Tunable C Band Coupled-C BPF with Resonators
Using Active Capacitor and Inductor

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in Electrical Engineering

By

YU WANG

B.S., Taiyuan University of Technology, China, 2013

2016

WRIGHT STATE UNIVERSITY
August 4, 2016

I HEREBY RECOMMEND THAT THE THESIS PREPARED UNDER MY SUPERVISION BY Yu Wang ENTITLED “Tunable C Band Coupled-C BPF with Resonators Using Active Capacitor and Inductor” BE ACCEPTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF Master of Science in Electrical Engineering

___________________________
Chien-In Henry Chen, Ph.D.
Thesis Director

___________________________
Brian Rigling, Ph.D.
Department Chair

Committee on Final Examination

___________________________
Chien-In Henry Chen, Ph.D.

___________________________
Yan Zhuang, Ph.D.

___________________________
Jiafeng Xie, Ph.D.

___________________________
Robert E.W.Fyffe,Ph.D.
Vice President for Research and Dean of the Graduate School
Abstract


In this thesis, a classic second-order coupled-capacitor Chebyshev bandpass filter with resonators using active capacitor and inductor is presented. The low cost and small size of CMOS active components makes the band pass filter (BPF) attractive in fully-integrated CMOS applications. The active capacitor is designed to compensate active inductor’s resistance for resistive match in the resonator. Meanwhile, adjusting design parameter of the active component provides BPF tunability in center frequency, pass band and pass band gain. Designed in 1.8V 180 nanometer CMOS process, the BPF has a tuning frequency range of 758-864 MHz, a controllable pass band of 7.1-65.9 MHz, a Q factor of 12-107, a pass band gain of 6.5-18.1dB and a stopband rejection of 38-50 dB.
TABLE OF CONTENTS

1 INTRODUCTION........................................................................................................................................1
  1.1 Background ...........................................................................................................................................1
  1.2 Past Work...............................................................................................................................................2
  1.3 Thesis objective and organization .........................................................................................................5

2 ACTIVE CAPACITOR WITH NEGATIVE RESISTANCE ..........................................................7
  2.1 Large signal analysis of AC ...............................................................................................................7
  2.2 Small signal analysis of AC ...............................................................................................................7
  2.3 AC simulations ......................................................................................................................................11

3 ACTIVE INDUCTOR ...............................................................................................................................12
  3.1 Gyrator-C active inductor (AI) .........................................................................................................12
    3.1.1 Lossless single-ended gyrator-C active inductor .........................................................................12
    3.1.2 Lossless differential (floating) gyrator-C active inductors .........................................................13
    3.1.3 Lossy Single-ended gyrator-C active inductors .........................................................................14
    3.1.4 Lossy differential (floating) gyrator-C active inductors ...............................................................15
    3.1.5 Controllable parameters of active inductors ...............................................................................16
      3.1.5.1 Tuning center frequency range ...........................................................................................17
      3.1.5.2 Inductance tunability ...........................................................................................................18
      3.1.5.3 Quality factor Q ....................................................................................................................18
    3.2 Practical active inductors based gyrator-C topology ......................................................................21
      3.2.1 Single-ended active inductor ...................................................................................................21
        3.2.1.1 Active inductor with feedback resistance ........................................................................21
        3.2.1.2 Active inductor with feed-forward path transistor ............................................................23
        3.2.1.3 Class AB active inductor ....................................................................................................23
        3.2.1.4 Active inductor with cascode topology .............................................................................25
      3.2.2 Differential active inductor .........................................................................................................26
        3.2.2.1 Differential active inductor with the inverter pair .............................................................26
3.2.2.2 Differential active inductor with cross-coupled transistors and feedback resistor .................................................................28

3.3 Large signal analysis of the proposed AI ..........................................29
3.4 Small signal analysis of the proposed AI .........................................30
3.5 AI simulation ..................................................................................31

4 ACTIVE BPF USING RESONATOR BASED ON ACTIVE INDUCTOR AND ACTIVE CAPACITOR ..........................................................33

4.1 Introduction ....................................................................................33
4.2 The proposed BPF ........................................................................37

5 BPF TUNABILITY AND PERFORMANCE EVALUATION .................43

6 CONCLUSION AND FUTURE WORK ............................................45

7 REFERENCE ....................................................................................46
LIST OF FIGURES

Figure 1.1 The classic second-order coupled-capacitor Chebyshev BPF ...........................................2
Figure 1.2 Second order BPF with tapped inductor and a shunt feedback inductor [2] ......................2
Figure 1.3 Second order BPF with switched inductor ........................................................................3
Figure 1.4 Transformer-based with Q improved technology inductor [8] .......................................3
Figure 1.5 BJT active inductor: (a) ..................................................................................................4
Figure 1.6 Tunable BPF with BJT active inductor [5]: (a) .................................................................5
Figure 2.1 The active capacitor and its equivalent circuit .................................................................7
Figure 2.2 The small signal modal of active capacitor .......................................................................8
Figure 2.3 AC capacitance ................................................................................................................10
Figure 2.4 AC negative resistance ....................................................................................................10
Figure 3.1 Lossless single-ended gyrator-C active inductor ............................................................12
Figure 3.2 Lossless differential gyrator-C active inductor ..............................................................13
Figure 3.3 Lossy single ended active inductor ..................................................................................14
Figure 3.4 Floating differential gyrator-C active inductor [17] ........................................................15
Figure 3.5 Bode plot of different frequency range [17] .................................................................18
Figure 3.6 Frequency response due to different resistive loss of AI .............................................19
Figure 3.7 Quality factor along with frequency response ...............................................................20
Figure 3.8 Cascade-grounded active inductor with a resistance feedback [9] ...............................22
Figure 3.9 Simulation result of quality factor and inductance [9] ...................................................22
Figure 3.10 Active inductor with improved technology [10] .............................................................23
Figure 3.11 Equivalent modal of class AB active inductor [11] .......................................................24
Figure 3.12 Class AB active inductor [11] .......................................................................................24
Figure 3.13 Tunable active inductor schematic [12] .......................................................................25
Figure 3.14 Small signal modal and its equivalent circuit [12] .............................................. 26
Figure 3.15 Differential active inductor [13]: (a) ................................................................. 27
Figure 3.16 Differential AI with feedback resistor [15]: (a) .................................................. 28
Figure 3.17 The proposed active inductor in this work ......................................................... 29
Figure 3.18 The small signal modal of active inductor .......................................................... 31
Figure 3.19 Inductance of active inductor ............................................................................. 32
Figure 3.20 Resistance of active inductor ............................................................................. 32
Figure 4.1 Classic Chebyshev N order BPF ......................................................................... 33
Figure 4.2 BPF with shunt feedback inductor [2] ................................................................. 34
Figure 4.3 BPF with shunt feedback capacitor [22] ............................................................... 35
Figure 4.4 BPF with shunt capacitor and grounding inductor [24] ...................................... 35
Figure 4.5 BPF separated into three parts [24] ................................................................. 35
Figure 4.6 Basic part of the second order BPF separated into L-match [24] ...................... 36
Figure 4.7 The proposed active BPF ...................................................................................... 37
Figure 4.8 Equivalent circuits of a resonator ......................................................................... 37
Figure 4.9 BPF performance ................................................................................................. 38
Figure 4.10 Capacitance vs. frequency (before and after using \( L_{DC} \) and \( C_{AC} \)) .......... 38
Figure 4.11 Conductance vs. frequency (before and after using \( L_{DC} \) and \( C_{AC} \)) .......... 39
Figure 4.12 Inductance vs. frequency (before and after using \( R_{AD} \)) .................................. 39
Figure 4.13 Resistance vs. frequency (before and after using \( R_{AD} \)) ............................... 40
Figure 5.1 Tunable BPF performance .................................................................................... 43
LIST OF TABLES

Table 5.1 The previously reported several works by using the same structure of classic Chebyshev band pass filter.......................................................... 44
Acknowledgement

I would like to express my sincere thanks to Dr. Henry Chen. Not only did he give me the academic guidance, but also endue me with the attitude how to treat the life.

