The Warehouse-Inventory-Transportation Problem for Multi-Echelon Supply Chains

A dissertation submitted
in partial fulfillment of the requirements for the degree of
Doctor of Philosophy

By

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ABSTRACT

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The Warehouse-Inventory-Transportation Problem for Multi-Echelon Supply Chains.

Warehouses play a vital role in mitigating variations in supply and demand, and providing value-added services in a supply chain. However, our observation of supply chain practice reveals that warehousing decisions are not included when developing a distribution plan for the supply chain. This lack of integration has resulted in substantial variation in workload (42%-220%) at our industry partner’s warehouse costing them millions of dollars. We address this real-world challenge by investigating the interdependencies between warehouse, inventory, and transportation decisions, integrate them in a mathematical programming model, and develop managerial insights based on solutions of industry-sized problem instances. Our three contributions to research in supply chain are as follows.

First, we introduce the warehouse-inventory-transportation problem (WITP), which determines the optimal distribution strategy from vendors to customers via one or more warehouses in order to minimize total distribution costs. We model WITP as a nonlinear integer programming model considering multiple vendors, stores, products, and time-periods, and one warehouse. The model also considers worker congestion at the
warehouse that could affect worker productivity. Our experiments indicate that the
distribution plans obtained via the WITP, as compared to a sequential approach of
solving an integrated inventory-transportation problem first and then solving the
warehousing problem, result in a substantial reduction in workload variance at the
warehouse, while considerably reducing the total distribution cost. These plans, however,
are sensitive to the aisle configuration and technology at the warehouse, and the level and
productivity of temporary workers. The state-of-the-art commercial solver could only
solve small problem instances.

Second, to solve industry-sized problems, we developed a heuristic framework.
This framework incorporates key features from the well-established Iterated Local Search
(ILS) meta-heuristic. The heuristic implements three sets of neighborhood moves
intended to improve warehousing, inventory, and transportation costs. It searches for a
better solution in two alternating phases, a local search phase and a perturbation phase.
We found that the solutions from the heuristic were close to optimal on small problem
instances. Additionally, the heuristic was able to solve efficiently industry-sized problems
with up to 500 stores and 1,000 products.

Third, we extend the WITP to model distribution decisions for supply chains that
manage the flow of products with varying life cycles. The varying demand patterns of
such products (e.g., basic and fashion) require different sets of decisions with different
objectives; cost-efficiency for basic products and time-effectiveness for fashion products.
These differences are typically handled by supply chains separately when planning for inventory and transportation. But these decisions are not necessarily separable from a warehousing perspective as both these product classes are simultaneously handled by identical resources at the warehouse (i.e., workers and technology). The extension of WITP (from single to multiple products classes) increased the complexity of the nonlinear model further when generating optimal distribution plans. We extend the ILS-based meta-heuristic framework, which we refer to as the Three Phase Heuristic (TPH), to solve industry-sized problems (e.g., 50 vendors, 200 stores, 1,000 products, and 28 time-periods). Experimental results demonstrate that TPH results in higher quality solutions with a reduction of up to 19% in total distribution costs when compared to an ad hoc policy. We also notice that the distribution plans are sensitive to the (i) duration of fashion window, (ii) product mix (basic vs. fashion), (iii) warehouse labor cost, and (iv) warehouse technology adopted for putaway and picking activities. Several managerial insights are presented to help supply chain managers in better aligning their processes to mitigate workload variation and reduce cost.

The algorithms and findings from this research are being embedded as part of an online tool, which is currently being tested in an upper level graduate class on Supply Chain, and will soon be piloted at our partnering facility.
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With the blessings of Lord Sainath Maharaj

Dedicated to the lotus feet of
Sadguru Sri Sainathuni Sarath Babuji
1. Introduction

The U.S. economy has been transforming from a manufacturing-oriented to a service-oriented infrastructure. Frequently, products are manufactured outside of the U.S., which has led to a significant emphasis on how they are delivered to U.S. ports and subsequently distributed to U.S. consumers. The success of a company depends on how effectively it manages the flow of products in its supply chain. The three logistical drivers (facilities, inventory, and transportation) and the three cross-functional drivers (information, sourcing, and pricing) play a key role in the successful management of the flow of products in the supply chain. A proper balance between the roles played by each of these drivers enhances the company’s supply chain performance in terms of responsiveness and efficiency (Chopra and Meindl, 2012). On the other hand, improper decisions in handling these key drivers would cost companies their competitive edge, especially in the current volatile economy, and contribute to significant job loss.

A variety of strategies have been proposed to design and optimize supply chains to help enterprises stay competitive in this volatile economy. Identifying various classes of products that flow through the supply chain and aligning different components of the supply chain to these product classes are considered to be effective strategies adapted by successful companies. Based on the duration of product-life, products are classified into the following three categories (Şen, 2008):
i. **Basic Products**: These are the products with a relatively long product-life. For example, apparel goods like denim pants and shirts, under garments, etc., or groceries like sugar, salt, etc., are sold throughout the year. Fisher (1997) refers to this class of products as functional products, as these products cater to the basic needs of a customer which do not change much over a period of time. These products generally have more or less stable and predictable demand, and low profit margins. Companies strive towards having an efficient supply chain in dealing with basic products with an objective of maximizing performance and minimizing physical costs of the supply chain such as purchase costs, inventory storage costs and transportation costs.

ii. **Fashion Products**: Products with an approximate life of 10 weeks are considered as fashion products. Fisher (1997) refers to this class of products as innovative products. As the profit margin is high for the fashion products, companies introduce a number of product varieties into the market during each fashion event of the year to gain more profits and to have a competitive edge. The shorter life cycle and higher product variety makes this group of products highly unpredictable. A high error in the prediction of demand would lead to either stock-outs or stock-overflow both of which incurs heavy loss to the companies. Thus the companies handling fashion products would tend to be more responsive in order to supply their customers right amount of products at right time to not only maximize their profits but also to minimize the market mediation costs such as costs incurred due to marked down / salvage costs and lost sales.
iii. **Seasonal Products**: Seasonal products are the products with approximately 20 weeks product life. These products tend to exhibit a mix of characteristic features of basic and fashion products.

A company would fall short of its goal if it does not identify and align its supply chain to its product groups; i.e., products with longer life (basic products) have efficient supply chain and products with shorter life (fashion products) have responsive supply chain. Whether it is a basic or fashion product, companies depend heavily on warehouses in satisfying their customer demands. Though the need for warehouses is increasing day-by-day but still the current approaches to supply chain planning are almost exclusively based on an integration of inventory and transportation decisions; e.g., the inventory-routing problem (IRP), and give little to no consideration to warehousing decisions (see Figure 1.5). The real example of this lack of integration is the warehouse in the Midwest US, which motivated our research. The lack of integration has affected the company’s bottom-line substantially.

**1.1 Motivation**

This research was motivated by observing the challenges faced by the Senior Director of logistics department at the warehouse of a U.S.-based company. This company builds world-class brands of fashion and related products while carrying basic products, and sells to consumers through retail and e-commerce channels of distribution.

The company’s supply chain consists of two echelons comprising of vendors, a warehouse, and stores (see Figure 1.1). The supply chain includes a single warehouse in the Midwest and manages the flow of 6,500 - 8,000 stock-keeping units (SKUs) supplied by over 100 domestic and overseas vendors. This warehouse replenishes over 300 retail
stores situated in nearly 40 states across the nation. The inbound shipments from overseas reach the warehouse via rail and road after entering the Los Angeles port on the West Coast. The outbound shipments from this warehouse are delivered to both the stores and consumers through one of two transportation modes (depending on the shipment weight), less-than-truck load (LTL) and parcel.

Figure 1.1 An illustration of the supply chain of the US-based apparel company

The company’s procurement department considers current inventory levels, expected demand, and lead times to decide what products to order, when to order, how much to order, and how to deliver from vendors to warehouse. The allocation department takes into account information about inbound shipments, inventory levels, and store demands when determining what products to deliver, when to deliver, how much to deliver, and how to deliver from warehouse to stores.

Although this warehouse serves as a hub in the supply chain, it operates in a reactive mode; that is, inventory and transportation plans are determined first and the
warehousing plans are determined later. This sequential approach results in the warehouse experiencing substantial variation in daily workload, which causes the warehouse manager to scramble for resources during peak times and experience resource under-utilization during drought times. Figure 1.2 shows the number of units picked per week at the company’s warehouse in the year 2011, where the weekly variation in the workload ranges from nearly 42% to 220% of that year’s weekly average. Data from another of our industry partners, a Fortune 100 grocery distributor, during August 29 – September 4 of 2011 indicated a variation in the number of units picked (76% and 153%) at one of their US warehouses (see Figure 1.3).

Figure 1.2 Weekly variation in the units picked at the US-based apparel warehouse
A close observation of Figure 1.2 also suggests another important point. Our industry collaborator being an apparel fashion retailer sells both basic and fashion products. The decisions for basic products follow a traditional approach where replenishment orders from store to warehouse (and in turn to vendor) are placed based on point-of-sale data. In contrast, decisions for fashion products, typically exhibiting more uncertain demand than the basic products, are managed much differently largely because of the strict requirement to make the product available at stores at predetermined times during the year to ensure a competitive edge. In addition, each fashion product at the warehouse arrives from the vendor as a single consolidated shipment and is deconsolidated based on a predetermined allocation quantity for each store — all this in a very short time-frame, typically 2 weeks. These fashion products are sold at stores within 3-4 weeks, before the next month’s fashion products arrive. This closely aligns with our observation of increased outbound activity for about 1-2 weeks at the warehouse for every 3-4 weeks during the year 2011 (see Figure 1.2). The workload variation

![Figure 1.3 Variation in the units picked at a warehouse of a Fortune 500 grocery distributor](image-url)
experienced by the warehouse in handling both the basic and fashion products with varied inbound and outbound schedules and the resulting inefficiencies clearly indicates a need for considering both product classes simultaneously in developing supply chain plans.

The key point here is that such a sequential planning approach might lead to significant workload imbalance at the warehouse, which could potentially increase (i) difficulty in workforce management and scheduling, (ii) increased need for overtime and temporary workers, and (iii) improper utilization of technological resources. All of these result in decreased productivity leading to substantial operational inefficiencies at the warehouse and can cost a company millions of dollars annually. In order to model these decisions, one must understand the role of warehousing in a supply chain.

1.2 Role of Warehousing in Supply Chain

Warehouses have emerged from their traditional passive role of serving as buffers to mitigate supply-demand variations to a more active role of providing value-added services such as consolidation/deconsolidation, assembling, kitting, etc. Considering the success of Amazon.com and alike, and the role their warehouses play in the supply chain, warehouses are often referred to as distribution centers (DCs). Warehousing costs in 2012 were nearly $120 billion across 600,000 small and large warehouses in the U.S., which is over 9% of the $1.28 trillion of the U.S. logistics cost (CSCMP, 2013).

The operations at the warehouse can be broadly divided into two broad categories:

i. Putaway - refers to the activity of moving products from the point of unloading to the storage/picking area of a warehouse.
ii. Picking - refers to the activity of fulfilling a customer order by picking and packing products from the picking area.

Other key activities include inspection and quality control, inventory management and return processing, and administrative functions. Figure 1.4 depicts the flow of products and the operations within a warehouse. Cross-docking, which refers to the activity of moving products directly from the receiving stage to the shipping stage avoiding the intermediary steps of putaway, storage and picking is also employed at many warehouses. All these operations are important and need to be well-coordinated and effectively operated for a warehouse to process orders quickly, effectively, and accurately (Tompkins et al., 2010).

Figure 1.4 Common warehousing activities
The productivity and throughput of a warehouse is influenced by several factors such as implementation of various policies and strategies, configuration of aisles, usage of different material handling equipment and assisting technologies. Table 1.1 presents the list of decisions involved with each of those factors for a pallet storage and case picking system. The decisions associated with each factor not only affect warehouse throughput and costs, but also affect one another. Warehouse managers often struggle to determine the right combination of all these factors that would result in optimal putaway or pick rates, thereby, helping them to effectively handle inflow and outflow of products. We call such a combination of decisions as technology. As the decisions in each factor are interdependent, it is essential that the resulting combination of all of these decisions is practically feasible to be implemented at an operational level. An example of a feasible technology would be a pallet storage system at a warehouse with wide aisle configuration using block stacking storage system with randomized storage policy, where pallets are putaway using a direct strategy to the storage area via a counter balanced lift truck and paper based technology. Similarly, a feasible technology in an order picking system would be batch picking cases from case flow racks with class-based storage policy using pallet jacks and pick-to-light technology.

These decisions have a significant bearing on warehouse throughput and costs, which, in reality, impact other supply chain decisions, such as inventory and transportation. For instance, narrow-aisles in the picking area of a warehouse reduce the required space (thus, space cost), but increase the probability of worker congestion. The resulting decrease in worker productivity reduces the warehouse’s throughput capacity, impacting the inbound and outbound transportation decisions. Similarly, the ability to
cross-dock at the warehouse may require advanced material handling equipment and information technology infrastructure; both come at a cost. Notice that cross-docking is associated with no inventory in the warehouse — an inventory decision is impacted by a warehousing decision.

Table 1.1 Factors affecting putaway and picking activities at a warehouse

<table>
<thead>
<tr>
<th>Key Factors</th>
<th>Putaway</th>
<th>Picking</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Unit of measure</strong></td>
<td>Pallet / Case</td>
<td>Case / Piece</td>
</tr>
<tr>
<td><strong>Strategy</strong></td>
<td>Direct</td>
<td>Discrete</td>
</tr>
<tr>
<td></td>
<td>Directed putaway</td>
<td>Batch</td>
</tr>
<tr>
<td></td>
<td>Batched and sequenced</td>
<td>Zone</td>
</tr>
<tr>
<td></td>
<td>Interleaving and</td>
<td>Bucket brigade</td>
</tr>
<tr>
<td></td>
<td>continuous moves</td>
<td></td>
</tr>
<tr>
<td><strong>Aisle Configuration</strong></td>
<td>Length, height, width, number, orientation</td>
<td>Length, height, width, number, orientation</td>
</tr>
<tr>
<td><strong>Storage System</strong></td>
<td>Floor or bulk storage</td>
<td>Carousals</td>
</tr>
<tr>
<td></td>
<td>Rack storage</td>
<td>Pallet racks</td>
</tr>
<tr>
<td></td>
<td>(stacking frames, pallet rack, double deep rack, drive-in rack, drive-thru rack, pallet flow rack)</td>
<td>Static shelves</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Case flow racks (gravity)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Bin shelving</td>
</tr>
<tr>
<td><strong>Storage Policy</strong></td>
<td>Randomized</td>
<td>Randomized</td>
</tr>
<tr>
<td></td>
<td>Class-based</td>
<td>Class-based</td>
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<tr>
<td></td>
<td>Volume-based</td>
<td>Volume-based</td>
</tr>
<tr>
<td></td>
<td>Cube-per-Order Index (COI)</td>
<td>Cube-per-Order Index (COI)</td>
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<tr>
<td></td>
<td>Shared storage</td>
<td>Shared storage</td>
</tr>
<tr>
<td><strong>Material Handling Equipment</strong></td>
<td>Walkie stacker</td>
<td>Tote</td>
</tr>
<tr>
<td></td>
<td>Counter Balance Lift Truck (CBLT)</td>
<td>Cart</td>
</tr>
<tr>
<td></td>
<td>Straddle truck</td>
<td>Pallet jack</td>
</tr>
<tr>
<td></td>
<td>Side loading truck</td>
<td>Order picker truck</td>
</tr>
<tr>
<td></td>
<td>Turret truck</td>
<td>Pick-to-belt</td>
</tr>
<tr>
<td></td>
<td>Hybrid storage/retrieval</td>
<td>Miniload ASRS</td>
</tr>
<tr>
<td></td>
<td>Automated Storage and Retrieval Systems (ASRS)</td>
<td>A-Frame</td>
</tr>
<tr>
<td><strong>Assist Technology</strong></td>
<td>Paper-based</td>
<td>Paper-based</td>
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<tr>
<td></td>
<td>Put-to-light</td>
<td>Put-to-light</td>
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<td></td>
<td>Voice-directed</td>
<td>Voice-directed</td>
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</table>
A supply chain would fall short of its goals if its warehouse is not successful in processing customer orders quickly, effectively, and accurately. There is no doubt that the growing importance of warehouses in modern supply chains has been acknowledged, but we have observed an apparent disconnect between inventory, transportation, and warehousing decisions. Optimizing warehousing decisions has almost always been considered secondary to optimizing inventory and transportation decisions. This lack of integration can, and does, cost millions of dollars in operational inefficiencies at warehouses. In the current volatile economy these costs can cost companies their competitive edge and contribute to significant job loss.

![Diagram of warehousing, inventory, and transportation decisions](image)

**Figure 1.5 Integration of warehousing, inventory, and transportation decisions**

The observed inefficiencies at the warehouse of our industry partners beg the following question: *how would a supply chain benefit if it proactively accounted for warehousing decisions at the tactical planning phase, instead of reacting passively every*
day? This question motivated us to introduce the warehousing-inventory-transportation problem (WITP) to the supply chain literature. The WITP integrates decisions regarding warehouse, inventory, and transportation, and identifies an optimal distribution strategy for a multi-echelon, multi-product, and multi-period supply chain such that the total distribution chain cost is minimized.

1.3 Research Contributions

Our research is an out-growth of the needs of our industry partners and the lack of approaches in academic literature, with an emphasis on incorporating warehousing decisions during supply chain planning. Below we indicate our research contributions.

1.3.1 Contribution 1

We introduce to the supply chain literature the integrated warehousing-inventory-transportation problem (WITP) that jointly considers warehouse utilization and capacities, along with inventory and transportation decisions to identify the optimal distribution strategy. We develop nonlinear models to address WITP for multi-echelon supply chains. The key aspect we capture in our model is a critical operational element of worker dynamics modeled via picker blocking, which has been a hot topic of discussion and analysis in recent articles on warehouse operations. We also consider other strategic and tactical decisions such as aisle configuration and layout (wide and narrow), warehouse technology, allowable number and productivity of temporary workers, and study their impact on (i) warehouse workload variation and workforce cost and (ii) inventory and transportation decisions. The optimal solutions obtained from the
integrated WITP for small problem instances has resulted in a savings of up to 28% compared to the optimal solutions obtained from sequential approach.

