I, Aditya Deshpande, hereby submit this original work as part of the requirements for the degree of Master of Science in Mechanical Engineering.

It is entitled:
Robot Swarm Based On Ant Foraging Hypothesis With Adaptive Levy Flights

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Robot Swarm based on Ant Foraging Hypothesis with Adaptive Lèvy Flights

A thesis submitted to the Graduate School of the University of Cincinnati in partial fulfillment of the requirements of the degree of

Master of Science

in the Department of Mechanical and Materials Engineering of the College of Engineering and Applied Sciences by

Aditya Milind Deshpande

B.E. University of Pune
August 2014

Committee Chair: Dr. Manish Kumar, Ph.D.
Abstract
Robot Swarm based on Ant Foraging Hypothesis with Adaptive Lévy Flights
by
Aditya Milind Deshpande

Submitted to the Graduate Faculty as partial fulfillment of the requirements for the
Master of Science Degree in Department of Mechanical and Materials Engineering
University of Cincinnati
June 2017

Design of robot swarms inspired by self-organization in social insect groups is currently an
active research area with a diverse portfolio of potential applications. This thesis is focused on
the development of control laws for swarm of robots inspired by ant foraging. Particularly, this
work presents control laws for efficient area coverage by a robot swarm in a 2D spatial domain,
inspired by the unique dynamical characteristics of ant foraging. The novel idea pursued in
the effort is that dynamic, adaptive switching between Brownian motion and Lévy flight in
the stochastic component of the search increases the efficiency of the search and area coverage.
The study is motivated by behaviors of certain biological studies who exhibit searching patterns
modeled using Lévy flight. Influence of different pheromone (the virtual chemotactic agent that
drives the foraging) threshold values for switching between Lévy flights and Brownian motion
is studied using two performance metrics - area coverage and visit entropy. The results highlight
the advantages of the switching strategy for the control framework, particularly in cases when
the object of the search is scarce in quantity or getting depleted in real-time.
Thesis Supervisor: Manish Kumar
Title: Associate Professor
To my loving parents, even though we never discussed about robots.
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Finally, I want to dedicate my graduate research thesis to my parents and younger brother. I thank them for their blessing and support.

Aditya Deshpande
## Contents

**Abstract**  ii  

**Acknowledgments**  v  

**Contents**  vi  

**List of Figures**  ix  

**List of Abbreviations**  xi  

### 1 Introduction  

#### 1.1 Motivation  1  

#### 1.2 Why Area Coverage with Robot Swarm?  1  

#### 1.3 Swarms in Nature  4  

##### 1.3.1 School of Fish  4  

##### 1.3.2 Flock of Birds  5  

##### 1.3.3 Swarming in Ants  6  

#### 1.4 Ant Foraging  7  

#### 1.5 Lévy flights  9  

#### 1.6 Objectives  9  

#### 1.7 Contributions  10  

#### 1.8 Organization of Thesis  11  

### 2 Robot Swarm Based Area Coverage: Literature Review  12  

#### 2.1 Why Area Coverage?  12  

#### 2.2 Area Coverage Based on Multi-Robot Systems  13  

#### 2.3 Bio-inspired Robot Swarm and Area Coverage  17
2.4 Other Applications of Swarm Intelligence ........................................ 22
2.5 Motivation For This Work .......................................................... 22

3 Swarm Control Law For Area Coverage ........................................ 24
3.1 Evolution of Ant Foraging Models .............................................. 24
3.2 Development of Swarm Control Law for Area Coverage ............... 26

4 Mathematical Model Of Lévy Flights ............................................ 30
4.1 Properties of Lévy Flights ......................................................... 32
4.2 Probability desnisty function of Lévy Distribution ......................... 33

5 Computational Implementation and Performance Metrics .................. 36
5.1 Implementation of the Control Law ............................................ 36
  5.1.1 Pheromone Diffusion .......................................................... 36
  5.1.2 Deposition and Evaporation of Pheromone ............................ 39
  5.1.3 Robot Movement ............................................................... 39
5.2 Performance Metrics ............................................................... 43
  5.2.1 Area Coverage Integral ...................................................... 43
  5.2.2 Visit Entropy ................................................................. 45
  5.2.3 Pop-up Threat Detection ................................................... 46

6 Search and Retrieval With Ant Inspired Robot Swarm ...................... 47

7 Numerical Simulations And Results ............................................ 50
7.1 Results For Area Coverage ...................................................... 52
  7.1.1 Area Coverage Performance ............................................. 52
  7.1.2 Area Coverage Integral .................................................... 56
  7.1.3 Visit Entropy ............................................................... 59
  7.1.4 Number Of Lévy Steps ................................................... 62
  7.1.5 Pop-up Threat Detection .................................................. 63
7.2 Search and Retrieval of Target ............................................... 68
8 Discussion, Conclusion and Future work

8.1 Discussion and Conclusions ......................................................... 73

8.2 Future Works ................................................................. 74

Appendices ................................................................. 84
## List of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-1</td>
<td>Centralized Swarm</td>
<td>3</td>
</tr>
<tr>
<td>1-2</td>
<td>Decentralized Swarm</td>
<td>3</td>
</tr>
<tr>
<td>1-3</td>
<td>Distributed Swarm</td>
<td>4</td>
</tr>
<tr>
<td>1-4</td>
<td>School of Fish</td>
<td>5</td>
</tr>
<tr>
<td>1-5</td>
<td>Flock of Eurasian Cranes</td>
<td>5</td>
</tr>
<tr>
<td>1-6</td>
<td>Ant Bridge</td>
<td>6</td>
</tr>
<tr>
<td>1-7</td>
<td>Ant Trail</td>
<td>7</td>
</tr>
<tr>
<td>1-8</td>
<td>Ant Foraging</td>
<td>9</td>
</tr>
<tr>
<td>2-1</td>
<td>Illustration of Voronoi diagram</td>
<td>14</td>
</tr>
<tr>
<td>2-2</td>
<td>Illustration of disk placement pattern for area coverage with pattern-based genetic algorithm</td>
<td>17</td>
</tr>
<tr>
<td>2-3</td>
<td>Magnetic Termite Mounds in Litchfield National Park in the Northern Territory, Australia</td>
<td>19</td>
</tr>
<tr>
<td>2-4</td>
<td>A termite nest built on tree trunk</td>
<td>20</td>
</tr>
<tr>
<td>4-1</td>
<td>Lévy Flights: 3000 steps with step-length chosen from Lévy Distribution</td>
<td>30</td>
</tr>
<tr>
<td>4-2</td>
<td>Brownian Motion: 3000 steps with step-length chosen from Normal Distribution</td>
<td>31</td>
</tr>
<tr>
<td>4-3</td>
<td>Probability density function of Lévy Distribution with varying scale parameter and shift parameter of zero compared with Standard Normal Distribution</td>
<td>34</td>
</tr>
<tr>
<td>5-1</td>
<td>Illustration of function discretization</td>
<td>37</td>
</tr>
<tr>
<td>5-2</td>
<td>Lévy Step by generated using Mantegna’s Algorithm</td>
<td>42</td>
</tr>
<tr>
<td>5-3</td>
<td>Histogram of Lévy random numbers generated using Mantegna’s Algorithm</td>
<td>42</td>
</tr>
<tr>
<td>5-4</td>
<td>Typical Area Coverage Performance Curve</td>
<td>43</td>
</tr>
</tbody>
</table>
7-1 Typical pheromone distribution by robot swarm at the end of 500 iterations with parameter values as $D_b = 0.5, \gamma = 0.01, \sigma = 0.1, \eta = 0.1$ and $\beta = 1.1$.

7-2 Corresponding robot positions to Figure (7-1).

7-3 Plot of area coverage performance (ACP) vs. Time where $D_b = 0.001, \gamma = 1e^{-5}, \sigma = 0.1$ and $\beta = 1.5$ (NOTE: $\eta = \infty$ for Brownian Noise plot and all other parameters remain same).

