ABSTRACT

INVESTIGATION OF THE FRACTURE RESISTANCE OF PAPER UTILIZING A MODIFIED LINEAR ELASTIC FRACTURE MECHANICS MODEL

by Ji Li

Although a significant controversy exists on the application of Linear Elastic Fracture Mechanics (LEFM) to paper materials, a simple modified LEFM model was shown to be capable to characterize and predict the fracture resistance of variety of commercial papers. The modified LEFM equation can be used as a simple method to evaluate the fracture sensitivity of paper. A series of double edge-notched tests (DENT) of different commercial papers were conducted and analyzed in this paper. Differences in the fracture sensitivity of several commercial paper materials based on structural and material response were provided. The ability to represent fracture data for a range of papers with the modified LEFM model indicates the tensile strength and fracture process zone are sufficient for characterization. Furthermore, measured tensile strength in paper is affected by inherent flaws and a method of indicating the intrinsic tensile strength of paper materials with the use of deeply notched DENT specimens and extracting it from the fitted modified LEFM equations was presented. Finally, it was concluded that the intrinsic tensile strength of paper is higher than the traditional tensile strength, which is affected by edge flaws after cutting.

INVESTIGATION OF THE FRACTURE RESISTANCE OF PAPER UTILIZING A MODIFIED LINEAR ELASTIC FRACTURE MECHANICS MODEL

Thesis

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Table of Contents

List of Tables

List of Figures

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1 INTRODUCTION

It is well known that paper possesses an open and inhomogeneous structure due to the constituent fibers and the manufacture process. This structure greatly impacts the mechanical properties of paper. For example, when a test coupon of paper is subjected to an increasing tensile load, failure may occur at some inherent structural flaw. We would consider this as a defect where stress concentrates to a higher level at or around this region. The structural inhomogeneity or inherent flaws may impact the measured tensile strength of paper. Most papers exhibit a sensitivity to cracks and/or induced notches, but tend to be relatively tough materials[1]. The sensitivity derives from developed stress concentrations around the tips of the cracks or notches, and the toughness comes from the network structure of the paper and the plasticity of the materials.

This thesis contains the results of an investigation into the fracture sensitivity of paper using the modified linear elastics fracture mechanics model presented in[1]. The thesis author is a co-author of reference [1], which is included in Appendix A for reference. The work of this thesis is to delve further into understanding the fracture behavior of paper and help establish the applicability and limitations of the theory presented in [1]. A thorough literature review is given in [1] and a summary is given in the following.

1.1 Background

Material discontinuities such as flaws, cracks and notches can reduce the strength of a sheet structure. For centuries, scientists have developed different mechanics theories in order to give a better explanation for fracture mechanism such like what mentioned in references[2, 3]. In 1920, Griffith[4] first presented a fracture mechanics theory based on an overall energy balance. In his theories, fracture energy is consumed by creating new fracture surface area. However, this theory is limited to linear elastic materials. For real materials, the stress concentrations around the crack tip would cause the localized non-linear behavior, such as plasticity or damage[5]. In order to extend the validity of Linear Elastic Fracture Mechanics (LEFM), Irwin (1960)[6] introduced a plastic correction factor by adding the initial crack length to an estimation of the plastic zone

size. Because most materials exhibit nonlinear inelastic behavior prior to failure, LEFM typically is applicable only under situations of "small scale yielding", i.e. such nonlinear zone size surrounding the crack tip is small enough in comparison to the crack length. Seth and Page [7] determined the stress intensity factor by using Double Edges Notches Test (DENT) based on LEFM theory. By considering paper as an orthotropic isotropic continuum, they concluded that LEFM can be applicable for paper materials only when the sample has relatively large width and crack length. Swinehart and Broek[8] showed that the predictability of failures for notched paper can be determined by laboratory testing and pointed out that LEFM provided a good prediction. They did not adjust the LEFM method to account for a Fracture Process Zone (FPZ), but their good predictions are only for relatively large sheets. By doing DENT test of newsprint specimens at cryogenic and room temperature, Donner [9] separated the FPZ into material part and structural part. Under cryogenic condition, the material component in FPZ was removed and a linear elastic and brittle behavior was observed. At cryogenic temperature, the FPZ size was 0.5mm in MD and 1.1mm in CD which would be the size of pure FPZ structural component length. The size of FPZ at room temperature is 1.5mm in MD and 3.7mm in CD, both of which are larger than those in cryogenic temperature.

In fracture mechanics, there are three basic modes of loading a crack tip, which are shown in Figure 1. In this thesis, only Mode I, tensile loading, is investigated. For Mode I failure of a sheet material, three notch configurations are typically used in fracture tests as shown in Figure 2.

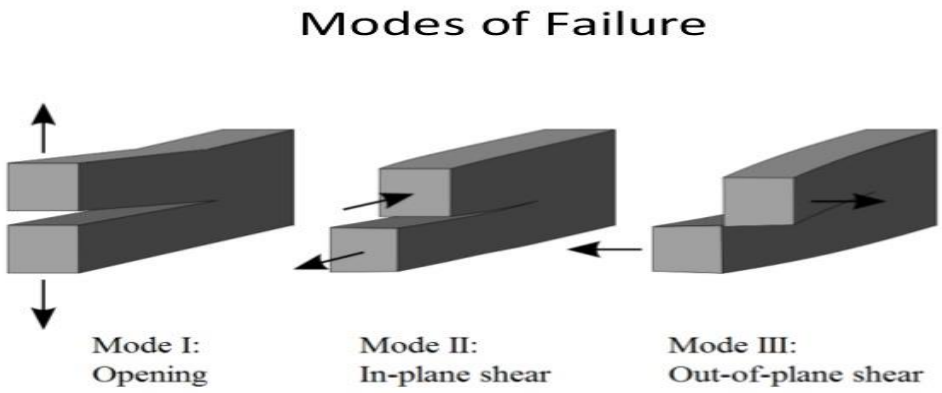

Figure 1 The Three modes of Fracture

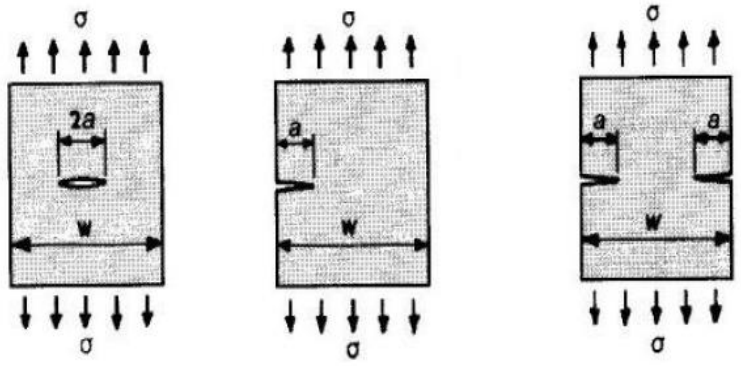

Figure 2 Three Notch Geometries of Mode I Fracture[1]

In Figure 2, the left one is a center-notched test(CNT) specimen, the middle one is a single edge-notched test(SENT)specimen and the right one has the double edge-notched test specimen (DENT).

With the limited applicability of LEFM theory, even with Irwin's correction, more rigorous theories were developed. Rice(1968)[10] first proposed the J-integral for Non-Linear Fracture Mechanics(NLFM) material analysis and J-integral was first applied as a crack tip parameter for paper by Uesaka et al. (1979)[11] ; The essential work of fracture(EWK) was first presented by Cotterell and Reddel(1977)[12] and it was first introduced to paper materials by Seth et al.(1993)[13]. Wellmar et al. (1997)[14] developed an NLFM theories based on an orthotropic non-linear deformation theory. In the recent years, cohesive zone model was proposed by Mäkelä and Östlund (2007)[15] as a way to accurately predict failures of notched paper. However, complex calculations make these more rigorous models quite cumbersome to use.

The LEFM analysis is much simpler than NLFM and its simplicity makes it more attractive and valuable than NLFM, which contains many numerical or tabled values. Swinehart and Broek

[8]made a strong argument for preferring stress intensity factor to the J-Integral method and a good correlation of limit load and notch size were shown in their paper, which indicates a strong predictive capability by using stress intensity factor. Mäkelä, Nordgagen and Gregersen [16] showed that fracture loads for samples with a width of 800-1000mm wide sample fracture toughness could be predicted based on the fracture toughness taken from a 50mm narrow samples by using NLFM. Although Mäkelä and Fellers[17] showed an explicit prediction equation of fracture resistance, it still contains the need for tabled values limiting its applicability. Coffin, Li and Li [1] demonstrated that by modifying LEFM with the addition of a FPZ produced predictions as good as that given in [9], yet with the simplicity of one explicit expression with only two measured quantities, tensile strength and FPZ. Thus, there may not be the need for cumbersome NLFM to characterize the fracture resistance of paper.

The idea contained in the modified LEFM [1] is that paper is inherent-flawed material due to its discrete web structure and this in turn produces a minimum nonzero FPZ. The inherent structure of paper means there is some size-scale below which the concept of a continuum does not exist. The inherent flaw size for papers is reported to be a few millimeters according to previous research [9, 18, 19]. There is certain size-scale, due to an inherent structure, below which the stresses cannot concentrate. In theory for a perfect elastic material, there is a stress singularity at the crack tip. But for a real material, the stress is limited to a certain finite level at the notch tip. In the stress concentration area around the crack root, several mechanisms may occur to limit the stress magnification. T. Yamauchi and K. Murakami have successfully used infrared thermography technology to observe the paper plastic deformation under loading[20, 21]. The plasticity in the crack tip area appears during loading process and it will diminish the stress concentration in this area to prevent further crack propagation. The cohesive mechanism[15] in the area will also limit the stress level at crack tip.

Due to the simplicity of computation and operation of LEFM method, a modified LEFM equation was proposed to characterize the fracture toughness and predict the failures in large webs[1]. For most papers, a 50mm sample width was shown to be wide enough to characterize crack sensitivity. This previous work also indicated that the modified LEFM equation can yield predictive capability for large web width by using data collected from the results of narrow width

samples. In reference [1], the Fracture Process Zone (FPZ) was presented as a measurement for stress concentration ability of a paper at a notch tip. The larger the FPZ the less the ability to concentration stress around crack tip and the lower the relative sensitivity of the paper to fracture. The method used in this thesis is based on the modified LEFM equation developed previously[1] and also given in Equation (8), which introduced the FPZ length d composed of both structural and material components. The aim of present work is to investigate the structural and material components effects on the fracture mechanical behavior of different commercial papers based on modified LEFM method via DENT fracture toughness tests. We are also seeking for a better and thorough explanation for the question why several commercial paper materials show different fracture sensitivities based on their structural and material property in this paper.

2 HYPOTHESIS

Paper materials have an inhomogeneous and an inherently flawed structure. When one cuts an edge in paper, these flaws are opened up. For a tensile test, some flaw along the edge of the sample will most likely lead to tensile failure. For a notched tensile test, the size of the inherent flaw will affect the fracture strength. The ability to represent fracture data for a range of papers with the modified liner elastic fracture mechanics model[1], suggests that tensile strength and fracture process zone are sufficient for characterization. Furthermore, the theory suggests that for many papers the intrinsic tensile strength of the network is significantly higher than the measured tensile strength. The results of deeply notched tensile specimens may provide a measure of intrinsic tensile strength.

3 EXPERIMENTAL

3.1 MATERIAL

3.1.1 Commercial Papers

In this paper, several commercial papers and cellulose film were used as the testing materials. Fracture resistance tests were operated on the following commercial paper specimens: newsprint, paper towel, copy paper, paperboard and a polymer film. The physical properties of the commercial papers are shown in Table 1 below. All the samples were conditioned under 50% relative humidity and a temperature of 22℃ before testing.

Table 1 Physical Properties of Commercial Paper

	Grammage, g/m^2	*Caliper(mm)*	*Density, kg/m^3*
Paper Towel	27	0.086	300
Copy paper	77	0.100	770
Newsprint	46	0.072	657
Paperboard	506	0.756	666
Cellophane	31	0.020	1550
Polypropylene film	25	0.024	1000

The papers listed in Table 1 represent a large range of structures from low grammage and low density to high grammage and high density.

A cellophane film is made from regenerated cellulose, which has very small size-scale structural level. The fracture resistance tests for the polypropylene film in this paper provide another fairly homogenous sheet structure for comparison.

3.1.2 Specimen Preparation

All the paper samples were stored under environmental condition of 50% relative humidity and a temperature of 22℃. For the Double Edge-Notched Test (DENT), a series of samples were cut into strips with a length of 7 inches and a width of 3 inches using a guillotine cutter. For the DENT testing, two cracks with the same length of *a*, were cut symmetrically at the midpoints on both edges of the strip and the width of the specimen is 2*w*. The geometry of the DENT specimen is shown in Figure 3 below. Every specimen was notched carefully with a sharp blade and fixed with a plane paper board to avoid pre-damage before loading.

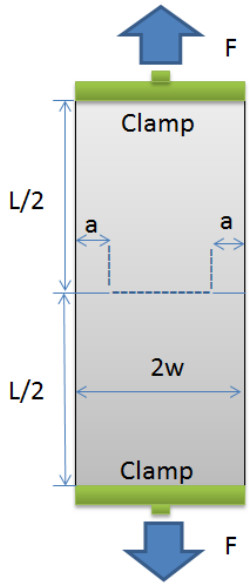

Figure 3 DENT specimen

3.2 Instrumentation

The fracture toughness tests for a series of DENT specimens were made with a pair of pneumatic clamps mounted on an Instron 3344 universal testing machine with a span distance of 180mm and crosshead speed of 25.4mm/min. The grips are 76.2mm wide with serrated faces.

Sample dimension used in this paper is 76.2 mm with a gage length of 180 mm. Before mechanical testing, the physical dimensions were measured. For the samples with very small ligament length or small crack length, the length of the crack was measured after the test.

Masking tape was used to protect the specimen from breaking under the grips. The masking tape did not protrude past the grips.

At least five repetitions were tested repeatedly for each ligament ratio. All our tests were operated under the constant laboratory environmental conditions of 50% relative humidity and a temperature of 22℃.

4 METHODS

Paper materials show a crack or notch sensitivity when they are under loading and tend to be relatively tough materials due to the network microstructure. When a paper sample is under loading, stress concentrations occur around the defects or cracks. In this stress concentration zone at the roots of crack, the material has yielded and plastic deformation appears. During the loading process, the load in this high stress zone would transfer from fibers inside the zone to fibers outside the zone even to remote field via fiber-to-fiber bonds in order to diminish the stress level in this area. As the load increases, at a certain level of loading, the maximum load is reached and the crack begins to propagate.

A modified LEFM equation developed in [1] was used in this paper. The DENT sample was shown in Figure 1. The length of specimen is L, width $2w$, and thickness t, has two notches symmetrically on the edges with each notch length a. The sample is loaded in a tension F. The net force across the ligament must equal F. If the specimen is notch sensitive at the notch tips, the stress concentration is higher at the notch roots than the middle parts of the ligament. For this reason, the load ratio F/F_0 in the DENT test where F_0 is the un-notched specimen tensile force, would be lower than the ligament ratio change $(1-a/w)$.

So when the relation is

$$\frac{\mathbf{F}}{\mathbf{F_0}} < \mathbf{1} - \frac{\boldsymbol{a}}{\boldsymbol{w}} \tag{1}$$

the sample is notch sensitive. If the sample has no sensitivity, the equality sign would hold in Equation (1). A greater than sign in Equation (1) implies the notched specimen is stronger than the sample with a width equal to the ligament length.

According to LEFM, the stress intensity factor can be calculated by Equation (2)

$$\boldsymbol{K_I = f(\frac{a}{w})\sigma\sqrt{\pi a}} \tag{2}$$

Where $f(a/w)$ is the shape correction factor for the finite dimension specimen and σ is the remote field stress. For DENT samples, the geometric correction factor is given as follow after Tada[22]

$$\textbf{DENT:}\quad \boldsymbol{f(\xi) = [1 + 0.122(\cos\frac{\pi\xi}{2})^4]\sqrt{\frac{2\tan(\pi\xi/2)}{\pi\xi}}} \tag{3}$$

and for center–edge notched tests

$$\textbf{CNT:}\quad \boldsymbol{f(\xi) = (1 - 0.025\xi^2 + 0.06\xi^4)\sqrt{\sec(\pi\xi/2)}} \tag{4}$$

where ξ is the ratio of the total length of the notches ($2a$) to the width of the sample ($2w$), i.e. $\xi = \frac{2a}{2w}$.

In LEFM theory, the stress in y-direction on the x-axis can be expressed as a function of the length from current position to notch tip (x).

$$\boldsymbol{\sigma_y = \frac{K_I}{\sqrt{2\pi x}}} \tag{5}$$

If we consider that the crack begins to propagate when the stress at the notch tip reaches some break level $\boldsymbol{\sigma_{ys}}$, the stress intensity factor K_I is constant. Then Equation (6) for the failure load of the un-notched sample can be transformed from Equation (2) after we introduce an inherent fracture process zone length, d.

$$\mathbf{F_0}=\boldsymbol{\frac{K_I A_0}{f(\frac{d}{w})\sqrt{\pi d}}} \tag{6}$$

where A_0 is specimen cross section area, i.e $A_0=2wt$, t is the thickness. Then we can get the limit load ratio with notch length, a, as follow.

$$\frac{\mathbf{F}}{\mathbf{F_0}} = \frac{\mathbf{f}(\frac{\mathbf{d}}{\mathbf{w}})}{\mathbf{f}(\frac{(\mathbf{a+d})}{\mathbf{w}})}\sqrt{\frac{\mathbf{d}}{(\mathbf{a+d})}} \text{ , where a < w-2d} \quad (7)$$

In reference[1], the inherent characteristic fracture process zone (FPZ) length d which is introduced in Equation (7) can be decomposed into both a structural part and a material part, $d=d_s+d_m$. The structural part component d_s can be defined as a microstructural defect size due to the discrete nature of fiber network. And the material component can be attributed to material plasticity. Furthermore, according to equations (3) and (4), the DENT correction factor f(ξ) goes to 1.122 and CNT correction factor goes to 1 for ξ going to 0. When ξ=0, if we apply these two correction factors into Equation (6), we could find that DENT specimen needs lower level failure load than CNT specimen needs. For this reason, the tradition un-notched tensile test specimen which have small opening web structures along both edges would tend to break at edges more like DENT rather than breaking inside of the web-structure like CNT.

After dividing inherent length d into d_s and d_m, a further recasting can be applied on Equation (7) as below.

$$\frac{F}{F_0} = \begin{cases} 1, \ a \le d_s \\ \frac{f(d/w)}{f(d-d_s+a)}\sqrt{\frac{d}{(d-d_s+a)}}, \ d_s < a < w-2d \end{cases} \quad (8)$$

In Equation (8), when the notch length is less than d_s, the failure load F should be equal to the un-notched specimen load F_0. In addition, we can also get the remote field stress σ_f from equation (8).

$$\sigma_f = \sigma_{TS}\begin{cases} 1, a \le d_s \\ \frac{f(d/w)}{f(d-d_s+a)}\sqrt{\frac{d}{(d-d_s+a)}}, d_s < a < w-2d \end{cases} \quad (9)$$

where σ_{TS} is the tensile strength. The Equation (9) shows that the remote field fracture stress with notches can be predicted by the traditional un-notched tensile test.

Both Equation (8) and Equation (9) were presented in[1] as a new modified LEFM equation for fracture characterization and prediction. By using this modified LEFM equation, firstly, we can easily utilize the specimen tensile strength to characterize the relative notch sensitivity of different papers by introducing FPZ=d. In addition, the stress singularity at the root of notch in the classic LEFM is removed by doing the stress ratio. What is more, it shows good convergence to tensile strength as the crack goes to zero.

5 RESULTS AND DISCUSSION

5.1 DENT Fracture Toughness Testing

During the DENT test, a series of commercial papers, cellophane paper and polypropylene film were tested under a condition of 50% Relative humidity and 22℃ for fracture resistance test. The DENT tests of different paper materials were used to verify whether Equation (8) could successfully characterize the fracture behavior of paper materials. Furthermore, the FPZ length d, structural zone length d_s and their functions in paper fracture mechanism were also investigated in this paper.

As shown from Figure 4 to Figure 10 below, Equation (8) which modified from Linear Elastic Fracture Mechanics (LEFM) provides an adequate fit for these DENT results of different commercial papers with relatively wide range of crack sizes.

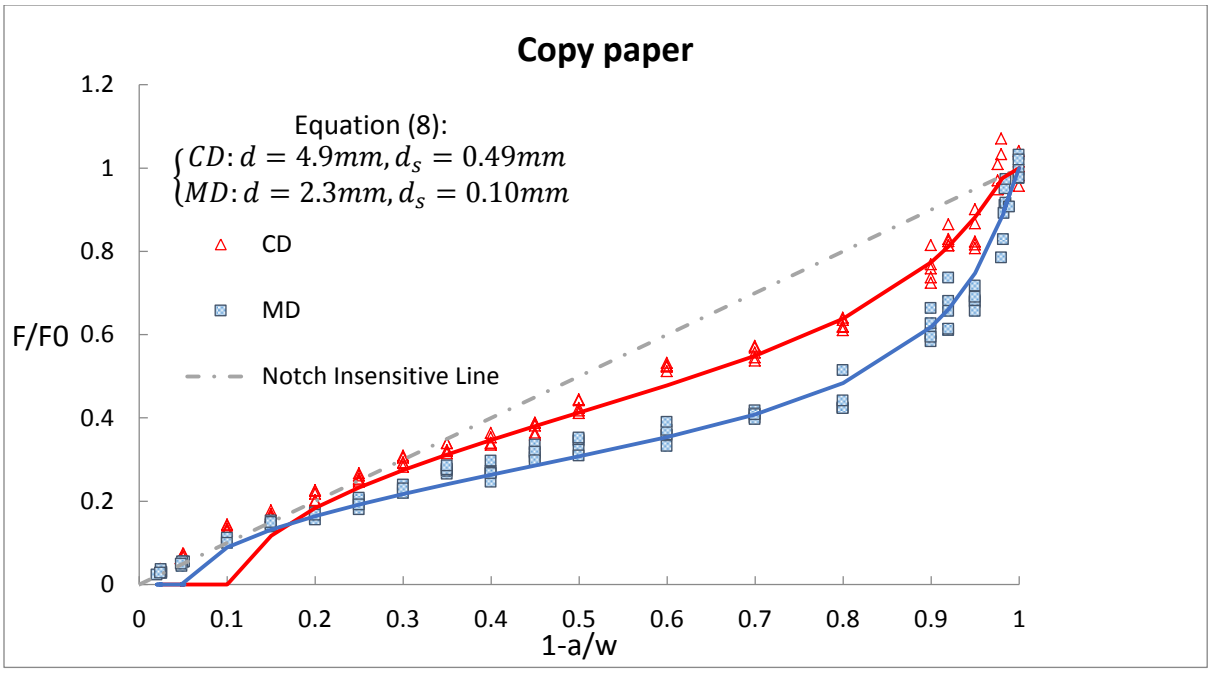

Figure 4 DENT load ratio versus relative ligament length for Copy paper. The solid lines are the Equation (8) and the dash line is notch-insensitive line.

Figure 4 shows the Equation (8) provides a good fitting curve for DENT result of copy paper. In Figure 4, MD specimen shows more sensitive for notches than CD specimen. For small ligament ratios or deeply cut notches, Equation (8) gives an under prediction at this range, i.e. (1-a/w) <0.1 for MD specimen and (1-a/w) <0.1 for CD specimen. As shown in Figure 6, if the ligament length is below some certain level, the two small stress concentration areas in front of notch tips begin to merge together which causes the ligament be fully loaded and then the DENT notch test would not relate to notch sensitivity. In addition, we can find that some points with small relative ligament ratios are higher than the notch-insensitive line in Figure 4. It means the specimen with deep notches is stronger than the un-notched specimen with the same ligament length. This is due to the overlap of two stress concentration zones at crack tips causes a relatively higher stress distribution that makes the average stress on ligament or stress per ligament length higher than the average tensile strength of un-notched specimen. The relative stress concentration area size depends on the effective FPZ size d and therefore the magnitude of d is indicative the stress concentration area. Furthermore, we can see that there are some loads ratios at small notch length also go above the notch-insensitive line. Figure 7 suggests that when the cut at the edge is smaller than a certain size, the cut will have little or no effect on the load limit. Our test shows

for copy paper CD samples with notch size shorter than 0.9mm, the samples do not break at the notches but rather at other points along the edges. In another words, this minimum critical notch size a_m is the physical meaning of the inherent characteristic structural length, d_s and it was found to be consistent with the theoretical value of d_s=1.0mm calculated by fitting Equation (8) into the experimental data. The comparisons of d_s with experimental critical minimum notch length a_m for different commercial papers are summarized in Table 2. The theoretical value of d_s which highly agrees with the experimental value also proves that Equation (8) can be used to characterize the fracture behavior.

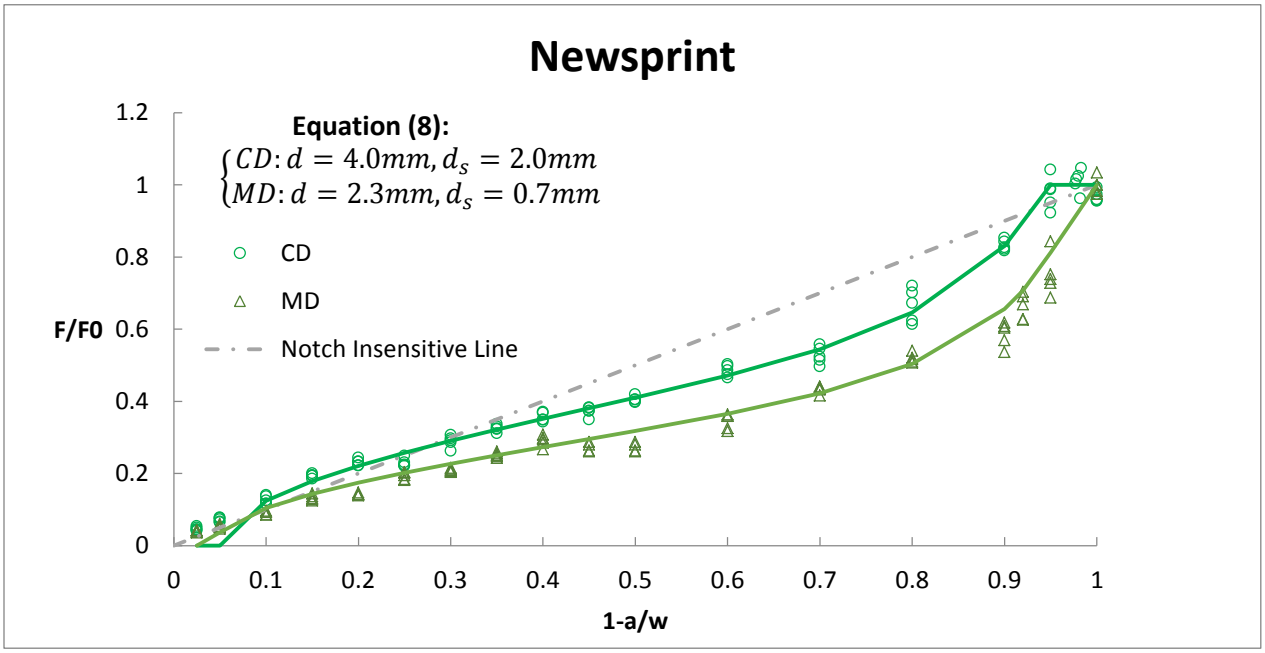

Figure 5 DENT load ratio versus relative ligament length for newsprint. The solid lines are the Equation (8) and the dash line is notch-insensitive line.

