchmark system and prototype cost equations**

Table 4 presents the results of all cost equations in terms of percent error for all part families as well as an average across all families. Though all prototype versions have been previously discussed, one can see by the data that (Equation 50) clearly has the smallest margin of error of both the prototype equations as well as the benchmark models. The best benchmark model (scaled and making use of the prototype machining equations) has an error level of 22.55 percent with is over seven percent greater than version 4.2b of the prototype cost equations.

# CONCLUSION

In summary, the critical mathematical variable in estimating the cost of a forging process are the surface area of the part to be forged, the average pressure needed to forge a given part, the complexity of the part shape when compared with a simple cylinder, and the cost of labor. These variables can be combined in the following equation:

$$\text{process cost} = K\left(AP_{ave}\right)^{n} EF_{com}$$

**(Equation 50)**

Where:

K, n = error factors

If :

$$C = \begin{bmatrix} \log\left(\dfrac{P_c}{E \cdot F_{com}}\right) \\ \vdots \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & \log AP_{ave} \\ 1 & \vdots \end{bmatrix}$$

Then:

$$\begin{bmatrix} \log K_1 \\ n \end{bmatrix} = \left[A^T A\right]^{-1} A^T C$$

**(Equation J51)**

A = surface area of the part to be forged

$$\mathrm{A} = \pi \cdot R_{ave}^{2}$$

**(Equation 32)**

$$\mathrm{A} = length \cdot 2 \cdot R_{ave}$$

**(Equation 33)**

Where:

$R_{ave}$ = the volumetric radius of the work piece
Length = length of a work piece parallel to the die

(Equation 32) should be used if the part is cylindrical in shape, (Equation 33) if the part is lengthy with respect to the dies.

$P_{ave}$ = average pressure needed to forge a given part using “slab” method

$$P_{ave} = Y + \frac{Y \cdot D}{6h}$$

**(Equation 34)**

Where:

Y = material yield stress
D = work piece diameter
H = work piece height

E = cost of labor ($102.57 in this study)
$F_{com}$ = shape complexity factor

$$F_{com} = 0.3\frac{S^{\frac{1}{2}}}{V^{\frac{1}{3}}}$$

**(Equation 35)**

Where:
S = surface area of the work piece in contact with the die
V = Volume of the work piece

Surface area and Volume should be calculated as follows dependant on the shape of the work piece as shown in Figure 12.

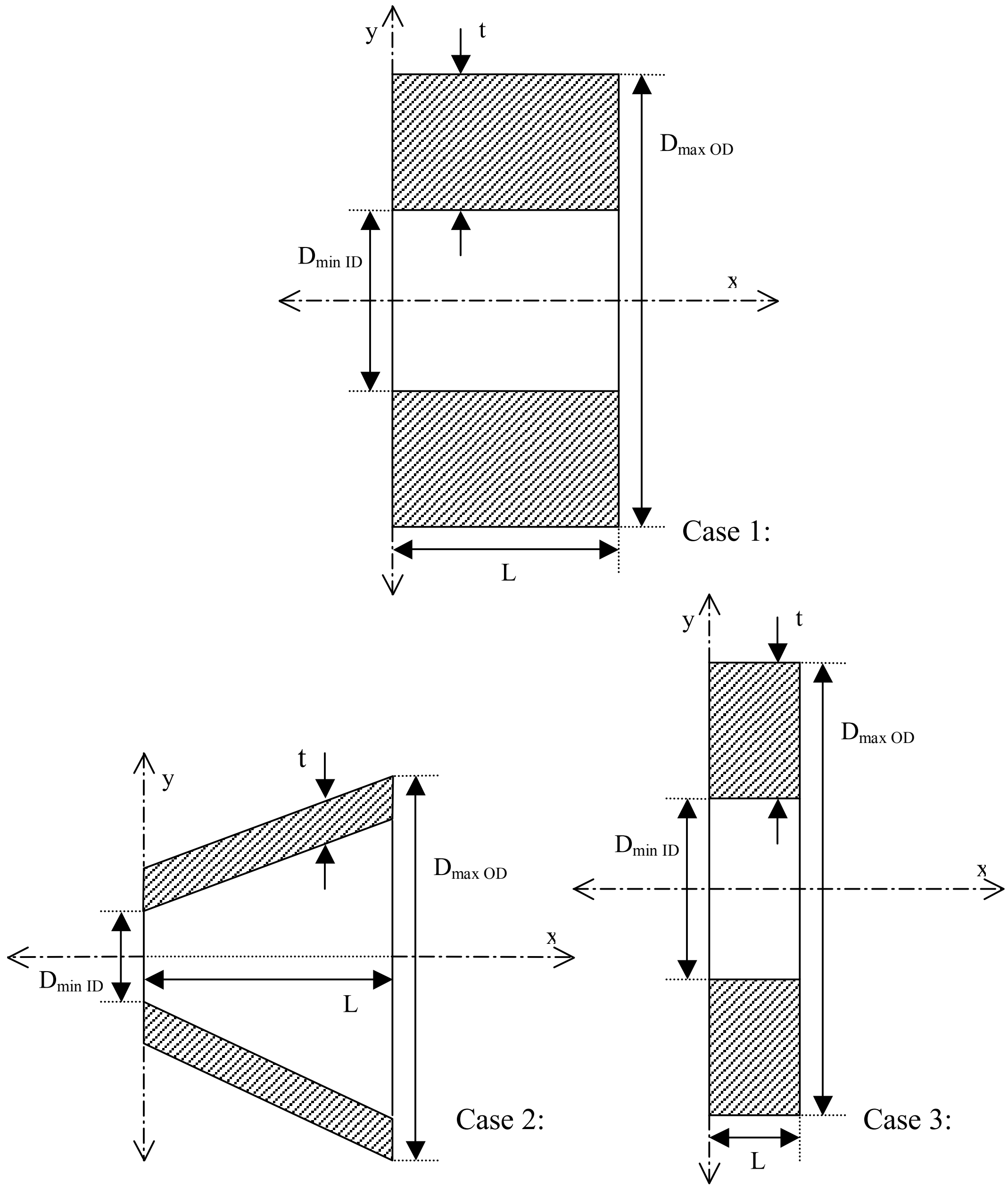


**Figure 12: volume and area case summary (cases 1 – 3)**

Where $D_{avg}$ and $R_{diff}$ can be defined as:

$$D_{avg} = \frac{D_{\max OD} + D_{\min ID}}{2}$$

**(Equation 22)**

$$R_{diff} = \frac{D_{\max OD} - D_{\min IN}}{2}$$

**(Equation 23)**

Note that in cases 1 and 3: $t = R_{diff}$

Thus:

$$V = \pi L t D_{avg}$$

**(Equation 24)**

$$A_{TB} = 2\pi D_{avg}\sqrt{\left(R_{diff} - t\right)^2 + L^2}$$

**(Equation 25)**

$$A_{FA} = 2\pi D_{avg} t$$

**(Equation 26)**

Where:

V = volume
$A_{TB}$ = surface area of the top and bottom or "side" surfaces
$A_{FA}$ = surface area of the fore and aft surfaces

The equation calculating the cost of the individual forging process should be combined with the larger equation as follows in order to calculate the total cost of a forging which includes, in addition to the metal compression process, material costs, rough turning for inspection processes, vendor profit markups and additional administrative costs.

$$\text{Total Cost} = (\text{material} + \text{forging} + \text{machining}) \cdot \text{vendor markup} \cdot \text{admin costs}$$

**(Equation 42)**

And,

$$\text{material cost} = BW$$

**(Equation 2)**

Where:

B = material cost per pound
W = billet weight = billet volume $\times$ material density

$$\text{machining cost} = T \cdot E$$

**(Equation 27)**

Where:

$$T = \frac{V}{R_R}$$

**(Equation 28)**

And:

$$R_R = \omega \cdot \pi \cdot D \cdot f \cdot d = 12 \cdot S \cdot f \cdot D$$

**(Equation 29)**

And:

$$S = \frac{\omega \cdot \pi \cdot D}{12}$$

**(Equation 30)**

Where:

T = machining time
E = escalation factor
V = volume to be removed (difference between forged and sonic volumes)
$R_R$ = material removal rate
S = turning speed
f = feed rate
ω = rotational velocity in RPM
D = part diameter
d = depth of cut

Individually, (Equation 50) has an error level of 33 percent. This is high but also a vast improvement over the benchmark 71 percent error. Furthermore, using (Equation 42) in conjunction with (Equation 50), to calculate total cost, nets a total error of 15 percent when compared to actual costs. This level of error is 7.5 percent improved over the comparison benchmark method.

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# Appendix A: Cursory Explanation of the Benchmark Forging Cost Model Flowchart

Figure A1 on the following page is a color-coded flow chart designed to simplify the benchmark forging cost estimation process so that it may be contrasted using a similar chart in the design of a new cost estimation process.

Before beginning to explain the flow chart itself it is important to notice several key details. Namely, that the mention of all calculation details is omitted in favor of displaying only key flow paths including: volume calculations, database information, and forging cost contributing operations.

Regarding the volume calculations, this flow chart is designed to break several key variables out of their corresponding cost equations in order to display from whence they came and to what larger processes they may contribute. In this way they may be deemed critical or expendable variables for further cost estimation process designs. Regarding database information and forging cost contributing operations, in order to better track variables and their respective flow lines, the flow chart is broken down into color coded flow paths including, volume calculations (blue), material (brown), G.E. (green), and forging databases (orange), and larger processing operations (black).

The volume calculations path includes the steps to get to the two different volume variables, billet and sonic. All contributing variables to this end are simply lumped into "feature attributes" due to the simplicity of such inputs (dimensions, density, etc).

The material database path includes three key variables, material density, forging factor, and machinability factor. These variables are constants inherent to every material. Contrasting this are the forging database variables, vendor markup, machining difficulty factors, configuration factor, and process factor. The composition and usefulness of these variables are questionable at best and seem to stray more onto the side of being "fudge" factors developed through many trials of individual portions of the overall cost estimation equation. Hence, many of these variables are not used in further cost estimation developments. The manufacturing database houses the variables: escalation factor and market factor. These will vary between forging firms and market conditions that determine the value of a forging in terms of being a sought after good.

Finally, all the variables under these four flow paths filter into on of the main forging cost estimation processes, material, process, or machining costs. The results of these calculations the sum into the total forging cost. Below is a summary of the individual variables mentioned above and where they fit into the major flow paths.

| | |
|---|---|
| Volume calculations: | billet volume, sonic volume |
| Material database: | material density, forging factor, machinability factor |
| Manufacturing database: | escalation factor, market factor |
| Forging database: | vendor markup, machining difficulty factor, configuration factor, process factor |

