A Computational Study of a Photovoltaic Compound Parabolic Concentrator

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in Engineering

By

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I HEREBY RECOMMEND THAT THE THESIS PREPARED UNDER MY SUPERVISION
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Abstract

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Routines have been written and added to the Wright State developed solar system simulation program called Solar_PVHFC to model incident solar radiation for a compound parabolic concentrator (CPC) that uses solar panels (photovoltaic panels) to produce electrical energy. Solar_PVHFC is a program that models a solar energy system composed of solar panels to produce electricity from the sun, hydrogen storage tanks to chemically store the energy produced by the solar panels, and fuel cells to convert between electrical and chemical energy when required. Solar_PVHFC features several adjustable parameters to model a solar panel, hydrogen storage, and fuel cell system. Now Solar_PVHFC can model CPC solar panels. The CPC portion of this program allows for building and modifying CPCs based on three input variables: the concentration ratio, the degree of truncation, and the absorber width. Included in the program is a crude cost analysis that can be used as an economic means of comparing variations of CPCs and comparing CPCs against conventional solar panels.

Solar_PVHFC models available solar radiation impinging on a solar panel using TMY3 data files. Inputs include the tilt and azimuthal angle of the panels, the geographical location of the panels, and the time period of the analysis. Because of this thesis work, Solar_PVHFC can now model panels that track the sun for any configuration of one or two axes of rotation and can even incorporate rotational limits. This thesis investigated panels using a fixed orientation, three different single axis tracking orientations, and two axis tracking. Any manufactured module with known specifications can be used as the solar panel, and the program calculates the current-voltage curve and maximum power point for that module on an hourly basis. This thesis investigated CPC and conventional solar panels with module efficiencies of 15.2%, 20.4%, 21.5%, and 28.3%.

CPC solar panel simulations were run for concentration ratios of 2, 5, and 10. The degree of truncation ranged from no truncation with a truncated height ratio of 1, low truncation with a truncated height ratio of 0.75, moderate truncation with a truncated height ratio of 0.5, and high truncation with a truncated height ratio of 0.25. The absorbing width is the width of the solar cells at the bottom of the CPC and was scaled with the concentration ratio in an effort
to maintain almost consistent total opening apertures. The absorbing width had values of 0.1657 m, 0.06627 m, and 0.03313 m with concentration ratios of 2, 5, and 10 respectively. Different mirror reflectivities and the use of cooling were also investigated.

The standard of comparison between different configurations of CPCs and conventional panels was the LCE (levelized cost of energy). This was calculated based on inputs of solar cell price and reflector material price on a per unit area basis. The LCE analysis used in this work only accounts for the solar cell and reflector costs and does not include costs associated with framing, tracking, wiring, inverters, maximum power point tracking, etc. It was thought that these costs would be similar for both the conventional solar panel system and the CPC solar panel system. This thesis’ cost analysis only looks at the part of the system where the analysis used provides differences between the CPC and conventional solar system. The electric output per unit area was also used as a means of comparison between the two systems.

Many results are shown and discussed in the main body of this thesis with an exhaustive collection of results found in the Appendix. East-west, north-south, and two axis tracking showed that CPCs could significantly reduce the LCE of higher priced conventional panels using the same solar cell module. Fixed and vertical axis tracking did not prove very effective for CPCs. The only CPC system to achieve a lower LCE than the low efficiency, low cost conventional panel was the low efficiency CPC, and this was conditional on two axis tracking and high reflectivity. The most effective CPCs generally utilized high degrees of truncation and low to moderate concentration ratios, with the exception of CPCs utilizing high concentration in combination with cell temperature control. All CPCs showed a drop in electric output per unit area compared to conventional panels; with north-south tracking showing the least drop.

A decrease in reflector costs and in high efficiency solar cell costs could enhance the potential of CPC solar panels. A higher mirror reflectivity could also make CPCs more competitive with conventional panels. CPCs show promise when compared to expensive, high efficiency conventional panels, but cannot compete with inexpensive, lower efficiency conventional panels. A more thorough cost analysis may bring more clarity to the comparisons between CPC solar panels and conventional solar panels.
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<td>$A_{PV}$</td>
<td>Area of photovoltaic material</td>
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<tr>
<td>$A_R$</td>
<td>Ratio of absorbing aperture to reflector area of a CPC, subscript T denotes for a truncated CPC</td>
</tr>
<tr>
<td>$A_{ref}$</td>
<td>Area of reflector material</td>
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<td>$a$</td>
<td>Aperture half-width of CPC, subscript T denotes for a truncated CPC</td>
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<tr>
<td>$a,b$</td>
<td>Coefficients in empirical relationships used to calculate circumsolar diffuse radiation</td>
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<td>$a'$</td>
<td>Solar cell (absorber) half-width of a CPC, subscript T denotes for a truncated CPC</td>
</tr>
<tr>
<td>$B$</td>
<td>Function of the day of the year</td>
</tr>
<tr>
<td>$C$</td>
<td>Geometric concentration ratio of a CPC, subscript T denotes for a truncated CPC</td>
</tr>
<tr>
<td>$C_{PV}$</td>
<td>Cost of PV material per unit area</td>
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<tr>
<td>$C_{ref}$</td>
<td>Cost of reflector material per unit area</td>
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<tr>
<td>$C_%$</td>
<td>Ratio of the cost of a CPC over the cost of a conventional panel of equivalent aperture area and PV price</td>
</tr>
<tr>
<td>$E$</td>
<td>Equation of time</td>
</tr>
<tr>
<td>$\hat{E}$</td>
<td>Unit vector for east</td>
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<tr>
<td>$E_c$</td>
<td>The direction of the rotational axis of a collector</td>
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<td>$F$</td>
<td>Control function for beam radiation entering a CPC</td>
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<td>$F_{1,2}$</td>
<td>Brightness coefficients</td>
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<td>$f$</td>
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<td>Empirically derived coefficients used to calculate the brightness coefficients</td>
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<td>GMT</td>
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<td>Height ratio, truncated CPC height over full CPC height</td>
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<td>$h$</td>
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<td>$I_b$</td>
<td>Hourly beam (direct) radiation on a horizontal surface with atmosphere</td>
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<td>Hourly radiation that makes it to CPC solar cell from the opening aperture, subscripts b, d, and g denote beam, diffuse, and ground components respectively</td>
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<tr>
<td>$I_d$</td>
<td>Hourly diffuse radiation on a horizontal surface with atmosphere</td>
</tr>
<tr>
<td>$I_e$</td>
<td>Total hourly irradiation on a horizontal surface (the ground) with atmosphere</td>
</tr>
<tr>
<td>$I_o$</td>
<td>T hourly irradiation on a horizontal surface without atmosphere</td>
</tr>
</tbody>
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$I_T$  Hourly irradiation on a tilted surface with atmosphere, subscripts b, d, and g denote beam, diffuse, and ground respectively

$K$  Extinction coefficient of glazing

$K_{\alpha}$  Incident angle modifier, subscripts b, d, and g denote beam, diffuse, and ground contributions respectively

$L$  Glazing thickness

$L_{loc}$  Local meridian of location in question

$L_{st}$  Standard meridian of location in question

$LCE$  Levelized cost of energy

$M$  Air mass modifier

$m$  Air mass

$n$  Day of the year (0-365); average number of reflections for a CPC, subscript T denotes for a truncated CPC

$n_1$  Index of refraction of air

$n_2$  Index of refraction of glass

$R_b$  Ratio of beam radiation on a tilted surface to beam radiation on a horizontal surface

$r_{\parallel}$  Parallel component of unpolarized radiation

$r_{\perp}$  Perpendicular component of unpolarized radiation

$S$  Radiation absorbed by photovoltaic cells per unit area for a conventional panel

$\hat{S}$  Unit vector for south

$S_c$  Direction defining the collector aperture orientation (normal to $V_c-E_c$ plane)

$S_{abs}$  Radiation absorbed by photovoltaic cells per unit area by a CPC

$S_{CPC}$  Radiation that makes it from the CPC opening aperture to the receiver surface, subscripts b, d, and g denote beam, diffuse, and ground components respectively

$T_1$-$T_5$  Beam, isotropic diffuse, circumsolar diffuse, diffuse from ground, and ground reflected components of $I_T$

$\hat{V}$  Unit vector that is normal to the earth’s surface

$V_c$  Direction normal to collector absorbing aperture surface

$x$  Intermediate equation used in calculating the average number of reflections for a CPC, subscript T denotes for a truncated CPC
Greek

\( \alpha \)  Collector absorptivity, subscripts b, d, and g denote beam, diffuse, and ground components respectively

\( \alpha_s \)  Solar altitude angle

\( \beta \)  Collector tilt, specifically the angle between a horizontal surface and collector surface

\( \beta_a \)  Tilt of collector’s axis of rotation

\( \gamma \)  Collector azimuth angle

\( \gamma_a \)  Azimuth angle of collector’s axis of rotation

\( \gamma_s \)  Solar azimuth angle

\( \delta \)  Declination

\( \varepsilon \)  Longitudinal slant angle

\( \eta \)  Efficiency

\( \theta \)  Angle of incidence, defined between a surface normal and incident radiation, lack of subscript denotes beam, subscripts d and g denote diffuse and ground respectively

\( \theta_c \)  Acceptance half angle of a CPC

\( \theta_e \)  Effective angle of incidence, subscripts d and g denote diffuse and ground respectively

\( \theta_l \)  Angle projected in the longitudinal plane of a CPC

\( \theta_r \)  Angle of refraction, lack of subscript denotes beam, subscripts d and g denote diffuse and ground respectively

\( \theta_t \)  Angle projected in the transverse plane of a CPC

\( \rho \)  Reflector material reflectivity

\( \rho_g \)  Ground reflectivity

\( \tau \)  Transmittance

\( \tau_c \)  Transmittance through a cover over the CPC aperture, subscripts b, d, and g denote beam, diffuse, and ground components respectively

\( \tau_{\text{CPC}} \)  Transmittance for a CPC due to reflection losses, subscripts b, d, and g denote beam, diffuse, and ground components respectively

\( \tau_{\alpha} \)  Transmittance-absorbance product, subscripts b, d, g, and n denote beam, diffuse, ground, and normal components respectively
φ  Latitude of location in question; Angle between parabola axis and parabola end point, subscript T denotes for a truncated CPC

ω  Hour angle

ω_s  Sunset hour angle
Chapter 1. Introduction

1.1. Objectives of Project

The objective of this project is to computationally investigate compound parabolic concentrators (CPCs) coupled with solar cells (photovoltaic cells) as a means of producing electricity. Power output, as well as costs, are studied. Ultimately it is hoped that using CPCs in combination with photovoltaic cells can be made more cost effective than conventional solar panels of equivalent aperture area. In this work, a great deal of performance and cost data on CPCs and conventional solar panels is presented so that comparisons can be made. Comparisons are made based on several variables that influenced cost and solar energy collection.

In order to produce these results, a computer model was created that simulates the operation of a CPC with photovoltaic cells. This is a detailed model that considers all aspects of a CPC-photovoltaic cell system (In the rest of this thesis this will simply be called a CPC). This model is built on top of a currently available computer code called Solar_PVHFC. Solar_PVHFC is a Wright State computer program that models a solar energy system composed of solar panels to produce electricity from the sun, hydrogen storage tanks to chemically store the energy produced by the solar panels, and fuel cells to convert between electrical and chemical energy when required. Solar_PVHFC is a detailed analysis of these systems that features several adjustable parameters for the solar panel, hydrogen storage, and fuel cell system. In this work we are not interested in the hydrogen storage or fuel cell portion of this program, but heavy use has been made of the photovoltaic, PV, portion of Solar_PVHFC.
1.2. Motivation of Project

The cost of PV panels has been decreasing dramatically as cell efficiency has been increasing; see Figure 1 and Figure 2 respectively. Concentrating systems have been used for thermal solar applications for millennia [1], but have not been used on PV systems until recently. The question arose of how CPCs might accommodate PV applications. Using the concentration of a CPC, the area of the PV cell can be reduced, thus reducing a costly component of the system. The goal of making solar generated electricity cheaper than conventional panels was the motivation for this project. It was thought that replacing some of the PV cells with less expensive reflecting materials would bring the cost of solar generated electricity down.

Figure 1: Price of crystalline silicon photovoltaic cells ($/watt) over the course of several decades.[2] Image reused with permission from The Economist.
Figure 2: Photovoltaic efficiency has been increasing for several decades. [3]
1.3. Current Status of Concentrating Photovoltaics

Concentrating photovoltaic (CPV) systems are usually divided into two categories: low and high concentration. There may be some discrepancy in the strict definitions, but low concentration systems (LCPV) usually range from 2 to 100 suns while high concentration systems (HCPV) are usually anything greater than 100 suns (1 sun = 1000 W/m²). Sometimes high concentration systems are further divided into medium and high, but this is not common since there are other underlying characteristics to help categorize low and high concentrations. LCPVs typically use conventional silicon solar cells, require simple cooling techniques (i.e. fins), and can often be effective with limited tracking. On the other hand, HCPVs typically call for multi-junction cells, require more aggressive cooling, and usually demand two axis tracking. In either case, it is widely accepted that CPVs become more attractive with increasing cell efficiency. As shown in Figure 2, tremendous gains have been made in improving cell efficiency. The current world record, as of Dec. 1, 2014, is a multi-junction cell operating at 46.0% efficiency. [4]

The dominate forms of HCPVs are Fresnel lenses and parabolic mirrors. Perhaps the most prolific HCPV design is Soitec’s CX-M500, of which 13 MW have been installed in 14 countries to date. Their module uses a Fresnel lens and a concentration ratio of 500 suns with a multi-junction cell boasting an efficiency of 40%, giving the whole module an efficiency of 31.8%. It does require dual axis tracking, but its high efficiency demands recognition. [4] Receiving the most attention for parabolic concentrators is IBM Research as they develop a hybrid photovoltaic and thermal module operating at 2000 suns. The innovation of their design is in its removal of waste heat from its multi-junction solar cells. Between generated electricity and thermal energy, they hope to claim 80% efficiency and to bring the cost of energy down to less than $0.10/kWh. [5] In general, high efficiency multi-junction solar cells are becoming more available and are expected to become more affordable, a good indicator for the success of HCPVs.

LCPV designs go hand-in-hand with larger acceptance angles, meaning they are not as dependent on tracking systems as HCPVs. They also can make use of efficient conventional solar cells and thus avoid the substantial costs of multi-junction cells. Really, a low concentration system can be made as easily as adjoining a mirror to a solar panel and thus
increase the impinging radiation. A more sophisticated design would be a trough concentrator; this can take on all sorts of geometries and may even prove useful in building integrated photovoltaics. The most prominent LCPV design on the market is SunPower’s C7 tracker. The C7 uses a parabolic mirror to reflect sunlight onto a solar cell at a concentration ratio of 7 suns. It also utilizes a single axis tracking system. It boasts a module efficiency of 20% and also claims to be the lowest levelized cost of energy for utility-scale deployment. [6] SunPower has recently revealed plans for a 300 MW C7 Tracker system in China. [7] Another solar company, Cogenra, produces the T14 module that uses a concentration ratio of 14 suns. Cogenra believes the T14 will bring CPV energy down to $0.05/kWh, [8] which would be a significant milestone for any energy technology.

One additional concentrating technology that bears mentioning is luminescent solar concentrators. These low concentration collectors receive incident light and rely on total internal reflection to deliver solar radiation to the edges of the aperture area where it is collected by solar cells. This technology could potentially fill a huge need as building façades, collecting energy while still permitting some light to transmit through, except that its record efficiency is stumped at 7.1%. [9]

It should be noted that the success of concentrating systems is largely dependent on the solar resource at the location. HCPVs can only be implemented in areas with significant direct normal radiation, whereas LCPV might find a niche environment where it can outperform conventional and HCPV systems. A report given at the 4th International Conference on Solar Concentrators for the Generation of Electricity or Hydrogen in 2007 analyzed the cost per watt for solar thermal, HCPV, and LCPV. The conclusion was that each technology stood a chance based on the scale and location of the system. [10]

CPVs have been under consideration since the 1970s, but the technology as a legitimate contender to conventional PV and other energy sources is relatively new. Despite this, there is a lot of optimism about CPVs and the economy is beginning to reflect this. Solar industry analysts IHS report that CPVs experienced 37% global market growth in 2014, culminating in roughly 250 MW of installations. This growth, which includes low and high concentrating systems, is expected to continue for several years. [11] Karl Melkonyan, an analyst for IHS, predicts the cost per watt for CPVs to decrease to $1.59/W by 2017, and reports that the
levelized cost of energy in 2013 was $0.14/kWh and is expected to decrease by 12% for the next few years. [12] The current status of concentrating photovoltaics is very promising indeed.
Chapter 2. Solar_PVHFC

Solar_PVHFC is a program written in MATLAB by Michael Gustafson. It models available solar radiation for a flat PV panel and joins it to an energy demand, as well as reversible fuel cells and hydrogen storage. This program provides the backbone for the CPC model discussed in Chapter 4. Since Solar_PVHFC is the backbone of the CPC program written for this work, this chapter gives a brief overview of Solar_PVHFC. For a more complete account, see Gustafson’s “A Computational Approach to Simulating the Performance of a 24-Hour Solar Hydrogen Fuel Cell Electric Power Plant”. [13]

2.1. Program Functions

Solar_PVHFC uses GUI interfaces to run simulations of available solar radiation in various locations and compares it to an input demand. The program also models a fuel cell and hydrogen storage system that can store solar energy in the form of hydrogen chemical energy and then releases this chemical energy in the form of electrical energy in order to meet the input demand. The outputs from this program provide valuable information for correct component sizing for solar, hydrogen, and fuel cell systems.

Upon initiating the program, the user is able to either load previous files or run a new simulation. After choosing to run a new simulation, the user selects the location of the simulation. Dayton, OH and Yuma, AZ are preset options, but the user is also able to load a TMY3 file for any location for which this data is available. TMY3 files are available for several cities in every state in the U.S., and many locations outside the country as well. Next, the user chooses a demand. There are many preset options ranging from energy efficient or base houses to regional power plants. A demand file can also be created in an Excel spreadsheet, so as to model any potential demand that runs on an annual cycle.

Next comes the GUI for the photovoltaic array design. From this GUI the user selects a PV module to use. There are several modules already loaded into the program, or the user
can load a custom module from a spreadsheet with the correct format. Solar_PVHFC reads specifications about the module from its spreadsheet:

- Reference temperature and irradiance
- Short circuit current
- Open circuit voltage
- Current at max power
- Voltage at max power
- Short circuit current temperature coefficient
- Open circuit voltage temperature coefficient
- Band gap for semiconductor material
- Length and width of a single panel

Solar_PVHFC uses the module specifications to plot an I-V curve (current-voltage curve) and calculates a maximum power point at which to operate. The user can create an array of panels by entering how many panels are in parallel and series. The mounting can also be set as a ground or roof mount, which affects temperature coefficients for the simulation, the ground reflectance can be adjusted, and the tilt and azimuth angle of the array can be input (see Figure 3).
The subsequent GUI in Solar_PVHFC entails the input parameters for the reversible fuel cell. The user can detail cathode and electrode properties, the exchange current density, transfer coefficient, operating temperature, and the balance of plant energy requirement. The GUI calculates and displays I-V and I-P (current-power) curves for fuel cell and electrolyzer operation. The user can also input how many fuel cells are in series and parallel, and adjust the cell surface area.

The user is then prompted to enter the maximum storage capacity of the hydrogen tank in kilograms at 50 bars of pressure. Finally, the time frame for the simulation is established by entering the starting month, day, and hour, and how many days the simulation should run. The simulation can be run for up to 20 years (that’s 7300 days).

Once Solar_PVHFC has completed its simulation, it presents four plots of data showing:

- Total available solar radiation for the duration of the simulation
- Energy outputs from the solar array and fuel cells along with energy drains from the electrolyzer and demand
- Amount of hydrogen in storage in kilograms
- Energy deficit or surplus

Figure 4: Output plots from Solar_PVHFC.

All of the data in the plots is on an hourly basis. The program also “prints” outputs in the command window. These outputs include totals of solar energy available and collected, and electricity generated. The outputs also include the amount of heat generated by the fuel cell and the percentage of the demand that was met. A final GUI that appears at the end of the simulation gives the user the option of saving the run.
2.2. Mathematical Model Used

The mathematical models that Solar_PVHFC uses are presented in detail in Gustafson’s “A Computational Approach to Simulating the Performance of a 24-Hour Solar Hydrogen Fuel Cell Electric Power Plant”. [13] Of particular importance to this investigation is the model for solar radiation on a tilted surface. Gustafson’s Solar_PVHFC makes use of an anisotropic diffuse radiation model that breaks the diffuse radiation into three components: isotropic diffuse, circumsolar diffuse, and diffuse from the horizon. For a compound parabolic concentrator, a simpler one-part isotropic diffuse model is used. This is elaborated on in Section 4.2.2.

