INVESTIGATION INTO THE BEHAVIOR OF BOLTED JOINTS

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in Engineering

by

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I HEREBY RECOMMEND THAT THE THESIS PREPARED UNDER MY SUPERVISION BY Steven M. Page ENTITLED Investigation into the Behavior of Bolted Joints BE ACCEPTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF Master of Science in Engineering.

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ABSTRACT

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Models to capture the physics of jointed structures have been proposed for over 40 years. These models approximate the behavior of the joint under carefully developed operating conditions. When these conditions change, the model has to be changed. Recent developments in numeric codes like finite elements have created interest in incorporating joint models into the design process but joint models need to represent the joined structure over a broader operating range.

This work investigates the dynamic response of a structure with a joint. Isolation of a few dominant effects may give way to a model able to capture a broader operating range. To isolate the effects of the joint two specimens were created. A specimen that is without a joint serves as a control. The second specimen is geometrically similar and contains a double lap joint with a bolt fastener. The differences between the specimens represent the effects of the bolt.

Control variables of bolt tension, excitation level and sampling time were chosen. Amplitude response and hysteresis curves were recorded. This data was used to examine the non-linear response of the bolted specimen. Qualitative observations are included.

The control specimen shows little effect from non-linear behavior in the frequency response. The bolted specimen shows non-linear behavior in the frequency response. When the joint is introduced to the geometry the system drops in amplitude, drops in resonant frequency, and demonstrates a non-linear softening effect. As the initial bolt tension is re-
duced the magnitudes of these changes increase. In addition when the system is allowed to dwell with a single sine wave at resonance the amplitude of the response often increases.

Hysteresis curves reveal that more than a softening non-linearity affects the response. The curve shows a softening affect when displacing in one direction and a hardening affect when displacing in the opposite direction. This may be affected by the geometry as the control specimen demonstrates a tri-linear stiffness.

It is evident that previous joint models do not capture all of the effects observed. Additional research to link the physical cause to the observed affect will aid in adjusting or creating a joint model to be used in numeric codes.
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<table>
<thead>
<tr>
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<tbody>
<tr>
<td>$F_s$</td>
<td>Joint slip force</td>
</tr>
<tr>
<td>$\alpha_i$</td>
<td>AIBE - Stiffness after macro slip</td>
</tr>
<tr>
<td>$f_yi$</td>
<td>AIBE - Force level for macro-slip</td>
</tr>
<tr>
<td>$\beta_i$</td>
<td>AIBE - half width of density function for $f_yi$</td>
</tr>
<tr>
<td>$u(t)$</td>
<td>Input force</td>
</tr>
<tr>
<td>$\dot{x}$</td>
<td>Time derivative of displacement</td>
</tr>
<tr>
<td>$\ddot{x}$</td>
<td>Second time derivative of displacement</td>
</tr>
<tr>
<td>$F(x, \dot{x})$</td>
<td>Restoring force</td>
</tr>
<tr>
<td>$m$</td>
<td>Mass</td>
</tr>
<tr>
<td>$A$</td>
<td>Wen model - stiffness</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Wen model - width of hysteresis loop</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Wen model - softening and hardening ranges of hysteresis curve</td>
</tr>
<tr>
<td>$K$</td>
<td>Stiffness matrix</td>
</tr>
<tr>
<td>$k$</td>
<td>Spring stiffness</td>
</tr>
<tr>
<td>$U$</td>
<td>Iwan joint relative displacement</td>
</tr>
<tr>
<td>$X(t, \phi_i)$</td>
<td>Jenkin element displacement</td>
</tr>
</tbody>
</table>

## Chapter 3

<table>
<thead>
<tr>
<th>Symbol</th>
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<tr>
<td>$R_g$</td>
<td>total resistance of the strain gage</td>
</tr>
<tr>
<td>$R_i$</td>
<td>Resistance of the Wheatstone bridge resistors</td>
</tr>
<tr>
<td>$V_{out}$</td>
<td>Voltage output from Wheatstone bridge</td>
</tr>
<tr>
<td>$V_{in}$</td>
<td>Voltage input to excite Wheatstone bridge</td>
</tr>
<tr>
<td>$G_{amp}$</td>
<td>Gain of the amplifier</td>
</tr>
<tr>
<td>$\Delta R_g$</td>
<td>Change in resistance of due to deforming strain gage</td>
</tr>
<tr>
<td>$R_{nom}$</td>
<td>Nominal resistance of undeformed strain gage</td>
</tr>
<tr>
<td>G.F.</td>
<td>The gage factor relating relative change in resistance to strain</td>
</tr>
</tbody>
</table>
σ  Axial stress
E  Modulus of elasticity
P  Force applied
A  Cross sectional area
R1  Resistance
C1  Capacitance matching piezoelectric patch
C2  Capacitance of voltage dividing capacitors

Chapter 5

Nm  Bending moment
N − m  Torque applied
x  Displacement of system
ẋ  Time derivative of displacement
x¨  Second time derivative of displacement
ω_n  Single degree of freedom natural frequency
ζ  Linear damping ratio
γ  Non-linear cubic damping coefficient
k  Linear spring stiffness
α  Non-linear cubic stiffness coefficient
U  Force amplitude divided by mass
Ω  Forcing frequency
t  Time
φ  Phase
a_m  Coefficient for m^{th} harmonic amplitude in harmonic series
a  First coefficient in harmonic series
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Dedicated to

Murry and Bonnie Page
Introduction

1.1 Motivations

Joint models have been proposed for over 40 years. These models focused reproducing the system response for a particular instance. Iwan [1] notes that the parameters for his model change after dynamically loading a physical specimen. Numeric codes like finite elements (FE) have become standard tools in design and analysis. Codes like FE have pushed design and analysis to a new level of fidelity, but the addition of joints to a structure leads to either extreme complexity or intuitive guessing.

Any joint interface consists of multiple contact surfaces. Structures held together by even a single bolt have interactions between the bolt head and structural surface, the shank and the hole, the mating surfaces of the structure, the nut to the structural surface, and the threads and nut. Modeling in FE requires each contacting surface to have boundary conditions delegating how the surfaces interact with each other. In these contact regions high stresses occur in small areas like the tips of threads and at surface asperities. Fine meshes resolve the stresses in these small areas, but are very computationally expensive. The systems complexity is multiplied every time a joint is encountered.
Simplified joint models trade off computational expense for accuracy. They simulate the effects of a joint, but have only a fraction of the elements or boundary conditions of a detailed joint model. Several desired characteristics for joint models are that their parameters have physical meaning, they are simple to implement, and easy to identify. However, even promising models like Iwan’s [1] have problems approximating the physics of the joint under varying circumstances [2].

The inability to approximate a joint under different operating conditions could stem from a lack of understanding the physics in the joint interface. An investigation into the behavior of bolted joints was carried out to give understanding to the complex physics involved with joints. With an understanding of the physics a better joint model can be made that does approximate the joint under different loading conditions.

1.2 Background research

Several joint models attempt to predict the behavior of structures. Joint models can generally be categorized as either (i) physics-based, (ii) experiment-based (which are sometimes known as model-free methods), or (iii) joint parameter identification-based sub-structuring methods[3].

1.2.1 Physics-Based Models

Two primary classes of physics-based models have been identified in the literature: phenomenological, which define a global relation between friction force and displacement, and
Iwan Model

Iwan[1] developed an elasto-plastic model that can be used to account for the nonlinear stiffness and energy dissipation at the joint interface. His model is composed of a parallel system of spring-slider (i.e., Jenkins) elements, as shown in Figure 1.1. The joint’s slip force is based on the average slip force of the population of sliders.

Segalman’s [4] formulation of the Iwan joint model is completely characterized by four model parameters. Joint slip force, $F_s$, is obtained directly from force-deflection measurements, two other parameters are obtained by fitting the model to measured cyclic en-
ergy dissipation, and the last parameter is discovered after inserting the previous three into a system of equations.

Song et al. developed an adjusted Iwan model and a two-dimensional adjusted Iwan beam element (AIBE). Their adjusted Iwan model takes into account the experimental fact that the joint possesses some stiffness even when all spring-slider elements slip; i.e., when macro-slip occurs. A spring added in parallel to the Iwan model accounts for the joint stiffness during macro-slip. The AIBE employs two of these adjusted models corresponding to the usual degrees of freedom for beam elements (transverse displacement and rotation at each node).

Song et al. used a multi-layer feed-forward (MLFF) neural network to solve the inverse problem presented in the AIBE model. The element has six parameters that describe the force-displacement relation, three for each adjusted Iwan model. The parameters \( \alpha_i \) and \( f_{yi} \) (\( i = \{1, 2\} \)) represent the stiffness after macro-slip and the force level at which macro-slip occurs, respectively. The initial force-deflection curve is a function of the population density obeyed by the individual slider elements. This density is assumed to be uniform with width \( 2\beta_i \) and mean \( f_{yi} \). Experiments show that some micro-slip is observed even for very small loads, so \( \beta_i \) is chosen to allow some of the sliders to have vanishing strength; hence, it is assumed that \( \beta_i = f_{yi} \) so that only two parameters are independent for each Iwan element.

When identifying AIBE parameters, classical optimization methods become problematic owing to the lack of smoothness of the objective function. The MLFF neural network provides a more robust method of solving for the parameters. Song, et al. initially set
$\alpha_1 = \alpha_2, \ f_{y1} = f_{y2} \ \text{and} \ \beta_i = 1.0$ to increase efficiency of the identification procedure. The identification requires training the network on the decay envelopes obtained computationally using a range of model parameters, and then giving it a measured decay envelope to identify.

This model is considered promising for future development but still has some weaknesses. Iwan [1] recorded that his physical test joint changed with time but he could use averaged identified parameters to model the system. Shiryayev et al. [2] demonstrated this model does not accurately predict the response of the joint for different types of excitation. This model does not take into account the variability associated with the joint though it does address the hysteresis curve shape and demonstrate a spring softening type of nonlinearity.

**Wen’s Model**

In Wen’s model [6] the restoring force is governed by a nonlinear differential equation that describes a hysteretic relationship between the force and the relative displacement. It is important to note that the restoring force in Wen’s model depends not only on the history of displacement and the instantaneous displacement but on the history of the restoring force as well. There are four parameters that can be varied in Wen’s model. By adjusting these factors, one can change the shape and size of the hysteresis loop, which quantifies energy dissipation.

Wen’s model has great flexibility with the energy dissipation, but the parameters lack physical meaning. Again, the parameters are considered constant, so effects like bolt loosening would not enter into the formulated. Wen’s model has been implemented into finite
elements by Yue [3].

**LuGre (Lund-Grenoble) Model**

Dahl created a dynamic model to describe the observed spring-like phenomena during stiction. The benefits and drawbacks of Dahl’s model are in its simplicity. In Dahl’s model, the friction is solely based on displacement. The LuGre model[7] is based on Dahl’s model but adds velocity dependence. It visualizes the contact surface as having irregularities that act as bristles. Each bristle deflects like a spring until it reaches a limit and then slips to a new position. The LuGre model uses the average properties of the population of bristles to estimate joint dynamics. A velocity profile, stiffness, damping coefficient, and viscous friction coefficient describe the dynamic friction force in the joint. The model accounts for phenomena like stiction and lubrication but may not show dissipation if the damping coefficient is not related to the velocity.

