INNOVATIVE FORCED RESPONSE ANALYSIS METHOD APPLIED TO A
TRANSONIC COMPRESSOR

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By

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ABSTRACT


A set of inlet guide vane (IGV) unsteady surface pressure measurements of a transonic compressor is presented. Using a flexible pressure sensor array, unsteady IGV suction-surface and pressure-surface pressures are acquired for six spanwise by five chordwise locations for various speed lines and throttle settings. Measurements from this sensor array are used to investigate unsteady vane/blade interaction aeromechanical forcing functions in a modern, highly loaded compressor stage. A significant effect is shown on the unsteady forced response of the IGV with changes in compressor operating point and IGV/rotor axial spacing for various span and chord locations. In particular, variations in the compressor operating point (i.e., mass flow rate and pressure ratio) cause change in both the magnitude and phase of the forced response, with the near-stall operating point producing the highest response. Changes in the axial spacing between the IGV and rotor rows from 12% to 26% of the IGV chord resulted in a 50% reduction in the magnitude of the forced response. A significant variation in the forced response with span is noted, especially at the 5% span location where the rotor relative flow is subsonic. In this region, changes in the operating point and axial spacing had a negligible effect on the forced response of the IGV.
An innovative data reduction/analysis method is presented to quantify and statistically analyze the degree of blade-to-blade variations in the measured aerodynamic forcing functions obtained by turbomachinery experimentation. This method is used to analyze experimental data of IGV surface unsteady pressure response due to the aerodynamic forcing function produced by the downstream transonic compressor rotor with (1) factory-whole blades and (2) trimmed (blended) blades resulting from the repair of crack damage on two of the rotor blades. Results from the variation metric and $\ell_2$-norm analysis indicate that the scaled metric possesses large magnitude change versus blade index for the trimmed rotor compared to that of the untrimmed rotor, with the largest values occurring near the trimmed blades. Each method is nearly always able to detect the trimmed blades. Using the distance between cluster centroids from the $K$-means cluster analysis as a metric of variation within each rotor, cluster distances increased by as much as a factor of 4 for the trimmed rotor compared to the untrimmed rotor. Therefore, correctly identifying the trimmed rotor data as having a significantly higher amount of blade-to-blade variability. Finally using the cluster distance as a goodness parameter for variability, the non-trimmed data was investigated for trends with changes in compressor operating conditions. This analysis showed an increase in blade-to-blade variability with increases in the compressor flow rate. Therefore, this data reduction/analysis method has the potential to be utilized as an indicator of the compressor operating point for control methods.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Chapter</th>
<th>page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>1.1 Motivation</td>
<td>1</td>
</tr>
<tr>
<td>1.2 Previous Research</td>
<td>2</td>
</tr>
<tr>
<td>1.3 Present Research</td>
<td>5</td>
</tr>
<tr>
<td>2. EQUIPMENT</td>
<td>7</td>
</tr>
<tr>
<td>2.1 Research Facility</td>
<td>7</td>
</tr>
<tr>
<td>2.2 Rotor Modification</td>
<td>10</td>
</tr>
<tr>
<td>2.3 Instrumentation and Data Acquisition</td>
<td>13</td>
</tr>
<tr>
<td>3. DATA ANALYSIS</td>
<td>18</td>
</tr>
<tr>
<td>3.1 Raw Data</td>
<td>18</td>
</tr>
<tr>
<td>3.2 Preliminary Data Analysis</td>
<td>19</td>
</tr>
<tr>
<td>3.3 Validation with Previous Data</td>
<td>23</td>
</tr>
<tr>
<td>4. TYPICAL FORCED RESPONSE ANALYSIS RESULTS</td>
<td>24</td>
</tr>
<tr>
<td>4.1 Variation with Operating Point</td>
<td>24</td>
</tr>
<tr>
<td>4.2 Variation with Span and Chord</td>
<td>29</td>
</tr>
<tr>
<td>4.3 Variation with Row Spacing</td>
<td>30</td>
</tr>
<tr>
<td>5. BLADE-TO-BLADE VARIATION</td>
<td>41</td>
</tr>
<tr>
<td>5.1 Background and Understanding</td>
<td>41</td>
</tr>
<tr>
<td>5.2 Blade-to-Blade Variation</td>
<td>43</td>
</tr>
<tr>
<td>5.2.1 Variation Metric</td>
<td>44</td>
</tr>
<tr>
<td>5.2.2 K-means Clustering Analysis</td>
<td>45</td>
</tr>
<tr>
<td>5.3 Treatment Validations</td>
<td>46</td>
</tr>
<tr>
<td>5.3.1 Least Squares Fit</td>
<td>46</td>
</tr>
<tr>
<td>5.3.2 Clustering Error</td>
<td>51</td>
</tr>
<tr>
<td>6. VARIATION WITHIN ROTOR RESPONSE</td>
<td>54</td>
</tr>
<tr>
<td>6.1 Metric formation</td>
<td>54</td>
</tr>
<tr>
<td>6.2 Parameter Investigation</td>
<td>57</td>
</tr>
<tr>
<td>6.3 Clustering Investigation</td>
<td>61</td>
</tr>
<tr>
<td>6.3.1 Cluster Identification</td>
<td>61</td>
</tr>
<tr>
<td>6.3.2 Cluster Dirstance</td>
<td>63</td>
</tr>
</tbody>
</table>
LIST OF TABLES

Table 5.1 Factors influencing shape of second period ................................................................. 47

Table 5.2. Example output of LM method for four parameters and $\ell_2$-norm examining tests of $D$ on a series of 33 periods having 40 points per period. ............................................. 49

Table 5.3. Output values showing cluster distances based on the clustering output of the four topple \{A,B,C,D\} and its $\ell_2$-norm. No error introduced by using $\ell_2$-norm. .. 52
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure 2.1. Schematic of the compressor rig flow path (SMI).</td>
<td>8</td>
</tr>
<tr>
<td>Figure 2.2. Image of SMI section, case removed.</td>
<td>8</td>
</tr>
<tr>
<td>Figure 2.3. Individual IGV blades.</td>
<td>9</td>
</tr>
<tr>
<td>Figure 2.4. Crack in rotor blades 16 (left) and 15 (right).</td>
<td>10</td>
</tr>
<tr>
<td>Figure 2.5. Blending of rotor blade 16 (left) and 15 (right).</td>
<td>11</td>
</tr>
<tr>
<td>Figure 2.6. Total pressure ratio map for 100% design speed.</td>
<td>12</td>
</tr>
<tr>
<td>Figure 2.7. Efficiency map for 100% design speed.</td>
<td>12</td>
</tr>
<tr>
<td>Figure 2.8. Locations for Kulite sensors.</td>
<td>13</td>
</tr>
<tr>
<td>Figure 2.9. Transducer locations for the IGV flex pressure sensor array.</td>
<td>15</td>
</tr>
<tr>
<td>Figure 2.10. Flex circuit (left) and installed IGV (right).</td>
<td>16</td>
</tr>
<tr>
<td>Figure 3.1. Rotor revolutions of two testing regimes.</td>
<td>22</td>
</tr>
<tr>
<td>Figure 3.2. Magnitude of the unsteady pressure response harmonics.</td>
<td>22</td>
</tr>
<tr>
<td>Figure 3.3. Flex sensor validation.</td>
<td>23</td>
</tr>
<tr>
<td>Figure 4.1. Differential pressures at 95% span.</td>
<td>26</td>
</tr>
<tr>
<td>Figure 4.2. Differential pressures at 50% span.</td>
<td>27</td>
</tr>
<tr>
<td>Figure 4.3. Differential pressures at 5% span.</td>
<td>28</td>
</tr>
<tr>
<td>Figure 4.4. Sketch of various shock positions with mass flow.</td>
<td>29</td>
</tr>
<tr>
<td>Figure 4.5. Peak-to-peak amplitudes of differential pressure.</td>
<td>30</td>
</tr>
<tr>
<td>Figure 4.6. Differential Pressures at 95% Span, near-stall operating point.</td>
<td>32</td>
</tr>
<tr>
<td>Figure 4.7. Differential Pressures at 50% Span, near-stall operating point.</td>
<td>33</td>
</tr>
</tbody>
</table>
Figure 4.8. Differential Pressures at 5% Span, near-stall operating point ................. 34

Figure 4.9. Frequency spectrum of differential pressure at 90% chord, near-stall operating point ............................................................................................................................................................................ 36

Figure 4.10. Peak-to-peak amplitudes of differential pressure vs. meridional chord distance, near-stall operating point ............................................................................................................................................................................ 37

Figure 4.11. Normalized, 1st harmonic differential pressure variation with span and chord for three spacings, Near-stall operating point ............................................................................................................................................................................ 39

Figure 4.12. Condensed, normalized, 1st harmonic differential pressure variation with chord for three spacings compared to linear theory prediction .......................................................................................................................................................... 40