7-4 Plot of area coverage performance (ACP) vs. pheromone threshold $\eta$ where $D_b = 0.001, \gamma = 1e^{-5}, \sigma = 0.1$ and $\beta = 1.5$.

7-5 Area Coverage Performance Plot.

7-6 Area coverage integral plot with noise magnitude variation. $RED$ for $\sigma = 0.1$, $GREEN$ for $\sigma = 0.3$ and $BLUE$ for $\sigma = 0.5$.

7-7 Area coverage integral plot with noise magnitude $\sigma$ variation for $D_b = 0.01, \gamma = 0.1, \eta = 1.5, \beta = 1.5$.

7-8 Visit entropy $H$ Against Pheromone Threshold $\eta$.

7-9 Visit entropy against Lévy index $\beta$.

7-10 Number of Lévy flights taken by each swarm agent vs. pheromone threshold $\eta$.

7-11 Pop-up threat detection with no sensor failures.

7-12 Pop-up threat detection with sensor failures probability $p_f = 0.1$.

7-13 Pop-up threat detection with sensor failures probability $p_f = 0.2$.

7-14 Pop-up threat detection with sensor failures probability $p_f = 0.5$.

7-15 Typical Search and Retrieval of Target by the Swarm at time $t = 35$ seconds.

7-16 Typical Search and Retrieval of Target by the Swarm at time $t = 222$ seconds.

7-17 Typical Search and Retrieval of Target by the Swarm at time $t = 736$ seconds.

7-18 Typical Search and Retrieval of Target by the Swarm at time $t = 1085$ seconds.

7-19 Target Quantity vs. Time.

7-20 Target Quantity vs. Time plot for comparing (1) Augmented search based on proposed area coverage law ($Green$) and (2) Random search ($Blue$).
# List of Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Full Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>LHS</td>
<td>Left Hand Side</td>
</tr>
<tr>
<td>RHS</td>
<td>Right Hand Side</td>
</tr>
<tr>
<td>FDM</td>
<td>Finite Difference Method</td>
</tr>
<tr>
<td>PDE</td>
<td>Partial Differential Equation</td>
</tr>
<tr>
<td>ACP</td>
<td>Area Coverage Performance</td>
</tr>
<tr>
<td>ACI</td>
<td>Area Coverage Integral</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction

1.1 Motivation

With the advancements in robotics, there is enormous potential for the application of autonomous robots in the number of domains. But as the problem complexity increases, a single autonomous robot is not enough for the completion of task. This gives rise to the need of robot swarm. To address such a problem, this thesis is focused on development of a swarm control law for robots.

1.2 Why Area Coverage with Robot Swarm?

Area coverage comes under the category of persistent tasks. For small area with global knowledge, a single robot is enough for monitoring. But as the area under consideration increases, it is very difficult to monitor environment with a single robot. If the environment is static, a single robot is sufficient, but it will consume a lot of time. If the environment is unknown or there are frequent changes in it, then single robot is not advisable. Robot swarm is one of the very important tools which can be used for these kind of persistent tasks. Here, swarm of robots need to keep the track of changes in the area under consideration and also need to search for the source of changes in that area [1]. Mapping and area exploration [2], large-scale search and rescue [3] comprise of area coverage as a major component. For application of robot swarm to carry out this task, one has to confront with difficulties in the swarm control.

Conventionally, the swarm of robots is controlled using a central computer or a leader robot
in the swarm. This technique suffers with drawbacks as follows:

- The central control depends only on single machine. Failure of central machine renders the whole system useless. Thus, this system lacks redundancy.

- A computer comes with limited computational capacity. So if each agent is performing some complex task, the central computer may lack the capacity to accomplish such a task. This leads to limited complexity of the task assigned to the swarm.

- Due to computational limitations, this swarm may also become incapable to handle indefinite number of agents. Therefore, if number of robots in the swarm cannot be increased without evaluating the computational capabilities. This leads to the issues in scalability of the swarm system.

To overcome these limitations, there are two methods which can be used to control the swarm. One is decentralized approach and other is distributed approach. These two swarm control approaches can be explained as follows:

1. Decentralized Approach: All the swarm agents are divided in subgroups. Each group has its own leader. All the group leaders in the swarm, have a super-leader which is controlling group leaders. This allows scalability of the swarm to some extent. But limitation of each group leaders capacity comes into picture, if the number of swarm agents in a particular group keep increasing. In this system, the failure of super-leader leads to formation of multiple centralized swarms.

2. Distributed Approach: Every agent acts independently based on its local knowledge. This system has infinite scalability and high robustness. Although there are advantages of this approach, there are difficulties in its development and implementation. One has to consider the lower level architecture of each swarm agent with details such as resource allocation, communication etc.

The figures (1-1), (1-2) and (1-3) show the illustrative schematics of the centralized, decentralized and distributed swarms of unmanned aerial vehicles (UAVs) respectively.
Figure 1-1: Centralized Swarm

Figure 1-2: Decentralized Swarm
1.3 Swarms in Nature

Despite the difficulties in distributed approach, the gregarious species observed in nature like ants, bees, birds, fishes are able to coordinate and form swarms with large population. Here, a single agent does not show any explicit intelligence, but the swarm as a whole can accomplish incredible tasks. The biological intelligence observed in these species cater for the drawbacks of swarm control techniques.

1.3.1 School of Fish

It is observed that various species of fish, which are involved in schooling, do so because of the risk of being eaten by other predators in ocean [4, 5]. Because of schooling, these fish species are able to scare away or confuse the predators. Figure (1-4) captures a swarming behavior observed in fish.
1.3.2 Flock of Birds

Swarming is also observed among birds. Birds form flocks while migrating. One of the primary reason for migration in birds is food. As per the studies, the cost of migration is reduced because of this swarming behavior in birds [7]. The fig. (1-5) shows the example of bird swarming observed in Eurasian Cranes.
1.3.3 Swarming in Ants

Ants show social behavior. It is observed that these insects live in fundamental family like unit and their lifecycle is organized around this unit known as ant colony. Although a single ant does not show any particular intelligence, ant colony as a whole is able to perform complex tasks as food foraging and building huge nests. Figures (1-6) and (1-7) show the examples of the complex behavior ants can show in a swarm.

![Ant Bridge](image)

Figure 1-6: Ant Bridge

In fig. (1-6), it can be seen that ants form a bridge like structure. Figure (1-7) captures some safari ants in a trail. In this trail, some soldier ants form a tunnel to protect the worker ants.

The speciality of all these behaviors is that there is no leader or central control for the swarm. But there is emergence of intelligence and desired patterns with simple rules which each swarm agent follows. Thus, to overcome difficulties in the design of distributed approach, the swarms intelligence inspired from these biological species can be used.

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3Photograph [9], distributed under a [CC-BY 2.0 license](https://creativecommons.org/licenses/by/2.0/)
1.4 Ant Foraging

Organism mobility in response to a chemical stimulus is known as chemotaxis. One of the well known system where chemotaxis is observed and well studied is *E. coli* [11]. It is observed that various bacteria, single-cellular and multi-cellular organisms direct their movements according to a particular chemical in their environment. For bacterial and micro-organisms, this is significant to find food in the surrounding environment. Many insect species follow a similar approach for food foraging.

Ant colony is one of the well studied systems following this approach for foraging and network formation. A single ant is not particularly intelligent but the as a group, ants come up with incredible complex patterns [12].

At the beginning, each participating ant starts moving randomly. While ant colony forages
for food, the ant which finds the food source returns to the nest with the food. While returning, it secretes the chemical pheromone in the environment along its path which results in formation of the pheromone trail. If other ants in the colony who are still foraging and have not found the food source, sense this pheromone, they follow the trail and find the food source. These ants while returning to the nest reinforce the trail with the pheromone. Ants detect the pheromone and its relative concentration to make their decision to move in particular direction. The direction with higher pheromone concentration along the trail is likely chosen. This gives rise to the biased random walk in ant colony. Mean while, this pheromone is a volatile chemical and it has the properties of diffusion and evaporation. Diffusion leads to spread of this pheromone in the surrounding area and evaporation leads to depletion of the pheromone trail.