2.3. Meeting Demand with Fuel Cells and Hydrogen Storage

Solar_PVHFC was initially created to study solar power working in conjunction with reversible hydrogen fuel cells to meet a demand. As covered in Section 2.1, the user is able to input what solar panels they want to use, how large the array is, and how many fuel cells are to be used. The solar panels power the demand while solar radiation is available. When there is insufficient solar radiation to meet the demand (for example, it’s cloudy or nighttime), then the fuel cells can work in conjunction with the solar panels or on their own to produce the power demand. The fuel cells may not be able to meet the demand due to a depletion of
hydrogen or too small a fuel cell system. In this case, when neither the solar panels nor fuel cells can totally meet the electric demand, the model calls this an energy deficit and it is presumed that electricity will be purchased from the grid to settle the demand.

A solar array that is more than large enough to meet the demand will produce excess electricity. This excess electricity is used to run the system’s electrolyzers; water is separated and hydrogen is stored in a 50 bar storage tank. The maximum capacity of the tank is a user input and is specified in kilograms of hydrogen. When the storage tank reaches its max capacity, excess electricity is called a surplus and is presumed to be sold back to the grid.

Deficit and surplus electricity are plotted with the program’s outputs and can be seen in the bottom plot of Figure 4. These values help assess how well the solar-fuel cell system did in meeting the demand, and can provide insights to better component sizing.
Chapter 3. Alterations to Solar_PVHFC

The original Solar_PVHFC program has undergone some alterations that contribute to its accuracy and capabilities. Significant alterations are listed in this chapter. The most updated version of Solar_PVHFC was used for all work that is presented in this thesis.

3.1. Modified Perez Brightness Coefficients

Solar_PVHFC models available solar radiation by reading a TMY3 file for a specific location and calculating the total radiation impinging on a tilted surface from three sources: direct (beam) radiation, diffuse radiation, and radiation reflected from the ground. Furthermore, it uses the Perez model to further divide and calculate the diffuse radiation into three components: isotropic diffuse, circumsolar diffuse, and horizon brightening, as shown in equation (1):

\[ I_{d,T} = I_d \left[ (1 - F_1) \left( \frac{1 + \cos \beta}{2} \right) + F_1 \frac{a}{b} + F_2 \sin \beta \right] \]

Details and explanations of this equation can be found in Gustafson’s thesis [13] or Duffie and Beckman [14] among other sources. The emphasis is on the brightness coefficients \( F_1 \) and \( F_2 \). \( F_1 \) and \( F_2 \) are functions of the zenith angle \( \theta_z \), a clearness index \( \varepsilon \), and a brightness parameter \( \Delta \), and are calculated from tabulated values of empirical data \( (f_1x \text{ and } f_2y) \):

\[ F_1 = \max \left[ 0, \left( f_{11} + f_{12} \Delta + \frac{\pi \theta_z}{180} f_{13} \right) \right] \]

and

\[ F_2 = \left( f_{21} + f_{22} \Delta + \frac{\pi \theta_z}{180} f_{23} \right) \]

The original Solar_PVHFC program made use of tabulated f values from a 1988 Sandia National Laboratories report [15]. These values were updated in a 1990 paper by Perez et al. [16] and the updated values are currently being used in Solar_PVHFC. The brightness coefficients affect how much each component of diffuse radiation contributes to the overall diffuse. Using the updated tabulated values increases the accuracy of the model.
3.2. Tracking

The original Solar_PVHFC program allowed the user to run simulations for a conventional flat-plate solar panel in a fixed position. The user could specify a tilt and azimuth angle of the collector for the duration of the simulation. In anticipation of modeling concentrating solar panels and to increase the program’s functionality, a tracking option has been added. Solar_PVHFC’s tracking capabilities include all varieties of single and dual axis tracking, as well as a setting for a rotation limit. Figure 6 provides a visual to aid in understanding how tracking works. Tracking affects the panel’s tilt and/or its azimuth angle, as summarized in Error! Reference source not found..

![Diagram of solar panel tracking](image)

Figure 6: A solar panel may be in a fixed position or track along one or two axes. Image taken from PVWatts Calculator with a modification made to the definition of the azimuth angle. [17]

An explanation of terms may help avoid confusion. The reference for angles follows that of Duffie and Beckman [14]. That is, a panel in the northern hemisphere that is facing the equator perfectly due south will have an azimuth angle of $\gamma = 0^\circ$. A panel facing perfectly east will have an azimuth angle of $\gamma = -90^\circ$, and perfectly west $\gamma = +90^\circ$. The tilt of a panel is given by $\beta$, where $\beta = 0^\circ$ is a horizontal panel (parallel to the ground), $\beta = 90^\circ$ is a vertical panel, and $180^\circ < \beta < 90^\circ$ is a panel that is actually facing downwards towards the ground. A subscript ‘a’ indicates the azimuth and tilt of the axis of rotation. Thus $\gamma_a$ and $\beta_a$ define what axis the panel rotates about. In relation to the sun, the zenith angle, $\theta_z$, is the angle between the sun and
a vertical line from the surface; $\theta_z = 90^\circ$ at sunrise and sunset when the sun lies on the horizon, and $\theta_z = 0^\circ$ when the sun is directly overhead. The solar azimuth angle, $\gamma_s$, is the angle between the sun and a line due south from the collector; $\gamma_s = 0^\circ$ when the sun is perfectly due south. A deviation eastward is negative, and a deviation westward is positive.

The various tracking methods can be categorized as follows:

i) Single axis tracking

(1) Vertical axis of rotation: The collector has a fixed tilt ($\beta =$ constant) and changes its azimuth angle ($\gamma = \gamma_s$) to follow the sun from east to west throughout the day.

(2) Axis of rotation in plane of the collector

(a) The axis of rotation is horizontal (parallel to the ground)

(i) The axis of rotation runs east to west: The collector’s azimuth angle ($\gamma = 0^\circ$) is fixed, but the tilt ($\beta$) adjusts throughout the day.

(ii) The axis of rotation runs north to south: The collector’s azimuth flips from due east ($\gamma = -90^\circ$) in the morning to due west ($\gamma = 90^\circ$) in the afternoon. The tilt ($\beta$) changes throughout the day.

(iii) The axis of rotation is aligned some other way: The collector’s azimuth ($\gamma$) flips at most once a day. The tilt ($\beta$) changes throughout the day.

(b) The axis of rotation sits at an angle to the ground (such as the middle picture in Figure 6) with a fixed tilt ($\beta_a$) and fixed azimuth angle ($\gamma_a$). The tilt of the collector ($\beta$) and the collector’s azimuth angle ($\gamma$) are both changing throughout the day.

ii) Two axes of rotation

(1) The collector perfectly tracks the sun so that its azimuth angle is equal to the solar azimuth angle ($\gamma = \gamma_s$) and its tilt is equal to the zenith angle ($\beta = \theta_z$).
Table 1: Formulas for a tracking module.

<table>
<thead>
<tr>
<th>Tracking Option</th>
<th>Collector azimuth angle</th>
<th>Collector tilt</th>
<th>Model source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed (no tracking)</td>
<td>$\gamma = \text{constant}$</td>
<td>$\beta = \text{constant}$</td>
<td>N/A</td>
</tr>
<tr>
<td>Vertical axis of rotation</td>
<td>$\gamma = \gamma_s$</td>
<td>$\beta = \text{constant}$</td>
<td>Braun and Mitchell [18]</td>
</tr>
</tbody>
</table>
| Axis of rotation in plane of the collector—Horizontal axis | $\gamma = \gamma_a + 90^\circ \text{ if } \gamma_s - \gamma_a \geq 0$
|                                  | $\gamma = \gamma_a - 90^\circ \text{ if } \gamma_s - \gamma_a < 0$ | $\beta = \beta_o + \sigma_\beta \times 180^\circ$ where,
|                                  |                         | $\beta_o = \tan^{-1}(\tan \theta_z \cos(\gamma - \gamma_s))$
|                                  |                         | $\sigma_\beta = \begin{cases} 0 & \text{if } \beta_o \geq 0 \\ 1 & \text{otherwise} \end{cases}$ | Braun and Mitchell [18] |
| Axis of rotation in plane of the collector—Sloped axis | $X = \frac{\sin \theta_z \sin(\gamma_s - \gamma_a)}{\sin \theta_z \cos(\gamma_s - \gamma_a) \sin \beta_a + \cos \theta_z \cos \beta_a}$
|                                  | $\Psi = \begin{cases} 0^\circ, \text{if } X = 0, \text{or if } X > 0 \text{ and } (\gamma_s - \gamma_a) > 0, \text{or if } X < 0 \text{ and } (\gamma_s - \gamma_a) < 0 \\ +180^\circ, \text{if } X < 0 \text{ and } (\gamma_s - \gamma_a) > 0 \\ -180^\circ, \text{if } X > 0 \text{ and } (\gamma_s - \gamma_a) < 0 \end{cases}$
|                                  | $R = \tan^{-1}(X) + \Psi$ | $\beta = \cos^{-1}(\cos R \cos \beta_a)$ | Marion and Dobos [19] |
| Two axes of rotation             | $\gamma = \gamma_s$    | $\beta = \theta_z$ | Braun and Mitchell [18] |

The original PVGUI in Solar_PVHFC has been altered to include the tracking options shown in Table 1.
Duffie and Beckman [14] created plots of horizontal collectors tracking along single axes over the course of a day. Figure 7 compares those plots with plots produced from Solar_PVHFC’s tracking models. The y-axis shows extraterrestrial solar radiation using a solar constant of 1367 W/m² impinging on the collector’s surface. The x-axis shows the hours of a day; the x-axis for Solar_PVHFC has been chopped on both sides to conserve space. E-W (east-west) (or HEW (horizontal east west)) corresponds to a horizontal panel with its axis of rotation aligned east to west. N-S (north-south) (or HNS (horizontal north-south)) corresponds to a horizontal panel with its axis of rotation aligned north to south. A line labeled with $\beta$ or called “Fixed” is a non-tracking panel with a slope of $\beta = 45^\circ$ and $\gamma = 0^\circ$. In the picture on the left, the dashed lines are for the winter solstice (Dec. 21) and the solid lines are for the summer solstice (June 21). The plots are also set for a latitude of 45°. The plot on the right shows lines connecting with the x-axis; these simply retain a value of zero when the sun is not up while the plot on the left does not plot these values. Two horizontal lines have been drawn connecting the two plots and showing their congruence. This provides a good check for Solar_PVHFC’s tracking abilities, at least for a single horizontal axis of rotation.

Figure 7: Duffie and Beckman’s plots of horizontal single axis tracking are compared to Solar_PVHFC’s tracking model outputs. On the left, the dashed lines correspond to the winter solstice while the solid lines correspond to the summer solstice. E-W refers
to a horizontal axis aligned east to west and N-S refers to a horizontal axis aligned north to south. Lines labeled with $\beta$ refer to a fixed (non tracking) panel with a tilt of $\beta=45^\circ$.

3.3. Glazing Absorption

The reference conditions measured for a solar panel, such as the reference temperature and irradiance, are configured for a solar cell without its protective glazing covering its surface. The original Solar_PVHFC program used radiation incident on the surface to calculate its electric output. The current version of the program has been altered to incorporate transmission and absorption losses of radiation as it interacts with a panel’s glazing. This is called the absorptance, $S$, and depends on the incidence angle, air mass, and incident radiation [14].

The angle of incidence for radiation on the tilted surface of a panel is $\theta$. Subscripts ‘b’, ‘d’, and ‘g’ denote beam, diffuse, and ground radiation respectively. $\theta_b$ is the angle between the panel’s surface and the sun and is simply referred to as $\theta$. It is more useful to portray $\theta_d$ and $\theta_g$ as effective angles of incidence. Brandemuehl and Beckman [20] developed equations for $\theta_{e,d}$ and $\theta_{e,g}$ as functions of the panel’s tilt $\beta$,

\[
\theta_{e,d} = 59.7 - 0.1388\beta + 0.001497\beta^2 \\
\theta_{e,g} = 90 - 0.5788\beta + 0.002693\beta^2,
\]

where $\beta$ is in degrees. As the radiation passes from air to the glazing, Snell’s law is used to find the angles of refraction: $\theta_r$, $\theta_{r,d}$, and $\theta_{r,g}$,

\[
\theta_r = \sin^{-1}\left(\frac{n_1\sin\theta}{n_2}\right). \quad (6)
\]

The index of refraction for the mediums is taken to be $n_1=1$ for air and $n_2=1.526$ for glass (glazing). $\theta_{r,d}$, and $\theta_{r,g}$ are found by the same method as $\theta_r$ using equation ((6)). Note that the interface between the glass and the photovoltaic cells is usually well matched and can be neglected. The transmittance-absorptance product, $\tau\alpha(\theta)$, can then be calculated for beam, diffuse, and ground reflected radiation as,

\[
\tau\alpha(\theta) = e^{-\left(\frac{KL}{\cos\theta_r}\right)} \left[1 - \frac{1}{2}\left(\frac{\sin^2(\theta_r-\theta)}{\tan^2(\theta_r+\theta)} + \frac{\tan^2(\theta_r-\theta)}{\tan^2(\theta_r+\theta)}\right)\right]. \quad (7)
\]
In equation ((7)), K is the glazing extinction coefficient (typically 4 m\(^{-1}\)) and L is the glazing thickness (typically 2 mm) [14]. In this way, the transmittance-absorptance product can be found for the incident angles of the beam, diffuse, and ground radiation, providing \(\tau\alpha(\theta)\), \(\tau\alpha(\theta_{c,d})\), and \(\tau\alpha(\theta_{c,g})\). These will be written as: \((\tau\alpha)_b\), \((\tau\alpha)_d\), and \((\tau\alpha)_g\) for beam, diffuse, and ground–reflected radiation respectively. One important exception to equation ((7)) is made for radiation at normal incidence, for which Snell’s law cannot be used. The transmittance-absorptance product for normal incidence is

\[
(\tau\alpha)_n = e^{-KL} \left[ 1 - \left( \frac{n_1-n_2}{n_1+n_2} \right)^2 \right].
\]  

When the above mentioned values for K, L, \(n_1\), and \(n_2\) are used in equation ((8)), \((\tau\alpha)_n=0.9490\).

The air mass is a function of the solar altitude angle, \(\alpha_s\), (the compliment of the zenith angle) and is presented in the work by Kasten and Young [21]:

\[
m(\alpha_s) = \sin(\alpha_s) + 0.1500(\alpha_s + 3.885^\circ)^{-1.253}^{-1}.
\]  

The air mass is then used to determine the air mass modifier, M, taken from Duffie and Beckman [14]:

\[
M = 0.935823 + 0.054289m(\alpha_s) - 0.008677m(\alpha_s)^2 + 0.000527m(\alpha_s)^3
\]

\[
-0.000011m(\alpha_s)^4
\]

The radiation in the program can be split in to five components: direct beam, isotropic diffuse, circumsolar diffuse, diffuse from the horizon (horizon brightening), and ground reflected radiation. In Solar_PVHFC, these five components are called \(T_1\), \(T_2\), \(T_3\), \(T_4\), and \(T_5\) respectively, and are defined as:

\[
T_1 = I_b R_b
\]  

\[
T_2 = I_d (1 - F_1) \left[ \frac{1 + \cos \beta}{2} \right]
\]  

\[
T_3 = I_d F_1 \left( \frac{a}{b} \right)
\]  

\[
T_4 = I_d F_2 \sin \beta
\]  

\[
T_5 = I_e \rho_g \left[ \frac{1 - \cos \beta}{2} \right]
\]

More information on how the radiation is modeled can be found in Gustafson [13] and Duffie and Beckman [14].
The above contributions of incidence angles, air mass, and incident radiation come together to determine the equation for the absorbed radiation, $S$:

$$S = M[T_1(\tau\alpha)_b + T_2(\tau\alpha)_d + T_3(\tau\alpha)_b + T_4(\tau\alpha)_d + T_5(\tau\alpha)_g].$$

(16)

It is worth noting that while there are three components of diffuse radiation, $(\tau\alpha)_d$ is only used with the isotropic diffuse, $T_2$, and the horizon brightening, $T_4$. This is because $\tau\alpha$ is related to the radiation incidence angle. The circumsolar diffuse, $T_3$, strikes the panel surface at an angle similar to that for beam radiation, so $(\tau\alpha)_b$ is used with $T_3$. Horizon brightening diffuse, $T_4$, is not as obvious, but Duffie and Beckman recommend using $(\tau\alpha)_d$ rather than $(\tau\alpha)_g$.

Solar_PVHFC has been altered to include the calculation of the absorbed radiation. In the interface PVGUI, the user is able to select if they would like to simulate a panel with or without glazing, although the default is with glazing. If glazing is selected, the user is able to change the default $K$ and $L$ values. If the user chooses to run a simulation for a panel without glazing, they may enter and absorbance between 0 and 1, 1 being all the incident radiation is absorbed. This option bypasses the calculation for $S$. The updated PVGUI is displayed in Figure 8.
Figure 8: The altered version of PVGUI in Solar_PVHFC. Additions include tracking and glass absorption. An option to run a cost comparison was also added, and this is discussed in Error! Reference source not found..

3.4. Adjusting to Solar Time

The solar radiation data contained in TMY3 files is given on an hourly basis. The original Solar_PVHFC program assigned this hour’s worth of constant energy to the end of each hour. This created an occasional problem at sunrise and sunset. Many values were calculated for the end of an hour, but the radiation is for the entirety of the hour. In particular, the ratio of beam radiation on a tilted surface to beam radiation on a horizontal surface, $R_b$,

$$R_b = \frac{\text{beam radiation on a tilted surface}}{\text{beam radiation on a horizontal surface}}$$  \hspace{1cm} (17)

is very sensitive at sunrise and sunset. At sunrise or sunset, the beam radiation on a horizontal surface becomes very small causing $R_b$ to grow very big. This works out because this large $R_b$
value is paired with a very small amount of radiation, $I_b$ (see equation (11)). $R_b$ is a negative number before sunrise and after sunset. If the sun sets towards the beginning of the hour, but $R_b$ was calculated at the end of the hour, then a certain amount of radiation would count negatively towards a panel’s absorbed radiation. The solution to this problem was to calculate the sunrise and sunset times and have the program use those as cutoffs for radiation exposure. The equation for the sunset hour, $\omega_s$, is 

$$\omega_s = \cos^{-1}(-\tan \varphi \tan \delta)$$

where $\varphi$ is the location’s latitude and $\delta$ is the earth’s declination for that day [14]. The program treats times before solar noon as negative, and times after solar noon as positive. Therefore, the sunrise hour is simply $-\omega_s$. This begs the question: “What is solar noon?” Solar time is a time based on the sun’s angular movement throughout the day, with solar noon being when the sun crosses the observer’s meridian. Solar time does not coincide with local clock time. The original Solar_PVHFC program was not adjusted to solar time; the current version is. Using solar time enhances simulation accuracy since all sun-angle relationships used to calculate available radiation are based on solar time.

The location’s longitude and difference from Greenwich Mean Time must be used for solar time calculations. Fortunately, TMY3 files specify both of these. The equation for solar time is

$$\text{Solar time} = \text{Standard time} + 4(L_{st} - L_{loc}) + E$$

where $L_{st}$ is the location’s standard meridian, $L_{loc}$ is the location’s local meridian, and $E$ is a parameter that adjusts the solar time based on variances in Earth’s rotational speed as a function of the time of year,

$$E = 229.2(0.000075 + 0.001868 \cos B - 0.032077 \sin B - 0.014615 \cos 2B - 0.04089 \sin 2B)$$

where $B$ is a function of the $n^{th}$ day of the year:

$$B = (n - 1) \frac{360}{365}.$$

The ‘4’ in equation ((19)) comes from the sun taking four minutes to traverse 1° in the sky. The model for solar time, which comes from Duffie and Beckman [14], is based on numbering longitude in degrees west, that is, $0^\circ \leq \text{longitude} < 360^\circ$. However, TMY3 files usually record
the location’s longitude as \(-180^\circ \leq \text{longitude} \leq +180^\circ\) (negative being west of the prime meridian, and positive being east). So it is necessary to convert longitude coordinates into degrees west. Special attention should be given to making sure unfamiliar TMY3 files record their location’s longitude as \(-180^\circ \leq \text{longitude} \leq +180^\circ\) to avoid miscalculations. The standard meridian for a location is found by multiplying its time difference from Greenwich Time (\(\Delta\text{GMT}\)) by 15°. Once again, the model is set up to read \(0 \leq \Delta\text{GMT} < 24\), but the TMY3 file typically records it as \(-12 \leq \Delta\text{GMT} \leq +12\) (negative being west of the prime meridian, and positive being east). A similar conversion is done. Once the location’s local longitude and \(\Delta\text{GMT}\) are converted to the right scale, equation (19) can be applied. It will add or subtract minutes from the standard time, which are converted into decimals. Solar\_PVHFC then uses this solar time to calculate all the necessary sun-angle relationships.