Figure 5.1. Generated data used for calibration of the Levenberg-Marquardt (LM) Algorithm. For a base set of 33 periods (three shown above), the second period was scaled or translation as shown ............................................................................................................................................................................ 48

Figure 5.2. Absolute value of regression parameters output by the LM method .......... 50

Figure 5.3. Error in LM output with respect to known values ................................... 51

Figure 5.4. Cluster identification showing 20 tests of A,B,C, and D provided identical results ............................................................................................................................................................................ 53

Figure 6.1. Blade Passage 4 (2001 data). 90% Chord, 50% Span ............................... 56

Figure 6.2. Blade Passage 21 (2001 data). 90% Chord, 50% Span ............................. 56

Figure 6.3. Blade Passage 21 (2001 data). 95% Chord, 50% Span ............................. 57

Figure 6.4. Regression parameters (top) for all 33 blade passages and their corresponding scaled values (bottom) ............................................................................................................................................................................ 58

Figure 6.5. Several span locations at 90% chord ......................................................... 60

Figure 6.6. Cluster memberships for 2001 data, near stall, close spacing data .......... 62

Figure 6.7. Cluster memberships for 2001 data, peak efficiency, close spacing data ...... 62

Figure 6.8. 1997 cluster distances showing spacing effects ........................................ 65

Figure 6.9. 1997 cluster distances showing throttle effects ........................................ 66

Figure 6.10. 2001 cluster distances for near stall throttle and three spacings ............ 69

Figure 6.11. 2001 cluster distances for peak efficiency throttle and three spacings ..... 70

Figure 6.12. 2001 cluster distances for open throttle and three spacings .................. 71
Figure 6.13. 2001 cluster distances for close spacing and three throttles. ....................... 72
Figure 6.14. 2001 cluster distances for mid spacing and three throttles. ........................... 73
Figure 6.15. 2001 cluster distances for far spacing and three throttles. ........................... 74
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1. INTRODUCTION

1.1 Motivation

Developers of aircraft gas turbine engines continually strive for greater efficiency and higher thrust-to-weight ratio designs. To meet this goal, the trend in gas turbine designs has been to reduce the size and weight of engines by decreasing the number of compressor stages and the axial spacing between blade rows. However, these reductions result in significantly increased aerodynamic loading of the blades and unsteady interaction between blade rows. In addition, advanced compressor designs generally feature thin, low aspect airfoils, which offer increased performance but are highly susceptible to flow-induced vibrations. As a result, high cycle fatigue (HCF) has become a universal problem throughout the gas turbine industry.

In response to the HCF problems of modern gas turbine engines, a considerable portion of recent research in compressor and turbine design has focused on the unsteady aerodynamic interaction of adjacent vane/blade rows. While a fair amount of work has been performed in predicting the vane/blade interaction through Computational Fluid Dynamic (CFD) models, a limited amount of experimental data exists for verifying the CFD predictions of this complicated phenomenon. The execution of time accurate, multistage CFD simulations is usually computationally intensive. To reduce the computational work, rather than the entire flow annulus, only a sector of it is modeled based on the geometric symmetry of the blade rows. Periodic boundary conditions are
enforced at the circumferential faces of this reduced computational domain. Therefore an inherent modeling assumption is that the flow of the modeled sector identically repeats to cover the complete annulus. Desired is a detailed understanding of the ramifications of such an assumption. Experimental data available for study provided a unique opportunity to examine vane/blade interactions with both a tuned rotor and a rotor with a non-uniform blading configuration.

1.2 Previous Research

Probasco et al. investigated IGV-rotor interaction in the Stage Matching Investigation (SMI) transonic compressor rig at Wright-Patterson AFB using unsteady IGV surface pressure measurements at 50% and 75% span.\textsuperscript{1,2} A computational analysis was made using the quasi-3-D Navier-Stokes code, 2DVBI. Significant unsteady interaction was measured with a 7.5 psi unsteady pressure fluctuation at the 75% span, 95% chord location, for the 12% IGV chord axial spacing, 105% design speed and near-stall operating point. The unsteady forces were affected by changes in axial spacing and compressor back-pressure, with the conditions of closest spacing and lowest mass flow rate causing the most interaction. Comparison of the computational predictions with the measured unsteady surface pressures showed good agreement in phase and less agreement in the magnitude of the harmonic content of the vane/blade interaction.

With the same test article, Koch et al. investigated the vortical forcing function generated by the IGV row acting on the rotor.\textsuperscript{3,4} Total pressure measurements just downstream of the IGV assembly were made using a circumferential transverse assembly (CTA), allowing a detailed 2-D mapping of the wake generated by these IGVs to be made at three downstream axial locations: 12%, 26%, and 56% of the IGV chord.
Comparisons to steady-state solutions from both 2DVBI and the three dimensional version, 3DVBI, CFD codes were made, with good agreement at 12% IGV chord and less agreement at the 26% and 56% locations. An important outcome of this research was the discovery of a significant decrease in the vortical forcing function in the hub and shroud regions, demonstrating the strong effect of the end wall boundary layers extending 5–10% span into the flow from either end wall.

Other transonic compressor research was performed by Richman and Fleeter.\textsuperscript{5} They performed a 3-D Navier-Stokes analysis of a transonic compressor at the RAMP laboratory of Purdue University using the CFD code TURBO. The results were compared to measurements made by Sanders et al.\textsuperscript{5,6} These measurements included IGV unsteady surface pressures at 90% span and particle image velocimetry (PIV) data for the IGV passage. The computational results showed good agreement with both the PIV and surface pressure measurements. In addition, the CFD results demonstrated that the IGV-rotor interaction is highly three dimensional in nature. However, the CFD analysis tended to over-predict the steady surface pressure magnitudes while under-predicting the unsteady components, primarily in the trailing-edge region. This was attributed to the need for improved modeling of the viscous effects to properly simulate the interaction at the IGV trailing edge.

Regarding the blade-to-blade variability issue, mainly experimental data has been used to investigate the variability in measured flow quantities (e.g., pressure, velocity). Cherrett and Bryce\textsuperscript{7} presented unsteady total pressure measurements at the rotor exit of a transonic compressor. Time traces of both average and RMS pressures obtained by the phase-locked ensemble averaging technique showed considerable blade-to-blade
variability, though the degree was not quantified.\textsuperscript{8} Observed unsteadiness at three to four times blade passing frequency was attributed to rotor shock system oscillation. The oscillations were stable in the rotor relative frame (i.e., phase locked) and characterized by stable differences from one passage to another. Strazisar\textsuperscript{9} investigated the flow field of a transonic fan rotor using laser anemometry.\textsuperscript{9} Increased blade-to-blade variation in the rotor relative Mach number were observed at the peak efficiency operating point compared to smaller variation at the near stall point. The measured blade spacing was uniform to within 2\%, with blade motion perturbing the spacing by less than 0.1\%. The increased variation at the peak efficiency point was attributed to unstable transition between oblique and normal shock structures, with assumed higher sensitivity to geometric variations and small flow perturbations at this flow point. Strazisar cites similar work by Harvey et al.\textsuperscript{10} who reported blade-to-blade flow variability from laser velocimetry measurements in a transonic fan, “Some rotor blade passages displayed nearly shock-free flow, while other passages displayed the expected normal shock characteristics.” The pattern was locked to the rotor.

Limited work directed toward aeromechanics research has been conducted wherein blade-to-blade variability effects in both the forcing function and blade response have been studied. Sanders and Fleeter\textsuperscript{6} studied the blade-to-blade variability of rotor wakes measured in a high-speed fan stage and its effect on the measured unsteady lift coefficient of the downstream vanes.\textsuperscript{11} Small blade-to-blade wake variations were found to produce large variations in the vane row response. At the design point, the variability of the velocity due to the wakes is a maximum of $\pm 5\%$, representing twice the standard deviation ($2\sigma$) of the phase-locked ensemble average, and is sizeable compared to an
average velocity deficit of 13%. The $2\sigma$ variability of vane response is nearly half the maximum average response. Boyd and Fleeter$^{12}$ studied the blade-to-blade variability of wakes generated by the inlet guide vanes of a low speed compressor and its effect on the surface pressure response of the downstream rotor blade.$^{12}$ Four parameters were used to characterize the measurement signals (delay, width, amplitude, and energy); these parameters were then used to statistically analyze the data. Less than 50% of the rotor-based ensemble averaged wakes were found to be similar to the passage-based ensemble average of the rotor-based ensemble averaged wakes.

1.3 Present Research

These research studies have provided important steps towards quantifying and understanding the basic physics of the unsteady aerodynamic flow interactions in transonic compression systems, but further research is required. In particular, detailed experimental data is vital to the validation and improvement of modern CFD design tools. The objective of this research is to address the need for high spatial-resolution unsteady surface pressure measurements in transonic turbomachinery. This is accomplished by obtaining unsteady pressure measurements at six spanwise and five chordwise locations for three compressor operating points at 100% design speed. The test article is a highly loaded transonic compressor system. The significant effects caused by variations in the compressor operating point and IGV/rotor axial spacing will be shown.