In ant colony system, pheromone acts as an implicit mode of communication [13]. This mechanism leads to the emergence of social behavior and network formation in the colony. This phenomenon of a consensus through indirect coordination is also known as stigmergy.

Stigmergy in ant colony system leads to the formation of distributed network. As the communication among ants in the colony is dependent on the amount of pheromone they can observe in their locality, there is no central governing agent. Indefinite number of ants can be added or removed from the colony making the swarm scalable. Each ant follows rules which depend on the pheromone which it observes in the surrounding making the network simple and solving the complexity issues.

Figure (1-8) explains the ant foraging in picturesque manner. It demonstrates how the trail formation from the food source to the nest happens because of the pheromone gradient along the trail. Depleted food source resulting in the decaying pheromone trail and the formation of the new trail as the foraging ants find a new food source are also depicted in the image.
1.5 Lévy flights

With section 1.3 describing various swarm behaviors observed in nature, it is also observed that many species perform searching task in random manner. When in search of food, an agent in the swarm explores the surrounding environment randomly. During this random search many animals like albatross, marine predators and even some micro-organisms show specific pattern of movements which involve a lot of small steps interspersed with occasional large excursions [14]. This behavior was specially observed when there was scarcity of food. This mix of small steps and occasional very large excursions is known as Lévy flights.

The Lévy flights present an efficient approach for search. These result into fractal movement trajectories giving rise to optimal search in complex topographies. This promotes their application for search with actual robots in unknown area as well.

1.6 Objectives

The objective of this work is to develop an efficient biology inspired control law for robot swarm which can be implemented in area coverage. The control law is based on ant foraging
hypothesis combined with Lévy flights. This offers the idea of improved performance in area coverage task. In the case of mobile robots, coverage is often achieved as a result of the robots physically moving throughout the area of interest. Thus, this problem can also be extended to area search and rescue task.

Objectives of this thesis work are highlighted as follows:

- Area coverage and its analogy to ant foraging.
- Mathematical modelling of ant foraging behavior and development of swarm control law.
- Mathematical modelling of Lévy flights.
- Development of ant foraging inspired robot swarm control law with adaptive Lévy flights for area coverage.
- Introduction of target search and retrieval problem as well as its mathematical modelling based on ant foraging inspired robot control law.
- Validation of concepts by numerical simulations.

1.7 Contributions

The main contribution of this work is development of the robot swarm control framework for area coverage inspired by the ant foraging hypothesis with adaptive Lévy flights. The mathematical model for the proposed swarm control law is presented in this work. The work is further extended to solve the target search and retrieval problem with robots. Comparison and benefits of adaptive Lévy strategy for swarm with conventional random search approach is discussed. Parameter variation study of the proposed swarm control models using numerical simulations is done. The results obtained show the improved area coverage performance with this approach.
1.8 Organization of Thesis

Chapter 2 covers the literature review for the swarm techniques. Chapter 3 gives the derivation of the swarm control law. Followed by the mathematical description for Lévy flights in chapter 4. Chapter 5 discusses the computational implementation of the approach and the adaptive transition between Brownian motion and Lévy flights in ant foraging robots. In chapter 6, presents the mathematical model for search and rescue task in combination with the derived control law. Chapter 7 represents numerical results for the proposed strategies. Chapter 8 includes conclusion and future work.
Chapter 2

Robot Swarm Based Area Coverage: Literature Review

With the development in computer intelligence and robotics, robot swarms have gained special attention. Multi-robot systems have potential to perform complex tasks in various scenarios ranging from security applications such as intruder detection and environment monitoring to household applications such as floor cleaning. The common element in these scenarios is area coverage. In area coverage, the goal of the team of robots is to cooperatively observe the whole area with the available resources on the agents in the robot swarm. In this case, the map of the area under consideration in this task may be known beforehand or unknown [15].

2.1 Why Area Coverage?

Electronics miniaturization has made it possible to monitor many different scenarios which was not possible before. The work in [16] presents the feasibility of use of mobile and static sensor networks for use in environmental sciences. This kind of sensor based monitoring can be of immense help for monitoring climate changes around the globe. This idea also gives rise to the possibility of application of mobile robots with suitable sensors to be used in unknown dynamic environment for the source of pollution. In this case, as the size of the area grows, robot swarm proves to be a perfect solution.

Swarm robots provide a solution of redundancy issue in hazardous tasks [17]. Though the failure of a single robot in a swarm will reduce the speed of the task at hand, it will still be able
to complete the task. For example as if the swarm is used for the work like “Volcan Project” [18], then monitoring task will not be hindered by a single robot failure. (“Volcan Project” is for aerial surveillance of volcanic areas to analyze volcanic gases.)

In the application of multiple UAVs for environment monitoring as described in [19], the ground control station was responsible for the coordination of 3 UAVs. But if the task involved more agents, the human control is infeasible, as the system cannot self-sustain with the loss of human factor.

Swarm intelligence can also be used in underground mining [20], planetary explorations [21], detecting and tracking wildland fires [22], nanoscale drug delivery [23].

To address these limitations and exploit the potential of multi-robot systems, today’s research tries to build machine which can learn from failures and build swarms which can self-sustain. Swarm intelligence provides this pathway.

### 2.2 Area Coverage Based on Multi-Robot Systems

Number of different approaches are proposed in literature for control of multi-agent systems and area coverage. The planning and control framework for multi-agent systems can be classified in three broad classes, viz., artificial potential field methods, roadmap approaches and cell decomposition methods [24]. In all these methods, a continuous problem is discretized within the sample space to find the feasible solution.

Potential field based methods are generally applied to solve pattern formation problem. The work in [25] presents an example with this approach where the centralized multi-agent system ensures global asymptotic convergence to particular formation with collision avoidance. The distributed control for collision free formation stabilization presented in [26] is also based on artificial potential field associated with the desired formation. The path planning approach for multi-robot formation using potential-based genetic algorithm is proposed in [27]. Although the approach presented with safe collision free paths for robot swarm, the algorithm requires a global path planner and a motion planner. The method based on mixed integer linear programming [28, 29] is also proposed for multi-agent formations. This is a centralized approach and
with high computational cost.

The roadmap approach are used for the task of area exploration with multi-robot systems. One of the examples of this kind is Voronoi diagrams (Voronoi Tessellation) based techniques. Figure 2-1 illustrates the Voronoi diagram.

The Voronoi diagrams are often applied to robotic exploration. This method can be thought of as divide and conquer approach. The environment is segmented and the critical points (usually centroids of the tessellations) are extracted. These critical points are allocated to each exploring robot in multi-agent system as the target [31]. Other methods using Voronoi diagrams are presented in [32, 33, 34].

There are also probabilistic roadmap approaches used for exploration purposes. The work in [35] is a great example of this approach. The authors have described and proved experimentally that a team of mobile robots can explore the environment and perform localization. In this approach, the robot agent in the team can detect other team members during localization. The probability density function is employed by each robot to estimate the location of other agents. This work shows that the global localization is possible even with the robots not knowing their initial positions. But this approach has limitations with respect to certainty of detecting other robot agent. The case when a robot has a false positive reading, i.e., when it erroneously detects

1Photograph [30], distributed under a CC BY-SA 4.0 license
another robot agent when there is none. Authors have discussed the limitation of the approach in such a case.

The work on sensor based coverage present in [36, 37]. In [36], the authors have presented an explicit communication based approach for coverage control with realistic model of multi-robot communication network. In this network model, each agent in the network has limited communication/sensing range which is smaller than the area under consideration. With this constraint, the work proposes two control laws which are compliant for implementation on multi-robot system derived with combination of tools from system theory, graph theory and locational optimization. These gradient ascent control laws are in: (1) continuous time setting, (2) discrete time setting. To perform the stability analysis of control laws and smoothness analysis of the objective function in this approach, the authors have proved extensions of the discrete-time LaSalle Invariance Principle and the conservation of mass law from fluid dynamics respectively.

