Impact of SAR Image Formation Quality on Target Separability

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in Engineering

by

Cody A. Lawyer
B.S.E.E., Wright State University, 2013

2014
Wright State University
I HEREBY RECOMMEND THAT THE THESIS PREPARED UNDER MY SUPERVISION BY Cody A. Lawyer ENTITLED Impact of SAR Image Formation Quality on Target Separability BE ACCEPTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF Master of Science in Engineering.

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ABSTRACT


The polar format algorithm (PFA) allows the use of computationally efficient fast Fourier transforms in synthetic aperture radar (SAR) image formation, but introduces phase errors when making the far-field approximations that facilitate this approach. The phase errors cause spatially variant distortion and defocus in the formed image. These effects may complicate target recognition applications. To limit the impact of defocus, scene size is usually limited such that the maximum quadratic phase error within an image falls below some threshold. This thesis looks at how distortion and defocus affects the classification of targets, with the hope of developing an application-driven scene size limit.
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Acknowledgment

First, I would like to thank Dr. Brian Rigling for all the support and guidance he provided me during this process. Next, I would like to thank the Center for Surveillance Research for allowing me to perform this research. I would also like to thank the other students in the Wright State Sensors Exploitation Lab for their support. Finally, I would like to my parents and Kathryn for providing support throughout my entire education.
Dedicated to

Kathryn
Chapter 1

Introduction

1.1 Introduction

Synthetic aperture radar (SAR) was introduced in [1] as a method to image objects using electromagnetic waves. It allows imaging in many weather conditions where optical sensors would fail. A potentially large amount of data can be generated by synthetic aperture radar sensors, so there is a need to process it efficiently. Efficient methods for image formation typically come at the price of some degree of image degradation in terms of defocus and distortion. Such image degradation may adversely affect the performance of automatic target recognition (ATR) algorithms, which seek to classify detected objects without human input.

The polar format algorithm (PFA) is commonly used in forming SAR images. It uses a planar approximation to the curved radar wave front. Compared to the back projection algorithm (BPA), which is an exact method for forming SAR images and has computational complexity $O(N^3)$, the far-field approximation reduces computational complexity to $O(N^2 \log_2 N)$, through the introduction of FFTs. However, the approximation introduces phase errors, which cause spatially variant distortion and defocus. Targets at different locations in an image will see different amounts of distortion and defocus. A maximum
allowable scene size is typically selected to bound the defocus errors in the scene, without consideration for performance in applications such as ATR. It is generally unknown whether selection of an arbitrary bound yields image quality that meets or exceeds the needs of different applications.

Previous work on SAR ATR performance prediction can be segmented into three types. The first is empirical studies using real SAR data. In [2], different methods of feature extraction were used on SAR images then tested with different methods of classification. In [3], a non-parametric method of error estimation was used. In [4], a variety of maximum likelihood classifiers were tested. In [5], the effect of image resolution on classification performance was studied. In [6], statistical separability tests were used to estimate classification performance. In all of those papers, the Moving and Stationary Target Acquisition and Recognition (MSTAR) dataset was used.

The next type of work on performance prediction is using synthetically generated SAR data. In [7], a study independent of ATR algorithm was performed using synthetic data and optimal Bayesian methods for error probability estimation. The final type of work on performance prediction is using mathematical models. In [8], models were developed for targets, and the upper bound on error was estimated using a vote-based ATR algorithm. In [9], Gaussian models were developed for the radar signals and used to estimate the type and pose of the target. In [10], models that capture the performance of different ATR algorithms were developed and tested using a score-based method.

The effects of the distortion and defocus caused by the PFA have been previously studied. In [11], scene size limits for a linear flight path were derived using approximations for the distortion and defocus in an image. In [12], scene size limits for a circular flight path were derived as well as a post-processing method to correct for phase errors introduced by the PFA. In [13], another post-processing method was developed to correct distortion and defocus.

In this thesis, we extend previous work on SAR ATR performance prediction by using
synthetically generated images that emulate image formation with the PFA without having corrected for distortion and defocus errors. This allows a study of ATR driven scene size limits for the PFA. The effects of image degradation on ATR performance are characterized through a variety of statistical separability tests, independent of a specific ATR algorithm. The expectation is that the separability between two targets will decrease away from scene center due to the distortion and defocus seen by the targets. This expectation is caused by the fact that distortion and defocus makes it more difficult for a human to visually classify the targets.

1.2 Outline

An outline of the thesis is as follows. In the rest of chapter 1, the signal model and image formation procedure for synthetic aperture radar is described. In chapter 2, probabilistic descriptions of image formation errors are derived. In chapter 3, the tests used to measure the separability between two datasets are described. The method of generating simulated data and applying errors to existing data is described in chapter 4. In chapter 5, the method of testing and the results of the tests are described. Finally, conclusions on the impact of SAR image formation quality on target separability are reached in chapter 6.

1.3 Signal Model & Image Formation

A SAR system transmits pulses as it travels along a path, and those pulses are reflected by scatterers located in the scene. Assuming no platform measurement errors, and assuming that the radar and image are located in the same planes for simplicity, this allows the radar position to be defined as \( \mathbf{r_a}(\tau) = [x_a(\tau), y_a(\tau)]^T \) and a scatterer position defined as \( \mathbf{r_k} = [x_k, y_k]^T \). The received signal can then be represented as a sum of scattering center
responses

\[ S(f, \tau) = \sum_k A_k \exp \left( -\frac{j4\pi f (d_k(\tau) - d_a(\tau))}{c} \right) \]  \hspace{1cm} (1.1)

with the range from the radar to the scatterer defined as

\[ d_k = \| \mathbf{r}_a - \mathbf{r}_k \| = \sqrt{(x_a - x_k)^2 + (y_a - y_k)^2} \]  \hspace{1cm} (1.2)

and with

\[ d_a = \| \mathbf{r}_a \| = \sqrt{x_a^2 + y_a^2} \]  \hspace{1cm} (1.3)

defined as the range from the radar to the scene center. For compactness, we have suppressed the dependence on the slow time variable \( \tau \). This leads to the differential range term being defined as

\[ \Delta R_k = \sqrt{(x_a - x_k)^2 + (y_a - y_k)^2} - \sqrt{x_a^2 + y_a^2} \]  \hspace{1cm} (1.4)

The BPA image formation algorithm is computed as

\[ I_{BPA}(\tau) = \frac{1}{N_p K} \sum_{n=1}^{N_p} \sum_{k=1}^{K} S(f_k, \tau_n) \exp \left( \frac{j4\pi f_k \Delta R(\tau_n)}{c} \right) \]  \hspace{1cm} (1.5)

for each pixel location \( \tau = [x, y]^T \). This can be implemented with computational complexity \( O(N^3) \) and provides an exact image, formed by the matched filter of the phase history data where \( f_k \) are the frequency samples at each time \( \tau_n \).