To address the issues of signal variability, a method is presented to quantify the degree of blade-to-blade variations in unsteady flow measurements of turbomachinery experimental data. Experimental data of vane surface pressure response to the downstream forcing function produced by a transonic compressor rotor are analyzed. In
the first experiment the rotor geometry was nominally uniform. However, in the second experiment two adjacent blades of the rotor were trimmed at the tip leading edge to eliminate cracks, and consequently introduced non-uniformity in unsteady forces acting on the vane row. The blade-to-blade variability as manifested on the surface pressure of the upstream vane row is quantified and statistically analyzed, making use of regression parameter sets, which describe the surface pressure signal variability.
2. EQUIPMENT

2.1 Research Facility

Data presented herein was obtained at the Compressor Aero Research Lab (CARL) Stage Matching Investigation (SMI) facility at Wright-Patterson Air Force Base in Dayton, Ohio. This facility is a 1.5 stage (IGV/rotor/stator) axial compressor rig designed for the testing of high-speed, highly loaded compressor rotors. The research lab consists of an open- or closed-loop (currently open) tunnel system with an upstream venturi flow meter to measure the mass flow rate. Performance of the rig is driven by a 2000-hp electric motor capable of producing variable speeds of 6000 to 21,500 rpm.

The test apparatus, Fig. 2.1, was manufactured by Pratt & Whitney Aircraft Engines. This apparatus is of modular design, capable of employing an array of test sections focusing on various aspects of compressor technology. Designed by CARL personnel, the SMI rig is shown in Figure 2.2. The rig comprises a single-stage core compressor consisting of a rotor and stator with 33 and 49 airfoils, respectively, each with a 19-in. tip diameter. The SMI’s core compressor design results in a transonic rotor.
The primary intent for this research compressor is for a stage matching investigation characterizing the effect of adjacent compressor stages on compressor performance. To study the influence of different upstream stages, a special IGV assembly was placed upstream of the rotor section to simulate blockage created by
upstream stages. A pair of testing regimes in 1997 and 2001 were conducted to record the upstream, unsteady pressure response on the IGV.

The IGV’s (Fig. 2.3) were designed by Pratt & Whitney with the purpose of creating wakes consistent with those of a modern technology, highly loaded, low aspect ratio, upstream stage. The IGV’s do not turn the flow as is the normal case. They have a constant solidity along the span and no net steady aerodynamic loading. For these tests there are 24 vanes in the IGV row with an axial spacing between the IGV and rotor rows of 12%, 26%, and 56% of the IGV chord (i.e., close, mid, and far spacing).

Figure 2.3. Individual IGV blades
2.2 Rotor Modification

A few years after completion of the first experiment in 1997, two blades on the rotor blisk were found to have cracks forming on their leading edge. The cracks ran from the leading edge near the tip, back along the chord. Figure 2.4 shows the size of the cracks on each blade. The expense of a new test rotor was balanced against the cost of the repair and rebalancing of the cracked test article.

To repair these cracks, the two damaged rotor blades were blended (i.e., trimmed) to remove the cracked portion of each blade. Therefore these two blades are different from the other rotor blades. Figure 2.5 shows how much of each blade was removed (8% Span and 15% Chord for blade 15 and 19% Span and 23% Chord for blade 16). Because blade 16 had a larger crack more of the tip leading edge of blade 16 was removed.

Figure 2.4. Crack in rotor blades 16 (left) and 15 (right).
After these blades were blended, the performance of the SMI compressor was mapped for the close-IGV configuration and compared with previous (non-blended) rotor data. Figures 2.6 and 2.7 show the total pressure ratio and efficiency curves for both the modified and original configurations. The initial performance tests were complete in October of 1996, while the blended rotor was tested in February of 2001. Uncertainty bars are shown for each data point and the difference in the performance of the SMI compressor caused by blending the rotor is within the experimental uncertainty. It was concluded that the performance of the compressor had not significantly changed.
Figure 2.6. Total pressure ratio map for 100% design speed.

Figure 2.7. Efficiency map for 100% design speed.
2.3 Instrumentation and Data Acquisition

Data of the first experiment, conducted in 1997, was acquired using traditional Kulite LQ-125 miniature pressure transducers.\textsuperscript{1,2} These were installed on two IGV’s for two spanwise locations, 50\% and 75\% span, and at chordwise locations 95\%, 89\%, 83\%, 70\%, and 50\% chord. A diagram of the transducer locations is shown in Figure 2.8. The size and space characteristics with this configuration are such that a far amount of surface area is necessary for the installation of even a single sensor. While more than capable of performing their task, these transducers come with a high price tag. Consequently, the number of sensor locations used was determined by budget considerations rather than the scope of the experiment.

Figure 2.8. Locations for Kulite sensors.
Following this initial study other means of data acquisition were examined. For the second experiment, conducted in 2001 by Leger et al, a high spatial resolution, high frequency response flexible (flex) pressure sensor array was designed, fabricated, and validated to measure the IGV surface unsteady pressures.\textsuperscript{13,14} Again, two adjacent IGV’s were instrumented each with, in this instance, pressure sensor dies mounted on a flexible (“flex”) circuit substrate. The flex circuit substrate serves several purposes: first, it provides the electrical connections from the sensors (dies) to the outside of the rig in a slim, clean package, and without the bundle of wires and machined trenches generally associated with instrumentation. Second, the flex circuit allows the sensor array to conform to the shape of the surface to which it is adhered. In this case, a simple shallow face 0.030 inches deep was machined into the IGV surface to compensate for the thickness of the flex substrate to allow flush mounting with the vane surface.

The flexible circuit substrate also enables the sensors to be packed to give a high spatial resolution because the wire locations for each sensor are precisely controlled (i.e., individual trenches are not machined into the blade surface). The incorporation of this new technology is cost efficient on several considerations: first, the cost of blade instrumentation on a per-sensor basis is decreased. Second, the cost associated with the added complexity of connecting four individual wires for each sensor to the DAQ system is significant when compared to utilizing the flexible circuit substrate, which only requires two connections. Exact dollar figures for labor of facility operators cannot be calculated; the savings in terms of time, potential errors, and failure modes are significant. The final sum of funding provided for instrumentation and installation of the
flex sensors amounted to $42,000 or $700/sensor. Previous instrumentation of Probasco et al. totaled $1,250/sensor.

Earlier investigations indicated that the highest pressure fluctuations occur near the trailing edge of the IGV. Hence sensors were clustered near the trailing edge at the 95%, 90%, 85%, 77%, and 60% chord locations. In light of results obtained by Koch et al.\(^3\) spans located near both of the outer walls (5% and 95% span) were instrumented to capture as much of the radial flow effects as possible. For both of the two instrumentation methods, 1997 and 2001, there is a sensor placed at 50% span, and 95% chord as a means of comparison. Three other spans were instrumented at 25%, 65%, and 80% span to complete the high-resolution array. A schematic of the IGV sensor layout is shown in Figure 2.9.

![Figure 2.9. Transducer locations for the IGV flex pressure sensor array.](image)

The two flex circuits were then adhered with 3M brand adhesive film to the shallow faces machined into the vanes. The vanes were sent to the manufacturer of the pressure dies, Endevco, for mounting of the sensors. The dies feature a sealed cavity on the backside of the diaphragm, allowing the sensors to measure absolute pressures via a reference vacuum-sealed in this cavity during production. The sensors are fixed in place
via conductive epoxy joining connection pads to corresponding pickups on the electrical substrate. Elevated from the surface, the sensor face is leveled by a lamination of a thin sheet rubber and Kapton tape. Silicon gel eliminated any voids between layers and sensors which yields a final package that is smooth on the surface and stable in its construction. An image of the flex substrate with the thirty pressures dies follows in Figure 2.10 at the left.

![Image](image.jpg)

**Figure 2.10.** Flex circuit (left) and installed IGV (right).

Before installation, calibrations were performed on all test apparatus and data collection systems. At temperatures of 55°F and 75°F, an electronic pressure regulator induced atmospheric conditions from 6 to 16 psia in 1-psi increments. Resulting studies show precisely the fully linear, ~1 volt/psi sensitivity as quoted from the manufacturer. An effective gain of 300 was used with the sensor DC output voltage zeroed at 11 psia, the anticipated mean pressure at the vane during compressor operation. After calibration, the instrumented vanes were installed in the SMI rig, as shown at the right of Figure 2.10.
Once in place the neck of the substrate extends through a slot allowing for edge board connectors to port data to the data acquisition system. Signals from the pressure sensor array were recorded to a 28-track Datatape model 3700J analog tape deck operating at 120 in./sec. Data was digitized from the tapes at a frequency of 125 KHz with a tape speed of 30 in./sec., effectively quadrupling the available digitization rate. The number of rotor revolutions digitized varies with rotor speed; approximately 200 revolutions consisting of 2,300 discrete points are available for analysis.
3. DATA ANALYSIS

The reduction and analysis method of data acquired during tests evaluating the use of the flexible pressure sensor array will now be presented. From the flex array experiments a tremendous amount of data was recorded but unprocessed. The following section relates methods of digitization and ensembling of data collected in 2001. This analysis is then applied to both the data of 1997 and 2001 to ensure comparable tested yielded comparable data reduction methods so as to enable comparisons of the two data sets.