I would like to express my sincere thanks to my parents who have been supporting me with their love and care.

Finally, thanks to the committee members for their professional knowledge and services for my thesis defense.
1 INTRODUCTION

1.1 Background

The rapid development of complementary metal-of-semiconductor (CMOS) endues the integrated circuit with small size and low cost in both digital and analog applications. A wireless communication system mainly consists of three components: mixer, band pass filter, and low noise amplifier. The band pass filter blocks unwanted signals, selects desirable signal matched to different criterions along with the mixer, for example, 1,920-1,980 MHz of WCDMA, 890-960 MHz of GSM, 1,575 MHz of GPS, and 2,400-2,483 MHz of 802.11b/g. So, a band pass filter with high Q and good selectivity of center frequency and bandwidth is desirable in today’s applications. The LC based passive band pass filter has been used for several decades, however, when applied to the nanotechnology CMOS integrated circuit it confronts two limitations: 1) the degraded performance of CMOS spiral inductor due to its significant resistive loss limits the BPF quality factor and restrains its gain and bandwidth [1, 2] and 2) the off-chip inductors are bulky and expensive, significantly increasing the instability of integration and manufacturing cost. Active capacitor (AC) and active inductor (AI) provide a feasible solution to the limitations. In previous work, a tunable AC can achieve in a wide capacitive range from 40 fF to 1 pF [1, 3] and a tunable AI can achieve in an inductive range from 1 nH to 300 nH [4]. Therefore, using AC and AI to produce a small size and low cost BPF with tunable gain, tunable center frequency and tunable bandwidth is in high demand. For this reason, eliminating resistive loss and improving quality factor of AI becomes a main factor in improving the active BPF.
According to our simulation when Q factor of AI is greater than 30 then the inductor behaves as a small resistive loss component; when Q factor of AI is greater than 100 then the inductor behaves as a resistive less component.

1.2 Past Work

Figure 1.1 The classic second-order coupled-capacitor Chebyshev BPF

![Diagram of classic second-order coupled-capacitor Chebyshev BPF](image)

Figure 1.2 Second order BPF with tapped inductor and a shunt feedback inductor [2]

Reducing resistive loss of the resonator in the classic Chebyshev band pass filter will improve the BPF performance in gain, bandwidth and center frequency [1-2, 5-8]. On the other hand, improving Q factor of the inductor is an effective approach to eliminating the resistive loss in it. For example, the tapped-inductor with a feedback shown in Fig. 1.2 is utilized to compensate the resistive loss from inductor [2]. Also, it
adds an additional shunt feedback inductor between input and output in order to make the BPF operate in the desirable frequency range and have a good selectivity. This results in a wideband BPF operating at K-band with a relative large 17% bandwidth from the center frequency. This direction can be demonstrated as shown in Fig. 1.3 to switch the inductor in the resonator.

Figure 1.3 Second order BPF with switched inductor

A transformer-based with Q improved technology inductor shown in Fig. 1.4 was presented to produce a frequency-dependent negative resistance for resistive loss compensation [8]. It operates at a center frequency of 2,368 MHz and a bandwidth of
60 MHz. Both are passive BPFs. Their center frequencies and bandwidths are not tunable and sizes are much larger than active BPFs [5-7].

Inserting a gyrator-C based active inductor with high Q into a resonator demonstrates BPF applications at different frequency ranges [6-7] as shown in Fig. 1.5. Nevertheless, it is not a tunable design. A tunable design is proposed in [5] as shown in Fig. 1.6. Even though the resonator with a low loss switch in each cell is
implemented into the design to compensate frequency dependent resistive loss for the purpose of tunability, the complex structure consumes large area and power consumption.

Figure 1.6 Tunable BPF with BJT active inductor [5]: (a) Basic three-cell filter and (b) Schematic of AI

Another design incorporates an active capacitor with self-negative resistance to offset the resistive loss, which gives a considerable large 32% bandwidth. However, it is lack of tunability due to poor match between frequency dependent negative resistance and the positive resistance of CMOS spiral inductor [1].

1.3 Thesis objective and organization

In this thesis, a new BPF using active capacitor and active inductor is presented.
Self-negative resistance of active capacitor is designed to compensate the positive resistance of active inductor. Meanwhile, adjusting parameters of active component can control tunability of the BPF for center frequency, gain and bandwidth. Active BPF using active capacitor and active inductor is explored first in our study. The thesis is organized as follows. Section 2 and 3 discuss the principle design and operation of active capacitor and active inductor. Section 4 presents a compensation structural resonator using active capacitor and active inductor. Section 5 unfolds the performance of the BPF and discuss its tunability. Finally, summary of this work and comparison with previous work is presented in Section 6.
2 ACTIVE CAPACITOR WITH NEGATIVE RESISTANCE

2.1 Large signal analysis of AC

The first active capacitor (AC) with negative resistance was demonstrated in [3]. The paper [1] adopted this AC structure and designed it in 0.18 µm CMOS technology. In this Section we extend the AC design principle to make it tunable and compensate resistive loss of the resonator in BPF. The active capacitor and its equivalent circuit are shown in Fig. 2.1. The AC is designed by the cross-coupled pair of $M_2$ and $M_3$ and the resistive load $M_1$. $I_{M_2}$ is controlled by $V_D$. $V_{CC}$ is determined by $I_{M_2}$ and $V_G$ and $I_{M_3}$ is controlled by $V_{CC}$. In our design principle, we keep $V_{CC} > V_D - V_t$ to make $M_2$ in saturation and keep $V_D > V_{CC} - V_t$ to make $M_3$ in saturation. So, $V_D - V_t < V_{CC} < V_D + V_t$.