### 1.2.2 Experiment-Based Models

Experiment-based models provide an alternative approach based on fitting generalized regression models to experimental data from specific classes of joints. The most common of these models depend only on Newton’s 2nd law to estimate the joint restoring force, \( F(x(t), \dot{x}(t)) \), based on the measured input force, \( u(t) \), and the acceleration, \( \ddot{x} \), of the joint. Consequently, these models are often referred to as restoring force methods. For a joint with known mass, \( m \), the restoring force can be written directly as

\[
F(x, \dot{x}) = u - m\ddot{x}. \tag{1.1}
\]
The various methods differ primarily in the type of regression function used to represent the restoring force. The advantages of this approach include its simplicity and the ability to capture strongly nonlinear behavior. The disadvantages include the potential requirement of large data sets and the possibility of over-fitting if the data set is too small to separate higher-order physics from noise and random measurement errors. The ability to generalize their results to new cases is also questionable.

1.2.3 Substructure-Based Identification Methods

The objective of these methods is to identify the properties of joints by measuring how the presence of a joint affects known substructures[3]. The joint typically is assumed to be an elastic linear substructure with viscous damping and it is assumed the frequency response functions (FRF’s) of the separate substructures are known.

Yue[3] recently extended the ideas of Park and Fellipa to generate localized nonlinear models of joints based on a generalization of Wen’s model. Yue defined the parameters needed to identify the Wen’s model as $A$, $\gamma$, and $\delta$, which represent the stiffness, width of hysteresis loop, and the softening and hardening ranges of the hysteresis curve, respectively. For systems described by two or more degrees of freedom, the parameters represent matrices of appropriate dimension. Identification starts with modeling the experimental system as linear and discovering the stiffness matrix, $K$. The approximation begins with setting $K = A$ and then adjusting $A$ based on jointed system eigen-analysis while $\gamma = \delta = 0$. The parameters $\gamma$ and $\delta$ are calculated by holding $A$ constant and correlating the simulated FRF to the experimental FRF.
1.2.4 General Observations on Joint Models

Each of the hysteretic model classes described here has been employed primarily from a deterministic calibration perspective. Efforts to quantify their variability does not appear to have been published.

1.2.5 Variability in Bolted Joints

Bolted joints experience a vast array of changes. There can be as much variation as $\pm 30\%$ of the expected tension in a bolt when using the torque tightening method [8]. Besides the variation involved with the initial joint constitution, bolt tension can drop more than 40% over a short length of time [9], [10], and [11]. Then there are cases when bolt tension can increase rather than decrease based on excitation frequency and level [12].

1.3 Thesis Overview

The first chapter discusses the problem with modeling bolted joints. The complexity of joints has been the motivation for several models. These models approximate the response of the joint at a particular operating condition, but fail to do so away from the design point. A new model may be able to capture the physics of the joint better if the dominant features introduced by a mechanical joint were known. This work looks at using a lap joint with a single bolt to identify the dominant effects of the bolted joint.

Chapter two walks through the experimental hardware. Preliminary design work is
presented for the test specimen and the instrumented bolt. Their manufacturing is also described. Particular attention was given to specimen and transducer mounting methods to support the assumption of free-free boundary conditions. The data acquisition system is also described.

Chapter three describes the hardware from an experimental point of view. Calibrations are performed for the instrumented bolt and the piezoelectric patches. Support wires are tested for interaction with the specimen. The room, table, and Uni-strut frame are tested for possible responses that may interact with later experiments.

Chapter four describes responses from the control specimen tests. Impact testing is done and the modal parameters are identified. Sine sweep tests and hysteresis curves are taken for later comparison with the bolted specimen.

Chapter five investigates the dynamic effects of a bolted lap joint. Initial bolt tension is varied to look at the immediate effect on the dynamic amplitude response. Amplitude response and time are added to the experiment to track the resonant frequency and amplitude response. A non-linear least squares curve fit routine is implemented to identify parameters of the amplitude response. Hysteresis curves are also recorded for varying time, tension, and amplitude.

Chapter six concludes with a description of what was accomplished. It also mentions some of the visual observations from the experiments. The chapter is concluded with suggestions for future work.

The appendices include information for specimen preparation. Appendix A goes through the procedure for preparing the bolted specimen for an experimental run. Ap-
Appendix B contains the material data and geometry needed to estimate bolt tension due to torque. Appendix C discusses how different transducers were applied. Appendix D contains the MATLAB code for implementing the identification presented in Chapter 5.
Experiment Design

In order to separate the effects of instrumentation and mounting from those of the joint, two specimen were made: A control specimen made of one integral piece to give a baseline for observed behavior and a two-piece specimen with a double lap joint, sometimes called a butt splice, that can be fixed with a single bolt. The change in behavior by adding a bolted lap joint was expected to be small so care was taken in choosing instrumentation, test frames, and apparatus to reduce unwanted interactions like electronic noise and parametric excitation of the test frame.

2.1 Specimen

The specimens are rectangular beams with integral masses at either end (see Figure 2.1). The integral masses were included to lower the natural frequency of the extensional mode. The flanges of the bolted specimen were made flexible so the fraying surfaces would have the same normal load as the tension in the bolt.

Three different sized holes exist in the bolted specimen, but only the 12.7 mm hole was used. Only one instrumented bolt was created and used with the structure. Previous
Figure 2.1: Dimensions apparatus and instrumentation locations (dimensions in inches).
work by Song et al. [5] used a similar method, but with a double butt splice. They modeled two bolt interfaces as only one for their numeric simulations. The single bolt was used in this work to model the effect of individual joint interfaces that may later be combined to model the overall effect of several fasteners.

Aluminum 7075-T651 was chosen for the apparatus material. This aluminum alloy and temper is used for aircraft framing and various other aircraft components. The use of this material should allow for the findings to be quickly applied to structures that also use this material.

The first seven modes of the two-piece specimen were estimated by finite elements using commercially available software called Abaqus (see Figures 2.2, 2.3, 2.4, 2.5, 2.6, 2.7, and 2.8). These results do not account for structural damping or joint interaction. Free-free boundary conditions were assumed for this modeling. This model was useful for making sure a finite element model could capture the dynamics of the beam, determine the nodal locations for the first bending mode, and to compare the ideal case of no joint interaction to the control specimen.

Previous to this modeling, the specimen had been mounted using holes drilled at the top of the integral masses (shown but not dimensioned in Figure 2.1). This mounting location was changed after the finite element model was compared to the specimen and shown to be a valid solution for the first several modes. The change in dynamic characteristics can best be seen in Section 4.2.1. The new mounting method uses $\frac{1}{4} - 20$ Unified Fine (UNF) socket cap screws threaded into the specimen at the nodal locations.

A single $\frac{1}{2} - 20$ UNF bolt, 50 washers, and 13 nuts were made from the same material
Figure 2.2: First bending mode. Node locations used for mounting.

Figure 2.3: Second mode is close to first, but is not excited with piezoelectric patches.
Figure 2.4: Third mode, bending in out of plane direction and not excited by hammer or patches.

Figure 2.5: Fourth Mode could be excited, but was well separated from first mode.
Figure 2.6: Fifth mode was excited during hysteresis testing.

Figure 2.7: Sixth mode was rarely excited.
Figure 2.8: Seventh mode was not excited for this testing.

type and temper as the apparatuses. The material was matched to account for thermal expansion. The bolt has a specially designed shoulder and head to allow strain gages and lead wires to be applied without interfering with the structure (See Figure 2.9) and to have a Von-Misses stress less than the yield stress for 7075-T651 aluminum when up to 27 Nm of torque and 1500 N of tensile load is applied.

Four foil strain gages are applied to the shoulder of the bolt. Three are uniaxial gages applied axially on the outside bolt shoulder $120^\circ$ radially. The fourth foil gage is a half bridge shear gage. These gages give the ability to measure strain in bending, axial load, and shear.
2.2 Experimental setup

A test frame was made to allow the specimen to hang from 45-inch long 0.01-inch diameter stainless steel wire. The length of the wire reduced the rigid body pendulum motion of the specimen to 0.5 Hz. The rigid body vertical motion (bouncing) from the strings acts at 5 Hz. The length of the wires isolates the wire resonances from the excitation frequencies for the specimen. The wires were connected to the structure by holes drilled through the edges of the integral masses. This attachment method was changed to bolts located at the nodes of the first resonance, Figure 2.10. This simulates the structure having a free-free boundary condition with minimal effect from the support mechanism for the first bending mode.

Tri-axial accelerometers were placed on the left and right, top and bottom of the bolted joint (Figures 2.1 and 2.12). They allowed for the loss in acceleration to be calculated.
Figure 2.10: Thin stainless steel wires and bolts were used to support the specimen.
Figure 2.11: Transducer wires are bent to be restrained but flexible.

across the joint. Strain gages were also applied adjacent to or collocated with the accelerometers. Strain data was collected to estimate the force acting in the beam, which would allow for the calculation of the change in force across the joint. The result was a way to generate hysteresis curves for the joint. All transducers were mounted with Loctite 420 adhesive.

Transducer cable management focused on reducing the motion of the cable while still allowing the structure to move freely. Reduction in cable motion decreases damping from the cables while often introducing stiffness to the structure. A compromise was found by bending the transducer wires 90° (Figure 2.11) to be compliant to the motion of the structure but constrain the motion of the cable itself.

Piezoelectric patches from Ferroperm type Pz29 were mounted to excite the structure. They measure 50mm by 50mm by 0.5 mm thick. A conductive epoxy was used to fasten the actuator to the specimen. A jig was made for applying the patches uniformly on each
specimen. The patches were attached so that the polarity favored the extensional mode configuration, i.e. the patches located by one integral mass shared the negative pole. In doing this the patches experienced a greater voltage potential in extension than when driven in bending.

The entire test setup was placed inside of a sound control chamber. The chamber consisted of three layers. The closest layer to the specimen was a light foam often called “egg-crate foam.” Its most common use is for adding comfort to beds. It was selected for its porous texture. The pores help to reduce the transmission of high frequency sound. This first layer was fixed in place using nails pounded through the second layer into the third. The second layer is more densely packed foam often used for carpet padding. A contact adhesive was used to hold this to the third layer. The third layer was $\frac{3}{4}$ inch plywood. Rather than act as an absorber, this layer acted as a sound barrier. It kept low frequency sound (bellow 500 Hz) from entering or exiting the experimental area. This sound box was deemed necessary to protect the operator from high sound levels and the experiment from the noisy environment it was in. After the box was assembled the experiment was
later moved from a lab with a noisy air handler to a quieter lab. Within the new lab the experiment was no longer excited at frequencies close to hearing resonances, 1 kHz to 3 kHz, so the box was removed.

For data acquisition Three Spectral Dynamics Siglabs, model 22-20a, were used. The Siglab units can be daisy chained to handle six channels of simultaneous data acquisition with up to 20 kHz bandwidth. The unit has built in anti-aliasing filters and digital signal processors. Software includes a sine sweep utility that incorporates a tracking bandwidth filter for more precise measurements and has function generation for broadband excitation.
Validation Experiments

3.1 Validating Bolt Load

Getting accurate bolt tension data from a bolted joint is difficult. Strain gage sensitivity to bolt tension was estimated from the part geometry and gage information supplied. An experiment using an Instron tensile test machine was devised to compare the calculated estimate to the experimentally observed.

