ABSTRACT

EXAMINING THE EFFECTS OF A FRACTION INTERVENTION ON SIXTH GRADE STUDENTS' RATIONAL NUMBER SENSE

by Allison Louise Perkins

Competency with fractions is a pre-requisite for success in algebra and beyond. However, recent data suggest that over 50% of middle and high school students in the United States cannot solve fraction computation questions on an elementary school level. Research indicates that the difficulty with fractions exists with all students, not just students with a mathematics learning disability. Some programs have been established to target these needs. Further research to evaluate the efficacy of interventions and programs targeting fraction computation and reasoning is needed. Programs using the concrete-representational-abstract sequence (CRA) have shown to increase ability levels of students with and without learning disabilities. CRA targets conceptual understanding by progressing from concrete manipulatives, to representational drawings, to abstract symbols. The purpose of the current study was to evaluate the efficacy of a fractions intervention using the CRA sequence in combination with video instruction on fraction computation and mathematical rational number sense in sixth grade students over a ten-week period. Students watched video lessons using the CRA sequence and completed practice problems. Overall, findings suggest that the intervention was effective in increasing fraction computation and strategy use, a component of number sense.

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Introduction

In 2009, the United States Department of Education adopted a new set of rigorous curriculum standards for public school education called the Common Core State Standards Initiative. These high standards were developed with the intent to prepare all students for college and the workforce. Clear expectations of what students should be able to achieve are provided at each grade level and new readiness assessments have been created to accompany them. State proficiency standards are expected to be aligned with national proficiency standards. Many states have adopted the Common Core State Standards, but true implementation is a long process that has yet to be accomplished (Peterson & Ackerman, 2015).

Despite clear goals and expectations, many students still struggle to achieve academic benchmarks. Recent results from the 2015 National Assessment of Educational Progress report an increase in math scores for 4th and 8th grade students between 1990 and 2015, but an overall decrease between 2013 and 2015 (NAEP, 2015). Per the NAEP Standards, only 40% of fourth grade students were considered proficient (e.g. meeting benchmark) or above compared to 33% of eighth grade students. As is evidenced by the data, the discrepancy between the expectations of the mathematics standards and student performance is still a very relevant issue. However, there is also a concern in regard to specific aspects of mathematics.

According to the National Math Advisory Panel Final Report (NMAP, 2008), proficiency and the knowledge base to meet state and national standards with fractions is severely underdeveloped in K-8 mathematics education in the United States. Additionally, the report found that many students have very poor preparation in regard to operations with rational numbers and fractions in general. The lack of preparation is problematic, as knowledge and proficiency of fractions is one of the three critical foundations of algebra in addition to flexibility of whole number operations and knowledge of geometry. Without competency in the area of fractions, students are unable to successfully progress through higher level math courses The lack of advancement in math has further repercussions as many states require passing grades in Algebra courses or exit exams as a graduation requirement (Ohio Department of Education, 2015; Kentucky Department of Education, 2015).

Further, gaining post-secondary employment is dependent upon mathematics competency as 21st century jobs become more technical (with a greater focus on STEM education – Science, Technology, Engineering, and Math) and require more advanced degrees and training. Research suggests that jobs that are considered middle skill occupations will soon make up around 45% of national job openings, and high skills positions will account for another 33% of the national jobs. (Achieve, 2010). Jobs that are considered middle skill occupations require at least some training or degree post-secondary while high skills jobs require at least a four-year college degree. These statistics imply that over 75% of the nation's jobs will soon require some sort of education after high school which will likely require skills in higher level mathematics.

Children who experience difficulties with fractions and other rational numbers at a young age continue to do so throughout the rest of their education (Mazzocco & Devin, 2008). Despite the challenges fractions hold for students, there are strategies to support skill development among learners. Through the use of benchmark formative assessment and targeted math intervention, students can quickly and efficiently develop the foundation for rational number sense and lessen the gap between them and their peers, both in the US and overseas.

Statement of the Problem

Fractions have been referred to as one of the most difficult areas of mathematics for students (Behr, Wachsmuth, Post, & Lesh, 1984; Hiebert, 1985; McLeod & Armstrong, 1982; Ni, 2001). Recent research suggests that over 50% of middle and high school students are unable to compute fraction problems on an elementary school level (NMAP, 2008). While fraction computation has been discussed many times (such as in the literature noted previously), little research supports interventions that have a positive impact on the math achievement of middle school age students with identified math difficulties.

According to "What Works Clearinghouse" (Institute of Educational Statistics, 2015), 255 targeted interventions exist to support general literacy development, while only 149 interventions exist to develop general mathematic skills. Specifically, in regard to fractions, only one intervention was found on WWC that sought to support fraction competency. While the intervention was effective, it can only be generalized to fourth grade students and is limited to addition and subtraction with fractions. As nearly 50% of 8th grade students are unable to order fractions from least to greatest (National Center for Educational Statistics, 2007) the lack of support in schools is highly alarming.

Due to changing standards and increasing focus on more complex mathematical computation and reasoning, it is imperative to examine how students reason and interpret mathematics. Doing so is crucial because understanding student rationale for solving a problem and observing the use of appropriate/inappropriate strategies allows the educator to identify misconceptions in theory (Black & William, 2009). The use of formative assessment strategies that promote the verbalization of student thinking can make it easier to identify which interventions can support the development of mathematical comprehension.

In addition to the use of formative assessment strategies to identify specific learning targets to support struggling math students, it is also important that teachers and researchers examine the components of effective instruction. According to the NMAP (2008) effective mathematics instruction contains explicit instruction, step-by-step explanations and demonstration of strategies, corrective feedback, frequent practice, and the use of concrete and visual representations. Specifically, special education research has suggested the concrete-representational-abstract sequence (CRA), is highly effective at teaching math skills to struggling learners and those with identified learning disabilities (Butler, Miller, Crehan, Babbitt and Pierce, 2003; Watt, 2013). Instruction using CRA focuses on initial exposure with concrete manipulatives, followed by representational drawings, and ends with abstract instruction using numbers and symbols (Witzel, 2005). Though research has indicated high efficacy for the use of the CRA sequence, little research has examined the use of the CRA sequence specifically for instructions.

Purpose of the Study

The purpose of the current study is to assess and evaluate rational number sense in middle school aged children. Specifically, the current study examines the efficacy of a systematic mathematics intervention that incorporates a gradual instruction sequence (CRA) used to strengthen students' conceptual and procedural computations of fractions.

Definition of Terms

There are many key terms relevant to the study mentioned throughout the literature. The most important items are listed and defined below:

Formative Assessment is a term used to describe assessment procedures that take place during the learning process and allow for constructive feedback and revision for the betterment of the learning process (Taras, 2005).

Number Sense refers to an individual's comprehension of the relationship between numbers and how they can be changed by operations (McIntosh, Rays, B.J. & Rays, R.E., 1992).

Rational Number Sense refers to one's understanding of fractions, how they relate to each other, and the ability to apply rules regarding computation with fractional numbers.

Research Question/Hypotheses

For the purpose of the current study, the following questions will be addressed:

- 1. What effect will the Perceptions intervention have on students' rational reasoning scores determined by the Math Reasoning Inventory (MRI)?
- 2. What effect will the Perceptions intervention have on student's rational mathematical reasoning?
- 3. What effect will the Perceptions intervention have on student's accurate completion of the Aimsweb fraction computation items?

As part of the current study, the following hypothesis will be investigated:

- 1. Null hypothesis: Students will not answer more Math Reasoning Inventory interview question items correctly after the Perceptions fraction intervention. Alternate hypothesis: Students will answer more Math Reasoning Inventory interview question items correctly after the Perceptions fraction intervention.
- Null hypothesis: Students will not exhibit higher levels of rational reasoning on the Math Reasoning Inventory Fractions interview questions following the Perceptions fraction intervention. Alternate hypothesis: Students will exhibit higher levels of rational reasoning on

the Math Reasoning Inventory Fractions interview questions following the Perceptions fraction intervention.