2.2 Small signal analysis of AC

![Figure 2.1 The active capacitor and its equivalent circuit](image)

The AC small signal model and its equivalent circuit are depicted in Fig. 2.2. $V_G$ is almost the sum of $V_{CC}$ and $V_D$ as $V_t$ is small and $I_{M_1} = I_{M_2}$, which expresses the relationship between $V_{CC}$ and $V_{in}$. An easier way to analyze the small signal model is to set $V_{CC} = \rho V_{in}$ and $\rho$ is controlled by transistor parameters.
Continue to analyze the small signal model shown in Fig. 2.2.

\[ V_{gs1} = V_{g1} - V_s = V_{ds1} = -V_{CC} \]  

(2.1)

Therefore,

\[ -g_{m1}V_{gs1} = -g_{m1}(-V_{CC}) = g_{m1}V_{CC} \]  

(2.2)

So, the current source \( g_{m1}V_{gs1} \) can be flipped to opposite direction without changing the symbol. Also, \( V_{gs2} = V_{gs3} = V_{CC} \). The admittance from the input port is determined by \( I_{in}/V_{in} \):

\[ I_{i1} = (V_{in} - V_{CC})sC_{gd2} + (V_{in} - V_{CC})sC_{gd3} \]

(2.3)

\[ I_{i2} = V_{in}sC_{gs2} + g_{m3}V_{in} \]  

(2.4)

\[ I_{in} = I_{i1} + I_{i2} \]

\[ = (V_{in} - V_{CC})s(C_{gd2} + C_{gd3}) + V_{in}(sC_{gs2} + g_{m3}) \]  

(2.5)

\( V_{CC} \) is the reference voltage shown in Fig. 2.1. The branch current \( I_{o1} \) and \( I_{o2} \) is:

\[ I_{o1} = V_{CC}g_{m1} + V_{CC}sC_{gs1} \]  

(2.6)

\[ I_{o2} = V_{CC}g_{m2} + V_{CC}sC_{gs3} \]  

(2.7)
\[ I_{out} = I_{o1} + I_{o2} \]
\[ = V_{CC}(g_{m1} + g_{m2}) + V_{CC}(sC_{gs1} + sC_{gs3}) \]
\[ = V_{CC}(g_{m1} + g_{m2} + sC_{gs1} + sC_{gs3}) \]  \hspace{1cm} (2.8)

At the reference point,

\[ I_{i1} = I_{out} \]  \hspace{1cm} (2.9)

Therefore,

\[ (V_{in} - V_{CC})s(C_{gd2} + C_{gd3}) \]
\[ = V_{CC}(g_{m1} + g_{m2} + sC_{gs1} + sC_{gs3}) \]  \hspace{1cm} (2.10)

So

\[ Y_{in} = \frac{I_{in}}{V_{in}} \]
\[ = \left( \rho g_{m1} + \rho g_{m2} + g_{m3} \right) + s\left( \rho C_{gs1} + C_{gs2} + \rho C_{gs3} \right) \]
\[ = g_{ac} + sC_{ac} = \frac{1}{R_{neg}} + sC_{ac} \]  \hspace{1cm} (2.11)

From Eq. (2.11) expresses the negative resistance is controlled by the transconductance \( g_{m1}, g_{m2} \) and \( g_{m3} \) and the capacitance is determined by the gate-to-source capacitance of NMOS transistors. Adjusting these parameters will produce different negative resistance and capacitance values, which can be used to compensate the resistive loss of inductor.
Figure 2.3 AC capacitance

Figure 2.4 AC negative resistance
2.3 AC simulations

Fig. 2.3 and 2.4 unfold that tuning $V_G$ (from 1.6V to 2.3V) produces different capacitance values (from 128 fF to 175 fF) and negative resistance values (from -183Ω to -338Ω). For our applications, increasing $V_G$ will increase the capacitance value and decrease the negative resistance value. In the meantime, both capacitance and negative resistance values are stable and almost constant in a specific frequency range. For example, when $V_G$ varies from 1.6 V to 1.7 V, the capacitance increases from 128 fF to 148 fF and the negative resistance decreases from -338 Ω to -228 Ω (i.e., the corresponding negative conductance increases from -2.96 mS to -4.38 mS). The capacitance and negative resistance values are stable and constant in 3,859 MHz and 2,486 MHz frequency range, respectively.
3 ACTIVE INDUCTOR

3.1 Gyrator-C active inductor (AI)

Several active inductors have been proposed [4, 9-17]. Most are designed on the base of the gyrator-C topology: 1) single-ended active inductors [9-12], and 2) differential active inductors [13-16]. In the following, the basic theory of gyrator-C active inductor is first discussed in which an equivalent circuit is presented to demonstrate how it performs as an inductor and can spiral inductors used in many BPFs. Next, single-ended accompanying differential gyrator-C active inductors are discussed for different applications. The proposed active inductor is shown in Fig. 3.16. Its structure is on the base of the single-ended gyrator-C and its tunable active inductor [4]. Fig. 3.17 shows its small signal model.

3.1.1 Lossless single-ended gyrator-C active inductor

![Figure 3.1 Lossless single-ended gyrator-C active inductor](image)

In Fig. 3.1, there are two transconductors connected back to back to constitute this single-ended gyrator-C active inductor. A lossless active inductor can be realized when the input and output admittance of the transconductor approximates to zero and transconductance of the gyrator is constant. In the other word, there is no current flowing into $G_{m2}$. From the following analysis, an equivalent inductance value can be
obtained,

\[ V_B = G_{m1} V_A \frac{1}{sC} \]  \hspace{1cm} (3.1)

\[ I_o = -G_{m2} V_B \]  \hspace{1cm} (3.2)

\[ I_o = -I_A \]  \hspace{1cm} (3.3)

\[ I_A = \frac{G_{m1} G_{m2}}{sC} V_A \]  \hspace{1cm} (3.4)

\[ Z_{in} = \frac{V_A}{I_A} = \frac{sC}{G_{m1} G_{m2}} \]  \hspace{1cm} (3.5)

\[ L_{equ} = \frac{C}{G_{m1} G_{m2}} \]  \hspace{1cm} (3.6)

From Eq. (3.6), it is found that the equivalent inductance value is determined by the capacitance value C and two transconductance \( G_{m1} \) and \( G_{m2} \).

### 3.1.2 Lossless differential (floating) gyrator-C active inductors

![Diagram of Lossless differential gyrator-C active inductor](image)

Figure 3.2 Lossless differential gyrator-C active inductor

The differential gyrator-C active inductor is built by switching the single-ended transconductor in single-ended gyrator-C active inductor with differential transconductor. As shown in Fig. 3.2, C represents the load capacitance at the B node. From the following analysis, an equivalent inductance value can be obtained,
\[ V_{B^+} = -\frac{g_{m1}}{sC} (V_{A^+} - V_{A^-}) \] (3.7)

\[ V_{B^-} = \frac{g_{m1}}{sC} (V_{A^+} - V_{A^-}) \] (3.8)

\[ I_o = -I_A \] (3.9)

\[ I_o = G_{m2} (V_{B^+} - V_{B^-}) \] (3.10)

\[ I_A = \frac{2G_{m2} g_{m1}}{sC} (V_{A^+} - V_{A^-}) \] (3.11)

\[ Z_{in} = \frac{v_{A^+} - v_{A^-}}{I_A} = \frac{sC}{2g_{m1} g_{m2}} \] (3.12)

\[ L_{equ} = \frac{C}{2G_{m1} G_{m2}} \] (3.13)

From Eq. (3.13), it is found that the equivalent inductance value is dependent on the load capacitance \( C \) and two transconductance \( G_{m1}, G_{m2} \) and is half of Eq. (3.6) of the single-ended gyrator-C active inductor.

