A NEW SIMULATION OF MULTI – STATE FADING CHANNELS

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Abstract

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We have developed a computer simulation for a new multi-state fading channel model. Multi-state models can yield more accurate fading amplitude time series than can single state models. Our initial multi-state model is aimed at mobile satellite channels. We use the simulation to verify a new multi-state model proposed in [1], but the simulator can be used for general-purpose simulation of any multi- (or single-) state fading. We validate this simulation against both theory and measured data [2], using the second – order statistics of average fade duration and level crossing rate.

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Chapter 1: Introduction

1.1 Introduction

Digital communications is becoming increasingly attractive because of the evergrowing demand for data communications and due to the fact that digital transmission offers greater data processing options and flexibilities compared to analog transmission. A functional block diagram of a typical digital communication system is shown in Fig. 1.1.



Figure 1.1. Block diagram of a typical digital communication system

The upper row of blocks in Fig. 1.1 [1] depicts the various signal transformations from the information source to the transmitter output. The lower row of blocks denotes the signal transformations from the receiver input to the information sink. It can be observed that the processes undergone from the receiver input to the sink are basically opposite to the transformations undergone by the signal from the source to the transmitter output. These transformations are done to enable the transmission of the source signals across a communication channel. The communication channel is the physical medium used to send the signal from the transmitter to the receiver, e.g., a cable or pair of wires for wired systems, and the atmosphere for wireless systems. The transformations that have to be applied to the source signal before transmission depend greatly on the characteristics of this physical channel (e.g., center, or "carrier" frequency, frequency response, attenuation, etc.). The design of spectrally efficient communication systems requires a detailed understanding of radio propagation environment. Knowledge of the characteristics of the communication channel is therefore of particular importance in the analysis and design of communication systems, especially in the case of wireless mobile channel transmissions, because this channel is usually time varying in nature due to motion between transmitter and receiver, resulting in propagation path changes.

This thesis deals mainly with the study of the communication channel and some methods to simulate and model this channel. Emphasis is placed on channel modeling of mobile satellite channels and rural/open area terrestrial radio channels. It can also be made applicable to cellular radio channels if we take into account dispersion characteristics. A typical mobile radio system consists of a fixed number of base stations that define the radio coverage area for a specific region. Characteristics of radio channels vary greatly with the operating frequency and the mode of operation, i.e., line of sight (LOS) radio links and non-LOS (NLOS) links, and in many cases must take into account electromagnetic propagation effects such as reflection, diffraction, and scattering.

1.2 Propagation Review

In most modern mobile communication systems, there are two major effects imposed upon transmitted signals:

- <u>Large-scale path loss</u>: This path loss is basically a function of the distance between transmitter and receiver. Since the transmitted signal has finite energy, the energy of the signal is reduced as the distance increases. This loss in signal energy as a function of distance is also referred to as attenuation, spreading loss, or basic transmission loss.
- <u>Small scale fading</u>: This is usually a "local" phenomenon and is nearly independent of the distance between the transmitter and the receiver. This fading category is used to describe the rapid fluctuations of the amplitudes, phases, or multipath delays of a radio signal over a short period of time or travel distance.

Radio signals transmitted to/from mobile-radio base stations (terrestrial or satellite) are not only subject to significant propagation path losses that are encountered in atmospheric propagation, but are also subject to the distorting effects of terrestrial propagation. In a typical urban environment it's quite often that only non-line of sight

radio propagation paths exist between the base station (BS) and the mobile station (MS) because of natural and man-made objects present between the BS and a MS. As a consequence of this, the radio waves propagate via scattering, reflection, and diffraction, which are the root cause of distorted radio propagation. We refer to these as the three basic physical mechanisms that impact signal propagation in a mobile communication system:

- <u>Reflection</u>: Occurs when an electromagnetic wave impinges on a smooth surface with very large dimensions relative to the radio frequency (RF) signal wavelength.
- ii. <u>Diffraction</u>: This occurs when a dense body with dimensions that are large relative to the signal wavelength obstructs propagation path between transmitter and receiver, and the propagation wave encounters an edge of the object, causing secondary waves to be formed behind the obstructing body.
- iii. <u>Scattering</u>: Occurs when a radio wave impinges on large rough surfaces whose dimensions are on the order of the RF signal wavelength causing the energy to be spread out (scattered) or reflected in all directions. In an urban environment the obstructions that cause scattering include lampposts, foliage, street signs, etc.

Fading is a general term for the phenomenon of the signal variability. Fading in mobile radio channels is usually characterized by two distinct phenomena:

- Constructive and destructive addition of multiple arrivals of the transmitted signal (multipath propagation)
- Obstruction of the LOS path (also called shadowing).

Due to these phenomena the plane waves arriving at the BS/MS from many different directions arrive with different delays. This effect is called multipath propagation. Fig. 1.2 shows an illustration of multipath propagation.



Figure 1.2. Illustration of multipath fading.

It can be observed from Fig. 1.2 that the signal received at the mobile station could be due to reflections, scattering, and diffractions or could have a LOS transmission with the base station. The signals from the base station could reach the mobile station

with different time delays due to different transmission path lengths. These signals also often have different amplitudes, depending upon the path they travel.

Propagation between a mobile unit and a base station is most susceptible to the effects of multipath fading phenomena, because all communication is essentially at ground level. The effects of multipath phenomena are not as significant in air-to-ground or satellite-to-earth station communications because the high angle of propagation typically avoids the types of interferences caused by surrounding natural land features and man-made structures.

The multipath phenomena cause multiple waves to combine vectorially at the receiver antenna to produce a composite received signal. The carrier wavelength used in mobile radio applications is usually in the UHF band, which typically ranges from 225MHz to 1GHz. Therefore small changes in differential propagation delays due to MS mobility can cause large changes in the phase of the individually arriving plane waves [2]. The arriving plane waves at the MS and BS antennas will experience constructive and destructive addition depending on the location of the MS. It is apparent that a receiver at given location can experience a signal level (power) that is several tens of dB different from that at another location a short distance away where the phase relationships between the incoming waves has changed [3].

If the MS is moving or there are changes in the scattering environment then the spatial variations manifest themselves as time variations; this phenomenon of signal fluctuations is called *envelope fading*. The short-term fluctuations caused by the local multipath environment are known as fast fading to distinguish them from the much longer-term variation in mean signal level, known as slow fading. This latter effect is

caused by movement over distances large enough to produce variation in the overall path length between the transmitter and the receiver. Since the mobile station moving into the shadow of hills or buildings often causes these variations, slow fading is often called *shadowing*.

1.3 Popular Fading Distribution Overview

An exact analysis of these multipath channels would be very complex since it is would require isolation and identification of each part of a reflected wave while the scatterers are in motion; hence we usually use a *statistical* approach to model these channels. Statistical models have been shown to exhibit good agreement with observed parameter values. There are several probability distributions that can be used in attempting to model the statistical characteristics of fading channels. These distributions are frequently used in mobile radio to represent the short-term amplitude distribution of mobile radio signals. These statistical distributions are normally used to describe the signal envelope variations; they are also used to evaluate the fade margins required for both the uplink and downlink budgets. A brief description of some of the popular fading models is given in this chapter and a more detailed explanation of these models is given in the next chapter. Some of the popular fading models to describe the channel characteristics are as follows.

1.3.1 Rayleigh

In urban environments local scatterers usually surround a mobile station so that the plane waves will arrive from many directions without a direct LOS component. Two–dimensional isotropic scattering where the arriving plane waves arrive from all directions with equal probability is a very commonly used scattering model in a macrocellular environment. For this type of scattering the received envelope is *Rayleigh distributed* at any time, and is said to exhibit *Rayleigh fading* [2].

1.3.2 Rician

In practical applications the movement of the mobile often causes the mobile to switch from a LOS path to a NLOS path and vice versa. Even in the absence of LOS propagation path, there often is a dominant reflected or diffracted path between the base station and the mobile station. The LOS or dominant reflected or diffracted path produces the *specular* component and a multitude of weaker secondary paths contribute to *scattered* components of the received envelope. In this type of propagation environment, the received signal envelope still experiences fading, but the presence of the specular component changes the received envelope distribution, and very often a Rician distributed envelope is assumed. The received envelope is said to exhibit *Rician fading*. It is intuitively to be expected that there will be a fewer deep fades compared to the Rayleigh fading due to the LOS propagation path, or at least a dominant specular component, and that the specular component will be a major feature of the channel power spectrum.

1.3.3 Nakagami–*m*

The Nakagami-*m* distribution was introduced by Nakagami in the early 1940's to characterize rapid fading in long distance high frequency channels. This distribution was selected to fit empirical data and is one of the most versatile, in the sense that it has greater flexibility and accuracy in matching some experimental data than Rayleigh, log-normal, or Rician distributions. The distribution has been found to be the best fit for some data signals received in urban radio multipath channels [2]. The *m* parameter is known as the *shape factor* of the Nakagami distribution, and via the variation of the parameter *m*, the Nakagami distribution can model conditions from Rayleigh to Rician and beyond, so it is often used to model fading in terrestrial environments as well as satellite environments. This choice offers more flexibility to the modeler than the previous two distributions. These appealing features account for the widespread application of the Nakagami distribution to theoretical and applied research in wireless communication [5].

1.4 Thesis Scope

From the above description of fading in mobile radio channels, we have noted that the general phenomenon of fading is often characterized by two distinct phenomenona: multipath fading and shadowing. In this thesis we focus on the development of different fading generators for modeling multipath envelope fading. We first develop (computer simulations) fading generators for the popular Rayleigh and Rician distributions, and then based on some recent work [6], develop a generator for a Nakagami-*m* distribution. These fading generators yield the channel amplitude time series, i.e., a sequence of samples, according the fading distribution and its parameters. These fading generators are used to model what is often known as a single state fading channel. In many cases, the channel exhibits distinct propagation characteristics over distinct time periods. These distinct characteristics are often classified as channel "states." The simplest non-trivial example is a two state case, in which the channel can be said to be in a "good," (viz. unshadowed) or "bad" (viz. shadowed) state.

In our thesis the main aim is to develop a multi-state fading model that yields more accurate amplitude time series representations than can be obtained with the simpler single-state models. Multi-state models also offer greater flexibility in modeling satellite communication channels compared to single state models. After proper validation of the several single state generators, we then employ these single state generators to develop a multi-state fading generator. We use some recent results [11] to validate our multi-state fading generator against both theory and measured data [12], using the second-order statistics of average fade duration and level crossing rate.

1.5 Outline of Thesis

In this chapter we gave a brief summary of the purpose of studying fading channels, different popular existing fading models and an outline of what we plan to achieve from this research. Chapter 2 describes the single state fading models in more detail and the simulation procedures we have used along with the results of the simulation procedures. In Chapter 3 we introduce multi-state fading and give a brief description of some popular multi-state fading models. Chapter 4 describes the new multi-state model,

the simulation procedure used and results of the simulation. Finally in Chapter 5 we conclude the thesis by summarizing the results obtained and by suggesting some future work that can be done to further this research.

Chapter 2: Single State Fading Models

In this chapter we shall first give a more detailed description of the popular single state fading models noted in the previous chapter. We begin with a brief mathematical introduction to the fading phenomenon and then describe the generation and simulation of the three popular fading models described in the previous chapter.

