A DIRECTION FINDING SYSTEM USING LOG PERIODIC DIPOLE ANTENNAS IN A SPARSELY SAMPLED LINEAR ARRAY

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in Engineering

By

JONATHAN ANDREW WELDON
B.S., Texas Christian University, 2006

2010
Wright State University
I HEREBY RECOMMEND THAT THE THESIS PREPARED UNDER MY SUPERVISION BY Jonathan Andrew Weldon ENTITLED A Direction Finding System Using Log Periodic Dipole Antennas in a Sparsely Sampled Linear Array BE ACCEPTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF Master of Science in Engineering.

Ray Siferd, Ph.D.
Thesis Director

Kefu Xue, Ph.D.
Department Chair

Committee on Final Examination

Ray Siferd, Ph.D.

Ronald Riechers, Ph.D.

Saiyu Ren, Ph.D.

Marian Kazimierczuk, Ph.D.

John A. Bantle, Ph.D.
Vice President for Research and Graduate Studies and Interim Dean of Graduate Studies
ABSTRACT


This thesis explores the use of wide band log periodic dipole array (LPDA) antennas in direction finding systems. A wide band log periodic antenna will be constructed and tested to ensure hardware capability. A novel approach utilizing non-uniform spacing in a linear array will be used to improve the spatial resolution of the direction finding system. These specialized linear arrays are known as minimum redundancy or non-redundant linear arrays.
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DEDICATION

I dedicate this thesis to my wife, Jessica, for her endless efforts in motivating my success, to my mentor, Dr. Ronald Riechers, for his friendship and guidance along my journey and to my cat, Charlotte, for keeping me company into the wee hours of the morning.
1. Introduction

1.1. Thesis Motivation

Development of a relatively small conformal broadband direction finding system for use with airborne or ground assets is an area of interest in current military and civilian applications. The clandestine tracking of enemy combatants via their wireless communications is a high priority in the war on terror and will continue to be in the future. As our assets and wartime scenarios change, a flexible antenna is needed to perform this surveillance and reconnaissance mission. Development of a customizable antenna system offering broad frequency ranges and accurate direction finding capabilities would be of great utility to these applications.

1.2. Thesis Objectives

1.2.1. Define a flexible antenna system as characterized in the motivation.

1.2.2. Describe the capabilities and characteristics of the antenna system.

1.2.3. Discuss the development of the antenna subcomponents and system.

1.2.4. Define a theoretical model of the antenna subcomponents and system.

1.2.5. Compare the theoretical results with the measured results.
2. Phased Array Antennas

2.1. Theory of Phased Array Antennas

The aforementioned need for a flexible, customizable, and small broadband direction finding system poses a problem not easily solved by a singular antenna. Monopulse techniques are one way to solve this problem; but they generally require parabolic dish antennas which have limited flexibility with regard to both bandwidth and size. Antennas with modal characteristics can also be utilized to meet this end but they lack the overall flexibility required, should antenna elements need to be variably spaced.

An alternative solution is the phased array, which provides similar directional gain characteristics to a parabolic dish or modal antenna without the restrictions to bandwidth or size. Phased arrays are also customizable to meet the spatial restrictions of a platform and can allow for its antenna subcomponents to be sparsely located; which if done correctly can further enhance angular resolution. Modern antenna manufacturing techniques allow for production of highly compact wideband antennas capable of being placed conveniently on a platform with minimal effect on space.

It is necessary to discuss the concept of phased arrays in order to build a full understanding of this thesis. A phased array is a compilation of multiple antennas linked together into a system whereby the radiated energy from each source combines such that it can be directed to a desired angle. Once the phased array has been designed to focus the energy in a particular direction it is simple to steer the energy to different angles. All that is needed to direct the steering angle is an adjustment to the phase of each antenna.
The same conclusion can be drawn regarding passive detection by utilizing the bi-directional property of electromagnetics. The angle at which a detected signal is oriented can be found through determining the phase difference of a received signal in the various elements of a phased array.

Angular resolution is the measure of interest for a direction finding system. Because angular resolution is dependent upon directivity, it is necessary to discuss the latter before examining the former. Imagine an antenna with an isotropic radiated field. This theoretical antenna would have uniform field and gain in all directions. If a signal was received by this isotropic radiator it would be impossible to determine the direction of arrival. Now imagine if the radiated field was consolidated spatially as it is with a parabolic dish antenna or a phased array. It is now obvious to conclude that a detected signal lies in a spatial region defined by the radiated field pattern. This antenna pattern is now considered to be directive and it is defined as the ratio of gain in decibels versus an isotropic radiator. The higher the directivity of an antenna system, the more narrow the radiated field. Higher directivity therefore produces an increase in angular resolution by limiting the spatial extent to which field is sampled.

The benefit of phased arrays to direction finding is that the spatial field being sampled is defined by the directivity of each antenna element while the resolution is determined by the directivity of the array. The end result is a fully sampled space with angular resolution relative to the directional gain of the phased array.

2.2 Design of a Phased Array Antenna

Designing a phased array is a fairly simple feat. It only requires that an arbitrary number of antennas be set up in a deterministic fashion such that the phase variation
between each element can be found. The real trick is in determining both the layout and also the antenna types. Various applications drive antenna requirements for linear or circular polarizations. Other considerations such as physical restraints may also play a role in the design of a phased array. This could include aerial platform constraints or simply space limitations. In any situation, a phased array is still customizable to meet the demands.

It is of interest to this thesis to implement an improved resolution design. In order to enhance the resolution it is necessary to first explore what options are both available and possible. One area of research applies the idea of non-redundancy. The intent is to design the phased array such that the element spacing provides sampled field without overlap. This lack of spatial over sampling is what gives the name non-redundant or in some cases, minimum redundancy. The result is a sparse array where the elements are spaced in such a fashion as to minimize or completely remove redundancy in their field patterns. The end benefit is improved sampling capability without increased aperture size, element numbers, or cost.

In order to demonstrate the array field patterns a simple calculation is done using the free space Green’s function in an array setup. Each element will radiate as an isotropic point source, so the only things to adjust will be the phase and intensity. Balanis [1] sums these point source radiators and calls the resulting function the array factor (AF). This array factor demonstrates the directivity gained by an array of isotropic radiators versus a singular point source. The follow equation will generate an array factor from the number of elements, spacing, phase and directed angle.
\[ AF = \sum e^{i(n-1)(kd\cos\theta)+\beta} \]

Equation 1

Where \( k \) is the wave number, \( d \) is the spacing interval in wavelengths, \( \theta \) is the directed angle and \( \beta \) is the phase excitation between elements. This array factor can then be plotted, which will render the equivalent output as the array of standard isotropic radiators, or it can be multiplied with the gain pattern of any particular antenna. Obviously the directional gains will increase similarly for both the isotropic case and other highly directional antennas.

