IMPLEMENTATION OF HIGH SPEED AND LOW POWER RADIX-4 8*8 BOOTH MULTIPLIER IN CMOS 32nm TECHNOLOGY

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in Electrical Engineering

By

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I HEREBY RECOMMEND THAT THE THESIS PREPARED UNDER MY SUPERVISION BY Rishit Patel ENTITLED Implementation of High Speed and Low Power Radix-4 8\*8 Booth Multiplier in CMOS 32nm Technology BE ACCEPTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF Master of Science in Electrical Engineering.

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ABSTRACT


According to Moore’s law, number of transistors integrated on a single chip double every 18 months with a lot new functionality embedded, which results the increasing of delay and power consumption of a chip. To improve the performance of a more complicated digital circuit design, faster and power efficient digital sub-components are in urgent need. Multipliers are the key components in the field of DSP, GPU and CPU which compute enormous amount of binary data. A radix-4 8*8 booth multiplier is proposed and implemented in this thesis aiming to reduce power delay product. Four stages with different architecture are used to implement this multiplier rather than traditional 8*8 booth multiplier. Instead of using adder in stage-1, it is replaced with binary-to-access one converter circuit and 10-bit MUX 2:1 to reduce power consumption by 23.76% and increase speed by 12.02% compared to stage-1 of traditional 8*8 booth multiplier. This proposed design is implemented in CMOS 32nm technology at 1.0 voltage supply. The worst-case delay of the proposed radix-4 8*8 booth multiplier at 2 Giga data rate is 423 picosecond and power consumption of 0.274 milli-watt with transistor count of 2860.
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Rishit Patel, MSEE 2017
1. Introduction

1.1 Digital Matched Filter (DMF)

The application of digital matched filters are in the field of communication such as code division multiple access (CDMA) and direct sequence spread spectrum (DSSS), radar technology, global position system (GPS), image processing, etc. [1] [2] [3] [4]. In signal processing, a matched filter is obtained by correlating a known signal (i.e. transmitted signal), or template, with an unknown signal to detect the presence of template in the unknown signal (i.e. received signal) [5] [6]. There are two basic types of matched filters available depending upon the area of application such as analog matched filters (AMF) and digital matched filters (DMF). Both the types of matched filters are used in the receiver part of the communication discipline. Analog matched filters are usually used before analog-to-digital converter (ADC) and digital matched filters are used after ADC which produces digital signal. The block diagram depicting the place of digital matched filters in the field of communication is shown in figure 1 [7].

The matched filter is the optimal linear filter for maximizing the signal to noise ratio (SNR) in the presence of additive stochastic noise. Matched filtering is a demodulation technique with LTI (linear time invariant) filters to maximize SNR. The filter input, \( x(t) \) consists of a pulse signal \( g(t) \) corrupted by additive channel noise \( w(t) \), as shown in below equation 1.
\[ x(t) = g(t) + w(t), \quad 0 \leq t \leq T \quad (1) \]

where T is an arbitrary observation interval. The pulse signal g(t) may represent a binary 0 or 1 in a digital communication system. Since the filter is linear, the resulting output y(t) is expressed as shown in below equation (2):

\[ y(t) = g(t) + n(t) \quad (2) \]

Where g(t) and n(t) are produced by the signal and noise components of the input x(t), respectively.

Figure 1 Block diagram of telecommunication field showing position of DMF
DMF is used in the acquisition of PN-code in the spread spectrum systems to achieve the synchronization and the de-spreading [1]. Traditional architectures for digital matched filters are based on either the direct form structure or the transposed form structure [1]. The equation of n-tap direct-form DMF is expressed as below

\[ y[k] = \sum_{i=0}^{n-1} C_i \times x[k - i] \]  

(3)

As we can see from equation (3) that filter co-efficient is multiplied with delayed input value and after the computation of each tap, the final step is to add all outputs from each tap. So, we can conclude that the basic components for implementing digital matched filter are multipliers, adders and delay element (i.e. register or flip-flop).

1.2 Multipliers

Multipliers are the key component in DMF and many other application such as digital signal processing (DSP), graphics processing unit (GPU), image processing, video processing etc. [8]. Convolution, filtering and inner products are the vital processes of digital signal processing which uses the multiply and accumulate (MAC) unit [9] [10] [11]. Multipliers are more complex than adders and sub-tractors, so the speed of a multiplier usually determines the operating speed of a DSP system and the application where it is used [12] [13]. In most high speed DSP, multimedia applications and GPU, the multiplier plays an important role because it dominates the chip power.
consumption and operation speed. So, research has concentrated on high-speed and low-power multipliers for computationally fast portable devices. Any kind of multiplier is divided into three stages such as partial product generation, partial products addition, final-addition stage [12].

The speed of multiplication can be increased by reducing the number partial products. Many high-performance algorithms and architectures have been proposed to accelerate multiplication speed and keeping low power consumption in mind. Various multiplication algorithms such as Booth, Braun, Wallace, Baugh-Woolley, modified booth have proposed but each has its own importance and based upon that they have specific applications [14].

1.2.1 Array Multiplier

The simplest form of multiplier is array multiplier [15]. It uses bunch of half adders (HA), full adder (FA) and AND gates depending upon the multiplicand and multiplier bits. The simplest way to perform multiplication is to use single two input adders. Each bit of multiplier is AND-ed with multiplicand bit to produce partial products and each time partial products are left shifted. The main drawback of array multiplier is that it has large latency as it takes more clock cycles to compute the output [15]. The number of partial products to be added is determined by the number of bits that the multiplicand and multiplier have used. In order to get fast performance, one of the methods is to arrange the circuit to generate all the partial products in parallel and organize in an array.
1.2.2 Booth Multiplier

The second and widely used algorithm which leads to faster performance is booth algorithm [16]. Booth multiplication algorithm is fast compared to lot of other algorithms. It was invented by Andrew Donald in 1950 and main importance of booth algorithm is that it can multiply to signed binary numbers [17]. Various encoding styles are available depending upon number of bits in the group such as radix-2, radix-4, radix-8, radix-16, etc. [18]. The number of bits in each group for different encoding styles are mentioned in table 1.