2.1 Fading Channel Impulse Response

Multipath fading arises physically from the addition of a large number of multipath reflections at the receiver. These reflected signals (from buildings, hills, the ground, etc.) are often nearly equal amplitude, but random in phase. It can be shown [4] that the complex baseband channel impulse response corresponding to this type of fading is

$$h(t;\tau) = \sum_{k=0}^{N-1} \alpha_k(t) \exp\left\{j\left[\omega_{D,k}(t-\tau_k(t)) - \omega_c \tau_k(t)\right]\right\} \delta[t-\tau_k(t)]$$
(2.1)

where $\alpha_k(t)$ represents the k^{th} received amplitude, the exponential term represents the k^{th} received phase, and the k^{th} path is delayed by a time- varying delay $\tau_k(t)$. The δ function is a Dirac delta, and $\omega_c = 2\pi f_c$, where f_c is the carrier frequency. The term $\omega_{D,k} = 2\pi f_{D,k}$ represents the Doppler shift associated with the k^{th} received multipath echo. The Doppler shift represents the shift in frequency of the received signal due to motion of the transmitter and/or receiver. The Doppler shift will be discussed in more detail in the next section when we describe the Rician model.

It is to be noted that in the work that we have done we are considering a nondispersive channel in which case the time delays τ_k are very closely spaced in time and much smaller than any signal symbol duration. In this case we approximated all τ_k as τ , and when all amplitudes are equal, the sum of the exponentials is our multipath-fading envelope. For the Rician case, one of the amplitudes is much larger than the others.

2.2 Popular Fading Models

2.2.1 Rayleigh Fading

This distribution is usually used to model a channel when there exists no significant LOS component and radio propagation is usually achieved by local scattering. When there are a large number of scatterers in the channel that contribute to the signal at the receiver (i.e., no prominent LOS path), then the composite received signal consists of a large number of equi-amplitude plane waves. This kind of fading is commonly encountered in urban areas, for instance a mobile user among many high-rise buildings.

If the number of received waves N is sufficiently large, from (2.1) (theoretically infinite, but in practice greater than 6 [2]) and by the Central Limit Theorem the complex received envelope can be modeled as a wide-sense stationary Gaussian random process. The real and imaginary parts of the complex received envelope are independent and identically distributed zero-mean Gaussian random variables, thus the envelope, the square root of the sum of the squared in-phase and quadrature (I & Q) zero-mean Gaussian processes, is said to be Rayleigh distributed. These I and Q processes are completely characterized by their mean value and autocorrelation function. When the time delays $\tau_k(t)$ are on the order of $1/f_c$ and larger, the random phase terms $\exp(-j\omega_c \tau_k(t))$ are essentially uniformly distributed over the interval $[0,2\pi)$, and vary rapidly (the path delays themselves vary slowly, but the delays multiplied by the carrier frequency vary rapidly [1]). Since the means of the *I* & *Q* channel processes are zero, the variance of the quadrature components equals the mean-squared value (the mean power). The Rayleigh probability density function (pdf) is completely characterized by this mean square value. As noted, under these conditions the envelope of the channel response at any time instant has a Rayleigh probability distribution and the phase is uniformly distributed in the interval $(0, 2\pi)$. This translates to the following: a Rayleigh process is the envelope of two zero-mean Gaussian processes, where by envelopes we mean the square root of the sum of the squares. That is the envelope r(t) of the complex received signal is given by

$$r(t) = \sqrt{I^2(t) + Q^2(t)}$$
(2.2)

and the *pdf* is given by

$$p_{R}(r) = \begin{cases} \frac{2r}{\Omega} \exp\left(-\frac{r^{2}}{\Omega}\right) &, r \ge 0\\ 0 &, elsewhere \end{cases}$$
(2.3)

where $\Omega = E(R^2)$.

The probability distribution of the phase (θ) can be obtained by integrating the joint *pdf* equation over *r*, which results in a uniform distribution [3].

Shown in Fig 2.1 and Fig 2.2 is a time series plot of Rayleigh faded signal envelope as a function of time and the Rayleigh *pdf*, for $E(r^2) = I$.



Figure 2.1. Time series of Rayleigh fading samples

Fig. 2.1 is the output of a simulation that uses the above-mentioned Gaussian processes, and these processes are filtered with a filter of normalized bandwidth B=0.1, to yield the time correlation. Bandwidth is relative to the sampling frequency of the simulation.



Figure 2.2. Rayleigh fading probability distribution function

2.2.2 Rician Fading

There are many radio channels in which fading is encountered that are basically LOS communication links with multipath components arising from secondary reflections, or signal paths, from surrounding terrain or other obstacles. In such channels, the number of multipath components is usually small and hence the channel may be modeled in a manner somewhat similar to the Rayleigh model but with an important difference: the presence of the specular component and the presence of a Doppler shift in the frequency associated with this LOS component or specular component. Whenever relative motion exists between the transmitter and receiver, there is a shift in the frequency of the received signal due to the Doppler Effect. The Doppler shift represents the frequency shift of the received signal due to motion of the transmitter and/or receiver. The Doppler frequency parameter f_m is the *maximum* Doppler shift that the signal undergoes. Waves arriving from ahead of the mobile have a positive Doppler shift, i.e., an increase in frequency, while the reverse is the case for waves arriving from behind the mobile. Waves arriving from directly ahead of, or directly behind the vehicle are subjected to the maximum rate of change of phase, giving [3]

$$f_m = \frac{v}{\lambda} \tag{2.5}$$

where f_m – Maximum Doppler shift, Hz

v – velocity of the mobile unit, m/s

 λ – wavelength of the carrier, m

Fig. 2.3 shows an illustration of the mechanism causing the Doppler shift in frequency. Let the n^{th} reflected wave with amplitude *c* and phase ϕ_n arrive from an angle α_n relative to the direction of the motion of the antenna.



Figure 2.3. Illustration of Doppler shift

The Doppler shift of this wave is then

$$\Delta f_n = \frac{v}{\lambda} \cos \alpha_n \tag{2.6}$$

where v is the speed of the antenna and α_n is the angle of arrival.

Referring back to Eq (2.1), the Doppler frequency, for terrestrial velocities $f_{D,k}$ is most often much smaller than 1/T where T is the shortest baseband signal duration (symbol, bit, or chip time). Thus usually the maximum Doppler shift is much smaller than the signal bandwidth. The phase terms $\exp\{j\omega_{D,k}[t-\tau_k(t)]\}$ associated with the Doppler shift of the k^{th} path generally vary much more slowly than the random phase terms, as $f_{D,k}$ (which can itself be a function of time) is in general much smaller than f_c . The *pdf* of the Rician distribution is given by [2]

$$p_r(r) = \frac{r}{\sigma^2} \exp\left\{-\frac{r^2 + s^2}{2\sigma^2}\right\} I_0\left(\frac{rs}{\sigma^2}\right) \qquad r \ge 0$$
(2.6)

where s^2 - power in the dominant component,

 σ^2 - power in the scattered components.

In the literature a Rician process is often characterized by 2 parameters: its maximum Doppler frequency and its Rice-factor or "*K*–factor". The Rice Factor is defined as follows:

$$K = \frac{\text{Power in LOS component}}{\text{Power in scattered components}}$$

Interpreting the Rice factor in mathematical form we have, in dB

$$K = 10\log\frac{s^2}{2\sigma^2} \quad \text{dB}$$
(2.7)

The envelope distribution can be rewritten in terms of the Rice factor and the average envelope power $E[r^2] = \Omega_p = s^2 + 2\sigma^2$ by noting that

$$s^{2} = \frac{K\Omega_{p}}{K+1}, \qquad 2\sigma^{2} = \frac{\Omega_{p}}{K+1}$$
(2.8)

The Rician *pdf* in terms of the Rice factor (k –numeric value (not dB)) is

$$p_r(r) = \frac{2r(k+1)}{\Omega_p} \exp\left\{-k - \frac{(k+1)r^2}{\Omega_p}\right\} I_0\left(2r\sqrt{\frac{k(k+1)}{\Omega_p}}\right), \quad r \ge 0$$
(2.9)

It can be observed that for K = 0 the channel exhibits Rayleigh fading, and when $K = \infty$ the channel does not exhibit any fading at all. The *pdf* of the envelope $p_r(r)$ is shown in Fig 2.4 for various values of *K*. From the plots it can be observed that for K = 0 the *pdf* is a Rayleigh distribution and for K >>1 the *pdf* becomes approximately Gaussian with a mean square value (power) s^2 . In Fig. 2.4 the mean square values of the *pdf* have been normalized to one.

Similar to the Rayleigh distribution, when the time delays $\tau_k(t)$ are on the order of $1/f_c$ and larger, the random phase terms $\exp(-j\omega_c \tau_k(t))$ are essentially uniformly distributed over the interval $[0,2\pi)$, resulting in a uniformly distributed random phase for the scattered components.



Figure 2.4. Rician PDF's for different K values

An example time series of the Rician fading samples is shown in Fig. 2.5 for K= 5 dB. As expected, the presence of the specular or the LOS component reduces the number of deep fades when compared to the Rayleigh distribution time series in Fig. 2.1. For the simulations used to generate Fig. 2.4 we used N = 100,000 samples with the mean square value for each case set equal to one. The filter bandwidth was set to 0.2 and the maximum Doppler frequency was set equal to 0.05. For Fig. 2.5 the parameters for the simulation were same as that of Fig. 2.4 except that we just 200 samples to generate the time series.



Figure 2.5. Time series of Rician fading samples, K = 5 dB

2.2.3 Nakagami-*m* Fading

As explained in the previous chapter, the Nakagami distribution is very popular due to its versatility in providing greater flexibility and accuracy in matching some experimental data, and also due to the fact that the distribution has been found to provide a very good fit for the mobile radio channel. Beyond its empirical justification, the Nakagami distribution is often used because the distribution can model fading conditions that are either more or less severe than Rayleigh fading. When m = 1, the Nakagami distribution is the Rayleigh distribution, when m = 1/2 it is a one-sided Gaussian distribution, and when $m \to \infty$ the distribution becomes an impulse (no fading) [2].

Two useful relations in our case are those relating the Nakagami-*m* shape factor *m* and the Rician *k* factor and σ^2 (the power of the scattered waves), given by [6]

$$m \sim \frac{(1+k)^2}{2k+1}$$
 (2.10)

$$k \sim \frac{\sqrt{m^2 - m}}{m - \sqrt{m^2 - m}}$$
 $m > 1$ (2.11)

Note that the above relations between m and k and not exact but approximations. Since the Rice distribution contains a Bessel function while the Nakagami distribution does not, the Nakagami distribution often leads to convenient closed-form analytical expressions that are otherwise unattainable. The Nakagami-*m* probability density function p(r) of the envelope *r* is given by [6]

$$p(r) = \frac{2m^{m}r^{2m-1}}{\Gamma(m)\Omega^{m}} \exp\left(-\frac{mr^{2}}{\Omega}\right)$$
(2.12)
where $m = E^{2}[r^{2}]/\operatorname{var}(r^{2}), \ \Omega = E[r^{2}],$
 $E[r^{\nu}] = \frac{\Gamma(m+\nu/2)}{\Gamma(m)} \left(\frac{\Omega}{m}\right)^{\nu/2}$

and

$$\Gamma(m) = \int_{0}^{\infty} x^{m-1} \exp(-x) dx$$

is the Gamma function. Fig. 2.6 shows the Nakagami distribution for several values of m. It can be observed from the Fig 2.6 that the Nakagami-m pdf for m = 1 resembles the Rayleigh pdf. For the simulations used to generate Fig 2.6 we have used N = 100,000 samples with a filter bandwidth of 0.1.