The spatially-distributed approach in [36] for group of mobile robots was extended with distributed implementation of deterministic annealing (DA) technique in [37] to avoid the local optimum for the objective functions. This local optimum is one of the major drawbacks with the gradient descent based techniques as these are greatly dependent on initial condition. Therefore, the use of DA. Although, DA is less prone to local optima, it does not guarantee the convergence to global optimum.

For area coverage using robot swarm, often a cell decomposition approach is used. In this case, the continuous space is divided into discrete contiguous regions. The work in [38] presents a real time and robust coverage approach with physical vacuum cleaning robots. It proposes a On-line Robust Multi-robot Spanning Tree Coverage (ORMSTC) algorithm. But with this approach, the robot visits each cell only one time making this suitable for static environment. Although with no agent failure in the system, ORMSTC guarantees complete area coverage, it fails to address the case of dynamic environment. The work with single robot with similar approach is also available in [39] where the robustness with respect to the ability to filter bad sensors readings is considered.

Another work on cell decomposition [40], proposes the approach where the environment is
considered static and moment of all the robots are planned which results in considerable amount of complexity. This work is built on the concept of Boustrophedon cell decomposition which allows the representation of diverse and non-polygonal obstacles in configuration space of the robot. This approach is scalable for $N$ robots and two robots can communicate only when one is in line of sight of other robot. This approach was designed specifically for the static environment. Therefore, the authors have tried to address the issues of repetitive area coverage.

In [41], area coverage with pattern-based genetic algorithm where the environment is partitioned among robots considering their sensing capabilities is presented. It performs routing and partitioning of the area concurrently among all the robots. This approach has the advantage over two-stage (routing followed by partitioning) planning based multi-robot area coverage. The authors have provided the experiment results to support their claim. Figure 2-2 shows the overlapping disk pattern. The colored area is to depict the obstacles. The area coverage task assigned using this disk pattern lives no area out of reach for robot team. Genetic algorithm is then used to decide the order in which each disk area is visited.
The dynamic aspect of area coverage due to mobile agents is studied in [42]. The authors have presented the results of the study on intrusion detection capability of this network. This work emphasize on the effectiveness of mobile sensor network to detect the intrusions/changes that can never be detected in stationary network.

### 2.3 Bio-inspired Robot Swarm and Area Coverage

To make distributed swarm control possible, nature provides the best option where many social species can be observed. Groups of different animals like birds, fish, ants, bees work together to perform some particular tasks as discussed in previous chapter. Lot of research in swarm robotics is also inspired inspired by the behavior of these species. One can infer that an individual agent does not plan its actions for future before hand, but the decision making is done with the sensory inputs it currently perceives in its surroundings. Thus, these swarms appear to be robust, flexible and easily scalable.
In [43], authors have examined *Polistes* wasp colonies and task assignment among the agents in those colonies. Based on the observations, they proposed the simplified swarm model which can be applied in robotics. This model involves the interactions between individuals and their neighborhood (local information) resulting in emergence of certain form of task distribution within this gregarious group. The individuals are mobile and have local computational capabilities. When two individual meet, the interaction and information transfer between them is modelled using Fermi function depending on their force variables. This force variable for each agent indicates its influence value. The effect of one agent over the other is judge based on some threshold for the force variable. Generation of a collective pattern was observed in this model with each agent. This model can also be applied to accomplish multiple tasks if the corresponding multiple threshold values are introduced in the system.

Craig W. Reynolds’ model [44] is also one of the famous nature inspired swarm model. This model was inspired from the flocking behaviors observed in birds. In this swarm, each boid (or swarm agent) follows the three rules stated briefly, and in order of decreasing precedence: (1) Collision Avoidance - avoid collisions with neighboring boids; (2) Velocity Matching - each boid tries to match velocity with nearby boids; (3) Flock Centering - a boid always tries to stay close to nearby boids.
Swarm robots presented in [46] are inspired form the collective behavior observed in cockroaches. Using a probabilistic finite state machine approach these robots are able to collectively aggregate in an arena circular in shape.

Self-organization observed in ants is has been of great interest to robotics and swarm intelligence community. As discussed in [1,3,3] ants can perform complex tasks by following simple rules based on only the local information. Figures (2-3) and (2-4) show some of the amazing and complex structures by ant colonies.

To emulate this behavior in robots, scientists have modelled various agent based models for ant colony. Ant trail formation based on partial differential equation is presented in [47]. In this model, author has considered the dispersion of ants and the pheromone. The pheromone is responsible for inducing the attractive force on the ant concentration in the model which results in trail formation. The work studies the effect of the variation of attractive force of pheromone.

\footnote{Photograph [45] distributed under a CC BY-SA 3.0 license}
on the ants.

Figure 2-4: A termite nest built on tree trunk

The work on ant colony inspired algorithms for robot swarms is presented in [49]. In this work author has not considered pheromone based information transfer, but direct communication between the robots. When this swarm starts area exploration, some of the robots become beacons (remain stationary) after dispersing. These beacons simulate the pheromone for the ant based system and transfer information to mobile agents in the swarm. The failure of any of these beacons will result in the disruption of swarm communication network and the area where the failure has occurred will remain unexplored. The author has not explored the effects of variation in pheromone deposition and evaporation rate on the system performance. The work on random walks in this research is also limited to uniformly distributed random walks.

The work on area coverage using ant foraging robot swarm model is available in [50][51][52][53]. In [50], authors have developed the generic theoretical model which can emulate the ant foraging behavior. The model accounts for the the emergent behaviors observed in ants.

\(^3\)Photograph [48] distributed under a CC BY-SA 2.0 license
In the simulation results of this work showed the trail behavior of ant colony is an emergent phenomenon. The model emulated the experimental results observed in the ant species *P. megacephala*.

The work in [50] was continued in [51] where the authors came up with the control laws in continuous time for 2D robot swarm based on ant foraging. Analysis of numerical stability for the proposed continuous time control law model was done. The results of this analysis advocate the critical dependence of dynamic stability on the physical parameters of control laws. The study of noise intensities variation and its effect on the system is also done in this work demonstrating the importance of noise. White Gaussian noise was considered.

The ant foraging based for robot swarm is further investigated by [52, 53]. In these, authors have concentrated on area coverage. The robot swarm was used for area coverage. Instead of trail formation as observed in ants, the control law as used in [51] was modified to cover as much area as possible by swarm robots. The robots used digital pheromone. The work in [52] investigates area coverage capabilities of the swarm with variation in randomness. A metric was developed for measuring the performance. The diffusion and evaporation constants of the robot pheromone were varied and system performance was analyzed. The noise variation study is also provided. Normal distribution was used for this study. More detailed noise variation analysis was conducted in [53]. Here, the swarm inspired from ants was implemented with Lévy flights. The work measured the swarm performance with three different performance metrics: area coverage performance; visit entropy; pop-up threat detection. The authors observed the advantage of Lévy flights over the simple Gaussian noise.

Lévy flights were introduced in section 1.5. In literature, there is not much work available in robotics with implementation of Lévy flights. This is mainly because of mathematical convenience of using Gaussian noise. Lévy flights are observed in many natural systems. Especially, while foraging many species including fruit flies, wandering albatross, jackals, spider monkeys, reindeer, dinoflagellates are observed to execute Lévy flights/walks [14]. It has also been shown that this movement pattern is more efficient while foraging rather than just random movements based on white noise [54]. This naturally observed efficient behavior is the motivation behind the research in this thesis.
In [55], the authors have addressed the Lévy foraging hypothesis in robot search. The research provides the comparative study of Lévy flights and Brownian motion for moving target detection in the unknown environment. The metric of detection rate was considered. With the results of this study author has confirmed the superiority of Lévy flights over the conventionally used random walks. It was observed that Lévy flight outperforms the random walk if the sensor detection radius is not too small or too large.

The work in [56] presents an approach for random search with adaptive switching from Brownian to Lévy motion based on biological fluctuation. But this work is limited only to a single robot. The mathematical model used in the robot is inspired from the simple and noise exploiting bacterial movements.