3.1 Raw Data

Data was acquired at several speeds and operating points, consistent with the previously recorded data by Probasco et al.\textsuperscript{1,2} During testing, the signals from the 60 pressure-sensor array were multiplexed and recorded to a 28-track Datatape model 3700J analog tape deck operating at 120 in./sec. When the signals were digitized after testing, the tape was played back at 30 in./sec., effectively quadrupling the digitization rate capability. Once digitized the data are not in a form representative of the physical pressure for analysis purposes.

Signals are originally recorded in a system of whole number counts representing sensor voltages. These counts are a linear transformation of voltage which are translated using a calibration signal recorded at the time of the original experiments. A signal generator creates a simple sine wave of known amplitude and passes this through the
system in the same manner as the experimental readings. In the case of the 2001 data, a sine wave of 5 volts peak-to-peak amplitude having no DC offset is recorded on the tape for each channel used in experimentation. Once digitized, a channel specific count to volts ratio as well as offset are determined. Following this procedure, a second linear transformation is used to determine pounds per square inch values for the pressure sensors. The derivation of this second transformation was done in 2001 as part of the standard pretest procedures for any set of measurement devices. Sensors and all pertinent electronics were calibrated by Leger et al, leading to a sensor specific database of volt to psi scaling and psi offset values. These two sets of calibration measurements are employed in the next step of data reduction.

3.2 Preliminary Data Analysis

Following the digitization of the signals, the digitized data were ensemble averaged on the time basis of the rotor revolution period, using the time traces of approximately 200 rotor revolutions. The specific number or revolutions available for average is a function of sampling frequency, the speed of rotor revolution, and the amount of user specified data recorded. Generally, ensemble averaging eliminates the fluctuations due to incoherent unsteadiness associated with turbulence (and any unsteadiness non-synchronous with the rotor) and thereby clarifies only the stationary periodic unsteadiness of interest. Because the ensemble average is based on the rotor revolution period (as opposed to the blade passage period), blade-to-blade differences in the forcing function is preserved.

Phase-locked ensemble averaging is a signal enhancement technique used by experimentalists to eliminate random unsteadiness to clarify periodic unsteadiness in the
measured time variant flow quantities \(^8\). Suppose \(Y_{ijk}\) is a time variant measurement, measured from a fixed location in space. The subscripts \(i,j,k\) represent integer indices for the rotor blade, data sample, and rotor revolution. An increase in any index represents an increase in time, with \(j\) the fastest running index, and \(k\) the slowest. Let \(I, J, K\) be integers which represent \(I\) blades on the rotor, \(J\) samples per blade passage, and \(K\) rotor revolutions of acquired data. Therefore, with a constant data sampling frequency which is an integral multiple of the blade passing frequency, data is acquired at discrete times \(t_m\) where index \(m = j + (i-1)J + (k-1)IJ\), and \(i = 1,2,\ldots,I\) \(j = 1,2,\ldots,J\) \(k = 1,2,\ldots,K\). In general, the sampling frequency is not an integral multiple of the blade passing frequency, so that in practice \(J\) is the number of equal-interval bins per blade passage which is a rounding of the number of samples per blade passage. The ensemble average based on the rotor revolution period, abbreviated \(R\ e.a.\) is given by

\[
\overline{Y}_j = \frac{1}{K_j} \sum_{k=1}^{K_j} Y_{jk}
\]

3.1.

where \(K_{ij}\) is the population of bin \(ij\) and \(K_{ij} \sim K\).

For example, Figure 3.1 depicts the \(R\ e.a.\) applied to the IGV surface pressure data of 1997 and 2001. Data was collected at the 100% speed, peak efficiency throttle point for sensor location 50% span, and 95% chord. The curves have been shifted along the ordinate axis for clarity. The trimmed blades are indicated by the dashed box. Among the blade passages, variance in peak amplitudes and peak-to-peak period is obvious. Close visual examination shows variance is not confined to the region of the trimmed blades.

Of the 33 blades of the SMI rotor, the response data produced by the forces of blades 4 and 5 (in the numbering scheme of Fig. 3.1) were chosen for all analyses.
presented in the study of Chapter 4. These blades lie on the opposite side of the wheel from the two blended blades. The aerodynamic effect caused by the blended blades are presumed to represent a consistent bias which will not affect the studied parametric trends of the IGV response produced by a given rotor blade.

The data can be further processed through an additional ensemble average operation based on the blade passing period, abbreviated B e.a., and is given by

\[ \overline{Y}_j = \frac{1}{T} \sum_{i=1}^{T} \overline{Y}_{ij} \]  

3.2.

The B e.a. reduces the data of the R e.a. to a single blade passage. Figure 3.2 compares the B e.a to the thirty-three blade passes making up the R e.a. of Figure 3.1. The B e.a. curve is Fourier decomposed as well as the \( T=33 \) blade passage curves that piecewise form the R e.a. of Figure 3.1. The first harmonic magnitudes of the 1997 data (top) have nearly 20% maximum variation with respect to blade passage index, with all harmonics showing variation. The trimmed rotor (labeled 2001 in Fig. 3.2) data show over 40% maximum variation in the first harmonic, with variation levels in the first three harmonics generally higher than those of the untrimmed rotor (1997) data.
Figure 3.1. Rotor revolutions of two testing regimes.

Figure 3.2. Magnitude of the unsteady pressure response harmonics.
3.3 Validation with Previous Data

A comparison of the data of the present study with the previous experimental data of Probasco et al was performed. The flex circuit surface unsteady pressure data from one sensor is superimposed with the previously acquired Kulite data for a duplicate test point, Figure 3.3. The 95% chord, 50% span location for the near-stall, 105% design speed operating point was chosen for comparison. The corrected mass flow was matched between the two experiments. Except for a phase shift caused by slightly different circumferential mounting locations of the transducers, the 1997 and 2001 signals compare favorably, therefore, validating the new sensors and showing the insignificant effect of the blending of the rotor on the IGV response due to the force of rotor blades 4 and 5.

Figure 3.3. Flex sensor validation.
4. TYPICAL FORCED RESPONSE ANALYSIS RESULTS

The detailed forced response measurements obtained with the high resolution flex sensor array is presented using the industry standard analysis method. In particular the variation of the forced response with changes in operating point and blade row spacing are shown.

4.1 Variation with Operating Point

Figures 4.1, 4.2, and 4.3 show chordwise time traces of the differential pressure of the IGV surfaces at 95% span (near case), 50% span, and 5% span (near hub) locations, respectively, for the close axial spacing and three operating points (near-stall, peak efficiency, and open throttle) at 100% design speed. The time traces from top to bottom correspond to the 60%, 77%, 85%, 90%, and 95% chord locations. Note that in Fig. 4.1, data is not presented at the 60% and 85% chord locations because of failed electrical conductors.

Figure 4.1 represents the data at the 95% span location. At all chordwise locations, the near-stall throttle setting produces the largest unsteady forced response. The overall magnitude of the unsteady forced response is the highest at the 95% chord location where it is a normalized value of 0.53. This translates into a pressure variation of 6.5 psid.

Evident in Figure 4.1 is the response’s monotonic change in amplitude and phase with compressor operating point. As the mass flow rate is decreased while wheel speed is held constant, the rotor’s oblique leading edge shock detaches and migrates forward.
forming a less oblique, stronger bow shock. This change in shock strength and location is responsible for the amplitude and phase change in the IGV response with operating point change. This concept is depicted in Figure 4.4. This trend is consistently observed in the data.

In Figure 4.1, moving forward from the trailing edge of the IGV shows the forced response amplitude to decrease. Also the steep rise in response caused by the shock develops a more gradual slope as the disturbance moves forward. This decreased amplitude along the vane passage is apparent in all subsequent data analysis.

At the 50% span location, Fig. 4.2, the same trends are observed, but the forced response is of greater amplitude. However, in Fig. 4.3, representing the 5% span location, the time traces of $\Delta P$ are nearly pure-tone sinusoidal from the trailing edge location and forward. The signal is more nearly symmetric about its peaks, unlike the signals of Figs. 4.1 and 4.2 which result from the propagating shock wave and expansion fan train, producing a signal asymmetric about its peaks. The absence of significant higher harmonic energy content is thought to indicate that the bow shock does not extend to this spanwise location and that the pressure response is caused by the shock-free potential field of the rotor. Velocity triangles indicate that the relative Mach number sonic boundary is at approximately 10% span. Therefore, this response is consistent with a subsonic forcing function in the relative frame.

