ABSTRACT

COMPARING THE EFFECT OF PEDAGOGICAL APPROACHES TO TEACHING MULTIPLICATION AND DIVISION OF FRACTIONS

by Meggan Holli Dingus

The National Mathematics Advisory Panel (NMAP) regards fractions as the most important foundational mathematical skill that is developed, unfortunately fractions have been one of the most difficult mathematical skills for students with and without disabilities to master due to their lack of conceptual understanding of fractions. The following study examined the impact of visual models used in inclusive and gifted math settings to teach the division and multiplication of fractions in order to assess conceptual understanding of fractions. This study compared an inclusive and gifted classroom with similar instruction methods and found significant differences between in their overall posttest scores (F = 32.49; p = .00). However, when comparing their use of visual representations there was no significant difference (F = 2.166; p = .144).

COMPARING THE EFFECT OF PEDAGOGICAL APPROACHES TO TEACHING MULTIPLICATION AND DIVISION OF FRACTIONS

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Dedication

I dedicate this thesis to all of the students, like myself who have struggled in math. Without conceptual understanding, students of all abilities struggle with math concepts and it is my hope that this research will help future student struggle less in math and lead to more students feeling successful.

Acknowledgements

First and foremost, I would like to thank my fiancé, parents, family, and friends for supporting me throughout the process of completing my thesis. I would also like to thank Miami University's School Psychology program for giving me the opportunity to expand my learning and work with their faculty on a project I am passionate about. I would like to especially thank Miami University's faculty that have supported me throughout my thesis journey. Specifically, I am grateful for the incredible guidance Dr. Sarah Watt provided me throughout this project and for her role as a mentor during my time at Miami University. I would also like to thank Dr. Katy Mezher and Dr. Dana Cox for the wonderful insights and advice throughout this process. I would not have been able to complete this project or had the confidence to create a project from scratch without the support of everyone around me. I am humbled and grateful to those who have helped me through this process.

Introduction

Mathematics is a fundamental part of any educational experience for all students. No Child Left Behind era led to increasingly difficult mathematical success or proficiency that had become more difficult to achieve for all students due to the increasingly high-stakes assessments tied to success (Bottge, Rueda, Serlin, Hung, & Kwon, 2007). In response to this act the National Council of Teachers of Mathematics (NTCM; 2001) called for curricular reform that focuses on conceptual understanding within mathematical proficiency and for teachers to help students "develop mathematical habits of mind" through a variety of pedagogies or teaching approaches (Bottge, Heinrich, Chan, & Serlin, 2001; Bottge et. al, 2007). According to the NCTM, mathematical proficiency is constituted by the development of five interrelated strands: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition (National Council of Teaching of Mathematics [NCTM], 2014). Although the NTCM has called for instruction that specifically emphasizes conceptual understanding, it is important to note that students attain overall proficiency by gaining proficiency in each of the five strands; when proficiency in one strand is missing, difficulties in mathematics occur for students and overall proficiency is not complete (NCTM, 2014).

Teaching methods that tend to focus on one of the five strands and neglects other strands may lead to student confusion and worsen learning outcomes (NCTM, 2014). For instance, according to the NCTM (2014), tasks designed to encourage students to use procedures and formulas are not necessarily linked to having an active meaning; therefore, these tasks promote and can even rush procedural fluency among students. Although students are gaining a procedural fluency within a mathematical topic, students are not engaging in conceptual understanding of the same mathematical topic which causes difficulties for students when the foundation of a previous conceptual understanding becomes the basis of higher-level mathematics (NCTM, 2014 p.19 & 42). The NCTM has called for math curriculum reform and has challenged math teachers, special educators, and curriculum researchers to learn new pedagogical approaches for instructing students with and without disabilities in order to have all students gain overall proficiency in mathematical topics (Bottge et. al, 2007). The goal of new pedagogies is to promote success in math, especially foundational areas of math, while continuing to hold all students to high standards set by standardized testing and educational laws.

Mathematics is a subject that has foundational areas (i.e. computations, place value, fractions, etc.) that are necessary for success in other—more difficult—areas of math; this has been demonstrated by longitudinal studies which show that difficulties experienced by students in math foundational areas persist across years (Lewis, 2016). When a student does not fully understand key math concepts when they are initially taught, it may present problems in comprehension and generalization of skills later in the math curriculum (Zhang, Stecker, & Beqiri, 2017). In 2008, the National Mathematics Advisory Panel (NMAP) regarded fractions as the most important foundational mathematical skill that is developed, unfortunately a significant number of school-aged children struggle with learning fractions (Zhang, et al., 2017 p. 225). Historically, fractions have been one of the most difficult mathematical skills for students with and without disabilities (Misquitta, 2011). In order to help students, attain the foundational skills necessary to progress in math, new interventions and approaches to fraction instruction have been developed and researched over time, however further research is still warranted.