In Figure 5, Equation (8) also well predicts the fracture sensitivity for both MD and CD of newsprint papers. Comparing Figure 5 to Figure 4, Equation (8) shows a better fit for newsprint paper than copy paper, especially the specimens with deep cuts. Such a better prediction for small relative ligament of newsprint than copy paper is due to higher lignin content in mechanical pulping process of newsprint which makes newsprint more brittle. Since newsprint is more brittle, the material plasticity character plays a less important role for newsprint. This plasticity contribution in FPZ can be related to the ratio of d_m to d, which equals to $(d-d_s)/d$ and the plasticity zone size was related to the magnitude of d_m. The newsprint has a smaller

contribution from the plasticity than the copy paper , i.e. For CD newsprint, $d_m/d=(d-ds)/d=(4.0mm-2.0mm)/4.0mm=0.50$ is much smaller than CD copy paper $d_m/d=(4.9mm-0.5mm)/4.9mm=0.90$. Additionally, for the reason that CD newsprint has $d_m=2mm$ comparing $d_m=4.4mm$ of CD copy paper, the size of CD newsprint plasticity zone is relatively smaller than that of CD copy paper. Thus, for the same small ligament of DENT for both CD newsprint and CD copy paper, copy paper may reach the full loaded status earlier than newsprint paper as the notches go deeper.

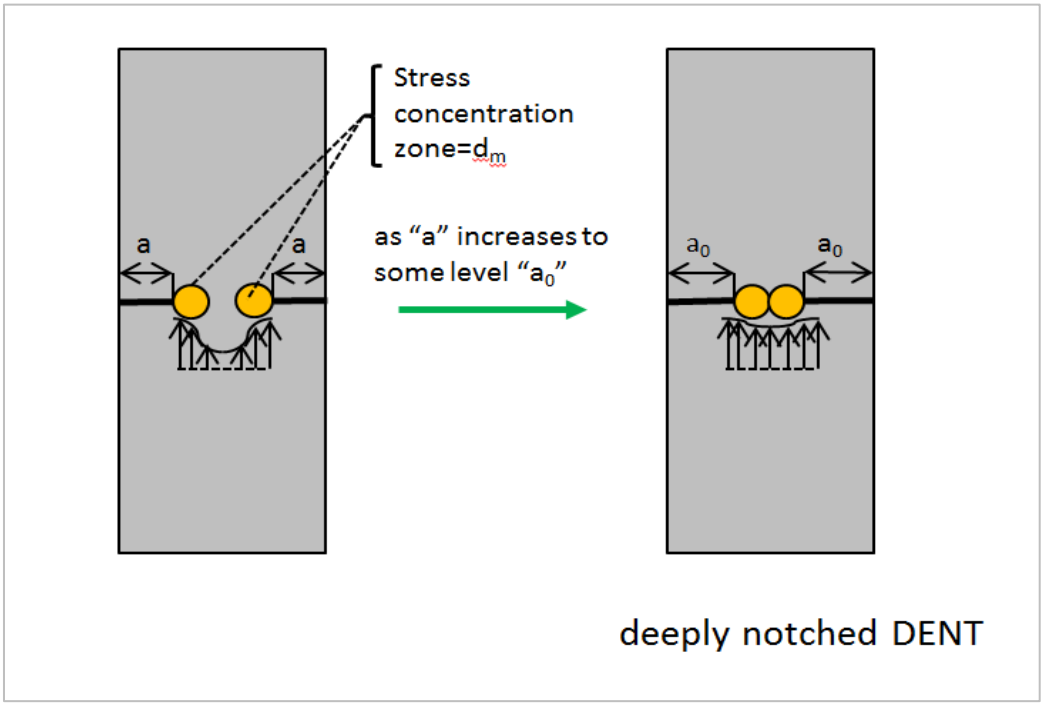

Figure 6 deeply notched DENT specimen shows notch insensitivity character

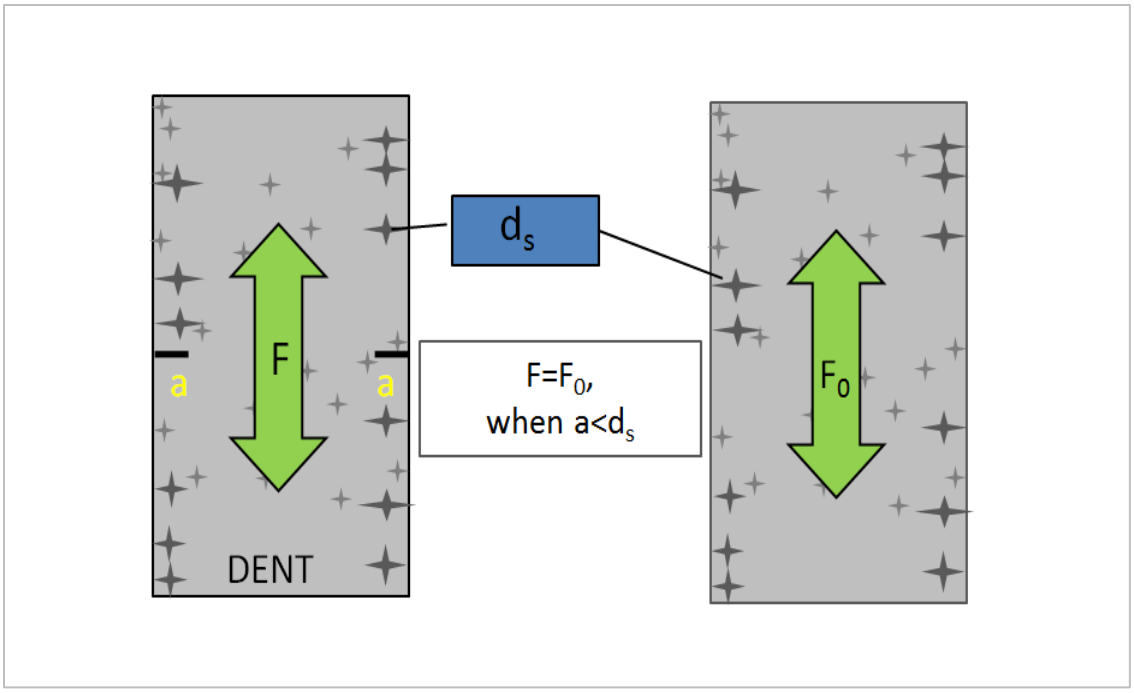

Figure 7 DENT specimen with small notches shows notch insensitivity character

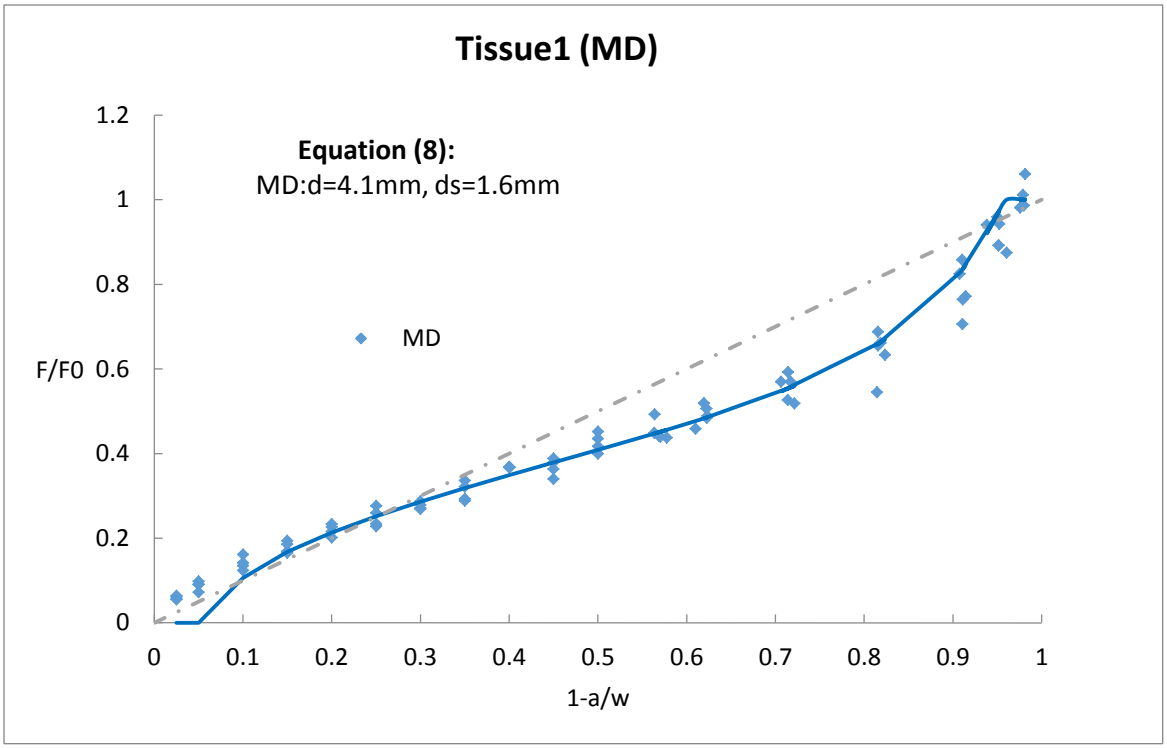

Figure 8 DENT load ratio versus relative ligament length for Tissue MD direction. The solid lines are the Equation (8) and the dash line is notch-insensitive line.

In Figure 8, Equation (8) was used to fit the DENT data for tissue paper in MD direction. For tissue papers, the MD structural zone length is calculated to be 1.6mm which is much larger than that of MD copy paper and MD newsprint. Take a further look into Figure 6, it shows the slope of the load ratios at these small notches are relatively small, which means the sample edge of tissue paper would be not effective in carrying load. Therefore, tissue paper material is dominated by the network structure which shows relatively low notch sensitivity. Associating with the information in Table 2, the lowest value of (d-ds)/d=0.6 for MD tissue paper also provides a proof from another point to show that the low fracture sensitivity for MD tissue is not due to its plasticity.

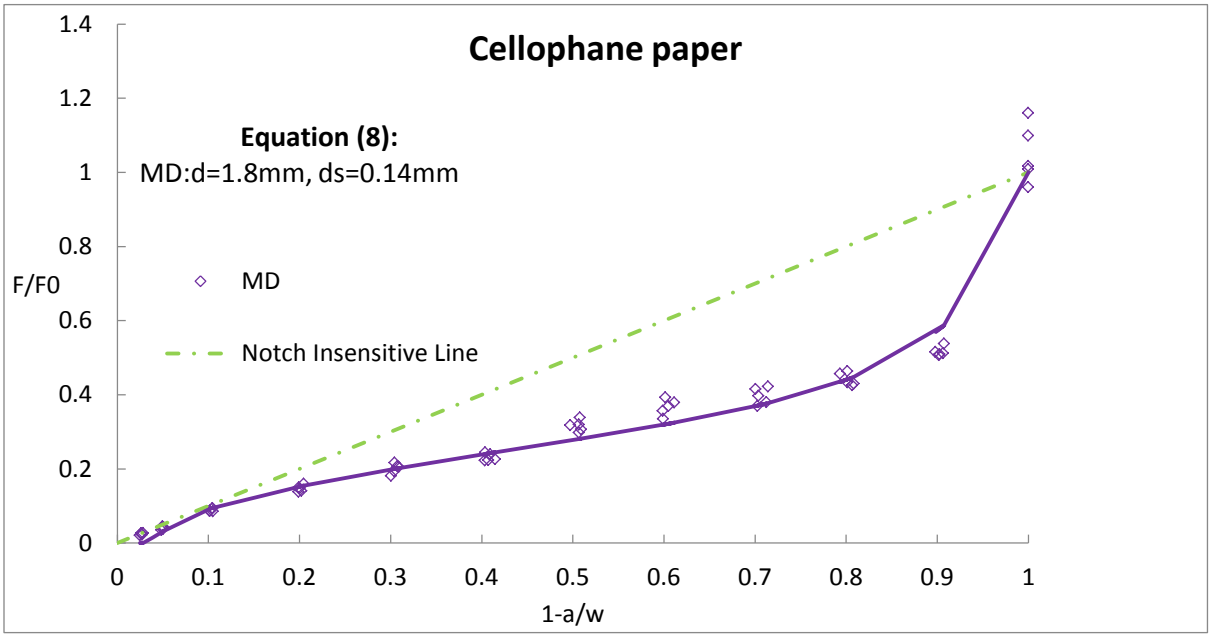

Figure 9 DENT load ratio versus relative ligament length for cellophane film MD direction. The solid lines are the Equation (8) and the dash line is notch-insensitive line.

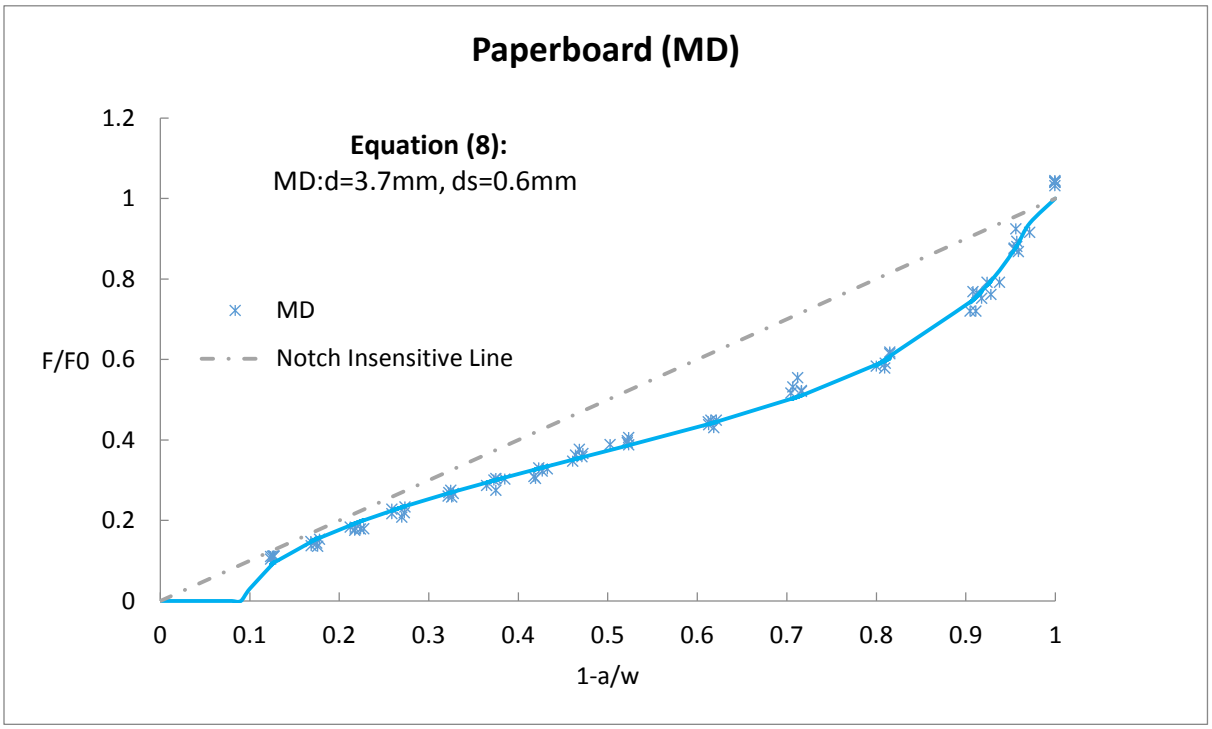

Figure 10 DENT load ratio versus relative ligament length for paperboard in MD direction. The solid lines are the Equation (8) and the dash line is notch-insensitive line.

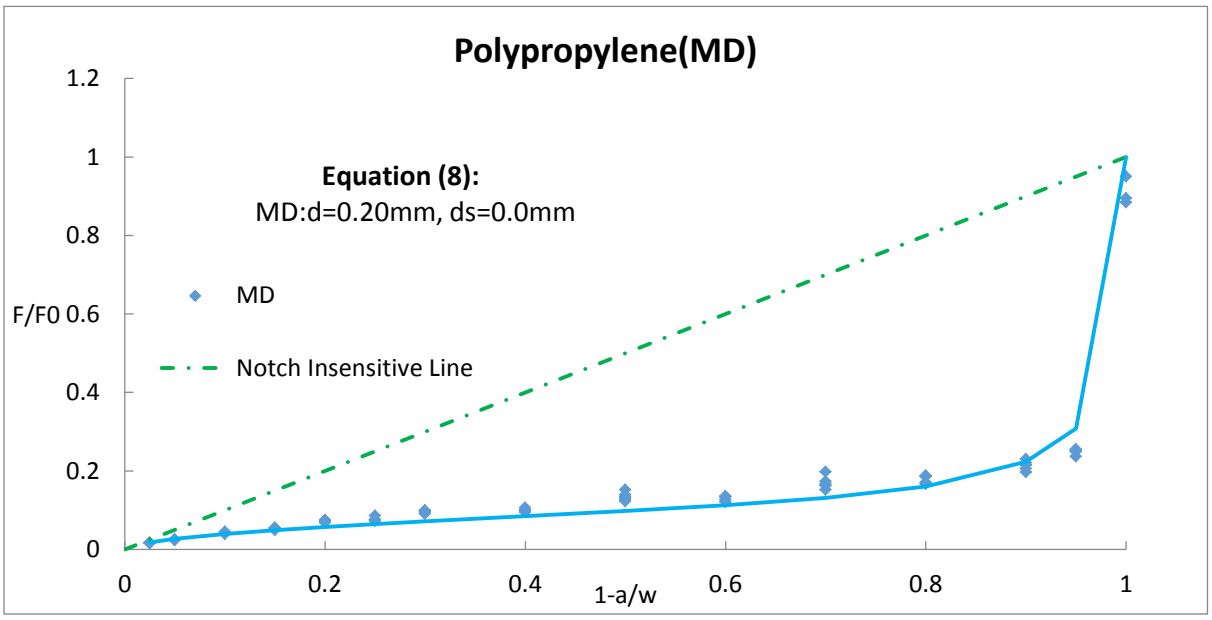

Figure 11 DENT load ratio versus relative ligament length for polypropylene film in MD direction. The solid lines are the Equation (8) and the dash line is notch-insensitive line.

Figure 9-11 show that the Equation (8) provides a reasonable fit for fracture sensitivity for MD cellophane film, MD paperboard and even for polypropylene film. In Figure 9, cellophane paper which is made of regenerated cellulose was tested and it shows larger notch sensitivity than the previous paper materials. Due to the special manufacture process, the structural scale is very small in cellophane which equals to 0.14mm according to Equation (8). Table 2 also shows that the plasticity is the dominating character in cellophane rather than the network structure of the sheet.

In Figure 10, due to the high thickness and stiffness of paperboard, it is very difficult to do some really deep cuts (i.e. ligament ratio<0.12) by using a scissor or a blade and it is impossible to keep the small ligament from pre-damage when you install the samples on the tensile tester. For this reason, data was not collected at these ligament length ratios below 0.12 in Figure 8. As

shown in Figure 10, paperboard shows less notch sensitivity than MD newsprint and it is likely attributed to plasticity of fibers according to Table 2.

The graph of MD polymer film in Figure 11 was used as a comparison test here to show a material with extremely small structure scale and fully plastic property. As shown in Figure 9, the polypropylene film specimen with small notches has a sharp drop in load ratios and therefore it shows the most significant fracture sensitivity among all the paper materials tested so far. For cellophane and polypropylene film, the limit load ratios with deep cuts go under the notch-insensitive line because the effective fracture process zone sizes for both materials are so small that they can concentrate load easily and cause the failure of the deeply notched specimen at relatively low remote field stresses.

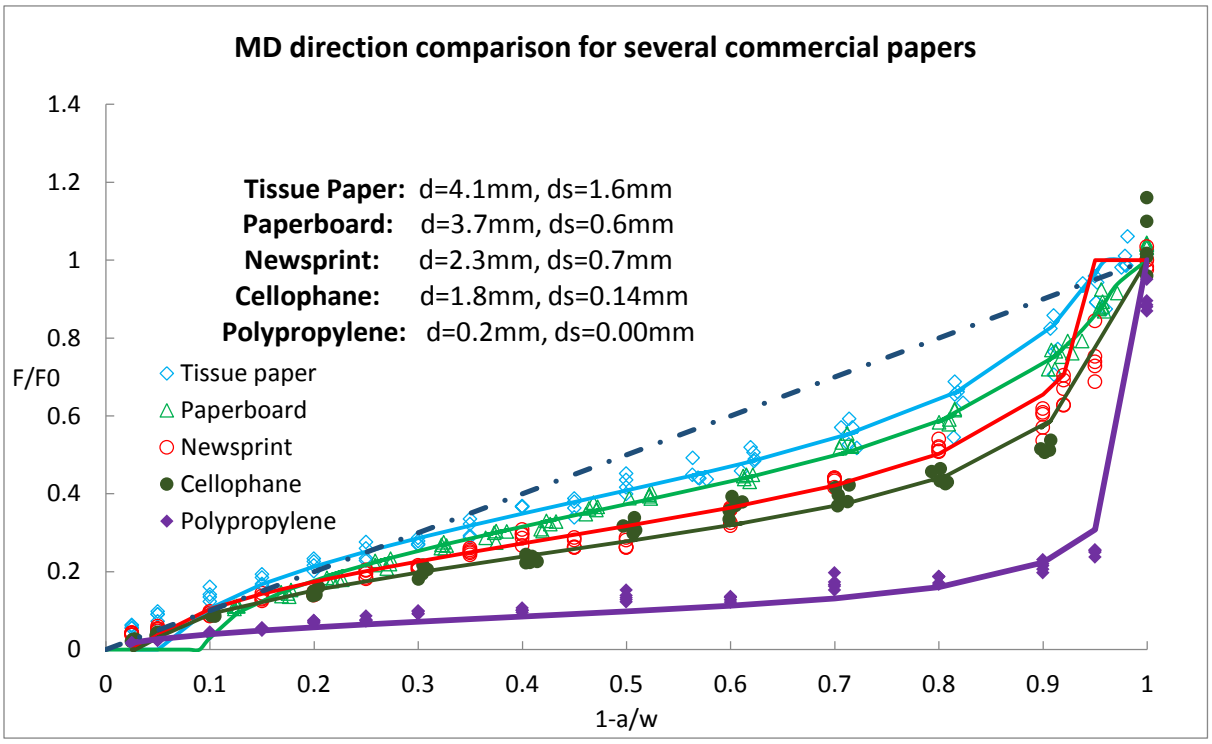

Figure 12 The comparison for several commercial papers in MD direction. The solid lines are the Equation (8) and the dash line is notch-insensitive line.

Figure 12 shows Equation(8) was able to give proper fitting curves for variety of commercial papers in MD direction. Among all types of paper materials, tissue paper shows the least sensitivity to cracks, followed by paperboard and then the newsprint. Cellophane paper has the greatest notch sensitivity among the cellulose-based materials. However, polypropylene film was

tested to be the most sensitive to fracture in all the testing materials. Accoding to reference[1], the DENT results of the handsheets suggested the FPZ size is highly dependent on the sheet efficiency; the larger magnitude of d, the lower sheet efficiency is. Thus, the different FPZ sizes of d in Figure 12 also reveal the internal load concentration efficiency, i.e. the value of FPZ length of polypropylene d=0.2mm shows that this material can easily concentrate load at the crack tip and fails at a relative low far-field stress. According to the SEM pictures in previous research of Kun Li [23], we could tell that the paperboard has the second less imhomogenous structure than paper towel. But the structural zone size d_s=0.6mm of MD paperboard is even smaller than that d_s=0.7mm of MD newsprint in Figure 12 above. For this problem, if we do not consider the material heterogeneity and other scale issues, this result may be attributed to the large thickness of paperboard which is composed of mutiple fiber layers and this special multiple fiber layer structure makes fiber density relatively high in z-direction of the sheet comparing to other lower thickness paper materials. This multple-layer strcuture of fibers in z-direction may form fibers bonding in z-direction and therefore minimize the structure defect zone length of "each layer" to make the bulk structral zone size smaller than what we expected from the 2D SEM images of material surfaces in Kun's research. What is more, according to results above, d_s is the material property which highly depends on the loading direction, therefore, its magnitude cannot be simply estimated by visible structure information from the surface SEM images.

Table 2 Inherent Fracture Process Zone size, d & Structural Zone size ds from Equation(8) vs. the experimental critical notch size a_m

		d,mm	*d_s ,mm*	*d_m/d**	*a_m ,mm*
Copypaper	**MD**	2.3	0.10	0.96	<0.42
	CD	4.9	0.50	0.90	0.60
Newsprint	**MD**	2.3	0.70	0.69	0.82
	CD	4.0	2.00	0.50	1.90
Paperboard	**MD**	3.7	0.60	0.83	0.70
Tissue	**MD**	4.1	1.60	0.60	1.50
Cellophane	**MD**	1.8	0.14	0.92	–
Polypropylene	**MD**	0.2	0.00	1	–

***$d_m/d=(d-d_s)/d$**

Table 2 shows that all the parameters of FPZ in Equation(8) and experimental critical notch size a_m within the possible operating scale. For cellophane and polypropylene, it is impossible to find the critical notch length a_m by hand-cut operation due to the small lengths required. The value of d_m/d is used in this paper as a plasticity dominate factor whose magnitude is related to whether plasiticity or network structure dominate the inherent characteristic zone. When the value of d_m/d is closed to 1, it means more plastisicity features dominate this FPZ; on the other side, the more d_m/d close to 0, the more network character dominates the FPZ. In Table 2, according to the MD DENT results, polypropylene shows the most significant plasticity dominate feature and newsprint shows the least plastic character which means there are more contribution from the inherent network structrue for tissue paper comparing to other commercial papers.

5.2 Ligament Stress Distribution of DENT Specimen

For a further investgation of DENT fracture , we need to focus on the stress distribution on ligament part. When the DENT sample is loaded in tension with load F, the tension across the

ligament must be equal to F. The value of fracture ligament average stress for each specimen was calculated in order to evaluate the stress status on the ligament. All values of ligament stress under different notch ratios were normalized by the tensile strength of the un-notched specimen for further comparing. Figure 13-23 show the stress status on the DENT ligament for different commercial papers, regenerated cellophane paper and polypropylene film. σ_L is the ligament stress for DENT specimen and σ_0 is the tensile strength of un-notched paper sample. The x-axis represents the proportion of total notch length 2a in total width length 2w. i.e. 2a/2w=a/w.

