Implementation of RF Steganography Based Joint Radar/Communication LFM Waveform Using Software Defined Radio

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By

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Abstract


As communication and radar technology continue to become increasingly sophisticated, the sophistication of technologies used by unintended parties to acquire transmitted information increases in direct proportion. As a result, entities such as the military and commercial communication industries require methods to protect transmitted information from undesired recipients. Furthermore, the frequency spectrum, a finite resource, is becoming increasingly congested due to inefficient utilization. This thesis presents a novel RF steganography concept that uses linear frequency modulated (LFM) radar signals capable of optimizing the use of the frequency spectrum and hiding digital communication within the LFM to covertly transmit to legitimate recipients. Finally, this work demonstrates that these joint radar/communication waveforms can be designed, transmitted, and received, using software defined radio.
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Dedication

I dedicate this paper to my mom and dad, my sister, my closest friends, and future scholars, that they may find the knowledge that they seek.
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1 Introduction

1.1 Motivation

There is a strong desire by not only military, but commercial parties to protect the transmission and receiving of information. It is not uncommon to see news of a hostile entity hacking military or industrial databases and exploiting the information they obtain for destructive purposes. In response to this, efforts have been put forth to ensure that any data being received and transmitted is done so in a way that no hostile, or any other unintended parties are involved.

While today’s methods to protect information are sophisticated, hostiles in response develop more and more robust ways of working around these protection measures. This is especially concerning in terms of defense. Communication plays a very important role in military defense operations, and if communication channels are compromised, that could determine which way the pendulum swings in terms of military advantage, which could potentially lead to undesirable outcomes. It is because if this that innovative ways to protect communication signals are sought.

The proposed method of communication signal protection outlined is this paper is an attempt to compensate for the less effective methods of signal protection. Here, propose that we take resources already available in normal military operation practices, in an attempt to simultaneously alleviate stress on other limited resources, while protecting information contained within communication signals.
1.2 Communication Signal Discretion/RF Steganography

In the most recent years, researchers have developed increasingly efficient ways to transmit and procure higher and higher volumes of data. By manipulating the electromagnetic spectrum, there are those who can use these methods, with intentions that might include: Electronic attacks, such as jamming communications or radar functions; Electronic protection, which might entail designing jamming resistant systems; and Electronic support, which includes supplying necessary intelligence and threat recognition to allow effective attack and protection [1]. These methods are commonly known as *electronic warfare* (EW). EW, as suggested, can be used with either hostile or amicable intent. Specifically in military operations and other covert operations, it is imperative that countermeasures are taken to prevent hostiles, or any other parties from detecting the presence of radio frequency (RF) communication [2].

One very commonly researched method of maintaining communication discretion is developing *low probability of detection* (LPD) RF waveforms by making it such that the power spectrum density of the waveform is lower than the ambient noise floor [2]. This method utilizes spread spectrum technologies such as direct-sequence spread spectrum or frequency hopping [3]. One other common method is exploiting noise like signals, such as chaotic signals, to carry information [4].

In this experiment, we use a different, less investigated approach to hiding communication. With this method, rather than hiding the waveform itself using LPD designs, we propose to hide the communication signal in another form of RF transmissions. This method is called *RF steganography* [1]. For the purpose of this experiment, we will be performing our RF steganography by embedding a communication signal within a LFM signal, as the LFM is a commonly used signal in modern radar systems. We will be using a newly developed modulation scheme as our communication signal (which will be discussed later in this paper) to optimize on the
effectiveness of the steganography. This newly designed joint radar/communication waveform will still be an effective radar signal, which will provide range and Doppler measurements to radar operators, while concurrently carrying a communication signal to intended recipients [1].

1.3 Thesis Outline

Chapter 1 outlines the motivation behind the research and the main beneficiaries. A brief background of the method used in this experiment is covered as well.

Chapter 2 will start off with a description of our main method of analyzing our waveforms, known as second-order cyclostationary analysis. The chapter will then provide a more in depth background of the mathematics of the carrier RF waveform of interest, as well as all of the appropriate derivations of the waveform and the new combined RF/communication waveforms. The radar performance of the modulation types will be discussed as well. The chapter will go on to briefly describe SDR implementation, and the capabilities of our own Universal Software Radio Peripheral (USRP) equipment.

Chapter 3 will outline the methods and procedures used for conducting the experiment. It will start by describing how theoretical analysis was done, then move on to methods used to manipulate the waveforms for our USRP transmit and receive interface. Finally, The chapter will discuss the methods of analyzing our real world received data.

Chapter 4 will show the results of the experiment. It will display graphs of the spectral correlation functions of the different modulation types. The final results of the different waveforms will also be compared to each other. Lastly, the methods of demodulation, as well as a BER performance of the final modulation type will be discussed.

Chapter 5 will be a final inspection of the data. It will discuss possible alternatives
to the methods used throughout this experiment, as well as any future work and applications that may come from this research.
2 Background

2.1 Second-Order Cyclostationary Analysis

In this experiment it was determined that using robust methods to identify characteristics of our signals would be essential in understanding how to manipulate those characteristics in order to avoid detection. Here, second-order cyclostationary analysis is used to outline these characteristics.

Conventional signal processing methods assume that random processes are statistically stationary, as in the parameters used to generate the signals do not vary with time. However, most man made signals seen in communication, radar, and other systems, contain parameters that vary periodically with time [5–7]. This is known as a cyclostationary process. In some cases even multiple, not harmonically related periodicities are involved. Examples include: sinusoidal carriers in amplitude, phase, and frequency modulation systems; periodic keying of the amplitude, phase, or frequency in digital modulation systems; or even periodic motion in rotating machinery [5].

While there might be some instances where signal processors will ignore cyclic characteristics of received signals, more advanced detectors can use these cyclostationary features to detect the presence of a signal, determine its parameters, and uncover and decrypt any underlying communication signal [5]. To understand how second-order cyclostationary analysis allows for the identification of these parameters, it is important to look at the cyclic autocorrelation function and spectral correlation function.
2.1.1 Cyclic Autocorrelation Function

The limit cyclic autocorrelation is a generalization of the conventional limit autocorrelation and limit spectrum, which is a fundamental parameter when determining random data from constant phenomena \[7,8\]. First, assume we have a complex signal, \(x(t)\). The mean value of \(x(t)\), is given by \[9,10\]

\[
M_x(t, \tau) = E[x(t + \tau)], \quad (2.1)
\]

where \(\tau\) is the lag parameter.

For cyclostationary signals, the mean value is independent of \(\tau\). The autocorrelation function (AF) (also known as the temporal lag product series) is the correlation of a random signal with itself at two different time instances. This is written as,

\[
R_x(t_1, t_2) = E[x(t_1) x^*(t_2)] \quad (2.2)
\]

To centralize this around some time \(t\), this can be rewritten as,

\[
R_x(t, \tau) = E[x\left(t + \frac{\tau}{2}\right) x^*\left(t - \frac{\tau}{2}\right)] \quad (2.3)
\]

where,
\[
\begin{align*}
t_1 &= t + \tau/2 \\
t_2 &= t - \tau/2
\end{align*}
\]

and
\[
\begin{align*}
t &= (t_1 + t_2)/2 \\
\tau &= t_1 - t_2
\end{align*}
\]

For cyclostationary signals, the AF can be a periodic function or a nearly periodic
function. For both of these cases, a Fourier series can be used to represent the autocorrelation,

$$R_x(t, \tau) = \sum_{\alpha} R_x^\alpha(\tau)e^{j2\pi\alpha t} \quad (2.4)$$

where $R_x^\alpha(\tau)$ is a Fourier coefficient called the cyclic autocorrelation function (CAF), and $\alpha$, is referred to as the cycle frequencies (CF). The CAF can be defined as,

$$R_x^\alpha(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} R_x(t, \tau) e^{-j2\pi\alpha t} dt \quad (2.5)$$

where $T$ is the period, and $\alpha = m/T$.