### 3.1.3 Lossy Single-ended gyrator-C active inductors

![Figure 3.3 Lossy single ended active inductor](image)

In Fig. 3.3, \( G_{m1} \) and \( G_{m2} \) is the transconductance; \( G_1 \) and \( G_2 \) is the total conductance at node B and A, respectively. So, \( 1/G_1 \) is the sum of the output impedance of \( G_{m1} \) and the input impedance of \( G_{m2} \). Similarly, \( 1/G_2 \) is the sum of the output impedance of \( G_{m2} \) and the input impedance of \( G_{m1} \). \( C_1 \) and \( C_2 \) is the total capacitance at node B and A, respectively.
At node A:
\[-G_{m_1}V_A + V_B (sC_1 + G_1) = 0\]  (3.14)

At node B:
\[-G_{m_2}V_B + V_A (sC_2 + G_2) - I_A = 0\]  (3.15)

From node A, the input impedance equals to:
\[Y_{in} = \frac{I_A}{V_A} = G_2 + sC_2 + \frac{1}{s\left(\frac{C_1}{G_{m_1}G_{m_2}}\right) + \frac{G_1}{G_{m_1}G_{m_2}}}\]  (3.16)

Compared with the simplified model of RLC circuit,
\[R_p = \frac{1}{G_2}\]  (3.17)
\[C_p = C_2\]  (3.18)
\[L_{equ} = \frac{C_1}{G_{m_1}G_{m_2}}\]  (3.19)
\[R_s = \frac{G_1}{G_{m_1}G_{m_2}}\]  (3.20)

### 3.1.4 Lossy differential (floating) gyrator-C active inductors

Analysis of the floating differential gyrator-C active inductor is similar to that of
the single-ended gyrator-C active inductor. $C_1$ and $C_2$ is the total capacitance at node 1 and 2; $G_{o1}$ and $G_{o2}$ is the total conductance at node 1 and 2. So, the equivalent circuit considering the capacitance and conductance is depicted in Fig. 3.4.

$$-G_{m1}(V_2^+ - V_2^-) + \left(\frac{SC_1 + G_{o1}}{2}\right)(V_1^- - V_1^+) = 0$$

$$I_{in} + G_{m2}(V_1^+ - V_1^-) + \left(\frac{SC_2 + G_{o2}}{2}\right)(V_2^- - V_2^+) = 0 \quad (3.21)$$

The input admittance from the port 2 of the gyrator-C active inductor can be calculated as follows,

$$Y_{in} = \frac{I_{in}}{V_2^+ - V_2^-} = \frac{G_{o2}}{2} + s \left(\frac{c_2}{2} + \frac{G_{o1}}{2(G_m_1 G_m_2)}\right)$$

$$= \frac{G_{o2}}{2} + s \left(\frac{c_2}{2} + \frac{G_{o1}}{2(G_m_1 G_m_2)}\right) \quad (3.22)$$

Then, in comparison with the equivalent RLC parallel circuit, the following values can be obtained,

$$R_p = \frac{2}{G_{o2}} \quad (3.23)$$

$$C_p = \frac{C_2}{2} \quad (3.24)$$

$$L_{equ} = \frac{C_1/2}{G_{m_1} G_{m_2}} \quad (3.25)$$

$$R_s = \frac{G_{o1}/2}{G_{m_1} G_{m_2}} \quad (3.26)$$

### 3.1.5 Controllable parameters of active inductors

To make active inductor practical in different applications, three parameters in AI are made controllable. They are: 1) tuning center frequency range, in which the AI performs normally as an inductive device, 2) inductance tunability, in which the AI gives a wide range of equivalent inductance value, and 3) quality factor Q, in which resistive loss of the active inductor is determined. The RLC circuit in Fig. 3.3 is used
as an example to discuss the three controllable parameters

3.1.5.1 Tuning center frequency range

From the analysis of the lossy and lossless active inductors, the lossless active inductor has a full inductive frequency range while lossy active inductor can only operate in a specific frequency range. In Fig. 3.3, a lossy active inductor is equivalent to a RLC circuit. By extracting the impedance function and depicting the bode plot, the frequency range are obtained based on the poles and zeros. The self-resonant frequency (SFR) is an important frequency shown in the figure. Below SFR, the active inductor operates as an inductor. Above SFR, the active inductor operates as a capacitor. So, SFR is the break point between inductor and capacitor operation.

\[
Z = \left(\frac{R_s}{C_p}\right) \frac{L}{s^2 + \frac{\frac{1}{R_p C_p}}{1 + s^2 \left(\frac{R_s}{L}\right) + \frac{R_p + R_s}{R_p C_p L}}} \quad (3.27)
\]

Based on the above equation, the pole SFR is obtained,

\[
w_p = \sqrt{\frac{R_p + R_s}{R_p C_p L}} \quad (3.28)
\]

Given the parallel resistor is much greater than the series resistor, then

\[
w_p = w_o = \sqrt{\frac{1}{LC_p}} \quad (3.29)
\]

\(w_o\) is the SFR.

From Eq. (3.27), a zero is obtained as

\[
w_z = \frac{R_s}{L} = \frac{G_{o1}}{C_1} \quad (3.30)
\]

As shown in Fig. 3.5, the bode plot gives three different frequency spectrum. This gyrator-C operates as a resistor in the frequency range \([0, w_z]\), an inductor in the frequency range \([w_z, w_o]\), and a capacitor in the frequency range \([w_o, \infty]\). So, the inductive frequency range is decided by \(w_z\) and \(w_o\). \(w_z\) is determined by \(G_{o1}\) and \(C_1\). \(w_o\) is determined by \(C_p\) and \(C_p\) is determined by \(C_2\). So, adjusting \(G_{o1}\), \(C_1\)
and $C_2$ can maximize the inductive frequency range $[w_z, w_o]$.

![Bode plot of different frequency range](image)

**Figure 3.5 Bode plot of different frequency range [17]**

### 3.1.5.2 Inductance tunability

From the above analysis, it is shown that the inductance of gyrator-C active inductor is tunable. Adjusting the load capacitance or transconductance in the gyrator can produce different inductances. Following this, in our approach we’ll add a resonator in the circuit to adjust the capacitance.

### 3.1.5.3 Quality factor Q

An overflow resistance in the resonator produces a compressed shape (good selectivity) curve in BPF [1]. When the passive inductor quality factor Q decreases, the resistive loss in the inductor becomes significant. When Q decreases, the gain of BPF decreases and the bandwidth increases. Frequency response due to different
resistive loss of AI is shown in Fig. 3.6.