### 2.4 Other Applications of Swarm Intelligence

Many heuristic approaches for optimization methods are also inspired from these nature inspired swarm behaviors. Particle swarm optimization [57] is inspired from bird flocking, firefly algorithm [58] is inspired by the behavior of fireflies. Ant colony optimization [59] is a heuristic approach which is inspired by the ant behavior and can be used to find optimized paths through graphs. Cuckoo search [60] is another optimization approach inspired by the obligate brood parasitic behaviour of some cuckoo species. The authors have also applied Lévy flights in this work as a perturbation term and showed the increase in the performance of the algorithm in search for optima. Cat swarm optimization [61] is swarm intelligence algorithm based on cat behavior.

### 2.5 Motivation For This Work

The approach compliant with the Lévy foraging strategy observed in nature suggests the transition of foraging from Brownian motion to Lévy flights in case of sparse environment is beneficial for searching food source.

The analogous approach of adaptive switching between Brownian motion and Lévy flights has never been applied to robot swarm inspired from ant colony behavior performing area
coverage tasks. The results in this work show the advantages of this approach. Here, the performance of the swarm is measured with three distinct metrics based on percent area coverage, visit entropy and pop-up threat detection. The superiority of adaptive switching between Brownian motion and Lévy flights for area coverage confirms the suitability of its application which involve search and monitoring in unknown environments.
Chapter 3

Swarm Control Law For Area Coverage

As discussed in section 1.4, ant colony provides a potential solution for solving the problem of distributed swarm control design. In this work, to design the control law based on ant foraging we start with Keller-Segel model for chemotaxis [62].

3.1 Evolution of Ant Foraging Models

In [63], a mathematical model of foraging of ants with the assumption that a fixed number of trails exist was proposed. This model can be represented with the equations (3.1) and (3.2).

\[
\frac{db_i}{dt} = p_i \Lambda q - \gamma b_i \quad (3.1)
\]

\[
p_i = \frac{(k + b_i)^n}{\sum_{i=1}^{N} (k + b_i)^n} \quad (3.2)
\]

Equation (3.1) stands for the evolution of the pheromone concentration \( b_i \) along the trail \( i \) with time \( t \). \( p_i \) is the probability of the ant to choose one of the \( N \) available trails. \( \Lambda \) is the stochastic variable which takes the value 1 based on the value of probability \( p_i \) otherwise it is zero. \( \gamma \) is the evaporation rate of the pheromone on the trail.

Equation (3.2) gives the probability of choosing trail \( i \) out of \( N \) available trails. \( k \) in this equation is value of pheromone attributed to unmarked trail, i.e., the trail which remains unexplored by the ants and therefore having no pheromone laid on it. Value of \( n \) incorporates the non-linearity in this ant foraging model. Even if the pheromone quantity on trail \( i \) is slightly
higher than some other trail \( j \), the high value of \( n \) significantly increases the probability of an ant choosing trail \( i \) with higher pheromone concentration.

Although the model from equations \(3.1\) and \(3.2\), explain the evolution of ant population in case of fixed number of trails, the main problem of emergent trail formation in ant foraging is not addressed.

The Keller-Segel model was introduced in [50] to address the limitations in prior ant foraging model. This model can be explained with equations \(3.3\) and \(3.4\).

\[
\frac{\partial a(r, t)}{\partial t} = \nabla \cdot (D_a \nabla a(r, t) + \rho_s a(r, t) \nabla b(r, t) + \rho_o a(r, t) \nabla N(r)) \tag{3.3}
\]

Equation \(3.3\) represents the evolution of concentration of ants with time \( t \) at space coordinate \( r \in \mathbb{R}^2 \). First term in \textit{RHS} stands for diffusion of ant population which gives rise to random component of ant movement. \( D_a \) is the diffusion coefficient of ant concentration. Second term stands for the pheromone gradient based motion of ants towards the food source. \( \rho_s \) is proportion of ants in food search mode. Third term on \textit{RHS} stands for proportion of ant population carrying food \( \rho_o \). The ants carrying food move along the nest gradient which is given by the equation \(3.7\).

\[
\frac{\partial b}{\partial t} = D_b \nabla^2 b(r, t) + \rho_o q a(r, t) - \gamma b(r, t) \tag{3.4}
\]

Equation \(3.4\) represents the evolution of pheromone in the environment with time. The equation comprises of pheromone diffusion term with diffusion coefficient \( D_b \), pheromone deposition term by food carrying ants with deposition rate \( q \) and pheromone evaporation term with evaporation rate \( \gamma \). The pheromone deposition rate \( q \) was taken as per equation \(3.5\).

\[
q = \frac{q_o}{1 + d_T} \tag{3.5}
\]

where \( q_o \) is a constant and \( d_T \) is the distance of the ant location \( a(r, t) \) from the food source.

The following equation \(3.6\) gives the relationship between \( \rho_o \) and \( \rho_s \).

\[
\rho_o + \rho_s = 1 \tag{3.6}
\]
As per this model, ants are in search mode when they are not carrying food.

\[ N(r) = \frac{1}{1 + (r - r_N)^2} \]  

(3.7)

where \( r_N \) is the location of ant nest.

With this model authors showed the emulation of ant foraging is possible and emergent trail formation was observed. With the coupled equations (3.3) and (3.4), the limitations of prior model discussed in equations (3.1) and (3.2) were also addressed.

In the following section, the model of area coverage control law is derived using ant foraging model described in [50].

### 3.2 Development of Swarm Control Law for Area Coverage

Keller-Segel Minimal Model can be used for derivation of ant foraging based area coverage control law for swarm of robots [52, 53, 64]. It is based on the reaction of a swarm agent to local pheromone concentration averaged over the swarm with infinite population. This model therefore accounts for the coupled evolution of the robot and pheromone concentrations with equations (3.8) and (3.10) respectively.

\[
\frac{\partial a(r, t)}{\partial t} = \nabla \cdot \left( D_a \nabla a(r, t) - \chi a(r, t) \nabla b(r, t) \right) + k(r, t) 
\]  

(3.8)

where \( D_a \) is diffusion coefficient for robot concentration \( a(r, t) \), \( b(r, t) \) is pheromone concentration, \( \chi \) is repulsiveness (positive value \( \chi \)) or attractiveness of pheromone (negative value \( \chi \)).

LHS in equation (3.8) represents rate of change of robot concentration with respect to time. The first term in RHS represents the robot diffusion flux while the second term constitutes the effect of the pheromone diffusion flux on the robot concentration. This term reflects the coupling in the evolution of the two dynamic components of the system, viz., robots and pheromone. The third term in this equation constitutes the death of the robot agents.

In developing the chemotaxis model in [62], the authors have considered the group of living cells. But here, we consider the robot swarm and also assume the number of robots remain
constant in the swarm. Therefore, simplifying equation (3.8):

\[
\frac{\partial a(r,t)}{\partial t} = \nabla \cdot \left( D_a \nabla a(r,t) - \chi a(r,t) \nabla b(r,t) \right) \tag{3.9}
\]

In equation (3.9), the third term \(k(r,t)\) from (3.8) is eliminated.

\[
\frac{\partial b(r,t)}{\partial t} = \nabla \cdot D_b \nabla b(r,t) + g(r,t) - h(r,t) \tag{3.10}
\]

where \(D_b\) is pheromone diffusion coefficient, \(g(r,t)\) and \(h(r,t)\) are pheromone variation functions, \(t\) is time and \(r \in \mathbb{R}^2\) is the space coordinate in two dimensions.

In equation (3.10) \(LHS\) represents the rate of change of pheromone concentration. In \(RHS\), the first term represents the diffusion of the pheromone, the second term accounts for the creation of the pheromone in the environment and the third term is representing the depletion of pheromone.