### 2.1.2 Spectral Correlation Function

The spectral correlation function (SCF) is the most exploited characterization of cyclostationarity of random signals. Much like how a power spectrum is the spectral density of variance, the spectral correlation function is the spectral density of covariance. The SCF is found by taking the Fourier transform of the CAF, which is called the Wiener-Khintchine theorem [9–12],

$$S_x^\alpha(f) = \int_{-\infty}^{\infty} R_x^\alpha(\tau) e^{-j2\pi\alpha f} d\tau \quad (2.6)$$

where $\alpha$ is the cyclic frequency, and $f$ is the spectrum frequency.

An important formula, which will be used for the derivations seen later in this chapter, is SCF of a compound signal. The compound signal $y(t)$ is found to be the product of two statistically independent signals $x(t)$ and $\omega(t)$ [12]. From here, the SCF of the compound signal can be written as:
\[ S_y^\alpha(f) = \sum_\beta \int_{-\infty}^{\infty} S_x^{\alpha-\beta}(f-v)S_{\omega}^\beta(v)dv \] (2.7)

where \( v = f - \alpha/2 \), and \( S_y^\alpha(f) \), \( S_x^\alpha(f) \), and \( S_{\omega}^\alpha(f) \) are the spectral correlation functions of the signals \( y(t) \), \( x(t) \), and \( \omega(t) \) respectively. This formula, which is referred to as \textit{spectral correlation convolution} [12], will be used later in chapter two to derive the SCFs of the communication signal embedded radar signal.

\subsection*{2.2 Linear Frequency Modulation Waveform}

The \textit{linear frequency modulation} (LFM), otherwise known as the chirp signal, is one of the first bio-inspired signals, and is commonly used in RF applications such as radar and sonar [13]. The term “chirp” is in reference to the sound made by birds when they communicate. Birds can use these chirp noises to convey sophisticated communication signals to other birds. It is this that inspired the LFM signal [14,15]. As opposed to standard continuous wave, \( x(t) = \cos(2\pi f_c t) \), where the frequency is fixed, the instantaneous frequency, \( f(t) \) of a chirp signal varies linearly with time. Here, [12,14]

\[ f(t) = f_0 + kt \] (2.8)

where \( f_0 \) is the starting frequency, and \( k \) is the rate of frequency increase. The equation for the LFM is expressed as:

\[ x(t) = A_c \cos \left( 2\pi f_0 t + 2\pi \frac{k}{2} t^2 + \phi_0 \right), 0 \leq t \leq T \] (2.9)
where, $A_c$ is the amplitude of the signal, $\phi_0$ is the phase offset of the signal, and
\[ k = \frac{f_1 - f_0}{T} \]
where $f_1$ is the ending frequency, and $T = t_1 - t_0$ is the chirp duration, where $t_1$, and $t_0$ are the times corresponding to the ending and starting frequencies respectively. It should be noted that when the rate of frequency increase, $k$, is larger than 0, the frequencies are changing in a positive direction, which is known as an up-chirp, as shown in figure 2.1. When $k$ is less than 0, this is known as a down-chirp, as shown in figure 2.2.

![Figure 2.1: Up-chirp Signal.](image1)

![Figure 2.2: Down-Chirp Signal.](image2)

The LFM provides the capability of interference rejection, as well as low Doppler sensitivity, which makes it an area of interest for the purposes of communication [14].

### 2.2.1 Autocorrelation and Fourier Coefficients/Cyclic Autocorrelation of LFM Signal

Let’s assume that the LFM waveform can be represented as:

\[ c(t) = A_c \cos \left( 2\pi \psi_0 t + \pi \psi' t^2 + \phi_0 \right), -\frac{T_c}{2} \leq t \leq \frac{T_c}{2} \]  

(2.10)
where $A_c$ is the amplitude of the signal, $\psi_0$ and $\phi_0$ are the center frequency and initial phase (respectively) when $t = 0$, and $\psi'$ is the frequency modulation rate.

The complex form of the signal can be given as [12]:

$$c(t) = 0.5A_c e^{j[2\pi\psi_0 t + \pi\psi' t^2 + \phi_0]} + 0.5A_c e^{-j[2\pi\psi_0 t + \pi\psi' t^2 + \phi_0]}$$  \hspace{1cm} (2.11)$$

Note that the single side band spectrum of the signal can be approximated as a rectangular function of frequency.

$$F_c(f) \approx \sqrt{\frac{1}{2\psi'}} W\left(\frac{f - \psi_0}{\psi' T_c}\right) e^{j[\phi_0 + \Phi_{\psi'}(f-\psi_0)]}$$  \hspace{1cm} (2.12)$$

where,

$$W(f) = \begin{cases} 
1 & -0.5 \leq f \leq 0.5 \\
0 & otherwise. 
\end{cases}$$

and,

$$\Phi_{\psi'}(f) = \frac{\pi}{4} - \frac{\pi f^2}{\psi'}$$  \hspace{1cm} (2.13)$$

This approximation will be used for the SCF derivations that will be done later in this chapter [12]. From here, the CAF can be found from the AF, $R_c(t, \tau)$ which is
The four different components of the AF can be denoted as: $R_{c,1}(t,\tau)$, $R_{c,2}(t,\tau)$, $R_{c,3}(t,\tau)$, and $R_{c,4}(t,\tau)$. Next, we can obtain the CAF by taking the Fourier transform on $R_c(t,\tau)$. Since $c(t)$ is of a finite length of $T_c$, the nonzero values of the AF will be as follows [12]:

$$
R_c(t,\tau) = c(t + \tau/2) c(t - \tau/2)
$$

$$
= 0.5A \left\{ e^{j[2\pi\psi_0(t+\tau/2)+\pi\psi'(t+\tau/2)^2+\phi_0]} + e^{-j[2\pi\psi_0(t+\tau/2)+\pi\psi'(t+\tau/2)^2+\phi_0]} \right\}
$$

$$
* 0.5A \left\{ e^{j[2\pi\psi_0(t-\tau/2)+\pi\psi'(t-\tau/2)^2+\phi_0]} + e^{-j[2\pi\psi_0(t-\tau/2)+\pi\psi'(t-\tau/2)^2+\phi_0]} \right\}
$$

$$
= 0.25A_c^2 e^{j[2\pi\psi_0 t + 2\pi\psi'(t^2 + \tau^2/4) + 2\phi_0]} + 0.25A_c^2 e^{-j[2\pi\psi_0 t + 2\pi\psi'(t^2 + \tau^2/4) + 2\phi_0]}
$$

$$
+ 0.25A_c^2 e^{j[2\pi\psi_0 \tau + 2\pi\psi'\tau]} + 0.25A_c^2 e^{-j[2\pi\psi_0 \tau + 2\pi\psi'\tau]}
$$

(2.14)

This makes it so that the integral range of the Fourier transform is set from $-(T_c - |\tau|)/2$, to $(T_c - |\tau|)/2$. The Fourier transform of $R_c(\tau)$, which is gives the
CAF, is written as [12]:

\[
R_c^\alpha(\tau) = \frac{1}{T_0} \int_{-(T_c-|\tau|)/2}^{(T_c-|\tau|)/2} R_c(t, \tau) e^{-j2\pi\alpha t} dt
\]

\[
= \frac{0.25 A_c^2}{T_0} e^{j[\pi(\psi'/2)\tau^2 + 2\phi_0]} \ast \int_{-(T_c-|\tau|)/2}^{(T_c-|\tau|)/2} e^{j[2\pi2\psi_0t + \pi2\psi't^2]} e^{-j2\pi\alpha t} dt
\]

\[
+ \frac{0.25 A_c^2}{T_0} e^{-j[\pi(\psi'/2)\tau^2 + 2\phi_0]} \ast \int_{-(T_c-|\tau|)/2}^{(T_c-|\tau|)/2} e^{-j[2\pi2\psi_0t + \pi2\psi't^2]} e^{-j2\pi\alpha t} dt
\]

\[
+ \frac{0.25 A_c^2}{T_0} e^{j(2\pi\psi_0\tau)} \ast \int_{-(T_c-|\tau|)/2}^{(T_c-|\tau|)/2} e^{j(2\pi\psi't\tau)} e^{-j2\pi\alpha t} dt
\]

\[
+ \frac{0.25 A_c^2}{T_0} e^{-j(2\pi\psi_0\tau)} \ast \int_{-(T_c-|\tau|)/2}^{(T_c-|\tau|)/2} e^{-j(2\pi\psi't\tau)} e^{-j2\pi\alpha t} dt
\]  \hspace{1cm} (2.16)