![Resistive Loss Diagram](image)

Figure 3.6 Frequency response due to different resistive loss of AI

The quality factor can be calculated by measuring the energy loss in one oscillation cycle [17].

\[
Q = 2\pi \frac{\text{Net magnetic energy stored}}{\text{Energy dissipated in one oscillation cycle}}
\]  
(3.31)

The power consumed in one cycle is

\[
P(jw) = I(jw) \times V(jw) = Re[Z]|I(jw)|^2 + jIm[Z]|I(jw)|^2
\]  
(3.32)

From the RLC equivalent circuit in Fig. 3.3, Re[Z] is the resistance of the inductor and Im[Z] is the inductance of the inductor. The quality factor can be expressed by
\[ Q = \frac{Im[Z]}{Re[Z]} \quad (3.33) \]

So, the quality factor in this equivalent circuit can be obtained by combining the Eq. (3.33) and Eq. (3.17) - (3.20).

\[ Q = \left(\frac{w_{Lequ}}{R_s}\right) \frac{R_p}{R_p + R_s \left[ 1 + \left( \frac{w_{Lequ}}{R_s}\right)^2 \right]} \left( 1 - \frac{R_s^2 C_p}{L_{equ}} - w^2 L_{equ} C_p \right) \quad (3.34) \]

From Eq. (3.34) it can be found that the quality factor consists of three parts.

\[ Q_1 = \frac{w_{Lequ}}{R_s} \quad (3.35) \]

\[ Q_2 = \frac{R_p}{R_p + R_s \left[ 1 + \left( \frac{w_{Lequ}}{R_s}\right)^2 \right]} \quad (3.36) \]

\[ Q_3 = \left( 1 - \frac{R_s^2 C_p}{L_{equ}} - w^2 L_{equ} C_p \right) \quad (3.37) \]

All these quality factors are depicted in Fig. 3.7.
3.2 Practical active inductors based gyrator-C topology

3.2.1 Single-ended active inductor

3.2.1.1 Active inductor with feedback resistance

An improved quality factor active inductor is presented with a feedback resistance in [9]. As shown in Fig. 3.8, the active inductor uses the cascade-grounded structure in order to limit the conductance \( g_{ds} \) at the output terminal. Connecting \( M_2 \) to \( M_3 \) in the cascaded-grounded structure can decrease the conductance. From Eq. (3.38) ~ (3.41), it is found that decreasing the conductance of \( M_3 \) is an approach to reducing the resistive loss and improving the equivalent inductance. This will increase the quality factor. A feedback resistance \( R_f \) performs an additional inductance value at terminal of \( M_1 \).

\[
C_{eq} = C_{gs_3} \quad \text{(3.38)}
\]

\[
G_{eq} = \frac{2g_{ds_2} + R_f g_{ds_2}}{R_f g_{ds_2} + 1} \quad \text{(3.39)}
\]

\[
R_{eq} = \frac{g_{m_1} g_{ds_2} g_{ds_3} + w^2 [g_{m_2} c_{gs_1}^2 - g_{m_1} c_{gs_1} c_{gs_2} (R_f g_{ds_2} + 1)]}{g_{m_1}^2 g_{m_2} g_{m_3} + w^2 g_{m_2} g_{m_3} c_{gs_1}^2} \quad \text{(3.40)}
\]

\[
L_{eq} = \frac{g_{m_1} g_{m_2} c_{gs_1} + w^2 c_{gs_3}^2 c_{gs_2} (R_f g_{ds_2} + 1)}{g_{m_1}^2 g_{m_2} g_{m_3} + w^2 g_{m_2} g_{m_3} c_{gs_1}^2} \quad \text{(3.41)}
\]

From the above analysis, adding a feedback resistance introduces \( R_f g_{ds_2} + 1 \) into \( R_{eq} \) and \( L_{eq} \). \( R_{eq} \) is reduced and \( L_{eq} \) is increased by increasing \( R_f \), which improves the quality factor.
Figure 3.8 Cascade-grounded active inductor with a resistance feedback [9]

Figure 3.9 Simulation result of quality factor and inductance [9]
3.2.1.2 Active inductor with feed-forward path transistor

![Active inductor with feed-forward path transistor](image)

Figure 3.10 Active inductor with improved technology [10]

As shown in Fig. 3.10, the active inductor is based on gyrator-C active inductor but has an additional capacitor $C$. $M_2$ is in the feed-through path. $r_o$ is the transistor’s output resistance. Compared with Fig. 3.10(c), the following equations are obtained from the equivalent circuit,

$$L = \frac{2C}{g_{m1}g_{m3}}$$

(3.42)

$$C_p = \frac{1}{2}C_{gs1} + C_{gd1} + C_{gd3} + C_{dv3}$$

(3.43)

$$R_s = \frac{2}{g_{m1}g_{m3}r_{o2}}$$

(3.44)

$$R_p = r_{o3}$$

(3.45)

From the above equations, it can be found that equivalent inductance is determined by the capacitance $C$ and transconductance of $M_1$ and $M_3$. In order to reduce the resistive loss and improve the quality factor, $M_2$’s output resistance $r_{o2}$ is reduced and the capacitance is increased.

3.2.1.3 Class AB active inductor

As shown in Fig. 3.11, an equivalent modal of the class AB active inductor [11]
consists of two parts: 1) delay network and 2) gain stage. The delay network is used to change amplitude and phase which are delivered to the voltage terminal. The relationship between the voltage and current at the gain stage introduces an admittance value, which decides if the active inductor operates as a capacitor or an inductor.

\[ Z_{in} = \frac{V_n}{I_n} \]

Figure 3.11 Equivalent modal of class AB active inductor [11]

Figure 3.12 Class AB active inductor [11]

In Fig. 3.11, it is known that RC network leads to a capacitive property when compensating the phase unbalance. The amplifier at the gain stage can give the current phase a 180-degree change to operate as an inductor. Seen from the input port,
the impedance can be obtained,

$$Z_{in} = \frac{1+j\omega RC}{g_m+j\omega C} \quad (3.46)$$

$$Im[Z_{in}] = \frac{\omega g_m RC - \omega C}{g_m^2 + \omega^2 C^2} \quad (3.47)$$

From Eq. (3.47), it can be found that the inductive property depends on the delay network on the condition that $g_m R > 1$. And, the peak inductance when $w = g_m / C$.

Fig. 3.12 shows the practical Class AB active inductor.

### 3.2.1.4 Active inductor with cascode topology

![Figure 3.13 Tunable active inductor (TAI) schematic][12]

As shown in Fig. 3.13, the TAI (tunable active inductor) is comprised of three current source $I_1$ to $I_3$ and four transistors $M_1$ to $M_4$. There are two feedback loops inside the TAI. The first loop is constituted by $M_1$ and $M_2$ and the second loop is constituted by $M_3$ and $M_4$. Two loops constitute the cascade structure for two purposes: 1) reducing the output conductance and 2) increasing the gain. $M_4$ is added to improve the effect of the cascade, which does not affect the frequency response because it does not change the current through $M_1$ to $M_3$. 

---

[12]: https://example.com/tunable-active-inductor-schematic.png
The input admittance shown in Fig. 3.13 can be obtained through the small signal modal and its equivalent circuit shown in Fig. 3.14.

\[ C_{eq} = C_{gs1} \]  (3.48)

\[ G_p = \frac{1}{R_p} = \frac{g_{m1}}{1-w^2(c_{gs2}c_{gs4}/g_{m3}g_{m4})} = \frac{g_{m1}}{1-w^2/\omega t_3 \omega t_4} \]  (3.49)

\[ L_{eq} = \frac{c_{gs2}}{g_{m1}g_{m2}} \left( 1 - \frac{w^2c_{gs2}c_{gs4}}{g_{m3}g_{m4}} \right) = \frac{1}{g_{m1}\omega t_2} \left( 1 - \frac{w^2}{\omega t_3 \omega t_4} \right) \]  (3.50)

\[ R_{eq} = \frac{w^2c_{gs2}c_{gs3}}{g_{m1}g_{m2}g_{m3}} = \frac{w^2c_{gs2}}{g_{m1}g_{m2}\omega t_3} = \frac{w^2}{g_{m1}\omega t_2 \omega t_3} \]  (3.51)

where \( \omega t \) is defined as the cutoff frequency of each transistor. From Eq. (3.48) \sim (3.49), it can be found that the equivalent inductance is determined by \( g_{m1}, \omega t_2, \omega t_3, \omega t_4 \).

