INVESTIGATION OF CYLINDRICALLY-CONFORMED FOUR-ARM SPIRAL ANTENNAS

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in Engineering

By

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ABSTRACT


A four-arm spiral antenna offers broadband frequency response, wide beamwidths, reduced size compared to other antenna designs, and the ability to determine the relative direction of an incident signal with appropriate mode-forming. The reduced overall area projection of the four-arm spiral antenna compared to other antenna designs and the ability to be manufactured in a planar format allows the antenna to reside within an Unmanned Air Vehicle (UAV) fuselage. This thesis investigates the effects of cylindrically-conforming two different designs of a four-arm spiral antenna to reside within the fuselage of a medium-sized UAV. Theoretical predictions of antenna performance were created using the Numerical Electromagnetics Code (NEC) package and compared to measured results of flat and cylindrically-conformed four-arm spiral antennas with and without ground plane apertures.
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1. INTRODUCTION

1.1. Thesis Motivation

Developing a low-cost (disposable) and light-weight direction-finding system for use with an Unmanned Air Vehicle (UAV) is the motivation of this thesis. As the battlefield evolves around the world, the ability to locate communications of enemy combatants is one of the most relevant tasks for supporting the war-fighter. Developing an antenna for a direction-finding system with an emphasis on modularity for different mission types and broad frequency response would demonstrate the utility of such a system to the military.

1.2. Thesis Objectives

The objectives of this thesis are the following:

- Define a suitable antenna as defined by the thesis motivation.
- Describe the characteristics of the antenna.
- Discuss the development of an antenna capable of direction-finding.
- Define a theoretical model capable of describing the antenna.
- Provide a comparison of the theoretical and measured results.
2. FREQUENCY-INDEPENDENT ANTENNAS

2.1 Theory of Frequency Independence

Due to the ever-expanding utilization of the electromagnetic spectrum, antennas that can operate over wide frequency ranges have become increasingly desirable. As stated in Balanis [1], prior to the 1950s, antenna technology could only provide bandwidths of 2:1 or less. Afterwards though, the concept of specifying antenna geometries purely by angles instead of characteristic lengths allowed for bandwidths of 40:1 or more. If an antenna is only defined by angles, attributes such as the characteristic input impedance, field pattern, and polarization remain constant as the size of the antenna is scaled.

The theory of frequency-independence, as described by the work of Rumsey [2] and Elliot [3], begins by assuming an antenna geometry within spherical coordinates \((\rho, \theta, \phi)\), where each terminal (feed point) of the antenna surface is infinitesimally close to the origin and is confined to rotation about the \(\theta = 0\) axis. To further simplify the problem, surfaces are assumed to be perfectly conducting, and the region surrounding the antenna is assumed to be infinite in extent, homogenous, and isotropic. Such a surface is described by a single curve (for a wire) or a set of curves (to define the edges of a strip)

\[ r = F(\theta, \phi) \]  

(2.1-1)
where \( r \) represents the radial distance along the curve. To define the property of scaling, start by assuming an antenna were to be scaled to operate at a frequency that is \( K \) times lower than the original antenna. The physical size of the antenna would need to be increased by a factor of \( K \). The scaled curve is then described by

\[
r' = KF(\theta, \phi)
\]

(2.1-2)

where both surfaces superimpose one another after the second curve is rotated by an angle \( \phi_c \) about the \( \theta = 0 \) axis. This is shown by the following

\[
F(\theta, \phi + \phi_c) = KF(\theta, \phi)
\]

(2.1-3)

where the rotated angle \( \phi_c \) depends only on \( K \). The following figure depicts three spirals that correspond to a single-arm spiral, a scaled single-arm spiral, and a scaled single-arm spiral that has been rotated to visualize congruence of the two spirals.

![Comparison of Scaled Spirals](image)

Figure 2.1-1 Comparison of Scaled Spirals
To prove congruence between the two curves, begin by differentiating both sides of equation (2.1-3) with respect to both $\phi_c$ and $\phi$.

$$\frac{d}{d\phi_c} [KF(\theta, \phi)] = \frac{dK}{d\phi_c} [F(\theta, \phi)] = \frac{\partial}{\partial\phi_c} [F(\theta, \phi + \phi_c)] \quad (2.1-4)$$

$$\frac{d}{d\phi_c} [KF(\theta, \phi)] = \frac{\partial}{\partial(\phi + \phi_c)} [F(\theta, \phi + \phi_c)] \quad (2.1-5)$$

$$\frac{\partial}{\partial\phi} [KF(\theta, \phi)] = K \frac{\partial}{\partial\phi} [F(\theta, \phi)] = \frac{\partial}{\partial\phi} [F(\theta, \phi + \phi_c)] \quad (2.1-6)$$

$$\frac{\partial}{\partial\phi} [KF(\theta, \phi)] = \frac{\partial}{\partial(\phi + \phi_c)} [F(\theta, \phi + \phi_c)] \quad (2.1-7)$$

Equating (2.1-5) and (2.1-7) results in

$$\frac{dK}{d\phi_c} [F(\theta, \phi)] = K \frac{\partial}{\partial\phi} [F(\theta, \phi)]. \quad (2.1-8)$$

Substituting (2.1-1) into (2.1-8) gives

$$\frac{1}{K} \frac{dK}{d\phi_c} r = \frac{1}{r} \frac{\partial r}{\partial\phi}. \quad (2.1-9)$$

Due to the spatial independence of the left-hand side of (2.1-7), the general solution of the curve becomes

$$r = F(\theta, \phi) = e^{\alpha \phi} f(\theta) \quad (2.1-10)$$

where $\alpha$ is

$$\alpha = \frac{1}{K} \frac{dK}{d\phi_c} \quad (2.1-11)$$

and $f(\theta)$ is a completely arbitrary function that describes the curve or curves that define the surface of the antenna.
2.2 Types of Frequency-Independent Antennas

Frequency-independent antennas within the context of this thesis can be categorized into three types: biconical, log-periodic, and spiral. The following sections describe the variants, attributes, limitations, and the basic antenna theory behind each type.

2.2.1 Biconical Antennas

Biconical antennas have several geometric variants such as the planar biconical (bowtie), finite biconical, and the discone. This section focuses on the infinite biconical antenna to describe the general theory. Although it is impractical to assume the existence of a biconical antenna of infinite extent, it can be shown that the current diminishes to a negligible amount after a finite distance. It is suitable to define the length of a finite biconical antenna by this truncated distance, where the performance of the antenna approaches the infinite case. To describe the theory of biconical antennas, we start with the well known fact that the bandwidth of a dipole antenna can be increased by enlarging the diameter of the conductors. Expanding on this concept, if we instead flared the diameter of the conductors outward as the distance from the feed point increases, the geometry of a biconical antenna is defined. The structure can be analyzed as a transmission line where the voltage source produces a current distribution that flows radially outwards. This current distribution creates a magnetic field that encircles each cone. The electric field, which is perpendicular to the magnetic field, is $\theta$ directed. The following figure depicts the geometry of the infinite biconical antenna.
From Stutzman and Thiele [4], the normalized field pattern of the antenna is

$$ F(\theta) = \frac{\sin(\theta_h)}{\sin(\theta)} $$

(2.2.1-1)

where $\theta_h$ is the cone half angle and $\theta$ is the angle from the z-axis. From this relation, it can be determined that the larger the cone angle, the greater the field strength will be within the available free-space. The following figure depicts the normalized field pattern of the infinite biconical antenna.
The input impedance of the antenna is defined as

\[
Z_{in} = 120 \ln \left( \cot \left( \frac{\theta}{2} \right) \right)
\]  \hspace{1cm} (2.2.1-2)

which has the unique property of being invariant with respect to frequency. The same frequency invariant properties can also be extended to the field pattern. The following figure depicts the input impedance versus the cone half angle. It should be noted that for the case of the finite biconical antenna, the ends of the cones cause a reflection of the incident waves produced by the voltage source to produce a standing wave within the antenna structure. This will create a complex input impedance at the antenna terminals.
Figure 2.2.1-3 Normalized Impedance of the Biconical Antenna for Different $\theta_h$

As a candidate for the antenna system, the biconical antenna would need to be utilized in its planar form, the bowtie, to satisfy the geometry requirements. The bowtie antenna would also need to be a part of a phased array to provide the modal characteristics necessary to provide direction-finding capabilities. These inadequacies preclude the biconical antenna from being considered for the antenna system defined by the motivation of this thesis.

2.2.2 Log-Periodic Antennas

The second type of frequency-independent antenna is the log-periodic. As with the biconical antenna, there are several geometric variants of the log-periodic, such as the toothed planar, the toothed wedge, the trapezoid wire, the zig-zag wire, and the dipole array. The following section will focus on the log-periodic dipole array to describe the
general theory. The geometry of the log-periodic dipole array antenna, like the biconical, is based on the dipole antenna. We begin by defining the log-periodic dipole array as a linear distribution of dipoles with increasing length that are connected together by alternating the feed polarity to each dipole. The following figure depicts the geometry of the log-periodic dipole array.

![Figure 2.2.2-1 Geometry of the Log-Periodic Dipole Array Antenna (From Stutzman and Thiele [4])](image)

It can be seen from the geometry of the log-periodic antenna that it cannot be solely described by angles as prescribed by the characteristics of the frequency-independent antenna. It should be noted, though, that the performance of the log-periodic antenna approaches the characteristics of the frequency-independent antenna.

As derived in Stutzman and Thiele [4], the bandwidth of the log-periodic dipole array is defined by the following equations

\[
L_1 \approx \frac{\lambda_{\text{LowerFrequency}}}{2} \quad \text{and} \quad L_N \approx \frac{\lambda_{\text{UpperFrequency}}}{2} \quad (2.2.2-1)
\]
where $L_1$ and $L_N$ are the longest and shortest dipoles respectively. The following equations define the design parameters used to produce log-periodic dipole arrays of varying performance.

\[ \tau = \frac{R_{n+1}}{R_n} = \frac{L_{n+1}}{L_n} = \frac{d_{n+1}}{d_n} \]  

(2.2.2-2)

Where $\tau$ is the geometric ratio of the antenna, it defines the length growth rate for each of the successive dipoles used in the array.

\[ \sigma = \frac{d_n}{2L_n} \]  

(2.2.2-3)

Where $\sigma$ is the spacing factor, it describes the distance between each successive dipole within the array.

\[ \alpha = 2 \tan^{-1}\left(\frac{1-\tau}{4\sigma}\right) \]  

(2.2.2-4)

Finally, $\alpha$ is the angle that is formed from the intersection of the lines extending from both ends of the dipoles along the array. The following figure depicts design curves for different log-periodic dipole arrays and their respective maximum gains.
As seen in the figure, the best performing antennas have a very slow transition in length from one dipole to the next. The active region, or where the current is concentrated on the antenna, is around the dipole where the length is approximately one half of a wavelength of the operating frequency. It should also be noted that, in practice, dipoles are often added to either end of the antenna to ensure the strength of the active region at either end of the antenna. The impedance, like other frequency-independent antennas is nearly constant over frequency. The following figure depicts a general impedance variation with respect to frequency for a log-periodic dipole array.
As a candidate for use with the antenna system, the log-periodic antenna would need to be utilized in any of its planar forms to satisfy the geometry requirements. The log-periodic antenna would also need to be a part of a phased array to provide the modal characteristics necessary to provide direction-finding capabilities. These inadequacies preclude the log-periodic antenna from being considered for the antenna system defined by the motivation of this thesis.

2.2.3 Spiral Antennas

The final type of frequency-independent antenna to describe within this discussion is the spiral. Like the biconical and log-periodic antennas, the spiral also has several geometric variations in design. The variants can be categorized into three types: equiangular, logarithmic, and Archimedean. Each spiral variant also has a planar, conical,
and spherical form. Previously, the conical form was the most prevalent where the planar form has become the popular choice for research due to its ability to be manufactured by printed circuit techniques. To describe the general theory of each type of spiral antenna, we will focus on the two-arm planar versions of each. The two-arm versions of the spiral antenna can be thought of as a long dipole whose conductors have been wound around the feed axis.