Table 1. Number of bits encoding in various encoding styles

<table>
<thead>
<tr>
<th>Encoding Style</th>
<th>Number of bits in Each group</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radix - 2</td>
<td>2</td>
</tr>
<tr>
<td>Radix – 4</td>
<td>3</td>
</tr>
<tr>
<td>Radix – 8</td>
<td>4</td>
</tr>
<tr>
<td>Radix – 16</td>
<td>5</td>
</tr>
<tr>
<td>Radix - 32</td>
<td>6</td>
</tr>
</tbody>
</table>

1.2.2.1 Radix-2 Booth algorithm

In radix-2 algorithm, zero is appended to right most side to the multiplier bits and group the multiplier in such a way that each group consists of 2 bits. So that the first pair consists of appended zero and least significant bit (LSB) of
multiplier and the next pair is the overlapping of the first pair in which most significant bit (MSB) of the first pair will be the LSB of the second pair. So that for n-bit * n-bit multiplication, n partial products are obtained. Radix-2 booth algorithm produces same number of partial products as array multiplier so number of cycles to compute the result is almost similar.

1.2.2.2 Radix-4 Booth algorithm

To further decrease the number of partial products, algorithms with higher radix value are used [18]. In radix-4 algorithm grouping of multiplier bits is done in such a way that each group consists of 3 bits as mentioned in table 1. Similarly the next pair is the overlapping of the first pair in which MSB of the first pair will be the LSB of the second pair and other two bits. Number of groups formed is dependent on number of multiplier bits. By applying this algorithm, the number of partial product rows to be accumulated is reduced from n in radix-2 algorithm to n/2 in radix-4 algorithm [19]. The grouping of multiplier bits for 8-bit of multiplication is shown in figure 2.
Figure 2. Grouping of multiplier bits in Radix-4 Booth algorithm

For 8-bit multiplier the number groups formed is four using radix-4 booth algorithm. Compared to radix-2 booth algorithm the number of partial products obtained in radix-4 booth algorithm is half because for 8-bit multiplier radix-2 algorithm produces eight partial products. The truth table and the respective operation is depicted in table 2. Similarly when radix-8 booth algorithm is applied to multiplier of 8-bits each group will consists of four bits and the number of groups formed is 3. For 8x8 multiplication, radix-4 uses four stages to compute the final product and radix-8 booth algorithm uses three stages to compute the product. In this thesis, radix-4 booth algorithm is used for 8x8 multiplication because number components used in radix-4 encoding style is much less compared radix-8 encoding style as discussed in table 3 which decreases consumption and delay as well.
Table 2. Truth Table for Radix-4 Booth algorithm

<table>
<thead>
<tr>
<th>$B_{i+1}$</th>
<th>$B_i$</th>
<th>$B_{i-1}$</th>
<th>Operation</th>
<th>$Y_{i+1}$</th>
<th>$Y_i$</th>
<th>$Y_{i-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>+0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>+A</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>+A</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>+2A</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>-2A</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>-A</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>-A</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>-0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

The multiplier bits are provided to booth encoder which produces 3-bit output which will perform operations like 0, ±A or ±2A depending upon inputs. Operations can be performed with the help of multiplexer (MUX) and adder/sub-tractor blocks. For

Table 3. Comparison of components between radix-4 and radix-8 encoding style

<table>
<thead>
<tr>
<th>Stages</th>
<th>Radix-4 encoding</th>
<th>Radix-8 encoding</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4</td>
<td>3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Booth encoder</th>
<th>3-bit</th>
<th>4-bit</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Multiplexers</th>
<th>3:1 MUX</th>
<th>5:1 MUX</th>
</tr>
</thead>
</table>

| 1-bit adder/sub-tractor | 64(16 in each stage) | 72 (24 in each stage) |
example, as shown in table 2, if 100 bit is provided to booth encoder, then it will produce 101 output. The output of this MSB ($Y_{i+1}$) is used to select adder or sub-tractor and two LSB ($Y_i$ and $Y_{i-1}$) is used to select 2A from multiplexer. From the above discussion it is clear that the essential components for implementing radix-4 booth algorithm are booth encoder, adder/sub-tractor and multiplexer (MUX). The gate level implementation of radix-4 booth encoder is shown in figure 3.

![Figure 3. Gate level implementation of Radix-4 Booth encoder](image)

1.3 Adder/Sub-tractor

The main function of adder block is to add two binary numbers. Half adder adds two bits at a time while, full adder adds three bits at same instance. Full adder can be implemented using two half adders as shown in figure 4. The inputs to full
adder are A, B and carry-in (Cin) and the outputs are sum (S) and carry-out (Co). The Boolean equation for sum (S) and carry-out (Co) are as follows:

\[
S = A \text{xor} B \text{xor} Cin \tag{4}
\]

\[
Co = (A \text{ xor} B) \text{ nand} Cin \text{ nand} (A \text{nand} B) \tag{5}
\]

Truth table of 1-bit full adder is as shown in table 4. It uses 8 different combinations of input and based upon that it produces outputs related to sum and carry bit.

Figure 4. Gate level implementation of 1-bit full adder
### Table 4. Truth Table for 1-bit full adder

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>Cin</th>
<th>S</th>
<th>Cout</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
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<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

In many computer and other kinds of processors, adders are used not only in the arithmetic logic unit (ALU), but also in other parts of the processor, where they are used to calculate addresses, table indices, increment and decrement operations, and similar operations.

In cases where two’s complement and one’s complement is being used to represent negative numbers, it is very important to convert adder into adder-subtractor block. Adder block can be converted to adder-subtractor block using carry-in (Cin) as control line such that if Cin = 0 the block will perform addition and if Cin = 1 the block will perform subtraction. An additional gate i.e. XOR is added to the block to
perform addition and subtraction using single block. The implementation of adder subtractor block is as shown in figure 5.

![Diagram of 1-bit adder/subtractor](image)

**Figure 5. Block diagram of 1-bit adder/subtractor**

### 1.4 Multiplexer

In VLSI, the multiplexer is a device that selects one of several analog or digital input signals and forwards the selected input into a single line. A multiplexer of $2^n$ inputs has $n$ select lines, which are used to select which input line to send to the output as shown in the figure 6.
Depending upon the inputs to select lines, specific input is selected and provided to output. Booth multiplier uses 2:1 MUX and 3:1 MUX of various bit length. The simplest multiplexer is 2:1 MUX which has two inputs, one select line and one output is selected. For example, A and B are the two inputs, S is the select line and Z is the output for 2:1 MUX then Boolean equation of 2:1 MUX is as shown below:

\[ Z = A \cdot S' + B \cdot S \] \hspace{1cm} (6)

The truth table of 2:1 MUX is listed in table 4 and gate level implementation of 2:1 MUX is as shown in figure 7.
Table 5. Truth Table for 2:1 MUX

<table>
<thead>
<tr>
<th>S</th>
<th>A</th>
<th>B</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
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<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Figure 7. Gate level implementation of 2:1 MUX
1.5 Objective & Motivation

1.5.1 Objective

The objective of this thesis is to design high-speed and low-power multiplier for digital matched filters. Booth multiplication algorithm is used in this thesis to implement high speed and low power multiplier for digital matched filters. Multipliers use adders to add the partial products generated at each stages to compute the final product. So the need of high speed and low power adders are the basic necessary to implement booth multipliers.