1.3.2 Contribution 2

From a solution perspective, WITP could be considered as NP complete (discussed in detail in Chapter 3). It is also analogous to a two-stage capacitated lot-sizing problem, which typically has weak linear programming (LP) bounds and lacks strong cutting planes (Bitran and Yanasse, 1982). Our preliminary experiments show that though the LP relaxation of WITP can be solved easily, it is difficult to obtain an optimal or a near optimal solution within 6 hours, even for small problem instances. For example, the best solution obtained for a problem instance with 1 vendor, 1 warehouse, 20 stores, 1 product, and 5 time-periods using the Xpress MIP solver has an optimality gap of over 10%. A multi-echelon supply chain can have over 100 vendors, more than 1 warehouse, over 100 stores, and over 1,000 products. The total number of integer variables for WITP instances of this size is over a billion. To obtain near-optimal solutions for industry-sized problems, we develop an Iterated Local Search (ILS) based meta-heuristic optimization framework to effectively and efficiently solve the deterministic WITP for large problem instances.

The experimental results on smaller datasets show that the heuristic solutions either match or lie within 1% of the optimal solutions for most of the problem instances. The heuristic even outperformed the exact solutions that could not always reach optimality. Moreover, the heuristic was so fast that we noticed a considerable difference in the runtime between the heuristic and optimal solutions. The variance in the daily warehouse workload obtained by the proposed heuristic is comparable to that obtained
through the optimal solution suggesting that the heuristic is able to balance warehouse workload. The heuristic was then used to solve the industry-sized problems with 10 vendors, 500 stores, 1000 products, and 5 time-periods. Further analysis on such problems indicated that the WITP plans were sensitive to other warehousing decisions such as aisle configuration (which affects worker congestion), technology (which determines the worker productivity), and allowable level and productivity rate of temporary workers.

**1.3.3 Contribution 3**

We extend the WITP to account for two product classes: basic and fashion. As indicated earlier, the varying demand patterns and life-cycles associated with each product class requires different sets of decisions with different objectives; cost-efficiency for basic products and time-effectiveness for fashion products. These differences are typically handled by supply chains separately when planning for inventory and transportation. But these decisions are not necessarily separable from a warehousing perspective as both these product classes are simultaneously handled by the same warehouse resources (i.e., workers and technology). The substantial differences in supply and demand patterns for these two product classes, combined with their warehousing needs has led to high workload variation and operational inefficiencies at the warehouse of our industry partners.

The WITP extension includes two product classes and determination of technology at the warehouse. The resulting model is a nonlinear MIP. As a solution approach to this complex nonlinear problem, we modify substantially the ILS-based meta-heuristic framework developed for WITP for a single product class in order to
address the decisions related to the distribution of fashion products in addition to the basic products. We refer to this framework as Three Phase Heuristic (TPH).

The TPH was efficient in solving industry-sized problems. The experimental results show that as the proportion of fashion products flowing through the warehouse increases the variation in the workload increases substantially. The TPH was also efficient in generating solutions with best suitable technologies for putaway and picking activities at the warehouse that leads to minimum total supply chain costs. When compared with a naïve fashion policy such as Basic First Fashion Next (BFFN) the TPH solution has shown a reduction of 19% in the total distribution costs.

1.4 Dissertation Outline

The remainder of this proposal is organized as follows. Chapter 2 presents a comprehensive literature review on various integrated models for supply chain planning with an emphasis on warehousing. Contributions 1 and 2 are detailed in Chapter 3. The details of Contribution 3 are elaborated in Chapter 4. Finally, Chapter 5 summarizes the conclusions we draw from this research, and also presents avenues for future research.
This chapter reviews existing literature in the area of supply chain planning that integrate key drivers of the supply chain like production, inventory, transportation, and warehousing. Section 2.1 focuses on single/multi-echelon supply chains handling basic products. Literature on the design and planning of supply chains handling multiple product classes are discussed in Section 2.2. Section 2.3 identifies important questions, yet to be addressed in the area of WITP, which serve as objectives of our research.

### 2.1 Supply Chains Handling Basic Products

In this section we discuss integrated models for a single/multi-echelon supply chain such as inventory-routing problem, inventory-location problem, production-inventory-distribution-routing problem, and models pertaining to warehouse location, design, and operation.

#### 2.1.1 Integrated Models for Inventory-Routing Problem

Past research on developing integrated supply chain/distribution models have focused on integrating production, inventory, and transportation decisions. A popular integrated problem in this area is the *inventory-routing problem* (IRP), which refers to developing a repeatable distribution strategy that minimizes transportation costs and the number of stock-outs. Both deterministic and stochastic IRP-versions have been introduced in the literature (Campbell et al., 1998; Kleywegt et al., 2004; Lin and Chen, 2008).
Abdelmaguid and Dessouky (2006) argue that IRP primarily focuses on minimizing the total transportation costs, with little consideration for inventory costs. Consequently, they propose an integrated inventory-distribution problem (IIDP) that considers inventory and transportation costs, allowing backorders in a supply chain consisting of one warehouse and multiple customers with deterministic demand in multiple time-periods. They propose a non-linear mixed integer programming model for IIDP and solve it using a genetic algorithm.

Çetinkaya et al. (2006) present a renewal theoretic model to compute parameters of an integrated inventory-transportation policy where demand follows a general stochastic process. They consider two-echelon supply chain with one vendor, one customer, and one product, unit transportation cost that includes handling (i.e., loading a trailer), and inventory-related costs at the vendor’s warehouse. They compute simultaneously the optimal order quantity for inventory replenishment at the vendor and the optimal dispatch quantity for outbound shipments and study the impact of shipment consolidation on the expected long-run average costs. However, workforce requirements associated with other warehousing activities, such as unloading, put-away, picking, cross-docking, worker congestion, and worker stratification are not captured in this paper.

Parthanadee et al. (2006) propose MIP model to solve a multi-product, multi-depot periodic distribution problem. The model assumes that the product supplies are limited at some depots. So the model, along with backorders, also allows depots to operate interdependently. The paper claims that the use of long term memory in the diversification process of the tabu search provides effective solutions for large problem
instances. Also, the interdependent operations yielded better savings compared to the independent operations among depots.

Zhao et al. (2008) address a deterministic inventory routing problem (DIRP) with frequency (periodic and long-term operations) approach. The two-echelon logistics system they consider comprises of one supplier, one warehouse, and multiple customers with deterministic customer-specific demand rate for a single product. They propose a fixed partition and power-of-two strategy to solve the problem and try to improve the obtained solution with variable large neighborhood search (VLNS) algorithm. The approach was tested on 50 and up to 75 customers. One of the limitations of this model is that it does not impose inventory constraints either on warehouse or on the customers.

Çetinkaya et al. (2009) propose an MIP model for a large-scale, integrated multi-product inventory lot-sizing and vehicle-routing problem. The model considers direct (plant-to-store) and interplant (plant-to-plant) deliveries. The problem being NP-hard, they try to solve with a heuristic that decomposes the overall problem into inventory and routing sub-problems. The heuristic improves only the outbound efficiency of the supply chain and does not consider the inbound.

2.1.2 Integrated Models for Inventory-Location Problem

The integration of inventory and warehouse location decisions has been addressed before (Daskin et al., 2002; Shen et al., 2009; Üster et al., 2008). Shen and Daskin (2005) study the trade-offs between customer service quality and cost in a joint inventory-location problem. Üster et al. (2008) consider a three-tier distribution system with one vendor, one intermediate warehouse, and multiple customers. They propose an integrated location-inventory model to determine the optimal location of the warehouse with an objective of
minimizing the system-wide transportation and inventory-related costs. The results in this paper demonstrate the impact of integrated approach over the traditional approach of solving the location and inventory related decisions separately.

Ozsen et al. (2009) extend their earlier integrated deterministic location-inventory model to minimize the sum of fixed warehouse location, transportation, and inventory costs. They analyze the impact of multi-sourcing on location and inventory decisions, and indicate that multi-sourcing becomes a more valuable option as transportation costs increase.

2.1.3 Integrated Models for Production-Inventory-Distribution-Routing Problem

Lei et al. (2006) consider the production-inventory-distribution-routing problem (PIDRP), where the focus is on coordinating the production and transportation schedules between a set of vendors and a set of customers (which could be warehouses). A two-phase sequential approach is used to solve a multi-plant, multi-DC, and multi-period PIDRP. In Phase I the transportation routings are restricted to direct shipments and solve the original MIP model. The potential inefficiency of direct shipments is corrected by solving the associated consolidated problem in Phase II using a heuristic procedure. The results show that this two-phase approach requires less than 1 minute of CPU time for most of their test problems and in 70% of cases generates solution better than the one generated by CPLEX MIP solver in a 4-hour CPU time.

Bard and Nananukul (2008) follow a similar methodology to that of Lei et al. (2006) to solve a one-plant, multi-customer PIDRP assuming a single mode of transportation. In Phase I they use an allocation model to determine optimal production and delivery quantities. The good feasible solutions obtained in Phase I are subsequently
improved in Phase II by employing a reactive tabu search algorithm with path-relinking. Their study differs from the traditional IRP as it considers the trade-off between production decisions and inventory levels at the facility.

Boudia et al. (2009) propose memetic algorithm with population management (MA|PM) to solve integrated production-distribution problem (IPDP) with one facility, one product, and multiple customers with deterministic demand for multiple time-periods. Their test results on problem instances with 20 time-periods and 50, 100 and 200 customers show that MA|PM outperforms the two-phase heuristic and greedy randomized adaptive search procedure (GRASP).

2.1.4 Models on Warehouse Location, Design, and Operation

In the area of warehousing academic literature has focused primarily on warehouse location, design, and operation. White and Francis (1971) were probably the first researchers to develop quantitative models to decide between private and leased warehouses. Since then numerous models have been developed to assist in warehouse design, more specifically sizing (Goh et al., 2001; Heragu et al., 2005; Ng et al., 2009), aisle-layout (Roodenberg and Vis, 2006; Gue et al., 2009), and operational aspects (Ratliff and Rosenthal, 1983; Parikh and Meller, 2010).

Warehousing cost include the costs related to the utilization of workforce, space, equipment, and utilities. Decreased worker productivity leads to increased labor cost. One of the key factors to reduce the productivity of the workers is the warehouse congestion caused due to the worker interaction in the aisles (Parikh and Meller, 2010). Gue et al. (2006) use worker blocking as a surrogate for congestion. They were the first to propose analytical models to estimate picker blocking in narrow-aisle order picking system
(OPS). Their results conclude that blocking increases with increase in the number of pickers but decreases with increase in the number of pick columns.

Parikh and Meller (2009) estimated worker blocking in the wide-aisle systems. They propose analytical models for non-deterministic pick times and conclude that blocking increases monotonically with an increase in pick density. Recently, Parikh and Meller (2010) show that non-deterministic pick-times (caused due to variation in the number of picks at a pick-column) result in higher blocking than previously observed in narrow aisles.

2.2 Supply Chains with Multiple Product Classes

In this section we discuss literature that i) align product to supply chain strategies, and ii) integrate and coordinate approaches in the fashion industry.

2.2.1 Alignment of Supply Chains to Product Classes

Many supply chains experience problems because of the mismatch between the type of products and type of supply chain (Fisher, 1997). Fisher (1997) classified products based on their demand patterns into two categories, functional and innovative. The right approach for the companies is to match their functional and innovative products with physically efficient and market responsive supply chains, respectively.

The grouping of products extended further based on their structural complexities (Lamming et al., 2000; Li and O’Brien, 2001). According to Lamming et al. (2000), a product could be unique due to its technological contents, handcrafting, customized design, or by its brand reputation. As the degree of uniqueness increases the supply chain shifts from a volume-driven approach to value-driven one (Brun et al., 2008).
According to Aitken et al. (2003), the success of a company depends upon its ability to classify products and re-engineer its supply chain to accommodate the impact of product life-cycles. They grouped the products into four clusters based on the product characteristics proposed by Christopher and Towill (2002). Depending upon the product’s stage in its life-cycle and the cluster to which it belongs, the product is routed through either one of its four supply chain strategies; push system, Kanban (pull system), leagile, and agile. Through a case study they demonstrated how a company can become successful by implementing such a process in its supply chain management.

According to Khan et al. (2008) the unprecedented shift in the supply chain strategies in the fashion industry over the last decade from product-centric to customer-centric had a major impact on the changing risk profile and responsiveness of fashion retailers. The product-centric strategy is oriented towards supply chain’s efficiency and the customer-centric strategy is designed to close the gaps between supply chain planning and execution. But the customer-centric supply chain particularly, the last mile of retail supply chain, from distribution center to the retail stores, has typically faced challenges in the last few years (Baird, 2008). In Retail System Research report Baird (2008) claims that the last mile of retail execution has the potential to deliver significant differentiation, or become an enormous bottleneck in customer service. The need for the alignment of product design with such supply chain strategies and their impact on supply chain resilience and responsiveness is illustrated through a case study by Khan et al. (2012).

2.2.2 Literature on Integrated/Coordinated Approaches in the Fashion Industry
In order to become more responsive and reduce the risk for loss, companies in the fashion industry started to coordinate with upstream as well as downstream components of their
supply chains. Weng (1999) studied the power of coordination and strategic alliances within a supply chain system comprising of one manufacturer and one distributor. The paper analyses the roles of information sharing, attitude toward risk, and coordination between manufacturer and distributor in operating products with shorter life-cycle that has price-sensitive random demand. The paper derives useful managerial insights by comparing optimal coordinated production and pricing policies and the distributor’s production and pricing policies in the absence of coordination. The results suggest that such a coordination becomes important when the attitude towards risk is neutral, random demand is very sensitive to the distributor’s sale price, and the distributor’s unit purchase price is much higher than the manufacturer’s unit cost.

Researchers have identified that with the advancement in the information technology supply chain structures in the fashion industry tend towards forming virtual organizations, which are characterized by flexibility, fast responsiveness, and high efficiency (Hughes et al., 2001; Khalil and Wang, 2002; Lin and Lu, 2005). Wang and Chan (2010) investigated two multinational textile enterprises, one integrating upstream with a brand owner on market side and the other integrating downstream with suppliers on manufacturing side. They demonstrated that through a virtual organization approach the responsiveness of the supply chains has improved and the flexibility in responding to the market demand was satisfactory.

**2.2.3 Analytical Models in Fashion Industry**

The supply chains for fashion products should not only be responsive but also need to be accurate in meeting the demand. The merchandise has to be marked down if the supply exceeds demand and sold at a price even less than the cost. On the other hand, if supply is
less than demand, the company incurs lost sales. To address this issue significant research has been conducted in developing analytical models to optimize inventory replenishment of retail fashion products (Fisher et al., 2001; Weng and McClurg, 2003; Li et al., 2009; Patil et al., 2010). The common features in all those models are:

- All models are stochastic
- Consider a finite selling period and so the inventory at the end of period is marked down in price and sold at a loss
- Consider multiple production commitments such that sales information is obtained and used to update demand forecasts between planning periods

Fisher et al. (2001) proposed a heuristic to solve a two-stage stochastic dynamic program that determines a retail product’s initial and replenishment order quantities minimizing the cost of lost sales, back orders, and obsolete inventory. They differ from other stochastic inventory models by allowing their method to choose the optimal reorder time, quantifying the benefit of lead time reduction, and choosing the best replenishment contract. Li et al. (2009) generalized the models proposed by Fisher et al. (2001) by taking into consideration time-dependent inventory holding and backorder costs.

Patil et al. (2010) studied the impact of quantity discounts and transportation cost structures on procurement, shipment, and clearance pricing decisions via a stochastic programming with recourse formulation. They claim that under some business settings (such as low inventory and procurement costs), the conventional strategy of placing and transporting a single large order is a better option.
2.3 Gaps in the Literature

From our review of existing literature we observe that the following issues are not addressed in the literature related to integrated supply chain models:

1. Warehouses have almost exclusively been treated as nodes in a supply chain with a known capacity. Little to no work exists that evaluates the impact of executing supply chain plans on warehouse operational efficiency — we call this the forward impact.

2. Warehouse design decisions (e.g., aisle layout, workforce requirement, and technology use) and operational impacts (e.g., workload variation and worker congestion) have a significant bearing on its throughput and cost. For example, a new picking technology that improves worker throughput could change the schedule of shipments and inventory requirements. A fundamental understanding of the implications of warehouse design and operational decisions on inventory and transportation decisions is lacking — we call this the reverse impact.

3. Lack of research on supply chain optimization models that jointly consider products with differing life-cycles, such as basic and fashion.

4. Lack of research that explores the impact of distribution of products with differing life-cycles on warehouse’s design and operational decisions, such as technology and workforce (permanent and temporary).

To address the above gaps in the academic literature and concerns expressed by our industry partners, we introduce the warehousing-inventory-transportation problem (WITP). As mentioned earlier, the WITP jointly considers warehousing design and operational decisions, along with inventory and transportation decisions, with the
objective of developing optimal supply chain plans in a multi-echelon, multiproduct, and multi-period setting. The decisions addressed by WITP are compared and summarized in Table 2.1.

Table 2.1 Comparison of traditional problems and WITP based on decisions addressed

<table>
<thead>
<tr>
<th>Decisions Considered</th>
<th>Inv-Loc</th>
<th>IRP</th>
<th>PIDRP</th>
<th>WITP</th>
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<tr>
<td><strong>Warehousing</strong></td>
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<tr>
<td>Capacity (rate of units flowing in and out)</td>
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<td>Workforce planning (permanent and temporary)</td>
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<td>Worker congestion</td>
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<td>A single or multiple source for stores</td>
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<td>Warehouse and store location</td>
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<td><strong>Inventory</strong></td>
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<td>Replenishment policy</td>
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<td>Backlogging, lost-sale</td>
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<td>Number of shipment and quantities</td>
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<td>Multi-stop routes</td>
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Note: x* indicates that these decisions can be easily included in the proposed basic model for WITP.
3. The Warehouse-Inventory-Transportation Problem for Supply Chains

3.1 Introduction

Modern supply chains rely heavily on warehouses for rapidly fulfilling customer demand through retail, web-based, and catalogue channels. Warehousing costs in 2010 were nearly $112 billion across 600,000 small and large warehouses in the U.S., which is over 9% of the $1.2 trillion of the U.S. logistics cost (CSCMP, 2011). Warehouses, now often referred to as distribution centers (DCs), have emerged from their traditional passive role of serving as buffers to mitigate supply-demand variations to a more active role of providing value-added services such as consolidation/deconsolidation, assembling, kitting, etc. The operations of Amazon.com illustrate the importance of careful warehouse management in modern supply chains (Curtis, 2013; Lee, 2013). Figure 3.1 illustrates the key functions of a warehouse, which are receiving, inspection and quality control, repackaging, putaway, storage, order-picking, sorting, packing and shipping, and cross-docking (Tompkins et al., 2010).