The first type of spiral to examine is the equiangular; the following figure depicts the geometry of the equiangular spiral.

![Diagram of the equiangular spiral antenna](image)

As stated above, all spiral antennas can be defined purely by angles. We begin by defining the design equations from Stutzman and Thiele [4]. There are four equations that define the radius of the inner and outer curves of each conductor’s surface.

\[ r_i = r_0 e^{a \phi} \]  

(2.2.3-1)
The flare rate $a$ is more easily represented by the expansion ratio

$$\varepsilon = e^{a2\pi}. \quad (2.2.3-5)$$

When designing equiangular spirals, it has been experimentally shown that the best performing designs have one and one-half turns ($\phi = 0$ to $3\pi$) and have a flare rate equal to 0.221. The typical bandwidth of this design is 8:1, where $r_o$ is equal to $\lambda_{UpperFrequency}/4$, and the maximum radius $R$ is equal to $\lambda_{LowerFrequency}/4$. It should be noted that the previous figure can also be described as a self-complementary antenna. Self-complementary structures are comprised of a conductor surface whose compliment is identical to the original surface. This occurs when the conductor surface occupies the same area as the free space surface. Self-complementary antennas have the remarkable property of having a nearly constant impedance across their bandwidth regardless of shape. The two-arm self-complementary antenna has a theoretical input impedance of 189 ohms where experimentation has determined that this number should be reduced to approximately 164 ohms.

Other forms of the equiangular spiral are shown in the following figures, the first is the conical spiral, which was one of the first spiral antennas geometries to be evaluated.
The other geometric conformation is the spherical spiral. Practically speaking though, this specific spiral would need to be cut in half along the equatorial plane to create a viable antenna. It does provide additional application possibilities, such as conforming to the nose of a guided munition.
The second type of spiral to examine is the logarithmic; the following figure depicts the geometry.
Like the equiangular spiral, the radius of each arm is given by an exponential. The radius equations as defined by Johnson [5] are

\[ r_1 = r_0 e^{2\phi} \]  
\[ r_2 = r_0 e^{a(\phi-\delta)}. \]

The other radii, \( r_3 \) and \( r_4 \), can be created from \( r_1 \) and \( r_2 \) by subtracting \( \pi \) from the exponent. The pitch angle \( \psi \) is related to the flare rate \( a \) by

\[ \tan \psi = \frac{1}{a} \]

and the design ratio \( \tau \) is the ratio of the radius of the spiral arm after one turn.

\[ \tau = e^{-2\pi|a|} = e^{\left(\frac{2\pi}{\tan \psi}\right)} \]

The final type of spiral, which will be discussed throughout the remainder of this thesis, is the Archimedean spiral. The following figure depicts the geometry of the planar form.
As defined by Johnson [5], the arms of the Archimedean spiral can be described by the centerline curve

\[ r = a \phi \]  \hspace{1cm} (2.2.3-10)

where either side of the strip is defined as

\[ r = a \left( \phi \pm \frac{\delta}{2} \right) \]  \hspace{1cm} (2.2.3-11)

and the second conductor can be described by rotating the first arm by \( \pi \).

The width of the strip is defined as

\[ W = a \delta \]  \hspace{1cm} (2.2.3-12)

and the spacing between the centerlines of each strip is

\[ S = 2 \pi a \]. \hspace{1cm} (2.2.3-13)
It should be noted that the self-complementary case occurs when $W/S$ is equal to 0.25. Because the Archimedean spiral is only defined by a constant instead of an angle, it does not truly conform to the frequency-independent principle defined earlier. This should not diminish the fact that the Archimedean spiral performs extremely well compared to other spiral designs and dominates in terms of the level of research that has been devoted to its design.

2.3 Modal Behavior of the Spiral Antenna

The property of multi-arm spiral antennas that is appealing to the motivation of this thesis is the ability to provide a direction-finding capability to the antenna system. This direction-finding capability is made possible by feeding the spiral antenna in different configurations or *modes*. Unlike the biconical or log-periodic antenna, the multi-arm spiral allows other feed configurations to be used rather than the balanced line feed of the two-arm antenna. To understand the principles of feed configurations, we begin by discussing the general theory of mode-forming and the properties of the two-arm antenna feed. A two-arm antenna radiates by feeding each arm with an excitation that is 180 degrees out of phase with one another to create what is called a balanced feed. A balanced feed is defined as an antenna feed structure where the vector sum of all of the arm excitations is equal to zero. It should be noted that a balanced feed is different from a balanced line, where a balanced line is defined as a transmission line that has equal and opposite currents traveling along the two conductors. Extending this principle to an antenna with four arms as described by Corzine and Mosko [7], a balanced feed would
consist of a set of feed excitations whose relative phases would be 0, 90, 180, and 270 degrees with respect to the first terminal.

As described by the two and four-arm antenna cases, the total phase progression around the feed points of the antenna is 360 degrees. To understand the mode of an antenna, it is possible to allow the total phase progression around the feed points of an antenna be any integer multiple of 360 degrees, where the integer multiplier is known as the mode number. An antenna that has a phase progression of 360 degrees or \(2\pi\) radians is said to be operating in mode one, where a phase progression of 720 degrees or \(4\pi\) radians is operating in mode two. The number of modes an antenna is capable of supporting is

\[
N_{\text{Modes}} = (N_{\text{Arms}} - 1)
\]  

(2.3-1)

where \(N_{\text{Arms}}\) is the number of antenna arms. The following equation describes the feed excitation phase progression for any mode.

\[
P(N) = 360\pi \left( \frac{(N - 1)}{N_{\text{Arms}}} \right) M
\]

(2.3-2)

\(N\) is an integer between 1 and \(N_{\text{Arms}}\), \(P\) is the phase of the \(N^{th}\) arm in degrees, and \(M\) is the mode number. The following figure depicts what the feed excitation phase progressions are for each mode of a spiral antenna that has up to six arms. It should be noted that the following phase excitations are not exclusively for a spiral antenna; any antenna geometry with multiple arms will suffice.
The fundamental property of direction-finding is the ability to exploit the distinctive field patterns and phasing of the different modes. The following figure depicts a vertical cut of the normalized field pattern of a generic four-arm spiral antenna.

Figure 2.3-1 Feed Excitation Phase Progression for each Mode of a Multi-arm Antenna (From Corzine and Mosko [7])
Figure 2.3-2 Normalized Field Patterns for Different Modes of a Four-Arm Spiral Antenna (From Penno and Pasala [8])

The mode one field pattern is always a single lobe oriented along the boresight of the antenna and has a typical beamwidth of 70 degrees or more. Mode two causes a null along the boresight of the antenna that creates a toroid-like field pattern that has a maximum gain at approximately 38 degrees off of boresight. Mode three creates a similar pattern to mode two, but the maximum gain occurs approximately 45 degrees off of the boresight. As described by Penno and Pasala [8], the field pattern of each mode can be reproduced by replacing the spiral with an infinite set of current loops that range in circumference from the feed point loop to the outer loop of the spiral. Each loop of current has a circumference of one wavelength for mode one, two wavelengths for mode two, and so on. The phase progression on each loop goes from 0 to 360 degrees as it traverses around the loop for mode one, 0 to 720 degrees for mode two and so on. This principle can be extended to understand that the minimum frequency that a mode two
field pattern can exist is at least twice the frequency of the lowest mode one field pattern. Therefore, for a four-arm spiral antenna, the lowest frequency that a mode three field pattern can exist is at least three times the frequency of the lowest mode one field pattern.

To understand the direction-finding properties of the mode-former, we start by defining the requirements of direction-finding. At a minimum, an elevation and azimuth angle measured from the boresight of the antenna is needed to define the relative orientation of a received signal to the antenna. The elevation angle of the detected signal with respect to the boresight of the antenna is found by comparing the amplitudes of the mode one and mode two output signals, where every elevation angle provides a unique set of output amplitudes for each of the modes. Examining figure 2.3-2, it is apparent that at elevation angles near boresight, the mode two field pattern strength is considerably smaller than the mode one field pattern strength. The azimuth angle of the detected signal with respect to the boresight of the antenna is found by comparing the phase of the mode one and mode two output signals. The following figure illustrates how the azimuth angle is determined by subtracting the mode one and mode two output phases. It should be noted that the antenna surface is in the x-y plane and the mode one ring has a circumference of one wavelength and the mode two ring has a circumference of two wavelengths.
It can be concluded from the figure that the mode two output phase is double that of the mode one phase. This difference between the two output phases is *unique* at every azimuth point. The following figure depicts the modal phase difference as a function of azimuth angle.
Figure 2.3-3 Azimuth Determination by Modal Phase Difference (From Corzine and Mosko [7])
3. ANTELLA SYSTEM DEVELOPMENT

3.1 Spiral Antenna Development

The following sections provide the details of the design and fabrication of the square and Archimedean four-arm spiral antennas developed for this thesis. As noted previously, the focus of this thesis is the Archimedean spiral, but a square spiral was also developed to understand the effects of different spiral designs on the performance of the antenna system.

3.1.1 Spiral Antenna Design

Defining the upper and lower frequencies of operation for the spiral antenna provides a general starting point for calculating the overall dimensions. To achieve the best results, the antennas were etched on the thinnest microwave substrate available. Microwave substrate is similar to printed circuit board material, but is designed for radio and microwave frequency applications where the relative permittivity and thickness of the dielectric material sandwiched between the two layers of copper has to be known to create circuits that operate properly at these higher frequencies. Due to the dielectric core, the thinner the substrate, the less the pattern would be affected due to the rise of the effective permittivity caused by adding a dielectric to the free space surrounding the
antenna. The limitations on antenna size were set by the thinnest substrate available, which was 0.010 inches thick and approximately 7.500 inches wide. To provide enough room around the edges of the antenna to prevent any coupling between the antenna and the enclosure, the maximum size of the antenna to be etched on the microwave substrate was limited to just under six inches. The following equation provides a relation between the outer diameter of the spiral and the lowest operating frequency

\[ f_{\text{Lower}} = \frac{c}{\pi D_{\text{Outer}}} \]  

where \( D_{\text{Outer}} \) is the outer diameter of the spiral in meters, \( c \) is the speed of light within a vacuum in meters per second, and \( f_{\text{Lower}} \) is the lower frequency in Hertz. The upper frequency of the antenna is generally limited by the diameter of the coaxial cables used to feed the antenna. If a bundle of four coaxial lines are used to feed a four-arm antenna, the distance between the center conductors of each coaxial line is limited by the outer diameter of the coaxial line. The maximum operating frequency of a four-arm spiral antenna feed by a bundle of coaxial lines is limited by the following equation

\[ f_{\text{Upper}} = \frac{c}{\pi \sqrt{2} D_{\text{Inner}}} \]  

where \( D_{\text{Inner}} \) is the diameter of the coaxial cable in meters and \( f_{\text{Upper}} \) is the upper frequency in Hertz. A general design rule for determining the distance between feed points at the center of the spiral dictates the gap between arm surfaces should be equal to the arm width.

To determine the number of rotations a spiral arm will make as each arm is drawn out, the spiral strip width and arm spacing must be calculated beforehand. For self-complementary antennas, the strip width is one half of the arm spacing. This creates a
characteristic input impedance of 95 ohms for mode one of a four-arm antenna. To reduce the magnitude of the impedance mismatch of a 50 ohm transmission line terminating into a 95 ohm antenna, the strip width can be increased to reduce the impedance of the antenna. As described by Caswell [9], the effect of increasing the strip width does come with the penalty of causing the completely real-valued impedance to acquire a negative reactive component where narrowing the strip width causes an increase in the real part of the impedance and a slight variation to the reactive component. Even though widening the strip width causes the impedance to change, the overall magnitude is still less than the self-complementary case. The previous design equations for the Archimedean spiral hold for the square spiral; the only difference between the two designs is the square spiral does not have an analytic function that defines the arc of each arm. Each arm of the square spiral has to be drawn out by hand or by drafting software such as AutoCAD to create. The following parameters were used to design the square and Archimedean spiral antennas.

