Timing-Pulse Measurement and Detector Calibration for the OsteoQuant®

A thesis submitted in partial fulfilment of the requirements for the degree of Master of Science in Engineering

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ABSTRACT

Enchakalody, Binu, M.S. Egr., Department of Biomedical, Industrial and Human Factors Engineering, Wright State University, 2009. Timing-Pulse Measurement and Detector Calibration of the OsteoQuant®.

The OsteoQuant® is a second-generation pQCT scanner, which can provide precise density assessment of bone. The scanner is being upgraded with an x-ray tube radiation source and a semiconductor CZT detector. This thesis provides solutions for two issues: motion-detector timing and dead-time/beam-hardening corrections.

During translation, the motion-control system of the scanner repositions the source and detector by small intervals. However, the detector collects photon counts asynchronously with respect to the motor-timing pulses and reads photon counts (frames) at regular intervals based on its own clock. Since there needs to be correspondence between source/detector location and detector readout, the first part of this project deals with relating the motor- and detector-timing pulses to each other. The goal is to implement a system capable of registering these pulses in microsecond resolution using a common time base. These time stamps can then be used to accurately relate each detector frame to a motor position. The measurement of the timing pulses is achieved by using two 32-bit counters, controlled by a common time base and supervised by a LabVIEW® program. This counter system is capable of measuring a signal with a time period of 2.22 ms with a maximum error of ±3 µs.
The second part of this project deals with methods to correct the effects of dead time and beam hardening. The dead time is related to the properties of the detector system, whereas beam hardening is a result of the attenuation of the poly-energetic spectrum of the x-ray beam. The aim is to correct the photon-count loss due to dead time to an error level of less than 0.5% of the maximum expected photon counts and the non-linearity of the projection values due to beam hardening to an error level of less than 1% of the expected maximum projection value. The measured photon counts vs. current curves were described using a fourth-degree polynomial and then linearized to their expected counts. This provided corrected photon counts within the set error level.

For the beam-hardening correction, the absorber was simulated by aluminum / Plexiglas slabs up to the equivalent thickness of the forearm and the lower part of the leg. The projection-vs.-thickness curves were mathematically modeled using a fifth-degree polynomial and a bimodal-energy model and then linearized. Both corrections supplied satisfactory results if applied to data sets measured the same day. However, the measurement of slabs to calculate the correction parameters is tedious. To avoid daily measurement of all the slabs, a simplified approach was developed by applying the primary corrections from one particular date to the data sets collected on other dates followed by a secondary correction based only on a few plates measured on the specific date. This secondary correction, based on a third-degree polynomial, resulted in residuals within the desired error range. The largest of these residuals for the bimodal primary corrections were less than 0.012 projection value units and those for
the polynomial primary corrections less than 0.017 projection value units for the 10-slab data set, which simulated the forearm thickness. For the 19-slab data-set, which simulated the leg thickness, the maximum residuals were 0.03 and 0.04 projection value units for the bimodal and polynomial primary corrections, respectively. Thus, the bimodal-energy model performed better than the polynomial model for the beam-hardening correction.
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Osteoporosis, which translates to ‘porous bone’, is a disease that deteriorates the amount and quality of bone tissue, causing increased bone fragility. This deterioration of bone may be localized or distributed throughout the body. The diagnosis of osteoporosis is based on the measure of bone mineral density (BMD). Abnormalities in bone, aging and prolonged bed rest are reasonable causes for this decrease in bone density. Studies reveal that bone-mineral loss is an inevitable part of aging [Smith and Shoukri 2000]. Low BMD values are inversely related to the risk of bone fracture. The BMD values can be measured from the spine, hip, wrist or finger of the human skeleton.

Quantitative computed tomography (QCT) is a non-invasive method used to measure BMD. Peripheral quantitative computed tomography (pQCT) is a form of QCT, where the scanned region is restricted to the peripheral skeleton of the human body. The principle of scanning used in pQCT is similar to that of regular CT.
A scanner generally consists of a radiation source and a detector, mounted on a gantry assembly, which can be moved around the object to be scanned. The radiation source emits x-ray photons towards an object of interest, and photons emerging from this object are captured using a radiation detector. The detector and its associated electronics convert these x-ray photons into electronic signals to estimate the transmitted photon count. The transmitted photon count represents the number of photons attenuated by the object of interest, and the counts for different profiles of the object are collected by translating and rotating the scanner. Attenuation coefficients corresponding to the density of the object are calculated by reconstructing this profile information using a reconstruction algorithm. The BMD values are measured from an area of interest in the reconstructed image.

The OsteoQuant® [Hangartner 1993] is a pQCT scanner that can provide precise density assessment of the trabecular and cortical regions of bone. It is a second-generation scanner, which works on the translate-rotate principle. The scanner is being upgraded with an x-ray tube source and a cadmium zinc telluride (CZT) semiconductor detector array. This array consists of 64 pixilated-detector elements. CZT detectors are capable of converting high-energy photons (>200 keV) into electron-hole pairs. The detector can be operated at room temperature. The combination of cadmium, zinc and telluride provides a relatively high density and atomic number, which is advantageous in efficiently converting x-ray energy into electrical energy.

The scanner can cover an arc of 180 degrees with 16 incremental rotations spanning a maximum measurement radius of 100 mm. The mechanical motion of the scanner
is a blend of three orthogonal movements: the translation and rotation of the source and detector and the axial positioning of the gantry. These movements are achieved through a combination of high-precision mechanical hardware and stepping motors.

For each axis of motion, the motor moves the scanner at a required speed and acceleration. These movements have to be well synchronized to assure correlation between the data collected and the measurement location. The motion-control system generates the necessary pulses to rotate the motor shaft for the required measurement interval. The accumulation time for data collection during the measurement interval is controlled by the detector. The data collected is irrelevant unless the motor and data collection timing pulses are correlated.

There are certain properties of a detector such as dead-time characteristics that cause loss in photon counts of the collected data. Beam-hardening is another concern, since attenuation of a poly-energetic x-ray spectrum causes wrongly interpreted attenuation coefficients, which results in reduced projection values $\ln\left(\frac{I_0}{I}\right)$ in the reconstructed image. The effects of dead-time and beam-hardening can produce erroneous BMD values.

There are two goals in this project, which are implemented in two different phases. The first part of this project deals with correlating the independent motor- and detector-timing pulses. The goal is to implement a system capable of recording the time-stamps for these pulses in microsecond resolution. This level of precision is required since the motor and detector events occur at millisecond intervals.
The second part of this project deals with methods to correct the effects of dead-time and beam-hardening. The aim is to correct the photon-count loss due to dead-time to an error level of less than 0.5% of their expected photon counts and correct the miscalculated projection-values to an error level of less than 1% of the expected projection values. For most of these corrections, different mathematical models of the detector data were linearized, and their error statistics were analyzed. The stability of the dead-time and beam-hardening corrections was analyzed using data collected over a period of nine months.

The following sections describe the principle physics involved in these problems, followed by the approach and methods to achieve the desired objectives. The analyses show the expected and observed results, and they are discussed in detail. The thesis is split into two sections based on the main goals of this project.
Background: Timing-Pulse System

The OsteoQuant® is a computed tomography scanner used for bone density assessment. An I-125 isotope had been used as the photon source, but supplier problems necessitated a change to an x-ray tube. At the same time, it was decided to upgrade the photomultiplier detection system to a cadmium zinc telluride (CZT) semiconductor array.

The translation and rotation of the source and detector and the axial positioning of the gantry is achieved through a combination of high-precision mechanical hardware and stepping motors. The linear motion employs linear guides and ball screw assemblies that allow a tolerance of 0.01 mm. The rotation uses a parallel-shaft helical gear to minimize the backlash.

The x-ray beam is collimated into a fan that covers the detector array. Fig 2.1 illustrates the major components of the scanner with the different positioning modes. Translation involves the source-detector assembly moving across the patient while photons are being emitted and measured. There are seven rotations and eight trans-
lations involved for the collection of a complete data set, providing 128 projection profiles for a span of 180 degrees. Rotation involves angling the source-detector assembly about the patient with respect to the x- and y-axes. The axial positioning of the scanner is parallel to the patient axis, usually called the z-axis.

Figure 2.1: Front and lateral views of the OsteoQuant®, where the source (S) and detector (D) assemblies undergo translation (T), rotation (R) and axial positioning (A). The lateral positioning (L) and vertical movement (H) are for patient comfort. [Hangartner 1993]

This axial positioning defines the slice location. There is also a provision for an up-down movement of the limb holder and lateral movement of the scanner for patient comfort, both depicted in Fig 2.1. Each axis of motion requires the motor to move the scanner at a required speed and acceleration. These movements have to be well synchronized to assure correlation between the data collected and the measurement location. The motion control system should be capable of generating the necessary timing pulses according to measurement interval and interval duration. These timing pulses must also coincide with the data collection pulses of the detector. The motion-
control system assumes the position of the scanner gantry to be at its origin prior to a scan.

2.1 Motion Control System

The quality of the reconstructed image of a scanned object depends on the precision of the mechanical devices that position the scanner. Stepper motors are used to reposition the scanner for all axes. Stepper motors are electromechanical devices that convert electrical pulses into mechanical movements. The shaft of the motor rotates under discrete angles depending on the pulses it receives. The sequence at which these pulses are applied determines the rotation of the stepper motor shaft. The number of pulses required to complete one rotation of the motor shaft is a core characteristic of the stepper motor. The frequencies of these pulses are directly proportional to the speed of the shaft rotation. For controlled movements, stepper motors can provide precise positioning and repeatability, and errors are non-cumulative from one step to another.

Depending on the movement intended, the scanner requires its motors to provide rotations at varying length and speed. In order to complete a scan, the OsteoQuant® requires its motors to undergo multiple axis movements at varying velocities. The system that governs these motor movements is called a motion controller. A motion control system would generally include motors, amplifiers and encoders. The amplifiers provide enough current to drive the motor. The encoder translates motion information into step pulses which are fed back into the controller. The motion
control system generates a string of pulse sequences to produce the desired motion profile. The motor position information may be fed back into the motion controller to produce a feedback control loop. The motion controller can be configured to produce full-step, half-step or micro-step pulses.

The motion control system used in the OsteoQuant® is a DMC-1500 by Galil Motion Control. It is designed for coordinated motion profiling and has the flexibility to work with multiple axes. The DMC-1500 accepts over 100 BASIC-like commands for specifying motion and machine parameters that help control independent or multiple axes. The control system is provided with a data table that contains the command to be executed, followed by number of step pulses and frequency. As a precautionary measure, each motion is provided with a set of switch-controlled relays to avoid high speed end crashes from erroneous table values. Each translation and rotation covers the radius of the scanner in equally spaced intervals.

2.2 Motor Shaft Velocity Phases During Translation

At the start of a translation, the motor shaft cannot immediately rotate at the required speed due to inertia. The motor shaft is ramped up from rest to the required speed, and the time to achieve the required speed varies from one motor to another. Figure 2.2 illustrates the trapezoidal velocity profile of the three phases involved in a translation.

Once the required speed is achieved, the motor runs at a constant velocity. The constant velocity is achieved by dropping the acceleration of the shaft rotation to zero.
2.3. DATA COLLECTION

Data collection occurs during this phase of translation. The final phase occurs towards the end of a translation. Due to inertia, bringing the motor shaft to a complete halt requires a gradual slow down. This is achieved by decreasing the velocity, and this deceleration continues until the motor shaft reaches a complete stop. The motion-control system is capable of accepting commands to accelerate and decelerate by the required number of pulses.

2.3 Data Collection

The scanner undergoes translational and rotational motion about the object to measure projections based on the attenuation of x-ray photons. The attenuated photons are captured using the CZT semiconductor detector. To reconstruct the object, we require a good understanding of the material distribution of the object being scanned. This can be achieved by using short durations of photon accumulation as the source and detector arrangement moves about the volume of the object.
Figure 2.3: The source-detector assembly moves across the object along the diameter of the scanner.

Figure 2.3 illustrates the source-detector assembly movement during a translation, during which time the control system repositions the source and detector by small intervals.