The four additive components of the CAF can be denoted as: \( R_{c,1}^\alpha(\tau) \), \( R_{c,2}^\alpha(\tau) \), \( R_{c,3}^\alpha(\tau) \), and \( R_{c,4}^\alpha(\tau) \). Taking the first component, \( R_{c,1}^\alpha(\tau) \), can be defined as [12]:

\[
R_{c,1}^\alpha(\tau) = \frac{0.25 A_c^2}{T_0} e^{j[\pi(\psi'/2)\tau^2 + 2\phi_0]} \ast \int_{-(T_c-|\tau|)/2}^{(T_c-|\tau|)/2} e^{j[2\pi2\psi_0t + \pi2\psi't^2]} e^{-j2\pi\alpha t} dt \hspace{1cm} (2.17)
\]

The integration of the \( R_{c,1}^\alpha(\tau) \) component is the Fourier transform of a complex LFM with a center frequency of \( 2\psi_0 \) and a frequency modulation rate of \( 2\psi' \). The result of which can be approximated as rectangular along the \( \alpha \) axis for a particular value of \( \tau \). The \( \alpha \) axis is centered at \( \alpha = 2\psi_0 \), with a width of \( 2\psi' \). From here, \( R_{c,1}^\alpha(\tau) \) can be approximated to be [12]

\[
R_{c,1}^\alpha(\tau) = \begin{cases} 
\approx \frac{0.25 A_c^2}{T_0} e^{j[\pi(\psi'/2)\tau^2 + 2\phi_0]} \sqrt{1 \over 4\psi'} * W \left( \frac{\alpha - 2\psi_0}{2\psi'(T_c - |\tau|)} \right) \\
* e^{j2\psi'(\alpha - 2\psi_0)} & -|\tau| \leq T_c \\
0 & \text{otherwise.}
\end{cases}
\]  \hspace{1cm} (2.18)
Similarly, \( R_{c,2}^\alpha(\tau) \) can be approximated to be,

\[
R_{c,2}^\alpha(\tau) = \begin{cases}
\approx \frac{0.25 A^2}{T_0} e^{-j \left[ \pi (\psi'/2)^2 + 2\phi_0 \right]} \sqrt{\frac{1}{4\psi'}} W \left( \frac{\alpha + 2\psi_0}{2\psi' (T_c - |\tau|)} \right) & -|\tau| \leq T_c \\
0 & \text{otherwise.}
\end{cases}
\] (2.19)

Next, the component \( R_{c,3}^\alpha(\tau) \) is defined as,

\[
R_{c,3}^\alpha(\tau) = \frac{0.25 A^2}{T_0} e^{j(2\pi \psi_0 \tau)} \ast \int_{-(T_c - |\tau|)/2}^{(T_c - |\tau|)/2} e^{j(2\pi \psi' \tau)} e^{-j2\pi \alpha t} \, dt \] (2.20)

and the integral part of \( R_{c,3}^\alpha(\tau) \) can be rewritten as

\[
\int_{-(T_c - |\tau|)/2}^{(T_c - |\tau|)/2} e^{-j[2\pi(\alpha - \psi') \tau]} \, dt \\
= (T_c - |\tau|) \frac{\sin \left[ \pi (\alpha - \psi' \tau) (T_c - |\tau|) \right]}{\pi (\alpha - \psi' \tau) (T_c - |\tau|)} \\
= (T_c - |\tau|) \text{sinc} \left[ \pi (\alpha - \psi' \tau) (T_c - |\tau|) \right] \] (2.21)

similarly, for \( R_{c,4}^\alpha(\tau) \), this can be expressed as

\[
\int_{-(T_c - |\tau|)/2}^{(T_c - |\tau|)/2} e^{-j[2\pi(\alpha + \psi') \tau]} \, dt \\
= (T_c - |\tau|) \frac{\sin \left[ \pi (\alpha + \psi' \tau) (T_c - |\tau|) \right]}{\pi (\alpha + \psi' \tau) (T_c - |\tau|)} \\
= (T_c - |\tau|) \text{sinc} \left[ \pi (\alpha + \psi' \tau) (T_c - |\tau|) \right] \] (2.22)
These equations come out to be a group of sinc functions which are centered at a linear line where \( \alpha = \psi' \tau \), for any given \( \tau \). Figure 2.3 below shows a graph of the CAF in the \( \alpha - \tau \) plane [12].

![Figure 2.3: CAF of LFM](image)

The SCF of the function \( c(t) \) can now be calculated by taking the Fourier transform of the calculated CAF, \( R_c^\alpha(\tau) \) [12].

\[
S_c^\alpha(f) = \frac{1}{T_0} \int_{-T_c}^{T_c} \int_{-(T_c-|\tau|)/2}^{(T_c-|\tau|)/2} R_c(t, \tau)e^{-j2\pi\alpha t}e^{-j2\pi ft}dtd\tau \\
= \frac{0.25A_c^2}{T_0} \int_{-T_c}^{T_c} \int_{-(T_c-|\tau|)/2}^{(T_c-|\tau|)/2} \left\{ e^{j[2\pi2\psi_0 t+2\pi\psi'(t^2+\tau^2/4)+2\phi_0]} + e^{-j[2\pi2\psi_0 t+2\pi\psi'(t^2+\tau^2/4)+2\phi_0]} \\
+ e^{j(2\pi\psi_0 \tau+2\pi\psi' \tau)} + e^{-j(2\pi\psi_0 \tau+2\pi\psi' \tau)} \right\} e^{-j2\pi\alpha t}e^{-j2\pi ft}dtd\tau 
\]  

(2.23)
Of whose components come out to be $S_{c,1}^{\alpha}(f)$, $S_{c,2}^{\alpha}(f)$, $S_{c,3}^{\alpha}(f)$, and $S_{c,4}^{\alpha}(f)$. The figure below shows an image of the SCF of an LFM [12].

The four components are dimensionally identical to each other. Components $S_{1,c}^{\alpha}(f)$ and $S_{2,c}^{\alpha}(f)$ are symmetrical to each other about the $f$ axis. Similarly, components $S_{3,c}^{\alpha}(f)$ and $S_{4,c}^{\alpha}(f)$ are symmetrical to each other about the $\alpha$ axis.

2.2.2 Restricted SCF of LFM Waveform

Here, the lag value to get $R_c(t, \tau)$ is restricted to a small range. The expression of the restricted AF is given as,

$$ R'_c(t, \tau) = \begin{cases} 
R_c(t, \tau) & |\tau| \leq \tau_\omega, \\
0 & |\tau| > \tau_\omega.
\end{cases} \quad (2.24) $$

where $\tau_\omega$ is a constant that is less than $T_c$. The restricted AF may yield a restricted CAF, $R'_C^\alpha(t, \tau)$, which may be thought of as a restricted version of $R_c^\alpha(\tau)$. The
restricted CAF, $R_{c}^\alpha(t, \tau)$, is made up of four components: $R_{c,1}^\alpha(\tau)$, $R_{c,2}^\alpha(\tau)$, $R_{c,3}^\alpha(\tau)$, and $R_{c,4}^\alpha(\tau)$, which correspond to the limited versions of $R_{c,1}^\alpha(t, \tau)$ to $R_{c,4}^\alpha(t, \tau)$ in $R_{c}^\alpha(t, \tau)$ [12]. Figure 2.5 below depicts the restricted CAF of an LFM signal.

