Effects of the Kinematic Model on Forward-Model Based Spotlight SAR ECM

A Thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in Electrical Engineering

by

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ABSTRACT

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Spotlight synthetic aperture radar (SAR) provides a high-resolution remote image-formation capability for airborne platforms. SAR image formation processes exploit the amplitude, time, and frequency shifts that occur in the transmitted waveform due to electromagnetic propagation and scattering. These shifts are predictable through the SAR forward model which is dependent on the waveform parameters and emitter flight path. The approach to develop an electronic countermeasure (ECM) system that is founded on the SAR forward model implies that the ECM system should alter the radar’s waveform in a manner that produces the same amplitude, time, and frequency shifts that a real scatterer would produce at a desired location. A collection of such scatterers would be capable of forming a larger collective energy distribution in the final image. However, since the forward model is dependent on the radar platform’s kinematic model, the jamming energy distribution created from a forward-model based ECM system will inherently have some level of sensitivity to kinematic error. This thesis discusses a forward-model based ECM modulation scheme and provides an assessment of its sensitivity through Monte Carlo simulations and an entropy-based image similarity distance.
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Dedicated to my amazing wife, Emily.
Introduction

1.1 Overview

Spotlight synthetic aperture radar (SAR) allows for remote formation of high-resolution images of a localized area of the earth. This mode is facilitated by a radar carried by an aerial vehicle that illuminates the desired area for some amount of time and collects backscatter signals from different angles. SAR image formation processes (IFP) exploit the amplitude, time, and frequency shifts that occur in the transmitted waveform due to electromagnetic propagation and scattering. The magnitude and hysteresis of each shift is dependent on the relative kinematics between the emitter and the scene, which is observed through analysis of the forward model. The approach to develop an electronic countermeasure (ECM) system that is founded on the SAR forward model implies that the ECM system should alter the radar’s waveform in a manner that produces the same amplitude, time, and frequency shifts that a real scatterer would produce. This approach increases confidence that the ECM waveform results in the desired energy distribution in the final image regardless of the waveform or IFP utilized by the SAR system. However, since the forward model is dependent on the radar platform’s kinematic model, the jamming energy distribution created from a forward-model based ECM system will inherently have some level of sensitivity to kinematic error. This thesis discusses a forward-model based ECM modulation scheme and provides an assessment of its sensitivity through Monte Carlo trials and image similarity calculations.
1.2 Challenges

A major challenge of SAR ECM development is the complex dependency of the time-variant geometry between the radar, the ECM platform, and the illuminated scene. In this work, the SAR ECM system is assumed to have the capability to determine a SAR platform’s location and trajectory to some degree of accuracy. The SAR ECM system is assumed to have basic digital radio frequency memory (DRFM) capabilities, which include the capability to record the waveform transmitted by the radar, perform basic signal processing tasks to the data, and transmit a response waveform at a power level sufficient to achieve a jamming-to-signal ratio of one [1, 2, 3]. Also, the radar’s IFP and the utilization of auto-focus techniques are expected to be unknown to the ECM system, which may blur the intended energy distribution in an undesirable manner. There are numerous other challenges, such as ECM technique effectiveness assessments, but they are specific to an ECM technique and are not considered here. Also any multi-path effects are ignored.

1.3 Hypothesis

SAR IFPs are fundamentally dependent on an estimate of the geometric relationship of the antenna and the illuminated area in order to exploit the amplitude, time, and frequency shifts that are present in the received waveform to produce a facsimile of the scene. Therefore, the hypothesis presented is that the quality of the resulting jamming energy distribution in a spotlight SAR processed image is limited by the accuracy of the forward-model based ECM system’s estimation of the radar platform’s kinematic model.
1.4 Outline

The development approach for SAR ECM in this work is inspired by recent literature on the subject. Following the primary assumptions often presented in other works, Chapter 2 discusses spotlight SAR theory and presents the forward model with a linear-frequency modulated (LFM) signal model. Chapter 2 continues with kinematic and general SAR-processing models which explain how the scene is estimated from the collected waveform data and results in an image. Phase analysis of the LFM signal model is then used to express the forward model in a manner which clearly exposes the kinematic relationships. The spotlight SAR theory section concludes by discussing common errors in SAR IFPs and the effects of moving targets.

The second half of Chapter 2 discusses general ECM theory with a focus on an ECM system’s required knowledge of the radar’s kinematic model in order to control a jamming energy distribution. ECM waveform design is assessed as an extension of the SAR phase analysis in order to identify a forward-model based modulation scheme that a DRFM may utilize in order to embed the ECM user’s desired false target information into the returned waveform.

Chapter 3 presents the methodology that is followed in order to assess the sensitivity of the jamming energy point spread function to errors in the ECM system’s kinematic estimate. This includes a discussion of Monte Carlo trial accuracy and the Jaccard Distance. Finally, chapter 4 presents the results of the experiment detailed in chapter 3.
Background

2.1 Literature Review

SAR ECM has been studied from multiple perspectives. The authors of [4] approached noise and deception jamming requirements with respect to effective radiated power levels with the assumption that the jammer has knowledge of the radar’s location, timing, and waveform details. In [5], the authors analyzed noise jamming techniques against a general SAR ground moving target indicator process and determined the necessary jamming-to-signal ratio required for noise jamming to effectively hide a moving target’s Doppler shift. The authors of [6] studied active deception jamming for a single false target, multiple false targets, and scene generation. In [7], the authors provided an analysis of the effects on inverse SAR jamming through sinusoidal phase modulation. The authors of [8] provided an ECM signal generation method to counter space-born SAR emitters through parallel computing. In [9], the authors demonstrated SAR ECM jamming through the utilization of micro-Doppler perturbations, which result in vibrating-target effects [16]. In [26] and [27] the authors present a DRFM-based image generation method which utilize individual false targets to create a jamming image. These studies provide much in regards to ECM techniques, however their effectiveness may be limited due to the emitter’s unknown or poorly estimated flight path and possible use of auto-focus. While many studies assume a straight emitter flight path in order to simplify analysis, in practice the estimates of a flight path distort the intended ECM energy distribution.
2.2 Spotlight SAR Theory

Spotlight SAR systems persistently illuminate a local area and exploit the properties of electromagnetic propagation and scattering to form images from the amplitude, time, and frequency shifts of the microwave spectrum. The scattering of a pulse $p$ from the scene reflects back some portion of the pulse’s energy which is determined by the scene’s reflectivity density $\rho$ at time $t$. A 2-dimensional estimate of $\rho$ is formed by the radar from the 1-dimensional collection of pulse returns [11]. Each scatterer reflects back the transmitted waveform with amplitude, time, and frequency shifts unique to that scatterer’s reflectivity density and location. To demonstrate, allow the $m^{th}$ LFM pulse transmitted from the emitter to be modeled as

$$s_{Tx}(t - m\text{PRI}) = A_0 p(t - m\text{PRI}) e^{j2\pi f_c(t - m\text{PRI}) + j\pi k(t - m\text{PRI})^2},$$  \hspace{1cm} (2.1)$$

where $A_0$ is the pulse amplitude, $f_c$ is the carrier frequency, PRI is the pulse repetition interval, $M$ is the number of pulses, $k$ is the frequency-modulation factor, and time $t$ is proportional to range $r$ through the standard range equation

$$t = \frac{2r}{c},$$ \hspace{1cm} (2.2)$$

where $c$ is the speed of light\(^1\). Throughout propagation the waveform is incident with the ground in a manner that provides a range $r$ and cross-range $r'$ grid as shown in Fig. 2.1. A portion of the signal energy is reflected back towards the radar, and at the face of the antenna the forward model is the projection of $\rho$ convolved with $s_{Tx}$

$$s_{Rx}(t) = s_{Tx}(t) * \int_c \rho(t, r') dr' + w(t),$$ \hspace{1cm} (2.3)$$

\(^1\)A more thorough derivation of the LFM forward model, processing, and its spectrum is provided in App. E.
where $w$ is additive white Gaussian noise (AWGN) which is omitted from the remainder of the derivations (App. E, Eq. (E.3)) [11, 13, 14] et al. Since $\rho$ is made up of $N$ scatterers, the projection of $\rho$ may be modeled as

$$
\rho(r) = \sum_{n=0}^{N-1} A(r, r' - r'_n). \quad (2.4)
$$

Note that the superposition of $r'$ at every $r$ may also be expressed in terms of angle $\theta_n$, where