Equation (3.10) can be further simplified to have a linear pheromone deposition or production rate \(q\) and linear evaporation rate \(\gamma\) as follows:

\[
g(r,t) = qa(r,t) \tag{3.11}
\]

\[
h(r,t) = \gamma b(r,t) \tag{3.12}
\]

The equations (3.11) and (3.12) are the result of the following reasons respectively: (1) The creation/deposition of pheromone \(g(r,t)\) will happen only at the locations where the robots are present. (2) The depletion of the pheromone \(h(r,t)\) because of evaporation which is assumed as constant value \(\gamma\).

\[
\frac{\partial b(r,t)}{\partial t} = D_b \nabla^2 b(r,t) + qa(r,t) - \gamma b(r,t) \tag{3.13}
\]

Here, the rate of change of pheromone concentration with time is the function of diffusion,
deposition and evaporation. The first term for diffusion on the RHS is diffusion coefficient and Laplacian of the pheromone field, the second term incorporates pheromone deposition rate of each robot and robot concentration and the third term incorporates pheromone evaporation rate and pheromone concentration.

The diffusion coefficient $D_b$ in equation (3.13) is a proportionality constant between the diffusion of pheromone and pheromone gradient. This can be thought of as the value which determines the speed at which the pheromone spreads in the environment. Its dimension is area per unit time. In case of the deposition rate $q$, linearity means a constant amount of pheromone is created or deposited by a swarm agent per unit time in the environment. For evaporation rate $\gamma$, the units of measurement can be thought of as the amount of pheromone depleted per unit time.

Equations (3.9) and (3.13) make the Fokker-Plank equation to outline the change in robot concentration with time. But for application in actual robots, discretization of these equations is required.

The coupling in the robot and pheromone concentration results into biased random walk which is presented in (3.9). For the individual robot the approximation of motion can be represented with summation of gradient of pheromone concentration and noise term. Equation (3.14) describes the discrete Langevian equation derived from continuum description of robots in equation (3.9). This equation can be used for actual robots.

$$\dot{R}_i = v_i = -\chi \nabla b(r,t)|_{R_i} + \sigma dW$$

(3.14)

It should be noted that this kinematic model was designed neglecting the inertial components of the swarm agents so as to use robot velocity as the function of pheromone gradient and constant magnitude noise only. In equation (3.14), the fist term is the pheromone gradient following component and second term is the noise term with constant noise magnitude $\sigma$ and $dW$ representing white noise. $R_i$ and $v_i$ are the position and velocity of the robot $i$ respectively. The direction of noise is sampled from uniform distribution.

As the final equation relating the continuous system description for robot population in the
swarm in (3.9) and the discrete description presented in equation (3.14) is presented as follows:

\[ a(r, t) = \sum_{i=1}^{N} \delta(r - R_i(t)) \quad (3.15) \]

where total number of robots in the swarm are denoted by \( N \). Therefore, RHS represents the summation of the Dirac delta(\( \delta \)) functions for each swarm agent.

The combination of kinematic model presented in the equation (3.13) and (3.15) are together applied for simulating the swarm system.

It should be noted that the negative sign for the first term on the RHS of equation (3.14) facilitates the area coverage. If the sign is reversed, it results in the rendezvous. In this case, the pheromone being attractive will result in the robots attract each other.

In case of ants, this strategy of attraction to the pheromone results in attracting the agents who are searching for food by the ant who has found food.
Chapter 4

Mathematical Model Of Lévy Flights

In nature, if one observes the animal movements, it can be seen that the movement in various species is because of number of intentions like in search for possible mate, food and water foraging, escaping predators. In case of search as described in section 1.5 animals are observed to conduct unoriented search with little or no prior information about the environment. With the research in resent years, it has been evident that many species tend to explore the unknown environment with Lévy flights. The Lévy flights are the random walk where each step length is chosen from Lévy distribution. Figure 4-1 illustrates the Lévy flights.

Figure 4-1: Lévy Flights: 3000 steps with step-length chosen from Lévy Distribution
Comparing figures (4-1) and (4-2), one can see the difference between the Lévy flight scheme and Brownian motion scheme. During Lévy flights some of the steps taken are very large, which makes it possible to cover larger area.

Figure 4-2: Brownian Motion: 3000 steps with step-length chosen from Normal Distribution

For generating the figures (4-1) and (4-2), the simple point models were used. These are given in equation (4.1) and (4.2). Equation (4.1) explains the Brownian motion and (4.2) explains the Lévy flights.

\[
P_r(t + dt) = P_r(t) + N(0, \sigma) \tag{4.1}
\]

where \( N(0, \sigma) \) generates a random vector from normal distribution.

In equation (4.2) and (4.3), \( \theta_{\text{rand}} \) is the random number generated from uniform distribution between [0, 1]. LF is the random number generated using Lévy distribution.

\[
P_r(t + dt) = P_r(t) + \nu_{\text{rand}} \times LF \tag{4.2}
\]
\[ v_{\text{rand}} = \begin{bmatrix} \cos(2\pi \theta_{\text{rand}}) \\ \sin(2\pi \theta_{\text{rand}}) \end{bmatrix} \]  

(4.3)

4.1 Properties of Lévy Flights

Lévy distribution is distribution of sum of \( N \) identically and independently distributed random variables having high variability.

The properties of the Lévy flights can be enumerated as:

1. The steps in Lévy process are stationary.
2. Lévy flights have independent increments.
3. Lévy flights have infinite standard deviation.

In mathematical terms, if \( X = X_t : t \geq 0 \) is a stochastic process which results in Lévy flights then:

Stationary process

It is must be stationary process, i.e., for any time \( t \) and some \( h > 0 \), the value of \( X_{t+h} - X_t \) is independent of \( t \) and the difference is equal in distribution to \( X_h \).

Independent increments

It must have independent increments, i.e., for times \( 0 \leq t_1 < t_2 < \cdots < t_n < \infty \), the differences \( X_{t_2} - X_{t_1}, X_{t_3} - X_{t_2} \) and so on are independent.

Infinite Standard Deviation

Lévy distribution is a stable distribution \([65]\), i.e., if \( X_1, X_2, \ldots, X_n \) are random numbers chosen from Lévy distribution \( X \), then there exist \( c > 0 \) and \( d \in \mathbb{R} \) as constants which make equation (4.4) possible.

\[ X_1 + X_2 + \cdots + X_n = cX + d \]  

(4.4)
Therefore, if these random variables $X_i$ have the standard deviation $\sigma$, the value of $c$ and the value of the standard deviation of the resultant summation \[65\] can be given as:

\[
\begin{align*}
  c &= n^{\frac{1}{\alpha}} \quad (4.5) \\
  \sigma_{\Sigma} &= n^{\frac{1}{\alpha}} \sigma \quad (4.6) \\
  \alpha &< 1, \quad \text{for Lévy flights} \quad (4.7)
\end{align*}
\]

The dispersion of the sample mean of these random variables $X_i$ is:

\[
\sigma_{(\Sigma/n)} = \frac{\sigma_\Sigma}{n} = n^{-\frac{1}{\alpha}} \sigma \quad (4.8)
\]

When $n \to \infty$, the value of $\sigma_{(\Sigma/n)} \to \infty$.

Hence, the proof of the third property mentioned above.

### 4.2 Probability desntisty function of Lévy Distribution

The mathematical description of Lévy distribution \[65, 66, 67, 68\] can be given as shown in equation \(4.9\).

\[
L(s, \gamma, \mu) = \begin{cases} 
\sqrt{\frac{\gamma}{2\pi}} \exp(-\frac{\gamma}{2(s-\mu)}) \frac{1}{(s-\mu)^{1/2}}, & \text{if } 0 < \mu < s < \infty \\
0, & \text{if } s \leq 0
\end{cases} \quad (4.9)
\]

where $\mu$ is shift parameter and $\gamma > 0$ is the parameter which controls the scale of the distribution. $\gamma$ is responsible for the width and $\mu$ for the shift of the distribution peaks. In figure \(4-3\), the difference between the standard normal distribution and the Lévy distribution can be clearly observed.

The reason for the frequently long steps in the Lévy distribution can be visualized from the figure \(4-3\). As it can be seen, the Gaussian distribution settles down on both the sides of x-axis. The area below this distribution is near to zero towards the tail. Unlike normal
distribution, $pdf$ of Lévy distribution has a heavy tail, i.e., the significant area falls under the tail of the distribution. It can also be seem that as the scaling factor $\gamma$ increases, the area under the tail also increases and the tail of the distribution becomes more fat. Intuitively, fat tails in Lévy distributions makes the occurrence of large numbers more probable as compared to Gaussian distribution.