3. Null hypothesis: Students will not answer more Aimsweb fraction items correctly after the Perceptions fraction intervention.

Alternate hypothesis: Students will answer more Aimsweb fraction items correctly after the Perceptions fraction intervention.

Literature Review

Math is a part of everyday life and is considered an important gateway skill to postsecondary options and successful employment. Though everyday math can be defined as simple computations, knowledge of more complex mathematics is important as well. Despite the importance, many adults and children in the United States experience difficulty with complex math concepts, especially in the area of fractions (DeWolf, Grounds, Bassok & Holyoak, 2013; Mazzocco & Devin, 2008). While a few assessments have been created to identify areas of weakness in regarding to fraction competency, very few effective interventions exist to support the development of these skills.

Why Is Math Important?

Using mathematical principles and skills is a key component of daily life for children and adults alike. Knowledge of mathematics is needed to budget expenses, pay bills, and determine sales tax and appropriate tips, as well as numerous other aspects of life (Murnane & Levy, 1996). Phillips' research (2007) found that a startling 78% of adults surveyed are unable to calculate interest rates on loans, and over 50% do not know how to determine the appropriate tip at a restaurant. His findings suggest that many adults have not acquired basic understanding of math skills that impact everyday life.

However, knowledge of math is important for reasons other than daily functioning. Additionally, competency in mathematics is important for student success in high school and beyond. New national education standards require that students are proficient in specific areas before they are allowed to move onto the next grade (Common Core State Standards Initiative, 2010). Historically, students in the United States have tested lower than students in other countries (Baldi, Jin, Shemer, Green, Hergert & Xie, 2007), which may have implications on our ability to compete in the global job market as a nation. The nation responded to the historically low worldwide performance by developing more rigorous standards that support students' depth of knowledge across multiple domains. Along with the development of the standards comes increased student expectations. Many states require students to pass numerous courses in advanced mathematics (algebra I&II, geometry) and/or exit exams before getting their high school diploma (Ohio Department of Education, 2015; Kentucky Department of Education, 2015).

It is well established that knowledge of mathematics is a necessity for all students in high school and beyond in college and the workforce. Success in Algebra II and further courses is widely considered to be linked to college attendance and success beyond (Horn and Nunez, 2000). The push for higher standards and a greater sense of competency has implications for even entry level positions (Achieve, 2010). However, only 23% of the nation's students are proficient in Algebra by grade 12 (U.S. Department of Education, 2005). For a country that seeks to become a true global competitor, that leaves much to be desired. Overall, the high prevalence of low achievement in math is a matter of major concern for the entire country (Gersten, et al., 2009).

The Trends in International Mathematics and Science Study (TIMMS) has been used to evaluate performance of fourth and eighth grade students across the world in mathematics and science every four years since 1995. The assessment provides timely and reliable information regarding performance of students in the United States with that of other countries. According to the TIMMS results, many European and Asian countries have continued to outperform the US in the area of mathematics. As a result, the President initiated the National Math Advisory Panel in 2006 to research best practices in math education with existing research. Specifically, this committee was called to advise the Department of Education on ways to make the United States more competitive in mathematics.

As a result of the Panel's findings and Executive Summary (2008), educators have shifted their focus in regard to math competency. The National Mathematics Advisory Panel was created to evaluate trends in students in the United States' performance in mathematics and identify areas of concern (NMAP, 2008). In 2008, the NMAP published a Final Report detailing the state of the country in regard to mathematics and the implications of such. The report highlighted the importance of not only Algebra, but the three Critical Foundations that it is composed of: whole numbers, fractions, and geometry/measurement. A particular concern was competence in

fractions. According to the report, difficulty with fractions is highly pervasive in the United States and not nearly enough attention is being focused on effective fraction instruction and remediation. As stated in the report, almost half (approximately 40%) off all middle school students tested expressed difficulty with fractions, and half of middle and high school students struggled with fraction questions taken from the elementary standards. In addition, the Panel discussed the importance of students to understand both procedural and conceptual underpinnings of mathematical concepts.

The Common Core State Standards were developed to address the startling discrepancies that exist in comparison of international achievement rates in core content areas (Common Core, 2010). The premise of the new standards in the area of mathematics is to "provide specific standards focused on conceptual understanding and incorporating practices in which students must participate to develop conceptual understanding," (Chandler, Fortune, Lovett, & Scherrer, 2016). Therefore, the standards aim to develop reasoning skills that can prepare students for college and the "real world". Students must develop a sense of mathematical literacy that involves being able to understand problems and know how to solve them, think abstractly and quantatively, use appropriate strategies when solving problems. However, in order for the goals set by the standards to be achieved, difficulties in areas such as fractions must be addressed.

Documented difficulties with fractions existed long before the first TIMSS report and have been cited numerous times since then. The results of The National Assessment of Educational Progress' second mathematics assessment indicated these difficulties in both 9 and 13-year-old students (Carpenter, Kepner, Corbitt, Lindquist & Reys, 1980). While 60% of 13-year-old students were able to add two fractions with common denominators, only 30% of these students were able to add fractions with unlike denominators, a skill that is expected of all 5th grade students (CCSSM, 2010). The large amount of data indicating that students in high school have not yet mastered skills required of 5th graders is particularly alarming.

One study sought to examine differences in scores on tests of rational numbers for children diagnosed with math learning disabilities, those who were considered low achieving math students, and those considered typically achieving (Mazzocco & Devin, 2008). The task required all students to rank order items presented to them on four different tests – pictorial representations of rational numbers, decimals, fractions, and a mix of fractions and decimals. The results of Mazzocco and Develin's study indicated that children diagnosed with mathematical learning disabilities (MLD) are not the only students who struggle with fractions. While those with MLD did obtain much lower scores across the board, those identified as low achieving students performed well below the norm. In fact, 78% of the low achieving students failed to rank 100% of items correctly on the test in which there was a mix of decimals and fractions.

In addition to the variety of scores obtained by each group examined, Mazzocco and Devin's research also has many implications for long term evaluation of rational number sense. The same measures were given to each student every year from sixth to eighth grade. Students who did poorly on the task in 6th grade also did poorly in 8th grade, and the low scores were not unique to students in the Math Learning Disability group. The results indicate that if deficits in rational number sense are not targeted and remediated, they can lead to lifelong problems with fractions.

Some researchers have examined specific aspects of fractions that students struggle with the most, regardless of whether or not they are identified as having a math learning disability. Many have found that adding across numbers is one of the most frequently occurring mistakes and occurs most often when the fractions do not have common denominators (Byrnes & Wasik, 1991; Newton, 2008). Students who "add across" simply add both of the numerators and both of the denominators to make a new fraction, without attempting to find common denominators. The misconception indicates that these students do not have a good conceptual understanding of fractions and the many rules that go along with fraction computation. Weak conceptual understanding leads to the belief that adding across is an appropriate way to solve the problem.