Despite the importance of fraction knowledge for advanced mathematics there is still limited research on effective techniques for improving fraction understanding (Watt & Therrien, 2016), particularly for students with disabilities. A relatively recent review conducted by Watt, Watkins, and Abbitt (2014) outlined five effective instructional methods for improving proficiency in fractions for students with disabilities or difficulties in math: (1) concrete-representational-abstract (CRA) sequence, (2) cognitive strategy instruction, (3) enhanced anchored instruction (EAI), (4) tutoring, and (5) the use of graphic organizers. While Watt and colleagues' (2014) review focused on algebra, it included interventions to support fraction proficiency given that fractions are a foundational skill for success in algebra as defined by the National Math Advisory Panel, 2008. The outcome of this review demonstrated that the CRA sequence had the largest effect on student fraction acquisition and positively impacted students' conceptual understanding of fractions.

The CRA sequence is a gradual instructional sequence that allows a student to interact with material through manipulatives (concrete), pictures (representational), and then eventually working with numbers/symbols (abstract) . This sequence has been proven effective as an intervention to allow students to improve their fraction knowledge by first developing a concrete, conceptual meaning of fractions (Witzel, Mercer, & Miller, 2003). The goal of this sequence is for students to move between the abstract and concrete phases of fraction understanding in a fluid manner, suggesting they have knowledge of both procedures and the meaning behind the math (Witzel, 2005).

Although the CRA sequence is an effective pedagogy for improving proficiency in fractions, there are significant gaps in the current literature because the CRA sequence is primarily used as an intervention for students with disabilities and those struggling in math to improve overall learning outcomes (Agrawal & Morin, 2016; Misquitta, 2011; Shin & Bryant, 2015; Watt & Therrien, 2016; Witzel et al, 2003; Witzel, 2005). There has not been a specific study looking at how all student learning outcomes, among those with and without disabilities, are impacted by using the CRA model. The purpose of the current study is to further examine the use of the CRA sequence within inclusive, general education settings compared to students in a gifted or accelerated math classroom, and the impact on all student learning. The study will further this research by examining the impact of conceptual learning, and more specifically the use of the CRA teaching sequence, within sixth grade gifted and inclusive classrooms. The goal of this study was to examine whole class learning outcomes (procedural and conceptual) within two groups (gifted and inclusion classrooms) and if the type of whole class instruction may have impacted the student learning outcomes. The research questions that will be examined in this study are the following:

1. What is the relationship between all students' accuracy of visual representations and multiplying and dividing fraction problems?
2. What associations exist between the type of classroom (gifted or inclusive classroom environments) and immediate student learning outcomes?
3. What relationship exists between knowledge of visual representations and overall learning outcomes?
4. What associations exist between the type of instruction and immediate student learning outcomes?

Review of Literature

Fractions: The Gateway to Advanced Mathematics

Algebra is often referred to as the gatekeeper to higher mathematics (National Mathematics Advisory Panel [NMAP], 2008). Knowledge of algebra is necessary for many careers, completion of Algebra I and II is a requirement for most traditional diplomas, and it can provide greater access to higher education (Watt et al., 2014). Research has found success in algebra to be linked to understanding rational numbers--especially fractions (Hunt, Welch-Ptak, and Silva, 2016; NMAP, 2008; Shin & Bryant, 2015).

Although fractions are considered an essential building block for algebra, they are historically one of the most difficult mathematical skills to master (Misquitta, 2011; Shin & Bryant, 2017). Fractions have been identified as a foundational area for success in algebra because although fractions are taught in elementary school, many adults—including teachers and school-aged children alike struggle to conceptually understand fractions (Cady, Hodges, & Collins, 2015; Misquitta, 2011; Zhang et al., 2017). The National Center for Education Statistics in 2013 indicated that about half of 8th and 12th-grade students lack the conceptual understanding to develop competence in fractions (Shin & Bryant, 2017).

Not only do students struggle with fractions, but teachers do not always have an adequate understanding of fraction concepts therefore they rely on the procedures set forth in textbooks for teaching fractions to their students (Cady et al., 2015). When teachers rely on the curriculum and textbooks on a surface level then they are not fully understanding the curriculum themselves (Cady et al., 2015). A study conducted by Doğan & Tertemiz, (2019) studied how preservice teachers understood fractions and found out that many teachers do not have adequate knowledge of fractions. Fractions are historically one of the most difficult mathematical skills to master because unlike working with whole numbers it is not completely intuitive (Misquitta, 2011). Teachers without sufficient knowledge of fractions will not be able to develop sufficient knowledge for how to teach fractions on multiple levels (Doğan & Tertemiz, 2019). The NMAP report shows that because fractions serve as a foundation for advanced math, when students struggle with fractions, they continually struggle with throughout their education which has also been demonstrated through longitudinal studies that difficulties in fractions persist over time (Lewis, 2016). The NMAP report demonstrates how important fraction learn is for success in higher math yet both teachers and students alike continually struggle with fractions (NMAP, 2008; Misquitta, 2011) The NMAP reports that "at least 40 percent of middle school students experienced difficulty with fractions and nearly 50 percent of middle and high school students struggled with elementary level fractions content" (NMAP, 2008; Misquitta, 2011 p. 109). Student success rates in fractions are directly impacted by teachers and teaching styles in fractions, since many teachers do not understand fractions themselves it would be safe to think that maybe something with teacher knowledge and teaching styles must change in order to help more students be successful.