The first thing to look at is the case of minimum redundancy sparse aperture enhanced detection. This is essentially the case of sparse aperture detection where there is minimal redundancy in the field sampling. Moffet [2] first approaches this by looking at the function and form behind minimum redundancy arrays. The way to consider these arrays is as an optimization to a uniform linear array or a large monolithic aperture. The idea is that by simply removing some of the elements or aperture there is no degradation to the spatial resolution of an antenna or array. With this in mind it is easy to construct an array of non-uniformly spaced elements and test the improved resolution versus an equivalent uniform array. The following table illustrates some of the minimum redundancy cases as posed by Moffet. The non-redundant case will be covered shortly with information from Duan’s [3] paper along with a comparison of Duan’s and Moffet’s minimum redundancy cases.
Recall the aforementioned array factor equation. In order to apply the array factor equation to the general array spacing shown it is necessary to first discuss the spacing factor \( d \). It is fairly simple to calculate the output of the array factor with regards to \( d \). Set a starting point for \( d \) at the origin and then simply increment the spacing factor \( d \) to fit the desired configuration. In the uniform array scenario \( d \) will start at zero for the first element, increment to one for the second element, then two and so on. In the minimum and non-redundant cases it will need to be incremented according the configuration spacing. Computationally the sum of the configuration spacing must be applied to the spacing factor such that \( d \) is the total sum of values for each element. Once this array factor is established it is easy to both simulate and manufacture the array. The following table contains spacing factors from Duan’s paper for both minimum redundancy and non-redundant cases. Simulation will be performed later comparing these spacing factors to those obtained by Moffet.
The interesting fact is that by comparing Moffet’s and Duan’s tables it is easy to see that Duan utilized Moffet’s restricted case for the minimum redundancy arrays. The following plot demonstrates the output of several cases portrayed in Duan’s table; a MatLab simulation will be demonstrated later to validate these results and also verify a modeled antenna system.

<table>
<thead>
<tr>
<th>$L$</th>
<th>$K_{ULA}$</th>
<th>INTER-SENSOR SPACING OF MRA (d)</th>
<th>$K_{MRA}$</th>
<th>INTER-SENSOR SPACING OF NRA (d)</th>
<th>$K_{NRA}$</th>
</tr>
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<td>3</td>
<td>0.7071</td>
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<tr>
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<td>1.3</td>
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<td>1.3</td>
<td>0.2097</td>
</tr>
<tr>
<td>5</td>
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<td>0.1304</td>
<td>1.3.5</td>
<td>0.1031</td>
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<td>6</td>
<td>0.2390</td>
<td>1.5.3.2</td>
<td>0.0841</td>
<td>1.3.6.2</td>
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</tr>
<tr>
<td>7</td>
<td>0.1890</td>
<td>1.3.6.2.3</td>
<td>0.0690</td>
<td>1.3.6.8.5.2</td>
<td>0.0390</td>
</tr>
<tr>
<td>8</td>
<td>0.1543</td>
<td>1.3.6.6.2.3</td>
<td>0.0413</td>
<td>1.3.5.6.7.10.2</td>
<td>0.0280</td>
</tr>
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<td>9</td>
<td>0.1291</td>
<td>1.3.6.6.2.3</td>
<td>0.0310</td>
<td>1.4.7.13.2.8.6.3</td>
<td>0.0206</td>
</tr>
<tr>
<td>10</td>
<td>0.1101</td>
<td>1.2.3.7.7.4.4.1</td>
<td>0.0231</td>
<td>1.5.4.13.3.8.7.12.2</td>
<td>0.0162</td>
</tr>
</tbody>
</table>

**Table 2 Duan’s table of minimum and non-redundant array configurations**
In considering the original motivation it now becomes obvious that a phased array utilizing resolution enhancing non-redundant spacing will meet or exceed the requirement for a flexible and accurate direction finding system. The next track is that of the individual antenna design. The elements of the phased array must also meet the requirements set forth in the motivation or all of these advantages will be lost. The main consideration beyond antenna size and directivity is also its customizability and bandwidth. This being the case, it follows that a wide band planar log periodic antenna would be a perfect fit for such a system. It can be built to meet particular bandwidth requirements while also retaining wide band operation all while remaining small, rigid.
and inexpensive. The following section will discuss the theory and operation behind a log periodic dipole antenna, also known as a log periodic dipole array, and will define a method by which to design and manufacture an antenna within certain parameters.
3. Log Periodic Dipole Array

3.1 Theory of the Planar Log Periodic Dipole Array (LPDA)

The log periodic dipole array (LPDA) offers good flexibility with regard to bandwidth and a moderate level of directivity. These characteristics make it an ideal choice for a direction finding phased array. The following section will focus on the planar log-periodic dipole array and discuss both the theory of operation and also the design procedures.

The LPDA, as seen in Figure 2, is defined as a linear array of dipoles connected with an alternating feed line whereby varying lengths of dipole elements are used to establish the low, high and incremental wavelengths within the design constraints. This type of antenna is classified as frequency independent as the antenna pattern and input impedance vary negligibly over a band of frequencies within the designed bandwidth [4]. The bandwidth, frequencies and gain of the antenna can be arbitrarily defined to meet the needs of the design. This application in particular will use a 3 GHz bandwidth operating between 1 GHz and 4 GHz.
3.2 Design of a Log Period Dipole Array

Utilizing C. Peixeiro’s [6] nomograph in Figure 3, which corrects for a miscalculation of the radiation pattern in Robert Carrel’s [4] original design procedure, a suitable directivity for this application can be chosen. In this design, 8 dBi of directive gain will be used and although the choice appears arbitrary at first, physical restrictions created by manufacturing processes drive this decision. These restrictions will be discussed in detail later. After choosing the desired directivity, the scale factor $\tau$ and space factor $\sigma$ for the dipole elements can then be determined from the nomograph. The scale and space factors are simply a determination of the logarithmic variation of dipole element sizes and also their linear separation within the array. The dotted lines located on the nomograph are Carrel’s lines of constant directivity. Peixeiro’s corrected contours are seen as solid lines. Using the nomograph and choosing 8 dBi of constant directivity results in a scale factor of 0.86 and a space factor of 0.165.
Figure 3 Constant Directivity Contours

$\alpha$, which is first noted in Figure 1 is the half angle for the geometric angle formed by the dipole array, must also be found. It is calculated using equation 24 from Carrel’s paper and the predetermined values for the scale factor and space factor.

\[
\tan \alpha = \frac{1 - \tau}{4\sigma}
\]

Equation 2

$\alpha$ is then easily determined to be .2090. This will be important later when determining the other dimensions of the antenna including boom length and element sizes.