Other researchers have also completed error analyses to examine why students make mistakes in fraction computation. In 2006, Brown & Quinn evaluated error patterns students make in a series of questions regarding fraction computation. High school students were given six categories of fraction problems to solve: algorithmic applications, applications of basic fraction concepts in word problems, elementary algebraic concepts, specific arithmetic skills that are prerequisite to algebra, comprehension of the structure of rational numbers, and computation fluency. The researchers found that most students lacked experience with even basic fraction problems. When presented with a problem, most tried to use abstract algorithms that appeared to be incorrectly memorized, which led to completely incorrect answers. The incorrect use of algorithms demonstrated that they did not understand the basic concept of fractions or have knowledge of concrete strategies to solve the problem. Though Brown and Quinn's sample is older than the target sample of the current study, it has many implications. By the time students enter high school, many have forgotten the abstract shortcuts to do mathematics which suggests that very few students used pictorial representations of fractions to answer questions. Therefore, it is imperative to stress the concrete basics of fractions and understanding of number sense before even attempting to teach students how to do the abstract shortcuts. Fluency is not possible if the foundation to fraction competency is not properly established.

The National Math Advisory Panel advises a balanced between procedural and conceptual knowledge during instruction as these concepts are thought to support and complement one another (NMAP, 2008). With regard to fractions, NMAP associated conceptual knowledge with "comparing and judging the magnitude of fractions, understanding fraction representation, and determining fraction equivalence." Procedural knowledge, on the other hand, involves the actual computation of fractions (addition, subtraction, multiplication, and division) and use of algorithms so to solve problems. In order to facilitate the development of both types of knowledge, NMAP recommends the following methods of instruction: explicit instruction, step-by-step explanations and demonstration of strategies, corrective feedback, frequent practice, and the use of concrete and visual representations. The recommended methods of instruction are important to develop intervention programs.

Other research also indicates that a balance of procedural and conceptual knowledge is crucial for skill mastery. Mazzocco, Devlin & McKenney's 2008 study with students from three levels indicated that children in the low achieving group made more errors than the typically achieving group, but those errors were similar to those of the typically achieving group. The pattern of errors suggests that that the children in the low achieving group have a good grasp of conceptual knowledge, but made errors when applying procedural knowledge. Children with math related disabilities, however, made more overall errors than either group and different types of errors. They often demonstrated poor conceptual understanding of the items and used fewer appropriate procedural strategies (finger counting instead of automatic mental math) which suggests that a balance of both conceptual knowledge and procedural knowledge is especially important for students with math learning disabilities. The findings regarding a balance of knowledge supports the recommendation of NMAP (2008).

Research on Fraction Interventions

The research on fraction interventions for struggling learners or those with math disabilities is small. However, the research that does exist supports findings from studies examining overall math interventions for the population of learners. Researchers suggest the use of mnemonic strategies, enhanced anchored instruction using video-based modules, explicit instruction, and the concrete-representational-abstract teaching sequence has positive effects on math achievement among learners with disabilities (Gersten et al., 2009).

Mnemonic strategies have been created to help students learn how to compute fractions. Test and Ellis created the LAPs method in 2005. The LAPs method was composed of eight parts, focusing on the mnemonic: **L**-ook at the sign of denominator; **A**-sk yourself the question: Will the smallest denominator divide into the largest denominator an even number of times?; and **P**-ick your fraction type. Based on the student answers to the second question, they were to choose one of the three memorized response types that determines how they solve the question. The LAP mnemonic method was used with six 8th grade students in special education and was found to be effective for 5 of the students. While the use of the mnemonic strategy is not necessarily effective to all students, it does indicate that mnemonics may be helpful for remembering the steps to fraction computation. r, Study limitations, however, indicate the strategy is only useful for addition and subtraction, and students may forget the fraction types they are to choose from.

Researchers have also used video instruction as a form of fraction intervention. In 1989, researchers developed a videodisc technology system targeted for students with learning disabilities who were placed in a general education classroom (Miller & Cooke). Each day, the students took written quizzes along with their classmates over the concept from the previous day followed by video narration of new topics. The video posed comprehension questions that students answered orally and there were accompanying assignments. Teachers rewarded good behavior and modeled appropriate responses to the video and made sure students were taking notes throughout and working the accompanying practice problems. The posttest score revealed that the intervention was reasonably effective, as students with learning disabilities scored a 72% and general education students scored a 78% and all but one student gave the intervention a positive rating.

Further research examining video-based instruction was evaluated using an "enhanced anchored instruction" approach (Bottge, 2001). Enhanced anchored instruction (EAI) is an instructional strategy used to each student in a multimedia format and have them generalize what they learned to real life by applying it to hands-on problems. Bottge, Rueda, Serlin, Hung, and Kwon tested this strategy's implications on mathematics achievement with a group of seventh grade students with mixed abilities (2007). Students were exposed to two EAI problems in which they watched a video that included a scenario in which math calculation occurred. For example, in one of the clips, the characters wanted to build a skateboard ramp. Students in the class watched as the actors solved the math problems required to estimate the cost of materials. Next, students completed activities similar to those in the clip by measuring length of boards and viewing store ads to estimate prices. Finally, they used the skills they acquired for a hands-on building project. Differences on pre and posttest measures showed growth for all students. While those who had learning disabilities scored higher than the typical peers, there were no differences with regard to improvement sizes which suggests that video based instruction is effective in helping all students achieve.

In the last few decades, research on fractions instruction has shifted from a focus on tools that work best for struggling learners to robust interventions for all students. Early research on

the concrete-semi concrete-abstract sequence (CSA; also referred to as CRA) examined the effectiveness of the CSA sequence on fraction computation abilities of fourth grade students in a general education setting (Jordan, Miller, & Mercer, 1999). During the concrete phase, students used construction paper circles cut into pieces to represent parts of fractions. The semi concrete phase (now known as representational) relied upon drawing lines to represent fractions, while the abstract phase included worksheets without representations. For each phase, students were shown demonstrations, experienced guided practice, and solved intendent practice problems. Results indicated that students who received CSA instruction made significant gains from pretest to post-test, compared to those in the control group. The findings suggest that the CSA sequence may be more effective in teaching fractions than the traditional text-based materials, which are primarily abstract in nature.

Other researchers have applied the concrete-representational-abstract sequence to mathematics instruction in an inclusion setting. Witzel (2005) used a pre-post-follow-up design to evaluate differences in students' ability to solve multiple-step linear equations after receiving instruction using either abstract instruction or the CRA sequence. All students complete an algebra pre-test, post-test, and follow up post-test three weeks after the post-test. Regardless of the model used, each class included students with and without disabilities and all students received 19 scripted lessons with the same instructional steps, covering five math skills. For the CRA group, concrete representations were created using small sticks for numbers, minus signs, numbers for coefficient markers, a printed "x", a plus symbol, a large stick, and an equal sign. Pictorial representations resembled the concrete stage and were solved in the same manner. The abstract stage involved using Arabic symbols to solve the problem. All three stages were consecutively covered for each skill. Post-test and follow-up results of the study demonstrated that students in the CRA had mean scores that were nearly double the scores of students in the control group. The increase in scores for students in the CRA group were consistent for students with disabilities and those who were considered high achieving. The results imply that using pictorial examples and hands-on activities can lead to a deeper conceptual understanding of the topic and retention of skills.

Watt (2013) also used a pre-post-follow up design to evaluate the efficacy of the CRA sequence on math achievement in the area of equations and fractions in two phases. For each phase, sixth grade students were split into treatment (CRA) and control groups. The first phase focused on solving equations and the second focused on solving fractions. Students in the treatment group for the equations unit were in the control group for the fractions unit and vice versa. As well, those in the treatment group received ten sessions of pre-training prior to the intervention that targeted pre-requisite skills and use the CRA sequence. The pre-training involved vocabulary instruction, modeling, and practice. Students in the control group did not receive pre-training prior to the unit on equations or fractions. For both areas, students in the treatment group made significant growth from pre-test to post-test. The growth validates the effectiveness of using the CRA sequence to teach higher level mathematics.