The detector system consists of an eValuator-2000 model, 64-channel, high-count rate, CZT linear array detector (eV Products, Saxonburg, PA). The data-collection interface offers user-adjustable parameters for the detector, such as the number of frames and accumulation time. During translation, the detector produces photon count readouts (frames) at regular intervals. The frequency of these frames is independent of the step pulses produced by the motion control system, i.e., there is no common frame of reference between the step and data collection pulses.
2.3. DATA COLLECTION

The accumulation time parameter sets the duration in milliseconds for the detector to collect photons by generating a frame or photon-count readout. For a given translation, the longer the duration, the higher the counts and the lower the number of frames. The accumulation time can be set from 1 to 50 milliseconds, and the number of frames up to a maximum of 1,000,000.

The collected data are only useful if there is synchronization between the motor- and detector-timing pulses, i.e., if each frame can be accurately related to a motor position. These motor- and detector-timing pulses are both preset in a millisecond time period range before each scan. As we do not know the relative phase of the two pulse trains and do not want to assume consistency between the two frequencies, we must measure the time of the occurrence of each pulse using a common time base. Measuring time stamps of these pulse trains using a common time base provides a reference between the two pulse events, which can be used to relate a motor position to a detector frame.

The purpose of the present project is to create a timing circuit that can measure the motor- and detector-timing pulses and register every occurrence of these timing pulses at microsecond resolution.
3

Methods: Timing-Pulse Measurement

The motor-detector synchronization was not an issue with the previous scanner model, since the motor- and detector-timing pulses were generated by a common clock.

One method of approaching this problem is to use the same timing source to generate both timing pulses similar to the previous version of the scanner; another would be to externally synchronize these timing pulses. Synchronizing the motor- and detector-timing pulses is more feasible since it does not require modification of the timing source.

The objective of this project was to create a system capable of registering time stamps at the event of timing pulses. In this approach, the motor- and detector-timing pulses are treated as two independent pulse sources, which are monitored by a common time base. The two independent sources are connected to two counter modules that share a common time base, and they provide interrupt signals to these...
counter modules. After initial synchronization of the two timing modules, time stamps corresponding to the occurrences of the motor and detector pulses are generated for each interrupt signal received.

Since the time periods for both timing pulse sources are in milliseconds, the operating frequency of the circuit should be capable of clocking an event in microsecond resolution, requiring high-frequency counting devices.

Microsecond resolution requires counters working at clock frequencies of 1 MHz or above. They should also have data transfer speeds fast enough to register the timing pulse events within a microsecond. This kind of data transfer can be achieved using a counter with a universal serial bus (USB) interface combined with a temporary storage buffer. The event notification speed from the counters to the computer must be in milliseconds or less to register all events. The specifications of the USB-4301 module (Measurement Computing Corporation, Norton, MA) satisfy our requirements [Measurement Computing 2006]. The counter module is interfaced to a PC using a LabVIEW® program.

3.1 USB-4301

The USB-4301 is a low-power USB-2.0-compliant, 16-bit, 5 channel, up-down binary counter, capable of operating at frequencies as high as 5 MHz. This counter module can be used in event counting and pulse generating applications. The heart of this module is the 9513A counter-timer chip.
Apart from the 9513A chip, a micro-controller provides an internal interrupt. The module is configured using InstaCal, a Windows-based driver and calibration program, which configures the base frequency and the rising or falling interrupt edge. Multiple counter modules are represented by board numbers in InstaCal, i.e., the first 4301 module is Board 0, and the second module is Board 1.

The terminal count (TC) is the maximum count value for a given counter size, which is 65,235 ($2^{16} - 1$) for a 16 bit counter. There are various counting modes that can be software programmed using this module. The programmable parameters can be accessed through a package of 9513A library functions provided with the USB-4301. The key components of the 4301 module are illustrated in Figure 3.1; only the components used in this project are described below, which are extracted from the USB-4301 specification sheet.

**Clock Generator (OSC Out)** - The module provides four base frequencies of 1 MHz, 1.66 MHz, 3.33 MHz and 5 MHz. The selected base frequency can be internally divided by a factor (frequency dividing parameter) of 1 to 5 (FREQ1 - FREQ5). This internally divided frequency is available at the OSC Out pin.

**Digital Input (DI0-DI7) and Output (DO0-DO7)** - The module provides eight digital input and eight digital output ports. These input terminals can detect the state of a given 5V/TTL level input while the output terminals can produce 5V outputs.

**Interrupt Input (INT)** - This input terminal interrupts the micro-controller operation to execute one or more of several firmware routines. It can be programmed
Chapter 3

Functional Details

USB-4301 block diagram

USB-4301 functions are illustrated in the block diagram shown here.

Figure 3.1: Functional block diagram of the USB-4301 module [Measurement Computing 2006]
3.1. USB-4301

to trigger off the rising- or falling-edge clock signal. The interrupt generates an event notification, which will be sent to the computer within 1 to 33 ms. A system-dependant interrupt latency can create a delay of up to 100 µs between successive interrupts, but typical values vary from 9 µs to 40 µs.

Counter Terminals - The module is divided into 5 counter groups, with each group having its own input, output and gate. The counter terminals are the input (CTR1 Input - CTR5 Input), gate (CTR1 Gate - CTR5 Gate) and output (CTR1 Output - CTR5 Output) terminals. Each counter group has a set of software-programable parameters for count source, gating, counting, input and output parameters, BCD or binary counting, etc.

Counter Input - The count source for the input can be any of the five internal input frequencies (FREQ1 - FREQ5) or the external input from any of the five counter-group inputs, gates or outputs.

Counter Output - Each counter group has an individual tri-state, low impedance programmable output port. The output of each counter group can be programmed to respond to different modes:

- inactive output low: always active low
- high pulse on TC: generates an active-high pulse on TC
- toggle last state on TC: active state, toggles on TC
- inactive output with high impedance: default state of output has high impedance
3.2. COUNTER MODES

- active low on TC pulse: generates an active-low pulse on TC

**Gate Input** - Each counter group is provided with a gate, which allows it to externally trigger the counter and control the behavior of the counter. The gate may be used as a count enable, count inhibit or as a trigger to start the count sequence. The counter can be gated using level or edge gating. Level gating starts or resumes the counting sequence for the active high level (AHL) or active low level (ALL) of the input signal. Edge gating can be configured to respond to an active high edge (AHE) or active low edge (ALE) of the input signal.

**Registers** - Each counter group has two 16-bit read/write registers called load and hold registers. The load register stores initial and temporary counter values, whereas the hold register stores instantaneous or current counter values.

3.2 Counter Modes

Counter groups can be programmed to form different counting modes. The next section describes only the counting modes required for this project.

**Mode B** - In Mode B, the counter is only active when the selected gate input is active. The counter ceases counting during the inactive period and resumes counting during the active period of the input.

**Mode X** - This is a hardware, edge-triggered mode capable of instantaneously reading counter values without interrupting the counting process. In this mode, the
counter may be programmed to commence counting with or without a hardware trigger.

**Gating with Interrupt** - Interrupt latency may affect the stored count value when the counter tries to save the current count while the counter is counting. By combining the X and B modes, the counter responds only to the active gate level of an interrupt, and this can be used to our advantage. Using active level gating and an interrupt whose edge triggers to the inactive level of the pulse, the module has ample time to store the current counter value without affecting ongoing counting, because the counter is inactive during readout. We only need to make sure that the inactive period of the pulse is long enough to allow readout under the worst condition (> 40 µs) and that the duration of the inactive period is highly repeatable and exactly defined.

The proposed readout scheme can be achieved with two of the four gating-with-interrupt modes, the ALL gating with a rising-edge interrupt and the AHL gating with a falling-edge interrupt. All four modes are explained below (Fig. 3.2).

**ALL Gating and Falling-Edge Interrupt** - In this mode, the counter counts only during the active-low level of the interrupt signal. Since the interrupt edge precedes the low level of the pulse, the counter counts while saving the current count, causing interrupt latency to affect the stored counts.

**ALL Gating and Rising-Edge Interrupt** - In this mode, the counter counts during the active-low level. The rising-edge interrupt signal precedes the
### 3.2. COUNTER MODES

Figure 3.2: Active level gating with clock-edge interrupt latency of 9 to 40 µs.

 inactive level, and the counter ceases to count while the counter content is read. The inactive period provides sufficient time for the module to store the counts before resuming the count sequence, and the interrupt latency does not affect the count values.

**AHL Gating and Rising Edge Interrupt** - Here, the counter counts only during the active-high level after the rising-edge interrupt signal. Similar to the ALL Gating and Falling-Edge Interrupt mode, the interrupt latency problem affects the stored count values in this mode, too.

**AHL Gating and Falling-Edge Interrupt** - In this mode, the counter counts during the active-high level. Similar to the ALL gating with rising-edge inter-
3.2. **COUNTER MODES**

interrupt, the interrupt edge precedes the inactive period, providing ample time to store count values between successive active gate levels.

Figure 3.2 shows an example of how the four modes treat 2 cycles of an interrupt signal with a time period of 5 ms. It can be observed how the counters measure the time periods of only the active level of the interrupt signal, neglecting the time period of the signal during the inactive level.

Initially, the intention was to simply hardwire the motor- and detector-timing pulses to the interrupt pin and store the count values as time periods for every interrupt event by configuring the modules to counter mode 'Mode X'. To test the precision of the measured time periods, the interrupt signals were simulated using a function generator for time periods ranging from 1 ms to 20 ms. The difference between the expected and measured time periods showed an unacceptable error.

This issue was resolved using the counter mode 'Gating with Interrupt'. The measured time periods using AHL gating with falling-edge interrupt and ALL gating with rising-edge interrupt were within an acceptable error range. However, the drawback of using level gating is that the counter neglects all events during the inactive gate level period, and the measured time period accounts for only the active level period of the interrupt signal.

A solution to this problem is to break up the interrupt signal into a measured and unmeasured segment. The measured segment is the measured time period during the active gate level; the unmeasured segment is the time period of the interrupt signal during the inactive gate level. The unmeasured segment can be measured only if
there is prior information about the duration of the inactive part of the interrupt signal. Since measuring the duty cycle of every interrupt would require additional circuitry, it was decided to reshape the inactive portion of the interrupt signal with an accurately defined duration, irrespective of the time period of the original interrupt. Summing the measured and unmeasured segments gives the actual time period of the interrupt signal.

The pulse-shaping circuit was designed to replace the inactive segment of every interrupt signal with a time period of $128 \, \mu s$. The ALL gating with rising-edge interrupt mode was preferred over the AHL gating with falling-edge mode, because the motor- and detector-timing pulses are short positive pulses that have an active-high time period less than $128 \, \mu s$.

3.3 Description of the System

The independent motor- and detector-timing pulses are assigned to Module 1 (Board 0) and Module 2 (Board 1), respectively. The common time base or master clock for the motor- and detector-timing pulses is internally generated by configuring Module 1 to FREQ1 (1 MHz), which is externally available at the OSC Out terminal of Module 1 (Fig. 3.3).

The scan time for the OsteoQuant® is approximately 3 minutes. A 16-bit counter running at 1 MHz achieves TC in 65 ms. This requires cascading the first two counters in Module 1 and Module 2 to form a 32-bit counter. A 32-bit counter running at 1 MHz achieves TC in approximately 71 minutes, providing more than the required
scan time. CNTR1 in Module 1 and Module 2 represents the lower 16 bits (least significant counter or LSC), CNTR2 in Module 1 and Module 2 represents the higher 16 bits (most significant counter or MSC). The 1 MHz master clock is the input source to the LSC’s of Module 1 and Module 2. The LSC increments for every master clock pulse, whereas the MSC increments on every LSC TC. The motor and detector pulses are connected to the interrupt and the gate terminal of the LSC counters.

The modules, along with the synchronization and pulse-shaping circuits, are designed to register the events of the motor- and detector-timing pulses as time stamps. The following section describes the external circuit components involved in the system. The section thereafter describes the software steps used in generating the time stamps.

### 3.3.1 External Circuit Components

The external circuit ensures that Module 1 and Module 2 receive the shaped motor and detector pulses as interrupts, using the master clock as their common time source. The counting sequence of both modules is synchronized by an internally generated start pulse from Module 1, which, after a delay, also disables an interrupt mask.

![Figure 3.3: The wiring diagram for Module 1 and Module 2](image)

Figure 3.3: The wiring diagram for Module 1 and Module 2
3.3. DESCRIPTION OF THE SYSTEM

Initially set for both interrupt signals. The pulse shaping, start pulse, synchronization and delay circuit, as well as the interrupt mask, represent the external circuit components and are described below.