Determining the largest and smallest element sizes is the next order of business. It is important to note that although the desired bandwidth of 3 GHz is defined as the ratio of the largest wavelength over the shortest wavelength these do not directly determine the largest and smallest elements of the LPDA. Truncation coefficients must
be used to fulfill the directivity requirements of the design criteria. The result is an effective bandwidth which is less than the initial design although the end result has minimal effect on the overall bandwidth of the antenna. Carrel discusses application of a bandwidth ratio using the desired bandwidth and an active region bandwidth. This has been replaced by Peixeiro with a nomograph and truncation coefficients which modify the large and small dipole lengths, although the end result is the same. Figure 4 displays Peixeiro’s nomograph for truncation coefficient $K_1$, which is relative to the longest dipole element. Figure 5 displays the nomograph for $K_2$, and it applies to the shortest dipole element. Note the nomographs use constant curves for $Z_0$. This is the input impedance of the LPDA, however with the planar LPDA design the truncation coefficients have almost no effect on the input impedance as the impedance is defined by the planar line widths. This will be discussed in more detail later on. Following the 8 dB directivity curve and utilizing the space factor it can be determined that the truncation coefficient $K_1$ is 0.55.
Again, following the 8 dB directivity curve in Figure 5 and relating it the spacing factor produces the truncation coefficient $K_2$, which is 0.3.
Utilizing the truncation coefficients $K_1$ and $K_2$ it is simple to determine the new dipole lengths for the small and large element dipoles as defined by $L_1$ and $L_N$.

\[ L_1 \geq K_1 \lambda_{\text{max}} \quad \text{Equation 3} \]

\[ L_N \leq K_2 \lambda_{\text{min}} \quad \text{Equation 4} \]

The resulting values are 0.165 meters and 0.0225 meters, respectively. As this accounts for the bandwidth of the active region it is now possible to determine the number of dipole elements needed.

\[ N = 1 + \frac{\log(L_1/L_N)}{\log(1/\tau)} \quad \text{Equation 5} \]

Inputting in the now known $L_1$ and $L_N$ along with the previously determined scale factor it is easy to obtain a value for the number of dipole elements $N$, which comes out to be 14.2122. Obviously partial dipoles do not exist so this number is rounded down to 14.

Now that the number of dipole elements has been determined it is appropriate to find the boom length, or the overall length of the LPDA. However, it is necessary that the impedance values for the boom and dipole elements be determined first. This
produces considerable issue, as a planar design utilizes dielectrics and microstrip lines to produce planar arrays. The implementation of planar methodologies creates a whole new set of problems. This design is based on Campbell’s [7] planar LPDA as seen in Figure 6. The antenna is built around a copper stripline inside of a dielectric board which feeds the LPDA etched on the outside of the same dielectric boards. The resulting antenna elements and boom are formed using microstrip lines. The antenna section, as discussed separately from the stripline, will be referred to as the antenna plane. Unfortunately the transverse electromagnetic field traveling down the stripline encounters a 1:4 balun transform, seen in Figure 7, at the interface of the board and the open air. This balun causes the impedance seen by the stripline to be a quarter the size of the actual impedance value due to the transform. Pantoja [8] noticed this design flaw and redesigned Campbell’s planar LPDA. Pantoja used a material with a dielectric constant of 2.5 which allowed him to compensate and design a high impedance antenna so that the balun transform resulted in a matched load.
This thesis design uses Rogers 3010, adjusting the impedance of the array elements and boom to account for the 1:4 balun seen at the stripline boundary is all but impossible. The line widths necessary to produce higher impedances on the order of 200 Ohms can not be achieved using available photo etching techniques on a material such as Rogers 3010 with a dielectric constant of 10.2. As such, the impedance values for the boom microstrip and dipole elements must be chosen to provide both desirable size and impedance characteristics as large as possible. The balun induced impedance mismatch will then be managed with the use of a Klopfenstein Taper.
The Klopfenstein taper, as seen in the cross section cut of Figure 8, is an impedance matching Dolph-Tchebycheff transmission line taper designed to minimize reflections over a particular pass-band. The benefit of this design versus other transformers is the wide band capability, which in this application is ideal. There is no wavelength dependent length requirement which also enables the taper design to be tailored to physical constraints of a transmission line structure. This particular taper also works best with a primary mode transmission line. The primary mode criteria are met with the use of microstrip and stripline transmission lines as they are forms of TEM waveguides. Therefore use of the Klopfenstein taper as an impedance matching transformer is ideal in this planar antenna structure. The Klopfenstein taper will be used in this design after the impedance of the antenna is found.
Recall the impedance mismatch seen due to the reflection induced balun. Using free online software, TXLINE by Applied Wave Research, it is easy to calculate relative dielectric constants and line widths for given impedances of planar structures such as striplines and microstrips. Knowing a relative minimum line width limit and the width of the stripline allows for a good starting point for this process. The following equation determines the input impedance of the antenna plane. $Z_0$ is the impedance of the antenna boom and $Z_a$ is the impedance of the dipole elements. The photoetching process used in this design reaches a physical limit just below three tenths of a millimeter. This is advantageous as the line width for a stripline with a characteristic impedance of 50 Ohms is approximately three tenths of a millimeter; while the line width for a microstrip with characteristic impedance of 80 Ohms is roughly the same. The advantage occurs because
at 80 Ohms line widths are on the verge of becoming too small to etch but provide a high value for $R_0$. The closer $R_0$ becomes to 200 Ohms the easier it is to match the antenna plane to the stripline characteristic impedance of 50 Ohms.

\[
R_0 = \frac{Z_0}{\sqrt{1 + \frac{Z_0}{4\sigma Z_a}}}
\]

Equation 6

\[
\sigma' = \frac{\sigma}{\sqrt{\tau}}
\]

Equation 7

Utilizing the above equation and the previously determined values for $\sigma$ and $\tau$ it is easy to obtain a value for $R_0$ of 51.5 Ohms. The 1:4 balun transform as seen by the stripline results in a value of 13 Ohms.

At this point it is possible to jump back and determine the boom length, dipole lengths, and separation distances for the dipole elements. Revisiting the boom length it makes sense to look back at Carrel’s design paper. As discussed prior, the $L_1$ and $L_n$ values determine the active region bandwidth as determined by the truncation coefficients $K_1$ and $K_2$. This active region bandwidth helps determine the boom length
for the array. $B_s$ is the structure bandwidth and also the active region bandwidth. It is determined by the ratio of the longest dipole element to the shortest dipole element.

$$B_s = \frac{L_1}{L_N} \quad \text{Equation 8}$$

Once $B_s$ is determined Carrel’s length equation can be used to solve for the boom length of the antenna.

$$\frac{L}{\lambda_{\text{max}}} = \frac{1}{4} \left(1 - \frac{1}{B_s}\right) \cot \alpha \quad \text{Equation 9}$$