<table>
<thead>
<tr>
<th>Spiral Antenna Type</th>
<th>Archimedean</th>
<th>Square</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outer Diameter (in)</td>
<td>5.684</td>
<td>5.684</td>
</tr>
<tr>
<td>Lower Frequency (GHz)</td>
<td>0.661</td>
<td>0.661</td>
</tr>
<tr>
<td>Inner Diameter (in)</td>
<td>0.282</td>
<td>0.282</td>
</tr>
<tr>
<td>Upper Frequency (GHz)</td>
<td>13.323</td>
<td>13.323</td>
</tr>
<tr>
<td>Strip Width (in)</td>
<td>0.141</td>
<td>0.141</td>
</tr>
<tr>
<td>Arm Spacing (in)</td>
<td>0.203</td>
<td>0.203</td>
</tr>
<tr>
<td>Number of Rotations</td>
<td>3.25</td>
<td>3.375</td>
</tr>
</tbody>
</table>

Table 3.1.1-1 Design Parameters for Archimedean and Square Four-Arm Spiral Antennas

### 3.1.2 Spiral Antenna Fabrication

The fabrication of the antenna structures started with creating AutoCAD drawings of the square and Archimedean spirals defined by table (3.1.1-1). After the drawings were
created, each spiral was printed out in actual size on clear 8½ by 11 inch sheets of plastic
designed to be used with the Dalpro Benchtop Dry Film Etching System. The spirals
were etched on 0.010 inch thick microwave substrate with a $\varepsilon_r = 3.2$. The opposite
copper layer was also etched off to produce a printed-wire antenna without a ground
plane on the back of the dielectric. After the spirals were etched, they were immersed in a
tinning solution to protect the copper from corrosion.

The next step in fabricating the antenna was to create two enclosures to shield the
rearward radiation and provide support to the antennas. The two enclosures were
constructed with twenty gauge aluminum sheet to form a cavity that was 7.25 inches
square and 5 inches deep. One enclosure was fabricated with a flat face and the other
with a cylindrically-conformed face that had a 7.874 inch (20 cm) radius of curvature.
The top edge of each enclosure was formed with a lip to provide enough room to allow
holes to be drilled and tapped for number eight nylon screws. These nylon screws were
used to attach the antenna to the front of the enclosure. To provide attenuation to the
rearward traveling radiation from the antenna, which reduces interference between the
waves reflecting from the back wall of the enclosure, Emerson & Cuming AN-77
microwave absorber was used to line the back wall of the cavity. The AN-77 microwave
absorber provides reflectance attenuation of 20 dB or more at frequencies above 1.2 GHz.
The following figure shows a finished flat-faced enclosure with absorber lining.
The feed structure for the antenna was constructed with a bundle of four 0.141 inch copper coaxial lines with bulkhead SMA connectors on one end of each of the cables to allow the coaxial lines to be attached to the enclosure and provide a connection point for the antenna through the enclosure. The four coaxial lines were soldered together by two pieces of 0.050 inch thick microwave substrate with holes drilled in each to align the coaxial lines for each antenna. This substrate acted as a support structure for the bundle and also provided a grounding mechanism for the coaxial bundle as they extended away from the enclosure back wall. The coaxial lines were cut to length to align with the top of the enclosure and connect to the etched antenna. The outer conductor of each coaxial line was removed from the last 0.070 inches to provide a better impedance match between the 50 ohm coaxial lines and the antenna surface. The following figure shows the finished feed structure attached to the Archimedean spiral surface.
The following figure shows a finished Archimedean spiral mounted in an enclosure with a cylindrically-conformed face. The entire assembly is being held by a frame used to attach the antenna to the measurement system that will be described in a measured data chapter.
The final step in fabricating the antenna was constructing two ground plane apertures: one flat and one cylindrically-conformed with the purpose of holding the antenna enclosures during testing. The ground plane structures were constructed using 0.500 inch by 0.500 inch thick pine studs for the frame and 20 gauge aluminum sheet for the ground plane. To allow enough room for the cabling on the back of each antenna, each ground plane structure was 12 inches tall. The flat ground plane structure was 24 inches long by 24 inches wide and the curved structure was 24 inches long by 16 inches wide. The shorter width on the curved structure is due to the ground plane wrapping around the curvature of the structure. The following figures show the finished flat and cylindrically-conformed ground planes apertures with the antenna enclosures mounted within them.
Figure 3.1.2-4 Flat Ground Plane Aperture Mounted with Empty Antenna Enclosure

Figure 3.1.2-5 Cylindrically-Conformed Ground Plane Aperture Mounted with Empty Antenna Enclosure
Figure 3.1.2-6 Flat Ground Plane Aperture Mounted with the Square Spiral

Figure 3.1.2-7 Cylindrically-Conformed Ground Plane Aperture Mounted with the Archimedean Spiral
The following figure is a closer view of the square spiral mounted within the flat ground plane aperture to show the copper tape used to provide an electrical bond between the ground plane and the antenna enclosure.

![Figure 3.1.2-8 Closer View of Square Spiral Mounted in Flat Ground Plane Aperture](image)

3.2 Mode-Former Development

The following sections provide the details of the design and fabrication of the mode-formers developed for this thesis.

3.2.1 Mode-Former Design

The purpose of a mode-former for a four-arm antenna is to produce a set of four feed excitations of equal amplitude and phase progressions of 360 degrees for mode one, and 720 degrees for mode two. The design of the mode-former began with producing
four equal-amplitude excitations. A Wilkinson power divider was realized in microstrip lines with a corporate feed arrangement to provide the four excitations. The next step was to produce the phase progression needed for each mode. For simplicity and due to the fact that the motivation of this thesis was to produce an antenna and not the mode-former, a fixed line length design was utilized to provide an inexpensive, but narrow-banded solution to phasing the excitations. The phasing of the feed arms was calculated to provide proper phase progression at a frequency of 1.75 GHz. The wavelength can be used to determine the phase length by noting that a quarter wavelength is equal to 90 degrees, a half wavelength is equal to 180 degrees and so on. After the phasing of each arm was calculated, the additional line lengths were calculated for microstrip transmission lines. It should be noted that an ideal mode-former provides a phase progression that is constant across all frequencies. This mode-former design only provides the proper phasing at one frequency, where the phase error is proportional to the difference between the measurement frequency and the design frequency. The frequency was chosen due to the proximity of the PCS cellular frequency band as a possible detection signal for direction-finding.

3.2.2 Mode-Former Fabrication

The fabrication of the two mode-formers started with creating AutoCAD drawings of each. As before, the drawings were printed out in actual size on clear plastic for use with the etching system. The microwave substrate used for the mode-formers was a 0.050 inch thick RO3010 series high-frequency laminate from Rogers Corporation. After the mode-formers were etched, SMA end-launchers were attached to both sides of
the board to provide connection points between the measurement system and the mode-former and between the mode-former and the antenna enclosure. The following figures show the finished mode-formers.
As the figures show, the left side of each mode-former provides a connection point to the measurement system. This connection point is split into four arms where additional microstrip line has been added to each arm to provide the proper phase progression at 1.75 GHz. The four arms are numbered in ascending order starting from the top of each picture. The figures also depict the hand-tuning of the connection points on the right side of the mode-formers to ensure that phase angle of each arm was within +/-0.5 degrees of the design and to provide compensation for the four (not phase matched) cables that were used to connect the mode-former to the antenna. The measured results chapter provides the phase error versus frequency for both of the mode-formers.
4. THEORETICAL MODEL

4.1 Maxwell’s Equations

As a starting point to understanding the theoretical model, the magnetic vector potential will be derived from Maxwell’s equations to define one of the possible solution methods of Maxwell’s equations. We begin with Maxwell’s equations in the time-harmonic form within a homogeneous medium as defined by Balanis [10]

\[ \nabla \times \vec{E} = -j \omega \vec{B} \]  \hspace{1cm} (4.1-1)

\[ \nabla \times \vec{H} = \vec{J} + j \omega \vec{D} \]  \hspace{1cm} (4.1-2)

\[ \nabla \cdot \vec{D} = \rho \] \hspace{1cm} (4.1-3)

\[ \nabla \cdot \vec{B} = 0 \] \hspace{1cm} (4.1-4)

with the following constitutive parameters

\[ \vec{D} = \varepsilon \vec{E} \] \hspace{1cm} (4.1-5)

\[ \vec{B} = \mu \vec{H} \] \hspace{1cm} (4.1-6)

Based on the vector identity

\[ \nabla \cdot (\nabla \times \vec{\Omega}) = 0 \] \hspace{1cm} (4.1-7)

where \( \vec{\Omega} \) is an arbitrary vector, the magnetic field strength is defined as

\[ \vec{H} = \frac{1}{\mu} \nabla \times \vec{A} \] \hspace{1cm} (4.1-8)
Substituting (4.1-6) and (4.1-8) into (4.1-1) results in
\[ \nabla \times \vec{E} = -j \omega \nabla \times \vec{A} \]  
(4.1-9)
where bringing both terms to one side results in
\[ \nabla \times (\vec{E} + j \omega \vec{A}) = 0. \]  
(4.1-10)
Using the vector identity
\[ \nabla \times (-\nabla g) = 0 \]  
(4.1-11)
where \( g \) is a scalar function, (4.1-10) becomes
\[ \vec{E} = -j \omega \vec{A} - \nabla \phi \]  
(4.1-12)
where \( \phi \) is the electric scalar potential. Inserting (4.1-8) and (4.1-12) into (4.1-2) results in
\[ \nabla \times (\nabla \times \vec{A}) = \mu \vec{J} + j \omega \mu \varepsilon (\nabla \cdot \vec{A} - j \nabla \phi) \]  
(4.1-13)
and using the vector identity
\[ \nabla \times (\nabla \cdot \vec{A}) = \nabla (\nabla \cdot \vec{A}) - \nabla^2 \vec{A} \]  
(4.1-14)
reduces (4.1-13) to
\[ \nabla^2 \vec{A} + \omega^2 \mu \varepsilon \vec{A} = -\mu \vec{J} + \nabla (\nabla \cdot \vec{A} + j \omega \mu \phi). \]  
(4.1-15)
Using the Lorentz Gauge
\[ \nabla \cdot \vec{A} = -j \omega \mu \phi \]  
(4.1-16)
now \( \phi \) can be written as
\[ \phi = -\frac{1}{j \omega \mu} \nabla \cdot \vec{A} \]  
(4.1-17)
and (4.1-15) reduces to the inhomogeneous vector Helmholtz Equation for the magnetic vector potential.
\[ \nabla^2 \vec{A} + \omega^2 \varepsilon \mu \vec{A} = -\mu \vec{J} \]  

(4.1-18)

where \( \vec{E} \) can now be written as

\[ \vec{E} = -j \omega \vec{A} - j \frac{1}{\omega \varepsilon \mu} \nabla (\nabla \cdot \vec{A}) \].  

(4.1-19)

4.2 Numerical Electromagnetics Code Discussion

The theoretical model utilized within this thesis is the Numerical Electromagnetics Code (NEC) package developed by the Lawrence Livermore Laboratory. NEC utilizes the Method of Moments (MoM) solution method to solve for the current distribution on a wire antenna structure that has been segmented. The calculated current distribution can then be used to find the field pattern, impedance, and polarization of the antenna. The following sections provide the general theory necessary to understand the operation of NEC.