A total of four gage patterns were installed on the bolt used in these experiments. Three of these gages were installed independently along the bolt axis, and one was a 90-degree shear rosette pattern. The axially aligned gages were tested using as a Vishay bridge amplifier and Instron test machine. Each gage was tested for resistivity (350 ohms nominal) and for possible shorts to ground.

The expected voltage from a Wheatstone bridge is expressed as equation 3.1. Figure 3.1 references the variables used to describe the Wheatstone bridge equations.

\[
V_{out} = \left[ \frac{R_g}{R_3 + R_g} - \frac{R_2}{R_1 + R_2} \right] V_{in} \times G_{amp} \tag{3.1}
\]
The change in resistance of the gage $\Delta R_g$ is linearly related to the strain experienced with a coefficient called the Gage Factor (G.F.). Strain is related to stress, $\sigma$, and with the cross sectional area $A$ and modulus of elasticity $E$ an expression for output voltage to input load $P$ can be made.

$$\text{strain} = \frac{\Delta R_g}{R_{nom}} = \frac{\sigma}{E} = \frac{P}{AE}$$ (3.2)

$$\Delta R_g = \frac{P(G.F)R_{nom}}{AE}$$ (3.3)

$$V_{out} = \left[ \frac{R_{nom} + \frac{P(G.F)R_{nom}}{AE}}{R_3 + R_{nom} + \frac{P(G.F)R_{nom}}{AE}} - \frac{R_2}{R_1 + R_2} \right] V_{in} \times G_{amp}$$ (3.4)

In this case, $R_1 = R_2 = R_3 = 350\Omega$, and $R_g = R_{nom} + \Delta R_g$, where $R_{nom} = 350\Omega$. The cross sectional area where the foil gages are applied is approximately $A = 12.6\pi\text{mm}^2$, $E$ is listed in Table B.2, the Gage Factor (G.F.) is given from the manufacturer as 2.105, $V_{in}$ is 10Vdc, $G_{amp} = 1000$. These values result in equation 3.5, where P is the tensile load in Newtons.

$$V_{out} = \left[ \frac{3 \times 10^{-4}P + 350}{3 \times 10^{-4}P + 700} - \frac{1}{2} \right] 10000$$ (3.5)
Putting a load of $P = 1000$ N into Equation 3.5 results in an output voltage of 2.14V. This is the expected voltage from the amplifier.

The instrumented bolt was then tested to find the sensitivity to tensile loading. An Instron tensile test machine was configured to apply an 890 N tensile load to the bolt and then return to zero. The bolt was held by the head using a block of steal with a $\frac{1}{2}$ inch slot. The threaded end of the bolt was screwed into the coupler connected to the machine. A tensile load of 890 N was chosen as the upper limit because the threaded coupler only engaged five threads. The estimated max load from torquing the bolt to 11.30 Nm was 1,112 N.

The resulting calibration curves do not match the linear assumption used for calibrating. It can be seen from Figures 3.2, 3.3, and 3.4 that all three axial gages experienced a
Figure 3.3: Calibration of gage 2

Figure 3.4: Calibration of gage 3
hysteresis affect during the loading cycle. Besides having voltages twice what was expected, the third gage responded with a negative voltage (Figure 3.4). All of the Wheatstone bridges were wired to produce a positive voltage when in tension. In-lab experiments demonstrate that all return signals are positive when the bolt is tightened.

One reason for the hysteresis may be that the Instron tensile test machine that was used has a 100,000 N load cell model 2518-100. Only 0.89% of the full scale was used in testing the bolt load. Increasing the load beyond this could damage the bolt. The load uncertainty is ±25 N over 0.4% of full scale. It would seem that the hysteresis in the data might be an issue with the load cell and not with the bolt gage installation.

Other possibilities, like hysteresis of the gages, poor installation, bad connection, could also exist. However, none of these explain the negative voltage from the third gage. It is very likely that the bolt was pulled with an eccentric load. This would account for a negative load on one gage and higher than expected in the other two. All bolt load results are presented as percent of maximum voltage seen, or as an estimate based on gage 2 which showed the most consistent response.

### 3.2 Testing the Support Structure

A test was devised to examine external influences on the test specimen. Specifically the transmission of energy from the support structure to the specimen and from specimen to support structure was examined.

To determine if the support structure was being excited during testing, a PCB ac-
celerometer (model 352C22) was mounted to the support wire using paraffin wax. The monolithic specimen was excited at $9.14 \times 10^{-2}$ N-m and the auto spectral density of the accelerometer signal was observed to be broadband noise.

The square Uni-strut pieces were not tested with an accelerometer because the stainless steel lines that attached them to the specimen did not produce any noticeable motion. For the Uni-strut frame a test for electrical isolation was performed before each test. If the specimen had a path to ground other than through the piezoelectric patches then the assumed input moment would not be correct. Isolation was considered acceptable if a handheld multi-meter reached overload, approximately 10 M-Ohms.

A Newport optics table supported the Uni-strut frame. Although the table does have the ability to levitate on air, this option was not used because the table was unable to remain level. To check for mechanical isolation, the floor was impacted with 750 Newtons and the response from the accelerometers mounted to the specimen was observed. It was noted that there were observable spikes in the signal when impacts occurred, indicating that occupants in the room should refrain from walking during a test. Also, the table was impacted with a rubber mallet and the auto-spectral density from the specimen-mounted accelerometers was observed. There was little more than noise in the testing ranges of 65 Hz, 80 Hz, and 100-120 Hz.
3.3 Testing the Output from Piezoelectric Exciter

The desired force excitation for the specimen was a single frequency sinusoid. Dosch et. al [13] uses piezoelectric constitutive laws to derive a closed form solution relating voltage across the piezoelectric device to moment applied on the beam. These equations assume a linear relationship. To test the assumption that the input matched a sinusoid, a test circuit was created to measure the strain observed by the patches. This process is also outlined in [13].

The piezoelectric patches have an equivalent capacitance of 43.6 nF measured at 100 Hz. The measurement was accomplished with a Hewlett Packard LCR meter (a device used to measure inductance, capacitance, and resistance) and used 0.30 m long, powder coated, 30 gage copper wire to connect to the piezoelectric patches. The patches were mounted to the specimen during the measurement and the average of all the patches was used to estimate the equivalent capacitance.

The strain measuring circuit (Figure 3.5) was used on the bolted two-piece specimen. The strain at the piezoelectric patches was not a sinusoid but was like a triangle function. The triangular shape of the strain is a result of the non-linearity inherent in the bolted system. A duplicate test with just monolithic specimen was attempted but the hardware failed to work properly. The signal was compared to a finite element model of the jointed structure with a non-linear Iwan joint interface; the two were similar under the same sinusoidal excitation. Additionally, using the formula for bending moment from the piezoelectric patches yielded almost identical acceleration responses from the finite element model. This test indicated that the patches were exciting the structure in such a way that an FE model was
Figure 3.5: Circuit design for measuring the strain in a piezoelectric patch.

able to capture the dominant response of that excitation.
Control Specimen

The control specimen is physically to the bolted specimen without the effects of the bolted lap joint. The tests in this chapter create a baseline for later comparison to the bolted specimen. It is assumed that the differences between the response of the control specimen and the bolted specimen are due solely due to the bolt.

4.1 Specimen Comparison

The control specimen is geometrically similar to the jointed specimen. It should be noted that the 1-inch by 2-inch portion of the control was 0.1 inch shorter, and that the three bolt holes drilled into the jointed specimen are not present in the control.

The material used for the control was ordered and shipped from the same supplier of the bolted specimen. Both were machined and finished at the same time, with the exception of a through hole in the control located at the tongue and groove interface on the bolted specimen. This hole, shown in Figure 4.1, was machined at a Wright State’s Instrumentation Shop using end mills and shapers rather then a plasma cutter. The residual stress from the mill and shaper are typically greater than those from plasma cutting. Also, the surface
Figure 4.1: Control specimen had material removed to match bolted specimen. Finish is smoother for the plasma cut specimen.

The instrumentation was placed onto the specimen by measuring set distances from the integral masses, maintaining the symmetry of the system. The same sensors were used for both apparatuses and the piezoelectric exciters were all made from one batch of material. Piezoelectric material can vary due to manufacturing process. The assumption that the excitation moment is the same on each corner of each side is more reasonable when the piezoelectric material was made at the same time. The same jig (Figure 4.2) was used for applying the piezoelectric exciters on both apparatuses.

The test stand, signal conditioning, and data acquisition are the same for both control and bolted specimen. The control was tested first and then again after some tests had been performed on the bolted specimen. The control did not experience the same number of cycles the bolted specimen experienced. Excitation levels were not the same because the control specimen’s amplitude response was over twice that of the bolted specimen. At
excitations above $3.74 \times 10^{-2}$ Nm and close to resonance the Loctite 420 adhesive attaching the sensors began to crack.

### 4.2 Quantitative results

Three separate impact tests were performed on the control specimen: directly after receiving the unit, after changing the mounting location and removing some material, and after all the instrumentation was added to the specimen. Each impact was performed using a modal hammer, PCB 086C03, with a soft tip for exciting 0 - 1 kHz. The hammer delivered approximately 220 N of force per blow. Each impact location was uniformly distributed down the length of the beam. One PCB 352C22 accelerometer was mounted on the top face, far corner of an integral mass. A Siglab unit was used to record the impact and response. Five frequency domain averages were taken per impact location. Observing the effect of each averaged FRF until little change was observed chose the number of averages. The system changed very little after three averages, so five were used.
Table 4.1: Resonances and damping ratios using top mount with un-notched control.

<table>
<thead>
<tr>
<th>Resonance #</th>
<th>Frequency (Hz)</th>
<th>Damping Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>129.42</td>
<td>0.0036</td>
</tr>
<tr>
<td>2</td>
<td>360.52</td>
<td>0.0013</td>
</tr>
<tr>
<td>3</td>
<td>817.47</td>
<td>0.0006</td>
</tr>
</tbody>
</table>

The frequency response was calculated using the Siglab’s built in $H_1$ routine with boxcar windowing. Boxcar windowing was chosen since accurate damping information was desired. Spectral leakage from the relatively clean signals was less of an issue than inflated damping from using an exponential window. Once the frequency response plots were recorded, the Eigensystem Realization Algorithm (ERA) was used to identify the state matrix and extract the frequencies and damping ratios.

### 4.2.1 Effect of mounting

When the control specimen was first received a series of impact tests were performed to identify the resonant frequencies and damping ratios. The control was impacted every 2 cm on the surface of the 5.08 cm wide beam. Table 4.1 shows the data for the first three bending modes excited. The torsion, extensional, and the out of plane transverse bending modes where not excited. This initial test used small holes drilled at the top of the integral masses to support the structure. The mounting location can be seen in Figure 2.10 along with the newer mounting method of a screw located at the first bending frequency node.

These results matched well with the finite element model, though when the bolted
specimen was modeled with finite elements the missing material adjacent to the bolt location changed the tensional mode significantly. It was decided to remove the material in the control specimen to make it similar to the jointed specimen. While this operation was being performed another investigation into mounting method was started.