Where $L$ is the boom length, $\lambda_{\text{max}}$ is the maximum wavelength, $B_s$ is the structure bandwidth and $\alpha$ is the previously calculated half angle for all the dipoles on one side. Using this equation $L$ is calculated to be 30 centimeters. This can be validated by using Peixeiro’s equations for both spacing and element length. The only information needed is the space and scale factors. Utilizing Peixeiro’s equations one and two it is easy to solve for the overall length and element sizes using multiple iterations. The same applies to the dipole element separation distance.
\[ \tau = \frac{L_{N+1}}{L_N} \]  \hspace{2cm} \text{Equation 10}

\[ \sigma = \frac{d_N}{2L_N} \]  \hspace{2cm} \text{Equation 11}

Once the iterative process is complete a table of dipole element lengths and separation distances is produced. The process is trivial and will not be shown but Table 3 shows a list of values for the 14 dipole element lengths and 13 values for separation distances. Summing the separation distances renders the overall boom length. Fortunately the value is approximately 30 centimeters, which validates the previously calculated boom length.
It is important to note that this is the free space length for the boom and that in order to compensate for the microstrip manufacturing an effective dielectric scaling factor must be used in order to scale the length. As the dielectric material surrounding the antenna is half air and half glass epoxy it is no easy task, fortunately TX LINE can calculate the effective dielectric constant for both a microstrip and stripline. The effective dielectric constant is six for the microstrip and 10.2 for the stripline. This makes sense given one is a dielectric half-space and the other is fully engulfed in a dielectric. Applying the scaling factor renders the following values for both dipole element length and element separation distance.
<table>
<thead>
<tr>
<th>Dipole Element Length [Centimeters]</th>
<th>Element Separation Distance [Centimeters]</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.0935</td>
<td>2.0881</td>
</tr>
<tr>
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<td>1.7957</td>
</tr>
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</table>

Table 4 Modified Dipole Element Length and Separation Spacing

Now that everything is calculated and the input impedance of the antenna is known, it is time to design a Klopfenstein taper to fix the impedance mismatch caused by the balun. Recall the impedance of the antenna is 50 Ohms, so to correct the mismatch it is required to compensate for the balun and match a 50 Ohm line to the down converted 12.86 Ohms. Utilizing the design in Klopfenstein’s [5] paper and MatLab, impedance values can be readily calculated for the taper line. The only notable difference in calculation versus Klopfenstein is how the modified Bessel function is handled. Klopfenstein used standard tables for calculating the Bessel function but MatLab handles this just as any other special function. Calculating the impedance is then simple. The following equations demonstrate the key inputs for solving the taper design problem.
\[
\ln(Z_0) = \frac{1}{2} \ln(Z_1Z_2) + \frac{\rho_0}{\cosh(A)} \left\{ A^2 \phi(2x/l, A) + \frac{1}{2} + U(x - l/2) + U(x + l/2) \right\}, |x| \leq l/2
\]

Equation 12

\[
= \ln(Z_2), x > l/2, \quad U(z) = 0, z < 0,
\]

\[
= \ln(Z_1), x < -l/2. \quad U(z) = 1, z \geq 0,
\]

Equation 13

\[
\phi(z, A) = -\phi(-z, A) = \int_0^z \frac{I_1(A\sqrt{1 - y^2})}{A\sqrt{1 - y^2}} dy |z| \leq 1
\]

Equation 14

\[Z_0\] is the calculated impedance value along the taper and represents the function. \[Z_1\] and \[Z_2\] are the high and low impedance values intended to be matched, \[A\] is a constant derived from minimizing reflections in the pass band and \[\phi\] is the Bessel function integral. Notice the unit step functions included in the equation, these are simply in place to handle the beginning and end of the taper design as there will be a small mismatch between the taper contour and the actual impedance values for the lines. Figure 9 is a plot of the impedance curve generated using Klopfenstein’s method. The impedance mismatch of the contour is very noticeable at the ends of the curve.
Converting the impedance values to line widths is also a relatively simple task, done by iteratively calculating incremental line widths using TXLINE. In this design the impedance curve was segmented into 10 values which were then placed in TXLINE rendering 10 line widths. These line widths were then placed at equal intervals to the end of the stripline creating a smooth solid metal taper. The taper does not look much more than a simple expanded line but the values are correct and correspond with a Bessel function solution. Note also that there is not a length requirement associated with this taper design as it is wideband and any physical length to wavelength relationship is not relevant. Therefore an arbitrary and reasonable length was chosen to fit the design. The previously mentioned mismatches at the ends of the taper are handled by smoothing the
edge. Although there is no requirement to smooth the discontinuity it is easily accomplished and can help further mitigate reflections seen by the boundary.

At this junction all aspects of design for the planar log periodic dipole array have been covered. The antenna elements have been tailored to fit the initial design parameters the boom length and spacing has been calculated and the dielectric scaling factors have been implemented. The only step necessary at this point is the manufacturing. Standard photoetching techniques are used and in order to produce a quality photoetching image it is necessary to use a schematic tool. AutoCad was used to draw the schematics to include the correct lengths and line widths for both the planar antenna arrays and also the stripline. The etching process is fairly simple once this is done. The only complicated portion to this design is the handling of the stripline. Two boards must be etched, one will have an antenna plane and a stripline plane while the other will have only an antenna plane and a completely conductor free back side. When placed together the two boards must align such that the stripline, microstrip antenna booms and microstrip dipoles all align appropriately. Great care must be taken to align the sub-millimeter lines and ensure that the dipole elements line up in an agreeable fashion. It is also very important to ensure enough line is left at the ends to allow for a stripline launcher to be connected to the rigid structure without overlapping with the antenna elements. Extra transmission line does not alter the impedance characteristics of the antenna. Figure 10 displays the two antenna planes and Figure 11 displays the center stripline. The second image of Figure 11 is the zoomed in version of the Klopfenstein taper which adorns the end of the stripline running between the antenna planes. Note that
the appearance of variation in line widths for the antenna elements is an artifact of the image processing only and not a realizable variation to element line width.

Figure 10 Planar LPDA Antenna Planes
Figure 11 Klopfenstein Taper with Zoomed Taper End
4. Modeling and Simulation

4.1 Log Periodic Dipole Array Modeling

Antenna modeling is a useful method to obtain the designed antennas anticipated field patterns. Although the antenna for this thesis is planar and dielectric effects are important, the free space dimensions are still useful. Modeling an LPDA is best done using a free space computational method. Numerical Electromagnetic Code, or NEC for short, was developed to computationally solve for antenna parameters and field patterns. The code is based entirely on the moment method, as devised by R.F. Harrington [12], and is freely available online. NEC allows a user to graphically or textually build an antenna and then apply an excitation to test the resulting parameters. Given that the NEC does not account for the dielectric boards or the planar transmission lines it is not the best tool for determining input parameters for this design. However, the information obtained by the free space model are accurate as they are produced by generating the sum of thousands of individual radiating current elements. These summed elements can be used to produce the field pattern, impedance and polarization of an antenna. The field pattern results are also independent of the input impedance. Therefore, if the input excitation has a single unit value then the radiated field produced is normalized, rendering accurate antenna gain patterns.