1.5.2 Motivation

With the increasing requirement for huge number of functionalities performed by a single chip, semiconductor industries are trying to encompass billions of transistors in a small space. Speed of the operations is in the great need while reducing the power consumption. The goal for every designer is to design a low power, high speed and low cost chips. Some applications require more speed while others require less power consumption and lower area to build more functionalities on the chip. But we cannot achieve all the parameters at the same time because there is always a trade-off between this components [20]. For example, the ripple carry adder is simple to design but the computation speed is very low. Some parallel prefix adders like kogge-stone adder, Brent-kung adder, etc. provides fast output but consumes high area as well as high power. Performance specifications in digital designs can be achieved by scaling the transistor sizes. Technology is scaling to atomic sizes. Technology scaling along with
novel and efficient techniques for implementation of digital design helps reducing the size and power consumption while keeping the speed of operation almost same.

There are many research undertaken for implementation of high-speed and low-power multipliers. Growing demands for portable, battery-powered systems necessitates low power matched filters to reduce the power consumption of code acquisition circuits. Low-power matched filters can be implemented by using either analog or digital technology. For short and fast matched filters, analog implementation is more power efficient than digital counterpart [21]. As complementary metal-oxide semiconductor (CMOS) technology advances rapidly and the reference code length in modern wireless communication systems increases, significant attention has focused on high-speed and low-power DMF’s. For high-speed and low-power DMF’s, we need multipliers with high-speed and low-power consumption.

1.6 Thesis Organization

The rest of the thesis is organized as follows: Chapter 2 discusses the proposed booth multiplier using CMOS technology to design high-speed and low-power 8x8 booth multiplier. The implementation of the proposed 8x8 booth multiplier is custom-designed in SAED32nm technology and simulated using wave-viewer provided by Synopsys, which are detailed in Chapter 3. Finally, conclusion and future work are included in Chapter 4.
2 Proposed Radix-4 8x8 Booth Multiplier

The proposed radix-4 8x8 booth multiplier uses same architecture as traditional booth multiplier but eliminates the circuit which is not used for computation. For 8x8 multiplication, both traditional and proposed booth multiplier uses four stages to compute the final product. Traditional 8x8 booth multiplier uses same type of component in all the four stages shown in figure 8. In each stage it uses components like booth encoder, 16-bit adder/sub-tractor block and 16-bit 3:1 MUX. The multiplier bits with appended is zero is applied to booth encoder of each block. The input to booth encoder of stage-1, stage-2, stage-3 and stage-4 are \( B_1-B_0 \), \( B_3-B_2-B_1 \), \( B_5-B_4-B_3 \) and \( B_7-B_6-B_5 \) respectively and generate output depending upon inputs. The operation of all the four stages is similar. The MSB of the output of booth encoder is applied to adder/sub-tractor block which will select adder if input to block is 0 and select sub-tractor if input to block is 1. The two LSB of the output of booth encoder is applied to multiplexer block which will select either 0, ‘A’ or ‘2A’ depending upon the structure of 3:1 MUX. If the input to multiplexer block is 00 it will select 0 input, if 01 it will select ‘A’ and if 10 it will select ‘2A’ as an output.
Figure 8. Block diagram of traditional radix-4 8x8 booth multiplier
Figure 9. Block diagram of the proposed radix-4 8*8 booth multiplier

After selection of which input to select i.e. 0, ‘A’ or ‘2A’, the output of MUX is provided to the adder/subtractor block. The other input to adder/subtractor block is the output from previous block. For the 1st stage, the inputs
to adder/sub-tractor block are zero and output from multiplexer. The proposed radix-4 8*8 booth multiplier uses 10-bit adder/sub-tractor in stage-2, stage-3 and stage-4 while in stage-1 it uses 10-bit modified BEC instead of adder/sub-tractor block to reduce the power consumption and latency. The block diagram for the proposed radix-4 8*8 booth multiplier is shown in figure 9. The comparison of components used in traditional and proposed booth multiplier is listed as shown in table 5. As seen in the table 6 that traditional 8x8 booth multiplier uses booth encoder, 16-bit adder/sub-tractor block and 16-bit 3:1 MUX in all the four stages. The proposed radix-4 8*8 booth multiplier uses booth encoder, 10-bit adder/sub-tractor and 10-bit 3:1 MUX in stage-2, stage-3 and stage-4 while stage-1 uses booth encoder, 10-bit 3:1 MUX, 10-bit modified BEC and 10-bit 2:1 MUX. The proposed 8x8 Booth multiplier uses different set of components in all stages as compared to traditional 8x8 Booth multiplier. The proposed 8x8 Booth multiplier uses booth encoder, 10-bit 3:1 MUX, 10-bit modified binary-to-excess-1 converter BEC and 10-bit 2:1 MUX in stage-1. Stage-2/3/4 of proposed 8x8 Booth multiplier is implemented using booth encoder, 10-bit adder/sub-tractor block and 10-bit 3:1 MUX. So in total proposed design uses four booth encoder, four 10-bit 3:1 MUX, one 10-bit 2:1 MUX, one 10-bit modified BEC and three 10-bit adder/sub-tractor. From table 5 we can conclude that we saved area by using less bit length of components, which in turn saves power consumption. The operation of each stage is explained in this section.
Table 6. Comparison of components used in traditional and proposed 8x8 Booth multiplier