Figure 2.6. Nakagami – *m* fading probability distribution function

2.3 Simulation of Fading Models

In this section we shall discuss the generation methods for the Rayleigh distribution, the Rician distribution and the Nakagami-m distribution. At the end of the section we shall compare the results from the simulations generated with the analytically obtained results.

2.3.1 Generation of a Rayleigh Process

To generate a Rayleigh process, one often begins with the "traditional" definition: A Rayleigh process is the square root of a central chi-squared variate with two degrees of freedom. This translates to the following: a Rayleigh process is the envelope of two zeromean Gaussian processes, where by envelope we mean the square root of the sum of the squares [12].

To aid in the explanation, we show in Fig. 2.6 a diagram of the Rayleigh process generator. The inputs to the left of Fig. 2.6 are white Gaussian processes of zero mean. The two branches can be treated as the in-phase and quadrature channel of a complex Gaussian random process. A convenient normalization is to set the channel power to one $(E(r^2) = 1)$. Since the Gaussian processes are zero mean, the variance equals the power and thus we normalize the variances of the Gaussian processes to one.
The power of the Rayleigh variable r_k is equal to the expected value of r_k^2 , i.e.,

$$E(r^{2}) = E(v_{k1}^{2} + v_{k2}^{2}) = 2\sigma^{2} + \mu^{2}$$
(2.13)

where σ^2 - power in each of the I & Q channels (variance),

 μ^2 - mean of the Gaussian processes ($\mu = 0$ in our case).



Figure 2.7. Illustration of the components used to generate the Rayleigh fading process

The filtering and scaling operation are used to reduce the fade rate of the white processes to equal the desired maximum fading rate. The filtering operation is performed as the power spectral density of the white process is the Fourier transform of its autocorrelation, and is a constant value. What this means is that the Gaussian process is rapidly varying (or can be), contrary to what we want our model to be. Thus we need to filter the white Gaussian process to slow down the variation to something approximating what is seen in practice. Filtering narrows the output power spectrum and widens the resulting autocorrelation function [1]. The scaling operation is performed after the filter operation for the outputs of the filter to have the same mean and variance (power) as the input. The two different noise sources must have the same power spectral density (PSD) to produce a Rayleigh faded envelope.

Often the ideal is a low pass filter with a cut off frequency of f_0 ($f_{D, \text{max}}$). For the model developed, we have used a conventional low pass filter, with a variable order (2nd - 9th): a Chebyshev type I filter. The filter order is adjusted depending on the desired cutoff frequency. For filter cutoff frequencies below 0.1 ($f_{sample}/2$) we generally use a 3rd or 4th order filer and a 9th order filter for cutoff frequencies above this value. This is done because with very narrowband filters, some of the filter coefficients become extremely small (e.g., 10⁻⁹), where these coefficients are those generated by the Matlab built-in function denoted "cheby1." The filter stability is not always guaranteed in the cases of small coefficients.

With a sampling rate of f_s , the value of the scale factor *b* is then approximated as $\sqrt{f_s/f_o}$ for outputs v_k that have the same mean and variance (power) as the input. In our model the sampling frequency is set to one. The outputs after scaling are then squared and added to generate the Rayleigh faded samples.

2.3.1.2 Verification of Model

This model was used to generate N = 100,000 random samples of the Rayleigh fading process. To verify our model we compare the histogram of the generated samples with the analytical Rayleigh *pdf*. The plot of the comparison of the two histograms is shown in Fig. 2.7. We have also shown a histogram to demonstrate the phase uniformity over $[0,2\pi)$ in Fig 2.8. From Fig. 2.7 we can observe that model developed exhibits a good agreement with the analytical Rayleigh *pdf*. From Fig. 2.8 we can observe the phase uniformity between 0 and 360 degrees.



Figure 2.8. Rayleigh analytical *pdf* & histogram.



Figure 2.9. Phase histogram of the Rayleigh fading samples.

For the plots shown in Figures 2.7, and 2.8 we have used a 9th order Chebyshev filter with a filter cut off frequency of 0.1 Hz. The full N=100000 samples were used to generate both the histograms of the amplitude and phase.

2.3.2 Generation of a Rician Process

The main distinction between the Rayleigh and Rician distribution is the presence of LOS or specular component in the Rician, absent in the Rayleigh case. Due to the presence of the LOS or specular component one has to take into account Doppler shift in frequency of the component.

Similar to the Rayleigh case, the generation of the Rician process begins with the "traditional" definition: a Rician process is the square root of a non-central Chi-square process with two degrees of freedom. Thus essentially the Rician process is the envelope of two *non-zero mean* Gaussian processes. In the mobile radio context, this isn't completely correct, as the result of generating the process in this way is a process that has a LOS component with a Doppler shift of identically zero. For non-zero LOS Doppler shifts, we need to add a tone of the desired frequency to create the non-zero mean Gaussian process [12]. To aid in this discussion we show in Fig 2.9 a diagram of the Rician process generator.

The Rician process is often characterized by these two parameters:

1. Maximum Doppler frequency.

2. Rice-factor or "K-factor"

To these two parameters we could also add the Doppler frequency of the LOS component, which in general is time varying [12].



Figure 2.10. Illustration of the components used to generate the Rician fading process

In the Rician case, we assume that the first-arriving signal is a constant amplitude μ . Thus the power in the LOS component is μ^2 . The power in the diffuse component is conventionally denoted $2\sigma^2$, where the power in each of the channels (in-phase and quadrature) is σ^2 . Therefore our expression for *k* where $K = 10\log(k)$, is:

$$k = \mu^2 / \left(2\sigma^2 \right) \tag{2.14}$$

From the illustration of the model in Fig 2.9 we see that except for the Doppler frequency component, the model is identical to the Rayleigh sample generation model discussed in section 2.3.1.

Referring to Fig. 2.9, the random processes v_{k1} and v_{k2} equal the corresponding filtered Gaussian processes plus the tone at the Doppler frequency of the LOS components, f_{DL} . The power of the Rician variable r_k is equal to the expected value of r_k^2 , i.e.,

$$E(r^{2}) = E(v_{k_{1}}^{2} + v_{k_{2}}^{2}) = 2\sigma^{2} + \mu^{2}$$
(2.15)

where σ^2 - power in each of the I & Q channels (variance),

μ^2 - mean of the Gaussian processes.

If we set $f_{DL} = 0$, the mean of each Gaussian process v_{k1} and v_{k2} is $\mu/\sqrt{2}$, and the variance of each is σ^2 . In this case the mean power $E(r^2)$ is identical to that in (2.15). Using again the normalization $E(r^2)=1$ and combining equations (2.14) and (2.15), we can obtain equations for μ and σ in terms of k using Eq.(2.8). Therefore given a desired value of k (or K), we can set the mean and variance parameter in the model.

A simple Rician model assumes that the means of the Gaussian processes are constant. This will certainly yield a Rician distributed envelope, but will not realistically model the higher order envelope statistics for a particular scattering environment. For our model we have used a better approach suggested by [12]. In this case the means of the $m_I(t)$ and $m_Q(t)$ corresponding to means of the in phase and quadrature components of the LOS signal are given by

$$m_I(t) = \mu \cdot \cos(2\pi f_{DL}t + \phi) \tag{2.16}$$

$$m_O(t) = \mu \cdot \sin(2\pi f_{DL} t + \phi) \tag{2.17}$$

where f_{DL} and ϕ are the Doppler shift and random phase offset associated with the LOS or specular component, respectively.

2.3.2.1 Verification of Rician Model

This model was used to generate N = 200 random samples of the Rician fading process *r* for two values of the Rice factor *K*: K = 5dB and K = 10 dB. Additional parameters used for this simulation are a 9th order Chebyshev filter with cutoff of 0.1 Hz, and a maximum Doppler frequency (f_m) of 0.01Hz.



Figure 2.10. Rician fading vs. time for two different 'K' values

To verify our model we compare the histograms of the Rician generated samples for the two *K* values with the analytical Rician *pdf* for the same *K* values and the plots are shown in Fig. 2.11. The parameters for this simulation is same as the above, except that to get an accurate histogram we have used N = 100,000 samples.



Figure 2.12 Rician analytical *pdf* & histogram for two different 'K' values

It can be observed from Fig. 2.11 that the model developed exhibits a good agreement with the analytical Rician *pdf*. We have also created a histogram of the phase of the Rician samples, and shown in Fig. 2.12. As stated above it can be observed that the phase is uniform between 0 and 2π .



Figure 2.13. Phase histogram of the Rician fading samples

2.3.3 Generation of a Nakagami-*m* fading Process

The Nakagami-*m* model has no physical foundation unlike the previous two described models, and is essentially an empirical formula selected to fit observed data. We have used the reference [6] to develop this model. In this model the shape factor *m* can take on values m = n/2, with *n* a non-zero positive integer. Though developed for discrete values of *m* this model works for any integer multiple of 0.5, greater than or equal to 0.5.

We review the method developed in [6]. Let $x_i(t)$ and $y_i(t)$ be Gaussian random processes, corresponding to in-phase and quadrature components, respectively. We set the mean of the Gaussian processes to zero and the variances to σ^2 . Let $r_0^2 = x_0^2$, or equivalently $r_0^2 = y_0^2$, and $r_i^2 = x_i^2 + y_i^2$, i = 1, 2... We note that r_0 is semi-positive Gaussian distributed whereas r_i , i = 1, 2... are Rayleigh distributed. The fading model of the envelope *r* is defined as

$$r^{2} = r_{0}^{2} + \sum_{i=1}^{(n-1)/2} r_{i}^{2}$$
, *n* odd (2.18)

$$r^{2} = \sum_{i=1}^{n/2} r_{i}^{2}$$
 , *n* even (2.19)

This model has been shown to fit the Nakagami-*m* distribution in an exact manner [6]. An illustration of the Nakagami-*m* generator is shown in Fig. 2.13.



Figure 2.14. Illustration of the components used to generate the Nakagami-m fading process.

2.3.3.1 Verification of Nakagami Model

This model was used to generate N = 200 random samples of the Nakagami-*m* fading process *r* for two values of *m*: m = 1 and m = 3.5. We have shown the time series of the Nakagami-*m* fading envelope for the above values of *m* in Fig. 2.14. As expected, the number of deep fades is greater for m = 1 compared to m = 3.5. A 9th order Chebyshev filter with cutoff of 0.1 Hz was used in the simulation.



Figure 2.15. Nakagami-*m* fading vs. time for two different '*m*' values

To verify our model we compare the histograms of the Nakagami-m generated samples with the analytical Nakagami-m pdf for the same m values and the plot is shown in Fig. 2.15. It can be observed from Fig. 2.15 that the model developed exhibits a good agreement with the analytical Nakagami-m pdf.



Figure 2.15 Nakagami-*m* analytical *pdf* & histogram for two different *m* values

Summarizing what we discussed in this chapter, we have described three popular single state fading models, their simulation procedures and validated the results from the simulation with analytical results. In the next chapter we introduce multi-state fading concepts and describe some popular multi-state fading models.