Applying the current design to NEC code should provide an accurate and adequate gauge of the field patterns expected from the log periodic dipole array. The only notable difference in the antenna design, besides the obvious planar design and size
disparity, is the feed point. The free space LPDA is fed at the small element end while the planar antenna is fed at the large element end. However, this is a visual difference only. The planar log periodic array is fed using a stripline. Therefore in reality there should be no difference in a coaxial feed line or the stripline transmission line. This is important because the free space antenna model for the LPDA does not show a center conductor stripline. Instead it is simply a free space wire model of the antenna elements and boom which is excited with a unit input at the small element end. This model can be seen in a front and isometric view in Figures 12 and 13, respectively.
One of the less obvious features about this antenna is the alternating elements along the feed boom. Each individual boom only has half of the half-wave dipole element along the feed line. The other half of the half-wave dipole element lies along the other feed line. The individual elements then alternate booms in a crisscross pattern. This is called a crisscross feed and it can be seen in Figure 14.
Figure 14 Alternating Elements along the Two Feed Lines

It is apparent that the actual feed lines of the LPDA do not crisscross to match the antenna elements. This does not pose a problem however as Balanis [9] details the reason behind a crisscross log periodic design and the benefits generated by a coaxial feed as demonstrated above. Mechanically crisscrossing the feed between adjacent elements provides a 180 degree phase shift added to the terminal of each antenna element. Since the phase between the next closest adjacent half-element is almost in opposition, very little energy is radiated by them and their interference effects are
negligible. At the same time, the longer and larger spaced elements radiate. This is to say that the phase shift introduced by the crisscross prevents mutual coupling between elements while helping the smaller dipoles act as directors and the larger elements as reflectors, similar to a Yagi-Ulda antenna. The mechanical phase reversal between these elements produces a phase progression so that the energy is beamed end fire in the direction of the shorter elements. The feed arrangement noted above with the coaxial feed is convenient as it not only provides a balanced feed line but it also introduces a 180 degree phase reversal between elements. This phase reversal is due to the broad band balun introduced by the transmission line, as the center conductor feeds one boom transmission line and the outer conductor feeds the other boom transmission line. The same desirable phase shift is then created without the use of a mechanical crisscross feed line.

One other factor that should be discussed before producing field patterns using NEC is the transition from planar conductors to wire conductors. The width of a conductor provides for its impedance characteristics. Variations to the length of a conductor adjust the phase and resistive attenuation. Although the intent is not to calculate the input parameters for the log periodic array it is still of benefit from an accuracy standpoint to adjust the wire diameter to an appropriate size. Caswell [6] addressed this issue and determined the transformation between flat strips to wires to be,

\[ R_w = \frac{W_s}{4} \]  

Equation 15
where $W$ is the wire radius and $W_s$ is the strip width. Given that the boom and dipole elements all have the same impedance value it is then easy to apply the transform uniformly across the entire antenna model. Previously the line widths were determined to be roughly three tenths of a millimeter. In order to simplify things the NEC model was done using a wire diameter of one tenth of a millimeter. This was then applied to both the boom wires and dipole elements.

Now that the free space antenna has been designed, the modeling issues discussed and parameters calculated it is simply a matter of application to model the wire antenna and create field patterns. The following figures will display both the antenna orientation and relevant field patterns. Four patterns will be shown for both the horizontal and vertical polarization at the center frequency of 2.5 GHz. Recall Figures 12 and 13, they show the front and isometric view of the modeled antenna. The Z-axis indicates the upward direction and it is the axis of rotation for the $\phi$ parameter. The two images shown for vertical polarization will have a $\phi$ orientation of 0 degrees and 90 degrees. This will demonstrate the field radiated from end fire emission, the back lobes and the side lobes. The horizontal polarization case will be demonstrated with a horizontal cross section and the end fire case. The cross polarized case will show strong isolation between polarizations.
Figure 15 Field Pattern of Antenna Gain @ 2.5 GHz.

$\phi = 0$ indicating an end fire scenario toward the right in the X-axis direction. Notice the back lobe falls below -10 dBi while the end fire gain is near 7 dBi. The area above and below the dipole elements is a minimum as expected.
Figure 16 Field Pattern of Antenna Gain @ 2.5 GHz.

\[ \phi = 90 \] indicating an end fire scenario into the page. Notice the side lobes are symmetrical about the Z-axis and appear as an expected dipole field pattern. The maximum gain is around 1 dBi.
Figure 17 Field Pattern of Antenna Gain @ 2.5 GHz.

$\theta = 0$ indicating an end fire scenario toward the right in the X-axis direction. Notice the gain is uniform at a floor of -25 dBi. This is the horizontal scenario indicating cross polarization. There is good cross polarized isolation given there was 7 dBi of gain in the co-polarized field pattern.
Figure 18 Field Pattern of Antenna Gain @ 2.5 GHz.

\( \theta = 90 \) indicating an end fire scenario toward the right in the X-axis direction. Notice the back lobe falls below -10 dBi while the end fire gain is near 7 dBi. This is a horizontal cut therefore the nulls seen in the vertical cut don’t exist.
4.2 Phased Array Simulation

The array factors discussed earlier are simulated using only five elements for ease of computation. The following plot is the array factor in dB as plotted on a linear graph versus the steering angle. The red array factor is the standard five element uniform array while the blue array factor is the general minimum redundancy array per Moffet’s paper. Utilizing this information along with the prior discussion regarding resolution it is easy to see the increased angular resolution achieved by using a minimum redundancy array.

![Figure 19 Element Uniform Array vs. 5-Element Minimum Redundancy Array](image)

This should by no means be considered the only array factor which will render minimum redundancy results. Duan proposes an array factor that is somewhat different
and generates equally as interesting an output. The following plot will compare the two minimum redundancy arrays. Moffet’s array factor can be seen in blue while Duan’s array factor can be seen in red. The results depict an obvious difference whereby the general minimum redundancy array from Moffet’s paper provides increased resolution capability versus the minimum redundancy case posed by Duan.

![Figure 20 5-Element General Minimum Redundancy Array vs. 5-Element Minimum Redundancy Array](image)

As it follows the final case to consider is of course that of the non-redundant case. Using Duan’s table of non-redundant array configurations it is easy to compare the various array setups. The following plot will show the three various minimum and non-redundant array factors versus one another along with the original uniform linear array.
The color scheme will follow that the uniform array is blue, Moffet’s minimum redundancy array is red, Duan’s minimum redundancy array is cyan and the non-redundant array is magenta.

![Array factors for four 5-Element Arrays](image)

**Figure 21** Array factors for four 5-Element Arrays detailing the Uniform, Non-Redundant and both Minimum Redundancy cases.

It’s quite obvious that the non-redundant array, seen in magenta, offers the narrowest array factor. This will then provide the highest resolution for all the various enhanced arrays.

The following plot was generated by multiplying the array factor, AF, with the field of the log periodic dipole array as modeled in NEC. The result is the antenna...
pattern of a phased array of log periodic dipole antennas. It is easily seen that there is slight improvement with a highly directive antenna, and as expected at higher gain array factors the contribution of the log periodic antennas is less noticeable.

Figure 22 NEC Log Periodic Dipole Array Pattern Multiplied with Sparse Array Factors
5. Results

5.1. Planar Log Periodic Dipole Array Results

Efforts were made to manufacture five antennas such that a fully realized phased array could be constructed. Ultimately only two antennas were successfully made operational. The following section discusses the manufactured antennas and their properties.