<table>
<thead>
<tr>
<th>Stage number</th>
<th>Traditional 8x8 Booth Multiplier</th>
<th>Proposed 8x8 Booth Multiplier</th>
</tr>
</thead>
<tbody>
<tr>
<td>STAGE-1</td>
<td>Booth Encoder</td>
<td>Booth Encoder</td>
</tr>
<tr>
<td></td>
<td>3:1 MUX</td>
<td>2:1 MUX</td>
</tr>
<tr>
<td></td>
<td>16-bit adder/sub-tractor</td>
<td>10-bit BEC</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3:1 MUX</td>
</tr>
<tr>
<td>STAGE-2</td>
<td>Booth Encoder</td>
<td>Booth Encoder</td>
</tr>
<tr>
<td></td>
<td>3:1 MUX</td>
<td>3:1 MUX</td>
</tr>
<tr>
<td></td>
<td>16-bit adder/sub-tractor</td>
<td>10-bit adder/sub-tractor</td>
</tr>
<tr>
<td>STAGE-3</td>
<td>Booth Encoder</td>
<td>Booth Encoder</td>
</tr>
<tr>
<td></td>
<td>3:1 MUX</td>
<td>3:1 MUX</td>
</tr>
<tr>
<td></td>
<td>16-bit adder/sub-tractor</td>
<td>10-bit adder/sub-tractor</td>
</tr>
<tr>
<td>STAGE-4</td>
<td>Booth Encoder</td>
<td>Booth Encoder</td>
</tr>
<tr>
<td></td>
<td>3:1 MUX</td>
<td>3:1 MUX</td>
</tr>
<tr>
<td></td>
<td>16-bit adder/sub-tractor</td>
<td>10-bit adder/sub-tractor</td>
</tr>
</tbody>
</table>

2.4 Working of Stage-1

The operational block diagram of stage-1 of the proposed 8x8 booth multiplier is as shown in figure 10.
Figure 10. Block level implementation of stage-1.

As shown in figure 10, the inputs to 10-bit 3:1 MUX of stage-1 are 8-bit zero, 8-bit of multiplicand A and 9-bit of 2A. The inputs to booth encoder of stage-1 are appended zero and two LSB of multiplier bit. Depending on the output of booth encoder, stage-1 decides to get 0, A or 2A from 10-bit 3:1 MUX and whether to select output from 10-bit 3:1 MUX or 10-bit modified BEC. The input to 10-bit modified BEC is inverted output from 10-bit 3:1 MUX. The input to 10-bit 2:1 MUX is output from 10-bit modified BEC and output from 10-bit 3:1 MUX. As one input to stage-1 is all zero so
adder/sub-tractor block is replaced by 10-bit modified BEC and 10-bit 2:1 MUX. So the output of stage-1 will be either output from 10-bit modified BEC or from 10-bit 3:1 MUX.

### 2.4.1 10-bit Modified BEC

The function of binary-to-excess-1 converter is to add 1 to whatever the input is provided. The main advantage to using BEC is in carry select adder (CSA) to reduce the number of gates so we can decrease area consumption and by doing so power consumption can also be reduced. For example, 4-bit BEC has four inputs and 5-bit output if it has carry. The Boolean equation for 4-bit of BEC is written below:

\[
X_0 = B_0'
\]

\[
X_1 = B_1 \text{ xor } B_0
\]

\[
X_2 = (B_1 \text{ and } B_0) \text{ xor } B_2
\]

\[
X_3 = (B_2 \text{ and } B_1 \text{ and } B_0) \text{ xor } B_3
\]

Where B0, B1, B2 and B3 are the input to 4-bit BEC and X0, X1, X2 and X3 are respectively the output of 4-bit BEC.

The gate level implementation of regular 10-bit BEC is as shown in figure 11. As it can be seen from figure 11 that each AND_2 gate have to wait for previous AND_2 gate. The inputs to each AND_2 are present input (Bi) and output from previous AND_2 gate. So the total delay of 10-bit regular BEC is as shown in equation below:
\[ T_d = 8 \times (\text{Delay of AND2}) + \text{Delay of XOR2} \]

Figure 11. Gate level implementation of 10-bit regular BEC

The delay of regular 10-bit BEC can be reduced by replacing all AND_2 gates with repeating structure of AND_2, AND_3 and AND_4 i.e. 2-3-4 from left to right.

The gate level implementation of 10-bit modified BEC is as shown in figure 12.

Figure 12. Gate level implementation of 10-bit modified BEC
As shown in the figure 12, structure 2-3-4 is used to implement 10-bit modified BEC. Here every time present AND gate have not to wait for previous AND gate. The changes made are instead of using 8 AND_2 it uses 3 AND_2, 3 AND_2 and 2 AND_4. The input to AND_4 gate will be present input (B_i) and three previous inputs. The output of AND_4 is applied to AND_2 and AND_3 to its left as shown in figure 12. So the delay of 10-bit modified BEC will be given by equation below:

\[
Td = 2 \times (\text{Delay of AND}_4) + \text{Delay of AND}_3 + \text{Delay of XOR}_2
\]

### 2.4.2 10-bit 3:1 MUX

The purpose of using 3:1 MUX is to select either 0, A or 2A. The select lines for 3:1 MUX are two, which is provided from the output from booth encoder. We are using 10-bit 3:1 MUX because as we are giving 8-bit multiplicand, so if we left shift it we will get 9-bits for 2A and the output provided to 2:1 MUX and modified BEC is also 10-bit. Gate level implementation of 3:1 MUX for function 7 by using three NAND_3 gates and two inverters are shown in figure 13.

Figure 13. Gate level implementation of 3:1 MUX
\[ Z = (A \text{nand } S_0') \text{nand } (B \text{nand } S_0') \text{nand } (C \text{nand } S_0 S_1') \quad (7) \]

To implement 10-bit 3:1 MUX, ten 1-bit 3:1 MUX are connected in parallel to produce 10-bit of output as shown in figure 14. The 8-bit zeros (i.e. 00000000) is applied to top input of 3:1 MUX, 8-bit multiplicand is applied to middle input of 3:1 MUX and left shifted multiplicand is provided to bottom input of 3:1 MUX. For two remaining 3:1 MUX in 10-bit 3:1 MUX two top inputs will be 00 and two middle inputs will be MSB of multiplicand. And for the 10th 3:1 MUX the bottom input will be the MSB of multiplicand left shifted by 1. 10-bit 3:1 MUX produces 10-bit output which is provided to modified BEC and 10-bit 2:1 MUX.