![Figure 2.5: Restricted CAF of LFM](image)

The SCF can be derived from the CAF, which is written as $S_{c}^\alpha(f)$, and is referred to as the restricted SCF [12]. The restricted SCF can be broken into four parts: $S_{c,1}^\alpha(f)$, $S_{c,2}^\alpha(f)$, $S_{c,3}^\alpha(f)$, and $S_{c,4}^\alpha(f)$, which correspond to $R_{c,1}^\alpha(\tau)$ to $R_{c,4}^\alpha(\tau)$. Looking
first at \( S'_{c,1}(f) \), it can be derived as [12],

\[
S'_{c,1}(f) = \int_{-\tau_0}^{\tau_0} R_{c,1}^\alpha(\tau) e^{-j2\pi f \tau} d\tau
\]

\[
\approx \int_{-\tau_0}^{\tau_0} \frac{0.25A_e^2}{T_0} e^{j[\pi(\psi'/2)\tau^2 + 2\phi_0]} e^{j2\pi f \tau} d\tau
\]

\[
\approx \frac{0.25A_e^2}{T_0/4\psi'} e^{j[\pi(\psi'/2)\tau^2 + 2\phi_0]} e^{j2\pi f \tau} d\tau
\]

\[
\approx \frac{0.25A_e^2}{2T_0\psi'} e^{j[\pi(\psi'/2)\tau^2 + 2\phi_0]} e^{j2\pi f \tau} d\tau
\]

(2.25)

where,

\[
\tau_b = \begin{cases} 
0 & \alpha \leq 2\psi_0 - \psi' T_c, \\
\frac{\alpha - 2\psi_0}{\psi'} + T_c & 2\psi_0 - \psi' T_c < \alpha \\
\tau_0 & \leq 2\psi_0 - \psi' T_c + \psi' \tau_\omega \\
\psi_0 - \psi' T_c + \psi' \tau_\omega & 2\psi_0 - \psi' T_c + \psi' \tau_\omega < \alpha \\
\leq 2\psi_0 + \psi' T_c - \psi' \tau_\omega & \leq 2\psi_0 + \psi' T_c - \psi' \tau_\omega \\
(2\psi_0 - \alpha)/\psi' + T_c & 2\psi_0 + \psi' T_c - \psi' \tau_\omega < \alpha \\
\leq 2\psi_0 + \psi' T_c & \leq 2\psi_0 + \psi' T_c \\
0 & \alpha > 2\psi_0 + \psi' T_c 
\end{cases}
\]

(2.26)

It can be stated that \( S'_{c,1}(f) \) Fourier transformed from \( R_{c,1}^\alpha(\tau) \) with respect to a particular \( \alpha \) value is the spectrum of an LFM signal. The maximum range of \( \tau \) is \([-\tau_\omega, \tau_\omega]\), so the maximum width of the double sideband spectrum is contained within the boundaries, \([-\psi' \tau_\omega, \psi' \tau_\omega]\). Therefore, \( S'_{c,1}(f) \) can be seen as a restricted version of \( S_{c,1}(f) \) within the limits of \( f = \pm \psi' \tau_\omega \). The same can be assumed for \( S'_{c,2}(f) \). When
It is shown that \( S'_{c,1}(f) \) and \( S'_{c,2}(f) \) are approximately two rectangles with a width of \( 2\psi'\tau\omega \) on the \( \alpha \) and a height of \( \psi'T_c \) on the \( f \) axis [12].

Looking at the \( S'_{c,3}(f) \) and \( S'_{c,4}(f) \) components, recall that they are the transforms of \( R'_{c,3}(f) \) and \( R'_{c,4}(f) \), which are sinc functions. The main lobes of these sinc functions are centered at approximately \( \tau = \alpha/\psi' \). Any restriction on \( \tau \) will also result in a restriction on \( \alpha \). When the parameters of \( \tau \) are limited to \( \tau = \pm \tau\omega \), \( R'_{c,3}(f) \), and therefore, \( S'_{c,3}(f) \), will be limited by \( \alpha = \tau\omega\psi' + 1/(T_c - \tau\omega) \), and \( \alpha = -\tau\omega\psi' - 1/(T_c - \tau\omega) \). This is also the case for \( S'_{c,4}(f) \). Figure 2.6 shows an \( \alpha \) versus \( f \) plot of the limited LFM SCF [12]. The image shows the different components of \( S_{c}(f) \) and their limits.

![Figure 2.6: Restricted LFM SCF](image)

### 2.2.3 Radar Performance of LFM Waveform

The proposed RF steganography technique applies some modifications to the LFM chirp signal. As these modifications affect the performance of the radar signal,
it is important to see if and by how much the radar performance is degraded. One common way to determine the radar performance is by looking at its ambiguity function, which is essentially the Fourier transform of the cross-correlation of the radar pulse with its matched filter [14,16]. The ambiguity function shows the measure of the responding matched filter to a finite energy signal affected by a time delay $\tau$ and a Doppler frequency shift, $v$. The periodic ambiguity function is expressed as [14]:

$$|\chi(\tau, v)| = \left| \int_0^T u(t)u^*(t-\tau)e^{2\pi jvt} dt \right|$$  (2.27)

Figure 2.7 below shows the ambiguity function of a generic, unmodulated LFM waveform.

Figure 2.7: Unmodulated LFM Ambiguity Function

This will serve as a basis for comparison against the ambiguity functions of the LFM radar waveforms with different modulation types. These different types will be discussed later in this chapter.
2.3 Modulation of LFM with Communication Signal

The concept here is to use digital modulation to embed the communication signal into the LFM waveform. This will be aptly referred to as a *joint radar/communication signal*. [2, 14, 16]. Most radar systems are *monostatic*. That is, their transmitter and receiver are collocated. Therefore, the transmitted wave must make its way to its intended target, and then back to the receiver before the radar can perform any functions. Because of the two-way propagation of the wave (egressing and ingressing), the signal suffers significant attenuation [14]. As a way to compensate for this, most radar transmitters have a high transmit gain in comparison to standard communication transmitters. However, an embedded communication signal would be traveling to a remote receiver, and therefore would not need to make a round trip. So the communication would be receiving a high signal to noise ratio (SNR) communication signal, courtesy of the radar carrier wave.

![Figure 2.8: Joint Radar/Communication Signal Path](image)

Figure 2.8: Joint Radar/Communication Signal Path

The next few sections will cover the different types of modulation that can be exploited for the sake of sufficiently hiding the communication signal within the LFM radar waveform. The modulation types will include a standard binary phase shift keying modulation (BPSK), and the newly developed reduced binary phase shift keying (RBPSK), and the varying symbol duration RBPSK (VSDRBPSK).
Given common applications of radar and radar waveforms, the signals will be designed as a pulsed waveform, illustrated in figure 2.9.

\[ s(t) = a(t) \ast x(t) = \sum_{i=0}^{N-1} b_i p(t - iT_b) \ast x(t) \]

\[ = \sum_{i=0}^{N-1} b_i p(t - iT_b) \ast A_c \cos \left( 2\pi f_0 t + 2\pi \frac{k}{2} t^2 + \phi_0 \right) \] (2.28)

where \( a(t) \) is the baseband 2PAM signal of \( N \) bits, \( b_i \) is the \( i^{th} \) information bit and is of the set \( \{+1, -1\} \), \( p(t) \) is a rectangular pulse with unit height and a duration of \( T_b \). It should be noted that multiplying a baseband 2PAM signal with an LFM chirp signal is the same as introducing a phase offset of either 0 or \( \pi \) radians to the signal.
waveform at different data symbols [12,14]. Now:

\[
s(t) = \sum_{i=0}^{N-1} p(t - iT_b) * A_c \cos \left( 2\pi f_0 t + 2\pi \frac{k}{2} t^2 + \theta_i + \phi_0 \right)
\]  (2.29)

where \( \theta_i = 0 \) if \( b_i = 1 \), and \( e^{\theta_i} = \pi \) if \( b_i = -1 \).

Figure 2.10 depicts the signal constellation map of BPSK modulation. Figure 2.11 shows the difference between an unmodulated and a BPSK modulated LFM waveform. As it shows, there is a phase shift whenever two bits with different polarities are adjacent to each other. Because of this, it is easy for anyone observing the signal to see the digital modulation in the LFM.