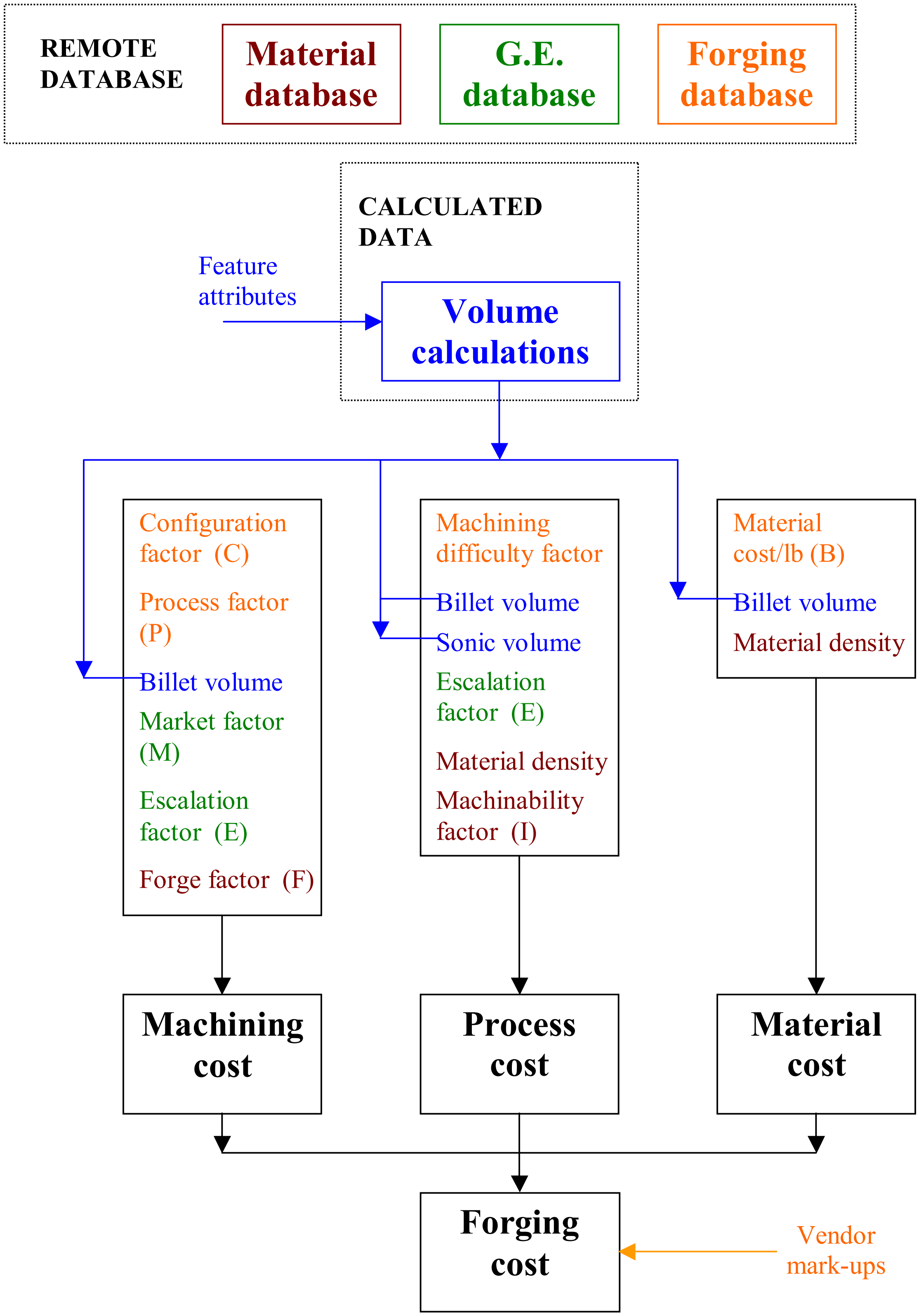


**Figure A1: benchmark forging cost model**

# Appendix B: Cursory Explanation of the Prototype Forging Cost Model Flowchart

Figure B1 on the following page is a color-coded flow chart designed to simplify a prototype forging cost estimation process in contrast to a similar flow chart in Appendix A documenting the benchmark process. Similar to that flowchart, this one is broke down into important variables and color-coded flow lines as well.

Different from Figure A1, volume calculations now include generic volume calculations arriving at a billet volume variable as well as several other calculations (shape complexity factor, work-piece area in contact with die, and average die pressure) that involve basic material data and dimensions and later filter into the process cost.

The three database flow paths remained the same as in the benchmark program. However, only variables that were understood and made significant contributions to either the physics of the forging process (which could later be translated into cost) or sales/market figures (that were simply a matter of market documentation) were left to contribute to the new cost estimation design. All unknown or suspected "fudge" factors were removed.

Again, all the variables under these four flow paths filter into one of the main forging cost estimation processes. However, sonic inspection cost and machining cost were broken out of the general flow as these calculations are not subject to the data in this document, follow conventional machining calculation methods, and simply appear as "plug in" values in forging cost data. Instead all charted variable make contributions solely to their process and/or material costs. Below is a summary of the individual variables mentioned above and where they fit into the major flow paths.

| | |
|---|---|
| Volume calculations: | billet volume |
| Material database: | material density, forge factor |
| GE database: | escalation factor, market factor |
| Forging database: | material cost per pound, configuration factor, vendor markup |

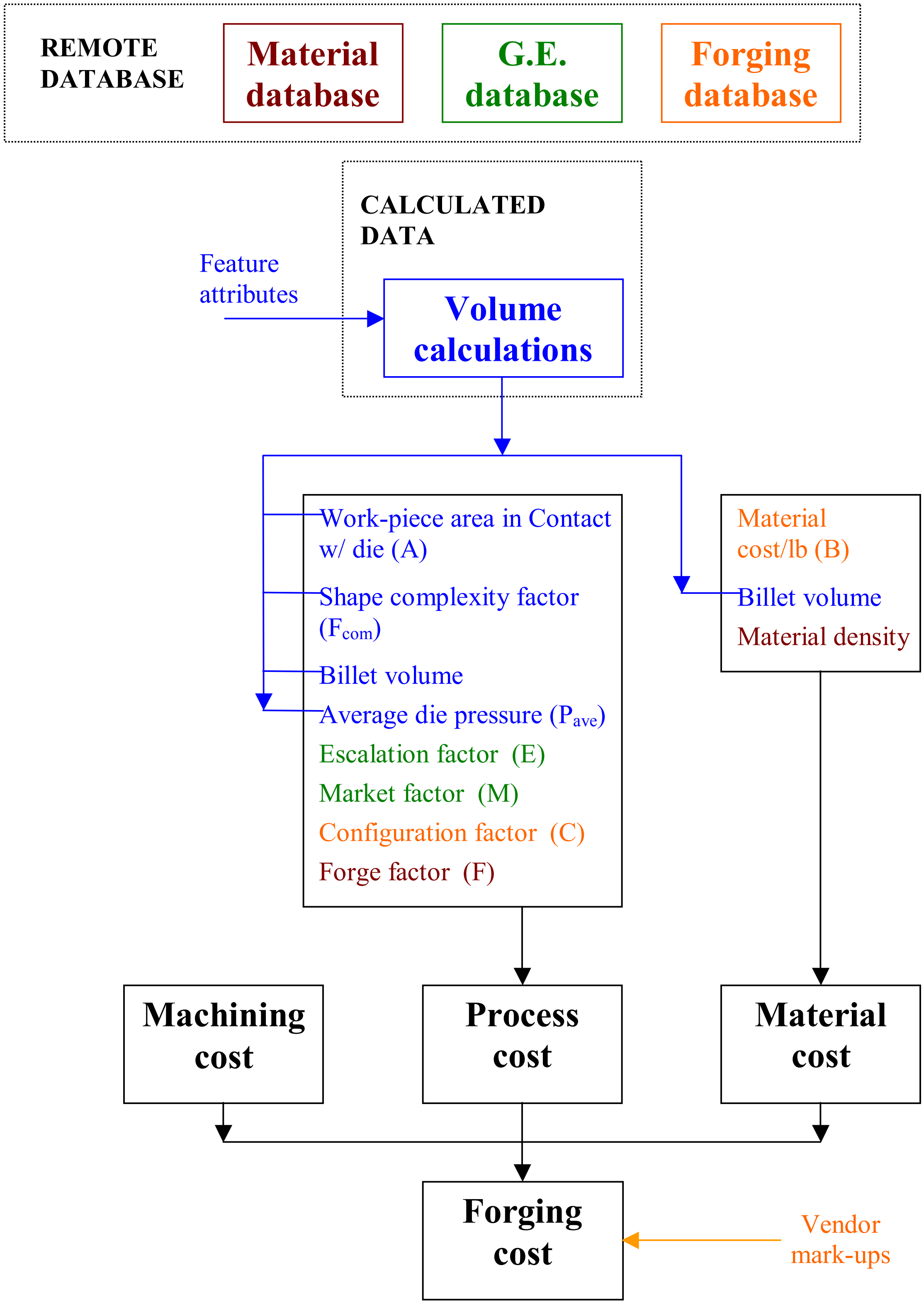


**Figure B1: prototype forging cost model**

# Appendix C: Volume Estimation Using the Benchmark Model

The benchmark model of estimating the volume of a forged part does not use a stand-alone mathematical model. Instead, it relies on a set of part attributes around which a set of variable volume equations revolve. Due to the attribute nature of these variable equations it would be very difficult to expound on any final volume equation as the component mathematics vary between part types. This then forces all final volume equations to differ significantly from part to part. The following will attempt to examine shaft attribute volume equations in order to illustrate the analysis process. Though these exact equations are not valid for every type of part they still provide valuable insight into the logical processes and complexity involved in the benchmark volume estimation model.

$$V_{shaft} = V_{cyl} + V_{cone}$$

**(Equation C1)**

Note that the following systems of equations have been recorded similar to the form of computer logic. The proper way to review them is to examine a part and use the volume equations based on the part's features according to the "if" statements. "if" statements generally ask whether or not a part has a certain feature and/or the location of such a feature. In the following equations A is equal to 0.1, 0.175, or 0.25 depending on the shape of the part and if the desired result is for the forged volume or the sonic volume.

## *Volume of a Cylinder*

$$V_{cyl} = V_1 - V_2 + V_3 - V_4$$

**(Equation C2)**

Where:

$V_1$ = flange outer forging volume
$V_2$ = flange inner forging volume
$V_3$ = shaft outer forging volume
$V_4$ = shaft inner forging volume

$V_1$, $V_2$, $V_3$, and $V_4$ are all attribute specific equations that are utilized depending on part feature scenarios.

If: $L_{shaft} < 4.0$ and cone = "none" then,

$V_1 = 0$

$V_2 = 0$

$$V_3 = \frac{\pi}{4} D^2 H$$

$D = OD_{shaft} + (2*0.05)$

$H = L_{shaft} + (2*0.05)$

$V_4 = 0$

Else:

If: flange shaft = "none" or "ID" then,

$V_1 = 0$

$V_2 = 0$

$$V_1 = \frac{\pi}{4} D^2 H$$

$D = OD_{shaft} + 2 + 2A$

$H = 2A + 0.175$

$$V_2 = \frac{\pi}{4} D^2 H$$

$D = OD_{shaft} + 2A$

$H = 2A + 0.175$

$$V_3 = \frac{\pi}{4} D^2 H$$

$D = OD_{shaft} + 2A$

$H = L_{shaft} + 2A$

If: $ID_{shaft} \leq 6.0$ then,

$V_4 = 0$

$$V_4 = \frac{\pi}{4} D^2 H$$

If: shaft flange = "OD" or "none" then,

$D = OD_{shaft} - 2A$

If: shaft flange = "ID" then,

$D = OD_{shaft} - 2 - 2A$

$H = L_{shaft} + 2A$

### *Volume of a Cone*

$$V_{cone} = V_1 - V_2 + V_3 - V_4 + V_5$$

**(Equation C3)**

Where:

$V_1 - V_2$ = cone flange volume
$V_3 - V_4$ = inside/outside appendage volume
$V_5$ = cone forging volume

$V_1$, $V_2$, $V_3$, $V_4$, and $V_5$ are all attribute specific equations that are utilized depending on part feature scenarios.

If: appendage configuration = "none" then,

If: cone flange = "none" or "ID" then,

$V_1 = 0$
$V_2 = 0$

$$V_1 = \frac{\pi}{4} D^2 H$$

$D = OD_{cone\ max} + 2 + 2A$
$H = Thickness_{flange} + 2A(Thickness_{flange} = 0.175)$

$$V_2 = \frac{\pi}{4} D^2 H$$

$D = OD_{cone\ max}$
$H = Thickness_{flange} + 2A$

If: cone flange = "OD" or "none" then,

$$V_3 = \frac{\pi}{4} D^2 H$$

$D = OD_{cone\ max}$
$H = thickness_{flange} + 2A$

$$V_4 = \frac{\pi}{4} D^2 H$$

$D = OD_{shaft} + 2A$
$H = thickness + 2A$

$V5 = V_{5A} - V_{5B}$

$$V_{5A} = \frac{\pi}{4}\left[OD^2 + (ID)(OD) + ID^2\right]L$$

$OD = OD_{cone\ max}$
$ID = ID_{cone\ min}$
$L = length_{cone} - thickness_{flange}$

$$V_{5B} = \frac{\pi}{4}\left[OD^2 + (ID)(OD) + ID^2\right]L$$

OD = $OD_{cone\ max}$
ID = $ID_{cone\ min}$
L = $length_{cone}$ – thickness

If: cone flange = "ID" then,

$V_3 = V_{3A} + V_{3B}$

$$V_{3A} = \frac{\pi}{4} D^2 H$$

$$V_{3B} = \frac{\pi}{12}\left[D^2 + (ID)(D) + ID^2\right]L$$

D = $OD_{cone\ max}$ + 2A
ID = $ID_{cone\ max}$
H = $thickness_{flange}$ + 2A
L = $length_{cone}$ – $thickness_{flange}$

$$V_4 = \frac{\pi}{12}\left[D^2 + (ID)(D) + ID^2\right]L$$

D = $OD_{cone\ max}$ – 2 – 2A
ID = $ID_{cone\ min}$

$$L = \frac{\left(length_{cone} - thickness_{flange}\right)\left(OD_{cone\max} - 2 - 2A\right)}{\left(OD_{cone\max} + 2A\right)}$$

$V_5 = V_{5A} - V_{5B}$

$$V_{5A} = \frac{\pi}{12}\left[OD^2 + (ID)(OD) + ID^2\right]L$$

OD = $OD_{cone\ max}$
ID = $ID_{cone\ min}$
L = $length_{cone}$ – $thickness_{flange}$

$$V_{5B} = \frac{\pi}{12}\left[OD^2 + (ID)(OD) + ID^2\right]L$$

OD = $OD_{cone\ max}$
ID = $ID_{cone\ min}$
L = $length_{cone}$ – thickness

If: appendage configuration = "inside" then,

$$V_3 = \frac{\pi}{4} D^2 H$$

D = $OD_{cone\ max}$ + 2A
H = $thickness_{flange}$ + 2A

$$V_4 = \frac{\pi}{4} D^2 H$$

D = $OD_{shaft}$ + 2A
H = $height_{cone}$ + 2A

$$V_5 = \frac{\pi}{12}\left[OD^2 + (ID)(OD) + ID^2\right]L$$

$OD = OD_{cone\ max}$
$ID = ID_{cone\ min}$
$L = length_{cone} - thickness_{flange}$

If: appendage configuration = "outside" then,

$$V_3 = \frac{\pi}{4}D^2H$$

$D = OD_{cone\ max}$
$H = length_{cone} + 2A$

$$V_4 = \frac{\pi}{4}D^2H$$

$D = OD_{shaft} + 2A$
$H = thickness + 2A$

$$V_5 = \frac{\pi}{12}\left[OD^2 + (ID)(OD) + ID^2\right]L$$

$OD = OD_{cone\ max}$
$ID = ID_{cone\ min}$
$L = thickness_{flange} - length_{cone}$

If: appendage configuration = "both" ("inside" and "outside") then,

$$V_3 = \frac{\pi}{4}D^2H$$

$D = OD_{cone\ max}$
$H = length_{cone} + 2A$

$$V_4 = \frac{\pi}{4}D^2H$$

$D = OD_{shaft} + 2A$
$H = length_{cone} + 2A$
$V_5 = 0$

Following the completion of this progression of equations, or a similar set depending on if the part in question is not a shaft, one should have two volumes derived from the components of (Equation C2) and (Equation C3), respectively. These volumes should then be summed in equation (Equation C1) in order to arrive at the estimated volume of the forged part prior to any machining.