Decisions around warehouse design and operations include aisle layout, material handling selection, workforce planning and scheduling, and information technology infrastructure. These decisions have a significant bearing on the warehouse’s throughput.

and cost, and impact other supply chain decisions such as inventory and transportation. For example, a new picking technology such as pick-to-light or robotic picking (e.g., Kiva robots) that alters (actually, improves) worker productivity may mean that inbound and outbound shipment schedules, and inventory requirements at the warehouse, would get modified due to this change in the warehouse’s throughput.

Figure 3.1 Common warehousing activities

This research is motivated by the current practice of distribution planning, specifically at our industry partner, a US-based apparel supply chain. This supply chain sells to consumers through retail and e-commerce channels. Their warehouse, the only one in the supply chain, manages the flow of 6,500 - 8,000 products supplied by over 100 domestic and overseas vendors, and replenishes over 300 retail stores situated in nearly 40 states across the nation. Although this warehouse serves as a hub in the supply chain,
it operates in a reactive mode; that is, inventory and transportation plans are determined first and the warehousing plans are determined later. This sequential approach results in the warehouse experiencing substantial variation in daily workload, which causes the warehouse manager to scramble for resources during peak times and experience resource under-utilization during drought times. Figure 3.2 shows the number of units picked per week at the company’s warehouse in the year 2011, where the weekly variation in the workload ranges from nearly 42% to 220% of that year’s weekly average. Data from another of our industry partners, a *Fortune* 100 grocery distributor, during August 29 – September 4 of 2011 indicated a variation in the number of units picked (76% and 153%) at one of their US warehouses (see Figure 3.3).

![Figure 3.2 Weekly variation in the units picked at the US-based apparel warehouse](image)

The key point here is that such workload imbalances create substantial operational inefficiencies at the warehouse and can cost a company millions of dollars annually. From a warehouse operations perspective, a relatively balanced workload across all time-periods is preferred because it leads to (i) easier worker management and scheduling, (ii)
reduced need for overtime hours and/or temporary workers, and (iii) effective utilization of technological resources leading to increased worker productivity.

Figure 3.3 Variation in units picked at a warehouse of a Fortune 100 US grocery distributor

The observed inefficiencies at the warehouse of our industry partners beg the following question: *how would a supply chain benefit if it proactively accounted for warehousing decisions at the tactical planning phase, instead of reacting passively every day?* This question motivated us to introduce the warehousing-inventory-transportation problem (WITP) to the supply chain literature. The WITP integrates decisions regarding warehouse, inventory, and transportation, and identifies an optimal distribution strategy for a multi-product, and multi-period supply chain such that the total distribution chain cost is minimized.

The remainder of this chapter is organized as follows. We first summarize relevant literature in Section 3.2 and then introduce a nonlinear integer programming model for the WITP in Section 3.3. In Section 3.4 optimal solutions generated by the linearized version of WITP model are compared to solutions generated by a sequential
approach observed in the industry. In Section 3.5 we provide details of a heuristic designed to solve industry-sized problem instances (e.g., 500 stores and 1,000 products) followed by few valuable managerial insights that are derived based on our experiments and sensitivity analyses. Finally, we discuss conclusions and future work in Section 3.6.

3.2 Background Literature

In recent years we have witnessed a significant thrust on integrating transportation decisions with inventory in distribution planning. The objective is to balance inventory and transportation costs. A well-studied problem is the inventory-routing problem (IRP), which refers to developing a repeatable distribution strategy that minimizes transportation costs and the number of stock-outs. Both deterministic and stochastic versions of IRP have been studied (Campbell et al., 1998; Kleywegt et al., 2004; Zhao et al., 2008; Lin and Chen, 2008). Other approaches to integrate inventory and transportation decisions have also been explored; e.g., Parthanadee and Logendran (2006) and Çetinkaya et al. (2008).

Abdelmaguid and Dessouky (2006) introduce the integrated inventory-distribution problem (IDP) for multi-period systems considering both inventory and transportation costs, and allowing for backlogging. Lei et al. (2006) consider the production-inventory-distribution-routing problem (PIDRP), where the focus is on coordinating production and transportation schedules between vendors and customers. Bard and Nananukul (2008) solve a one-plant, multi-customer PIDRP with the assumption of single-mode transportation. Research on the integration of inventory and warehouse location decisions address identifying optimal location for the warehouse
while minimizing system-wide transportation and inventory costs (Daskin et al., 2002; Shen et al., 2003; Üster et al., 2008; Ozsen et al., 2009).

Literature on warehousing is massive and has primarily focused on location, design, and operation. Numerous models have been developed to assist in various aspects of warehouse design; e.g., sizing (Goh et al., 2001; Heragu et al., 2005; Ng et al., 2009), aisle-layout (Roodenberg and Vis, 2006; Gue and Meller, 2009), and operational aspects (Ratliff and Rosenthal, 1983; Parikh and Meller, 2010a). One of the key areas of focus in warehousing has been on order picking as it contributes about 50% of the total warehousing costs (Tompkins et al., 2010). Worker congestion during order picking has been identified as a key factor that causes decreased warehouse productivity and increased costs (Gue et al., 2006; Parikh and Meller, 2009; Parikh and Meller, 2010b).

From our review of the existing literature, studies on supply chain planning have primarily focused on integrating inventory and transportation decisions. Warehouses, in the context of supply chain planning, have almost exclusively been treated as nodes with known capacity. Little to no work exists that evaluates the impact of executing distribution plans on warehouse operations, more specifically the variation in warehouse workload. Additionally, warehouse design decisions (e.g., layout, workforce, and technology) and operational impacts (e.g., worker congestion) have a significant bearing on its throughput and cost. A fundamental understanding of the implications of warehouse design and operations on inventory and transportation decisions is lacking.

This research addresses these fundamental gaps in the academic literature and concerns expressed by our industry partners by introducing the warehouse-inventory-transportation problem (WITP). The decisions addressed by WITP are compared and
summarized in Table 3.1. We now present a nonlinear integer programming model for the WITP and an efficient heuristic to solve industry-sized problem instances.

Table 3.1 Comparison of traditional problems and WITP based on decisions addressed

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Note: x* indicates that these decisions can be easily included in the proposed basic model for WITP.

3.3 The Warehouse-Inventory-Transportation Problem

The WITP is intended to determine the optimal distribution of products from vendors to stores via a warehouse with the objective of minimizing the total distribution cost. A distribution planning problem is typically defined as determining the quantity and schedule of both inbound shipments (vendor to warehouse) and outbound shipments (warehouse to stores), along with the inventory levels at the warehouse and stores (Chopra and Meindl, 2012). The WITP extends this definition by incorporating several warehousing decisions that help address the following questions:

- What level of permanent workforce should be used at the warehouse during the planning horizon?
- How many temporary workers are required during each time-period in the planning horizon to respond to variations in product flow through the warehouse?
To what extent does the allowable level of such workers, and their reduced productivity, affect the warehouse workload?

- How do different warehousing technologies, which determine theoretical worker productivity, impact the workforce plans, and eventually transportation and inventory decisions?
- How does aisle configuration, which has direct implications on worker congestion and eventually determines the actual worker productivity, affect the workforce plans?

The WITP model that we present focuses on integrating the three sets of decisions, warehousing, inventory, and transportation, at both the tactical and operational levels. That is, we seek to derive an optimal, repetitive, distribution plan for a prespecified time-horizon (say, 3-6 months). From this perspective, decisions such as warehouse location, aisle configuration, and technology employed --- all of which are strategic and/or tactical --- are assumed to be given. However, we do capture certain aspects of these decisions indirectly via sensitivity analysis. For instance, the impact of aisle configuration and (wide and narrow) on workforce dynamics is captured via congestion (see Section 3.4). The impact of warehouse technology, which includes picking strategies, storage configuration, material handling equipment, and pick-assist methods, is captured by varying a worker’s productivity (see Section 3.5). Additionally, the impact of changes in the number of allowable temporary workers, and their reduced productivity, is also analyzed.

To model the warehouse workforce mix, we consider workforce requirements for two major activities, putaway and picking. Putaway refers to the activity of moving
products from the point of unloading to the storage/picking area of a warehouse. Picking refers to the activity of fulfilling a customer order by picking and packing products from the picking area.

A critical aspect we capture in our model is worker congestion modeled as blocking. We define picker blocking as the dynamic interaction between workers during the picking operation in the picking area. It is assumed that replenishment to the picking area, either directly from the inbound docks (as part of the putaway activity) or from the reserve area (as a separate replenishment activity), is conducted at a different time from the picking activity. We next discuss how blocking impacts warehouse productivity.

3.3.1 Impact of Picker Blocking on Warehouse Productivity

Blocking can be substantial in picking systems, especially in narrow-aisle systems, depending on the pick density, storage policy, routing method, and the number of workers picking simultaneously in the system. It leads to reduced worker productivity, which in turn increases the number of pickers required to carry out the picking operations effectively and meet the desired throughput (Gue et al., 2006; Parikh and Meller, 2009). In wide aisle systems, pickers experience blocking due to their inability to pick at the same pick-column simultaneously. This form of blocking is referred to as *pick-column blocking* (see Figure 3.4(a)). In contrast, blocking in narrow aisle systems is experienced due to a picker’s inability to pass other pickers in the same aisle. This form of blocking is referred to as *in-the-aisle-blocking* (see Figure 3.4(b)). Both types of blocking result in increased picker idle time, thus affecting worker productivity and subsequently increasing labor costs.
Parikh and Meller (2010) developed a simulation model to estimate in-the-aisle blocking for a narrow-multi-aisle order picking system which includes number of pick-faces, pick-density, number of pickers, and pick to walk ratio. Blocking can occur during putaway as well as picking, but putaway blocking is of relatively little practical significance (per our industry partner’s experience), and is therefore omitted from the models developed here.

Figure 3.4 Two forms of picker blocking: (a) pick-column blocking in a wide-aisle and (b) in-the-aisle blocking in a narrow-aisle

We next present a nonlinear integer programming model for the WITP with picker-blocking in a two-echelon, multi-product, and multi-period supply chain, consisting of multiple vendors and stores that are connected by one warehouse.
3.3.2 A Nonlinear Integer Programming Model for the WITP

We make the following assumptions in our mathematical model: (i) vendors have sufficient supplies to meet the demand at the warehouse; (ii) backorders are not allowed; and (iii) putaway and picking activities at the warehouse are considered as they both frequently are labor intensive. We first present the model parameters and decision variables in Tables 3.2 and 3.3, respectively, followed by a nonlinear integer programming model for the WITP.

The objective function (1) in the model is to minimize the total distribution cost, which includes costs related to warehouse workforce (permanent and temporary), inventory holding at the warehouse and stores, and transportation (fixed and variable). Constraints (2)-(4) ensure that sufficient numbers of permanent and temporary workers (considering their productivity factors) are available for putting away inbound products and picking outbound products at the warehouse. Constraints (3) consider picker blocking, where \( b(k_t) \) denotes the average blocking experienced by \( k_t \) pickers in period \( t \). Constraints (5) and (6) restrict the number of temporary workers to be below an allowable fraction of permanent workers (largely due to limited availability and reduced productivity). Constraints (7)-(10) specify the inventory levels at the warehouse and stores. Considering a cyclic distribution strategy, Constraints (8) and (10) ensure that inventory at the end of the current time horizon is identical to the inventory at the beginning of the next time horizon. Constraints (11) impose space restriction at each store. The weight-based transportation capacities for shipments from vendor to warehouse and from warehouse to store are modeled through Constraints (12) and (13). The bounds on the decision variables are specified by Constraints (14) and (15).
### Table 3.2 Parameters in the WITP Model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>v</td>
<td>index for vendor; ( v = 1, 2, \ldots, V )</td>
</tr>
<tr>
<td>s</td>
<td>index for retail store; ( s = 1, 2, \ldots, S )</td>
</tr>
<tr>
<td>p</td>
<td>index for product; ( p = 1, 2, \ldots, P )</td>
</tr>
<tr>
<td>t</td>
<td>index for period; ( t = 1, 2, \ldots, T )</td>
</tr>
<tr>
<td>( \Omega_v )</td>
<td>set of products ( p ) that are sourced from vendor ( v )</td>
</tr>
<tr>
<td>( D_{spt} )</td>
<td>demand for product ( p ) at store ( s ) in period ( t )</td>
</tr>
<tr>
<td>( W_p )</td>
<td>weight of each item of product ( p ); lbs</td>
</tr>
<tr>
<td>( Q )</td>
<td>truck capacity; lbs</td>
</tr>
<tr>
<td>( V_p )</td>
<td>volume of each item of product ( p ); ft(^3)</td>
</tr>
<tr>
<td>( K_s )</td>
<td>maximum physical space at store ( s ); ft(^3)</td>
</tr>
<tr>
<td>( A_1(A_2) )</td>
<td>rate at which a worker can putaway (pick) products; items/time-period</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>fraction of permanent workers that can be employed as temporary workers</td>
</tr>
<tr>
<td>( \phi_1 (\phi_2) )</td>
<td>factor that captures the reduction in productivity of temporary workers for putaway (picking); ( \phi \in {0,1} )</td>
</tr>
<tr>
<td>( C_p (C_{sp}) )</td>
<td>holding cost at warehouse (store ( s )) for product ( p ); $/item/period</td>
</tr>
<tr>
<td>( C^\alpha )</td>
<td>labor cost for a permanent warehouse worker for the entire time-horizon (( T ) periods); $</td>
</tr>
<tr>
<td>( C^\beta )</td>
<td>labor cost for a temporary warehouse worker per period ( t ); $/period</td>
</tr>
<tr>
<td>( C^f_v (C^f_s) )</td>
<td>fixed cost of shipment from vendor ( v ) (warehouse) to warehouse (store ( s )), accounting for distance between them; $/shipment</td>
</tr>
<tr>
<td>( C^v_p (C^v_s) )</td>
<td>variable weight-based cost of shipment from vendor ( v ) (warehouse) to warehouse (store ( s )), accounting for the distance between them; $/lbs</td>
</tr>
</tbody>
</table>

### Table 3.3 Decision Variables in the WITP Model

<table>
<thead>
<tr>
<th>Decision Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_1 (\alpha_2) )</td>
<td>number of permanent workers required for putaway (picking) activities</td>
</tr>
<tr>
<td>( \beta_{1t} (\beta_{2t}) )</td>
<td>number of temporary workers required for putaway (picking) activities in period ( t )</td>
</tr>
<tr>
<td>( k_t )</td>
<td>total number of workers required for picking at the warehouse in period ( t )</td>
</tr>
<tr>
<td>( b (k_t) )</td>
<td>average blocking experienced by ( k_t ) pickers in period ( t )</td>
</tr>
<tr>
<td>( x_{vp} (x_{sp}) )</td>
<td>quantity of product ( p ) inbound from vendor ( v ) to warehouse (outbound from warehouse to store ( s )) in period ( t )</td>
</tr>
<tr>
<td>( y_{sp} )</td>
<td>inventory of product ( p ) at the warehouse (store ( s )) at the end of period ( t )</td>
</tr>
<tr>
<td>( n_{vt} (n_{st}) )</td>
<td>number of shipments from vendor ( v ) (warehouse) to the warehouse (store ( s )) in period ( t )</td>
</tr>
</tbody>
</table>
Minimize
\[ C^a(\alpha_1 + \alpha_2) + \sum_t C^\beta (\beta_{1t} + \beta_{2t}) + \sum_{pt} C^h y_{pt} + \sum_{spt} C^h y_{sp} \]
\[ + \sum_{vt} C^f n_{vt} + \sum_{st} C^f n_{st} + \sum_{vpt} C^v (W_p x^i_{vpt}) \]
\[ + \sum_{spt} C^v (W_p x^o_{spt}) \] \hspace{1cm} (1)

Subject to

Warehouse:
\[ A_1 (\alpha_1 + \phi_1 \beta_{1t}) \geq \sum_{vpt} x^i_{vpt} \hspace{1cm} \forall t \] \hspace{1cm} (2)

\[ A_2 k_t(1 - b(k_t)) \geq \sum_{sp} x^o_{spt} \hspace{1cm} \forall t \] \hspace{1cm} (3)

\[ \alpha_2 + \phi_2 \beta_{2t} \leq k_t \hspace{1cm} \forall t \] \hspace{1cm} (4)

\[ \beta_{1t} \leq \gamma \alpha_1 \hspace{1cm} \forall t \] \hspace{1cm} (5)

\[ \beta_{2t} \leq \gamma \alpha_2 \hspace{1cm} \forall t \] \hspace{1cm} (6)

\[ y_{pt} = y_{p(t-1)} + \sum_v x^i_{vpt} - \sum_s x^o_{spt} \hspace{1cm} \forall p, t > 1 \] \hspace{1cm} (7)

\[ y_{p1} = y_{pT} + \sum_v x^i_{vpt} - \sum_s x^o_{spt} \hspace{1cm} \forall p \] \hspace{1cm} (8)

Store:
\[ y_{spt} = y_{sp(t-1)} + x^o_{spt} - D_{spt} \hspace{1cm} \forall s, p, t \] \hspace{1cm} (9)

\[ y_{sp1} = y_{spT} + x^o_{sp1} - D_{sp1} \hspace{1cm} \forall s, p \] \hspace{1cm} (10)

\[ \sum_p v_p y_{spt} \leq K_s \hspace{1cm} \forall s, t \] \hspace{1cm} (11)

Transportation:
\[ \sum_{pt \in \Gamma_v} W_p x^i_{vpt} \leq Q n_{vt} \hspace{1cm} \forall v, t \] \hspace{1cm} (12)

\[ \sum_p W_p x^o_{spt} \leq Q n_{st} \hspace{1cm} \forall s, t \] \hspace{1cm} (13)

Bounds:
\[ x^i_{vpt}, x^o_{spt}, y_{pt}, y_{spt}, n_{vt}, n_{st}, \alpha_1, \alpha_2, \beta_{1t}, \beta_{2t}, k_t \in \{0, Z_+\} \hspace{1cm} \forall v, s, p, t \] \hspace{1cm} (14)

\[ b(k_t) \geq 0 \hspace{1cm} \forall t \] \hspace{1cm} (15)

Constraints (3) are nonlinear because \( b(k_t) \) is a discretely valued function of \( k_t \).