Two antenna planes were manufactured one had the copper ground plane completely removed and the other board was etched down to the stripline with taper. Careful effort was taken to ensure the stripline and boom lines of the antenna planes lined up perfectly. This is necessary to ensure a proper TEM waveguide with the predetermined impedance characteristics. Figure 23 displays one antenna plane, the other antenna plane is simply the conjugate. This achieves the necessary alternating dipole elements.
Figure 23 Planar Log Period Dipole Array

Figure 24 is a completed set of log periodic dipole antenna planes lined up with a stripline equipped with a Klopfenstein taper. Notice the line widths are roughly the same and the taper is at the small element end of the dipole array. The small bit of solder noticeable on the taper is necessary for the wire over the edge which shorts the stripline to one of the antenna planes. The obvious holes present on the board are artifacts from the stripline launcher and nylon screws. The stripline launcher acts to connect both antenna planes together and provide a rigid mechanical connection while the nylon screws pull the two antenna planes together minimizing the air gap between the dielectric
and the stripline. This is necessary not only for purposes of physical solidarity, but the
design was calculated for a dielectric constant of 10.2. Variations to the dielectric half
space that are an order magnitude different can drastically alter the impedance of the
stripline.

![Figure 24 Completed Log Periodic Dipole Array with Stripline](image)

The two boards were then sandwiched together with extreme care taken to ensure
proper alignment of the stripline with the microstrip antenna planes. Figure 25 shows the
firing end of the antenna as it was put together. You can notice the metal piece over the
edge of the top layer which is the shorting connection to one of the antenna planes. The blue foam in the image is standard blue insulation foam which has electromagnetic properties outside the spectrum of interest. The same goes for the white nylon screws used for mechanically securing the boards together. These components therefore do not introduce any noticeable variations in to the resulting data.

Figure 25 Image of the Firing End of the Antenna

Figure 26 displays the connector end of the log periodic array. Notice the tape placed over the edge of the board, this was done to stabilize the connection. The stripline launcher was attached directly to the glass epoxy boards which are fairly flexible. This flexibility caused a small amount of “slop” in the connection which caused variations to the input impedance and rendered erroneous results. The tape holds the connection stable to ensure minimal shifts to the input impedance.

Figure 26 Image of Stripline Launcher Connected to the Antenna
After a long iterative process making small changes to the connector, nylon screws and foam, reliable results were achieved. The completed log periodic dipole array, although not attractive, is mechanically stable and renders repeatable measurements. The process for achieving these results was repeated for the only other functional antenna. The voltage standing wave ratio (VSWR) measurement for the first antenna can be seen in Figure 27. This covers the full range of the intended design frequency band from 1-4 GHz. Notice the scale is 10 units per division on the vertical axis. At low frequencies the antenna gives highly periodic results which are not necessarily uncharacteristic of a log periodic antenna. As the frequency increases the increasing and decreasing trends appear to level off rendering a nice clean operating band after about 2.5 GHz. Figure 28 displays similar results for the second antenna. Take note that the vertical axis has a scale of 5 units per division. The same nice clean operating band after 2.5 GHz is realized with the second antenna as well as the first. The next step is to look at a smaller band of interest between 2.5 and 3.5 GHz and then make antenna pattern measurements to compare their similarities. It would be of benefit to a phased array to have antenna elements with matching field patterns. Therefore identifying similar patterns will be of importance.
Figure 27 Voltage Standing Wave Ratio Measurement for the First LPDA from 1-4 GHz
Figure 28 Voltage Standing Wave Ratio Measurement for the Second LPDA from 1-4 GHz

Figure 29 is a zoomed in version of the VSWR between 2.5 and 3.5 GHz for the first antenna and Figure 30 is the zoomed in version for the second. This is the most stable frequency band on these particular antennas. Notice the vertical scale is now limited down to five units per division. Over the bulk of this band the antennas keep a VSWR below 3.5 which is very important for functional operation. VSWR measurements below this level indicate a minimal amount of reflection over the band and reasonable transmission. This is important because this band was used for measuring antenna patterns for the log periodic dipole array.
Figure 29 Voltage Standing Wave Ratio Measurement for the First LPDA from 2.5-3.5 GHz
After taking the VSWR measurements, which indicate the relationship of the antenna impedance to the characteristic impedance of the driver, it only makes sense to sample the radiated field to get a visual of the patterns created by the antennas. The measurements displayed in the following images look at various frequencies throughout the band of interest. The antennas were set up to be vertically polarized, as it was modeled in Figure 18. Note the side lobes do not necessarily match the modeled field exactly but the trend is similar with minimal back lobes. The data was collected taking into account system loss and the receive antenna gain was properly considered. The low values of gain are simply the result of a limited aperture for the antennas.
Figure 31 First Antenna at 2.6 GHz

Figure 31 is the first antennas gain pattern. This pattern was taken from the maximum gain output of the antenna at the frequencies between 2.5 and 3.5 GHz. It is important to note that the primary lobe is off bore sight, which is merely an artifact of the data collection method being misaligned.
Figure 32 Second Antenna at 2.52 GHz

Figure 32 is the second antenna gain pattern. It too was taken at the maximum gain between 2.5 and 3.5 GHz. This pattern is less than 100 MHz off of the same maximum gain frequency as the first antenna. The designed center frequency was 2.5GHz for both antennas and it is obvious they work optimally at said frequency. The following antenna patterns are simply to demonstrate both antennas function across the band.
Figure 33 First Antenna at 2.5 GHz
Figure 34 Second Antenna at 2.5 GHz
Figure 35 First Antenna at 3 GHz
Figure 36 Second Antenna at 3 GHz
Figure 37 First Antenna at 3.25 GHz
Figure 38 Second Antenna at 3.25 GHz
5.2. Phased Array Results

The field pattern for the log periodic dipole array at 2.525 GHz was imported into MatLab, for modeling of the various sparse array configurations. As before, this was done by simply multiplying the field pattern point by point with the array factor. The result is the output from the array relative to the antennas used. This pattern shows strong similarities to the modeled antenna pattern when it was multiplied with the generated array factor.

![Figure 39 Log Periodic Dipole Array Pattern Multiplied with Sparse Array Factors](image)

Although it was not feasible, given the constraints, to manufacture an entire array to validate this output a small phased array of two elements was constructed using the
two functional antennas. The following antenna patterns will demonstrate the phased array output at 3.25 GHz. Figure 40 demonstrates the output with a primary lobe at roughly 135 degrees. This was due mainly to bore sighting errors. After this pattern was collected an additional length of coaxial line was added to the input of the second antenna to act as a phase shifter. This additional phase acts to steer the primary lobe of the phased array. Figure 41 shows the primary lobe shifted nearly 90 degrees clockwise indicating that the phased array does in fact function and is deterministically dependent on phase. As stated before due to the bidirectional nature of electromagnetics it is now possible to not only steer transmitted energy but also determine the phase of a received signal. This is the necessary criteria for a direction finding system.
Figure 40 Two Antenna Phased Array at 3.25 GHz
Figure 41 Two Antenna Phased Array at 3.25 GHz
6. Conclusions

Looking back and considering the original premise for this thesis work, it was postulated that a non-redundant linear array of log periodic dipole antennas would be a viable option for a wide band direction finding system. The antennas would provide not only adequate bandwidth to attack the problems of interest but also provide increased directivity contributing to higher levels of angular resolution.