The PFA can be derived through a first order Taylor expansion of the differential range as shown in [15]. Performing the Taylor expansion and rewriting the linear terms in polar coordinates gives \( \Delta R = -x \cos(\theta_a) - y \sin(\theta_a) \). Representing the received data in \( k \)-space by assigning \( k_x = (4\pi f/c) \cos(\theta_a) \) and \( k_y = (4\pi f/c) \sin(\theta_a) \) leads to the PFA being
implemented as

\[
I_{PFA}(r) = \frac{1}{N_x N_y} \sum_{k_x, k_y} S(k_x, k_y) \exp(-j(x k_x + y k_y))
\]  \hspace{1cm} (1.6)

where \(N_x\) and \(N_y\) are the number of samples in \(k_x\) and \(k_y\) and FFTs are used to efficiently compute the final result with computational complexity \(O(N^2 \log_2(N))\).

\[
y \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad
\[ \Omega_{X-DPE} = \frac{y_c^2}{2r_c^3} x^2 - \frac{x_c y_c}{r_c^2} x y + \frac{x_c^2}{2r_c^3} y^2 \] (1.7)

\[ \Omega_{Y-DPE} = \left( \frac{y_c}{r_c^2} - \frac{y_c^3}{2x_c^2 r_c^2} \right) x^2 + \left( \frac{2y_c^2}{x_c r_c^2} - \frac{x_c}{r_c^2} \right) x y + \left( \frac{-3y_c}{2r_c^2} \right) y^2 \] (1.8)

and maximum quadratic phase error

\[ \Phi_{DPE} = \frac{\pi f_c}{c} \left( \frac{9 \cos^2(\theta_s) - 7}{2x_c \cos(\theta_s)} x^2 + \frac{-9 \cos^2(\theta_s) + 6}{2x_c \cos(\theta_s)} x y + \left( \frac{2 \sin^3(\theta_s)}{x_c \cos^2(\theta_s)} - \frac{7 \sin(\theta_s)}{x_c} \right) y^2 \right) \left( \frac{\cos^4(\theta_s) L_a^2}{2r_c^4} \right) \] (1.9)

are defined by evaluating the differential range error caused by the first order Taylor expansion.

The differential range error uses the exact error. Using the differential range error, approximations for the \( x \) distortion,

\[ \Omega_{X-DRE} = \begin{align*} x \cos(\theta_s) - y \sin(\theta_s) - \sqrt{(x_c - x)^2 + (y_c - y)^2} & - \sqrt{x_c^2 + y_c^2} \end{align*} \] (1.10)

\[ \Omega_{Y-DRE} = \begin{align*} x \tan(\theta_s) - y & + \frac{(y_c - y)^2}{\sqrt{(x_c - x)^2 + (y_c - y)^2}} \frac{r_c}{x_c^2} + \frac{y_c}{\cos^2 \theta_s} \end{align*} \] (1.11)

\( y \) distortion,
and maximum quadratic phase error

\[
\Phi_{DRE} = \frac{\pi f_c}{c} \left( \frac{1}{\cos^3(\theta_s)} \right) \\
- \frac{r_c^2}{\cos^4(\theta_s)} \frac{(x_c - x)^2}{\sqrt{(x_c - x)^2 + (y_c - y)^2}} \\
+ \frac{x_c^4}{r_c \cos^4(\theta_s)} \left( \frac{\cos^4(\theta_s) L_a^2}{2r_c^2} \right)
\]  

(1.12)

are defined by evaluating the differential range error caused by the first order Taylor expansion.
Chapter 2

Probabilistic Descriptions of Image Formation Errors

Probabilistic descriptions of the image formation errors describe the amount of distortion and defocus a randomly placed target in an image formed by the PFA will experience. These probability distribution functions would allow the expected value of distortion and defocus for a given scene to be calculated.

2.1 Attempt at Theoretical Derivation

Theoretical derivation of the probabilistic descriptions of the image formation errors would allow for general results that are not for a specific scene setup. Using (1.7) - (1.12), and having $x$ and $y$ distributed as uniform random variables, derivation of the probability distribution functions for each was unsuccessfully attempted. We turn instead to empirical estimates of these distributions.
2.2 Empirical Method

A Monte Carlo simulation was used to generate random locations throughout a given scene size. The distortion and defocus were calculated at all the random locations. Histograms were then generated for each method of calculating the distortion and defocus. Finally, MATLAB’s curve fitting tool was used to get a curve fit for each histogram. The following results were generated for the specific scene parameters as listed in Table 2.1. The $x$ and $y$ locations were generated as uniformly distributed random variables over the independent range $[-r_{max}, r_{max}]$, where $r_{max}$ is the maximum scene radius.

Table 2.1: Scene parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Center Frequency</td>
<td>10 GHz</td>
</tr>
<tr>
<td>Resolution</td>
<td>0.308 m</td>
</tr>
<tr>
<td>QPE Free Scene Size</td>
<td>50 m</td>
</tr>
<tr>
<td>Iterations</td>
<td>1000000</td>
</tr>
<tr>
<td>Scene Size</td>
<td>300 m</td>
</tr>
<tr>
<td>Squint Angle</td>
<td>0°</td>
</tr>
</tbody>
</table>

2.2.1 DPE Distortion

For the DPE $x$ distortion, a $7^{th}$-order polynomial of the form

$$f_{DPE-x}(x) = p_1 x^7 + p_2 x^6 + p_3 x^5 + p_4 x^4 + p_5 x^3 + p_6 x^2 + p_7 x + p_8$$  \hspace{1cm} (2.1)

provided the best fit for the generated histogram. The coefficients for the fit are listed in Table 2.2.
Figure 2.1: DPE $x$ distortion empirical histogram and fit

Table 2.2: DPE $x$ distortion fit coefficients

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_1$</td>
<td>$-9.537 \times 10^{-8}$</td>
</tr>
<tr>
<td>$p_2$</td>
<td>$5.371 \times 10^{-6}$</td>
</tr>
<tr>
<td>$p_3$</td>
<td>$-0.0001223$</td>
</tr>
<tr>
<td>$p_4$</td>
<td>$0.001442$</td>
</tr>
<tr>
<td>$p_5$</td>
<td>$-0.009345$</td>
</tr>
<tr>
<td>$p_6$</td>
<td>$0.03267$</td>
</tr>
<tr>
<td>$p_7$</td>
<td>$-0.05655$</td>
</tr>
<tr>
<td>$p_8$</td>
<td>$0.04512$</td>
</tr>
</tbody>
</table>