One additional alteration that was made to enhance Solar\_PVHFC’s accuracy was to read an hour by its middle instead of at its end. As mentioned, the program used to assign radiation from the TMY3 file to the end of each hour, and calculated values at the end of each hour. The program now uses the median of each hour to give better balance to each calculation.

### 3.5. Comparing to PVWatts

PVWatts® is an online solar calculator created and operated by the National Renewable Energy Laboratory (NREL) [17]. It allows the user to specify the tilt and azimuth of a panel, to size the panel and select an efficiency, and to utilize fixed, single, or dual axis tracking. It then models solar radiation based on a TYM3 data set. In many ways, PVWatts is very similar to Solar\_PVHFC. It varies in that Solar\_PVHFC utilizes customized demand files and has the freedom of using whatever solar module is loaded in to it, among other differences. As far as modeling solar radiation, the main difference is that the two programs have different cell temperature models. Therefore, PVWatts provided an excellent checking method to verify the accuracy of Solar\_PVHFC in modeling available solar radiation.

PVWatts presents its outputs as average daily radiation per area over the entire month. So a fixed panel with a tilt of 39.8° and azimuth angle due south in Dayton, OH (using WPAFB TMY3 data set) collects a daily average energy of 2.9 kWh/m² for the month of January.
radiation modeled by Solar_PVHFC was compared to PVWatts version 2 for a fixed orientation in Dayton, OH in Figure 9.

![Incident Radiation, Fixed Orientation](image)

Figure 9: Comparing results from the solar radiation models PVWatts and Solar_PVHFC for a fixed orientation. The location is Dayton, OH.

The radiation comparison shows how the lines for the two models nearly overlap. This is a good indication of accurate modeling for a fixed system. The electric output is compared in Figure 10. The electric output appears to be very similar for the two models and follows an identical trend. The difference in the electric output results is most likely due to the different temperature models used by each program. Cell efficiency is a function of cell temperature, and electric output is directly related to cell efficiency.
Results for single axis and two axis tracking are also presented. For a single axis tracking system, Solar_PVHFC duplicated PVWatt’s tracking option by assigning an axis of rotation azimuth angle of $\gamma=0^\circ$ (due south) and an axis of rotation tilt angle of $\beta=39.8^\circ$ (that’s Dayton’s latitude). Solar_PVHFC also used a rotational limit of $\pm45^\circ$ to match PVWatt’s rotational limit, this prevents a panel from following the sun all the way to the horizon (ultimately to prevent a self-shading array). Figure 11 shows how the available solar radiation compares, and Figure 12 shows how the electric output compares. Two axis tracking is compared in Figure 13 and Figure 14.
Figure 11: Comparing the incident solar radiation for a single axis tracking system with PVWatts. The location is Dayton, OH.

Figure 12: Comparing the electric output for a single axis tracking system with PVWatts. The location is Dayton, OH.
Figure 13: Comparing the incident solar radiation for a two axis tracking system with PVWatts. The location is Dayton, OH.

Figure 14: Comparing the electric output for a two axis tracking system with PVWatts. The location is Dayton, OH.
PVWatts compiled its system losses into a DC-to-AC derate factor with a default setting of 0.92. In order to best simulate this compiled loss, Solar_PVHFC adjusted its MPPT efficiency to 0.92 (default is 0.95). MPPT efficiency is not the same as DC-to-AC losses, but is believed to be sufficient for matching the losses modeled in each program.

Additional differences in the two programs include the treatment of ground reflectivity. PVWatts version 2 claimed to have read an albedo from the TMY3 file or to assume a default value of 0.2. Upon inspection, the albedo values recorded in TMY3 files seem unreliable. Solar_PVHFC used a ground reflectance of 0.2 for the entire year. Also, PVWatts uses a different method for reducing incident radiation based on the angle of incidence, $\theta$, from Solar_PVHFC.

Despite the differences between PVWatts and Solar_PVHFC, the two programs tend to model available solar radiation and electric output very similarly. The disparity between them is, for the most part, recognized and does not subtract from either programs’ validity.
Chapter 4. Compound Parabolic Concentrator Model

As stated in the objectives of this thesis, a model for CPCs has been added to Solar_PVHFC. This chapter explains the details of this model. A crude cost analysis model is also presented.

4.1. CPC Geometry

The geometry of a CPC is critical to its function and performance. A schematic of a CPC is shown in Figure 15.

Figure 15: A diagram of a compound parabolic concentrator (CPC). Taken from Duffie and Beckman [14].
A CPC is made from two parabolic mirrors that are cut off at their lower ends where they intersect with an absorber plate. For our purposes, this absorber plate is a photovoltaic cell. The focal point of the left parabola in Figure 15 is the intersection of the absorber plate and the right parabola, and vice versa. Error! Reference source not found. contains a list of the parameters and their descriptions.

Table 2: Parameters used to define CPC geometry.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_c$</td>
<td>Acceptance half angle</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Angle between parabola axis and parabola end point</td>
</tr>
<tr>
<td>$a$</td>
<td>Half aperture width through which radiation enters CPC</td>
</tr>
<tr>
<td>$a'$</td>
<td>Half width of absorber plate</td>
</tr>
<tr>
<td>$h$</td>
<td>Height of CPC from the absorber plate to the parabola end point</td>
</tr>
<tr>
<td>$f$</td>
<td>Focal length of each parabola</td>
</tr>
<tr>
<td>$C$</td>
<td>Concentration ratio</td>
</tr>
<tr>
<td>$A_R$</td>
<td>Ratio of reflector area to absorbing aperture</td>
</tr>
<tr>
<td>$n$</td>
<td>Average number of reflections undergone by an entering beam of radiation</td>
</tr>
<tr>
<td>Subscript T</td>
<td>Denotes parameter for a truncated CPC, no subscript denotes parameter for a full CPC</td>
</tr>
</tbody>
</table>

A CPC’s acceptance half angle, $\theta_c$, is also its cutoff angle. All radiation entering at an angle greater than $\theta_c$ will be reflected back out of the CPC through the aperture. Most concentrating solar collectors have a very small cutoff angle. CPCs are an exception and this is one of the qualities that make them appealing. A moderate cutoff angle reduces the need to track the sun accurately and therefore reduces the complexity and cost of the collector system.

The concentration ratio of an ideal CPC is a function of its acceptance angle [14]:

$$ C = \frac{1}{\sin \theta_c}. \quad (22) $$

CPCs are used at low concentrations typically between $1 < C < 12$. An important point to realize is that CPCs have a very large reflecting area; the parabola end points (see Figure 15) extend until they are parallel. It was discovered that the parabolas could be truncated; that is,
the CPC height could be shortened from $h$ to $h_T$. Truncation dramatically reduces the required reflector area without having much effect on the concentration. In fact, it is standard practice to truncate CPCs and denote their truncated dimensions with a subscripted $T$. Truncating a CPC has very little effect on the acceptance angle and is treated this way throughout the model. The following equations are used to completely define a full CPC,

$$f = a'(1 + \sin \theta_c), \quad (23)$$

$$a = \frac{a'}{\sin \theta_c}, \quad (24)$$

$$h = \frac{f \cos \theta_c}{\sin^2 \theta_c}, \quad (25)$$

$$C = \frac{a}{a'} = \frac{1}{\sin \theta_c}, \quad (26)$$

$$A_R = -\frac{f}{a} \left\{ \cos(\varphi/2) \right\}_{\theta_c + \pi/2}, \quad (27)$$

$$n = \max \left\{ \frac{A_R}{2} - \frac{x^2 - \cos^2 \theta_c}{2(1 + \sin \theta_c)}, 1 - \frac{1}{C} \right\}, \quad (28)$$

and

$$x = \left( \frac{1 + \sin \theta_c}{\cos \theta_c} \right) \left( -\sin \theta_c + (1 + \cot^2 \theta_c)^{1/2} \right) \quad (29)$$

The following equations completely define a truncated CPC,

$$a_T = \frac{f \sin(\varphi_T - \theta_c)}{\sin^2(\varphi_T/2)} - a' \quad (30)$$

$$h_T = \frac{f \cos(\varphi_T - \theta_c)}{\sin^2(\varphi_T/2)} \quad (31)$$

$$C_T = \frac{a_T}{a'} \quad (32)$$

$$A_{RT} = -\frac{f}{a_T} \left\{ \cos(\varphi/2) \right\}_{\theta_c + \pi/2} \quad (33)$$

$$n_T = \max \left\{ \frac{A_{RT}}{2} - \frac{x_T^2 - \cos^2 \theta_c}{2(1 + \sin \theta_c)}, 1 - \frac{1}{C_T} \right\} \quad (34)$$

$$x_T = \left( \frac{1 + \sin \theta_c}{\cos \theta_c} \right) \left( -\sin \theta_c + \left( 1 + \frac{h_T}{h} \cot^2 \theta_c \right)^{1/2} \right) \quad (35)$$
The focal length, $f$, is the same for a truncated CPC as a full CPC. Equations $(23) - (26)$ and $(30) - (32)$ come from Duffie and Beckman [14]; equations $(27)$ and $(33)$ come from Welford and Winston [22]; and equations $(28)$, $(29)$, $(34)$, and $(35)$ come from Rabl [23].

When running simulations for a CPC, the user is able to input three variables that define the rest of the CPC geometry:

1. The concentration ratio, $C$, of a full CPC.
2. The absorbing plate width, $2a'$. The user actually inputs the total width which is equal to $2a'$. The program divides this width by 2 to solve for the other parameters.
3. The height ratio, $h_1/h$. The height ratio must be between 0 and 1. Choosing a height ratio of 1 will run the simulation for a full CPC.

These inputs can be seen in the CPC GUI in Figure 16. In addition to defining the CPC geometry, the user can also build the CPC array. CPCs may be thought of as a trough (not to be confused with trough concentrators), of shallow walls and narrow absorbers extending a certain length. The assumption is made that CPCs will be much narrower than a conventional solar panel, so the user is able to specify a number of them running parallel on a single panel. This input is called “Number of troughs per panel” in the GUI. Figure 17 shows how a CPC panel may have multiple CPC troughs. The user is also able to specify the length of the panels.
Figure 16: The CPC GUI in Solar_PVHFC allows the user to specify all necessary parameters to define a CPC.
Another user input is the concentrator reflectivity. This input ranges between 0 and 1. 0 means the parabolic mirrors absorb all radiation that strike them, 1 means they perfectly reflect all incoming radiation. Aluminum, a favored reflector material, typically has a reflectivity ranging between 0.8 and 0.9 depending on the radiation wavelength [24]. A company called Nielsen Enterprises sells Mylar mirror sheeting that they claim has a reflectivity between 0.92 and 0.99 [25]. The default setting for Solar_PVHFC is a conservative reflectivity of 0.8.

The CPC GUI also has an option for the CPC to have a cover. A cover may add to a CPC’s stability and protect its solar cells from weather damage. Solar_PVHFC assumes the covering is a pane of glass and calculates transmission losses accordingly, based on the radiation incident angle. It does not model the cover’s possible influences on the CPC’s heat transfer abilities.

Other changes that come with the CPC GUI from the PV GUI are the different preloaded available panels. As discussed in section 1.3, concentrating solar systems become
increasingly more attractive with increasing PV efficiency. The preloaded panels in the CPC GUI include four panels:

1. Astronergy NMC 250 with a rated efficiency of $\eta = 15.2\%$. This represents typical medium efficiency conventional silicon panels. [26]
2. SunPower E20 327 with a rated efficiency of $\eta = 20.4\%$. This panel represents high efficiency for conventional silicon panels. [27]
3. SunPower X21 335 with a rated efficiency of $\eta = 21.5\%$. This panel is a current leader in high efficiency for conventional silicon panels. [28]
4. Spectrolab UTJ with a rated efficiency of $\eta = 28.3\%$. This ultra triple junction cell is a leader in multi-junction cell efficiency without concentration (multi-junction cells have achieved much greater efficiencies, but require high concentrations of hundreds of suns to be cost competitive, see section 1.3). This cell was also chosen because it is available on the market. [29]

Of course, other modules can be loaded into the program.

### 4.2. Modeling Incident Radiation

The model for calculating the radiation for a CPC is presented by Duffie and Beckman [14], where $S_{\text{CPC}}$ is the radiation passing through the CPC and absorbed by the solar cells (see Figure 18),

$$S_{\text{CPC}} = S_{\text{CPC},b} + S_{\text{CPC},d} + S_{\text{CPC},g} \quad (36)$$

where

$$S_{\text{CPC},b} = I_{b,\text{CPC}} \tau_{cb} \tau_{\text{CPC,b}} \alpha_{b} C \quad (37)$$

$$S_{\text{CPC},d} = I_{d,\text{CPC}} \tau_{cd} \tau_{\text{CPC,d}} \alpha_{d} C \quad (38)$$

and

$$S_{\text{CPC},g} = I_{g,\text{CPC}} \tau_{cg} \tau_{\text{CPC,g}} \alpha_{g} C \quad (39)$$

In these equations $C$ is the concentration ratio of the CPC; $C_T$ is used for truncated CPCs. $I_{b,\text{CPC}}$, $I_{d,\text{CPC}}$, and $I_{g,\text{CPC}}$ are the beam, diffuse, and ground radiation contributions to the CPC and are
treated in equations (41) – (43). \( \tau \) is the transmittance of the beam, diffuse, and ground radiation through the CPC cover. \( \tau \) is treated in equations (45) – (51). \( \tau \) will equal one for a CPC with no cover. \( \tau_{CPC,b} \), \( \tau_{CPC,d} \), and \( \tau_{CPC,g} \) are transmittances that account for specular reflectance and the average number of reflections within the CPC trough. They are usually treated as being the same for each type of radiation and are approximated as

\[
\tau_{CPC} = \rho^n \tag{40}
\]

where \( \rho \) is the concentrator reflectivity (an input parameter) and \( n \) is the average number of reflections (\( n_T \) is used for a truncated CPC). \( \alpha \) is the absorbance of the collector and is generally very close to one for manufactured solar cells. In Solar_PVHFC \( \alpha_b=\alpha_d=\alpha_g=1 \).

\( I_{b,CPC}, I_{d,CPC}, \) and \( I_{g,CPC} \) use the beam \( (I_b) \), diffuse \( (I_d) \), and ground components \( (I_e) \), which is total radiation incident on a horizontal surface i.e. the ground) of solar radiation. \( I_b, I_d, \) and \( I_e \) are the same beam, diffuse, and ground components used to calculate available radiation for a conventional flat panel. \( I_{b,CPC}, I_{d,CPC}, \) and \( I_{g,CPC} \) are given by

\[
I_{b,CPC} = F I_b R_b \tag{41}
\]

\[
I_{d,CPC} = \begin{cases} \frac{I_d}{C} & \text{if } (\beta + \theta_c) < 90^\circ \\ \frac{I_d}{2} \left( \frac{1}{C} + \cos \beta \right) & \text{if } (\beta + \theta_c) > 90^\circ \end{cases} \tag{42}
\]

and

\[
I_{g,CPC} = \begin{cases} 0 & \text{if } (\beta + \theta_c) < 90^\circ \\ \frac{I_e \rho_g}{2} \left( \frac{1}{C} - \cos \beta \right) & \text{if } (\beta + \theta_c) > 90^\circ \end{cases} \tag{43}
\]

The beam contribution, \( I_{b,CPC} \), contains the control function, F. The control function determines when the beam radiation is outside of the acceptance half angle, \( \theta_c \), and is given by Pinazo, Cañada, and Arago [30] as

\[
F = \begin{cases} 1 & \text{if } |\theta_t| \leq \theta_c \\ 0 & \text{if } |\theta_t| > \theta_c \end{cases} \tag{44}
\]

The angle \( \theta_t \) is the transverse angle, the sun’s projection in the CPC’s transverse plane, and is presented in equation (53) and Figure 20. This control function effectively turns the beam component on or off.
The diffuse and ground contributions, $I_{d,CPC}$ and $I_{g,CPC}$, are related by a view factor based on whether or not the CPC can “see” the ground. They also contain the CPC’s concentration ratio (which is $C_T$ for a truncated CPC). The ground component contains $\rho_g$, which is the ground’s average reflectivity, as an input parameter.

Figure 18: Impinging radiation on a CPC. Radiation impinging on the optional CPC cover is $I_t$. Radiation experiences transmission losses through the optional cover and reflective losses from the parabolic mirrors. Radiation impinging on the cover of the solar cells is called $S_{CPC}$. Radiation that is absorbed by the solar cells is called $S_{abs}$. $S_{abs}$ is used to calculate the electricity produced.

For some locations, it might be advantageous to place a cover over the CPC aperture. This helps protect the parabolic mirrors and solar cells, and it may also play a role in the CPC’s heat transfer processes. This thesis does not investigate motives for adding a cover, nor does Solar_PVHFC account for the cover influencing the CPC’s heat transfer abilities. Solar_PVHFC only deals with the cover’s effect on radiation transmittance via the parameter $\tau_c$.

To calculate $\tau_{c,b}$, $\tau_{c,d}$, and $\tau_{c,g}$, the angle of incidence of each type of radiation must be known. Because $\tau_c$ is calculated at the opening aperture of the CPC (before undergoing
reflections), \( \theta \) (the angle of incidence for beam radiation) can be used to find \( \tau_{c,b} \). First, Snell’s law is used to calculate the angle of refraction (also used in equation (6)):

\[
\theta_r = \sin^{-1}\left(\frac{n_1 \sin \theta}{n_2}\right)
\] (45)

The cover is assumed to be glass, having a refractive index of \( n_2 = 1.526 \), interfacing with air having a refractive index of \( n_1 = 1 \). Duffie and Beckman [14] provide the method for calculating the transmittance of unpolarized radiation through a cover. Unpolarized radiation passing between two different mediums is broken into perpendicular and parallel components:

\[
r_{\perp} = \frac{\sin^2(\theta_r - \theta)}{\sin^2(\theta_r + \theta)}
\] (46)

\[
r_{\parallel} = \frac{\tan^2(\theta_r - \theta)}{\tan^2(\theta_r + \theta)}
\] (47)

The radiation transmissivity is a function of both reflective and absorption losses (\( \tau_r \) and \( \tau_a \) respectively):

\[
\tau_r = \frac{1}{2} \left( 1 - \frac{1 - r_{\parallel}}{1 + r_{\parallel}} \right) \left( 1 - \frac{1 - r_{\perp}}{1 + r_{\perp}} \right)
\] (48)

\[
\tau_a = \exp\left( -\frac{KL}{\cos \theta_r} \right)
\] (49)

Recall that \( K \) is the extinction coefficient and \( L \) is the cover thickness (introduced in equation (7)). For the case when \( \theta = 0^\circ \) the incident beam radiation is perfectly normal to the CPC’s aperture and

\[
\tau_r = \frac{1 - (n_1 - n_2)^2}{1 + (n_1 - n_2)^2}.
\] (50)

Finally, the transmittance of the cover (for beam radiation) can be determined by

\[
\tau_c \approx \tau_a \tau_r
\] (51)

It should be noted that this method is for a single cover. Multiple covers would require dealing with multiple reflections between the covers.

The transmittance for diffuse and ground radiation follows the same procedure as the beam radiation, with the only difference being in its angle of incidence. Beam radiation enters the CPC like a ray and is easily “turned on or off” by the control function, \( F \); but a CPC is
always exposed to diffuse radiation. However, only a portion of the diffuse and ground radiation effectively enters the CPC. Because of this, the effective diffuse and ground angles used for a conventional panel (in equations (4) and (5)) are not used. A new effective angle is used. CPCs tend to maximize their effectiveness with impinging beam radiation, and thus the ground and diffuse components are not as important. Perhaps a shortcoming in the model is that the new effective angle of incidence for diffuse radiation on a CPC is assumed to be the same as that for ground radiation; a view factor (dependent on the panel’s tilt) will still determine whether or not the panel sees the ground. At any rate, this is the best that can be done. This new effective angle of incidence for diffuse and ground radiation, $\theta_{c,\text{dg}}$, is presented by Brandemuehl and Beckman [20]:

$$\theta_{e,\text{dg}} = 44.86 - 0.0716\theta_c + 0.00512\theta_c^2 - 0.00002798\theta_c^3.$$  \hfill (52)

Note that $\theta_{c,\text{dg}}$ is only a function of the CPC’s acceptance half angle, $\theta_c$, and its dependence can be seen in Figure 19. Once $\theta_{c,\text{dg}}$ is known, the diffuse and ground radiation can be treated in the same way as beam radiation. That is, cover transmittances $\tau_{c,d}$ and $\tau_{c,g}$ can be found by the same method as $\tau_{c,b}$ (with $\theta_{c,\text{dg}}$ replacing $\theta$).
Figure 19: The effective angle of incidence for diffuse and ground radiation for a CPC, $\theta_{e,dg}$. The x axis is the acceptance half angle, $\theta_c$. Using $\theta_{e,dg}$ allows the diffuse and ground radiation to be treated like beam radiation. Plot taken from Brandemuehl and Beckman [20].