In 2003, Butler, Miller, Crehan, Babbitt and Pierce examined the differences between concrete-representational-abstract (CRA) instructional sequences and representational-abstract (RA) instructional sequences in two groups of students with learning disabilities. The lesson sequence for the CRA group began with instruction of fraction equivalence with concrete manipulatives. Lesson four phased out the use of manipulatives and emphasized the use of representational drawings to determine equivalencies. Lessons seven through ten taught students abstract algorithms to determine the answer. Students in the RA group were not exposed to

manipulative but instead used representational drawings for lessons one through seven. Students in the CRA group were found to have greater improvement on the posttest measure than students in the RA group, though the scores were only statistically significant for problems involving understanding of fraction equivalency. Overall, however, the students in both groups performed as well as students in the general education setting. The finding suggests that while the physical manipulatives may not be statistically significant, visual representatives of fractions are effective in helping students with disabilities learn on the same level as their general education peers. To date very little research has been conducted to evaluate the efficacy of the CRA sequence specifically for teaching fractions to struggling learners (Butler et al., 2003 & Watt, 2013).

Recently, however, a new intervention curriculum focusing on rational number sense has been developed by a group of educators called "Teach 4 Mastery" (2015). Perceptions- Blue Book employs many different aspects of fraction intervention that have been used in previous literature, such as explicit lesson by lesson video instruction, student workbooks with practice problems, and manipulatives as visual representations of fractions. The intervention is aligned to the Common Core State Standards and was developed with an empirical basis of explicit instruction that includes visual representations that are "systematic, cumulative, and multisensory" (Teach 4 Mastery, 2015; Gersten et al, 2009). However, no research was found to support the efficacy of the Perceptions intervention and the current study seeks to determine that.

Choice of Assessment Measures

A few formative assessments have been created to target student strength and weakness in the area of mathematics and adapt teaching accordingly. The Math Reasoning Inventory (MRI) was created to target these strengths and weaknesses, only specifically in the area of rational numbers (MRI, 2012). The MRI is a two-part assessment (interview and computation) that examines student rational number sense and knowledge of computation of whole numbers, decimals, and fractions.

A recent pilot study of the MRI fraction section demonstrated the applicability of findings derived from student responses to classroom instruction as a whole (Boyer, 2014). In Boyer's study, teachers completed the MRI at the beginning and end of the fractions unit as a pre and posttest measure and answered questions and responded to interviews to determine whether they amended their teaching as a result of data garnered from the assessment. Additionally, the experimenter conducted observations in the classroom to examine if change occurred.

While no change was observed during the MRI pilot study in regard to improvement of scores between the pre and posttest, there was a change in instruction. Some teachers that used the MRI in their classrooms increased the amount of group instruction and group work time spent in class discussion of the work assigned. Additionally, most of the teachers increased the number of probing questions they asked, indicating that many teachers sought to determine whether the students understood the concepts. Despite a few changes in instruction, none of the hypotheses in the study were supported. Though teachers obtained information about how their students perform and their underlying conceptualization of the topic, they did not know how to apply the knowledge in a way that changed outcomes. Boyer's study is the first evaluating the efficacy and usefulness of the Math Reasoning Inventory.

Though the pilot study of the MRI did not result in change, data obtained from the Math Reasoning Inventory has many important applications. Educators can learn the types of questions with which students struggle, whether computation or interview types. Specifically, the three types of interview questions (understanding how to compute fraction problems, determining how to solve word problems, and estimating relationships between fractions) revelas areas in which students could use further assistance. The information learned from the MRI informs which intervention is chosen to remediate the skills.

As is demonstrated by the literature, formative assessment and intervention have been shown to work hand in hand to increase academic achievement for students. There is a need for more targeted math intervention, particularly in the area of fractions. Though fractions are taught and believed to be mastered in elementary school, many students struggle with rational number sense. The Math Reasoning Inventory, a formative assessment that tests students' rational number sense and reasoning skills, was created to identify the weaknesses in students' rational number sense and provide an opportunity for remediation. Based upon the information obtained from the MRI, many interventions exist to support knowledge of fractions computation and facilitate competency. However, it is currently unclear which interventions and strategies are the most effective for struggling learners. Therefore, further research into the field of formative assessment and intervention in the area of rational numbers is necessitated to evaluate the effectiveness of both the MRI and the Perceptions intervention curriculum. The current study seeks to use an assessment to evaluate rational number sense in regard to fractions, thus informing and determining the efficacy of a targeted math intervention for fractions.

Methods

Participants

The specific population used for the current study was a class of sixth grade students at an urban elementary school in the Midwest. All students in the class were invited to participate in the study based on their need for intensive fraction intervention identified through their progress on the school wide progress monitoring math test. Students were excluded if they did not return their signed consent form prior to the beginning of the study. Nineteen of twenty-six students returned their consent forms and participated fully in the study. The study took place within the first period mathematics class during the second half of the school year. The school in which the participants attended groups core classes based upon ability level. Participants in the study were enrolled in a core math class for students who struggle or at risk for failure in math. At the time of the study, the class was focusing on a beginning geometry unit and focused on topics such as identifying polygons, and determining perimeter and area. Students are traditionally instructed using a problem-based math curriculum aligned to the CCSSM.

Of the 19 students, 11 were female (57.9%) and 8 were male (42.1%). All students (100% of sample) were eligible for free/reduced lunch. The average age of the students was 12 years old while the oldest was 14 years old and the youngest was 11 years old. Most of the students (47.4%) identified as Hispanic American, but 25.0% were Caucasian, 15% were Asian American, and 10% were African American. Per student report, 40% of students reported speaking only English as home, 35% reported speaking both English and another language at home, and 20% reported only speaking Spanish at home.

Materials and Dependent Variables

Math Reasoning Inventory. The Math Reasoning Inventory (MRI; Burns, 2010) is an online tool developed by a team of teachers and other educators. The assessment is aligned to the Common Core State Standards and through both quantitative and qualitative data collection assesses students conceptual and procedural understandings of mathematical concepts. There are

three subtests of the MRI used to estimate a student's rational number sense: Whole Numbers, Decimals, and Fractions. Each assessment is composed of two parts: Interview and Written Computation measures. For the purposes of the current study, only the Fractions interview subtest was administrated. The Written Computation from the MRI was only composed of four questions, which was determined by the researchers to not be a sufficient number of questions to accurately represent fraction computation abilities.

During the interview portion of the study, students individually asked 12 questions. The questions required students to compare and compute fractions using mental math. Each question will be read aloud and also presented in written form. Students were not allowed to compute the problems on paper, but were given as much time as they needed to think of a response. Some questions were in a multiple choice format, but others were open-ended. After a response was given, students were asked how they solved the problem (for example, if they converted a fraction to a decimal, used common denominators, etc.).

The interview section of the MRI was given as both a pre and posttest measure to evaluate rational number sense. Student responses were first scored for incorrect or correct (0 = incorrect; 1 = correct) answers, and then for use of a strategy (0=no strategy use (included guessing); 1 = strategy use). A variable was created for each "Total Correct Response" and a "Total Strategy Usage". Though the MRI assessment tool is fairly new and has not been used extensively, reliability and validity measures have previously been obtained. Cronbach's alpha was used to correlate consistency between pairs of items and a score of 0.87 was found for the Fractions subtest. Construct validity was established by correlating student scores on all three subtests of the MRI, which resulted in high correlations, indicating that the measure is measuring the target construct (rational number reasoning/sense). The technical adequacy of the measure however should be interpreted with caution because it was established by the publisher and not through independent research.