### 3.3.1.1 Pulse Shaping Circuit

The task of this circuit is to create a pulse of 128 $\mu$s duration upon occurrence of the positive edge of an incoming pulse. The pulse shaping circuit consists of a monostable multivibrator, set-reset (SR) flip-flop and an 8-bit binary counter, which runs on the master clock frequency (Fig. 3.4). The input timing pulse triggers the multi-vibrator circuit into its monostable or unstable state, which eventually flips back into its stable state. The time period of the unstable state can be designed using a resistor-capacitor network. The multi-vibrator is designed to produce an inverted (Q') narrow pulse. The negative edge of Q' sets the SR flip-flop, which in turn, enables the 8-bit counter. The MSB output (Q) of the 8-bit counter toggles every half TC ($\frac{2}{8} \text{ bits} = 128 \mu$s), which resets SR and disables the counter.

![Figure 3.4: Pulse shaping circuit.](image-url)
3.3. DESCRIPTION OF THE SYSTEM

3.3.1.2 Start Pulse

The start pulse enables synchronization and unmasking of the external interrupt sources. The start pulse is generated by CNTR3 in Module 1 by counting up to the first TC at a frequency of 1 MHz. The output mode of CNTR3 is set to a high pulse on TC. It takes approximately 65 ms for the counter to count from zero to TC and generate the start pulse. This pulse is externally available at the CNTR3 Output terminal. This 65 ms delay before starting the counting sequence provides ample time for the counter groups to initialize and configure the necessary modes, such that both modules are in the same state when synchronization occurs.

3.3.1.3 Synchronization Circuit

The synchronization circuit consists of a pulse-shaping circuit followed by a delay circuit (Fig. 3.5). The input to this pulse-shaping circuit is the start pulse. The purpose of the pulse shaping is to toggle the high state of the start pulse (CNTR3 MSB) after 128 µs. The pulse-shaped start pulse (synchronization pulse) provides the first interrupt signal, common to both modules, allowing synchronization of the two counters.

To make sure that no motor or detector interrupts are registered before the occurrence of the synchronization pulse, motor and detector interrupts are masked until a short time after the synchronization pulse has been generated. The unmasking of the interrupt sources is achieved by a signal generated by the delay circuit. The delay circuit consists of a mono-stable multi-vibrator and an SR flip-flop. The purpose of
3.3. DESCRIPTION OF THE SYSTEM

Figure 3.5: Synchronization circuit

the multi-vibrator is to provide a delay of about 0.5 ms after the first interrupt signal.
The pulse-shaped start pulse is the input to the delay circuit. The delayed output of
the multi-vibrator sets the flip-flop. Both counter modules can receive timing inter-
rupts as long as this flip-flop is in its set state. Initially this flip-flop is reset, disabling
the counter modules from accepting interrupts. This reset is achieved using a digital
pulse from the D0 terminal of Module 1. This internally generated high-to-low state
pulse ensures the flip-flop remains reset till the end of the program.

Figure 3.6 illustrates the timing diagram of the pulse-shaped start pulse and the
delay pulse, which goes to its high state approximately 0.5 ms after the start pulse.
During its low state, the delay pulse acts as an interrupt mask to the pulse shaped
motor and detector pulses.

3.3.1.4 Interrupt Mask

The purpose of the interrupt mask is to avoid the modules from receiving any timing
interrupts before the synchronization pulse has been generated. The interrupt mask
consists of two, two-input AND gates with the delay pulse as a common input and
3.3. DESCRIPTION OF THE SYSTEM

3.3.1.5 Combined Circuit

Figure 3.7 shows the wiring diagram for the pulse shaping, synchronization and interrupt mask of the external circuit. Each output of the two AND gates is connected to its respective counter module through two OR gates, allowing the synchronization pulse to be fed to the two counter modules simultaneously.
3.3. DESCRIPTION OF THE SYSTEM

Figure 3.7: The external circuit components.
Figure 3.8: Timing diagram for the pulse-shaped start pulse (a), initial timing pulses (b), inverted output of the multi-vibrator (c), pulse-shaped timing pulses (d) and interrupt pulses (e). The labels (a) to (e) correspond to the labels in Figure 3.7.
3.3. DESCRIPTION OF THE SYSTEM

The timing diagram of the synchronization circuit along with the motor and detector pulses before and after pulse shaping is shown in Fig 3.8. The labels (a) to (e) correspond to the labels from the wiring diagram in Figure 3.7. The final interrupt signal, shown in (e), represents the pulse-shaped start pulse and the pulse shaped timing pulses.

Figure 3.9 shows the wiring diagram of the external circuitry components. The monostable multivibrator MV (MV1-MV4) is an LM74123 chip, the quad flip-flop SR (SR1-SR4) is an 74279 chip and the 8-bit counter C8 is a 74F269 chip. The buffer chip 1SL8491 acts as an interface between the detector timing pulses and the input circuit.

3.3.2 Counter Module Programming

Using CNTR1-CNTR3 in Module 1 and CNTR1-CNTR2 in Module 2 requires programming the counter modes and groups. A GUI in LabVIEW® was designed to set the programmable parameters for the required counter groups. Both modules have to be initialized, cascaded and configured before receiving the pulse-shaped motor- and detector-timing pulses as interrupts. The steps involved in each stage of the LabVIEW® program are detailed below (Fig. 3.10).

3.3.2.1 Initialization

The first step is to initialize CNTR1-CNTR3 in Module 1 and CNTR1-CNTR2 in Module 2. The 9513 counter is initialized using the library function ‘C9513Init,’ which specifies the clock frequency and the frequency-dividing parameters. CNTR1-CNTR3
Figure 3.9: Wiring diagram of the external circuitry components.
in Module 1 are assigned to Board0, while CNTR1-CNTR2 in Module 2 are assigned to Board1. The master clock is generated by setting the frequency to FREQ1 and the frequency-dividing parameter to 1.

3.3.2.2 Configuration

At this stage, CNTR1 and CNTR2 of both modules are cascaded into the LSC and MSC to create the 32-bit counters. Besides cascading, this stage configures the counter groups and registers of the modules. The individual counter group input, gating and output are configured using the library function 'C9513Config'. The input to the LSCs and CNTR3 Input of Module 1 is the master clock. The input to the MSCs are set as the output of the LSCs. The LSCs are set to the 'ALL gating' mode whereas the remaining counters are set to the 'no gating' mode. The output of the MSCs and CNTR3 of Module 1 are set to the 'high output at TC' mode, whereas the remaining counters are set to the 'inactive output low' mode.

3.3.2.3 Counter Save

During this stage of the LabVIEW® program, the counting modes for the LSCs and MSCs are configured to 'gating with interrupt'. The counting mode of the remaining counters have to be disabled because they are enabled by default. An interrupt handler is installed using the 'CStoreInt' library function. At the event of an interrupt, current count values of the LSCs and MSCs are pushed into their respective hold registers. These 32-bit counter data are transferred to a temporary buffer on the PC using the USB port.
3.3. DESCRIPTION OF THE SYSTEM

Figure 3.10: Flow chart describing the system algorithm
3.3. DESCRIPTION OF THE SYSTEM

The program loops for a specified number of interrupt signals. The background of this program is monitored by the 'GetStatus' library function, which keeps track of the number of interrupts handled. A 'StopBg' library function stops the interrupt handler on completing the required number of interrupts, and this forces D0 to reset the SR-2 flip-flop.

3.3.2.4 Time Stamp

This part of the program converts the saved counter values in the temporary buffer into a list of time stamps. The first counter value in the buffer corresponds to the start pulse. This counter value is the only common point of reference in the buffer between both modules. To synchronize subsequent time stamps, the first LSC and MSC readouts for both counters are subtracted from all other readouts.

The saved counter values reflect the time periods of the measured segments of the interrupt signals and need to be augmented by the non-measured but constant segment of 128 $\mu$s. After this correction, the modified buffer is pushed into an Excel spreadsheet as the final list of time stamps.

The time-stamp spreadsheet can be used to relate the detector frame to the exact motor position. The next chapter discusses the analysis of the precision of the measured time periods.
4

Results: Timing-Pulse Measurement

This chapter discusses the performance and results of the synchronization circuit for the motor- and detector-timing pulses. The performance is based on the analysis of the error statistics and precision of the registered time stamps. The circuit was never tested on the motor pulses, as the OsteoQuant® is being remodeled; the detector timing pulses were tested using this circuit. Simulated signals from a crystal-driven function generator are used during experimentation since a consistent and reliable pulse source is required to test the errors of the circuit.

4.1 Error Statistics

The pulse-shaping circuit reshapes the interrupt signal such that the period of inactive gate level is restricted to a certain time period. The measured and fixed periods are
summed to provide the time period of the interrupt signal. An experiment was conducted to check the consistency of measuring time periods using the AHL gating with rising edge interrupt.

Table 4.1: Analysis of time-period precision for the USB-4301. The table shows the individual duty cycles and measured time-periods for each module along with the mean, maximum, minimum and the standard deviation of the sum of both modules over 100 readings.

<table>
<thead>
<tr>
<th>Module I Duty Cycle (%)</th>
<th>Module II Duty Cycle (%)</th>
<th>Module I Time Period (µs)</th>
<th>Module II Time Period (µs)</th>
<th>Module I + Module II (µs) Mean</th>
<th>Standard Deviation</th>
<th>Maximum</th>
<th>Minimum</th>
</tr>
</thead>
<tbody>
<tr>
<td>88%</td>
<td>12%</td>
<td>1292</td>
<td>183</td>
<td>1475.25</td>
<td>0.71</td>
<td>1477</td>
<td>1474</td>
</tr>
<tr>
<td>85%</td>
<td>15%</td>
<td>1260</td>
<td>215</td>
<td>1474.82</td>
<td>0.55</td>
<td>1476</td>
<td>1474</td>
</tr>
<tr>
<td>74%</td>
<td>26%</td>
<td>1087</td>
<td>388</td>
<td>1474.82</td>
<td>0.71</td>
<td>1477</td>
<td>1474</td>
</tr>
<tr>
<td>63%</td>
<td>37%</td>
<td>934</td>
<td>540</td>
<td>1474.01</td>
<td>0.54</td>
<td>1475</td>
<td>1473</td>
</tr>
<tr>
<td>47%</td>
<td>53%</td>
<td>689</td>
<td>786</td>
<td>1474.75</td>
<td>0.68</td>
<td>1477</td>
<td>1473</td>
</tr>
<tr>
<td>25%</td>
<td>75%</td>
<td>361</td>
<td>1115</td>
<td>1475.88</td>
<td>0.80</td>
<td>1477</td>
<td>1475</td>
</tr>
<tr>
<td>10%</td>
<td>90%</td>
<td>153</td>
<td>1323</td>
<td>1476.22</td>
<td>0.80</td>
<td>1478</td>
<td>1475</td>
</tr>
</tbody>
</table>

Both USB-4301 modules were used with the first counter module configured to an AHL gate with a falling-edge interrupt and the the second module configured to an ALL gate with a rising-edge interrupt. The first module registers the time duration of the high segment of the input signal; the second module does the same for the low segment. By only changing the duty cycle but not the signal frequency, the sum of the measured time periods of both modules is expected to be constant under all conditions. The standard deviation of the repeatedly measured sum provides an indicator of the precision of the system.
4.1. ERROR STATISTICS

The function generator was set to a frequency of approximately $\frac{1}{1.47}$ ms; the duty cycles were varied from 10% to 90%. Table 4.1 shows the various duty cycles and the mean time period measured from 100 cycles. Also shown are the maximum, minimum and standard deviation of the sum for the 100 measured time periods.

4.1.1 Linearity of the Time Period Readout

An experiment was conducted to test the linearity of the measured time periods. This was done by varying the duty cycle from 10% to 90% of an interrupt input signal of time period 1.47 ms. This signal was provided as the interrupt input signal to both USB-4301 modules. One of the modules was configured to AHL gating with active-low interrupt; the other was configured to ALL gating with active-high interrupt. With this configuration, one module measures the duration of the high part of the signal, and the other module measures the duration of the low part. As the signal frequency is not changed, only the duty cycle, the sum of both signal durations is expected to remain constant. The individual signal durations, however should vary linearly with the duty cycle.