These constraints can be linearized as follows. First, for each possible number of pickers \( k_t \), \( b(k_t) \) is replaced by discrete values \( B_l \), the average blocking corresponding to \( l \) pickers in the system. Note that \( B_l \) can be calculated offline using our simulation model.
for a warehouse with narrow aisles (Parikh and Meller, 2010b). Second, a binary decision variable, $k_{tl}$, is introduced representing the number of pickers to be selected; i.e., $k_{tl}$ equals 1 if the number of pickers in time $t$ is $l$, and 0 otherwise. We replace nonlinear constraints in (3) with the following set of linear constraints:

$$A_2 \sum_t (1 - B_l)(lk_{tl}) \geq \sum_{sp} x_{sp}^{pl} \quad \forall \ t \quad (16)$$

$$\sum_l k_{tl} = 1 \quad \forall \ t \quad (17)$$

$$\alpha_2 + \phi_2 \beta_{2t} = \sum_l k_{tl} \quad \forall \ t \quad (18)$$

Bounds:
$$k_{tl} \in \{0,1\} \quad \forall \ t, l \quad (19)$$

We next compare distribution plans, in terms of the total distribution cost and the variance in warehouse workload, generated by solving the WITP directly with those generated by a sequential approach.

### 3.4 Comparison of WITP with a Sequential Approach

We refer to the sequential approach for designing distribution plans as first solving the joint inventory-transportation problem (ITP) and then solving the corresponding warehousing problem (WP); we denote this approach as ITP+WP. Such a sequential approach was observed during our industry work experience and discussions with our industry collaborators (and many other companies that we know).

The ITP+WP approach implies that warehousing decisions have to react to inventory and transportation decisions made a priori. That is, in this sequential approach we first solve optimally the joint inventory-transportation problem (ITP) and then, using this solution as an input to the warehousing problem (WP), we find the optimal
workforce level at the warehouse. The model for the ITP includes the inventory and transportation constraints (7)-(13), and associated cost terms in the objective function. The model for WP includes the warehousing constraints (2), (4)-(6), (16-19), and associated workforce cost terms in the objective function. For a given dataset, the optimal solution to the ITP provides information on inbound and outbound quantities (i.e., $x_{vpt}^i$ and $x_{vpt}^o$), warehouse and store inventory levels (i.e., $y_{pt}$ and $y_{spt}$), and inbound and outbound shipments (i.e., $n_{vt}$ and $n_{st}$). These inbound and outbound quantities are used as inputs to the WP model to determine the workforce level for putaway and picking activities (i.e., $\alpha$ and $\beta$, respectively). The total distribution cost is then calculated as the sum of inventory, transportation, and warehousing costs obtained from both the ITP and the WP models.

It is worth noting that the ITP is similar to the One Warehouse Multiple Retailer (OWMR) problem studied by Federgruen and Tzur (1999), Levi et al. (2008), Shen et al. (2009), and Solyali and Sural (2012). The OWMR problem is to find a distribution plan such that the total cost of ordering and inventory, both at the warehouse and individual retailers, is minimized. It captures the fixed and variable ordering/transportation costs between vendor-warehouse and warehouse-retailer. The ITP that we derive from WITP is set up in a similar way, except that we consider cyclical inventory constraints due to the practical need for generating a repeatable distribution plan as indicated by our industry collaborators. These inventory constraints ensure that the final inventory level during a time horizon becomes the initial inventory for the next time horizon (and is usually non-zero). OWMR problems have typically been studied assuming initial inventory to be zero.
OWMR has been shown to be NP complete by the above references, and hence it is reasonable to deem ITP, and so the more complex WITP, as NP complete. The heuristics that have been proposed to solve OWMR have largely focused on a single-item multi-retailer setting, except for Federgruen and Tzur (NRL, 1999) where they attempt to solve up to 10 items and 10 retailers. Because the scalability of such heuristics has not been established, and our focus is on solving industry-sized problems and developing managerial insights, we resort to numerically quantifying the benefits of WITP over the sequential approach of ITP+WP.

Before discussing the heuristic to solve industry-sized problems, we first show a comparison of the optimal distribution plans generated by solving the models for WITP and ITP+WP on a series of relatively small problem instances. Doing so helped us evaluate the tradeoff between the three decisions and generate insights that were eventually used in developing a heuristic to solve industry-sized problems. The impact of aisle configuration at the warehouse (narrow vs. wide), which affects blocking and worker productivity, was also analyzed using this comparison.

As indicated earlier, picker blocking can be considerable in warehouses with narrow aisles. However, many warehouses employ wide aisles that allow workers to pass each other in the aisle in order to reduce congestion. Note that wide aisles require a larger area, compared to narrow aisles, for the same amount of storage and, hence, may or may not be a viable option financially for warehouses located near urban areas or in regions where space comes at a premium. To analyze the benefits of the WITP approach for supply chains that have a warehouse with wide aisles, in which blocking has minimal effect on worker productivity in most cases (Parikh and Meller, 2009), we remove the
blocking function, $b(k)$, and replace Constraints (3) in the WITP model presented earlier with the following:

$$kA_1 \geq \sum_{sp} x_{sp}^o \quad \forall t.$$

We considered a supply chain with one vendor, one warehouse, one product, and five time-periods. Several problem instances were randomly generated by varying the number of stores ($S$), and putaway/picking rates ($A_1/A_2$). We generated four levels for $S$ (2, 5, 10, 20), and three levels for $A_1$ and $A_2$ (200 items/hr, 300 items/hr, 500 items/hr). Product demand for each store was uniformly generated between 0 and 3,000 items per time-period, and the unit holding cost at the warehouse and stores were $0.01$/item and $0.1$/item, respectively. The labor cost for permanent and temporary workers were $15$/hr and $10$/hr, respectively. The models for ITP+WP and WITP were solved using a commercial solver, FICO Xpress – MP 7.0, on a personal computer with a Pentium 4 3.2 GHz processor and 1 GB RAM.

The impact of aisle configuration (narrow vs. wide) on optimal distribution plans are shown in Tables 3.4 and 3.5, which also shows a comparison of ITP+WP and WITP plans. In these tables, each problem instance is indexed by a pair of elements corresponding to stores ($S$) and putaway/picking rates ($A_1/A_2$), respectively. The rates, $A_1$ and $A_2$ are set to be equal in our experimentation. The columns “V-W” and “W-S” refer to the numbers of shipments from a vendor to the warehouse and from the warehouse to a store, respectively. The columns “P” and “T” represent the number of required permanent workers and temporary workers for the entire time horizon. The total distribution cost is
represented by “ΣC.” The column “Gap” represents percentage difference between the best solution found after 6 hr and best bound.

From a cost perspective, in 10 out of 12 instances the total cost of WITP plans was lower compared to ITP+WP plans even though the WITP could not be solved to optimality in 2 of those instances (#3 and #7). Cost savings via WITP in these 10 instances ranged between 4.6% and 28.6%. For the 2 other instances (#8 and #12), WITP had a much larger %-gap in the best solution compared to the ITP+WP solutions. Interestingly though, the cost differences were relatively small even with such high %-gap in the WITP solutions.

Observe that in most problem instances, the number of shipments was larger in the WITP case than ITP+WP; for instances, when they were identical the time-periods of shipments were not identical. Similarly, the number of workers (putaway and picking) was much larger in ITP+WP case than WITP. That is, given the inclusion of warehouse workforce cost in the objective function, the WITP model was better positioned to tradeoff this cost component against the transportation and inventory costs yielding a lower total cost solution compared to the sequential ITP+WP approach. The inventory cost increase depended on how much the schedule of inbound and outbound shipments changed between ITP+WP and WITP solutions. The rescheduling of the shipments in WITP, more spread out than ITP+WP, also helped reduce the workload variation at the warehouse; see Figure 3.6.

For example, consider dataset (DS) #3 that had 10 stores, putaway and picking rates of 200 items/hr, and a planning horizon of 5 days (one work-week). The optimal ITP+WP solution had 4 inbound shipments during Days 1-3 from the vendor to the
warehouse and 11 outbound shipments from the warehouse to the 10 stores during Days 1-3. The schedules for inbound and outbound quantities (57,354 units each) during the 5-day planning horizon were such that they occurred during the first three days of the planning horizon (see Figure 3.5). The numbers of permanent workers required in the putaway and picking areas were 12 and 22, respectively. In contrast, the optimal WITP solution ensures that both inbound (5) and outbound (11) shipments were distributed reasonably evenly across the 5-day horizon. That is, the variance in the daily warehouse workload had been reduced substantially, from 0%-209% to 94%-110% of the mean workload value of 115 hr/day. Only 7 and 8 permanent workers were required in the putaway and picking areas, respectively, in the optimal solution (see Table 3.4). This reduction in the labor cost at the warehouse ($21,040 to $9,480) offset an increase in the transportation ($19,365 to $20,169) and inventory ($8,610 to $8,949) costs, leading to an overall cost savings of 21.25% over ITP+WP (see Table 3.4).
Figure 3.5 Comparison of optimal distribution plans (ITP+WP vs WITP) for DS3: (a)-(c) correspond to the ITP+WP and (d)-(f) to the WITP

Table 3.4 Comparison between optimal plans from ITP+WP and WITP
Table 3.5 Comparison between optimal plans from ITP+WP and WITP (wide-aisle case)

<table>
<thead>
<tr>
<th>Problem Instance</th>
<th>ITP+WP</th>
<th>WITP</th>
<th>Savings</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Shipments</td>
<td>Put</td>
<td>Pick</td>
</tr>
<tr>
<td>DS (S, Λ₁=Λ₂)</td>
<td>V-W</td>
<td>W-S</td>
<td>P</td>
</tr>
<tr>
<td>1 (2, 200)</td>
<td>1 2 7 1 7 1 14,202</td>
<td>1 0</td>
<td>2 4 4 0 2 0 10,559</td>
</tr>
<tr>
<td>2 (5, 200)</td>
<td>2 5 9 2 7 2 23,231</td>
<td>1 0</td>
<td>3 5 5 3 4 0 20,132</td>
</tr>
<tr>
<td>3 (10, 200)</td>
<td>4 11 12 3 12 3 42,855</td>
<td>227 0</td>
<td>5 11 7 1 7 2 38,306</td>
</tr>
<tr>
<td>4 (20, 200)</td>
<td>4 20 10 8 11 6 58,917</td>
<td>21,600</td>
<td>4.1%</td>
</tr>
<tr>
<td>5 (2, 300)</td>
<td>1 2 4 1 4 1 10,602</td>
<td>1 0</td>
<td>2 4 3 0 2 0 9,796</td>
</tr>
<tr>
<td>6 (5, 300)</td>
<td>2 5 6 1 4 2 19,551</td>
<td>1 0</td>
<td>3 5 4 0 3 0 18,692</td>
</tr>
<tr>
<td>7 (10, 300)</td>
<td>4 11 8 2 8 2 37,895</td>
<td>211 0</td>
<td>5 11 5 4 4 5 35,121</td>
</tr>
<tr>
<td>8 (20, 300)</td>
<td>4 20 7 4 8 3 54,757</td>
<td>21,600</td>
<td>4.17%</td>
</tr>
<tr>
<td>9 (2, 500)</td>
<td>1 2 3 0 3 0 9,242</td>
<td>1 0</td>
<td>1 4 3 0 1 0 8,526</td>
</tr>
<tr>
<td>10 (5, 500)</td>
<td>2 5 4 1 3 0 17,591</td>
<td>1 0</td>
<td>2 5 4 0 2 0 17,352</td>
</tr>
<tr>
<td>11 (10, 500)</td>
<td>4 11 5 1 5 1 34,135</td>
<td>205 0</td>
<td>4 11 4 0 3 0 32,841</td>
</tr>
<tr>
<td>12 (20, 500)</td>
<td>4 20 4 4 5 1 50,997</td>
<td>21,600</td>
<td>4.17%</td>
</tr>
</tbody>
</table>

Even when the warehouse had wide aisles, it is evident from Table 5 that the cost savings were substantial with WITP compared to the sequential ITP+WP. However, these savings reduced compared to the narrow aisle case. This is because although the ITP solution remained unchanged in both wide and narrow aisle cases (e.g., column “Shipments” in Tables 3.4 and 3.5 are identical), blocking in wide aisles was substantially lower. So when the WP was optimized, the number of workers required in wide aisles was lower compared to narrow aisles for the same ITP solution; e.g., for DS #3 in Tables 3.4 and 3.5, the total pickers was 27 (22 permanent and 5 temporary) for narrow aisles and 15 (12 permanent and 3 temporary) for wide aisles. Consequently, the total cost of plans with ITP+WP for the wide aisle case was lower than for the narrow aisle case ($42,855 vs. $49,015). The corresponding WITP plans for both narrow and
wide also exhibited a reduction in the number of pickers; i.e., 13 (narrow) and 9 (wide). Given that warehouse workforce cost now has a relatively lower contribution in the objective function of WITP for wide aisles (compared to narrow aisles), the corresponding optimal distribution plans were largely driven by transportation and inventory cost reductions. The resulting effect was that the optimal plans for the warehouse with wide aisles may sometimes have a higher workload variation compared to narrow aisles, but still much lower than the ITP+WP plans (see Figure 3.6).

Figure 3.6 Comparison of variances in daily warehouse workload between ITP+WP vs. WITP. Note that a, b, c represent ITP+WP, WITP (narrow-aisles, NA) and WITP (wide-aisles, WA), respectively.

From a solution perspective, we discussed earlier that WITP is NP complete. It is also analogous to a two-stage capacitated lot-sizing problem, which typically has weak linear programming (LP) bounds and lacks strong cutting planes (Bitran and Yanasse, 1982). Our preliminary experiments show that though the LP relaxation of WITP can be
solved easily, it is difficult to obtain an optimal or a near optimal solution within 6 hours, even for small problem instances. For example, the best solution obtained for a problem instance with 1 vendor, 1 warehouse, 20 stores, 1 product, and 5 time-periods using the Xpress MIP solver has an optimality gap of over 10%. A two-echelon supply chain can have over 100 vendors, more than 1 warehouse, over 100 stores, and over 1,000 products. The total number of integer variables for WITP instances of this size is over a billion. To generate near-optimal distribution plans in a reasonable amount of time, we designed a heuristic algorithm that uses insights gained from the above experiments.

3.5 A Heuristic for the WITP

Our proposed heuristic algorithm considers the impact of advancing and/or delaying inbound and outbound shipments, and swapping of product quantities in these shipments on both total distribution costs and workload variation at the warehouse. The heuristic incorporates key features from the well-established iterated local search, which consists of two alternating phases, a local search phase and a perturbation phase (Loürenco et al., 2002). The heuristic implements three sets of moves, intended to reduce transportation, warehousing and inventory costs. The high level structure of the heuristic is as follows, after which we explain each step in detail:

Initial solution (s), incumbent solution (s**)

\[ s^{**} = s \]

Repeat

Inbound phase

\[ s^* = \text{local search (s) (making Moves 1 and 2)} \]

\[ s^{*'} = \text{perturbation (s*) (applying inbound swap)} \]

Acceptance criteria

\[ s^{**} = s^*, \text{ if } s^{*'} < s^* \]

\[ s^{**} = s^*, \text{ if } s^{*'} > s^* \]
Outbound phase
\( s^* = \text{local search} (s) \) (making Moves 3 and 4)
\( s^{*'} = \text{perturbation} (s^*) \) (applying outbound swap)

Acceptance criteria
\[
\begin{align*}
    s^{**} &= s^{*'}, \text{ if } s^{*'} < s^* \\
    s^{**} &= s^*, \text{ if } s^{*'} > s^*
\end{align*}
\]

Until the stopping rule is met
End

1. **Initial Solution:** Let \( s \) refer to a feasible solution to the WITP. A feasible solution will provide values to the inbound and outbound shipment schedules and product quantities in each shipment, the required warehouse workforce (permanent and temporary), and inventory levels at the warehouse and stores. We derive a feasible solution \( s \) by ensuring that the total demand is met at both the warehouse and the stores across all time-periods.

2. **Inbound Phase:** For the given initial solution \( (s) \) we iteratively improve the inbound solution (which consists of inbound product quantities, inbound shipment schedules, inventory at the warehouse, and warehouse workforce for putaway) using Moves 1 and 2 (described later). After each iteration, a new solution \( (s') \) is accepted based on an acceptance criterion; the superior solutions are always accepted; i.e., \( s' < s \), and the inferior solutions are accepted with a probability \( p \). From our initial experiments we set the value of \( p \) as 0.05. The search stops if the stopping rule is met and the best solution \( s^* \) found so far is recorded. We perturb this solution by implementing Move 5 (described later) for a pre-specified number of iterations. The new solution \( (s^{*'}) \) is accepted only if it is better than the previous best solution (i.e., \( s^{*'} < s^* \)). Otherwise, \( s^* \) remains the best inbound solution and the heuristic progresses to the outbound phase.
3. **Outbound Phase**: Given the best inbound solution found thus far, we improve the outbound solution (which consists of outbound product quantities, outbound shipment schedules, store inventory, and warehouse workforce for picking) iteratively using Moves 3 and 4 (described later). The new solution is accepted based on the acceptance criteria mentioned above. If the stopping rule is met, then the best solution found so far is perturbed using Move 5. Again, as in the inbound phase, only the best solution is considered for the next step in the search process. Repeat Steps 2 and 3 until the stopping rule is met.

4. **Stopping Rule**: The algorithm stops if the maximum number of iterations is reached or if, for a prespecified number of iterations, the newly found solutions fall within $\delta\%$ of the incumbent solution. Based on initial experimentation, we set the value of $\delta$ as $\pm0.25\%$.