Aimsweb fraction probes. Aimsweb is a system managed by Pearson that is used for universal screening, progress monitoring, and data management. There are many assessments available and a variety of question probes for each grade. For the purposes of the current study, forty-two fraction computation questions were selected from the Aimsweb Sixth Grade Mathematics Computation Progress Monitoring probes (M-COMP). The questions were provided to students in a packet (randomly selected from a larger probe, copied and pasted in a Word document) and the students were given as much time as needed to complete the items. They also had the option to skip any questions they did not know how to complete. Aimsweb measures are normally timed, but as fluency was not an area being examined in the current study, therefore, the researcher and teacher removed the time limit from those items.

All of the questions pulled from the Aimsweb probes were fraction computation problems in which the student was required to generate a response to the problem. For the current study, the responses were scored in a variety of ways. First, the total number of questions calculated correctly was scored (answers that were not completely simplified were counted as correct). Second, the number of correct items were calculated for specific types of problems (simplification, multiplication, division, addition of common denominators, addition of unlike denominators, subtraction of common denominators, and subtraction of unlike denominators). There were 4 questions devoted to simplification, 6 multiplication questions, 9 division questions, 2 addition of common denominators, 11 addition of unlike denominators, 4 subtraction of common denominators, and 6 subtraction of unlike denominators. Aimsweb items have been found to show consistent levels of reliability and validity between probes. The average correlation between probes for the 6th Grade Math Computation was found to be 0.89. To determine validity, total scores on the G-COMP were correlated with those on Group Mathematics Assessment and Diagnostic Evaluation. The validity study found a correlation of r = 0.76, indicating that the assessments measure the same construct.

Perceptions - Changing the Way You Look at Math (Blue Book). Perceptions is a supplemental math intervention focusing on multiplication, division, and fractions created by Teach 4 Mastery. The intervention uses explicit video anchored lessons combined with teacher modeling and adequate student practice to support conceptual and procedural understandings of the math concepts. Each lesson is highly scripted and centered around the video component, which is structured identically for every lesson. The videos range in length, from seven to twelve minutes. The video begins by introducing the new topic with an example that links the new information to previously learned material. The on-screen teacher then walks the students through an example problem, using all aspects of the concrete-representational-abstract sequence to illustrate the concept. Students are encouraged to follow along with their personal manipulative kit. The video demonstrates how to solve several example problems using each part of the CRA sequence. After, supplemental worksheets are provided for further practice.

The Perceptions program is unique in that it moves students from concrete learning using manipulatives, to representational instruction with pictures, and then replaces the visual aids with abstract numbers and symbols. The concrete-representational-abstract sequence (CRA) has a strong research base for its effectiveness in special education (e.g. Gersten et al., 2009). As part of each lesson, the students were taught how to use a fraction manipulative kit to work through sample problems. Each kit contained clear overlays that allowed students to easily compare and find equivalent fractional parts. The kits contained sets of overlays to teach halves, thirds, fourths, fifths, sixths, and tenths. After practice with the concrete manipulative models, students moved on to the representational stage of drawing squares split into parts, and finally ended at the abstract stage without any sort of visual aid.

While the intervention program covers whole numbers, decimals, and fractions, only lessons pertaining to fractions were used within the current study, which occurred during the second half of the school year. The second half of the school year is a very busy time due to state testing and led to a time constraint in regard to length of the intervention. Therefore, only a selection of the lessons was used. The specific lessons targeted were chosen by the researcher and the classroom teacher prior to the start of the intervention and reflected areas the teacher perceived as an area of difficulty. See Table 1 for objectives, example problems, and phase of the CRA sequence used for each lesson.

During the fractions intervention, the students were split into three groups as determined by the teacher. The groups were similar in ability but were chosen primarily on how students in the group related to each other. Students in the classroom struggled with challenging behaviors and certain peer's groups did not work well together. During intervention sessions, the researcher remained in the classroom with the group, and the classroom teacher and her aide pulled the two remaining groups into another area to work.

Dependent variables. There were three dependent variables explored in the current study. The first dependent variable was the number of MRI interview questions answered correctly. Correct answers included answers left in non-simplified form. The second dependent variable was the number of MRI interview questions answered using an appropriate strategy.

Appropriate strategy use was defined as using strategies such as converting fractions to decimals, approaching the problem in terms of currency, converting to common denominators, or drawing visual representations. Inappropriate strategy use was defined as guessing, evaluating size in terms of the largeness of the denominator, or any other strategy not conducive to solving the problem correctly. The last dependent variable was the number of Aimsweb computation questions answered correctly. Correct answers included responses left in non-simplified form, with the exception of questions specifically asking for answers in simplified form.

Procedures

After the completion of the pretest for all students, the intervention began. Eight intervention sessions occurred. The class was split into three groups and the starting order rotated each week. The sequence recommended by the intervention was followed. The Perceptions intervention was very scripted and the same procedures occurred each day of the intervention. First with their groups, students watched the videos targeting specific math concepts related to fractions. During the video the students followed along with the instruction using their personal manipulative kit that was distributed and collected at the beginning and end of each lesson. The experimenter stopped the video after each overlay was shown on the screen, modeled the concrete tools, and then would give the students time to replicate the example.

After the video, supplemental worksheets from the Perceptions materials were used for additional practice. Due to time constraint, the researcher chose the first five examples from each worksheet and had the students complete them independently. An example of practice problems and the specific skills targeted can be seen in Table 1. They were encouraged to use their manipulative kits and also asked to draw the representations on their paper alongside the numeric problem. After the students finished each problem, students would independently work the problem out on the board by drawing the representation and explaining their rationale for solving it. Following the completion of all of the intervention sessions, the students were pulled one-byone to complete the same posttest measures by the experimenter and an assistant (present for only one post-test data collection session). Last, all scores were coded and entered into an IBM SPSS Statistics software spreadsheet.

Experimental design and analysis. The purpose of the current study was to determine the causal effects of an intervention on the math reasoning ability of a group of students. These students were grouped by their classroom teacher but received the same intervention. For that reason, a quasi-experimenter research design was chosen in combination with a pretest-posttest design in which a series of measures were administered before and after the intervention. The Math Reasoning Inventory Fractions Interview and the Aimsweb Fraction Computation probes were administered immediately before and after the Perceptions intervention to all students with a signed consent form. All of the students in the study received approximately eight thirty minute instructional sessions over a two-month period. These sessions were conducted by the experimenter and occurred twice per week during the regular mathematics class period. The teacher continued the same curriculum and work that normally occurs during the class on other days.

To examine growth from pre to posttest, a paired samples *t*-test was conducted for the following variables: Total Correct for MRI Interview, Total Reasoning Used for MRI Interview, Total Correct Aimsweb Questions, Total Correct Simplification...Subtraction with Different Denominators.

Results

The first research question asked what the effect of the Perceptions intervention would be on correct scores as determined by the Math Reasoning Inventory Fraction Interview. The second question asked what the effect of the Perceptions intervention would be on strategy use in regard to answering the MRI Fraction Interview Questions. The third question asked what the effect of the Perceptions intervention would be on the accurate completion of Aimsweb fraction computation items. Complete data was available for all students that participated in the study.

Descriptive Analysis

Preliminary correlational analyses of the dependent measures were conducted prior to the paired samples *t* test. The primary findings are discussed here, but all correlations can be found in Table 2. The number of pre-test Aimsweb questions answered correctly was positively correlated with the number of pre-test MRI fraction interview questions answered correctly, r = 0.491, n = 19, *p* <0.05, indicating a moderate correlation. The pre-test total strategy use was also positively correlated with the number of pre-test MRI fraction interview questions answered correctly, r = 0.495, n = 19, *p* <0.05, indicating a moderate correlation. The pre-test strategy use was correctly, r = 0.495, n = 19, *p* <0.05, indicating a moderate correlation. The pre-test strategy use was correlated with the number of Aimsweb computation questions answered correctly, r = 0.564, n = 19, *p* <0.05, indicating a strong correlated with the number of Aimsweb questions answered correctly was positively correlated with the number of Aimsweb computation for the pre-test NRI fraction interview questions answered correctly, r = 0.564, n = 19, *p* <0.05, indicating a strong correlated with the number of Aimsweb questions answered correctly was positively correlated with the number of Aimsweb questions answered correctly was positively correlated with the number of Aimsweb questions answered correctly was positively correlated with the number of Aimsweb questions answered correctly, r = 0.670, n = 19, *p*<0.005.