# Appendix D: Stick Method Example Sketches

The following figures display example of how stick figures should be sketched based on finished part shape. There are four examples; each of which consists of a drawing of the cross-section of an actual part followed by a sketch of how the stick drawing might look. Stick sketches should consist of horizontal or angled sticks, which, if given thickness, would form an approximation of the forged part. Connections between sticks are shown by lighter dotted lines. It may be noted that some connections would prevent sticks from touching if given thickness. Such connections would not actually appear in a scaled sketch. Instead they are present simply to show delineation between different stick segments and their relative positions.

As the forging process tends to leave excess material on all part dimensions and especially cavities and curves, the stick sketch should not too much resemble the final part for fear of underestimating the forged volume and/or over complicating volume calculations. Such excess forging material will be removed later during sonic volume estimation and machining stages.

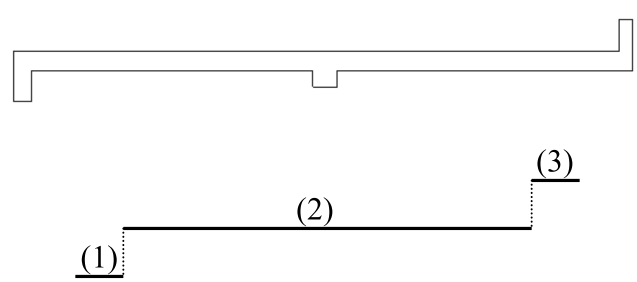


**Figure D1: sketch and stick diagram of a cutaway drum-shaft**

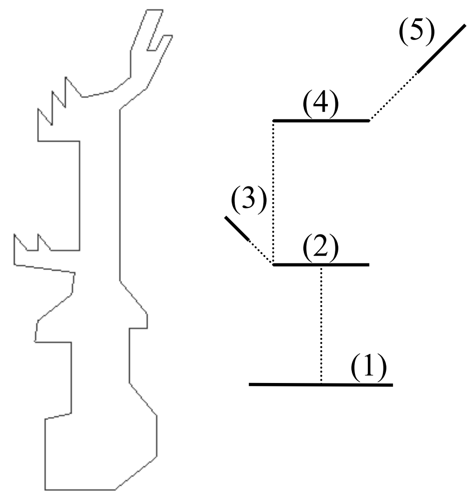


**Figure D2: sketch and stick diagram of a cutaway seal**

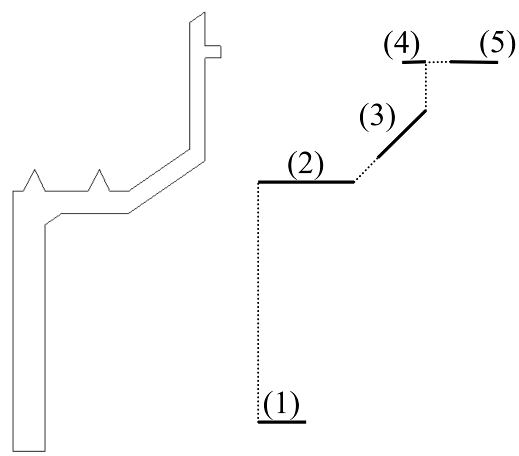


**Figure D3: sketch and stick diagram of a cutaway seal**

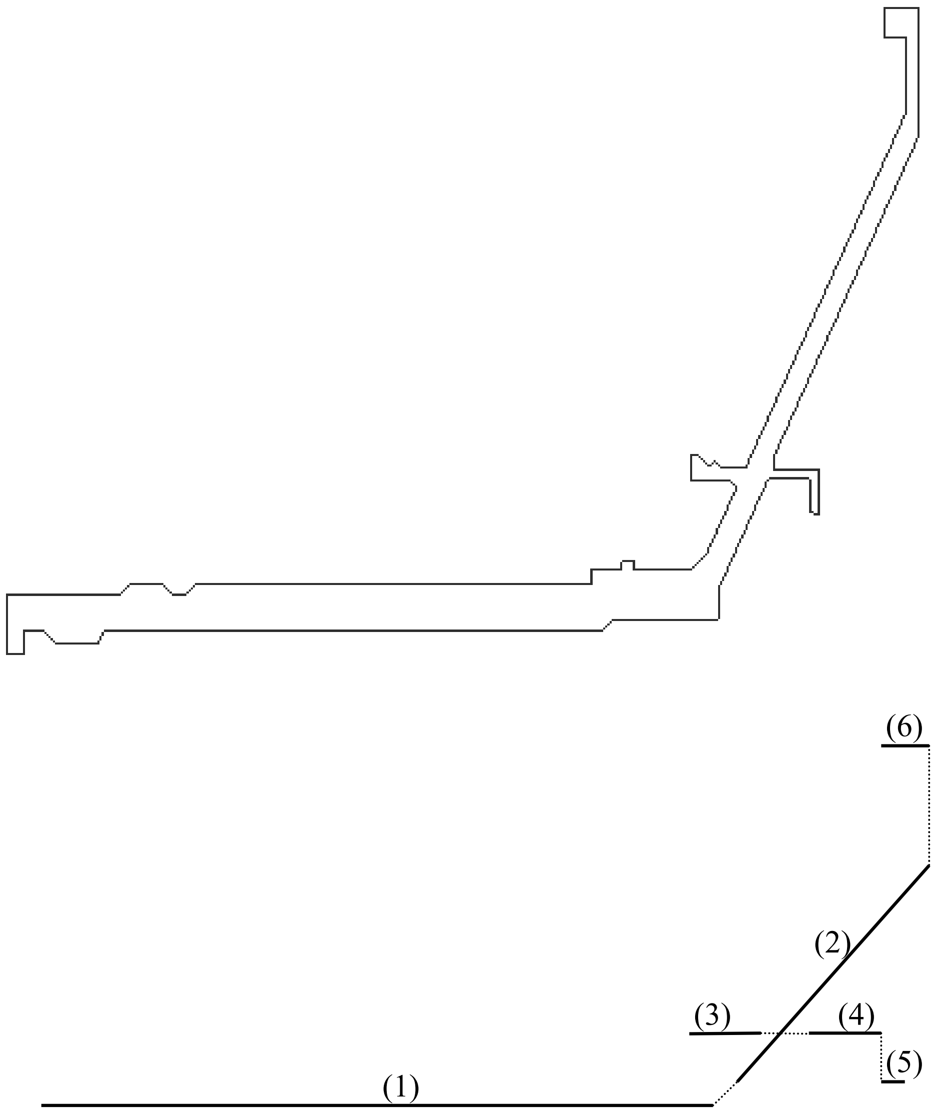


**Figure D4: sketch and stick diagram of a cutaway short-shaft with a cone**

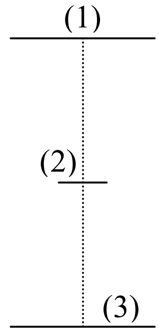


**Figure D5: stick diagram of a cutaway disk seal**

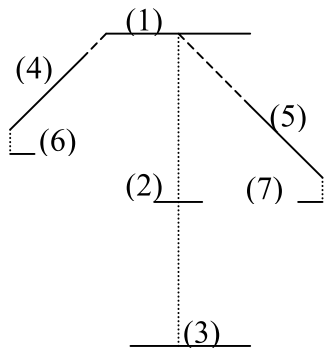


**Figure D6: stick diagram of a cutaway disk seal**

# Appendix E: Stick Method Mathematical Proofs

The following appendix seeks to walk through the mathematics upon which the stick method of volume estimation is based. This will be done by looking at the equations behind critical shape configurations and dimensioning methods. In general, these shapes will first be explained using conventional geometric volume equations with some manipulation. Then, these same figures will be analyzed using calculus integration techniques. In all cases the resulting volumetric equations are equal through some simplification. Note that the notation of V = volume is used repeatedly to signify the volume of the figure appropriate to the method currently receiving analysis.

## *Volume of a Frustum of a Right Circular Cone*

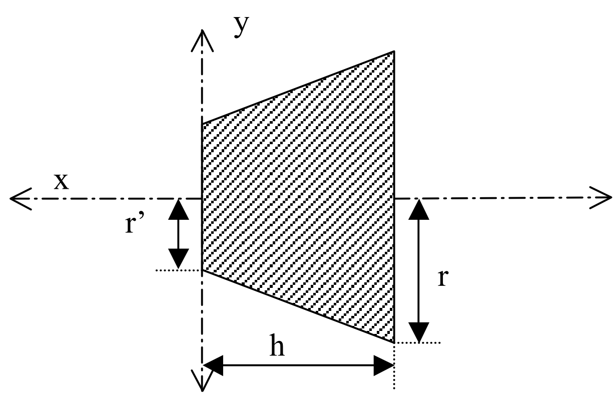


**Figure E1: cutaway of a frustum of a right circular cone using conventional dimension notations**

By convention it is known:

$$V = \frac{\pi h}{3}\left(r^2 + rr' + r'^2\right)$$

**(Equation E1)**

# Geometric Analysis

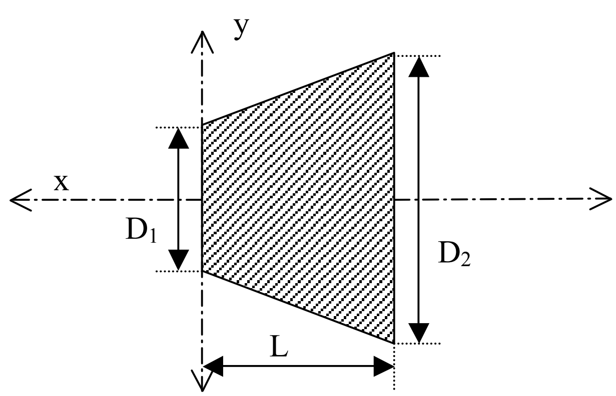


**Figure E2: cutaway of a frustum of a right circular cone using stick notation**

Based on (Equation E1) substituting the stick notation seen in Figure E2 for the more conventional notation shown in Figure E1:

$$V = \frac{\pi L}{3}\left(\frac{D_2^2}{4} + \frac{D_1 D_2}{4} + \frac{D_1^2}{4}\right)$$

simplified:

$$V = \frac{\pi L}{12}\left(D_2^2 + D_1 D_2 + D_1^2\right)$$

**(Equation E2)**

# Calculus Analysis

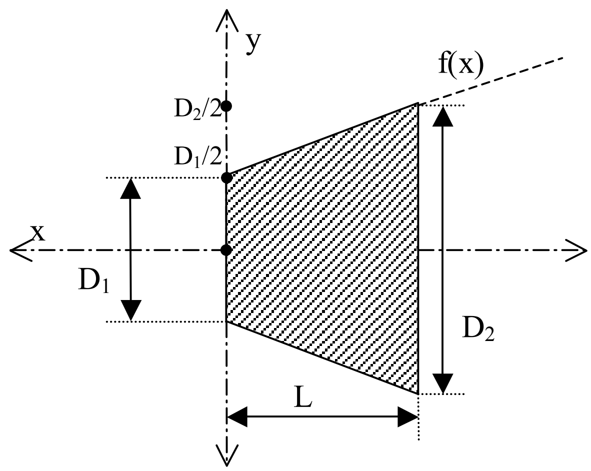


**Figure E3: cutaway of a frustum of a right triangle**

The volume of the frustum shown in Figure E3 can be described by revolving the following equation about x-axis between zero and L using the convention set forth in (Equation E3).