Since the finite element model matched well with the experimental specimen, the nodal locations of the bolted model were used to locate new mounting points. The holes were drilled and tapped and strung as shown in Figure 2.10. A new modal analysis was performed.

It can be seen from Table 4.2 that the first bending frequency was changed by less than two percent, but higher frequencies were affected more dramatically, 10.5% and 12.2% respectively. Damping reduced by an order of magnitude for each bending mode. These changes are attributed to the mounting rather than the removal of material. Moving the wire attachment point to the first nodal location drastically reduced relative motion between the wire and specimen, reducing energy lost through transmission and friction.

The previous impact tests had been performed with only one transducer attached to the specimen. The control specimen was instrumented with the same transducers as the

<table>
<thead>
<tr>
<th>Resonance #</th>
<th>Frequency (Hz)</th>
<th>Damping Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>127.2</td>
<td>0.00012</td>
</tr>
<tr>
<td>2</td>
<td>322.5</td>
<td>0.00009</td>
</tr>
<tr>
<td>3</td>
<td>717.6</td>
<td>0.00005</td>
</tr>
</tbody>
</table>

Table 4.2: Resonances and damping ratios using new mounting with notched control.
Table 4.3: Resonances and damping ratios with instrumentation.

<table>
<thead>
<tr>
<th>Resonance #</th>
<th>Frequency (Hz)</th>
<th>Damping Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>126.6</td>
<td>0.00019</td>
</tr>
<tr>
<td>2</td>
<td>322.1</td>
<td>0.00014</td>
</tr>
<tr>
<td>3</td>
<td>709.3</td>
<td>0.00021</td>
</tr>
</tbody>
</table>

jointed specimen and tested for resonant frequencies and damping ratios (Table 4.3). The damping ratios increased between 58%-76%. Some increase was expected from transducer interaction with the specimen, and with energy transmission and dissipation in the transducers wires. Fully instrumented the test articles have eight transducer wires and four leads to the piezoelectric exciters. Unlike the support wire, these are not located at nodal locations and do experience some excitation from the specimen. The effect of the transducers to the specimen in terms of damping is significantly smaller than that of the mounting.

4.2.2 Linearity of control specimen

Direct comparison of the control specimen to the bolted specimen was difficult because the same regimes of excitation were not used (See Section 4.1). The change in excitation may have caused the structure to behave differently. To test the effect of excitation amplitude four sine sweeps of varying amplitude were performed.

Figure 4.3 shows that the overlaid magnitude plots of the FRF did not change based on excitation amplitude. There is a small discrepancy at the frequency response peak between up and down sweeps (Figure 4.4) but the correlation also systematically dipped at these
Figure 4.3: The magnitude responses for several excitation levels overlay each other.

Figure 4.4: There is a small discrepancy at the peak.
locations. This is because the system has such light damping that when passing through resonance the response beats longer than the five-second delay used for the data capture. If the damping were higher then the off band frequencies would decay more. Other than at the peak amplitude response the systems increasing and decreasing frequency sweeps are a match. These results indicate that the linear effects of the structure dominate the non-linear ones, and a linear assumption is made for the control specimen.

4.2.3 Well Separated Modes

The torsion mode is not listed in Tables 4.1, 4.2, and 4.3, but was affected by the removal of material from the control. The change in frequency for the torsion mode brought it closer to the first bending mode. For analysis the modes are still considered well separated since the piezoelectric excitation does not excite this mode when exciting the bending mode, as shown in Figure 4.5.

The closest mode below the first bending resonance is about 14 Hz. This appears to be a rigid body motion, most likely the vertical bouncing of the structure on the support wires. This mode is only excited when using the impact hammer because the hammer impacts in the same direction as the rigid body motion. The piezoelectric patches do not excite this mode because they apply force as a coupled moment. Since this rigid body bouncing is a decade away from the first bending mode it is not considered to have significant influence on the first bending mode shape.
4.2.4 Hysteresis Curves

The energy dissipated per cycle is of great importance for the bolted joint case. For the resonance case the excitation was picked to be $1.03 \times 10^{-2}$ Nm so as to maximize the response without loosing transducers. The measured strain and acceleration signals are averaged for 10,000 windows of data. The acceleration is then shifted to remove the dc bias and the data sequence truncated to only contain whole periods of the signal. This signal is then integrated twice to get displacement. The bending moment is estimated from the piezoelectric strain gages located on the top and bottom of the structure.

The hysteresis curves do form a symmetric shape about the origin (Figures 4.6 and 4.7). It was thought that the strange result might be the effect of gravity loading. The specimen was turned over and the test was run again. The result was unexpected. The
Figure 4.6: Hysteresis curve of control specimen at 65 Hz and $4.57 \times 10^{-2}$ excitation.

flipped over curves, Figures 4.8 and 4.9, are not even the same shapes as the previously acquired ones. It is believed that the originally taken data may be suspect because the curves are not symmetric about the origin.

### 4.3 Qualitative results

The control specimen emits high intensity sound when excited close to the extension frequency. Initially it was thought that both the control and bolted specimen would need to be operated within a sound barrier for safety. A sound barrier box was created for this purpose. It was removed when the bolted specimen only produced a low audible hum.

The control produced amplitudes that could be observed with the unaided eye. The
Figure 4.7: Hysteresis curve of control specimen at 65 Hz and $9.14 \times 10^{-2}$ excitation.

Figure 4.8: Hysteresis curve of control specimen at 65 Hz and $4.57 \times 10^{-2}$ excitation.
observer could see the residual positions of the beam, like looking at a vibrating string or a rubber band stretched and plucked. Chladni figures [14] were used to estimate the nodal lines. The sand granules used were rather large making it difficult for the sand to accumulate.

A finger was placed on the 5.08 cm surface of the beam while the system was being excited at resonance. The shape of the beam did not appear to change, though the amplitude did change. First mode nodal lines were felt close to the location of the mounting points, but this method did not give a precise location.

Though neither qualitative method to find the nodal lines of the structure yielded precise results, they did agree with the locations predicted by the FE model. This increased confidence in both the FE model and the locations used for supporting the structure.
Bolted Joint Dynamics

A set of experiments was designed to investigate how a bolted joint changes in its dynamic behavior. Some of these tests look at autonomous parameters, while others focus non-autonomous. Autonomous parameters are those that do not change during the experiment. Such parameters would be bolt size or initial bolt torque value. Non-autonomous parameters are those that do change with time.

This work was performed using bending moment amplitudes of $4.57 \times 10^{-2}$ Nm and $9.14 \times 10^{-2}$ Nm. Three bolt torques, 6.78 N-m, 9.04 N-m, and 11.30 N-m, were used for the majority of tests. The units of bolt torque are expressed as N-m, whereas the units for bending moment are expressed as Nm. This change in units is to help clarify which parameter is being discussed.

5.1 Affect of Bolt Tension on FRF

Although bolt tension is the desired control variable, torque is the controllable variable. This test consisted of applying a torque to the bolt of 13.56 N-m, 11.30 N-m, 9.04 N-m, 6.78 N-m, and 4.52 N-m. At each torque level a sine sweep from 100 Hz to 120 Hz was
Figure 5.1: An increase in spring softening is observed as initial bolt torque is reduced. This frequency range encompasses the first bending mode of the structure. Each sweep was approximately 30 minutes long. The result demonstrates a change in frequency response based on initial tension in the bolt. The sweep excitation amplitude was $9.14 \times 10^{-2}$ Nm.

A separation between up and down frequency sweeps is observed at the peak. As tension decreases the system continues to soften. The natural frequency appears to shift to a lower frequency, along with a decrease in the response amplitude of the beam. At the lowest levels of bolt tension, the up and down sweep peaks appear to separate more (Figure
5.1. It is not clear why the peaks separate as bolt torque is reduced.

At the lowest torque level, 4.52 N-m, a small, heavily damped peak appears after the primary bending resonance. The higher bolt tensions do not show any signs of this second peak. Future experiments were conducted above 4.52 N-m of torque to ensure the bending mode was isolated from all other modes.

5.2 Sine Dwell and Sweep Tests

A set of experiments was performed to observe the change in the frequency response of the first bending moment. A series of sine sweeps was used to document system behavior as it varied in time. Between these sweeps the system was excited with a single frequency sinusoid. The frequency of the sinusoid was chosen from the previous sine sweeps maximum amplitude frequency.

The control variables were bolt torque and bending moment. Three bolt torque levels were used: 11.30 N-m, 9.04 N-m, and 6.78 N-m. Levels above 11.30 N-m put the bolt at risk for shear failure, and values below 6.78 N-m changed the dynamics of the system to include a heavily damped peak close to the bending moment peak (see Figure 5.1 in Section 5.1). Excitation consisted of two bending moments. The first set of experiments were completed for the $9.14 \times 10^{-2}$ Nm excitation, then for $4.57 \times 10^{-2}$ Nm.

Before starting this test the bolted specimen was separated and cleaned. It was then reassembled using two unused washers and one unused nut. The bolt was tightened until the desired torque was achieved. Data was collected from the vertical direction acceleration.
and strain located on the top surface of the flange, the strain gages on the bolt, and the return signal from the piezoelectric amplifier.

The structure was sine swept with fixed force amplitude of $9.14 \times 10^{-2} \text{ Nm}$ or $4.57 \times 10^{-2} \text{ Nm}$. Each sine sweep took 20 minutes. The sweeps were performed as up and down sweeps. A data point was taken every tenth of one Hz over a 5 Hz band that contained the amplitude response peak. A three second delay was used between frequency changes, and 5 averages with a tracking filter bandwidth of 2 Hz for each point.

A tracking filter is a band pass filter that moves in frequency with the output signal. This filter effectively removes the harmonics that may be non-linearly excited. The primary benefit from this type of filter is that the amplitude reported is the amplitude of the response frequency matching the input frequency [15]. If this type of filter were not used then the amplitude and phase would be shifted and changed from the summation of multiple signals.

Between sine sweeps the structure underwent a sine dwell at the peak amplitude frequency determined from the previous sine sweep. The same excitation level was used for the dwell as for the sweep. The dwell times varied in length based on the amount of time that had passed since reconstitution. Shortly after joint reconstitution the dwell time between sweeps averaged 10 minutes. After six sweep-dwell cycles, the average dwell time is increased to 30 minutes. Similarly after several sweep-dwell cycles of 30 minutes the dwell time was lengthened again. Data intervals were increased in this manner because the majority of the change happened in the first 60 minutes.

Each time a sine dwell was started the voltages from the strain gages on the bolt were manually recorded. When the change in strain was less than 0.1% or there was a slight
increase in voltage the test was stopped and a different torque level was selected.

When applying a new bolt torque the system was left intact. The specimen was set on blocks, the bolt was loosened then tightened to the desired level. The nut and washers were left in contact with the same surfaces. There was no attempt to clean the specimen or to remove the aluminum powder produced from the surfaces grinding together.

There are some anomalies with testing. In particular these tests were not all conducted contiguously. That is, at the end of the day the specimen was allowed to set idly. The number of minutes the structure was excited does not indicate this idle time. The number of minutes excited only counts the minutes the system was excited with the sine dwell. The result of this can be observed in Figure 5.2 at about 50-70 minutes where there is an instant drop in the response.