Inferential Analyses

MRI fraction interview questions answered correctly. As part of the analysis, the pre and posttest values of total MRI fraction interview questions correct were analyzed using a paired samples *t*-test. Student responses that contained a correct answer (even if it was not simplified) were counted as correct and the total number of responses correct for a student were calculated. Specifically, responses considered incorrect were coded as a 0 and responses considered correct were coded as a 1. The results of the paired samples *t*-test indicated that there was a significant change in the number of items answered correctly as part of the MRI fractions interview portion between the pre (M = 4.26, SD = 1.28) and post-test (M = 5.63, SD = 2.22), t (18) = -2.56, *p* < 0.05 (see table 3). There was a significant change noted in the correctness of questions 2, 5, 7, 8, 9, and 11 as well. Question 2 was a question comparing size of two fractions, questions 5 and 7 were addition questions with unlike denominators, questions 8, 9, and 11 were multiplication questions. Item level statistics can be found in Table 4.

MRI interview question strategy use. Though one purpose of the study was to examine change between number of pre and posttest MRI fraction interview questions answered correctly, another was to go one step further and study the change in the levels of rational thinking as evidenced by their use of strategies. After students initially responded to the MRI questions, they were also asked if they used a strategy to answer the question. They were coded for use an incorrect strategy/no strategy (a score of 0) or appropriate strategy (a score of 1). A second paired samples *t*-test was conducted to evaluate the difference between strategy use. The results of the paired samples *t*-test indicated that there was a significant change in the number of items in which students used an appropriate strategy to answer the questions between the pre (M = 3.00, SD = 2.19) and post-test (M = 8.00, SD = 2.60), t (18) = -7.45, p < 0.05 (see Table 3). The

data were further analyzed to examine the change in strategy use by question and type of question. There was a significant change in strategy use for the following questions: 1, 2, 3, 8, 9, 10, and 11. Questions 1, 2, 3, and 10 were questions that asked the student to determine the relationship between the size of two numbers (larger, same, etc.). Questions 8 and 11 were fractions computations regarding multiplication. Question 9 asked students to fill in a missing number to solve a multiplication computation problem. See Table 5 for item level differences.

Aimsweb fraction computation questions answered correctly. The third research question focused on the effect of the Perceptions intervention on the number of Aimsweb fraction computation items that students were able to answer correctly. With the exception of the "simplification" category, responses were considered correct if they were not simplified. Specifically, responses considered incorrect were coded as a 0 and responses considered correct were coded as a 1. Paired samples t -tests were conducted on the overall number of questions answered correctly, as well as for each type of question: simplification, addition with like denominators, addition with unlike denominators, subtraction with like denominators, subtraction questions answered correctly between the pre (M = 7.79, SD = 1.19) and post-test (M = 13.58, SD = 8.90), t (18) = -3.01, p < 0.01 (see table 3).

Further analyses were conducted for each question type and there were significant findings for simplification, addition with common denominators, addition with unlike denominators, and division. A significant change in the number of addition questions with common denominators between the pre (M = 0.89, SD = 0.94) and posttest (M = 1.68, SD = 0.75), t (18) = -3.34, p < 0.005. Similarly, a change was observed in the number of addition questions with unlike denominators between the pre (M = 0.00, SD = 0.00) and post-test (M = 2.10, SD = 3.45), t (18) = -2.66, p < 0.05. Last, a significant difference was explored in division questions between the pre (M = 1.42, SD = 2.12) and post-test (M = 3.00, SD = 3.40), t (18) = -2.29, p < 0.05. Detailed statistics can be viewed in Table 6.

Discussion

Overview

The previous section highlighted the main results from the study. Overall, the results imply that the Perceptions intervention was effective at increasing the number of Fraction interview and computation questions that participants could correctly answer as well as increase the strategy use to answer those questions. The following section will summarize and further interpret the findings. It will also explore possible limitations of the study, implications, and areas of future research.

Summary and Interpretation of the Findings

The first research question asked what the effect of the Perceptions intervention would be on the number of MRI fraction interview questions answered correctly. The second question asked what the effect of the Perceptions intervention on strategy use when answering the MRI fraction interview questions. The third research question asked what effect the intervention would have on the number of Aimsweb questions answered correctly. The following section will review each of the research questions in regard to the findings.

Effects of Perceptions intervention on number of MRI fraction interview questions answered correctly. Overall, there was a significant increase in the number of MRI Fraction Interview questions answered correctly after the Perceptions intervention. An increase was noticed for all students except one. The student in question was an English Language Learner (ELL) who had been in the United States for less than year and spoke minimal English. Research suggests that of the two types of language that ELL's must learn (social and academic). The academic language takes much longer to develop (Scarcella, 2005) and affects their academic skills development. Therefore, though the student may have been capable of performing on grade level with her peers, her language status hindered her ability to comprehend the material and apply it. For the rest of the class, however, the results indicate that the Perceptions intervention was successful in teaching and re-mediating content to the extent that students were able to accurately answer questions using mental math. The finding is consistent with the research on the Concrete - Representational - Abstract sequence of instruction. Students who are sequentially taught concepts in three stages (starting with something concrete such as a block, moving to representational drawings, and ending with numerical equations) are able to develop better conceptual understanding of the concept/skill at an abstract level (Anstrom, 2006). Therefore, it makes sense that after repeated exposure to fraction concepts in all three stages of CRA, most of the students in the study had a heightened ability to think abstractly and compute mental math problems regarding fractions.

Specifically, a significant change in correctness was noted for following questions: 2, 5, 7, 8, 9, and 11. As was mentioned in the Results section, question 2 was a question comparing size of two fractions, questions 5 and 7 were addition questions with unlike denominators, questions 8, 9, and 11 were multiplication questions. These changes could be explained by an increase in conceptual understanding. However, there was not an increase in correctness for questions 1, 3, 4, 6, 10, and 12. For the MRI portion, students were required to compute the answers to these questions in their head. It is possible that it was too challenging for the students to perform mental math to complete some of the multi-step questions in their head.

Effects of Perceptions intervention on strategy use during MRI fraction interview. The results of the study indicated a significant change in the use of strategy to answer the Math Reasoning Inventory Fraction Interview questions. As was stated previously, the use of the CRA sequence to build conceptual understanding of the topic was instrumental in achieving higher levels of abstract thinking and computation. Without full understanding the concept, students would not be able to answer the question correctly and explain an appropriate strategy that was used to answer the question. However, another key component of the Perceptions intervention was the introduction of relevant vocabulary at the beginning of every unit, and the emphasis on explaining the steps to solving the problem by using key terminology.

The importance of learning relevant vocabulary was highlighted during the pre-test. When students were asked to determine which of a pair of fractions were larger, many said "the number on the bottom is bigger, so that fraction is bigger". That logic indicates a lack of understanding of what a fraction even means. Others would get the answer correct but become unable to explain their rationale and strategy because they did not have vocabulary to express it. However, after the Perceptions intervention, students were able to use words such as "numerator" and "denominator" to explain the relationship between two fractions and ascertain which fraction was truly later. They were also able to explain strategies that they used, such as turning a fraction into a decimal, whereas they may have previously replied "money" and were unable to elaborate further.