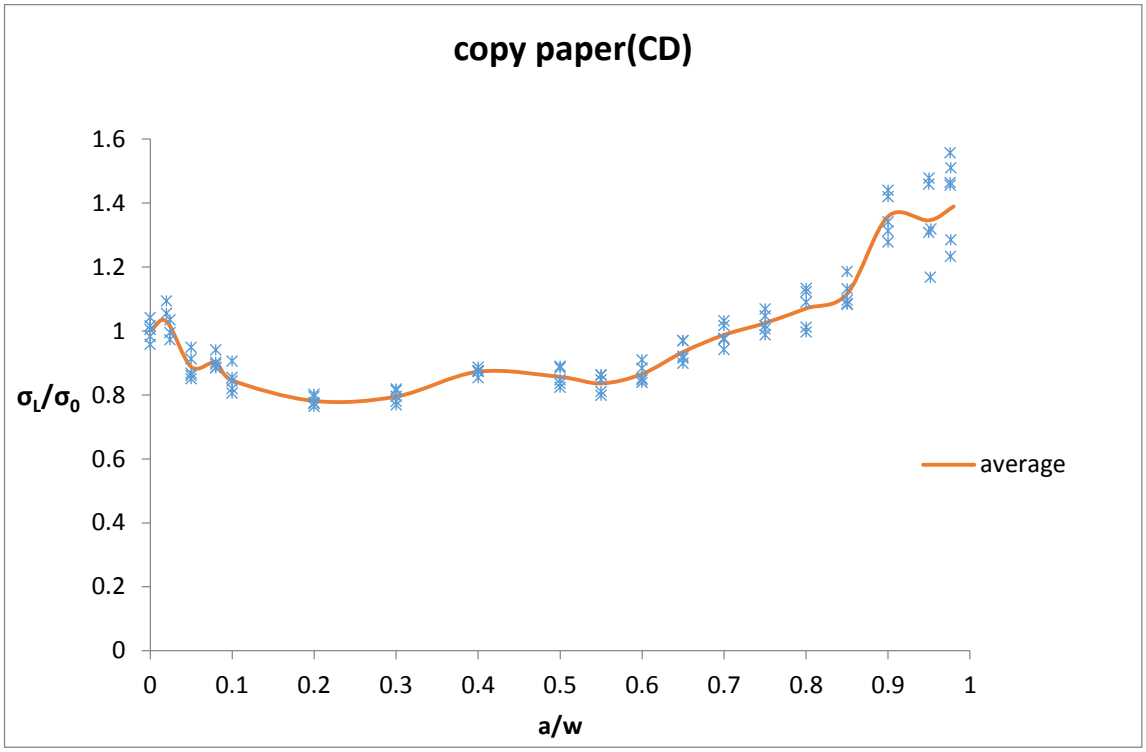

Figure 13 Average ligament stress normalized by tensile strength for CD copy paper. Red line represents the average value of the experimental data

In Figure 13 above, the ligament stress ratio begins with one and then goes slightly above one under certain small crack sizes. After that, the curve starts decreasing as notch length increases. Till a certain point a/w=0.2, the ligament stops decreasing and begins to climb up. In addition, the ligament stress ratio rises as the notches go deeper. When the notch length increases to some scale, the ligament stress ratio goes above 1 and keeps on increasing. At last, at the deepest cut (a/w is closed to 1), the ligament stress ratio can reach to 1.3.

All of the features of the stress distribution curve above illustrate that:

a) For small cracks, when the notch length is smaller than the structral zone length d_s, the ligament stress is greater than the tensile strength of the un-notched sample. In this case, the DENT specimen shows the insensitive feature to notches which corresponds to the data points in CD direction when (1-a/w)>0.95 in Figure 4.
b) During the process of the fracture, the ligament stress ratio decreased to some minimum and then increased to one. In this range, the average ligament stress of the DENT speciment is smaller than the tensile strength of un-notched specimen because the DENT samples show a sensitivity to notches. This domain corresponds to the data points in CD direction below the Notch-Insensitive Line in the Figure 4.
c) For the deepest notches, the ligament stress ratio is greater than one and reached 1.3 at some finite ligament length. This indicate that, for some deeply notched DENT specimens, when the ligament is small enough to cause the effective FPZ merging together, the ligament stress of deeply DENT sample is larger than the tensile strength of un-notched specimen. In another words, this status symbolizes deeply notched DENT specimens show notch insensitive character corresponding to the points above notch-insensitive line with small (1-a/w) value in the Figure 4.

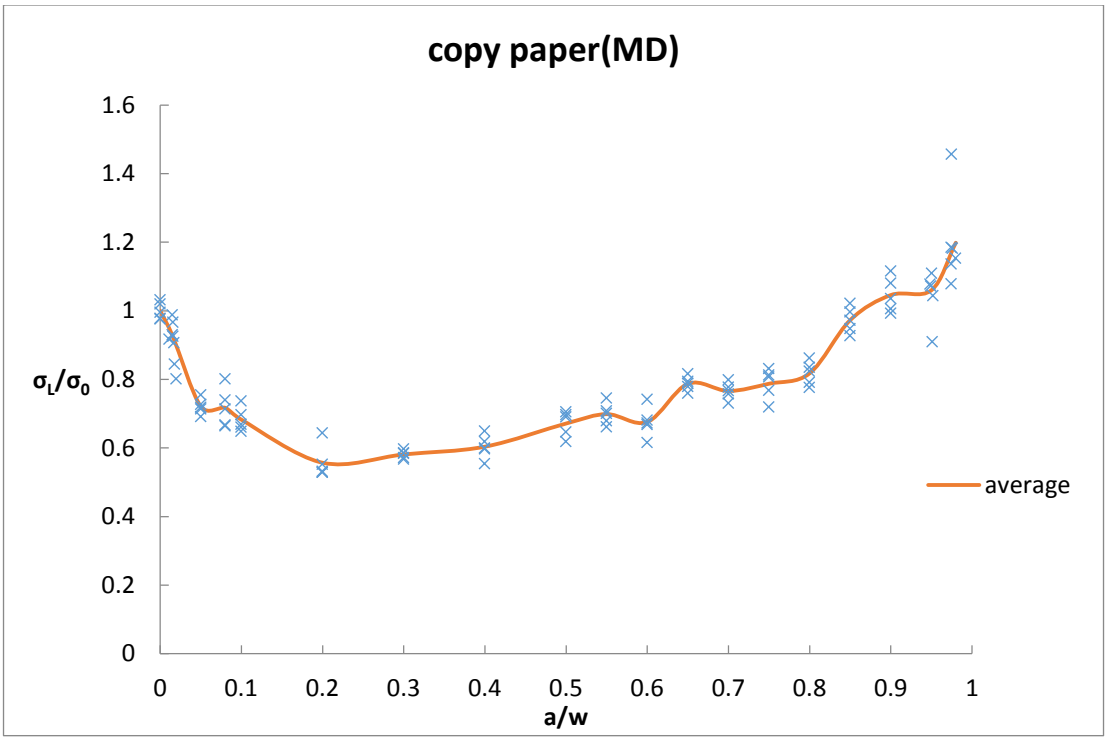

Figure 14 Average ligament stress normalized by tensile strength for MD copy paper. Red line represents the average value of the experimental data

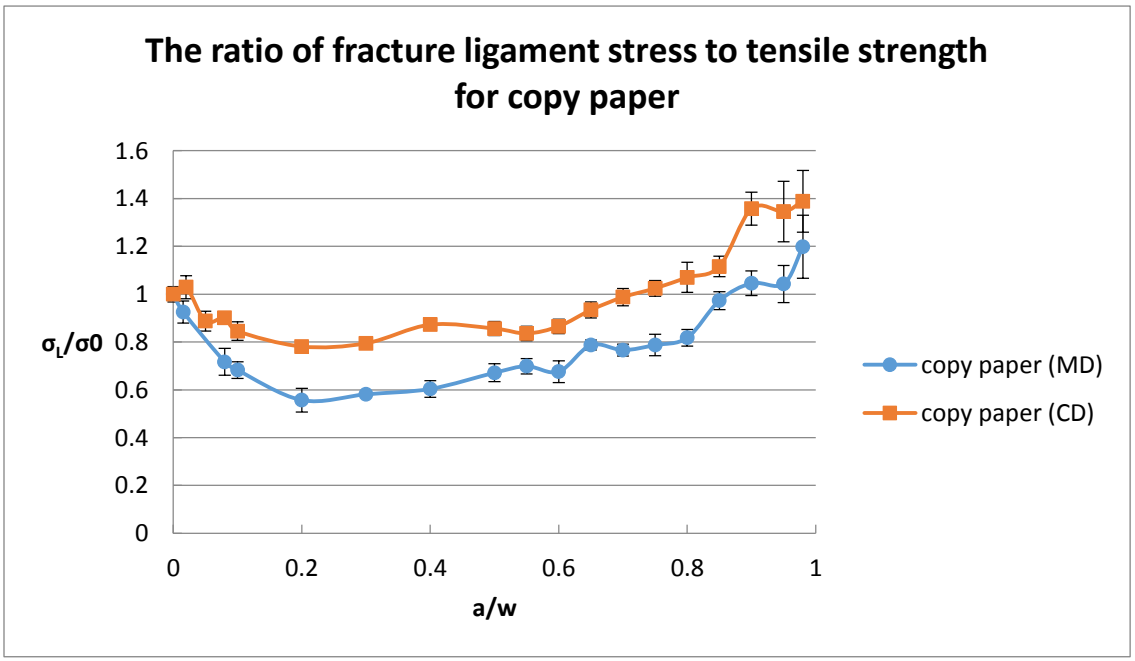

Figure 15 The ratios of fracture ligament stress to tensile strength for copy paper in both machine direction and cross machine direction

Figure 14 shows a similar trend to the curve in Figure 13 but for MD rather than CD. The slope change in this figure also corresponds to the MD data points in different range in Figure 5. Comparing to ligament stress ratio in CD direction, the MD ligament stress ratios value with deeply cuts are much lower and the comparison between the ligament stress ratios of two directions is shown in Figure 15. This difference is attributed to that, for deeply notched CD specimen, more fibers at the crack tips are orientated in CD direction and the more CD direction fibers means more load carrying fibers pass through the ligament. In this case, the stress around the notch tips would be able to extend into the remote horizontal zone more efficiently via fiber-to-fiber bonds on CD fibers. Additionally, CD copy paper has a larger FPZ size than that of MD and it indicated the stress concentration zone size of CD copy paper is larger. Thus the stress ligament ratio of CD is larger than that of MD at the same notch width ratio. More importantly, there are more fiber-to-fiber bonds on a CD fiber than those on a MD fiber to make CD fiber have higher load transfer efficiency and it will certainly generate a larger effective stress distribution zone around the notch tips in CD DENT specimen. Due to the larger stress distribution zone for CD DENT samples, the same amount of fibers at crack tips in CD specimen can transfer more load from notch tip zone to a remote area. Comparing the DENT specimen in CD direction to MD direction specimen, CD specimen has a larger FPZ zone and also can create larger effective stress distribution zone than MD specimen. For the reasons above, the CD ligament stress ratio in Figure 15 is larger than MD ligament ratio at the same a/w value.

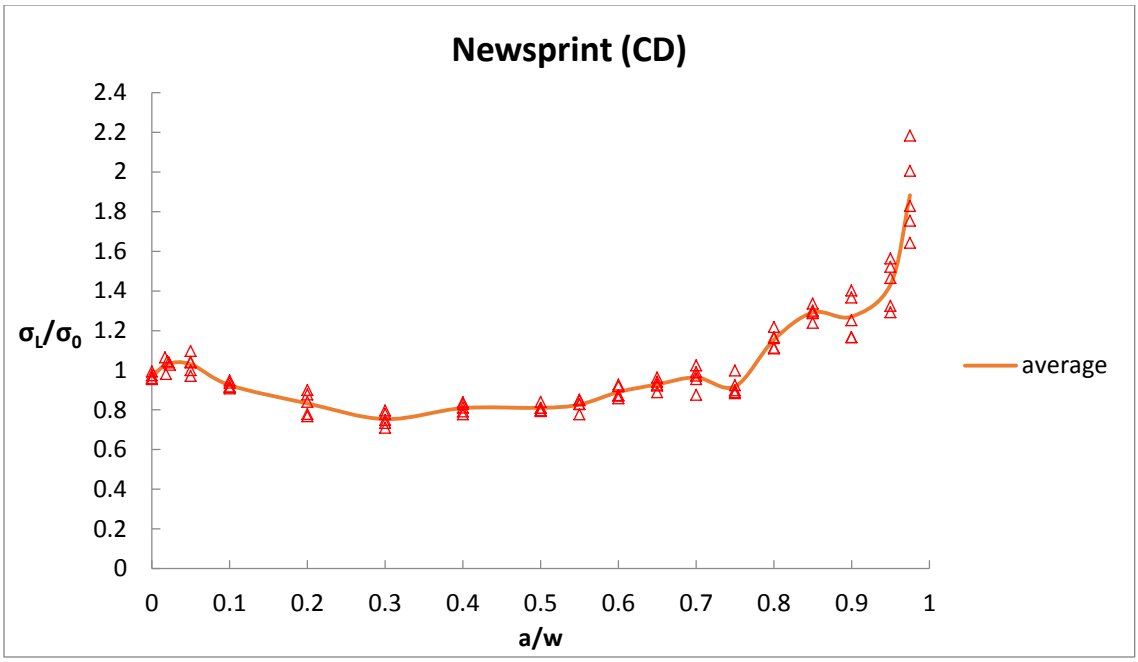

Figure 16 Average ligament stress normalized by tensile strength for CD newsprint paper. Red line represents the average value of the experimental data

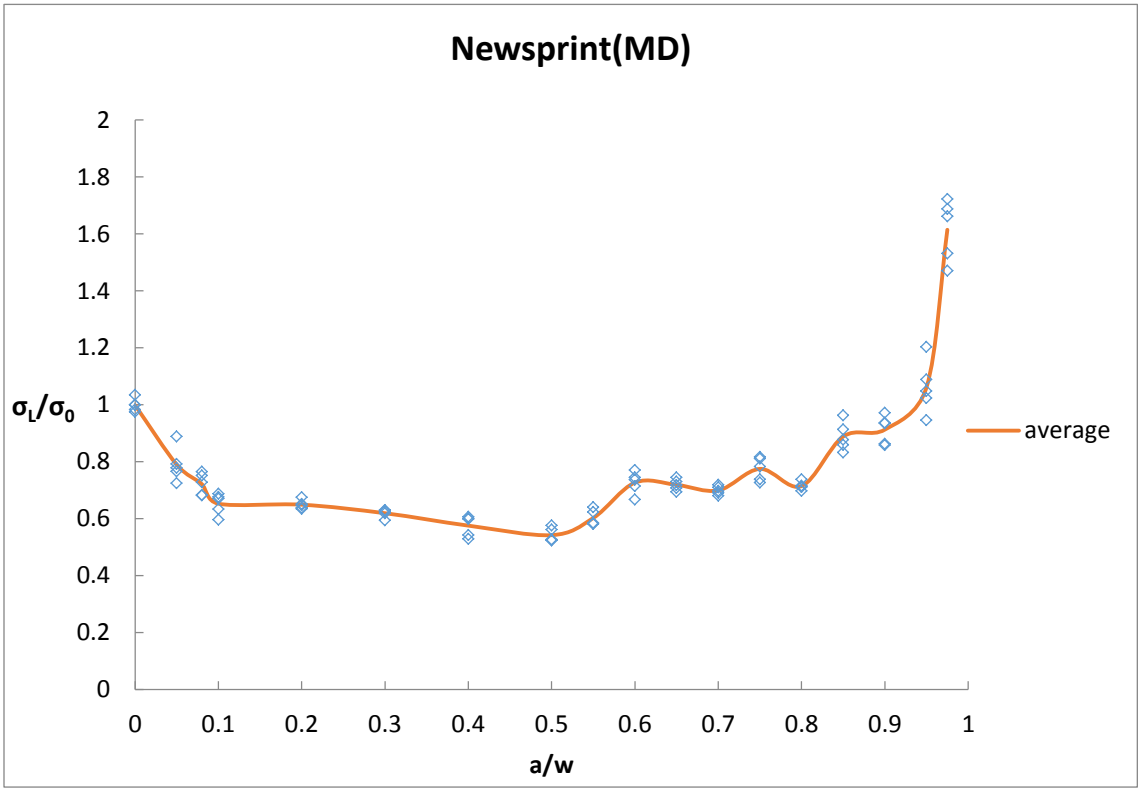

Figure 17 Average ligament stress normalized by tensile strength for MD newsprint paper. Red line represents the average value of the experimental data

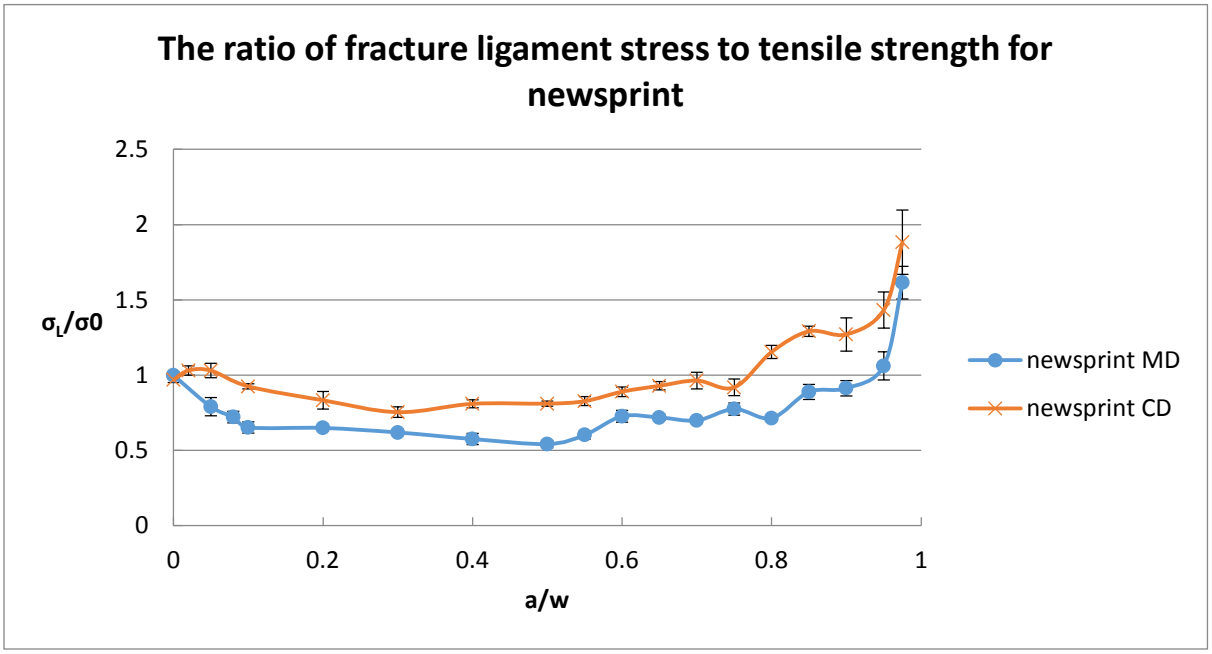

Figure 18 The ratio of fracture ligament stress to tensile strength for newsprint in both machine direction and cross machine direction

The results of newsprint in Figure 16 and 17 are similar to that of copy paper in Figure 13 and 12. Comparing newsprint paper to copy paper in each direction, respectively (i.e. ligament stress ratio in MD in Figure 23), MD newsprint shows a higher ligament stress ratio than MD copy paper. Although they have the same FPZ which suggests their stress concentration zone size would be the same, a smaller plasticity factor 0.69 of MD newsprint means it can disperse load easier and faster than copy paper. It can be attributed to more network structure dominating the FPZ of MD newsprint and it has a more appropriate fiber length for carrying load or form stronger fiber-to-fiber bonds than copy paper. The higher load dispersing efficiency of newsprint comparing to copy paper makes its DENT specimen less sensitive to notches and this less notch sensitive character of newsprint is due to the dominant network structure. The magnitudes of network feature of newsprint and copy paper can be evaluated by using data in Table 2, i.e. the network dominate factor $=1-d_m/d=0.04$ for MD copy paper and 0.31 for MD newsprint. Apparently, the network factor of MD newsprint is larger than that of MD copy paper and newsprint will exhibit less notch sensitivity than copy paper.

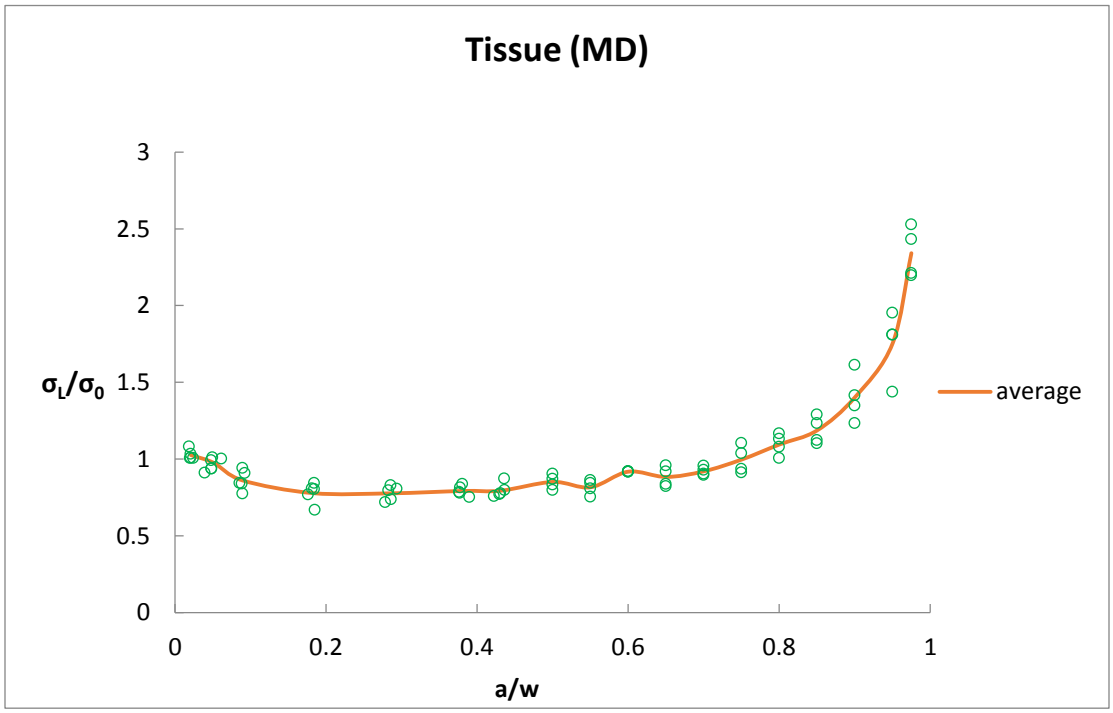

Figure 19 Average ligament stress normalized by tensile stress for MD tissue paper. Red line represents the average value of the experimental data and trend line is also plotted.

As shown in Figure 19, the slope decrease rate of average ligament stress ratio for tissue paper is the slowest as the notches start to grow and the stress ratio under deep notch is the highest (σ_L/σ_0 =2.5 at a/w=1) among all the testing materials. The highest ligament stress ratios indicate tissue paper have little sensitivity to cracks. According to what we mentioned above, this feature shows a high network structure contribution in fracture.

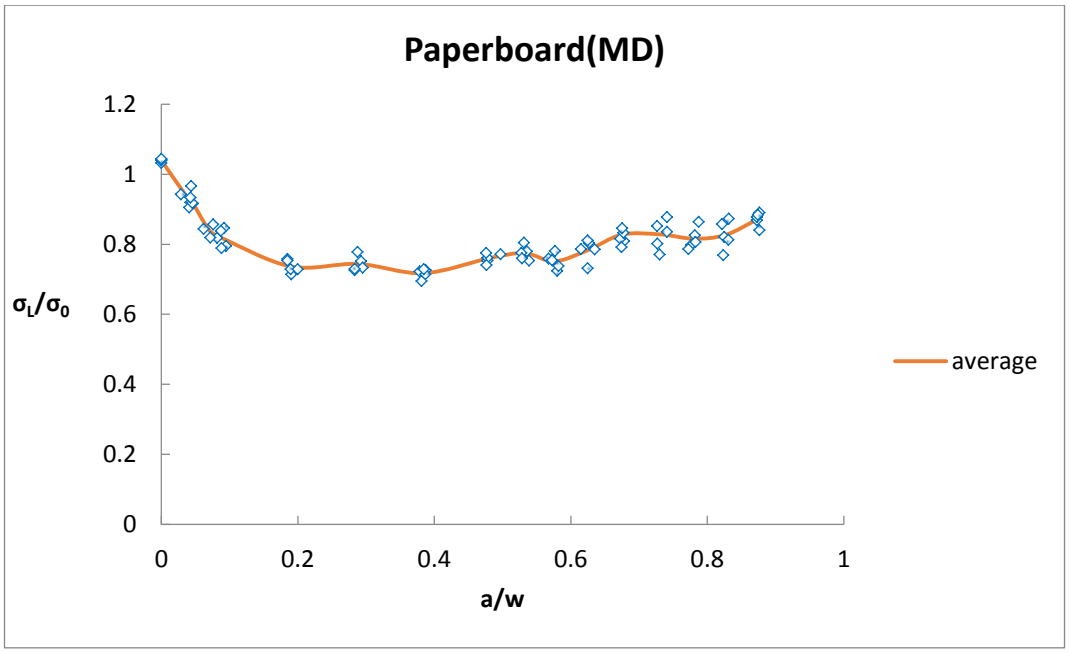

Figure 20 Average ligament stress normalized by tensile strength for MD paperboard. Red line represents the average value of the experimental data

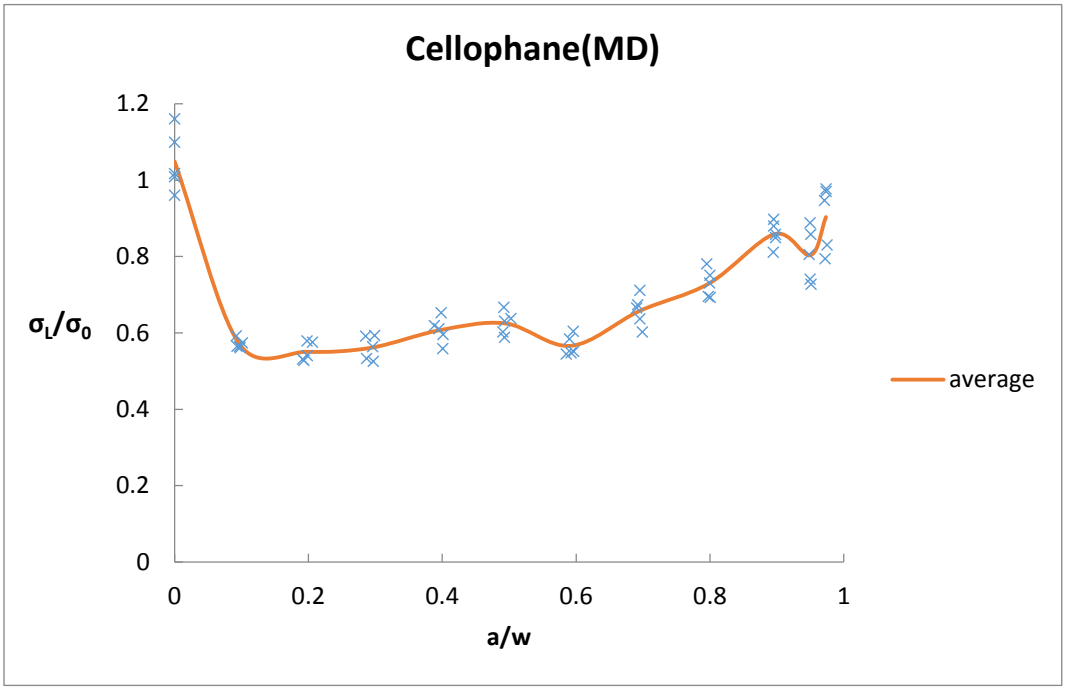

Figure 21 Average ligament stress normalized by tensile strength for MD regenerated cellophane. Red line represents the average value of the experimental data

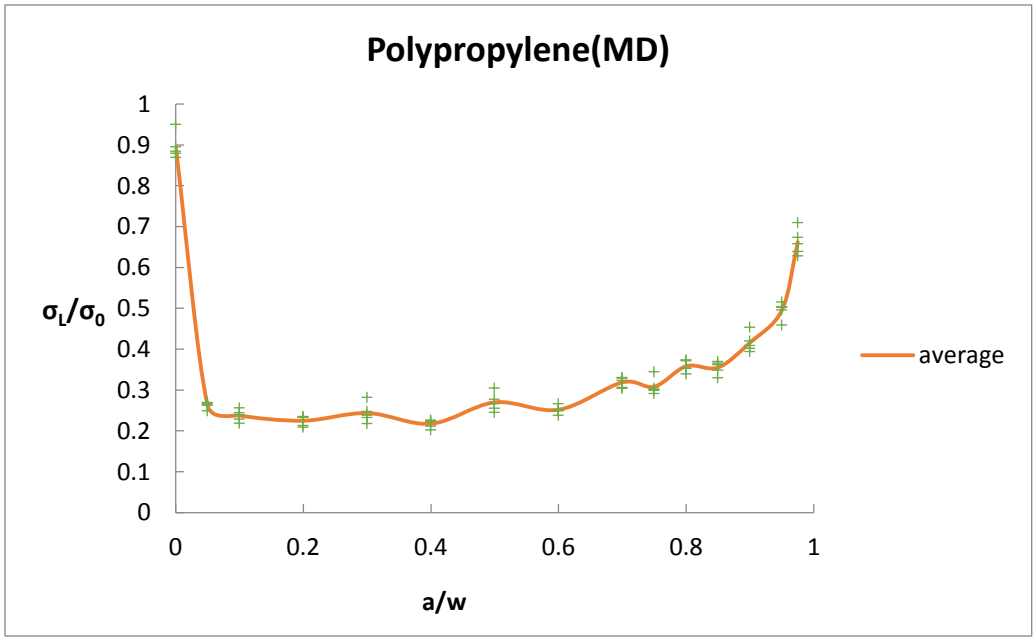

Figure 22 Average ligament stress normalized by tensile strength for MD polypropylene film. Red line represents the average value of the experimental data

Figure 20, 21, 22 show the ligament stress ratios of paperboard, regenerated cellophane and polypropylene film, respectively. In both figure 21 and figure 22, the notch sensitivity increases dramatically with small notches and the stress ratios are lower than 1 for the deep notches. The high sensitivity to cracks represents these two material behaviors are dominated by plasticity rather than the structure of sheet.