The log periodic antenna posed unique challenges in terms of manufacturing and calculation. The various sources of information never clearly defined the precise methodology for designing a planar log periodic array. Multiple papers including Carrel’s and Campbell’s did not properly consider all facets of the problems and even had a few errors. After thoroughly vetting the antenna design against all available sources it was obvious that an impedance mismatch between the transmission line and antenna planes was an inevitable end. Accepting this mismatch was not an option, so finding a solution was necessary. Utilizing a Klopfenstein taper was a novel approach to correcting the impedance issue and it appears from the results that it was effective for a good part of the band. This could potentially be a new method for correcting the impedance mismatch found in all planar log periodic antennas.

The simulation results for the various sparse array configurations demonstrated not only good but excellent increases in the overall angular resolution of a phased array direction finding system. Even in the scenarios where a low directivity antenna is used the resultant field pattern is quite narrow and drastically exceeds the direction finding
resolution of a normal linear array. The array factor coupled with the log periodic antenna provides increased resolution over other less directive options and is ultimately a good choice for a direction finding sparse aperture array.

Through modeling and manufacturing of a wide band log periodic antenna and simulation of a sparse aperture phased array, all aspects of the original notion have been satisfied. The end result was a planar conformal linear polarized compact antenna capable of being phased into an accurate and resolution enhanced direction finding system.
Bibliography


Appendix A -- MatLab Code

Log Periodic Dipole Array Design Code

%LogPeriodic Equations
clear all;
close all;
%Low Frequency(in GHz)
freqmin = 1;
%High Frequency(in GHz)
freqmax = 4;
%Scale Factor(tau)
tau = 0.86;
%Space Factor(sigma)
sigma = 0.165;
%Use Nomographs to Find K1 and K2 Scale Factors
K1 = 0.55;
K2 = 0.3;
%Calculate Alpha
alpha = atan((1-tau)/(4*sigma));
%Maximum Wavelength(meters)
lamdamax = 3e8/(freqmin*1e9);
%Minimum Wavelength(meters)
lamdaN = 3e8/(freqmax*1e9);
%Calculate the L1 and LN Boom Lengths with Scale FActors
L1 = K1*lamdamax;
LN = K2*lamdaN;
%Calculate the number of antenna elements
N = 1+(log(L1/LN)/log(1/tau));
N = round(N);
lengtharray = zeros(N,1);
lengtharray(1) = L1;

%--------------------------------------------------------------------------
%--------------------------------------------------------------------------
%Calculate Antenna Plane Impedance as Seen by Stripline
%--------------------------------------------------------------------------
%--------------------------------------------------------------------------
%Desired boom impedance
zo = 80;
%Desired dipole impedance
za = 80;
%Calculate Sigma Prime
sigmaprime = sigma/sqrt(tau);
%Calculate Ro, the Antenna Plane Impedance
Ro = zo/(sqrt(1+(zo/(4*sigmaprime*za))));
%------------------------------------------------------------------------
%Calculate Antenna Elements Size and Spacing (use TXLINE)
%------------------------------------------------------------------------
%Enter Dielectric Constant of the Planar Board Material
epsilon = 10.2;
%Use TXLINE to find Epseff or the effective permittivity of the boom
Epseff = 6.8;
%Calculate Ed the effective permittivity of the dipoles
Ed = (epsilon*1.167)/2.2;
%Calculate Structure Bandwidth(Bs)
Bs = L1/LN;
%Calculate Structure Length(L in Meters)
L = lamdamax*.25*(1-(1/Bs))*cot(alpha);
%Calculate Length and Separation Sizes for Dipoles
for i = 2:N;
    lengtharray(i) = lengtharray(i-1)*tau;
end
separationarray = lengtharray(1:(N-1)).*2.*sigma;
%Modify Lengths and Separations Accounting for Dielectric/switch to
centimeters as well
modifiedlengtharray = 100.*lengtharray./sqrt(Ed);
modifiedlengtharray
modifiedseparationarray = 100.*separationarray./sqrt(Epseff);
modifiedseparationarray
Ro
%------------------------------------------------------------------------
%Calculate Klopfenstein Taper which acts as a Chebychev Filter for
%Impedance Matching. Uses a modified bessel function to calculate the
%idealized taper
%------------------------------------------------------------------------
%Calculate Apparent Load Due to 1:4 Balun Note that we Divide by 3 to
%achieve a reasonable VSWR within physical limits of the substrate.
Rload = Ro/4;
%Desired Impedance
Zo = 50;
%Calculate Reflection Coefficient
rhoo = .5*log(Rload/Zo);
%Desired Reflection Coefficient
rhom = rhoo/20;
%Calculate A, the passband reflection coefficient
A = acosh(rhoo/rhom);
\[ y = 0:-.1:-.9; \]
\[ z = 0:.1:1; \]
\[ \text{fncy} = A.*\text{sqrt}(1-(y).^2); \]
\[ \text{xy} = \text{besseli}(1,\text{fncy})./\text{fncy}; \]
\[ \text{phiy} = \text{cumtrapz}(y,\text{xy}); \]
\[ \text{intfuncy} = \exp(0.5*\text{log}(\text{Zo}*\text{Rload})+(\text{rhoo}/\cosh(A)).*(A^2).*\text{phiy}); \]
\[ \text{fncz} = A.*\text{sqrt}(1-(z).^2); \]
\[ \text{xz} = \text{besseli}(1,\text{fncz})./\text{fncz}; \]
\[ \text{phiz} = \text{cumtrapz}(z,\text{xz}); \]
\[ \text{intfuncz} = \exp(0.5*\text{log}(\text{Zo}*\text{Rload})+(\text{rhoo}/\cosh(A)).*(A^2).*\text{phiz}); \]
\[ \text{z1} = \log(\text{Zo}); \]
\[ \text{z2} = \log(\text{Rload}); \]
\[ \text{plot}(y,\text{intfuncy}) \]
\[ \text{hold on} \]
\[ \text{plot}(z,\text{intfuncz}) \]