$$f(x) = \frac{(D_2 - D_1)x}{2L} + \frac{D_1}{2}$$

$$V = \pi \int_L^0 f(x)^2 \, dx$$

**(Equation E3)**

$$V = \pi \int_L^0 \left[ \frac{(D_2 - D_1)x}{2L} + \frac{D_1}{2} \right]^2 dx = \left. \frac{\pi[(D_1 - D_2)x - D_1 L]^3}{12L^2(D_1 - D_2)} \right|_0^L$$

Through substitution and simplification the resulting equation is equal to (Equation E2).

$$V = \frac{\pi L}{12}\left(D_2^2 + D_1 D_2 + D_1^2\right)$$

## *Volume of a Frustum of a Right Circular Cone Shell*

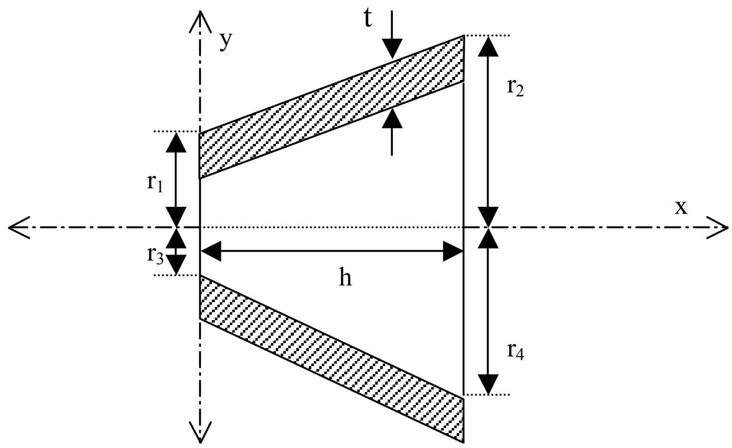


**Figure E4: cutaway of a frustum of a right circular cone shell using conventional dimension notations**

By convention it is known:

$$V_f = \frac{\pi h}{3}\left(r_2^2 + r_1 r_2 + r_2^2\right)$$

Thus, if:

$$V = V_{f_1} - V_{f_2}$$

Then:

$$V = \frac{\pi h}{3}\left(r_2^2 + r_1 r_2 + r_1^2 - r_4^2 - r_3 r_4 - r_3^2\right)$$

If:

$$r_3 = r_1 - t$$
$$r_4 = r_2 - t$$

Then, by substitution:

$$V = \frac{\pi h}{3}\left[r_2^2 + r_1 r_2 + r_1^2 - (r_2 - t)^2 - (r_1 - t)(r_2 - t) - (r_1 - t)^2\right]$$

$$V = \frac{\pi h}{3}\left[(3r_1 + 3r_2)t - 3t^2\right]$$

$$V = \pi h t\left(r_1 + r_2 - t\right)$$

**(Equation E4)**

## Geometric Analysis

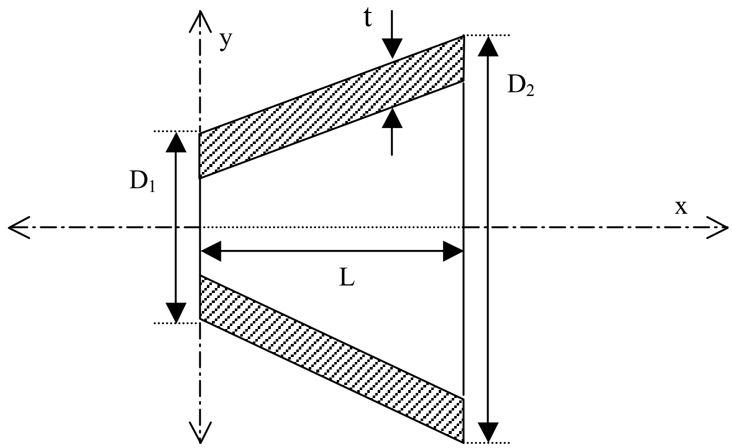


**Figure E5: cutaway of a frustum of a right circular cone shell using stick notation from endpoints**

Combining stick and conventional equations for substitution into (Equation E4):

$$r_1 = \frac{D_1}{2}$$

$$r_2 = \frac{D_2}{2}$$

$$h = L$$

$$V = \frac{\pi h}{3}\left[\left(3r_1 + 3r_2\right)t - 3t^2\right] = \frac{\pi L}{3}\left[\left(\frac{3D_1}{2} + \frac{3D_2}{2}\right)t - 3t^2\right]$$

$$V = \frac{\pi L t}{2}\left(D_1 + D_2 - 2t\right)$$

**(Equation E5)**

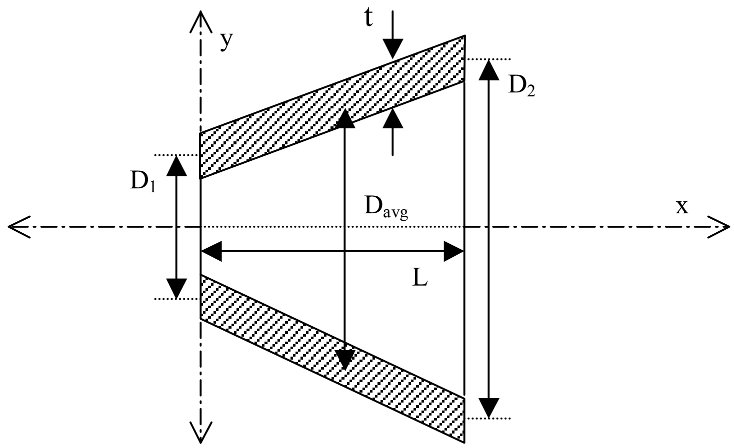


**Figure E6: cutaway of a frustum of a right circular cone shell using stick notation from midpoints**

Notice that all D values are measured from the midpoint of t.

Combining endpoint dimensioning from Figure E7 with midpoint notations:

$$D_{1_{old}} = D_{1_{new}} + t$$
$$D_{2_{old}} = D_{2_{new}} + t$$

Thus:

$$V = \frac{\pi L t}{2}\left[\left(D_{1_{new}} + t\right) + \left(D_{2_{new}} + t\right) - 2t\right]$$

Dropping the subscript "new" and simplifying further:

$$V = \frac{\pi L t}{2}\left(D_1 + D_2\right)$$

If:

$$D_{avg} = \frac{\left(D_1 - D_2\right)}{2}$$

**(Equation E6)**

Then:

$$V = \pi L t D_{avg}$$

**(Equation E7)**

# Calculus Analysis

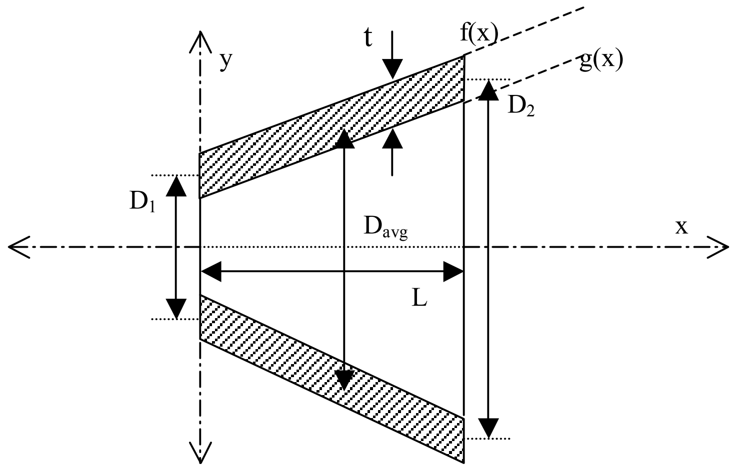


**Figure E7: cutaway of a frustum of a right circular cone shell using stick notation from endpoints**

Notice that all D values are measured from the midpoint of t. Similar to previous examples, the volume of a shell may be found by revolving f(x) and g(x) around the x-axis from zero to L using an adaptation of (Equation E3).

$$f(x) = \frac{(D_2 - D_1)x}{2L} + \frac{D_1}{2} + \frac{t}{2}$$

$$g(x) = \frac{(D_2 - D_1)x}{2L} + \frac{D_1}{2} - \frac{t}{2}$$

$$V = \pi \int_0^L \left[ f(x)^2 - g(x)^2 \right] dx$$

$$V = \pi \int_0^L \left\{ \left[ \frac{(D_2 - D_1)x}{2L} + \frac{D_1}{2} + \frac{t}{2} \right]^2 - \left[ \frac{(D_2 - D_1)x}{2L} + \frac{D_1}{2} - \frac{t}{2} \right]^2 \right\} dx$$

$$V = \frac{\pi L t}{2} (D_1 + D_2)$$

**(Equation E8)**

When dimensions are not measured from endpoints:

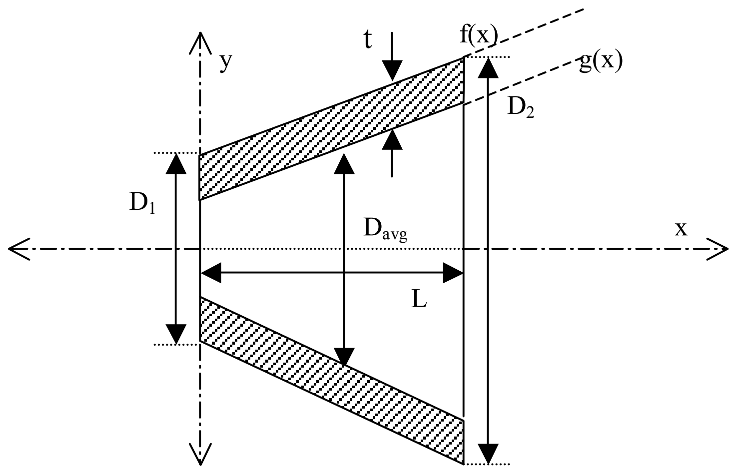


**Figure E8: cutaway of a frustum of a right circular cone shell using stick notation from endpoints**

$$f(x) = \frac{(D_2 - D_1)x}{2L} + \frac{D_1}{2}$$

$$g(x) = \frac{(D_2 - D_1)x}{2L} + \frac{D_1}{2} - t$$

$$V = \pi \int_0^L \left[ f(x)^2 - g(x)^2 \right] dx$$

$$V = \pi \int_0^L \left\{ \left[ \frac{(D_2 - D_1)x}{2L} + \frac{D_1}{2} \right]^2 - \left[ \frac{(D_2 - D_1)x}{2L} + \frac{D_1}{2} - t \right]^2 \right\} dx$$

$$V = \frac{\pi L t}{2}(D_1 + D_2 - 2t)$$

**(Equation E9)**

Both results calculate out to the same answer using different systems of measurement. This accounts for the difference in the appearance of the equations.