For the sake of future derivations, the BPSK modulated LFM will use the representation defined by Zhang et al. [12]:
\[ \tilde{z}(t) = \begin{cases} 
\sum_m b_m(t - mT_0)c(t - mT_0) & \text{if} \quad mT_0 - T_c/2 \leq t \leq mT_0 + T_c/2, \\
0 & \text{if} \quad mT_0 + T_c/2 < t < (m + 1)T_0 - T_c/2, \\
m = 0, \pm 1, \pm 2... 
\end{cases} \tag{2.30} \]

Where, \( T_0 \) is the period of the pulses, \( b_m(t - mT_0) \) is the baseband signal, and \( c(t - mT_0) \) is the carrier LFM signal. The signal \( b_m(t) \) can be described as:

\[ b_m(t) = \sum_{n=0}^{N-1} b^m_n q(t - nT_b) \quad -T_c/2 \leq t \leq T_c/2 \tag{2.31} \]

Here, \( b^m_n \) represents the \( n \)-th bit that is embedded into the \( m \)-th pulse with a shaping filter of \( q(t) \). \( T_b \) represents the bit duration, and \( N = \lfloor T_c/T_b \rfloor \) is the total number of bits contained within one pulse.

Lastly, the signal model may be written as [12]:

\[ z(t) = b_0(t)c(t) = \left[ \sum_{n=0}^{N-1} b^0_n q(t - nT_b) \right] c(t), \quad -T_c/2 \leq t \leq T_c/2 \tag{2.32} \]

where \( c(t) \) is the LFM carrier waveform.

### 2.3.2 SCF of BPSK Modulated LFM

Recall from equation (2.7) that the SCF of \( z(t) \) can be found by convolving the SCFs of \( b_0(t) \), denoted as \( S_{b_0}^\alpha(f) \), and \( c(t) \), denoted as \( S_c^\alpha(f) \). Looking first at \( S_{b_0}^\alpha(f) \),
the SCF of a BPSK signal \( b(t) \) with an infinite length, can be written as [12,17]:

\[
S^\alpha_b(f) = \frac{1}{T_q} \left[ Q(f + \alpha/2)Q^*(f - \alpha/2)S^\alpha_a(f) \right].
\] (2.33)

where

\[
S^\alpha_a(f) = \begin{cases} 
1 & \alpha = k/T_b, \quad k = 0, \pm 1, \pm 2, \ldots \\ 
0 & \text{elsewhere},
\end{cases}
\] (2.34)

By inspection, it is seen that the BPSK SCF exhibits flat peaks at \( \alpha = k/T_b \). The amplitude of these peaks attenuate as \( |k| \) increases. A BPSK signal with a finite length, \( b_0(t) \) can be seen as a section of \( b(t) \) taken using a rectangular window \( g(t) \) with a length of \( T_c \). The SCF of \( b_0(t) \) can be derived as the convolution of \( S^\alpha_b(f) \), and the SCF of \( g(t) \), or \( S^\alpha_g(f) \) [12].

\[
S^\alpha_{b_0}(f) = S^\alpha_b(f) \ast S^\alpha_g(f),
\] (2.35)

where,

\[
S^\alpha_g(f) = G(f + \alpha/2)G^*(f - \alpha/2)
\]

\[
G(f) = T_c \text{sinc}(\pi f T_c).
\] (2.36)

The convolution done on \( S^\alpha_b(f) \) makes the peaks wider in the \( \alpha \) dimension. The width of the peaks in \( \alpha \) become \( 2/T_c \). Now, with \( S^\alpha_{b_0}(f) \) and \( S^\alpha_c(f) \) (calculated earlier in this chapter), and using equation (2.7), The SCF of a BPSK modulated LFM can
be written as the convolution of the SCF of the BPSK signal and the SCF of the LFM waveform [12, 17]:

\[ S_\alpha^x(f) = S_\alpha^{b_0}(f) \otimes S_\alpha^c(f) \]  

(2.37)

Keep in mind that here, \( S_\alpha^c(f) \) is derived from the restricted AF \( R_c^\prime(t, \tau) \) derived earlier. The image below graphically depicts what is happening when the BPSK SCF is convolved with the SCF of the LFM. As shown, the width of the peaks from the LFM spanning the \( \alpha \) domain are affected by the BPSK modulation. In the final product, evidence of the modulation is shown by peaks occurring in the frequency domain at intervals of \( 1/T_b \) or \( f_b \) [12].

Figure 2.12: BPSK SCF Convolved with LFM SCF

2.3.3 Radar Performance of BPSK Modulated LFM Waveform

The radar performance of the BPSK modulated LFM can be shown in the figure below [16].

As shown, although the time delay remains the same, there is a clear degradation in the Doppler tolerance. This, in combination with the obvious features associated with modulation in the spectral correlation function, suggest that this radar/communication waveform may not exhibit acceptable radar performance. For that reason, a novel modulation type will be used to mitigate these undesired effects.
2.3.4 Reduced Phase Shift Keying

Here, a modulation scheme is introduced where, instead of using 0 to $\pi$ in the signal constellation representing binary data, two constellation points with a significantly smaller phase difference is used. Figure 2.14 illustrates the constellation map of the proposed modulation scheme [2,12,14,16].

The constellation points now become $\phi$, and $-\phi$ to represent the binary data. This new modulation scheme is referred to as reduced phase shift keying (reduced PSK, or RPSK) or reduced binary phase shift keying (RBPSK) [12]. The formula for the RPSK modulated LFM is written as [14]:

$$s(t) = \sum_{i=0}^{N-1} p(t - iT_b) * A_c \cos \left(2\pi f_0 t + 2\pi \frac{k}{2} t^2 + \theta_i + \phi_0\right)$$

$$= \sum_{i=0}^{N-1} p(t - iT_b) * A_c \cos \left(2\pi f_0 t + 2\pi \frac{k}{2} t^2 + b_i * \phi + \phi_0\right) \quad (2.38)$$
Now the phase offset introduced by the $i$th data symbol has become $\theta + i = b_i \times \phi_0$. Therefore, if $b_i = 1$ then a positive phase, $\phi$ is introduced. Likewise, if $b_i = -1$, a negative phase, $-\phi$ is introduced. It should be noted, that when $\phi = \frac{\pi}{2}$ radians, the RPSK modulation is the same as a conventional BPSK modulation.

As a result of using a smaller phase difference, the RPSK modulation scheme exhibits a poorer bit to error ratio (BER) performance in comparison to the BPSK modulation. The BER of the BPSK in an additive white Gaussian noise (AWGN) channel is given by [14]:

$$Q\left(\frac{d_{\text{min}}/2}{\sigma}\right) = Q\left(\frac{A}{\sigma}\right) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$  \hspace{1cm} (2.39)

where $d_{\text{min}}$ is the phase difference, $A = \sqrt{E_b}$, $N_0/2$ is the power spectral density of the AWGN, and $E_b$ is the bit energy. However, the BER performance of the RPSK
modulation is:

\[ Q\left(\frac{d_{\text{min}}/2}{\sigma}\right) = Q\left(\frac{A\sin(\phi)}{\sigma}\right) = Q\left(\sqrt{\frac{2E_b\sin^2(\phi)}{N_0}}\right) \] (2.40)

The evaluated \( \sin(\phi) \) term can only be a maximum value of 1, and a minimum value of 0. At best, the BER performance of the RPSK modulation is only as good as the BER performance of the BPSK modulation. Any value of the \( \sin(\phi) \) term evaluated to be less than 1 will obviously decrease the BER performance. As a result, the SNR will need an increase in inverse proportionality to the decrease in phase in order to yield a BER performance as efficient as the original BPSK modulation [14]. However, thanks to the high SNR of the LFM carrier wave, this performance loss can be overlooked.

In the images below, the results of various phase differences are shown in a time domain analysis. Note that when the phase difference is smaller, evidence of modulation becomes less obvious.