In order to understand why students are struggling in fractions, it is important to understand how students are taught fractions. There are two specific aspects of fraction learning that have been identified that can impede the success of students' understanding and learning fractions. When students do not learn to interpret fractions as the "part of a whole" it can lead to difficulty manipulating fractions and comparing their value and equivalence (Hunt et al., 2016; Misquitta, 2011) "Part of a whole" strategy occurs when students are introduced to fractions as a "part of a whole"--typically using imagery as part of a pizza or pie (Misquitta, 2011). The part of a whole instructional method relies on teachers using a visual representation that helps students visualize fractions in the very beginning stages of learning. The work by Misquitta (2011) claims using a "part of a whole" representation is not intuitively harmful to students learning once they are taught to multiply, divide, and use improper fractions, however the visual representation typically used for this teaching method is a pie part of whole which makes it nearly impossible to represent improper fractions or manipulate fractions through multiplication and division with this method Misquitta's work addresses the importance of proper visual representations within fraction instruction at all phases of learning, but particularly when students are first introduced the concept of a fraction. By using procedural tricks and methods alone to solve advanced fraction computations, children not only are more likely to make procedural errors, but they also make errors in conceptual understanding of the basics of the mathematical concept (Misquitta, 2011). Therefore, the "part of a whole" method often results in students developing conceptually based errors when manipulating fractions versus procedurally based errors.

A second area identified in the research that can impede student success with fractions, is the overall level of fraction acquisition students' accomplish within typical class instruction (Hunt et al., 2016). Hunt and colleagues' study examined specific levels of understanding that children go through when they are introduced to fractions. Hunt and colleagues' (2016) study identified four problem-solving levels of fractions:

- Level 0: No Fractions, where students do not understand fractions--they only understand their previous knowledge of numbers and do not know whole numbers.

- Level 1: Emergent sharing, student utilized guess and check or a whole number based “build up” for reasoning--they understand that there is such a thing as a whole number and that it can be split into parts making it not whole.
- Level 2: Half, students began to coordinate making equal shares with exhausting the whole numbers--they understand that a whole number can be split into two equal halves and they utilize this.
- Level 3: Emergent relation or coordination, students understood equal shares with exhausting the whole and using it as planned--they understand that multiple types of fractions go together to make the whole. They utilize fractions in their designed way.

The above study identified that only 35% of students in their study that either had a specific learning disability (SLD) in math or were given additional math support were in this level 3 category where they understand fractions initially (Hunt et al., 2016). This study indicated that 65% of the students did not gain full understanding of fractions which will impact their future success in higher level mathematics. By identifying potential misconceptions students have for learning fractions, educators are able to figure out ways for helping students have different visual understandings of fractions and help guide students through the levels of understanding. Understanding the levels of fraction understanding introduced by Hunt and colleagues (2016) allows educators to better scaffold and implement proper visual representations within their instruction.

Conceptual vs. Procedural Learning

Currently the field of mathematics is at a crossroads where educators and researchers alike are evaluating students’ ability to retain both the procedures and conceptual understandings of various math content. Researchers have determined specific parts of math like algebra and fractions that are quintessential for success in higher mathematics and are essential building blocks to succeed in math. However, these approaches are commonly evaluated by effectiveness of supplemental interventions that are added for students that are struggling in math. After a careful review of the literature, it is clear there is a strong correlation between conceptual teaching techniques and higher student performance on post tests and on follow up assessments (e.g. Agrawal & Morin, 2016; Maccini, Mulcahy, & Wilson, 2007; Watt & Therrien, 2016 & Witzel, 2005). Making procedural errors are common for anyone learning math and occur for people who have both conceptual and procedural understandings of a topic. However, those who have only the procedural understandings will make both conceptual and procedural errors which will lead to worse test results on post tests and follow up measures. Watt and colleagues (2016) state in their review of interventions for teaching algebra to struggling learners: “a change in how we deliver core instruction needs to occur for all students to have access to grade level math content and the skills necessary to complete high level math courses.” This change in core instruction that the authors refer to is for curriculum to not only focus on procedures of math problems, and instead allow students to understand the conceptual basis of the problem. Core instruction should enable teachers to properly teach both the procedures of a fraction problem and the concepts of how to accurately accomplish mathematical problems within an applied context (Bottge et al., 2001; Bottge, Cohen, & Choi, 2017; Bottge, Toland, Gassaway, Butler, Choo, Griffen, & Ma, 2015; Witzel, 2005). Regardless of which approach previous researchers have used in their studies, common features of effective pedagogical approaches focus on teaching mathematical concepts through real-world problem application, multiple teaching methods, and use of visual representations or manipulatives. These pedagogical approaches that focus on meaningful information render more student students growth and better outcomes than students without those components in their mathematical learning (Bottge et al., 2001; Misquitta, 2011; Shin & Bryant, 2015; Watt et al., 2016; and Witzel, 2005).