### *Volume of a Hollow Cylinder*

## Geometric Analysis

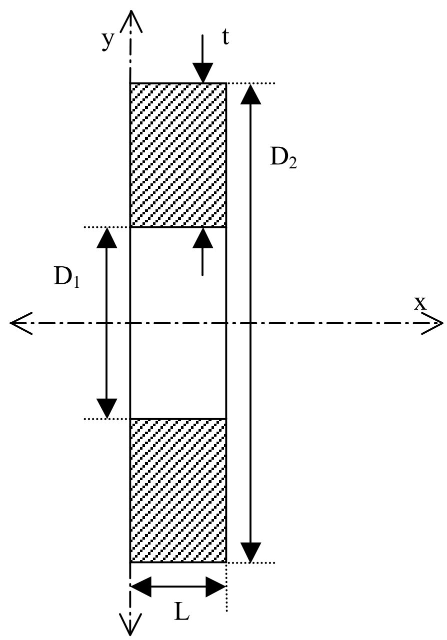


**Figure E9: cutaway of a hollow cylinder using stick notation from endpoints**

Using the conventional cylindrical volume (Equation E10):

$$V_{cylinder} = \frac{\pi}{4} L D^2$$

**(Equation E10)**

Where:

$V_{cylinder}$ = volume of outside ring of material
$V_{hollow}$ = volume of inside cylinder of absent material

Thus:

$$V = V_{cylinder} - V_{hollow}$$

$$V = \frac{\pi}{4} L D_2^2 - \frac{\pi}{4} L D_1^2$$

$$V = \frac{\pi L}{4}\left(D_2^2 - D_1^2\right)$$

**(Equation E11)**

$$V = \pi L\left(\frac{D_2 + D_1}{2}\right)\left(\frac{D_2 - D_1}{2}\right)$$

$$t = \frac{D_2 - D_1}{2}$$

**(Equation E12)**

By substituting (Equation E6) and (Equation E12):

$$V = \pi L t D_{avg}$$

Or, using if distances are measured from the midpoints:

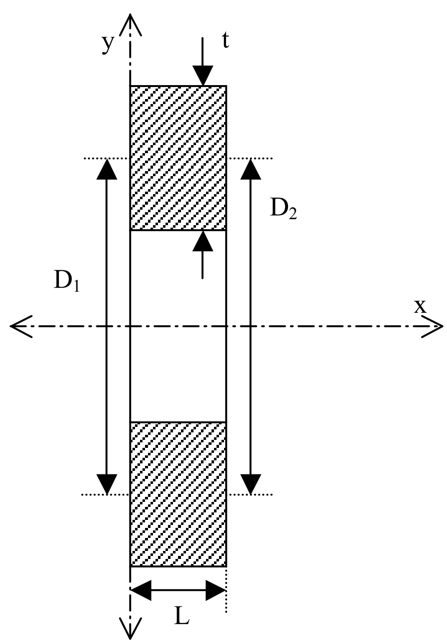


**Figure E10: cutaway of a hollow cylinder using stick notation from midpoints**

Substituting the midpoint equivalents of the endpoint dimensions in shown in Figure E10 into (Equation E11):

$$V = \frac{\pi L}{4}\left[(D_2 + t)^2 - (D_1 - t)^2\right]$$

$$V = \frac{\pi L}{4}\left[D_2^2 + 2(D_1 + D_2)t - D_1^2\right]$$

If: $D_1 = D_2$

$$V = \pi L t D_2 = \pi L t D_1$$

**(Equation E13)**

## Calculus Analysis

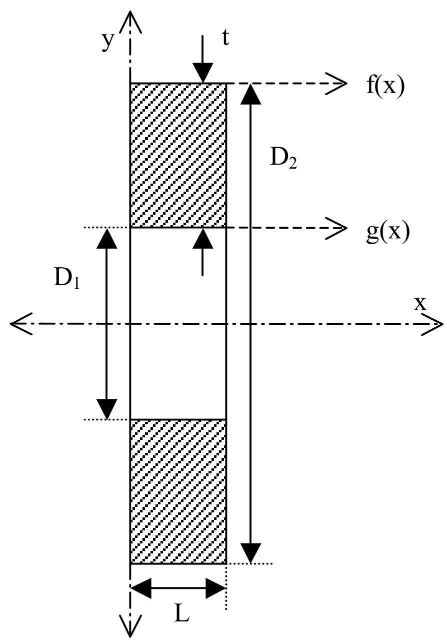


**Figure E11: cutaway of a hollow cylinder using stick notation from endpoints**

Using the same adaptation of (Equation E13) seen previously where:

$$f(x) = \frac{D_2}{2} \qquad\qquad g(x) = \frac{D_1}{2}$$

$$V = \pi \int_0^L \left[\left(\frac{D_2}{2}\right)^2 - \left(\frac{D_1}{2}\right)^2\right] dx$$

$$V = \frac{\pi L}{4}\left(D_2^2 - D_1^2\right)$$

$$V = \pi L t D_{avg}$$

**(Equation E14)**

Or, using if distances are measured from the midpoints:

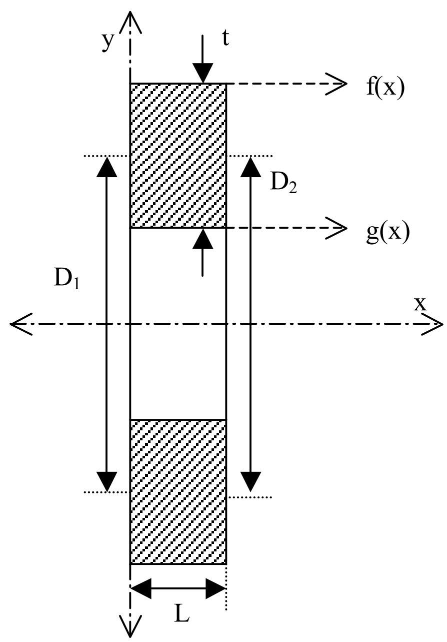


**Figure E12: cutaway of a hollow cylinder using stick notation from midpoints**

$$f(x) = \frac{D_1 + t}{2} \qquad g(x) = \frac{D_1 - t}{2}$$

$$V = \pi \int_0^L \left[ \left( \frac{D_1 + t}{2} \right)^2 - \left( \frac{D_1 - t}{2} \right)^2 \right] dx$$

$$V = \pi L t D_1 = \pi L t D_2$$

**(Equation E15)**

### *Surface Area of a Cone (Excluding Circular Ends)*

## Geometric Analysis

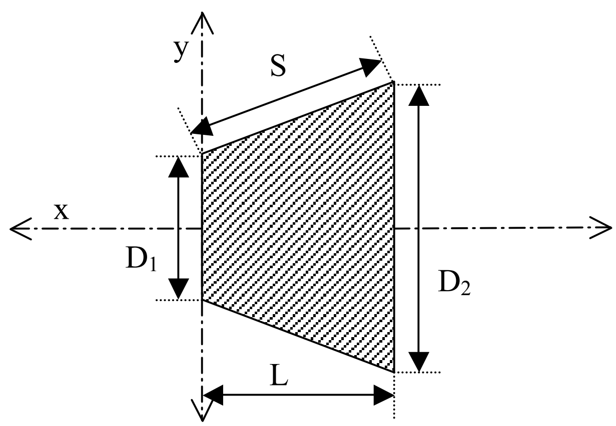


**Figure E13: cutaway of a frustum of a right triangle with S as a side length**

According to convention:

$$A = \pi S\left(\frac{D_2}{2} + \frac{D_1}{2}\right)$$

**(Equation E16)**

When:

$$S = \sqrt{\left(\frac{D_2}{2} - \frac{D_1}{2}\right)^2 + L^2} \qquad \text{and} \qquad D_{avg} = \frac{(D_2 + D_1)}{2}$$

Thus:

$$A_S = \pi S D_{avg}$$

**(Equation E17)**

# Calculus Analysis

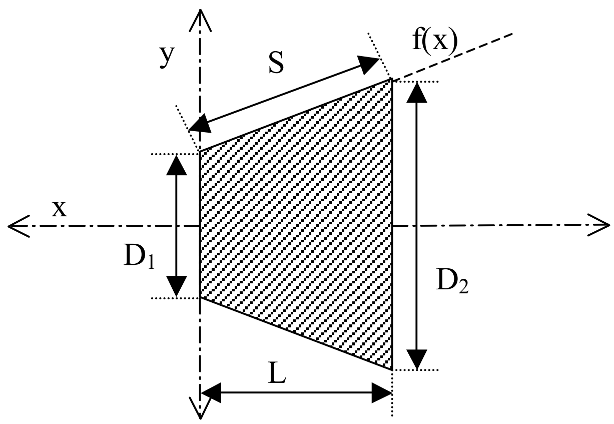


**Figure E14: cutaway of a frustum of a right triangle with S as a side length and line f(x)**

The surface area of a frustum of a right triangle (excluding fore and aft end areas) may be found by calculating the outside area of a revolved equation f(x) between zero and L using the following equation:

$$A_S = \int_A^B 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

**(Equation E18)**

Where: f(x) = y and:

$$y = \frac{(D_2 - D_1)x}{2L} + \frac{D_1}{2}$$

$$\frac{dy}{dx} = \frac{(D_2 - D_1)}{2L}$$

Through substitution:

$$A_S = \int_0^L \left\{ 2\pi \left[ \frac{(D_2 - D_1)x}{2L} + \frac{D_1}{2} \right] \sqrt{1 + \left( \frac{D_2 - D_1}{2L} \right)^2} \right\} dx$$

$$A_S = \frac{\pi}{4}(D_2 + D_1)\sqrt{4L^2 + (D_1 - D_2)^2}$$

$$A_S = \frac{\pi}{4}(D_2 + D_1)\sqrt{4L^2 + (D_1 - D_2)^2}$$

$$A_S = \pi\left(\frac{D_2}{2} + \frac{D_1}{2}\right)\frac{1}{2}\sqrt{4L^2 + (D_1 - D_2)^2}$$

$$A_S = \pi D_{avg}\sqrt{\left(\frac{D_1}{2} - \frac{D_2}{2}\right)^2 + L^2}$$

Since $\left(\frac{D_1}{2} - \frac{D_2}{2}\right)$ is squared:

$$A_S = \pi D_{avg}\sqrt{\left(\frac{D_2}{2} - \frac{D_1}{2}\right)^2 + L^2}$$

$$A_S = \pi D_{avg} S$$

**(Equation E19)**

## ***Surface Area of a Shell (Excluding Circular Ends)***

## Geometric Analysis

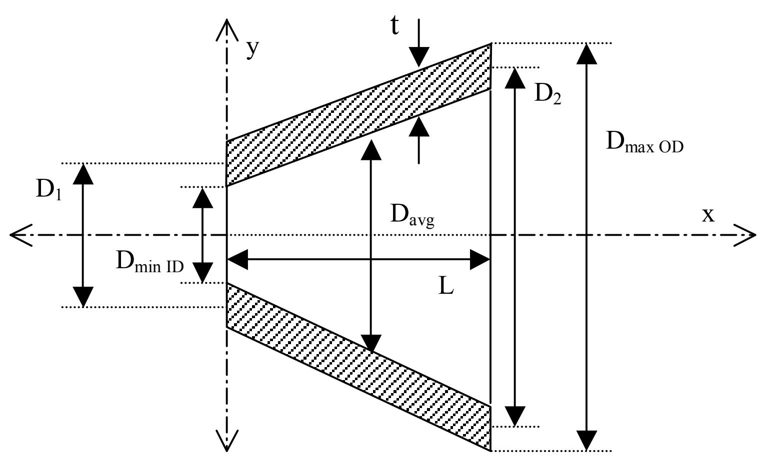


**Figure E15: cutaway of a frustum of a right circular cone shell using $D_{max}$ dimensions**

Using (Equation E17):

$$A_S = \pi D_{avg} S$$

$$A_S = A_{OD} + A_{ID}$$

Where:

$A_{OD}$ = outer conical surface area
$A_{ID}$ = inner conical surface area

By modifying (Equation E17):

$$A_{OD} = \pi\left(D_{avg} + t\right)S$$

$$A_{ID} = \pi\left(D_{avg} - t\right)S$$

And as an aside:

$$D_{\min ID} = D_1 - t$$

$$D_{\max OD} = D_2 + t$$

Thus:

$$D_{avg} = \frac{D_1 + D_2}{2} = \frac{D_{\min ID} + D_{\max OD}}{2}$$

Additionally:

$$D_2 - D_1 = \left(D_{\max OD} + t\right) - \left(D_{\min ID} - t\right)$$

$$D_2 - D_1 = \left(D_{\max OD} - D_{\min ID}\right) - 2t$$

If:

$$R_{diff} = \frac{\left(D_{\max OD} - D_{\min ID}\right)}{2}$$

**(Equation E20)**

Then:

$$D_2 - D_1 = R_{diff} - t$$

Therefore:

$$S = \sqrt{\left(R_{diff} - t\right)^2 + L^2}$$

So:

$$A_S = A_{ID} + A_{OD} = 2\pi D_{avg} S$$

$$A_S = 2\pi D_{avg}\sqrt{\left(R_{diff} - t\right)^2 + L^2}$$

**(Equation E21)**

## Calculus Analysis

Since by calculus it was already proven that $A_S = \pi D_{avg} S$ and since $A_S = A_{OD} + A_{ID}$ then

$$A_S = A_{ID} + A_{OD} = 2\pi D_{avg} S$$

# Appendix F: Volume Estimation Example

The following is an example volume estimation using both the stick and benchmark methods. The drum shaft whose cross-section is pictured in Figure F1 is the sample part to be analyzed.

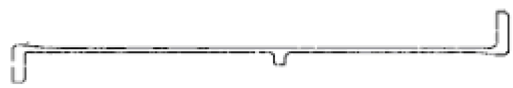

**Figure F1: stick/ benchmark example drum shaft**

## *Stick Method*

Step 1: Use horizontal and angled sticks to illustrate the general shape of the cross-section of the part to be analyzed, as shown in
Figure F2. The use of more sticks will likely increase the accuracy of the volume estimation to a point. However, it will also make analysis more difficult. Keep in mind that the formation of sticks should reflect how the part will look when forged not when production is complete. The newly forged part will have far more fill material that is later removed.