The effect of the operating point change on the response amplitude for the subsonic relative flow (Figure 4.3) is less pronounced than that of the supersonic relative flow (Figures 4.1 and 4.2). The phase changes, however, are comparable.
Figure 4.1. Differential pressures at 95% span.
Figure 4.2. Differential pressures at 50% span.
Figure 4.3. Differential pressures at 5% span.
4.2 Variation with Span and Chord

The peak-to-peak amplitudes of differential pressure versus span are presented in Fig. 4.5 with the chordwise location as the parameter. These results are for the close spacing, near-stall conditions. The peak-to-peak amplitude is defined as the difference in maximum and minimum values of the time trace over one blade passing period. The amplitudes are maximum at 50% or 80% span and attenuate near the endwalls. The attenuation near the case ranges 19–32% from its maximum. The attenuation begins well outside the endwall boundary layer, which extends approximately 6 percentage points of span.
4.3 Variation with Row Spacing

Figures 4.6, 4.7, and 4.8 show chordwise time traces of the differential pressure of the IGV surfaces at 95% span (near case), 50% span, and 5% span (near hub) locations, respectively, for the near-stall operating point and three axial spacings (close – 12% IGV Chord, mid – 26%, and far – 56%) at 100% design speed. The time traces from top to bottom correspond to the 60%, 77%, 85%, 90%, and 95% chord locations.

The upstream propagating bow shock effects on the unsteady forced response loading is evident in Figure 4.6 for the close and mid spacing, but is not nearly as evident
in the far data. The unsteady loading magnitude decreases with increased axial spacing between the rotor leading edge and the IGV trailing edge at all spanwise locations, Figures 4.6 through 4.8.

Increasing the axial spacing between the IGV and rotor results in a significant phase shift in the unsteady forced response loading that is consistent with the upstream propagating forcing function having to travel a greater distance to generate the forced response. This phase shift with spacing is not significantly affected by the variation in the spanwise location from 95% to 5% span.

The magnitude of the forced response of the IGV is affected more by changes in spacing when the response is dominated by the bow shock, 95% and 80% span, than in regions which are purely sinusoidal or subsonic in nature, 5% span. This is shown clearly when comparing the decrease in unsteady loading for the 5% span, Figure 4.8, with spacing at each chordwise location with Figures 4.6 and 4.7 for the 95% and 50% span locations respectively.
Figure 4.6. Differential Pressures at 95% Span, near-stall operating point.
Figure 4.7. Differential Pressures at 50% Span, near-stall operating point.
Figure 4.8. Differential Pressures at 5% Span, near-stall operating point.
Investigating spacing effects by considering harmonic content of the time traces is another effective method to investigate the experimental data. Figure 4.9 shows a plot of the first 4 harmonics of the near stall data at 90% chord for all of the spans and axial spacings. Going from 95% span to 5% span results in a significant decrease in the harmonic content of the unsteady forced response especially for the mid and far spacing configurations. The decrease in harmonic magnitude with span is directly related to the decrease rotor relative Mach number and the associated rotor bow shock generated when the relative flow is above the speed of sound. The decrease in magnitude with axial spacing is associated with the viscous dissipation of the bow shock caused by an increase in the distance the rotor forcing function has to travel to excite the vane row.

For the close spacing (12% IGV chord) configuration, the bow shock influence on the harmonic content is significant from 50% span to 95% span. The second and third harmonics are important factors. At the mid spacing (26% IGV chord), the higher harmonic effects begin at 65% span. Finally, for the far spacing (56% IGV chord), the effects do not start until 80% chord. This is consistent with an increasing rotor relative Mach number with span or radial position resulting in a stronger bow shock, which propagates upstream.

Depicted in Fig. 4.10 is the peak-to-peak amplitude of differential pressure for the three IGV spacings. The abscissa represents the distance that each sensor is located from the rotor leading edge plane in terms of rotor chord. The curves indicate a decay of loading strength with axial distance from the rotor, which is usually monotonic, but in some instances there is a slight increase followed by a decrease with axial distance.
Figure 4.9. Frequency spectrum of differential pressure at 90% chord, near-stall operating point.
Figure 4.10. Peak-to-peak amplitudes of differential pressure vs. meridional chord distance, near-stall operating point.
The data of Fig. 4.10 were Fourier analyzed to extract the magnitude of the first harmonic of blade passing frequency and are normalized by the magnitude of the sensor nearest the trailing edge (95% chord) and are presented in Fig. 4.11. This allows one to observe the chordwise distribution that is proportional to the loading per unit forcing function magnitude. The data collapses to a smaller variance band for the 50 to 95% span locations than for the locations nearer to the hub, with all showing the same trend of decay away from the rotor.

Figure 4.12 depicts the data of Fig. 4.11, but presented with a common ordinate scale and juxtaposed against the linear theory of Smith\textsuperscript{15} for the operating conditions of this data set: chordal reduced frequency 4.71, Mach number 0.411, interblade phase angle $-495$ deg, solidity 1.60, and stagger 0.0 deg. Predictions from the Smith analysis have been normalized in the same manner as the data to yield unity differential pressure at the 95% chord location. Clearly the linear theory analysis is conservative, over predicting the loading magnitude, and does not predict the rearward loading distribution.
Figure 4.11. Normalized, 1\textsuperscript{st} harmonic differential pressure variation with span and chord for three spacings, Near-stall operating point.
Figure 4.12. Condensed, normalized, 1st harmonic differential pressure variation with chord for three spacings compared to linear theory prediction.
5. BLADE-TO-BLADE VARIATION

5.1 Background and Understanding

Previous analyses detail the reduction of data to a set of time-based periods of either the rotor revolution or blade passing period. Generally time traces of the blade passing period are examined as evaluation of the response for the system as a hole. This is contrary to recent work which has shown that ensembling below the level of the rotor revolution removes significant characteristics from the experimental data. Insight as to an appropriate data reduction method and metric of the blade-to-blade variation is gained from the statistical analysis of variance (ANOVA) hypothesis test for equality of means.

Two hypotheses are posed for statistical testing:

1) For each blade passage, the measurement at each bin can be thought of as a mean blade passage measurement plus some noise;

2) The blade passages behave the same, on average.

Note: We have assumed that the effect of the revolutions, on average, behave the same. Define the model M as

\[ Y_{ijk} = \mu_i + \gamma_j + \epsilon_{ijk} \]

where \( \mu_i \) denotes the contribution to the mean from blade passage \( i \), and \( \gamma_j \) denotes the contribution to the mean from bin \( j \), and \( \epsilon_{ijk} \) denotes the random error associated with revolution \( k \) for blade \( i \) and bin \( j \).

Assumption (1) can be expressed as \( Y_{ijk} = \mu_i + \gamma + \epsilon_{ijk} \). In other words, within each blade, the different bin values can be attributed to randomness (as opposed to a bin
effect). In order to investigate this assumption, we perform the following test for each blade:

\[ H_0 : \gamma_1 = \cdots = \gamma_{65} = \gamma \]  

on model (M) using an ANOVA (Analysis of Variance) Hypothesis Test for equality of means. ANOVA is a statistical procedure that quantitatively compares the within sample variability to the among sample variability. The test statistic that quantifies this quantity has an F distribution. See Walpole, Myers, and Ye for more details. The statistical analysis was performed using SAS\textsuperscript{©} statistical software.

Assumption (2) can be expressed as \( Y_{ijk} = \mu + \gamma_i + \epsilon_{ijk} \). In other words, the 33 blade curves (see Figure 3.1) can be thought of as the sum of a single mean blade curve plus randomness. In order to test this, we test the hypothesis:

\[ H_0 : \mu_1 = \cdots = \mu_{33} = \mu \]  

on model (M) using ANOVA (Analysis of Variance) Hypothesis Test for equality of means.

The experimental data from the 50% span and 95% chord pressure sensor for both the 1997 tests (untrimmed rotor) and the 2001 tests (trimmed rotor) have been investigated with \( I = 33, J = 65 \), and \( K_{ij} \sim K = 305 \). Both assumptions were rejected at significance level 0.0001 for both data sets. An implication of rejecting the first assumption is that it is appropriate to model each blade passage as some non-linear function of bin. An implication of rejecting the second assumption is that it is not appropriate to assume that all blades in a single rotor can be modeled as one mean blade curve plus some noise.
It is interesting to note that the $F$-value for the second of the two hypothesis tests for the 2001 data was larger than the analogous quantity for the 1997 set, indicating a stronger rejection in the case of the trimmed blades. This is consistent with our intuition, as we know from observation of the blades that in fact the 33 blades in the trimmed rotor data set are not equal on average.