Promoting Meaningful Learning Approaches in Fractions

Conceptual curriculums allow students to interact with material in meaningful ways which impact learning in a positive way (Agrawal & Morin, 2016; Misquitta, 2011; Mudaly & Naidoo, 2015 & Shin & Bryant, 2015). Students that can connect their learning to meaningful problems that they care about have higher success in learning and retaining their knowledge after it has been learned (Bottge, 2001). After research supported conceptual teaching techniques over procedural teaching techniques, researchers changed their focus to looking at interventions for how to improve conceptual learning. In order to test interventions researchers studied alternative teaching strategies to evaluate their effectiveness in comparison to procedural techniques that have been used for many years. The most common of alternative and conceptual strategies used include direct instruction, tutoring, Enhanced Anchored Instruction (EAI), and Concrete-Representational-Abstract (CRA) instructional sequence.

Tutoring and Enhanced Anchored Instruction are two common strategies used to help improve a student's mathematical success. A study by Lewis (2016) evaluating errors in fraction problems with students that made different qualitative errors found that they did not improve with tutoring in fractions. The reason Lewis gave for students' lack of improvement with tutoring was due to a "persistent understanding of fractions." In other words, the results of Lewis' study concluded that once students have experienced error patterns in their conceptual understanding, they will continue to have them unless they are broken by a new conceptual understanding (Lewis 2016). Rather than tutoring students using their same basis of knowledge, students need to be given a complete change to their knowledge base instead of providing only tutoring to build on their current skills. EAI is the second most commonly studied intervention that focuses on having videos that show a complex and "real world" problem for children to use their knowledge to solve it. Having this real-world application makes these problems meaningful to students and they are willing to put forth more effort. However, in a study conducted in 2007 by Bottge, EAI was compared to typical teaching practices students taught with EAI did not have any significant differences in their pretest and posttest scores. Although problems were more meaningful, students did not have much improvement due to their conceptual understanding not being changed by EAI. Both interventions are used and yet still present problems to a student's overall conceptual understanding of mathematics.

In contrast, studies from researchers like Witzel investigated the CRA sequence where students are able to fluidly move between the steps of CRA and if they are struggling with an abstract application they can move to representational processes and then work their way back to abstract processes. The CRA sequence works by using concrete representation of mathematical problems--like blocks for addition problems. The sequence then moves from the concrete component to the visual representation component and then onto the abstract part for correct completion of a math problem. How this works is that once a student can do the problem with the blocks--understanding that one block plus one block equals two--the student will then move onto draw pictures to represent the problem. A student will represent the blocks by drawing pictures on paper to solve the addition problem. After a student can solve the problem with the pictures, they will move onto using abstract ways to solve the problem like numbers and formulas: 1+1=2. Another take on the CRA sequence was conducted by Watt and Theiran (2016) where they conducted preteaching by using the CRA sequence in their preteaching sessions. Watt and Theiren's (2016) study shows that the CRA sequence is both effective as a retroactive intervention and as a proactive preteaching strategy for helping students succeed with their fractions. Although there were occasional positive outcomes with tutoring and EAI, the CRA sequence was the most used teaching technique across studies in fractions and algebra to increase conceptual understanding. The CRA sequence rendered the most positive results in students' understanding of mathematical topics on conceptual and procedural levels. This very fluid transition from conceptual to procedural is easy for a school to adapt and it is why it is commonly adapted because it makes sense and has a wealth of research support. A review on the CRA sequence suggests that the reason this sequence is commonly used is because the CRA sequence "helps develop a clear link between conceptual and procedural knowledge" (Agrawal and Morin, 2016 p. 35).