The measured time periods were averaged over 100 interrupt cycles. Figure 4.1 shows the measured time periods plotted against the corresponding duty cycles for each module. Excellent linearity for the measured time periods for both modules is shown. The sum of the measured time periods from both modules is a constant, represented by a horizontal straight line. This indicates linear and accurate measure-
4.1. ERROR STATISTICS

Figure 4.1: Linearity of the measured time periods for Module 1 and Module 2 for the duty cycles in Table 4.1.

ement of the time periods by both modules from 153 $\mu$s to 1,323 $\mu$s. The measured time periods range from 2.220 to 2.225 ms, giving a maximum error of $\pm 3$ $\mu$s.

4.1.2 Error Range of the Measured Time Periods

To assess the consistency of the time periods, a histogram of 100 data points obtained from an interrupt signal at 2.22 ms interval was created (Fig. 4.2).

This error corresponds to a relative error of 0.13%, which is an acceptable error, considering that the error due to Poisson statistics for a readout interval of 2.222 ms is 1.94% (assume $1.2 \times 10^6$ counts/sec).
4.2 Spreadsheet

4.2.1 Data Transfer to PC

The spreadsheet from the buffer of the LabVIEW® program displays the time stamps corresponding to the events of the motor and detector timing pulses. Table 4.2 shows a sample spreadsheet, where the MSC and LSC readouts represent 32-bit time-stamps for each pulse source. The calculated time period between interrupts is the successive difference between two time stamps and serves here as a check to assess the consistency of the readout with a stable interrupt source.

The synchronized (Sync.) pulses are the raw time stamps minus the stamp of the start pulse. The first time stamp, being the start pulse, is ignored, and all events that follow the start pulse are time stamps of interest, because they were generated by the
motor or the detector. The spreadsheet transmitted to the user does not include the raw time periods nor the event numbers.

### 4.2.2 Spreadsheet Generation

Although the maximum number of interrupt events the counter may receive throughout an entire scan is $2^{16}$, the maximum number of events the module can accept is only $2^{15}$. This may limit the duration of a given scan. This problem can be alleviated, however, if multiple streams of $2^{15}$ counts can be concatenated. This requires the closing out of the current spreadsheet, restarting and resynchronizing the timing pulses and queuing a new spreadsheet. These tasks must be completed during a rotation, as data are collected only during translations. We estimate that the rotation time is in the order of 1 second. To test the feasibility of this approach, we needed to verify
that the time taken to close out of a spreadsheet, reset, restart and resynchronize the module is less than 1 second.

An experiment was set up to measure the time required for each segment of the program. During initialization the counter modules and registers are configured. This is followed by a combined fixed delay of 65.12 ms generated by the synchronization and delay pulses. The time stamps are then recorded and, at the end of the sequence, transferred into an instantaneously generated spreadsheet. Once the current spreadsheet is closed, D0 resets the modules. The final step is to restart and resynchronize the modules before the next sequence of data collection.

The statistics of 30 periods that initialized, transferred time stamps into a spreadsheet, reset and restarted the modules were measured. To consider the longest time period for data transfer, the module was programmed to record $2^{15}$ events. The mean, standard deviation as well as maximum and minimum time periods for individual and combined segments are shown in Table 4.3.

Table 4.3: Statistics for the time required to close out a spreadsheet, reset and restart the modules.

<table>
<thead>
<tr>
<th></th>
<th>Data Transfer to Spreadsheet (ms)</th>
<th>Reset (ms)</th>
<th>Restart (ms)</th>
<th>Initialization (ms)</th>
<th>Sync. Pulse Delay (ms)</th>
<th>Total Time (ms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>81.05</td>
<td>3.16</td>
<td>66.57</td>
<td>4.27</td>
<td>65.12</td>
<td>220.17</td>
</tr>
<tr>
<td>Std. Deviation</td>
<td>11.3</td>
<td>0.46</td>
<td>7.81</td>
<td>0.05</td>
<td>0</td>
<td>22.02</td>
</tr>
<tr>
<td>Maximum</td>
<td>111</td>
<td>4</td>
<td>103</td>
<td>4.6</td>
<td>65.12</td>
<td>287.72</td>
</tr>
<tr>
<td>Minimum</td>
<td>70</td>
<td>2</td>
<td>59</td>
<td>4.3</td>
<td>65.12</td>
<td>200.42</td>
</tr>
</tbody>
</table>
The maximum total time noted was 0.28 second, which is less than the allowable time of 1 second. It is, thus, feasible to go beyond the maximum count of $2^{15}$ by creating new counting streams during rotations.
Discussion: Timing-Pulse Measurement

A 2.22 ms signal was measured with an uncertainty of $2.22 \pm 0.003$ ms. The result of this experiment proved that the system is capable of measuring pulse trains in milliseconds with microsecond precision. The system was also tested to register events as short as 150 $\mu$s with an error of $\pm 4$ $\mu$s. The system was not tested for shorter events, as the resolution of the source was not dependable for signals generated less than 150 $\mu$s. The current run time of the system can be increased by further cascading the 2 unused counters in Module 1 and the 3 unused counters in Module 2.

Since the scanner is being remodeled, the motor-timing pulses were simulated using the function generator. The system was tested using these simulated motor pulses and the actual detector-timing pulses. After testing, the final circuit was mounted onto a general-purpose circuit board, and the LabVIEW® program was converted
into an executable file. The system hardware and software are scanner independent but can be used only on a Microsoft Windows® platform.

A disadvantage of the system is the interrupt latency of the counter module, which necessitates additional circuitry. However, the correction of the time stamps is performed in the executable module and is transparent to the user.

The current system was designed to handle the timing pulses generated for all phases of the motor movement. By using the time stamps in the spreadsheet, a system could be designed that excludes the motor- and detector-timing pulses that occur during the acceleration and deceleration phases. The successive differences of the time stamps in the spreadsheet should be constant during the constant velocity phase but should monotonically decrease and increase during acceleration and deceleration, respectively.
6

Background: Detector Calibration

There are certain general properties that apply to all types of radiation detectors. Some of these properties can cause loss in expected photon counts. These losses are often modeled as being caused by dead-time and may result in errors in the reconstructed image if left uncorrected.

6.1 Dead Time

Dead time can be defined as the minimum time required for a detector system to distinguish two successive counting events. Once such an event occurs at the detector, it is converted into an electrical signal depending on the intensity and duration of the event. This charge, which is created in the detector, is then collected by applying an electric field across the detector. The time required to collect this charge depends on certain characteristics (charge carrier, mobility, distance to collection electrodes, etc.) of the detector itself and of the subsequent electronics. A problem occurs if another event happens during the time required to create the electrical signal.
The occurrence of a radiation quantum is a random event governed by Poisson statistics. Due to its random nature, there is always a probability that the detector misses a true event that follows a recorded event because it happened during the dead-time period. The missed true events are called dead-time losses, and these losses become more severe for larger event rates.

There are two models that define the dead-time behavior in counting systems; the paralyzable and non-paralyzable response models. Figure 6.1 shows the response of both a paralyzable and non-paralyzable counting system. The example illustrates a detector that receives six true events, and every detector event is followed by a dead-time period \( \tau \).

![Figure 6.1: A sample counting system with paralyzable and non-paralyzable behavior [Knoll 1989].](image)

When a counting system receives a true event during the dead-time period, both paralyzable and non-paralyzable models do not record this event. The non-paralyzable model will accept new events after the expiration of the dead-time caused by the initial
event, independent of the number of photons hitting the detector during the dead-time period. In contrast, a paralyzable detector will extend its dead-time period for each true event by $\tau$. Low observed count rates in a paralyzable model can be the result of a low interaction rate or a very high interaction rate, where the dead-time period is repeatedly extended.

6.2 Modeling the Response Behavior

Assume a detector system responding to a steady flux of radiation, where $n$ is the rate of true interactions, $m$ is the recorded count rate and $\tau$ is the dead time of the system. A model can provide a relation between the dead-time and the interaction rates, which can help account for the dead-time losses. Modeling a non-paralyzable system [Knoll 1989] is easy, since $m \cdot \tau$ is the fraction of the period during which the detector is dead. The rate of true events lost is the product of $m$, $n$, and $\tau$. The rate of losses can also be expressed as $n - m$.

$$n - m = mn\tau$$ \hspace{1cm} (6.1)

The expected true count rate is then

$$n = \frac{m}{1 - m\tau}$$ \hspace{1cm} (6.2)

The same relation cannot be used to describe a paralyzable model [Knoll 1989], since the dead-time period is subjected to the number of true events $n$. The number of detected events $m$ can be expressed as

$$m = ne^{-n\tau}$$ \hspace{1cm} (6.3)
6.2. MODELING THE RESPONSE BEHAVIOR

This equation cannot be solved analytically for n; thus, many investigators apply the dead-time correction for a non-paralyzable system (Eq. 6.3) also to paralyzable systems.

Figure 6.2 shows the observed count rate $m$ plotted against the true count rate $n$. At low count rates, both paralyzable and non-paralyzable models behave alike; the difference can be noticed at higher interaction rates. At $\frac{1}{r}$, the interaction rate is so high that the detector has barely enough time to complete its dead-time period before the next event occurs. The highest observed count rate in the paralyzable model is observed when $m$ is $\frac{1}{r}$. For count rates higher than $\frac{1}{r}$, $m$ decreases with $n$. For the non-paralyzable model, the count rate becomes asymptotical once the value of $m$ approaches $\frac{1}{r}$. 

Figure 6.2: True counts vs. measured counts for paralyzable and non-paralyzable system models [Knoll 1989].
6.2. MODELING THE RESPONSE BEHAVIOR

An ideal counting system would follow the dashed line where \( m = n \). Linearization of the measured counts to the ideal or true counts is the method of correcting for dead-time. Both models predict high losses in measured events when the event rates are high. However, the described models in Figure 6.1 represent two extreme system behaviors. Practical systems often exhibit properties that are partially paralyzable and partially non-paralyzable.

Initial detector readouts were collected by operating the system at a tube voltage of 45 kVp, with eleven increments of tube current from 0 - 1 mA, using a photon accumulation time of 50 ms (Fig 6.3). The measured counts are plotted against the varying tube current. Ideally, the number of photons striking the detector is linearly proportional to the number of registered events. But here, the expected linearity is lost for event rates recorded above 0.3 mA. This loss in linearity for photon counts is a clear indication of the dead-time characteristics of the detector.

![Figure 6.3: The measured non-linear and the expected linear photon counts for varying tube currents of one detector element from a sample data set.](image)
Assuming the values below 0.3 mA are not affected by dead-time, a line can be established based on the counts up to 0.3 mA. This line, if extrapolated to higher tube currents, illustrates the extent of count losses with higher count rates. The slope of this line represents the change in counts for the change in tube current (slope \( \frac{\Delta \text{Counts}}{\Delta \text{Current}} \)). A mean slope can be calculated, using the slopes of all 64 detector elements, to obtain a common line, to which all the detector readouts will be corrected.

A common method used for dead-time correction is the modeling of the observed count rate vs. tube current through a polynomial, which provides the basis for an analytical correction expression to a straight line. This part of the project computes this correction and studies the stability of this correction using data collected over a span of nine months. The stability study analyzes the error statistics of the corrections calculated from the data of a certain date applied to the data from another date. Besides dead time, another area of concern is beam hardening.

Attenuation of the poly-energetic spectrum of the x-ray beam causes non-linearity in the projection value vs. thickness curve, which is described as beam hardening. The different types of interactions of x-ray photons with matter are explained below to better understand the concept of beam hardening.

### 6.3 Interaction of Ionizing Radiation with Matter

The basic physical principles involved in the interaction of ionizing radiation with matter are

- photoelectric absorption
6.3. INTERACTION OF IONIZING RADIATION WITH MATTER

- Compton scattering
- coherent scattering
- pair production

Most medical imaging related applications operate below 150 keV. Although coherent scattering exists at lower energies, photoelectric absorption and Compton scattering are the dominant sources of interaction in this energy range, resulting in

\[ \mu_{total} \cong \zeta_{\text{photoelectric}} + \sigma_{inc} \text{Compton} \] (6.4)

The attenuation coefficient \( \mu \) is material dependent and is the sum of the two dominant components (Eq. 6.4).