From Figure 13-22, the slope of the curve with small notch length is decided by the material's sensitivity to fracture. For those who have a relatively slow initial slope like tissue paper, it would indicate this kind of material is insensitive to crack. On the other hand, for those materials that have a steep slope with little crack size, it indicates that sheet has large notch sensitivity.

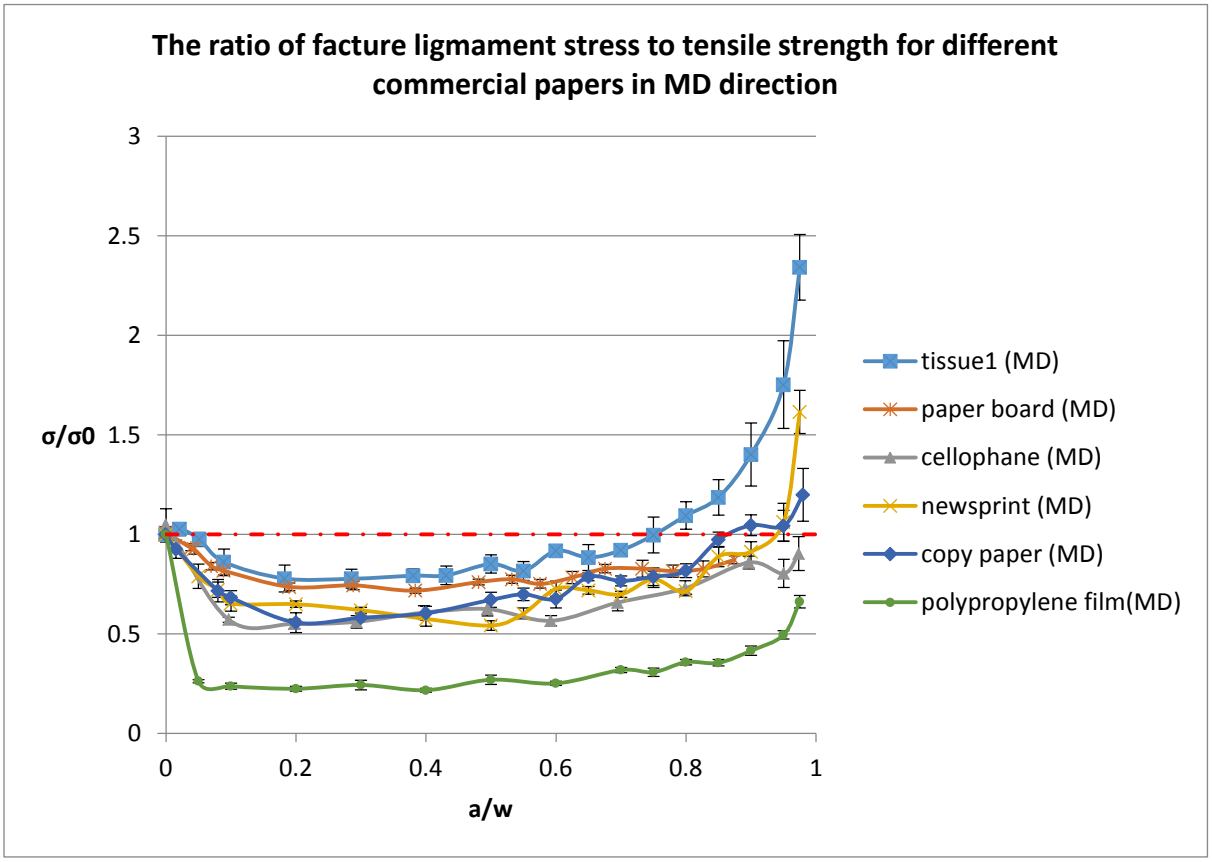

Figure 23 The ratios of ligament stress to tensile strength for different commercial papers in MD direction

Figure 23 gives us the comparison of the ligament stress to tensile strength ratio for some commercial papers in the MD direction. In Figure 23, first, it reveals that all the papers are

showing notch sensitivity at small notch to width ratios by decreasing tensile strength with the increased notch size. In addition, the tissue paper exhibits the smallest notch sensitivity compared to other papers by showing the slowest strength decrease at small notches. And the polypropylene film shows the largest notch sensitivity on the other side. What is more important, the notch sensitivities for all the papers tested showed a decreasing trend at large notch size and for some certain papers i.e. tissue, newsprint and copy paper, when the notch size increased large enough, the ligament stress can exceed the tensile strength and this tensile strength is limited by the edge defects. Last but not least, Figure 23 strongly indicated that, the different fracture resistances of these commercial papers are highly related to the material structural differences.

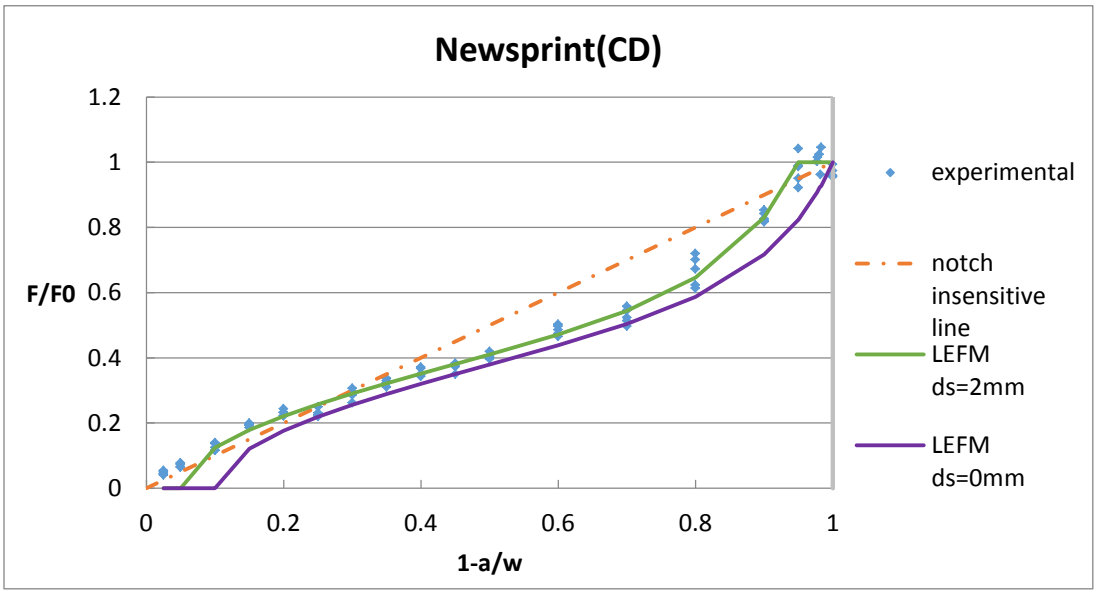

Figure 24 DENT load ratios versus relative ligament length for newsprint CD direction.

In order to investigate the effects of edge defects, the structural defects of the material was removed and compared with the original data. An example of newsprint CD is given in Figure 24. In Figure 24, the green curve from Equation (8) can provide a good prediction for the experimental data, especially for the deep notches. The green fitting curve showed that the deeply notched DENT specimen (1-a/w=0.15) has no notch sensitivity and the DENT specimen in this range is stronger than the specimen whose width equals to the ligament length. Interestingly, if the d_s is removed (purple curve in Figure 24), the curve in the same deep notches range goes below the Notch-Insensitive Line. This height change strongly indicated that after d_s

is removed, the deeply notched DENT specimen would exhibit nearly the same or weaker fracture toughness than the specimen with the width of the ligament length. This prediction reveals that the existence of structural defects ds is the reason for deeply notched DENT specimen exihibits no sensitivity. From this standpoint, it also explains our hypothesis why traditional tensile test cannot exhibit the true intrinsic tensile strength of paper materials. It can be concluded that the structrual defects at the edges will make the measured tensile strength lower than the true intrinsic tensile strength.

5.3 Estimation of the Intrinsic Tensile Strength

5.3.1 Estimation of intrinsic tensile strength by deeply notched DENT

For DENT specimens, when the ligament length is small, usually between d and 3d[1], the average ligament stress tends to be greater than the tensile strength of un-notched specimen for these network structure paper materials. This has been demonstrated from the data in the previous section 5.2. Figure 25-28 show the ratio of average ligament stress to tensile strength for CD and MD copy paper, CD and MD newsprint.

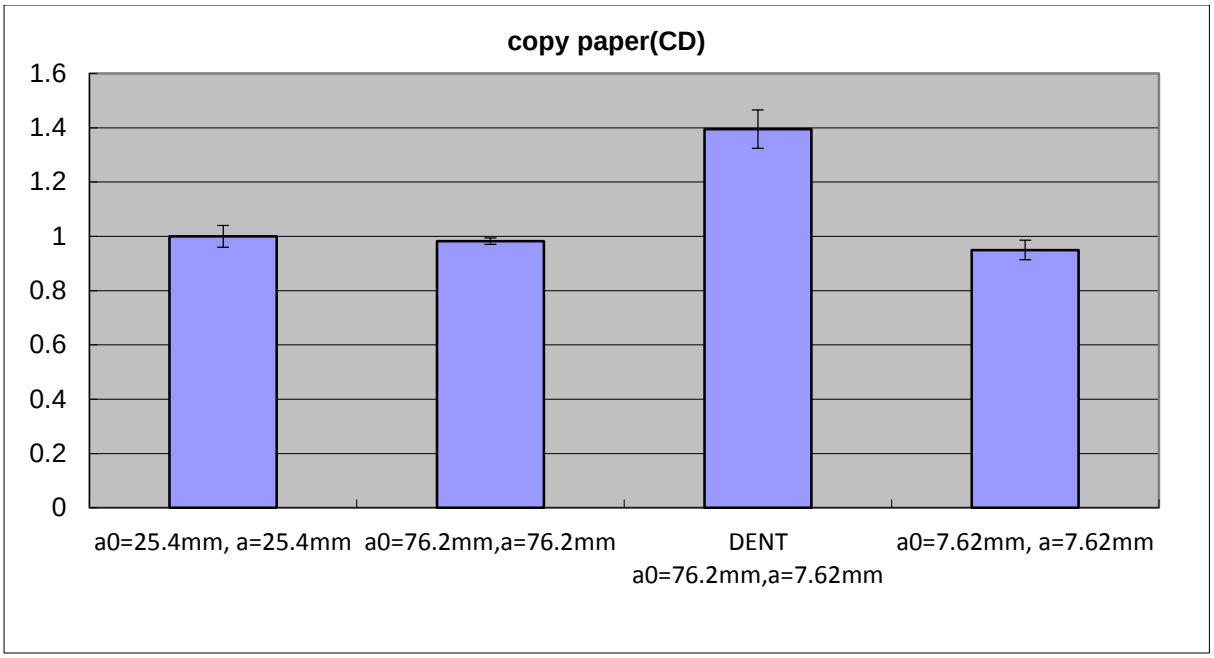

Figure 25 Comparison of average ligament stress versus tensile strength of deeply notched DENT specimen for CD copy paper

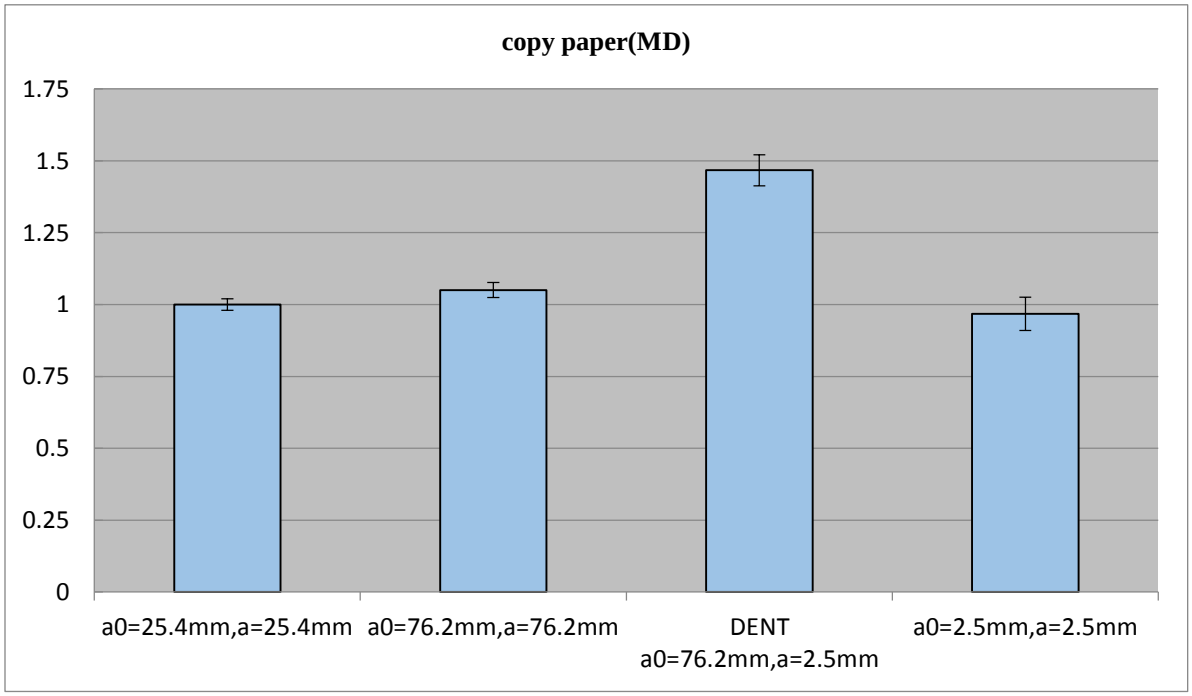

Figure 26 Comparison of average ligament stress versus tensile strength of deeply notched DENT specimen for MD copy paper

In Figure 25, three un-notched tensile samples are tested with the widths of 25.4mm, 76.2mm and 7.62mm. A deep-notched DENT specimen with ligament length of 7.62mm was also tested. The DENT specimen shows a strength which is 47% stronger than the tensile strength of 7.62mm specimen. That indicates that the intrinsic tensile strength of CD copy paper is higher than the tensile strength. The fracture of traditional tensile specimen is limited by the opening structures along the cutting edges. The cut at the edges of a sample would disable the load carrying and transferring ability of fibers along the edges. The situation is totally different for a DENT specimen. Comparing to tensile sample, the same amount of fibers crossing the same length of the ligament can create more paths to transfer load. In addition, for a small ligament length DENT specimen, the fracture process zones overlap each other, which may cause a lower stress concentration. This fracture tensile strength at deep notches is then less affected by stress concentrations and may give a better estimation of the material intrinsic tensile strength. Figure 26 shows for MD copy paper the ligament length of 2.5mm provided an average ligament stress which was 52% larger than the tensile strength.

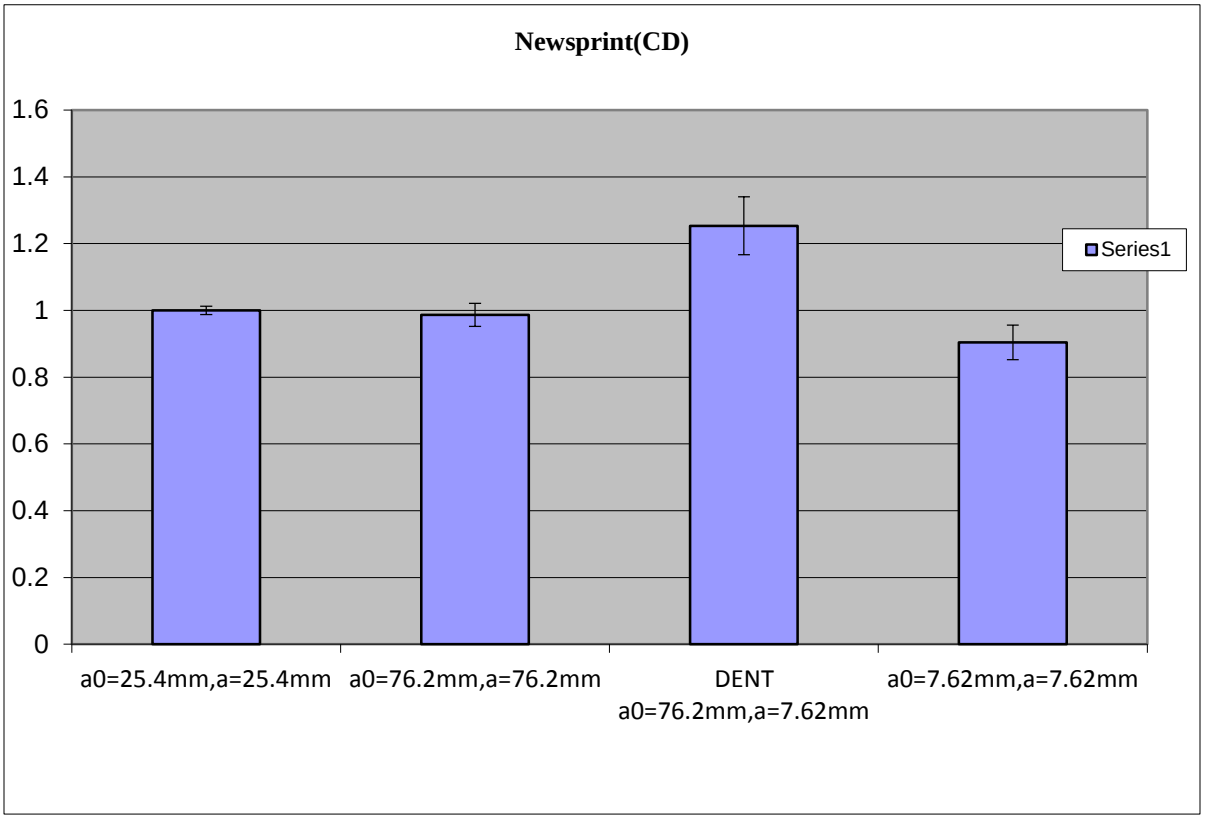

Figure 27 Comparison of average ligament stress versus tensile strength of deeply notched DENT specimen for CD newsprint paper

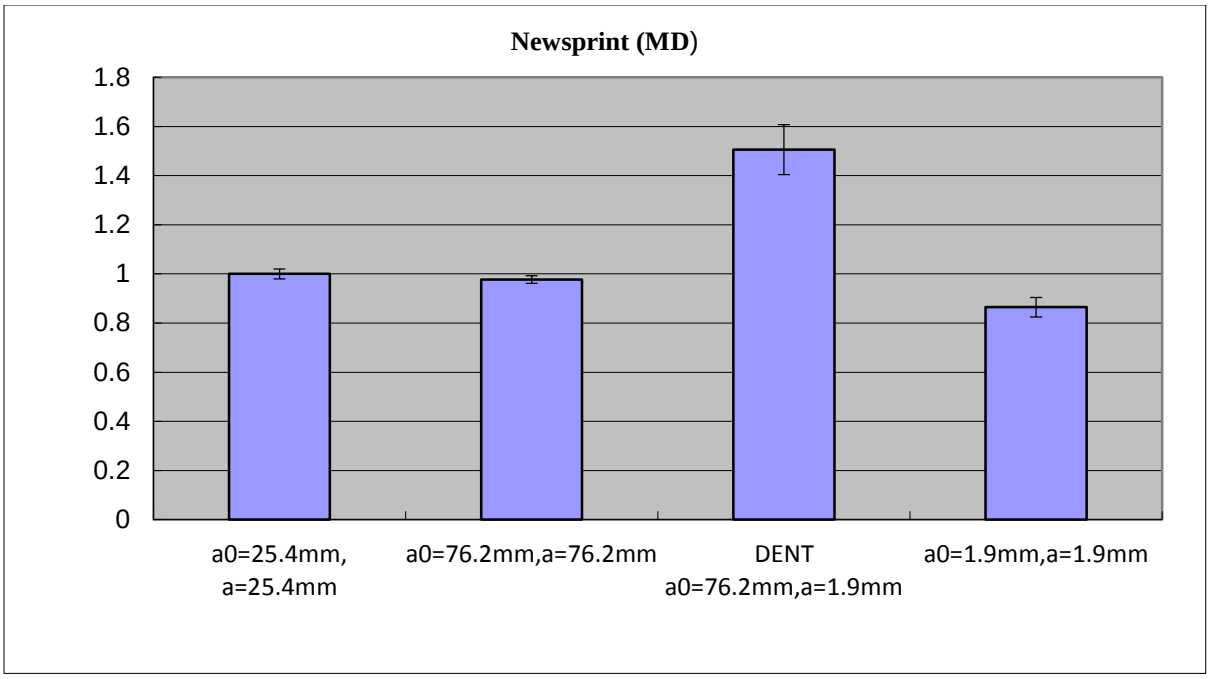

Figure 28 Comparison of average ligament stress versus tensile strength of deeply notched DENT specimen for MD newsprint

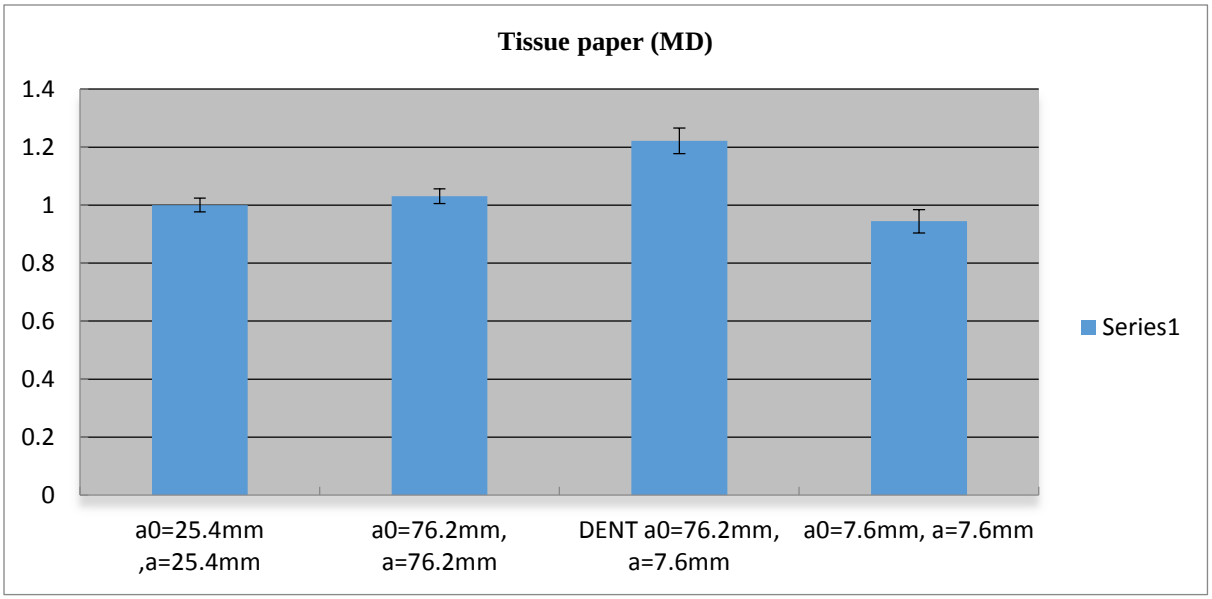

Figure 29 Comparison of average ligament stress versus tensile strength of deeply notched DENT specimen for MD tissue paper

As shown in Figure 27 and 28, similar to copy paper, newsprint with a ligament of 7.62mm gave an average ligament strength which was 1.39 the tensile strength in CD and newsprint coming with a 1.9mm ligament length provided ligament strength that equals to 1.76 the tensile strength

in MD. The four figures in this section suggest that the traditional un-notched specimen may break at the edges [1] at a lower load due to the opening of structure after the specimen was cut. The fibers in the ligament are more functional in transferring and carrying load resulting in deep-notched DENT specimens having greater strength than a regular tensile specimen. Therefore, the network structure character in a DENT specimen can be maintained to disperse the high stress at crack tips to a remote field. Similarly, the DENT specimen of MD tissue paper at ligament ratio of 0.1 exhibits 1.22 the tensile strength in Figure 29.

5.3.2 Estimation of the intrinsic tensile strength by modified LEFM equation

In order to evaluate the intrinsic tensile strength from our modified LEFM equation, the LEFM equation fitting curve of newsprint (CD) was plotted to illustrate the method due to newsprint is more brittle and therefore better fitting the modified LEFM equation comparing to other papers.