$$
\sum_{n=0}^{N-1} A(r, \theta_n) = \sum_{n=0}^{N-1} A\left(r, \arctan \left(\frac{r}{r'_{n}}\right)\right). \quad (2.5)
$$
The time shifts \( \tau_n \) and frequency shifts \( f_{dn} \) for each scatterer are dependent on their location \((x_n, y_n)\) and velocity \(v_n\) relative to the emitter through

\[
\tau_n = \frac{2\sqrt{x_n^2 + y_n^2}}{c} \quad (2.6)
\]

\[
f_{dn} = \frac{2v_nf_c \cos(\theta_n)}{c} = \frac{2v_nf_c \cos \left( \arctan \left( \frac{y_n}{x_n} \right) \right)}{c} = \frac{2vf_c y_n}{c \sqrt{x_n^2 + y_n^2}}, \quad (2.7)
\]

assuming \(x_n \neq 0\). Eqs. (2.4), (2.6), and (2.7) allow the forward model Eq. (2.3) to be modeled in the more common form, for one pulse,

\[
s_{Rx}(t) = A_0 \sum_{n=0}^{N-1} p(t - \tau_n) A_n e^{j2\pi(f_c + f_{dn})(t - \tau_n) + j\pi k(t - \tau_n)^2}, \quad (2.8)
\]

which shows the amplitude, time, and frequency shifts that represent the scene within the forward model of a LFM signal.

### 2.2.1 Kinematic and Processing Model

The scene reflectivity density is embedded in the waveform through the time, Doppler, and amplitude shifts of Eqns. (2.6) and (2.7), as shown in Eq. (2.8). However, Eqns (2.6) and (2.7) are not limited to a position and velocity model. Throughout a CPI, the transmitted waveform reflects from the scene at some time-varying distance defined by \(x\) and \(y\), while the platform is traveling at speed \(\vec{v}\) with acceleration \(\vec{a}\). A notation that is more commonly used in SAR signal models is

\[
\mathcal{K}(\vec{r}, \vec{v}, \vec{a}; t) = \begin{cases} 
\vec{r} & = \dot{x}(t) + \dot{y}(t) + \dot{z}(t) \\
\vec{v} & = \ddot{x}(t) + \ddot{y}(t) + \ddot{z}(t) \\
\vec{a} & = \dot{x}(t) + \dot{y}(t) + \dot{z}(t),
\end{cases} \quad (2.9)
\]
and could include modeling of roll, pitch, yaw, or other motions of the radar platform. A high fidelity model may utilize even more detail, such as the level of detail afforded by the \textit{Brawler} and \textit{BLUEMAX} models \cite{28, 29}.

An ideal $K$ for traditional spotlight SAR results in polar-format data with uniform sampling in frequency and angle, however errors induced by a non-ideal $K$ (such as range migration and antenna vibration) result in uneven sampling. Other polar coordinate and waveform considerations are discussed in Appendices A, B, and \cite{13, 14, 15, 16, 17, 23}.

Following the collection of $M$ pulse returns, exploitation of the amplitude, time, and frequency shifts is performed through an image formation process (IFP). Although there are numerous methods of processing spotlight SAR data into an image, they maintain the common goal of forming a “clear” image, and generally follow the steps discussed next. As shown in Fig. 2.2, the received signal is demodulated if necessary and then sampled by an analog-to-digital converter, after which the IFP forms an image as a 2-D map. An IFP, such as the down-sampled matched filter (DMF) or polar-format algorithm (PFA), is utilized to focus the scene data. Due to space-variant and invariant errors that may occur during the collection, some auto-focus technique may be applied to the image to focus the final image $	ilde{\rho}$ \cite{16, 13}. A more in depth discussion of this processing model and the DMF is provided in Appendix C and \cite{13, 14, 15, 16, 18}. Also, an example case for the matched-filter and deramp process of an LFM signal is provided in App. E.

Typical IFPs utilize demodulation techniques to isolate the time and Doppler shifts. For example a spotlight SAR system that utilizes the traditional PFA to process a received LFM waveform will result in an image which contains two well known sources of phase
error. Residual video phase will be present due to incomplete demodulation if only a de-ramp procedure is utilized (App. E, Eq. (E.19)) [11, 13, 14] et al. Error due to range curvature, which manifests as a quadratic phase change, will be present since the PFA’s matched-filter is based on the first-order Taylor series of the differential range between the radar and a reference range [35]². Both errors may be corrected in order to isolate the desired information in a form that results in a clear image [11, 13, 14, 35]. Regardless, residual artifacts and methods to correct them, which are specific to the waveforms and/or IFPs used, are well documented and expected to be utilized in order to produce a quality image [11, 13, 14, 18] et al.

The IFP used to simulate the examples in this paper is a Taylor-windowed DMF (discussed in Appendix C, Eq. (C.2), and [19]). While this IFP is known to be slower than some other IFPs, the DMF’s matched filter is calculated from the full forward model to include range-curvature. The simulated radar’s \( K \) is limited to

\[
\begin{align*}
\vec{r}(t) &= \vec{r}_0 + \vec{v}t \\
\vec{r}_0 &= \hat{x}x_0 + \hat{y}y_0 + \hat{z}z_0 \\
\vec{v} &= \hat{y}v_0
\end{align*}
\]

where \( x_0 \) and \( y_0 \) are the initial range and cross-range to the CRP, and \( v_0 \) is the initial velocity of the emitter platform. The radar’s waveform parameters are provided in Table 2.1, where \( f_c \), BW, PW, \( k \), and PRI are the carrier frequency, bandwidth, pulse width, linear-frequency modulation rate, and pulse repetition interval, respectively³. Figure 2.3 provides an example result of this IFP for a bed-of-nails scene. The examples provided in this paper are limited to individual point scatterers for convenience and to allow objective compar-

²Range curvature error will be noticeable if the scene extent is large enough, as detailed in [35].
³Although the PW used is unrealistic (3-60 \( \mu s \) is more common [18]), it is used throughout the examples in order to show blurring effects clearly.
Table 2.1: Radar kinematic and waveform parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_0$</td>
<td>−10</td>
<td>km</td>
</tr>
<tr>
<td>$y_0$</td>
<td>0</td>
<td>km</td>
</tr>
<tr>
<td>$z_0$</td>
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<td>km</td>
</tr>
<tr>
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<td>m/s</td>
</tr>
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<td>MHz</td>
</tr>
<tr>
<td>PW</td>
<td>.01</td>
<td>µs</td>
</tr>
<tr>
<td>$k$</td>
<td></td>
<td>BW/PW MHz/µs</td>
</tr>
<tr>
<td>PRI</td>
<td>5.6</td>
<td>ms</td>
</tr>
<tr>
<td>Pulses per CPI</td>
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<td></td>
</tr>
<tr>
<td>$\Delta\theta$ per Pulse</td>
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<td>Degrees</td>
</tr>
<tr>
<td>Taylor Sidelobe Level</td>
<td>−35</td>
<td>dB</td>
</tr>
</tbody>
</table>

Figure 2.3: DMF result for a bed-of-nails scene. The radar kinematic and waveform parameters are provided in Tab. 2.1.

isons of image quality. Note that the DMF does not deskew the contoured grid, as done in the PFA and other interpolation-based algorithms, but rather suppresses sidelobes through forward-model correlation and windowing. Since the forward model is largely based on $K$, any error of $K$ in the IFP degrades the desired “thumbtack” point-spread function. This degradation is demonstrated in Fig. 2.4. Note that in Figs. 2.3 and 2.4 the residual sidelobe
Figure 2.4: DMF result for a bed-of-nails scene with a 5 percent error in the range estimate.

energy does not follow the original contour grid or a Cartesian grid. Different IFPs may produce different sidelobe patterns, as discussed in Appendix C.