![10-bit MUX_3:1](image-url)
2.4.3 10-bit 2:1 MUX

The purpose of 10-bit 2:1 MUX in stage-1 is to select either 10-bit output from modified BEC or 10-bit output from 3:1 MUX. 10-bit 2:1 MUX uses MSB of booth encoder as its select line to choose from two inputs. The gate level implementation of 2:1 MUX is as discussed above in figure 7. 10-bit 2:1 MUX is implemented using ten 2:1 MUX in parallel. The inputs to 10-bit 2:1 MUX are 10-bit output from 3:1 MUX and 10-bit output from 10-bit modified BEC. 10-bit 2:1 MUX produces 10-bit output out of that two LSB is provided to final product, that will become P0 and P1 bit of final product and the remaining eight bits are provided to 10-bit adder/sub-tractor block of stage-2.

2.5 Working of Stage-2/3/4

As it is seen in figure 15, stage-2/3/4 are implemented using booth encoder, 10-bit 3:1 MUX and 10-bit adder/sub-tractor block. The input to booth encoder of stage-2, stage-3 and stage-4 are B3-B2-B1, B5-B4-B3 and B7-B6-B5 respectively. The function of stage-2, stage-3 and stage-4 are same, just the inputs to each stage changes, so working of stage-2 is discussed first and then overview of the functioning of stage-3 and stage-4 are discussed. The input to 3:1 MUX of traditional 8x8 booth multiplier of stage-2 is the multiplicand bits left shifted by 2 but in the proposed 8x8 booth multiplier, input to 10-bit 3:1 MUX is the same multiplicand bits. There is no need to left shift the multiplicand bits because of the architecture of proposed 8x8 Booth multiplier. Traditional Booth multiplier uses circuitry for left shift by 2 in each stages but proposed
8x8 Booth multiplier do not need that circuitry because of its architecture. The output of 10-bit 3:1 MUX is provided to 10-bit adder/sub-tractor block as discussed earlier.

![Block diagram of stage-2/3/4](image)

**Figure 15. Block diagram of stage-2/3/4**

### 2.5.1 10-bit Adder/Sub-tractor

As discussed earlier, the function of adder/sub-tractor circuitry is to add or subtract two binary numbers depending upon the Cin. If Cin = 0, it will perform
addition else if Cin = 1 it will perform subtraction. Depending upon the requirement, we can select which type of adders to use. For example, if the requirement is less area we can select ripple carry adder (RCA) because the complexity of it is low, but the drawback of it is that it has high delay. And if the requirement is high speed we can use parallel prefix adders Kogge-Stone, Brent-Kung, etc. but they consume very high compared to ripple carry adders. The optimal adder for high speed and minimal area consumption is carry select adders (CSA).

Carry select adders are one of the fastest adders available in VLSI industry and the operation of it is explained using 10-bit adder [22]. Usually there will be two blocks of full adders in each part except first part such that first part will have one block of n–bit full adders while remaining part will have two blocks of n-bit full adders [23]. The reason for using two blocks of full adders is that first block will compute sum and carry-out by assuming carry-in Cin = 0 and second block will compute sum and carry-out by assuming carry-in Cin = 1 in advance. As CSA adders compute sum and carry-out by assuming to different situation such as Cin = 0 and Cin = 1 so carry-out from previous block is used to select sum and carry-out for present block using multiplexer (MUX).

There are two main types of carry select adders such as linear carry select adders known as CSA and square root carry select adders (SQRT CSA). The main difference between these types of adders is that linear CSA uses equal number of bit blocks while SQRT CSA uses unequal number of full adders in each block. The block level implementation of 16-bit linear carry select adders is as shown in figure 16 and block level implementation of 16-bit square root carry select adders is as shown in figure 17.
From the above figure 16 it is can be stated that 16-bit linear carry select adders needs seven 4-bit RCA and three 5-bit 2:1 MUX’s. The number of transistor ratio of adder to 2:1 MUX it is 3:1 so 15 2:1 MUX will be equal to 5 full adders. According to above assumption, number of gates used in 16-bit linear CSA is 165 gates and the delay of it is 17 (11+6) gate delays which is almost 2.5 times faster than 16-bit RCA but area consumption of 16-bit linear CSA is almost 2 times that of 16-bit RCA.
To further increase speed of linear CSA, another type CSA is used i.e. square root carry select adder. SQRT CSA uses same architecture as linear CSA but the main difference is that SQRT CSA uses unequal number of full adders in each part [24]. In SQRT CSA, current stage will have equal or more numbers full adders with respect to previous stage and equal or less number of full adder than next stage. The optimal number of full adders in each stage will be 2-2-3-4-5 for 16-bit SQRT CSA as shown in Figure 17.

Figure 17. Block diagram of 16-bit SQRT CSA using RCA+RCA
figure 17. Gate delay for 16-bit SQRT CSA is 13 out of which 2-bit FA have gate delay of 5 and four multiplexer will have delay of 8 but it increases area consumption as well as power consumption compared to linear CSA. As per the requirement of high speed booth multiplier it is implemented using square root carry select adder.

To further decrease area and power consumption of SQRT CSA, dual RCA in each stage is replaced by one RCA and one BEC [25]. RCA will compute result for Cin=0 and output is applied to BEC and it will perform same operation as RCA with Cin=1 [26]. The block diagram of 16-bit SQRT CSA using RCA+BEC combination is as shown in figure 18.

The delay equation for N-bit linear CSA with M stages is given by:

\[ T = t(setup) + \left( \frac{N}{M} \right) \times t(carry) + M \times t(mux) + t(sum) \]

The delay equation for N-bit SQRT CSA with P stages is given by:

\[ T = t(setup) + P \times t(carry) + (2N)^{0.5} \times t(mux) + t(sum) \]

From the above discussion we can conclude that square root carry select adder using RCA+RCA provides you the higher speed compared to linear CSA and SQRT using RCA+BEC.

So, square root carry select adder using RCA+RCA is used to implement 10-bit SQRT CSA because it gives high speed compared to other techniques of carry select adders.
2.5.1.1 10-bit SQRT CSA using RCA+RCA

The block diagram of 10-bit SQRT CSA using RCA+RCA is shown in figure 19.

The summary and comparison of components used in 10-bit SQRT CSA using RCA+RCA and 10-bit SQRT CSA using RCA+BEC are shown in table 6.
Figure 19. Block diagram of 10-bit SQRT CSA using RCA+RCA

Table 7. Component comparison in two different styles of SQRT CSA

<table>
<thead>
<tr>
<th></th>
<th>10-bit SQRT CSA Using RCA+RCA</th>
<th>10-bit SQRT CSA Using RCA+BEC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-bit full adder</td>
<td>19</td>
<td>10</td>
</tr>
<tr>
<td>BEC</td>
<td>-</td>
<td>4</td>
</tr>
<tr>
<td>Multiplexer</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Delay (ps)</td>
<td>147</td>
<td>193</td>
</tr>
</tbody>
</table>
2.5.1.2 10-bit SQRT CSA using RCA+BEC

The block diagram of 10-bit SQRT CSA using RCA+BEC is shown in figure 20.