Limitations within CRA Research

One limitation for generalization of previous studies on math interventions is that tese studies focus on students with specific learning disabilities (SLD, often called learning disabilities LD), or those struggling in mathematics who begin receiving intervention after they are already struggling in mathematics. Previous research on effective math interventions studies tend to use interventions that highlight conceptual learning over procedural (tutoring, CRA, EAI etc.) in order to increase a student's conceptual knowledge of fractions and produce positive outcomes on posttest assessments (Watt et al., 2016). However, these studies do mention how students without SLDs would benefit from this instruction, they neglect to even mention students that do not have an SLD altogether. Furthermore, the majority of these studies take place outside of the general education classroom and are typically implemented by researchers and not the classroom teachers (Watt et al., 2016). More specifically, research on the CRA sequence is the most researched intervention within supplemental intervention settings, therefore researchers have not determined the impact of the CRA sequence on students who are accelerated in math (i.e. gifted) intensely compared to studied on a class wide level in comparison to same aged-peers, not receiving accelerated math instruction.

Current Study

The current study will expand on previous research that suggests a correlation between conceptual based learning and increased student outcomes, both procedural and conceptual. The study will further this research by examining the impact of utilization of the CRA sequence--

especially visual representations--for whole class learning within sixth grade inclusive classrooms compared to peers receiving similar instruction in an accelerated (i.e. gifted) mathematics classroom. Researchers will explore differences in students' outcomes dependent and independent of the level of services they are receiving. The research questions that will be examined in this study are the following:

1. What is the relationship between all students' accuracy of visual representations and multiplying and dividing fraction problems?
2. What associations exist between the type of classroom (gifted or inclusive classroom environments) and immediate student learning outcomes?
3. What relationship exists between knowledge of visual representations and overall learning outcomes?
4. What associations exist between the type of instruction and immediate student learning outcomes?

Methods

Participants

This study examines the instruction in two 6th grade classrooms in a suburban low-mid SES school district. The two classrooms consist of different student demographics, one group consists of students in the gifted program and the other student group is an inclusion classroom consisting of students of all abilities including students in special education. The sample size consists of 114 students, 37 students in the gifted classroom and 77 students in the inclusion classroom. Participants range in age from 11-13 years old.

The classrooms were selected through the University partnership office with a district willing to support faculty/student research. Participants were included in this study by being a student in each classroom that the study is being conducted in and consenting to participate in the research. The participants were only to be included in this study once their parents signed consent for their child's scores to be included and the students assent to their [the student's] pretest/posttest scores to be used for research purposes. Participants were excluded from this study if researchers are not given proper consent and assent to use data gathered in study. Students will also be excluded from data if they are not present for any of the repeated measure assessments. They are also excluded if they are not being taught by those teachers during the entirety of the fraction curriculum (an exclusion example would be if the student has an extended absence during the fraction unit), or if the student leaves the school district before data from the posttest and maintenance measures are obtained.

Researcher Generated pre- post- and follow-up Measure

A pretest, posttest, and maintenance assessment were developed by researchers and then assessed for internal reliability and validity (see appendix A). Only the pretest and posttest researcher generated (RG) assessments were used for data collection. The maintenance measure was scheduled with teachers, however due to the Ohio school closure in spring of 2020 caused by the COVID-19 pandemic, the researchers were unable to collect maintenance measure data. Internal reliability for the RG assessments were assessed by having content experts (e.g., math teachers) review all questions for consistency, validity in skills measured, and reliability.

The RG assessment consisted of two sections, one section focused on procedural learning outcomes and the other focused on conceptual learning outcomes. Procedural learning outcomes were measured by ten equation questions that instructed students to solve for the answer. The procedural questions are traditional questions that present numbers (abstract methods) and ask students to produce more numbers as answers. The second component of the RG assessment presented students with word problems to solve. When solving the word problems students were instructed to provide a visual representation, equation, and then the answer. When students are able to provide a visual representation then they have a better conceptual understanding than only providing route numbers as answers. Visual representations were determined to be a large contributing factor to student success because the visual model is what provides the link between the physical model and the abstract numbers. Visual representations alone are deeply connected to developing a conceptual understanding in fraction (Empson & Levi, 2011; Misquitta, 2011; Shin & Bryant, 2015). Therefore, in order to measure conceptual understanding, students were required to provide a visual representation of multiplication or division of fractions. Previous research established the connection between visual representations and conceptual understanding of fractions which allowed the research team to develop an assessment that was reasonable for whole class administration by the teacher and omitted students providing a concrete representation as part of their assessment.

Math Curriculum

Every district relies heavily on the state learning standards to guide their math instruction and will select curriculums that fulfil those standards. The implementation of these standards is integrated with the eight mathematical practices outlined by the NCTM (2014) and defined in the Common Core State Standards for Math (CCSSM; 2010). Both classrooms used the *Connected Mathematics: Let's Be Rational* curriculum (Lappan, Phillips, Fey, & Friel, 2014). The *Connected Mathematics Project* (CMP) is a problem-centered curriculum that focuses on real world problems (Lappan et al. 2014). CMP lessons are designed to allow children to be able to interact with their problems in a variety of ways; this curriculum allows for the opportunity to use picture representations either as supplemental material or explicit aspects of the curriculum taught with an approach like the CRA sequence (Lappan et al. 2014). The curriculum was not selected or developed by researchers instead, researchers are studying how the teacher instruction impacts student's knowledge of conceptual and procedural understanding of solving multiplication and division of fractions.