At this point, $S_{\text{CPC}}$ is known (equation (36)), $S_{\text{CPC}}$ being the amount of radiation that impinges on the solar cell at the bottom of the CPC (see Figure 18). The next task is to calculate how much of that radiation is absorbed ($S_{\text{abs}}$). Section 3.3 covers how the radiation absorption was handled for a conventional panel. It is very similar for a CPC, but the equation for $S_{\text{abs}}$

$$S_{\text{abs}} = (\tau \alpha)_n M \left[ S_{\text{CPC,b}} K_{\tau \alpha,b} + S_{\text{CPC,d}} K_{\tau \alpha,dg} + S_{\text{CPC,g}} K_{\tau \alpha,dg} \right]$$

(53)

looks slightly different from equation (16), namely in the use of incident angle modifiers ($K_{\tau \alpha}$). $(\tau \alpha)_n$ is the transmission-absorption coefficient for radiation impinging normal to the aperture surface and was first calculated in equation ((8)), repeated here for convenience,
\[(\tau\alpha)_n = e^{-KL \left[1 - \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2\right]}\].

\(M\) is the air mass modifier calculated by equation (10). \(S_{\text{CPC,b}}\), \(S_{\text{CPC,d}}\), and \(S_{\text{CPC,g}}\) are the beam, diffuse and ground reflected components from equations (37) - (39). The incident angle modifier is defined by Duffie and Beckman [20] as the transmission-absorption coefficient of radiation at the actual incident angle divided by the transmission-absorption coefficient at an incident angle normal to the surface:

\[K_{\tau\alpha} = \frac{\tau\alpha(\theta)}{\tau\alpha(0)} = \frac{\tau\alpha(\theta)}{(\tau\alpha)_n}.\]  

(54)

The equation for \(\tau\alpha\) was given in equation (7) and is repeated here for convenience

\[\tau\alpha(\theta) = e^{-\left(\frac{KL}{\cos \theta_r}\right)} \left[1 - \frac{1}{2} \left(\frac{\sin^2(\theta_r - \theta)}{\sin^2(\theta_r + \theta)} + \frac{\tan^2(\theta_r - \theta)}{\tan^2(\theta_r + \theta)}\right)\right].\]

For lack of a better model, the angle of incidence for diffuse and ground reflected radiation is taken to be the effective angle, \(\theta_{\text{e,dg}}\), put forth in equation (52); its angle of refraction is found by using Snell’s law (equation (45)). Thus \(K_{\tau\alpha,\text{dg}}\) is known and is used for both the diffuse and ground reflected radiation. The case for \(K_{\tau\alpha,\text{b}}\) is not as easy.

The beam radiation’s angle of incidence, \(\theta\), is known at the aperture, but after the radiation has undergone countless reflections in its journey to the absorber plate, this angle is very difficult to assess. In fact, it is necessary to implement ray tracing techniques over the whole spectrum of possible incidence angles (0 to \(\theta_c\)), but the results of the ray tracing will be different for every configuration of a CPC. Despite this dismal dilemma, McIntire and Reed [31] devised a delightful definition of the incident angle modifier for beam radiation as a function of two angles: the longitudinal angle and the transverse angle.

\[K_{\tau\alpha}(\theta_l, \theta_t) \sim K_{\tau\alpha}(\theta, 0)K_{\tau\alpha}(0, \theta_t)\]  

(55)

The longitudinal angle, \(\theta_l\), is a projection of the incident angle, \(\theta\), on to the longitudinal plane of the collector. The transverse angle, \(\theta_t\), is a projection of the incident angle, \(\theta\), on to the transverse plane of the collector. The definition of the longitudinal and transverse planes varies among sources; the most sensible definition coming from McIntire and Reed where the longitudinal plane runs the length of the CPC and the transverse plane runs between the two
Figure 20 shows that $\theta_i$ runs in the direction of $E_c$, and $\theta_t$ runs in the direction of $S_c$.

McIntire and Reed provide a model for calculating $\theta_i$ and $\theta_t$, but the model provided by Pinazo et al. [30] presented charts to compare with for accuracy. The main difficulty in using the work of Pinazo et al. is that they have defined the longitudinal and transverse planes opposite to McIntire and Reed. That said, Pinazo et al.’s convention has been converted to match McIntire and Reed. Also, Pinazo et al. measure $\theta_i$ against $E_c$ instead of $V_c$. For the sake of consistency in how incident angles are measured, this standard has been converted as well so that incident angles ($\theta_i$, $\theta_t$, and $\theta$) are measured relative to the aperture’s normal, $V_c$. Pinazo et al. also have the convention that mornings are represented by a positive hour angle, $\omega$ ($-\omega$ in
the afternoon), and east is represented by a positive azimuth, \( \gamma \) (-\( \gamma \) is west). This is exactly opposite from the conventions used in Solar_PVHFC, so Pinazo et al.’s parameters have been converted to match those used in the rest of the program. That is, \( \omega \) (the hour angle) is negative in morning and positive in afternoon, and \( \gamma \) (the azimuth angle) is negative when east of south and positive when west of south.

In Pinazo et al.’s model \( E_c \) is the vector running the length of the CPC and lies in the surface of the opening aperture. \( V_c \) is the vector normal to the aperture’s surface. \( S_c = E_c \times V_c \). That is, \( S_c \) is the vector running perpendicular to the CPC length and lying in the plane of the surface of the aperture (see Figure 20). The unit vectors \( \hat{E}, \hat{V}, \) and \( \hat{S} \) (no subscript \( c \)) denote the directions east, normal to a horizontal surface, and south respectively.

There are three direction cosines (equations (57)-(59)) that are used in defining the sun’s position (see Table 3 for a refresher in nomenclature),

\[
\mathbf{\hat{S}}_{SOL} = (\cos \sigma \mathbf{\hat{s}}, \cos \alpha \mathbf{\hat{v}}, \cos \pi \mathbf{\hat{e}})
\]

\[
\cos \sigma = -\sin \delta \cos \varphi + \cos \delta \sin \varphi \cos(-\omega)
\]

\[
\cos \alpha = \sin \delta \sin \varphi + \cos \delta \cos \varphi \cos(-\omega)
\]

\[
\cos \pi = \cos \delta \sin(-\omega)
\]

<table>
<thead>
<tr>
<th>Table 3: Nomenclature used to define the position of the sun and collector. Conventions taken from Duffie and Beckman [14].</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Angular Nomenclature</strong></td>
</tr>
<tr>
<td>( \theta )</td>
</tr>
<tr>
<td>( \varphi )</td>
</tr>
<tr>
<td>( \delta )</td>
</tr>
<tr>
<td>( \gamma )</td>
</tr>
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<td>( \beta )</td>
</tr>
<tr>
<td>( \omega )</td>
</tr>
<tr>
<td>( \varepsilon )</td>
</tr>
<tr>
<td>( \theta_z )</td>
</tr>
<tr>
<td>( \gamma_s )</td>
</tr>
</tbody>
</table>

\[
\theta_t = \tan^{-1}\left(\frac{S_c}{V_c}\right) \quad \text{(63)}
\]

and

\[
\theta_l = \tan^{-1}\left(\frac{E_c}{V_c}\right) \quad \text{(64)}
\]

Figure 21: A demonstration of the longitudinal slant angle, \( \varepsilon \). On the left, a CPC is orientated such that \( \varepsilon=0^\circ \); and on the right, a CPC is orientated such that \( \varepsilon=90^\circ \).
Several comparisons were made with Pinazo et al. to verify that the model was working. For ease of comparison, the model was put in EXCEL. The specific case of comparison presented here is between Figure 22 and Figure 23 for Valencia, Spain where \( \varphi=40^\circ \), \( \beta=50^\circ \), \( \varepsilon=10^\circ \), \( \gamma=-15^\circ \), and the date is Dec. 21. It is important to realize that Pinazo et al.’s \( \theta_t \) is Solar_PVHFC’s \( \theta_t \). For this comparison, \( h \) is the altitude angle of the sun (\( h=90^\circ - \theta_z \)). Also, Pinazo et al.’s angle of incidence, \( \theta \), is incorrectly plotted and should be translated lower along the y axis.

Figure 22: A plot taken from Pinazo et al. [31] \( \theta \) is incorrectly plotted here and should be translated lower along the y axis. This plot is meant for comparison with Figure 23. Pinazo et al.’s \( \theta_t \) is equivalent to Solar_PVHFC’s \( \theta_t \). \( h \) is the solar altitude angle.
Figure 23: This is an Excel plot of the model for calculating $\theta_l$ and $\theta_t$ presented by Pinazo et al. It is meant for comparison with Figure 22.

As Figure 22 and Figure 23 show, the calculations for $\theta_l$ and $\theta_t$ are in agreement with Pinazo et al. The model was then coded in Matlab and integrated into Solar_PVHFC. A key difference here is that Pinazo et al. had $\theta_l$ ranging from 0° to 180°, Solar_PVHFC has $\theta_l$ ranging from -90° to +90° (since it is measured relative to the aperture’s normal, $V_c$, instead of the longitudinal vector, $E_c$).

To clarify the nature of $\theta_l$ and $\theta_t$ in Solar_PVHFC, imagine a solar collector whose trough/length/longitudinal plane is aligned east to west and is facing south. As the sun rises in the east, $\theta_l$ and $\theta_t$ will both be positive ($\theta_l$ starting at 90° at sunrise, $\theta_t$ starting at some positive angle depending on the tilt of the collector). At solar noon, $\theta_l$ will be zero. $\theta_t$ will be positive if the sun’s altitude angle is lower than the angle between the collector’s normal and the Earth’s surface, but $\theta_t$ will be negative if the sun’s altitude angle is higher than the angle between the collector’s normal and the Earth’s surface. In the afternoon, $\theta_l$ will become negative (finishing at -90° at sunset). $\theta_t$ will be positive by sunset (though it may never have become negative depending on the collector’s tilt and what altitude angle the sun achieved).
Now that $\theta_l$ and $\theta_t$ are known, the incident angle modifiers $K_{\tau\alpha}(\theta_l,0)$ and $K_{\tau\alpha}(0,\theta_t)$ can be found. Rather than calculating a transmission-absorption product, $K_{\tau\alpha}(\theta_l,0)$ and $K_{\tau\alpha}(0,\theta_t)$ are fit to empirical data presented by King, Boyson, and Kratochvil [32] and shown here in Figure 24.

![Figure 24: The incident angle modifier as a function of the angle of incidence. This data and plot are taken from King et al. [32](#)](image)

Data points were read from King et al.’s plot and the trend for the incident angle modifier as a function of the angle of incidence was remade in EXCEL. A six degree polynomial was fit to the data giving

$$K_{\tau\alpha}(\theta_l,0) = c_1 + c_2 |\theta_l| + c_3 \theta_l^2 + c_4 (|\theta_l|)^3 + c_5 \theta_l^4 + c_6 (|\theta_l|)^5 + c_7 \theta_l^6$$  \hspace{1cm} (65)$$

and

$$K_{\tau\alpha}(0,\theta_t) = c_1 + c_2 |\theta_t| + c_3 \theta_t^2 + c_4 (|\theta_t|)^3 + c_5 \theta_t^4 + c_6 (|\theta_t|)^5 + c_7 \theta_t^6$$  \hspace{1cm} (66)$$

where the coefficients are the same for both $\theta_l$ and $\theta_t$ and are listed in Error! Reference source not found.. Notice that only positive $\theta_l$ and $\theta_t$ are used.
Table 4: A list of coefficients used for calculating $K_{r\alpha}(\theta_l,0)$ and $K_{r\alpha}(0,\theta_t)$ in equations (65) and (66).

<table>
<thead>
<tr>
<th>Coefficients for $K_{r\alpha}$ Polynomials</th>
<th></th>
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<tbody>
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<tr>
<td>$c_2$</td>
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</tr>
<tr>
<td>$c_3$</td>
<td>-1.522713x10^{-4}</td>
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<tr>
<td>$c_4$</td>
<td>9.1129769x10^{-6}</td>
</tr>
<tr>
<td>$c_5$</td>
<td>-2.4763091x10^{-7}</td>
</tr>
<tr>
<td>$c_6$</td>
<td>3.2344057x10^{-9}</td>
</tr>
<tr>
<td>$c_7$</td>
<td>-1.757796x10^{-11}</td>
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</tbody>
</table>

In the final follow through of equation (55), the incident angle modifier components are multiplied together:

$$K_{r\alpha,b} = K_{r\alpha}(\theta_l, \theta_t) = \min[K_{r\alpha}(\theta_l, 0) K_{r\alpha}(0, \theta_t), 1] \quad (67)$$

Notice that the maximum $K_{r\alpha,b}$ value is one because the collector should not have a higher transmission-absorption coefficient than that at normal incidence (recall equation (54)).

With that, everything is known to be able to solve for the radiation absorbed by the solar cell, $S_{abs}$ (from equation (53)). $S_{abs}$ is then used to calculate the electricity produced by the solar cell. For more on how the electrical output of the solar cells are determined once $S_{abs}$ is known the reader should refer to Gustafson’s thesis [13].

The ardent reader may have noticed an inconsistency difference between the model used for calculating radiation for a conventional panel (equation (16)) and that used for a CPC (equation (53)). The model for radiation on a tilted surface for a conventional panel uses five components, three of which are diffuse radiation (the other two being for beam and for ground reflected radiation). This model using three diffuse components is called the anisotropic model and breaks the overall diffuse (isotropic diffuse) into three coupled forms: horizon brightening, circumsolar, and isotropic diffuse, as mentioned in section 3.1. Unfortunately, the model for a CPC only accounts for isotropic diffuse and it is unknown how to handle circumsolar and horizon brightening components as they are intercepted, reflected, and absorbed by a CPC. The model for a CPC simply sets the brightness coefficients, $F_1$ and $F_2$, equal to zero so that
all diffuse radiation is isotropic (uniform over the entire sky dome) (see equations (2) and (3)). With this difference in how diffuse radiation is calculated, it would not be fair to compare a conventional panel using a three part diffuse model to a CPC using a one part diffuse model. So Solar_PVHFC has been updated to allow the user to select a one or three part diffuse model for conventional panels. The anisotropic model is considered more accurate, and the isotropic model tends to underestimate the total impinging radiation (see Figure 25).

![Impinging Radiation, percent increase from isotropic to anisotropic model](image)

**Figure 25:** A plot showing the percent increase in total impinging radiation from an isotropic model to an anisotropic model. It depicts this increase for two different cities, two different times of year, and for a fixed and two axis tracking panel.

The percent increase in impinging radiation from using an isotropic model to using an anisotropic model (see Figure 25) reveals some influencing variables. For instance, Dayton shows a slightly greater increase than Yuma. Similarly, the month of December has a greater percent increase than the month of June. Also, a panel tracking along two axes has a greater increase in impinging radiation than a fixed panel. The root in all these differences is mostly weighted in how the model calculates the circumsolar contribution and the isotropic model’s
inability to handle it. The discrepancy in Dayton’s impinging radiation is greater than that in Yuma because Dayton receives a significantly higher fraction of diffuse to total radiation than Yuma. There is an increase in transitioning from fixed to two axis tracking because tracking panels capitalize on circumsolar radiation and the isotropic model doesn’t handle that. The greatest disparity occurs when changing from June to December. This difference is due to the anisotropic model seeing a small increase in total diffuse radiation from June to December, while the isotropic model sees a moderate decrease in diffuse radiation from June to December. It all amounts to a different handling of the diffuse radiation. That said, using the isotropic model is currently the best option until an anisotropic model can be developed. This is mostly evened out throughout the year as CPCs are compared to conventional panels that use the isotropic model.

4.3. Cost Analysis

A rough cost analysis has been done to help evaluate the effectiveness and feasibility of CPCs compared to conventional panels. The cost analysis only accounts for the cost of the solar cells and the costs of the reflectors. These costs are those that are different between the CPC photovoltaic system and the conventional panels. The cost of framing, axis tracking (if used), other equipment required, operational costs, and maintenance costs are not included. Note that these costs are determined per kWh of electricity produced and thus the economic value from the power produced is included in the analysis by default. A cost input GUI was added to Solar_PVHFC (see Figure 26).

When using Solar_PVHFC to run a simulation for a conventional panel, the user is able to enter the cost of the solar cell per square meter. This is simply the cost of the panel divided by the panel area. The following output tells the user the total cost per panel and the total cost of the array. It also presents the cost per kWh ($/kWh) of electricity produced by the solar array. This is simply the total cost of the array divided by the total electric output for the duration of the simulation. This figure is only useful for inter-comparisons of simulations run for the same time duration: the longer the simulation, the more electricity produced, the better the cost per kWh.
When using Solar_PVHFC to run a simulation for a CPC (truncated or full), the user has two available options as far as cost comparisons are concerned. The user can either enter a cost ratio or use actual cost inputs. The cost ratio is useful when the actual cost isn’t known. A cost ratio is the cost of the reflector material, $C_{\text{ref}}$, divided by the cost of the PV cell, $C_{\text{PV}}$.

$$C_\% = \frac{C_{\text{PV}} A_{\text{PV}} + C_{\text{ref}} A_{\text{ref}}}{C_{\text{PV}} A_{\text{aperture}}} \times 100,$$

(68)

where $A_{\text{PV}}$ is the surface area of the solar cell per panel, $A_{\text{ref}}$ is the surface area of the reflector material per panel, and $A_{\text{aperture}}$ is the total opening aperture area per panel. $C_\%$ is printed in the command prompt upon completion of a simulation reading as “This CPC costs $C_\%$ of standard array of equivalent aperture area and PV price.” So a CPC with reflector material at $20/\text{m}^2$ and a solar cell at $100/\text{m}^2$ will have a cost ratio of 0.2. By assuming the cost of reflector material is $1/\text{m}^2$, the program takes the reciprocal of the cost ratio to get the cost of the solar cell per area, $C_{\text{PV}}$. With this equation the program can calculate how much a CPC costs compared to a conventional panel of equivalent aperture area and PV price. This is a crude comparison.
Figure 26: Solar_PVHFC’s cost input GUI. The user can either use a cost ratio (for a CPC) or enter actual known costs of reflector material and PV cells on a per unit area basis. A conventional panel will only have the “Cost of PV cells” available as an input. Alternatively to entering a cost ratio, the user can opt to input actual reflector and solar cell costs on a per unit area basis. The cost per area of reflector material should actually encompass the cost of the trough structure and any other material costs in excess of a conventional panel operating in the same fashion (i.e. same mounting, same tracking abilities, etc.). A senior design project carried out by Rob Shadix and Alec Blankenship in 2014 [33] investigated the cost of a CPC using Styrofoam cutouts as CPC trough structure and reflective sheeting from Nielson Enterprises [25]. The Styrofoam was found to be approximately $130/m² (that’s per m² of aperture) which is significant. The reflective sheeting was found to be approximately $3/m² (that’s per m² of reflector surface area). Further examination of Nielson Enterprises’ products showed that bulk orders could get reflective sheeting at around $1.55/m². It was decided that Styrofoam would have to be replaced as a structural support due to its price. It is believed that CPC walls could be made from stamped die-cast aluminum sheet metal and that bulk production would dramatically drive down the cost. For this reason, the
goal for reflector/trough cost per m$^2$ was set at $20/m^2$ (that’s per m$^2$ of reflector surface area) and this price was used for all the simulations conducted in this thesis.

The four panels that were used in this thesis are mentioned in section 4.1. The cost per area of solar panels is not typically advertised, so the Cost Inputs GUI in Solar_PVHFC recommends a price based on the solar cell efficiency. The price for the Astronergy ($\eta=15.2\%$) panel is $219 [34]$, putting it at $0.84/W or $133/m^2$. However, silicon panel costs have been on the decline [35] and in order to better represent the cost of panels operating around 15% efficiency, the price of $0.76/W was chosen. Working backwards, this puts a 15% efficient panel at $116/m^2 and this is the recommended value for this type of panel.

The price for SunPower panels E20 and X21 ($\eta=20.4$ and $\eta=21.5$ respectively) were provided via correspondence with Steve Ladelfa, an employee of Yellow Lite. The E20 module costs $689.97 per panel, putting it at $423/m^2. The X21 module weighs in at $779.70 per panel, putting it at $478/m^2. SunPower’s panels are a better representation of high efficiency silicon technology (as opposed to Astronergy representing moderate efficiency), and so, these prices were not altered.