### 2.2.2 Phase Analysis

The challenges due to the coupled range/cross-range information embedded in the waveform are observed through phase analysis. Equations (2.6) and (2.7) show that the $K$ information is coupled in both the time delay and Doppler shift. The importance of this coupling is provided through an inspection of the time delay of a single point scatterer between two pulses separated in time. As seen in Figure 2.5, as the emitter travels, both the range and cross-range of a point in the scene change, such that throughout a CPI the point scatterer is highly unlikely to be illuminated at the same range or cross-range more than once. This behavior demonstrates that both $x$ and $y$ are functions of time and that single scatterers will have relative velocity that maps one-to-one with angle [15, 22]. The phase history is formed by first collecting 1D signals at different angles. The entire 1D collection is then
Figure 2.5: The change in range and cross-range contours for two different points of a flight path. With a pulse transmitted at each angle, the target resides on different range and cross-range contours, resulting in a change in phase.

mapped into a 2D fast-time $t_f$, slow-time $t_s$ array, where for $M$ pulses

$$t_s \in (0, MPRI) \quad (2.13)$$

$$\Delta t_s = \text{PRI} \quad (2.14)$$

$$t_f \in (0, \text{PRI}) \quad (2.15)$$

$$\Delta t_f = \frac{\text{PW}}{\text{TBW}}, \quad (2.16)$$

where the time-bandwidth product TBW is a measure of pulse compression [11, 12]. An example of the phase argument response from a single scatterer throughout a synthetic aperture is provided in Fig. 2.6 which demonstrates that the phase arguments provide a
Figure 2.6: Ideal phase arguments with $x = 10$ km, $y$ from $-500 : 500$ m, similar to Fig. 2.5, and radar parameters consistent with Tab. 2.1.

phase history as a result of the changes in $x$ and $y$.

### 2.2.3 $\kappa$ Error, Moving Targets, and Auto-Focus

Spotlight SAR data collection is subject to numerous sources of error, most of which take the form of a phase error, such as range migration and antenna vibration. Phase errors may affect the resulting image globally or locally and there are both space-invariant and space-variant auto-focus techniques that serve to reduce the error to some degree. Auto-focus has been studied in depth by the authors of [13, 15, 16] et al.\(^4\)

\[^4\]Many space-variant auto-focus techniques are the result of block processing (or sub-scene processing) the resulting image with the same algorithms employed by space-invariant auto-focus techniques [16].
The phase of the received signal, written as a combination of Eqs. (2.6)-(2.8) as

\[
\phi(x_n, y_n, v_n)_m = j2\pi \left( f_c + \frac{2v_f c y_n}{c \sqrt{x_n^2 + y_n^2}} \right) \left( t - \frac{2 \sqrt{x_n^2 + y_n^2}}{c} \right) + j\pi k \left( t - \frac{2 \sqrt{x_n^2 + y_n^2}}{c} \right)^2,
\]

expresses the forward model’s phase in terms of \(K\). An error for each \(K\) parameter may be written as a fraction \(\delta\) of the true parameter value, such that \(\phi\) may be expressed as

\[
\phi(x_{n,m}, y_{n,m}, v_{n,m}) |_{x_{n,m}=X_{n,m}+\delta x, y_{n,m}=Y_{n,m}+\delta y, v_{n,m}=V_{n,m}+\delta v},
\]

where \(X, Y,\) and \(V\) are the true range, cross-range, and velocity of the emitter. Platform motion error causes quadratic phase error which results in defocus of the image [13, 15, 16, 35]. The example from Fig. 2.6 is expanded in Fig. 2.7 to demonstrate the changes in time delay and Doppler shift history due to a constant \(K\) error percentage. This phase error is also seen in the presence of a moving target, where the relative coordinates and velocity of the radar with respect to the moving target change differently than the rest of the stationary scene. Equivalent to the changes in the phase arguments in Eq. (2.18), this error results in a local smearing effect that would typically be corrected through a space-variant auto-focus technique. This smearing effect is detailed in depth by Carrara, and extends to any source of accumulating \(K\) error [13, 15, 16, 21].

If an ECM system is developed which modulates a received waveform based on forward-model calculations, the same error effects are expected to be present and independent of errors which are radar induced. For example, if an ECM system produces a response based on the true \(K\), and the emitter has some error that results from antenna vibration, the error would effect the jamming energy distribution as well as the normal return energy. Therefore, any processing steps that alter the focus of the scene data will also alter the focus of the jamming energy distribution. The development of an ECM modulation
Figure 2.7: Ideal phase arguments with an estimation error of $x = \text{Est} \times X$, where Est is the scale factor listed in the legend. The radar parameters used here are the same as used for Fig. 2.6.

scheme that is based on the forward model is presented next.

### 2.3 ECM Development

SAR ECM systems may exploit the same electromagnetic properties of reflection and scattering to change the radar’s waveform causing a deceptive image. In this section, theoretical analysis shows that the unknown processing, unknown depth of auto-focus used by the SAR system, and SAR’s inherent dependency on $K$ modeling, require the ECM system to estimate $K$ in order to appropriately modulate the radar’s waveform to achieve a desired jamming energy distribution.
An ECM system may receive a signal from the emitter, modify it, and transmit it back to the emitter in order to embed false information in the waveform. As previously stated, the jammer is assumed to have basic DRFM capabilities, such as those detailed by the authors of [1]. With the appropriate waveform modification this may result in the formation of a desired energy distribution in the final image. Additive jamming energy $J$ augments the received signal as

$$s_{Rx}(t) = s_{Rx}(\rho(t), s_{Tx}(t), K(t)) + s_{Rx}(\rho_J(t), s_{Tx}(t), K_J(t)),$$

where $\rho_J$ represents the scene density that the ECM signal embeds and $K_J$ is the jammer’s estimate of $K$.

The DRFM applies the amplitude, time, and frequency shifts to the waveform in different ways. First, the amplitude is likely to be held at a maximum in order to maintain the highest possible jamming-to-signal ratio. The time shift is accomplished through delaying the transmission of the recorded pulse for an appropriate amount of time. If the ECM system is capable of estimating the radar’s PRI then the target may also be effectively placed in front of the jammer in range [1, 2, 3]. The frequency shift may be applied through multiplication of the recorded signal with a designed coefficient or set of coefficients [2, 3, 26, 27].

An approach to ECM waveform development begins with the generation of a single false target at a desired point in the reconstructed image. As seen in Fig. 2.8, from the jammer’s perspective at location $(x_J, y_J)$ a received pulse has a specific angle of arrival and therefore requires different modulation coefficients in order to produce a false target displaced by some $\Delta x_1$ and $\Delta y_1$ from the jammer. Assuming the radar position is $x_r =$
Figure 2.8: Illumination grid geometry. The radar is located at \((x_r, y_r)\) moving in a \(+y\) direction, the jammer is at \((x_j, y_j)\), and the desired false target displacement is some \((\Delta x_1, \Delta y_1)\) from the jammer and some angle \(\theta\) from the \(+y\) velocity vector of the radar.

If \(y_r = 0\), the waveform received by the jammer is

\[
s_{Tx}(t - t_J) = A_0 p(t - t_J) e^{j2\pi (f_c + \epsilon f_J)(t - t_J) + j\pi k(t - t_J)^2}
\]

\[
t_J = \sqrt{x_J^2 + y_J^2} / c
\]

\[
f_J = vy_J f_c / c \sqrt{x_J^2 + y_J^2}
\]

\[
\epsilon = \begin{cases} 
1 & \text{if } y_j + \Delta y_1 - y_r \geq 0 \\
-1 & \text{otherwise}
\end{cases}
\]

Note that the pulses received by the jammer already have time and Doppler shifts as would be expected for the one-way propagation between the radar and the jammer. Based on the range between the radar and the jammer \(\sqrt{(x_J - x_r)^2 + (y_J - y_r)^2}\), as well as the desired displacements of \(n\) false targets relative to the jammer, the ECM system’s return signal \(s_J\) is a modulated copy of the received radar signal. The ECM system’s responding signal may be expressed as

\[
s_J(t) = s_{Tx}(t - t_J - \tau_n) e^{j2\pi (f_c + f_J + f_n)(t - t_J - \tau_n)}
\]
where the $K$ based time shift (Eq. (2.6)) and Doppler shift (Eq. (2.7)) are calculated through

$$
\begin{align*}
  x_1 &= x_J - x_r + \Delta x_1 \\
  y_1 &= y_J - y_r + \Delta y_1 \\
  v_1 &= v_r + \Delta v \\
  \tau_n &= 2 \left( \frac{\sqrt{x_1^2 + y_1^2}}{c} - t_j \right) \\
  f_n &= 2 \left( \frac{v_1 y_1 f_c}{c \sqrt{x_1^2 + y_1^2}} - f_J \right)
\end{align*}
$$