For the DPE $y$ distortion, a sum of 8 Gaussians of the form

$$ f_{DPE-Y}(x) = \sum_{i=1}^{8} a_i \exp \left( -\left( \frac{x - b_i}{c_i} \right)^2 \right) $$

provided the best fit for the generated histogram. The coefficients for the fit are listed in Table 2.3.
Figure 2.2: DPE $y$ distortion empirical histogram and fit

Table 2.3: DPE $y$ distortion fit coefficients

<table>
<thead>
<tr>
<th>$a_1$</th>
<th>$b_1$</th>
<th>$c_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.004439</td>
<td>0.004649</td>
<td>0.1034</td>
</tr>
<tr>
<td>$a_2$</td>
<td>$b_2$</td>
<td>$c_2$</td>
</tr>
<tr>
<td>0.002294</td>
<td>0.002294</td>
<td>4.29</td>
</tr>
<tr>
<td>$a_3$</td>
<td>$b_3$</td>
<td>$c_3$</td>
</tr>
<tr>
<td>0.004178</td>
<td>-0.2591</td>
<td>1.045</td>
</tr>
<tr>
<td>$a_4$</td>
<td>$b_4$</td>
<td>$c_4$</td>
</tr>
<tr>
<td>0</td>
<td>2.17</td>
<td>0.001803</td>
</tr>
<tr>
<td>$a_5$</td>
<td>$b_5$</td>
<td>$c_5$</td>
</tr>
<tr>
<td>-0.001891</td>
<td>-0.716</td>
<td>0.6544</td>
</tr>
<tr>
<td>$a_6$</td>
<td>$b_6$</td>
<td>$c_6$</td>
</tr>
<tr>
<td>0</td>
<td>-3.955</td>
<td>$1.97 \times 10^{-5}$</td>
</tr>
<tr>
<td>$a_7$</td>
<td>$b_7$</td>
<td>$c_7$</td>
</tr>
<tr>
<td>0.002022</td>
<td>3.131</td>
<td>14.88</td>
</tr>
<tr>
<td>$a_8$</td>
<td>$b_8$</td>
<td>$c_8$</td>
</tr>
<tr>
<td>0.0009926</td>
<td>-6.86</td>
<td>13.4</td>
</tr>
</tbody>
</table>

2.2.2 DRE Distortion

For the DRE $x$ distortion, a $7^{th}$-order polynomial of the form

$$ f_{DRE-x}(x) = p_1 x^7 + p_2 x^6 + p_3 x^5 + p_4 x^4 + p_5 x^3 + p_6 x^2 + p_7 x + p_8 $$ (2.3)
provided the best fit for the generated histogram. The coefficients for the fit are listed in Table 2.4.

![Figure 2.3: DRE $x$ distortion empirical histogram and fit](image)

Table 2.4: DRE $x$ distortion fit coefficients

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Value</th>
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<tbody>
<tr>
<td>$p_1$</td>
<td>$-2.634 \times 10^{-8}$</td>
</tr>
<tr>
<td>$p_2$</td>
<td>$1.791 \times 10^{-6}$</td>
</tr>
<tr>
<td>$p_3$</td>
<td>$-4.905 \times 10^{-5}$</td>
</tr>
<tr>
<td>$p_4$</td>
<td>$0.0006924$</td>
</tr>
<tr>
<td>$p_5$</td>
<td>$-0.005354$</td>
</tr>
<tr>
<td>$p_6$</td>
<td>$0.02225$</td>
</tr>
<tr>
<td>$p_7$</td>
<td>$-0.04555$</td>
</tr>
<tr>
<td>$p_8$</td>
<td>$0.04249$</td>
</tr>
</tbody>
</table>

For the DRE $y$ distortion, a sum of 8 Gaussians of the form shown

$$f_{DRE-Y}(x) = \sum_{i=1}^{8} a_i \exp \left( -\left( \frac{x - b_i}{c_i} \right)^2 \right)$$  \hspace{1cm} (2.4)
provided the best fit for the generated histogram. The coefficients for the fit are listed in Table 2.5.

![Histogram and fit](image)

Figure 2.4: DRE $y$ distortion empirical histogram and fit

Table 2.5: DRE $y$ distortion fit coefficients

<table>
<thead>
<tr>
<th>$a_1$</th>
<th>$b_1$</th>
<th>$c_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.004874</td>
<td>0.003983</td>
<td>0.2153</td>
</tr>
<tr>
<td>$a_2$</td>
<td>$b_2$</td>
<td>$c_2$</td>
</tr>
<tr>
<td>-0.002411</td>
<td>-0.605</td>
<td>2.388</td>
</tr>
<tr>
<td>$a_3$</td>
<td>$b_3$</td>
<td>$c_3$</td>
</tr>
<tr>
<td>0.005988</td>
<td>-0.1893</td>
<td>2.311</td>
</tr>
<tr>
<td>$a_4$</td>
<td>$b_4$</td>
<td>$c_4$</td>
</tr>
<tr>
<td>0</td>
<td>2.129</td>
<td>0.002029</td>
</tr>
<tr>
<td>$a_5$</td>
<td>$b_5$</td>
<td>$c_5$</td>
</tr>
<tr>
<td>0.0001243</td>
<td>-3.198</td>
<td>0.9514</td>
</tr>
<tr>
<td>$a_6$</td>
<td>$b_6$</td>
<td>$c_6$</td>
</tr>
<tr>
<td>0.0005079</td>
<td>-3.72</td>
<td>1.273</td>
</tr>
<tr>
<td>$a_7$</td>
<td>$b_7$</td>
<td>$c_7$</td>
</tr>
<tr>
<td>0.002747</td>
<td>1.278</td>
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</tr>
<tr>
<td>$a_8$</td>
<td>$b_8$</td>
<td>$c_8$</td>
</tr>
<tr>
<td>0.0005343</td>
<td>-6.401</td>
<td>12.85</td>
</tr>
</tbody>
</table>
2.2.3 DPE Phase Error

For the DPE phase error, a Fourier fit of the form

\[ f_{DPE-\phi}(x) = a_0 + \sum_{i=1}^{8} a_i \cos(i\omega x) + b_i \sin(i\omega x) \]  

(2.5)

provided the best fit for the generated histogram. The coefficients for the fit are listed in Table 2.6.