The Spectrolab UTJ panels represent top of the line multi-junction, nonconcentrating cells and are designed for special applications (i.e. satellites). With that in mind, Spectrolab reports its UTJ panel starting at $250/W [36]. This puts the UTJ panel at $93,669/m^2.
Once the actual costs are entered, Solar_PVHFC calculates several cost figures on a per panel basis and also for the entire array. It presents reflector costs, solar cell costs (PV material), total costs, costs if the CPC was not truncated, and the cost of a conventional panel that has the same aperture area and the same PV price. A representative sample of these results are shown in Figure 27.

As mentioned, the cost per kWh is not very good for comparisons. A standardized practice that is used throughout the energy industry is the comparison of the levelized cost of energy, LCE. If Solar_PVHFC is running the cost input GUI and the simulation is at least a year long (365 days), then it will calculate a simple LCE. The LCE accounts for the time value of money and is defined as

\[
LCE = \frac{\text{total annual costs}}{\text{total annual output}}. \quad (69)
\]

The total annual cost is the total capital costs of the solar array, \(P\), spread over the lifetime of the panels and adjusted over that lifetime by an interest rate. Typical lifetimes of solar panels (maintaining reasonable efficiency) is taken to be \(N=20\) years, and the interest rate of money is assumed to be \(i=5\%\). Thus the total annual costs can be represented by the annuity equation

\[
\text{Annuity} = P \frac{i(i+1)^N}{(1+i)^N - 1}. \quad (70)
\]

The total annual output is the total electrical energy produced by the solar array for a year. It does not matter when the year is started, as long as the simulation is run for at least one year. If the simulation is run for several years, the LCE is still calculated based on that first year’s output. This is because the same data is read into the program (from a TMY3 file) and each year’s solar output is identical to the next. The LCE is presented in $/kWh. To see how it appears in the command prompt see Figure 28.

![Figure 28: Solar_PVHFC output displaying the levelized cost of energy (LCE) in the last line. The LCE is only calculated for simulations that are run for at least one year.](image)
The LCE for most simulations is less than 1 $/kWh, and sometimes less than 0.1 $/kWh. Analyst company, Lazard, analyzed the levelized cost of energy (2014) for various renewable energy technologies and compared them with conventional energy technologies [37]. Their LCE assessment of the capital investment for solar PV is 0.168-0.248 $/kWh for residential applications and 0.065-0.079 $/kWh for utility scale applications. The LCE output for Solar_PVHFC often falls within this range which is good verification that Solar_PVHFC’s LCE model is reasonable. However, LCE outputs are best used for comparing between CPCs with varying parameters and may prove inaccurate when compared to professional reports like Lazard’s.
Chapter 5. Simulation Results

Hundreds of simulations were run with Solar_PVHFC in order to investigate the effects of various configurations on performance. Primary variables were the concentration ratio, truncation, absorbing aperture width, solar module used (cell efficiency), and tracking orientations. Secondary variables that were examined included mirror reflectivity, use of a cover, and cooling. The term “conventional” is used interchangeably with “standard”, both refer to a non-concentrating photovoltaic panel.

5.1. Organization of Simulations

All simulations were carried out for Yuma, AZ for a time period of 1 year, 365 days. Yuma was selected because it is known for having an excellent solar resource. The success of a solar technology must first prove effective in a region where solar technology is already feasible, as opposed to a region like Dayton, OH where grid parity is still a future event. Simulations were run for one full year in order to account for seasonal advantages and disadvantages of using concentrated solar power. TMY3 data is used to provide solar radiation on a horizontal surface for a typical year and running a simulation for longer than a year would just recycle the data and provide no new results.

Table 5 provides a brief summary of how selections were made for setting up simulations. A base house in Yuma was selected as the demand, but this selection was actually irrelevant since this study did not examine how well a demand could be met.

CPCs were compared to conventional panels that used the same orientation and solar cells. Quantities such as total incident radiation, total electric output, and levelized cost of energy were calculated on a per unit area basis so it was not important how large an array or panel was. Never-the-less, simulations were run with arrays of ten panels in parallel and ten panels in series so that a sufficient amount of electricity was produced to be visible on Solar_PVHFC’s output plots.
Table 5: A flow chart to help show how simulations were run using Solar_PVHFC.

<table>
<thead>
<tr>
<th>Flow chart for setting up simulations</th>
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<tr>
<td>GUI</td>
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<td>Panel Type and consequent PV GUI</td>
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Panel orientation included fixed, single axis, and dual (two) axis tracking. A fixed panel had an azimuth angle of $\gamma=0^\circ$ (due south) and a tilt of $\beta=32.667^\circ$ (Yuma’s latitude). There were three varieties of single axis tracking: vertical, east-west (E-W), and north-south (N-S). Vertical axis tracking rotates a panel on a vertical axis; the panel has a fixed tilt of $\beta=32.667^\circ$. E-W tracking has an axis of rotation that runs east to west so that it tracks the sun’s altitude; its axis has $\gamma=90^\circ$ and $\beta=0^\circ$. N-S tracking has an axis of rotation that is aligned north to south; its axis has $\gamma=0^\circ$ and $\beta=32.667^\circ$. Two axis tracking follows the sun for the entire day. None of the tracking options utilized rotational limits.

Most CPC simulations used a longitudinal slant angle of zero ($\varepsilon=0^\circ$). This means the CPCs’ troughs were orientated in a horizontal manner. Additional simulations were run for vertical axis trackers and north-south axis trackers using a CPC with $\varepsilon=90^\circ$ whose trough length points from the ground to the sky. Refer back to Figure 21. For results, assume $\varepsilon=0^\circ$ unless otherwise described as having $\varepsilon=90^\circ$.

CPCs are low concentrating systems. This thesis investigated concentration ratios of 2, 5, and 10; these concentration ratios are for full CPCs, not truncated CPCs. Truncating a CPC will decrease the concentration ratio by a small amount; the more heavily truncated, the more the concentration ratio is decreased.

The truncation ratios investigated were 0.25, 0.5, 0.75, and 1. Truncation is a height ratio $H$, truncated height divided by the height of a full CPC; so a full CPC has no truncation or a truncation ratio of $H=1$, while $H=0.25$ is described as significant truncation, and $H=0.75$ is low truncation.

The absorbing aperture was selected in a way that might keep the CPC total aperture similar to that of a conventional panel. The aperture varied with what concentration ratio was being used. For $C=2$, the absorber width was 0.1657 m, for $C=5$, the width was 0.06627 m, and for $C=10$, the width was 0.03313 m.

Most simulations were run with a low estimate for the concentrator reflectivity of $\rho=0.8$. Additional simulations for two axis tracking were run using $\rho=0.92$ and $\rho=1$. 0.92 was chosen as the lower limit of what was available from Nielson Enterprises [25], and 1 was used...
as the best possible reflectivity. Most simulations also incorporated a cover which resulted in transmission losses. Some additional simulations were run using two axis tracking that did not use a cover. Two axis tracking will optimize solar energy for any and every system, so these changes made to $\rho$ and the cover were done for two axis tracking to see how much “the best” could be improved.

The cost inputs were all run as actual inputs, not ratios. The justification for selected prices is covered in section 4.3.

As mentioned in section 1.2, the motivation for this project was to see if a CPC using high efficiency solar cells could be made cost competitive with a conventional panel using low efficiency solar cells. A CPC can accomplish this because PV area is reduced at the expense of reflector area being increased, and reflector material is cheaper than PV material. Figure 29 plots how the PV area varies with the reflector area. As the concentration ratio increases from 2 to 5 to 10, the reflector area increases and the PV area decreases. As the truncation varies from highly truncated 0.25 to no truncation 1 (a full CPC), the reflector area increases and the PV area decreases. The areas in Figure 29 are presented as fractions of the total aperture area.
Figure 29: Trends for how reflector area and PV area change for different concentration ratios and degrees of truncation. Areas are presented as fractions of the total aperture area. Solid lines correspond to reflector area on the left axis. Dashed lines correspond to PV area on the right axis.

The major category of division for simulations is the method of tracking. This is because this is a controlled variable within a set of CPC configurations as they are compared to a conventional panel. Also, it would not be fair to compare the levelized cost of energy (LCE) between tracking methods since the cost of tracking mechanisms is not included in the economic study. Within each tracking method, the four types of solar cell modules are presented. For each solar cell module, the concentration ratio and truncation are varied. The levelized cost of energy for a conventional (or standard) panel is subtracted from the levelized cost of energy for a CPC. Therefore, a CPC only breaks even relative to a conventional panel at ΔLCE=0 and must be plotted as negative if it becomes cheaper than a conventional panel. The electric output per area is also plotted and presented here.
5.2. Fixed Orientation

For fixed panels, the best performance for electric output per area came from the Spectrolab UTJ standard panel. The best CPC performance was the Spectrolab panel with a concentration ratio of $C=2$ and truncation of $H=0.25$. The CPC using the Spectrolab panel also showed the greatest gain in LCE over its equivalent standard panel. These results are shown in Figure 30 and Figure 31.

![Spectrolab UTJ Panel](image)

**Figure 30:** Spectrolab UTJ panel, rated at $\eta=28.3\%$. Electric output per area plotted for CPC and conventional panel for various concentration ratios and degrees of truncation. Orientation is fixed tilt, $\beta=32.667^\circ$, $\gamma=0^\circ$; mirror reflectivity $\rho=0.8$. 
Figure 31: Spectrolab UTJ panel, rated at $\eta=28.3\%$. Difference in CPC and standard levelized cost of energy (LCE) for various concentration ratios and degrees of truncation. Orientation is fixed tilt $\beta=32.667^\circ$, $\gamma=0^\circ$; mirror reflectivity $\rho=0.8$.

Despite the fact that the CPC Spectrolab panel showed the most gain against its conventional equivalent, the fact of the matter remains that for most applications a Spectrolab panel is still far too expensive relative to an Astronergy or SunPower panel. The Spectrolab panel will always show significant gain against its conventional equivalent and this reinforces the advantage of using concentrating systems with high efficiency, costly solar cells. However, this also demands an examination of the more affordable panels. Figure 32 plots the lowest LCE for each CPC module type using a fixed orientation and compares it to the LCE for a conventional panel of the same module. The plot is divided into a top and bottom portion. In the bottom, the LCE ranges from 0 to 0.16 $$/kWh and shows competitiveness between the Astronergy module and the two SunPower modules. The Spectrolab module’s LCE is cut off. The top portion of the plot shows the Spectrolab LCE in full scale, which eclipses the other three modules.
Figure 32: For each module type, the best configuration of concentration ratio and truncation (yielding the lowest LCE for that module) is compared to the module’s conventional panel. In the bottom plot, the Spectrolab panel is cut off. In the top plot, the Spectrolab panel’s LCE is shown in full and completely dwarfs the other three modules in cost.

Something to notice in Figure 32 is that the only module whose conventional panel had a lower LCE than its best CPC was the Astronergy module. This is because the cost of the
Astronergy solar cells are about a quarter of the cost of the SunPower solar cells. Using CPCs with the SunPower solar cells reduced the LCE for certain configurations, namely a concentration ratio of 2 and truncation of 0.5. However, the best CPC LCE (SunPowerE with LCE=0.083 $/kWh) is still nearly three times higher than the lower efficiency conventional Astronergy panel (with LCE=0.031 $/kWh).

It is also worth seeing the electric output per area for the various CPC configurations. All charts of electric output per area are similar to Figure 30; CPCs always produce less power than conventional panels, output decreases as concentration increases, and output decreases as truncation decreases (the greatest decrease in electric output per area being for a full CPC). This is true for every tracking orientation. There are a number of reasons for this behavior. The most significant cause for this drop in power relative to the conventional panels is concentrators do not perform well when they are not pointed at the sun. Because these are fixed CPCs the performance degrades substantially for those times the sun is not within the cutoff angle of the CPC. A second reason is higher concentration ratios cause higher cell temperatures, resulting in lower cell efficiencies. Thirdly, less truncation produces more reflector surface area which results in greater reflection losses as radiation passes from the top to the bottom of the CPC. With this in mind, a conventional panel will always out perform a CPC in electric output per area (assuming they use the same solar cells), and a CPC with a concentration ratio of 2 and truncation of 0.25 will always be the best performing CPC of the configurations that have been investigated here. Yet the greatest electric output per area is not always an indicator of the lowest LCE. Figure 33 shows the electric outputs per area that correspond to those best configured CPCs presented in Figure 32; best configured meaning those configurations resulting in the lowest LCE of their module type. It can be seen how the electric output per area increases noticeably with cell efficiency.

Plots of the electric output per area and LCE for all the simulated configurations of a CPC for every module type using a fixed orientation can be found in the Appendix, Section A.

These results do not encourage the use of a CPC with a fixed orientation. If high efficiency cells must be used, then a CPC can reduce the LCE compared to a conventional panel; but a cheaper, less efficient conventional panel will yield a lower LCE.
5.3. Single Vertical Axis Tracking

5.3.1. Single Vertical Axis Tracking, $\varepsilon=0^\circ$

For vertical axis tracking where $\varepsilon=0^\circ$, the panel with the highest electric output per area is of course the most efficient panel, the conventional Spectrolab UTJ panel, at 665 kWh/m². This is an increase from the fixed conventional Spectrolab panel at 535 kWh/m²; the effects of tracking can dramatically increase a panel’s electric output. The highest electric output per area for a CPC likewise belongs to the Spectrolab module at a concentration ratio of 2 and truncation of 0.25, producing 408 kWh/m² over the course of a year (an increase from its fixed orientation counterpart at 373 kWh/m²). As can be seen, in the transition from fixed to vertical axis tracking a conventional panel saw a greater percent increase (24.3%) than the reported CPC (9.4%) in electric output per area.
The lowest LCE for CPCs for each module type is depicted in Figure 34. The electric output per area corresponding to those CPCs with the lowest LCE is shown in Figure 35.

Figure 34: The LCE is presented for vertical axis tracking systems with $\epsilon=0^\circ$. Conventional panels are compared to the CPCs of each module type that had the lowest LCEs. Notice the top and bottom portions forming two plots; the bottom plot cuts off the Spectrolab results which are shown in full in the top portion.
From Figure 34 it can be seen that the Spectrolab module (conventional and CPC) completely dwarf the other three modules in terms of LCE. Also notice that the Spectrolab module was the only CPC to have a lower LCE than its conventional equivalent. That is, it is not cost effective (based on LCE) to use a CPC with vertical axis tracking and $\varepsilon=0^\circ$ for Astronergy or SunPower modules; and the Spectrolab CPC is not cost effective because its LCE is more than a hundred times greater than a lower efficiency conventional panel (i.e. Astronergy). These results imply that a conventional panel capitalizes on vertical axis tracking more so than a CPC, making CPCs a poor choice for this tracking method. The poorer performance of CPCs is likely rooted in the acceptance angle of the CPC geometry. It is possible that adjusting the fixed tilt of the axis might improve this orientation, but as it stands it should be said that vertical axis tracking with $\varepsilon=0^\circ$ is not beneficial for use with CPCs. The longitudinal slant angle, $\varepsilon$, was rotated by $90^\circ$ and is presented in the next subsection.

Plots of the electric output per area and LCE for all the simulated configurations of a CPC for every module type using vertical axis tracking can be found in the Appendix, Section B.
5.3.2. Single Vertical Axis Tracking, $\varepsilon=90^\circ$

Rotating the CPC to $\varepsilon=90^\circ$ so that it was in more of a vertical direction improved vertical axis tracking. In fact, the Spectrolab UTJ CPC (C=2, H=0.25) increased electric output per area by 26.7\% (to 517 kWh/m$^2$) from the vertical axis tracking $\varepsilon=0^\circ$ orientation, and increased 38.6\% from a fixed orientation equivalent CPC. It proved especially effective for the Spectrolab CPC LCE as can be seen in Figure 36 where the CPCs with lowest LCEs for each module type are compared to their conventional equivalents. It can also be seen that the Astonergy CPC (C=2, H=0.25) is nearly equivalent to its conventional equivalent in terms of LCE, and both SunPower modules show significantly reduced LCEs compared to their conventional equivalents. The SunPower E20 CPC’s (C=2, H=0.5) LCE is still an 82\% increase from the conventional Astonergy panel.
Figure 36: The LCE is presented for vertical axis tracking systems with $\varepsilon=90^\circ$. Conventional panels are compared to the CPCs of each module type that had the lowest LCEs. Notice the top and bottom portions forming two plots; the bottom plot cuts off the Spectrolab results which are shown in full in the top portion.

It should also be noticed that most of the best configured (lowest LCE) CPCs changed when rotating from $\varepsilon=0^\circ$ to $\varepsilon=90^\circ$. This is because an orientation of $\varepsilon=0^\circ$ caused a significant
amount of radiation to be rejected from CPCs with higher concentration ratios (smaller acceptance angles), but an orientation of $\varepsilon=90^\circ$ made higher concentration ratios more effective. Being able to make use of the higher concentration ratios is what drove the Spectrolab CPC’s LCE so low. In fact, it became so much lower that its best configuration produced 26% less electricity per area than the best configured vertical axis tracking Spectrolab CPC with $\varepsilon=0^\circ$. This can be seen by comparing Figure 35 with Figure 37, which show the electric output for those best configured CPCs (lowest LCE).

![Electric output per m² for conventional panels vs CPCs with lowest LCE for vertical axis tracking, $\varepsilon=90^\circ$](image)

Figure 37: The electric output per area for vertical axis tracking, $\varepsilon=90^\circ$. Conventional panels are compared to the CPCs with the lowest LCE (as shown in Figure 36).

Single vertical axis tracking utilizing an orientation of $\varepsilon=90^\circ$ proves an effective option for CPCs competing with conventional panels and is worth more detailed investigations.

### 5.4. Single East-West Axis Tracking

For an east-west (E-W) axis of rotation, the conventional Spectrolab panel produced 557 kWh/m² over the course of a year and the Spectrolab CPC ($C=2$, $H=0.25$) produced 424
kWh/m². The conventional Spectrolab panel saw a 4.1% increase in electric output per area from the fixed orientation, but the Spectrolab CPC saw a 13.9% increase from the fixed orientation. This demonstrates that a CPC can capitalize on E-W tracking more than a conventional panel. The reason for this likely lies in the horizontal orientation of the CPC troughs. The E-W Spectrolab CPC (C=2, H=0.25) produced 4.0% more electric per area from the Spectrolab CPC using vertical axis tracking and ε=0°, but produced 18% less than the CPC using vertical axis tracking and ε=90°.

As for the LCE, it is interesting that not a single CPC configuration for the Astronergy module yielded a lower LCE than the conventional Astronergy panel. However, nearly every configuration for the SunPower CPC modules and every configuration for the Spectrolab CPC modules yielded lower LCEs than their respective conventional panels. The ΔLCE for the SunPower X21 module and Spectrolab UTJ module is shown in Figure 38 and Figure 39 respectively.

![Graph showing SunpowerX (ΔLCE)](image)

**Figure 38:** SunPower X21 panel, rated at η=21.5%. ΔLCE is plotted for CPC and conventional panel for various concentration ratios and degrees of truncation. Orientation: Single E-W axis of rotation, axis orientation: β=0°, γ=90°; mirror reflectivity ρ=0.8.
Figure 39: Spectrolab UTJ panel, rated at $\eta=28.3\%$. $\Delta LCE$ is plotted for CPC and conventional panel for various concentration ratios and degrees of truncation. Orientation: Single E-W axis of rotation, axis orientation: $\beta=0^\circ$, $\gamma=90^\circ$; mirror reflectivity $\rho=0.8$.

When Figure 39 is compared to Figure 31 (the Spectrolab $\Delta LCE$ for a fixed orientation), it is observed that a concentration ratio of 10 was the least cost effective for a fixed orientation, but the most cost effective for an E-W axis tracking orientation. This is because the acceptance angle, $\theta_c$, is a function of the concentration ratio: the higher the concentration ratio, the smaller the acceptance angle. A concentration ratio of 10 has a small acceptance angle ($\theta_c=5.7^\circ$) compared to a concentration ratio of 2 ($\theta_c=30^\circ$). Tracking along an east-west axis allowed the CPC to capture far more radiation that is reflected out from a fixed orientation CPC.

Despite CPCs’ great success brought on by E-W tracking, CPCs are still not yielding an LCE lower than the lowest priced conventional panel (the Astronergy module) as seen in Figure 40. However, the difference in LCE between the Astronergy conventional panel and CPC is only 0.002 $$/kWh, and the LCE for the SunPower E20 CPC module (C=5, H=0.25) is only about twice that of the conventional Astronergy panel; not to mention the significant gain by the Spectrolab CPC (C=10, H=0.75) on its conventional equivalent.
Figure 40: The LCE is presented for an E-W axis tracking system. Conventional panels are compared to the CPCs of each module type that had the lowest LCE. Notice the top and bottom portions forming two plots; the bottom plot cuts off the Spectrolab results which are shown in full in the top portion.
Figure 40 shows that with a slight reduction in reflector cost or reduction in moderate cell efficiency costs (the SunPower modules), CPCs could overtake lower efficiency conventional panels.