**Phased Array Design Code**

```matlab
\%Antenna Array
clear all;
close all;
phi = 0:(2*pi/3649):2*pi;
c = 3e8;
theta = pi/2;
\%Wavelength
Freq = 2.525e9;
lamda = c/Freq;
\%Spacing Size
R = 1;
d = .1*lamda;
k = (2*pi)/lamda;
A = 1;
A1 = 1;
A2 = 1;
A3 = 1;
A4 = 1;
\%Array Elements: Starting at Element 1 and going to element 5.
\text{E} = A*\exp(i*k*(0)*\sin(theta)*\cos(phi)) + A1*\exp(i*k*d*\sin(theta)*\cos(phi)) +
A2*\exp(i*k*2*d*\sin(theta)*\cos(phi)) + A3*\exp(i*k*3*d*\sin(theta)*\cos(phi)) +
A4*\exp(i*k*4*d*\sin(theta)*\cos(phi));
\text{Edb} = 20*\log10(\text{abs}(\text{E})/\text{max}(\text{abs}(\text{E})));\%
\%Testing Array of Symmetric Minimum Redundancy Array (2-3 Element Arrays)
\%sharing a singular node at 0
\text{Esymm} = A*\exp(i*k*(0)*\sin(theta)*\cos(phi)) +
A1*\exp(i*k*2*(R*d)*\sin(theta)*\cos(phi)) + A2*\exp(i*k*(R*3*d)*\sin(theta)*\cos(phi)) +
A3*\exp(i*k*4*(R*d)*\sin(theta)*\cos(phi)) + A4*\exp(i*k*6*(R*d)*\sin(theta)*\cos(phi));
```
%General Array from Moffet Paper
Egmra = A*exp(i*k*(0)*sin(theta)*cos(phi)) + 
A1*exp(i*k*(R*4*d)*sin(theta)*cos(phi)) + A2*exp(i*k*(R*5*d)*sin(theta)*cos(phi)) + 
A3*exp(i*k*(R*7*d)*sin(theta)*cos(phi)) + A4*exp(i*k*(R*13*d)*sin(theta)*cos(phi));
Egmradb = 20*log10(abs(Egmra)/max(abs(Egmra)));

%Minimum Redundancy Array from Duan Paper
Emra = A*exp(i*k*(0)*sin(theta)*cos(phi)) + A1*exp(i*k*(R*d)*sin(theta)*cos(phi)) + 
A2*exp(i*k*(R*4*d)*sin(theta)*cos(phi)) + A3*exp(i*k*(R*7*d)*sin(theta)*cos(phi)) + 
A4*exp(i*k*(R*9*d)*sin(theta)*cos(phi));
Emradb = 20*log10(abs(Emra)/max(abs(Emra)));

%Non Redundant Array from Duan Paper
Enra = A*exp(i*k*(0)*sin(theta)*cos(phi)) + A1*exp(i*k*(R*d)*sin(theta)*cos(phi)) + 
A2*exp(i*k*(R*4*d)*sin(theta)*cos(phi)) + A3*exp(i*k*(R*9*d)*sin(theta)*cos(phi)) + 
A4*exp(i*k*(R*11*d)*sin(theta)*cos(phi));
Enradb = 20*log10(abs(Enra)/max(abs(Enra)));

polaraxis = (180/pi).*phi;

-0.56 ];
neclpda = interp(neclpda,50);

nec = 10.^(neclpda./20);
necdb = 20*log10(abs(nec)/max(abs(nec)));
tot = nec.*E;

nratot = nec.*Enra;
nratotdb = 20*log10(abs(nratot)/max(abs(nratot)));
dbtot = 20*log10(abs(tot)/max(abs(tot)));
mratot = nec.*Emra;
dbmra = 20*log10(abs(mratot)/max(abs(mratot)));

figure;
plot(polaraxis,dbtotal,'k');
hold on
plot(polaraxis,Edb,'r');
plot(polaraxis,necdb,'c');
plot(polaraxis,dbmra,'g');
plot(polaraxis,Emradb,'y');
plot(polaraxis,Enradb,'b');
plot(polaraxis,nratotdb,'m');
xlabel('Angle')
ylabel('DB')
title('Field of 5-Element Array Antenna')
realantenna = [0.007981912 0.007838519 0.007436433 0.005175086 0.006046502 0.004999171 0.003770343 0.002738195 0.001807799...
antennaaxis = linspace(0,2*pi,length(realantenna));
newphi = antennaaxis;
Enew = A*exp(i*k*(0)*sin(theta)*cos(newphi)) + A1*exp(i*k*d*sin(theta)*cos(newphi)) + A2*exp(i*k*2*d*sin(theta)*cos(newphi)) + A3*exp(i*k*3*d*sin(theta)*cos(newphi)) + A4*exp(i*k*4*d*sin(theta)*cos(newphi));
Enewdb = 20*log10(abs(Enew)/max(abs(Enew)));
Egmranew = A*exp(i*k*(0)*sin(theta)*cos(newphi)) +
A1*exp(i*k*(R*4*d)*sin(theta)*cos(newphi)) +
A2*exp(i*k*(R*5*d)*sin(theta)*cos(newphi)) +
A3*exp(i*k*(R*7*d)*sin(theta)*cos(newphi)) +
A4*exp(i*k*(R*13*d)*sin(theta)*cos(newphi));

Egmradbnew = 20*log10(abs(Egmranew)/max(abs(Egmranew))); 

Emranew = A*exp(i*k*(0)*sin(theta)*cos(newphi)) +
A1*exp(i*k*(R*d)*sin(theta)*cos(newphi)) +
A2*exp(i*k*(R*4*d)*sin(theta)*cos(newphi)) +
A3*exp(i*k*(R*7*d)*sin(theta)*cos(newphi)) +
A4*exp(i*k*(R*9*d)*sin(theta)*cos(newphi));

Emradbnew = 20*log10(abs(Emranew)/max(abs(Emranew))); 

Enranew = A*exp(i*k*(0)*sin(theta)*cos(newphi)) +
A1*exp(i*k*(R*d)*sin(theta)*cos(newphi)) +
A2*exp(i*k*(R*4*d)*sin(theta)*cos(newphi)) +
A3*exp(i*k*(R*9*d)*sin(theta)*cos(newphi)) +
A4*exp(i*k*(R*11*d)*sin(theta)*cos(newphi));

Enradbnew = 20*log10(abs(Enranew)/max(abs(Enranew)));

reallpda = 10.*realantenna./20;

Enratot = reallpda.*Enranew;

Enratotdb = 20*log10(abs(Enratot)/max(abs(Enratot))); 

Emratot = reallpda.*Emranew;

Emratotdb = 20*log10(abs(Emratot)/max(abs(Emratot))); 

reallpda = 20*log10(abs(reallpda)/max(abs(reallpda)));

totreal = reallpda.*abs(Enew);

realdbtotal = 20*log10(abs(totreal)/max(abs(totreal)));

figure;
plot(antennaaxis1,realdbtotal,'k');
hold on
plot(antennaaxis1,Enewdb,'r');
plot(antennaaxis1,realpda,'c');
plot(antennaaxis1,Enradbnew,'y');
plot(antennaaxis1,Emradbnew,'b');
plot(antennaaxis1,Emratotdb,'g');
plot(antennaaxis1,Enratotdb,'m');
xlabel('Angle')
ylabel('DB')
title('Field of 5-Element Array Antenna')
### Appendix B -- NEC Code

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