![Block diagram of 10-bit SQRT CSA using RCA+BEC](image)

Figure 20. Block diagram of 10-bit SQRT CSA using RCA+BEC

The inputs to 10-bit SQRT CSA using RCA+RCA are output from 10-bit 3:1 MUX from stage-2 and output from stage-1. It will perform addition or subtraction depending upon select line and produces 11-bit of output, of which MSB is the carry-out. From the remaining 10-bit, two LSB are directly sent to final output and they are P2.
and P3 bits of final product, and eight bits are sent to adder/sub-tractor block of stage-3.

Similarly, stage-3 produces 11-bits, of which MSB is the carry-out and two LSB are directly sent to final output and they are P4 and P5 bits of final product and remaining eight bits are provided to adder/sub-tractor block of stage-4. The output of stage-4 will become P15-P6 bits of final product. So the final product is made up of 16-bits i.e. P15-P0.

Figure 21. Block diagram of proposed 8x8 Booth multiplier
3 Implementation of proposed 8*8 Booth Multiplier in 32nm CMOS Technology

From the discussion in Chapter 2, it can be concluded that the main sub-components used in the implementation of proposed 8*8 booth multiplier are MUX 2:1, MUX 3:1 and 1-bit full adder. The above listed sub-components are used to implement higher level blocks like 10-bit MUX 2:1, 10-bit MUX 3:1 and 10-bit SQRT CSA using RCA+RCA respectively. The basic gates used in the implementation of booth encoder and 10-bit modified BEC with inverter are 2-input XOR, 2, 3, 4-input AND gate and inverter. MUX 2:1, MUX 3:1 and 1-bit full adder are implemented using pass transistor logic (PTL) while inverter, 2-input XOR gate and 2, 3, 4-input gate are implemented using static logic. All components are implemented in 32nm complementary metal oxide semiconductor (CMOS) technology. The main purpose of using PTL is to decrease area and delay in basic element so that overall area of the design is decreased and speed is increased. Static inverters are added at the output of PTL logic to boost the weak outputs caused by the pass transistor to be strong logic. By doing so we can overcome the disadvantage of PTL and keeping area low and faster. Pass transistor logic often uses fewer transistors, runs faster and requires less power than the same function implemented with the same transistors in fully complementary CMOS logic [27]. As the technology is scaling down, power supply of design is also decreasing with scaling technology. As integrated circuit supply voltages decreases, the disadvantages of pass transistor logic become more significant; the threshold voltage of transistors becomes large compared to supply voltage, severely limiting the number of sequential PTL stages.
3.1 Implementation of sub-components in 32nm CMOS technology

Schematic implementation in 32nm of MUX 2:1, MUX 3:1 and 1-bit full adder is explained here with their results (i.e. waveforms) showing delay and power consumption.

3.1.1 MUX 2:1

Schematic view of MUX 2:1 using PTL is shown in figure 22. As seen in figure 22, 2 NMOS transistors and 3 inverters are used to implement MUX 2:1. The gate of one NMOS transistor is connected to select line S and another with S’. Input A is connected to drain terminal of one NMOS and input B is connected to drain terminal of another NMOS transistor. Two inverters are connected at output to boost the output logic and increase the drive strength of MUX 2:1. Compared to CMOS logic, PTL saves 33% area based on the number of transistors.

Figure 22. PTL implementation of MUX 2:1
Simulation result of MUX 2:1 using PTL logic is shown in figure 23. The propagation delay $t_{HL}$ and $t_{LH}$ are 15.2 ps and 24 ps respectively and the power consumption is 0.567 uW at 1 GHz input frequency.

![Simulation result of MUX 2:1](image)

**Figure 23. Simulation result of MUX 2:1**

### 3.1.2 MUX 3:1

Schematic view of MUX 3:1 using PTL is shown in figure 24. As seen in figure, 6 NMOS transistors and 4 inverters are used to implement MUX 3:1. There are three pair of two NMOS transistors and gate of first pair is connected to $S0'$ and $S1'$ respectively. Similarly, gate of other two pairs are $S0'$ and $S1$, $S0$ and $S1'$ respectively. Input A is connected to drain terminal of first NMOS transistor in first pair. Similarly, inputs B and C are connected to drain terminal of second and third pair respectively. The
source terminal of all three pairs are connected to two inverters to increase the drive strength of MUX 3:1.

Simulation result of MUX 3:1 using PTL logic is shown in figure 25. The propagation delay $t_{HL}$ and $t_{LH}$ are 21.2 ps and 49.7 ps respectively and the power consumption is 1.33 uW at 1 GHz input frequency. The number of transistors used in MUX 3:1 with PTL is 14 which is less compared to static CMOS implementation with 28 transistors needed.

Figure 24. PTL implementation of MUX 3:1
3.1.3 1-bit Full Adder

The implementation of 1-bit full adder with PTL logic is shown in figure 26. Full adder uses 12 NMOS transistors and five inverters. Out of five inverters, three inverters are used to get inverted inputs and other two inverters are connected at outputs to increase the drive strength of PTL logic. As shown in the figure 26, the upper part is used to get the sum of full adder and lower part is used to get carry-out of 1-bit full adder.

The number of transistors used in 1-bit full adder with PTL is 22 which are less compared to 1-bit mirror adder static CMOS implementation (i.e. 28 transistors).
Simulation result of 1-bit full adder using PTL logic is shown in figure 27. The propagation delay $t_{HL}$ and $t_{LH}$ are 25.1 ps and 8.46 ps respectively and the power consumption is 1.63 uW at 1 GHz input frequency.
3.2 Results of Sub-components

This section includes the results of each sub-components of proposed 8*8 booth multiplier such as booth encoder, 10-bit modified binary-to-excess one converter (BEC) with inverter, 10-bit SQRT CSA using RCA+RCA. The result of 10-bit MUX 3:1 and 10-bit MUX 2:1 is usually same as 1-bit MUX 3:1 and 1-bit MUX 2:1 instead we are using 10 components in parallel. The delay of 10-bit MUX’s will be same as 1-bit MUX because computation is carried in parallel.