Chapter 3: Multistate Fading Models

In the last chapter we described various fading models and the simulation procedures used to produce those models. Channel fading models can be generally classified into the following two types [7]:

Single state models: These models are suitable for environments where the mobile receiving station is in a uniform environment where the propagation paths do not have abrupt changes. The models described in the previous chapter come under this category.

Multi-state models: When a mobile terminal travels in a large area or through non-uniform environments, the received signals may change abruptly, yielding for example different average power levels which are the case when a mobile user travels from an open area to densely populated urban areas. A more suitable model to describe this sort of a channel is a *multistate* model, as a single state model cannot characterize the slow variations corresponding to the large scale fading or a change in the mean power level. This model is usually a linear combination of several *single-state* fading models, each of which corresponds to a specific uniform environment.

In this chapter we describe *multistate* models and also give a brief description of two popular multistate models, the Lutz model [8] and the Loo model [10] and lastly describe some recent work by others in developing multi-state models.

3.1 Introduction

The mobile radio channel is characterized by rapidly changing channel characteristics which arise naturally and inevitably as a consequence of the mobility. As the amplitude of a signal received over such a channel fluctuates, the receiver will experience periods during which the signal can not be received reliably. If a certain minimum (threshold) signal level is needed for acceptable communication performance, in these channels the received signal will experience (at least) two distinct cases of received signal power:

- Sufficient signal strength or "non-fade intervals," during which the receiver can work reliably and at low bit error rate,
- Insufficient signal strength or "fades," during which the bit error rate inevitably is close to one half (randomly guessing ones and zeros) and the receiver's output is no more reliable.

A graphical illustration of the received signal amplitude vs. time is shown in Fig.

3.1.



Figure 3.1 Illustration of received signal amplitude vs. time.

Referring to Fig. 3.1 we could categorize the fade and non-fade periods as two *states*: we can refer to the fade period as the *bad state* and the non-fade period as the *good state*. The *good* state is also generally referred to as the *unshadowed state* and the *bad state* is referred to as the *shadowed state*. This two-state model is the simplest of the class of general *multistate* models. A bi-modal *pdf* is used to show the total distribution. The switching between the two states is often modeled with a finite state Markov chain. This two-state simplification of the wireless channel behavior is called a Gilbert-Elliot model [9]. This model is essentially a binary channel whose two states can be considered the good and bad states. The probability of the channel being in any state at any given time is quantified by the steady-state probabilities.

For our thesis we have developed a two-state model simulation (good and bad states). The good state can be related to the non-fade period or unshadowed state and the bad state correspond to the fade period or shadowed state. This two state model can be easily extended to an n-state model by partitioning the received signal envelope into n intervals and a finite state Markov model can be fitted for the physical channel model. In the last section of this chapter we briefly discuss a 3-state model [14]. The Markov model is described more in detail in the next section.

3.2 Markov chains

The name chain model is derived from one of the assumptions which allow this system to be analyzed, namely the Markov property. The Markov property states that given the current state of the system, the future evolution of the system is independent of its history. The controlling factor in a Markov chain is the transition probability; it is a conditional probability for the system to go to a particular new state, given the current state of the system.

A discrete time Markov chain $\{X_n | n = 0, 1...\}$ is a discrete time, discrete valued random sequence such that given $X_{0...} X_n$, the next random variable X_{n+1} depends only on X_n , through the transition probability

$$P[X_{n+1} = j | X_n = i, X_{n-1} = i_{n-1}, \dots, X_0 = i_0] = P[X_{n+1} = j | X_n = i] = P_{ij}$$
(3.1)

The transition probabilities P_{ij} of a Markov chain satisfy $P_{ij} > 0$ and $\sum_{i=0}^{\infty} P_{ij} = 1$.

The value of X_n summarizes all of the past history of the system needed to predict the next element X_{n+1} of the random sequence. We call X_n the *state* of the system at time n, and the sample space of X_n is called the *set of states* or *state space*. Thus there is a (usually constant) *transition probability* P_{ij} that the next state will be j given that the current state is i. When the Markov chains have a finite set of states $\{0, 1..., K\}$, we say we have a finite Markov chain. In this case, it is convenient to denote the set of one step transition probabilities by the matrix

$$P = \begin{bmatrix} P_{00} & P_{01} & \dots & P_{0K} \\ P_{10} & P_{11} & \dots & \vdots \\ \vdots & & & \\ P_{K0} & \dots & P_{KK} \end{bmatrix}$$
(3.2)

Since the contents of the matrix are probability values they are non-negative and the elements of any row must sum to 1. The matrix P is called a *state transition matrix*. We may alternatively represent the chain by a graph with nodes representing the sample space of X_n as directed arcs for all pairs of states (i, j) such that $P_{ij} > 0$.

Fig. 3.2 shows a graphical representation of a 2-state Markov chain. In this figure, the following notation applies:

 P_{gb} is the probability of transition from a good state to a bad state

 P_{gg} is the probability for the process in the good state to remain in the good state

 P_{bg} is the probability of transition from a bad state to a good state

 P_{bb} is the probability for the process in the bad state to remain in the bad state.



Figure 3.2 Graphical illustration of a Markov chain

Representing this in the matrix form discussed above we get

$$P = \begin{bmatrix} P_{gg} & P_{gb} \\ P_{bg} & P_{bb} \end{bmatrix}$$
(3.3)

Thus the only parameter required to implement the Markov two-state model is the matrix P and this model can be easily scaled to implement an *n*-state Markov model by providing an $n \ge n$ transition probability matrix.

3.3 Popular Multistate Models

3.3.1 Lutz Model

Lutz *et. al.* [8] introduced a two-state analog model to describe a land mobile satellite channel which can be readily be used for hardware and software fading simulation. This model was developed from data measured and recorded in different European areas between a satellite and a ground station (a cruising van) for different elevation angles and different environments. The results of this extensive statistical evaluation include spectra of the fading amplitude, probability density functions, distributions of the received signal power, and the percentage of time for fade and non-fade periods at a given fade level or fade depth.

In [8], they use a two-state Gilbert-Elliot model to represent the land mobile satellite channel. An important parameter of the model is the time-share of shadowing A, ranging from less than 1% on certain highways to 89% in some urban environments.

The authors divide the received signal envelope into two periods or states, the good and bad channel periods.

Good state: The good channel state corresponds to areas with unobstructed "view" of the satellite (unshadowed areas). This corresponds to periods when the received signal power is above a certain threshold value; below this value the received signal is no longer reliable. When no shadowing is present, the received signal is assumed to consist of a multipath signal superimposed on the direct LOS satellite signal, with the total received signal amplitude modeled as a Rician process.

Bad state: The bad channel state represents the case when the direct satellite signal is shadowed by obstacles. When this shadowing is present, it is assumed that no direct signal path exists and that the multipath fading has a Rayleigh characteristic.

The threshold value must be determined such that the time-share of when the received signal power is below the threshold is equal to the parameter A. In both shadowed and unshadowed cases, the signal components are received with independently time-varying amplitudes and phases

The resulting probability density function of the received signal power in terms of the time share of shadowing, A is

$$p(S) = (1 - A)p_{Rice}(S) + A \int_0^\infty p_{Rayl}(S|S_0)p_{LN}(S_0)dS_0$$
(3.4)

where A is the time share in the shadowed state,

$$p_{Rice}(S) = ce^{-c(S+1)}I_0(2c\sqrt{S}), \qquad (\text{Rician } pdf)$$

$$p_{Rayl}(S|S_0) = \frac{1}{S_0}\exp(-S/S_0), \qquad (\text{Rayleigh } pdf)$$

$$p_{LN}(S_0) = \frac{10}{\sqrt{2\pi}\sigma\ln 10}\frac{1}{S_0}\exp\left[-\frac{(10\log S_0 - \mu)^2}{2\sigma^2}\right], \qquad (\text{Lognormal } pdf)$$

Here *c* is the Rice factor, S_0 is the short-term mean received power in the shadowed state, μ is the mean power level decrease from unshadowed to shadowed states, and σ^2 is the variance of the amplitude due to shadowing. (The integral expression results from the theorem of total probability, and can't be analytically simplified.) The *pdf p(s)* is independent of the vehicle velocity, which is assumed constant. The parameters *A* and the various other parameters required for the Rayleigh, Rician and Lognormal fading processes were determined from the statistics of the recording by a least square curve-fitting procedure.

From (3.4) it can be see that the Lutz equation is a combination of the Rayleigh, Rician and lognormal processes. It can be observed that for the fraction of time in the shadowed state, the received power is described by a Rayleigh/lognormal distribution and for the fraction with a LOS component; the received power is described by the Rician process, which is quite intuitive.

The *pdf*'s obtained show good agreement with the statistics of the recorded signal power. Figure 3.3 shows a dynamic model of the land mobile satellite channel that reproduces the fading amplitude samples having the desired *pdf* of the received signal power, including the dynamic power of the fading and shadowing process.

Referring to Figure 3.3 it is seen that the transmitted signal s(t) is deteriorated by multiplicative fading a(t) and additive white Gaussian noise n(t) with power spectral density N_0 . The characteristics of the switching process between the shadowed and unshadowed sections have been approximated by a Markov model similar to the Markov model described earlier.

Referring to Figure 3.3 again we see that Rician fading is produced by attenuating the Rayleigh process to power of 1/*c* and adding a value of unity to represent the direct satellite signal component. The Rayleigh/lognormal fading samples are generated by multiplying the Rayleigh process with a slow lognormal shadowing process. This approach has the advantage that very few fades can be reproduced which is not possible when assuming constant multipath power.

A brief summary of this model is that the channel model developed is largely characterized by the time-share of shadowing, A, and the Rice factor, c, describing the channel during unshadowed periods. The authors claim that the model shows that reliable and efficient data transmission via the land mobile satellite channel should be achievable, if the transmission scheme is suitably adapted to channel behavior.



Figure 3.3 Dynamic model of land mobile satellite model

3.3.2 Loo Model

The authors of [10] have conducted channel measurements ranging from ultrahigh frequency to *Ka* band and have developed a statistical channel model from this data for land mobile satellite systems. This popular channel model assumes that the LOS component under shadowing is log-normally distributed and the multipath component is Rayleigh distributed. The two processes are additive and the channel model is given by the combination of log-normal and Rayleigh models. Thus the channel model is as given below

$$a(t) = \operatorname{Re}\{[y_c(t) + a_c(t) + j(y_s(t) + a_s(t))]\exp[j2\pi f_c t]\}$$
(3.5)

where $a_c(t)$ and $a_s(t)$ are white Gaussian random processes, and $y_c(t)$ and $y_s(t)$ are lognormal random processes. The signal envelope and signal phase are given by

$$r(t) = \sqrt{[y_c(t) + a_c(t)]^2 + [y_s(t) + a_s(t)]^2}$$
(3.6)

$$\phi(t) = \tan^{-1} \left(\frac{y_s(t) + a_s(t)}{y_c(t) + a_c(t)} \right)$$
(3.7)

The Loo model has also been shown to compare reasonably well with the measured data. This model can represent some of the most common known fading conditions, such as Rician, log-normal, and Rayleigh.

3.3.3 Other Multi-State Models

There are currently several others developing multi-state models for mobile satellite systems. We briefly discuss another multi-state model [14]. The main elements in the model were the direct signal and the multipath component.