### 3.2.2 Differential active inductor

#### 3.2.2.1 Differential active inductor with the inverter pair

In Fig. 3.15, a differential active inductor [13] with high quality factor is presented. The inverter pair is designed to add a negative resistance in order to improve the quality factor. And, the negative resistance can be adjusted by \( V_{ar3} \). Under assistance of the inverter pair, a path is created for an independent DC current to control the negative resistance. The DC current doesn’t pass through \( M_5 \) and \( M_6 \), which means...
transconductance of $M_5$ can be tuned for high inductance.

Figure 3.15 Differential active inductor [13]: (a) Schematic, (b) Small signal modal and (c) Equivalent circuit

The small signal modal is shown in Fig 3.15(b). The admittance between $V_1$ and $V_2$ can be obtained,

$$Y = \frac{1}{Z} = -\frac{g_{m_{A1}}}{2} + 1\left(sC_1 + sC_{gs1}\right)$$

$$+ \frac{1}{2\left(s^2C_{gs3}C_{gs5}/g_{m1}g_{m3}g_{m5}\right) + \left(sC_{gs3}/g_{m1}g_{m3}\right)} + \frac{1}{2\left(1/g_{m3}\right) + \left(sC_{gs3}/g_{m1}g_{m3}\right)}$$

(3.52)

The admittance of the equivalent circuit in Fig. 3.15(c) equals to

$$\frac{1}{Z} = G + j\omega C_p + \frac{1}{R_{s1} + j\omega L_1} + \frac{1}{R_{s2} + j\omega L_2}$$

(3.53)

From Eq.(3.52) and (3.53), it can be found that

$$G = -\frac{g_{m_{A1}}}{2}$$

(3.54)

$$C_p = \frac{C_1 + C_{gs3}}{2}$$

(3.55)
\[ L_{s1} = \frac{2C_{gs5}}{g_{m1}g_{m5}} \]  
(3.56)

\[ L_{s2} = \frac{2C_{gs3}}{g_{m1}g_{m3}} \]  
(3.57)

\[ R_{s1} = -\frac{2w^2C_{gs2}C_{gs5}}{g_{m1}g_{m3}g_{m5}} \]  
(3.58)

\[ R_{s2} = \frac{1}{g_{m1}} \]  
(3.59)

From the above analysis, it can be found that the negative resistance is added to G. Because of large \( R_{s2} \), \( L_{s1} \) contributes the main inductance more than \( L_{s2} \).

3.2.2.2 Differential active inductor with cross-coupled transistors and feedback resistor

![Diagram](image)

Figure 3.16 Differential AI with feedback resistor [15]: (a) Schematic and (b) Equivalent circuit

In Fig. 3.16, \( C_A, C_B, g_A, g_B \) are parasitic capacitance and conductance at node A and B. And, \( g_f \) is defined as conductance value of the feedback resistance \( R_f \). In
From above equations, it can be found that $g_{m3}$ can be tuned to a small value in order to obtain a large $R_p$. $L_{eq}$ can be tuned by changing the feedback resistance value $1/g_f$.

### 3.3 Large signal analysis of the proposed AI

Two pairs of current mirrors ($M_0$, $M_1$) and ($M_5$, $M_6$) are used in Fig. 3.17. Both gate voltages of $M_0$ and $M_5$ are controllable by tuning $R_1$, $R_2$, $M_1$, $M_6$ which controls current of $M_0$ and $M_5$. The $M_2$, $M_3$ and $M_4$ control small signal parameters like $g_{m2}$, $g_{m3}$, $g_{m4}$, $C_{gs2}$ and $C_{gs4}$.
3.4 Small signal analysis of the proposed AI

For the small signal model of the proposed active inductor, the input impedance equals to:

\[ Y_{in} = \frac{I_{in}}{V_{in}} = C_{gs3} s + g_{m3} + \frac{g_{m2} g_{m3} g_{m4} g_{R3}}{(C_{gs4} s + g_{m4} + g_2 + g_5) [(g_0 C_{gs4} s + g_{R3} C_{gs2}) s + g_{R3} g_0]} \] (3.38)

and

\[ Y_{in} = C_{gs3} s + g_{m3} + Y_{ins} \] (3.39)

\( Y_{ins} \) is extracted from Eq. (3.38),

\[ Y_{ins} = \frac{g_{m2} g_{m3} g_{m4} g_{R3}}{(C_{gs4} s + g_{m4} + g_2 + g_5) [(g_0 C_{gs4} s + g_{R3} C_{gs2}) s + g_{R3} g_0]} \] (3.40)

Assume \( g_7 = g_2 + g_5 \).

\[ Z_{ins} = \frac{1}{Y_{ins}} \]

\[ = \frac{C_{gs4} C_{gs2} (g_0 + g_{R3}) s^2 + (g_0 g_{R3} C_{gs4}) s^3 + (g_0 + g_{R3} C_{gs2}) s + (g_{m4} + g_7) g_0 g_{R3}}{g_{m2} g_{m3} g_{m4} g_{R3}} \] (3.41)

And,

\[ Z_{ins}(j\omega) = R_S + j\omega L_{equ} \] (3.42)

So,

\[ R_S = \frac{(g_{m4} + g_7) g_0 g_{R3} - C_{gs4} C_{gs2} (g_0 + g_{R3}) \omega^2}{g_{m2} g_{m3} g_{m4} g_{R3}} \] (3.43)

\[ L_{equ} = \frac{(g_{m4} + g_7) (g_0 + g_{R3}) C_{gs2} + g_0 g_{R3} C_{gs4}}{g_{m2} g_{m3} g_{m4} g_{R3}} \] (3.44)

Compared with the simplified model in Fig. 3.3, it is shown that \( R_p = \frac{1}{g_{m3}} \)

and \( C_p = C_{gs3} \).
From the above analysis, $L_{equ}$ and $R_s$ is function of $g_{m_2}$, $g_{m_3}$, $g_{m_4}$, $g_0$, $g_7$, $g_{R_3}$, $C_{gs_2}$ and $C_{gs_4}$. Both are controllable by changing the large signal bias conditions as discussed in Section 3.2.

### 3.5 AI simulation

Fig. 3.19 and 3.20 shows that by tuning large signal bias conditions, different inductance values (from 1 nH to 300 nH) and resistance values (from 43 Ω to 344 Ω) can be produced. As shown in the plot, the highest inductive frequency range is achieved at 5,156 MHz with a peak inductance of 23 nH. By means of adjusting the bias conditions, different inductance and resistance values can be produced for different applications, in a specific inductive frequency range. Fig. 3.20 shows the tunable resistance. For example, when the active inductance value is adjusted from 1 nH to 300 nH, the resistance value changes from 344 Ω to 107 Ω and the frequency range is from 275 MHz to 770 MHz.
Figure 3.19 Inductance of active inductor

Figure 3.20 Resistance of active inductor
4 ACTIVE BPF USING RESONATOR BASED ON ACTIVE INDUCTOR 
AND ACTIVE CAPACITOR

4.1 Introduction

The 2nd order active BPF is designed based on the classic Chebyshev BPF structure [18-20]. Fig. 4.1 shows the N-th order classic Chebyshev BPF. Typically, the higher order of Chebyshev BPF has higher selectivity than the lower order of Chebyshev BPF. However, the higher order of Chebyshev BPF consumes larger area and power. An advantage of the Chebyshev BPF is its simple structure can be repeatedly used as resonators.