Figure 2.5: DPE phase error empirical histogram and fit
Table 2.6: DPE phase error fit coefficients

<table>
<thead>
<tr>
<th>$a_0 = -4.244 \times 10^9$</th>
<th>$w = 0.1745$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1 = 6.446 \times 10^9$</td>
<td>$b_1 = 3.982 \times 10^9$</td>
</tr>
<tr>
<td>$a_2 = -2.404 \times 10^9$</td>
<td>$b_2 = -4.803 \times 10^9$</td>
</tr>
<tr>
<td>$a_3 = -2.676 \times 10^8$</td>
<td>$b_3 = 2.98 \times 10^9$</td>
</tr>
<tr>
<td>$a_4 = 7.703 \times 10^8$</td>
<td>$b_4 = -1.028 \times 10^9$</td>
</tr>
<tr>
<td>$a_5 = -3.822 \times 10^8$</td>
<td>$b_5 = 1.499 \times 10^8$</td>
</tr>
<tr>
<td>$a_6 = 9.069 \times 10^7$</td>
<td>$b_6 = 1.654 \times 10^7$</td>
</tr>
<tr>
<td>$a_7 = 9.64 \times 10^6$</td>
<td>$b_7 = -8.709 \times 10^6$</td>
</tr>
<tr>
<td>$a_8 = 2.404 \times 10^5$</td>
<td>$b_8 = 8.318 \times 10^5$</td>
</tr>
</tbody>
</table>

### 2.2.4 DRE Phase Error

For the DRE phase error, a Fourier fit of the form

$$f_{\text{DRE-} \Phi}(x) = a_0 + \sum_{i=1}^{8} a_i \cos(iwx) + b_i \sin(iwx)$$  \hspace{1cm} (2.6)

provided the best fit for the generated histogram. The coefficients for the fit are listed in Table 2.7.
Figure 2.6: DRE phase error empirical histogram and fit

Table 2.7: DRE phase error fit coefficients

<table>
<thead>
<tr>
<th>$a_k$</th>
<th>$b_k$</th>
<th>$w$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-6.931 \times 10^9$</td>
<td>$6.41 \times 10^9$</td>
<td>0.1745</td>
</tr>
<tr>
<td>$1.059 \times 10^{10}$</td>
<td>$6.173 \times 10^7$</td>
<td>1.338 $\times 10^6$</td>
</tr>
</tbody>
</table>
Chapter 3

Data Simulation

This chapter describes the method for generating synthetic images that approximate the result of using the polar format algorithm to form the image. The method used to take an existing image and apply distortion and defocus to it as if it were formed by the polar format algorithm is also described.

3.1 Generating Synthetic Images

First, a number of values need to be defined by the user or script generating the synthetic images. The size of the image (\(N\) pixels by \(N\) pixels) and the pixel resolution (\(\rho\)) (meters/pixel) are properties of the image that need to be defined. The squint angle of the radar needs to be defined and will be used with the pixel resolution to calculate the range to the radar. From [15],

\[
r_{\text{max}} = 2\rho \sqrt{\frac{r_e}{\lambda}}
\]  

(3.1)

can be rearranged to determine the needed radar range \(r_e\) for a given defocus free scene size \(r_{\text{max}}\) and resolution \(\rho\)
\[ r_c = \lambda \left( \frac{r_{\text{max}}}{\rho} \right)^2 \]  

(3.2)

From [14],

\[ \rho = \frac{\lambda r_c}{2L_a \cos(\theta_s)} \]  

(3.3)

can be rearranged to determine the needed aperture length \( L_a \) for a given resolution \( \rho \) and radar range \( r_c \)

\[ L_a = \frac{\lambda r_c}{2\rho \cos(\theta_s)} \]  

(3.4)

and the required bandwidth [14]

\[ B = \frac{c}{2\rho} \]  

(3.5)

to create images with the desired resolution.

The target is defined by three vectors which contain the range and cross range locations as well the amplitude of the points that make up the target. These points are centered around zero. These points are then rotated if necessary to a given angle, \( \theta_r \). For example, \( t_r = [1 \quad -1 \quad 1 \quad -1]^T \) and \( t_{rc} = [1 \quad 1 \quad -1 \quad -1]^T \) are the vectors used to define a target with points at \((1,1), (-1,1), (-1,-1), \) and \((1,-1)\). These can be rotated to an angle \( \theta \) using the standard rotation matrix

\[
\begin{bmatrix}
\cos \theta_r & -\sin \theta_r \\
\sin \theta_r & \cos \theta_r
\end{bmatrix}
\begin{bmatrix}
t_r \\
t_{rc}
\end{bmatrix}
\]  

(3.6)

After the points have been rotated, the points are shifted to the desired location in the image. The spatial distortion is calculated using (1.10) and (1.11) for each of the points and is applied. The quadratic phase error at the center of the image is calculated using (1.12)
and applied. The points are then shifted such that their centroid coincides with the origin.

The image can then be formed using the centered distorted image point vectors. First, using the calculated bandwidth $B$, frequency sample vectors are generated in range and cross range using

$$f_r = \left[ \frac{-B}{2}, \frac{B}{2}, \ldots \frac{B}{2} \right]$$

(3.7)

and

$$f_{cr} = \left[ \frac{-B}{2}, \frac{B}{2}, \ldots \frac{B}{2} \right]$$

(3.8)

Wave number vectors are then calculated using

$$k_r = \frac{4\pi f_r}{c}$$

(3.9)

and

$$k_{cr} = \frac{4\pi f_{cr}}{c}$$

(3.10)

The range and cross range wave number vectors are converted to two dimensional grid versions $k_{rG}$ and $k_{cGr}$ using MATLAB’s “meshgrid” function. The quadratic phase error is calculated for the center of the distorted image and the quadratic coefficient is calculated using

$$a_q = \frac{4\Phi_q(x, y)}{N^2}$$

(3.11)

and the defocus vector is defined as

$$D(n) = \exp(ja_q n^2)$$

(3.12)
where \( n = -N/2 \ldots N/2 \). This vector is converted into a grid \( D_G \) using MATLAB’s "repmat" function. Finally, the signal can be generated using

\[
S(k_r, k_{cr}) = \sum_{m} A_m (\exp(-j(r_m k_{rG} + c r_m k_{crG})) + D_G)
\]

for each of the \( m \) points in the target. The signal \( S(k_r, k_{cr}) \) can then be zero padded and a two-dimensional FFT used to form the image. An example of a formed image is shown in Figure 3.1.