This final clause statistically indicated a need to examine the periodic data to give weight to how variant the constituent blade passes are within a given rotor revolution. With this goal in mind, data analysis on a different level was initiated. The goal of this research was to develop a means of identifying blade passes as being either similar to much of the data or belonging to a minority set of blade passes being deviant from the 33. Of primary importance is the method’s ability to ascribe a level of variation to a specific revolution of data. Of secondary importance is the ability of the program to identify the response of the damaged rotor blades as deviant.

In brief this process is as follows: a basis function generated from the 33 blade passes is described and used as a defining mean. A comparative analysis is performed on each of the 33 blade passes using this basis function to derive a qualitative variable or set of variables which gauge an individual’s conformity to the basis. Finally this set of blade pass specific weighting parameters are separated into categories of variant or invariant using clustering software.

5.2 Blade-to-Blade Variation

In the previous section, two goals were stated: the identification of deviant blades, and the formation of a quantitative variable of rotor variation. There exists for each of these a set of procedures outlined in this section.
5.2.1 Variation Metric

Technically speaking the first step in the identification of variation is the Re.a algorithm. This section outlines other processes inherent to the method following this initial step.

First a basis function $Y(t)$ is formed that is at least $C^1$ continuous and periodic with the blade passing period $T_B$, i.e., $Y(t) = Y(t + T_B)$. We chose a cubic spline with periodic boundary conditions to form $Y(t)$ with the knots obtained from applying the slightly modified B e.a. to the data. The B e.a. employed here both forms the basis function and prepares the individual blade passes for comparison in the next step. For time variant flow quantities exhibiting no blade-to-blade variability, the signal period $T_S$ is equal to the blade passing period $T_B$ which is an integral multiple of the rotor revolution period $T_R$, viz., $T_B = T_R/I$. For data which manifests blade-to-blade variability, generally, the signal period varies with blade passage, $T_{S,i}$ is not equal to $T_B$. Consequently, for the calculation of the basis function $Y(t)$, $T_{S,i}$ was defined as the trough-to-trough period (the apparent local blade passage period) of the Re.a. Separation in manor allows for the examination of period width variations and better organizes the response of a single rotor blade to that of single period.

Next, by linearly scaling and translating the basis function along the abscissa and ordinate, we seek to fit the basis function to each of the $I$ piecemeal curves that form the Re.a. curve (3). A modified form of the Levenberg-Marquardt algorithm (Moré, 1977)\textsuperscript{17} is used to accomplish this fit which minimizes the sum of the square of residuals $r_{ij}$ for a given blade passage $i=\text{const}$,

$$r_{ij} = \bar{Y}_{ij} - [a_i + b_i \bar{Y}(\tau_{ij})]$$

5.4
\[ \tau_{ij} = c_i + d_i t_{ij} \]

where \( t_{ij} \) are the discrete times associated with bins \( ij \) for \( i = \text{const} \in [1, I], \ j = 1, 2, \ldots, J. \) Thus the regression parameter set \( \{a, b, c, d\}_i \) is solved for each blade passage \( i. \) In this form the solution set belonging to the identity comparison is \( \{a, b, c, d\}_i = \{0, 1, 0, 1\}. \) By transforming the scaling parameters \( b \) and \( d \) to be centered about zero, viz., \( D = 1/d - 1 \) and \( B = b - 1, \) their measure then becomes that of fractional period width or fractional peak-to-peak wave amplitude. In this form, a positive (negative) value would indicate an increase (decrease) in magnitude, and a value of 1.0 would indicate a doubling in magnitude. The translation parameters \( a \) and \( c \) are already centered about zero but have units of that of either the horizontal or the vertical axis. In order for all parameters to have similar meaning, \( a \) and \( c \) are transformed by \( A = a/(\text{peak-to-peak amplitude}) \) and \( C = c/(\text{period width}). \) The solution set belonging to the new identity comparison is \( \{A, B, C, D\}_i = \{0, 0, 0, 0\}. \) Therefore, the farther a scaled set is from the zero set, the larger the variance of that blade passage data from the basis function.

### 5.2.2 K-means Clustering Analysis

To gage the variability in any single set of rotor revolution data curves, a statistical clustering (or classification) scheme, the K-means clustering analysis, is used (see, for example, MacQueen\(^{18}\) or the more general introduction found in Everitt\(^{19}\)). The goal of a clustering scheme is to separate the members of the data set into clusters (classes) of similar objects. Beginning with a predetermined number of clusters, the data points are randomly assigned to the clusters, and a geometric centroid is calculated for each cluster. Membership reassignment is iteratively made to globally minimize a criterion function to within a specified tolerance. The criterion function is based on the Euclidean distance of
the members from their cluster centroid. Specifically, the algorithm clusters \( I \) data points into \( K \) disjoint sets \( S_n \) containing \( I_n \) data points so as to minimize the sum-of-the-squares criterion \( R \),

\[
R = \sum_{n=1}^{K} \sum_{i \in S_n} \left\| x_i - x_n^C \right\|
\]

where \( x_i \) is a vector representing the \( i \)th data point and \( x_n^C \) is the geometric centroid of the data points in \( S_n \). The scaled regression parameter set was chosen for analysis, \( x_i \in \{A,B,C,D\} \), and \( K = 2 \), using SAS® statistical software.

5.3 Treatment Validations

This section is subdivided into two subsections: one dealing with the nature of the least squares fit technique and the second analyzing the clustering procedures of SAS. For each subsection a series of generated data is defined to test the performance of the codes when simple datasets of known behavior are input. It is shown that the programs behaved as expected in all cases.

5.3.1 Least Squares Fit

The chosen least squares fit technique is formally introduced in the previous section. For review, a basis function is created, formed from the ensemble average based on the rotor revolution period, abbreviated R e.a. This curve is obtained by linearly scaling and translating the function along the abscissa and ordinate in order to fit the basis function to each of the \( I \) piecemeal curves that form the R e.a. curve. This is performed through an algorithm which minimizes the sum of the square of residuals \( r_{ij} \) where:
Simply stated the variables a, b, c, and d define the translation and scaling of the function \( \bar{Y} \). Artificial data is created to test the method’s ability to determine these values. This data is similar in nature to the experimental data but is translated or scaled by a predetermined amount. Four such datasets are created based on a string of thirty-three periods each having forty discrete points. In each test the second period is augmented to reflect a given translation or scaling along each axis, thereby determining the accuracy with which the method can distinguish changes in periodic data. The first three periods of the case designed to examine parameter \( a \) are plotted in Figure 5.1, where the altered data is represented by the solid line and superposed with it is the unaltered data shown by dashes.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Function</th>
<th>%Change</th>
<th>Dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a )</td>
<td>Translation in Y</td>
<td>-25%</td>
<td>Peak-to-Peak amplitude</td>
</tr>
<tr>
<td>( b )</td>
<td>Scale in Y</td>
<td>-50%</td>
<td>Peak-to-Peak amplitude</td>
</tr>
<tr>
<td>( c )</td>
<td>Translation in X</td>
<td>-12.82%</td>
<td>Period Width</td>
</tr>
<tr>
<td>( d )</td>
<td>Scale in X</td>
<td>-20%</td>
<td>Period Width</td>
</tr>
</tbody>
</table>

Table 5.1 Factors influencing shape of second period.
The results obtained for the artificial data were important on two levels. Primarily it proved that the method successfully identified the altered periods and quantified the amount of deviation in each case. Secondly, the artificial data provided important insight as to how to normalize the regression parameters and later combine them into a single measure of deviation. Table 5.2 shows a sample of parameter output derived from the analysis of $D$ of the above tests for three of the thirty-three periods.
<table>
<thead>
<tr>
<th>Period</th>
<th>$a$</th>
<th>$b$</th>
<th>$c$</th>
<th>$d$</th>
<th>$\ell_2$-norm</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.56E-03</td>
<td>0.998</td>
<td>7.48E-03</td>
<td>1.25683</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>4.06E-05</td>
<td>1.000</td>
<td>-1.43E-02</td>
<td>1.0012</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>4.06E-05</td>
<td>1.000</td>
<td>-1.43E-02</td>
<td>1.0012</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.2. Example output of LM method for four parameters and $\ell_2$-norm examining tests of $D$ on a series of 33 periods having 40 points per period.

Raw output from the code above shows regression parameter results for \{a,b,c,d\} in its unscaled form. The solution set belonging to the identity comparison here is \{a,b,c,d\} = \{0,1,0,1\}. The particular test above examines the code’s response to changes in “d.” This parameter was of particular interest due to its peculiar behavior. As shown below, an anticipated 20% decrease yields a value of 1.2568 for $d$. After extensive testing it was shown this value is actually 1/$d$. The reciprocal of above is then 0.7957, indicating 20% reduction in width. Using the transformation set introduced in section 5.2.1, the final value of \{A,B,C,D\} is \{1.78E-04, -1.59E-03, 1.92E-04, -0.20435\}; only 2.175% error is found in $D$. The $\ell_2$-norm incurs slightly more error from $A$, $B$, and $C$ but is still low at 2.18%.