5.2.1 Bolt Tension

It was assumed that the bolt would self loosen up to 50% while undergoing sine excitation [10], [12] and [9]. However, the tensile load of the bolt changed less than 6%. Bolt tension dropped rapidly at first, then slowed down after the first 100 minutes of excitation as shown in Figures 5.2 and 5.3.

These cases show the two higher bolt tensions eventually reached a steady tension level. The tension dropped about 28 N for the $9.14 \times 10^{-2}$ Nm excitation with 11.30 N-m and 9.04 N-m bolt torques before leveling out and 24 N for the $4.57 \times 10^{-2}$ Nm excitation with 11.30 N-m and 9.04 N-m bolt torques. The lowest bolt tension did not follow this
Figure 5.2: Bolt tension leveled out in first 100 minutes of $9.14 \times 10^{-2}$ Nm excitation.

The 6.78 N-m torque dropped to about 37 N for the $9.14 \times 10^{-2}$ Nm case, and did not level out for the $4.57 \times 10^{-2}$ Nm case.

The bolt tension took longer to reach steady state for the lower excitation level, and the drop in tension was consistent in the two higher bolt torque cases. These results do not indicate any significant bolt-nut loosening over the duration of the test.

### 5.2.2 Change in Amplitude Response

A drop in bolt tension was observed during the testing, as shown in Section 5.2.1. It is logical to assume that the response amplitude after 100 minutes of excitation would be smaller from the reduction of tension. However, the opposite result is observed. Figures
Figure 5.3: Bolt tension continues to drop after 100 minutes with $4.57 \times 10^{-2}$ Nm excitation.
Figure 5.4: Plot of response amplitudes for $9.14 \times 10^{-2}$ Nm excitation and 11.3 N-m torque. Figures 5.4, 5.5, 5.6, 5.7, 5.8, and 5.9 show the results from the tests. Figures 5.10, 5.11, 5.12 and 5.13 show that as excitation time increased, the response amplitude of the system increased for most cases.

This result demonstrates that the 4-6% drop in bolt tension has less affect on the dynamics than other phenomena that increase amplitude. If the drop in bolt tension were the dominant effect then from Figure 5.1 it would be assumed that the amplitude would drop. The opposite is observed where the amplitude increases. This means that something more dominant is happening to the system than the drop in bolt tension. It also appears that the system responses have a reduced spring softening affect. It should be noted that when the system sat unexcited the frequency response amplitude jumped both up and down, then returned to level close to those from the day before.
Figure 5.5: Plot of response amplitudes for $9.14 \times 10^{-2}$ Nm excitation and 9.04 N-m torque.

Figure 5.6: Plot of response amplitudes for $9.14 \times 10^{-2}$ Nm excitation and 6.78 N-m torque.
Figure 5.7: Plot of response amplitudes for $4.57 \times 10^{-2}$ Nm excitation and 11.3 N-m torque.

Figure 5.8: Plot of response amplitudes for $4.57 \times 10^{-2}$ Nm excitation and 9.04 N-m torque.
Figure 5.9: Plot of response amplitudes for $4.57 \times 10^{-2}$ Nm excitation and 6.78 N-m torque.

Figure 5.10: Plot of peak response amplitudes for $4.57 \times 10^{-2}$ Nm excitation.
Figure 5.11: Plot of peak response amplitudes for $9.14 \times 10^{-2}$ Nm excitation.

Figure 5.12: Plot of peak response frequencies for $4.57 \times 10^{-2}$ Nm excitation.
Figure 5.13: Plot of peak response frequencies for $9.14 \times 10^{-2}$ Nm excitation.

### 5.2.3 Quantification of Damping Ratio and Natural Frequency

**Non-linear Modeling**

In order to compare how the system changes over the course of the experiment, model identification was performed. It was desired for the parameters to represent commonly understood dynamic features, like damping ratio and natural frequency. This model is not supposed to be a predictive model, i.e. it is to be used as a metric for changes in the system and may not to have the necessary complexity to consider the autonomous and non-autonomous parameters in the system.

The experiment demonstrated a softening spring effect, but not drifting. A cubic stiffness nonlinearity matches this effect. The modes of the system are well isolated, so a
single degree of freedom model was adopted. Initial curve fits using the Duffing equation, Equation 5.1, did not give the desired results. The Duffing equation could fit the peak, but lacked the ability to fit either side away from the peak. Cubic damping was added to the Duffing equation giving Equation 5.2.

\[
\ddot{x} + 2\zeta \omega_n \dot{x} + kx + \alpha x^3 = U \cos(\Omega t) \quad (5.1)
\]

\[
\ddot{x} + 2\zeta \omega_n \dot{x} + \gamma \dot{x}^3 + kx + \alpha x^3 = U \cos(\Omega t) \quad (5.2)
\]

In order to solve for the steady state response of the system the method of harmonic balance is used [16]. Harmonic balance expresses a periodic signal (Equation 5.3). This solution is substituted back into the governing differential equation and the coefficients of each lowest \( M + 1 \) harmonics are set to zero.

\[
x = \sum_{m=0}^{M} a_m \cos (m\omega t + m\phi) \quad (5.3)
\]

The very narrowband-tracking filter used with the sine sweeps eliminates all but the fundamental response frequency. This filtering justifies the use of a single term harmonic balance solution to identify the damping ration and natural frequency. For the single term case the phase shift is accounted for in the excitation giving

\[
\ddot{x} + 2\zeta \omega_n \dot{x} + \gamma \dot{x}^3 + kx + \alpha x^3 = U \cos(\Omega t + \phi) \quad (5.4)
\]
with assumed solution

\[ x = a \cos(\Omega t) \] (5.5)

Substituting equation 5.5 into equation 5.4 and setting the coefficients of \( \sin(\Omega t) \) and \( \cos(\Omega t) \) equal to zero gives equations 5.6 and 5.7

\[
\omega_n^2 - \Omega^2 + \frac{3}{4} \alpha a^2 = \frac{U}{a} \cos(\phi) \tag{5.6}
\]

\[
-2\zeta \Omega \omega_n - \frac{3}{4} \gamma a^2 \Omega^3 = \frac{U}{a} \sin(\phi) \tag{5.7}
\]

Squaring the equations, adding them, then solving for the amplitude \( a \) yields six possible solutions. Three of those solutions are infeasible, as the amplitude for this system is always positive. The remaining solutions are valid only for real values of amplitude. The solution was found using Mathematica, and programmed into MATLAB to perform the identification. The code containing the solution is presented in Appendix D.

It is important to note that \( a \) is a function of \( \Omega \). The linear damping ratio, \( \zeta \), is a measure of the energy dissipated from the system. The natural frequency \( \omega_n \) is the same as if the system were linear, \( \alpha = 0 \) Figure 5.14. The non-linear coefficient \( \alpha \) determines the direction the backbone curves. It also determines the shape at the corner of the hysteresis curve. When the system is spring softening, that is \( \alpha < 0 \) as in Figure 5.15, the amplitude response curve looks like it leans to the left. The hysteresis loop also begins to bend at the edges of the loop as Figures 5.15 and 5.16 demonstrate. The nonlinear damping term \( \gamma \) adjusts the width of the response peak and height. It controls the shape of the amplitude response plot in a very similar way that the linear damping ratio does.
Figure 5.14: Hysteresis loops from non-linear equation: $\alpha$ is zero.

Figure 5.15: Hysteresis loops from non-linear equation: sign of $\alpha$ is negative.
Identification of the Non-linear Differential Equation

The amplitude function can be used as a cost function to optimize the parameters \( \omega_n, \zeta, \gamma, \) and \( \alpha \) with known parameters \( a(\Omega) \) and \( \Omega \). The phase, \( \phi \), is not used in finding the amplitude of the response. MATLAB’s optimization toolbox non-linear least squared error curve fit routine was used to optimize the unknown parameters. Appendix D contains one example routine for fitting data at 6.78 N-m.

A non-linear equation was used to curve fit each of the sine sweeps done to estimate the parameters of the system (Section 5.2.3). Particular interest was paid to the natural frequency and the damping ratio. Figure 5.17 demonstrates a typical curve fit of the response amplitude using the cubic damping and cubic stiffness nonlinear equation. The measured

Figure 5.16: Hysteresis loops from non-linear equation: sign of \( \alpha \) is positive.
and identified fit well at the peak and along the higher frequency side, but separate on the lower frequency side.

Natural Frequency

Figures 5.18 and 5.19 demonstrate that the natural frequency of the system did not noticeably change over time. The results obtained from changing the initial bolt torque indicate the natural frequency should drop as the bolt tension is decreased. The reduction in tension does not seem to be a major contributor to the changes seen in the response of the structure.
Comparison of the natural frequency vs. time for $9.14 \times 10^{-2}$ Nm excitation.

Figure 5.18: Change in $\omega_n$ for $9.14 \times 10^{-2}$ Nm excitation.

Comparison of the natural frequency vs. time for $4.57 \times 10^{-2}$ Nm excitation.

Figure 5.19: Change in $\omega_n$ for $4.57 \times 10^{-2}$ Nm excitation.
System Damping

Two damping terms are present in this model, non-linear cubic damping and linear damping. The original modeling used only the linear damping term to model the peaks, but issues arose with getting the width of the peak to match the experimental data. The cubic non-linear term was then added to account for this.

The linear damping ratio, shown in Figures 5.20 and 5.21, is difficult to interpret. For the $9.14 \times 10^{-2}$ Nm excitation the damping ratio for the higher initial bolt torques appears to drop, where as for the lowest bolt torque increases. When the non-linear damping ratios, shown in Figures 5.22 and 5.23, are considered it becomes apparent that the linear and non-linear terms are correlated to each other.

The lowest bolt torque for the $9.14 \times 10^{-2}$ Nm excitation case demonstrates this correlation well. The linear damping starts low and increases, where as the non-linear ratio starts high and drops. The other cases show this same behavior. Upon reviewing the cost function used for curve fitting there seems to be a duality in the solution where either the non-linear term is higher and the linear lower or visa-versa. The solution is highly dependent on the initial guess for these parameters, leading to the jagged fits for the lower amplitude excitation (Figures 5.21 and 5.23). The non-linear term is necessary because results obtained without the term did not converge to a solution that matched the data.
Figure 5.20: Change in $\zeta$ for $9.14 \times 10^{-2}$ Nm excitation.

Figure 5.21: Change in $\zeta$ for $4.57 \times 10^{-2}$ Nm excitation.
Figure 5.22: Change in cubic damping coefficient, $\gamma$, for the $9.14 \times 10^{-2}$ Nm excitation.

Figure 5.23: Change in cubic damping coefficient, $\gamma$, for the $4.57 \times 10^{-2}$ Nm excitation.
Non-linear Stiffness

From a visual examination of Figure 5.1 it might be assumed that as the bolt tension is reduced the system non-linearity becomes softer. That is, the non-linear coefficient would have a greater magnitude with negative sign. Figure 5.25 and 5.24 demonstrate that the non-linear coefficient does not follow this trend. Though $\alpha$ for the 6.78 N-m case has the greatest negative magnitude, the 11.30 N-m case does not have the smallest negative value for $9.14 \times 10^{-2}$ Nm excitation.

The model may not have the accuracy to compare between excitation levels. It would be expected that the initial sine sweep for each torque level should curve fit almost the same. The curve fit of the 11.30 N-m bolt torque for $9.14 \times 10^{-2}$ Nm and $4.57 \times 10^{-2}$ Nm differ by 70% initially.
Figure 5.24: Change in $\alpha$ for the $9.14 \times 10^{-2}$ Nm excitation.