Research on "math-talk" supports the change in vocabulary and length of response. Studies of "math-talking learning communities" have emphasized that when students are taught mathematical concepts through the use of vocabulary, manipulatives, representations, and opportunities to solve problems and share their ideas/insights/rationale for solving the problem, students are able to build a community of learning and build deeper understanding (Hufferd-Ackles, Fuson, & Sherin, 2004). By learning the vocabulary and practicing it during the videos and when working the practice problems, students further reinforced the concepts they were learning and demonstrated a higher level of understanding that when explaining the strategy, they used to solve a problem.

Over the 12 questions on the MRI Fraction Interview portion, there was a significant change in strategy use for the following questions: 1, 2, 3, 8, 9, 10, and 11. As was mentioned in the Results section, questions 1, 2, 3, and 10 were questions that asked the student to determine the relationship between the size of two numbers (larger, same, etc.). Questions 8, 9, and 11 dealt with multiplication. There was not a significant change for questions 4, 5, 6, 7, and 12. Questions 4, 5, 6, and 7 involved addition or subtraction of fractions with unlike denominators. Question 12 was a word problem in which students had to divide fractions. As was explained, it is possible that it may have been too challenging for students to compute mental math for the multi-step questions. As they did not know how to approach the problem, they were unable to provide an adequate or appropriate strategy to solve it.

Effects of Perceptions intervention on Aimsweb fraction computation questions answered correctly. Last, a significant change was noted in the number of Aimsweb Fraction Computation questions answered correctly after the Perceptions intervention. Overall, students were able to answer more questions correctly after the Perceptions Intervention, with the exception of the ELL student. Specifically, there was an increase in correctness for the following problem types: simplification, addition with common denominators, addition with unlike denominators, and division. There was not an increase for subtraction with common denominators, subtraction with uncommon denominators, and multiplication. Students were allowed to work these problems out on paper, using as much time as they needed. However, students were not allowed to use a calculator. When multiplying some of the fractions questions, the resulting denominators were typically 3 digits; it is possible that the students were not able to multiply large 2 digit numbers in their heads to get the appropriate denominator. The increased difficulty in mental math could explain the lack of change for multiplication. In principle, multiplication and division of fractions are very similar operations. Students were able to successfully divide fractions, but the resulting denominators were much smaller and easier to multiply by hand or in their heads than the regular multiplication problems.

The discrepancy between addition and subtraction is more difficult to explain. It appears as if the students were unable to generalize skills regarding fraction computation of addition to that of subtraction. As well, the current study focused only on teaching and re-mediating skills related to fraction computation. It is possible that the difficulty with subtraction stemmed from a misconception or lack of skill with subtraction and not necessarily subtraction with fractions. Concepts such as "borrowing" were not reviewed in regard to simple subtraction. Rather, the emphasis was more on forming common denominators than instruction on the fundamentals of subtraction. No evidence was found to explain this discrepancy.

Limitations

There were several limitations to the current study. First was the small sample size. The study only included nineteen participants. The class initially contained twenty-five students but the six who did not participate were either expelled, frequently suspended, moved out of district, or did not return their consent form. Further, all of the participants were from the same leveled mathematics class. Due to the sample, the classroom teacher did not feel as if the class should be further differentiated based upon ability. A larger sample with students from all groupings of the math classes would have provided a more diverse sample size in regard to ability. A larger sample would have allowed the researcher to use pre-test data to inform groupings during the intervention. Additionally, a larger sample size would have provided a smaller margin of error and allowed for greater generalizability.

A second limitation was the length of the study. The entire study was completed in twelve weeks, including the weeks of pre and post data collection. Only 10 intervention sessions occurred. A longer study would have allowed for a greater breadth of content to be covered and more time to review it in detail. With a longer window of time, it would have been possible to administer a needs assessment prior to the pre-test to identify aspects of rational numbers that students struggled with, instead of just honing in on fractions per teacher request. As well, there was not a follow-up to evaluate permanence over time.

The third limitation involves the uncertainty of educational research. Though the researcher followed a plan for intervention, there were unexpected changes throughout. The sessions initially occurred only on Monday and Friday, but occasionally had to be switched to Tuesday or Wednesday due to conflicts with the classroom teachers schedule and snow days. As well, the final day of post-test data collection occurred on the last day of school. After testing for three weeks prior, most of the students were experiencing some degree of apathy and fatigue, which may have influenced the results.

Implications

The findings of the current study have several implications. First, the results suggest that the Perceptions intervention may be effective in remediating and teaching concepts related to fractions. In addition to aiding computation of fraction problems, students also demonstrated an increase in the skills required to use an appropriate strategy to solve the problems. The results could be particularly salient due to the use of the CRA sequence of teaching mathematics. Research on the CRA sequence in teaching fractions and developing a higher conceptual understanding of content is supported by past research (Butler, Miller, Crehan, Babbitt and Pierce, 2003; Watt, 2013; Witzel, 2005), but warrants further investigation. As well, the CRA sequence is aligned with NMAP's suggested method of teaching mathematics (NMAP, 2008). By using the CRA sequence and other related instructional strategies such as step-by-step instructions, modeling, and corrective feedback to teach mathematics, teachers are able to check for understanding at each step, and facilitate a higher level of conceptual knowledge. Which, in turn, will allow for a solid foundation upon which procedural knowledge is then built.

The Perceptions intervention extended the CRA sequence one step further by adding the use of video instruction, which has also proven effective in past studies (Bottge, 1999). Previous research on the CRA sequence focused mostly on students with learning disabilities, but the current study suggests that the intervention can be beneficial for all students. In addition,

interventions such as the Perceptions intervention are easy to implement by anyone within education and are of low cost.

Lastly, the intervention window for the current study was only ten weeks total. During this time, a significant amount of growth was observed between the pre-test and post-test. If this much growth was observed in such a short time period, there is potential for even greater growth with a larger window of time. It is possible that given more time and the use of all included lessons, student growth may have been even more significant. These findings from such a short time span have significant implications on the way mathematics is taught in the United States.

Future Research

In conclusion, the current study was able to accomplish several things. First, it examined the efficacy of a new fractions intervention on re-mediating fraction skills. It also addressed the need for more specific math interventions designed for the general education classroom. The results of the current study supported the hypotheses that were proposed. The combination of instruction components included within the Perceptions math intervention increased the number of MRI Fraction interview questions and Aimsweb fraction computation questions answered correctly. It also increased the use of appropriate strategies to solve the MRI questions.

The current study highlights the need for further research in the field of mathematics intervention. It examined one fraction intervention that exists for struggling students in the general education classroom. However, when considering the number of interventions that exist for English/language arts versus mathematics, the numbers are not nearly comparable. The current study emphasized the importance of developing a concrete understanding of a topic before attempting to develop an abstract understanding. Generalizing a similar framework to the entire mathematics curriculum is important. As well, replicating the current study with a larger and more diverse population should be considered as well. By differentiating the groups within the intervention, the efficacy of the interventions can be evaluated in a manner that is more consistent with a true experimental design which would also allow the results to truly be generalizable to other populations of students.

Further research should also be done to examine the discrepancy between addition and subtraction of fractions. A larger sample of test items for each computation time could provide greater evidence to evaluate the discrepancy further. Last, future research is needed to determine whether the results remain stable over time. It is necessary to determine if students will continue to use the representational strategy if they forgot how to solve fraction computation problems in abstract manner.