During photoelectric absorption, an incoming x-ray photon knocks an electron out of its shell by transferring all of its energy to the electron. This effect is prominent at energies below 100 keV. Compton scattering involves the interaction between a photon and a free or outer-shell electron of an atom. These electrons possess lower binding energies than their inner-shell counterparts. When the photon strikes such an electron, part of its energy is transferred to the electron, whereas the photon retains the rest. On collision, the scattered photon and electron undergo both energy and directional change. The electron is called the recoil electron, and the photon is called the scattered photon. This interaction is most important in soft-tissue like materials. Figure 6.4 illustrates the linear attenuation coefficient of water and the contributions of the photoelectric effect and Compton scattering to the total attenuation coefficient.
X-ray photons, on interaction with an object, are attenuated depending on the material properties of the object. According to Beer’s law for a mono-energetic beam,

\[
\ln \left( \frac{I_0}{I} \right) = \mu d
\]  

(6.5)

where  
\[ I_0 : \text{number of transmitted photons} \]
\[ I : \text{number of incident photons} \]
\[ \mu : \text{linear attenuation coefficient} \]
\[ d : \text{thickness of the object} \]

According to Eq. 6.5, the projection value is the logarithm of the ratio of the number of photons emitted and photons attenuated, \( \ln \left( \frac{I_0}{I} \right) \). For an ideal case of photon interaction with homogeneous matter, the projection values are expected to be proportional to the thickness of the material. The interaction of transmitted photons of varying energy levels with matter leads to beam hardening.
6.4 Beam Hardening

X-ray beams used in CT are usually polychromatic with a finite spectrum. When this photon beam passes through material, it tends to predominantly lose its lower energy photons, hardening the beam in the process. Lower-energy x-rays are more prone to attenuation and, therefore, the average energy of a polychromatic beam increases with increasing thickness of the material. This effect produces a spectral shift in the transmitted photon beam depending on the material properties of the object it passed through.

In CT reconstruction, the reconstructed image values are assumed to be linearly proportional to the density of the object scanned, which is fulfilled if a mono-energetic x-ray source is used. However, the use of a poly-energetic x-ray source results in wrongly estimating density values for increasing thicknesses and attenuation coefficients in the scanned object. This problem has to be taken care of since most medical CT scanners, including the OsteoQuant®, use a poly-energetic x-ray source.

There are several methods used to counter the beam-hardening effect, which can be implemented before or after the scan. One pre-scan method employed is the water-bag technique. The primary goal of using this technique is to reduce the dynamic range of the detector, and this was employed in the first EMI CT scanner [Hounsfield 1995]. This technique used water bags to surround the head of the patient, producing equidistant x-ray path lengths. Water shows a similar attenuation coefficient as soft tissue, and the water-bag method reduces the effects of beam-hardening in the absence
of bone by acting as a filter. Bow-tie shaped filters have taken the place of water bags in modern CT scanners.

The post-scan method of correction linearizes a mathematical model of the projections. For a homogenously dense object, the expected projection values will be linearly dependant on object thickness. Linearization [Herman 1979] [Brooks and Chiro 1976] corrects the curvature of the observed projection vs. thickness curve to its expected linear behavior. To calculate the correction coefficients, slabs of materials, similar to those encountered in the patient scans, need to be measured. The material for skeletal sites consist mainly of bone and soft tissue, for which aluminum (Al) and plexiglas (Pl) are good substitutes. This part of the project uses slab pairs of these substitute materials to create the projection values, based on which the correction parameters are calculated.

![Graph](image)

Figure 6.5: The measured non-linear and expected linear projection values for varying absorber thicknesses.
6.4. BEAM HARDENING

Using these slab pairs, the dead-time corrected projection values were plotted as a function of slab pair thickness for an x-ray source setting of 45 kVp and 1 mA (Fig. 6.5). The expected linearity between projection values and thicknesses is lost after 3 slab pairs. The non-linear projection values that follow after the 3 slab pairs are the result of beam hardening. These non-linear projection values can be corrected to a line with an arbitrary slope, which is often chosen to reflect the slope of the first few data points in the plate measurements.

The objective of this part of the project is to correct the dead-time corrected projection values for beam hardening by linearizing individual mathematical models for each of the 64 detector elements to a common line. The error statistics of these corrections applied to the detector data collected over a span of nine months, were assessed to analyze the stability of the corrections. The data required for beam-hardening correction are acquired by collecting projection values for numerous increments in slab thicknesses; repeating this procedure before every scan can become a tedious process. Thus, a secondary beam-hardening correction method, which relies only on a small number of slab thicknesses, is also investigated in this project.
7

Methods: Detector Calibration

In order to study the characteristic properties of the source and detector, they were operated in a lead-shielded box, using the operating parameters to be used in the OsteoQuant®. This part of the project deals specifically with compensating the losses due to the dead-time and correcting for beam-hardening.

A common approach for correcting both dead-time and beam hardening is to create a mathematical model of the data and linearize the model to obtain the corrected values. Least-squares optimization is a common method for determining the best fit for a given data set.

7.1 Least-Squares Optimization and Linearization

The least-squares method is generally used to calculate the best fit to a set of data points. This best fit is calculated by choosing the mathematical model that gives the least sum of squared errors between the model function and the actual data points. Consider a set of $n$ data pairs of $(x, y)$, where the dependant variable $y$ is a function
of the independent variable \( x \). This can be expressed as:

\[
y_i = f(x_i) \tag{7.1}
\]

where \( i = 1, 2, 3, \ldots n \)

The least-squares method models a fit over these data points using a function \( y' = f(x, \beta) \), where \( \beta \) is a vector of adjustable parameter values. This method estimates the parameter values of \( \beta \) for the least sum of the squared errors (residual) between the dependant and predicted \( y \) values.

\[
\text{individual residual, } r_i = \text{observed } y_i - \text{predicted } y_i \tag{7.2}
\]

\[
\text{residual, } r = \sum_{i=1}^{n} (r_i)^2 \tag{7.3}
\]

Consider the least-squares fit to a data set to be a fourth-degree polynomial function. Eq. 7.2 can be rewritten as

\[
y_i = f(x_i, \beta) \tag{7.4}
\]

\[
y_i = \beta_4 x_i^4 + \beta_3 x_i^3 + \beta_2 x_i^2 + \beta_1 x_i + \beta_0 \tag{7.5}
\]

The predicted values \( y_i \) are determined by adjusting the parameters \( \beta_j \), until the model gives the least residual. Figure 7.1 shows the modeled variable \( y_i \) vs. the independent variable \( x_i \) along with the line \( Y(x) \).

Linearization is achieved by correcting the data points of \( y_i \) to the line \( Y_i \).

\[
Y_i = m x_i + c \tag{7.6}
\]
7.1. LEAST-SQUARES OPTIMIZATION AND LINEARIZATION

The slope of the line $Y$ is calculated based on the linear data pairs in $y$, i.e. $(x_1, y_1)$ to $(x_3, y_3)$. Considering an intercept of $c = 0$, Eq. 7.6 can be rewritten as:

$$x_i = \frac{Y_i}{m}$$

(7.7)

By replacing Eq. 7.7 in Eq. 7.5, the measured variable $y_i$ is expressed as a function of the variable $Y_i$ at the same location $x_i$.

$$y_i = Y_i^4 \left( \frac{\beta_4}{m^4} \right) + Y_i^3 \left( \frac{\beta_3}{m^3} \right) + Y_i^2 \left( \frac{\beta_2}{m^2} \right) + Y_i \left( \frac{\beta_1}{m} \right) + \beta_0$$

(7.8)

We need to reverse the equation and express $Y_i$ as a function of $y_i$.

$$Y_i^4 \left( \frac{\beta_4}{m^4} \right) + Y_i^3 \left( \frac{\beta_3}{m^3} \right) + Y_i^2 \left( \frac{\beta_2}{m^2} \right) + Y_i \left( \frac{\beta_1}{m} \right) + (\beta_0 - y_i) = 0$$

(7.9)

The linearized data points are estimated by solving Eq. 7.9 for $Y_i$. Because we deal with a fourth-degree polynomial, the recalculated $Y_i$ has four possible solutions.
Ignoring the negative and imaginary solutions, the solution closest to $Y_i$ for the corresponding $x_i$ is chosen.

The experimental setup used for collecting the data required to generate the correction equations for dead-time and beam hardening are detailed in the following section.

## 7.2 Experimental Setup

The source and detector were placed in the lead-shielded wooden box of size 85 cm x 64 cm x 30 cm. The source and detector are centrally aligned with a distance of 48.5 cm between them, maintaining the source to detector distance of the OsteoQuant®.

**Source**: The source is a radiation shielded, packaged x-ray tube (Oxford Instruments, Oxfordshire, England). The tube weighs 4 lb and can operate at a maximum anode voltage of 50 kVp and a maximum anode current of 1 mA, at a maximum operating temperature of 55°C. The anode material is a compound made of tungsten, molybdenum and rhodium. The tube has to be warmed up by running the tube first at 20 kVp and 0.5 mA for 10 seconds and then 40 kVp and 1 mA for 10 seconds.

**Detector**: The detector is a CZT semi-conductor cuboid with a volume of 16 mm x 4.7 mm x 1.75 mm (eV Products, Saxonburg, PA). The CZT crystal is composed of 50% tellurium, 5% zinc and 45% cadmium. Each of the two detector anodes is segmented into two strips of 16 elements with a size of 0.9 mm x 0.9 mm and
7.3. DEAD-TIME CORRECTION

a gap of 0.1 mm between the strips and 0.1 mm between the elements. Guard strips surround the sides of the detector elements. The detector can be operated at room temperature up to 32°C.

Slabs: Slabs are used in the beam hardening experiment as a substitute for the bone and soft-tissue in human-body. The bone and soft-tissue mimicking slabs are made of aluminum and Plexiglas (lucite), respectively. Each slab measures 10.5 cm x 6.5 cm, with the thickness of an individual slab being 1.62 mm for aluminum and 7.8 mm for Plexiglas. A total of 19 pairs of these slabs were used, creating a combined thickness of 30.78 mm of bone and 148.2 mm of soft-tissue equivalent material.

7.3 Dead-Time Correction

The objective was to correct the detector output for dead-time to an error less than 0.5% from the expected counts at 1 mA and study the stability. The expected counts in the scanner for a 50 ms collection interval at 1 mA are 72,000, so the residual of the corrected counts must be below 350 counts (0.5% of 72,000). The stability was studied by calculating the dead-time correction based on the data collected at a certain date and applying it to the data collected before and after that date.

The collected data represent the response of the 64 detector elements for 11 levels of tube current. A line of expected counts is established using the counts collected at tube currents below 0.3 mA. The slope of this line represents the expected change in counts for the change in tube current. The slope of counts vs. tube current for every
7.3. DEAD-TIME CORRECTION

individual detector element is calculated, and the mean of these 64 slopes is

\[ \text{slope, } s_{\text{dead}} = \frac{1}{64} \sum_{i=1}^{64} \frac{\Delta \text{Counts}_i}{\Delta \text{Current}_i}. \]  

(7.10)

The photon-count loss is evident by the difference between the measured counts and the line of expected counts. Since the correction method should be applicable to the entire data set collected for this project, the crucial part was to determine the degree of the polynomial fit that linearizes the data within the expected error level from the ideal slope.

In order to linearize the photon counts, the detector data have to be mathematically modeled. The first linearization attempt used a second-degree polynomial. However, this correction did not satisfy the required error criterion, and higher-degree fits were tried. As expected, lower individual residual values were obtained with higher-degree fits. An F test was used to compare the goodness of fit for the different polynomial models. The F ratio is defined as follows:

\[ F = \frac{(SS_1 - SS_2)/SS_1}{(DF_1 - DF_2)/DF_1} \]  

(7.11)

where,

- \( SS_1 \) : Sum of squared errors of the lower degree polynomial model
- \( SS_2 \) : Sum of squared errors of the higher degree polynomial model
- \( DF_1 \) : Degree of freedom of the lower degree polynomial fit model
- \( DF_2 \) : Degree of freedom of the higher degree polynomial fit model

Based on the F test, it was determined that the fourth-degree polynomial model best fit the data, yielding a significance level \( p < 0.01 \). To linearize the detector data, Equation 7.10 was substituted as the slope in Equation 7.9.

\[ Y_i^4(\frac{\beta_4}{s_{\text{dead}}^4}) + Y_i^3(\frac{\beta_3}{s_{\text{dead}}^3}) + Y_i^2(\frac{\beta_2}{s_{\text{dead}}^2}) + Y_i(\frac{\beta_1}{s_{\text{dead}}}) + (\beta_0 - y_i) = 0 \]  

(7.12)
7.4 Stability of the Dead-Time Corrections

Every pixel in the detector is corrected to a line $Y$ of slope $s_{\text{dead}}$. The corrected counts $Y_i$ were calculated by finding the real zeros of the polynomial in Eq. 7.12. The $n^{th}$ root is generally the real positive solution; but, if not the case, the $n - 1^{th}$ real positive solution is chosen. This was implemented using a Matlab® script that linearizes the individual detector element models to the same line. The input to this script are the detector data and the pre-determined slope for the line.