Figure 5.25: Change in $\alpha$ for the $4.57 \times 10^{-2}$ Nm excitation.
5.3 Hysteresis loops

A study was done to consider the change in the shape of the hysteresis loop based on torque, time, frequency, and excitation level. The specimen was cleaned before the test started. The torque levels and excitation levels were the same used for the sine dwell and sweep tests (Section 5.2). Unlike those tests, intermediate recordings during sine dwell were not taken. That is, data was only taken at the beginning of the test and at the end of the test. The vertical-direction acceleration and strain data were recorded from the top of the flange. Data was also recorded from the lower flange strain gage and the upper strain gage on the un-flanged side.

A sine sweep was performed from 110 Hz to 120 Hz using the same setup as Section 5.2. The sine sweep took 40-minutes to complete. Following the sweep the structure was set to dwell at resonance with the same excitation used for the sweep. Five thousand time averages were recorded. Another time history was recorded at 65 Hz. This frequency was chosen based on isolation from system support interactions (See Section 3.2). Interference from lights or supplied electricity (120 Hz and 60 Hz) was insignificant. The frequency was sufficiently far away from system resonances to make the low amplitude non-linear phenomena observable.

A resonant frequency sine dwell was also recorded, though the amplitude of the response made observing the non-linearly excited harmonics difficult. The system is allowed to dwell for 36 hours, then another set of impact, sweeps, and dwells are recorded. For analysis the measured strain from the specimen mounted strain gages was used to find moment collocated with the acceleration measured. The acceleration was integrated twice to
indicate the displacement of the system.

A direct comparison between loops taken at resonance and loops taken at 65 Hz is difficult, as shown in Figure 5.26. For this reason the results will be presented based on frequency of excitation and broken down further based on excitation level.

5.3.1 Hysteresis curves at resonance

It was hoped that results taken at resonance would show non-linear harmonics and an increase in energy dissipation because of the large response amplitude. This was not the case. In Figure 5.27 the hysteresis curves are dominated by the primary resonant frequency obscuring the non-linear effects of the system. However, these results do indicate a reduced amplitude response with reduced initial bolt torque. This is in agreement with the bolt torque frequency sweeps taken separately.
(a) Lower excitation, 6.78 N-m bolt torque.
(b) Higher excitation, 6.78 N-m bolt torque.
(c) Lower excitation, 9.04 N-m bolt torque.
(d) Higher excitation, 9.04 N-m bolt torque.
(e) Lower excitation, 10.3 N-m bolt torque.
(f) Higher excitation, 10.3 N-m bolt torque.

Figure 5.27: Subtle features are hidden in the hysteresis curves at resonance.
5.3.2 Hysteresis curves at 65 Hz

The curves recorded at 65Hz are different in appearance than those at resonance. The hysteresis curves at $4.57 \times 10^{-2}$ Nm excitation, Figures 5.28, 5.30, and 5.32, have a crescent like shape. The hysteresis curves at $9.14 \times 10^{-2}$ Nm, Figures 5.29, 5.31, and 5.33, excite an additional frequency. This non-linearly excited frequency correlates to the fifth bending mode, shown in Figure 2.6.

The hysteresis curves from 6.78 N-m of bolt torque, Figures 5.28 and 5.29, show a clockwise rotation in the hysteresis curve. The clockwise rotation denotes that for the same input more output is measured. At 9.04 N-m this changes to a slight counter clockwise rotation (Figures 5.30 and 5.31). When the bolt was given a torque of 11.30 N-m there was no noticeable rotation in the hysteresis curve (Figures 5.32 and 5.33).

Again, the results here do not contradict results from other experiments in this thesis. The displacements between bolt torque levels do not change by much, but this is to be expected since the specimen is being excited away from resonance. The introduction of harmonics from single sine excitation indicates a non-linear system.

5.4 Qualitative Observations

The response amplitude of the jointed specimen at the first resonance was not perceivable to the naked eye. The vibration of the specimen at this resonance could be felt. If the specimen were in close proximity to the naked ear (within a foot) and the excitation was greater than $6.85 \times 10^{-2}$ Nm then an audible hum could be heard. This is a noticeable
Figure 5.28: Hysteresis at 65 Hz with $4.57 \times 10^{-2}$ Nm excitation and 6.78 N-m torque.

Figure 5.29: Hysteresis at 65 Hz with $9.14 \times 10^{-2}$ Nm excitation and 6.78 N-m torque.
Figure 5.30: Hysteresis at 65 Hz with $4.57 \times 10^{-2}$ Nm excitation and 9.04 N-m torque.

Figure 5.31: Hysteresis at 65 Hz with $9.14 \times 10^{-2}$ Nm excitation and 9.04 N-m torque.
Figure 5.32: Hysteresis at 65 Hz with $4.57 \times 10^{-2}$ Nm excitation and 11.3 N-m torque.

Figure 5.33: Hysteresis at 65 Hz with $9.14 \times 10^{-2}$ Nm excitation and 11.3 N-m torque.
difference from the response of the control specimen. The jointed specimen did not appear to rock or swing while being excited by the piezoelectric patches, and the support wires felt still.

Depending on the level and frequency of excitation the thin flanges appeared to flap away from the tongued portion, then slap against it. This phenomenon was not perceivable at the frequencies used for testing the specimen. The compliance of the flanges also made joint assembly more difficult. The assembly was visually inspected to make sure the joint had been tightened without inadvertently bending the flanges. The bolt tension from using the calibrated torque method was very dependent on the flange fastening position.

When the joint was separated a coating of metal dust was present between the tongue and flanges. When cleaned using brake cleaner, the surface was polished and pitted (Figures 5.35 & 5.34). The polishing and pitting was not visible when first machined.

The wear in the center of the surfaces might be from a small ridge on the inside of the flanges. This ridge was formed during manufacturing, though it wasn’t noticed until shortly after initial experiments began.
Figure 5.34: Wear patterns down the center of the flanges and scratching/polishing marks from washers.
Figure 5.35: Polishing and pitting on one side of the tongue.
Conclusions

6.1 Joint Dynamics

Several experiments to characterize the dominant dynamic affects of adding a bolted joint have been performed. A control specimen and a jointed specimen were tested. Two types of tests were done. Frequency sweeps were performed to look at the effect of the joint in the frequency domain and hysteresis curves were taken to examine the energy dissipated per cycle. The difference in the response between the two specimens was attributed to the effect of the bolted joint.

The effect of initial bolt tension can be a major factor in joint behavior. The resonant frequency of the first bending mode shifted to a lower frequency as the initial bolt tension was reduced. Bolt tension dropped as the system was excited. The bolt tension reduced quickly just after tightening. This drop could be attributed to many different phenomena like embedding and creep though no particular phenomena have been isolated. This reduction does not appear to be the dominant effect in the dynamics of the system. In some cases the response amplitude increased 11-16% yet the bolt tension decreased up to 6%.
Two possibilities exist for this increase in magnitude. The first is a reduction in the softening effect, effectively unbending the peak. The second is a decrease in the damping of the joint. It could be reasoned that the linear stiffness could change but this term is directly related to linear system natural frequency, which does not change. The curve fit data using cubic damping and cubic stiffness does seem to suggest both the damping decrease and a softening effect decrease, but the results are unreliable.

Initial conditions made curve fitting difficult. Many local minima existed making it difficult to achieve a consistent solution. It also appears that the linear and non-linear damping coefficients are correlated to each other. When one is identified as lower, the other is identified as higher. The system non-linearity appears to need more or different terms than cubic stiffness and cubic damping to describe them.

From sine dwell hysteresis curves its evident the joint response does not behave in a manner predicted by joint models. The loops appear to be spring hardening in positive displacement and spring softening in negative displacement. The control specimen exhibited strange hysteretic behavior. The first time data was taken the system exhibited a tri-linear response, then when the system was literally flipped over and testing again the tri-linear region disappeared. It’s unclear why this happened, as is unclear why the jointed specimen’s hysteresis looks like a crescent.
6.2 Future Work

The initial thrust of this work was to adapt a joint model for non-deterministic approaches in predicting dynamic motion in bolted joints. This goal was not feasible with the current models found in the cited literature. More basic research needs to be performed to understand the physics involved at the joint interface. Finite elements can help but only in limited ways. A highly refined mesh of the joint element can give insight into the effect of geometry. Finite elements programs are limited in that they cannot accurately model the pitting and polishing seen in the experiment. Additional lab testing will need to be done.

If a model could capture the physics with some consistency then the original purpose of this work can be continued. Several washers were to be used as new joint surfaces. It is advised that this testing be automated. The automation would keep testing consistent between trials. Consistency would reduce operator variability. If a joint model were able to capture the dynamics of the joint, then its parameters could be identified and used as a metric. The statistical basis for the variability would be based on these identified parameters.
Bibliography


Preparing Specimen

To insure accurate, repeatable measurements the test specimen needs to be configured in a systematic fashion. This procedure focuses on the setup of the bolt-equipped apparatus, though the single piece structure is similar. Setting up the structure begins with the collection of the necessary parts, assembling the structure, and an initial test for proper assembly.

A.1 Collect Parts

Make sure the following instrumentation and parts are available

1. computer with DAQ
2. strain gage amplification systems
3. piezoelectric patch/accelerometer amplifiers
4. instrumented bolt
5. labeled washers and nut
6. both halves of instrumented jointed specimen
7. test frame
8. stainless steel wire
A.2 Assemble Specimen

Prior to assembly it is assumed that all transducers were calibrated and that the sensitivities are known. It is also assumed that transducers have been applied. If not see appendix C.

1. Clean a work area large enough to work with the test specimen. Examine the transducers and wires for any physical damage or looseness. Make sure that all transducer wires are secured. Move the two pieces of the test specimen together so that the lap joint can be secured with a bolt. The integral masses should rest on a stand so that the specimen can be hung with little movement.

2. Put 1 washer onto the instrumented bolt and insert the bolt into the lap joint. Put a washer followed by a nut on the exposed thread of the bolt and run the nut up to the specimen. *DO NOT* cinch the nut. The bolt should be able to ‘jiggle’ in the joint.

3. Make a ‘splint’ for the joint. Put one piece of flat stock on each side (parallel to hole) of the joint and hold with 2 clamps. Use caution to not rub, scrape, or pull the transducers or their wires.

4. Balance the strain gage amplifiers. Make sure the bolt is still loose enough to not bias the load.

5. Hold the *head of the bolt* with a wrench while using a torque wrench to tighten the nut. The head should remain stationary while the nut is rotated. *Do not exceed 20 lbs-ft of torque.* Apply the predetermined torque. Let the system set for 3-4 seconds, loosen nut and re-tighten.

6. Hang the specimen by looping stainless steel line around mounting bolts and through test frame. Attach any remaining transducers, and connect all the instrumentation to
the equipment. Try to isolate wires from noise sources and secure them to avoid vibration during testing.

7. remove the supports used to hold up the specimen. It should hang freely. Check to make sure the specimen is electrically isolated. Make sure all the equipment is on and has warmed up.

A.3 Testing the Installation

At this point the test rig and specimen are ready to be tested for proper installation.