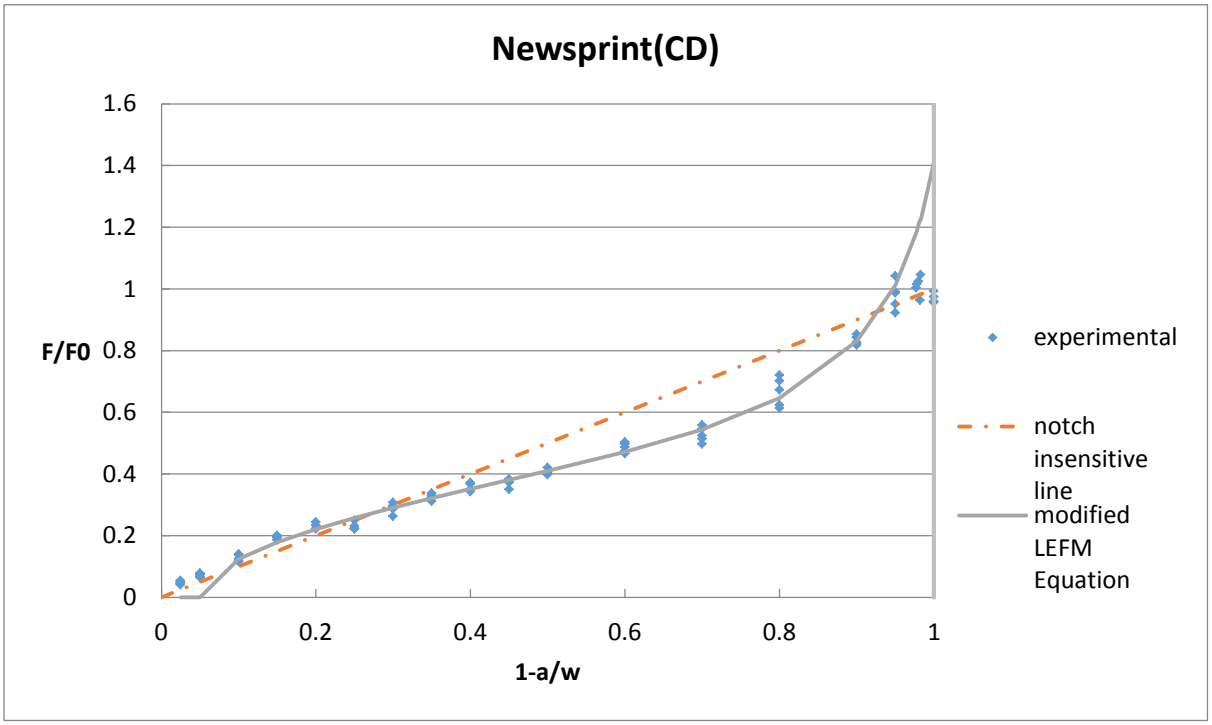

Figure 30 Fracture load ratios versus ligament to width ratio for DENT specimen of CD newsprint, ds=0mm.

As shown in Figure 30, similar to Figure 5, the modified LEFM Equation (8) was used to fitting the data of newsprint in CD direction. However, in order to investigate the intrinsic tensile strength here, the d_s was removed from the Equation (8) and the remove of structure component

makes the fitting curve convert to a certain value greater than one at zero notches. Therefore, this value at zero notches gives us the ratio of intrinsic tensile strength to experimental tensile strength with the same width at zero notches. For the reason that the structural defects at the edges are opened up when the tensile test strip was cut, the measurable tensile strength will be always lower than the intrinsic tensile strength (when d_s is removed) at zero notch length. In Figure 30, the intrinsic tensile strength was found to be 1.41 the tensile strength at zero notches.

Secondly, the intrinsic tensile strength at deeply notched DENT specimen was also evaluated from the modified LEFM equation in Figure 30. It clearly shows that the arch section of the fitting curve above the notch-insensitive line between the of ligament ratios of 0.1 and 0.3 exhibits the notch insensitivity character. In this case, the tensile strength measured during this section will not be related to the notch sensitivity but to the intrinsic tensile strength of the network. In this section with deep notches in Figure 29, the ratio of the arch height to the height of the notch-insensitive line at the same ligament ratio will indicate the ratio of intrinsic tensile strength to the measurable tensile strength whose width equals to the ligament length. Therefore, the intrinsic tensile strength can be also evaluated at deep notches and the result suggests that the intrinsic tensile strength can reach up to 1.24 the tensile strength at the ligament ratio of 0.1 in Figure 30.

Table 3 The ratios of intrinsic tensile strength to tensile strength for three different papers

	The ratio from Modified LEFM Equation(8) at zero notch	*The ratio from Modified LEFM Equation(8) at deep notches DENT*	*Experimental ratio*	d_m / d *from Table 2*
Newsprint(CD)	1.41	1.24	1.25	0.5
Tissue(MD)	1.27	1.12	1.22	0.6
Copy paper(CD)	1.06	0.93	1.40	0.9

The two ratios of intrinsic tensile strength to the tensile strength predicted from Equation (8) at deep and zero notches were compared with the experimental ratio for each paper in Table 3. As shown in Table 3, the true intrinsic tensile strength calculated from both zero notches and deep

DENT is close in magnitude. However, the intrinsic tensile strength calculated from zero notches is overestimated by 12.8% and the intrinsic tensile strength obtained from the deep notches DENT gives us the relatively accurate value comparing to the experimental value. At the same time, another two paper materials with different plasticity dominant factors are also investigated and the ratios are recorded in Table 3. In order to investigate the predictions for different commercial papers, more papers with different plasticity dominant factors were studied and the results with an increasing d_m/d were summarized in Table 4 below.

Table 4 The ratios of intrinsic tensile strength to tensile strength for commercial papers with the increasing dm/d value

	d_m/d from Table 2	*The intrinsic tensile strength ratio from Equation(8) at zero notch*	*The intrinsic tensile strength ratio from Equation(8) at deep notches DENT*	*Experimental intrinsic tensile strength ratio*
Newsprint (CD)	0.5	1.41	1.24	1.25
Tissue (MD)	0.6	1.27	1.12	1.22
Newsprint(MD)	0.69	1.41	1.24	1.50
Paperboard (MD)	0.83	1.09	–	–
Copy paper (CD)	0.90	1.06	0.93	1.40
Cellophane (MD)	0.92	1.04	–	–
Copy paper (MD)	0.96	1.02	0.89	1.08
Polypropylene film (MD)	1	1	–	–

In Table 4, since the special stiffness and thickness for paperboard, cellophane and polypropylene film, it is impossible to perform really deep cuts and keep the DENT specimen

from pre-damage before loading the sample on the tensile tester. Therefore, the experimental tensile strength ratio and the ratio from our Equation (8) at deeply notched DENT samples are not available for these specimens.

After comparing the results from the different papers in Table 3 and Table 4, it was found that our modified LEFM equation can provide a good prediction for the intrinsic tensile strength of the paper with small plasticity dominate factor. In addition, as the plasticity dominant factor increases, the intrinsic tensile strength ratio from Equation (8) at deep cuts decreases further from the experimental ratio because the greater the zone of plasticity, the greater the underestimation at deep notches predicted by Equation(8). This standpoint was observed and explained from Figure 4 to Figure 12. Last but not least, the results in Table 4 show that the intrinsic tensile strength ratio from the Equation (8) at zero notches can provide a better prediction of intrinsic strength than that from the Equation (8) at deep cuts. This is perhaps for deeper cuts, the structural defect effect at edges was gradually removed and the role of material plasticity increased. Therefore, the deeply notched DENT paper with large plasticity dominate factor has a larger plastic deformation zone area which can distribute more load before failure and this actual plastic deformation zone size is larger than that in our LEFM assumption which is that plastic deformation zone size should be small enough in comparison with crack length[24]. In this case, our LEFM Equation (8) provided a under prediction due to the underestimation of plastic deformation zone size to distribute the stress at tips for the papers with large plasticity dominant factors.

More importantly, according to the comparison between the intrinsic tensile strength ratios from the LEFM modified equation (8) and experimental results, the estimation method for intrinsic tensile strength ratio by deeply notched DENT is found to be a much simpler method which only requires two tensile tests comparing to the estimation method by zero notches which needs to plot through the whole ligament ratios from zero to one.

6 CONCLUSIONS

The modified linear elastic fracture mechanics (LEFM) model were used in this thesis research and the modified LEFM Equation (8) was successfully applied to characterize the fracture sensitivity and predict tensile strength for several commercial paper materials. In the LEFM model, an effective fracture process zone size d, a structural components d_s and a materials components d_m were introduced into the modified LEFM equation. It was found that the magnitude of FPZ size d can be applied to characterize the relative fracture sensitivity of paper; the larger value of d, the less sensitivity to fracture the paper possesses. In addition, different d and d_s values as indicated here characterize differences in the internal structural for different paper materials. The magnitude of d was found to be related to the sheet load concentration efficiency; the magnitude of d decreases as the load concentration efficiency increases. For this reason, although the polymer film (d=0.2mm) in this experiment had the largest fracture toughness which indicates the best load transfer efficiency, when a crack is introduced, the stress at the crack tip can be concentrated faster and fails at lower far field load.

In this thesis, a plasticity dominant factor (d_m/d) was introduced to evaluate the dominant character in fracture process zone. A smaller plasticity dominant factor close to zero suggests the network structural characteristic; on the other side, a greater plasticity dominant factor closed to one means the material plasticity characteristic in FPZ. A smaller magnitude of plasticity factor means a larger network factor, which indicates faster and higher load dispersion efficiency. The combination of plasticity dominant factor and the FPZ size can successfully explain the relation between the facture behavior and the stress distribution on a ligament. Additionally, it was found that whether the modified LEFM equation can give a better fit at the deep cuts highly depends on the relative magnitude of the plasticity dominant factor. As the plasticity dominant factor increases, the more plasticity characterized the fracture process zone and the more underestimation the LEFM equation (8) provided at deep cuts. This is due to the plasticity character dominates the FPZ zone as the notches goes deeper and when the notches go deep enough, the actual plasticity zone size exceeds the plasticity zone sized in our LEFM equation. More actual plastic zone size made the sample can tolerate more stress than the predicted one from LEFM model.

In order to take a further study of the DENT specimen under loading, the ratios of average ligament stress to tensile strength were calculated and the stress distribution across the ligament was investigated for all the tested commercial papers. The results suggests the ratio can successfully characterize the sensitivity of paper to fracture and interestingly at some level of deep notches, the fracture sensitivity of the material can be removed and the DENT sample showed a stronger intrinsic tensile strength (the stress ratio>1) which exhibited the material inherent property. What is more, the structural defects at the edges were demonstrated to be the main reason of the traditional tensile test fails to exhibit the true intrinsic tensile strength.

Therefore, further investigations of the deeply notched DENT were conducted with the purpose of obtaining the intrinsic tensile strength of the paper material. The intrinsic tensile strength obtained from the experimental deeply notched DENT specimen was compared to the theoretical intrinsic tensile strength calculated from modified LEFM equation (8) at zero notches and deep notches. The comparison result reveals that our LEFM equation (8) can provide a good prediction of the intrinsic tensile strength both at zero notches and deep notches for brittle newsprint paper which have a relatively small ratio of d_m/d. But for lager plasticity dominant factor papers, the calculative intrinsic strength of deeply notched DENT specimen under-predicts the experimental value due to the larger plasticity contribution than expected. However, the calculative intrinsic strength at zero notches showed relatively stable prediction ability through all range of plasticity dominant factors. All the comparisons of the intrinsic tensile strength pointed out a simple truth that the intrinsic tensile strength can be measured by a deeply notched DENT method and the simplicity of the application makes the modified LEFM method much more competitive than other methods.

Finally, it can be concluded that the structure and material components do play an important role in the fracture behavior of paper and the modified LEFM equation can be successfully used to characterize and predict the fracture behavior for variety of commercial papers. Furthermore, the intrinsic tensile strength of the network is much higher than the tensile strength and magnitude of the intrinsic tensile strength can be measured by the deeply notched DENT specimen.

Future work should be done to give a further investigation of d_m effects on deep notched DENT specimen since there is no convincible explanation of the underestimation of the LEFM equation at deep cuts in this paper. In addition, a highly sensitive infrared thermographic technique should be applied to observe the load concentration zone of DENT specimen of different papers and give a more clear correlation between the FPZ size d and stress concentration zone size. Last but not least, a simulation program should be developed to visualize the localized fracture activities under certain sheet fabrication parameters.

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8 APPENDIX A

UTILIZATION OF MODIFIED LINEAR ELASTIC FRACTURE MECHANICS TO CHARACTERIZE THE FRACTURE RESISTANCE OF PAPER

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ABSTRACT

Linear elastic fracture mechanics modified to account for an effective fracture process zone is sufficient to characterize and predict fracture resistance for a wide range of papers. The simplicity of the method, which only requires the tensile strength and a measure of the effective fracture process zone length, gives it great advantage over other existing approaches. The results presented here show that for a wide range of commercial papers, samples widths as narrow as 50 mm are sufficient to determine the effective process zone length, and that scaling holds well enough to allow prediction for fracture of wide webs. The results indicate that the tensile strength of paper is a result of a fracture process where the defect is most typically induced from cutting the network structure along the edges. As a consequence, the inherent tensile strength of the network can be significantly larger than the measured tensile strength. The effective fracture process zone length parameter is taken as a measure of the inability for the paper to concentrate load near the crack tip. This ability for network structures to concentrate load has significant impact on the fracture resistance of the sheet relative to its tensile strength.

1 INTRODUCTION

Imperfection limits the strength of paper, but this defect sensitivity is tempered by dissipative material and structural processes and features that limit the ability of the sheet to concentrate load in a small region. A substantial amount of literature has been devoted to understanding the fracture resistance of paper; see the reviews of Kortschot [1], Mäkelä [2], and Niskanen [3]. A recent account of the requirements, advantages, disadvantages, and applicability of Linear Elastic Fracture Mechanics [LEFM], nonlinear fracture mechanics, cohesive zone modeling and damage mechanics was provided by Östlund and Mäkelä [4]. It is clear that, LEFM in a strict sense cannot be applied to paper. Uesaka et al. [5] first showed that the J-integral method was better suited for characterization of fracture in paper compared to the stress-intensity factor. At the 10th FRS, Niskanen [3] concluded that LEFM cannot generally be applied to paper. The main fault being that crack sizes must be sufficiently large to render any plastic fracture process zone small and in turn unreasonably large sample sizes. Östlund et al. [6] point out that LEFM even with a plastic zone correction is not self-consistent, because a measure of rupture energy over predicts the stress intensity factor and a measure of stress intensity under predicts rupture energy. Despite this fact, there is still evidence in the literature [7- 9] that LEFM methods can be useful to characterize fracture resistance.

Andersson and Falk [10] used a Griffith-Irwin type fracture criteria to account for the undefined fracture process zone (FPZ) that precedes the well-defined crack [11]. They did not correct for the finite-width of their samples (15 mm), which is likely too small to fully capture the behavior [12] and it would seem that they under-predict the FPZ (0.6 mm for handsheets). They also conducted constant load fracture tests on larger width samples, and

the resulting FPZ seems to be approximately four times larger (on the order of 2.5 mm for handsheets.) Seth and Page [5] utilized LEFM to study the fracture behavior of paper and conclude that for LEFM to be applied to paper, the samples must be of sufficient sample width and crack length. Swinehart and Broek [7] showed that LEFM equations can predict fracture loads of large webs with large cracks with no modification for the FPZ.

Donner [6] continued along the same lines the previous work [10, 5] and separated the FPZ into a structural and material component. By conducting tensile tests on newsprint samples in conditions obtained from a cryogenic process, a very brittle and linear-elastic response was obtained. Fracture tests in the cryogenic process showed that an FPZ of about 0.5 mm for MD and 1.1 mm for CD. At room temperature, the zone FPZ was 1.5 mm for MD and 3.7 mm for CD.

Kortshot and Trakas [13] took a similar approach, point stress criteria (PSC), to describe the fracture resistance, utilizing both centered holes of various diameters and centered slits for newsprint, bond paper, and a copy paper. For holes they found the characteristic FPZ length to be in the range of 1.2 to 1.7 mm for MD and 3.3 to 3.5 mm for CD. For slits, the characteristic length in MD was smaller, 0.8 to 1.0 mm, which could have been affected by the expression they chose to use for the stress-distribution near the slit. Inspection of their results shows that the strength of a sheet with a 1 mm hole was not significantly different than the tensile strength indicating that the small hole did not affect the ability to the paper to effectively carry load; indicative of Donner's [6] structural FPZ. Considine et al [14] utilized the PSC and an average stress criteria (ASC) along with LEFM equations for the stresses near a hole for an orthotropic material. These authors do not report the FPZ, but do report inherent flaw sizes ranging from 0 to .88 mm for MD and 0 to 1.55 mm for CD for a range of papers.

The attractiveness of LEFM is the simplicity of its application; an argument proffered by Swinehart and Broek [7] for favoring stress intensity factor over the J-integral method. An abundance of explicit equations are available for LEFM, and if applicable one could apply them to characterize paper materials and predict other cases with relative ease. Implementing nonlinear fracture mechanics is complex; requiring a description of the constitutive behavior, a library of stored geometric correction factors, and numerical evaluation for each point of interest. To be useful the LEFM method should be capable of predicting the behavior of large samples from measurements made on small samples.

Using nonlinear fracture mechanics, Mäkelä, Nordhagen, and Gregersen [15] demonstrated that they could predict the behavior of wide samples (800-1000 mm) based on fracture toughness measurements of narrow samples (50 mm). Expanding on the approach of Swinehart and Broek [7] for a J-integral method, Mäkelä and Fellers [16] and Mäkelä [17] have presented procedures and explicit equations that can be used for prediction of fracture resistance. While it eliminates the need to complete a finite-element analysis, it still requires a library of correction values, and numerical inversion for each prediction.

Östlund and Mäkelä [4] state the following: "Many fracture mechanics models can be applied to paper materials and products depending on the problem and objectives of the analysis, but is it best to use the simplest possible model that has predictive capability." The simplicity of LEFM is too attractive to completely dismiss and despite its reported shortcomings it may still be valuable as a predictive tool that can be implemented with minimal testing and little computational difficulty. Although LEFM is stated to be adequate for large sheets with large cracks, it remains to explore its applicability for smaller cracks and predictions based on independent measures. That is one of the purposes of this contribution.

In the following, the LEFM equations are modified with the addition of an inherent FPZ and a normalization to the tensile strength. The previous literature is considered in light of this approach, and the predictive capability using the data from [15] is shown to be just as adequate as the nonlinear fracture mechanics approaches used in [15], [16], and [17]. By assuming that the tensile strength is governed by the same fracture process, the singularity for small cracks sizes is removed. The interpretation of the FPZ presented here is as a measure of the inability of the paper to concentrate load at the crack-tip. The larger the FPZ the less ability the structure has to concentrate load at the tip and the higher the fracture toughness relative to the tensile strength. Results are provided to demonstrate how the relative defect sensitivity of papers can be assessed. This provides a simple method that can be utilized to characterize the fracture toughness of materials and predict the behavior in large webs; at least with small cracks.

2 ANALYSIS

2.1 Modified LEFM

Because paper is a discrete network structure of fibers, it is inherently flawed. There is a scale level below which the assumptions of continuity are invalid. The discrete nature of paper is smoothed because it is stochastic and continuum models can be applied to great success as long at the dimensions of interest are relatively large. With regards to fracture, previous fracture studies [8, 14] indicate that this inherent flaw in on the order of a few millimeters. In an ideal elastic continuum, stresses can be singular, but in real materials they are limited. For a brittle response, we are limited by this minimum structural scale, and if the failure mechanism is a fracture process, we can cut-off the singularity by a normalization of the fracture loads.

Most papers exhibit sensitivity to cracks or notches, but tend to be relatively tough materials. Notch sensitivity can typically be attributed to a concentration of stresses near the tip of the notch. There is a zone around the notch where the material has yielded and/or undergoes partial failure. At some level of loading the material fails globally, typically starting near the notch tip. The maximum load could correspond to the point when the notch length begins to increase or shortly after that event. Inside the zone of influence near the crack tip, a multitude of mechanisms could be occurring to diminish the stress concentrations. Plasticity will limit magnification of stresses. Cohesive failure of the structure will allow reduction of stress levels. The inherent structural inhomogeneity will create some scale level below which stresses cannot concentrate. The literature includes successful application of theories that account for one of these aspects while ignoring others; for example see [15] for plasticity and [18] for material heterogeneity. In these models, some parameter is utilized to account for the effective behavior of the material regardless of the actual contribution from various different effects.

In-plane, fracture tests geometries are typically conducted as either a center-notched test (CNT), a double edge-notched test (DENT), or a single edge-notched test (SENT) with specimens as shown in Figure 1. The geometries are defined for each test such that the ligament length is 1-a/w. For the cases considered here the notch or crack, a, is considered a slit, with the tip as sharp as the limiting discrete size scale of the structure allows. The sample is loaded in tension with a load F. Force equilibrium requires that the net force on the remaining ligament must still equal F, but if stresses are higher at the notch tips the failure load reached in fracture will be reduced more so than the reduction in ligament length.

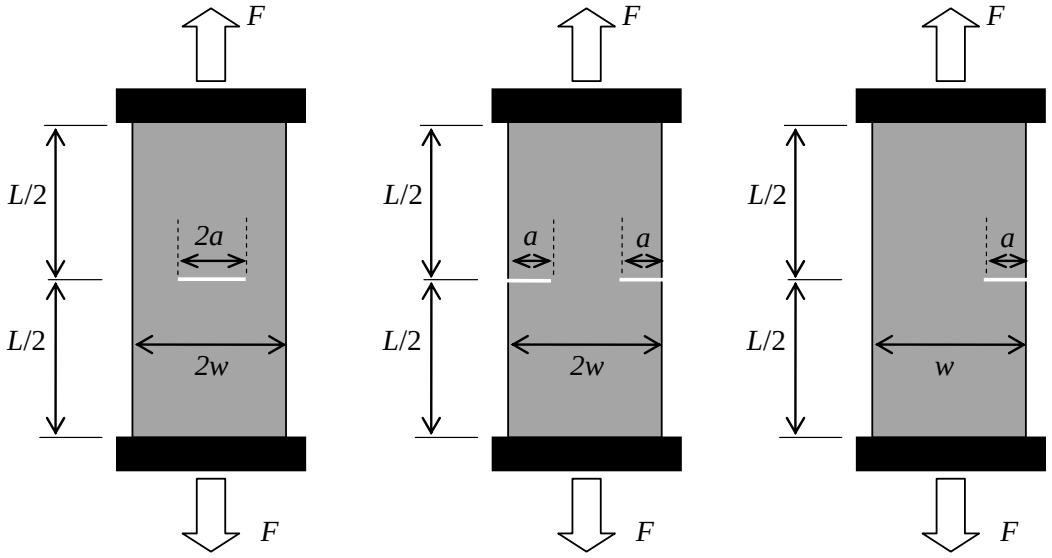


Figure 1. Typical geometries for in-plane fracture tests with paper.

Notch sensitivity can be assessed by comparing the ratio of ultimate load of the specimen, F, to that of the un-notched specimen, F_0. The criteria is

$$\frac{F}{F_0} < 1 - \frac{a}{w} \text{ implies specimen is notch sensitive.} \tag{1}$$

An equality sign in Equation (1) would imply no sensitivity to the notch, and a greater than inequality would imply the notched specimen is effectively stronger than a specimen whose width equals the ligament length. For a material, whose strength is determined by defects one would expect the load ratio to exceed one as the ligament length approaches 2*FPZ because if stresses are elevated at a crack tip, the inherent strength must be greater than the bulk tensile strength.

Consider a linear elastic material. Following classic LEFM, the stress intensity factor can be expressed as [19]

$$K_I = \sigma\sqrt{\pi a}\, f(a/w) \tag{2}$$

where σ is the far field stress and *f(a/w)* is a correction factor for finite width samples. In Equation (2), the length is assumed to be sufficiently long as to not influence the correction factor. Expressions for the correction factors for the three geometries given in Figure 1 are [19]:

for CNT: $f(x) = \sqrt{\sec(\pi x/2)} \;\; \forall\; x < 0.7$

for DENT: $f(x) = \left[1 + .122\cos^4(\pi x/2)\right]\sqrt{\frac{2\tan(\pi x/2)}{\pi x}} \;\; \forall\; x \le 1$ (3)

$$\text{for SENT:}\quad f(x) = 1.122 - 0.231x + 10.550x^2 - 21.70x^3 + 30.382^4 \;\; \forall\, x \le 0.6$$

Note the ratio of center to edge notched correction factor for x going to zero is 1.122. If we assume that failure occurs when the stresses at the tip reach some failure level, it implies that the stress intensity factor, K_I, is constant for all crack sizes and Equation (2) can be inverted for a>0. Now we assume that paper has an inherent characteristic fracture process zone length, d, such that the un-notched limit load can be obtained from Equation (1) as

$$F_0 = \frac{2K_I wt}{f(d/w)\sqrt{\pi d}} \tag{4}$$

where t is the thickness. Then the limit load ratio for at any notch $a>d$, the load limit ratio can be written as

$$\frac{F}{F_0} = \frac{f(d/w)}{f((a+d)/w)}\sqrt{\frac{d}{(a+d)}} \;\; \forall \;\; a < w - 2d \;\; \text{or limits from Equation (3).} \tag{5}$$

The length d is assumed to be composed of both a structural component and a material component $d = d_s + d_m$. The structural component can be considered as resulting from the fiber network structure and related to the discrete nature of the paper. Consider d_s resulting from a "flaw". In the presence of these flaws, edge failure would be more likely in a tensile test because load transfer structure is open at the edges. In addition, comparison of equation (3) shows that edge-notched specimens fail at a lower load then center-notched samples for small notches. In addition, both edges would have these flaws so the tensile strength should be similar to the DENT geometry except with small flaws.

For cracks $a<d_s$, the fracture load should remain equal to F_0. Thus equation (5) can be written as

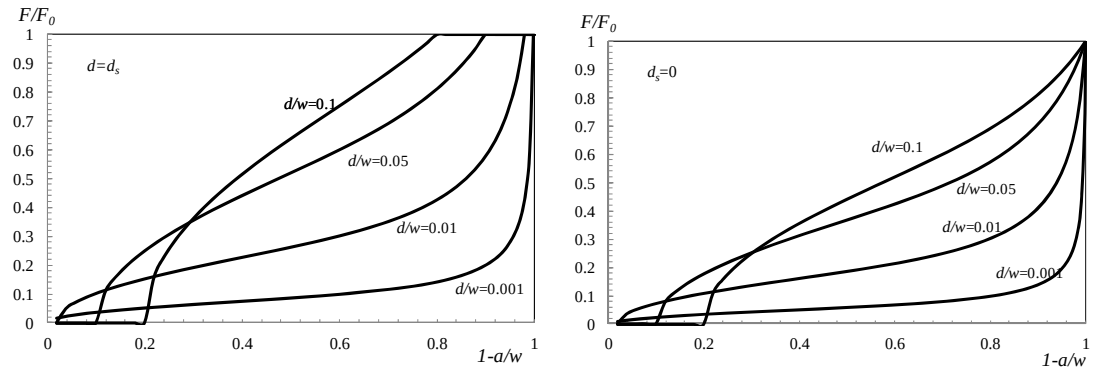
$$\frac{F}{F_0} = \begin{cases} 1 & \forall a \le d_s \\ \dfrac{f(d/w)}{f(a+d-d_s)/w)}\sqrt{\dfrac{d}{(a+d-d_s)}} & \forall d_s < a < w - 2d \end{cases} \tag{6}$$

The load ratio is equivalent to the average far field stress ratio and thus equation (6) provides a prediction for the fracture resistance of a paper. For edge cracked samples, the far-field fracture stress, σ_f can be predicted from the tensile strength, TS, the characteristic fracture process zone length, d and the structural limit, d_s. as

$$\sigma_f = TS\begin{cases} 1 & \forall a \le d_s \\ \dfrac{f(d/w)}{f(a+d-d_s)/w)}\sqrt{\dfrac{d}{(a+d-d_s)}} & \forall d_s < a < w - 2d \end{cases} \tag{7}$$

If center-notched specimens are used, the limiting ratio of failure stress to tensile strength at zero-notch length would be 1.122 because the zero-notch specimen is more likely to fail at an edge rather than the center.