7.4 Stability of the Dead-Time Corrections

Although the fourth-degree polynomial correction applied to the data, for which the correction coefficients were derived, produced adequate results, daily measurements of count vs. current curves maybe unnecessary if the correction coefficients derived on one day could be adequately applied to data measured on another day. This hypothesis was tested by applying the calculated correction of a given date to the data collected before and after this date (different-date corrections).

To set up the data required for the stability analysis, the coefficient vectors for the fourth-degree polynomial model of the detector data were saved in a matrix. There were coefficient vectors from 22 dates for data sets collected over nine months. The absolute values of the individual residuals resulting from the least-squares fits and subsequent corrections were averaged over the number of data points that contributed to the particular analysis. These means of individual residuals will be referred to as
average fitting error from here on.

\[
\text{Average Fitting Error} = \frac{1}{n} \sum_{i=1}^{n} |\text{expected } y_i - \text{corrected } y_i| \quad (7.13)
\]

where \( n \) : number of data points

The stability was analyzed by comparing the average fitting error of the corrections applied to the data collected at the same date (same-date corrections) with the average fitting error of the corrections applied to the data collected from a different-date (different-date corrections). The least expected average fitting errors are those from the same-date corrections.

### 7.5 Beam-Hardening Correction

The objective was to correct the dead-time corrected projection values for beam hardening to an error of less than or of \( 1\% \) of the expected projection values at 10 slab-pairs (9.42 cm). The expected projection value at this thickness is 5, so the error in the corrected projection values must be no greater than \( \pm 0.05 \) (1\% of 5). Again, the stability of these corrections was analyzed by assessing the error statistics calculated from same-date and different-date corrections.

The beam-hardening correction method is similar to that of dead-time correction, i.e. linearization of a mathematical model of the data. The data required to calculate these corrections are collected by taking count readouts for increments in object thickness at 45 kVp tube-voltage and 1 mA tube current. Each thickness increment is based on a pair of aluminum and Plexiglas slabs with thicknesses of 1.62 mm and
7.5. **BEAM-HARDENING CORRECTION**

7.8 mm, respectively. Whereas some data-sets were collected using 19 such slab pairs, the majority of the data was collected using 10 slab pairs, simulating the thickness of 10.62 mm of bone and 78 mm of soft-tissue, approximately the attenuation path through a forearm. Each slab pair was treated as a homogenous material, having an combined attenuation (effective) coefficient of aluminum and Plexiglas. Beer’s law (Eq. 6.5) can be used here to define the projection value.

\[
\ln\left(\frac{I_0}{I_i}\right) = \mu_{eff} d_i
\]  

(7.14)

where 
- \(i\): 1, 2, 3, ... 19 for the number of slab pairs used
- \(I_0\): counts collected with no object in the beam path
- \(I_i\): counts collected with \(i\) number of slab pairs
- \(\mu_{eff}\): effective linear attenuation coefficient for Al and Plexiglas
- \(d_i\): thickness of \(i\) number of slab pairs in the beam path

The projection values are calculated for all slab pairs and all 64 detector elements. The corrected projection-values are estimated by linearizing a mathematical model of the projection values vs. number of slab pairs for each detector element.

In the projection values vs. slab-pair thickness graph (Fig. 6.5), the non-linearity is visible for thicknesses over 1 slab pair. A line of expected projection values is established using the projection value for 1 slab pair. The slope of this line represents the expected change in projection-value for a given change in absorber thickness and represents the effective linear attenuation coefficient \(\mu_{eff}\). The slopes of the data from the detector elements are averaged and used as the common slope for the correction
7.5. **BEAM-HARDENING CORRECTION**

of the projection values for all 64 detector elements.

\[
\text{slope, } \mu_{\text{eff}} = \frac{1}{64} \sum_{i=1}^{64} \frac{\Delta\text{Projection Value}}{\Delta\text{Thickness}} \tag{7.15}
\]

The projection values were fitted against two different mathematical models; a fifth-degree polynomial and a bimodal-energy model [de Casteele et al. 2002]. The fifth-degree polynomial model was chosen by comparing the goodness of fit of different polynomial models using the F test shown in Eq. 7.11. The error statistics of the linearized projection values using both fit models were then compared against each other to select the better of the two correction methods.

### 7.5.1 Linearization Using the Polynomial Model

Judging the projection value vs. slab thickness plots, it was decided that an \(n^{th}\)-degree polynomial fit can model these data. The intent was to find the best polynomial model that satisfies the required accuracy after linearization, without modeling the noise in the data. Different degrees of polynomials were used to model the projection data.

Each detector element’s data was fitted using a second to a sixth-degree polynomial function. The robustness of the different polynomial fits was measured using the F-test for a 99% confidence interval. The fourth-degree polynomial function best fit the data, yielding a significance level \(p < 0.01\). However, it was the linearization using the fifth-degree model that satisfied the required error criterion. The fifth-degree polynomial model for fitting the projection data is written below.

\[
y_i = \ln \left( \frac{I_0}{I_i} \right) \tag{7.16}
\]

\[
y_i = \beta_5 x_i^5 + \beta_4 x_i^4 + \beta_3 x_i^3 + \beta_2 x_i^2 + \beta_1 x_i + \beta_0 \tag{7.17}
\]
The $y_i$ values are linearized to the line $Y_i$ of slope $\mu_{\text{eff}}$ and intercept $c=0$.

\[ Y_i = \mu_{\text{eff}} x + c \]  

(7.18)

Substituting the dependant variable with $\frac{y_i}{\mu_{\text{eff}}}$ in Eq. 7.17, we obtain:

\[ Y_i^5\left(\frac{\beta_5}{\mu_{\text{eff}}}ight) + Y_i^4\left(\frac{\beta_4}{\mu_{\text{eff}}^2}\right) + Y_i^3\left(\frac{\beta_3}{\mu_{\text{eff}}^3}\right) + Y_i^2\left(\frac{\beta_2}{\mu_{\text{eff}}^4}\right) + Y_i\left(\frac{\beta_1}{\mu_{\text{eff}}^5}\right) + (\beta_0 - y_i) = 0 \]  

(7.19)

The corrected values $Y_i$ are calculated by solving for the real zeros in Equation 7.19.

With a power of five, there are five possible roots. After eliminating the imaginary and negative roots, the root value closest to the expected projection value for the corresponding thickness in the line is chosen as the corrected projection value.

### 7.5.2 Linearization Using the Bimodal-Energy Model

According to Beer’s Law, the projection values of the x-ray beam passing through an object are linearly proportional to $\mu(E)$. The effective attenuation coefficient $\mu_{\text{eff}}$ is a function of the average energy of the x-ray spectrum (bremsstrahlung). The projection values are generally linearized to a line of slope $\mu_{\text{eff}}$. The bimodal-energy model [de Casteele et al. 2002] suggests that the attenuation is a function of predominantly two energies, a dominant energy ($E_2$) and a lower energy ($E_1$).

The projection values can be modeled as a function of these two energies. The intensity of the incoming photons can be expressed as a function of the energy spectrum $I'_0(E)$ and the detector efficiency $\gamma(E)$ (Eq. 7.20), whereas attenuated intensity $I$ can be expressed as a function of the energy spectrum $I'_0(E)$, the linear attenuation
7.5. **BEAM-HARDENING CORRECTION**

Coefficient \( \mu(E) \), and the detector efficiency \( \gamma(E) \) (Eq. 7.21).

\[
I_0 = \int I_0'(E) \gamma(E) dE
\]

\[
I = \int I_0'(E) \gamma(E) e^{-\mu(E)d} dE
\]

The energy dependence of all energy-dependent parameters can be approximated with two energies \( E_1 \) and \( E_2 \). The ratios \( \frac{I}{I_0} \) then yields:

\[
\frac{I}{I_0} = \frac{A e^{-\mu_1 d} + B e^{-\mu_2 d}}{A + B}
\]

where \( \mu_1 = \mu(E_1) \)

\( \mu_2 = \mu(E_2) \)

\( A = I_0'(E_1) \gamma(E_1) \)

\( B = I_0'(E_2) \gamma(E_2) \)

The parameters \( A \) and \( B \) in Eq. 7.22 can be defined as the ratio \( \alpha \) of the lower to higher energy

\[
\alpha = \frac{A}{B} = \frac{I_0'(E_1) \gamma(E_1)}{I_0'(E_2) \gamma(E_2)}
\]

Reformatting Eq. 7.22 yields

\[
A \left( \frac{I}{I_0} - e^{-\mu_1 d} \right) = -B \left( \frac{I}{I_0} - e^{-\mu_2 d} \right)
\]

and the ratios \( \alpha = \frac{A}{B} \) can be expressed as

\[
\alpha = \frac{e^{-\mu_2 d} - \frac{I}{I_0}}{\frac{I}{I_0} - e^{-\mu_1 d}}
\]

Solving for the attenuation ratio \( \frac{I}{I_0} \) results in

\[
\left( \frac{I}{I_0} \right) = \frac{e^{-\mu_2 d} - \alpha e^{-\mu_1 d}}{1 + \alpha}
\]

\[
\left( \frac{I}{I_0} \right) = e^{-\mu_2 d} \left( 1 - \alpha e^{-(\mu_1 + \mu_2)d} \left( \frac{1}{1 + \alpha} \right) \right)
\]
Inverting the ratio and taking the log on both sides of Eq. 7.27 provides

\[
\ln\left(\frac{I_0}{I}\right) = \ln(e^{\mu_2 d}) + \ln\left(\frac{1 + \alpha}{1 - \alpha e^{-(\mu_1 + \mu_2)d}}\right) \quad (7.28)
\]

\[
\ln\left(\frac{I_0}{I}\right) = \mu_2 d + \ln\left(\frac{1 + \alpha}{1 - \alpha e^{-(\mu_1 + \mu_2)d}}\right) \quad (7.29)
\]

The projection values can be modeled by Eq. 7.29, using the three parameters \(\mu_1\), \(\mu_2\) and \(\alpha\). Here, \(\mu_1\) represents a steep slope of the lower energy \(E_1\) of the projection versus attenuation-thickness curve for smaller thicknesses, and \(\mu_2\) is the slope caused by the dominant energy \(E_2\) for large thicknesses. Since the projection data were collected using up to 19 plate sets, there are 19 possible equations with three unknowns, making the equation system over determined. The three unknown fitting parameters were calculated using a non-linear least-squares approach. The equation system was iteratively solved using a Matlab script that assumes initial values for the unknown fitting parameters. To solve for the unknowns using the non-linear least squares method, \(\frac{\partial \epsilon}{\partial \alpha}\), \(\frac{\partial \epsilon}{\partial \mu_1}\) and \(\frac{\partial \epsilon}{\partial \mu_2}\) are equated to zero, in order to find the minimum squared error (\(\epsilon\)) associated with each unknown.

\[
\frac{\partial \epsilon}{\partial \alpha} = \frac{\partial}{\partial \alpha} \sum \left(\mu_2 d + \ln\left(\frac{1 + \alpha}{1 - \alpha e^{-(\mu_1 + \mu_2)d}}\right) - \ln\left(\frac{I_0}{I}\right)\right)^2 \quad (7.30)
\]

\[
\frac{\partial \epsilon}{\partial \mu_1} = \frac{\partial}{\partial \mu_1} \sum \left(\mu_2 d + \ln\left(\frac{1 + \alpha}{1 - \alpha e^{-(\mu_1 + \mu_2)d}}\right) - \ln\left(\frac{I_0}{I}\right)\right)^2 \quad (7.31)
\]

\[
\frac{\partial \epsilon}{\partial \mu_2} = \frac{\partial}{\partial \mu_2} \sum \left(\mu_2 d + \ln\left(\frac{1 + \alpha}{1 - \alpha e^{-(\mu_1 + \mu_2)d}}\right) - \ln\left(\frac{I_0}{I}\right)\right)^2 \quad (7.32)
\]