(2.25)  
(2.26)  
(2.27)  
(2.28)  
(2.29)

to embed the $K$-based false target facsimile into the ECM’s waveform. Note that Eqs. (2.28) and (2.29) are both offset by one propagation factor relative to the jammer, $t_j$ and $f_j$, in order to ensure that once the waveform propagates from the jammer to the receiver the time delay and Doppler shift specific to the jammer will no longer be in the waveform. To demonstrate, assume the jammer received an LFM waveform modeled by Eq. (2.20). Following Eq. (2.24), the modulated signal transmitted by the jammer is

$$
\begin{align*}
  s_J(t) &= p(t - t_J)e^{-j2\pi(f_c + f_J)(t - t_J) - jk(t - t_J)^2} \\
  &\times p(t - \tau_n)e^{-j2\pi f_n t} \\
  &= p \left( t - \frac{2 \sqrt{x_1^2 + y_1^2}}{c} + t_J \right) e^{-j\Phi} \\
  \Phi &= 2\pi \left( f_c + \frac{2v_1 y_1 f_c}{c \sqrt{x_1^2 + y_1^2}} - \frac{v y_J f_c}{c \sqrt{x_J^2 + y_J^2}} \right) \left( t - \frac{2 \sqrt{x_1^2 + y_1^2}}{c} + t_J \right) \\
  &\quad - \pi k \left( t - \frac{2 \sqrt{x_1^2 + y_1^2}}{c} + t_J \right)^2
\end{align*}
$$

(2.30)  
(2.31)  
(2.32)
such that the forward model at the face of the receiver becomes

\[
\begin{align*}
    s_{Rx} & = p \left( t - \frac{2\sqrt{x_1^2 + y_1^2}}{c} \right) e^{-j\Phi} \\
    \Phi & = 2\pi \left( f_c + \frac{2v_1y_1f_c}{c\sqrt{x_1^2 + y_1^2}} \right) \left( t - \frac{2\sqrt{x_1^2 + y_1^2}}{c} \right) \\
    & \quad - \pi k \left( t - \frac{2\sqrt{x_1^2 + y_1^2}}{c} \right)^2, \quad (2.34)
\end{align*}
\]

as would be expected from the pulse return of a scatterer at the false target’s location.

A numerical example is provided with the following radar, ECM, and false target location \((x_F, y_F)\) and displacement in meters:

\[
\begin{align*}
x_r, y_r & = (0, 0) \quad (2.35) \\
x_J, y_J & = (400, 300) \quad (2.36) \\
x_F, y_F & = (500, 1200) \quad (2.37) \\
\Delta x_1, \Delta y_1 & = (100, 900), \quad (2.38)
\end{align*}
\]

as shown in Fig. 2.8. The radar emits a pulse which propagates to the ECM location. The signal model changes according to Eq. (2.20) at the ECM’s reception of the pulse such that

\[
\begin{align*}
t_J & = \frac{500}{c} \quad (2.39) \\
f_J & = \frac{v_r f_c}{c} \frac{300}{500}. \quad (2.40)
\end{align*}
\]

The ECM system delays and modulates the received signal according to Eqs. (2.28) and (2.29)
such that

\[ \tau_n = \frac{1300 - 500}{c} \] (2.41)

\[ f_n = \frac{v_r f_c}{c} \left( \frac{1200}{1300} - \frac{300}{500} \right). \] (2.42)

The one-way propagation from the jammer back to the radar results in the forward model expressed in Eq. (2.34) with the following time and Doppler shifts

\[ \tau_n = \frac{2(1300)}{c} = \frac{2\sqrt{500^2 + 1200^2}}{c} \] (2.43)

\[ f_n = \frac{2v_r f_c}{c} \left( \frac{1200}{1300} \right). \] (2.44)

as would be expected from the pulse return of a scatterer at the false target’s location.

### 2.4 Summary

Equation (2.24) provides the $K$-based modulation which an ECM system may implement in order to ensure the desired jamming energy distribution forms in the final image. Where there are multiple ways for an ECM system to estimate the SAR platform’s $K$, the level of precision required to maintain confidence in the resultant jamming energy distribution quality may be found through modeling and simulation.
Methodology

3.1 Proposed Study

To support the hypothesis that the quality of the resulting jamming energy distribution is limited by the accuracy of the forward-model based ECM system’s estimation of the radar’s $K$, the sensitivity of jamming energy distributions to $K$ error is assessed through Monte Carlo simulation. These simulations have a defined accuracy as discussed in Sec. 3.2.2.

Since the $K$ variables $x_1$, $y_1$, and $v_1$, defined by Eqs. (2.25), (2.26), and (2.27), do not contribute to the phase equivalently, they are evaluated individually to observe any characteristics that are unique to each parameter. Three sets of Monte Carlo simulations are performed, each with one $K$ variable treated as a uniform random variable and the other two as constants. For each Monte Carlo set the random variable is given a standard deviation $\sigma$, individually denoted as $\sigma_{x_1}$, $\sigma_{y_1}$, and $\sigma_{v_1}$. Following signal modeling and DMF processing, the Jaccard Distance is used to quantify the final image’s similarity with an ideal image, discussed further in Sec. 3.2.3. The ideal image is generated through a zero-error $K$ calculation and a noiseless system. Note that the exact source of the $K$-estimation error is not of interest here.

Since any jamming distribution may be made through the use of a sufficient number of false point targets, the degradation of a single false target located at the CRP is evaluated. The results are presented in 2D graphs that compare each standard deviation of one $K$ parameter and the average Monte Carlo result of the Jaccard Distance to allow one to
form a conclusion of how sensitive the jamming energy distribution is to $K$ error within a specified confidence level.

### 3.2 Experiment Procedure

The combination of the emitter models, Monte Carlo accuracy derivation, and Jaccard distance metric, discussed next, provide the tools required to assess the sensitivity of an ECM system’s control of the jamming energy distribution to $K$ errors. The following steps are proposed:

1. Generate the reference image; an ideal and noiseless SAR image based on Tabs. 3.1 and 3.2.
2. Demonstrate the average Jaccard distance for a $-20$ to $20$ signal-to-noise ratio sweep.
3. For each standard deviation $\sigma_{x_r}$, $\sigma_{y_r}$, and $\sigma_{v_r}$, empirically find the equivalent Jaccard distance standard deviation $\hat{\sigma}$. This step and those that follow are noiseless. The standard deviations for $\sigma_{x_r}$ and $\sigma_{y_r}$ range between $[10^{-3}, 10^2]$ m with a logarithmic step size. The standard deviations for $\sigma_{v_r}$ range between $[10^{-3}, 10^2]$ m/s with a logarithmic step size.
4. Estimate the necessary number of Monte Carlo trials $N$ for each $\hat{\sigma}$, found in step 2, through Eq. (3.6) with the confidence coefficient $z_c$ that corresponds to the confidence level of 98%. These steps provide that with 98% confidence the estimated Jaccard distance for each specific $\sigma$ is accurate to within 4% of the true average. The values of 98% and 4% were arbitrarily chosen.
5. Run the Monte Carlo trials for each $N$ corresponding to the appropriate $\sigma$. MATLAB®’s $\text{rand}$ function is used to calculate the appropriate population samples (Sec. 3.2.2) [34].
6. Display the trends of each $\sigma$ versus the average Jaccard distance and discuss any key characteristics of the results.

3.2.1 Emitter Model

For the proposed assessment, the true kinematic model $K$ of the SAR emitter platform is limited to the range and velocity model provided in Eq. (2.10). The simulated SAR signal is structured within typical SAR waveform parameters as defined by Skolnik, and is modeled as an LFM for pulse compression and consistency with most SAR literature [13, 15, 18] et al. The radar’s parameters are listed in Tab. 3.1. It is also assumed that the radar has an ideal estimate of its own flight path to negate motion drift error in the IFP, and that the transmission power levels of both the radar and jammer are such that the signal power received at the face of the radar satisfies the jammer-to-signal ratio (JSR) value in Tab. 3.1.