Once it was shown the code could accurately determine parameters, several more datasets were created to determine what effect increasing the number of points and increasing the number of periods had on the data. The number of points was increased from 40 to 640 in increments of $2^{n-1}$ in five steps. Likewise the number of periods was increased from 33 to 528. Figure 5.2 shows four plots, were one period of generated data was translated or scaled in the vertical or the horizontal. Tests involved both the set \{A,B,C,D\} and calculated $\ell_2$-norm. For the parameters $A$, $B$, and $C$ there is a
convergence to the known value when the number of periods is increased. However, increasing the periods has no effect on parameter $D$. Inversely, increasing the number of points per period had no effect on the derived values of $A$, $B$, or $C$ but $D$ appears to be converging in an oscillatory manor. Figure 5.3 shows the relative error between the derived parameter values and the known parameter values. It is apparent the error associated with this program is dependant on the number of periods being used. In reality this equates to a number of rotor blades constituting a revolution. A number of blades less than 33 would have a larger associated error. Assuming a 40-point period and a revolution having 33 periods the total RMS error is 6.3%.

![Figure 5.2. Absolute value of regression parameters output by the LM method.](image-url)
Figure 5.3. Error in LM output with respect to known values.

5.3.2 Clustering Error

Similar to what was accomplished previously for the least squares fit program, a second set of data was created to test the SAS procedures. The SAS procedure uses output from above which is in the form of thirty-three sets of A, B, C, or D or thirty-three \( \ell_2 \)-norms. Consider Table 5.3 where five tests are present, which assign a value to one or all of the four parameters and the \( \ell_2 \)-norm. To negate errors introduced by the LM algorithm, all sets are taken to be zero with one of the thirty-three having a value in \( \{A, B, C, D\}_i \) and the \( \ell_2 \)-norm. Therefore, any error present in the clustering output solely belongs to SAS. As shown in the table the output value for “cluster distance”, based on \( \{A,B,C,D\}_i \), or the \( \ell_2 \)-norm, is identical to the input value. Cluster distance is simply the distance between centroids of two data clusters. SAS accurately identifies the single, non-zero value as the second cluster 100% of the time. The clustering technique employs
the use of an $\ell_2$-norm in calculating these distances so, as expected, there is no differentiation between $\ell_2$-norm and $\{A,B,C,D\}_i$ input for this simple input. However, there exists infinitely many combinations of $\{A,B,C,D\}_i$ for a given $\ell_2$-norm. For the method to perform optimally the input of the clustering method must be in the form of $\{A,B,C,D\}_i$ in order to function as desired. The $\ell_2$-norm is then only used as simplification of parameter output and not used later in the cluster analysis.

<table>
<thead>
<tr>
<th>Test</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>$\ell_2$-norm</th>
<th>Cluster Distance (ABCD)</th>
<th>Cluster Distance ($\ell_2$-norm)</th>
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Table 5.3. Output values showing cluster distances based on the clustering output of the four topple $\{A,B,C,D\}$ and its $\ell_2$-norm. No error introduced by using $\ell_2$-norm.

In the previous section a test regime was introduced based on increasing periods or points in the generated data. This is simply twenty unique datasets having similar form to that of Table 5.2 (i.e. a single period in a series of 33 was altered). This data was readily available and used as input for the SAS procedure. Results yielded cluster distances equal to their expected value in all cases. Likewise, the clusters assigned to the minority and majority groupings were identical. Figure 5.4 shows the clusters as reported by SAS where data is either the baseline case having cluster identity of one or the deviant case where the cluster identity is two. This is manifested as a field of black belonging to the first cluster and a string of lighter points at Period = 1. The clustering analysis
accurately depicts this period as being deviant and all other periods as being similar to the mean.

Figure 5.4. Cluster identification showing 20 tests of A,B,C, and D provided identical results.
6. VARIATION WITHIN ROTOR RESPONSE

Through the course of evaluating the blade-to-blade variation several consecutive steps are performed in order arrive at a final measure of variation. Primarily these steps involve the comparison of a basis to the separate blade passes to yield parameter output, \( \{A,B,C,D\} \) or the \( \ell_2\)-norm (i.e. the square root of the sum of the squares of \( \{A,B,C,D\} \)), and clustering of the output 4 tuple parameter set at each sensor location for all compressor data points. The following analysis was performed on a large population of data comprised of three spacings (close, mid, and far), three throttles (open, peak efficiency and near stall), two speeds (100% and 105% speed), data from the 2001 and 1997 tests, and the three chord locations nearest the trailing edge of the IGV (83%, 89% and 95% from 1997 or 85%, 90%, and 95% 2001 test).

The results from each of the above steps are presented followed by an examination of trends present in the data with respect to spacing, throttle, location etc. It is shown that the method is able to identify mistuning based on the parameter output and with this able to identify the known location of aerodynamic mistuning.

6.1 Metric formation

Results from the variation metric analysis applied to the 2001 data of blade passages 4 and 21 are presented in Figures 6.1 and 6.2, respectively. In these figures, the IGV unsteady surface pressure at midspan, 95% chord is presented. The graphs superimpose the segment of the R e.a. curve corresponding to the blade passage of
interest along with the basis function and the fitted basis function. This fourth blade passage is located on the rotor directly opposite the two trimmed blades. At this location, we expect the best agreement between the R e.a. blade passage curve and the basis function. Indeed, Figure 6.1 shows there is very little translation and scaling required for the basis function to match the R e.a. curve, and all the regression parameters have magnitudes near zero.

Figure 6.2 is representative of the data belonging to the region of trimmed blades. There is a marked increase in the regression parameter values, indicating larger deviation of the R e.a. blade passage curve from the basis function. The regression parameter of largest magnitude is the scaling coefficient, $D$, having a value of 1.9034; this is two order increase in the magnitude of $D$ in Figure 6.1. An example of an extremely deviant blade pass is seen in Figure 6.3. This data is of the same percent speed, throttle condition, and span location of the previous two examples but exists at the 95% chord location. In Figure 6.2 the $\ell^2$-norm of the four parameters is 1.950 that can be compared to the value of 10.684 for that of Figure 6.3. Notwithstanding these seemingly large values, the results represent converged values from the Levenberg-Marquardt analysis.

The method is successful in its ability to capture the essence of deviation in the form of the described regression parameters. Further, the method is robust: blade pass signals which were significantly different from the basis in no way hindered the completion of any of hundreds of total revolutions studied. Results having extremely large $\ell^2$-norm of $\{A,B,C,D\}$ (above 2.0) occur sporadically in the data, but upon close inspection they exist for blade pass signals of extraordinary shape, yet are well fitted by the linearly transformed basis.
Figure 6.1. Blade Passage 4 (2001 data). 90% Chord, 50% Span

Figure 6.2. Blade Passage 21 (2001 data). 90% Chord, 50% Span
6.2 Parameter Investigation

For each of the 1997 and 2001 data, 33 regressor parameter sets (one per blade passage) were calculated to describe the entire rotor revolution. A comparative analysis of parameter values \( \{A, B, C, D\} \) of both the trimmed and untrimmed rotor at similar compressor operating conditions is shown in the top plot of Figure 6.4. The nature of the more nearly uniform rotor is seen in the 1997 data characterized by a flat line parameter set. The data of 2001 shows variation primarily in the area of the trimmed blades (blade passes 20 and 21). This set of four parameters is reduced to a single parameter, the \( \ell^2 \)-norm of \( \{A, B, C, D\} \), to reduce the dimensionality of the metric and thus aid in the comprehension of the data. The middle plot of Figure 6.4 shows that the deviations of interest are preserved by the \( \ell^2 \)-norm metric.
Figure 6.4. Regression parameters (top) for all 33 blade passages and their corresponding scaled values (bottom).
The bottom plot of Fig. 6.4 is that of the root mean square error (RMSE) between the fitted basis curve and the R e.a. curve which form \{A,B,C,D\}_i. The RMSE is defined as,

$$RMSE_i = \sqrt{\frac{1}{J} \sum_{j=1}^{J} r_{ij}^2} \quad \text{for } j = 1,2,\ldots J \text{ number of data points} \quad 6.1$$

The largest RMSE occurs in the 2001 data and at the trimmed blade location. Yet here we see that the RMSE of both data sets is acceptable, being under 0.05.