In this equation system, convergence is only declared when the solutions for the fitting parameters are within a specified upper and lower bound. Unlike in de Casteele’s experiment [de Casteele et al. 2002], here the energies \(E_1\) and \(E_2\) are not pre-determined, and the expected values of the parameters \(\mu_1\), \(\mu_2\), and \(\alpha\) are unknown. To prove that
the appropriate relationship between \( E_2 \) and \( E_1 \) exists for our data, a random 19-plate data set was chosen to calculate the fitting parameters and estimate the energies. This was done by treating slab pairs as a homogenous material, and the range of \( \mu(E) \) was calculated using the thickness ratio for an Al and Pl slab pair.

\[
\text{Thickness of one slab pair} = 0.162 \text{ cm (Al)} + 0.781 \text{ cm (Pl)} = 0.942 \text{ cm}
\]

\[
\%\text{Thickness of Al} = \frac{0.162}{0.942} \times 100\% = 17.20\%
\]

\[
\%\text{Thickness of Pl} = \frac{0.781}{0.942} \times 100\% = 82.80\%
\]

\[
\mu(E) = 0.172 \mu_{\text{Al}}(E) + 0.828 \mu_{\text{Pl}}(E) \tag{7.33}
\]

The individual \( \mu_{\text{Al}} \) and \( \mu_{\text{Pl}} \) values were found from standard linear attenuation coefficient tables [Johns and Cunningham 1984] within an energy range of 6 keV to 50 keV. The \( \mu_{\text{Al}} \) and \( \mu_{\text{Pl}} \) values were plugged into Eq. 7.33 to calculate the \( \mu(E) \) values. These values of \( \mu(E) \) provide an estimate of the expected lower and upper bound for \( \mu_1 \) and \( \mu_2 \). The calculated values of \( \mu_1 \), \( \mu_2 \) and \( \alpha \) were verified by finding the associated energies \( E_1 \) and \( E_2 \), by interpolating from the plot of \( \mu(E) \) in \( \text{cm}^{-1} \) vs. energy in keV (Fig. 7.2).

Based on these calculations, the parameters \( \mu_1 \) and \( \mu_2 \) were bound between 0 to 2 cm\(^{-1}\). The bound values for \( \alpha \) were arbitrarily chosen between 0 and 2 based on the solutions from a few test data sets initially calculated without bounds.

The goodness of fit of the modeled projection data was assessed using the Chi-squared test, which yielded a significance level \( p < 0.01 \). The value for \( \mu_1 \) was calculated to be 0.972 cm\(^{-1} \) (28.635 keV), \( \mu_2 \) was 0.382 cm\(^{-1} \) (38.119 keV) and the
7.5. **BEAM-HARDENING CORRECTION**

Figure 7.2: Linear attenuation coefficient ($\mu(E)$) vs. energy for the Al/Pl slabs. The two data points identified in the figure are the result of applying the bimodal-energy model to a sample data set.

Intensity ratio $\alpha$ of the higher to lower energy was 0.8468. The calculated energies show that $E_2 > E_1$ and that these energies are close to the estimated average energy ($\sim \frac{45kV}{2}$).

The bimodal beam-hardening model was linearized by projecting the points represented by the fitted equation to a line $P_{\text{ideal}}$ (Eq. 7.34), which has a slope $\mu_{\text{eff}}$ and intercept $c = 0$. The variable $d$ (Eq. 7.29) is replaced by $\frac{P_{\text{ideal}}}{\mu_{\text{eff}}}$:

$$P_{\text{ideal}} = \mu_{\text{eff}} x + c$$

$$\left| \left( \frac{\mu_2}{\mu_{\text{eff}}} \right) P_{\text{ideal}} + \ln \left( \frac{1 + \alpha}{1 + \alpha e^{-(\frac{\mu_1 - \mu_2}{\mu_{\text{eff}}}) P_{\text{ideal}}}} \right) - \ln \left( \frac{I_0}{I_i} \right) \right| \leq 0.01$$

The corrected projection values were obtained by solving Eq. 7.35 until convergence, with an acceptable error less than or equal to 0.01. This error limit was arbitrarily set to correct the projection values to an error no greater than 1%. Linearization using both the polynomial and bimodal-energy models satisfy the required error criterion.
for the data sets that were collected over a period of nine months. This included
data sets that used both 10 and 19 slab pairs. The stability of the beam-hardening
corrections was analyzed by evaluating the error statistics of the corrected values
using the polynomial and bimodal-energy models.

Measuring the plates to calculate the parameters necessary for the primary cor-
rection every day is a tedious process. This process can be simplified by applying the
primary corrections from one particular date to the data sets collected from other
dates and following this with a secondary correction based only on a few plates mea-
sured on the specific date.

### 7.6 Secondary Correction

The secondary correction, based on a couple of strategically measured slab thicknesses
from the same date as the data set under consideration is expected to reduce the
average fitting error. First, a simple linear model was applied after the primary
correction. An arbitrary thickness of 5 slab pairs for the 10-plate data set was used
together with zero slabs for the intercept. However, the linear correction using this
model did not fulfill the error criterion, and higher-degree models were investigated.

For the 10-plate data sets, it was discovered that a second-order correction with
two slab-pair thicknesses was required to satisfy the desired accuracy, providing three
fixed points at 0, 4 and 9 slabs pairs. However, a third degree model was necessary to
fulfill the error criterion for the 19 plate data sets. Different-date primary corrections
produced deviations beyond the set error boundaries (Fig. 7.3(a)). The application of a secondary correction tightened the deviations considerably (Fig. 7.3(b)).

Since the objective was to implement a generalized secondary correction model, it was decided to use the third-degree model on both 10 and 19 plate data sets. The combination of the three slab-pair thicknesses that produced the least average fitting error was determined by trial and error. We found that 0, 3, 7 and 9 slab pairs for the 10-plate data set and 0, 6, 14 and 19 slab pairs for the 19-plate data sets provided adequate results. The stability of the secondary corrections using the polynomial and bimodal-energy models was then analyzed.
7.7 Stability Analysis for Beam-Hardening Correction

The beam-hardening correction was meant to be applied to data collected on the same date. The stability analysis assessed the drift in the detector-element responses and the effect of this drift on the beam-hardening correction. The data required for this were periodically collected for nine months under unchanged working conditions, from 07-17-2006 through 03-13-2007, with multiple readings on certain dates.

The error statistics of the corrections applied to the data for all the data sets were assessed using the average fitting errors for the same-date corrections and different-date corrections. If there was no drift, in the measured data, the detector responses should not vary with time. To test this hypothesis, the correction derived from each individual date was applied to all data sets. To set up the data for this analysis, measured projection values were fitted using the fifth-degree polynomial and the bimodal-energy model, and their corresponding fit coefficient vectors were saved in a matrix. There were data collected on 22 dates during the nine months, with multiple data sets collected on most of these dates, resulting in 134 coefficient matrices for the 10-plate data sets and 5 coefficient matrices for the 19-plate data sets. Each data set was linearized using every fit-coefficient matrix. The results and the analysis of the results of these experiments are discussed in the following chapter.
8

Results: Detector Calibration

This chapter analyzes the error statistics involved in dead-time and beam-hardening correction methods and discusses the results of the stability analysis experiments performed for the dead-time as well as the primary and secondary beam-hardening corrections.

8.1 Dead-Time Correction

The objective was to correct photon counts for dead-time at an error level of less than 0.5% from the expected counts at 1 mA tube current.

8.1.1 Comparison of Models

A sample data set was mathematically modeled using second-degree (Fig. 8.1(a)), third-degree (Fig. 8.1(b)), and fourth-degree (Fig. 8.2) polynomial functions and linearized to illustrate the effectiveness of this correction method. The desired accuracy was achieved by modeling the uncorrected counts by a fourth-degree polynomial and
linearizing them (Fig. 8.2). The deviations of the corrected from the expected counts were significantly reduced with higher-degrees of fit.

Figure 8.1: Dead-time corrected photon counts using the a) second-degree and b) third-degree polynomial model for a data set of 64 detector elements.

The histogram (Fig. 8.3) of the individual residuals between the expected and corrected counts of the fourth-degree model shows that most individual residuals are within ±350 counts. In this sample data set, most of the individual residuals of corrected counts of the 64 detector elements are within the set limits.

In the next step, a stability experiment was conducted to analyze the error statistics of the fourth-degree polynomial correction applied to all data sets.

8.1.2 Application of the Best Model

The performance of the dead-time corrections was analyzed using the error statistics in same-date and different-date corrections applied to the 22 data sets collected from
Figure 8.2: a) Fourth-degree polynomial model of uncorrected counts and b) linearization using the fourth-degree model for a data set of 64 detector elements.

Figure 8.3: Histogram of the individual residuals using a fourth-degree polynomial correction for a data set of 64 detector elements.
07/19/2006 through 03/16/2007. Each data set contains measured counts by the 64
detector elements for 11 increments of tube current. The same-date and different-date
corrections are assumed to be stable if their average fitting error is less than or equal
to 350.

The average fitting error of the same-date corrections applied to the 22 data-sets
are shown in Figure 8.4(a). An irregular response in a single detector element in the
09/05 and 01/09 data sets produced higher average fitting errors. Despite that, the
average fitting error is well below the maximum allowed average fitting error of 350
for all 22 data sets; this demonstrates the stability of the same-date fourth-degree
polynomial dead-time correction.

The average fitting error of the different-date corrections applied to the 22 data-
sets (Fig. 8.4(b)) were expected to be no larger than the average fitting error of the
same-date corrections and hopefully no greater than 350. However, different-date
corrections for dead time were proven to be not feasible, because the average fitting
errors are much greater than 350. Using a secondary correction of a lesser degree after
the different-date dead-time correction may lower the average fitting error. Despite
that possibility, it was decided to not further investigate a different-date primary
correction followed by a secondary correction, because the data collection required
for the same-date dead-time correction can be automated and is not overly time
consuming.
Figure 8.4: Analysis of dead-time correction using a) same-date correction and b) different-date correction
8.2 Beam-Hardening Correction

The objective was to correct the projection values to have individual residuals no greater than ±0.05 in units of the projection values.

8.2.1 Comparison of Models

Mathematical functions using the polynomial and bimodal-energy models were used to linearize the projection values. The results of the two models for a sample data set of 10-plates are shown in Figure 8.5. The modeled data using both mathematical functions show similar results that satisfy the chi-squared test for $p < 0.01$. These models are further compared after linearization.

Figure 8.5: Projection values modeled using (a) fifth-degree polynomial model and (b) bimodal-energy model for a data set.
8.2. **BEAM-HARDENING CORRECTION**

The same sample data set was mathematically modeled using the second, third, and fourth-degree (Fig. 8.6) polynomial models and linearized to illustrate the effectiveness of the same-date primary corrections. Most of the individual residuals of the corrected counts for the second to fourth-degree polynomial models exceed the expected range of ±0.05. However, the desired level of correction was achieved using the bimodal and fifth-degree polynomial models.

8.2.2 **Application of the Best Model**

The bimodal-energy model and the fifth-degree polynomial beam-hardening models were compared using a sample 10-plate and 19-plate data set. Same-date primary, different-date primary corrections, and different-date primary followed by secondary corrections were applied to these data sets.

8.2.2.1 **Same-Date Primary Correction**

Same-date primary corrections were applied to a sample data set that was modeled by polynomial and bimodal models (Fig. 8.7). The histograms of the individual residuals between expected and corrected projection values are also shown. Both models satisfy the desired error range of ± 0.05.

The same-date primary corrections were also applied to a sample 19-plate data-set modeled by the polynomial and bimodal model (Fig. 8.8). The histograms of the individual residuals for both models are also within the desired error level.
Figure 8.6: Beam-hardening-corrected projection values using a) second, b) third, and c) fourth-degree polynomial models for one data set of 64 detector elements.
Figure 8.7: Same-date primary corrected projection values and their histograms of individual residuals for a), b) the polynomial model and c), d) the bimodal-energy model, respectively, for a 10-plate data set including 64 detector elements.
8.2. BEAM-HARDENING CORRECTION

Figure 8.8: Same-date primary corrected projection values and their histograms of individual residuals for a), b) the polynomial model and c), d) the bimodal-energy model for a 19-plate data set for all 64 detector elements.
8.2. BEAM-HARDENING CORRECTION

8.2.2 Different-Date Primary Correction

Different-date primary corrections of the polynomial and bimodal-energy models were applied to the same 10-plate data-set (Fig. 8.9).