### 3.2 Applying PFA to Existing Images

This section details the method of applying the approximated distortion and defocus to an undistorted image. Undistorted images are distorted through a method involving bilinear interpolation using Delaunay triangulation.

First, the spatial location of each pixel is calculated using the pixel resolution and the given location of the center of the image. Figure 3.2 shows an example of undistorted pixel locations. The distortion for each pixel is calculated using equations (1.10) and (1.11). The
distortion is added to the undistorted pixel locations to give the distorted pixel locations an example of which is shown in Figure 3.3.

The Delaunay triangulation of the distorted pixel locations is then calculated. Delaunay triangulation takes a set of points and creates a smooth surface made of triangles. The points are connected by a set of triangles so that no other points are in the circumcircle of any triangle. Figure 3.4 shows the triangulation of the points in Figure 3.3. The undistorted pixel locations are shifted using the distortion calculated for the center of the image. The image data is interpolated from the distorted triangulation to the undistorted shifted pixel locations to create a rectangular image. This is done by determining which triangle each pixel location is located in and interpolating the image data using the image data for three points of the triangle. Figure 3.5 shows the triangulation with the shifted undistorted pixel locations overlaid. Figure 3.6 shows an example image from the MSTAR dataset. Figure 3.7 shows an example image from the MSTAR dataset with distortion applied using the
previously described method. This is implemented in MATLAB using the built-in "griddata" function. As seen in Figure 3.7, empty areas appear at the corners of the image. To eliminate any effect this may have on the separability tests, both the original image and distorted image are cropped to the same size to remove the empty areas. Figure 3.8 shows an example image from the MSTAR dataset with cropping applied. Figure 3.9 shows an example image from the MSTAR dataset with distortion and cropping applied.
Figure 3.4: Delaunay triangulation formed using distorted pixel location

Figure 3.5: Delaunay triangulation with undistorted pixel locations overlaid
Figure 3.6: Example image from MSTAR dataset

Figure 3.7: Example image from MSTAR dataset with distortion applied
Figure 3.8: Cropped example image from MSTAR dataset

Figure 3.9: Cropped example image from MSTAR dataset with distortion applied
Chapter 4

Separability Measures

Separability measures are used to estimate how difficult it is to correctly classify samples from two sets of data. They provide performance estimates that are not dependent on a particular ATR algorithm. In general, the separability tests take two sets of samples and compare them.

The separability measures use vector representations of the formed images. The generated images are transformed into vectors by sequentially taking each column and stacking them into one column vector. Let $A = \{\alpha_k | k = 1...m\}$ and $B = \{\beta_k | k = 1...n\}$ be defined as a collection of vectorized images of target $A$ and target $B$, respectively.

4.1 Two-Sample Energy Test

The two-sample energy test is a statistical test to determine whether two sets of samples are from the same distribution [17]. It uses the idea of potential energy between electrostatic charges to generate the test statistic. Individual samples from the two sets that have a small distance between them are weighted more than those that have a larger distance between them. The weighting is determined by a user defined, continuous, monotonically decreasing function of the distance between the samples. The test statistic is the sum of the
energy in each set of samples minus the interaction energy between the sets calculated as

\[ E(A, B) = \frac{1}{m^2} \sum_{i<j}^m - \log(||\alpha_i - \alpha_j||) \]
\[ + \frac{1}{n^2} \sum_{i<j}^n - \log(||\beta_i - \beta_j||) \]
\[ - \frac{1}{mn} \sum_{i=1}^m \sum_{j=1}^n - \log(||\alpha_i - \beta_j||) \]  (4.1)

where \( m \) and \( n \) are the number of samples in \( A \) and \( B \), respectively. Smaller values of the test statistic indicates the sets of samples are from the same distribution. To make all the figures in the results section consistent, \( 1 - E(A, B) \) is displayed so a decrease in value indicates an increase in separability.

### 4.2 Friedman-Rafsky Test

The Friedman-Rafsky test is another statistical test to determine whether two sets of samples are from the same distribution [18]. Given two sets of samples, they are combined into one set, and the minimum spanning tree is calculated. Any branches that connect samples from different sets are removed. The number of branches removed is known as the number of runs, \( R(A, B) \). The smaller the number of runs, the more likely the two sets of samples are from different distributions. The Henze-Penrose divergence is a normalization of the Friedman-Rafsky test [19] computed as shown

\[ H(A, B) = 1 - \frac{1 - R(A, B)}{m + n} \]  (4.2)

It normalizes the number of runs so that a value of 1 is completely separable and a value of 0 is unseparable.
A spanning tree is a graph where each pair of points is only connected through one path. A minimum spanning tree minimizes the total distance of the branches in the graph. Prim’s algorithm [20] is a method to generate the minimum spanning tree for a set of points. First, the Euclidean distance between each pair of points is calculated. A branch is added between the two points with the smallest distance between them. The two points are marked as connected. The pair of points with the next smallest distance is then found. If these points are not already connected, a branch is added. Branches are added until all the points in the graph are connected. An example minimum spanning tree for two sets of two dimensional Gaussian data points is shown in Figure 4.1.

### 4.3 Bayes Error Estimation

Bayes error estimation using a Parzen window is a method to estimate the upper and lower bound of the error in classifying two sets of samples [21]. Each sample is tested to see from which set’s probability distribution function the sample is more likely to have come. The sample is then classified as coming from that set. The estimated error is the number of mis-classifications divided by the total number of samples. There are two variations
to estimate the upper and lower bound, which are the re-substitution and leave-one-out methods, respectively.

Both methods use a kernel function to estimate the probability distribution function. This kernel function \[21\] is defined as

\[
k(X) = \frac{m \Gamma\left(\frac{n}{2}\right) \Gamma\left(\frac{n+2}{2m}\right)}{(n\pi)^{\frac{n}{2}}} \cdot \frac{1}{r^n |S|^{\frac{n}{2}}} \cdot \exp \left[- \left\{ \frac{\Gamma\left(\frac{n+2}{2m}\right)}{n\Gamma\left(\frac{n}{2m}\right)} X^T (r^2 S)^{-1} X \right\}^m \right]
\]

where \(m\) determines the rate of drop off for the kernel so \(m = 1\) produces a normal kernel, \(S\) determines the shape of the hyper-ellipsoid, \(r\) controls the size of the kernel, and \(n\) is the dimension of \(X\). For large \(n\) (> 35), this kernel causes overflow errors due to the \(\Gamma(x)\) function. Since in this error estimation method only the normal kernel is used \(m = 1\), the kernel can be reduced to

\[
k(X) = \frac{\Gamma\left(\frac{n}{2}\right) \Gamma\left(\frac{n+2}{2}\right)}{(n\pi)^{\frac{n}{2}}} \cdot \frac{1}{r^n |S|^{\frac{n}{2}}} \cdot \exp \left[- .5X^T (r^2 S)^{-1} X \right]
\]