1. Start the DAQ system and check for signals on all connected channels. Look at each channel and determine if the signal is appropriate, i.e. there are no unexplained spikes or bias voltages for the current conditions.

2. Input a signal\(^1\) to the piezoelectric patches and check that the strain gages on the specimen detect the change.

3. If wires have been moved or put on for the first time check to see that their effect has been minimized. Do this by loosening the wire and exciting the system at resonance. Hold the wire between your fingers at different locations and compare the resulting amplitude. Locations with higher amplitudes have less damping and are desirable. Now look for locations to fix the wire too.

4. Use the measured load from the bolt and compare with the estimated torque load. Appendix B has the associated data and equations.

5. If everything makes sense and checks out then the experimentation begins.

---

\(^1\)Only do steps 2 and 3 if the time history is not important for that run.
Torque to preload calculations

Table B.2 has the properties of aluminum needed to calculate the load. $\mu$ is listed for Al on Al contact. Figure B.1 shows the dimensions needed and Table B.1 has dimensions for 1/2-20 UNF thread.

![Figure B.1: Thread dimensions illustrated.](image)

The equation for estimated torque from a desired load is shown in B.

$$T = \left[ \left( \frac{d_m}{2d} \right) \left( \frac{\tan \lambda + \mu \sec \alpha}{1 - \mu \tan \lambda \sec \lambda} \right) + 0.625 \mu_c \right] F_i d \quad (B.1)$$

$\mu_c$ and $\mu$ represent the coefficient of friction between the head and body and the bolt and nut threads, respectively. Since this test case has only one material, $\mu_c = \mu$. Often the

Table B.1: Dimension information for 1/2-20 UNF thread.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$d$</td>
<td>0.500 in</td>
</tr>
<tr>
<td>$dm$</td>
<td>0.4374 in</td>
</tr>
<tr>
<td>$P$</td>
<td>0.050 in</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>30°</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>2.08°</td>
</tr>
</tbody>
</table>
Table B.2: material properties of AL 7075-T7351.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$ (Young’s Modulus)</td>
<td>72 GPa</td>
</tr>
<tr>
<td>$\sigma_y$</td>
<td>435 MPa</td>
</tr>
<tr>
<td>$D$ (Density)</td>
<td>2810 Kg/m$^3$</td>
</tr>
<tr>
<td>$\mu$ (Coeff. of friction)</td>
<td>0.40</td>
</tr>
</tbody>
</table>

The majority of equation B is given the variable $K$ because standard thread sizes have uniform values. $K$ and $T$ are then given as

\[
K = \left(\frac{d_m}{2d}\right) \left(\frac{\tan \lambda + \mu \sec \alpha}{1 - \mu \tan \lambda \sec \lambda}\right) + 0.625 \mu_c
\]

\[
T = K F_i d
\]

Since the data used for $\mu$ has been estimated from uncertain Internet sources, it would be advisable to figure out the preload and estimate the $K$ value for the system. It is also important to realize that the equation for torque to load is an estimate. The bolt torque method has been found to be off as much as $\pm 40\%$. 

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86
Application of transducers

Heisenberg was a German scientist who worked with quantum mechanics. He made a principle involving atoms stating “The more precisely the position is determined, the less precisely the momentum is known in this instant, and vice versa.” Every piece of equipment added to a test article changes its dynamics, making the measurement not the system in question, but the modified system. The best method for measurements requires instrumentation that does not change the system. This is either impractical or impossible, so methods have been introduced to reduce instrumentation impact and maintain precision.

C.1 Foil strain gages

Application of foil strain gages can be a tedious procedure. If care is taken then the gage life will be lengthened and its precision will be maintained. A recommended procedure exists in Bulletin 309D from Measurement Group. Gages are tested after installation for isolation and proper resistance. The use of M-bond glue dictates a max life span for the gage installation of one year.

C.2 Piezoelectric Patches

Piezoelectric patches were purchased from Ferro-perm for exciting the structure. They measure 2.54 cm x 2.54 cm x 0.05 cm using material type Pz-29. To make the installation
more uniform a simple jig was used. The jig was machined on a vertical mill to space the edge of the piezoelectric patch 2 inches from the closest integral mass.

The patches were attached to the structure using CircuitWorks®Conductive Epoxy, a product made by Chemtronics®. The epoxy is designed for high strength conductive bonding. During installation a hot air based paint stripper was used to heat the surrounding structure. The aluminum conducted the heat to the epoxy. In this manner the epoxy set faster, harder, and with optimal conductivity but without damaging the piezoelectric patches.

The patches were installed with the positive face exposed on the top and bottom of the beam. When exciting the extensional frequency the patches were in parallel and the voltage dropped across both upper and lower patches was equal to the excitation voltage. When exciting the beam in bending, the patches were put into series

C.3 Piezoelectric Strain Gages

PCB model 740B02 strain gages were applied at 4 locations on the test specimen. They were positioned so as to measure the axial strain in the specimen at almost the same location as the accelerometers were measuring acceleration. The specimen was prepared by sanding the surface with 400 grit sandpaper. The surface was then cleaned using common brake cleaner. This removed all the surface oils and particles from sanding. Using a ruler and straight edge a rectangle was drawn where the transducer was going to be mounted.

A piece of clear tape was placed over the sensor, with over an inch of tape over hanging the sensor. Loctite 420 was applied to the sensor and a razor blade was used to smooth the glue over the bottom of the sensor. The sensor was lined up with the previously made rectangle and pressed down. The tape was used to secure the transducer while the glue cured.

After 8 hours the sensor was tested for conductivity to the specimen. If the resistance was greater than 10 MΩ the sensor was considered successfully installed. If not the senor
was removed and the process repeated.

C.4 Piezoelectric Accelerometers

PCB accelerometers models 356A16 and 356A24 were mounted in 4 locations, physically close to 4 strain gages. The mounting for the accelerometers is the similar to the process described for the strain gages in Appendix C.3 but the surface of the specimen was not sanded.
Appendix D

Coded Harmonic Balance

Function for amplitude response: “har3.m”

%Harmonic balance 3 (har3) uses a 1 term harmonic balance solution to the
%SDOF GDEQ with both cubic damping and cubic stiffness. This file is
%intended to be used with a non-linear least squares curve fit routine.

function amp=har3(par,o)

w=par(1)*1e2;
z=par(2)*1e-3;
al=par(3)*1e11;
g=par(4)
P=par(5)*1e-2;

%The solution uses a lot of repetituous code. To reduce number crunching
%these repetitions have been labeled at p1, p1a, p2 and are used in
%the following code.
pl=alˆ2*gˆ2*o.ˆ6;
p1=3456*alˆ3*o.ˆ6 - 31104*al*gˆ2*o.ˆ12 + 34992*alˆ4*Pˆ2 + ...
69984*alˆ2*gˆ2*o.ˆ6*Pˆ2 + 34992*alˆ4*o.ˆ12*Pˆ2 + 10368*alˆ3*o.ˆ4*w*Pˆ2 + ...
93312*al*o.ˆ2 - 10368*alˆ3*o.ˆ2*w - 93312*al*o.ˆ2*w - 8*w*P + ...
3456*alˆ3*w*P + 31104*al*gˆ2*o.ˆ6 - 10368*alˆ2*g*o.ˆ8*w*z + ...
62208*gˆ3*o.ˆ14*w*z + 207360*alˆ2*g*o.ˆ6*w*3*z - 124416*al*o.ˆ12*w*3*z - ... 
103680*al*o.ˆ2*g*o.ˆ4*w*5*z + 62208*gˆ3*o.ˆ10*w*5*z - 124416*alˆ3*o.ˆ4*w*2*z + ...
207360*al*o.ˆ2*g*o.ˆ8*w*4*z - 124416*al*o.ˆ3*o.ˆ2*w*4*z - ...
207360*al*o.ˆ2*g*o.ˆ8*w*4*z + 248832*al*o.ˆ2*g*o.ˆ6*w*3*z + ...
27648*gˆ3*o.ˆ12*w*3*z -

p2=(p1a+(p1a.ˆ2+4*(-576*(al*o.ˆ2-al*o.ˆ2-wˆ2-2*g*o.ˆ4*w*z).ˆ2 ...
+ 432*(p1)*a.ˆ2*(o.ˆ4-2*o.ˆ2*wˆ2+4*o.ˆ2*wˆ2*zˆ2)).ˆ3/((1/2)).ˆ(1/3);

%Amplitude is determined from a sixth order polynomial, which breaks down
%to 3 possible solutions (amplitude is positive, eliminating 3 solutions)

a1=sqrt(8*al*o.ˆ2./(9*pl) - 8*al*o.ˆ2./pl - 16*o.ˆ4*w/z/((9*pl)... 
+ 16*w*(1/3)*al*o.ˆ2*o.ˆ4/((3*p1)*p2)...
- 16*o.ˆ2*(1/3)*o.ˆ2*o.ˆ10/((p1)*p2)...
- 32*o.ˆ2*(1/3)*al*o.ˆ2*o.ˆ2*w/((3*p1)*p2)...
+ 32*o.ˆ2*(1/3)*g*2*o.ˆ8*w*2./((p1)*p2)...
+ 16*o.ˆ2*(1/3)*al^2*w*4./((3*p1)*p2)...)
\[-16^2 \cdot (1/3) \cdot g^2 \cdot o.\cdot ^{6}w^4./ (p_1. \cdot p_2) \]  
\[-256^2 \cdot (1/3) \cdot a_1 \cdot g^2 \cdot o.\cdot ^{6}w^4z./ (3^3 \cdot p_1. \cdot p_2) \]  
\[+ 256^2 \cdot (1/3) \cdot a_1 \cdot g^2 \cdot o.\cdot ^{4}w^3z./ (3^3 \cdot p_1. \cdot p_2) \]  
\[-4^2 \cdot (1/3) \cdot a_1 \cdot ^{2}o.\cdot ^{2}w^2z^2./ (p_1. \cdot p_2) \]  
\[+ 64^2 \cdot (1/3) \cdot g^2 \cdot o.\cdot ^{8}w^2z^2./ (3^3 \cdot p_1. \cdot p_2) \]  
\[+ (p_2./ (27^2 \cdot (1/3) \cdot p_1)); \]