4.2.1 Integral Equations

The following section describes the formulation of the electric field integral equation from the inhomogeneous vector Helmholtz Equation that NEC uses to solve for the currents on each wire segment that comprises a wire antenna structure. We begin by simplifying the problem by assuming a source with a current density \( J_z \) is located at the origin of the coordinate system. Due to the infinitesimally small size of the source, it can be thought of as a delta function source as defined by Balanis [10], and Stutzman and Thiele [4]. We can convert (4.1-18) to
\[ \nabla^2 A_z + \beta^2 A_z = \mu J_z \]  
(4.2.1-1)

where the substitution for \( \beta \) is

\[ \omega^2 \varepsilon \mu = \beta^2. \]  
(4.2.1-2)

If the source is assumed to be a delta function, any point other than the origin of the coordinate system will be equal to zero. Assuming this delta function source allows the wave equation to be solved as a homogeneous equation for any point not at the origin.

\[ \nabla^2 A_z + \beta^2 A_z = 0 \]  
(4.2.1-3)

This is known as the scalar Helmholtz’s equation and has a solution in spherical coordinates of the form

\[ A_z = \frac{e^{-j\beta r}}{4\pi r} \]  
(4.2.1-4)

where \( r \) is the distance from the source point (at the origin) to the observation point. If we allow the source point to be at an arbitrary position other than the origin, the distance parameter \( R \) is defined as the distance between the source point and the observation points.

\[ A_z = \frac{e^{-j\beta R}}{4\pi R} \]  
(4.2.1-5)

As defined by the Green’s function method, if we assume a collection of source points, then the total contribution is equal to the integral of the source point responses due to (4.2.1-5) and weighted by the total distribution of the current density over a volume \( V \).

\[ A_z = \mu \iiint_{V} J_z \frac{e^{-j\beta r}}{4\pi R} dV \]  
(4.2.1-6)

If each of the other components are considered, the general solution for \( \tilde{A} \) is defined as
\[
\vec{A}(\vec{r}) = \mu \iiint_{V} \vec{J}(\vec{r}') e^{-jBR/4\pi R} dV
\]

where

\( \vec{r} \) is the vector from the origin to the observation point

\( \vec{r}' \) is the vector from the origin to the source point

\( R = |\vec{r} - \vec{r}'| \) is the distance from the source point to the observation point.

At this point, the free space Green’s Function can be defined as

\[
G(\vec{r}, \vec{r}') = \frac{e^{-jBR/4\pi R}}{4\pi R}
\]

reducing (4.2.1-7) down to

\[
\vec{A}(\vec{r}) = \mu \iiint_{V} \vec{J}(\vec{r}') G(\vec{r}, \vec{r}') dV.
\]

If we assume that the current density is confined to a wire of radius \( a \) that is perfectly electrically conducting (PEC), the current will only reside on the surface. The volume integral can then be converted into a surface integral that operates over the surface \( A \) of the wire.

\[
\vec{A}(\vec{r}) = \mu \iint_{A} \vec{J}_{s}(\vec{r}') G(\vec{r}, \vec{r}') dA
\]

If we define that the antenna surface is constructed with an arrangement of PEC wires, the total tangential electric field on the surface of the wire is equal to zero. If the problem is thought of in terms of electromagnetic scattering, the total tangential field can be equated to the summation of scattered and incident waves at the surface of the antenna

\[
\vec{E}^{s,T}_{\perp} = \vec{E}^{I}_{\perp} + \vec{E}^{S}_{\perp} = 0
\]

\[
\vec{E}^{I}_{\perp}(\vec{r} = \vec{r}_{s}) = -\vec{E}^{S}_{\perp}(\vec{r} = \vec{r}_{s})
\]
where $\mathbf{r}_s$ is the observation point that is now restricted to the surface of the wire. The incident field can be thought of inducing a current density. If reciprocity holds, a negative scattered field is then induced by the same current density. Substituting (4.2.1-10) into (4.1-19) and utilizing the reciprocity principle of (4.2.1-12), the electric field can be written as

$$E_i'(\mathbf{r} = \mathbf{r}_s) = \left[ j \omega \mu \int_A \int_s \mathbf{J}(\mathbf{r'}) G(\mathbf{r}_s, \mathbf{r'}) dA + j \frac{1}{\omega \varepsilon} \nabla \cdot \left[ \nabla G(\mathbf{r}_s, \mathbf{r'}) dA \right] \right]$$

where the subscript $t$ is the tangential component.

### 4.2.2 Thin Wire Approximation

The following sections provide a broad overview of the utilization of the electric field integral equation tailored for use with wire antennas as described in detail by Mittra [13]. The electric field integral equation of (4.2.1-13) can be simplified further by utilizing the thin-wire approximation. The first step as described by Caswell [12], is to bring the divergence and gradient within the integral after simplification,

$$E_i'(\mathbf{r} = \mathbf{r}_s) = \left[ j \omega \mu \int_A \int_s \mathbf{J}(\mathbf{r'}) G(\mathbf{r}_s, \mathbf{r'}) dA + j \frac{1}{\omega \varepsilon} \int_A \left( \nabla \cdot \mathbf{J}(\mathbf{r'}) \right) \nabla G(\mathbf{r}_s, \mathbf{r'}) dA \right]$$

where

$$\nabla' = \frac{\partial}{\partial x'} \mathbf{x}' + \frac{\partial}{\partial y'} \mathbf{y}' + \frac{\partial}{\partial z'} \mathbf{z}'$$

and $\nabla'$ operates only on the source coordinates. With the integral simplified, the thin-wire approximation can be applied. The thin-wire approximation is defined by the following assumptions:
- The current only flows in the direction of the wire axis
- The current can be represented by a filament on the wire axis
- The boundary condition on the electric field needs to be enforced in the axial direction only

The following figure illustrates the geometry of the thin wire approximation.

![Figure 4.2.2-1 Equivalent Source Conversion (from Stutzman and Thiele [4])](image)

The thin-wire approximation allows the current density on the surface $\vec{J}_s$, to be represented as an infinitely narrow current filament of length $L$ that has been divided by the circumference of a wire. This conversion is allowed due to the fact that the current remains azimuthally invariant around the circumference of the wire and is directed along the axis of the wire. As the figure shows, the observation points can either be on the axis, like (b) and (c), or on the radius of the wire.

$$\vec{J}_s(\vec{r}') = \frac{1}{2\pi a} I(s') [\hat{s}'(\vec{r}')]$$  \hspace{1cm} (4.2.2-2)
The following figure describes the local axes for the source and observation points.

![Figure 4.2.2-2 Local Axes Description](image)

Taking the tangential component of the electric field can now be defined as

\[
\hat{s}(\bar{r}) \cdot \hat{E} = \hat{s}(\bar{r}) \cdot \left[ j \omega \mu \int \tilde{J}(\bar{r}) G(\bar{r}, \bar{r'}) dA + j \frac{1}{\omega \epsilon} \int \left( \nabla' \cdot \tilde{J}(\bar{r'}) \right) \nabla G(\bar{r}, \bar{r'}) dA \right]. \quad (4.2.2-3)
\]

If the surface integral is separated into its components of one integral operating around the circumference of the wire and the other integral operating along the length of the wire, we can substitute the thin-wire approximation into the integrals resulting in
\[
\hat{s}(\vec{r}) \cdot \vec{E}' = j \omega \mu \int_\mathcal{L} I(s') (\hat{s}(\vec{r}) \cdot \hat{s}'(\vec{r}')) \left[ \frac{1}{2\pi a} \int_0^{2\pi} G(\vec{r}, \vec{r}') ad\psi' \right] ds'
\]

\[
+ j \frac{1}{\omega \varepsilon} \int_\mathcal{L} I(s') \frac{\partial^2}{\partial s \partial s'} \left[ \frac{1}{2\pi a} \int_0^{2\pi} G(\vec{r}, \vec{r}') ad\psi' \right] ds'
\]

(4.2.2-4)

where

\[
\hat{s}(\vec{r}) \cdot \nabla G(\vec{r}, \vec{r}') = \frac{\partial}{\partial s} G(\vec{r}, \vec{r}')
\]

(4.2.2-5)

\[
\nabla' = \hat{s}'(\vec{r}') \frac{\partial}{\partial s'}.
\]

(4.2.2-6)

If we evaluate the circumferential integral, all of the constants cancel to reveal the remaining Green’s function that is now confined to the wire segment instead of free space.

The new Green’s Function, which operates on the loop of radius \(a\) around the equivalent thin-wire current element

\[
G_\psi(\vec{r}, \vec{r}') = G(\vec{r}, \vec{r}')
\]

(4.2.2-7)

where \(R\) is now restricted to the wire segment

\[
R = \sqrt{(s-s')^2 + a^2}
\]

then (4.2.2-4) reduces to

\[
\hat{s}(\vec{r}) \cdot \vec{E}' = j \omega \mu \int_\mathcal{L} I(s') (\hat{s}(\vec{r}) \cdot \hat{s}'(\vec{r}')) G_\psi(\vec{r}, \vec{r}') ds' + j \frac{1}{\omega \varepsilon} \int_\mathcal{L} I(s') \frac{\partial^2}{\partial s \partial s'} G_\psi(\vec{r}, \vec{r}') ds'
\]

(4.2.2-8)

which defines the electric field integral equation used by NEC as described by Burke and Poggio [11].
4.2.3 Method of Moments Solution Method

The following section defines the general theory of the Method of Moments for solving operator equations. We begin by defining the linear operator equation

\[ Lf = e \]  \hspace{1cm} (4.2.3-1)

where \( L \) is a linear operator (which in the case of the electric field integral equation is an integral operator), \( f \) is the unknown response, and \( e \) is a known excitation. The unknown response can be expanded into a sum of constant coefficients multiplied by a basis function \( f_n \). The choice of basis function depends on the problem being solved. When the form of the solution is unknown, the piecewise constant (pulse) is the most straightforward. If the form of the solution is known beforehand, the basis function is chosen to match the response of the solution such as the triangular and sinusoidal basis functions. These particular functions are more capable of handling solutions that have a traveling wave form, such as antennas.

\[ f \approx \sum_{n=1}^{N} a_n f_n \]  \hspace{1cm} (4.2.3-2)

A set of equations for the unknown coefficient \( a_n \) is created by taking the inner product of (4.2.3-1) with a set of weighting functions

\[ \langle w_m, Lf \rangle = \langle w_m, e \rangle \quad ; \quad m = 1, \ldots, N \]  \hspace{1cm} (4.2.3-3)

where the inner product, which for this geometry, is not weighted, is defined as

\[ \langle u, v \rangle = \int_A u(\vec{r})v(\vec{r})dA \]  \hspace{1cm} (4.2.3-4)

Combining (4.2.3-3) and (4.2.3-2) produces

\[ \sum_{n=1}^{N} a_n \langle w_m, Lf_n \rangle = \langle w_m, e \rangle \quad ; \quad m = 1, \ldots, N \]  \hspace{1cm} (4.2.3-5)
which can be written in matrix form to produce Ohm’s Law

\[
[A][Z] = [V]
\]  \hspace{1cm} (4.2.3-6)

where \(A\) is a vector containing the unknown coefficients. \(Z\) is a matrix of the summed inner products of the integral operator and basis function with the weighting function and \(V\) is a vector of the inner products between the weighting function and the excitation. To calculate the values \(A\), the \(Z\) matrix is inverted

\[
[A] = [Z]^{-1}[V].
\]  \hspace{1cm} (4.2.3-7)

The weighting function, like the basis function, can be chosen for the problem type. The weighting and basis functions can be different depending on the problem geometry. The other possibility is if the weighting and basis functions are the same, where this is defined as Galerkin’s Method.