The characterization of the direct signal was a two-stage approach where the variations were divided into *very slow* which can be described by a steady-state model (Markov) and *slow*, which can be represented by a *log-normal* distribution. The model developed by the authors is a three- state model. This choice was made to accommodate the high dynamic range in the received signal. The following states were defined:

- S_1 LOS conditions;
- S₂-moderate shadowing conditions;
- S₃-deep shadowing conditions;

A first order Markov chain process is used for switching between the different states. The multipath component is characterized by the *average multipath power* parameter or, alternatively, the *carrier-to-multipath ratio*. To jointly model the behavior of the direct signal and the multipath component *within each state* (nor for overall received signal) the Loo distribution is proposed in this model.

Summarizing, in this chapter we briefly described multi-state fading and some popular multi-state fading models. In the next chapter we introduce the new multi-state fading model, describe the simulation procedures and validate the results obtained from this model with measured data.

Chapter 4: New Multi-State Fading Model

In this chapter we introduce the fading model developed in [11] and illustrate how it differs from the other multi-state fading models discussed in the previous chapter. This new model is developed from measured data [13]. We then describe the simulations we have developed to test this new model and compare the results obtained from these simulations with the original data of [13], thereby validating this new multi-state model.

4.1 Introduction

This new multi-state model aims to obtain a new analytical expression for the *pdf* of received signal envelope in a mobile satellite channel. The expressions for the *pdf* are derived from measured data in [13] for an urban environment, but the method yields insight into other environments too. This model used the *Average Fade Duration* (AFD) and *Level Crossing Rate* (LCR) to obtain a composite model similar to the other two-state models discussed in the previous chapter. The concepts of AFD and LCR will be discussed in more detail in this chapter. Since this new model only provides the *pdf*, our contribution is amenable to the development of computer simulations for this mobile satellite channel *pdf* model.

4.2 New Model Development

4.2.1 Level Crossing Rates and Average Fade Durations

Two important second order statistics associated with envelope fading are the *level crossing rate* (how often the envelope crosses a specified value) and the *average fade duration* (how long the envelope remains below a specified level). Fig 4.1 shows an illustration of the time-varying envelope used to help define these two parameters.



Figure 4.1. Illustration of LCR and AFD

Referring to Fig. 4.1, if z(t) is the received envelope then the LCR and AFD for a specified level *R* would be computed as follows, for a given realization of z(t)

$$LCR(R) = \frac{N}{\sum_{i=1}^{N} (t_{R,i+1} - t_{R,i})}$$
(4.1)

$$AFD(R) = \frac{1}{N} \sum_{i=1}^{N} \left(u_{R,i} - t_{R,i} \right)$$
(4.2)

where *N* is the total number of crossing at level *R* in a given direction (either in the positive or negative direction). The notation $t_{R,i}$ represents the n^{th} crossing of the signal below the threshold level *R* and $u_{R,i}$ represents the i^{th} crossing of the signal above the threshold level.

4.2.2 Model Derivation

In this section we review the new pdf model developed in [11]. Most models based upon empirical data that attempt to find a pdf must make some assumptions, partition the data and then apply some sort of curve fit. This model used the LCR and AFD data to obtain the pdf model. The LCR can easily be obtained from the measured data by creating thresholds. The AFD in this case has been derived from the LCR and the actual fading time series [13]. These two functions are directly related to the desired fading pdf via a first-order differential equation. The generic expression for the AFD for a fade below a level of R dB (beneath the mean amplitude level) is

$$AFD(R) = cdf(R)/LCR(R)$$
(4.3)

where LCR(R) is the level crossing rate at level R, and the distribution function cdf(R) is the integral of the $pdf \ p_z(z)$ from 0 to R. Here $R = 20 \log_{10}(z)$. Transforming R to $z = 10^{R/20}$, the domain of z is divided into two regimes states. The reasons for making a choice of two regimes are:

- i. The convenience and agreement with a visual "fit" observed in the data of[13]
- ii. To agree with physically–justified division of prior models into two states
- iii. After numerous attempts at fitting over the entire domain of z, it was found that the number of terms required to achieve a good fit in the "single –regime" approach was too large to render the resulting *pdf*'s convenient.

The explicit expression for the function $p_z(z)$, obtained by taking a derivative of (4.3) is

$$p_{z}(x) = \frac{d}{dx} [LCR(x)AFD(x)] = L(x)A'(x) + A(x)L'(x)$$
(4.4)

where LCR has been abbreviated by L, and AFD by A, and the primes denotes derivatives. Using two sets of data for LCR and AFD from [13] corresponding to two different elevation angles, were developed

The data from the *cumulative density functions* (cdf's), also given in [13], was used to obtain "state probabilities", the probabilities of being in a fading state. For the functional form of the *pdf* the author chose the Nakagami *pdf*, which has been discussed in Chapter 2. It is found that L(x) is of Nakagami form when the *pdf* was Nakagami (without any assumption on the scattering geometry). For additional flexibility, a linear combination of Nakagami *pdf's* was used

$$L(x) = \sum_{k=1}^{N} c_k p_N(x, m_k, P_k)$$
(4.5)

where the Nakagami pdf's are defined as

$$p_{N}(x,m,P) = \frac{2m^{m}}{\Gamma(m)P^{m}} x^{2m-1} \exp(-mx^{2}/P)$$
(4.6)

(see equation (2.12)) with P, the average power in the distribution equal to $E(x^2)$. The final result for the *pdf* $p_z(x)$ in each regime is made up of a sum of Nakagami *pdf's* similar to (4.5), and this can be derived by obtaining the solution of the following first order non-homogenous differential equation when we assume the use of (4.5) and a similar Nakagami form for the *pdf*:

$$A'(x) + A(x)a(x, \{m_k\}, \{P_k\}) = b(x, \{m_k\}, \{P_k\}),$$
(4.7)

which comes from (4.4), where the function $a(x, \{m_k\}, \{P_k\})$ is defined as

$$a(x, \{m_k\}, \{P_k\}) = \frac{\sum_{k=1}^{N} c_k p_N(x, m_k, P_k) h(x, m_k, P_k)}{L(x)}$$
(4.8)

$$b(x, \{m_k\}, \{P_k\}) = \frac{\sum \alpha_k p(x, m_k, P_k)}{L(x)}$$
(4.9)

The coefficients $\{\alpha_k\}$ in (4.9) are those for the *pdf* $p_z(x)$, and are analogous to the c_k 's in (4.5), that is, the *pdf* is

$$p_z(x) = \sum \alpha_k p_N(x, m_k, P_k)$$
(4.10)

The function $h(x, m_k, P_k)$ in the numerator of (4.8) arises from differentiation of the Nakagami *pdf*, and is defined as

$$h(x, m_k, P_k) = -2m_k x / P_k + (2m_k - 1) / x$$
(4.11)

Allowing the coefficients of $\{\alpha_k\}$ in the *pdf* sum to be distinct from those in the sum for L(x) allows more flexibility in fitting the functions for A(x) to the data. The solution for A(x) turns out to be

$$A(x) = \frac{1}{\mu(x)} \int_{0}^{x} \mu(y) b(y, \{m_k\}, \{P_k\}) dy$$
(4.12)

where $\mu(x) = \exp\left(\int_{0}^{x} a(y, \{m_k\}, \{P_k\})dy\right)$ [11]. These equations are solved numerically

since they are not solvable in closed from in the general case, except when N = 1. For insight, consider the case when N = 1. It can be observed that when A(x) is in the form of (4.12), the *pdf* $p_z(x)$ is *exactly* the form of (4.10). Using the data from [13], the author has obtained *pdf's* for two elevation angles. For each case he has divided the envelope

amplitude range into two regimes and obtained the individual *pdf's for each regime*. The *pdf's* from the two regimes are combined using the appropriate state probabilities, obtained by reading a single point from the graph of the corresponding *cdf's* in [13]. The curve fits for the resulting simulated L(x) and A(x) show good agreement with the measured data, as will be shown.

To summarize, the new model uses AFD and LCR data, along with the Nakagami assumption, to fit *pdf's* using equations (4.5), (4.10), and (4.12). The result is the set of *pdf's* of the form of (4.10), along with their constituent parameters ($\{\alpha\}, \{m\}, \{P\}$).

We then use *pdf* parameters in a simulation that produces the fading amplitude time series which is validated against the original measured data. The overall procedure is illustrated in Fig. 4.2.



Figure 4.2. Block Diagram of Procedure to Validate the Model

4.2.3 Simulation Development of New Multi-State Fading Model

In the previous section we described the analytical development of the new multistate fading model (NMSFM). We now aim to simulate this model to verify its analytical development. We can divide this simulation into four different modules

- i. Nakagami fading generators
- ii. Markov process generators
- iii. Level crossing rate calculator
- iv. Average fade duration calculator

Modules iii and iv are solely for model validation.

A block diagram of the simulation procedure is shown in Fig. 4.2. The shaded blocks in Fig. 4.3 represent the inputs to the simulations. We shall first explain the overall simulation development and then explain in detail the development of each of the individual modules.

From Fig. 4.3 it can be observed that we require two separate Nakagami fading generators. Each generator is used to generate the fading samples corresponding to one of the two regimes. Strictly, the same programs are used for both the Nakagami generator, with distinct parameters for each state. The parameters mLo and mHi denote the low-regime and high-regime m values of the Nakagami generators, respectively. Similarly the parameters PLo and PHi denote the mean square values for each of the Nakagami generators. After we have the fading samples for each of the two regimes we combine the fading samples according to their state probabilities. We shall explain later how we obtain the state probabilities.

The process of switching between the two regimes is accomplished using a Markov process generator as explained in an earlier chapter. The resulting samples obtained after the switching process are the desired multi-state fading samples.



Figure 4.3. Block Diagram of New Multistate Fading Model

In order to validate the obtained fading samples against the measured data we calculate the *LCR* and *AFD* from the obtained fading samples, and compare these with the measured LCR and AFD data.
A sample multi-state pdf is shown in Fig.4.4 for the following Nakagami generator parameters

 $mLo = 1.5 \qquad PLo = 0.9$

$$mHi = 14 \qquad PHi = 1.1$$



Figure 4.4 Multi- State Fading PDF

4.2.4 Parameters for the Nakagami Fading Generators

The Nakagami generator we have developed requires three input parameters, namely

- i. Number of samples required.
- ii. Value of the shape factor *m* (has to be a positive multiple of 0.5 as discussed in Chapter 2).
- iii. Mean square value of the fading samples (PLo or PHi).

From the measured data in [13] we have calculated of the fitting parameters required for the mobile satellite channel modeling. We took the *LCR* and *AFD* data from the measured data and then curve fit to equations (4.5), (4.10), and (4.12) to obtain the m values and the mean square values. The fitting parameters are shown in Table 4.1, where the Nakagami parameters and constants are given for the four streets for which data was given in [13].