A straight flight path with a side-facing emitter is utilized, and slant angle/elevation is ignored. With respect to Fig. 2.8, the radar vehicle travels in the $+y$ direction, and the beginning coordinates of the SAR system, ECM system, and false target are provided in Tab. 3.2. Note that $x_1$ and $y_1$ are located at the origin such that $\Delta x_1 = \Delta y_1 = 100$, as shown in Fig. 2.8. The ECM system is assumed to be residing within the illuminated area and

<table>
<thead>
<tr>
<th>Table 3.1: Experiment Radar Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carrier Frequency</td>
<td>11 GHz</td>
</tr>
<tr>
<td>PRI</td>
<td>1.374 s</td>
</tr>
<tr>
<td>Effective Range Gate</td>
<td>1.1 $\mu$s</td>
</tr>
<tr>
<td>Pulse Width</td>
<td>3$\mu$s</td>
</tr>
<tr>
<td>Bandwidth</td>
<td>100 MHz</td>
</tr>
<tr>
<td>Range to CRP</td>
<td>10 km</td>
</tr>
<tr>
<td>Velocity</td>
<td>200 m/s</td>
</tr>
<tr>
<td>Acceleration</td>
<td>0 m/s$^2$</td>
</tr>
<tr>
<td>Number of Pulses per CPI</td>
<td>100</td>
</tr>
<tr>
<td>JSR</td>
<td>1</td>
</tr>
</tbody>
</table>
Table 3.2: Scene Location List

<table>
<thead>
<tr>
<th>Coordinate</th>
<th>Meters from Origin</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x_r)</td>
<td>-10000</td>
</tr>
<tr>
<td>Initial (y_r)</td>
<td>-129.85</td>
</tr>
<tr>
<td>(x_J)</td>
<td>-100</td>
</tr>
<tr>
<td>(y_J)</td>
<td>-100</td>
</tr>
<tr>
<td>(x_1)</td>
<td>0</td>
</tr>
<tr>
<td>(y_1)</td>
<td>0</td>
</tr>
</tbody>
</table>

capable of transmitting pulses before or after the reception of any pulse once the first pulse has been received. It is assumed that independent pulse-to-pulse \(K\) estimates are made by the ECM system. Any ECM response delay due to waveform parameter estimation is ignored, as this would only be expected to slightly degrade the coherently-processed result in amplitude. The radar and ECM system’s noise figures are ignored, however the Jaccard distance for various SNR levels is generated for reference. Any other system-specific types of error, such as DRFM-playback jitter, pulse-recording accuracy, and hardware-induced spectral spurs, are ignored in order to isolate the hypothesis-supporting effects. The ECM system will modulate each pulse for the false target through Eq. (2.24).

### 3.2.2 Monte Carlo Accuracy

As shown by Driels and expanded on by Oberle, the number of Monte Carlo trials required to achieve a specific confidence level may be determined through random variable estimators [36, 37]. For \(N\) samples, when \(N\) is sufficiently large, the Central Limit Theorem provides that the distribution of a sample mean \(\hat{\mu}\) and sample variance \(\hat{\sigma}^2\), calculated from independent trials, has a Gaussian distribution with

\[
\hat{\mu} \approx \mu \quad (3.1)
\]
\[
\hat{\sigma}^2 \approx \frac{\sigma^2}{N}, \quad (3.2)
\]
Table 3.3: Confidence levels and \( z_c \) [36, 37].

<table>
<thead>
<tr>
<th>( z_c )</th>
<th>99.75 %</th>
<th>99 %</th>
<th>98 %</th>
<th>96 %</th>
<th>95 %</th>
<th>90 %</th>
<th>80 %</th>
<th>68 %</th>
<th>50 %</th>
</tr>
</thead>
<tbody>
<tr>
<td>( z_c )</td>
<td>3</td>
<td>2.58</td>
<td>2.33</td>
<td>2.05</td>
<td>1.96</td>
<td>1.645</td>
<td>1.28</td>
<td>1</td>
<td>0.6745</td>
</tr>
</tbody>
</table>

where \( \mu \) and \( \sigma \) are the population mean and standard deviation, respectively [33, 34, 36, 37].

Along with the well known confidence coefficients \( z_c \) (Tab. 3.3) for confidence levels of a normal distribution, a confidence interval (CI) is defined by

\[
\hat{\mu} - \frac{z_c \sigma}{\sqrt{N}} < \mu < \hat{\mu} + \frac{z_c \sigma}{\sqrt{N}}.
\] (3.3)

Further, subtracting Eq. 3.3 by \( \hat{\mu} \), the percentage of error of the mean \( \varepsilon \) is

\[
- \frac{100z_c \hat{\sigma}}{\sqrt{N}} < 100(\mu - \hat{\mu}) < \frac{100z_c \hat{\sigma}}{\sqrt{N}},
\] (3.4)

with a maximum of

\[
\varepsilon = \frac{100z_c \hat{\sigma}}{\sqrt{N}}.
\] (3.5)

Note that here both the population average \( \mu \) and the sample average \( \hat{\mu} \) are assumed to be in units of some normalized metric, therefore Eq. (3.5) differs from the results in [36, 37] by a normalization factor. Finally, the number of trials required for a desired \( \varepsilon \) is found by solving Eq. (3.5) for \( N \) such that

\[
N = \left( \frac{100z_c \hat{\sigma}}{\varepsilon} \right)^2.
\] (3.6)

### 3.2.3 Image Similarity Metric

In order to quantify the similarity between ideal and non-ideal results, the Jaccard distance (also referred to as the shared information distance) is utilized. While correlation-based...
measurements are valid, they are more sensitive to translational changes in the image due to their nature as a measurement of linear dependency between two objects than entropy-based methods. Euclidean distance and other Minkowski distances are spatially dependent and therefore also more sensitive to translational shifts than entropy-based methods. Therefore, an entropy-based measurement was chosen due to its increased sensitivity to structural changes in the compared images [32].

The authors of [31, 32] demonstrate that many entropy based measures used in the past to compare image similarity do not satisfy the fundamental requirements for a distance metric, commonly known as the identity axiom, the triangle inequality, the symmetry axiom, and that a distance is always positive (App. D). However, they provide the following definition of the Jaccard distance which does satisfy these requirements:

\[ d(A, B) = H_{AB} - MI, \]  
\( (3.7) \)

where \( H_{AB} \) is the joint entropy of images \( A \) and \( B \), and \( MI \) is their mutual information [32, 33]. Since “image similarity” is not an easily perceived distance, the Jaccard Distance is more clearly understood (and less subjective) when normalized such that

\[ D(A, B) = \frac{d(A, B)}{H_{AB}} \in [0, 1], \]  
\( (3.8) \)

where a value of zero denotes when \( A \) and \( B \) are the most similar, and a value of one denotes when \( A \) and \( B \) are the most dissimilar. Translational, rotational, and noise affects on the Jaccard Distance are demonstrated in App. D.

Note that while the Jaccard distance is being used for image similarity measurements, the images themselves represent cross-ambiguity functions of the transmitted signal and the scene, as expressed in Eq. (2.3). Small shifts in the scene may cause the sidelobes to act constructively or destructively, so it is possible for a small shift in the \( K \) estimate to
cause an increase or decrease in the Jaccard distance. However, as the K error continues to increase, a generally increasing (furthering) trend is expected in the Jaccard distance.
Results

Following step 1, as defined in Sec. 3.2, the ideal reference image was formed from the geometry shown in Fig. 4.1a, as described by the parameters given in Tabs. 2.1 and 3.2. The ideal range history for both the ECM location and the false target location is shown in Fig. 4.1b. The generated comparison image is shown in Fig. 4.2. Following step 2, as defined in Sec. 3.2, the average Jaccard distance for various SNR values is demonstrated in Fig. 4.3.

Following steps 3 and 4, the standard deviation of the Jaccard $\hat{\sigma}$ for each $K$ parameter was directly calculated at $\sigma_x$, $\sigma_y$, and $\sigma_v$. These values were used to calculate the required number of trials $N$. The resulting $N$ trials required to achieve a 98% confidence level that the Monte Carlo results for each step in standard deviation are within 4% of the true average is shown in Fig. 4.4.
Figure 4.1: a) Simulation scene. As the radar travels in the positive cross-range direction, a pulse echoes from the ECM Location along a blue line. The ECM system alters the waveform such that the data resembles a pulse return from the false target location along the appropriate path, shown as a red line. b) The range history of both the ECM system and the false target. This range history is equivalent to the length of the red and blue lines in (a) for each pulse.