The electric outputs per area for the best configured CPCs (according to their lowest LCE in Figure 40) are presented in Figure 41. It is interesting that among the CPCs the Astronergy module had the highest electric output per area and the Spectrolab module had the lowest. This is due to the Astronergy CPC having the lowest concentration ratio (C=2) and the Spectrolab CPC having the highest (C=10) (among those CPCs with the lowest LCEs). This demonstrates the point that higher concentration ratios lead to higher cell temperatures and therefore lower cell efficiencies. This is an important point and investigated in more detail later on in Section 5.6.2.

Plots of the electric output per area and LCE for all the simulated configurations of a CPC for every module type using E-W axis tracking can be found in the Appendix, Section D.

![Electric output per m² for conventional panels vs CPCs with lowest LCE for E-W axis tracking](image)

Figure 41: The electric output per area for E-W axis tracking. Conventional panels are compared to the CPCs with the lowest LCE (as shown in Figure 40).
5.5. Single North-South Axis Tracking

5.5.1. Single North-South Axis Tracking, $\varepsilon=0^\circ$

Using north-south axis tracking with $\varepsilon=0^\circ$ increased the electric output for conventional panels by about 30% from their fixed orientation counterparts, and N-S axis tracking CPCs with $C=2$ and $H=0.25$ increased by about 46% from their fixed orientation counterparts. The conventional Spectrolab panel utilizing N-S axis tracking produced 695.0 kWh/m$^2$ and the Spectrolab CPC ($C=2$, $H=0.25$) produced 545.2 kWh/m$^2$ over the course of a year. The reason for this tremendous increase in electric output from a fixed panel is that the sun’s azimuth angle changes more dramatically than the sun’s altitude angle over the course of a day; an E-W axis tracks the sun’s altitude angle, but a N-S axis is able to track the sun’s azimutal angle.

The CPCs with the lowest LCEs for each module using N-S axis tracking are depicted in Figure 43 and compared to conventional panels. The difference in the LCE for a conventional Astronergy panel and a CPC Astronergy panel ($C=2$, $H=0.25$) is practically negligible at $\Delta$LCE=0.0009 $$/kWh, as shown in Figure 42.
Figure 42: The difference in conventional and CPC Astronergy LCEs for N-S axis tracking, $\varepsilon=0^\circ$. A CPC with a concentration ratio of $C=2$ has a nearly identical LCE to its equivalent conventional panel.
Figure 43: The LCE is presented for a N-S axis tracking system, $\varepsilon=0^\circ$. Conventional panels are compared to the CPCs of each module type that had the lowest LCEs. Notice the top and bottom portions forming two plots; the bottom plot cuts off the Spectrolab results which are shown in full in the top portion.

Even though N-S axis tracking systems saw a tremendous gain in electric output per area, the gain in LCE was not as great. In fact, the LCEs for CPCs using N-S axis tracking are
very similar to those for E-W axis tracking. The N-S tracking Astronergy modules yielded lower LCEs than the E-W tracking Astronergy modules, but the opposite was true for the SunPower modules and the Spectrolab module (this is not referring to every configuration of CPC, only those with the lowest LCE as shown in Figure 43). On a promising note, the Astronergy CPC’s LCE is so similar to its conventional equivalent that a small reduction in reflector cost would put it ahead of the conventional panel.

The electric output per area for each module’s conventional panel and best configured CPC (the lowest LCE) is plotted in Figure 44. It is curious that the most efficient panel, Spectrolab UTJ, should have the lowest electric output per area. This is because its CPC configuration with the lowest LCE uses a concentration ratio of C=5 and truncation of H=0.75; the higher concentration ratio leads to higher cell temperatures and lower efficiencies, and the lesser truncation leads to more reflective losses off the parabolic mirrors. In fact, the overall efficiency (percentage of impinging radiation converted into electrical output) for the Spectrolab module fell from 18.1% to 7.4% from C=2 to C=5 for a truncation of H=0.75.

Plots of the electric output per area and LCE for all the simulated configurations of a CPC for every module type using N-S axis tracking and $\varepsilon=0^\circ$ can be found in the Appendix, Section E.

The longitudinal slant angle, $\varepsilon$, was rotated by 90° and is investigated in the next subsection.
5.5.2. Single North-South Axis Tracking, $\varepsilon=90^\circ$

The longitudinal slant angle, $\varepsilon$, was rotated by $90^\circ$ for single north-south axis trackers. The change resulted in very little difference from the $\varepsilon=0^\circ$ orientation. Concentration ratios of 2 saw negligible variations. Concentration ratios of 5 saw very little variations. Concentration ratios of 10 benefited from this rotation and saw a decrease in LCE. In particular, the Spectrolab CPC (C=5, H=0.75) saw its LCE decrease by 20.6% from the $\varepsilon=0^\circ$ orientation to 5.96 $$/kWh, and its electric output increase by 26% to 258.7 kWh/m². This is shown in Figure 45, which depicts the best configured CPCs (lowest LCE) for each module type and compares them to their conventional equivalents. Figure 45 is identical to Figure 43 with the exception of the Spectrolab CPC.

Figure 46 shows the electric output per area for those best configured CPCs depicted in Figure 45. It is identical to Figure 44 with the exception of the Spectrolab CPC.
Figure 45: The LCE is presented for a N-S axis tracking system, $\varepsilon=90^\circ$. Conventional panels are compared to the CPCs of each module type that had the lowest LCEs. Notice the top and bottom portions forming two plots; the bottom plot cuts off the Spectrolab results which are shown in full in the top portion.
Rotating $\varepsilon$ by 90° had very little effect for this tracking method except for the higher concentration ratios. The reason that higher concentration ratios were most affected is because a CPC with $\varepsilon=0°$ and C=10 has an acceptance angle around 5.7°, and a N-S axis tracker axis is tilted at 32.2° allowing the CPC to see to an altitude of around 40°. But the solar altitude in Yuma, AZ hovers around 80° in the summer, so the beam radiation is getting cut out in the summer for the middle of the day; a significant loss. The rotation to $\varepsilon=90°$ prevents this particular loss since the cutoff angle is based in an azimuthal plane and not in an altitude plane. This also explains why vertical axis trackers were able to improve when $\varepsilon$ was rotated 90°.

### 5.6. Two Axis Tracking

#### 5.6.1. Two Axis Tracking, $\rho=0.8$

As expected, modules using two axis tracking outperformed all other tracking methods. This section examines panels that do not use temperature control and CPCs that use a mirror reflectivity of $\rho=0.8$ so that the results can be compared to the other tracking methods. In terms
of electric output per unit area conventional panels increased by about 34% from a fixed orientation, and CPCs (with C=2 and H=0.25) increased by about 51% from a fixed orientation. A true comparison between the cost effectiveness of each tracking method would need to account for the cost of tracking mechanisms, so the comparison is most valid within a tracking method.

Two axis tracking aids both conventional and CPC panels, but CPCs are able to extract more benefit because they really capitalize on beam radiation which is what a two axis system tracks. The LCE for conventional panels is plotted in Figure 48 along with the CPC configured to have the lowest LCE for each module type. The Astronergy CPC (C=2, H=0.25) is very close to beating its conventional counterpart, as was the case with E-W and N-S tracking. A small reduction in reflector costs could make it more competitive. The SunPower modules (E20 and X21) each significantly outperform their conventional counterparts in terms of LCE, but their LCE is still nearly twice that of the lower efficiency conventional panel (Astronergy). A decrease in SunPower solar cell costs could make these CPCs cost competitive with the conventional Astronergy panel, but Figure 47 shows that this PV cost reduction would have to be significant. A SunPower E20 CPC’s (C=5, H=0.25) LCE would match the conventional Astronergy panel’s LCE (both using two axis tracking) at a PV cost around $165/m². This is a 60% decrease from the SunPower E20’s current PV cost of $423/m² (a PV cost of $116/m² was used for the Astronergy module). This is not an ultimatum though, because a lower PV cost might change which SunPower E20 CPC configuration had the lowest LCE and therefore could influence the lowest required PV cost to match the conventional Astronergy panel. This should be addressed in future investigations.
Figure 47: The LCE for the SunPower E20 CPC (C=5, H=0.25) panel is plotted as the cost of PV material ($/m^2) is decreased along the x-axis. This is compared to the LCE of a conventional Astronergy module using a constant PV cost of $116/m^2. The SunPower E20 CPC achieves the LCE of a conventional Astronergy panel (both using two axis tracking) at a SunPower E20 module cost of ~$165/m^2 for PV material. This is a 60% reduction from the current SunPower E20 module cost at $423/m^2. This is specific to CPCs using a mirror reflectivity of $p=0.8$.

The highest efficiency Spectrolab CPC (C=10, H=0.75) panel has a dramatically reduced LCE compared to its conventional counterpart, but it is still about a hundred times higher than the conventional Astronergy panel. Nonetheless, looking at the Spectrolab CPC’s LCE implies that it would be a crushing loss not to use concentrating techniques for this high efficiency panel.

The electric output per area corresponding to these CPCs with the lowest LCE is plotted in Figure 49. Once again it can be seen that the highest efficiency panel, Spectrolab UTJ, produces the lowest electric output per area due to its higher concentration ratio (C=10). This configuration still achieves the lowest LCE of the Spectrolab CPCs, because the Spectrolab module price is simply so high that it usually requires the minimum amount of PV area (thus the higher concentration ratio) to give it the lowest LCE.
Plots of the electric output per area and LCE for all the simulated configurations of a CPC for every module type using two axis tracking and $\rho=0.8$ can be found in the Appendix, Section G.

Figure 48: The LCE is presented for a two axis tracking system. Conventional panels are compared to the CPCs of each module type that had the lowest LCEs. Notice the
top and bottom portions forming two plots; the bottom plot cuts off the Spectrolab results which are shown in full in the top portion.

![Electric output per m² for conventional panels vs CPCs with lowest LCE for 2 axis tracking, ρ=0.8](image)

Figure 49: The electric output per area for two axis tracking. Conventional panels are compared to the CPCs with the lowest LCE (as shown in Figure 48). This is specific to CPCs using a mirror reflectivity of ρ=0.8.

### 5.6.2. Two Axis Tracking: Variations in Mirror Reflectivity

After several simulations were completed, it was decided that a mirror reflectivity of ρ=0.8 may have been too modest, and that a greater reflectivity might have been used without noticeably increasing the CPC reflector costs. As discussed in the beginning of this chapter (section 5.1) reflectivities of ρ=0.92 and ρ=1 are studied. In addition to changing the reflectivity, CPCs were assumed to have no transparent cover so as to eliminate transmission losses through the cover glass. This was only done for two axis tracking, and it should be noted that the increase in mirror reflectivity has no effect on conventional panels.
For a mirror reflectivity of \( \rho = 0.92 \), the electric output of a CPC with \( C = 2 \) and \( H = 0.25 \) increased by about 14\% from a two axis tracking CPC with a reflectivity of \( \rho = 0.8 \). A reflectivity of \( \rho = 1 \) saw a corresponding increase of about 17\% (from \( \rho = 0.8 \)). The CPCs with the lowest LCEs for each module are plotted in Figure 50 for \( \rho = 0.92 \) and Figure 51 for \( \rho = 1 \).

Figure 50: The LCE is presented for a two axis tracking system with mirror reflectivity \( \rho = 0.92 \). Conventional panels are compared to the CPCs of each module type that had
the lowest LCEs. Notice the top and bottom portions forming two plots; the bottom plot cuts off the Spectrolab results which are shown in full in the top portion.

Figure 51: The LCE is presented for a two axis tracking system with mirror reflectivity $\rho=1$. Conventional panels are compared to the CPCs of each module type that had the lowest LCEs. Notice the top and bottom portions forming two plots; the bottom plot cuts off the Spectrolab results which are shown in full in the top portion.
Increasing the mirror reflectivity from $\rho=0.8$ to $\rho=0.92$ caused the LCE to decrease by around 12%. But increasing the mirror reflectivity from $\rho=0.92$ to $\rho=1$ yielded a meager LCE decrease of about 1%. This is a curious happenstance since a mirror reflectivity of $\rho=1$ means that there are no reflective losses within the CPC geometry and one would expect a significant increase in performance. This goes to show how influential solar radiation is on cell temperature and how influential cell temperature is on cell efficiency. A higher mirror reflectivity allows more radiation to reach the bottom of the CPC and be absorbed. Figure 52 shows how the amount of impinging radiation impacts the solar cell temperature. There are variations in cell temperature for the same levels of radiation because the ambient temperature varies as well, but this does not conceal the trend of increasing radiation causing an increase in cell temperature.

Plots of the electric output per area and LCE for all the simulated configurations of a CPC for every module type using two axis tracking and $\rho=0.92$ or $\rho=1$ can be found in the Appendix, Sections H or I respectively.

![Cell Temperature vs. Radiative Energy Flux](image)

Figure 52: The hourly solar cell temperature is plotted against incident solar radiation. The above plot was done for a fixed orientation conventional Astronergy panel in
Yuma, AZ for a year. The reason for multiple cell temperatures for the same radiative energy is due to varying ambient temperatures.

An increasing cell temperature resulting from increasing radiation energy means that energy is being used to heat the cell as opposed to being converted to electricity. Figure 53 shows how simulations were run for each module type as conventional panels where the cell temperature was held constant at various temperatures. It can be seen how the cell efficiency decreased linearly with increasing cell temperature for each module. This is detrimental to high concentrating CPCs as they greatly increase the radiation flux on their solar cell surface and if it weren’t for this fact the CPCs with the lowest LCEs would be comprised of those with a concentration ratio of C=10.

![Cell Temperature vs. Cell Efficiency](image)

**Figure 53:** Cell temperatures were fixed for the duration of a simulation for conventional panels. Increasing that fixed cell temperature caused the cell efficiency to linearly decrease for each module.

The reader may be wondering how the various CPC configurations differ in cell temperature. Figure 54 depicts the cell temperature throughout a day for different CPC
configurations for a fixed panel, while Figure 55 shows this for a two axis tracker. The difference between the two orientations is substantial. The fixed tilt plot shows how only CPCs with a concentration ratio of C=2 are able to “see” beam radiation since the higher concentrating CPCs have too small an acceptance angle. Most of the lines overlap each other and can’t be distinguished. Meanwhile, the two axis tracker allows every CPC to “see” beam radiation from sunrise to sunset. The CPCs with a concentration ratio of C=10 were nearly 100 °C higher than those with C=2 at around the middle of the day, achieving a max cell temperature around 160 °C. Most solar cells are not rated for temperatures this high. For instance, the SunPower X21 module is rate for operating between -40 °C and +85 °C [28], and it is likely that the high temperatures incurred by concentration ratios could damage the cell. This goes to show how sensitive solar cells are to solar radiation, and this is exacerbated by higher concentration ratios. It should be no surprise then that CPCs using higher concentration ratios operate less efficiently than those at lower concentration ratios.

Figure 54: The cell temperature varies over the course of a day and for different CPC configurations. For a fixed tilt, only a standard/conventional panel and CPCs with a concentration ratio of C=2 are able to “see” beam radiation due to the small acceptance angle for higher concentrations. The max cell temperature did not exceed 50 °C. The simulations were run for Jan. 1 in Yuma, AZ.
Increasing the mirror reflectivity from $\rho = 0.8$ to $\rho = 0.92$ to $\rho = 1$ levels off in terms of LCE. This is due to higher radiation fluxes (with diminished reflective losses) causing higher cell temperatures and therefore lower cell efficiencies. The next course of action is to investigate CPCs utilizing these higher reflectivities, while controlling the cell temperature.

![Two Axis Tracking: Cell Temperature Change](image)

Figure 55: The cell temperature varies over the course of a day and with different CPC configurations. For two axis tracking, every CPC can "see" beam radiation and therefore exhibits cell temperature increases. The plots are almost divided into four sections: at the bottom is the ambient air temperature; the five lines above that are CPCs with C=2 and a standard panel; the next clump of four lines are the four CPCs with C=5, and the top four lines are for CPCs with C=10. The max cell temperature achieved was nearly 160 °C. The simulations were run for Jan. 1 in Yuma, AZ.

### 5.6.3. Two Axis Tracking: Variations in Cell Temperature

Figure 54 and Figure 55 express that cell temperature not only increases with concentration ratio, but also with tracking, namely two axis tracking. In order to investigate a niche where CPCs are expected to perform very well, cooling was applied to two axis tracking
simulations. Specifications and rated efficiencies for most panels are given for 25°C, so a modest temperature increase to about 37°C (or 310 K) was selected as the operating temperature. As seen in the above figures, this implies minimal cooling for CPCs with a concentration ratio of C=2, moderate cooling for a CPCs with C=5, and significant cooling for CPCs with C=10 (which can approach temperatures of 160 °C). To model the cooling, the cell temperature was simply set to not exceed 37 °C. There was no addition made to the cost of these systems, though adding cooling to a solar system will contribute to the cost and would need to be accounted for in a more thorough investigation. Also, cooling was only applied to two axis tracking conventional panels and CPCs with a reflectivity of ρ=0.92 and ρ=1.

Cooling increased a panel’s electric output as shown in Figure 57. A conventional panel’s electric output per area increased by about 4% due to the application of a maximum cell temperature, and a CPC with ρ=0.92 or ρ=1 increased by 7% from its non-cooled equivalent (C=2, H=0.25). The addition of cooling helped close the electric output gap between conventional panels and CPCs. This reveals that CPCs’ main efficiency loss in two axis tracking systems can be attributed to reflective losses and cell temperature rises.

The addition of cooling had an effect on the LCE for each module. Figure 58 shows the best performing (lowest LCE) CPCs for each module type using a reflectivity of ρ=0.92 and a max temperature of 37 °C, and Figure 59 shows the same for CPCs with a reflectivity of ρ=1. Looking at the CPCs with ρ=1 reveals the lowest LCEs throughout this investigation. A conventional Astronergy panel yielded an LCE of 0.022 $/kWh, which was finally beaten by an Astronergy CPC (C=5, H=0.25) with an LCE of 0.017 $/kWh. Both SunPower modules nearly closed the gap with equivalent LCEs of 0.026 $/kWh (both with C=10, H=0.25). The Spectrolab UTJ CPC achieved its lowest LCE at 1.204 $/kWh (for C=10, H=1). The use of cooling allowed higher concentration ratios to be more effective. The Astronergy CPC with the lowest LCE has used a concentration ratio of C=2 for all non-cooled systems, but was able to utilize C=5 for a cooled system. The same goes for the SunPower modules which used a concentration ratio of C=10 for cooled systems (whereas C=2 or 5 for all non-cooled systems). A cooled Spectrolab CPC was the only CPC to achieve its lowest LCE with a full CPC (no truncation). A full CPC usually adds more reflector area than it’s worth in collected energy, but not in this case. Figure 57 shows how module efficiency decreased with concentration, but the decrease was lessened by cooling. This trend is shown
for the SunPower E20 module with a truncation of H=0.25, but it is representative of the other module trends as well as the other truncations.

It should be noted that even a CPC with no reflection losses and a maximum temperature of 37°C still operates less efficiently than a conventional panel, as shown in Figure 56. Setting a maximum temperature at 37°C does not mean the cell always operates at this temperature, but simply that it won’t exceed it. CPCs with higher concentration ratios reach this max temperature sooner than conventional panels or lower concentration CPCs and therefore see lower efficiencies despite no reflection losses and the use of cooling.

Plots of the electric output per area and LCE for all the simulated configurations of a CPC for every module type using two axis tracking, temperature control, and $\rho=0.92$ or $\rho=1$ can be found in the Appendix, Sections J or K respectively.