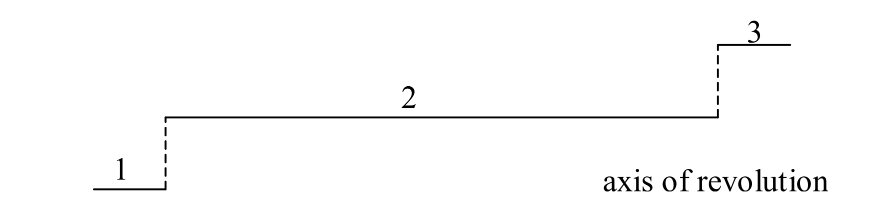


**Figure F2: shaft example as a stick sketch**

Step 2: For each of the created sticks measure minimum inside diameter ($D_{minID}$), maximum outside diameter ($D_{maxOD}$), length (L), and thickness (t). For each measurement be sure to consider the additional material that is removed from the forging volume. In the case of forged volumes this constant is 0.175, for sonic volumes 0.1. For example, two times the constant will be added to the maximum OD, length, and thickness. While two times the constant will be subtracted from the minimum ID. Creating a table like Table F1 may help in organizing the data for later use in calculations.

| | 1 | 2 | 3 |
|---|---|---|---|
| $D_{minID}$: | 10.9 | 12.11 | 12.66 |
| $D_{maxOD}$: | 13.51 | 12.985 | 14.87 |
| L: | 0.6 | 10.23 | 0.59 |
| t: | 1.305 | 0.788 | 1.455 |

**Table F1: Stick analysis specifications**

Step 3: Use the measurement values of the different stick sections in the following equations to calculate the volume of each segment.

$$V = \pi L t D_{avg}$$

If:

$$D_{avg} = \frac{D_{\max OD} + D_{\min ID}}{2}$$

Where:

D = diameter
L = axial length
t = radial thickness

Thus:

$$V = V_1 + V_2 + V_3$$

Step 4: Add all the volumes together to get the total estimated volume for the forged part.

$$V = \pi\left[L_1 t_1\left(\frac{D_{\max OD_1} + D_{\min ID_1}}{2}\right) + L_2 t_2\left(\frac{D_{\max OD_2} + D_{\min ID_2}}{2}\right) + L_3 t_3\left(\frac{D_{\max OD_3} + D_{\min ID_3}}{2}\right)\right]$$

$$V_1 = \pi(0.25)(0.955)\left(\frac{11.25+13.16}{2}\right) = 9.1544$$

$$V_1 = \pi(9.88)(0.438)\left(\frac{12.11+12.985}{2}\right) = 170.5839$$

$$V_1 = \pi(0.24)(1.105)\left(\frac{12.66+14.87}{2}\right) = 11.4683$$

Therefore:

V = 384.9126

Step 5: Calculate the sonic volume similar to the forging volume using the excess material constant of 0.1 instead of 0.175.

V = 298.2656

## *Benchmark Method*

Step 1: Decide which features the part being estimated has so that the proper equations may be used for volume estimation. As seen in Figure F3, the example part has both ID and OD flanges therefore, in benchmark terms as seen in Appendix C, shaft flange = "ID" and "OD."

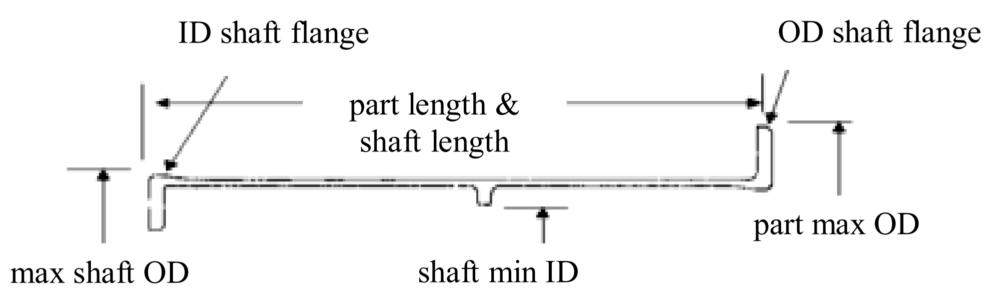


**Figure F3: shaft example labeled for benchmark analysis**

Step 2: Using the benchmark equations for the given featured part, step through the logical progression of steps to determine which measurements are crucial for the given shape. Table F2 illustrates the critical dimensions for a drum shaft with ID and OD flanges. Note that these same dimensions may not be critical for shapes with other features.

| | |
|---|---|
| $OD_{shaft\ max}$: | 12.635 |
| $L_{shaft}$: | 10.37 |
| $A_{forge}$: | 0.175 |
| $A_{sonic}$: | 0.1 |

**Table F2: Benchmark analysis specifications**

Step 3: Begin at the start of the correct benchmark analysis progression and note how the volume will be calculated. Step through the benchmark method carefully as each logical progression is different based on features and sizes. The following explains the movement through a shaft with an ID and OD flange.

$$V_{shaft} = V_{cyl} + V_{cone}$$

Since the example part has no cone features (cone = "none")

$$V_{shaft} = V_{cyl} = V_1 - V_2 + V_3 - V_4$$

Furthermore, since the example part has an OD flange (flange shaft = ID and OD):

$$V_1 = \frac{\pi}{4} D^2 H$$

Where:

$D = OD_{part\ max} + 2 + 2A$
$H = 2A + 0.175$

$$V_1 = \frac{\pi}{4}[12.985 + 2 + 2(0.175)]^2[2(0.175) + 0.175] = 92.5897$$

$$V_2 = \frac{\pi}{4} D^2 H$$

Where:

$D = OD_{part\ max} + 2A$
$H = 2A + 0.175$

$$V_2 = \frac{\pi}{4}[12.985 + 2(0.175)]^2[2(0.175) + 0.175] = 69.5237$$

$$V_3 = \frac{\pi}{4} D^2 H$$

Where:

D = $OD_{shaft}$ + 2A
H = $L_{shaft}$ + 2A

$$V_3 = \frac{\pi}{4}[12.985 + 2(0.175)]^2[10.37 + 2(0.175)] = 1419.608$$

$$V_4 = \frac{\pi}{4} D^2 H$$

Where, since the example part also has an ID flange (shaft flange = ID and OD):

D = $OD_{shaft}$ - 2 - 2A
H = $L_{shaft}$ + 2A

$$V_4 = \frac{\pi}{4}[12.985 - 2 - 2(0.175)]^2[10.37 + 2(0.175)] = 890.6217$$

V = 552.0527

Step 4: Calculate the sonic volume similar to the forging volume using the excess material constant of 0.1 instead of 0.175.

V = 386.5449

## *Results*

As noted above, the volume calculation for the example shaft using the stick method returned a value of V = 384.9126 while the benchmark method returned a value of V = 552.0527 resulting in a difference of nearly 100 percent. Needless to say this difference is significant. However, this difference is not a clear indication of which method is superior. In the presented example the shape of the shaft was relatively simple. Thus, the stick method was able to use an array of simple mathematical formulas to make, what would appear to be, a fairly accurate forging volume estimation. Whereas, the benchmark estimation was most likely adversely influenced by several assumptions in the utilized equations due to the example part's simplicity. For instance, benchmark assumed that the additional material diameter needed to forge a flange is always two units. The stick method uses the actual change in diameter from the shaft body to the tip of the flange to estimate additional material needs.

Similarly, benchmark assumes that when inside diameter features must be forged it is necessary to assume that the additional material needed will not only encompass the region of the feature but also the entire length of the inner shaft. This can mean the estimated addition of a significant amount of material in the case of a small, simple ID feature, as is the case in the above example. Conversely, the stick method limits the addition of forged material to the region of the given ID feature. However, his very same line of assumption can cause the stick method's estimation error to increase with the complexity of a given part while benchmark will, most likely, decrease.

However, as the method of forged volume estimation is not the focus of this document the behavior of the different methods of volume estimation will not be investigated further. Granted the use of one or the other of the methods may potentially alter the outcome of the forging cost estimation significantly. Hence, the superiority of the stick method was assumed during the course of data collection in building of an equitable forging cost estimation equation. This assumption by no means assumes perfection of the method of volume estimation used. Instead, the use of the same volume estimation method when comparing benchmark to the prototype forging cost model eliminates any volumetric error that may be inherent in either method. Thus the focus can be the forging process and not volume estimation.

# Appendix G: Record of Prototype Process Cost Equations

The following equations present a comprehensive tour of the four different forging process cost equations used in data trials as well as the original benchmark process equation.

## *Benchmark Forging Process Cost Equation*

The benchmark equation is composed of all material and market constants with the exception of billet weight. Hence, the equation's chief physical contribution to cost is material weight (W).

$$\text{process cost} = \mathrm{W}^{0.7}\, CP(F+M)E$$

**(Equation G1)**

W = billet weight = billet (forge) volume × material density
C = configuration factor
P = process factor
F = forge factor
M = market factor
E = escalation factor

## *Forging Process Cost Equation (Version 1)*

The first attempt at a forging cost equation used all of the benchmark constants with only slight alterations as to what physical factors contributed to the cost of forging. As loading weight has little to do with the cost of forging, assuming the correct size press is readily available, billet weight (W) was replaced by die contact area (A) and a multiplier indicating complexity of the forging shape ($F_{com}$). Additionally, process factor (P) was replaced with the physical variable of pressure needed to forge the part. This alteration was made due to the extensive unknowns used to make up the process factor.

$$\text{process cost} = A \cdot C \cdot P_{ave}(F+M)E \cdot F_{com}$$

**(Equation G2)**

A = work-piece contact area with die
C = configuration factor
F = forge factor for material (Battell)

M = market factor
E = escalation factor or labor costs per hour

$F_{com}$ = shape complexity factor $F = 0.3 * \frac{S^{\frac{1}{2}}}{V^{\frac{1}{3}}}$

S = forge surface area
V = forge volume

$P_{ave.}$ = average die pressure $P_{ave\,pres.} = Y + \frac{Y * D}{6 * h}$

Y = yield stress of material
D = die contact surface of work-piece
h = height of work-piece

## *Forging Process Cost Equation (Version 2)*

Version two or the forging process equation is equivalent to version 1 with only one addition. A constant was added to absorb some of the error that may have been compensated for using unknown process variables in the original benchmark equation. In this equation the error constant, K, is assigned as a simple multiplier.

$$\text{process cost} = A \cdot C \cdot P_{ave} K(F + M) E \cdot F_{com}$$

**(Equation G3)**

A = work-piece contact area with die
C = configuration factor
F = forge factor for material (Battell)
M = market factor
E = labor cost per hour

$F_{com}$ = shape complexity factor $F = 0.3 * \frac{S^{\frac{1}{2}}}{V^{\frac{1}{3}}}$

S = forge surface area
V = forge volume

$P_{ave}$ = average die pressure $P_{ave\,pres.} = Y + \frac{Y * D}{6 * h}$

Y = yield stress of material
D = die contact surface of work-piece
h = height of work-piece

K = error constant

With the addition of K as an error constant it was realized that all other constants could be combined to simplify the equation and remove reliance on various constants. Thus, K became a factor of configuration (C), forging (F), and market (M) factors or:

K(C,F,M) = error constant

This addition altered (Equation G3) as follows:

$$\text{process cost} = A \cdot P_{ave} KE \cdot F_{com}$$

**(Equation G4)**

## *Forging Process Cost Equation (Version 3)*

Due to the error inherent in an average pressure for such diverse and high-level forces that occur during the forging process, additional error factors were added to help compensate for both pressure and overall equation error. A slight variation on the same idea is also shown in (Equation G6) where the only change from (Equation G5) is the multiplier applied to the error factor $K_2$.

$$\text{process cost} = A\left(K_1 + K_2 \cdot P_{ave}\right) \cdot E \cdot F_{com}$$

**(Equation G5)**

$$\text{process cost} = \left(K_1 + K_2 A P_{ave}\right) E F_{com}$$

**(Equation G6)**

A = work-piece contact area with die
E = labor cost per hour

$F_{com}$ = shape complexity factor $\quad F = 0.3 * \dfrac{S^{\frac{1}{2}}}{V^{\frac{1}{3}}}$

S = forge surface area
V = forge volume

$P_{ave}$ = average die pressure $\quad P_{ave\,pres.} = Y + \dfrac{Y * D}{6 * h}$

Y = yield stress of material
D = die contact surface of work-piece
h = height of work-piece

$K_1$ = error constant 1
$K_2$ = error constant 2

## *Forging Process Cost Equation (Version 4)*

While keeping the same variable array of version 2, more complex error factors were added once again in an attempt to remove as much predictable error as possible. The Addition of n moved the process cost equation into higher order mathematics in the hopes of better curve matching. Both equations are similar in principle only the placement of the power factor n differing.

$$\text{process cost} = KAP_{ave}^{n}EF_{com}$$

**(Equation G7)**

$$\text{process cost} = K\left(AP_{ave}\right)^{n}EF_{com}$$

**(Equation G8)**

A = work-piece contact area with die
E = labor cost per hour

$F_{com}$ = shape complexity factor $\quad F = 0.3 * \dfrac{S^{\frac{1}{2}}}{V^{\frac{1}{3}}}$

S = forge surface area
V = forge volume

$P_{ave}$ = average die pressure $\quad P_{ave\,pres.} = Y + \dfrac{Y * D}{6 * h}$

Y = yield stress of material
D = die contact surface of work-piece
h = height of work-piece

K, $K_2$, n = error constants

# Appendix H: Calculation of Die Pressure Using Slab Method

The following is a mathematical explanation on the process equation variable $P_{ave}$ or the average die pressure on a forged part using the slab method of calculation. Note that all equations throughout the following process describe the pressure on and movement of the part described in Figure H1 and, given the level of mathematics; a prior knowledge of forging pressures is assumed.