\[a_2 = \sqrt{8 \cdot a_1 \cdot o.\cdot ^{2}/ (9 \cdot p_1) - 8 \cdot a_1 \cdot w^2./ (9 \cdot p_1) - 16 \cdot g \cdot o.\cdot ^{4}w^4z./ (9 \cdot p_1) \]  
\[-8 \cdot (1/3) \cdot a_1 \cdot ^{2}o.\cdot ^{6}w^4./ (3^3 \cdot p_1. \cdot p_2) \]  
\[+ 8 \cdot (1/3) \cdot a_1 \cdot ^{2}o.\cdot ^{4}w^3z./ (3^3 \cdot p_1. \cdot p_2) \]  
\[-8 \cdot (1/3) \cdot a_1 \cdot ^{2}o.\cdot ^{2}w^2z^2./ (3^3 \cdot p_1. \cdot p_2) \]  
\[+ 8 \cdot (1/3) \cdot ^{2}o.\cdot ^{8}w^2z^2./ (3^3 \cdot p_1. \cdot p_2) \]  
\[-8 \cdot (1/3) \cdot a_1 \cdot ^{2}o.\cdot ^{6}w^4z./ (3^3 \cdot p_1. \cdot p_2) \]  
\[+ 8 \cdot (1/3) \cdot ^{2}o.\cdot ^{4}w^3z./ (3^3 \cdot p_1. \cdot p_2) \]  
\[-8 \cdot (1/3) \cdot ^{2}o.\cdot ^{2}w^2z^2./ (3^3 \cdot p_1. \cdot p_2) \]  
\[+ 8 \cdot (1/3) \cdot ^{2}o.\cdot ^{8}w^2z^2./ (3^3 \cdot p_1. \cdot p_2) \]  
\[+ 256 \cdot (1/3) \cdot ^{2}o.\cdot ^{8}w^2z^2./ (3^3 \cdot p_1. \cdot p_2) \]  
\[-8 \cdot (1/3) \cdot ^{2}o.\cdot ^{6}w^4z./ (3^3 \cdot p_1. \cdot p_2) \]  
\[+ 8 \cdot (1/3) \cdot ^{2}o.\cdot ^{4}w^3z./ (3^3 \cdot p_1. \cdot p_2) \]  
\[-8 \cdot (1/3) \cdot ^{2}o.\cdot ^{2}w^2z^2./ (3^3 \cdot p_1. \cdot p_2) \]  
\[+ 8 \cdot (1/3) \cdot ^{2}o.\cdot ^{8}w^2z^2./ (3^3 \cdot p_1. \cdot p_2) \]  
\[+ (p_2./ (27^2 \cdot (1/3) \cdot p_1)); \]

\[a_3 = \sqrt{8 \cdot a_1 \cdot o.\cdot ^{2}/ (9 \cdot p_1) - 8 \cdot a_1 \cdot w^2./ (9 \cdot p_1) - 16 \cdot g \cdot o.\cdot ^{4}w^4z./ (9 \cdot p_1) \]  
\[-8 \cdot (1/3) \cdot a_1 \cdot ^{2}o.\cdot ^{6}w^4./ (3^3 \cdot p_1. \cdot p_2) \]  
\[+ 8 \cdot (1/3) \cdot a_1 \cdot ^{2}o.\cdot ^{4}w^3z./ (3^3 \cdot p_1. \cdot p_2) \]  
\[-8 \cdot (1/3) \cdot a_1 \cdot ^{2}o.\cdot ^{2}w^2z^2./ (3^3 \cdot p_1. \cdot p_2) \]  
\[+ 8 \cdot (1/3) \cdot ^{2}o.\cdot ^{8}w^2z^2./ (3^3 \cdot p_1. \cdot p_2) \]  
\[-8 \cdot (1/3) \cdot a_1 \cdot ^{2}o.\cdot ^{6}w^4z./ (3^3 \cdot p_1. \cdot p_2) \]  
\[+ 8 \cdot (1/3) \cdot ^{2}o.\cdot ^{4}w^3z./ (3^3 \cdot p_1. \cdot p_2) \]  
[-8 \cdot (1/3) \cdot ^{2}o.\cdot ^{2}w^2z^2./ (3^3 \cdot p_1. \cdot p_2) \]  
\[+ 8 \cdot (1/3) \cdot ^{2}o.\cdot ^{8}w^2z^2./ (3^3 \cdot p_1. \cdot p_2) \]  
\[+ 256 \cdot (1/3) \cdot ^{2}o.\cdot ^{6}w^4z./ (3^3 \cdot p_1. \cdot p_2) \]  
\[-8 \cdot (1/3) \cdot ^{2}o.\cdot ^{4}w^3z./ (3^3 \cdot p_1. \cdot p_2) \]  
[-8 \cdot (1/3) \cdot ^{2}o.\cdot ^{2}w^2z^2./ (3^3 \cdot p_1. \cdot p_2) \]  
\[+ 8 \cdot (1/3) \cdot ^{2}o.\cdot ^{8}w^2z^2./ (3^3 \cdot p_1. \cdot p_2) \]  
\[-8 \cdot (1/3) \cdot ^{2}o.\cdot ^{6}w^4z./ (3^3 \cdot p_1. \cdot p_2) \]  
\[+ 8 \cdot (1/3) \cdot ^{2}o.\cdot ^{4}w^3z./ (3^3 \cdot p_1. \cdot p_2) \]  
[-8 \cdot (1/3) \cdot ^{2}o.\cdot ^{2}w^2z^2./ (3^3 \cdot p_1. \cdot p_2) \]  
\[+ 8 \cdot (1/3) \cdot ^{2}o.\cdot ^{8}w^2z^2./ (3^3 \cdot p_1. \cdot p_2) \]  
\[+ (p_2./ (27^2 \cdot (1/3) \cdot p_1)); \]

% Amplitude is also real, so not all of these solutions are valid for the response frequencies used.

z1 = find(imag(a1) <= 0.0000001 & imag(a1) >= -0.0000001);

z2 = find(imag(a2) <= 0.0000001 & imag(a2) >= -0.0000001);

z3 = find(imag(a3) <= 0.0000001 & imag(a3) >= -0.0000001);

% Initialize amplitude array
amp = zeros(size(o));

% Fill array with possible solutions
amp(z1) = a1(z1);
amp(z2) = a2(z2);
\[ \text{amp}(z_3) = a_3(z_3); \]

Script for curve fitting amplitude response: “data60.m”

```matlab
file=[' 60inlbs_1day_0min.vss';
   ' 60inlbs_1day_10min.vss';
   ' 60inlbs_1day_20min.vss';
   ' 60inlbs_1day_32min.vss';
   ' 60inlbs_1day_48min.vss';
   ' 60inlbs_1day_58min.vss';
   ' 60inlbs_1day_68min.vss';
   ' 60inlbs_1day_80min.vss';
   ' 60inlbs_7day_0min.vss';
   ' 60inlbs_7day_10min.vss';
   ' 60inlbs_7day_20min.vss';
   ' 60inlbs_7day_32min.vss';
   ' 60inlbs_7day_48min.vss';
   ' 60inlbs_7day_58min.vss';
   ' 60inlbs_7day_68min.vss';
   ' 60inlbs_7day_80min.vss';
   ' 60inlbs_8day_0min.vss';
   ' 60inlbs_8day_38min.vss';
   ' 60inlbs_8day_68min.vss';
   ' 60inlbs_8day_113min.vss';
   ' 60inlbs_14day_0min.vss';
   ' 60inlbs_14day_60min.vss';
   ' 60inlbs_15day_0min.vss';
   ' 60inlbs_15day_62min.vss';
   ' 60inlbs_15day_122min.vss';
   ' 60inlbs_15day_182min.vss';
   ' 60inlbs_16day_0min.vss';
   ' 60inlbs_16day_14min.vss';
   ' 60inlbs_16day_24min.vss';
   ' 60inlbs_16day_34min.vss';
   ' 60inlbs_16day_44min.vss';
   ' 60inlbs_16day_54min.vss';
   ' 60inlbs_16day_64min.vss';
   ' 60inlbs_16day_74min.vss';
   ' 60inlbs_16day_84min.vss';
   ' 60inlbs_16day_94min.vss';
   ];
T=[0 10 20 32 48 68 80 80.1 110 220 220.1 258 288 333 333.1 393 393.1 455 515 575 575.1];
ub=[];
lb=[];
x0=[6.735 5 -12 10 61];
options=optimset('TolFun',1e-10,'LargeScale','off','TolX',1e-10, ...
    'maxFunEvals',3e4,'MaxIter',3e4);
for ind=1:size(file,1)
    load(file(ind,:),'-mat')
    %Curve fit duffing using Harmonic balance method type 2 - up sweep
    [x, resu]=lsqcurvefit(@har3,x0,Fvec(51:101)*2*pi,...
        abs(XferDat(51:101,2)./-(Fvec(51:101)*2*pi).^2*1000/1.057*9.75)*1000);
    %Curve fit duffing down sweep
    [y, resd]=lsqcurvefit(@har3,x0,Fvec(51:-1:1)*2*pi,...
        abs(XferDat(51:-1:1,2)./-(Fvec(51:-1:1)*2*pi).^2*1000/1.057*9.75)*1000);
    if ind>1
        frfu=[frfu XferDat(51:101,2)./-(Fvec(51:101)*2*pi).^2*1000/1.057];
        frfd=[frfd XferDat(51:-1:1,2)./-(Fvec(51:-1:1)*2*pi).^2*1000/1.057];
        resup=[resup; resu];
        resdn=[resdn; resd];
        frequ=[frequ Fvec(51:101)];
        freqd=[freqd Fvec(51:-1:1)];
        wduffu=[wduffu;x(1)*1e2];
        zduffu=[zduffu;x(2)*1e-3];
        wduffd=[wduffd;y(1)*1e2];
        zduffd=[zduffd;y(2)*1e-3];
        alphau=[alphau;x(3)*1e11];
        alphad=[alphad;y(3)*1e11];
        gammau=[gammau;x(4)*1e2];
        gammad=[gammad;y(4)*1e2];
    else
        frfu=[XferDat(51:101,2)./-(Fvec(51:101)*2*pi).^2*1000/1.057];
        frfd=[XferDat(51:-1:1,2)./-(Fvec(51:-1:1)*2*pi).^2*1000/1.057];
        resu=[resu];
        resd=[resd];
    end
end
```

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frequ=[Fvec(51:101)];
freqd=[Fvec(51:-1:1)];
wduffu=[x(1)*1e2];
zduffu=[x(2)+e-3];
wduffd=[y(1)*1e2];
zduffd=[y(2)*1e-3];
alphau=[x(3)*1e11];
alphad=[y(3)*1e11];
gammau=[x(4)*1e2];
gammad=[y(4)*1e2];
x1=x;
end

figure
plot(T,zduffu,'b',T,zduffd,'g--')
legend('Up Sweep','Down Sweep')
title(['\zeta vs time excited for 6.78 N-m torqued bolt, 100% excitation'])
xlabel('Time excited (min)'
ylabel('Damping Ratio')

figure
plot(T,wduffu/(2*pi),T,wduffd/(2*pi), 'g--')
legend('Up Sweep','Down Sweep')
title(['\omega_n vs time excited for 6.78 N-m torqued bolt, 100% excitation'])
xlabel('Time excited (min)'
ylabel('\omega_n (Hz)')

figure
plot(T,alphau,'b',T,alphad,'g--')
legend('Up Sweep','Down Sweep')
title('The non-linear coefficient \alpha vs. time excited, 6.78 N-m torque, 100% excitation')
xlabel('Time Excited (min)'
ylabel('\alpha')

figure
plot(T,gammau,'b',T,gammad,'g--')
legend('Up Sweep','Down Sweep')
title('Cubic damping coefficiant \gamma vs. time excited, 6.78 N-m torque, 100% excitation')
xlabel('Time Excited (min)'
ylabel('\gamma')

figure
plot(frequ(:,1),abs(fr(1,1))*.75,'g--',frequ(:,1),har3(x1,frequ(:,1)*2*pi)/1000,'b')
legend('Measured response','Identified response')
title('Comparison of measured to identified amplitude response at 0 min of excitation , 6.78 N-m torque, 100% excitation')
xlabel('Frequency (Hz)'
ylabel('Amplitude (m)')