Procedures

First, researchers obtained written consent from a parent/guardian of students in both classrooms and students gave written assent to become a participant in this study. Next, a researcher generated (RG) pretest assessment was administered to all students. Following the pretest, students were instructed by their cose math teacher in the fraction unit utilizing the CMP curriculum. Teachers recorded 3 complete lessons during this unit. Following the instructional unit, students were administered the RG posttest. Approximately four weeks after the completion of the fraction unit, researchers planned for students to be administered the RG follow-up measure. Unfortunately, this step in the procedures did not occur due to school closures. After the RG assessment data and video recordings are collected, research assistants blind-coded videos utilizing the researcher created coding schema. Fidelity between raters were scored for interrater reliability between video coders to determine differences in instruction between teachers. All other data will be analyzed once videos are coded.

Coding Instructional Videos

Coding the videos aims to answer what level of implementation of the CRA sequence teachers are using to promote conceptual understanding of multiplication and division of fractions. Video coding will be completed by four separate coders that are part of the research team. The coders were divided into teams of two in order to code videos for interrater reliability. Interrater reliability was assessed by looking at the kappa score of videos to make sure that there is a strong enough reliability for the videos to be valid.

The procedures of coding each video consisted of the video coder watching each video and tracking the frequency of times teachers utilized concrete models, visual representations, and abstract representations (e.g., CRA components). By coding the instructors use of CRA components within each class session, researchers can investigate if there is a difference in the instructors' teaching style.

Intersection of Data

After coding videos, researchers used the four assessment datasets collected (two pretest and two posttest RG assessments) to examine different levels of student understanding of fractions. The RG assessments were used to determine overall student understanding for how to solve multiplication and division of fractions as a whole. RG assessments were also used to determine student conceptual understanding separately from their procedural understanding. After assessing the students' levels of understanding, researchers used the coded data to determine if student understanding was impacted by the type of instruction and exposure to concrete examples, visual representations, and abstract representations provided to them.

Data Analysis

Coding Process

This study examines whether differences exist in learning outcomes of students in gifted and inclusive classroom settings when exposed to the CRA sequence, specifically visual representations, when solving multiplication and division of fraction problems. The independent variable examined was the type of class (e.g., inclusion versus gifted classroom) and the dependent variable is student learning outcomes. Teacher instruction methods will also be determined if differences exist. The data collection process of this study has students in both classroom settings participate in RG assessment, and teachers record their lessons during the multiplication and division of fractions unit to ensure fidelity of instructional methods and discover potential differences in instruction.

Videos were coded with an RG coding schema to and trained coders to maintain reliability and validity. Researchers should "first begin with a hypothesis and then design the coding system around the hypothesis" (Heyman, Lorberm Eddy & West, 2014, p. 15). To answer the research hypothesis, researchers coded to determine if the amount of exposure to visual representations, concrete models, or abstract problems (e.g., CRA components) differs between classrooms. The frequency of each CRA component was tracked to determine if one instructor used one CRA component more than others and then determine if that exposure impacted learning outcomes.

Statistical Measures

In order to analyze this data efficiently and answer the first two research questions, an ANCOVA was employed. Initially, an RM-ANOVA was determined to be the most appropriate measure, however without a RG maintenance assessment, it is impossible to complete an RM-ANOVA with only a pretest/posttest data design because there are only two data points and an RM-ANOVA requires three or more data collection points (Pallant, 2010; Tabachnick & Fidell, 2007). RM-ANOVAs use a covariate, which is the first data collection point, so that scores can be compared evenly across time (Pallant, 2010). Therefore, researchers decided to continue to use a covariate to control for the types of class students are in. Researchers employed an ANCOVA with pretests scores serving as the covariate to allow for measurement of growth of students and avoid a potential skew of gifted students outperforming peers in the inclusion classroom. With the pretest scores serving as a covariate, researchers were able to analyze how students in each classroom grew in their conceptual in addition to overall knowledge of solving multiplication and division of fractions.