Figure 56: The effects of the concentration ratio on module efficiency is plotted for the SunPower E20 module with a truncation of H=0.25. Notice how reflectivity changes the slope of the efficiency decrease, but there is little difference between $\rho=0.92$ and $\rho=1$. 
Figure 57: Electric output per area for conventional (standard) panels and CPCs for two axis tracking. CPC reflectivity is varied from $\rho=0.8$, 0.92, and 1. $T_{\text{max}}$ implies the use of cooling where the cell temperature does not exceed 37°C. The CPC configuration with the highest electric output per area is $C=2$ and $H=0.25$ and this is the only configuration presented here. Astro is short for Astronergy and Spectro is short for Spectrolab.
Figure 58: The LCE is presented for a two axis tracking system with mirror reflectivity $\rho=0.92$ and utilizing cooling so that the max cell temperature does not exceed 37°C. Conventional panels are compared to the CPCs of each module type that had the lowest LCEs. Notice the top and bottom portions forming two plots; the bottom plot cuts off the Spectrolab results which are shown in full in the top portion.
Figure 59: The LCE is presented for a two axis tracking system with mirror reflectivity $\rho=1$ and utilizing cooling so that the max cell temperature does not exceed 37°C. Conventional panels are compared to the CPCs of each module type that had the lowest LCE. Notice the top and bottom portions forming two plots; the bottom plot cuts off the Spectrolab results which are shown in full in the top portion.
5.7. Necessity for Low Cost Ratios

It is difficult to judge the cost competitiveness of CPCs. The SunPower modules were often successful in reducing their LCEs below that of their conventional counterparts; this was especially true of E-W and two axis tracking. But SunPower modules were never able to beat the lower efficiency conventional Astronergy panel. The only CPC to beat the conventional Astronergy panel was a high reflectivity two axis tracking Astronergy CPC. The Spectrolab UTJ panel dramatically reduced its LCE compared to its conventional counterpart in every tracking method, but its LCE was still more than fifty times greater than other panels at its best. One factor that could radically change the landscape of CPC competitiveness is the cost ratio. The cost ratio is the cost of reflector material divided by the cost of PV material. So a cost ratio of 0.5 means that the reflector material costs half as much as the PV material (on a per unit area basis).

Figure 60: Two ratios are plotted against each other. On the x axis is the cost ratio of reflector material to PV material. On the y axis is the ratio of the cost of a CPC over the cost of a standard (conventional) panel, both having the same aperture area and same PV price. Therefore, for a CPC to cost less than a conventional panel, it must stay below “1” on the y axis. This is plotted for several CPC configurations.
Figure 60 plots two different ratios. Along the x axis is the cost ratio of reflector material to PV material. Along the y axis is the ratio of the cost of a CPC over the cost of a conventional (standard) panel; both panels have the same opening aperture area and PV price. The cost of the conventional panel simply includes the cost of the PV material whereas the cost of the CPC includes the PV cost as well as the reflector cost. Changing the CPC configuration changes the PV area and the reflector area. In order for a CPC to cost less than a conventional panel, its ratio must be less than one. Looking at Figure 60 shows that every CPC configuration is greater than one for cost ratios of 0.4 and greater. The higher the concentration ratio, the steeper the slope, the lower the cost ratio needs to be. In order to see this better, Figure 61 zooms in on a smaller portion of the cost ratio. A “Break Even” line has been added to show that this is where the cost of a CPC equals the cost of a conventional panel. At a cost ratio (reflector to PV material) of 0.2, half of the CPC configurations are greater than the break even line. This demonstrates the need for CPCs to use relatively cheap reflector material to be competitive.

![Cost of CPC/Standard vs. cost ratio of reflector/PV](image)

Figure 61: A zoomed in plot of Figure 60. This shows that for a CPC to be cheaper than a conventional panel, its cost ratio of reflector to PV material must be kept small. It must be below the “Break Even” line.
Figure 61 only takes into account the material costs and does not look at the LCE. CPCs always experience greater efficiency losses than conventional panels, so Figure 61 is actually too optimistic for CPCs in terms of LCE competitiveness.

Achieving this low cost ratio is easy for expensive panels like the Spectrolab UTJ panel. That is why the Spectrolab panel experienced the most benefits from utilizing CPCs. The Spectrolab cost fraction used was ~0.0002. SunPower modules used a cost fraction around 0.05 which places them in a good position to benefit from CPCs. The Astronergy panel used a cost fraction around 0.17, which made it cost effective for a few configurations (but only in terms of bare material costs). It is reasonable to expect PV costs to decrease, as initially presented in Figure 1. Decreasing PV costs could allow SunPower CPCs to become competitive with conventional Astronergy panels, as demonstrated in Figure 47. But how much can the cost of reflector material be expected to decrease? A SunPower CPC could get caught in a Catch-22 where it becomes more competitive with conventional panels as the PV costs go down, but becomes less competitive as the cost ratio increases. This is indeed a concern for the Astronergy panel as well. Which is more likely to decrease faster, the cost of PV or the cost of the reflector? This is not as worrisome for the Spectrolab panel which is likely to always have a low cost ratio. However, Figure 62 shows how even when the cost of the Spectrolab PV material approaches zero, it is unable to yield a lower LCE than the conventional Astronergy panel. This is because the CPC shown in this figure is a full CPC (it had the lowest LCE for two axis tracking and $\rho=1$) which means it has a large area of reflector material; the cost of the reflector material alone keeps the Spectrolab’s LCE higher than the conventional Astronergy panel’s. To be fair, a PV cost this low for the Spectrolab module would make a different configuration more cost effective. Figure 63 shows a decreasing LCE for a Spectrolab CPC ($C=10$, $H=0.25$) compared to a conventional Astronergy panel. Everything in Figure 63 is the same as Figure 62 with the exception that the CPC now uses a truncation of $H=0.25$ (as opposed to $H=1$). This shows that a different CPC configuration yields a lower LCE than the conventional Astronergy panel, but this happens at a huge price drop in the Spectrolab module costs (from $93,669/m^2$ to around $500/m^2$).

The take away is that a low cost ratio is necessary for most configurations of CPCs. A cheap solar module will require far cheaper reflector material.
Figure 62: A decreasing cost of PV material for the Spectrolab UTJ module lowers its LCE, but never below that of a conventional Astronergy panel. The energy results used here to calculate LCE are for a two axis tracking Spectrolab CPC with C=10, H=1, $\rho=1$, and a max cell temperature of 37°C. $T_{\text{max}}$ also applies for the Astronergy panel.
Figure 63: The LCE decreases with PV cost. The energy results used here to calculate LCE are for a two axis tracking Spectrolab CPC with $C = 10$, $H = 0.25$, $\rho = 1$, and a max cell temperature of $37^\circ C$. $T_{max}$ also applies the for the Astronergy panel.
Chapter 6. Summary and Conclusions

6.1. Most Feasible CPC for each Tracking Type

After examining the performance of CPCs utilizing all kinds of tracking methods, solar cell efficiencies, concentration and truncation configurations, mirror reflectivities, and maximum temperature controls, it can be seen that some CPCs are more advantageous than others.

As mentioned in the beginning of Chapter 5, it is not reasonable to compare the LCE of one tracking method to that of another since the cost of tracking mechanisms were not accounted for in the total cost of the array. Therefore CPCs are compared to the conventional panel using the same tracking method. It is interesting to know how much better each CPC did compared to its conventional equivalent using the same solar cell efficiency, and it is also interesting to know how the highest performing CPCs (in terms of lowest LCEs) compared to a conventional panel using a lower efficiency solar cell (the Astronergy module).

For a fixed axis orientation, all modules except the Astronergy CPC yielded lower LCEs than their conventional equivalents. However, only the Astronergy CPC LCE was close to the conventional Astronergy panel. It is possible that higher mirror reflectivity ($\rho=0.92$) might make CPCs more competitive for this orientation, and this would be worth investigating in the future.

For vertical axis tracking with $\varepsilon=0^\circ$, only the Spectrolab CPC yielded a lower LCE than its conventional equivalent, but even this was not a significant gain. This tracking method proved to be the least effective for CPCs, due to the way the CPC’s acceptance angle rejected a good deal of radiation. However, rotating a CPC such that $\varepsilon=90^\circ$ proved a great success for CPCs using vertical axis tracking. All but the Astronergy CPC significantly reduced the LCE of their equivalent conventional panels, and the Astronergy CPC LCE was nearly equivalent to the Astronergy conventional panel. The SunPower modules helped close the gap between higher efficiency CPCs and lower efficiency conventional panels, but the best case scenario showed that a SunPower E20 CPC’s LCE ($C=5$, $H=0.25$) was still about 1.8 times greater than the conventional Astronergy panel. Still, this tracking orientation yielded the lowest LCE for
a non-Astronergy CPC for all fixed or single axis trackers; the SunPower E20 CPC (C=5, H=0.25) had an LCE of 0.045 $/kWh. Increasing the mirror reflectivity for these CPCs might prove them to be an even better competitor to low efficiency conventional panels.

All but the Astronergy CPC outperformed (in terms of LCE) their conventional panel equivalents for the east-west tracking method. While the Spectrolab CPC reduced its LCE by almost 4 times, it is more fruitful to examine the SunPower modules since they are closer to grid parity. The SunPower E20 CPC (C=5, H=0.25) reduced its LCE by almost 1.5 times. Its LCE is still almost twice as high as a conventional Astronergy panel. However, SunPower modules are known to produce more power than lower efficiency modules since they suffer less light-induced degradation [28] resulting in a longer lifetime, and they require less panels since they operate more efficiently. At first glance, it would seem that if a SunPower module could beat a lower efficiency panel (the Astronergy panel), and a SunPower CPC could beat a conventional SunPower panel, then a SunPower CPC would be much better than a conventional Astronergy panel. The rebuttal to this reasoning is that CPCs produce less electricity per area of aperture (see Figure 41); the gain made in LCE might be lost in the need for more panels to meet a demand. CPCs focus solar energy resulting in higher cell temperatures, resulting in lower cell efficiency. They also suffer reflective losses. That said, it would be beneficial to run simulations using E-W tracking for CPCs with a higher reflectivity (ρ=0.92) to reduce reflective losses, but the problem of heat gain in CPCs also needs to be addressed.

The panels using N-S tracking performed similarly to those using E-W tracking. In N-S tracking with ε=0°, the LCE for conventional panels was reduced more than it was for E-W tracking conventional panels, but the CPCs’ LCEs were about the same. Therefore, N-S tracking CPCs showed less gain (lower LCE) compared to their conventional equivalents. Rotating the CPCs such that ε=90° caused very little difference for N-S tracking CPCs, except that concentration ratios of 5 saw slight improvement and concentration ratios of 10 saw moderate improvement. As far as the best configured CPCs, this only effected the Spectrolab module.

A comparison between vertical axis (ε=90°), E-W, and N-S tracking is fairer than other tracking methods since all three are single axis trackers. If the price of tracking systems is comparable, it would be more cost effective to use N-S tracking for a conventional panel, but
for CPCs the most cost effective tracking orientation is the vertical axis tracker. However, the distinction is complicated by examining the electric output per area. An advantage of N-S tracking is that the disparity in electric output per area between conventional panels and CPCs is less than for all the other orientations (using a reflectivity of $\rho=0.8$ and no cooling). For the SunPower E20 module, the CPC (C=2, H=0.5) produced about 73% of its conventional equivalent’s electric output per area. This compares to the SunPower E20 CPCs producing about 56% of their conventional equivalent panel’s electric output per area for E-W (C=5, H=0.25), 58% for vertical axis tracking (C=5, H=0.25), 65% for fixed orientation (C=2, H=0.5), and 59% for two axis tracking (C=5, H=0.25). This is only among the best performing (lowest LCE) SunPower E20 CPCs for each tracking method.

Two axis tracking decreased the LCE for all modules, conventional and CPC alike. This is reasonable because the cost of tracking was not included in the analysis and two axis tracking allows the most energy to be collected. The Spectrolab and SunPower CPCs produced significantly reduced LCEs and the Astronergy CPC was similar to the Astronergy conventional panel.

The reflectivity was increased for two axis tracking CPCs, from 0.8 to 0.92 to 1. This resulted in more radiation reaching the absorber surface of the CPC. Tracking panels already receive more radiation than fixed panels, so this caused an increase in cell temperature and a corresponding decrease in cell efficiency. Thus the gain in LCE and electric output per area for higher reflectivity CPCs was not as great as might be expected. Nonetheless, the highest reflectivity ($\rho=1$) allowed the SunPower E20 CPC to produce about 70% of its equivalent conventional panel’s electric output per area (this was ~59% for $\rho=0.8$). Increasing the reflectivity did lower the Astronergy CPC’s LCE below the conventional Astronergy panel. Increasing the reflectivity for the vertical axis, E-W, and N-S tracking options increases their competitiveness too.

In response to the high cell temperature gain, cooling was applied to the two axis tracking systems. This lowered the LCE and raised the electric output per area. The SunPower modules utilizing cooling and $\rho=1$ nearly matched the conventional Astronergy panel’s LCE; and the SunPower modules (C=10, H=0.25) were producing about 77% of their equivalent conventional panels’ electric output per area.
Based on these simulations and the way that the levelized cost of energy was computed, the best performing CPCs were two axis trackers with perfectly reflective mirrors that used cooling. Realistically, the cost of a two axis tracking mechanism and cooling will detract from this promising performance. Furthermore, CPCs were first recognized as a promising concentrator because they wouldn’t require two axis tracking to be effective. Of the single axis trackers vertical axis tracking with ε=90° yielded the lowest LCE values, but N-S panels had a higher electric output per area than the vertical axis panels, so it is difficult to say which is actually better. The fixed orientation for CPCs did not show enough gain on conventional equivalents to be competitive.

As for the best CPC configurations, the best performing CPC usually had a high degree of truncation (H=0.25) and occasionally made use of H=0.5. Concentration ratios of 2 and 5 were most effective for non-cooled systems; higher concentrations (C=10) began to create problems with cell temperature, but were effective when combined with cooling.

6.2. Conclusions

The objective of this thesis was to investigate whether or not CPCs using photovoltaics could be made cost competitive with conventional panels of the same aperture area and photovoltaic price. It was seen that a CPC could lower a conventional panel’s levelized cost of energy, sometimes cutting it in half or more. However, because CPCs also result in less electricity produced per area, it is difficult to affirm that they are cost competitive with conventional panels. The fact of the matter is that a solar installation also requires labor, supportive framing, and maintenance among other costs, and these might not scale linearly with the number of panels needed. Less electric output per area means more panels are needed, which could lead to more of the other costs that don’t scale like PV and reflector costs. It is possible that the vertical axis (ε=90°), N-S, E-W, or two axis tracking CPCs could prove superior still, but this would require a more thorough cost analysis.

The drive for this investigation was to see high efficiency CPCs compete and beat out low efficiency conventional panels. The only CPC to beat the low efficiency conventional panel was the low efficiency CPC, and that was under the conditions of two axis tracking and high reflectivity. The SunPower modules got close under those same conditions. Ultimately, high efficiency CPCs could not compete with lower efficiency conventional panels.
Future paths of research related to this investigation would be to simulate CPC performance using a higher mirror reflectivity (i.e. $\rho=0.92$). In particular, this higher mirror reflectivity should be simulated for the vertical, E-W, and N-S axis CPCs since they show promise already. It would also be beneficial to expand the cost analysis to include the cost of tracking and cooling; this may support the use of two axis tracking. Higher concentration CPCs generate higher cell temperatures and it would be interesting to analyze the quality of the heat that must be rejected; combined photovoltaic and solar thermal systems are often seen leading the trends in high efficiency. An addition to the program, Solar_PVHFC, could be a solar thermal model and how a combined system might meet a demand.

In the future solar power can only keep moving forward and become cheaper. New techniques of utilizing our solar resource must always be considered. This thesis did just that. The development of a program to model and simulate CPC performance has enabled the investigation of CPCs using photovoltaics. CPCs have shown potential and implore further investigation. CPCs are not the only way to concentrate solar energy; there is still much to be learned. The industry can only grow. In the words of Thomas Edison, “I’d put my money on the sun and solar energy. What a source of power!”
References


Appendix

A listing of the results given in this Appendix are given below:

**Figures A.1-A.8:** Fixed orientation, electric output per area and levelized cost of energy.

**Figures B.1-B.8:** Vertical axis tracking with $\varepsilon=0^\circ$, electric output per area and levelized cost of energy.

**Figures C.1-C.8:** Vertical axis tracking with $\varepsilon=90^\circ$, electric output per area and levelized cost of energy.

**Figures D.1-D.8:** East-West axis tracking, electric output per area and levelized cost of energy.

**Figures E.1-E.8:** North-South axis tracking with $\varepsilon=0^\circ$, electric output per area and levelized cost of energy.

**Figures F.1-F.8:** North-South axis tracking with $\varepsilon=90^\circ$, electric output per area and levelized cost of energy.

**Figures G.1-G.8:** Two axis tracking for $\rho=0.8$, electric output per area and levelized cost of energy.

**Figures H.1-H.8:** Two axis tracking for $\rho=0.92$, electric output per area and levelized cost of energy.

**Figures I.1-I.8:** Two axis tracking for $\rho=1$, electric output per area and levelized cost of energy.

**Figures J.1-J.8:** Two axis tracking for $\rho=0.92$ and using $T_{\text{max}}$, electric output per area and levelized cost of energy.

**Figures K.1-K.8:** Two axis tracking for $\rho=1$ and using $T_{\text{max}}$, electric output per area and levelized cost of energy.
A. Fixed Orientation

Figure A.1: Astronergy NMC 250 panel, rated $\eta=15.2\%$. Yearly electric output per area for various concentration ratios. Orientation: Fixed tilt $\beta=32.667^\circ$, $\gamma=0^\circ$, mirror reflectivity $\rho=0.8$.

Figure A.2: Astronergy NMC 250 panel, rated $\eta=15.2\%$. Difference in CPC and Standard levelized cost of energy (LCE) for various concentration ratios. Orientation: Fixed tilt $\beta=32.667^\circ$, $\gamma=0^\circ$, mirror reflectivity $\rho=0.8$. 
Figure A.3: SunPower E20 327 panel, rated η=20.4%. Yearly electric output per area for various concentration ratios. Orientation: Fixed tilt β=32.667°, γ=0°, mirror reflectivity ρ=0.8.

Figure A.4: SunPower E20 327 panel, rated η=20.4%. Difference in CPC and Standard levelized cost of energy (LCE) for various concentration ratios. Orientation: Fixed tilt β=32.667°, γ=0°, mirror reflectivity ρ=0.8.
Figure A.5: SunPower X21 337 panel, rated $\eta=21.5\%$. Yearly electric output per area for various concentration ratios. Orientation: Fixed tilt $\beta=32.667^\circ$, $\gamma=0^\circ$, mirror reflectivity $\rho=0.8$.

Figure A.6: SunPower X21 337 panel, rated $\eta=21.5\%$. Difference in CPC and Standard levelized cost of energy (LCE) for various concentration ratios. Orientation: Fixed tilt $\beta=32.667^\circ$, $\gamma=0^\circ$, mirror reflectivity $\rho=0.8$. 
Figure A.7: Spectrolab UTJ panel, rated $\eta=28.3\%$. Yearly electric output per area for various concentration ratios. Orientation: Fixed tilt $\beta=32.667^\circ$, $\gamma=0^\circ$, mirror reflectivity $\rho=0.8$.

Figure A.8: Spectrolab UTJ panel, rated $\eta=28.3\%$. Difference in CPC and Standard levelized cost of energy (LCE) for various concentration ratios. Orientation: Fixed tilt $\beta=32.667^\circ$, $\gamma=0^\circ$, mirror reflectivity $\rho=0.8$. 
B. Single Vertical Axis Tracking, $\varepsilon=0^\circ$

Figure B.1: Astronergy NMC 250 panel, rated $\eta=15.2\%$. Yearly electric output per area for various concentration ratios. Orientation: Single axis tracking, vertical axis of rotation $\beta=32.667^\circ$, $\gamma=0^\circ$, $\varepsilon=0^\circ$, mirror reflectivity $\rho=0.8$.

Figure B.2: Astronergy NMC 250 panel, rated $\eta=15.2\%$. Difference in CPC and Standard levelized cost of energy (LCE) for various concentration ratios. Orientation: Single axis tracking, vertical axis of rotation $\beta=32.667^\circ$, $\gamma=0^\circ$, $\varepsilon=0^\circ$, mirror reflectivity $\rho=0.8$. 

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Figure B.3: SunPower E20 327 panel, rated \( \eta = 20.4\% \). Yearly electric output per area for various concentration ratios. Orientation: Single axis tracking, vertical axis of rotation \( \beta = 32.667^\circ \), \( \gamma = 0^\circ \), \( \epsilon = 0^\circ \), mirror reflectivity \( \rho = 0.8 \).

Figure B.4: SunPower E20 327 panel, rated \( \eta = 20.4\% \). Difference in CPC and Standard levelized cost of energy (LCE) for various concentration ratios. Orientation: Single axis tracking, vertical axis of rotation \( \beta = 32.667^\circ \), \( \gamma = 0^\circ \), \( \epsilon = 0^\circ \), mirror reflectivity \( \rho = 0.8 \).
Figure B.5: SunPower X21 337 panel, rated $\eta=21.5\%$. Yearly electric output per area for various concentration ratios. Orientation: Single axis tracking, vertical axis of rotation $\beta=32.667^\circ$, $\gamma=0^\circ$, $\varepsilon=0^\circ$, mirror reflectivity $\rho=0.8$.