Figure 2 illustrates the effect of d on the load ratio using equation (6). Figure 2a is for the case where the characteristic fracture zone length is $d=d_s$, and Figure 2b illustrates the case where $d_s=0$. Any combination between the two sets of curves can be obtained by adjusting the proportion of d_s to d.



(a) (b)

Figure 2. Fracture load ratio as a function of ligament length for various characteristic fracture lengths for DENT. (a) All structural $d=d_s$, (b) all material $d_s=0$.

By re-casting the LEFM equation as shown in Equation (7), two important features missing from classic LEFM are gained. First, instead of relying on a measure of released and consumed energy as a measure of fracture toughness, Equation (7) relies on the tensile strength of the sample for magnification and a determination of FPZ=*d* for the relative sensitivity of the material to defects. Second, the stress singularity at the crack tip is removed or rather irrelevant. The assumption of a nonzero *d* and constant stress intensity factor ensures that the predicted load will converge to the tensile strength as the crack length goes to zero.

Equation (6) or (7) is a modified LEFM model that can be used to characterize and predict the fracture sensitivity of paper. The FPZ parameter *d* provides a measure of fracture sensitivity relative to the strength of the material. For sample widths sufficiently larger than 2*d*, f(d/w) is approximately f(0) and the stress intensity factor can be defined as

$$K_I = 1.122TS\sqrt{\pi d} \tag{8}$$

The corresponding elastic energy release rate for an isotropic material is

$$R = \frac{K_I^{\,2}}{E} = 1.259\pi\frac{TS^2 d}{E} \tag{9}$$

For an orthotropic material, the effective modulus can be taken as $E = E_{11}^{\,3/4}E_{22}^{\,1/4}$, where E_{11} is the elastic modulus in the load direction and E_{22} is the elastic modulus in the direction perpendicular to the loading [6].

2.2 Fracture of a Flawed Elastic Lattice

To demonstrate the effect the discrete structure on the fracture sensitivity, a lattice model was developed using MATLAB. The model is shown in Figure 3. The elements were assumed to be linear springs, but large deformations were accounted for with a quasi-static time step updating the length and orientation of the elements with each incremental loading. The lattice is composed of nodes arranged in a square array with the characteristic length *c*. The springs are arranged to be horizontal, vertical, and diagonal. The diagonal elements are not connected where they cross. The stiffnesses are chosen to give an initial isotropic response. Along the

top edge the nodes are displaced with a uniform vertical displacement and free to move in the horizontal direction.

Along the bottom horizontal axis of symmetry, the last node is not connected to the line of symmetry. This is the initial flaw and renders the model a DENT. Additional crack lengths of length *a* are given by releasing the nodes to the right of *a* from being held to the line of symmetry. The vertical displacement along the top is incrementally increased until the reaction force at the crack tip (node furthest to the right being held to the horizontal line of symmetry) reaches a specified value. The model can easily be adjusted to be orthotropic, allow for plasticity of the elements or a cohesive release of the nodes fixed to the horizontal line of symmetry.

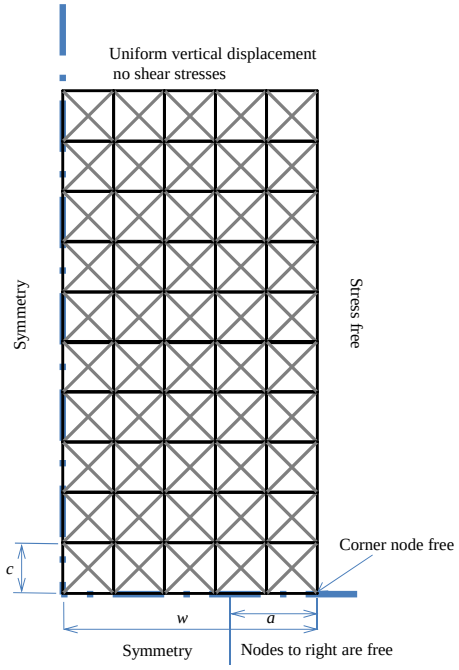
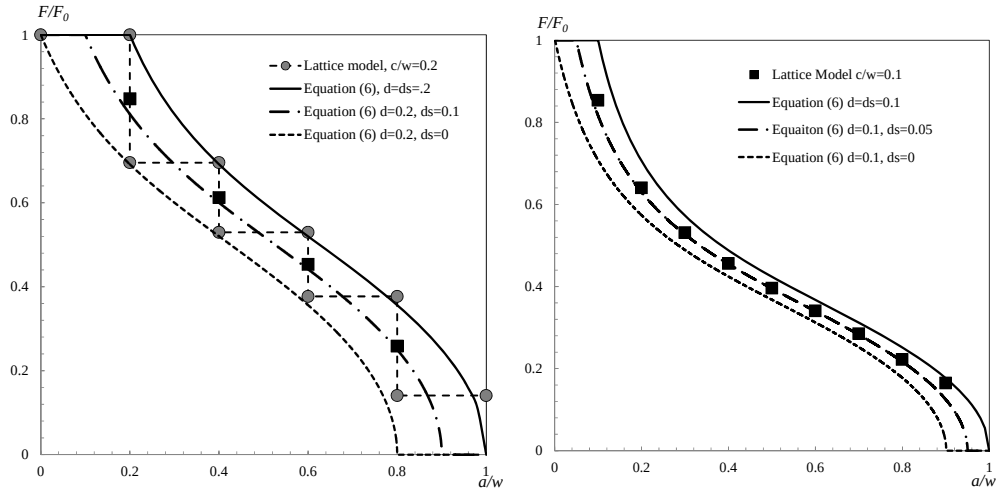


Figure 3. Flawed Lattice model (1/4 of DENT specimen)

The initial state is taken with just the right corner node released. The applied force is obtained by summing the reaction forces, which are vertical, along the top edge. Then the model is re-run, but the next node to the left is released, effectively doubling the crack size. The load ratio is then determined for each crack size. If one imagines the crack size to be continuous, then the results of load ratio versus crack length will give a stair-step function. The load remains constant for all crack lengths between two nodes, and has a step discontinuity at a crack length corresponding to a nodal location.

Figure 4 provides two results to show the behavior of the model. Figure 4a corresponds to a lattice with 5 unit cells in the half-width and 25 unit cells in the half-length so that the characteristic length ratio is c/w=0.2. Figure 4(b) corresponds to a model with a characteristic length ratio of c/w=0.1 by using 10 unit cells in the width direction and 50 in the half-length direction. In Figure 4a, the stair step response of the lattice model is shown. The circle markers represent the model result. The square markers represent the average of the two values at a node. The curves are representative of Equation (6) with three proportions of d_s to

d. The upper solid curve represents d_s=0, the bottom dashed curve represents d_s=*c*, and the middle dash-dot curve represents d_s=c/2. For all three lines, the characteristic FPZ is *d*=*c*.



(a) (b)

Figure 4. Load ratio versus crack length for flawed lattice model. Black squares represent the average load ratio that occurs before and after the release of a node at a crack length of *a*. (a) is for a lattice with *w*=5*c* and L=50*c*, and (b) is for *w*=10*c* and *L*=100*c*.

Figure 4b shows only the average points from the lattice model, but the three corresponding curves from Equation (6) for d_s=*c*, d_s=*c/2*, d_s=*c* are given. As the ratio *c/w* decreases, the three curves will converge and the sensitivity to cracking will increase. Equation (6) with $d=2d_s=c$ provides an excellent fit of the numerical results. A comparison of the two curves is given in Figure 5.

The length of *c* relative to the width determines the ability of the structure to concentrate load. Figure 6 compares the stress distribution for the case when the crack length is *a/w*=0.4 for the lattice models having *c/w*=0.1 and *c/w*=0.2. With the smaller lattice structure, the stresses can concentrate closer to the tip. This is why the smaller lattice exhibits more sensitivity to cracks.

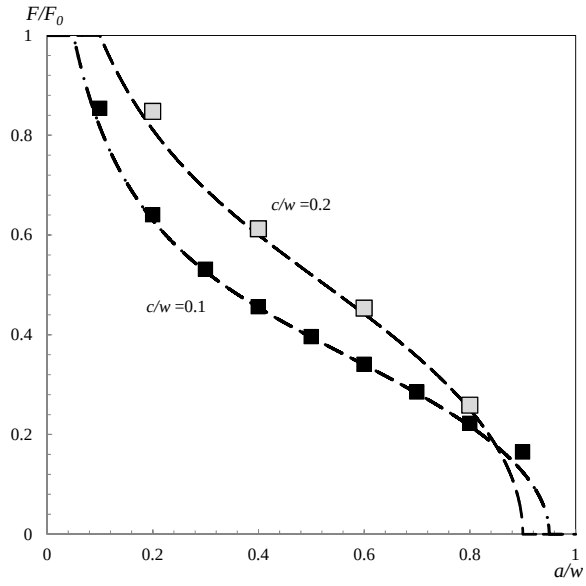


Figure 5. Effect of characteristic lattice structure size on sensitivity to fracture.

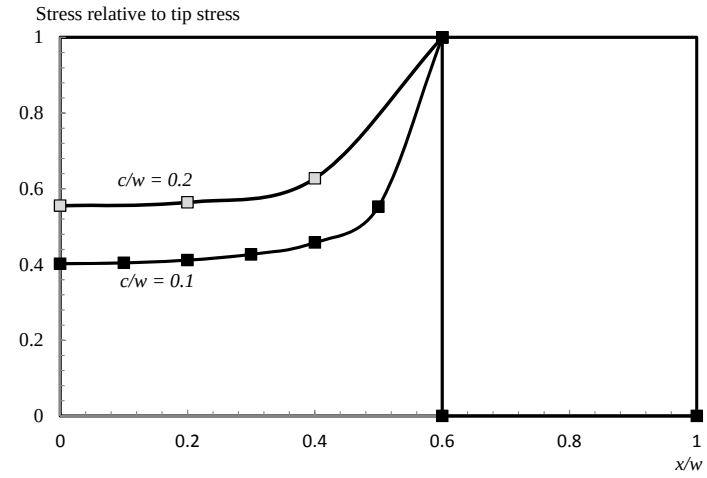


Figure 6. Stress distribution along ligament for two characteristic lattice sizes.

Thus, for an elastic material the parameter *d* represents a measure of the inability of the structure to concentrate load. Plasticity would further limit stress concentrations and increase the effective FPZ, *d*.

3 RELEVANCE TO THE LITERATURE

It remains to be seen if Equations (6) or equivalently (7) is useful to characterize the fracture sensitivity of a paper and is capable of predicting the fracture of large webs based on values obtained from small scale testing. It is worth re-examining the data in the literature to evaluate Equation (6) or (7). For most of these comparisons, the term d_s is set to zero because not enough information is available to distinguish it from d. This means that the data from the literature is typically fit with two parameters, tensile strength (*TS*), and the effective FPZ (*d).*

Östlund et al. [6] determine that LEFM could not be used to predict fracture. To make their argument, they used DENT specimens of a copy paper with 2, 4, and 6 mm notches and a sample width of either 100 or 50 mm. They report both the critical fracture stresses and the tensile strength of the samples. This fracture load to tensile strength ratio data is shown in Figure 7 along with a fit of Equation (7) where the parameter d is the fitting parameter. For MD was found to be d=2mm and for CD d=5 mm. Östlund et al. [6] calculated stress intensity factors from two methods. Directly from the fracture strength and from the fracture energy determined from a short span tensile test. They then determined values of d to minimize the error in the fracture stress calculation. A comparison of the current fit to that found in [6] is given in Table 2.

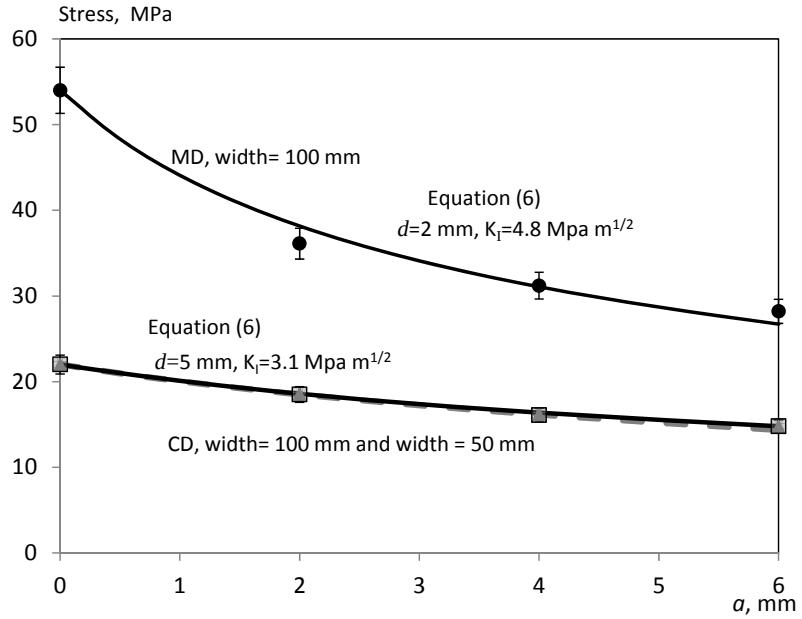


Figure 7. DENT fracture tests of a copy paper [6] compared to Equation (6).

Table 1. Comparison of parameters from Equation (7) and reference [4].

	d, mm		K_I, MPam$^{1/2}$			R, kJ/m^2	
	Eq. (7)	Ref. [6]	Eq (8)	Ref. [6] stress	Ref. [6] energy	Eq (9)	Ref. [6]
MD	2	8.0	4.8	3.2-4.4	7.4	4.3	10.2
CD	5	11.7	3.1	1.6 -2.3	4.7	2.8	6.5

Figure 7 shows that Equation (7) provides reasonable fit for the fracture strength. The fact that the same d fits for both the 50 and 100 mm lengths in CD suggests Equation (7) can be used when scaling to larger samples. In other words, for these crack lengths the shape factor is approximately constant as sample width increases beyond 50 mm. The fact that the tensile strength (zero notch) fits with the other fracture data suggests that one does not need a separate measure of fracture energy. The stress intensity factors determined from the short-span tensile test are large. The values of K_I= 4.8 and 3.1 MPam$^{1/}$2 from the current analysis are very similar to the results given by Swinehart and Broek [7] for papers with similar tensile strength in MD and CD. Clearly, using the fracture energy calculated from a short-span tensile test causes an over prediction of fracture resistance from LEFM methods for short cracks. The equivalent elastic fracture energy determined from Equation (9) is about 40% of that reported in [4]. There is no reason to expect the LEFM methods which has an effective FPZ to match the actual energy release which consumes energy to drive the plastic front. One would expect it to be less than such a measured value. Also, the short span measurement would be valid for deep notches and this could require higher energy than that required to propagate a small crack. This fit shown in Figure 7 does not require an input of fracture energy to be predictive. It requires the tensile strength and at least one fracture test to determine the value of d. The results shown in Figure (7) support the assumption that the tensile strength of an un-notched sample is also a result of fracture and that this measure gives us the necessary magnification factors to scale the load ratio factor. Thus, Equations (7) and (8) have validity. Donner [8] also found that the tensile strength was aligned with the fracture data.

Seth and Page [7] attributed the low energy calculated in the work of Anderson and Falk [10] to the small sample width. The one example of fracture load versus notch depth given by Anderson and Falk [10] was in their Figure 4, which is re-plotted in Figure (8). Anderson and Falk [10] plotted the stress squared as a function of 1/a, which should give a line. Anderson and Falk did not correct for the finite width of their samples. The circle markers in Figure 8 represent corrected values based on Equation (6). Figure 9 shows the stress versus notch length as well as the average ligament stress versus notch data. The stress is normalized to the fracture stress for the smallest notch. Figure 9 reveals that the average ligament stress for the two largest notches exceeds the average stress of the smallest notch. Because of the narrow samples used by Anderson and Falk [10] one might expect that the two deepest notches give results that are not indicative of the fracture resistance, but yielding across the entire ligament length would allow a higher load to be reached before failure. Anderson and Falk [10] reported a d=2.5 mm for handsheets tested with larger width. If this value is used in Equation (6), the two deepest notches are excluded, and the slope is adjusted to pass though (0,0), the stress intensity factor doubles (see Figure 8). This would then give a fracture energy at about 40% of that reported by Seth and Page [7]. This is similar to the % differences reported by Östlund et al. [4].

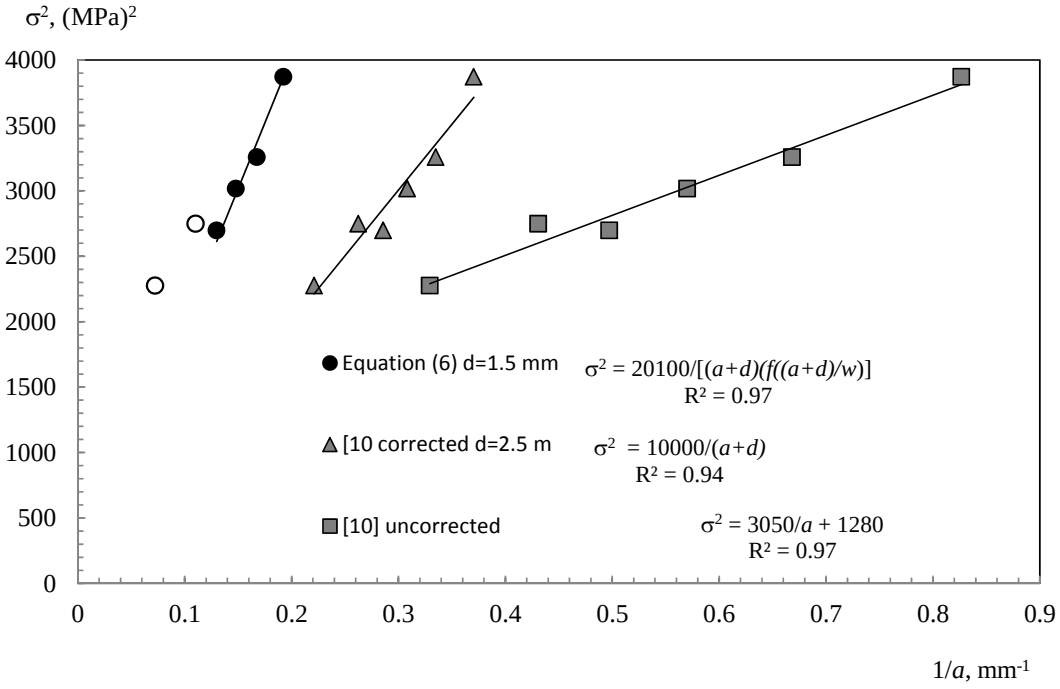


Figure 8. Fracture results from Fig 4 of Ref. [10] and Equation (6) with d=2.5 mm.

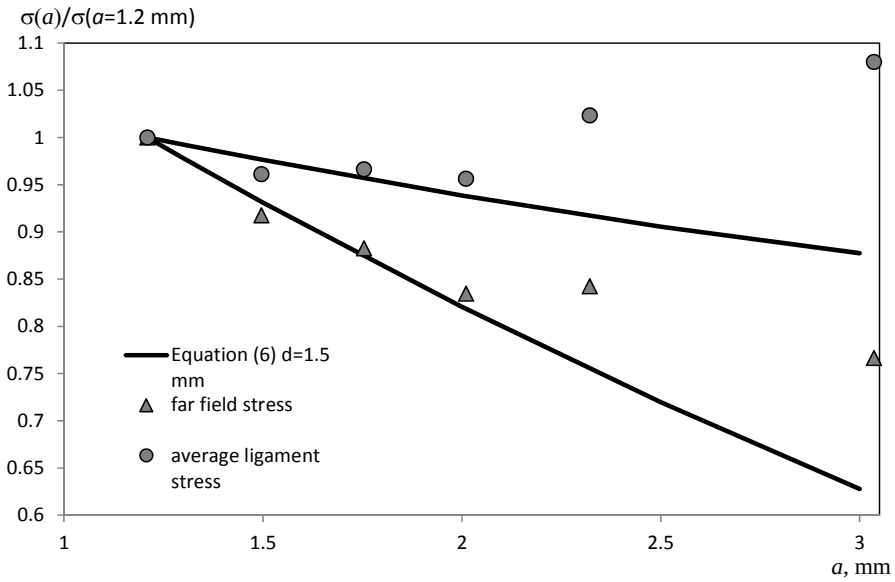


Figure 9. Relative far field stress level and average ligament stress for data from [10].

Seth and Page [7] reported a match in energy release rates from fracture tests and short-span notched tensile tests for large width DENT, having an aspect ratio *a/w*=0.4. In the short webs, they had a length to width ratio of three, but the ratio was one for the larger webs. Given that they tested in MD, the aspect ratio of one could give a sufficiently different response than the aspect ratio of three. It does not appear that they used a correction factor for length to width to account for the fact that a uniform far-field stress may not be obtained with the length to

width ratio of one. Thus, the agreement that Seth and Page observed may not hold for sufficiently long sample lengths.

Swinehart and Broek [20] present failure load versus crack length data from CNT for two papers in their Figure 4. This figure is recreated in Figure 10, where the load has been scaled to the tensile strength and the crack to half-width ratio is used. The dashed lines represent the LEFM fit from [20] and the solid lines represent the prediction from Equation (6). The values of d given in Figure 10 were calculated from Equation (8) using the stress concentration factors and the tensile strengths reported in [20]. Equation (6) fits they data as well or better at all points compared the LEFM fit. The largest improvement is for small cracks, where Equation (6) converges to a value of $1.122TS$. This comparison shows that the modified LEFM can improve the ability to describe the fracture data at small crack lengths.

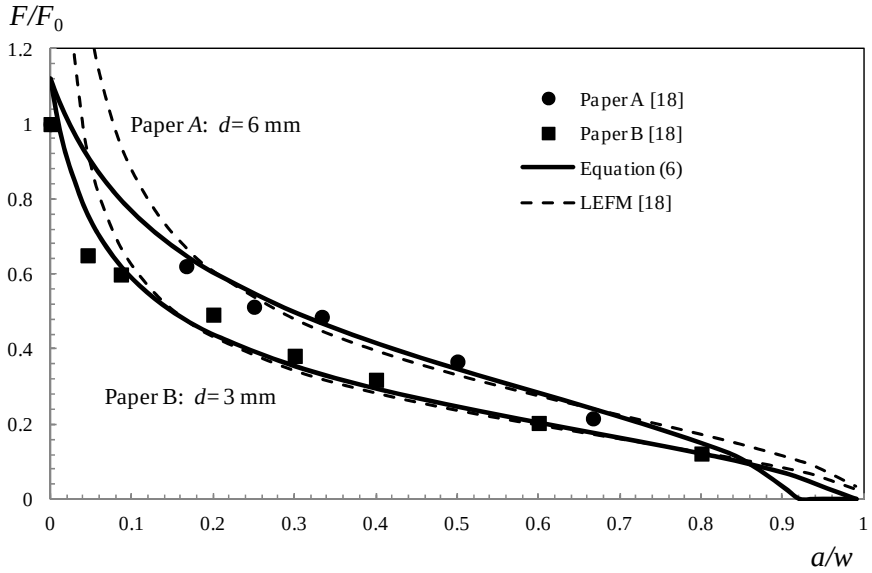


Figure 10. Comparison of Equation (6) to LEFM using data from [18].

The data of Mäkelä Nordhagen and Gregersen [15] can be used to determine if Equation (6) has predictive capabilities. Tensile and fracture test results for these papers was reported in [15,16,and 17]. The fracture test was on a 50 mm wide sample with a center notched crack, with a/w=0.4. Table 2 provides this data along with the value of d determined from Equation (6) and the fracture toughness index JI_{cr} reported in [17].

Table 2. MD Properties of papers from [16,17] and prediction of d from Equation (6).

	Fluting	Sack	News	Liner	MWC	SC
TI kN/kg	124	107	66	61.4	54.5	47
F, N	218	184	61.3	135	106	49.5
d mm	1.13	2.63	2.24	2.71	2.57	2.24
JI_{cr} Jm/kg	6.1	13.4	3.43	5.3	3.98	2.43

The measure of *d* given in Table 2 was determined by setting F/F_0=1.122F/*(grTIw)* and using the solver in Excel to determine the value of *d*, which satisfied the equality. Mäkelä et al. [15] completed fracture tests on large webs 800-100 mm wide for the papers listed in Table 2. Figure 11 a-f provides the predictions from Equation (6) compared to the data [15]. In addition, using the approach outlined in Mäkelä [17], four points on the curve can be obtained from the nonlinear fracture mechanics analysis. To determine these points, the tabled-factors given in [16, 17] along with the material properties shown in the corresponding graph of Figure [10], were used to determine the load ratio. For each point, the Excel solver was utilized to determine the load factor. Inspection of the graphs shows that the prediction of the modified LEFM equation is as good as the prediction from nonlinear fracture mechanics. Inspection of the predictions from nonlinear fracture mechanic combined with finite element analysis obtained in the original work [15] shows that the prediction from LEFM prediction is just as adequate.