An ANCOVA is thought of as an extension to an ANOVA that incorporates a covariate to statistically control for a third variable within the study that may provide additional skew ("One-way ANCOVA in SPSS Statistics," n.d). In order to run any ANCOVA specific assumptions for an ANOVA have to be met in addition to having the need to run a covariate. Out of the ten assumptions needed to be able to run an ANCOVA six of them have to be checked by running SPSS statistics to determine if an ANOVA is the correct test to run once the data is collected ("One-way ANOVA in SPSS Statistics," n.d). Without data collection, four assumptions can be determined without SPSS to ensure an ANOVA was an appropriate statistical test. The first assumption: dependent variables should be measured at the continuous level (ratio or interval variables), the repeated measure assessment (posttest/maintenance test) will serve as the continuous dependent variable ("One-way ANOVA in SPSS Statistics," n.d). The second assumption: independent variables should consist of two or more categorical variables ("One-way ANOVA in SPSS Statistics," n.d). The categorical variables can include both nominal and ordinal variables. The independent variable in this study is nominal. The nominal variable is the student group (gifted or inclusive). The third assumption: the study has one or more covariates ("One-way ANOVA in SPSS Statistics," n.d). The covariate used in this study will be pretest assessment scores. The fourth assumption: there is no relationship between the observations in each group meaning that groups (both classes and support level) do not overlap and that every participant stays in their individual group for the whole study ("One-way ANOVA in SPSS Statistics," n.d). The additional assumptions were checked via SPSS before data was ran and this data

After data was collected, students' scores will be entered for pretest and posttest assessments. Then the students' scores will be put into two separate ANCOVAs on SPSS with the covariate serving as the pretest scores. The first ANCOVA measured if differences exist between groups for overall student knowledge of multiplication and division of fractions. The second ANCOVA measures if differences exist between groups for conceptual student knowledge of multiplication and division of fractions.

In addition to running two ANCOVAs, a Pearson's Product-Moment Correlation test was run to determine if a correlation exists between students' conceptual understanding and their overall understanding of multiplication and division of fractions. There are four assumptions of this correlation test, the first one is checked without SPSS and the other three are checked after running SPSS. The first assumption is that the two variables are continuous either ratio or interval and the correlational variables are both continuous ("Pearson's Product-Moment Correlation using SPSS Statistics," n.d.). Therefore, all statistical measurements were appropriate means for this dataset and to answer the research questions.

Fidelity

Fidelity was assessed within the areas of coding, instruction, and potential differences within instruction. Coding fidelity was maintained by having two coders watch every video recording to ensure there were no biases in coding and that they were indeed coded with fidelity. Instruction fidelity was ensured by coding teacher recorded videos to ensure that each teacher was indeed using some CRA components (e.g., visual representations, concrete models, and abstract expressions) when they were instructing students on multiplication and division of fractions. The last fidelity check within this study occurred as a result of coding teacher videos researchers looked to see if students were instructed with different amounts of CRA components that may have impacted a student's learning outcomes especially in the area of conceptual understanding of multiplication and division of fractions. With examining fidelity in the area of coding, instruction, and potential differences in instruction researchers were able to ensure that data collected is adequately determining the result of student conceptual learning outcomes.

Results

Analysis of covariances were performed on two levels to determine if there were differences between group outcomes (i.e. accelerated vs. inclusive math classrooms) in their total score and between groups on their accuracy of using visual representations to solve problems. The covariate of pretest procedural scores was used for both ANCOVAs to control for differences in knowledge in the beginning of the curriculum. In addition to ANCOVAs a correlation test was run to determine if a correlation exists between visual representation knowledge and overall fraction knowledge.

The first analyses examined the differences between students in gifted (M = 1.46; SD =1.406) and inclusive (M=1.35; SD = 1.265) math classrooms in their use of accurate visual representations when answering word problems that required multiplication or division of fractions. However, when comparing their posttest visual representation scores the groups did not have a significant difference ($F = 2.166$; $p = .144$).

A second analysis examined the differences between students in gifted (M=18.05; SD=1.999) and inclusive (M=13.92; SD=3.940) math classrooms and their overall accuracy in dividing and multiplying fractions. The analyses showed that there were significant differences between groups ($F = 32.49$; $p = .00$) when their posttest total scores were compared while controlling for variance in their pretest scores.

A positive correlation ($r = 2.73$; $p = .003$) was found between the scores for the posttest visual representation score and posttest equation score. Two t-tests were run to determine if there was a difference in the use of visual representations during instruction in the gifted classroom (M = 1.125 ; SD = 2.244) compared to the inclusive classroom (M = .750; SD = 1.597). No differences were found (T = -.973; p = .213) in the teacher's use of visual representations which indicates that there is no difference in instructional methods.

Discussion

Implications

The overall findings in this study indicate multiple outcomes relating to student learning of fractions. Researchers aimed to answer three questions within this study. The first question aimed to answer: *"What is the relationship between all students' accuracy of visual* *representations and multiplying and dividing fraction problems?"* Researchers discovered how students in both classes answered the visual representation word problem to demonstrate their conceptual understanding. Students in both gifted and inclusion classrooms demonstrated difficulty in providing visual representations and therefore did not demonstrate high conceptual understanding for how to multiply and divide fractions. When comparing the visual representation scores, there was no difference in performance on visual representation problems between classes. All students performed poorly on visual representations, which demonstrates that all student groups are lacking conceptual understanding in these problems regardless of their classroom status.