### 3.5.1 Description of the Neighborhood Moves

This section describes the five moves that help the search process transition from a current solution, $s$, to a neighboring solution, $s'$. The first two moves operate on inbound product quantities, $x_{i, v, p, t}$, and the next two moves on outbound product quantities, $x_{o, z, p, t}$. The fifth move operates on either of these sets of variables. The moves are considered as complete or partial based on the quantity of product (inbound or outbound) moved from one period to another. The decision on the amount of quantity to be moved depends on the size of the shipment ($\psi_t$) scheduled in a period $t$. For example, the size of shipments scheduled from vendor $v$ is given by $\psi_t = \frac{\sum p_w p x_{o, v, p, t}}{q}$, where, $Q$ is the capacity of the truck.
Move 1 - Advance complete shipment: This move advances all shipments scheduled from vendor $v$ to the warehouse in period $t$ to period $t-1$ (i.e., advancing the entire quantity of each product scheduled in period $t$). We make this move only if there is a positive shipment in period $t$ and the total quantity being shipped from the vendor does not equal a truckload (i.e., $0 < \psi_t < 1$). The values of $x_{vpt}, n_{vt}$, and $y_{pt}$ are updated after this move. The motivation behind this move is two-fold: first, if a shipment is already scheduled from the vendor in $t-1$ then moving a shipment from $t$ would help in shipment consolidation and save on the fixed cost of shipment; and second, whether or not a shipment is scheduled in $t-1$, moving an entire shipment from period $t$ could help reduce the total number of inbound shipments at the warehouse in period $t$, which could reduce the number of required putaway workers and thus reduce the labor cost. Because of advancing shipment, the inventory cost might increase, which is traded off against likely decreases in transportation and warehousing costs.

Move 2 - Advance partial shipment: Instead of advancing an entire shipment, this move advances a fraction of a shipment from period $t$ to $t-1$, if $\psi_t > 1$. The fraction to be advanced depends on whether or not a shipment is scheduled in period $t-1$.

Condition 1: If no shipment is scheduled in period $t-1$ (i.e., $n_{vt(t-1)} = 0$), then advance half of the shipment from $t$ to $t-1$. For example, if 1.4 shipments (i.e., two shipments with the second shipment only 40% of a truckload) are scheduled from vendor in period $t$ and if $n_{vt(t-1)} = 0$, then advance 0.7 shipment to period $t-1$. This will not change the total transportation cost as there is no change in the total number of shipments from the vendor, still 1.4. But splitting bigger shipments would now spread the inbound quantities
across two time-periods, which reduces the required number of putaway workers at the warehouse in period $t$ and likely results in a balanced workload between period $t$ and $t-1$. 

Condition 2: If a shipment is scheduled in period $t-1$ (i.e., $n_{vt} > 0$), then advance a fraction of the shipment from $t$ to $t-1$ so that the $>1$ shipment may be rounded down to its nearest integer. For example, if 0.5 and 1.4 shipments are scheduled in period $t-1$ and $t$, respectively, then advance 40% of shipment from $t$ to $t-1$. After the move, the resulting inbound shipments in $t-1$ and $t$ are 0.9 and 1, respectively. Thus, this move helps in reducing the number of shipments from 3 to 2; i.e., saving one fixed shipment cost. The number of shipments is now being split equally, the workload at the warehouse is more balanced, which results in reduced labor costs. The values of $x^{t}_{vt}, n_{vt}$, and $y_{pt}$ are updated after this move.

Move 3 - Delay complete shipment: The implementation and motivation behind this move is similar to Move 1 except that instead of advancing inbound shipments this move would delay the entire outbound shipment scheduled from the warehouse to store $s$ from period $t$ to $t+1$. This move is implemented only if the condition $0 < \psi_t < 1$ holds true. The motivation behind this move is that if a shipment is already scheduled to the store $s$ in $t+1$, then moving a shipment from $t$ to $t+1$ would help in shipment consolidation and save on the fixed cost of shipment. Additionally, even if no shipments are scheduled in $t+1$, moving an entire shipment from period $t$ to $t+1$ could help reduce the total number of outbound shipments in period $t$, which in turn could reduce the number of required pickers and thus reduce the labor cost. The cyclical inventory constraint will ensure the feasibility of meeting the demand both at the warehouse and store in each period. The values of $x^{t}_{spt}, n_{st}, y_{pt}$ and $y_{spt}$ are updated after this move.
Move 4 - Delay partial shipment: This move is similar to Move 2 except that instead of advancing partial inbound shipments, we delay partial outbound shipments scheduled from warehouse to the store $s$ in period $t$ to $t+1$, if $\psi_t > 1$. We again use Conditions 1 and 2 described in Move 2, although in this move we delay the shipments. The values of $x_{spt}^o, n_{st}, y_{pt}$ and $y_{spt}$ are updated after this move.

Move 5 - Swap shipments in two periods: There are two types of swaps, inbound and outbound. For the inbound swap randomly select $m$ vendors and two periods having positive shipments. Then swap the shipment schedules of the $m$ selected vendors between the two periods. Based on our initial experiments $m$ is specified to be less than 50% of the total number of vendors scheduled in the selected two periods. The outbound swap is similarly constructed. The values of $x_{vpt}^l, n_{vt},$ and $y_{pt}$ are updated after an inbound swap and the values of $x_{spt}^o, n_{st}, y_{pt}$ and $y_{spt}$ are updated after an outbound swap.

We now compare the performance of our proposed heuristic with the optimal solution method in terms of solution quality and CPU time.

3.5.2 Performance of the Heuristic

Table 6 compares the optimal and heuristic solutions over the same 12 problem instances. Each problem instance corresponds to stores and putaway/picking rates. The rates for putaway ($A_1$) and picking ($A_2$) were set to be equal in our experimentation. The column ‘Difference’ corresponds to the percentage heuristic solution differs from the optimal solution. The CPU times are based on a personal computer with a Pentium 4 3.2 GHz processor and 1 GB RAM; the heuristic was coded in C#.

Notice that the heuristic solutions either match or lie within 1% of the optimal solutions for most of the problem instances. The heuristic outperforms the optimal
solutions that could not always reach optimality (see DS #4, #8 and #12). Moreover, notice the huge difference in the runtime between the heuristic and optimal solutions particularly for DS #3, #4, #7, #8, #11 and #12. The variance in the daily warehouse workload obtained by the proposed heuristic is comparable to that obtained through the optimal solution suggesting that the heuristic is able to balance warehouse workload.

To analyze the impact of technology (via putaway/picking rates, $Λ_1/Λ_2$), the allowable level of temporary workers ($γ$), and the productivity of a temporary workers ($φ$), 24 problem instances were considered with the following settings: three levels for $Λ_1$ and $Λ_2$ (200 items/hr, 300 items/hr, 500 items/hr), four levels for $γ$ (0, 0.5, 1, 2), and two levels for $φ$ (0.75, 1.0). All instances considered for this analysis had 10 vendors, one warehouse, 500 stores, 1,000 products, and 5 time-periods.

Table 3.6 Comparison between Exact and Heuristic solutions

<table>
<thead>
<tr>
<th>Problem Instance</th>
<th>Optimal Solutions</th>
<th>Heuristic Solutions</th>
<th>Cost Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$ΣC$</td>
<td>Time</td>
<td>Range (%) - Diff from Mean</td>
</tr>
<tr>
<td>DS ($S, Λ_1=Λ_2$)</td>
<td>$s$</td>
<td>s</td>
<td>%</td>
</tr>
<tr>
<td>1 (2, 200)</td>
<td>10,575</td>
<td>3</td>
<td>0% - 192%</td>
</tr>
<tr>
<td>2 (5, 200)</td>
<td>20,292</td>
<td>18</td>
<td>43% - 139%</td>
</tr>
<tr>
<td>3 (10, 200)</td>
<td>38,598</td>
<td>21,600</td>
<td>94% - 110%</td>
</tr>
<tr>
<td>4 (20, 200)</td>
<td>60,656</td>
<td>21,600</td>
<td>91% - 106%</td>
</tr>
<tr>
<td>5 (2, 300)</td>
<td>9,796</td>
<td>2</td>
<td>0% - 224%</td>
</tr>
<tr>
<td>6 (5, 300)</td>
<td>18,692</td>
<td>15</td>
<td>43% - 139%</td>
</tr>
<tr>
<td>7 (10, 300)</td>
<td>35,133</td>
<td>21,600</td>
<td>88% - 108%</td>
</tr>
<tr>
<td>8 (20, 300)</td>
<td>55,698</td>
<td>21,600</td>
<td>95% - 106%</td>
</tr>
<tr>
<td>9 (2, 500)</td>
<td>8,526</td>
<td>1</td>
<td>0% - 303%</td>
</tr>
<tr>
<td>10 (5, 500)</td>
<td>17,352</td>
<td>6</td>
<td>43% - 191%</td>
</tr>
<tr>
<td>11 (10, 500)</td>
<td>33,300</td>
<td>21,600</td>
<td>0% - 146%</td>
</tr>
<tr>
<td>12 (20, 500)</td>
<td>51,682</td>
<td>21,600</td>
<td>45% - 119%</td>
</tr>
</tbody>
</table>

Table 3.7 reports the total distribution cost, the daily workload at the warehouse, the %-variation from the average daily workload, and the CPU runtime of the heuristic
solutions, for the test instances. We discuss our observations from these experiments as managerial insights below. Figure 3.7 indicates the workload variation for various instances of technology and allowable levels of temporary workers.

Figure 3.7 The impact of technology and allowable level of temporary workers on workload variation; (a) and (b) correspond to the $\phi$ values of 0.75 and 1.0, respectively.
Table 3.7 Cost and variation in daily warehouse workload for different values of $A_1/A_2$, $\gamma$, $\varphi$

<table>
<thead>
<tr>
<th>Problem Instance</th>
<th>Heuristic Solutions</th>
<th>(\varphi = 75%)</th>
<th>(\varphi = 100%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Sigma C$</td>
<td>Putaway</td>
<td>Pick</td>
</tr>
<tr>
<td>DS ($A_1=\Lambda_2, \gamma$)</td>
<td>$\Sigma C$</td>
<td>$\Sigma C$</td>
<td>$\Sigma T$</td>
</tr>
<tr>
<td>1 200, 0</td>
<td>433,745</td>
<td>50</td>
<td>86</td>
</tr>
<tr>
<td>2 200, 0.5</td>
<td>423,934</td>
<td>38</td>
<td>64</td>
</tr>
<tr>
<td>3 200, 1</td>
<td>415,035</td>
<td>31</td>
<td>50</td>
</tr>
<tr>
<td>4 200, 2</td>
<td>411,682</td>
<td>24</td>
<td>119</td>
</tr>
<tr>
<td>5 300, 0</td>
<td>398,479</td>
<td>33</td>
<td>57</td>
</tr>
<tr>
<td>6 300, 0.5</td>
<td>386,686</td>
<td>28</td>
<td>44</td>
</tr>
<tr>
<td>7 300, 1</td>
<td>384,437</td>
<td>24</td>
<td>34</td>
</tr>
<tr>
<td>8 300, 2</td>
<td>382,890</td>
<td>15</td>
<td>85</td>
</tr>
<tr>
<td>9 500, 0</td>
<td>363,920</td>
<td>21</td>
<td>36</td>
</tr>
<tr>
<td>10 500, 0.5</td>
<td>361,062</td>
<td>17</td>
<td>26</td>
</tr>
<tr>
<td>11 500, 1</td>
<td>358,458</td>
<td>13</td>
<td>23</td>
</tr>
<tr>
<td>12 500, 2</td>
<td>354,333</td>
<td>11</td>
<td>18</td>
</tr>
</tbody>
</table>

3.5.3 Managerial Insights

The following insights are based on our experimentation, both on small- and industry-sized problems:

- The optimal WITP plan results in lower total distribution cost and lower workload variance compared to the optimal plan generated by the ITP+WP approach. As indicated earlier, the effect of considering warehouse workforce is that shipments tend to be consolidated less often and the inventory may be readjusted to reduce the total distribution cost. The resultant is that the optimal WITP plan has a more balanced workload at the warehouse compared to ITP+WP plan, which has positive practical implications when the warehouse manager plans for his workforce mix and level.

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• The impact of aisle configuration is that the benefits of WITP are higher when the warehouse has narrow aisles versus wide aisles. Blocking is typically higher in narrow aisles, which increases the required number of permanent and/or temporary workers, thus increasing the warehouse cost contribution in the objective function. Consequently, fewer shipments get consolidated and the workload is spread out more evenly in narrow aisle warehouses (see Tables 3.4 and 3.5).

• The use of a better warehousing technology may increase the workload variance. A better technology means higher productivity rates, which means fewer workers are required at the warehouse. Besides the fact that lower warehousing cost leads to lower total cost, the transportation and inventory costs tend to dominate the objective function, similar to ITP+WP, which may cause higher variance in warehouse workload (see Table 3.7 and Figure 3.7).

• An increase in the allowable level of temporary workers increases workload variation. More low-cost temporary workers allowed likely means less high-cost permanent workers required, thus reducing the warehousing cost contribution to the objective function. Similar to the previous reasoning, a low cost distribution plan with a relatively high workload variation is possible.

On one hand, workload variation at the warehouse has negative implications both from the workforce planning and the total distribution cost standpoints. On the other hand, under certain situations such as better warehousing technology and/or more low-productive temporary workers, a distribution plan with a relatively large workload variation may still yield a lower cost solution (see Table 3.7). This may have interesting practical implications. For instance, one of our industry partners indicated that they
would consider hiring only temporary workers if it saved them on the total distribution cost, even if these workers may lack the training, exposure, experience, and understanding of the work and culture of the company. This manager saw this as an opportunity to negotiate aggressively with agencies that supply such workers on a daily basis, even though he may be faced with increased workload variation.

3.6 Conclusions and Future Research

Our discussions with warehouse managers at several supply chains and the identification of a gap in the academic literature motivated us to introduce the warehouse-inventory-transportation problem (WITP) for supply chains. The proposed WITP balances warehousing, inventory, and transportation decisions such that the total distribution cost is minimized. We modeled the WITP as a nonlinear integer programming model, and considered several warehousing decisions. We also incorporated worker congestion in the WITP model as it is noticeably large in warehouses with narrow aisles.

Our experiments indicated that the WITP approach resulted in a substantial reduction in daily warehouse workload variation, compared to solutions generated by a sequential approach (ITP+WP). In addition, substantial savings in the total distribution cost was observed with the WITP approach. An efficient heuristic method that uses concepts from the Iterated Local Search approach was also outlined for efficiently solving industry-sized problem instances. Further analysis on such problems indicated that the WITP plans were sensitive to other warehousing decisions such as aisle configuration (which affects worker congestion), technology (which determines the worker productivity), and allowable level and productivity rate of temporary workers.
Our current work focuses on the selection of warehouse technology and consideration for strategies for distributing products with varying life cycles. Future work in this area could include extending the model to account for multiple warehouses in a supply chain. For such a network with multi-sourcing the decision of warehouse(s)-to-store allocation in each time-period would be relevant, but may substantially complicate the problem. Additional aspects such as the level of workforce cross-training and the decision of which warehouse technology to employ are worth investigating.

Acknowledgements

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4. Distribution Planning for Products with Varying Life Cycles

In the previous chapter, we demonstrated how a supply chain benefits by incorporating warehousing decisions during the planning phase along with inventory and transportation decisions. A nonlinear model was proposed that jointly considers all the three drivers of the supply chain that generated optimal distribution plans with minimum total supply costs along with significant reduction in the warehouse workload. This chapter focuses on warehouses dealing with two product classes and addresses various decisions related to each product class, while considering the warehouse technology.

4.1 Introduction

In apparel distribution, long life-cycle products are commonly known as basic products and short life-cycle products as fashion products. These two product classes exhibit varying degrees of demand uncertainty and require different sets of decisions with different objectives; cost-efficiency for basic products and time-effectiveness for fashion products (USOTA, 1987; Fisher, 1997; Şen, 2008; Patil et al., 2010).

At the warehouse of our industry partner, an apparel distributor, decisions for basic products follow a traditional approach where replenishment orders from store to warehouse (and in turn to vendor) are placed based on point-of-sale data. In contrast, decisions for fashion products, typically exhibiting more uncertain demand than the basic
products, are managed much differently largely because of the strict requirement to make
the product available at stores at predetermined times during the year to ensure a
competitive edge. In addition, each fashion product at the warehouse arrives from the
vendor as a single consolidated shipment and is deconsolidated based on a predetermined
allocation quantity for each store — all this in a very short time-frame, typically 2 weeks
(see Figure 4.1). These fashion products are sold at stores within 3–4 weeks, before the
next month’s fashion products arrive. Although the inventory and transportation
decisions for basic and fashion products may appear to be separable, they are not
separable from a warehousing perspective as both these product classes are
simultaneously handled by the same warehouse resources (i.e., workers and equipment).
This clearly indicates a need for considering both product classes simultaneously in
developing distribution plans.

The practice at our industry partner for fashion products, and often similar to
other supply chains, is that the vendor-buyer negotiations prescribe the arrival date of
fashion products to the warehouse. Furthermore, predetermined events during the year
prescribe the due date of these products at the stores. Based on a projected workload, the
warehouse manager would negotiate months in advance with the inventory allocation
department on a time-window for delivering fashion products to stores given each store’s
space constraints. This time-window serves as a lever for the warehouse manager to
modify the outbound date — affects holding time — to reduce workload variations. So
apart from other operational decisions, an additional decision for the warehouse manager
to make is the duration of time each fashion product should be held at the warehouse
(between receipt date from the vendor and shipment date to the store).
4.2 Literature Review

In this section we discuss literature related to i) alignment of product to supply chain strategies, ii) integration and coordinated approaches in fashion industry, and iii) analytical models in fashion industry.

4.2.1 Alignment of Supply Chains to Product Classes

Many supply chains experience problems because of the mismatch between the type of products and type of supply chain (Fisher, 1997). Typically, products are classified based on their demand patterns into two categories, functional and innovative (Fisher, 1997). The right approach for the companies is to match their functional and innovative products with physically efficient and market responsive supply chains, respectively.

The grouping of products was extended further based on their structural complexities (Lamming et al., 2000; Li and O’Brien, 2001). According to Lamming et al.
(2000), a product could be *unique* due to its technological contents, handcrafting, customized design, or by its brand reputation. As the degree of uniqueness increases the supply chain shifts from a volume-driven approach to a value-driven one (Brun et al., 2008).