Figure 4.2: Top: Upper 60 dB of comparison image generated with no noise and an ideal \( K \). Bottom: Upper 30 dB of local energy distribution region of the comparison image.
Figure 4.3: Jaccard estimates for noise-only error. 1000 Monte Carlo trials were run for each SNR.
Figure 4.4: Calculated number of trials required to meet the specified confidence level for the velocity and location random variables required to achieve a 98% confidence level that the result is within 4% of the true average, according to Eq. (3.6).
Finally, the resulting average Jaccard distance for each $K$ parameter is given in Fig. 4.5. Select results from the iterations along the range-axis (X) and cross-range axis (Y) are provided in Figs. 4.6 and 4.7, respectively.

Besides the expected translational shifts of the false targets, from a visual perspective the jamming energy distributions appear to undergo limited blurring until the $K$ estimate is greater than 50 meters in both range and cross-range at an approximate Jaccard distance of 0.6. The velocity estimate has no impact on the jamming energy distribution (assuming independent pulse-to-pulse $K$ estimates), as expected due to use of the DMF IFP.
4.1 Use Case Example

While the results shown are unique to the DMF IFP and an LFM waveform, they demonstrate the usefulness of the assessment method. Any other waveform or IFP will have similar but unique results which may be assessed through the same method. Further constraining the Jaccard calculation to a minimum desired dB from the peak of the mainlobe and isolating a region of interest (likely the mainlobe, regardless of translation) would improve the intuition afforded by the results. Also, evaluating the point spread function at every individual pixel would allow for the development of Jaccard contour maps, which would be valuable since the change in the jamming energy distribution is not likely symmetric about any axis. The Jaccard contour maps for all 3 \( \mathcal{C} \) parameter pairs, (x,y), (x,v), and (y,v), may assist in identifying ECM system regions of operability.
Figure 4.6: Select example results in range (X) dimension.

a) Left: DMF result of $\hat{\sigma}_x = 0.1172$ m, Jaccard = 0.0777. Right: Zoomed and scaled version of left image.

b) Left: DMF result of $\hat{\sigma}_x = 1.2690$ m, Jaccard = 0.4281. Right: Zoomed and scaled.

c) Left: DMF result of $\hat{\sigma}_x = 4.1750$ m, Jaccard = 0.7829. Right: Zoomed and scaled.

d) Left: DMF result of $\hat{\sigma}_x = 67.2336$ m, Jaccard = 0.9919. Right: Zoomed and scaled, note that the distribution resides near the scene extent.
Figure 4.7: Select example results in range (Y) dimension.
a) Left: DMF result of $\hat{\sigma}_y = 0.1172$ m, Jaccard = 0.0534. Right: Zoomed and scaled version of left image.
b) Left: DMF result of $\hat{\sigma}_y = 1.2690$ m, Jaccard = 0.3941. Right: Zoomed and scaled.
c) Left: DMF result of $\hat{\sigma}_y = 4.1750$ m, Jaccard = 0.6485. Right: Zoomed and scaled.
d) Left: DMF result of $\hat{\sigma}_y = 67.2336$ m, Jaccard = 0.9806. Right: Zoomed and scaled, note that the distribution resides near the scene extent.
Conclusion

Throughout this thesis it has been shown that the quality of the resulting jamming energy distribution is clearly limited to the ECM system’s $\mathcal{K}$ estimate accuracy. The modeling of the LFM forward model allowed for the development of the forward-model based false-target ECM method. The appropriately constrained Jaccard distance allowed for the change in jamming energy distributions to be measured. The Central Limit Theorem and Law of Large Numbers allowed for the Jaccard distance calculations to be bounded within a specified level of accuracy. Further, the assessment method was extended to a practical use-case example.

In general, this thesis has provided that an ECM system designer may assess the worst-case $\mathcal{K}$ estimation as a limit to the accuracy and quality of jamming energy distribution formation. Future work may include an extension to moving target indicators, optimum scene design with regards to resolvability, development of Jaccard contour maps, and the development of a tracking-jamming combined system.
Bibliography


Appendix A

Information Bounds and Waveform Parameters

With an understanding of the change that the information of the scene makes in the received waveform, limits of the information provide parameters for the radar. Through the projection-slice theorem, each projection (Fig. A.1) provides a slice of the 2D spectrum of the image which is offset by $f_c$.\(^1\) Multiple slices form an annulus (Fig. A.2) [14, 16].

Resolution

The length of a slice is provided through the range resolution

$$\Delta x = \frac{cPW}{2T_{BW}}$$ \hspace{1cm} (A.1)

where $T_{BW}$ is the time-bandwidth product and $PW$ is the pulse width. The slice length is therefore

\(^1\)The 2D spectrum of an image is typically represented in $K$ space. What will be seen here is that the Fourier data collected is a shifted and skewed version of the $K$ space of the scene.
Figure A.1: One projection samples the illuminated area in a polar format. Range is calculated based on time delay from the center Doppler line. Cross-range is mapped through the phase measurements for each isorange delay.

\[
\Delta x^{-1} = \frac{2TBW}{cPW} = \frac{2BW}{c} \text{cycles/m} = \frac{4\pi BW}{c} \text{rad/m}. \tag{A.2}
\]

which is centered at

\[
\frac{4\pi f_c}{c}. \tag{A.3}
\]

(Fig. A.2 (γ)) [15, 16, 17]. Similarly, the width of the collection of all of the slices through the entire \(\Delta \theta\) is based on the cross-range resolution
Figure A.2: A collection of slices in Fourier space. The annulus represents the skewed image K-space in the form of Doppler shifts, centered vertically on $f_r$ at $\gamma = 4\pi f_c/c$. Note that only the shaded portion of each slice is collected due to the bandlimited nature of radars.

\begin{align*}
\Delta y &= \frac{c}{2f_c\Delta \theta} \quad \text{(A.4)} \\
\Delta y^{-1} &= \frac{4\pi f_c\Delta \theta}{c} r_{rad}/m \quad \text{(A.5)}
\end{align*}

at Eq. A.3 [15, 16, 17]. Eqs. A.2 and A.5 provide the dimensions of the portion of image spectrum that is collected. As discussed and seen in figures A.1 and A.2, the resulting encoded data is in polar-format, and each sample contains identifiable range and phase information due to the geometric relationship between the emitter and the scene [11, 24].
With full knowledge of the radar parameters, the radar may extract the scene data from the encoding waveform and produce an image.

**Range Gating**

The return of $s_{Rx}$ allows for the scatterer range to be calculated. Since spotlight SAR is focused on the reconstruction of an image of a specific size, but standoff between the scene and the emitter is physically required, range gating is often utilized to minimize processing requirements. The distance to the radar, $x_0$, along with the desired scene size in range ($L_x$), provides the window time within the pulse repetition interval (PRI) to be

$$t_w = \frac{2x_0}{c} \pm \max \left( \frac{L_x}{c}, \frac{\text{PRI}}{2} \right),$$

as demonstrated in Fig. A.3.

For example, a comparison of desired image span in range versus time window size is provided in Fig. A.4.
Figure A.4: $T_w$ bounds for range gating based on desired scene size $L_x$, $x_0 = 10$ km.
Appendix B

Linear Frequency Modulated Waveform Parameters

SAR was first developed through the observation that at any time $t$, an individual point scatterer will provide a unique Doppler shift ($f_d$) due to the variation of the emitter velocity relative to each scatterer

$$f_d = \frac{2vf_c \cos(\theta_c)}{c} \quad (B.1)$$

where $v$, $\theta_c$, $f_c$, and $c$ represent the velocity, cone angle, carrier frequency, and speed of light, respectively; which is utilized by some $\tilde{A}$ [22]. Disregarding any moving scatterers in the scene relative to the ground, the desired scene size in cross-range ($L_y$) determines the angular deviation ($\Delta \theta$) for each cross-range bin and therefore the effective bandwidth ($\beta_{eff}$) for that scene (Fig. B.1(b)).

$$\Delta \theta = \sin^{-1} \left( \frac{L_y}{x_0} \right) \quad (B.2)$$

$$\beta_{eff} = \frac{2vf_c \cos(\theta_c \pm \Delta \theta)}{c} \quad (B.3)$$

Since longer collect ranges are inversely proportional to $\Delta \theta$, the desired $\Delta CR$ may
Figure B.1: Effective bandwidth extents for a given scene size ($L_x \times L_y$). Note that $\beta_{eff}$ is equivalent to twice the max $f_d$ deviation from $f_c$. $x_0 = 10$ km, $v = 340.29$ m/s, $f_c = 10$ GHz, and $\theta_c = \pi/4$ rad.