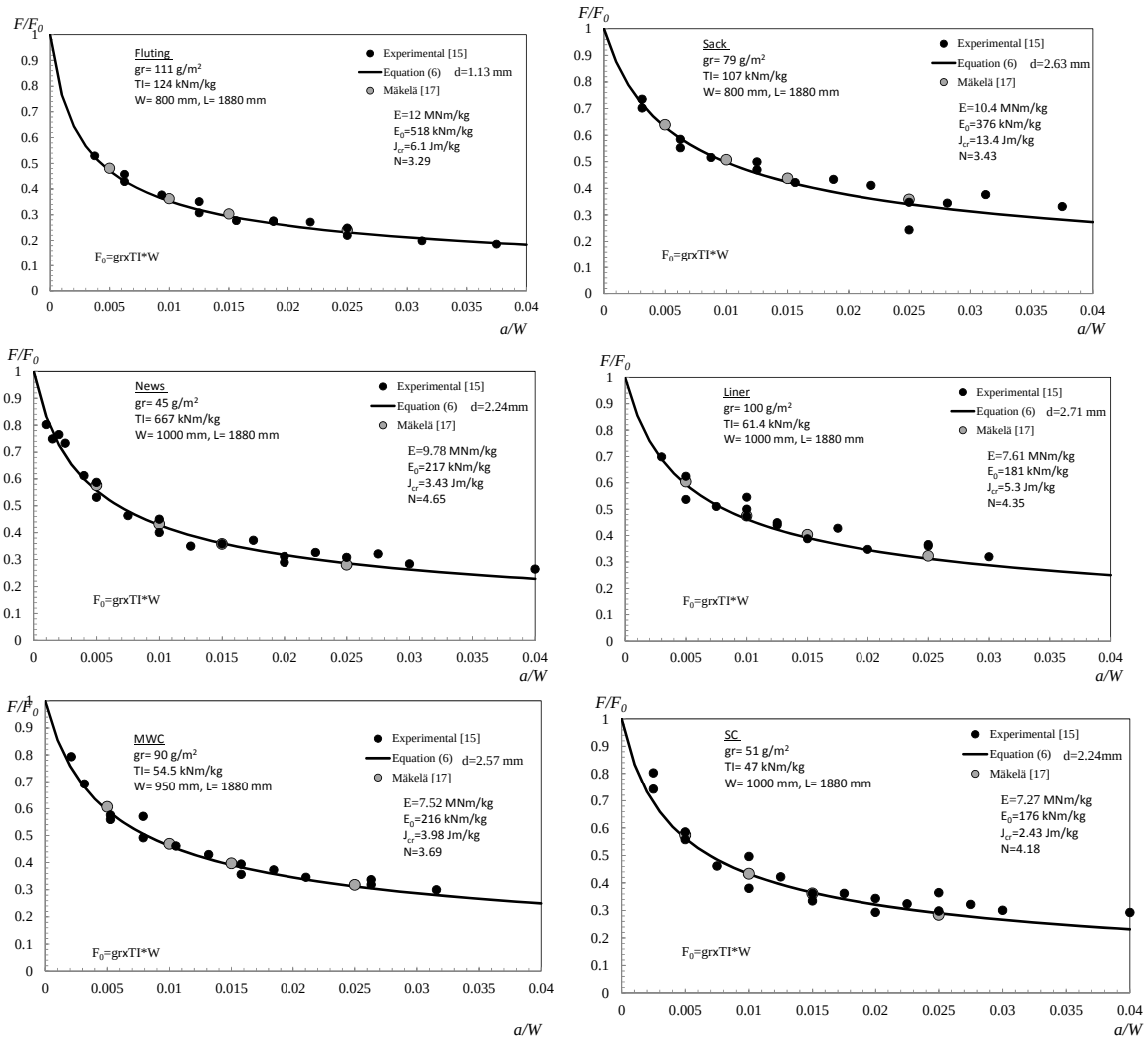


Figure 11. Comparison of LEFM results to experimental results from large webs [15], and the nonlinear fracture mechanics approach in [17].

Mäkelä [17] provided predictions for CD fracture tests for the same papers shown in Figure 11. The value of d was determined using the same process described above. For all the papers, the MD fluting had the lowest value of d=1.13 mm and the CD fluting had the largest value of d=8.8 mm. Figures 12 and 13 provide comparisons of the prediction from Equation (6) (vertical axis) and using the equations of Mäkelä [17] horizontal axis for SENT specimens. The various markers represent different width webs, each with a/w= 0.005, 0.01, 0.015, and 0.025. The unrealistic web of w= 100 meters is given to demonstrate that as the web width goes to infinity both solutions converge to that predicted from straight LEFM, which is shown as the dark dashed line. Once the web width gets small, the nonlinear fracture mechanics solution [17] diverges because the singularity at zero crack length remains in the solution. The current solution converges to the tensile strength for zero crack length.

Comparison of Figures 12 and 13 shows that the CD predictions is just as good as that for MD, even with a very large FPZ of d=8.8 mm. Figures 12 and 13 demonstrate that the modified LEFM theory is much better than classic LEFM, and is likely a better fit than the nonlinear fracture mechanics solution for small sample widths or small cracks.

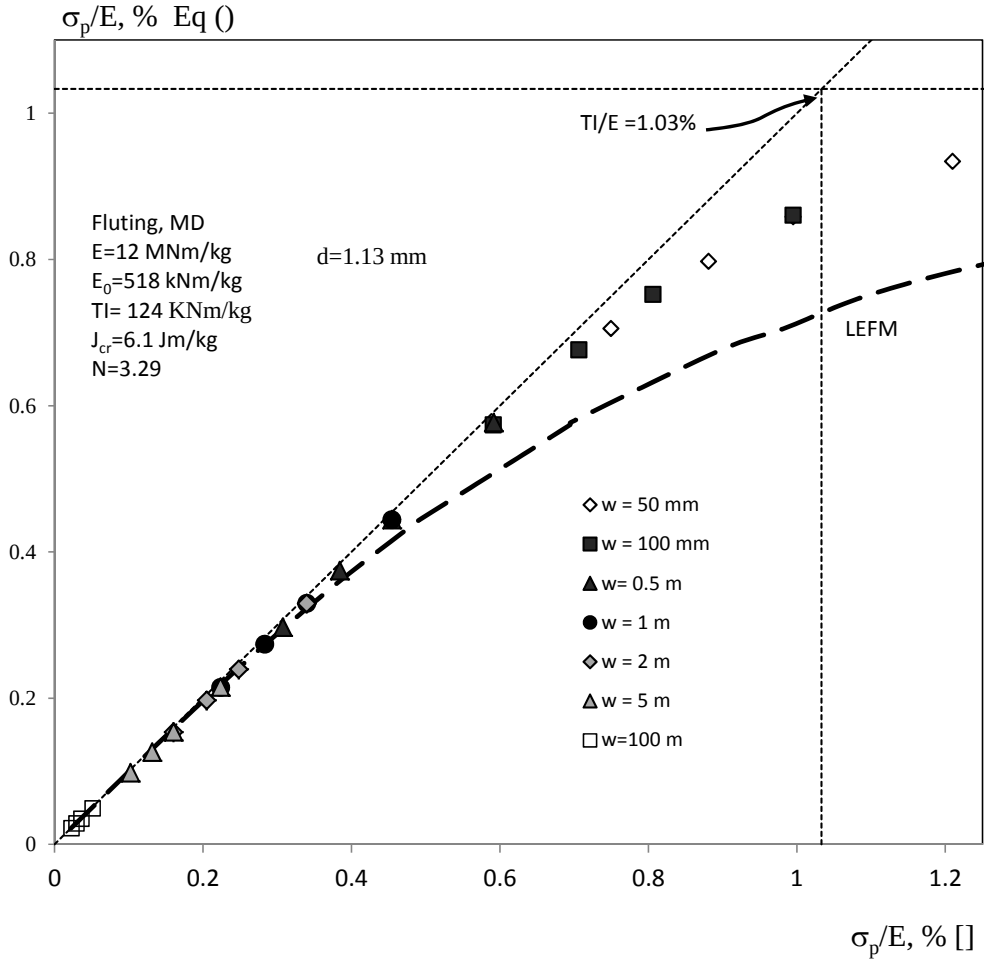


Figure 12. Comparison of Equation (6) with that from Reference [17] for MD Fluting.

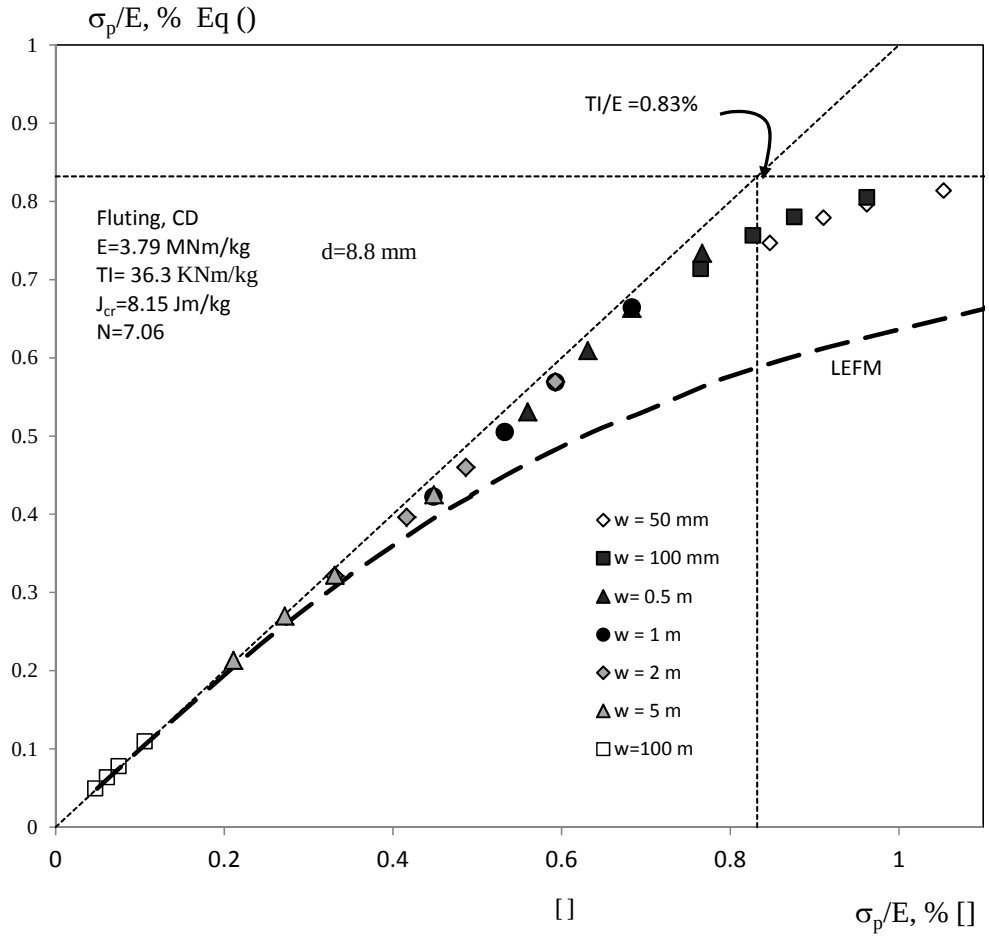


Figure 13. Comparison of Equation (6) with that from Reference [17] for CD Fluting.

The comparisons made in this section indicate that Equation (7) can be quite useful for characterization and prediction of the fracture sensitivity of papers. For the comparisons made here, only two parameters were needed, tensile strength, and the effective FPZ, *d*. The tensile strength is easily obtained from a standard tensile test, and *d* can be obtained from one fracture test. It appears that a 50 mm wide sample is sufficient for MD and CD at least up to a an FPZ of *d*=9 mm. Equation (7) has several advantages

- simplicity over methods of nonlinear fracture mechanics
- convergence to the tensile strength for small cracks
- predictive capabilities for a variety of commercial papers.

4 EXPERIMENTAL

4.1 DENT Testing

A series of DENT fracture tests were conducted to further elucidate the modified LEFM model. The testing was completed on an Instron model 3344 universal tester, with pneumatic clamping. The grips were 76.2 mm wide grips and had serrated faces. The constant rate of displacement was 25.4 mm/min.

For samples that showed a tendency to break at the clamps, masking tape was used to reinforce the paper under the grips.

Sample dimensions varied but typical test reported on here used a width of 76.2 mm and a gage length of 180 mm. In reflection, MD sample lengths should probably be larger to ensure that the far field stress is more uniform, but conclusions remain the same. Samples were cut with both rotary and guillotine cutters with no significant differences reported. Notches were cut prior to mounting with the use of either sharp scissors or a razor blade. Minimal differences in peak loads were found with different methods of sample preparation. For samples with small crack or ligament lengths, the size of the cut crack length was measured after the test.

All testing was conducted under constant environmental conditions of 50% Relative humidity and 22^{o} C.

4.2 Materials

A variety of commercial papers, a polymer film, and several handsheets were tested for fracture resistance. All samples were conditioned to 50% relative humidity and a temperature of 22^{o} C prior to testing. Properties of the commercial sheets are listed in Table 3. The papers represent a wide range of properties that one might expect from different grades. The grammage ranges from 22 to 200 g/m^2, the breaking length varies from 0.18 to 13.7 km, and the density varies from 170 to 830 kg/m^3.

Table 3. Physical Properties of Commercial sheets.

	Grammage, g/m^2	Density, kg/m^3	Breaking Length, km	
			MD	CD
Copypaper	77	769	5.2	3.5
Newsprint	46	638	5.9	1.2
Paperboard	200	638	7.6	2.9
Tissue Paper	22	170	0.18	0.14
Polypropylene	25	1000	13.7	----

The stress-strain curves for the materials listed in Table 3 are given in Figures 14 and 15. The stress is normalized with the elastic modulus. The normalized stress was determined by dividing load by the maximum slope evaluated from the load versus strain curve. Strain was determined as change in length divided by original length.

Figure 14 shows that the MD and CD curves for both Newsprint and the Copy paper are essentially the same, except MD is more brittle than CD. The copy paper is more ductile than the newsprint and shows more yielding. The MD paperboard has significantly less yielding and reaches a higher strength relative to its modulus than in the CD direction.

The tissue paper has high stretch (15%), and a very linear initial loading path for both MD and CD. This is because the tissue paper is in a bond-dominated regime with a very low breaking length and an equally low modulus. The polypropylene film has a well-defined yield point, followed by very slow strain hardening. The film is also quite ductile with a stretch of 150%. The CD paperboard curve is shown both in Figures 14 and 15 for reference.

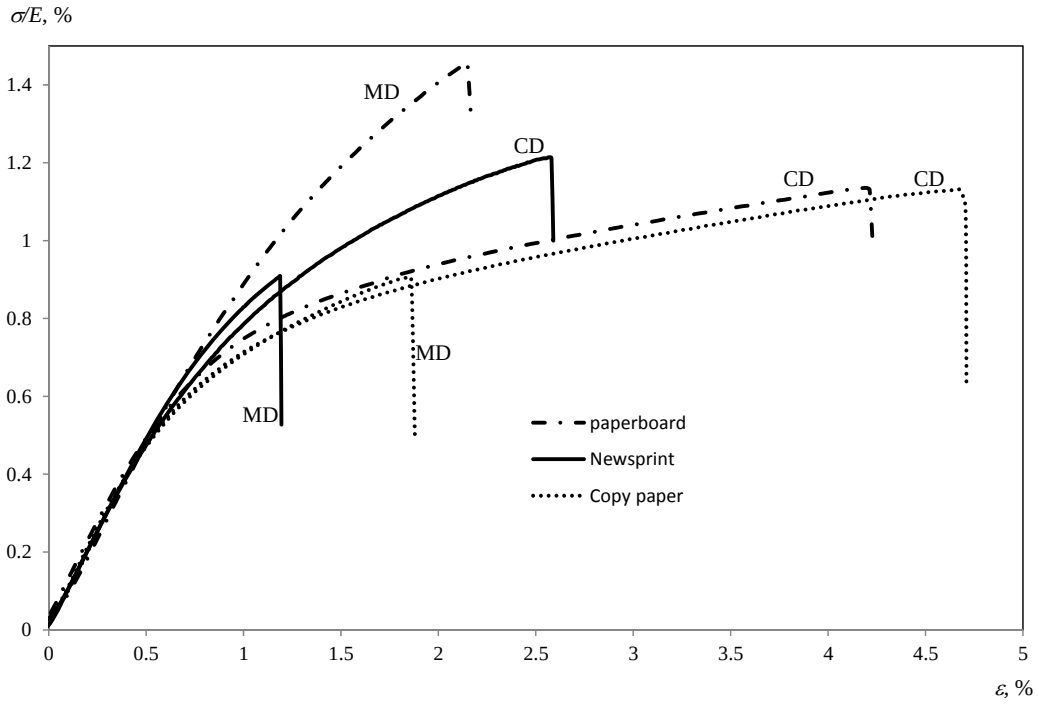


Figure 14. MD and CD Stress-strain curves for commercial papers. Stress is normalized to Elastic modulus.

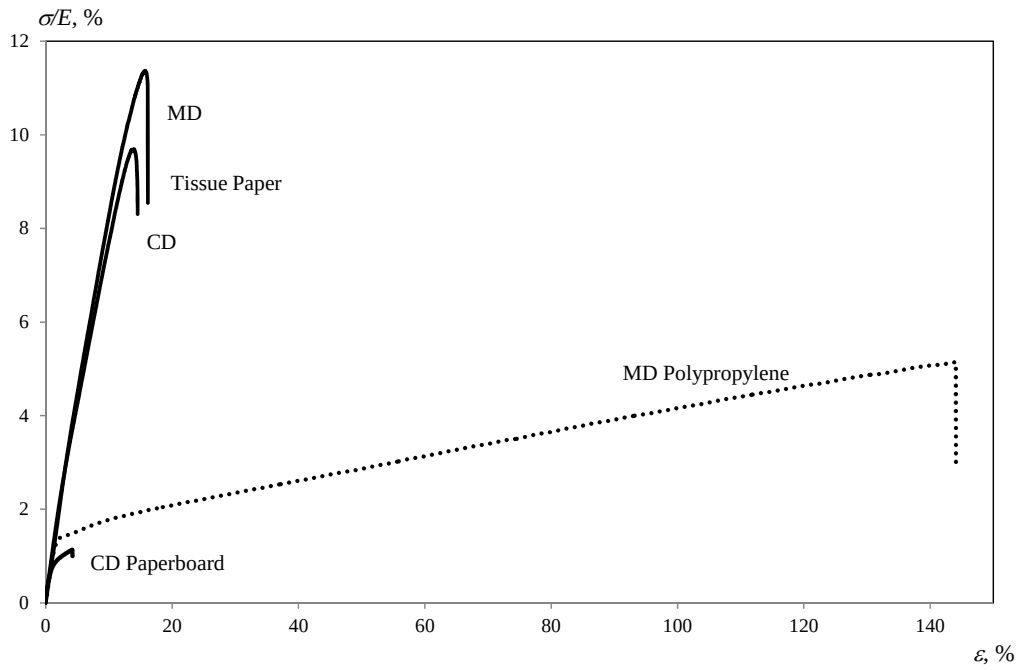


Figure 15. Stress-strain curves for tissue paper and polypropylene film. Stress is normalized to Elastic modulus.

Handsheets were produced on a 305 mm square Noble and Wood handsheet former. The pulp was NIST reference pulp 8495 (Northern Bleached Kraft Pulp). Beating was carried out in a Valley beater. Sheets were produced to three grammages, 25, 50, and 100 g/m^2. Pressing was carried out with a benchtop nip press and the sheets were dried on a drum dryer utilizing a tensioned fabric for restraint. Properties of the handsheets are given in Table 4. The focus of the handsheets was to further investigate fracture resistance with large fracture process zones from structure, the focus on no refining, low grammage, and no pressing.

Table 4. Properties of handsheets.

CSF	Grammage, g/m^2	Density, kg/m^3	Breaking Length, km
465	50	712	9.5
160	50	725	10.1
705	50	588	3.7
705	100	634	3.5
705	25	638	2.6
705	50	535	1.4

The stress-strain curves for the handsheets are given in Figure 16. Except of the CSF 160 and CSF 465 sheets, the sheets give a response where the efficiency is so low that the scaled curves do not superimpose as well as one might expect [21].

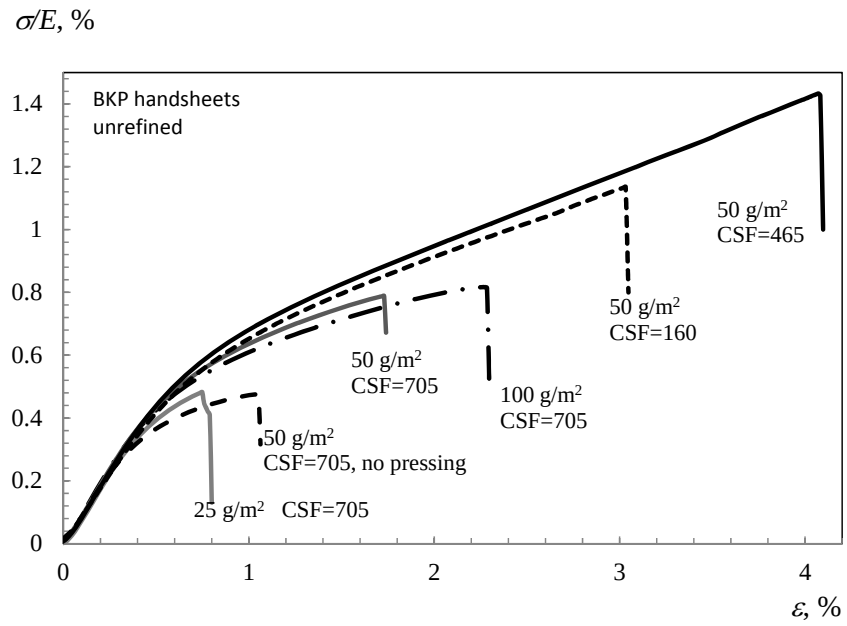


Figure 16. Stress-strain curves for handsheets.

5 RESULTS AND DISCUSSION

The focus of the DENT experiments was to determine if Equation (6) could be utilized to characterize the fracture behavior of a wide range of paper material responses, and if the structural contribution of the fracture process zone was necessary to explain the data. Figures 17-20 provide the results of DENT testing as well as fits using Equation (6). The parameters for the FPZ are given in Table 5. If the relative fracture resistance is due to inherent structure, one might expect $d=2d_s$, so that ratio is also given in Table 5.

Table 5. Effective Fracture process zone, d, and structural zone length d_s for commercial sheets.

		d, mm	d_s, mm	$(2d_s)/d$
Copy paper	MD	2.3	0.25	0.22
	CD	5.0	0.25	0.10
Newsprint	MD	1.8	0.8	0.89
	CD	4.0	2.0	1.00
Paperboard	MD	2.6	0.4	0.31
	CD	6.5	0.4	0.12
Tissue Paper	MD	4.2	1.7	0.81
	CD	7.0	2.5	0.71
Polypropylene	MD	0.23	0	0.00

Figures 17 and 18 show that Equation (6) represents the fracture sensitivity well for both MD and CD for a large range of crack sizes for both Newsprint and Copy paper. Equation (6) is a better fit for the Newsprint than the copy paper. For the copy paper, equation (6) under predicts the load ratio for deep notches or small ligament lengths as represented in the figures with 1-a/w. The newsprint appears to have a large contribution from the inherent structure rather than plasticity as observed by the large ratio of $2d_s/d$.

Figure 19 shows that for MD DENT Equation (6) gives a reasonable representation of the behavior the papers as well as the polypropylene film. As one might expect, even though the polypropylene is by far the most ductile material tested, is has the most significant sensitivity to fracture. The value of d=0.23 suggests that the film can easily concentrate load and failure occurs at low far-field stresses. The tissue on the other hand has the least relative sensitivity to fracture and it is likely not due to plasticity of the fibers but the structure of the sheet as observed by the relatively high ratio of $2d_s/d$.

As shown in Figure 20 for CD, the fit is also reasonable, expect perhaps for the paperboard, for which the data forms a curve that cannot be fit well with Equation (6). Perhaps the cohesive failure mechanism for this paperboard in CD are much more dominate and FPZ increases rather fast with crack size. Despite this poor fit, given the simplicity of the representation of Equation (6) is might be acceptable for practical considerations.

When one compares the MD and CD results, the clear trend is that MD is more sensitive to fracture than CD as indicted by the lower values of d. In addition, the inclusion of d_s is more important for CD compared to MD and for the tissue paper. For larger cracks sizes d_s can be ignored.

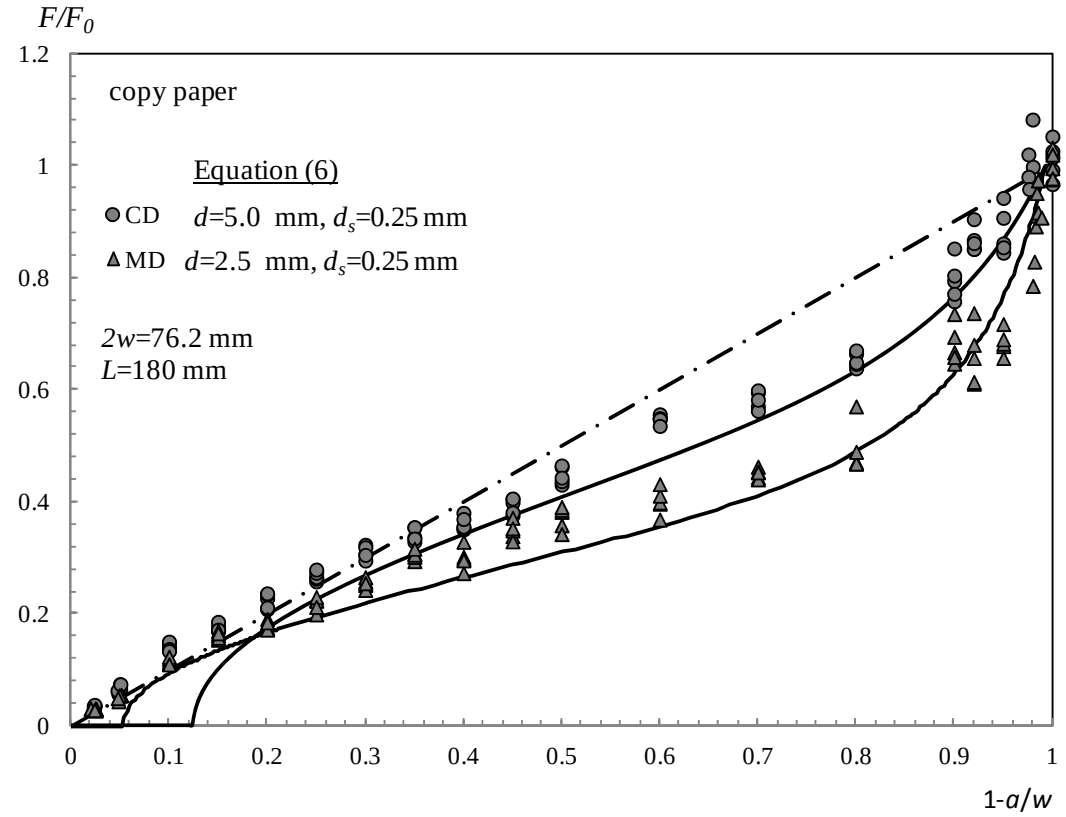


Figure 17. DENT results of load ratio versus relative ligament length for copy paper. Solid lines represent Equation (6). Dash-dot line represents notch-insensitive response F/F_0=1-a/w.

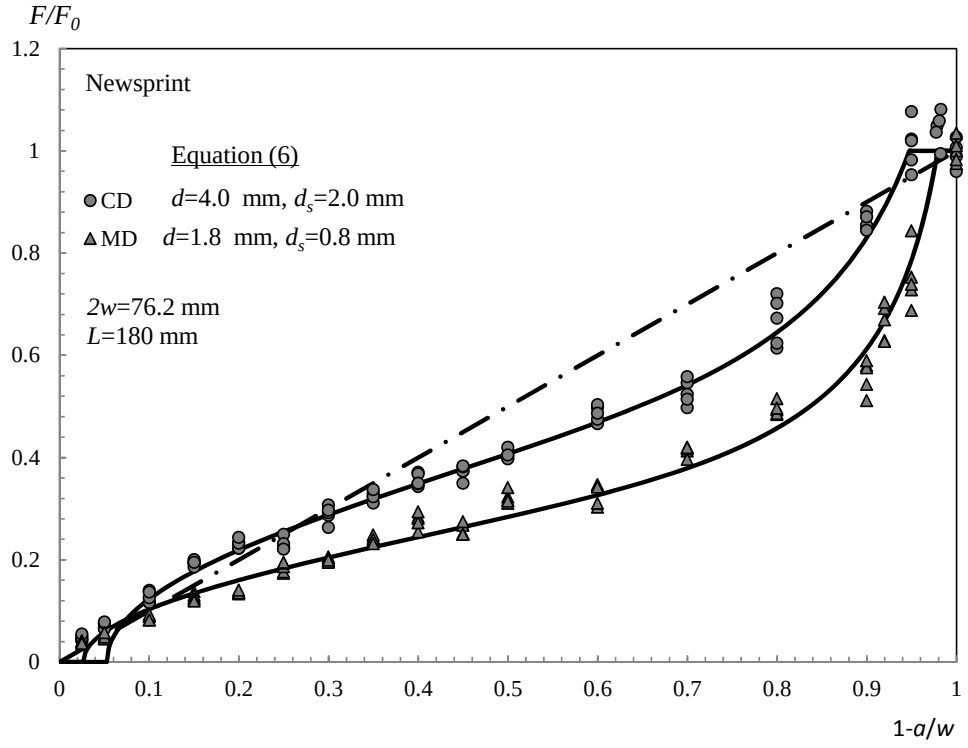


Figure 18. DENT results of load ratio versus relative ligament length for Newsprint. Solid lines represent Equation (6). Dash-dot line represents notch-insensitive response F/F_0=1-a/w.