Figure 2.15: \( \pm 15^\circ \) Representation, \( d_{\text{min}} = 30^\circ \)

Figure 2.16: \( \pm 30^\circ \) Representation, \( d_{\text{min}} = 60^\circ \)

Through smaller and smaller phases, both time domain analysis and spectrum analysis are unable to reveal the communication signal embedded within the LFM chirp signal. However, cyclostationary analysis still reveals the cyclic properties of this modulation.
2.3.5 SCF of RPSK Modulated LFM

Due to the special condition of a 90° phase representation for RPSK being the same as BPSK modulation, the SCF for RPSK can be thought of as a generalization for BPSK. Assume the signal model for the RPSK modulated LFM is represented as [12]:

\[
r(t) = \begin{cases} 
  \cos(\theta)c_I(t - mT_0) + \sin(\theta)b_m(t)c_Q(t - mT_0) & mT_0 - T_c/2 \leq t \leq mT_0 + T_c/2, \\
  0 & mT_0 + T_c/2 < t < (m + 1)T_0 - T_c/2, \\
  & m = 0, \pm 1, \pm 2, \ldots 
\end{cases}
\]

(2.41)

Where \( c_I(t) \) and \( c_Q(t) \) are the in-phase and quadrature components (respectively) of a LFM chirp signal, and are written as:

\[
\begin{align*}
  c_I(t) &= A_c \cos(\psi_o t + \psi' t^2 + \phi_0) \\
  c_Q(t) &= A_c \sin(\psi_o t + \psi' t^2 + \phi_0)
\end{align*}
\]

(2.42)

The single pulse RPSK-LFM equation is written as [12]:

\[
r(t) = \cos(\theta)c_I(t) + \sin(\theta)b_0(t)c_Q(t)
\]

(2.43)
The AF of this is given as:

\[
R_r(t, \tau) = \mathbb{E}[\tilde{r}(t + \tau/2)\tilde{r}(t - \tau/2)]
\]

\[
= \mathbb{E}[(\cos(\theta)c_I(t + \tau/2) + \sin(\theta)b_0(t + \tau/2)c_Q(t + \tau/2)]
\]

\[
\times [\cos(\theta)c_I(t - \tau/2) + \sin(\theta)b_0(t - \tau/2)c_Q(t - \tau/2)]]
\]

\[
= \cos^2(\theta)c_Q(t + \tau/2)c_Q(t - \tau/2)
\]

\[
+ \sin^2(\theta)E[b_0(t + \tau/2)b_0(t - \tau/2)]
\]

\[
\times c_I(t + \tau/2)c_I(t - \tau/2)
\]

\[
+ \cos(\theta)\sin(\theta)E[b_0(t + \tau/2)c_Q(t + \tau/2)c_I(t - \tau/2)]
\]

\[
+ \cos(\theta)\sin(\theta)E[b_0(t - \tau/2)c_Q(t - \tau/2)c_I(t + \tau/2)]
\]

Equation (2.44)

Much like the unmodulated LFM and BPSK modulated LFM, there are four additive components in the autocorrelation function. The first is the AF of a LFM signal, denoted as \(R_c(t, \tau)\), and has a weight of \(\cos^2(\theta)\). The second term is the AF of the BPSK-LFM signal, denoted as \(R_x(t, \tau)\), which has a weight of \(\sin^2(\theta)\). The third and fourth values can be ignored, as both \(E[b_0(t + \tau/2)]\), as well as \(E[b_0(t - \tau/2)]\) both drop to zero. So now, \(R_r(t, \tau)\) can be rewritten as [12]:

\[
R_r(t, \tau) = \cos^2(\theta)R_c(t, \tau) + \sin^2(\theta)R_x(t, \tau)
\]

Equation (2.45)

The autocorrelation of a RPSK modulated LFM waveform can be written as the weighted sum of the AFs of an LFM waveform and a BPSK modulated LFM. Since the conversion from autocorrelation to spectral correlation involves a linear transformation, the SCF of a RPSK-LFM can be written as the weighted sum of the SCF’s derived from the LFM waveform and the BPSK modulated LFM, as seen previously in this
\[ S_\alpha^\alpha(f) = \cos^2(\theta)S_\alpha^\alpha(f) + \sin^2(\theta)S_x^\alpha(f) \] (2.46)

The images below give a comparison between simulated SCF graphs of a standard BPSK modulated LFM and a RPSK modulated LFM with a phase representation of 30°, both in the \( \alpha \) vs amplitude plane.

As it shows, the peaks occurring at \( 1/T_b \), in the alpha domain are less noticeable with the RPSK modulation. This however, does not mean that a reduced phase makes the modulation completely undetectable through cyclostationary analysis. In order to achieve that, the concept of a variable symbol duration is introduced.

### 2.3.6 Variable Symbol Duration RPSK

With conventional modulation schemes, such as BPSK, symbols occur within the carrier wave at a fixed rate, \( T_b \), as demonstrated in the image below.
Here, the idea of a variable symbol duration is introduced. As the name suggests, rather than having a fixed duration for each symbol, a unique symbol duration, $T_{b_i}$, is assigned to the $i$th data symbol. No symbol duration is the same as any other, and the symbol durations are not multiples of each other. This effectively makes any cyclostationary feature associated with the modulation undetectable [14]. Figure 2.19 shows the concept of the variable symbol duration. The intended receiver will have knowledge of the different symbol durations, so demodulation would not be difficult.

![Variable symbol duration](image)

**Figure 2.20: Variable Symbol Duration**

The formula for the variable symbol duration RPSK (VSDRPSK) is as follows

$$s(t) = \sum_{i=0}^{i-1} p_i(t) * A_c \cos \left( 2\pi f_0 t + 2\pi \frac{k}{2} t^2 + \theta_i + \phi_0 \right)$$  \hspace{1cm} (2.47)

where $p_i(t)$ is the $i$th data symbol’s pulse with pulse width $T_{b_i}$.

$$p_i(t) = \begin{cases} 
1 & \sum_{l=0}^{i-1} T_{b_l} < t < \sum_{l=0}^{i-1} T_{b_l} + T_{b_i} \\
0 & \text{elsewhere}, 
\end{cases}$$  \hspace{1cm} (2.48)

The issue with this method is that a variable symbol duration means variable symbol energy $E_b$ for the different data symbols. The symbol energy for the $i$th symbol is $\frac{A_c^2}{2} * T_{b_i}$. $A_c$ is the amplitude of the chirp signal. Data symbols with longer
symbol durations will have higher symbol energy, and by extension, a better BER performance. This is an unwanted byproduct [14].

The solution to this problem is very simple. By manipulating the newly added phase difference parameter in the RPSK modulation, the variance in the BER performance brought about by the variable symbol duration can be rectified. Shorter symbol durations can be accounted for with larger phases to guarantee that the BER performance stays the same. With this formula [14]:

\[
T_{b_1} \sin^2(\phi_1) = T_{b_2} \sin^2(\phi_2) = T_{b_3} \sin^2(\phi_3) = \ldots T_{b_n} \sin^2(\phi_n) \tag{2.49}
\]

the BER performance per symbol will stay the same. From this, the phases can be found to be:

\[
\sin^2(\phi_n) = \frac{T_{bn-m}}{T_{bn}} \sin^2(\phi_{n-m}) \tag{2.50}
\]

\[
\sin(\phi_n) = \sqrt{\frac{T_{bn-m}}{T_{bn}}} \sin(\phi_{n-m}) \tag{2.51}
\]

\[
\phi_n = \arcsin \left( \sqrt{\frac{T_{bn-m}}{T_{bn}}} \sin(\phi_{n-m}) \right) \tag{2.52}
\]

\[m \in \mathbb{Z}, n \in \mathbb{Z}, n > m\]

Now, in figure 2.20, we see the variable symbol duration in association with the assigned phase.

![Figure 2.21: Variable Symbol Duration With Phases](image-url)
Now that the cyclostationary features of the modulation have been removed, the SCF of the VSDRPSK modulated LFM should behave almost identically to the unmodulated LFM waveform. This will be demonstrated in a later chapter \[12,14\].

### 2.3.7 Radar Performance of RPSK Modulated LFM Waveform

Once again taking a look at the ambiguity function for radar performance, clearly, the new RPSK modulation has a much less effect on the performance of the RF signal when compared to the BPSK modulation. When comparing the unmodulated LFM ambiguity function (figure 2.21) to the RPSK LFM ambiguity function (figure 2.22) \[16\],

![Figure 2.22: Unmodulated LFM Ambiguity Function](image1)

![Figure 2.23: ±15° RPSK-LFM Ambiguity Function](image2)

the two images look virtually identical. This shows that the RPSK modulation preserves the Doppler tolerance of the LFM waveform. The effectiveness of the RPSK waveform is bolstered with the implementation of the variable symbol duration.