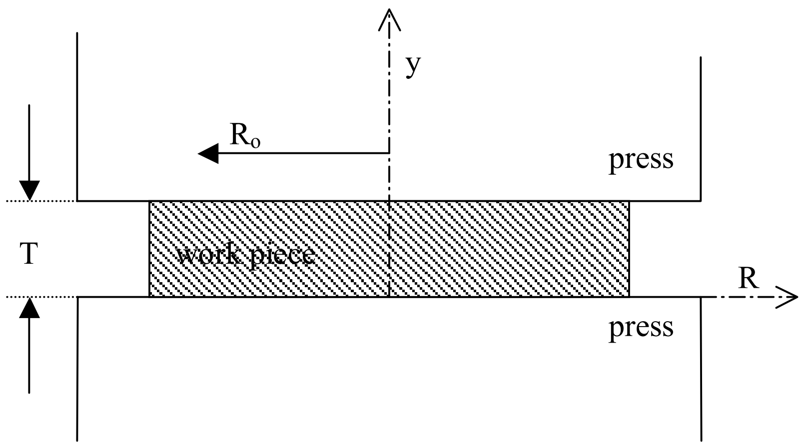


**Figure H1: cross-section of a cylindrical disk under forging compression**

Where, using Figure H1 and a cylindrical coordinate system, the velocity vector is:

$$\dot{U}_i\left(\dot{U}_R, \dot{U}_\theta, \dot{U}_y\right)$$

And strain components; $\dot{\varepsilon}_{ij}$, acquire the subscripts R, θ, and y.

$$\dot{\varepsilon}_{yy} = \frac{\partial \dot{U}_y}{\partial y} \qquad \dot{\varepsilon}_{RR} = \frac{\partial \dot{U}_R}{\partial R} \qquad \dot{\varepsilon}_{\theta\theta} = \frac{\dot{U}_R}{R} + \frac{1}{R}\frac{\partial \dot{U}_\theta}{\partial \theta}$$

$$\dot{\varepsilon}_{R\theta} = \frac{1}{2}\left(\frac{1}{R}\frac{\partial \dot{U}_R}{\partial \theta} + \frac{\partial \dot{U}_\theta}{\partial R} - \frac{\dot{U}_\theta}{R}\right)$$

$$\dot{\varepsilon}_{\theta y} = \frac{1}{2}\left(\frac{\partial \dot{U}_{\theta}}{\partial y} - \frac{1}{R}\frac{\partial \dot{U}_{y}}{\partial \theta}\right)$$

$$\dot{\varepsilon}_{yR} = \frac{1}{2}\left(\frac{\partial \dot{U}_{R}}{\partial y} - \frac{\partial \dot{U}_{y}}{\partial R}\right)$$

**(Equation H1)**

The equilibrium equations are then:

$$\frac{\partial \sigma_{R\theta}}{\partial R} + \frac{1}{R}\frac{\partial \sigma_{\theta\theta}}{\partial \theta} + \frac{\partial \sigma_{\theta y}}{\partial y} + 2\frac{\sigma_{R\theta}}{R} = 0$$

$$\frac{\partial \sigma_{Ry}}{\partial R} + \frac{1}{R}\frac{\partial \sigma_{\theta y}}{\partial \theta} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\sigma_{Ry}}{R} = 0$$

$$\frac{\partial \sigma_{RR}}{\partial R} + \frac{1}{R}\frac{\partial \sigma_{R\theta}}{\partial \theta} + \frac{\partial \sigma_{Ry}}{\partial y} + \frac{\sigma_{RR} - \sigma_{\theta\theta}}{R} = 0$$

**(Equation H2)**

Also pertaining to the above figure, the press is assumed to be a rigid body with the upper plate moving toward the lower plate at a velocity ($\dot{V}$) in the y direction. It is assumed that there is no rotation of the disk in the press or $\dot{U}_{\theta} = 0$ and that the cylinder being pressed remains concentric around the y-axis.

If

$$\Delta \dot{V} = 0 = 2\pi RT\,\dot{U}_{R} + \pi R^{2}\,\dot{U}$$

**(Equation H3)**

Then

$$\dot{U}_{R} = -\frac{1}{2}\frac{R}{T}\dot{U}$$

**(Equation H4)**

$$\dot{U}_{y} = \frac{y}{T}\dot{U}$$

**(Equation H5)**

Substituting these equations into (Equation H1) gives the strain-rate field:

$$\dot{\varepsilon}_{RR} = \dot{\varepsilon}_{\theta\theta} = -\frac{1}{2}\dot{\varepsilon}_{yy} = -\frac{1}{2}\frac{\dot{U}}{T} \qquad \dot{\varepsilon}_{R\theta} = \dot{\varepsilon}_{\theta R} = \dot{\varepsilon}_{yR} = 0$$

**(Equation H6)**

The internal power of deformation for the strain rate then becomes:

$$\dot{W}_i = \frac{2}{\sqrt{3}}\sigma_0 \int_V \sqrt{\frac{1}{2}\left(\dot{\varepsilon}_{RR}^2 + \dot{\varepsilon}_{\theta\theta}^2 + \dot{\varepsilon}_{yy}^2\right)}dV = \pi R_0^2 \sigma_0 \dot{U}$$

**(Equation H7)**

And, according to the constant shear assumption, the friction stress between press and disk:

$$\tau = m\frac{\sigma_0}{\sqrt{3}}$$

**(Equation H8)**

The total friction power loss is:

$$\Delta v = \dot{U}_R\Big|_{y=o,T} - 0 = \dot{U}_R\Big|_{y=0,T} = -\frac{1}{2}\frac{R}{T}\dot{U}$$

**(Equation H9)**

The external power supplied to the press through the upper plate is:

$$J^* = P\dot{U} = \frac{2}{\sqrt{3}}\sigma_0 \int_V \sqrt{\frac{1}{2}\dot{\varepsilon}_{ij}\dot{\varepsilon}_{ij}}\,dV + \int_{S_\Gamma} \tau\left|\Delta v\right|ds - \int_{S_t} T_i v_i ds$$

**(Equation H10)**

So that, using (Equation H7), (Equation H8), and (Equation H9):

$$P = \pi R_0^2 \sigma_0 \left(1 + \frac{2}{3}\frac{m}{3}\frac{R_0}{T}\right)$$

**(Equation H11)**

And, if friction, m, is assumed to be zero:

$$P_{ave} = \sigma_0 + \frac{2\sigma_0}{3\sqrt{3}}\frac{R_0}{T} \approx Y + \frac{YD}{6H}$$

**(Equation H12)**

Where k is the material's yield stress, D is the diameter of the cylinder and H is the height of the cylinder.

# Appendix I: Derivation of the Shape Complexity Factor

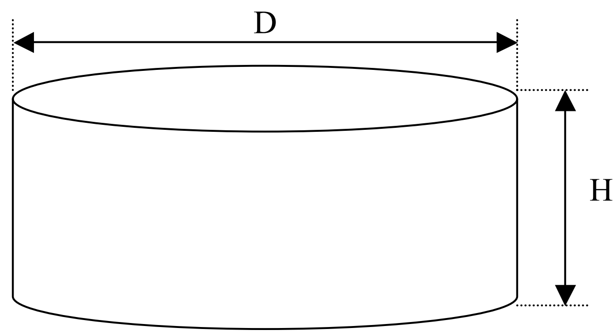


**Figure I1: simplest case forged shape with a complexity factor of one**

Assuming that the forging complexity factor is dependant on the volume and surface area of the part to be forged, the following is the projected complexity factor for the simplest forging case of a cylinder as shown in Figure I1. It is assumed that the complexity factor for the simplest case is equal to one.

$$F_{com} = k\frac{S^{\frac{1}{2}}}{V^{\frac{1}{3}}} = K\frac{\sqrt{2\pi D^2 + \pi DH}}{\sqrt[3]{\frac{\pi D^2 H}{A}}} = 1$$

Simplifying:

$$k = \frac{\sqrt[3]{\frac{\pi D^2 H}{A}}}{\sqrt{2\pi D^2 + \pi DH}}$$

**(Equation I1)**

If H = D then:

$$k = \frac{\sqrt[3]{\frac{\pi D^3}{4}}}{\sqrt{2\pi D^2 + \pi D^2}}$$

If D = 1 then:

$$k = \frac{\sqrt[3]{\frac{\pi}{4}}}{\sqrt{3\pi}} = 0.3$$

k = 0.3

Finally:

$$F_{com} = 0.3\frac{S^{\frac{1}{2}}}{V^{\frac{1}{3}}}$$

**(Equation I2)**

Where:

S = surface area of the part
V = volume of the part
A = area of part in contact with forging press

# Appendix J: Process Cost Equations Error Constant Solutions

The following presents the methods used in order to solve for the specified error constants given the differing versions of the forging process cost equation. The equation numbers referenced are those from the text.

### *(Equation 36)*

$$\text{process cost} = A \cdot P_{ave} \cdot K \cdot E \cdot F_{com}$$

As mentioned in the text, early versions of the process cost equation simply equated the calculated to the actual forging costs. In this way the value of the error constant was essentially the average of the actual divided by the calculated cost of forging. As seen in the text as (Equation 37).

$$K(C,F,M) = \frac{\sum_{n}^{i=1}\left[\frac{(\text{actual forging cost})_i}{(\text{projected forging cost})_i}\right]}{n}$$

**(Equation 37)**

### *(Equation 38)*

$$\text{process cost} = A(K_1 + K_2 P_{ave})EF_{com}$$

Let: $\text{process cost} = P_c$

$$\log P_c = K_1(AEF_{com}) + K_2[AP_{ave}EF_{com}]$$

$$[\log P_c] = [AEF_{com} \quad AP_{ave}EF_{com}] \times \begin{bmatrix} K_1 \\ K_2 \end{bmatrix}$$

Where:

$$C = \begin{bmatrix} P_c \\ \vdots \end{bmatrix}$$

$$A = \begin{bmatrix} AEF_{com} & AP_{ave}EF_{com} \\ \vdots & \vdots \end{bmatrix}$$

Thus:

$$\begin{bmatrix} K_1 \\ K_2 \end{bmatrix} = \left[A^T A\right]^{-1} A^T C$$

**(Equation J1)**

## *(Equation 39)*

$$\text{process cost} = K_1 A P_{ave}^n E F_{com}$$

$$\frac{\text{process cost}}{A \cdot E \cdot F_{com}} = K_1 P_{ave}^n$$

Let: $\text{process cost} = P_c$

$$\log\left(\frac{P_c}{A \cdot E \cdot F_{com}}\right) = \log K_1 + n \log P_{ave}$$

$$\left[\log\left(\frac{P_c}{A \cdot E \cdot F_{com}}\right)\right] = \left[1 \quad \log P_{ave}\right] \times \begin{bmatrix} K_1 \\ n \end{bmatrix}$$

Where:

$$C = \begin{bmatrix} \log\left(\dfrac{P_c}{A \cdot E \cdot F_{com}}\right) \\ \vdots \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & \log P_{ave} \\ 1 & \vdots \end{bmatrix}$$

Thus:

$$\begin{bmatrix} \log K_1 \\ n \end{bmatrix} = \left[A^T A\right]^{-1} A^T C$$

**(Equation J2)**

***(Equation 40)***

$$\text{process cost} = \left(K_1 + K_2 AP_{ave}\right) EF_{com}$$

Let: $\text{process cost} = P_c$

$$\log P_c = K_1\left(EF_{com}\right) + K_2\left[AP_{ave} EF_{com}\right]$$

$$\left[\log P_c\right] = \left[EF_{com} \quad AP_{ave} EF_{com}\right] \times \begin{bmatrix} K_1 \\ K_2 \end{bmatrix}$$

Where:

$$C = \begin{bmatrix} P_c \\ \vdots \end{bmatrix}$$

$$A = \begin{bmatrix} EF_{com} & AP_{ave} EF_{com} \\ \vdots & \vdots \end{bmatrix}$$

Thus:

$$\begin{bmatrix} K_1 \\ K_2 \end{bmatrix} = \left[A^T A\right]^{-1} A^T C$$

**(Equation J3)**

***(Equation 41)***

$$\text{process cost} = \left[K_1 \left(AP_{ave}\right)^n\right] EF_{com}$$

$$\frac{\text{process cost}}{E \cdot F_{com}} = K_1 AP_{ave}^n$$

Let: $\text{process cost} = P_c$

$$\log\left(\frac{P_c}{E \cdot F_{com}}\right) = \log K_1 + n \log AP_{ave}$$

$$\left[\log\left(\frac{P_c}{E \cdot F_{com}}\right)\right] = \left[1 \quad \log AP_{ave}\right] \times \begin{bmatrix} K_1 \\ n \end{bmatrix}$$

Where:

$$C = \begin{bmatrix} \log\left(\dfrac{P_c}{E \cdot F_{com}}\right) \\ \vdots \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & \log AP_{ave} \\ 1 & \vdots \end{bmatrix}$$

Thus:

$$\begin{bmatrix} \log K_1 \\ n \end{bmatrix} = \left[A^T A\right]^{-1} A^T C$$

**(Equation J4)**

# Appendix K: Benchmark Scaling Factors

It was known by the designers of the benchmark system that the built-in process equations provided poor accuracy when compared to actual costs. This error was only further compounded with the addition of varied process costs when calculating final costs. In order to solve this problem the total cost is multiplied by an error factor. The benchmark system calls this total cost adjustment process scaling.