According to Aitken et al. (2003), the success of a company depends upon its ability to classify products and re-engineer its supply chain to accommodate the impact of product life-cycles. They grouped the products into four clusters based on the product characteristics proposed by Christopher and Towill (2002). Depending upon the product’s stage in its life-cycle and the cluster to which it belongs, the product is routed through either one of its four supply chain strategies; push system, Kanban (pull system), leagile, and agile. Through a case study they demonstrated how a company can become successful by implementing such a process in its supply chain management.

According to Khan et al. (2008) the unprecedented shift in the supply chain strategies in the fashion industry over the last decade from product-centric to customer-centric had a major impact on the changing risk profile and responsiveness of fashion retailers. The product-centric strategy is oriented towards supply chain’s efficiency and the customer-centric strategy is designed to close the gaps between supply chain planning and execution. But the customer-centric supply chain particularly, the last mile of retail supply chain, from distribution center to the retail stores, has typically faced challenges in the last few years (Baird, 2008). In the Retail System Research report, Baird (2008) claims that the last mile of retail execution has the potential to deliver significant differentiation, or become an enormous bottleneck in customer service. The need for the alignment of product design with such supply chain strategies and their impact on supply
chain resilience and responsiveness is illustrated through a case study by Khan et al. (2012).

4.2.2 Integrated/Coordinated Approaches in the Fashion Industry

In order to become more responsive and reduce the risk for loss, companies in the fashion industry started to coordinate with upstream as well as downstream components of their supply chains. Weng (1999) studied the power of coordination and strategic alliances within a supply chain system comprising of one manufacturer and one distributor. The paper analyses the roles of information sharing, attitude toward risk, and coordination between manufacturer and distributor in operating products with shorter life-cycle that has price-sensitive random demand. The paper derives useful managerial insights by comparing optimal coordinated production and pricing policies and the distributor’s production and pricing policies in the absence of coordination. The results suggest that such a coordination becomes important when the attitude towards risk is neutral, random demand is very sensitive to the distributor’s sale price, and the distributor’s unit purchase price is much higher than the manufacturer’s unit cost.

Researchers have identified that with the advancement in the information technology supply chain structures in the fashion industry tend towards forming virtual organizations, which are characterized by flexibility, fast responsiveness, and high efficiency (Hughes et al., 2001; Khalil and Wang, 2002; Lin and Lu, 2005). Wang and Chan (2010) investigated two multinational textile enterprises, one integrating upstream with a brand owner on market side and the other integrating downstream with suppliers on manufacturing side. They demonstrated that through a virtual organization approach
the responsiveness of the supply chains has improved and flexibility in responding to the market demand was satisfactory.

4.2.3 Analytical Models in Fashion Industry

The supply chains for fashion products not only should be responsive, but also need to be accurate in meeting the demand. The merchandise has to be marked down if the supply exceeds demand and sold at a price even less than the cost. On the other hand, if supply is less than demand, the company incurs lost sales. To address this issue significant research has been conducted in developing analytical models to optimize inventory replenishment of retail fashion products (Fisher et al., 2001; Weng and McClurg, 2003; Li et al., 2009; Patil et al., 2010). The common features in all those models are:

- All models are stochastic
- Consider a finite selling period and so the inventory at the end of period is marked down in price and sold at a loss
- Consider multiple production commitments such that sales information is obtained and used to update demand forecasts between planning periods

Fisher et al. (2001) proposed a heuristic to solve a two-stage stochastic dynamic program that determines a retail product’s initial and replenishment order quantities that minimize the cost of lost sales, back orders, and obsolete inventory. They differ from the other stochastic inventory models by allowing their method to choose the optimal reorder time, quantifying the benefit of lead time reduction, and choosing the best replenishment contract. Li et al. (2009) generalized the models proposed by Fisher et al. (2001) by taking into consideration time-dependent inventory holding and backorder costs.
Patil et al. (2010) studied the impact of quantity discounts and transportation cost structures on procurement, shipment, and clearance pricing decisions via a stochastic programming with recourse formulation. They claim that under some business settings (such as low inventory and procurement costs), the conventional strategy of placing and transporting a single large order is a better option.

4.2.4 Gaps in the Literature:

From our review of the existing literature, we notice that little or no work exists on supply chain optimization models that

- Jointly considers products with differing life-cycles, and
- Studies the impact of such distribution plans on warehouse’s design and operational decisions, such as technology and workforce.

Table 4.1 indicates these gaps in light of existing literature. In this chapter we extend WITP proposed in Sainathuni et al. (2013) to address several decisions around distributing multiple product classes through a warehouse as indicated in the next section.
Table 4.1 Research gaps in the literature review

<table>
<thead>
<tr>
<th>Papers</th>
<th>Remarks/Decisions Addressed</th>
<th>Gaps</th>
</tr>
</thead>
<tbody>
<tr>
<td>Patil et al. (2010)</td>
<td>All are stochastic and address only fashion products.</td>
<td>1. No model jointly considers basic and fashion products</td>
</tr>
<tr>
<td>Webster and Weng (2008)</td>
<td></td>
<td>2. Warehousing workload and costs are not addressed</td>
</tr>
<tr>
<td>Chen et al. (2008)</td>
<td></td>
<td>3. Effect of warehousing technologies in handling multiple product classes is not studied</td>
</tr>
<tr>
<td>Weng and McClurg (2003)</td>
<td>The commonly addressed decisions are ordering/production quantities, inventory at manufacturer/retailer, back orders, marked-down/salvage/lost sale prices.</td>
<td></td>
</tr>
<tr>
<td>Weng and Parlar (2005)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Li and O'Brien (2001)</td>
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<td></td>
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<tr>
<td>Fisher et al. (2001)</td>
<td></td>
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<tr>
<td>Weng (1999)</td>
<td></td>
<td></td>
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<tr>
<td>Khan et al. (2012)</td>
<td>Aligning products with supply chains - product-centric and customer-centric</td>
<td></td>
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<tr>
<td>Wang and Chen (2010)</td>
<td>Virtual organization (upstream and downstream integration of fashion supply chain)</td>
<td></td>
</tr>
<tr>
<td>Brun and Castelli (2008)</td>
<td>Segmentation tree model based on product, brand</td>
<td></td>
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<tr>
<td>Bergvall-Forsberg and Towers (2007)</td>
<td>Agile supply chains in fashion industry</td>
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</tr>
<tr>
<td>Aitken et al. (2003)</td>
<td>Product clusters with supply chain strategies</td>
<td></td>
</tr>
<tr>
<td>Fisher (1997)</td>
<td>Matching products with supply chain</td>
<td></td>
</tr>
</tbody>
</table>

4.3 The Warehouse-Inventory-Transportation Problem for Two Product Classes (WITP-TPC):

The WITP-TPC for two product classes is intended to determine the optimal distribution of basic and fashion products from vendors to stores via a warehouse with the objective of minimizing the total distribution cost. The WITP-TPC incorporates the following decisions related to basic and fashion products:

- The timing of inbound and outbound shipments for each fashion product;
- The inventory holding time of each fashion product at the warehouse; and
- The technology to be employed at the warehouse to handle basic and fashion products.
Figure 4.2 depicts warehousing decisions for fashion products during a time horizon of \( t = 1 \) to \( T \) periods. The notations \( t_b \) and \( t_e \) in the figure denote beginning and ending periods for inbound and outbound of fashion shipments, respectively. The period between the two represents the length of the fashion window. Given the duration of the fashion window in a time horizon, a key decision variable to consider here is the determination of the timing of inbound and outbound shipments for each fashion product. These two times determine the resulting inventory holding time (\( HT \)) of each fashion product at the warehouse. When a fashion product arrives at the warehouse it can be shipped to the stores only after a processing time (\( PT \)). A fashion product should reach the store on or before the due date, \( t_d \), considering the lead time (\( LT \)) from warehouse to the stores.

![Diagram of warehousing decisions for fashion products](image)

**Figure 4.2** Warehousing decisions for fashion products

Before presenting the model for WITP-TPC we first discuss some of the key factors that significantly impact the worker’s productivity during putaway and picking activities.
4.3.1 Warehouse Technology

The putaway productivity, measured as putaway rate in a pallet storage system depends mainly on i) the type of strategy used to putaway pallets (e.g., direct putaway from receiving to storage area); ii) the configuration of aisles in the warehouse (e.g., number, length, height and width); iii) the type of storage systems used (e.g., floor and rack storage); iv) the storage policy implemented (e.g., randomized and class-based); v) the material handling equipment (MHE) used (e.g., pallet jack and counter balance lift truck) and vi) the assisting technology used to putaway (e.g., put-to-light, voice-picking, and RF). Similarly, the pick rate in an order picking system also depends on all the above mentioned factors except that because picking is typically done at a case or piece level the decisions involved within each factor might differ from that of a pallet storage system. Among these six factors, we consider storage systems, MHE, and assisting technologies as the major cost contributing factors for putaway and picking technologies at the warehouse.

The decisions associated with each factor affect not only warehouse throughput and costs, but also one another. Warehouse managers often struggle to determine the right combination of all these factors that would result in optimal putaway or pick rates thereby helping them to effectively handle inflow and outflow of products. We call such a combination of decisions as technology. As the decisions in each factor are interdependent it is essential that the resulting combination of all of these decisions is practically feasible to implement at operational level.

There are two difficulties in considering the technology as a part of the model. The first one is to find the list of all feasible technologies that can be employed at the
We discussed above that six different factors could potentially affect the productivity of putaway and picking activities at a warehouse. Each factor has at least 4 sub-factors or types that could impact warehouse productivity and costs (see Table 1.1 in Chapter 1). Hence, there would be at least 4,096 different combinations for each putaway and picking activities. From this list one needs to rule out the combinations that are not feasible to implement at the warehouse. The second task is to estimate the right cost structure for each single technology. Generally, most of the components in a technology has fixed and variable components. Though the fixed costs can be estimated it is difficult to estimate the variable costs as it depends on the factors like amount of usage, quality of maintenance, equipment’s life-cycle, etc. Factors like buying negotiations, discounts, etc further complicates the estimation of accurate cost structure for each feasible combination of technologies.

Based on the information extracted from our literature search (Frazelle, 2002; Napolitano, 2003; Pazour and Meller, 2011; Wulfrat, 2013) and interaction with many industry experts including directors and managers we developed a technology matrix with approximate productivity rates and costs for both putaway and picking activities. Table 4.2 presents the productivity rates and costs for six different putaway and picking technologies. All costs shown in the table have been annualized (i.e., if the actual cost of a technology is $30,000, the years of service is expected to be five, and the interest rate used is 20%, then $A/P = 3.0$, which then annualizes the cost of that technology to $10,000). The annualized costs for each putaway and picking technologies are assessed based on the type of storage system and assisting technology used at the warehouse. The MHE cost depends not only on the type of vehicle, but also on the number of vehicles
used, which is largely dependent on the number of putaway workers employed at the warehouse. So the MHE cost for putaway is the estimated daily cost for each type of vehicle used, which is then added to the daily worker cost. As we consider a piece picking system we do not assess MHE cost for picking technologies.

In Table 4.2 the technologies 1 to 6 for putaway and picking activities represent range of technologies from a manual to an automated system. For example technology 1 for putaway represents putting pallets directly to the storage area with wide aisles and block stacking storage system having randomized storage policy using counter balanced lift truck and paper based technology. Whereas technology 6 for putaway in the same table represent automated storage and retrieval system (ASRS). Similarly, technology 1 for the picking activity represent a manual order picking system with piece picking from gravity flow racks using paper-based technology. Technology 6 denotes an automated system such as A-frame. We now present the nonlinear model for WITP-TPC.

Table 4.2 Technology matrix for putaway and picking activities

<table>
<thead>
<tr>
<th>Technology</th>
<th>Putaway Rate</th>
<th>Annualized Cost</th>
<th>MHE Cost (added to worker cost)</th>
<th>Picking</th>
<th>Pick Rate</th>
<th>Annualized Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>θ₁₁</td>
<td>units/hr</td>
<td>$</td>
<td>$</td>
<td>θ₂₂</td>
<td>units/hr</td>
<td>$</td>
</tr>
<tr>
<td>1</td>
<td>600</td>
<td>0</td>
<td>9</td>
<td>1</td>
<td>100</td>
<td>8,333</td>
</tr>
<tr>
<td>2</td>
<td>1,200</td>
<td>77,500</td>
<td>27</td>
<td>2</td>
<td>200</td>
<td>44,167</td>
</tr>
<tr>
<td>3</td>
<td>2,400</td>
<td>258,333</td>
<td>37</td>
<td>3</td>
<td>300</td>
<td>100,000</td>
</tr>
<tr>
<td>4</td>
<td>3,600</td>
<td>350,000</td>
<td>87</td>
<td>4</td>
<td>400</td>
<td>150,000</td>
</tr>
<tr>
<td>5</td>
<td>4,800</td>
<td>750,000</td>
<td>0</td>
<td>5</td>
<td>500</td>
<td>233,333</td>
</tr>
<tr>
<td>6</td>
<td>6,000</td>
<td>1,000,000</td>
<td>0</td>
<td>6</td>
<td>1,000</td>
<td>666,667</td>
</tr>
</tbody>
</table>
4.3.2 A Nonlinear Integer Programming Model for the WITP-TPC

We make the following assumptions in our mathematical model:

(i) vendors have sufficient supplies to meet the demand at the warehouse;

(ii) quantity of fashion products arriving at the warehouse from the vendors is predetermined based on market research for the fashion trend and expected demand;

(iii) each fashion product is introduced to the market only once during a year;

(iv) all inbound and outbound shipments of fashion products are made during a single time-period;

(v) putaway and picking activities at the warehouse are considered as they both frequently are labor intensive.

(vi) lead time from warehouse to stores is given and deterministic.

We first present the model parameters and decision variables in Tables 4.3 and 4.4, respectively, followed by a nonlinear integer programming model for the WITP-TPC.
Table 4.3 Parameters in the Mathematical Model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V$</td>
<td>index for vendor; $v = 1, 2, \ldots, V$</td>
</tr>
<tr>
<td>$S$</td>
<td>index for store; $s = 1, 2, \ldots, S$</td>
</tr>
<tr>
<td>$P$</td>
<td>index for basic products; $p = 1, 2, \ldots, P$</td>
</tr>
<tr>
<td>$Q$</td>
<td>index for fashion products; $q = 1, 2, \ldots, Q$</td>
</tr>
<tr>
<td>$T$</td>
<td>index for time-period; $t = 1, 2, \ldots, T$</td>
</tr>
<tr>
<td>$t_q^b$ ($t_q^e$)</td>
<td>begin (end) time-period for fashion products in a time horizon; $1 \leq t_q^b \leq t_q^e \leq T-L_s$</td>
</tr>
<tr>
<td>$s_{sq}$</td>
<td>due date of fashion product $q$ at store $s$</td>
</tr>
<tr>
<td>$i$ ($j$)</td>
<td>index for the available technologies for putaway (picking) activities; $i$ ($j$) = 1, 2, \ldots, I</td>
</tr>
<tr>
<td>$\Omega_{vp}$ ($\Omega_{vq}$)</td>
<td>set of products $p$ ($q$) that are sourced from vendor $v$</td>
</tr>
<tr>
<td>$D_{sp}$ ($D_{sq}$)</td>
<td>demand for basic (fashion) product $p$ ($q$) at store $s$ in time-period $t$</td>
</tr>
<tr>
<td>$V_p$ ($V_q$)</td>
<td>volume per item of basic (fashion) product $p$ ($q$); ft$^3$</td>
</tr>
<tr>
<td>$W_p$ ($W_q$)</td>
<td>weight per item of basic (fashion) product $p$ ($q$); lbs</td>
</tr>
<tr>
<td>$L_s$</td>
<td>time to travel from warehouse to store $s$</td>
</tr>
<tr>
<td>$Q$</td>
<td>truck capacity; lbs</td>
</tr>
<tr>
<td>$A_{1i}$ ($A_{2j}$)</td>
<td>average putaway (picking) per worker using technology $i$ ($j$)</td>
</tr>
<tr>
<td>$\phi_1$ ($\phi_2$)</td>
<td>fraction of permanent worker productivity/rate attributed to a temporary worker for putaway (picking); $0 \leq \phi \leq 1$.</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>fraction of permanent workers that a warehouse can employ as temporary workers</td>
</tr>
<tr>
<td>$X_{vq}$</td>
<td>total quantity of product $q$ shipped from vendor $v$</td>
</tr>
<tr>
<td>$X_{sq}$</td>
<td>total quantity of product $q$ shipped to store $v$</td>
</tr>
<tr>
<td>$C^\alpha$</td>
<td>labor cost for a permanent warehouse worker for the entire time-horizon ($T$ periods);</td>
</tr>
<tr>
<td>$C^\beta$</td>
<td>labor cost for a temporary warehouse worker per period $t$; $/$time-period</td>
</tr>
<tr>
<td>$C_i^{MHE}$</td>
<td>cost of implementing technology $i$ used for putaway (picking) activities; $/$</td>
</tr>
<tr>
<td>$C_i^{B1}$ ($C_i^{B2}$)</td>
<td>cost of MHE used with technology $i$ implemented for putaway activity; $/$</td>
</tr>
<tr>
<td>$C_i^{hB}$ ($C_i^{sp}$)</td>
<td>holding cost for basic product $p$ at the warehouse (store $s$); $$/item/time-period</td>
</tr>
<tr>
<td>$C_i^{hq}$ ($C_i^{sq}$)</td>
<td>holding cost for fashion product $q$ at the warehouse (store $s$); $$/item/time-period</td>
</tr>
<tr>
<td>$C_i^f$ ($C_i^s$)</td>
<td>fixed cost of shipment from vendor $v$ (warehouse) to warehouse (store $s$), accounting for distance between them; $$/shipment</td>
</tr>
<tr>
<td>$C_i^v$ ($C_i^w$)</td>
<td>variable weight-based cost of shipment from vendor $v$ (warehouse) to warehouse (store $s$), accounting for the distance between them; $$/lbs</td>
</tr>
</tbody>
</table>

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Table 4.4 Decision Variables in the WITP Model