not be achievable without increasing the $TBW$. Although other techniques exist, this is almost exclusively achieved through the utilization of a linear FM waveform (LFM) [11, 14, 13, 15, 16, 17, 18]. With a chirp rate $k$ and pulse width $PW$, this alters the instantaneous frequency such that $\beta_{eff}$ increases,

$$\beta_{eff} = \frac{2v(f_c + k(t - nPW)) \cos(\theta_c \pm \Delta\theta)}{c}$$  \hspace{1cm} (B.4)

therefore increasing the $f_d$ deviation, demonstrated in Fig. B.2 where the Doppler shift relative to the CRP is slightly less than 1 kHz, versus the non-LFM result in Fig. B.1(b), where the Doppler shift relative to the CRP is approximately 150 Hz.
Figure B.2: Effective bandwidth extents for a given scene size $(L_x \times L_y)$ and LFM waveform. The same parameters were used here as for Fig. B.1, along with a $PW = 10$ us, and $K = 50$ MHz/us.
Appendix C
SAR Processing Model

1. **Sampling**: \( s_{\text{Rx}} \) is sampled at some rate \( \Delta t_f \) that satisfies the Nyquist sampling requirement of \( s_{\text{Tx}} \) such that the samples form a sampling index \( t_n \).

2. **Demodulation and Shaping**: Many image formation processes utilize demodulation-on-receive in order to minimize computational cost. Regardless, the data is then shaped into a 2D fast-time, slow-time map \( (s_{\text{Rx}}(t_f, t_s)) \). The combination of the PRI and \( t_n \) provides the indices for the fast-time and slow-time [11]. Although the emitter is constantly traveling, for simplicity it is assumed that for each individual PRI the emitter remains static. This implies a general physical position, velocity, and acceleration analysis being evaluated only along the \( t_s \) index. Further, this allows \( t_s \) and \( \theta \) to be used interchangeably. \( s_{\text{Rx}}(t_f, t_s) \) is formed in order to facilitate image formation through manipulation of the information history afforded by Fourier and/or correlation theory.

3. **Image Formation Algorithm**: An image formation algorithm \( (\tilde{A}) \) is utilized in order to extract the information encoded in \( s_{\text{Rx}}(t_f, t_s) \) and focus the data to a general 'top-down' viewpoint image \( (\psi) \) such that

\[
\psi(t_f, t_s) = \tilde{A}(s_{\text{Rx}}(t_f, t_s)).
\]  

(C.1)
The method used for the examples in this paper is the downsampled matched filter (DMF). This method generates the physical model response individually for a target at each pixel, such that for an \( N \times M \) image there will be \( NM \) forward model response generations. The value estimated for each pixel is the result of the squared dot product of the received fast-time, slow-time map and the corresponding individual forward model response \( (s_M) \) \(^{[19]}\). This value represents the radar cross section of the pixel area in proportion to the rest of the map. This method does not directly remap the coordinate system to Cartesian to utilize traditional processing, but calculates the correlation of the received data and a possible reflector at each pixel such that

\[
\psi(x_0, y_0) = \overrightarrow{s_{R_x}}(\rho, s_{T_x}, \mathcal{K}) \cdot \overrightarrow{s_{M}}(\delta(x - x_0, y - y_0), s_{T_x}, \mathcal{K}). \tag{C.2}
\]

4. **Autofocus:** It is common for errors to be present in the data which may be space-invariant and/or space-variant. Typically an autofocus technique \( (\hat{a}) \) is utilized in order to form the final focused image \( \tilde{\rho} \) through

\[
\tilde{\rho} = \hat{a}(\psi). \tag{C.3}
\]

For example, one common autofocus technique is the *Map Drift Algorithm* which is primarily utilized to account for errors in \( \mathcal{K} \). It functions through finding some correction coefficient \( \Delta_{err} \) by finding the displacement from zero of the maximum of

\[
\psi_1 \ast \psi_2 \tag{C.4}
\]

where \( \psi_1 \) and \( \psi_2 \) represent halves of the data (subapertures) with respect to the desired axis and \( \ast \) represents cross-correlation. For side-looking radar, since the
Table C.1: Processing Model Variable Listing

<table>
<thead>
<tr>
<th>Variable/Operation</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_{Rx}$</td>
<td>Signal received by emitter</td>
</tr>
<tr>
<td>$\Delta t_f$</td>
<td>Sampling frequency</td>
</tr>
<tr>
<td>$t_n$</td>
<td>1D sampling index</td>
</tr>
<tr>
<td>$t_f$</td>
<td>Fast-time index, equivalent to $t_n$ for one PRI relative to the CRP</td>
</tr>
<tr>
<td>$t_s$</td>
<td>Slow-time index, equivalent to $\theta$</td>
</tr>
<tr>
<td>$\tilde{A}$</td>
<td>Image formation algorithm resulting in an image $\psi$</td>
</tr>
<tr>
<td>$\hat{a}$</td>
<td>Auto-focus application resulting in a final image product $\tilde{\rho}$</td>
</tr>
<tr>
<td>$\tilde{\rho}$</td>
<td>Final image</td>
</tr>
</tbody>
</table>

quadratic phase error induced by $\mathcal{K}$ inaccuracies is symmetric about the time axis, $\Delta_{err}$ provides the appropriate error correction coefficient [16]. Further autofocus considerations are discussed in Section 2.2.3.

For further clarity, the previously stated variables and operations are restated in Table C.
Appendix D

Jaccard Distance Examples

The Jaccard Distance, or *shared information distance*, satisfies the fundamental requirements as an image similarity metric, which are the following: the *identity axiom*, the *triangle inequality*, the *symmetry axiom*, and its value is always positive [31, 32]. In order to provide some level of intuition into the magnitude of the Jaccard Distance between an image $A$ and a modified version of that image, $B$, as $B$ degrades from $A$, translational, rotational, and combined sweeps of the offsets between $A$ and $B$ are shown in Fig. D.1. Further, Fig. D.2 shows the histogram of the Jaccard Distance for 1000 Monte Carlo trials for

$$D(A, B)\bigg|_{B=A+n} \quad \text{(D.1)}$$

where

$$n \sim \mathcal{N}(0, 0.2^2) \quad \text{w.g.n.} \quad \text{(D.2)}$$
Figure D.1: Translational and rotational affect on the normalized Jaccard Distance for the top image.
Figure D.2: Monte Carlo result histogram of the Jaccard Distance between an image $A$ and $B = A + n$, where $n \sim \mathcal{N}(0, .02^2)$ w.g.n. for 1000 trials.
Appendix E

Linear Frequency Modulated Waveform Model and Processing

The transmitted LFM waveform

\[ s_{Tx}(t) = A_0 p(t) e^{-j\Phi(t)} \]  
\[ \Phi(t) = 2\pi f_0 t + \pi kt^2, \]

where \( t \), \( p \), \( A_0 \), \( f_0 \), and \( k \) denote time, pulse envelope, amplitude, carrier frequency, and chirp rate, respectively. \( s_{Tx} \) interacts with the scene reflectivity density \( \rho \) such that a portion of the signal energy is reflected back towards the emitter, and may be modeled with some time shift \( \tau \) such that the signal received by the radar \( s_{Rx} \) is

\[ s_{Rx} = \rho(t - \tau) * s_{Tx}(t - \tau). \]

Applying the standard 2-way range equation

\[ t = \frac{2r}{c}, \]
where \( r \) is the range and \( c \) is the speed of light, to Eq. (E.3) provides

\[
s_{Rx} = \rho(r) * s_{Tx}(t - \frac{2r}{c}). \tag{E.5}
\]

Showing the scene reflectivity density \( \rho \) as \( N \) scatterers

\[
\rho(r) = A_\delta (r - r_n) \tag{E.6}
\]

allows Eq. (E.5) to be represented as

\[
s_{Rx} = \rho(r) \cdot s_{Tx}(t) \tag{E.7}
\]

\[
= A_\delta (r - r_n) \cdot s_{Tx}\left(t - \frac{2r_n}{c}\right) \tag{E.8}
\]

\[
= A_\delta s_{Tx}\left(t - \frac{2r_n}{c}\right) \tag{E.9}
\]

\[
= A_\delta A_0 p\left(t - \frac{2r_n}{c}\right) e^{j2\pi f_0\left(t - \frac{2r_n}{c}\right) + j\pi k\left(t - \frac{2r_n}{c}\right)^2}. \tag{E.10}
\]