The second research question aimed to answer the following: *"What associations exist between the type of classroom (gifted or inclusive classroom environments) and immediate student learning outcomes?"* Researchers compared how well students did on their total posttest scores (procedural and conceptual problems combined) between groups. The gifted classroom significantly outperformed their inclusive classroom counterparts. This significant difference in scores when compared to the previous research question demonstrates that gifted students have a better procedural understanding of fractions, yet they continue to struggle with their conceptual understanding.

The third research question aimed to answer the following: *"What relationship exists between knowledge of visual representations and overall learning outcomes?"* Researchers found a positive correlation between students' correctly solving visual representations and their overall score improving. This correlation demonstrates that when students have a higher conceptual understanding, they are likely to have a higher procedural understanding of problems.

The final research question: *"What associations exist between the type of instruction and immediate student learning outcomes?"* was not answered in this study. When researchers analyzed coded video data, there was no difference in the amount of overall instructional methods in relation to their use of CRA components. Students in both a gifted and inclusive classroom were taught with similar instructional strategies that promoted the use of concrete models, visual representations in addition to abstract representations of fractions. Therefore, without a difference in instruction, it is not possible to answer this research question.

In summary this study builds upon and expands the use of visual representations for fractions. Previous studies have never examined the effects of visual representations at a class wide instructional level and with students without previous difficulties in math. This study provides additional evidence that suggests how all students have difficulty in conceptual understanding--even gifted students who outperform peers on assessments. When analyzing their conceptual understanding gifted students demonstrate similar levels of understanding to peers that have difficulties in math. In addition, when both gifted and inclusion students showed growth in their visual representation knowledge, it positively impacted their overall learning outcomes.

Limitations

Three main limitations existed within the current research study. The first limitation of this study was the absence of maintenance data. Maintenance data would have allowed for analysis of long-lasting effects of visual representations on overall learning outcomes. Another limitation of this study was the limited sample size of this data set. Only two classes were used to collect data and the pretest posttest results were compared to assess learning outcomes. A larger sample size would have allowed for greater generalizability of findings. A third limitation was noted within the collection of the classroom videos. The quality of the videos at times impacted the ability for researchers to code some time intervals within instruction.

Future Research

Limitations of this study indicate areas that future research should be used in order to gain a better understanding of student's learning of fractions. With the limited sample size available in this study, future research would benefit to have a larger sample size that includes many different classrooms in schools using different curriculums in order to track changes in instructional methods. Although this study expanded comparing the impact of visual representations in fractions by looking at a class wide instruction methods, there is room to continue this expansion to understand this concept in many additional classrooms, and curriculums. Previous research only focused on the visual representation of fractions as a tier 3 intervention instead of a general instructional method. More research should be completed to continue understanding the impact of visual representations on all students' learning outcomes. Research should also focus on the short- and long-term impact of student learning outcomes to understand how visual representations will impact learning for years to come.

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Appendix A: Researcher Generated Assessment

Name: ____________________ Date: ____________________

Teacher: ____________________

Pretest: Multiplication & Division of Fractions

1. $\frac{1}{4} \times \frac{7}{5}$

2. $6 \div 1\frac{2}{3}$

3. $\frac{4}{5} \div \frac{3}{8}$

4. $1\frac{1}{7} \times \frac{3}{4}$

5. $\frac{8}{5} \div \frac{3}{10}$

6. $\frac{3}{4} \times 12$

7. $\frac{5}{6} \times \frac{3}{8}$

8. $1\frac{2}{5} \times 2$

9. $3\frac{1}{2} \times 1\frac{2}{3}$

10. $10 \div \frac{1}{2}$

1

Instructions: In the following word problems choose which operation, multiplication or division, is necessary to solve. Draw a visual representation of the problem, write the equation you used to solve, and then write your answer to the problem.

1. Josh is training for a race. He is able to decrease his race time by $1\frac{3}{4}$ seconds each week. How many total seconds will be be able to decrease his race time by in 4 weeks?

Visual Representation	**Equation**	**Answer**

2. A box of pudding makes 4 cups of pudding. How many $\frac{3}{4}$ cup servings can you make from box?

Visual Representation	**Equation**	**Answer**

3. Annie has to write $2\frac{1}{2}$ pages in her journal for homework. She completed $\frac{1}{3}$ of the writing before she went to be. How much did she write before going to bed?

Visual Representation	**Equation**	**Answer**

4. Hank had $3\frac{1}{2}$ of lumber. It took $\frac{3}{4}$ of a piece of lumber to make one shelf. How many shelves could Hank build?

Visual Representation	**Equation**	**Answer**