The deviations of the corrected projection values from their expected values are more prominent when using different-date primary polynomial corrections than the different-date primary bimodal corrections. The histogram shows individual residuals exceeding ± 0.1, which is twice the desired error level. This error level indicates a drift in the detector pixel response with time. This drift can be a result of change in operating temperature or slight variations in source-detector operating parameters. In contrast, the error histogram for the bimodal-energy model shows lower levels of deviation, most of them within the desired error range of ± 0.05. Different-date primary corrections applied to the sample 19-plate data-set showed individual residuals for both models beyond the desired error level (Fig. 8.10), indicating the need for a secondary correction.

8.2.2.3 Secondary Correction on Different-Date Primary Correction

Secondary polynomial corrections were applied to the different-date primary corrected 10-plate data using the polynomial and the bimodal model (Fig. 8.11). The histograms show that the secondary corrections had a pronounced effect in reducing the individual residuals of the different-date primary polynomial and bimodal corrections to values mostly within the desired error level.
8.2. BEAM-HARDENING CORRECTION

Figure 8.9: Different-date primary corrected projection values and their histograms of individual residuals for a), b) the polynomial model and c), d) the bimodal-energy model for a 10-plate data set for all 64 detector elements.
8.2. BEAM-HARDENING CORRECTION

Figure 8.10: Different-date primary corrected projection values and their histograms of individual residuals for a), b) the polynomial model and c), d) the bimodal-energy model for a 19-plate data set for all 64 detector elements.
8.2. BEAM-HARDENING CORRECTION

Figure 8.11: Secondary correction applied to different-date primary corrections and their histograms of individual residuals for a), b) the polynomial model and c), d) the bimodal-energy model for a 10-plate data set for all 64 detector elements.
Secondary corrections were also applied to the different-date primary corrected 19-plate sample data-set (Fig. 8.12). The histograms reveal that most individual residuals for both models are within the ±0.05 range.

Figure 8.12: Secondary correction applied to different-date primary corrections and their histograms of the residuals, using the polynomial and bimodal-energy model for a 19-plate data set for all 64 detector elements.
8.2. BEAM-HARDENING CORRECTION

The analyses described above considered only one sample of the 10-plate and 19-plate data sets. For the following evaluations, we use all 123 10-plate data sets collected on 19 dates and all 5 19-plate data sets collected on three dates.

8.2.3 Correction Analysis Using 10-Slab-Pair Data-Sets

This section analyzes the error statistics of the same-date primary, different-date primary, same-date primary followed by secondary, and different-date primary followed by secondary corrections applied to all 10-plate data sets using the polynomial and bimodal models. These experiments helped compare the effectiveness of polynomial and bimodal corrections. Each 10-plate data set contains measured projection values for the 64 detector elements for 10 increments of slab thicknesses.

![Figure 8.13: Average fitting error for different-date primary and different-date primary followed by secondary corrections for the 10-plate data sets. The dates indicate the data set, from which the primary correction was derived and applied to all data sets.](image)
8.2. BEAM-HARDENING CORRECTION

Figure 8.13 shows the average fitting error for different-date primary corrected projections with and without secondary corrections. For all dates, the different-date bimodal corrections show lower average fitting errors than the polynomial corrections, both with and without secondary corrections applied. Most of the average fitting errors for the different-date bimodal corrections are lower than the maximum desired average fitting error of 0.05, but this is not the case for the different-date polynomial corrections. The effect of the secondary corrections applied to the different-date polynomial corrections scales down the average fitting error, and all of them are considerably lower than 0.05. The data also show that the secondarily corrected bimodal corrections are marginally better than the secondarily corrected polynomial corrections.

There is an abrupt change in average fitting error for the bimodal energy model after 11/16. Two detector elements show a different behavior during the two time periods and influence the fitting errors.

In another analysis, the same-date primary corrections were compared with the secondarily corrected same-date primary corrections (Fig. 8.14). For the polynomial model, the secondary correction, being of a lesser degree polynomial than the primary correction, is not expected to produce any improvement. This is reasonably well supported by the data shown in Figure 8.14, with the exception of one outlier on 9/13, which profits from the secondary correction. An unusual response from a single detector element produced the increased average fitting error. The bimodal-energy
model experiences a small but consistent improvement. The average fitting error of the same-date primary corrections for all 19 dates are less than 0.05.

Figure 8.14: Average fitting error for same-date primary and same-date primary followed by secondary corrections for the 10-plate data sets.

A final analysis compared the secondarily corrected same-date and different-date primary corrections based on the polynomial and bimodal models (Fig. 8.15). The secondarily corrected same-date corrections would be expected to show the lowest average fitting error. The data show that all average fitting error involving secondary corrections for the 10-plate data sets are below 0.02, which is considerably below the set limit of 0.05. The secondarily corrected same-date corrections are marginally better than the different-date corrections for the polynomial model. For the bimodal-energy model, about half the dates show smaller average fitting errors for the secondarily corrected same-date primary corrections, and the remaining dates show smaller average fitting errors for the secondarily corrected different-date primary corrections.
8.2. BEAM-HARDENING CORRECTION

8.2.4 Correction Analysis Using 19-Slab-Pair Data-Sets

Similar analyses as for the 10-slab data sets were executed to assess the error statistics of the 19-plate data-sets. Figure 8.16 shows the average fitting error for different-date primary corrections with and without secondary corrections for both polynomial and bimodal models. The average fitting errors of the different-date primary polynomial model clearly exceed the maximum desired average fitting error of 0.05. The different-date bimodal primary corrections behave better, as the average fitting errors are almost within the desired error range. The secondarily corrected primary corrections for both models show average fitting errors below 0.05. The secondary correction considerably reduces the average fitting errors of the primary polynomial corrections,
whereas the average fitting errors of secondarily corrected primary bimodal corrections are only marginally improved. This was also observed for the 10-plate data sets.

![Figure 8.16: Average fitting error for different-date primary corrections and different-date primary followed by secondary corrections for the 19-plate data sets.](image)

The influence of the secondary corrections on the same-date primary corrections is shown in Figure 8.17. As observed before with the 10-plate data, the polynomial model shows little difference, and the bimodal-energy model demonstrate some improvement.

The secondarily corrected same-date and different-date primary corrections were also compared (Fig. 8.18). For both models, the secondarily corrected different-date corrections appear to be as good as the secondarily-corrected same-date corrections. However, the bimodal-energy model behaves slightly better than the polynomial model. Both models achieve average fitting errors below 0.05.
8.2. BEAM-HARDENING CORRECTION

Figure 8.17: Average fitting error for same-date primary and same-date primary followed by secondary corrections for the 19-plate data sets.

Figure 8.18: Same-date and different-date primary corrections followed by secondary corrections using the polynomial and bimodal-energy models for the 19-plate data sets.
8.2.5 Summary of Beam-Hardening Analyses

The different-date primary bimodal correction method outperformed the polynomial method for both 10- and 19-plate data sets. Secondarily correcting primary bimodal corrections showed further improvement.

A summary of the performance of the beam-hardening-correction methods using the polynomial and bimodal-energy models is shown in Table 8.1. The checked cells in the table represent the methods that produced average fitting errors lower than 0.05.

The same-date primary corrections perform similarly well for both models, in all cases achieving the set error level.

Table 8.1: Summary of the same-date and different-date primary (Prim) corrections using the fifth-degree polynomial and bimodal-energy models, and the same-date and different-date primary corrections using both models followed by the secondary (Sec) corrections. The checked cells represent the methods that produced average fitting errors lower than 0.05.

<table>
<thead>
<tr>
<th>Data-Set</th>
<th>Fifth-Degree Polynomial Model</th>
<th>Bimodal-Energy Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Same Date</td>
<td>Different Date</td>
</tr>
<tr>
<td></td>
<td>Prim</td>
<td>Prim+Sec</td>
</tr>
<tr>
<td>10 Plates</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Worst Error</td>
<td>0.039</td>
<td>0.019</td>
</tr>
<tr>
<td>19 Plates</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Worst Error</td>
<td>0.040</td>
<td>0.041</td>
</tr>
</tbody>
</table>

However, the bimodal-energy model performs markedly better by showing lower average fitting errors for the different-date primary corrections than the polynomial model. Once the secondary correction is applied, the difference between the two
8.2. BEAM-HARDENING CORRECTION

models shrinks. With the secondary correction, both models achieve the set error requirement of less than 0.05. Higher attenuation thickness (19 plates) creates a larger challenge in beam-hardening correction compared to lower attenuation thickness. This difference is consistent for all correction approaches.

The various analyses showed that the bimodal-energy model consistently produces lower average fitting errors than the fifth-degree polynomial model. Consequently, it was decided to use the bimodal-energy model for beam-hardening correction applied as a different-day correction followed by a third-degree polynomial, secondary correction, which relies on a reduced set of 3 plate thicknesses measured the same day.
Discussion: Detector Calibration

The aim of this part of the project was to correct photon counts for dead-time at an error level of less than 0.5% from the expected counts at 1 mA tube current and to correct the projection values for beam hardening to an error level of less than 1% from the expected projection value for a thickness of 10 Al/Pl slab pairs.

The dead-time correction was achieved by linearizing a fourth-degree polynomial model of the data. The stability of this correction method was tested by applying same-date and different-date corrections on data sets that were collected over a period of nine months. The same-date dead-time corrections were all within the expected average fitting error value of 350 counts, whereas the average fitting errors of the different-date corrections were well above 350. The results of this analysis suggest the need for a secondary dead-time correction, if one desired to use the different-date primary corrections.

The average fitting errors produced by the different-date corrections suggest a drift in the detector response behavior over nine months. Since the data collection required
for the dead-time corrections can be automated, the need for different-date primary corrections was not investigated any further. Based on these results, it was concluded that linearization using the fourth-degree polynomial model is a reliable method for same-date primary dead-time correction.

The non-linear projection values due to beam hardening were described by a fifth-degree polynomial model, reflecting a purely mathematical approach, and a bimodal-energy model, reflecting the physics of beam hardening. The linearization of these models was assessed for the same- and different-date primary and secondary corrections. The corrections were applied to 10- and 19-plate data sets to emulate the thickness of the human forearm and the lower part of the leg. The same-date primary corrections using the fifth-degree polynomial and bimodal-energy models were applied to the 134 10-plate data sets collected over 7 months and the five 19-plate data sets collected over two weeks.

Same-date primary corrections consistently produced corrected projection values that were well within the expected average fitting error of 0.05. Most of the average fitting errors for the different-date bimodal corrections were below 0.05, whereas the average fitting errors for the different-date polynomial corrections were above 0.05. However, applying a third-degree polynomial secondary correction reduced the average fitting error of the different-date bimodal-energy and polynomial corrections to values within the desired error level.

Based on the results of these experiments, it was concluded that the different-date primary and secondary corrections using the bimodal-energy model performed better
than the polynomial model. These results also show that the linearization using the bimodal-energy model is a reliable method for primary and secondary beam-hardening corrections for 10 slab-pair and 19 slab-pair thicknesses.

There were not enough 19-plate data sets available to do a fair comparison between the 10- and 19-plate data sets. The corrections were all applied to the data collected using the CZT detector using consistent operating parameters, but they were not tested under any other conditions or with other detectors.

This research did not investigate the possibility of alternative mathematical models that can describe the count vs. current curve. Different dead-time correction models, e.g. the non-paralyzable dead-time model, could also be investigated.

A study of the dead-time characteristics for x-ray tube voltages other than 45 kVp is a potential area of interest, as the timing of the conversion of x-ray energy into electrical energy might depend on the energy of the x-ray photon. Similarly, beam-hardening corrections should be studied with different tube voltages, as the photon energy is a major factor in defining the attenuation properties.

Another area of interest is the stability of the corrections over a shorter time period, i.e. hours and days. The current project involved data collection over nine months in irregular intervals. Data collected in an hourly, daily, weekly, and monthly pattern can be used to closely investigate the time-specific effects of using corrections from one data set applied to another data set.
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