Street Name	Elevation Angle (degrees)	Regi (-35 dl dl	me 1 B to -7 B)	$lpha_1$	Regi (-7 dB d	me 2 8 to +3 B)	$lpha_2$
		mLo	PLo		mHi	PHi	
Zaimi St.	80	1.312	0.09	0.083	7.536	1.076	0.952
	60	0.946	0.085	0.616	6.651	1.021	0.412
Bouboulinas St.	80	2.499	0.275	0.047	5.646	1.213	0.987
	60	1.276	0.069	0.6	14.124	1.102	0.404
Ippokratous St.	80	23.161	1.074	1.035	X	X	X
	60	1.09	0.069	0.8	12.798	1.203	0.209
Askpliou St	80	16.786	1.068	1.004	X	Χ	X
	60	1.029	0.047	0.723	0.882	0.847	0.416

Table 4.1 Summary of Fitting Parameters

For the 80-degree elevation angle, the Ippokratous Street and Askpliou Street data yielded only a single regime. The reason we have a single state is because from the measured data for these two streets we could get a good fit with the data using only one regime compared to using two regimes, unlike the remaining cases. The range for Ippokratous street 80 degrees was -4.5dB to 1.5dB, and for Askpliou street 80 degrees the range was -6.5dB to 2.5dB. The parameters α_1 and α_2 are the coefficients over the respective regimes, used to normalize the resulting total *pdf* so that its area is unity (see (4.10)).

For the simulations we used N=100,000 samples. The reason we have generated so many samples is to get a smooth *LCR* and *AFD* curve for even low envelope values, which occur infrequently. We have also approximated the *m* values in the table, as our Nakagami generator can provide only *m* values that are positive multiples of 0.5.

4.2.5 Markov Process Generator

As mentioned in the previous section, the process of switching between the two states is accomplished using Markov chains. We obtain the steady state probabilities of being in a particular state from the *cumulative distribution function* (CDF) plots. Since we have already divided the available data into two regimes we can just look up the steady state probabilities of being in each regime from the *cdf* plot given in [13]. An illustration of calculating the steady state probabilities from the *cdf* plot is shown in Fig. 4.5.



Figure 4.5 Illustration of calculation of steady state probabilities

We obtain the steady-state probability from the CDF. This gives us the probability of being in the shadowed (bad) or unshadowed (good) state. Let P(shadowed)=A. We then run the Markov generator by itself to obtain plots of this probability A as functions of both transition probabilities (p_{gb} and p_{bg}). A plot of the steady state probabilities for different transition probabilities is shown below in Fig. 4.6.

The transition probabilities are determined empirically with the curves of Fig.4.6 serving as one constraint (so that the desired steady-state probability is obtained). The final values of the transition probabilities are chosen via iteration, in order to yield good agreement between the simulation and measured data. More will be said regarding this procedure subsequently.



Figure 4.6. Steady State Probabilities for Different Transition Probabilities

The relationships between these probabilities are $P_{gb} = 1 - P_{gg}$ and $P_{bg} = 1 - P_{bb}$.

4.2.6 Level Crossing Rate and Average Fade Duration Calculator

After we have obtained the multi-state fading samples given $\{m\}$, $\{P\}$ and A, we calculate the LCR and AFD to validate the model against the measured LCR and AFD data of [13]. We have already introduced the concepts of LCR and AFD in Section 4.2.1. For our simulation we have calculated all the fades that cross the reference value in the negative direction. The range of the reference values that we have considered in the simulations varies from -40 dB to +20 dB from the mean square value.

To test the *LCR* and *AFD* module, we first test it for a single state case. Shown in Figure 4.7 is the *LCR* plot for Ippokratous Street from [13] which we have divided into only one regime. Similarly, Figure 4.8 shows the *AFD* plots for the same street. The normalized *LCR* on the ordinates of the *LCR* plots represent the *crossings per wavelength* for the threshold indicated by the abscissa value. For the *AFD* plots the normalized *AFD* represents the duration of fades in wavelengths for the abscissa value, which in turn corresponds to the received signal level in dB normalized to the LOS value. From [13] we know that the measurements were made at 1.8 GHz, therefore the wavelength of the signal is

$$\lambda = \frac{c}{f_c} = \frac{3 \times 10^8}{1.8 \times 10^9} = 0.1667m \tag{4.13}$$

where c is the propagation speed of light and f_c is the carrier frequency.



Figure 4.7 LCR Plot for Ippokratous Street 80 degrees.

The fitting parameters for the Ippokratous Street 80-degree case were m = 23 and P = 1.074. From the plots it can be observed that the simulated plots for both the *LCR* and *AFD* exhibit very good agreement with the measured data.



Figure 4.8. AFD Plot for Ippokratous Street -80 degrees

4.3 Results of Simulation

We now validate the results for all the multi-state streets in Table 4.1 using the multi-state simulator developed with the measured data from [13].

Figures 4.9 and 4.10 are the LCR and AFD plots respectively of Bouboulinas

Street at 60-degrees elevation angle. The parameters of the simulation are:

```
mLo = 1
mHi = 14
PLo = 0.069
PHi = 1.102
```



Figure 4.9. LCR Plot for Bouboulinas Street 60 – degrees



Figure 4.10. AFD Plot for Bouboulinas street 60 degrees.

The transition probabilities for Bouboulinas Street (60 degrees) are

$$\mathbf{P} = \begin{bmatrix} P_{gg} & P_{gb} \\ P_{bg} & P_{bb} \end{bmatrix} = \begin{bmatrix} 0.99 & 0.01 \\ 0.016 & 0.984 \end{bmatrix}.$$

Figure 4.11 shows the *pdf* for Bouboulinas street 60-degree elevation angle case, comparing it with the analytical *pdf* that is obtained by combining two single state analytical *pdf*'s according to each regime's respective steady state probabilities.



Figure 4.11. Multi-State PDF of Bouboulinas Street 60 degrees.





Figure 4.12 LCR Plot for Askpliou Street 60 degrees

The transition probabilities for Askpliou Street (60 degrees) are

$$\mathbf{P} = \begin{bmatrix} P_{gg} & P_{gb} \\ P_{bg} & P_{bb} \end{bmatrix} = \begin{bmatrix} 0.99 & 0.01 \\ 0.025 & 0.975 \end{bmatrix}.$$



Figure 4.13 AFD Plot for Asklipiou Street 60 degrees

Figure 4.14 is a plot of the histogram of multi-state simulated fading samples compared with the analytical *pdf*.



Figure 4.14. Multi-State PDF of Asklipiou street 60 degrees.

Figures 4.15, 4.16 and 4.17 are the LCR, AFD, and *pdf* comparison plots respectively for Ippokratous street 60-degrees elevation angle case. . The parameters used for the simulation are:

$$mLo = 1$$

 $mHi = 13$
 $PLo = 0.069$
 $PHi = 1.102$



Figure 4.15. LCR Plot for Ippokratous Street 60 degrees



Figure 4.16. AFD Plot for Ippokratous Street 60 degrees

The transition probabilities for Ippokratous Street (60 degrees) are

$$\mathbf{P} = \begin{bmatrix} P_{gg} & P_{gb} \\ P_{bg} & P_{bb} \end{bmatrix} = \begin{bmatrix} 0.99 & 0.01 \\ 0.025 & 0.975 \end{bmatrix}.$$



Figure 4.17. Multi-State PDF of Ippokratous street 60 degrees.

Figures 4.18, 4.19 and 4.20 are the LCR, AFD, and *pdf* comparison plots respectively for Zaimi street 60-degrees elevation angle case. The parameters used for the simulation are:

$$mLo = 1$$

 $mHi = 6.5$
 $PLo = 0.085$
 $PHi = 1.021$



Figure 4.18. LCR Plot for Zaimi Street 60 degrees



Figure 4.19. AFD Plot for Zaimi Street 60 degrees.

The transition probabilities for Zaimi Street (60 degrees) are

$$\mathbf{P} = \begin{bmatrix} P_{gg} & P_{gb} \\ P_{bg} & P_{bb} \end{bmatrix} = \begin{bmatrix} 0.99 & 0.01 \\ 0.016 & 0.984 \end{bmatrix}.$$



Figure 4.20. Multi-State PDF of Zaimi street 60 degrees.

All the above multi-state LCR and AFD plots are for different streets in the Table 4.1 for an elevation angle of 60 degrees. From the *pdf*'s of the streets with 60 degrees elevation angle it can be observed that the majority of the time the received signal was in the shadowed state.

We shall now validate our simulation results for the streets with 80 degrees elevation angles in Table 4.1. Figures 4.21, 4.22 and 4.23 are the LCR, AFD and *pdf* comparison plots respectively for Bouboulinas Street at 80 degrees elevation angle.



Figure 4.21. LCR Plot for Bouboulinas Street 80 degrees

The parameters used for the simulation are:

$$mLo = 2.5$$

 $mHi = 5.5$
 $PLo = 0.275$
 $PHi = 1.213$



Figure 4.22. AFD Plot for Bouboulinas Street 80 degrees



Figure 4.23. Multi-State PDF of Bouboulinas Street 80 degrees.

The transition probabilities for Bouboulinas Street (80 degrees) are

$$\mathbf{P} = \begin{bmatrix} P_{gg} & P_{gb} \\ P_{bg} & P_{bb} \end{bmatrix} = \begin{bmatrix} 0.99 & 0.01 \\ 0.465 & 0.535 \end{bmatrix}.$$

Figures 4.24, 4.25 and 4.26 are the LCR, AFD and *pdf* comparison plots respectively for Zaimi street 80 degrees.



Figure 4.24. LCR Plot for Zaimi Street 80 degrees

The parameters used for the simulation are:

$$mLo = 1.5$$

 $mHi = 7.5$
 $PLo = 0.09$
 $PHi = 1.076$



Figure 4.25. LCR Plot for Zaimi Street 80 degrees

The transition probabilities for Zaimi Street (80 degrees) are

$$\mathbf{P} = \begin{bmatrix} P_{gg} & P_{gb} \\ P_{bg} & P_{bb} \end{bmatrix} = \begin{bmatrix} 0.99 & 0.01 \\ 0.17 & 0.83 \end{bmatrix}$$



Figure 4.26. Multi-State PDF of Zaimi Street 80 degrees.

From the LCR and AFD plots for the different streets and elevation angles it can be observed that there is a good agreement between the measured data and the simulated data.

Comparing the multi-state pdf's of the 60 degrees elevation angle and the 80 degrees elevation angle we observe that the amount of time in the shadowed state for the

60 degrees case is much higher when compared to the 80 degrees cases. The opposite holds true when comparing the time spent in the unshadowed state. This is rather intuitive as you would expect more fading for a lower elevation angle when compared to a higher elevation angle. It can be observed in some cases (Figures 4.12 and 4.15) the simulated LCR is higher when compared to the measured value for part of the domain; this is due to a high number of transitions observed in the time series. If we use a high value of transition probabilities between the two states then we induce "artificial" level crossings which increase the LCR values at *all* levels. Therefore given the steady state probabilities for each state we have to choose the transition probabilities such that the transition between the states is kept relatively low. The steady state transition values we have thus far used can be considered as an approximate upper bound to our choice of transition probabilities until we come up with a better method of deriving these transition probabilities. Based upon some values of these transition probabilities in another set of data [8], we have found that the transition probabilities should be less than approximately 0.125

To summarize, we have simulated all the multi-state fading cases in Table 4.1 and we have seen that the results from the new simulation model show good agreement with the measured data [13]. The modeling and validation procedure is summarized again in Fig 4.27.



Figure 4.27 Schematic of Model Validation

Chapter 5: Summary and Future Work

In this chapter we give a brief summary of the work done on this thesis and suggest some areas for future work.