The following equations describe how NEC utilizes the Method of Moments to solve for the current of (4.2.2-8). Starting with an expansion for the current on each of the \(n\) segments of wire that defines the antenna surface

\[
I(s') \approx \sum_{n=1}^{N} a_n f_n(s')
\]  \hspace{1cm} (4.2.3-8)

and substituting into (4.2.2-8) results in

\[
\sum_{n=1}^{N} a_n f_n(s') (\hat{s}(\vec{r}) \cdot \hat{s}'(\vec{r}')) G_{\psi} (\vec{r}, \vec{r}') ds'
\]

\[
+ j \frac{1}{\omega \varepsilon} \sum_{n=1}^{N} a_n f_n(s') \frac{\partial^2}{\partial \vec{r} \cdot \partial \vec{r}'} G_{\psi} (\vec{r}, \vec{r}') ds'
\]  \hspace{1cm} (4.2.3-9)

The next step is to employ the weighting and basis functions. NEC uses

\[
w_m = \delta(s-s_m)
\]  \hspace{1cm} (4.2.3-10)

\[
f_n(s') = A_n + B_n \sin(\beta(s'-s_n)) + C_n \cos(\beta(s'-s_n))
\]  \hspace{1cm} (4.2.3-11)
where the weighting function effectively samples the integral at the midpoint of each source location $s_m$ of segment length $L_m$ and the basis function operates over the midpoint of each segment $s_n$. Substituting (4.2.3-10) into (4.2.3-9) results in the following equations that constitute the matrix elements and vectors of (4.2.3-7).

$$V_m = \int_{L_m} \left( \hat{s}(\bar{r}) \cdot \bar{E}' \right) \delta(s - s_m) ds \quad (4.2.3-12)$$

$$Z_{mn} = j \alpha \mu \int_{L_m} \delta(s - s_m) \left[ \sum_{n=1}^{N} a_n f_n(s') \hat{s}(\bar{r}) \cdot \hat{s}'(\bar{r}') G_{\nu}(\bar{r}, \bar{r}') ds' ds \right. + \left. j \frac{1}{\omega \varepsilon} \int_{L_m} \delta(s - s_m) \left[ \sum_{n=1}^{N} a_n f_n(s') \frac{\partial^2}{\partial s \partial s'} G_{\nu}(\bar{r}, \bar{r}') ds' ds \right] \quad (4.2.3-13)$$

Utilizing the definition of the integrated delta function, (4.2.3-12) and (4.2.3-13) reduce to

$$V_m = \hat{s}(\bar{r}(s_m)) \cdot \bar{E}'(s_m) \quad (4.2.3-14)$$

$$Z_{mn} = j \alpha \mu \int_{L_m} f_n(s') \hat{s}(\bar{r}(s_m)) \cdot \hat{s}'(\bar{r}') G_{\nu}(\bar{r}(s_m), \bar{r}') ds' \quad (4.2.3-15)$$

where

$$\sum_{n=1}^{N} Z_{mn} a_n = V_m \quad m = 1, \ldots, N \quad (4.2.3-16)$$

The $Z$ matrix will be diagonally dominant due to the source segment terms being the largest component of the observation segment terms. This dominance is due to the distance between each source and observation segment. The diagonal terms of the $Z$ matrix represent situations where the source segment is at the same location as the observation segment. As the observation segment moves away from the source segment, its location in the $Z$ matrix moves further away from the diagonal therefore decreasing its
contribution to the overall calculation of the current. The $V$ vector will be mostly sparse except for the segments where there is a voltage source, which in the case of a four-arm spiral antenna, will be four locations.

After the current for each segment has been found, the field pattern can be calculated from the following magnetic vector potential far-field relations for the electric field.

$$\vec{E}_A \approx -j\omega \vec{A} \quad (4.2.3-17)$$

The far-field of the electric field occurs when the distance from the current source becomes great enough that the difference between the elevation angle of a ray extending from the center of the antenna to the observation point and a ray extending from a point along the antenna to the observation point approaches zero. This principle defines a region where the angular field response is nearly independent of the distance to the antenna. If (4.2.3-17) is used to find the field of each current segment, all of the individual field contributions can be vector-summed to produce the field response of the entire wire antenna structure. The following figure illustrates the antenna coordinate geometry.
After the electric field is known, the power at each angle can be defined as

\[ P(\theta, \phi) = \frac{1}{2} \sqrt{\frac{\mu_0}{\varepsilon_0}} r^2 \left( \hat{E} \cdot \hat{E}^* \right) \]  (4.2.3-18)

and the power gain as

\[ G(\theta, \phi) = 4\pi \left[ \frac{P(\theta, \phi)}{P_{\text{in}}} \right] \]  (4.2.3-19)

where the \( P_{\text{in}} \) is calculated from the current and voltage at the source

\[ P_{\text{in}} = \text{Re}(V^*) \]  (4.2.3-20)

Now that the theory of operation for NEC has been explained, understanding how to create wire antenna models for NEC can be discussed. Due to the fact that NEC is a wire antenna simulation, creating the input files for NEC starts with converting the flat
strips of the etched antenna into a wire with a specific radius. As shown in Caswell [9], the widely accepted transformation between flat strips and wires is

\[ r_w = \frac{1}{4} W_s \]  

(4.3-1)

where \( r_w \) is the wire radius and \( W_s \) is the strip width. To create the wire model for each antenna design, MATLAB was used to calculate the length and location of each wire segment. The calculated wire segments were used to create an input file that was properly formatted to be read by NEC. As a general design rule, the length of each segment should be at least six times the radius of the wire to conform to the thin-wire approximation. Within NEC, each wire segment has an associated wire radius and a set of Cartesian coordinates \((x,y,z)\) that describe the starting and ending points of the wire. This information is used by NEC to calculate the relative orientation of each wire segment and to determine where the observation point due to wire radius is located on each wire segment. It should be noted that the wire segments must be orientated so that the starting point of the first segment is at the feed point of each antenna arm and the end point is directed away from the origin. Each successive segment should use the end point of the previous segment as the starting point, and the end point should follow the spiral arm as it progresses away from the origin.

The final step to creating an NEC input file is to designate the feed excitations and their relative phases. NEC allows the user to designate any single wire segment to provide a location for the delta-gap voltage source to exist on. The length of the wire segment provides the value of delta needed to produce an electric field from a voltage potential divided by a distance. Along with the ability to designate which segments
constitute the feed structure of the antenna, the phase angle and amplitude of each voltage source can be modified to allow for different modes of the spiral antenna to be simulated.

4.3 Numerical Electromagnetics Code Results

The following sections provide the results obtained from the NEC simulations that were performed using the 4NEC2X version of NEC. As a clarification, only the vertically polarized power patterns will be shown, where the spiral antenna is generally circularly polarized. This is to avoid confusion with the measured results chapter where the measurement system was limited to vertically polarized results due to the vertically polarized receive antenna.

The NEC results will be displayed in spherical coordinates \((\rho, \theta, \phi)\) where the angle \(\theta\) is measured from the z-axis. A \(\theta\) angle of zero degrees equates to the boresight of the antenna and a positive angle corresponds to a counter-clockwise rotation looking from an observation point along the positive y-axis. The following figure illustrates the local geometry used to display the power pattern plots of each antenna design.
4.3.1 Archimedean Spiral

The first spiral to examine is the flat Archimedean design. The following figure depicts the segmented spiral used by NEC.
Figure 4.3.1-1 NEC Geometry of Archimedean Spiral

The voltage sources of the spiral are designated by the four pink bands near the origin. Each voltage source is located on the second segment in from the beginning of each arm. The following figure depicts the mode one maximum vertically polarized gain of the spiral antenna versus frequency.
As the figure shows, the maximum gain of the antenna reaches a nearly steady value after 3 GHz. The following figures depict the power patterns of the spiral for mode one and mode two. It should be noted that the power patterns have been plotted in a logarithmic scale where the unit of dBi is the relative gain of the antenna compared to an isotropic radiator.
Figure 4.3.1-3 Mode 1 Archimedean Spiral Power Pattern (dBi) $\phi$ Cuts (1 GHz)

Figure 4.3.1-4 Mode 1 Archimedean Spiral Power Pattern (dBi) $\theta$ Cuts (1 GHz)
Figure 4.3.1-5 Mode 1 Archimedean Spiral Power Pattern (dBi) $\phi$ Cuts (1.75 GHz)

Figure 4.3.1-6 Mode 1 Archimedean Spiral Power Pattern (dBi) $\theta$ Cuts (1.75 GHz)
Figure 4.3.1-7 Mode 1 Archimedean Spiral Power Pattern (dBi) $\phi$ Cuts (2.5 GHz)

Figure 4.3.1-8 Mode 1 Archimedean Spiral Power Pattern (dBi) $\theta$ Cuts (2.5 GHz)
Figure 4.3.1-9 Mode 1 Archimedean Spiral Power Pattern (dBi) $\phi$ Cuts (5 GHz)

Figure 4.3.1-10 Mode 1 Archimedean Spiral Power Pattern (dBi) $\theta$ Cuts (5 GHz)
Figure 4.3.1-11 Mode 2 Archimedean Spiral Power Pattern (dBi) $\phi$ Cuts (1 GHz)

Figure 4.3.1-12 Mode 2 Archimedean Spiral Power Pattern (dBi) $\theta$ Cuts (1 GHz)
Figure 4.3.1-13 Mode 2 Archimedean Spiral Power Pattern (dBi) $\phi$ Cuts (1.75 GHz)

Figure 4.3.1-14 Mode 2 Archimedean Spiral Power Pattern (dBi) $\theta$ Cuts (1.75 GHz)
Figure 4.3.1-15 Mode 2 Archimedean Spiral Power Pattern (dBi) $\phi$ Cuts (2.5 GHz)

Figure 4.3.1-16 Mode 2 Archimedean Spiral Power Pattern (dBi) $\theta$ Cuts (2.5 GHz)
Figure 4.3.1-17 Mode 2 Archimedean Spiral Power Pattern (dBi) $\phi$ Cuts (5 GHz)

Figure 4.3.1-18 Mode 2 Archimedean Spiral Power Pattern (dBi) $\theta$ Cuts (5 GHz)
The previous figures illustrate that the mode one beamwidth decreases as the frequency increases. There is also a slight “squaring” of the mode one $\theta$ cuts which is most likely due to the antenna becoming electrically large enough to support the beginning of a mode three power pattern. The figures also depict that the power pattern for mode two does not fully develop until after 2.5 GHz where the $\theta$ cuts become nearly concentric. At 5 GHz, the mode two power pattern is nearly perfect, agreeing with the work of Penno and Pasala. As a final point, the effects of scaling that were discussed in chapter two can now by seen as the rotation of the power pattern around the $z$-axis as the frequency is increased.

4.3.2 Cylindrically-Conformed Archimedean Spiral

The following section characterizes the results of the cylindrically-conformed Archimedean spiral. The following figure depicts the segmented spiral used by NEC.

Figure 4.3.2-1 NEC Geometry of Cylindrically-Conformed Archimedean Spiral
The following figure depicts the mode one maximum vertically polarized gain of the antenna versus frequency.

![Graph showing mode one maximum vertically polarized gain versus frequency](image)

Figure 4.3.2-2 Mode 1 Cylindrically-Conformed Archimedean Spiral Maximum Vertically Polarized Gain versus Frequency

As with the flat case, the maximum gain versus frequency approaches a nearly steady solution after 3 GHz. The following figures depict the power patterns of the spiral for mode one and mode two.
Figure 4.3.2-3 Mode 1 Cylindrically-Conformed Archimedean Spiral Power Pattern (dBi) $\phi$ Cuts (1 GHz)

Figure 4.3.2-4 Mode 1 Cylindrically-Conformed Archimedean Spiral Power Pattern (dBi) $\theta$ Cuts (1 GHz)
Figure 4.3.2-5 Mode 1 Cylindrically-Conformed Archimedean Spiral Power Pattern (dBi) $\phi$ Cuts (1.75 GHz)

Figure 4.3.2-6 Mode 1 Cylindrically-Conformed Archimedean Spiral Power Pattern (dBi) $\theta$ Cuts (1.75 GHz)
Figure 4.3.2-7 Mode 1 Cylindrically-Conformed Archimedean Spiral Power Pattern (dBi) $\phi$ Cuts (2.5 GHz)

Figure 4.3.2-8 Mode 1 Cylindrically-Conformed Archimedean Spiral Power Pattern (dBi) $\theta$ Cuts (2.5 GHz)
Figure 4.3.2-9 Mode 1 Cylindrically-Conformed Archimedean Spiral Power Pattern (dBi) $\phi$ Cuts (5 GHz)

Figure 4.3.2-10 Mode 1 Cylindrically-Conformed Archimedean Spiral Power Pattern (dBi) $\theta$ Cuts (5 GHz)
Figure 4.3.2-11 Mode 2 Cylindrically-Conformed Archimedean Spiral Power Pattern (dBi) $\phi$ Cuts (1 GHz)

Figure 4.3.2-12 Mode 2 Cylindrically-Conformed Archimedean Spiral Power Pattern (dBi) $\theta$ Cuts (1 GHz)
Figure 4.3.2-13 Mode 2 Cylindrically-Conformed Archimedean Spiral Power Pattern (dBi) $\phi$ Cuts (1.75 GHz)

Figure 4.3.2-14 Mode 2 Cylindrically-Conformed Archimedean Spiral Power Pattern (dBi) $\theta$ Cuts (1.75 GHz)
Figure 4.3.2-15 Mode 2 Cylindrically-Conformed Archimedean Spiral Power Pattern (dBi) $\phi$ Cuts (2.5 GHz)

Figure 4.3.2-16 Mode 2 Cylindrically-Conformed Archimedean Spiral Power Pattern (dBi) $\theta$ Cuts (2.5 GHz)
Figure 4.3.2-17 Mode 2 Cylindrically-Conformed Archimedean Spiral Power Pattern (dBi) $\phi$ Cuts (5 GHz)

Figure 4.3.2-18 Mode 2 Cylindrically-Conformed Archimedean Spiral Power Pattern (dBi) $\theta$ Cuts (5 GHz)
As with the flat Archimedean spiral, the previous figures illustrate that the mode one beamwidth decreases as the frequency increases. The difference between the flat and conformed case appears to be the warping of the $\theta$ cuts to produce an ellipse instead of a circle. Also, the circularly polarized antenna in the flat case has changed to an elliptically polarized antenna seen by the $\phi$ cuts not connecting at boresight. Mode two appears to also suffer from conforming the antenna where the power pattern has warped slightly.