### 2.4 Software Defined Radio Implementation

For the purposes of this study, a software defined radio (SDR) environment will be used to manipulate, transmit, and receive these proposed waveforms. *Software defined radio* provides more ease and options for the user compared to conventional radio systems, as most of the modulation process is done through software \[18\]. Traditional
hardware based radio systems are very limited in terms of cross functionality, and can only be modified by means of physical intervention. SDR however, can be modified remotely, and removes some of the limitations brought about by hardware. This makes SDR a more efficient and inexpensive method to perform radio technology based studies [19].
3 Method/Procedure

3.1 Theoretical Analysis of Modulated LFM Waveforms

The contents of this chapter will outline the methods/procedure of this experiment. These steps were selected to ensure that the experiment was done correctly and efficiently. The first step is the generation of the different modulated LFM waveforms using the programming platform, MATLAB. The different waveforms are then passed through a spectral correlation function algorithm, also written in MATLAB, to see what the simulated cyclostationary properties are.

3.2 Waveform Generation, Transmit/Receive GUI

A series of graphical user interfaces (GUIs) were made to allow for the manipulation of the different waveforms in a software environment. Starting with the unmodulated LFM waveform GUI, each GUI after provides more options as the modulation types become more robust. The images below show the different GUIs and how they are set up. Figure 3.1 is the universal GUI that calls the GUIs for the different waveforms.
From the different push buttons, the user can select between the unmodulated LFM, the BPSK modulated LFM, the fixed RPSK modulated LFM, and the VSDRPSK modulated LFM. Figure 3.2 shows the GUI for the unmodulated LFM.
From the options to select, the “Starting Freq Time(s),” corresponds to the $t_0$ value when the starting frequency begins. Likewise, the “Ending Freq Time(s),” corresponds to the $t_1$ value when the ending frequency ends. The absolute difference between these two values is the duration of each pulse.

The “Delay” option sets how long the listening period is between each pulse. The duration of the listening period is determined by taking the input number, multiplying by the pulse duration, and subtracting the original pulse duration from that product.

The “Starting Cutoff Freq(kHz),” and “Ending Cutoff Freq(kHz),” as suggested, correspond to the beginning and ending frequencies of the LFM waveform respectively. Note that the beginning and ending frequencies can be either low to high (up-chirp), or high to low (down-chirp). Lastly, the “number of pulses,” option allows the user to select the number of pulses they want in their LFM pulse train.
Figure 3.3 below shows the GUI for the BPSK modulated LFM.

![BPSK LFM Waveform GUI](image)

**Figure 3.3: GUI for the BPSK modulated LFM Waveform**

The set up for the BPSK modulated LFM GUI is almost the same as the unmodulated LFM GUI. The difference now is the “Bit Freq (kHz)” $F_b$ option. This allows the user to select how frequently, and by extension how many symbols occur throughout the carrier LFM waveform. It should be noted that the bit frequency must be less than the lower cutoff frequency of the carrier LFM waveform. The bit duration, $T_b$, is inversely proportional to the set bit frequency.

Figure 3.4 below shows the GUI for the fixed RPSK modulated LFM.
Again, the fixed RPSK GUI is similar to the previous two GUIs, with the exception of the “Phase(deg)” option. This allows the user to set the absolute value of the phase, $\phi$, representation of each bit in degrees. This number will be mapped to each bit, with a different sign corresponding to the sign of each bit, ($+\phi$ to ‘1’, and $-\phi$ to ‘−1’).
Lastly, figure 3.5 shows the GUI for the VSDRPSK modulated LFM.

![Figure 3.5: GUI for the VSDRPSK Modulated LFM Waveform](image)

As shown, most of the options are the same as the other GUIs. The main difference is that instead of a set phase option as seen in the fixed RPSK, there is now an upper and lower phase limit options “Max Phase Limit(deg)” and “Min Phase Limit(deg)” respectively. Now, each bit will be randomly assigned to a phase value (±φ for ±1 respectively) within that upper and lower limit. It should be noted that the set bit frequency value now is simply the initial value corresponding to the first phase corresponding to the first symbol, and will no longer be a fixed value.

The in-phase and quadrature (IQ) data from these waveforms are stored in a .txt file and loaded into the transmitter function block set up made using GNU Radio. The image below shows the transmit interface set up using GNU Radio.
Here, parameters such as: Transmit gain, Sample rate, as well as time and frequency analysis sinks can be set. The “File Source” block can be used to load in the .txt file that contains the IQ data for the generated waveforms. From there, the information is uplinked to the hardware, and is transmitted to the receiver. The hardware used in the experiment are two USRPx300 SDRs. Either one can be used as the transmitter or the receiver. The images below depict the USRPs used as the transmitter and receiver:
The function block set up in GNU Radio for the SDR can be seen in figure 3.9.
On the receiver side, the user has the option to separate the received IQ data into files containing real and imaginary data, or keep the IQ data together. The received data can be loaded into a .txt or .mat file using the “File Sink” blocks to be analyzed in MATLAB. Note that the transmitter and receiver must be set to the same sample rate and center frequency in order for the waveform to be successfully transmitted and received.

3.3 Data Analysis

After the waveforms have been successfully transmitted and received. The received data is then analyzed in MATLAB. The real data is taken for cyclostationary analysis, as the algorithm is written to process real data only.

The received VSDRPSK modulated waveform is run through a demodulator algorithm written in MATLAB, and compared to the transmitted signal to gauge BER performance.

3.3.1 Cyclostationary Analysis of Generated Waveforms

The received signals are processed through a receive signal GUI that displays the received waveform in the time domain, as well as the 3-D plot and a summed fast Fourier transform (FFT) plot of the SCF of the waveforms. The time domain and SCF graphs of the received signals are then compared to the transmitted signals generated using the MATLAB waveform generator GUls. The image below shows the receive signal GUI that was developed for this analysis.
In the panel, there are two options. The first option is the “receive data” option. This will load in the data of the waveform that has been specified in the MATLAB code. The window on the top right will show a real and imaginary (if applicable) plot overlaid on top of each other. The second option is the “analyze data” option. This will begin the cyclostationary analysis on the received signal, and return one plot that is the 3-D SCF graph of the signal (bottom right window), and a 2-D version of the SCF of the signal (bottom left window).

### 3.3.2 Demodulation of Communication Signal

Lastly, a demodulation scheme written in MATLAB is used to extract the VSDRPSK signal. A BER analysis is done by comparing the transmitted bits to the received bits.
4 Results/Data Analysis

For the sake of consistency, the number of pulses, starting and ending time, starting and ending frequencies, bit duration, and delay factor were kept the same for each waveform. The differences come with the different modulation types. The bits for the modulated waveform are pseudo random. A different bit sequence is generated per pulse, per modulated waveform. The following sections will contain a complete table that lists the specific parameters used for the corresponding graphs.

4.1 Unmodulated LFM Measured SCF

Table 4.1 below shows the parameters selected for the unmodulated LFM waveform test. The image that follows shows the received signal, as well as the 2-D and 3-D plots of the corresponding SCF.

Table 4.1: Parameters for Unmodulated LFM

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Starting Freq Time(s)</td>
<td>0</td>
</tr>
<tr>
<td>Ending Freq Time(s)</td>
<td>.5</td>
</tr>
<tr>
<td>Starting Cutoff Freq(kHz)</td>
<td>20</td>
</tr>
<tr>
<td>Ending Cutoff Freq(kHz)</td>
<td>50</td>
</tr>
<tr>
<td>Delay Factor</td>
<td>2</td>
</tr>
<tr>
<td>Number of Pulses</td>
<td>5</td>
</tr>
</tbody>
</table>
Figure 4.1: Full Received LFM Signal

The image shows the received real signal, as well as a 2-D and 3-D image of the SCF of the unmodulated LFM. Figure 4.2 gives a closer look at the 2-D SCF of the unmodulated LFM.
As expected, we see a cyclic frequency response spanning from $2 \ast F_0$ to $2 \ast F_1$. 
Later on, this will be compared to the SCF of the novel VSDRPSK modulated LFM.

### 4.2 BPSK Modulated LFM Measured SCF

Table 4.2 below shows the parameters selected for the BPSK modulated LFM waveform test. The image that follows shows the received signal, as well as the 2-D and 3-D plots of the corresponding SCF.