5.1 Summary

We have developed a new simulator for a multi-state fading channel. This simulator is general, and can be configured to yield existing models such as "Loo's model" and "Lutz model" and even simple single state models like Rayleigh, Rician and Nakagami-*m* distributions. The simulator has been validated against known theoretical models, and provides excellent agreement. It can also compute the statistics of measured data such as *level crossing rate and average fade duration*. The required inputs for the multi-state model include Nakagami *pdf* parameters (*m* and *P*), and the *cdf* data point(s) to determine the state boundaries. This simulation is useful in developing computer simulations of mobile satellite and terrestrial fading channel amplitude time series realizations, and with further analytical study and additional measured data, may provide additional insight into the physical channel character.

5.2 Future Work

As mentioned in Chapter 4, we obtain the probabilities of switching between states from the steady state probabilities, which give us an approximate upper bound on the transition probabilities and not exact values of the transition probabilities. We thus need to devise a better method to obtain the exact transition probabilities, rather than via a "trial and error" method.

For this model and simulation development we have used only experimental data from [13]. To better fine tune the model and make it more widely applicable, we would need to gather more experimental data for different propagation environments and validate the results from the simulator for the different fading environments.

Another topic for future work would be to include more non-stationary features in the model. In our model we have assumed the filter bandwidth, corresponding to the Doppler frequency shift, is constant throughout the simulation. This is not generally true in practice, for instance in a moving vehicle the Doppler frequency is not constant and would change according to the velocity of the vehicle.

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Appendix: Matlab Code

```
%
%
                GENERALIZED FADING SIMULATOR
                                                                    %
%
                                                                    %
%
       This program can simulate fading samples of the following distribution:
                                                                    %
%
                                                                    %
               1. Rayleigh fading samples
                                                                      %
%
                 2. Rician fading samples
%
                 3. Nakagami-m fading samples
                                                                  %
                                                                   %
%
%
     The common parameters required for all the three fading generators are:
                                                                   %
%
               1. Number of fading samples required
                                                                    %
%
              2. Filter bandwidth or maximum fading rate required
                                                                    %
  %
% Start of program -----
                                 ----- %
% Selecting fading distribution required
Process = input('\n Enter 1 for Rayleigh fading \n
                                          2 for Rician fading n
                                                               3 for Nakagami fading :');
if((Process \sim = 1) & (Process \sim = 2)& (Process \sim = 3))
  fprintf('Invalid Selection');
 break;
end
% Entering input parameters
No samples = input('\n Enter number of samples required :');
FadingRate = input('\n Enter the filter bandwidth = max fading rate, \leq 1:');
% Selecting the appropriate fading process generator
if(Process == 1)
 fading_samples = Rayleigh(No_samples, FadingRate); % Calling the Rayleigh
elseif (Process == 2)
    fading_samples = Rician(No_samples,FadingRate); % Calling the Rician
elseif(Process == 3)
    fading samples = Nakagami(No samples, FadingRate); % Calling the Nakagami
else printf('\nInvalid selecton');
end
% End of program ------- %
```

% % Nakagami Fading Generator % % % % % Input parameters to the generator are: % % 1. Number of fading samples required % 2. Filter bandwidht or maximum fading rate % % 3. Shape factor 'm' % function fading samples = Nakagami(No samples, FadingRate); % Start of program-----% m val = input('Enter Nakagami shape factor m : '); % Start of program-----% Generating the filter parameters [b,a] = cheby1(5,0.05,FadingRate); % Obtaining the filter coefficients % Generating the Gaussian samples % When m = 0.5 the samples are equal to the square of a single Gaussian Process if (m val == 0.5)gaussx = randn(1, No samples + 10000);gfilt1 = filter(b,a,gaussx); gfiltx = gfilt1./sqrt(mean(gfilt1.^2)); gfiltx = gfiltx(10001:No samples + 10000); $r = gfiltx.^2$; % Nakagami faded samples for m = 0.5elseif(m val == 1)% We use a different if loop for m = 1 since for m > 1 we have a m - dimensional array % The array is summed it across m dimensions. For m = 1 we have just a 1-D array % Hence we don't sum the array twice % Gaussian generator x and y padded with 10000 samples for transient gaussx = randn(1, No samples + 10000);gaussy = randn(1, No samples + 10000);gfilt1 = filter(b,a,gaussx); % Filtering the Gaussian output gfilt2 = filter(b,a,gaussy);gfiltx= gfilt1./sqrt(mean(gfilt1.^2)); % Scaling the output of the filter $gfilty = gfilt2./sqrt(mean(gfilt2.^2));$ % Removing the transient portion of the samples gfiltx1 = gfiltx(10001:No samples + 10000);gfilty1 = gfilty(10001:No samples + 10000); $r = (gfiltx1.^2 + gfilty1.^2);$ % Nakagami faded samples for even m = 1elseif(m val == 1.5)

% For m = 1

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```
gaussx = randn(1,No_samples + 10000);
gaussy = randn(1,No_samples + 10000);
gfilt1 = filter(b,a,gaussx); % Filtering the samples
gfilt2 = filter(b,a,gaussy);
gfiltx = gfilt1./sqrt(mean(gfilt1.^2)); % Sacling the output of the filter
gfilty = gfilt2./sqrt(mean(gfilt2.^2));
gfiltx1 = gfiltx(10001:No_samples + 10000);
gfilty1 = gfilty(10001:No_samples + 10000);
```

```
% For m = 0.5
```

```
gaussx0 = randn(1, No_samples + 10000);

gfilt = filter(b,a,gaussx0);

gfilt = gfilt./sqrt(mean(gfilt.^2));

gfilt1 = gfilt(10001:No_samples+10000);

r = (gfiltx1.^2 + gfilty1.^2);
```

 $r = r + gfilt1.^{2};$

```
% Even values of 'n' (m = n/2)
```

```
elseif(mod(2*m_val,2)==0)
```

```
% Gaussian generator x and y padded with 10000 samples for transient
gaussx = randn(m_val,No_samples + 10000);
gaussy = randn(m_val,No_samples + 10000);
```

```
% Filtering and scaling the squared Gaussian values
```

gfiltx(i,:)= gfilt1(i,:)./sqrt(mean(gfilt1(i,:).^2));

 $gfilty(i,:) = gfilt2(i,:)./sqrt(mean(gfilt2(i,:).^2));$ $gfiltx1(i,:) = gfiltx(i,10001:No_samples + 10000);$ $gfilty1(i,:) = gfilty(i,10001:No_samples + 10000);$

for i = 1:m_val; gfilt1(i,:) = filter(b,a,gaussx(i,:)); gfilt2(i,:) = filter(b,a,gaussy(i,:));

% Scaling the output of the filter

end

 $r = sum(gfiltx1.^2 + gfilty1.^2);$ % Nakagami faded samples for even m

else

```
% For odd values of 'n'
```

```
% For the even portion of 'n' m1 = m val - 1/2;
```

% Gaussian generator x and y padded with 10000 samples for transient gaussx = randn(m1,No_samples + 10000 gaussy = randn(m1,No_samples + 10000);

```
for i = 1:m1;
gfilt1(i,:) = filter(b,a,gaussx(i,:)); % Filtering the Gaussian output
gfilt2(i,:) = filter(b,a,gaussy(i,:));
gfiltx(i,:) = gfilt1(i,:)./sqrt(mean(gfilt1(i,:).^2)); % Scaling the output of the filter
gfilty(i,:) = gfilt2(i,:)./sqrt(mean(gfilt2(i,:).^2));
```

```
% Removing the transient response
     gfiltx1(i,:) = gfiltx(i,10001:No_samples + 10000);
    gfilty1(i,:) = gfilty(i,10001:No_samples + 10000);
 end
% For the odd portion of 'n'
% Extra addition of Gaussian samples done when 'n' is odd;
  gaussx0 = randn(1, No samples + 10000);
  gfilt = filter(b,a,gaussx0);
                                 % Filtering the Gaussian output
  gfilt = gfilt./sqrt(mean(gfilt.^2)); % Scaling the Gaussian output
  gfilt1 = gfilt(10001:No samples + 10000);
  r = sum(gfiltx1.^2 + gfilty1.^2);
  r = r + gfilt1.^2;
                               % Nakagami samples
end
   r=sqrt(r);
                          % Scaling the output to have E(r^2)=1
   rrms = sqrt(mean(r.*r));
  r = (r./rrms);
  fading_samples = r;
% End of main program-----%
```

% **Rician Fading Generator** % % % % % Input parameters to the generator are: % 1. Number of fading samples required % % 2. Filter bandwidth or maximum fading rate % % 3. Rice factor 'K' in dB % % % 4. Maximum Doppler frequency of LOS component function fading samples = Rician(No samples, FadingRate); % Start of program-----% K = input('Enter K factor in dB : ');Dopp freq = input('\nEnter Doppler Frequency of LOS component, $\leq fM$: '); % Start of program-----% Generating the Gaussian samples gauss1 = randn(1,No samples + 10000); % White Gaussian generator 1 gauss2 = randn(1,No samples + 10000); % White Gaussian generator 2 % Filtering the Gaussian output [b,a] = cheby1(9,0.5,FadingRate);% Obtaining the filter coefficients gfilt1 = filter(b,a,gauss1);gfilt2 = filter(b,a,gauss2) ; % Removing the transient portion of the filter output and scaling the output of the filter % Scaling factor s = sqrt(1./FadingRate);gfiltx = gfilt1(10001:No_samples+10000).*s gfilty = gfilt2(10001:No samples+10000).*s;% Generating the Rician fading samples $k = 10.^{(K/10)}$: % Converting the 'K' value to normal ratio from dB mean val = sqrt(k./(k + 1)); % mean amplitude of the random processes var root = 1./(sqrt(2.*(k+1))); % square root of variance of the random processes % Normalize filter outputs to have variance var root^2 (pre-filter values) gfiltx1 = gfiltx./sqrt(var(gfiltx)).*var root; gfilty1 = gfilty./sqrt(var(gfilty)).*var root; % Add the Doppler frequency-shifted LOS signal to the filtered Gaussian samples sample_no = 1:No samples; theta = rand(1,1)*2*pi; % Random phase between (0.360)Icomp doppshift = mean val.*cos(2*pi*Dopp freq*sample no + theta); Qcomp doppshift = mean val.*sin(2*pi*Dopp freq*sample no + theta);

% Generate the Ricean samples

Icomp = gfiltx1 + Icomp_doppshift; Qcomp = gfilty1 + Qcomp_doppshift; 105

$r = sqrt(Icomp.^2 + Qcomp.^2);$	% Filtered Ricean amplitude vector
rrms=sqrt(mean(r.*r));	% Compute $E(r^2)$, and its square-root
r=r/rrms;	% Normalize to obtain $E(r^2) = 1$
fading_samples = r;	

% End of main program-----%

0 _{/0} ************************************	
% Rayleigh Fading Generator	%
%	%
% Input parameters to the generator are:	%
% 1. Number of fading samples required	%
% 2. Filter bandwidth or maximum fading rate required %	
0/0 ***********************************	
function fading_samples = Rayleigh(No_samples,FadingRate);	
% Start of program%	
% Generate Gaussian samples	
gauss1 = randn(1,No samples + 10000); % White Gaussian generator 1	
gauss2 = randn(1,No_samples + 10000); % White Gaussian generator 2	
% 10000 samples taken extra to compensate for the transient effect of the filter	er
% Filtering the Gaussian outputs	
[b,a] = cheby1(7,0.5,FadingRate); % Obtaining the filter coefficients	
gfilt1 = filter(b,a,gauss1);	
gfilt2 = filter(b,a,gauss2);	
0/ Domaxing the transient parties of the filter sutput and seeling the sutput of	the filter
% Kemoving the transient portion of the finter output and scaling the output of	
h = sart(1 / Fading Rate); % Scaling factor	
σ squ(1.1 admigRate), σ scaling ratio afilty = afilt1(10001:No samples + 10000) *h:	
$gintx = gfilt2(10001:No_samples + 10000) * b;$	
ginty gint2(10001.100_sumples + 10000). 0,	
% Generating the Rayleigh fading samples	
% Normalizing the mean square value to obtain $E(r^2) = 1$	
\mathbf{J}_{1}	
$r = sqrt(gfiltx.^2 + gfilty.^2);$ % Rayleigh faded amplitude samples	
rrms = sqrt(mean(r.*r)); % Compute $E(r^2)$, and its square-root	
$r = r/rrms$; % Normalize to obtain $E(r^2) = 1$	

fading_samples = r;

% End of main program-----%

% % MULTI STATE FADING SIMULATOR % % % % This program simulates multi-state fading samples using Nakagami-m distribution % % % % % The functions required by this program are % 1. MultistateNakagami.m % % 2. Markov.m % % 3. LcrAfd.m % % % % % Start of program ------ % No samples = input('\nEnter number of samples required :'); init state = input('\nEnter initial state of the process (0-bad, 1-good) :'); goodm = input('\nEnter m value of the good state :'); goodP = input('\nEnter mean power of the good state :'); badm = input('\nEnter m value of the bad state :'); badP = input('\nEnter mean power of the bad state :'); Pgg = input('\nEnter probability of transition from good to good state :'); Pbb = input('\nEnter probability of transition from bad to bad state :'); % Markov switching matrix trans prob = [Pgg 1-Pgg ; 1-Pbb Pbb]; % Generating the Markov swithing samples switch samples = Markov(No samples,trans prob,init state); % Generating Nakagami fading samples for the good and bad state good samp = MultistateNakagami(No samples,goodm,goodP); bad samp = MultistateNakagami(No samples,badm,badP); % Switching between the states for i = 1:No samples; if switch samples(i) $\sim = 0$; fading samples(i) = bad samp(i); else fading_samples(i) = good_samp(i); end end % Calculating the mean square value of the composite fading sample sequences mean $sq = mean(fading samples.^2);$ % Calculating the LCR and AFD values lcrafd = LcrAfd(fading_samples.^2,mean_sq); lcrsamples = lcrafd(1,:); afdsamples = lcrafd(2,:);
% End of main program -----%