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Figure B.7: Spectrolab UTJ panel, rated $\eta=28.3\%$. Yearly electric output per area for various concentration ratios. Orientation: Single axis tracking, vertical axis of rotation $\beta=32.667^\circ$, $\gamma=0^\circ$, $\epsilon=0^\circ$, mirror reflectivity $\rho=0.8$.

Figure B.8: Spectrolab UTJ panel, rated $\eta=28.3\%$. Difference in CPC and Standard levelized cost of energy (LCE) for various concentration ratios. Orientation: Single axis tracking, vertical axis of rotation $\beta=32.667^\circ$, $\gamma=0^\circ$, $\epsilon=0^\circ$, mirror reflectivity $\rho=0.8$. 
C. Single Vertical Axis Tracking, $\varepsilon=90^\circ$

Figure C.1: Astronergy NMC 250 panel, rated $\eta=15.2\%$. Yearly electric output per area for various concentration ratios. Orientation: Single axis tracking, vertical axis of rotation $\beta=32.667^\circ$, $\gamma=0^\circ$, $\varepsilon=90^\circ$, mirror reflectivity $\rho=0.8$.

Figure C.2: Astronergy NMC 250 panel, rated $\eta=15.2\%$. Difference in CPC and Standard levelized cost of energy (LCE) for various concentration ratios. Orientation: Single axis tracking, vertical axis of rotation $\beta=32.667^\circ$, $\gamma=0^\circ$, $\varepsilon=90^\circ$, mirror reflectivity $\rho=0.8$. 
Figure C.3: SunPower E20 327 panel, rated η=20.4%. Yearly electric output per area for various concentration ratios. Orientation: Single axis tracking, vertical axis of rotation β=32.667°, γ=0°, ε=90°, mirror reflectivity ρ=0.8.

Figure C.4: SunPower E20 327 panel, rated η=20.4%. Difference in CPC and Standard levelized cost of energy (LCE) for various concentration ratios. Orientation: Single axis tracking, vertical axis of rotation β=32.667°, γ=90°, ε=0°, mirror reflectivity ρ=0.8.
Figure C.5: SunPower X21 337 panel, rated η=21.5%. Yearly electric output per area for various concentration ratios. Orientation: Single axis tracking, vertical axis of rotation β=32.667°, γ=0°, ε=90°, mirror reflectivity ρ=0.8.

Figure C.6: SunPower X21 337 panel, rated η=21.5%. Difference in CPC and Standard levelized cost of energy (LCE) for various concentration ratios. Orientation: Single axis tracking, vertical axis of rotation β=32.667°, γ=0°, ε=90°, mirror reflectivity ρ=0.8.
Figure C.7: Spectrolab UTJ panel, rated $\eta=28.3\%$. Yearly electric output per area for various concentration ratios. Orientation: Single axis tracking, vertical axis of rotation $\beta=32.667^\circ$, $\gamma=0^\circ$, $\epsilon=90^\circ$, mirror reflectivity $\rho=0.8$.

Figure C.8: Spectrolab UTJ panel, rated $\eta=28.3\%$. Difference in CPC and Standard levelized cost of energy (LCE) for various concentration ratios. Orientation: Single axis tracking, vertical axis of rotation $\beta=32.667^\circ$, $\gamma=0^\circ$, $\epsilon=90^\circ$, mirror reflectivity $\rho=0.8$. 
D. Single East-West Axis Tracking

Figure D.1: Astronergy NMC 250 panel, rated η=15.2%. Yearly electric output per area for various concentration ratios. Orientation: Single axis tracking, east-west axis of rotation γ=0°, mirror reflectivity ρ=0.8.

Figure D.2: Astronergy NMC 250 panel, rated η=15.2%. Difference in CPC and Standard levelized cost of energy (LCE) for various concentration ratios. Orientation: Single axis tracking, east-west axis of rotation γ=0°, mirror reflectivity ρ=0.8.
Figure D.3: SunPower E20 327 panel, rated $\eta=20.4\%$. Yearly electric output per area for various concentration ratios. Orientation: Single axis tracking, east-west axis of rotation $\gamma=0^\circ$, mirror reflectivity $\rho=0.8$.

Figure D.4: SunPower E20 327 panel, rated $\eta=20.4\%$. Difference in CPC and Standard levelized cost of energy (LCE) for various concentration ratios. Orientation: Single axis tracking, east-west axis of rotation $\gamma=0^\circ$, mirror reflectivity $\rho=0.8$. 

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Figure D.5: SunPower X21 337 panel, rated η=21.5%. Yearly electric output per area for various concentration ratios. Orientation: Single axis tracking, east-west axis of rotation γ=0°, mirror reflectivity ρ=0.8.

Figure D.6: SunPower X21 337 panel, rated η=21.5%. Difference in CPC and Standard levelized cost of energy (LCE) for various concentration ratios. Orientation: Single axis tracking, east-west axis of rotation γ=0°, mirror reflectivity ρ=0.8.
Figure D.7: Spectrolab UTJ panel, rated $\eta=28.3\%$. Yearly electric output per area for various concentration ratios. Orientation: Single axis tracking, east-west axis of rotation $\gamma=0^\circ$, mirror reflectivity $\rho=0.8$.

Figure D.8: Spectrolab UTJ panel, rated $\eta=28.3\%$. Difference in CPC and Standard levelized cost of energy (LCE) for various concentration ratios. Orientation: Single axis tracking, east-west axis of rotation $\gamma=0^\circ$, mirror reflectivity $\rho=0.8$. 
E. Single North-South Axis Tracking, $\varepsilon=0^\circ$

Figure E.1: Astronergy NMC 250 panel, rated $\eta=15.2\%$. Yearly electric output per area for various concentration ratios. Orientation: Single axis tracking, north-south axis of rotation $\beta=32.667^\circ$, $\gamma=0^\circ$, $\varepsilon=0^\circ$, mirror reflectivity $\rho=0.8$.

Figure E.2: Astronergy NMC 250 panel, rated $\eta=15.2\%$. Difference in CPC and Standard levelized cost of energy (LCE) for various concentration ratios. Orientation: Single axis tracking, north-south axis of rotation $\beta=32.667^\circ$, $\gamma=0^\circ$, $\varepsilon=0^\circ$, mirror reflectivity $\rho=0.8$. 
Figure E.3: SunPower E20 327 panel, rated $\eta=20.4\%$. Yearly electric output per area for various concentration ratios. Orientation: Single axis tracking, north-south axis of rotation $\beta=32.667^\circ$, $\gamma=0^\circ$, $\varepsilon=0^\circ$, mirror reflectivity $\rho=0.8$.

Figure E.4: SunPower E20 327 panel, rated $\eta=20.4\%$. Difference in CPC and Standard levelized cost of energy (LCE) for various concentration ratios. Orientation: Single axis tracking, north-south axis of rotation $\beta=32.667^\circ$, $\gamma=0^\circ$, $\varepsilon=0^\circ$, mirror reflectivity $\rho=0.8$. 

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Figure E.5: SunPower X21 337 panel, rated $\eta=21.5\%$. Yearly electric output per area for various concentration ratios. Orientation: Single axis tracking, north-south axis of rotation $\beta=32.667^\circ$, $\gamma=0^\circ$, $\varepsilon=0^\circ$, mirror reflectivity $\rho=0.8$.

Figure E.6: SunPower X21 337 panel, rated $\eta=21.5\%$. Difference in CPC and Standard levelized cost of energy (LCE) for various concentration ratios. Orientation: Single axis tracking, north-south axis of rotation $\beta=32.667^\circ$, $\gamma=0^\circ$, $\varepsilon=0^\circ$, mirror reflectivity $\rho=0.8$. 
Figure E.7: Spectrolab UTJ panel, rated $\eta=28.3\%$. Yearly electric output per area for various concentration ratios. Orientation: Single axis tracking, north-south axis of rotation $\beta=32.667^\circ$, $\gamma=0^\circ$, $\epsilon=0^\circ$, mirror reflectivity $\rho=0.8$.

Figure E.8: Spectrolab UTJ panel, rated $\eta=28.3\%$. Difference in CPC and Standard levelized cost of energy (LCE) for various concentration ratios. Orientation: Single axis tracking, north-south axis of rotation $\beta=32.667^\circ$, $\gamma=0^\circ$, $\epsilon=0^\circ$, mirror reflectivity $\rho=0.8$. 
F. Single North-South Axis Tracking, $\varepsilon=90^\circ$

Figure F.1: Astronergy NMC 250 panel, rated $\eta=15.2\%$. Yearly electric output per area for various concentration ratios. Orientation: Single axis tracking, north-south axis of rotation $\beta=32.667^\circ$, $\gamma=0^\circ$, $\varepsilon=90^\circ$, mirror reflectivity $\rho=0.8$.

Figure F.2: Astronergy NMC 250 panel, rated $\eta=15.2\%$. Difference in CPC and Standard levelized cost of energy (LCE) for various concentration ratios. Orientation: Single axis tracking, north-south axis of rotation $\beta=32.667^\circ$, $\gamma=0^\circ$, $\varepsilon=90^\circ$, mirror reflectivity $\rho=0.8$. 
Figure F.3: SunPower E20 327 panel, rated $\eta=20.4\%$. Yearly electric output per area for various concentration ratios. Orientation: Single axis tracking, north-south axis of rotation $\beta=32.667^\circ$, $\gamma=0^\circ$, $\epsilon=90^\circ$, mirror reflectivity $\rho=0.8$.

Figure F.4: SunPower E20 327 panel, rated $\eta=20.4\%$. Difference in CPC and Standard levelized cost of energy (LCE) for various concentration ratios. Orientation: Single axis tracking, north-south axis of rotation $\beta=32.667^\circ$, $\gamma=0^\circ$, $\epsilon=90^\circ$, mirror reflectivity $\rho=0.8$. 
Figure F.5: SunPower X21 337 panel, rated $\eta=21.5\%$. Yearly electric output per area for various concentration ratios. Orientation: Single axis tracking, north-south axis of rotation $\beta=32.667^\circ$, $\gamma=0^\circ$, $\varepsilon=90^\circ$, mirror reflectivity $\rho=0.8$.

Figure F.6: SunPower X21 337 panel, rated $\eta=21.5\%$. Difference in CPC and Standard levelized cost of energy (LCE) for various concentration ratios. Orientation: Single axis tracking, north-south axis of rotation $\beta=32.667^\circ$, $\gamma=0^\circ$, $\varepsilon=90^\circ$, mirror reflectivity $\rho=0.8$. 

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Figure F.7: Spectrolab UTJ panel, rated $\eta=28.3\%$. Yearly electric output per area for various concentration ratios. Orientation: Single axis tracking, north-south axis of rotation $\beta=32.667^\circ$, $\gamma=0^\circ$, $\varepsilon=90^\circ$, mirror reflectivity $\rho=0.8$.

Figure F.8: Spectrolab UTJ panel, rated $\eta=28.3\%$. Difference in CPC and Standard levelized cost of energy (LCE) for various concentration ratios. Orientation: Single axis tracking, north-south axis of rotation $\beta=32.667^\circ$, $\gamma=0^\circ$, $\varepsilon=90^\circ$, mirror reflectivity $\rho=0.8$. 
G. Two Axis Tracking, \( \rho=0.8 \)

Figure G.1: Astronergy NMC 250 panel, rated \( \eta=15.2\% \). Yearly electric output per area for various concentration ratios. Orientation: Two axis tracking, mirror reflectivity \( \rho=0.8 \).

Figure G.2: Astronergy NMC 250 panel, rated \( \eta=15.2\% \). Difference in CPC and Standard levelized cost of energy (LCE) for various concentration ratios. Orientation: Two axis tracking, mirror reflectivity \( \rho=0.8 \).
Figure G.3: SunPower E20 327 panel, rated $\eta=20.4\%$. Yearly electric output per area for various concentration ratios. Orientation: Two axis tracking, mirror reflectivity $\rho=0.8$.

Figure G.4: SunPower E20 327 panel, rated $\eta=20.4\%$. Difference in CPC and Standard levelized cost of energy (LCE) for various concentration ratios. Orientation: Two axis tracking, mirror reflectivity $\rho=0.8$. 
Figure G.5: SunPower X21 337 panel, rated $\eta=21.5\%$. Yearly electric output per area for various concentration ratios. Orientation: Two axis tracking, mirror reflectivity $\rho=0.8$.

Figure G.6: SunPower X21 337 panel, rated $\eta=21.5\%$. Difference in CPC and Standard levelized cost of energy (LCE) for various concentration ratios. Orientation: Two axis tracking, mirror reflectivity $\rho=0.8$. 
Figure G.7: Spectrolab UTJ panel, rated $\eta=28.3\%$. Yearly electric output per area for various concentration ratios. Orientation: Two axis tracking, mirror reflectivity $\rho=0.8$.

Figure G.8: Spectrolab UTJ panel, rated $\eta=28.3\%$. Difference in CPC and Standard levelized cost of energy (LCE) for various concentration ratios. Orientation: Two axis tracking, mirror reflectivity $\rho=0.8$. 
H. Two Axis Tracking, $\rho=0.92$

Figure H.1: Astronergy NMC 250 panel, rated $\eta=15.2\%$. Yearly electric output per area for various concentration ratios. Orientation: Two axis tracking, mirror reflectivity $\rho=0.92$.

Figure H.2: Astronergy NMC 250 panel, rated $\eta=15.2\%$. Difference in CPC and Standard levelized cost of energy (LCE) for various concentration ratios. Orientation: Two axis tracking, mirror reflectivity $\rho=0.92$. 
Figure H.3: SunPower E20 327 panel, rated $\eta=20.4\%$. Yearly electric output per area for various concentration ratios. Orientation: Two axis tracking, mirror reflectivity $\rho=0.92$.

Figure H.4: SunPower E20 327 panel, rated $\eta=20.4\%$. Difference in CPC and Standard levelized cost of energy (LCE) for various concentration ratios. Orientation: Two axis tracking, mirror reflectivity $\rho=0.92$. 
Figure H.5: SunPower X21 337 panel, rated η=21.5%. Yearly electric output per area for various concentration ratios. Orientation: Two axis tracking, mirror reflectivity ρ=0.92.

Figure H.6: SunPower X21 337 panel, rated η=21.5%. Difference in CPC and Standard levelized cost of energy (LCE) for various concentration ratios. Orientation: Two axis tracking, mirror reflectivity ρ=0.92.
Figure H.7: Spectrolab UTJ panel, rated η=28.3%. Yearly electric output per area for various concentration ratios. Orientation: Two axis tracking, mirror reflectivity ρ=0.92.

Figure H.8: Spectrolab UTJ panel, rated η=28.3%. Difference in CPC and Standard levelized cost of energy (LCE) for various concentration ratios. Orientation: Two axis tracking, mirror reflectivity ρ=0.92.
I. Two Axis Tracking, $\rho=1$

Figure I.1: Astronergy NMC 250 panel, rated $\eta=15.2\%$. Yearly electric output per area for various concentration ratios. Orientation: Two axis tracking, mirror reflectivity $\rho=1$.

Figure I.2: Astronergy NMC 250 panel, rated $\eta=15.2\%$. Difference in CPC and Standard levelized cost of energy (LCE) for various concentration ratios. Orientation: Two axis tracking, mirror reflectivity $\rho=1$. 
Figure I.3: SunPower E20 327 panel, rated $\eta=20.4\%$. Yearly electric output per area for various concentration ratios. Orientation: Two axis tracking, mirror reflectivity $\rho=1$.

Figure I.4: SunPower E20 327 panel, rated $\eta=20.4\%$. Difference in CPC and Standard levelized cost of energy (LCE) for various concentration ratios. Orientation: Two axis tracking, mirror reflectivity $\rho=1$.  

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Figure I.5: SunPower X21 337 panel, rated $\eta=21.5\%$. Yearly electric output per area for various concentration ratios. Orientation: Two axis tracking, mirror reflectivity $\rho=1$.

Figure I.6: SunPower X21 337 panel, rated $\eta=21.5\%$. Difference in CPC and Standard levelized cost of energy (LCE) for various concentration ratios. Orientation: Two axis tracking, mirror reflectivity $\rho=1$. 
Figure I.7: Spectrolab UTJ panel, rated η=28.3%. Yearly electric output per area for various concentration ratios. Orientation: Two axis tracking, mirror reflectivity ρ=1.

Figure I.8: Spectrolab UTJ panel, rated η=28.3%. Difference in CPC and Standard levelized cost of energy (LCE) for various concentration ratios. Orientation: Two axis tracking, mirror reflectivity ρ=1.
J. Two Axis Tracking, $\rho=0.92$, $T_{\text{max}}$

Figure J.1: Astronergy NMC 250 panel, rated $\eta=15.2\%$. Yearly electric output per area for various concentration ratios. Orientation: Two axis tracking, mirror reflectivity $\rho=0.92$, $T_{\text{max}}=37^\circ\text{C}$.

Figure J.2: Astronergy NMC 250 panel, rated $\eta=15.2\%$. Difference in CPC and Standard levelized cost of energy (LCE) for various concentration ratios. Orientation: Two axis tracking, mirror reflectivity $\rho=0.92$, $T_{\text{max}}=37^\circ\text{C}$.
Figure J.3: SunPower E20 327 panel, rated $\eta=20.4\%$. Yearly electric output per area for various concentration ratios. Orientation: Two axis tracking, mirror reflectivity $\rho=0.92$, $T_{\text{max}}=37^\circ\text{C}$.

Figure J.4: SunPower E20 327 panel, rated $\eta=20.4\%$. Difference in CPC and Standard levelized cost of energy (LCE) for various concentration ratios. Orientation: Two axis tracking, mirror reflectivity $\rho=0.92$, $T_{\text{max}}=37^\circ\text{C}$. 

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Figure J.5: SunPower X21 337 panel, rated η=21.5%. Yearly electric output per area for various concentration ratios. Orientation: Two axis tracking, mirror reflectivity ρ=0.92, T_{max}=37°C.

Figure J.6: SunPower X21 337 panel, rated η=21.5%. Difference in CPC and Standard levelized cost of energy (LCE) for various concentration ratios. Orientation: Two axis tracking, mirror reflectivity ρ=0.92, T_{max}=37°C.
Figure J.7: Spectrolab UTJ panel, rated $\eta=28.3\%$. Yearly electric output per area for various concentration ratios. Orientation: Two axis tracking, mirror reflectivity $\rho=0.92$, $T_{\text{max}}=37^\circ \text{C}$.

Figure J.8: Spectrolab UTJ panel, rated $\eta=28.3\%$. Difference in CPC and Standard levelized cost of energy (LCE) for various concentration ratios. Orientation: Two axis tracking, mirror reflectivity $\rho=0.92$, $T_{\text{max}}=37^\circ \text{C}$.
K. Two Axis Tracking, $\rho=1$, $T_{\text{max}}$

Figure K.1: Astronergy NMC 250 panel, rated $\eta=15.2\%$. Yearly electric output per area for various concentration ratios. Orientation: Two axis tracking, mirror reflectivity $\rho=1$, $T_{\text{max}}=37^\circ\text{C}$.

Figure K.2: Astronergy NMC 250 panel, rated $\eta=15.2\%$. Difference in CPC and Standard levelized cost of energy (LCE) for various concentration ratios. Orientation: Two axis tracking, mirror reflectivity $\rho=1$, $T_{\text{max}}=37^\circ\text{C}$.
Figure K.3: SunPower E20 327 panel, rated η=20.4%. Yearly electric output per area for various concentration ratios. Orientation: Two axis tracking, mirror reflectivity ρ=1, T_{max}=37°C.

Figure K.4: SunPower E20 327 panel, rated η=20.4%. Difference in CPC and Standard levelized cost of energy (LCE) for various concentration ratios. Orientation: Two axis tracking, mirror reflectivity ρ=1, T_{max}=37°C.
Figure K.5: SunPower X21 337 panel, rated $\eta=21.5\%$. Yearly electric output per area for various concentration ratios. Orientation: Two axis tracking, mirror reflectivity $\rho=1$, $T_{\text{max}}=37^\circ \text{C}$.

Figure K.6: SunPower X21 337 panel, rated $\eta=21.5\%$. Difference in CPC and Standard levelized cost of energy (LCE) for various concentration ratios. Orientation: Two axis tracking, mirror reflectivity $\rho=1$, $T_{\text{max}}=37^\circ \text{C}$.
Figure K.7: Spectrolab UTJ panel, rated $\eta=28.3\%$. Yearly electric output per area for various concentration ratios. Orientation: Two axis tracking, mirror reflectivity $\rho=1$, $T_{\text{max}}=37^\circ\text{C}$.

Figure K.8: Spectrolab UTJ panel, rated $\eta=28.3\%$. Difference in CPC and Standard levelized cost of energy (LCE) for various concentration ratios. Orientation: Two axis tracking, mirror reflectivity $\rho=1$, $T_{\text{max}}=37^\circ\text{C}$. 