![Figure 4.1 Classic Chebyshev N order BPF](image)

The Chebyshev BPF has inferior selectivity due to the poor rejection level. To improve selectivity in wide bandwidth, techniques of introducing transmission zeros to increase stopband by adding shunt capacitor, serial inductor or shunt inductor have been presented [21-24].

In Fig. 4.2, adding a shunt feedback inductor introduces three additional transmission zeros to frequency response [21]. One transmission zero is added to the lower frequency spectrum and two transmission zeros are added to the higher frequency spectrum. Admittance matrix helps locate the transmission zeros. The $y_{12}$ of the basic BPF in Fig. 4.2 (a) equals to

$$y_{12} = \frac{s^3C_1^2C_s}{(sC_a + \frac{1}{sL_a})^2 - s^2C_s^2} \quad (4.1)$$
\[ C_a = C_1 + C_p \]  
(4.2)

\[ L_a = \frac{L_p}{L_p s^2 C_s + 1} \]  
(4.3)

The admittance matrix of the BPF in Fig. 4.2 (b),

\[
Y = \begin{bmatrix}
\frac{1}{sL} + y_{11} & -\frac{1}{sL} + y_{12} \\
-\frac{1}{sL} + y_{21} & \frac{1}{sL} + y_{22}
\end{bmatrix}
\]  
(4.4)

Figure 4.2 BPF with shunt feedback inductor [2]

From the admittance matrix in Eq. (4.4), positions of three transmission zeros can be obtained by

\[-\frac{1}{sL} + y_{12} = 0 \]  
(4.5)

By Eq. (4.5),

\[ s^6 C_1^2 L_a^2 C_s L + s^4 L_a^2 \left( C_a^2 - C_s^2 \right) + 2s^2 L_a C_a + 1 = 0 \]  
(4.6)

As shown in Eq. (4.6), it is sixth order equation with three positive and three negative zeros. The three positive zeros are three additional transmission zeros. A direct approach to locating these three transmission zeros is using graphic method.

In Fig. 4.3, adding a shunt feedback capacitor [22] introduces two transmission zeros to the frequency response. One transmission zero is added to the low frequency spectrum and the other one is added to the high frequency spectrum. Same approach is use to locate the transmission zeros.
In Fig. 4.4, adding a shunt feedback capacitor and a ground inductor [24] in a second order BPF introduces two additional transmission zeros. One is added to the low frequency spectrum and the other one is added to the high frequency spectrum. This BPF can be divided into three parts, as shown in Fig. 4.5. The first network shown in Fig. 4.5(a) is a second order BPF without the feedback capacitor and the ground inductor. The second network in Fig. 4.5(b) has a ground inductor. The third network in Fig. 4.5(c) has a feedback capacitor.
Figure 4.6 Basic part of the second order BPF separated into L-match [24]

(a) Schematic (b) L-match analysis

The admittance matrix can be calculated in the following.

\[
Z_{s1} = Z_{(a)} + Z_{(b)} = \begin{bmatrix}
Z_{(a)11} & Z_{(a)12} \\
Z_{(a)21} & Z_{(a)22}
\end{bmatrix} + \begin{bmatrix}
sL_g & sL_g \\
-sL_g & sL_g
\end{bmatrix}
\]

\( (4.7) \)

\[
Y_{s2} = Y_{s1} + Y_c = \begin{bmatrix}
sC_c & -sC_c \\
-sC_c & sC_c
\end{bmatrix} + \begin{bmatrix}
Z_{s1(11)} & Z_{s1(12)} \\
Z_{s1(21)} & Z_{s1(22)}
\end{bmatrix}^{-1}
\]

\( (4.8) \)

So, \( Y_{21} \) can be obtained,

\[
Y_{21} = sC_c + \left\{ \frac{s^3C_2^2}{L_2 + L_1(2 + s^2C_1L_2 + s^2C_2L_2)} \right\}
\times \frac{L_2L_g + 2L_1L_g(1 + s^2C_1L_2) + L_1^2(1 + 2s^2C_1L_g + s^4C_1^2L_2L_g)}{1 + s^2C_2(L_1 + 2L_g) + s^2C_1L_1(1 + 2s^2C_2L_g)}
\]

\( (4.9) \)

By letting \( Y_{21} = 0 \), the locations of transmission zeros can be obtained.
4.2 The proposed BPF

![Diagram of the proposed active BPF]

Figure 4.7 The proposed active BPF

![Diagram of equivalent circuits of a resonator]

Figure 4.8 Equivalent circuits of a resonator
Figure 4.9 BPF performance

Figure 4.10 Capacitance vs. frequency (before and after using $L_{DC}$ and $C_{AC}$)
Figure 4.11 Conductance vs. frequency (before and after using $L_{DC}$ and $C_{AC}$)

Figure 4.12 Inductance vs. frequency (before and after using $R_{AD}$)
The active BPF proposed in this research is shown in Fig. 4.7. Two resonators are designed using active capacitor and active inductor in which the negative resistance of active capacitor compensates the resistive loss of active inductor. The performance is optimized at the center frequency of 758MHz, it achieves 18.1 dB gain, 7.1 MHz 3-dB bandwidth, 107 Q factor and 50 dB rejection ratio. The BPF performance is shown in the Fig. 4.9. Compared with the circuit in section 4.1, there is no need to add additional shunt capacitor or inductor to add additional transmission zeros to improve BPF selectivity.

In Fig. 4.7, $L_{DC}$ is added to produce DC bias voltage and block the AC signal; $C_{AC}$ is added to bypass the AC signal and block the DC current. Fig. 4.10 depicts capacitance vs. frequency (before and after using $L_{DC}$ and $C_{AC}$). Fig. 4.11 depicts conductance vs. frequency (before and after using $L_{DC}$ and $C_{AC}$). As shown in Fig. 4.10 and 4.11, after adding $L_{DC}$ and $C_{AC}$ the capacitance and negative conductance
values are stable and almost constant in the frequency range [758 MHz, 864 MHz]. In Fig. 4.7, after applying a DC supply voltage $V_{LD}$ and a resistor $R_{AD}$, a DC bias voltage (0.9V) is obtained at $V_X$. Fig. 4.12 depicts inductance vs. frequency (before and after using $R_{AD}$) and Fig. 4.13 shows resistance vs. frequency (before and after using $R_{AD}$). As shown in Fig. 4.12 and Fig. 4.13, after adding $R_{AD}$, the inductance and its positive resistance are slightly changed. The reason is illustrated as below.

From the analysis of the small signal equivalent model in Fig. 4.8, $L_{DC}$, $C_0$ and $R_{neg_0}$ constitute a RLC parallel circuit.