```
% Function Markov2.m generates a random binary vector S, with elements in set \{0,1\}
% where each element represents the state of a 2-state Markov process at time k, k=1,2,...N
% This function uses the binary random variable generator function b01(L,p0), where here L=1,
% and p0 is one of the diagonal elements of the Markov state transition matrix P.
                 (b01.m syntax is y=b01(L,p0), where y0 is zero or one)
%
%
        P=[P(1,1) P(1,2); P(2,1) P(2,2)] where P(i,j)=transition probability from state i to state j
%
        so p0 must be either P(1,1) or P(2,2)
% The other input is the starting state, 0 (corresponding to 1) and 1 (corresponding to 2)
function S = Markov2(N,P,s0)
if s0 ~= 1
 if s0 \sim = 0
 sprintf('%s','s0 must be either 0 or 1')
 return;
 end
end
Ptest=sum(P');
Ptest2=sum(Ptest-[1 1]);
if Ptest2 \sim = 0
 sprintf('%s','Each row of P must sum to one')
 return:
end
S(1)=s0;
                  %Starting state is s0
for kk=1:N
 x=b01(1,P(S(kk)+1,S(kk)+1));
 if x == 1
   S(kk+1) = S(kk);
  else
   S(kk+1) = \sim S(kk);
 end
end
function n=baseM2dec(s,M,L)
% This function converts a base M number to its decimal equivalent
fact=1: n=0:
for ii=L:-1:1
 n=n+s(ii)*fact;
 fact=fact*M;
end
```

%	***************************************	/ 0
%	Nakagami Fading Generator	%
%		%
%	Input parameters to the generator are:	%
%	1. Number of fading samples required	%
%	2. Mean square value of the samples required	%
%	3. Shape factor 'm'	%
%	***********	ó

function fading_samples = multinakagami(No_samples,m_val,P_val)

% Start of program-----%

% Generating the filter parameters

FadingRate = 0.066; [b,a] = cheby1(5,0.05,FadingRate); % Obtaining the filter coefficients

% Generating the Gaussian samples

% When m = 0.5 the samples are equal to the square of a single Gaussian Process

if (m_val == 0.5)
gaussx = randn(1,No_samples + 10000);
gfilt1 = filter(b,a,gaussx);
gfiltx = gfilt1./sqrt(mean(gfilt1.^2));
gfiltx = gfiltx(10001:No_samples + 10000);

 $r = gfiltx.^2$; % Nakagami faded samples for m = 0.5

 $elseif(m_val == 1)$

% We use a different if loop for m = 1 since for m > 1 we have a m - dimensional array % The array is summed it across m dimensions. For m = 1 we have just a 1-D array % Hence we don't sum the array twice

```
% Gaussian generator x and y padded with 10000 samples for transient
gaussx = randn(1,No_samples + 10000);
gaussy = randn(1,No_samples + 10000);
gfilt1 = filter(b,a,gaussx); % Filtering the Gaussian output
gfilt2 = filter(b,a,gaussy);
gfiltx= gfilt1./sqrt(mean(gfilt1.^2)); % Scaling the output of the filter
gfilty = gfilt2./sqrt(mean(gfilt2.^2));
```

% Removing the transient portion of the samples

gfiltx1 = gfiltx(10001:No_samples + 10000); gfilty1 = gfilty(10001:No_samples + 10000);

 $r = (gfiltx1.^2 + gfilty1.^2);$ % Nakagami faded samples for even m = 1

 $elseif(m_val == 1.5)$

% For m = 1

```
gaussx = randn(1,No_samples + 10000);
gaussy = randn(1,No_samples + 10000);
gfilt1 = filter(b,a,gaussx); % Filtering the samples
gfilt2 = filter(b,a,gaussy);
gfiltx = gfilt1./sqrt(mean(gfilt1.^2)); % Sacling the output of the filter
gfilty = gfilt2./sqrt(mean(gfilt2.^2));
gfiltx1 = gfiltx(10001:No_samples + 10000);
gfilty1 = gfilty(10001:No_samples + 10000);
```

% For m = 0.5

 $gaussx0 = randn(1, No_samples + 10000);$ gfilt = filter(b, a, gaussx0); $gfilt = gfilt./sqrt(mean(gfilt.^2));$ $gfilt1 = gfilt(10001:No_samples+10000);$ $r = (gfiltx1.^2 + gfilty1.^2);$

 $r = r + gfilt1.^2;$

% Even values of 'n' (m = n/2)

 $elseif(mod(2*m_val,2)==0)$

% Gaussian generator x and y padded with 10000 samples for transient

gaussx = randn(m_val,No_samples + 10000); gaussy = randn(m_val,No_samples + 10000);

% Filtering and scaling the squared Gaussian values

```
for i = 1:m_val;

gfilt1(i,:) = filter(b,a,gaussx(i,:));

gfilt2(i,:) = filter(b,a,gaussy(i,:));

gfiltx(i,:) = gfilt1(i,:)./sqrt(mean(gfilt1(i,:).^2));

gfilty(i,:) = gfilt2(i,:)./sqrt(mean(gfilt2(i,:).^2));

gfiltx1(i,:) = gfiltx(i,10001:No_samples + 10000);

gfilty1(i,:) = gfilty(i,10001:No_samples + 10000);
```

end

 $r = sum(gfiltx1.^2 + gfilty1.^2);$ % Nakagami faded samples for even m

else % For odd values of 'n'

```
% For the even portion of 'n'
m1 = m val - 1/2;
```

% Gaussian generator x and y padded with 10000 samples for transient

gaussx = randn(m1,No_samples + 10000); gaussy = randn(m1,No_samples + 10000);

for i = 1:m1; gfilt1(i,:) = filter(b,a,gaussx(i,:)); % Filtering the Gaussian output

```
 \begin{array}{l} gfilt2(i,:) = filter(b,a,gaussy(i,:));\\ gfiltx(i,:) = gfilt1(i,:)./sqrt(mean(gfilt1(i,:).^2)); \% \ Scaling the output of the filter\\ gfilty(i,:) = gfilt2(i,:)./sqrt(mean(gfilt2(i,:).^2));\\ gfiltx1(i,:) = gfiltx(i,10001:No\_samples + 10000);\\ gfilty1(i,:) = gfilty(i,10001:No\_samples + 10000);\\ end \end{array}
```

% For the odd portion of 'n'

% Extra addition of Gaussian samples done when 'n' is odd;

```
gaussx0 = randn(1,No_samples + 10000);
gfilt = filter(b,a,gaussx0); % Filtering the Gaussian output
gfilt = gfilt./sqrt(mean(gfilt.^2)); % Scaling the Gaussian output
gfilt1 = gfilt(10001:No_samples + 10000);
r = sum(gfiltx1.^2 + gfilty1.^2);
r = r + gfilt1.^2; % Nakagami samples
end
```

```
r=sqrt(r); % Scaling the output to have E(r^2)=P_val
rrms = sqrt(mean(r.*r));
r = (r./rrms).*sqrt(P_val);
fading samples = r;
```

```
% End of main program-----%
%
%
    Level Crossing Rate and Average Fade Duration Calculator
%
                                                 %
%
                                             %
         Input parameters to the generator are:
%
    1. Fading samples whose LCR and AFD need to be calculated
                                               %
%
                                              %
    2. Mean square value of the fading samples
%
                                              %
```

function LcrAfd_val = LcrAfd(fading_samples,P_val)

% Start of program-----%

index = 1; % Indexing variable dbpower = 10 .* log10(P_val); % Converting to dB the mean square value

% Calculating LCR and AFD for +20db to -20db of the power

for range = (dbpower-20):(dbpower+10); ref_val = 10.^(range/10); % Converting the threshold from dB to normal value fflag = -1; % To check for initial value above reference count = 1; % Total number of crossings in downward directions temp = 0; % To store the crossing across the reference

% Calculating LCR and AFD for each threshold value defined by the range

for sample_num = 1:length(fading_samples);

% To ensure that the count begins after sample value is above the reference

```
if (fading_samples(sample_num)>ref_val & fflag == -1);
fflag = 1;
end
```

% Checking if value below the reference value

```
if((fading_samples(sample_num) < ref_val) & (fflag == 1))
    temp(count,1) = sample_num;
    fflag = 0;</pre>
```

% Checking if value greater than the reference value

% Calculating the LCR and AFD

lcrsum = 0; % Initializing LCR afdsum = 0; % Initializing AFD

% Reduced count to compensate in case last crossing was below the threshold total = count - 2; for t = 1:total; lcrsum = lcrsum + (temp(t+1,1) - temp(t,1)); afdsum = afdsum + (temp(t,2)- temp(t,1));

end

% Storing LCR and AFD value for each thresold value

```
LcrAfd_val(1,index) = total./lcrsum;
LcrAfd_val(2,index) = afdsum./total;
index = index + 1;
end
% End of program ------%
```