A further approximation described in [21] is

\[
\frac{\Gamma(x + \delta)}{\Gamma(x)} \approx x^\delta
\]

which is accurate when \(x\) is large and when \(\delta\) is small. This allows a further reduction of the kernel to

\[
k(X) = \left(\frac{1}{2\pi}\right)^{\frac{n}{2}} \cdot \frac{1}{r^n |S|^{\frac{n}{2}}} \cdot \exp \left[- .5X^T (r^2 S)^{-1} X \right]
\]

which allows the kernel function to be used for very large dimensionalities.
4.3.1 Estimating Lower Error Bound for Bayes Error

The resubstitution method uses all the samples from each set when estimating the probability that a sample comes from a given set. Let \( A \) and \( B \) be the two sets that are being tested. When testing samples from \( A \), the resubstitution likelihood ratio test is

\[
- \ln \frac{1}{N_1} \sum_{j=1}^{N_1} k_1(A_i - A_j) w_1 \leq t \leq \frac{1}{N_2} \sum_{j=1}^{N_2} k_2(A_i - B_j) w_2
\]

(4.7)

where each sample of \( A \) from \( i = 1 \ldots N_1 \) is tested. \( k_1(X) \) is the kernel function described in (4.6) with \( S_A \) being the estimated covariance matrix for \( A \) and \( k_2(X) \) is the kernel function described in (4.6) with \( S_B \) being the estimated covariance matrix for \( B \). When the left-hand side is larger than \( t \), the sample is classified as coming from \( B \) and \( A \) otherwise. A similar test is used when testing samples from \( B \),

\[
- \ln \frac{1}{N_1} \sum_{j=1}^{N_1} k_1(B_i - A_j) w_1 \leq t \leq \frac{1}{N_2} \sum_{j=1}^{N_2} k_2(B_i - B_j) w_2
\]

(4.8)

After testing all the samples from both sets, the estimation of the lower bound for the Bayes error is the total number of misclassifications divided by the total number of samples.

4.3.2 Estimating Upper Error Bound for Bayes Error

The leave-one-out method uses all the samples from each set except the sample being tested when estimating the probability that a sample comes from a given set. When testing samples from \( A \), the leave one out likelihood ratio test is

\[
- \ln \frac{1}{N_1-1} \left[ \sum_{j=1}^{N_1} k_1(A_i - A_j - k_1(0)) \right] w_1 \leq t \leq \frac{1}{N_2} \sum_{j=1}^{N_2} k_2(A_i - B_j) w_2
\]

(4.9)

When testing samples from \( B \), the leave one out likelihood ratio test is
\[
- \ln \frac{1}{N_1} \sum_{j=1}^{N_1} k_1 (A_i - A_j) \quad \frac{u_1}{w_1} \leq t
\]

After testing all the samples from both sets, the estimation of the upper bound for the Bayes error is the total number of misclassifications divided by the total number of samples.

4.4 Template Matching

Template matching is a simplistic ATR algorithm to classify samples by comparing them to templates and selecting the best match. Two sets of vectors are used with template matching. There are template vectors and the vectors to be classified. First, all the vectors are normalized by dividing each vector by the 2-norm of the vector. To classify a sample, the inner product of the sample and each template is calculated. The sample is matched to the template for which the dot product is maximum.

When testing the effect of distortion and defocus on template matching, the following method is used. Let \( A_T = \{ \alpha_k | k = 1...m \} \) and \( B_T = \{ \beta_k | k = 1...n \} \) be defined as a collection of noiseless vectorized images of target 1 and target 2, respectively. These are the templates for the template matching process. Let \( A_N = \{ \alpha_k | k = 1...m \} \) and \( B_N = \{ \beta_k | k = 1...n \} \) be defined as a collection of noisy vectorized images of target 1 and target 2, respectively. These are the samples being classified by the template matching process. All collections of images have the same amount of distortion and defocus applied. Each sample from \( A_N \) and \( B_N \) is classified by taking the inner product of the sample with each sample from \( A_T \) and \( B_T \). The sample is matched to the template for which the inner product is maximum.

If a sample is matched to a template of the same type, it is counted as a correct classification. Noise is added to cause the undistorted and focused case to have non-zero classification error. This allows an increase or decrease in the classification error potentially
caused by the distortion and defocus to be seen.
Chapter 5

Results

As previously described, the separability measures use vector representations of the formed images. The generated images are transformed into vectors by sequentially taking each column and stacking them into one column vector. Each generated image is an image of a rotation of the original target. Let $A = \{\alpha_k | k = 1...m\}$ and $B = \{\beta_k | k = 1...n\}$ be defined as a collection of vectorized images of target 1 and target 2, respectively.

5.1 Method of Testing

Two targets are chosen for comparison. If the dataset does not include images of the target at multiple aspect angles, they are generated. Each sample is the image amplitudes in a vector form for each aspect angle. For the Friedman-Rafsky test and Bayes error estimation, due to the complexity of the calculations, each sample was reduced to dimensionality 100 via the discrete cosine transform. All four measures of separability are used on the undistorted sets to get a baseline for the target separability. Using a Monte Carlo simulation, locations are chosen throughout a given scene. At each location, the sets of targets are distorted and defocused based on the approximations in [11]. The measures of separability are used on the distorted sets of targets. This is done for a large number of locations
to characterize how distortion and defocus will affect separability. As the range to scene center increases, the amount of distortion and defocus increases.

The following tables show the parameters used to generate the undistorted and focused sets of data and the Monte Carlo simulation parameters, respectively. Any exceptions will be noted in that dataset’s section.