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Table 1								
Objectives,	Example	Problems,	and	CRA	Phase of	f Percep	otions L	esson

Lessons	Objectives	Example Problem	CRA Phase
Parts of a Fraction and Fraction of a Number	 Understand a fraction expresses how many of what kind Understand the parts of a fraction Understand how a fraction is a division problem Learn terms (numerator/denominator) 	What is ½ of 8? Construct and draw to help you solve the problem	Concrete and representational
Types of Fractions and Comparing/ Combining Fractions	 Understand difference between proper and improper fraction Understand that fractions can be less than, equal to, or greater than the whole unit or entity Understand kinds and names of basic fractions Understand how fractions could be written Learn terms (proper/improper fraction) 	Construct the fraction with MasterFractions, then draw it here. $\frac{1}{4} = $	Concrete and representational
Add and Subtract Fractions with Same Denominators & Decomposing Fractions	 Understand how to combine two fractions with same denominator Understand why fractions with same denominator can be combined Understand why the denominator is never combined inaddtion or subtraction of fractions Understand how to decompose a fraction Learn terms (decompose) 	(1) Construct with MasterFractions, record the problem, and give your answers. (2) Draw in the correct fraction, then solve. + = + = + = + = + =	Concrete and representational
Equivalent Fractions	 Understand equivalent fractions Understand same amount - difference size pieces Understand how to draw equivalent fractions Learn terms (equivalent) 	Construct, then draw and record finding equivalencies for each set. 1 = - = 3	Concrete and representational

Finding Equivalencies, Simplifying, and GCF	 Understand how skip counting helps find equivalent fractions Understand how reversing the skip count pattern helps to simplify/reduce fractions Understand the greatest common factor and how it works with fractions. Understand how to reduce fractuions with large numerators or denominators Learn terms (simplifying/reducing GCF) 	Find fractional equivalencies by drawing or use the MasterFractions if needed. 2 = 4 = -3	Concerete <u>or</u> representational
Adding and Subracting Fractions	 Understand how to add and subtract fractions with uncommon denominators Understand how to build equivalent fractions to find common denominator Understand the least common multiple Learn terms (multiple, least common multiple, lowest common denominator, multiplier) 	Find the least common multiple by skip counting. Circle the two common multiples, and write your answer in the blank. The LCM of 3 and 4 is 3,,,,,	Abstract, then representational
Changing Improper Fractions and Mixed Numbers	 Understand the difference between a mixed number and improper fraction Understand how to change from a mixed number to an improper fraction Understand how to change from an improper fraction to a mixed number Learn terms (mixed number) 	Circle the correct answer. $2\frac{1}{3}$ Improper Fraction Draw each fraction. $3\frac{1}{3}$ = 100000000000000000000000000000000000	Abstract and representational
Multiplication with Fractions	 Understand the concept of making "groups of" the second fraction Understand the the word "of" means to multiply when used between two fractions 	Multiply. Prove your answer by drawing. Simplify your answer if possible. a	Abstract, then representational
Division with Fractions	 Understand why division of fractions if the inverse of multiplication Understand how to divide two fractions using the <i>MasterFractions</i> 	Divide. Provide your answer by drawing. Simplify your answer if possible.	Abstract then representational

Table 2

Means, Standard Deviations, and Correlations of Pre-Test and Post-Test Measures

1	2	3	4	5	6	М	SD
1						4.26	1.28
.495*	1					3.00	2.19
.491*	.564*	1				7.79	1.19
.212	.355	.368	1			5.63	2.22
.050	.264	.223	.270	1		8.00	2.60
.340	.422	.485*	.670**	115	1	13.58	8.91
	1 .495* .491* .212 .050 .340	1 2 1 .495* 1 .491* .564* .212 .355 .050 .264 .340 .422	1 2 3 1 .495* 1 .491* .564* 1 .212 .355 .368 .050 .264 .223 .340 .422 .485*	1 2 5 4 1 . . .495* 1 . .491* .564* 1 .212 .355 .368 1 .050 .264 .223 .270 .340 .422 .485* .670**	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	1 2 3 4 5 6 M 1 . . 4.26 .495* 1 . 3.00 .491* .564* 1 7.79 .212 .355 .368 1 5.63 .050 .264 .223 .270 1 8.00 .340 .422 .485* .670** 115 1 13.58

Note: **p* < .05, ***p* < .01, ****p* < .001

Table 3Comparison of Pre-Test and Post-Test Measures

	MRI interview correct pretest	MRI Interview correct posttest	MRI strategy use pretest	MRI strategy use posttest	Aimsweb correct pretest	Aimsweb correct posttest
	Mean (SD)	Mean (SD)	Mean (SD)	Mean (SD)	Mean (SD)	Mean (SD)
	4.26 (1.28)	5.63 (2.21)	3.00 (2.19)	8.00 (2.60)	7.79 (1.19)	13.58 (8.91)
T value	2.577*		7.451***		3.006**	
N7 (• т		1 1	* .0 5 **	.0.01

Note: MRI = Math Reasoning Inventory; SD = Standard Deviation; * *p* <0.5, ***p*<0.01, ****p*<0.001

Table 4Comparison of Pre-Test and Post-Test MRI Questions Answered Correctly by Question

	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12
Pre- test Mean (SD)	1.632 (.496)	1.526 (.513)	2.105 (1.150)	1.737 (.452)	1.579 (.507)	1.579 (.507)	.790 (.631)	.947 (.405)	.790 (.491)	1.316 (.582)	.842 (.602)	1.105 (.459)
Post- test Mean (SD)	1.739 (.452)	1.316 (.478)	1.737 (.452)	1.842 (.375)	1.579 (.507)	1.684 (.478)	1.158 (.375)	1.263 (.562)	1.105 (.459)	1.579 (.507)	1.368 (.684)	1.105 (.072)
T Value	1.455	2.191*	1.439	1.000	2.689*	.622	2.348*	2.882**	2.364*	1.564	2.379*	.000

Note: MRI = Math Reasoning Inventory; SD = Standard Deviation; * *p* <0.5, ***p*<0.01, ****p*<0.001

Table 5	
Comparison of Pre-Test and Post-Test MRI Strategy	Use by Question

	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12
Pre-test Mean (SD)	.210 (.419)	.263 (.452)	.158 (.375)	.316 (.478)	.526 (.513)	.526 (.513)	.211 (.419)	.526 (.513)	.000 (.000)	.316 (.478)	.211 (.419)	.211 (.419)
Post-test Mean (SD)	.737 (.452)	.790 (.419)	.684 (.478)	.579 (.507)	.368 (.456)	.737 (.452)	.526 (.513)	.947 (.405)	.632 (.597)	.842 (.501)	.682 (.582)	.474 (.513)
T Value	3.750***	3.750***	3.750 ***	2.041	.900	1.714	1.837	3.024**	4.609***	3.293**	2.673*	2.041

Note: MRI = Math Reasoning Inventory; SD = Standard Deviation; * p < 0.5, **p < 0.01, ***p < 0.001

Com	Comparison of Pre-Test and Post-Test Aimsweb Questions Answered Correctly by Type											
	Simplification	Addition - like denominator	Addition - unlike denominator	Subtraction - like denominator	Subtraction - unlike denominator	Multiplication	Division					
Pre-test Mean (SD)	2.053 (1.544)	.895 (.037)	.000 (.000)	2.053 (1.649)	.158 (.688)	.895 (1.329)	1.421 (2.116)					
Post-test Mean (SD)	2.632 (1.116)	1.684 (.794)	2.105 (3.446)	2.739 (1.661)	.474 (1.172)	.842 (.765)	3.000 (3.399)					
T Value	1.934	3.336**	2.663*	1.277	.972	.170	2.290*					

Table 6Comparison of Pre-Test and Post-Test Aimsweb Questions Answered Correctly by Type

Note: MRI = Math Reasoning Inventory; SD = Standard Deviation; * *p* <0.5, ***p*<0.01, ****p*<0.001