3.2.1 Result of 10-bit modified BEC with inverters at input

10-bit modified BEC with inverter at input is used in the stage-1 as discussed in chapter 2, which removes the need of using 10-bit adder/sub-tractor. The functionality of 10-bit modified BEC with inverters at input is same as that of 10-bit adder in stage-1 because there is no second input coming from previous block. Using 10-bit modified BEC with inverters at input in stage-1 and 10-bit MUX 2:1 instead of 10-bit adder/sub-tractor decreases number of transistor counts and finally area consumption is decreased. The schematic of 10-bit modified BEC with inverters at input is same as shown in figure 12. As you can see from figure it is implemented using 2-3-4 AND structure which helps in reduction of delay compared to regular 10-bit BEC. As seen from the block diagram of stage-1 and stage-2/3/4, we see that stage-1 consist 10-bit modified BEC with inverters at input and 10-bit MUX 2:1 instead of 10-bit SQRT CSA using RCA+RCA. The comparison of result including delay, power and transistor count between 10-bit BEC and 10-bit adder/sub-tractor is shown in table 7.
Table 8. Comparison of result between 10-bit modified BEC and 10-bit SQRT CSA

<table>
<thead>
<tr>
<th></th>
<th>Delay (ps)</th>
<th>Power consumption (uW)</th>
<th>Transistor count</th>
</tr>
</thead>
<tbody>
<tr>
<td>10-bit modified BEC (&amp;10-bit MUX 2:1)</td>
<td>139 (+ 24) = 163</td>
<td>25.22</td>
<td>190 (+ 80) = 270</td>
</tr>
<tr>
<td>10-bit SQRT CSA Using RCA+RCA</td>
<td>171</td>
<td>60.52</td>
<td>634</td>
</tr>
</tbody>
</table>

The worst-case propagation delay of 10-bit modified BEC with inverters at input can be performed using 0000000000 input stimulus, so inverters at input gives 1111111111. The purpose of getting 1111111111 after inverter is that AND gates have to wait for other input to compute final output. So, 0000000000 is worst case input stimulus which gives 139 pico-second of delay at 1 GHz input frequency as indicated in figure 28. The input to the circuit is 1111111111 to calculate WPD and the expected output is 0000000000. Figure 28 consist of input B0 and outputs X0-X9 and delay is calculated between B0 and X9.
3.2.2 Results of Booth Encoder

The booth encoder is the key element in selecting the function to be performed depending upon multiplier bits. The truth table and schematic of booth encoder is same as discussed in chapter 1. The result of booth encoder for radix-4 algorithm is shown in figure 29 and delay calculation for the worst case is measured. The worst-case propagation delay of booth encoder is 27.9 pico-second and power
consumption is 1.45 micro-watt at 1 GHz input frequency. The transistor count of booth encoder is 32 for radix-4 algorithm. Suppose, the input to booth encoder is 101, which means it needs to produce \(-A\) depending upon the truth table of radix-4 booth encoder so the expected output should be 110. Out of 110, 10 is used to select A from MUX 3:1 and MSB (i.e. 1) is decide subtraction operation.

![Simulation results of radix-4 booth encoder](image)

**Figure 29. Simulation results of radix-4 booth encoder**

### 3.2.3 Result of 10-bit SQRT CSA using RCA+RCA

As the main aim is to design high speed 8*8 booth multiplier, we selected 10-bit SQRT CSA using RCA+RCA over other adders such ripple carry adder and linear carry select adder. The main drawback of using SQRT CSA using RCA+RCA structure over
RCA+BEC structure is that RCA+RCA consumes more area and power compared to the other structure listed above. 1-2-3-4 structure is used to implement 10-bit SQRT CSA such that computation of 1-bit, 2-bit, 3-bit and 4-bit respectively is done four different stages as shown in figure 19. The result of 10-bit SQRT CSA using RCA+RCA is shown in figure 30 and delay calculation for worst case is shown as well.

Figure 30. Simulation results and delay calculation of 10-bit SQRT CSA using RCA+RCA

The worst case for calculating delay is \( A = 0000000000, B = 0000000000 \) and \( C_{in} = 1 \) which produces the delay of 170 pico-second and power consumption of 60.52 micro-watt at 1 GHz input frequency. But if \( A = 1111111111, B = 1111111111 \) and \( C_{in} = 1 \)
3.3 Results of proposed 8*8 Booth Multiplier

The result of proposed 8*8 booth multiplier for worst case and best case propagation delay is discussed in this section. First we will discuss about worst case stimuli and worst case propagation delay with waveforms for proposed 8*8 booth multiplier. Second we will discuss about the best case stimuli and best case propagation delay with waveforms for proposed 8*8 booth multiplier. Then the same worst case and best case stimuli is applied to proposed 8*8 booth multiplier with DFF at output and worst propagation delay (WPD) and best propagation delay (BPD) is calculated.

3.3.1 Worst case propagation delay calculation without DFF

$0_{10}$ (0000 0000) and $-86_{10}$ (10101010) generates the worst case propagation delay for proposed 8*8 booth multiplier. Here 00000000 is applied to multiplicand bits and 10101010 is applied to multiplier bits. Waveform for proposed 8*8...
booth multiplier with WPD calculation is shown in figure 31. The reason for selecting above given stimuli is because as we fed 00000000 as multiplicand bit and 100 is applied to booth encoder of stage-1 which will produce 101 out of which 01 will select ‘A’ and 1 at MSB will select the output from 10-bit modified BEC with inverters at input through 10-bit MUX 2:1. In the stage-2 101 is applied to booth encoder which produces 110 out of which 10 will select ‘A’ and 1 at MSB will select subtraction operation which the worst case for power as well because every component is having activity.

Figure 31. Simulation results and WPD calculation for proposed 8*8 booth multiplier
Similarly, 101 is applied to booth encoder of stage-3 and stage-4 which produces 110 and depending upon these bits ‘A’ and subtraction operation are selected. So the above selected stimuli produces worst propagation delay. The power consumption for worst case stimuli 0.274 milli-watt at 1 GHz.

**3.3.2 Best case propagation delay calculation without DFF**

$0_{10}$ (00000000) and $85_{10}$ (01010101) generates the best case propagation delay for proposed 8*8 booth multiplier. Here 00000000 is applied to multiplicand bits and 01010101 is applied to multiplier bits. Waveform for proposed 8*8 booth multiplier with BPD calculation is shown in figure 32. The reason for selecting above given stimuli is because as we fed 00000000 as multiplicand bit and 010 is applied to booth encoder of stage-1 which will produce 010 out of which 10 will select ‘2A’ and 0 at MSB will select directly the output from 10-bit MUX 3:1.