![Probability density function of Lévy Distribution](image)

**Figure 4-3:** Probability density function of Lévy Distribution with varying scale parameter and shift parameter of zero compared with Standard Normal Distribution

In terms of Fourier transform Lévy distribution is given as follows:

$$F(k) = \exp(-\alpha |k|^{\beta}), \quad 0 < \beta \leq 2$$  

(4.10)
where $-1 \leq \alpha \leq 1$ represents skewness. Negative value of $\alpha$ indicates left-tailed or left skew direction, positive value of $\alpha$ indicates right-tailed or right skew and zero value indicates that distribution is symmetric. $\beta$ represents the Lévy index. It is responsible for the shape of tail region of the Lévy distribution. For small values of $\beta$ longer step sizes are observed in the distribution i.e. longer tail whereas for large $\beta$ it has a short tail.

In case of Lévy flights as it can be observed in figure (4-3), the distribution has long tail and it is positively skewed (i.e., a long tail towards the right side of the distribution).

The analytical solution of inverse Fourier transform of equation (4.10) is not available except for few special cases [67, 66]. For $\beta = 1$, the inverse Fourier transform of (4.10) takes the form of Cauchy probability distribution and for $\beta = 2$ it takes the form of normal distribution.
Chapter 5

Computational Implementation and Performance Metrics

This chapter presents the computational implementation details of the derived control law in chapter 3. Details of the robot kinematic model in the swarm and the intricacies involved in simulating the model are discussed. The model for simulating the Lévy flights in this work is also presented. The chapter also presents various performance metrics used in this work and their implementation details.

5.1 Implementation of the Control Law

For the control law developed in chapter 3, one has to look at the implementation of two terms. One is the kinematic model of the robots as derived in equation (3.14) and other is the implementation of the diffusion for the digital pheromone which is used in this model as specified in equation (3.13).

5.1.1 Pheromone Diffusion

Diffusion is movement of particles of from a region of high concentration to a region of lower concentration. In this case these particles are digitized pheromone concentrated at the location where it was deposited by the robot agent.

For simulating the pheromone diffusion as stated in partial differential equation (3.13), it is necessary to discretize the equation for practical computational implementation. This dis-
cretization is possible with the use of numerical methods available for solving partial differential equations.

Finite difference methods (FDM) are the dominant tools in mathematics for solving differential equations where the finite differences approximate the derivatives. The two well known methodologies for FDM are known as explicit method and implicit method. In explicit method the state of the system is calculated at time \( t + dt \) from the current state at time \( t \). Whereas implicit method solution is based on solving an equation involving current state of the system at time \( t \) as well as the state at later time \( t + dt \). Implicit methods need more computational power as compared to explicit. But in case of explicit methods, the numerical stability of the system is not acceptable as the solutions by this approach requires the step size to be taken extremely small. Therefore, to avoid this, implicit methods are preferred.

To put it simply, FDM is based on discretization of the function under consideration on a grid. The figure (5-1) explains the simple discretization of a function \( f(x) \) with grid size \( h \) along \( x \) – axis. Taylor series expansion is used to approximate the derivative of the function in this discrete space as shown in equation (5.1).

\[
f(x_i + h) = f(x_i) + \frac{f'(x_i)}{1!} h + \frac{f''(x_i)}{2!} h^2 + \cdots + \frac{f^{(n)}(x_i)}{n!} h^n + \text{HOT} \tag{5.1}
\]

Here \( \text{HOT} \) stands for higher order terms in the series. This equation be rewritten in simplified manner by including all the terms with power of \( h \geq 2 \) in \( \text{HOT} \):

\[
f(x_i + h) = f(x_i) + \frac{f'(x_i)}{1!} h + \text{HOT} \tag{5.2}
\]
By neglecting $HOT$ for small values of $h$, the resulting equation can be used to approximate the first derivative of the function $f(x)$ at $x = x_i$ as in equation (5.4):

$$f(x_i + h) = f(x_i) + \frac{f'(x_i)h}{1!}$$  
(5.3)

$$f'(x_i) = \frac{f(x_i + h) - f(x_i)}{h}$$  
(5.4)

Crank-Nicolson method is an implicit method available in FDM. This method is unconditionally stable for solving partial differential equations like diffusion [69, 70]. Hence, this method was chosen to solve the pheromone diffusion. This method is based on trapezoidal rule [70] used for calculating the integral of PDEs. The continuum representation of pheromone diffusion in first term on the RHS of equation (3.13) can be represented in a discrete form as follows:

$$b_{i+1}^r - b_i^r = \frac{D_b}{2} [B_i + B_{i+1}]$$  
(5.5)

$$B_i = b_i^{r+1} - 2b_i^r + b_i^{r-1}$$  
(5.6)

$$B_{i+1} = b_{i+1}^{r+1} - 2b_{i+1}^r + b_{i+1}^{r-1}$$  
(5.7)

Here, the discretization of the second derivative of the function $b(r, t)$ w.r.t. $x$ is presented.

An example for diffusion for a 5x5 grid is as follows:

$$
\begin{bmatrix}
  e_- & -D_b & 0 & 0 & 0 \\
  -D_b & D_b & -D_b & 0 & 0 \\
  0 & -D_b & D_b & -D_b & 0 \\
  0 & 0 & -D_b & D_b & -D_b \\
  0 & 0 & 0 & -D_b & e_+ 
\end{bmatrix}
\begin{bmatrix}
  b_{i+1}^1 \\
  b_{i+1}^2 \\
  b_{i+1}^3 \\
  b_{i+1}^4 \\
  b_{i+1}^5 
\end{bmatrix} =
\begin{bmatrix}
  d_i^1 \\
  d_i^2 \\
  d_i^3 \\
  d_i^4 \\
  d_i^5 
\end{bmatrix}
$$  
(5.8)
Here, the term $D_{b_1}$ in LHS of equation (5.8) is defined as shown in equation (5.9). And the term $d'_i$ on the RHS is defined as in equation (5.10).

\[
D_{b_1} = 1 + 2D_b \quad (5.9)
\]

\[
d'_i = D_b b'_{i+1} + (1 - 2D_b)b'_i + D_bb'_i \quad (5.10)
\]

and $e_-$ and $e_+$ are boundary conditions on negative side and positive side of axes respectively. For each step change from $i$ to $i+1$, the corresponding change in time is from $t$ to $t + dt$.

5.1.2 Deposition and Evaporation of Pheromone

With calculation of the pheromone quantity $b'_{t+dt}$ with diffusion as specified in previous section, the pheromone deposition and evaporation also needs to be considered.

To implement deposition and evaporation of pheromone, the values derived in equations (3.11) and (3.12) are to be added in $b'_{t+dt}$. This is shown in equation (5.11):

\[
b'_{t+dt} = qa(r,t) - \gamma b'_t \quad (5.11)
\]

Therefore, by following 5.1.1 and 5.1.2, it is possible to evaluate the resultant amount of pheromone at any time in the system.

5.1.3 Robot Movement

In area coverage task for the swarm of robots, the aim of each individual in the swarm is to go in the area which is not being monitored. To accomplish this, a robot kinematic model is developed in (3.14).

This model promotes the use of pheromone quantity at the robot’s current location for decision making. Based on pheromone gradient in the vicinity of the agent, it makes the decision

\footnote{Note: with this description in mind, the term $b'_{t+dt}$ is equivalent to $b'_{t+1}$}
to go in a particular direction. As per the law, the path which robot chooses is based on the repulsive action towards the pheromone.