Table 5.1: Data generation parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Center Frequency</td>
<td>10 GHz</td>
</tr>
<tr>
<td>Rotations</td>
<td>195</td>
</tr>
<tr>
<td>Image Size</td>
<td>256 by 256</td>
</tr>
<tr>
<td>Resolution</td>
<td>0.308 m</td>
</tr>
<tr>
<td>QPE Free Scene Size</td>
<td>50 m</td>
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</tbody>
</table>

Table 5.2: Monte carlo parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Iterations</td>
<td>5000</td>
</tr>
<tr>
<td>Scene Size</td>
<td>300 m</td>
</tr>
<tr>
<td>DCT Coefficients</td>
<td>100</td>
</tr>
<tr>
<td>Squint Angle</td>
<td>0°</td>
</tr>
</tbody>
</table>

Figure 5.1 shows the DRE quadratic phase error as a function of location in the scene. This is shown as reference because a number of the results show a similar pattern.
Figure 5.1: DRE quadratic phase error as a function of location in scene

5.2 Results For Synthetic Data

This section shows the results for the comparison of the two synthetically generated point targets. Figure 5.2 shows the rectangle target and Figure 5.3 shows the rectangle with curved side target. These two targets were created with the idea they would be similar and difficult to separate. Overall, the separability tests show an increase in separability as the range to scene center increases. With the exception of the Friedman-Rafsky test, the tests appear to be largely influenced by the defocus the targets receive.
5.2.1 Two-Sample Energy Test

The two-sample energy test shows the separability of the rectangle target and the rectangle with curved side target increasing with range to scene center. Figure 5.4 plots the result as a function of location in the scene. This is very similar to the way in which the quadratic phase error increases across the scene indicating defocus is the dominant factor in results for this test. Figure 5.5 plots the result as a function of range to the center of the scene.
5.2.2 Friedman-Rafsky Test

The Friedman-Rafsky test shows the separability of the rectangle target and the rectangle with curved side target increasing with range to scene center, but it is very noise like. Figure 5.6 plots the result as a function of location in the scene. There is no discernible pattern to the result. Figure 5.7 plots the result as a function of range to the center of the scene. This
run was for 1000 iterations.

Figure 5.6: Friedman-Rafsky test estimated upper error bound results as a function of location in scene where blue indicates increase in separability

Figure 5.7: Friedman-Rafsky test estimated upper error bound results as a function of range to scene center where decrease in value indicates increase in separability

Due to the noisy results of the Friedman-Rafsky test, it is necessary to verify that the test is working as intended. This is done by generating two sets of two-dimensional Gaussian data points with a number of different mean separations. Each set consists of 20 points. This is done 50 times and the results are averaged. Figure 5.8 plots the results of the Friedman-Rafsky test as a function of mean separation. As expected, the separability
of the two sets increases as the difference between the means of the two sets increases.

![Graph showing Friedman-Rafsky test estimated upper error bound results as a function of mean separation.](image)

Figure 5.8: Friedman-Rafsky test estimated upper error bound results as a function of mean separation where decrease in value indicates increase in separability

### 5.2.3 Bayes Error Estimation

Bayes error estimation shows the separability of the rectangle target and the rectangle with curved side target increasing with range to scene center. Figure 5.9 plots the estimated lower error bound as a function of location in the scene. Figure 5.10 plots the estimated upper error bound as a function of location in the scene. These both show a similar pattern to that of the quadratic phase error indicating defocus is the dominant factor in the results of this test. Figure 5.11 plots the results as a function of range to the center of the scene.
Figure 5.9: Bayes estimated lower error bound results as a function of location in scene where blue indicates increase in separability

Figure 5.10: Bayes estimated upper error bound results as a function of location in scene where blue indicates increase in separability
5.2.4 Template Matching

Template matching shows an increase in separability between the rectangle target and the rectangle with curved side target as the range to scene center increases. Figure 5.12 plots the result as a function of location in the scene. This also shows a pattern similar to that of the quadratic phase error indicating defocus is the dominant factor in the results of this test. Figure 5.13 plots the result as a function of range to the center of the scene.

Figure 5.11: Bayes estimated error bound results as a function of range to scene center where decrease in value indicates increase in separability
Due to the non intuitive results of template matching, it is necessary to verify that the test is working as intended. This is done by generating two sets of two-dimensional Gaussian data points with a number of different mean separations. Each set consists of 20 points. This is done 50 times and the results are averaged. Figure 5.14 plots the template matching classification error as a function of mean separation. As expected, the separability of the two sets increases as the difference between the means of the two sets increases.
5.3 Results For MSTAR Data

The MSTAR dataset contains SAR images of various military vehicles at many aspect angles. For this section, the BMP2 and the T72 vehicles were compared. These were chosen because a previous paper [6] studied the separability of them in the undistorted and focused case. The MSTAR image size is 128 by 128 pixels. Overall, the two-sample energy test and Bayes error estimation both predict an increase in separability as the range to scene center increases while template matching shows a decrease in separability. The Friedman-Rafsky test once again produces very noise like results.

5.3.1 Two-Sample Energy Test

The two-sample energy test shows the separability of the BMP2 and the T72 increasing with range to scene center. It follows a pattern similar to that of the quadratic phase error across the scene. Figure 5.15 plots the result as a function of location in the scene. Figure 5.16 plots the result as a function of range to the center of the scene. This run was for 1000
iterations.

5.3.2 Friedman-Rafsky Test

The Friedman-Rafsky test shows very noise like results. There is no apparent pattern to the variation in separability across the scene. Figure 5.17 plots the result as a function of location in the scene. Figure 5.18 plots the result as a function of range to the center of the scene.
scene. This run was for 1000 iterations.

Figure 5.17: Friedman-Rafsky test estimated upper error bound results as a function of location in scene where blue indicates increase in separability

Figure 5.18: Friedman-Rafsky test estimated upper error bound results as a function of range to scene center where decrease in value indicates increase in separability

5.3.3 Bayes Error Estimation

Bayes error estimation shows the separability of the BMP2 and the T72 increasing with range to scene center. Figure 5.19 plots the estimated lower error bound as a function of location in the scene. Both the upper and lower estimated error bounds show a pattern
similar to that of the quadratic phase error across the scene. Figure 5.20 plots the estimated upper error bound as a function of location in the scene. Figure 5.21 plots the results as a function of range to the center of the scene. This run was for 1000 iterations.

Figure 5.19: Bayes estimated lower error bound results as a function of location in scene where blue indicates increase in separability

Figure 5.20: Bayes estimated upper error bound results as a function of location in scene where blue indicates increase in separability
Figure 5.21: Bayes estimated error bound results as a function of range to scene center where decrease in value indicates increase in separability

5.3.4 Template Matching

Template matching shows a decrease in separability between the BMP2 and the T72 as the range to scene center increases. The pattern across the scene is similar to that of the quadratic phase error across the scene. Figure 5.22 plots the result as a function of location in the scene. Figure 5.23 plots the result as a function of range to the center of the scene. This run was for 1000 iterations.