The kinematic model of the radar \( K \) is represented through

\[
r - r_n = \sqrt{(x - x_n)^2 + (y - y_n)^2 + (z - z_n)^2} \tag{E.11}
\]

and therefore the scene \( \rho \) may be shown as relative to the radar's \( K \) as

\[
\rho(x, y, z) = A_\delta (x - x_n) \delta(y - y_n) \delta(z - z_n). \tag{E.12}
\]
Processing the received signal with a matched filter provides the result $y(t)$ through

$$y(t) = s_{Rx}(t) * s_{Tx}^*(t - \frac{2r_0}{c})$$ (E.13)

$$= A_n A_0 p(t - \frac{2r_n}{c}) e^{j\Phi(t - \frac{2r_n}{c})} \cdot A_0^{-1} p(t - \frac{2r_0}{c}) e^{-j\Phi(t - \frac{2r_0}{c})}$$ (E.14)

$$= A_n p(t - \frac{2r_0}{c}) e^{j2\pi f_c (t - \frac{2r_0}{c}) + j\pi k (t - \frac{2r_0}{c})^2} \times e^{j2\pi f_c (t - \frac{2r_0}{c}) + j\pi k (t - \frac{2r_0}{c})^2}$$ (E.15)

$$= A_n p(t - \frac{2r_0}{c}) e^{j\left(\frac{4\pi f_c}{c}(r_0 - r_n) + \frac{4\pi k}{c}(r_0 - r_n) + \frac{4\pi k}{c}(r_n^2 - r_0^2)\right)}$$ (E.16)

Expanding the range terms provides that

$$(r_n^2 - r_0^2) = (r_n^2 - 2r_n r_0 + r_0^2) + 2r_n r_0 - 2r_0^2$$

$$= (r_n - r_0)^2 + 2r_n r_0 - 2r_0^2$$ (E.17)

$$y(t) = A_n p\left(t - \frac{2r_0}{c}\right) e^{j\left(\frac{4\pi f_c}{c}(r_0 - r_n) + \frac{4\pi k}{c}(r_0 - r_n) + \frac{4\pi k}{c}(r_n^2 - r_0^2)\right)}.$$ (E.18)

The deramp process produces

$$y(t) = A_n p\left(t - \frac{2r_0}{c}\right) e^{j\frac{4\pi}{c}(f_c + k t)(r_0 - r_n) + k((r_n - r_0)^2 + 2r_n r_0 - 2r_0^2)}.$$ (E.19)

which may be simplified and written to isolate the time dependent terms through

$$t' = t - \frac{2r_0}{c}$$ (E.20)

$$\Delta r = r_n - r_0$$ (E.21)

$$y(t') = \left(p(t')e^{-j\frac{4\pi k t'}{c}\Delta r}\right)\left(A_n e^{-j\frac{4\pi f_c}{c}\Delta r + j\frac{4\pi k}{c}\Delta r^2}\right).$$ (E.22)
The spectrum of the processed signal provides the superimposed Doppler shifts which are typically mapped to cross-range locations. The spectrum of $y$ is found through

$$Y(\zeta) = \mathcal{F}\{y(t')\}$$

$$= \int_{-\infty}^{\infty} y(t') e^{-j\zeta t'} dt'$$

$$= A_n e^{-j \frac{4\pi fc}{c} \Delta r + j \frac{4\pi k}{c^2} \Delta r^2} \int_{-\infty}^{\infty} p(t') e^{-j \frac{4\pi k}{c} \Delta r} e^{-j \zeta t'} dt'.$$  \hspace{1cm} (E.25)

The spectrum $Y$ is further simplified by allowing

$$\zeta_n = \frac{4\pi k}{c} \Delta r \quad \text{s.t.}$$

$$Y(\zeta) = A_n e^{-j \frac{4\pi fc}{c} \Delta r + j \frac{4\pi k}{c^2} \Delta r^2} \int_{-\infty}^{\infty} p(t') e^{-j \zeta_n t'} e^{-j \zeta t'} dt'.$$  \hspace{1cm} (E.27)

If a rectangular pulse is assumed,

$$Y(\zeta) = A_n e^{-j \frac{4\pi fc}{c} \Delta r + j \frac{4\pi k}{c^2} \Delta r^2} \text{sinc}(\zeta) \delta(\zeta - \zeta_n)$$

$$= A_n e^{-j \frac{4\pi fc}{c} \Delta r + j \frac{4\pi k}{c^2} \Delta r^2} \text{sinc}(\zeta - \zeta_n),$$

and a single $\zeta_n$ produces

$$|Y(\zeta_n)| = A_n \text{sinc}(\zeta - \zeta_n).$$

The width of the $\text{sinc}(\zeta)$ mainlobe is the range resolution $\delta r$ as shown in Eq. (A.1), which may also be shown with Eq. (E.26) through

$$\delta \zeta = \frac{2\pi}{PW c} = \frac{4\pi k \delta r}{c},$$

$$\delta r = \frac{c}{2kPW} = \frac{c}{2BW}.$$  \hspace{1cm} (E.31) \hspace{1cm} (E.32)

The velocity of a scatterer $\vec{v}_g$ relative to the velocity of the radar $\vec{v}_r$ creates a Doppler shift $f_d$ in $s_{Tx}$ when incident with the scatterer. The phase of the received signal, from
Eq. (E.2) with a Doppler shift is then

\[ \Phi(t) = 2\pi \left( f_c + \frac{2f_c}{c} |\vec{v}_g - \vec{v}_a| \left( \frac{\vec{v}_g - \vec{v}_a}{|\vec{v}_g - \vec{v}_a|} \cdot \frac{\vec{r}_g - \vec{r}_a}{|\vec{r}_g - \vec{r}_a|} \right) \right) t + \pi \left( k + \frac{2f_c}{c} |\vec{v}_g - \vec{v}_a| \left( \frac{\vec{v}_g - \vec{v}_a}{|\vec{v}_g - \vec{v}_a|} \cdot \frac{\vec{r}_g - \vec{r}_a}{|\vec{r}_g - \vec{r}_a|} \right) \right) t^2, \]  

(E.33)
given the model shown in Fig. E.1, where \( \vec{v}_r \) and \( \vec{r}_r \) are the velocity and range of the radar, \( \vec{v}_g \) and \( \vec{r}_g \) are the velocity and range of a point on the ground, and \( \vec{k} \) is the range from the radar to the point on the ground. Provided that the Doppler shift

\[ f_d = \frac{2f_c}{c} |\vec{v}_g - \vec{v}_a| \left( \frac{\vec{v}_g - \vec{v}_a}{|\vec{v}_g - \vec{v}_a|} \cdot \frac{\vec{r}_g - \vec{r}_a}{|\vec{r}_g - \vec{r}_a|} \right), \]  

(E.34)
equation (E.33) is simplified as

\[ \Phi(t) = 2\pi (f_c + f_d) t + \pi (k + f_d) t^2. \]  

(E.35)

Noting that the Doppler shift effects both the carrier frequency and the LFM rate, Eq. (E.22)
may be used to show both the time and frequency shifts as

\[ y(t') = A_n p(t') e^{-j \frac{4\pi (k + f_d) t'}{c}} \Delta r - j \frac{4\pi (f_c + f_d) \Delta t}{c} + j \frac{4\pi (k + f_d) \Delta r^2}{c^2} \]  

(E.36)

\[ = A_n p(t') e^{-j 2\pi (k + f_d) t' \Delta t} - j 2\pi (f_c + f_d) \Delta t + j \pi (k + f_d) \Delta r^2 \]  

(E.37)

\[ = A_n p(t') e^{-j 2\pi (f_c + f_d) \Delta t - j 2\pi (k + f_d) t' \Delta t} + j \pi (k + f_d) \Delta t^2. \]  

(E.38)

Equation (E.38) provides a typical expression for the matched filter result of an LFM signal, where the left third of the phase contains all of the needed scene information to locate the scatterer. The remaining phase terms make up the well known residual video phase, which may be corrected [13, 15, 16, 18].