The below discussed stimuli produces best propagation delay because in stage-1 it does not have to wait for output from 10-bit modified BEC with inverters at input. For stage-2, 010 is applied to booth encoder which produces 010 out of which 10 selects ‘A’ and 0 at MSB selects adder operation. Same steps are performed at stage-3 and stage-4 because same inputs are provided to booth encoder. The power consumption of best case stimuli for proposed 8*8 booth multiplier is 0.343 milli-watt at 1 GHz.
Figure 32. Simulation results and BPD calculation for proposed 8*8 booth multiplier

3.3.3 Result comparison of components to implement proposed design

This section compares various metrics such as delay, transistor counts and power consumption of components used in implementing proposed 8*8 radix booth multiplier.
### Table 9. Metric comparison of components

<table>
<thead>
<tr>
<th>Component</th>
<th>Transistor count</th>
<th>Delay (ps)</th>
<th>Power consumption (uW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Booth Encoder</td>
<td>32</td>
<td>25.5</td>
<td>1.36</td>
</tr>
<tr>
<td>10-bit MUX 2:1</td>
<td>80</td>
<td>24</td>
<td>4.38</td>
</tr>
<tr>
<td>10-bit MUX 3:1</td>
<td>140</td>
<td>49.7</td>
<td>11.02</td>
</tr>
<tr>
<td>10-bit BEC with inv.</td>
<td>190</td>
<td>139</td>
<td>25.22</td>
</tr>
<tr>
<td>10-bit SQRT CSA (RCA + RCA)</td>
<td>634</td>
<td>171</td>
<td>60.52</td>
</tr>
<tr>
<td>Stage - 1</td>
<td>442</td>
<td>183</td>
<td>49.13</td>
</tr>
<tr>
<td>Stage -2/3/4</td>
<td>806</td>
<td>208</td>
<td>64.38</td>
</tr>
<tr>
<td>Proposed 8*8 radix Booth multiplier</td>
<td>2860</td>
<td>424</td>
<td>274.89</td>
</tr>
</tbody>
</table>

#### 3.3.4 Comparing proposed Booth multiplier with other multipliers in literature

The proposed 8*8 radix-4 booth multiplier is compared for same metrics discussed above with other 8*8 multipliers available in literature. The comparison is stated in below as shown in table 9.
Table 10. Comparison of proposed booth multiplier with other multipliers

<table>
<thead>
<tr>
<th></th>
<th>Reference - 12</th>
<th>Reference - 8</th>
<th>Proposed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Technology (nm)</td>
<td>90</td>
<td>65</td>
<td>32</td>
</tr>
<tr>
<td>Supply Voltage (V)</td>
<td>1.2</td>
<td>1.32</td>
<td>1.0</td>
</tr>
<tr>
<td>Temperature (°C)</td>
<td>25</td>
<td>25</td>
<td>25</td>
</tr>
<tr>
<td>Propagation Delay (ns)</td>
<td>1.96 (0.69)</td>
<td>1.04 (0.51)</td>
<td>0.42</td>
</tr>
<tr>
<td>Power Consumption (mW)</td>
<td>0.3 (0.588)</td>
<td>0.358 (0.510)</td>
<td>0.275</td>
</tr>
<tr>
<td>PDP (pJ)</td>
<td>0.405</td>
<td>0.185</td>
<td>0.113</td>
</tr>
</tbody>
</table>

Reference 1 - [12]
Reference 2 - [8]

Values mentioned in the brackets are the normalized values with respect to CMOS 32nm technology
4.0 Conclusion and Future work

4.1 Conclusion

- The proposed design of 8x8 booth multiplier is implemented using radix-4 booth algorithm so it uses four stages out of which 1st stage is implemented using booth encoder, 10-bit MUX 3:1, 10-bit modified BEC and 10-bit MUX 2:1 and 2nd, 3rd and 4th stages are implemented using booth encoder, 10-bit MUX 3:1 and 10-bit adder/sub-tractor SQRT CSA using RCA+RCA structure.

- As the input from previous block is always zero to stage-1, so there is no need of adder/sub-tractor block in 1st stage. In this thesis, 1st stage’s adder/sub-tractor block is replaced by 10-bit modified BEC and 10-bit MUX 2:1. By doing this we saved power as well as area by 23.76% and 45.16% respectively with decrease in delay by 12.02%.

- The worst-case delay of 1st stage at data rate of 1 GHz input frequency is 183 picosecond and power consumption of 49.13 micro-watt with transistor count of 442.

- Adder/Sub-tractor block of stage-2/3/4 is implemented using 10-bit SQRT CSA with RCA+RCA structure to boost the speed of operation compared to 10-bit SQRT CSA using RCA+BEC structure.

- The worst-case delay of stage-2/3/4 at data rate of 1 GHz input frequency is 208 picosecond and power consumption of 64.38 micro-watt with transistor count of 806.
• Traditional 8x8 booth multiplier uses 16-bit adder/sub-tractor blocks in each stages. The proposed radix-4 8x8 booth multiplier is designed with 10-bit SQRT CSA using RCA+RCA. The principle of the design is as following. Two LSB of output of 1st stage becomes LSB of final product (i.e. P1 and P0). Similarly, two LSB of output of 2nd and 3rd stage will become P2-P5 bits respectively. The output of stage-4 will become P6-P15 bits of final product, which is computed by the 10-bit SQRT CSA with RCA+RCA architecture.

• The worst-case delay of proposed 8x8 booth multiplier at data rate of 1 GHz input frequency is 424 picosecond and power consumption of 0.274 milli-watt with transistor count of 2860.

4.2 Future Work

Future work includes:

• Novel structures for high speed and low-power digital matched filters can be developed based on the proposed multiplier in this thesis.

• Delay elements (i.e. register and flip-flop) are to be optimized for performance and techniques should be developed for low-power consumption.

• Similarly adders can be implemented using new architecture which can operate at higher speed and consume low power as well. New techniques can be employed to adder structure so that they consume less area. The reason for decreasing area of adder is that adders are also used in implementation of multipliers. So the overall area consumption of DMF decreases.
• Resource sharing can also be implemented in booth multiplier so that area consumption of the design decreases.

• Testing the robustness of the designs by doing PVT variations and doing Monte Carlo analysis.
References