<table>
<thead>
<tr>
<th>Decision Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_{1t} (\theta_{2j})$</td>
<td>binary indicator of whether technology $i$ ($j$) is employed for putaway (picking) activities</td>
</tr>
<tr>
<td>$\lambda_1 (\lambda_2)$</td>
<td>average putaway (picking) rate of a worker corresponding to the technology deployed</td>
</tr>
<tr>
<td>$\alpha_1 (\alpha_2)$</td>
<td>number of permanent workers required for putaway (picking) activities</td>
</tr>
<tr>
<td>$\beta_{1t} (\beta_{2t})$</td>
<td>number of temporary workers required for putaway (picking) activities in time-period $t$</td>
</tr>
<tr>
<td>$\tau_{sq}^h$</td>
<td>holding time at the warehouse for fashion product $q$ destined to store $s$</td>
</tr>
<tr>
<td>$\rho_{vqt} (\rho_{sqt})$</td>
<td>binary indicator of whether the entire quantity of product $p$ ($q$) is shipped to the warehouse (store $s$) in time-period $t$</td>
</tr>
<tr>
<td>$x_{vpt}^i$</td>
<td>quantity of product $p$ <em>inbound</em> from vendor $v$ to warehouse in time-period $t$</td>
</tr>
<tr>
<td>$x_{spt}^o$</td>
<td>quantity of product $p$ <em>outbound</em> from warehouse to store $s$ in time-period $t$</td>
</tr>
<tr>
<td>$z_{spt}^i (z_{sqt}^j)$</td>
<td>quantity of product $p$ ($q$) <em>inbound</em> to store $s$ from warehouse in time-period $t$</td>
</tr>
<tr>
<td>$y_{pt} (y_{sp t})$</td>
<td>inventory of product $p$ at the warehouse (store $s$) at the end of time-period $t$</td>
</tr>
<tr>
<td>$y_{sqt}$</td>
<td>inventory of product $q$ at the store $s$ at the end of time-period $t$</td>
</tr>
<tr>
<td>$n_{vt} (n_{st})$</td>
<td>number of shipments from vendor $v$ (warehouse) to the warehouse (store $s$) in time-period $t$</td>
</tr>
</tbody>
</table>
Minimize \[ C^a(\alpha_1 + \alpha_2) + \sum_t C^b(\beta_{1t} + \beta_{2t}) + \sum_i C_i^{\theta_1} \theta_{1i} + \sum_{it} C_{i}^{MHE}(\alpha_1 + \beta_{1t}) \]
\[ + \sum_j C_j^{\theta_2} \theta_{2j} + \sum_{pt} C_p^{h} y_{pt} + \sum_a C_q^{h} X_q^{o} \tau_q^{h} \]
\[ + \sum_{spt} C_{sp}^{h} y_{spt} + \sum_{sqt} C_{sq}^{h} y_{sqt} + \sum_{vt} C_{v}^{f} n_{vt} + \sum_{st} C_{s}^{f} n_{st} \]
\[ + \sum_{vpt} C_v^{v} W_p x_{vpt}^i + \sum_{vqt} C_v^{v} W_q X_{vq}^{o} \rho_{vqt} + \sum_{spt} C_s^{v} W_p X_{spt}^{o} \rho_{spt} \]
\[ + \sum_{sqt} C_s^{v} W_q X_{sq}^{o} \rho_{sqt} \] (1)

subject to

Warehouse:
\[ \sum_i \theta_{1i} = 1; \sum_j \theta_{2j} = 1 \] (2)

Technology
\[ \sum_i \Lambda_{1i} \theta_{1i} = \lambda_1; \sum_j \Lambda_{2j} \theta_{2j} = \lambda_2 \] (3)

Workforce
\[ \lambda_1 (\alpha_1 + \phi_1 \beta_{1t}) \geq \sum_{vpt} X_{vpt}^i + \sum_{vq} X_{vq}^{o} \rho_{vqt} \quad \forall t \] (4)
\[ \lambda_2 (\alpha_2 + \phi_2 \beta_{2t}) \geq \sum_{sp} X_{sp}^{o} + \sum_{sq} X_{sq}^{o} \rho_{sqt} \quad \forall t \] (5)

Arrival and dispatch of fashion items
\[ \tau_{sq}^h = \sum_t t \rho_{sqt} - \sum_t t \rho_{vqt} \quad \forall v, s, q \] (7)

\[ \tau_{sq}^h \geq 1 \quad \forall s, q \] (8)
\[ \sum_t t \rho_{sqt} \leq t_{sqt}^v - L_s \quad \forall s, q \] (9)
\[ \sum_t \rho_{vqt} = 1; \sum_t \rho_{sqt} = 1 \quad \forall v, s, q \] (10)

Lead time consideration
\[ z_{sp(t+L_s)}^i = X_{spt}^{o} \quad \forall s, p, 1 \leq t \leq (T - L_s) \] (11)
\[ z_{sp(t-(T-L_s))}^i = X_{spt}^{o} \quad \forall s, p, (T - L_s + 1) \leq t \leq T \] (12)
\[ z^i_{sq(t+t_s)} = X^o_{sq} \rho_{sq} \quad \forall \ s, q, \tau^b_q \leq t \]
\[ \leq \tau^o_q \] (13)

Inventory:

Warehouse

\[ y_{pt} = y_{p(t-1)} + \sum_v x^i_{vp} - \sum_s x^o_{sp} \quad \forall \ p, t > 1 \] (14)
\[ y_{p1} = y_{p(t-1)} + \sum_v x^i_{vp1} - \sum_s x^o_{sp1} \quad \forall \ p \] (15)

Store

\[ y_{sp} = y_{sp(t-1)} + x^i_{sp} - D_{sp} \quad \forall \ s, p, t > 1 \] (16)
\[ y_{sp1} = y_{sp(t-1)} + x^i_{sp1} - D_{sp1} \quad \forall \ s, p \] (17)
\[ y_{sq} = y_{sq(t-1)} + x^i_{sq} \quad \forall \ s, q, \tau^b_q + L_s \]
\[ \leq t \leq \tau^v_{sq} \] (18)
\[ \sum_p V_p z^i_{sp} + \sum_q V_q z^i_{sq} \leq K_s \quad \forall \ s, t \] (19)

Transp:

\[ \sum_{p \in \Omega_v} W_p x^i_{vp} + \sum_{q \in \Omega_v} W_q X^o_{vq} \rho_{vq} \leq Q n_{vt} \quad \forall \ v, t \] (20)
\[ \sum_{p \in \Omega_v} W_p x^o_{sp} + \sum_{q \in \Omega_v} W_q X^o_{sq} \rho_{sq} \leq Q n_{st} \quad \forall \ s, t \] (21)

Bounds:

\[ x^i_{vp}, x^o_{sp}, z^i_{sp}, y_{pt}, y_{sp}, \alpha, \beta, \beta_1, \beta_2, \lambda_1, \lambda_2 \]
\[ \in [0, Z_+] \quad \forall \ v, s, p, t \] (22)
\[ z^i_{sq}, z^h_{sq}, y_{sq}, n_{vt}, n_{st} \in [0, Z_+] \quad \forall \ v, s, q, t \] (23)
\[ \theta_{1i}, \theta_{2j}, \rho_{vq}, \rho_{sq} \in [0, 1] \quad \forall \ i, j, s, q, t \] (24)

The objective function (1) in the above model is to minimize the total distribution cost, which includes the cost related to warehouse workforce (permanent and temporary), technology, inventory holding at the warehouse and stores, and transportation (fixed and variable).

Constraints (2) imply that only one type of putaway/picking technology can be employed at the warehouse, while Constraints (3) determine putaway/picking rates corresponding to the selected technology. Constraints (4) and (5) ensure that sufficient
numbers of permanent and temporary workers are available for putting away the inbound products and picking the outbound products at the warehouse (notice the quadratic terms in the left hand side). Constraints (6) restrict the number of temporary workers to be below an allowable fraction of permanent workers (largely due to limited availability and reduced productivity). Constraints (7) calculate the holding time of fashion products at the warehouse. Note that if the value of $\tau_{sq}^h$ is too small, then it will cause substantial workload variation in handling fashion products during a very short duration (see Figure 4.1). One day processing time required to handle fashion products at the warehouse is satisfied by Constraints (8). Constraints (9) ensure that the fashion products are shipped to the stores considering the travel time and due date. Constraints (10) jointly specify the situation when the entire quantity of a fashion product destined from vendor to the warehouse or from warehouse to a store must be shipped out during a single time-period. Constraints (11)-(13) considering the lead time determine the time at which the basic and fashion products are available at the store after outbound from the warehouse.

Constraints (14)-(18) specify the inventory levels at the warehouse and stores. Considering cyclic distribution strategy for basic products, Constraints (15) and (17) ensure that inventory at the end of the current time horizon is identical to the inventory at the beginning of the next time horizon. Constraints (19) impose space restriction at each store. The weight-based transportation capacities for shipments from vendor to warehouse and from warehouse to store are modeled through Constraints (20) and (21). The bounds on the decision variables are specified by Constraints (22)-(24).
4.4 A Three Phase Heuristic (TPH) for the WITP-TPC

Our proposed heuristic method considers the impact of advancing and/or delaying inbound and outbound shipments, and swapping of product quantities in these shipments on both total distribution costs and workload variation at the warehouse. The heuristic incorporates key features from the well-established Iterated Local Search as the heuristic consists of two alternating phases, a local search phase and a perturbation phase (Loürenco et al., 2002). The heuristic implements three sets of moves, intended to reduce warehousing, inventory, and transportation costs. The high level structure of the heuristic and its framework are shown in the Figures 4.3 and 4.4, respectively. We then explain each step in detail.

**Figure 4.3 High level structure of the ILS-based meta-heuristic**
Figure 4.4 Flow chart of ILS-based meta-heuristic framework for WITP-TPC
1. **Initial solution**: Let $s$ refer to a feasible solution to the WITP. A feasible solution will provide values to the inbound and outbound shipment schedules for both basic and fashion products, the product quantities in each shipment, the technology to be used at the warehouse for putaway and picking activities, the required warehouse workforce (permanent and temporary), and inventory levels at the warehouse and stores. We derive a feasible solution $s$ by ensuring that the total demand is met at both the warehouse and the stores across all time-periods.

2. **Inbound Phase**: For the given initial solution ($s$) we iteratively improve the inbound solution (which consists of inbound basic and fashion product quantities, inbound shipment schedules, inventory at the warehouse, and warehouse technology and workforce for putaway) in three steps.
   a. **Select Technology**: First, we implement *technology* move (described later) to select an appropriate technology for putaway at the warehouse to handle inbound basic and fashion shipments.
   b. **Improve Fashion**: As fashion products have narrow inbound and outbound time-windows and have higher priority compared to the basic products, we first iteratively improve the fashion inbound schedules using *warehouse* and *transportation* moves (described later). After each iteration, a new solution ($s'$) is accepted based on an *acceptance criterion*; the superior solutions are always accepted; i.e., $s' < s$, and the inferior solutions are accepted with a probability $p$. From our initial experiments we set the value of $p$ as 0.05. If the stopping rule is met the algorithm moves to the next step.
c. Improve Basic: The inbound solution is improved further by iteratively improving inbound shipment schedules and quantities for basic products by implementing warehouse and transportation moves. The acceptance of new solutions is same as described above. The search stops if the stopping rule is met and the best solution $s^*$ found so far is recorded. We perturb this solution by implementing inventory move (described later) for a pre-specified number of iterations. The new solution ($s^{*'}$) is accepted only if it is better than the previous best solution (i.e., $s^{*'} < s^*$). Otherwise, $s^*$ remains the best inbound solution and the heuristic progresses to the outbound phase.

3. **Outbound Phase**: Given the best inbound solution found thus far, the outbound solution (which consists of outbound basic and fashion product quantities, outbound shipment schedules, store inventory, and warehouse technology and workforce for picking) is iteratively improved in three steps similar to the ones described above. The three steps are repeated until the stopping rule is met.

4. **Stopping Rule**: The algorithm stops if the maximum number of iterations is reached or if, for a prespecified number of iterations, the newly found solutions fall within $\delta\%$ of the incumbent solution. Based on initial experimentation, we set the value of $\delta$ as $\pm0.25\%$.

4.4.1 **Description of the Neighborhood Moves**

This section describes the three sets of moves that help the search process transition from a current solution, $S$, to a neighboring solution, $S'$. 

**Move 1: Technology Move.** This move is intended to select appropriate technology at the warehouse for putaway and picking activities. When implemented during the inbound
phase, this move randomly selects one from a list of putaway technologies ($\theta_{1t}$). Selection of new technology leads to new productivity rate and cost to handle putaway activity. So, the algorithm at the time of computing the warehousing costs calculates the required permanent and temporary workforce for putaway ($\alpha_1, \beta_{1t}$) as per the productivity rate of the selected technology and then computes workforce and associated technology costs. Similarly, when implemented during the outbound phase, a new technology is selected randomly from a list of picking technologies ($\theta_{2j}$) in each iteration and the warehousing costs for workforce ($\alpha_2, \beta_{2t}$) and technology for picking are calculated based on the selected picking technology.

**Move 2: Warehouse Move.** The motivation behind this move is to improve warehousing cost by reducing the required workforce for putaway and picking activities and balancing warehouse workload by either advancing or delaying a fraction of inbound basic shipment. As we assume that the inbound and outbound shipments for fashion products are made during a single time-period, this move is only implemented on the inbound shipments of basic products. The decision on whether to make a partial or complete move depends on the size of the shipment ($\psi_t$) scheduled in a period $t$. For example, the size of shipments scheduled from vendor $v$ is given by

$$\psi_t = \frac{\Sigma w_p x_{vp}^t}{Q},$$

where, $Q$ is the capacity of the truck.

If $\psi_t > 1$ then a fraction of the shipment is moved to the period that has minimum workload. It has two benefits. First, splitting a large shipment would reduce the required number of permanent workers there by reducing the workforce cost and second, moving
those partial shipments to a period with minimum workload would result in workload balance or reduce workload variation.

Move 3: Transportation Move. The motivation behind this move is to improve transportation costs by consolidating shipments of both inbound and outbound basic and fashion products.

a. Complete Fashion Move: We assume that inbound and outbound of fashion products are made in a single time-period. So during the inbound and outbound phases this move allows entire quantity of scheduled fashion product in period \( t \) to a period \( t' \) that has minimum warehouse workload. The values of \( \rho_{vt}, n_{vt}, \tau_{sq}, \alpha_1, \) and \( \beta_{1t} \) are updated after a fashion inbound move and the values of \( \rho_{sq}, n_{st}, \tau_{sq}, y_{sq}, \alpha_2, \) and \( \beta_{2t} \) are updated after a fashion outbound move.

b. Complete Basic Move: With regard to the basic inbound products we make this move only when the total quantity being shipped from the vendor is less than a shipment i.e., if \( \psi_t < 1 \) then the entire shipment scheduled from that vendor in period \( t \) is moved to a period \( t' \) that has minimum warehouse workload for putaway. The values of \( x_{vpt}^i, n_{vt}, y_{pt}, \alpha_1, \) and \( \beta_{1t} \) are updated after a basic inbound move. With regard to the outbound basic shipments from warehouse to stores instead of making a move based on the size of the shipment we move entire shipment quantity scheduled from store \( s \) in period \( t \) to either a period with minimum number of pickers or to a period that has an outbound shipment scheduled from the warehouse to that store \( s \). The values of \( x_{spt}^o, n_{st}, y_{pt}, y_{spt}, \alpha_2, \) and \( \beta_{2t} \) are updated after a basic outbound move. The motivation behind these moves is two-fold: first, if a shipment is already scheduled from the vendor
or to the store in $t'$ then moving a shipment from $t$ would help in shipment consolidation and save on the fixed cost of shipment; and second, whether or not a shipment is scheduled in $t'$, moving an entire shipment from period $t$ to a period with minimum workload would result in similar benefits that are achieved through *Warehouse Move*.

**Move 4: Inventory Move.** As the above three moves are more intended towards improving warehousing and transportation costs we use swap move to improve inventory costs. We implement two types of swaps; inbound swap and outbound swap on the basic products.

a. **Inbound Swap Move:** For the inbound swap randomly select $m$ vendors and two periods having positive shipments. Then swap the shipment schedules of the $m$ selected vendors between the two periods. Based on our initial experiments $m$ is specified to be less than 50% of the total number of vendors scheduled in the selected two periods. The values of $x_{vpt}^I, n_{vt}$, and $y_{pt}$ are updated after an inbound swap.

b. **Outbound Swap Move:** The outbound swap is similarly constructed. The values of $x_{spt}^O, n_{st}, y_{pt}$, and $y_{spt}$ are updated after an outbound swap.

We now present results from our experimentation with the TPH when solving industry-sized problems.

**4.5 Results from Three Phase Heuristic (TPH)**

We generated a dataset of a supply chain size of 50-vendors, 200-stores, 1000 products, and 28 time-periods. Demand for basic and fashion products for each store was uniformly
generated between 0 and 1 item per time-period, and the unit holding cost at the
warehouse and stores were $0.01/item and $0.1/item, respectively. The labor cost for
permanent and temporary workers were $40/hr and $30/hr, respectively. We assume that
only 50% of the permanent workers are allowed to be hired as temporary workers. The
putaway and pick rates were 1200 and 200 items/hr, respectively.

The last 2 weeks in the 4 week time-horizon is considered as the fashion window
for arrival and shipping of fashion products at the warehouse. Any inbound product
requires a processing time of 1 day at the warehouse. So a product arriving at the
warehouse on any day would only be available for shipping to the stores from the
following day. The last day of the time-horizon is considered as the due date for all the
fashion products to reach the stores.

Several experiments were run to analyze the effects of the following on the supply
chain distribution strategy and workload variation at the warehouse: i) duration of fashion
window, ii) proportion of fashion products, iii) labor cost, and iv) technology. To assess
the quality of the solution we first compare TPH solution with that of a solution obtained
using a naïve policy which we refer to as Basic First Fashion Next (BFFN).

4.5.1 Comparison of TPH with BFFN

In the fashion industry we have observed that different companies adopt different policies
to maximize their profits and to stay competitive in the market. One of the policies being
considered at the warehouse of our industry partner is that before the fashion event starts
the warehouse would first handle all the inbound and outbound shipments of basic
products and then during the fashion window the warehouse would use all its resources to
handle fashion shipments. We refer to such a policy as Basic First Fashion Next (BFFN).
Table 4.5 shows that TPH not only results in 19%-savings in the total distribution cost compared to BFFN policy, but also has lower workload variation.