In Fig. 4.8(b), the admittance of this RLC parallel circuit equals to:

$$Y_{imp} = \frac{l_{in}}{V_{in}}$$

$$= \frac{1}{R_{neg_0}} + j\omega C_0 + \frac{1}{j\omega L_{DC}}$$

$$= \frac{1}{R_{neg_0}} + j(\omega C_0 - \frac{1}{\omega L_{DC}})$$  \hspace{1cm} (4.10)

If $L_{DC}$ is large enough, $\frac{1}{\omega L_{DC}}$ can be neglected. In Fig. 4.8(c), if $C_{AC}$ is large enough, then

$$C_{equ} = C_{AC}\frac{C_1}{C_{AC}} + C_1$$  \hspace{1cm} (4.11)

$$R_{negequ} = (1 + \frac{C_1}{C_{AC}})R_{neg_1}$$  \hspace{1cm} (4.12)

It means the effect of $C_{AC}$ can be neglected. On the other hand, $L_{DC}$ and $C_{AC}$ do not take part in the performance of BPF.

By adjusting $R_{AD}$, the input DC voltage of the active inductor can be adjusted to a desirable bias value accordingly. $R_{AD}$ is in parallel with $L_{AI_0}$ and $R_{S_0}$. Giving a large $R_{AD}$, $L_{AI_1} \approx L_{AI_0}$ and $R_{S_1} \approx R_{S_0}$, as shown in Fig. 4.8(c).

In order to find a match between the negative resistance and the positive resistance, $L_{AI_1}$ in serial with $R_{S_1}$ is changed to $R_p$ in parallel with $L_{equ}$ where Q
is quality factor of the active inductor.

\[ R_p = (1 + Q^2)R_{s1} \]  \hspace{1cm} (4.13)

\[ L_{equ} = (1 + \frac{1}{Q^2})L_{AI_1} \]  \hspace{1cm} (4.14)

\[ \omega_0 = \frac{1}{\sqrt{L_{equ}C_{equ}}} \]  \hspace{1cm} (4.15)

\[ R_{neg_{equ}} = R_p \]  \hspace{1cm} (4.16)
The active inductor in this application provides a fixed value of inductance and resistance. By adjusting the bias voltage $V_G$, a tunable capacitance of the active capacitor is obtained, which makes this BPF tunable. Fig. 5.1 displays the BPF tunability for the center frequency [758 MHz, 864 MHz], the 3-dB bandwidth [7.1MHz, 65.9MHz], the gain [6.5dB, 18.1dB], the rejection level [38dB, 50dB], and the quality factor [12, 107].

It is observed from Fig. 5.1 that when $V_G$ is decreased (from the lower bound to the upper bound), the gain is decreased, and the 3-dB bandwidth is increased. At the
center frequency of 758 MHz (red plot), the resistance loss of the active inductor is nearly cancelled by the negative resistance of the active capacitor, leading to an ideal resonator in the circuit.

Table 1 summarizes results of this work and comparison with past work using the same structure of classic Chebyshev band pass filter. As shown in this table, after applying active capacitor and active inductor, this BPF has a fairly high gain while others are about 1 dB. A rejection level of 50 dB is achieved, which is much higher than that of others, which provides a good selectivity and a high quality factor of 107. Meanwhile, tuning DC bias voltage makes the AC BPF tunable. The BPF structure proposed in this work is much simpler than that of [5].

<table>
<thead>
<tr>
<th>Reference</th>
<th>[1]</th>
<th>[2]</th>
<th>[5]</th>
<th>[6]</th>
<th>[7]</th>
<th>[8]</th>
<th>This work</th>
</tr>
</thead>
<tbody>
<tr>
<td>Technology process</td>
<td>CMOS 0.18 µm</td>
<td>CMOS 0.18 µm</td>
<td>BJT BFP420</td>
<td>BJT BFR92A</td>
<td>BJT BFR92A</td>
<td>CMOS 0.18 µm</td>
<td>CMOS 0.18 µm</td>
</tr>
<tr>
<td>Compensation technology</td>
<td>Active capacitor</td>
<td>Tapped inductor</td>
<td>RC based network</td>
<td>RC based network</td>
<td>RC based network</td>
<td>Transformer feedback</td>
<td>Active capacitor</td>
</tr>
<tr>
<td>Active component</td>
<td>Active capacitor</td>
<td>−</td>
<td>Active inductor</td>
<td>Active inductor</td>
<td>Active inductor</td>
<td>−</td>
<td>Active capacitor/inductor</td>
</tr>
<tr>
<td>Order</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>Center Frequency(MHz)</td>
<td>5300</td>
<td>23500</td>
<td>1950</td>
<td>2100</td>
<td>600</td>
<td>2368</td>
<td>758</td>
</tr>
<tr>
<td>BW(MHz)</td>
<td>1700</td>
<td>4000</td>
<td>300</td>
<td>0.1</td>
<td>60</td>
<td>7.1</td>
<td></td>
</tr>
<tr>
<td>Gain(dB)</td>
<td>0.77</td>
<td>1.65</td>
<td>-8</td>
<td>0</td>
<td>30</td>
<td>1.8</td>
<td>18.1</td>
</tr>
<tr>
<td>Rejection level(dB)</td>
<td>36.8</td>
<td>15.2</td>
<td>−</td>
<td>−</td>
<td>−</td>
<td>−</td>
<td>30</td>
</tr>
<tr>
<td>$P_{DC}$(mW)</td>
<td>2.2</td>
<td>4.2</td>
<td>4</td>
<td>−</td>
<td>120</td>
<td>8.8</td>
<td>50</td>
</tr>
<tr>
<td>Quality factor</td>
<td>3/6</td>
<td>195</td>
<td>140</td>
<td>2</td>
<td>40</td>
<td>25.5</td>
<td></td>
</tr>
<tr>
<td>Tunability</td>
<td>Center freq.(MHz)</td>
<td>−</td>
<td>1800–2050</td>
<td>−</td>
<td>−</td>
<td>−</td>
<td>758–864</td>
</tr>
<tr>
<td>Gain(dB)</td>
<td>−</td>
<td>−</td>
<td>−8</td>
<td>−</td>
<td>−</td>
<td>−</td>
<td>6.5–18.1</td>
</tr>
<tr>
<td>BW(MHz)</td>
<td>−</td>
<td>−</td>
<td>10</td>
<td>−</td>
<td>−</td>
<td>−</td>
<td>7.1–65.9</td>
</tr>
<tr>
<td>Quality factor</td>
<td>−</td>
<td>−</td>
<td>180–205</td>
<td>−</td>
<td>−</td>
<td>−</td>
<td>12–107</td>
</tr>
</tbody>
</table>
6 CONCLUSION AND FUTURE WORK

In this research, a classic 2nd order Chebyshev BPF, adopting both active capacitor and active inductor for tunability, low cost and smaller size, is presented. The negative resistance of the active capacitor compensates the resistive loss of active inductor, contributing to an optimized narrow band BPF at center frequency 758 MHz with a gain of 18.1 dB, a rejection level of 50 dB. Next, the tunability can be achieved by adjusting the active capacitance, leading to a tunable frequency range from 758 MHz to 864 MHz.

It is very challenging to find an exact match between active capacitance and active inductance so that negative resistance of active capacitor and positive resistance of active inductor are exactly cancelled at a high frequency range. To realize a low inductance in the high frequency range is also challenging, which also limits the performance of the tunable BPF.
7 REFERENCE