Figure 5.22: Template matching classification error results as a function of location in scene where blue indicates increase in separability
Figure 5.23: Template matching classification error results as a function of range to scene center where decrease in value indicates increase in separability

5.4 Results For CVDomes Data

The CVDomes dataset consists of simulated SAR data generated using CAD models of vehicles with an electromagnetic scattering simulation [22]. Three of the CVDomes targets were tested. The Honda Civic 4 door was tested against the Toyota Camry because they are similar small sedans. The Honda Civic 4 door was also tested against the Toyota Tacoma because they are less similar than the Civic and the Camry. Overall, both the two-sample energy test and Bayes error estimation show increasing separability with increasing range to scene center while template matching shows a decrease. The Friedman-Rafksy test once again shows noise like results.

5.4.1 Civic vs. Camry

This section presents the results of the Honda Civic 4 door versus the Toyota Camry case.
Two-Sample Energy Test

The two-sample energy test shows the separability of the Civic and the Camry increasing with range to scene center. There is a sharp drop in separability near scene center before increasing. Figure 5.24 plots the result as a function of location in the scene. It follows a pattern similar to that of the quadratic phase error over the scene. Figure 5.25 plots the result as a function of range to the center of the scene.

Figure 5.24: Two-sample energy test results as a function of location in scene where blue indicates increase in separability

Figure 5.25: Two-sample energy test results as a function of range to scene center where decrease in value indicates increase in separability
Friedman-Rafsky Test

The Friedman-Rafsky test once again shows very noise like results. There is no apparent pattern to how the separability varies over the scene. Figure 5.26 plots the result as a function of location in the scene. Figure 5.27 plots the result as a function of range to the center of the scene.

Figure 5.26: Friedman-Rafsky test estimated upper error bound results as a function of location in scene where blue indicates increase in separability

Figure 5.27: Friedman-Rafsky test estimated upper error bound results as a function of range to scene center where decrease in value indicates increase in separability
Bayes Error Estimation

Bayes error estimation shows the separability of the Civic and the Camry increasing with range to scene center. They appear to follow the pattern of the quadratic phase error in the scene. Figure 5.28 plots the estimated lower error bound as a function of location in the scene. Figure 5.29 plots the estimated upper error bound as a function of location in the scene. Figure 5.30 plots the results as a function of range to the center of the scene.

Figure 5.28: Bayes estimated lower error bound results as a function of location in scene where blue indicates increase in separability

Figure 5.29: Bayes estimated upper error bound results as a function of location in scene where blue indicates increase in separability
Figure 5.30: Bayes estimated error bound results as a function of range to scene center where decrease in value indicates increase in separability

**Template Matching**

Template matching shows a decrease in separability between the Civic and the Camry as the range to scene center increases. The template matching error sharply increases away from scene center then approaches 0.5. This could be caused by the fact that the Camry and Civic are very similar targets and any amount of defocus causes them to be easily confused. This leads to template matching becoming essentially a coin flip. Figure 5.31 plots the result as a function of location in the scene. Figure 5.32 plots the result as a function of range to the center of the scene.
5.4.2 Civic vs. Tacoma

This section presents the results of the Honda Civic 4 door versus the Toyota Tacoma case.
Two-Sample Energy Test

The two-sample energy test shows the separability of the Civic and the Tacoma increasing with range to scene center overall. There is a sharp drop in separability near scene center before increasing. A potential cause for this behavior is given with the template matching results. The separability is greater than the Civic versus Camry case as would be expected. Figure 5.33 plots the result as a function of location in the scene. It follows a pattern similar to that of the quadratic phase error over the scene. Figure 5.34 plots the result as a function of range to the center of the scene.

![Two-sample energy test results as a function of location in scene where blue indicates increase in separability](image-url)

Figure 5.33: Two-sample energy test results as a function of location in scene where blue indicates increase in separability
Figure 5.34: Two-sample energy test results as a function of range to scene center where decrease in value indicates increase in separability

**Friedman-Rafsky Test**

The Friedman-Rafsky test for the Civic and the Tacoma case returned that the two were completely separable over the entire scene.

**Bayes Error Estimation**

Bayes error estimation for the Civic and the Tacoma case returned that the two were completely separable over the entire scene. This corroborates the results of the Friedman-Rafsky test.

**Template Matching**

Template matching shows a decrease in separability between the Civic and the Tacoma as the range to scene center increases. The template matching error starts by sharply increasing away from scene center. Next there is a small period of decreasing error and finally the error approaches 0.5 towards the edge of the scene. A potential cause for this behavior is that initially template match struggles to classify the two targets. The separability then
increases in areas with small amounts of defocus and some distortion. Finally, in areas with large amounts of defocus the separability decreases. The overall pattern across the scene is similar in shape to that of the defocus. Figure 5.35 plots the result as a function of location in the scene. Figure 5.36 plots the result as a function of range to the center of the scene.

Figure 5.35: Template matching classification error results as a function of location in scene where blue indicates increase in separability

Figure 5.36: Template matching classification error results as a function of range to scene center where decrease in value indicates increase in separability

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Chapter 6

Conclusion

In this thesis, the impact of SAR image formation quality on target separability was studied. First, probabilistic descriptions of image formation errors caused by the approximations in the PFA were generated empirically. An attempt was made at a theoretical derivation of them but was unsuccessful. Next, four different tests to measure the separability between two sets of data were discussed and were implemented in MATLAB. To measure the effects of the PFA on target separability, undistorted and distorted data needed to be generated. A MATLAB tool that generates undistorted and distorted images of a target consisting of a user defined set of points was developed. Also a MATLAB tool that takes preformed images and distorts them based on the effects of the PFA was developed. Finally, using the previously described tools, the impact of the PFA on target separability was studied by using the separability measures on undistorted and distorted generated data, MSTAR data, and CVDomes data.

For the two synthetically generated targets, the two-sample energy test, Bayes error estimation and template matching classification error were in agreement. They all showed an increase in separability as the range to scene center increases. The Friedman-Rafsky test did not show a pattern and was very noise like. This contrasts with the results for the more complex CVDomes and MSTAR datasets. The two-sample energy test and Bayes error
estimation both predicted an increase in separability as the range to scene center increases. Template matching showed a decrease in separability as the range to scene center increased. Once again the Friedman-Rafsky test results did not show a pattern and was very noise like.

Overall, the two-sample energy test and Bayes error estimation predict an increase in separability for larger amounts of distortion and defocus. Template matching shows an increase in separability for the comparison of the two simplistic targets. For the more complex target comparisons using CVDomes and MSTAR data, template matching shows a decrease in separability with larger amounts of distortion and defocus. The template matching results for the CVDomes and MSTAR data were the only results that match the expectation of decreased separability away from scene center. The rest of the results show an increase in separability away from scene center. This makes it difficult to generate image scene size bounds using target separability.
Bibliography