4.3.3 Square Spiral

The following section characterizes the results of the square spiral. The following figure depicts the segmented spiral used by NEC.

Figure 4.3.3-1 NEC Geometry of Square Spiral

The following figure depicts the mode one maximum vertically polarized gain of the antenna versus frequency.
Differing from the Archimedean spiral, the maximum gain versus frequency begins to oscillate after 3 GHz. The following figures depict the power patterns of the spiral for mode one and mode two.
Figure 4.3.3-3 Mode 1 Square Spiral Power Pattern (dBi) $\phi$ Cuts (1 GHz)

Figure 4.3.3-4 Mode 1 Square Spiral Power Pattern (dBi) $\theta$ Cuts (1 GHz)
Figure 4.3.3-5 Mode 1 Square Spiral Power Pattern (dBi) $\phi$ Cuts (1.75 GHz)

Figure 4.3.3-6 Mode 1 Square Spiral Power Pattern (dBi) $\theta$ Cuts (1.75 GHz)
Figure 4.3.3-7 Mode 1 Square Spiral Power Pattern (dBi) $\phi$ Cuts (2.5 GHz)

Figure 4.3.3-8 Mode 1 Square Spiral Power Pattern (dBi) $\theta$ Cuts (2.5 GHz)
Figure 4.3.3-9 Mode 1 Square Spiral Power Pattern (dBi) $\phi$ Cuts (5 GHz)

Figure 4.3.3-10 Mode 1 Square Spiral Power Pattern (dBi) $\theta$ Cuts (5 GHz)
Figure 4.3.3-11 Mode 2 Square Spiral Power Pattern (dBi) $\phi$ Cuts (1 GHz)

Figure 4.3.3-12 Mode 2 Square Spiral Power Pattern (dBi) $\theta$ Cuts (1 GHz)
Figure 4.3.3-13 Mode 2 Square Spiral Power Pattern (dBi) $\phi$ Cuts (1.75 GHz)

Figure 4.3.3-14 Mode 2 Square Spiral Power Pattern (dBi) $\theta$ Cuts (1.75 GHz)
Figure 4.3.3-15 Mode 2 Square Spiral Power Pattern (dBi) $\phi$ Cuts (2.5 GHz)

Figure 4.3.3-16 Mode 2 Square Spiral Power Pattern (dBi) $\theta$ Cuts (2.5 GHz)
Figure 4.3.3-17 Mode 2 Square Spiral Power Pattern (dBi) $\phi$ Cuts (5 GHz)

Figure 4.3.3-18 Mode 2 Square Spiral Power Pattern (dBi) $\theta$ Cuts (5 GHz)
The previous figures illustrate that the mode one power pattern of the square spiral agrees with the Archimedean spiral up to 5 GHz where the power pattern begins to look like the head of a Phillips screwdriver. Mode two power patterns of the square spiral also agree with the results of the Archimedean spiral where at 5 GHz, the square spiral is nearly concentric.

### 4.3.4 Cylindrically-Conformed Square Spiral

The following section characterizes the results of the cylindrically-conformed square spiral. The following figure depicts the segmented spiral used by NEC.

![Figure 4.3.4-1 NEC Geometry of Cylindrically-Conformed Square Spiral](image)

The following figure depicts the mode one maximum vertically polarized gain of the antenna versus frequency.
The frequency response of the conformed square spiral agrees with the flat where the same oscillation of gain is seen after 3 GHz. The following figures depict the power patterns of the spiral for mode one and mode two.
Figure 4.3.4-3 Mode 1 Cylindrically-Conformed Square Spiral Power Pattern (dBi) $\phi$ Cuts (1 GHz)

Figure 4.3.4-4 Mode 1 Cylindrically-Conformed Square Spiral Power Pattern (dBi) $\theta$ Cuts (1 GHz)
Figure 4.3.4-5 Mode 1 Cylindrically-Conformed Square Spiral Power Pattern (dBi) $\phi$ Cuts (1.75 GHz)

Figure 4.3.4-6 Mode 1 Cylindrically-Conformed Square Spiral Power Pattern (dBi) $\theta$ Cuts (1.75 GHz)
Figure 4.3.4-7 Mode 1 Cylindrically-Conformed Square Spiral Power Pattern (dBi) $\phi$ Cuts (2.5 GHz)

Figure 4.3.4-8 Mode 1 Cylindrically-Conformed Square Spiral Power Pattern (dBi) $\theta$ Cuts (2.5 GHz)
Figure 4.3.4-9 Mode 1 Cylindrically-Conformed Square Spiral Power Pattern (dBi) $\phi$ Cuts (5 GHz)

Figure 4.3.4-10 Mode 1 Cylindrically-Conformed Square Spiral Power Pattern (dBi) $\theta$ Cuts (5 GHz)
Figure 4.3.4-11 Mode 2 Cylindrically-Conformed Square Spiral Power Pattern (dBi) $\phi$ Cuts (1 GHz)

Figure 4.3.4-12 Mode 2 Cylindrically-Conformed Square Spiral Power Pattern (dBi) $\theta$ Cuts (1 GHz)
Figure 4.3.4-13 Mode 2 Cylindrically-Conformed Square Spiral Power Pattern (dBi) $\phi$ Cuts (1.75 GHz)

Figure 4.3.4-14 Mode 2 Cylindrically-Conformed Square Spiral Power Pattern (dBi) $\theta$ Cuts (1.75 GHz)
Figure 4.3.4-15 Mode 2 Cylindrically-Conformed Square Spiral Power Pattern (dBi) $\phi$ Cuts (2.5 GHz)

Figure 4.3.4-16 Mode 2 Cylindrically-Conformed Square Spiral Power Pattern (dBi) $\theta$ Cuts (2.5 GHz)
Figure 4.3.4-17 Mode 2 Cylindrically-Conformed Square Spiral Power Pattern (dBi) $\phi$ Cuts (5 GHz)

Figure 4.3.4-18 Mode 2 Cylindrically-Conformed Square Spiral Power Pattern (dBi) $\theta$ Cuts (5 GHz)
The previous figures illustrate that the mode one power patterns of the conformed square spiral agrees with the conformed Archimedean spiral results. In both designs, the pattern is warped creating a slight elliptical polarization. Mode two power patterns of the conformed square spiral also agree with the results of the conformed Archimedean spiral, where the $\theta$ cuts of the pattern settle into a nearly concentric circle even though the surface has been conformed.

### 4.3.5 Radius of Curvature Analysis of the Archimedean Spiral

The following section compares the results of conforming the Archimedean spiral at different radiiuses of curvature. The purpose of this analysis is to determine the maximum radius of curvature that still permits the use of the antenna for direction-finding. The following figures provide the mode one and mode two power patterns of an Archimedean spiral antenna that has been conformed at the following radiiuses of curvature: infinity (flat), 80 cm (31.5 inches), 40 cm (16 inches), 20 cm (7.9 inches), 14 cm (5.5 inches), and 6 cm (2.4 inches). These radiiuses of curvature correspond to air platforms of different sizes ranging from the 707 to a Predator down to one of the smallest UAVs used by the Army, the Raven.
Figure 4.3.5-1 Radius of Curvature Analysis Mode 1 Cylindrically-Conformed Archimedean Spiral Power Pattern (dBi) $\phi$ Cuts (1.75 GHz)

Figure 4.3.5-2 Radius of Curvature Analysis Mode 1 Cylindrically-Conformed Archimedean Spiral Power Pattern (dBi) $\theta$ Cuts (1.75 GHz)
Figure 4.3.5-3 Radius of Curvature Analysis Mode 1 Cylindrically-Conformed Archimedean Spiral Power Pattern (dBi) $\phi$ Cuts (5 GHz)

Figure 4.3.5-4 Radius of Curvature Analysis Mode 1 Cylindrically-Conformed Archimedean Spiral Power Pattern (dBi) $\theta$ Cuts (5 GHz)
Figure 4.3.5-5 Radius of Curvature Analysis Mode 2 Cylindrically-Conformed Archimedean Spiral Power Pattern (dBi) $\phi$ Cuts (1.75 GHz)

Figure 4.3.5-6 Radius of Curvature Analysis Mode 2 Cylindrically-Conformed Archimedean Spiral Power Pattern (dBi) $\theta$ Cuts (1.75 GHz)
Figure 4.3.5-7 Radius of Curvature Analysis Mode 2 Cylindrically-Conformed Archimedean Spiral Power Pattern (dBi) $\phi$ Cuts (5 GHz)

Figure 4.3.5-8 Radius of Curvature Analysis Mode 2 Cylindrically-Conformed Archimedean Spiral Power Pattern (dBi) $\theta$ Cuts (5 GHz)
The previous figures illustrate the near invariance of the power pattern to the radius of curvature. Even when the antenna is conformed at a radius of curvature of 14 cm, the pattern shape is almost indistinguishable from the flat case. Only when the pattern is conformed to a radius of curvature equal to 6 cm does the power pattern shape begin to change. Even with this slight change in antenna power pattern shape, if the antenna pattern is known, the direction-finding calculations could be compensated to make up for these distortions.
5. MEASURED RESULTS

5.1 Measurement Method

The following sections describe the components of the antenna measurement system that was used to gather the measured results of this chapter. The following figure provides an overview of the components comprising the antenna measurement system.

Figure 5.1-1 Antenna Measurement System Component Layout
5.1.1 Vector Network Analyzer Discussion

The Vector Network Analyzer (VNA) used to provide the measured results of this thesis was the two-port Anritsu 37347C which has an operating frequency range from 40MHz to 20 GHz. The two-port VNA provides simultaneous transmission and reflection measurements at and between each of the ports. The transmission and reflection of the device under test is found by measuring of the ratio of the transmitted and received voltages at each of the ports. The transmission and reflection terms are categorized as scattering parameters, or s-parameters. Each of the four possible measurements are given by the following s-parameters:

- \( S_{11} \) is the ratio of the received voltage at port one to the transmitted voltage from port one
- \( S_{22} \) is the ratio of the received voltage at port two to the transmitted voltage from port two
- \( S_{21} \) is the ratio of the received voltage at port two to the transmitted voltage from port one
- \( S_{12} \) is the ratio of the received voltage at port one to the transmitted voltage from port two

Each of the s-parameters can be used to provide information about the frequency response of a device under test such as the insertion loss (IL), voltage standing wave ratio (VSWR), and group delay. For the antenna measurement system, the VNA is used to measure the transmission, or \( S_{21} \), across a range of frequencies between the two antennas.
5.1.2 Reference Antenna Discussion

The reference antenna used by the antenna measurement system was the Antenna Design & Manufacturing Corporation DHR-118/A 1-18 GHz Double Ridged Horn. Within the antenna measurement system, the reference antenna acts as both the receive antenna during a measurement and is also the calibrated antenna used to determine the final gain of the antenna being measured. The following figures provide the maximum gain versus frequency and antenna power pattern for the reference antenna.