Table 4.2: Parameters for BPSK Modulated LFM

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Starting Freq Time(s)</td>
<td>0</td>
</tr>
<tr>
<td>Ending Freq Time(s)</td>
<td>0.5</td>
</tr>
<tr>
<td>Starting Cutoff Freq (kHz)</td>
<td>20</td>
</tr>
<tr>
<td>Ending Cutoff Freq (kHz)</td>
<td>50</td>
</tr>
<tr>
<td>Delay Factor</td>
<td>2</td>
</tr>
<tr>
<td>Number of Pulses</td>
<td>5</td>
</tr>
<tr>
<td>Bit Frequency (kHz)</td>
<td>10</td>
</tr>
</tbody>
</table>
Similar to the unmodulated LFM, the real signal, as well as the corresponding SCF graphs are shown. Figure 4.6 shows a closer look at the 2-D SCF plot of the BPSK LFM waveform. Figures 4.7 and 4.8 show the frequency vs. alpha axis plots of the SCF, and the amplitude vs. alpha plots respectively.
Note that here, the SCF exhibits behavior that is indicative of the BPSK modulation. As opposed to the unmodulated LFM SCF, the BPSK modulated LFM SCF shows peaks occurring at $\alpha = \pm \frac{N}{F_b} = \pm N \ast F_b$. In this test, $F_b = 10$kHz, and it clearly shown that there are other peaks occurring at multiples of $F_b$ with attenuating amplitude.
4.3 RPSK Modulated LFM Measured SCF

Table 4.3 below shows the parameters selected for the fixed phase RPSK modulated LFM waveform test. Figure 4.9 shows the received signal, as well as the 2-D and 3-D plots of the corresponding SCF.

Table 4.3: Parameters for RPSK Modulated LFM

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Starting Freq Time(s)</td>
<td>0</td>
</tr>
<tr>
<td>Ending Freq Time(s)</td>
<td>.5</td>
</tr>
<tr>
<td>Starting Cutoff Freq(kHz)</td>
<td>20</td>
</tr>
<tr>
<td>Ending Cutoff Freq(kHz)</td>
<td>50</td>
</tr>
<tr>
<td>Delay Factor</td>
<td>2</td>
</tr>
<tr>
<td>Number of Pulses</td>
<td>5</td>
</tr>
<tr>
<td>Bit Frequency(kHz)</td>
<td>10</td>
</tr>
<tr>
<td>Phase(°)</td>
<td>30</td>
</tr>
</tbody>
</table>
Figure 4.9: Full Received RPSK LFM Signal

The next graph depicts a closer look at the 2-D plot of the SCF of the fixed RPSK modulated LFM. Figures 4.11, 4.12, and 4.13 show the frequency vs. alpha axis plots of the SCF, the amplitude vs. alpha plot, and a zoomed in version of the amplitude vs. alpha plot respectively.
For this test, the phase representation associated with the symbols has been reduced from ±90° (BPSK), to ±30°. The result of this is a clear reduction in amplitude in the peaks associated with the symbol duration at $\alpha = N \cdot F_b$. However,
the peaks are not completely hidden, and therefore, the communication signal is not perfectly hidden within the carrier.

4.4 VSDRPSK Modulated LFM Measured SCF

Lastly, Table 4.4 below shows the parameters selected for the VSDRPSK modulated LFM waveform test. The image that follows shows the received signal, as well as the 2-D and 3-D plots of the corresponding SCF. Note that now, instead of a phase representation, there is a phase range. Each symbol is represented by a different phase within the phase range (with the proper corresponding sign). Furthermore, the bit frequency becomes an arbitrarily selected initial value (less than the minimum cut off frequency) to satisfy the algorithm used to calculate the following phases and symbol durations.
Table 4.4: Parameters for VSDRPSK Modulated LFM

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Starting Freq Time(s)</td>
<td>0</td>
</tr>
<tr>
<td>Ending Freq Time(s)</td>
<td>.5</td>
</tr>
<tr>
<td>Starting Cutoff Freq(kHz)</td>
<td>20</td>
</tr>
<tr>
<td>Ending Cutoff Freq(kHz)</td>
<td>50</td>
</tr>
<tr>
<td>Delay Factor</td>
<td>2</td>
</tr>
<tr>
<td>Number of Pulses</td>
<td>5</td>
</tr>
<tr>
<td>Bit Frequency(kHz)</td>
<td>10</td>
</tr>
<tr>
<td>Maximum Phase Limit(°)</td>
<td>15</td>
</tr>
<tr>
<td>Minimum Phase Limit(°)</td>
<td>10</td>
</tr>
</tbody>
</table>

Figure 4.14: Full Received VSDRPSK LFM Signal

Figure 4.15 shows a closer view of the 2-D SCF of the VSDRPSK modulated LFM.

Now, since the symbol duration, and by extension, phase representation has
been randomized per symbol, the cyclostationary aspects of the communication signal have successfully been removed from the joint radar/communication waveform. As a result, the SCF of this new waveform exhibits behavior that is virtually identical to an unmodulated LFM.

Figures 4.16 and 4.17 show the frequency vs. alpha axis plots of the SCF, and the amplitude vs. alpha plots respectively.
The following images show a comparison between the SCFs of the unmodulated and VSDRPSK modulated LFM signals.

Figure 4.16: Frequency vs Alpha SCF of VSDRPSK Modulated LFM

Figure 4.17: Amplitude vs Alpha SCF of VSDRPSK Modulated LFM

Figure 4.18: 2-D Unmodulated LFM SCF

Figure 4.19: 2-D VSDRPSK Modulated LFM SCF
With this, the communication signal has successfully been concealed within the radar signal, and cannot be detected by second order cyclostationary analysis.

### 4.5 Demodulated Communication Signal

Now, the signal can be demodulated and the communication signal extracted by the intended receiver. With the recipient having knowledge of the various bit durations, a demodulation scheme can be written to search for phase information for the corresponding bits at those corresponding durations. For this experiment, a preamble signal is appended to the beginning of the transmitted signal in order to establish synchronization between the transmitter and receiver. This is necessary, as a shift in time between transmit and receive will cause a phase offset. The phase
offset will greatly reduce the accuracy of the demodulator.

Once the transmitted signal is received, and demodulated, a comparison is done between the received demodulated bits and the original transmitted bits to determine the BER performance. The BER performance of the received VSDRPSK modulated signal has exhibited an error rate of about 1 percent in multiple runs.

\[
BER = \frac{R_x \text{Bits}_{\text{correct}}}{T_x \text{Bits}} \times 100
\]  

(4.1)

The BER performance is determined by comparing the received, demodulated bits to the original transmitted bits. The number of correctly demodulated bits is divided by the number of transmitted bits and then multiplied by 100 to get a percentage.
5 Conclusion

In this thesis, various modulation types, including a novel modulation type (VSDRPSK) is applied to a Linear Frequency Modulated radar waveform to create joint radar/communication signals. The characteristics of these signals are analyzed through second order cyclostationary analysis. As the more complicated modulation types are applied, the analysis shows less and less influence from the communication component of the joint signal. Eventually, it is demonstrated that the novel modulation type is not detectable using second order cyclostationary analysis, thus effectively concealing a transmitted communication signal within a radar waveform.

The parameters of the newly generated waveforms are then manipulated using different graphical user interfaces (GUIs), and then transmitted and received using software defined radio (SDR) technology. Once received, the VSDRPSK modulated waveform is then demodulated to show that the transmission of the communication signal is still sufficient by inspection of its BER performance.

As a follow up to the concepts discussed in relation to the VSDRPSK modulation, a phase window utilizing smaller phases will yield a longer bit rate per bit. While this may seem less than ideal, the trade off is that the information contained within is secure. Inversely, phase windows using larger phases yield a faster bit rate, but at the risk of a easier to detect signal.

Lastly, Future work might include analysis on the waveforms using higher order cyclostationary analysis. If this method were to reveal aspects of the waveforms that second order cyclostationary analysis couldn’t, the task to manipulate the waveforms so that they deceive higher order analysis would emerge. Other possibilities might be
using a different carrier waves, such as those used in commercial, navigation, etc. to see how the performance varies.
REFERENCES