On a larger scale of the flow field, Figure 6.5 shows the $\ell_2$-norm analysis results of the 2001 data at all available IGV span locations at 90% chord for the scaled regression parameters \{A,B,C,D\}. Evident is large blade-to-blade variability detected at the lower spans (5–50% span); here the trimmed blade locations are prominent. However, the upper spans (65–95% span) exhibit a more spatially uniform behavior in undulation magnitude. At the 80% and 95% spans the location-specific increase in variation is lost inasmuch as many of the blade passes exhibit equally marginal amounts of deviation.
Figure 6.5. Several span locations at 90% chord.
6.3 Clustering Investigation

6.3.1 Cluster Identification

The results of the $K$-means cluster analysis applied to the regressor parameters is presented in Figure 6.6 and 6.7 for the 2001 data for close vane axial spacing, 50% span, for three chord locations. Figures 6.6 and 6.7 represent the near stall and peak efficiency operating points, respectively. Plotted are cluster results obtained using both the unscaled \{a,b,c,d\} and scaled \{A,B,C,D\} forms of the parameters. The two clusters are designated 1 and 2, with cluster 1 possessing the largest blade membership for all three chord locations. In Fig. 6.6, results based on the unscaled (top) and scaled parameters (bottom) both show that deviations exist in one or two blade passages in the area of the trimmed blades. For this case there is little gained in detection ability from applying the scaling to the parameters. This is contrasted by Fig. 6.7 where a majority of the variation occurring away from the damaged blades is reduced or even eliminated by applying the scaling. The scaled plots identify the trimmed blades as the source of much of the variation.
Figure 6.6. Cluster memberships for 2001 data, near stall, close spacing data.

Figure 6.7. Cluster memberships for 2001 data, peak efficiency, close spacing data.
6.3.2 Cluster Distance

The primary output of clustering is the new parameter of cluster distance. As has been the approach, a goal of this study is to derive a single, meaningful value which can be shown to measure variation within a revolution. Previously the $l_2$-norm simplification was performed on the parameter set $\{A,B,C,D\}$ in an attempt to reduce the order of output. For the method of clustering we return to the four parameter formulation and reduce the 33x4 array of data to a single measure of cluster distance. This single factor organizes all variations into categories of having either greater or lesser magnitude and by determining distance between categories, defines the variation of a revolution.

Here it is appropriate to begin by examining cluster distances obtained from tests in 1997. Shown is data from the 100% speed for three throttles, three chord location (95%, 89%, and 83% chord), and the three available spacings of close, mid and far IGV settings. Data collected at the far spacing and 50% locations failed and are not present. The data is arranged by span and chord location.

Figure 6.8 is a study of axial spacing effects. Here are three pairs of data from the 50% and 75% span locations increasing in throttle from bottom to top. For this chart, bars of increasing width are indicative of increasing spacing. From the results of chapter 5 there is a decrease in signal amplitude as IGV/rotor distances increase. It is found here that there is an increase in variation due to the increase in axial spacing. Shown are results from the three throttle settings. From top to bottom we have near stall, peak efficiency, and open throttle for both spans, 50% and 75%.

This last group of data from 1997 shows effects related to increasing the throttle setting from near stall to fully open. The layout of Figure 6.9 is similar to that of 6.8 where three sets of the two spans are arrayed. Bars of increasing width are indicative of
increasing throttle and from the bottom most plot to the top is increasing spacing. In chapter 4 it was shown that increases in throttle changes are responsible for decreasing signal strength. In studying cluster distances, it is shown that an increase in variation exists with increasing flow. These results are consistent with results reported by Strazisar\textsuperscript{9} reviewed in the first chapter.
Figure 6.8. 1997 cluster distances showing spacing effects.
Figure 6.9. 1997 cluster distances showing throttle effects.
Similar results from data acquired in 2001 are less clear. Presented for completeness are similar figures, having more spans, but describing the characteristics detailed previously. Presented is data from the 100% speed line for various spacing and throttle settings. Figure 6.10 shows close, mid, and far spacing results at the near stall throttle point. Similarly Figures 6.11 and 6.12 are spacing investigations at the peak efficiency and open throttle conditions, respectively. Figure 6.13 examines the data using another approach: maintaining close spacing for the three throttles. Likewise, Figures 6.14 and 6.15 are from mid spacing and far spacing.

The first set of three figures (Figures 6.10, 6.11 and 6.12) are from a constant throttle setting examining how spacing changes effect variation. All patterns seen clearly in the previous data are specific to an undamaged rotor; these could perhaps be representative of noise levels. Here trimming of the blades has induced variations that are indicative of what results from a non-uniform bladed rotor. Variations are four or as much as forty times larger than that of the 1997 data, but this is not necessarily the case at all locations. Large amplitude variations are not constrained to any particular span or chord but occur without provocations.

Results for throttle investigations also elicit little insight. Figures 6.13 - 6.15 are shown for completeness. Trends found in Figure 6.9 are not found with any degree of certainty. It can only be said that factors leading to variation here are quite different than those of a normally tuned rotor. Changes unique to two of the blades lead to increased variability in the corresponding response measured upstream on the IGV which overshadow variation inherent to all compressor rotors.
These results are neither discouraging nor completely unwarranted. The method does extremely well in both indicating a high level of variation existing in the 2001 data as well as in indicating this variation is specific to certain blade passes of the revolution. Cluster identifications studies indicate that a majority of the time it is possible to identify the trimmed blade passes by the increase in variation of the upstream unsteady pressure measurements. Lacking is a means of comparing the variation from case to case in the 2001 data. Data is shown to be dramatically deviant but increased cluster distance alone is not representative of greater variation. The linear transformation of the basis function is highly dependant on blade pass shape which is found not to act in any specific manner given location, speed and throttle setting.
Figure 6.10. 2001 cluster distances for near stall throttle and three spacings.
Figure 6.11. 2001 cluster distances for peak efficiency throttle and three spacings.
Figure 6.12. 2001 cluster distances for open throttle and three spacings.
Figure 6.13. 2001 cluster distances for close spacing and three throttles.
Figure 6.14. 2001 cluster distances for mid spacing and three throttles.
Figure 6.15. 2001 cluster distances for far spacing and three throttles.
7. SUMMARY AND CONCLUSIONS

A set of inlet guide vane unsteady surface pressure measurements was presented. Unsteady IGV surface pressures were acquired for six spanwise and five chordwise locations for a 100% or 105% speed line and three compressor operating points for three axial spacings using a high spatial resolution sensor array. Investigations of characteristics previously unmeasured were performed detailing effects sensitive to location, spacing, and operating condition.

Due to a crack formation of the experimental rotor two of the thirty-three rotor blades were blended. This reduction in material at the tip of the rotor’s leading edge manifests itself as non-uniform periods in the experimental data. An automated method of variation detection and quantification was developed. This method involves the comparison of blade passes to an overall average blade pass.

7.1 Physical Effects

Comparison of the 2001 measurement trends to those previously reported with 1997 data showed favorable agreement, therefore validating the performance of the new sensors. A significant effect was shown on the unsteady forced response of the IGV caused by changes in compressor operating point and IGV/rotor row axial spacing for various span and chord locations. In particular, variations in the operating point caused both a magnitude and phase change in the forced response with the near-stall operating point yielding the highest response. At the 5% span location where the rotor relative flow
is subsonic, the forced response is less sensitive to the parametric changes controlled in this study. There exists at high spans a limiting agent (be it the case wall itself or interactions thereabouts) which affects the nature of the response. With decreasing flow rate stronger bow shock formations producing increased response amplitude with decreasing throttle. Changes in the IGV/rotor row axial spacing from 12% to 26% of the IGV chord results in a nearly 50% reduction in the magnitude of the forced response at mid-span.

Upon comparison with previous results it was demonstrated that the blending of the two rotor blades did not produce a significant effect on the experimental results when limited to the forces from select rotor blades. The response belonging to blades on the opposite side (blade 4) of the rotor was presented for study.

### 7.2 Statistical Analysis

Presented was a method of analyzing compressor, vane unsteady surface pressure data with interest in the forcing function non-uniformity caused by blade-to-blade variability. As mentioned, data was analyzed from two experiments conducted with a compressor rotor having (1) factory uniform blades and (2) two trimmed blades. A regression algorithm was introduced to obtain variation metrics which quantify the degree of blade-to-blade variation present in the data. This metric begins as a collection of thirty-three, 4-parameters sets of data and is reduced to one number via statistical clustering analysis.

Results show that variability exists to a minimum in the untrimmed rotor, and parameters vary greatly due to the trimmed blades. From the 1997 data to the 2001 collection there is an increase in variability (cluster distance) by a factor of four and, in
some cases, as much as forty. Within the vane passage the response exhibited greatest variation at spans near the hub region. The described method provides a means of determining overall dissimilarities within a rotor revolution. Non-uniformities are more populous throughout the revolution (as opposed to localized perturbation in the blended region) with increasing span and increasing throttle. Finally, a K-means clustering analysis provided a method of detecting the location of disturbances which further indicates variation is not restricted to the area of the blended blades but certainly reaches a maximum in this location.
LIST OF REFERENCES