The following is the methodology used by the benchmark system to calculate the cost of a new or unknown part:

$$\text{Actual}_{\text{new}} = (\text{Est}_{\text{new}})(\text{Scale Factor})$$

**(Equation K1)**

Where:

$\text{Actual}_{\text{new}}$ = the benchmark calculated total cost after scaling
$\text{Est}_{\text{new}}$ = the benchmark calculated total cost before scaling
Scale Factor = error factor

The scale factor is made up of a set of ratios relating the actual cost to the estimated cost of a set of best part guesses from the benchmark library. Thus:

$$\text{Scale Factor} = \frac{\text{Actual}_{\text{known}}}{\text{Est}_{\text{best}}}$$

**(Equation K2)**

So:

$$\text{Actual}_{\text{new}} = (\text{Est}_{\text{new}})\left(\frac{\text{Actual}_{\text{known}}}{\text{Est}_{\text{best}}}\right)$$

**(Equation K3)**

From the previous equation $\text{Actual}_{\text{new}}$ and $\text{Est}_{\text{new}}$ are known. However, since all scale factors are added to the final benchmark equation as opposed to individual operational equations, this factor is unknown. Thus, in order to find a scale factor it will be necessary to work backwards from the actual forging operation cost – a value that is known.

$$\text{Forging Est} = \text{material} + \text{process} + \text{machining}$$

**(Equation K4)**

or

$$\text{Forging Est} = \text{x} + \text{y} + \text{z}$$

**(Equation K5)**

If: a, b, c = operational scale factors

Then:

$$\text{Forging Actual} = x(a) + y(b) + z(c)$$

**(Equation K6)**

And:

$$\begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ \vdots & \vdots & \vdots \\ x_n & y_n & z_n \end{bmatrix} \begin{Bmatrix} a \\ b \\ c \end{Bmatrix} = \begin{Bmatrix} \text{Actual}_1 \\ \text{Actual}_2 \\ \vdots \\ \text{Actual}_\text{n} \end{Bmatrix}$$

**(Equation K7)**

Solving the previous equation for a, b, and c:

$$\begin{Bmatrix} a \\ b \\ c \end{Bmatrix} = \left[A^T A\right]^{-1} A^T C$$

**(Equation K8)**

Where:

$$C = \begin{Bmatrix} \text{Actual}_1 \\ \text{Actual}_2 \\ \vdots \\ \text{Actual}_\text{n} \end{Bmatrix}$$

$$A = \begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ \vdots & \vdots & \vdots \\ x_n & y_n & z_n \end{bmatrix}$$

It may be noted that in the above (Equation K5) vendor markup and administration factors are not considered in the total forging cost as is the proper way to calculated total cost. This is because the vendor markup and administration factors are both essentially error factors that are combined with the scale factor when calculated as described above. The scaled results are exactly the same whether markups are used or not.

# Appendix L: Non-focal Areas of Study

Ring rolling, flash welding, and inertial welding were three areas of additional, less focused study during the forging cost equation trials. As all are included in the same overall study, many of the same history and ideas discussed in forging sections of this document still apply. However, as it was also not a focal point of the study and dealt with an even smaller portion of an already small data pool, statistically speaking no definitive conclusions can be drawn from any of the following data. Instead data results point to probable trends but insufficient sample size prohibits solid, universal conclusions. More testing is necessary.

## *Ring Rolling Equations*

Ring Rolling as a process is much simpler in terms of shape and part complexity than most forgings so the proposed cost equation is less complex. Similar to the thought process that went into designing the forging model, the cost or the ring rolling process is chiefly dependant on the surface area and theoretically the complexity of the part to be rolled. Therefore, a larger part and/or a more complex cross-section likely means more difficulty, processing time, and expense

The following ring rolling equations should be used similarly to the forging process cost discussed above. Two different ring rolling process cost equations were studied. The following presents a brief explanation of each followed by the results of applying data to each.

### Version 1

(Equation L1) shows the first iteration toward a viable equation to explain the costs of ring rolling. It is dependant only on the surface area of a given part. Initial hypotheses would suggest that this equation would rely on the development of its two error constants to be reliable. Additionally, the lack of any monetary multiplier leads one to believe that there must be more to any equation that would forecast the cost of ring rolling.

$$\text{ring rolling process cost} = KS^n$$

**(Equation L1)**

Where:

K, n = constants
S = part surface area
Cost = manufacturing material \$ - bulk material \$ - coating \$

Constants are solved for using the format outlined in Appendix H and use the following additional equations:

$$\begin{bmatrix} \ln(k) \\ n \end{bmatrix} = \left([A]^T [A]\right)^{-1} [A]^T [C]$$

$$[C] = [\ln(\cos t)]$$

$$[A] = [1 \quad \ln(S)]$$

## Version 2

The second ring rolling equation, presented in (Equation L2), seems to fill some of the logical holes left in version 1, most obviously the addition of a monetary escalation factor. Furthermore, a shape complexity factor was added with the theory that ring rolling will get more expensive as the shape grows more advanced in complexity as opposed to a simple ring.

$$\text{ring rolling process cost} = KS^n FE$$

**(Equation L2)**

Where:

K, n = constants
S = part surface area
F = shape complexity factor = $0.3 * \frac{S^{1/2}}{V^{1/3}}$
E = escalation factor = \$102.57

Constants are solved for using the format outlined in Appendix H and use the following additional equations:

$$\begin{bmatrix} \ln(k) \\ n \end{bmatrix} = \left([A]^T [A]\right)^{-1} [A]^T [C]$$

$$[C] = \left[ \ln\left( \frac{\cos t}{F \bullet E} \right) \right]$$

$$[A] = [1 \quad \ln(S)]$$

## Results

The following Table L1 presents the results from versions 1 and 2 of the ring rolling cost equations. Looking at the upper portion of the table one can see that one of the six parts tested is a clear outlier in terms of results. This accounts for the error levels in excess of 100 percent. Clearly, the poor results of one case have skewed the average error. The alternative results present the average error for each equation if the single outlier part were removed. Versions 1 and 2 each have an error level of 13.97 and 13.63 percent, respectively. As one might have hypothesized, version 2 had a smaller average error than version 1 – but not significantly so. Due both to the insignificant difference between results and the small number of parts tested it is impossible to determine which equation will better predict the cost of ring rolling.

**Ring rolling data summary**

| | error: | |
|---|---|---|
| | **Version 1:** | **Version 2:** |
| part 1: | 523.46% | 615.09% |
| part 2: | 41.90% | 42.74% |
| part 3: | 47.14% | 49.21% |
| part 4: | 17.27% | 19.27% |
| part 5: | 18.92% | 21.59% |
| part 6: | 22.15% | 24.03% |
| average: | 111.81% | 128.66% |
| standard dev: | 202.06% | 238.61% |
| **Alternate Results:** | | |
| average: | 29.47% | 31.37% |
| standard dev: | 13.97% | 13.63% |

**Table L1: Cost estimation results from ring rolling equations**

### *Flash Welding Equations*

Like ring rolling, the flash welding equations parallel the formation of the forging process equation. Like previous equations, the flash welding equations are formed from intuitive cost increasing factors. Error is then brought under control through the use of one or more constants. The two factors projected to chiefly effect the cost of the welding process are part weight and surface area to be welded. In the case of the former, a part needs to positioned properly and held in order to insure a tight weld free of warp or excess material hardening. Hence, the heavier the part to be welded the higher the projected cost. Additionally, the larger the surface area of the weld, the longer the weld bead will be required to form a proper joint. Logically, as the length of the weld bead increases, so will the cost of the welding process. The following equations denote the process costs associated with flash welding. Due to the small differences between versions and the ancillary nature of this research version discussions are primarily mathematical in nature.

## Version 1

$$\textit{flash welding process} \cos t = k\left(WA_{CS}\right)^{n}$$

**(Equation L3)**

Where:

$A_{CS}$ = cross-section area at point of flash weld

$$A_{CS} = \frac{Vol}{\pi \bullet \left(\frac{ID - OD}{2}\right)}$$

**(Equation L4)**

k, n = constant
W = part weight

Constants are solved for using the format outlined in Appendix H and use the following additional equations:

$$\begin{bmatrix} \ln(k) \\ n \end{bmatrix} = \left([A]^{T}[A]\right)^{-1}[A]^{T}[C]$$

$$[C] = [\ln(\cos t)]$$

$$[A] = [1 \quad \ln(W \cdot A_{CS})]$$

## Version 2

$$\text{\textit{flash welding process}} \cos t = k(W)^{m}(A_{CS})^{n}$$

**(Equation L5)**

Where:

k, n, m = constant
W = part weight
$A_{CS}$ = cross-section area at point of flash weld

Constants are solved for using the format outlined in Appendix H and use the following additional equations:

$$\begin{bmatrix} \ln(k) \\ m \\ n \end{bmatrix} = \left([A]^{T}[A]\right)^{-1}[A]^{T}[C]$$

Where:

$$[C] = [\ln(\cos t)]$$

$$[A] = [1 \quad \ln(W) \quad \ln(A_{CS})]$$

## Results

As can be seen in Table L2, the results from the calculation of the cost of flash welding is fairly clear based on the current data pool. As is logical, the more complex version 2 had a significantly lower level of error, 10.64 percent, when compared to version 1, 64.38 percent. This large difference in errors leads one to hypothesize that version 2 should adequately describe the cost of flash welding. However, as with ring rolling, the data pool is too small to adequately judge the true worth of either version of the flash welding cost equation.

**Flash welding data summary**

| | error: version 1: | version 2: |
|---|---|---|
| part 1: | 183.29% | 1.01% |
| part 2: | 36.97% | 13.72% |
| part 3: | 27.48% | 14.77% |
| part 4: | 51.27% | 21.39% |
| part 5: | 27.36% | 1.78% |
| part 6: | 59.91% | 11.18% |
| average: | 64.38% | 10.64% |
| standard dev: | 59.69% | 7.92% |

**Table L2: Cost estimation results from flash welding equations**

## *Inertial Welding Equations*

Inertial welding is a process whereby two parts are welded together using the frictional heat derived from compressing one rotating part to another, usually larger, stationary part. As the work in the process is focused in the positive and negative rotational acceleration of the part in question, it is logical that the only critical value in the proposed inertial welding cost equation should be part weight. The larger the part the more energy needed to start and stop the part rotation and thus increased costs. (Equation L6) shows the proposed inertial welding cost equation. However, the data that had been developed by the end of the forging project was unclear as to the weight of different parts that underwent inertial welding. Therefore, (Equation L6) received no testing whatever.

$$inertial\ welding\ process\ \cos t = kW^{n}$$

**(Equation L6)**

Where: k, n = constant

W = part weight

Constants are solved for using the format outlined in Appendix H and use the following additional equations:

$$\begin{Bmatrix} \ln(K) \\ n \end{Bmatrix} = \left([A]^T[A]\right)^{-1}[A]^T[C]$$

Where: $[C] = [\ln(\mathrm{C})]$ $\qquad [A] = [1 \quad \ln(W)]$