Figure 5.1.2-1 Reference Antenna Maximum Vertically Polarized Gain versus Frequency
5.1.3 Desktop Antenna Measurement System Discussion

The antenna measurement system utilized for measured results of this thesis was the Diamond Engineering Desktop Antenna Measurement System (DAMS) that operates from 0 to 18 GHz. The DAMS system is comprised of a software application that controls the VNA and the rotation platform controller. Along with the software, the system provides a tripod to hold the receive antenna and a second tripod to hold the rotation platform. The rotation platform is designed to hold the antenna under test while it is rotated across a set of azimuth angles and/or elevation angles.

The measurement of an antenna under test starts with mounting two identical reference antennas to the tripods supplied by the DAMS system. The reference antennas are then aligned so that the boresights of each antenna are pointing directly at each other.
After alignment, the reference antennas are then connected to the VNA where the port two reference antenna acts as the receive antenna and the port one reference antenna acts as the transmit antenna. Once both of the antennas have been connected to the VNA, a frequency range is designated and the VNA measures the transmission between the two antennas. This reference antenna transmission measurement is used to calculate the isotropic antenna transmission between the two antennas by subtracting the known gain with respect to frequency of one of the reference antennas from the transmission measurement. After the reference antenna transmission has been taken, the distance between the two antennas is recorded and the reference antenna that was mounted to the rotation platform is removed.

The antenna under test is then mounted to the rotation platform and is situated to be at the same distance from the receive antenna as the transmit antenna was. This allows the free space path loss differences between the reference antenna transmission and the antenna under test transmission to be ignored. After both antennas have been mounted and aligned again, the DAMS software is used to control the rotation platform and VNA to record the angular and frequency dependent transmission results of the antenna under test. As the DAMS software moves the rotation platform around one azimuth or elevation step at a time, the VNA measures the transmission of the two antennas across a frequency range. Each angle is then recorded by the DAMS software and stored to memory. After all of the angles and frequencies of interest have been measured, the data is exported into a text file that can be read by MATLAB or any spreadsheet software. After the data has been exported, the transmission measurements of the antenna under test is compared to
the reference antenna transmission results to calculate the relative gain of the antenna under test to the known gain of the reference antenna.

5.2 Mode-Former Results

The following section provides the results of the mode-formers utilized to create the different power patterns of the antennas. The following figure depicts the transmission loss of each arm of the mode one mode-former with respect to frequency.

![Figure 5.2-1 Mode 1 Mode-Former Transmission Loss versus Frequency](image)

An ideal mode-former would provide only 6 dB of loss, but due to the utilization of the Wilkinson power divider, some of the transmission is lost due to the bridging resistors. The following figure depicts the maximum variation in transmission between each arm of the mode one mode-former.
The figure shows that up to 2 GHz, the transmission balance between all of the arms of the mode-former is within 0.3 dB. The following figure shows the relative phase difference between each arm of the mode one mode-former.
The following figures depict the relative phase error with respect to frequency of the mode one mode-former compared to an ideal mode one mode-former.
The following figures depict the transmission loss and maximum variation in transmission loss between of each arm of the mode two mode-former with respect to frequency.
The following figure shows the relative phase difference between each arm of the mode two mode-former.
Figure 5.2-8 Mode 2 Mode-Former Arm Phase versus Frequency

The following figures provide the relative phase error with respect to frequency of the mode-former relative to an ideal mode one mode-former.

Figure 5.2-9 Mode 2 Mode-Former Arm Phase Error versus Frequency
The results show that the mode-formers perform well around 1.75 GHz. The phase error begins to degrade the quality of the mode-former after a frequency of 50 MHz on either side of 1.75 GHz. As a low-cost solution, several of these mode-formers could be fabricated to operate at different frequencies to provide a modular solution to the frequency coverage of spiral antennas.

**5.2.1 Power Pattern Phase Balance Comparison**

The following section compares the results of the NEC model that has been feed with the actual phase errors of each mode-former to the measured results of the Archimedean spiral antenna. Only $\phi$ cuts were measured on antennas that were not mounted within a ground plane aperture. The following figures depict measured and simulated $\phi$ cuts of each mode-former across a set of frequencies.
Figure 5.2.1-1 Measured and Simulated Mode 1 Archimedean Spiral Power Pattern (dBi) $\phi$ Cuts (1.25 GHz) (NEC is Phase and Amplitude Compensated)

Figure 5.2.1-2 Measured and Simulated Mode 1 Archimedean Spiral Power Pattern (dBi) $\phi$ Cuts (1.5 GHz) (NEC is Phase and Amplitude Compensated)
Figure 5.2.1-3 Measured and Simulated Mode 1 Archimedean Spiral Power Pattern (dBi) φ Cuts (1.75 GHz) (NEC is Phase and Amplitude Compensated)

Figure 5.2.1-4 Measured and Simulated Mode 1 Archimedean Spiral Power Pattern (dBi) φ Cuts (2 GHz) (NEC is Phase and Amplitude Compensated)
Figure 5.2.1-5 Measured and Simulated Mode 1 Archimedean Spiral Power Pattern (dBi) $\phi$ Cuts (2.25 GHz) (NEC is Phase and Amplitude Compensated)

Figure 5.2.1-6 Measured and Simulated Mode 2 Archimedean Spiral Power Pattern (dBi) $\phi$ Cuts (1.25 GHz) (NEC is Phase and Amplitude Compensated)
Figure 5.2.1-7 Measured and Simulated Mode 2 Archimedean Spiral Power Pattern (dBi) $\phi$ Cuts (1.5 GHz) (NEC is Phase and Amplitude Compensated)

Figure 5.2.1-8 Measured and Simulated Mode 2 Archimedean Spiral Power Pattern (dBi) $\phi$ Cuts (1.75 GHz) (NEC is Phase and Amplitude Compensated)
Figure 5.2.1-9 Measured and Simulated Mode 2 Archimedean Spiral Power Pattern (dBi) \( \phi \) Cuts (2 GHz)

(NEC is Phase and Amplitude Compensated)

Figure 5.2.1-10 Measured and Simulated Mode 2 Archimedean Spiral Power Pattern (dBi) \( \phi \) Cuts (2.25 GHz) (NEC is Phase and Amplitude Compensated)
The previous figures illustrate the increased elliptical polarization that occurs when the mode-former is not providing the ideal phase progression to the antenna. The mode one simulated results loosely agree with the measured results for frequencies at and below 1.75 GHz. Mode two begins with a stronger power pattern than what NEC predicts but settles into a closer agreement by 1.75 GHz.

5.3 Spiral Antenna Results

The following sections describe the results of each antenna that was fabricated for this thesis. All of the measured results have been plotted with the results of the NEC simulation (with perfect phase) to provide a comparison between the measured and theoretical results. It should be noted that the NEC results are for an antenna in free space as depicted in the theoretical section. The best situation for comparing the theoretical results to the measured results would be to construct a model that includes strips printed on a dielectric sheet instead of wires, an enclosure lined with microwave absorber instead of free space, and the ability to create a ground plane aperture. The availability of the electromagnetics software due to cost or licensing rules prevented these characteristics from being modeled and therefore is not a part of the comparison. It should also be noted that only $\phi$ cuts were measured for antennas not mounted in ground plane apertures.
5.3.1 Archimedean Spiral

The following section compares the measured results of the Archimedean spiral to the NEC simulated results. The following figure depicts the Archimedean spiral mounted on the DAMS platform.

![Figure 5.3.1-1 Archimedean Spiral Mounted in Test Fixture](image)

The following figure compares the mode one maximum vertically polarized gain of the measured antenna and the NEC simulated antenna versus frequency.
As the figure shows, the maximum gain of the antenna oscillates about the theoretical curve up to 2 GHz and then falls off afterward. This is most likely due to the Standing Wave Ratio (SWR) of the input impedance oscillating around 50 ohms and the depth of the antenna enclosure favoring a lower frequency response. The following figures depict the power patterns of the spiral for mode one and mode two.
Figure 5.3.1-3 Measured and Simulated Mode 1 Archimedean Spiral Power Pattern (dBi) $\phi$ Cuts (1 GHz)

Figure 5.3.1-4 Measured and Simulated Mode 1 Archimedean Spiral Power Pattern (dBi) $\phi$ Cuts (1.75 GHz)
Figure 5.3.1-5 Measured and Simulated Mode 1 Archimedean Spiral Power Pattern (dBi) $\phi$ Cuts (2.5 GHz)

Figure 5.3.1-6 Measured and Simulated Mode 2 Archimedean Spiral Power Pattern (dBi) $\phi$ Cuts (1 GHz)
Figure 5.3.1-7 Measured and Simulated Mode 2 Archimedean Spiral Power Pattern (dBi) φ Cuts (1.75 GHz)

Figure 5.3.1-8 Measured and Simulated Mode 2 Archimedean Spiral Power Pattern (dBi) φ Cuts (2.5 GHz)
The previous figures illustrate the increased elliptical polarization at lower frequencies for the mode one power pattern. Not surprisingly, the strongest agreement between measured and theoretical results occurred at 1.75 GHz.

### 5.3.2 Cylindrically-Conformed Archimedean Spiral

The following section compares the measured results of the cylindrically-conformed Archimedean spiral to the NEC simulated results. The following figure depicts the cylindrically-conformed Archimedean spiral mounted on the DAMS platform.

![Archimedean Spiral Mounted in Test Fixture](image)

**Figure 5.3.2-1 Archimedean Spiral Mounted in Test Fixture**

The following figure compares the mode one maximum vertically polarized gain of the measured antenna and the NEC simulated antenna versus frequency.
Like the flat geometry, the maximum gain of the antenna oscillates about the theoretical curve up to 2 GHz and then falls off afterward. The following figures depict the power patterns of the spiral for mode one and mode two.
Figure 5.3.2-3 Measured and Simulated Mode 1 Cylindrically-Conformed Archimedean Spiral Power Pattern (dBi) $\phi$ Cuts (1 GHz)

Figure 5.3.2-4 Measured and Simulated Mode 1 Cylindrically-Conformed Archimedean Spiral Power Pattern (dBi) $\phi$ Cuts (1.75 GHz)
Figure 5.3.2-5 Measured and Simulated Mode 1 Cylindrically-Conformed Archimedean Spiral Power Pattern (dBi) $\phi$ Cuts (2.5 GHz)

Figure 5.3.2-6 Measured and Simulated Mode 2 Cylindrically-Conformed Archimedean Spiral Power Pattern (dBi) $\phi$ Cuts (1 GHz)
Figure 5.3.2-7 Measured and Simulated Mode 2 Cylindrically-Conformed Archimedean Spiral Power Pattern (dBi) $\phi$ Cuts (1.75 GHz)

Figure 5.3.2-8 Measured and Simulated Mode 2 Cylindrically-Conformed Archimedean Spiral Power Pattern (dBi) $\phi$ Cuts (2.5 GHz)
The previous figures again illustrate the strong agreement between measured and theoretical results at 1.75 GHz. It is also seen that, like the theoretical model predicted, the power patterns between the flat and conformed cases have not been altered.

5.3.3 Square Spiral

The following section compares the measured results of the square spiral to the NEC simulated results. The following figure depicts the square spiral mounted on the DAMS platform.