Table 4.5 Comparison of costs and variation in the warehouse workload between TPH and BFFN solutions

<table>
<thead>
<tr>
<th>Policy</th>
<th>TPH</th>
<th>BFFN</th>
</tr>
</thead>
<tbody>
<tr>
<td>Warehouse Cost</td>
<td>$896,720</td>
<td>$1,111,840</td>
</tr>
<tr>
<td>Inventory Cost</td>
<td>$692,380</td>
<td>$696,584</td>
</tr>
<tr>
<td>Transportation Cost</td>
<td>$1,294,353</td>
<td>$1,627,482</td>
</tr>
<tr>
<td>Total Cost</td>
<td>$2,883,453</td>
<td>$3,435,906</td>
</tr>
<tr>
<td>Average Working Hrs</td>
<td>304</td>
<td>304</td>
</tr>
<tr>
<td>%-Range from Mean</td>
<td>12% - 343%</td>
<td>0% - 442%</td>
</tr>
<tr>
<td>%-Savings</td>
<td>19%</td>
<td></td>
</tr>
</tbody>
</table>

Figure 4.5 shows the workload distribution at the warehouse for both the policies. Analyzing further we could notice that the TPH solution has about 43% of time-periods lie within the range of 30% from the average workload whereas the BFFN solution has only 18% of time-periods that lie within that range. Thus, TPH results in higher quality solutions both in total distribution costs and workload variation at the warehouse compared to an ad hoc policy such as BFFN. We next present the TPH results with varying basic and fashion product-mix ratios.
4.5.2 Warehouse Workload Variation with Varying Product-Mix

We represent the proportion of basic and fashion products handled by the warehouse in a given time horizon as a product-mix ratio. A ratio of 1:0 denotes 100% basic and 0% fashion products. In order to understand the warehouse operational dynamics with varying proportions of basic and fashion products we consider 4 product-mix ratios across a total of 1000 products for our preliminary experiments.

- 1:0 (1000 basic and 0 fashion)
- 3:1 (750 basic and 250 fashion)
- 1:1 (500 basic and 500 fashion)
- 1:3 (250 basic and 750 fashion)

The results show that the increase in the proportion of fashion products increases the variation in the daily workload substantially (see Figure 4.6 (a)-(d)). In the absence of fashion products the best solution of TPH resulted in a well-balanced workload across all periods during the entire time-horizon. For each of the solution we calculated the percentage of time-periods in which the required working hours is within the range of

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Figure 4.5 Comparison of workload distribution between the TPH and BFFN solutions
30% from the average daily workload. We noticed that the percentage of time-periods for each of the solutions were 100%, 57%, 43% and 0%, respectively.

![Figure 4.6 Workload variations at the warehouse with various product-mix ratios](image)

**4.5.3 Effect of the Length of Fashion Window**

It was mentioned earlier in the chapter that the duration of time that the warehouse receives consolidated fashion shipments from the vendors and deconsolidates them based on the predetermined allocation quantity to each store happens in a very short time usually within 2 weeks. We did a sensitivity analysis on the duration of the fashion window to analyze the effect of the length of fashion window on the workload variation at the warehouse. Figure 4.7 compares workload distribution between 1 and 2-week fashion window solutions. We noticed that 1-week fashion window solution has only 22% of time-periods were within the range of 30% from the average workload when
compared to the 2-week solution that has about 43% of time-periods within that desired range. But the total distribution cost of the 2-week window solution was 7% more compared to the 1-week window solution.

![Fashion Window - 1 Week](image1)

![Fashion Window - 2 Weeks](image2)

Figure 4.7 Comparison in the daily workload variation between solutions of one and two weeks of fashion window

### 4.5.4 Effect of Worker Cost on the Workload Variation

We understand that the solutions obtained from TPH are cost driven. So we run a problem instance of 500:500 with permanent and temporary worker costs as $25/hr and 20/hr, respectively. Figure 4.8 compares the workload distribution at the warehouse between solutions obtained with different worker costs. The workload variation at the warehouse with lower worker cost is more compared to the higher worker costs. We noticed that the solution with lower worker cost has only 14% of time-periods were within the range of 30% from the average workload when compared to the high worker cost solution that has about 43% of time-periods within that range. The total distribution cost of the solution with high worker cost is 6% higher than the lower worker cost solution.
Figure 4.8 Comparison of workload distribution at the warehouse between the two worker costs solutions

4.5.5 Effect of Technology

It was mentioned earlier in this chapter that technology plays a major role in determining the warehouse throughput and cost. In order to understand the workload dynamics at the warehouse for different product-mix the above experiments were conducted assuming that the warehouse is set up with a particular technology for putaway and picking activities and the corresponding putaway and picking rates are 1,200 and 200 units/hr, respectively. In order to find the best technology combination for putaway and picking for a given problem instance and to assess the effect of technology on the distribution strategy and workload variation at the warehouse we ran 36 experiments with 36 different combinations of putaway and picking technologies on a problem size of 50 vendors, 200 stores, 750 basic and 250 fashion products, and 28 time-periods. The results are depicted in the Figure 4.9.

It can be observed that the lowest cost solution is obtained with putaway rate and picking rates of 2,400 items/hr and 1,000 items/hr, respectively. We notice that there is a huge difference of about 70% in the total distribution costs when compared to the
solution obtained through a manual system with putaway and picking rates of 1,200 items/hr and 100 items/hr. When compared to the total cost of an automated system with putaway and picking rates of 6000 items/hr and 1000 items/hr, the difference is around 3.5%. The results show that the lower cost solutions are obtained with pick rate of 1000 items/hr suggesting that the automated systems would better suit systems with high throughput. We can notice in the results the best picking technology for a given putaway technology and vice versa. The results also emphasize that as the technology increases from manual to automated it is not necessarily decrease total distribution costs. One should find the right technology combination that best fits its supply chain. As the solutions are cost-driven we intend to do sensitivity analysis on the technology costs.

Figure 4.9 Cost curves for respective putaway and picking technologies

4.5 Conclusions and Future Research

Decisions around warehouse design and operations become increasingly complex with warehouses handling products with varying life cycles; long (basic) and short (fashion). The varying demand patterns and life-cycles associated with each product class requires
different sets of decisions with different objectives; cost-efficiency for basic products and time-effectiveness for fashion products. These differences are typically handled by supply chains separately when planning for inventory and transportation. But these decisions are not necessarily separable from a warehousing perspective as both these product classes are simultaneously handled by the same warehouse resources (i.e., workers and technology). The substantial differences in supply and demand patterns for these two product classes, combined with their warehousing needs has led to high workload variation and operational inefficiencies at the warehouse of our partnering industry.

This chapter focused on warehouses dealing with two product classes and addresses various decisions related to each product class, while considering the warehouse technology. We extended WITP to address this problem and referred to this as Warehouse-Inventory-Transportation Problem for Two Product Classes (WITP-TPC). The WITP extension includes two product classes and determination of technology at the warehouse. The resulting model was a nonlinear MIP. As a solution approach to this complex nonlinear problem, we modified substantially the ILS-based meta-heuristic framework developed for WITP for a single product class in order to address the decisions related to the distribution of fashion products in addition to the basic products. We referred to this meta-heuristic framework as Three Phase Heuristic (TPH).

The TPH was efficient in solving industry-sized problems. The experimental results showed that as the proportion of fashion products flowing through the warehouse increases the variation in the workload increases substantially. The TPH was also efficient in generating solutions with best suitable technologies for putaway and picking.
activities at the warehouse that leads to minimum total supply chain costs. We noticed a substantial difference in the total costs when compared with the datasets that were run considering that the warehouse has predetermined technologies for both putaway and picking technologies. This would suggest that a supply chain might incur excess costs for not adopting best technology at its warehouse to handle products with varying life-cycles.

It was mentioned earlier in this chapter that the typical time-window for receiving and shipping of fashion products at the warehouse would be 1-2 weeks. We considered one and two week fashion window for our preliminary experimentations. A sensitivity analysis can be done with different time-windows (for example 3, and 4 weeks) to explore the impact of holding time of fashion products on the workload variation. In our model we assume that the inbound and outbound for fashion products are made in a single shipment. This constraint may be relaxed to see the effect of multiple inbound and outbound shipments on the workload variation and total costs. In this chapter we analyzed the effect of technologies adopted at the warehouse and compared the total supply chain costs on a single product mix ratio (750:250). A sensitivity analysis with various product mix ratios can be done to see the effect of technologies on the warehouse workload variation and costs.

Our discussions with several warehouse managers indicated that different policies are adopted by different companies in delivering fashion products to the stores. An interesting future avenue of this research is to consider few important policies and use the TPH to compare those strategies to derive some valuable managerial insights that could help warehouse managers effectively utilizing their resources (technology and workforce) in handling basic and fashion products.
Acknowledgements

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5. Conclusions and Future Research

This research was motivated by observing the challenges faced by the Senior Director of logistics department at the warehouse of a U.S.-based apparel distributor. The company has one warehouse and over 300 stores that offer more than 6,000 products for women with varying product life-cycles. Long life-cycle products are replenished to stores from its warehouse based on point-of-sale data, while short life-cycle products are pushed to stores once every four weeks prior to 13 predetermined events in the year (e.g., Christmas and Independence Day).

The warehouse operates in a reactive mode resulting in the substantial variation in daily workload. Another important factor that needed to be considered is that the warehouse of the apparel company handles both basic and fashion product classes. The varying demand patterns and life-cycles associated with each product class requires different sets of decisions with different objectives; cost-efficiency for basic products and time-effectiveness for fashion products. These differences are typically handled by supply chains separately when planning for inventory and transportation. But these decisions are not necessarily separable from a warehousing perspective as both these product classes are simultaneously handled by the same warehouse resources (i.e., workers and technology). The substantial differences in supply and demand patterns for these two product classes, combined with their warehousing needs has led to high workload variation and operational inefficiencies at the warehouse of our industry
partner. These two factors; the integration of warehouse with inventory and transportation and incorporation of varying life-cycle products led to this research. Below we indicate our research contributions.

5.1 Contribution 1

We introduced to the supply chain literature the integrated *warehousing-inventory-transportation problem* (WITP) that jointly considers warehouse utilization and capacities, along with inventory and transportation decisions to identify the optimal distribution strategy (Sainathuni et al., 2013). We developed nonlinear models to address WITP for multi-echelon supply chains. The key aspect we capture in our model is a critical operational element of worker dynamics modeled via picker blocking, which has been a hot topic of discussion and analysis in recent articles on warehouse operations. We also consider other strategic and tactical decisions such as aisle configuration and layout (wide and narrow), warehouse technology, allowable number and productivity of temporary workers, and study their impact on (i) warehouse workload variation and workforce cost and (ii) inventory and transportation decisions.

Our experiments indicated that the WITP approach resulted in a substantial reduction in daily warehouse workload variation, compared to solutions generated by a sequential approach (ITP+WP). In addition, substantial savings in the total distribution cost was observed with the WITP approach. The results based on optimal solutions to relatively small problem instances via Xpress-MP solver provided the following managerial insights that could help warehouse managers to efficiently and effectively manage and utilize their resources (technology and workforce):
The optimal WITP plan results in lower total distribution cost and lower workload variance compared to the optimal plan generated by the ITP+WP approach.

The impact of aisle configuration is that the benefits of WITP are higher when the warehouse has narrow aisles versus wide aisles.

The use of a better warehousing technology may increase the workload variance.

An increase in the allowable level of temporary workers increases workload variation.

5.2 Contribution 2

From a solution perspective, WITP could be considered as NP complete (discussed in detail in Chapter 3). It is also analogous to a two-stage capacitated lot-sizing problem, which typically has weak linear programming (LP) bounds and lacks strong cutting planes (Bitran and Yanasse, 1982). Our preliminary experiments show that though the LP relaxation of WITP can be solved easily, it is difficult to obtain an optimal or a near optimal solution within 6 hours, even for small problem instances. For example, the best solution obtained for a problem instance with 1 vendor, 1 warehouse, 20 stores, 1 product, and 5 time-periods using the Xpress MIP solver has an optimality gap of over 10%. A multi-echelon supply chain can have over 100 vendors, more than 1 warehouse, over 500 stores, and over 1,000 products. The total number of integer variables for WITP instances of this size is over a billion. We realize a need to develop new solution techniques to obtain near-optimal solutions for industry-sized problems. We develop an Iterated Local Search (ILS) based meta-heuristic optimization framework that consists of two alternating phases, a local search phase and a perturbation phase. The heuristic iteratively improves solution by implementing three sets of moves, intended to reduce
transportation, warehousing and inventory costs. The heuristic effectively and efficiently solves the deterministic WITP for large problem instances with a run time of less than an hour.

The experimental results on smaller datasets show that the heuristic solutions either match or lie within 1% of the optimal solutions for most of the problem instances. The heuristic even outperformed the optimal solutions that could not always reach optimality. Moreover, the heuristic was so fast that we noticed a huge difference in the runtime between the heuristic and optimal solutions. The variance in the daily warehouse workload obtained by the proposed heuristic is comparable to that obtained through the optimal solution suggesting that the heuristic is able to balance warehouse workload. The heuristic was efficiently used to solve a problem size of 20 vendors, 500 stores, 1000 products and 5 time-periods. Further analysis on such problems indicated that the WITP plans were sensitive to other warehousing decisions such as aisle configuration (which affects worker congestion), technology (which determines the worker productivity), and allowable level and productivity rate of temporary workers.

5.3 Contribution 3

We extended the WITP to account for two product classes, basic and fashion, and to determine technology at the warehouse. The resulting model is a nonlinear MIP. As a solution approach to this complex nonlinear problem, we modified substantially the ILS-based meta-heuristic framework developed earlier for WITP for a single product class in order to address the decisions related to the distribution of fashion products in addition to the basic products and technology selection. We refer to this framework as Three Phase Heuristic (TPH) as the heuristic improves a solution in three phases (technology, fashion
and basic) and the local search and perturbation components of the ILS are implemented hierarchically in two stages, inbound and outbound. Thus, the meta-heuristic framework is enhanced from a basic ILS to a hierarchical ILS. The TPH efficiently solves a problem size of 50 vendors, 200 stores, 1000 products, and 28 time-periods with a run time of about 90 minutes.

The experimental results show that as the proportion of fashion products flowing through the warehouse increases the variation in the workload increases substantially. The TPH was also efficient in generating solutions with best suitable technologies for putaway and picking activities at the warehouse that leads to minimum total supply chain costs. We noticed a substantial difference in the total costs when compared with the datasets that were run considering that the warehouse has predetermined technologies for both putaway and picking technologies.

5.4 Future Research

We first present possible research questions that have emerged out of this dissertation research.

In Contribution 1 we developed optimization-based approaches for WITP considering supply chains with one warehouse. However, large multi-echelon supply chains typically would have multiple warehouses. Our model can easily be extended to account for multiple warehouses and could potentially explore the benefits of multi-sourcing over single-sourcing. Single-sourcing refers to the condition that a store’s demand is satisfied by a single warehouse. Multi-sourcing means that two or more warehouses could fulfill store demand depending upon the inventory availability, warehouse utilization, and routing considerations. For such a network with multi-
sourcing the decision of warehouse(s)-to-store allocation in each time-period would be relevant, but may substantially complicate the problem.

In our model we assumed that a third-party transportation company was employed to deliver both inbound and outbound shipments. Routing of trucks, typical in private fleet, is not considered. Considering routing in a supply chain network with multiple warehouses would be an interesting area for future research.

Another assumption we made in our model was that the warehouse workers are not cross-trained and so can only perform either putway or picking activities. Cross-training is an important concept that many warehouses consider implementing to increase the flexible worker pool. With some training, workers in the putaway areas can be trained on the picking process (including the operation of required material handling equipment), and vice versa. In so doing, the true underlying cost of workforce is captured accurately via required worker hours. It is not difficult to add appropriate constraints to calculate the worker hours and modify the corresponding cost terms in the objective function. The problem complexity may increase slightly, but the structure will likely remain amenable to our proposed solution strategy. Understanding the dynamics between the level to which the workers can be cross-trained and its impact on the warehouse operational aspects such as worker congestion, workload variation, throughput, and costs would be an interesting extension of our research.

Cross-docking is an alternative distribution approach where products from inbound trailers are directly transferred to outbound trailers. The potential benefits of cross-docking are a reduction in inventory and handling of products. However, changes in the layout, additional material handling equipment, and advanced information
technology infrastructure may be required to support cross-docking, which come at a cost. Including cross-docking and exploring its impact on the optimal distribution strategies in a multi-echelon supply chain is another interesting avenue for future research.

In Contribution 3 we developed analytical model for WITP with multiple product classes. In our study we considered only basic and fashion products. The study can be extended further considering decisions related to seasonal products. Our model assumes that the inbound and outbound shipments of fashion products are made in a single time-period. Relaxing such constraints for multiple inbound and outbound shipments for fashion products and studying its impact on the distribution strategies and warehouse workload would be an interesting area for future research.

We assumed that the typical time-window for receiving and shipping of fashion products at the warehouse would be 1-2 weeks. A sensitivity analysis can be done with different time-windows (for example 2, 3, and 4 weeks) and to explore the impact of holding time of fashion products on the workload variation. In our model we assumed that the inbound and outbound for fashion products are made in a single shipment. This constraint may be relaxed to see the effect of multiple inbound and outbound shipments on the workload variation and total costs. In Chapter 4 we analyzed the effect of technologies adopted at the warehouse and compared the total supply chain costs on a single product mix ratio (750:250). A sensitivity analysis with various product-mix ratios can be done to see the effect of technologies on the warehouse workload variation and costs.
Finally, a critical factor to consider in generating supply chain plans is to account for uncertainty inherent in model parameters, such as product demand at stores and worker productivity at warehouses. Success and efficiency of an enterprise often depends on how effectively the variations in these parameters are managed. Although a deterministic mathematical model can generate an optimal solution for the expected values of the parameters, the optimal solution may turn out to be unreliable if there is significant variability in these parameters. To effectively address the uncertainties, developing a stochastic version of WITP is an interesting avenue for future research.
Bibliography