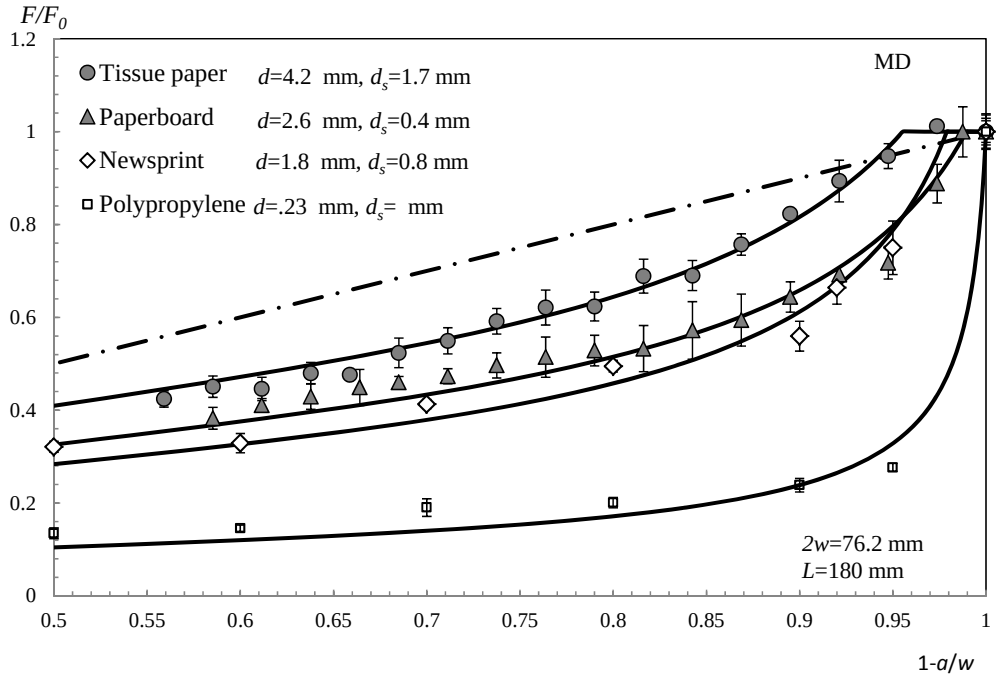


Figure 19. DENT results of load ratio versus relative ligament length for various papers in MD direction. Solid lines represent Equation (6). Dash-dot line represents notch-insensitive response F/F_0=1-a/w.

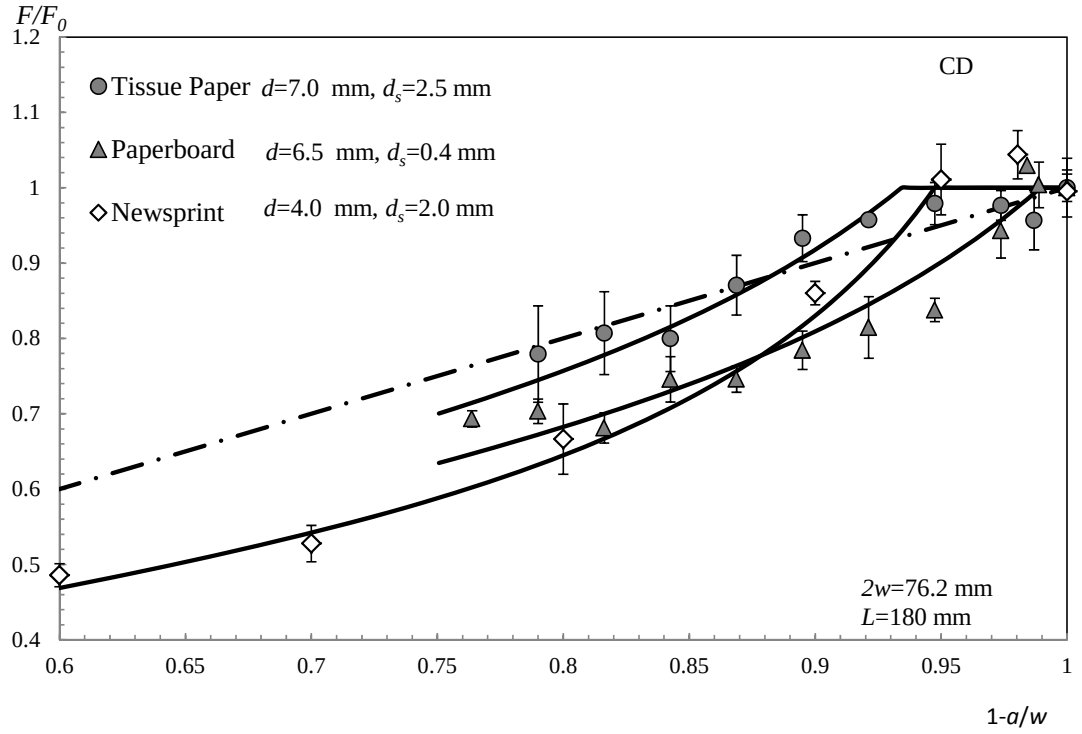


Figure 20. DENT results of load ratio versus relative ligament length for various papers in CD direction. Solid lines represent Equation (6). Dash-dot line represents notch-insensitive response F/F_0=1-a/w.

Bither and Waterhouse [22] showed that handsheets produced from unbeaten pulps showed little fracture sensitivity, but as the pulp was beaten handsheets the sensitivity to fracture increased. Their sample width was rather small, 25.4 mm, so the fracture process zone could have been too large for the unbeaten pulps. Seth and Page [21] effectively demonstrated that beating and wet pressing increased the efficiency in load transfer in the sheet. Low bonding leads to inefficiency. Conversely, as the efficiency in which the sheet carries load increases, the ability of the sheet to concentrate load would also likely increase. Therefore, relative fracture sensitivity would also likely increase. So even though fracture toughness might increase with beating, the fracture process zone would likely decrease. Therefore unbeaten sheets would be less sensitive to defects.

The DENT results for the handsheets, as shown in Figures 21 to 23 reinforce the concept that the loss of efficiency increases the area of paper activated in a fracture process, and thus, the relative sensitivity to fracture decreases. For well bonded sheets, the FPZ is smaller and the relative sensitivity increases. Figure 21 shows that the two sheet made from beaten pulp have small effective FPS of about d=2 mm, while the unbeaten pulp has an effective FPZ of d=5 mm. If one considers the stress-strain curves previously given in Figure 16, the two beaten pulps have much better developed stress-strain curves, and represent good transfer of load to fibers. Following the same line of reasoning, Figure 22 demonstrates that un-pressed sheets further increases FPZ and the structural contribution d_s.

Figure 23 demonstrates that lower grammage sheets have increased relative fracture resistance. One would expect that with low grammage sheets, coverage is low, the bonded

area is low, surface fibers make up a significant portion of the sheet, and thus, load transfer is impeded. This increases the FPZ and thus increases relative fracture resistance.

The results given above suggest that the effective fracture process zone, *d*, can be used as an indication of the relative fracture sensitivity of the sheet. As the load transfer efficiency is increased by means of improved bonding through beating and pressing, the stress-strain curve can be developed but relative fracture resistance decreases. The decrease in fracture process zone is indicative of an increase in the sheet's ability to concentrate load. The stress intensity factor would be affected by both the tensile strength and the magnitude of the fracture process zone.

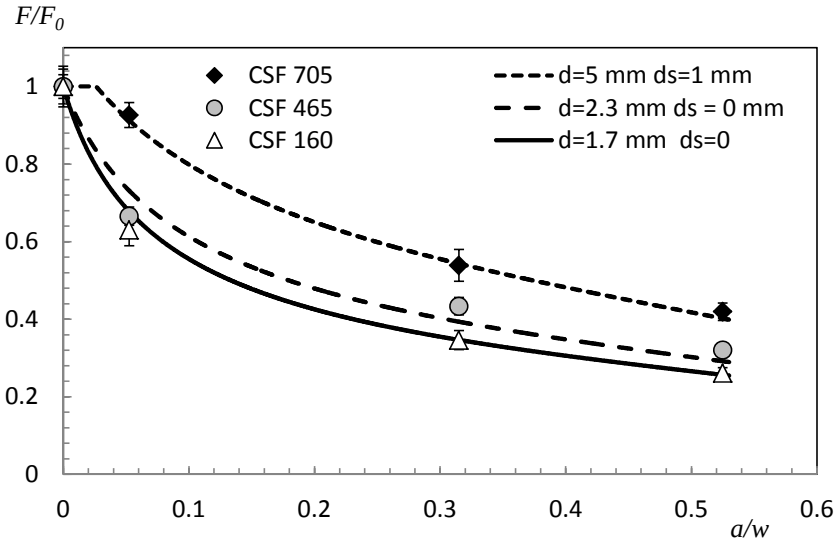


Figure 21. Effect of beating on DENT fracture sensitivity for handsheets, 50 g/m^2.

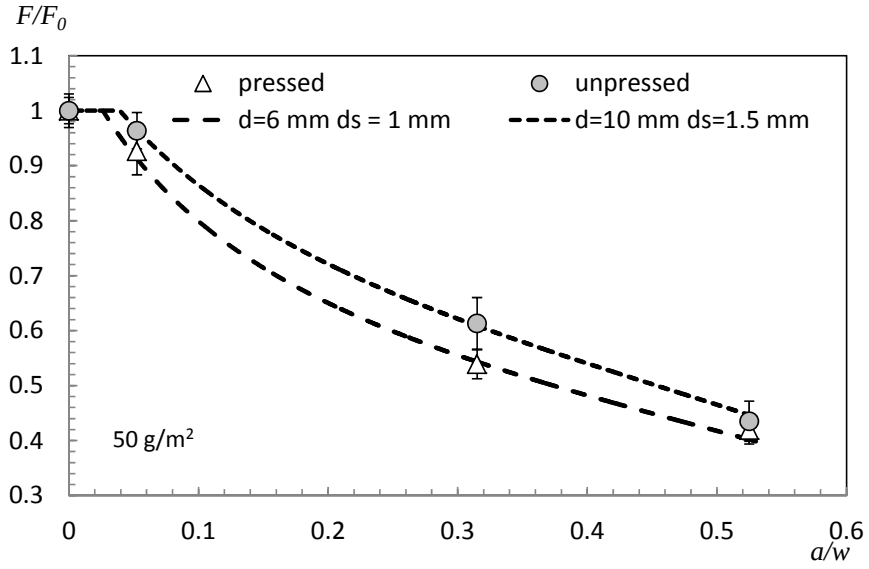


Figure 22. Effect of pressing on fracture sensitivity of handsheets made from unbeaten pulp.

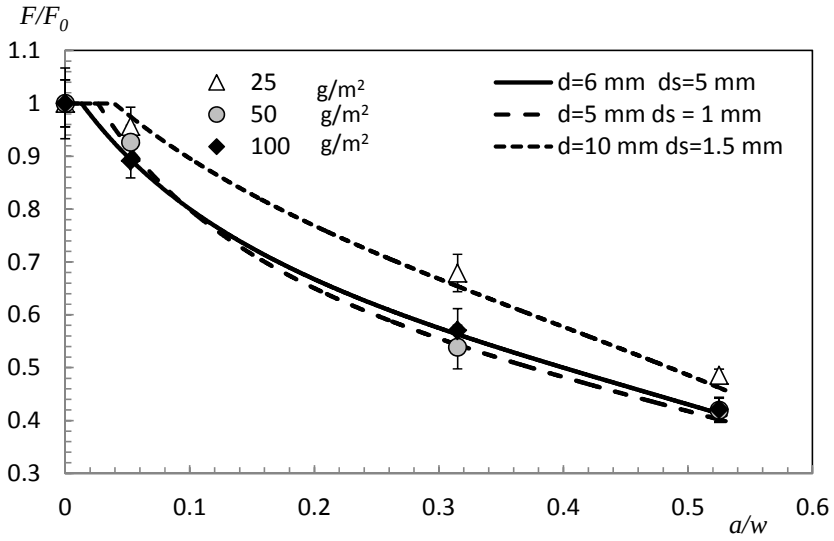


Figure 23. Influence of grammage on fracture sensitivity of handsheets from unbeaten pulp.

Swinehart and Broek [9] showed that sample scaling with LEFM held at least for wide webs and large cracks. The results from [6] as shown in Figure 7 suggest that scaling holds for narrow widths too. The results given in Figure 11 demonstrated that calculating the effective process zone from a 50 mm wide sample was sufficient to predict fracture loads for small cracks in large webs. The ability of a narrow width samples to provide an estimate of the fracture process zone depends on the magnitude of the FPZ. If the zone is small, say d=1.0 mm, then even a sample width of 15 mm should be adequate for cracks up to a=4mm, a width of 25 mm should be valid for cracks up to a=10 mm. For a large fracture process zone, d=10mm, a sample width of 50 mm should be valid for cuts up to a=8 mm.

Figure 24 shows results for MD specimens of copy paper with widths of 25.4, 50.8, and 76.2 mm width along with the curves given by Equation (6). These results demonstrate that with d=2.3 mm, the scaling predicted from Equation (6) is reasonable, and this should hold for larger webs. Figure 25 shows the results for CD. With d=5mm, Equation (6) does not hold as well for the 25.4 mm width, but it is adequate for the two larger widths. Even for the 25.4 mm wide web, Equation (6) is reasonable for cracks less then 6mm. Figure 17 shows that this fit is adequate for the 76.2 mm wide web for cuts up to a=34 mm or a ligament length of about 8 mm. The actual CD fracture load is larger than that predicted by Equation (6) for these deep cracks. For larger webs, it is likely that the prediction from Equation (6) would be valid for deep cuts and would most likely be a conservative under-estimate of the fracture strength.

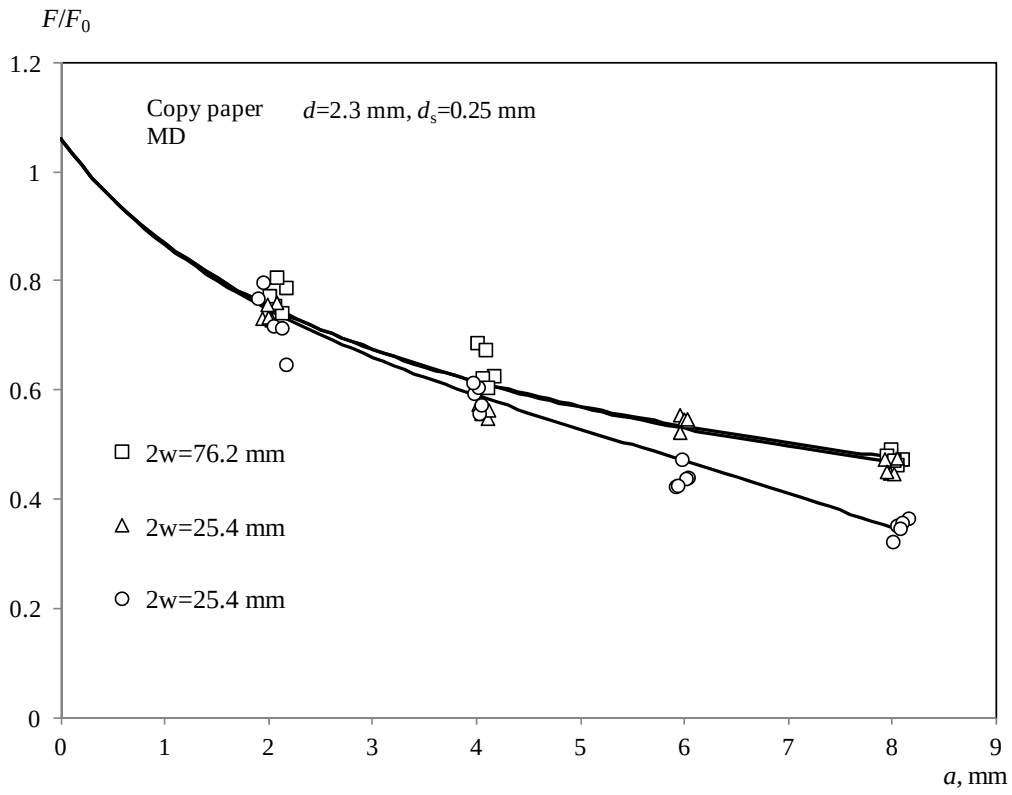


Figure 24. Fracture versus crack length for three widths. MD copy paper.

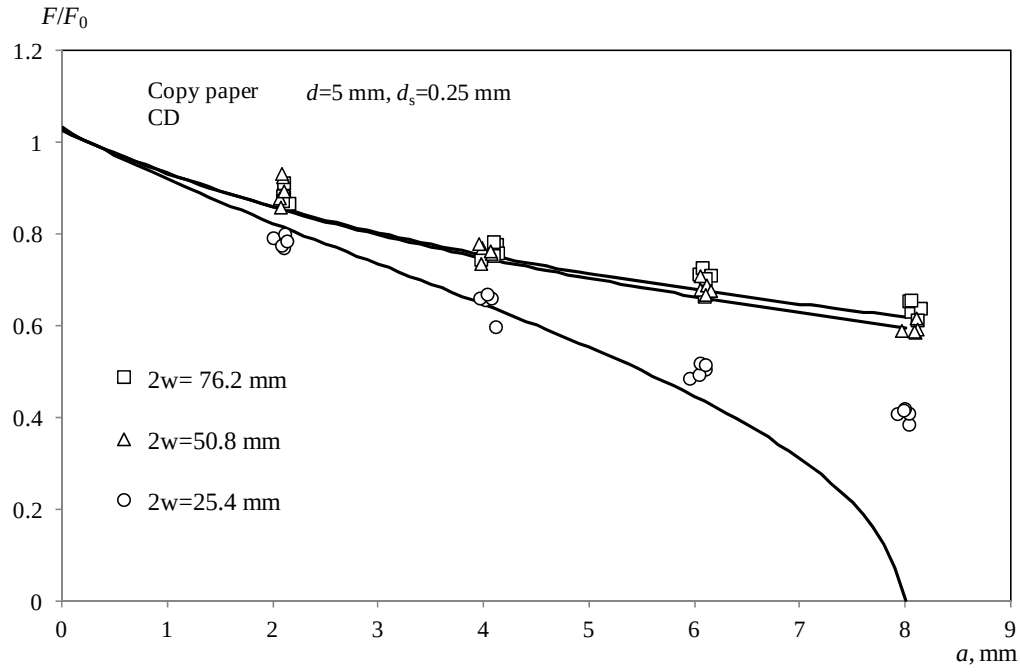


Figure 25. Fracture load versus crack length for three widths. Copy Paper CD

For DENT samples where the ligament length is in the range of d to $3d$, the average ligament stress likely exceeds the tensile strength of the material as demonstrated by the results of Tanaka and Yamauchi [23]. The plastic zone length determined by Tanaka and Yamauchi [23] from DENT tests with the ligament length one third the sample width can be recalculated to give the ratio of average ligament stress to tensile strength. They varied the width from 3 to 63 mm and their results show that the average ligament strength can exceed the tensile strength by an additional sixty percent. For example, for newsprint a ligament length of 2 mm gave a ligament stress that was 1.3 the tensile strength in MD and 1.5 the tensile strength in CD. This indicates that the intrinsic strength of the sample is higher than the measured tensile strength and that tensile strength is limited by fracture due to the cutting of the structure at the edges. With a DENT sample, fibers crossing the ligament form a path for load transfer. The same fiber cut at the edge of a sample would lose much of its ability to carry load. For smaller ligament lengths, the fracture process zones superimpose, stress concentrations are lower and the measure of fracture load is a better estimate of intrinsic tensile strength of the network.

Figure 26 provides the ratio of fracture ligament stress to tensile strength for four sample types of MD copy paper. Three of the samples are tensile strips (a=0) with three widths, 25.4, 75.2, and 2.5 mm. The fourth sample is a DENT with a ligament length of 2.5 mm. The DENT sample has a strength that is 47% larger than the tensile strength of the sample. This suggests that without a notch, the sample fractures at the edges because of inherent flaws in the structure, which are opened up when the edges are cut. The structure in the ligament of the DENT sample is intact and can carry significantly more load.

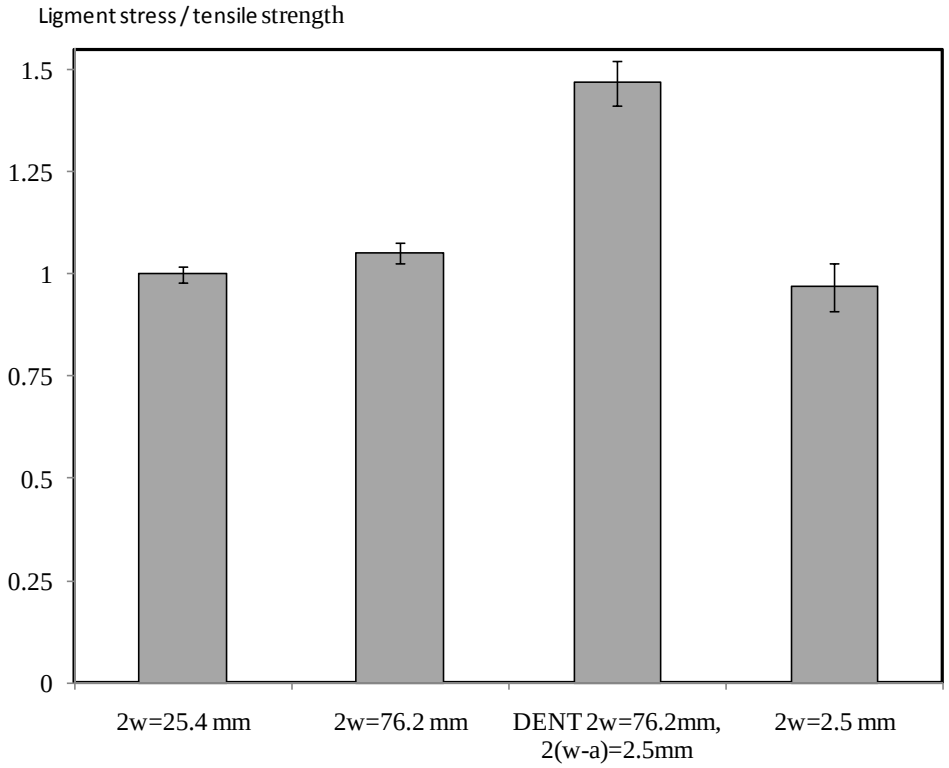


Figure 26. Comparison of tensile strength versus deep-notched DENT strength for MD copy paper.

6 CONCLUSIONS

Contrary to statements in the literature, it was found that a modified linear elastic fracture mechanics (LEFM) model can be applied to paper for both materials characterization and prediction. By using a ratio of fracture loads from LEFM equations, fracture resistance can be determined from the tensile strength and an effective fracture process zone, d. The fracture process zone can further be split to a structural, d_s, and a materials component, although this separation is not needed for the majority of cases to obtain reasonable predictions. Equation (6) proved useful in characterizing a wide range of papers from tissue to paperboard for both MD and CD. For most papers, a 50 mm sample width should be sufficient to characterize the materials fracture sensitivity. Results from small samples should scale to large webs at least until the crack depth is quite deep. Simplicity of application is the great advantage offered by the modified LEFM compared to other available methods.

The current results support the previous work of Donner [8] linking the tensile strength directly to the fracture behavior and suggesting that the inherent network structure of paper contributes to fracture toughness. As sheet efficiency decreases, tensile strength decreases, but the effective fracture process zones increases, thus the relative fracture toughness increases. In many cases the actual fracture toughness would decrease because the loss of strength exceeds the gains from an increased fracture process zone.

For a wide range of commercial papers, the effective fracture process zone was in the range of 1 to 3 mm for MD and only larger, about 4 mm, for tissue papers. In CD, the fracture process zone was found to be in the range of 4 to 9 mm for all papers investigated. For tissue papers, which tend to be low grammage and bond-strength dominated, the fracture toughness appears to be structural, a result of a large fracture process zone resulting from poor transfer of load. For newsprint, structure also appears to dominate the fracture toughness as indicated by the ratio of $2d_s/d$ near unity. For other papers, plasticity of the fibers probably plays a larger role in fracture toughness.

Although material plasticity plays an important role in fracture toughness, the material with the largest sensitivity to fracture was the polymer film with a stretch of 150%. That is because the ability of the sheet to concentrate load plays an even greater role in determining fracture toughness. The polymer film can concentrate load much better than paper's fiber network, and thus when a crack is introduced in the film, the stresses near the tip reach failure loads when the far field load is still quite low. In paper, the network structure impedes the ability of the sheet to concentrate stress and as a result the relative resistance to fracture is much higher. The effective fracture process zone can be considered as an indicator of how well the sheet can concentrate load. The smaller the value of d, the better the sheet can concentrate load. Even if the material were perfectly elastic and brittle, increasing the characteristic length of the structure would improve the relative fracture resistance.

The edges of a paper are inherently flawed because the structure is disrupted by cutting fibers that cross the edge. The tensile strength is then a result of fracture resulting from concentrated loads as some point where the edge flaw is largest (This assumes that no larger defects like a large shive or a hole are in the interior of the sheet). The notches or cracks introduced in a DENT cause the average stress over the ligament to be high and failure imitates at one or both of the notch tips. The process of cutting a slit induces little damage to the network structure remaining in the ligament. Thus the inherent strength of the sheet can be determined from deeply notch specimens and can easily be 50% greater than the tensile strength.

The second advantage to using the modified LEFM equation presented here, Equation (7), is that the singularity at small crack lengths is eliminated. The reason that LEFM was dismissed is that it over predicts the fracture strength for small cracks as evidenced by the literature where LEFM does a better job of predicting CD compared to MD even though CD has more plasticity associated with it. The modification presented here ratios the load to the tensile strength, which is determined from the effective fracture zone and thus insures reasonable convergence for small crack lengths. This actually provides a better estimate then other models that include plasticity but leave the singularity at zero-crack length. It is important to note that the current LEFM modification does not make use of the yield stress but rather assumes the tensile strength is also a result of fracture. The comparison to experimental results supports this assumption.

Finally, we conclude that by embracing the use of LEFM to describe the fracture resistance of paper, one can obtain new insights into the role of materials and structure to the observed mechanical behavior of paper.

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