![Square Spiral Mounted in Test Fixture](image)

Figure 5.3.3-1 Square Spiral Mounted in Test Fixture

The following figure compares the mode one maximum vertically polarized gain of the measured antenna and the NEC simulated antenna versus frequency.
Like the previous antenna geometries, the maximum gain of the antenna oscillates about the theoretical curve up to 2 GHz and then falls off afterward. The following figures depict the power patterns of the spiral for mode one and mode two.
Figure 5.3.3-3 Measured and Simulated Mode 1 Square Spiral Power Pattern (dBi) $\phi$ Cuts (1 GHz)

Figure 5.3.3-4 Measured and Simulated Mode 1 Square Spiral Power Pattern (dBi) $\phi$ Cuts (1.75 GHz)
Figure 5.3.3-5 Measured and Simulated Mode 1 Square Spiral Power Pattern (dBi) $\phi$ Cuts (2.5 GHz)

Figure 5.3.3-6 Measured and Simulated Mode 2 Square Spiral Power Pattern (dBi) $\phi$ Cuts (1 GHz)
Figure 5.3.3-7 Measured and Simulated Mode 2 Square Spiral Power Pattern (dBi) $\phi$ Cuts (1.75 GHz)

Figure 5.3.3-8 Measured and Simulated Mode 2 Square Spiral Power Pattern (dBi) $\phi$ Cuts (2.5 GHz)
The previous figures again illustrate the strong agreement between measured and theoretical results at 1.75 GHz, where the square spiral has surprisingly provided the most concentric mode one power pattern of all of the antenna geometries.

### 5.3.4 Cylindrically-Conformed Square Spiral

The following section compares the measured results of the cylindrically-conformed square spiral to the NEC simulated results. The following figure depicts the cylindrically-conformed square spiral mounted on the DAMS platform.

![Figure 5.3.4-1 Cylindrically-Conformed Square Spiral Mounted in Test Fixture](image)

The following figure compares the mode one maximum vertically polarized gain of the measured antenna and the NEC simulated antenna versus frequency.
The shape of the maximum gain curve of the antenna with respect to frequency remains unchanged compared to the other measured antenna results which show that, for at least lower frequencies, the square and Archimedean spiral generally perform equally well. This also happens to be the conclusion found within the theoretical results. The following figures depict the power patterns of the spiral for mode one and mode two.
Figure 5.3.4-3 Measured and Simulated Mode 1 Cylindrically-Conformed Square Spiral Power Pattern
(dBi) $\phi$ Cuts (1 GHz)

Figure 5.3.4-4 Measured and Simulated Mode 1 Cylindrically-Conformed Square Spiral Power Pattern
(dBi) $\phi$ Cuts (1.75 GHz)
Figure 5.3.4-5 Measured and Simulated Mode 1 Cylindrically-Conformed Square Spiral Power Pattern (dBi) \( \phi \) Cuts (2.5 GHz)

Figure 5.3.4-6 Measured and Simulated Mode 2 Cylindrically-Conformed Square Spiral Power Pattern (dBi) \( \phi \) Cuts (1 GHz)
Figure 5.3.4-7 Measured and Simulated Mode 2 Cylindrically-Conformed Square Spiral Power Pattern (dBi) $\phi$ Cuts (1.75 GHz)

Figure 5.3.4-8 Measured and Simulated Mode 2 Cylindrically-Conformed Square Spiral Power Pattern (dBi) $\phi$ Cuts (2.5 GHz)
The previous figures again illustrate the strong agreement between measured and theoretical results at 1.75 GHz, although the conforming of the antenna has created a slightly elliptical polarization on the mode one pattern and the maximum gain at boresight is reduced.

5.3.5 Square Spiral within Ground Plane Aperture

The following section compares the measured results of the square spiral mounted within a ground plane aperture to the NEC simulated results. The following figure depicts the square spiral mounted within the ground plane aperture.

The following figure compares the mode one maximum vertically polarized gain of the measured antenna and the NEC simulated antenna versus frequency.
Figure 5.3.5-2 Measured and Simulated Mode 1 Square Spiral Maximum Vertically Polarized Gain versus Frequency

The shape of the maximum gain curve has smoothed out during the ground plane aperture case which suggests that the enclosure is affecting the maximum gain power patterns of the previous antenna geometries. The following figures depict the power patterns of the spiral for mode one and mode two.
Figure 5.35-3 Measured and Simulated Mode 1 Square Spiral within Ground Plane Aperture Power Pattern (dBi) $\phi$ Cuts (1 GHz)

Figure 5.35-4 Measured and Simulated Mode 1 Square Spiral within Ground Plane Aperture Power Pattern (dBi) $\theta$ Cuts (1 GHz)
Figure 5.3.5-5 Measured and Simulated Mode 1 Square Spiral within Ground Plane Aperture Power Pattern (dBi) $\phi$ Cuts (1.75 GHz)

Figure 5.3.5-6 Measured and Simulated Mode 1 Square Spiral within Ground Plane Aperture Power Pattern (dBi) $\theta$ Cuts (1.75 GHz)
Figure 5.3.5-7 Measured and Simulated Mode 1 Square Spiral within Ground Plane Aperture Power Pattern
(dBi) \( \phi \) Cuts (2.5 GHz)

Figure 5.3.5-8 Measured and Simulated Mode 1 Square Spiral within Ground Plane Aperture Power Pattern
(dBi) \( \theta \) Cuts (2.5 GHz)
Figure 5.3.5-9 Measured and Simulated Mode 2 Square Spiral within Ground Plane Aperture Power Pattern (dBi) $\phi$ Cuts (1 GHz)

Figure 5.3.5-10 Measured and Simulated Mode 2 Square Spiral within Ground Plane Aperture Power Pattern (dBi) $\theta$ Cuts (1 GHz)
Figure 5.3.5-11 Measured and Simulated Mode 2 Square Spiral within Ground Plane Aperture Power Pattern (dBi) \( \phi \) Cuts (1.75 GHz)

Figure 5.3.5-12 Measured and Simulated Mode 2 Square Spiral within Ground Plane Aperture Power Pattern (dBi) \( \theta \) Cuts (1.75 GHz)
Figure 5.3.5-13 Measured and Simulated Mode 2 Square Spiral within Ground Plane Aperture Power Pattern (dBi) $\phi$ Cuts (2.5 GHz)

Figure 5.3.5-14 Measured and Simulated Mode 2 Square Spiral within Ground Plane Aperture Power Pattern (dBi) $\theta$ Cuts (2.5 GHz)
The previous figures illustrate that along with smoothing out the maximum gain with respect to frequency, the ground plane aperture has smoothed out the power patterns at each frequency and mode. The pointed tip at boresight for mode one is caused by the antenna not being rotated when the boresight measurement was made. This forced a single measurement to be used on the $\phi$ cuts where there should have been separate measurements for each angle to depict the polarization difference between each cut.

5.3.6 Cylindrically-Conformed Archimedean Spiral within Ground Plane Aperture

The following section compares the measured results of the cylindrically-conformed Archimedean spiral mounted within a ground plane aperture to the NEC simulated results. The following figure depicts the cylindrically-conformed Archimedean spiral mounted within the ground plane aperture.
The following figure depicts the mode one maximum vertically polarized gain of the measured antenna and the NEC simulated antenna versus frequency.
Figure 5.3.6-2 Measured and Simulated Mode 1 Cylindrically-Conformed Archimedean Spiral within Ground Plane Aperture Maximum Vertically Polarized Gain versus Frequency

The shape of the maximum gain curve agrees with the square spiral case which further suggests that the ground plane aperture drives the measured results. The following figures depict the power patterns of the spiral for mode one and mode two.
Figure 5.3.6-3 Measured and Simulated Mode 1 Cylindrically-Conformed Archimedean Spiral within Ground Plane Aperture Power Pattern (dBi) $\phi$ Cuts (1 GHz)

Figure 5.3.6-4 Measured and Simulated Mode 1 Cylindrically-Conformed Archimedean Spiral within Ground Plane Aperture Power Pattern (dBi) $\theta$ Cuts (1 GHz)
Figure 5.3.6-5 Measured and Simulated Mode 1 Cylindrically-Conformed Archimedean Spiral within Ground Plane Aperture Power Pattern (dBi) $\phi$ Cuts (1.75 GHz)

Figure 5.3.6-6 Measured and Simulated Mode 1 Cylindrically-Conformed Archimedean Spiral within Ground Plane Aperture Power Pattern (dBi) $\theta$ Cuts (1.75 GHz)
Figure 5.3.6-7 Measured and Simulated Mode 1 Cylindrically-Conformed Archimedean Spiral within Ground Plane Aperture Power Pattern (dBi) \( \phi \) Cuts (2.5 GHz)

Figure 5.3.6-8 Measured and Simulated Mode 1 Cylindrically-Conformed Archimedean Spiral within Ground Plane Aperture Power Pattern (dBi) \( \theta \) Cuts (2.5 GHz)
Figure 5.3.6-9 Measured and Simulated Mode 2 Cylindrically-Conformed Archimedean Spiral within Ground Plane Aperture Power Pattern (dBi) $\phi$ Cuts (1 GHz)

Figure 5.3.6-10 Measured and Simulated Mode 2 Cylindrically-Conformed Archimedean Spiral within Ground Plane Aperture Power Pattern (dBi) $\theta$ Cuts (1 GHz)
Figure 5.3.6-11 Measured and Simulated Mode 2 Cylindrically-Conformed Archimedean Spiral within Ground Plane Aperture Power Pattern (dBi) $\phi$ Cuts (1.75 GHz)

Figure 5.3.6-12 Measured and Simulated Mode 2 Cylindrically-Conformed Archimedean Spiral within Ground Plane Aperture Power Pattern (dBi) $\theta$ Cuts (1.75 GHz)
Figure 5.3.6-13 Measured and Simulated Mode 2 Cylindrically-Conformed Archimedean Spiral within Ground Plane Aperture Power Pattern (dBi) \( \phi \) Cuts (2.5 GHz)

Figure 5.3.6-14 Measured and Simulated Mode 2 Cylindrically-Conformed Archimedean Spiral within Ground Plane Aperture Power Pattern (dBi) \( \theta \) Cuts (2.5 GHz)
The previous figures illustrate that at frequencies below 1.75 GHz, the measured antenna power pattern trends closer to the simulated case in each geometry that was discussed within this thesis.
6. CONCLUSIONS

6.1 Summary of Work

Within this thesis, different types of frequency-independent antennas were discussed along with the theory of frequency-independence. It was found that a four-arm spiral antenna offered the best design choice for being integrated into a direction-finding system. Two different spiral types were used to design antennas that had a materials cost of less than $300 and could be fabricated within a week by a technician. If a higher quality mode-former was designed to be used in conjunction with this antenna design, a viable low-cost and light-weight solution would be realized.

Each of the antenna geometries were modeled in NEC to show that the difference between spiral types does not change the power pattern of either antenna at lower frequencies. On the other hand, at higher frequencies, the square spiral begins to diverge away from the results of the Archimedean spiral. The theoretical model also proved that changing the radius of curvature for each spiral type does not change the shape of the power pattern until a very small radius of curvature (6 cm) is used. The theoretical model did show that the power pattern changes more rapidly when the phase progression at the feed of the antenna is not ideal.

After fabricating each antenna type, power patterns of each antenna geometry was measured. These measured results demonstrated that an antenna provides better results
when a mode-former is producing a phase progression that is too short for a given frequency. At higher frequencies where the phase was much longer than needed, the power pattern quickly deteriorates. This conclusion agrees with the theoretical model to suggest that spiral type and conformation of the antenna surface does not change the overall power pattern as much as connecting the antenna to a less than an ideal mode-former.

6.2 Recommendations for Further Study

As with any antenna study, the recommendations for further study are almost limitless. One possible path of research might be to create antennas of different radiiuses of curvature to continue proving the invariance of spiral antennas to most conformations. Another path of research might be to explore different antenna geometry types, such as the logarithmic or equiangular spiral. Along with different antenna types, the number of arms could be explored along with different frequencies of operation. One particular antenna of interest to the author was the eight-arm self-complementary spiral antenna which has a characteristic input impedance of nearly 50 ohms. Such an antenna design would not need an impedance matching circuit and therefore offer an excellent SWR response with respect to frequency.
REFERENCES