Area coverage can be thought of as information search task. Pheromone is used as the quantifier for the information availability of robot’s current location. If new information is available to the robot, i.e., pheromone concentration at its current location is less than pheromone threshold value $\eta$, then the robot executes search with gradient following Brownian motion as per equation (3.14).

$$R_i(t + dt) = R_i(t) + \dot{R}_i dt$$

But there is still a way that this system’s performance can be improved. As specified in the importance of Lévy flights, many animal species use Lévy foraging strategy for food search. With the analogy of swarm robot area coverage to animal food foraging, information search can be thought of as the food search. With the same analogy if the robot agent in the swarm does not get any new information, it will take a Lévy flight.

Lévy flights induce perturbations in the system. Because of higher frequency of longer step lengths in those, the system may eventually achieve coverage of area in the remote distance sooner than expected which might eventually lead to superior area coverage than the approach specified in (5.12).

In mathematical terms which can be applied to actual robots, this can be specified as follows: If no new information is found by a robot agents in the surroundings means pheromone concentration at robot’s current location is high. This simply states the fact that one or more of the swarm agents have recently visited the area and therefore, the pheromone detection is high. To quantify high value of pheromone, threshold value $\eta$ is introduced. Therefore, robot agent chooses its next destination from Lévy distribution (i.e. it takes a Lévy Flight $LF$) and move towards that destination with constant velocity.

$$R_i(t + dt) = R_i(t) + LF$$

This kind of transition for robot movement from conventional ant foraging strategy to Lévy
flights is practical because it is based on the local information only. It considers robot agent and the pheromone it can observed in its neighborhood and no global knowledge is needed. Dependence on locally available information of this strategy makes it suitable to be used in case of communication constraints.

For Lévy destination, the direction of robot heading is drawn randomly from uniform distribution. But generating the random numbers which follow Lévy distribution is difficult. It is a challenge to manage the theoretical and numerical Lévy stable process. The work in [71], tackles this issue. Mantegna’s algorithm proposed in this work is chosen to simulate Lévy distribution [67, 72]. In this method, Lévy step size $s_i$ is drawn for robot $i$ as per equation (5.14). It should be noted that the algorithm presented in [71] generates symmetric Lévy stable distribution. The modulus of this taken for convenience in this work.

$$s_i = \left| \frac{u}{|v|^{\frac{1}{\beta}}} \right|$$  \hspace{1cm} (5.14)

The terms in the numerator and denominator of the equation (5.14) are taken from Gaussian distributions. These Gaussian distributions are –

$$u = N(0, \sigma_u^2)$$  \hspace{1cm} (5.15)

$$v = N(0, \sigma_v^2)$$  \hspace{1cm} (5.16)

The value of the standard deviations $\sigma_u$ and $\sigma_v$ in equations (5.15) and (5.16) are given by the following equations (5.17) and (5.18).

$$\sigma_u = \left\{ \frac{\Gamma(1 + \beta) \sin(\frac{\pi \beta}{2})}{\Gamma(\frac{1+\beta}{2}) \beta 2^{\frac{\beta}{2}}} \right\}^{\frac{1}{\beta}}$$  \hspace{1cm} (5.17)

$$\sigma_v = 1$$  \hspace{1cm} (5.18)

In equation (5.17), $\Gamma(x)$ is the gamma function and $0 \leq \beta \leq 2$ is the index of Lévy distribution.
The figures (5-2) and (5-3) illustrate the output of a MATLAB program for Mantegna’s Algorithm. Figure (5-2) shows the step length for 1000 random number generated using this algorithm and figure (5-3) represents the histogram of the corresponding step lengths.
5.2 Performance Metrics

If the ideal state of the process is identified analytically, it can be expressed as a metric for the process. Here, different metrics have been proposed to express the performance if the swarm in accomplishing the task at hand. This section defines and explains the significance of each metric in detail. These metrics are inspired from the work in [52, 53].

5.2.1 Area Coverage Integral

Area Coverage Performance

![Typical Area Coverage Performance Curve](image)

Figure 5-4: Typical Area Coverage Performance Curve

Area Coverage Performance (ACP) is a simple metric to measure the area covered by the swarm at given time. For the measurement purpose, area visited by swarm agents at least once should be marked as visited. For ACP, this area is counted and compared with the total area as
per the equation (5.19).

\[
ACP(t) = \frac{A_v(t)}{A_{\text{total}}} \quad (5.19)
\]

where \(A_v(t)\) is the area that has been visited at least once at time \(t\) and \(A_{\text{total}}\) is the total area under consideration.

Figure (5-4) shows the typical area coverage performance graph with respect to time for the swarm of 10 robots and for the area of dimensions 100x100 square units.

**Area Coverage Integral**

In figure (5-4), each point gives the value of ACP at any given time \(t\). Additionally, the slope of the curve at time \(t\) gives the instantaneous rate of area coverage. Both of these quantities form an important measure to quantify the comprehensiveness of the area coverage task. To represent both as a single number, area coverage integral (ACI) is introduced in equation (5.20).

\[
ACI = \int_{0}^{t_{\text{final}}} ACP(\tau)d\tau \quad (5.20)
\]

This metric makes it convenient to effectively compare the performance of swarm in different scenarios. It measures area under the curve in figure (5-4).

For implementation in real world where a discrete system is considered, the area is divided in discrete grids and is marked as visited if any of the swarm agents visit this grid.

\[A_v(t) = \sum_{x,y} ob(x,y,t), \quad \begin{cases} ob(x,y,t) = 1, & \text{if visited at-least once} \\ ob(x,y,t) = 0, & \text{otherwise} \end{cases} \quad (5.21)\]

Thus, the performance metric ACI can be written as:

\[
ACI = \sum_{t=0}^{t_{\text{final}}} A_v(t) \quad (5.22)
\]
5.2.2 Visit Entropy

If the area coverage is planned, i.e., the map of the area is already available and swarm movements are planned according to the map, number of times each area is visited is exactly equal. While in foraging task where an agent is searching at random locations as it moves, some of the areas will not be visited at all. Despite of this unevenness, similar visit frequency throughout the search space, which insinuates ordered area coverage and less entropy, is desirable unless the search is biased. The visit entropy metric was introduced in [53] as measure of area coverage performance.

To measure performance of the swarm robots based on the control law developed in this work, area under consideration is discretized in a grid. Each location in the grid is a cell. As each location is visited by a robot, we define a visit frequency $\psi(r, t)$ for each location as:

$$\psi(r, t) = \frac{n_{visit}}{t}$$  

(5.23)

where $n_{visit}$ is count of number of times a location in the area under consideration is visited at time $t$.

At any time $t > 0$, each location in the area under consideration is visited with different frequencies (if the location is not visited, the frequency is zero). Note that there can be more than one locations which have same visitation frequency. $k$ in $\psi_k(t)$ represents $k$th frequency value available which can be same at two or more locations at time instance $t$. $Pr(\psi_k(t))$ is the probability of being at a location with visitation $\psi_k(t)$.

$$Pr(\psi_k(t)) = \frac{\int_R A_{\psi_k}(r, t) dr}{A_{total}}, \text{ where } \begin{cases} A_{\psi_k}(r, t) = 1, & \text{if visited with frequency } \psi_k \\ 0, & \text{otherwise} \end{cases}$$  

(5.24)

$H(\psi(t))$ is the visitation entropy at time $t$ for given area as per equation (5.25).

$$H(\psi(t)) = -\int \sum_k Pr(\psi_k(t)) \log(Pr(\psi_k(t))) dk$$  

(5.25)
As frequency $\psi_k$ is a discrete value, the entropy in equation (5.25) is simplified to summation of all $k$ frequency values in equation (5.26).

$$H(\psi(t)) = - \int_k Pr(\psi_k(t)) \log(Pr(\psi_k(t))) dk$$  \hspace{1cm} (5.26)

The system visit entropy for whole simulation can be calculated by summing the value of $H(\psi(t))$ over the simulation time as shown in equation (5.27). This total simulation time is designated $H_{sys}$.

$$H_{sys} = \sum_{t}^{t_{final}} H(\psi(t))$$  \hspace{1cm} (5.27)

### 5.2.3 Pop-up Threat Detection

Detection of the pop-up threats the swarm is measured using a simple metric. For this metric random threats were introduced in the system. By random, it means a threat was introduced while the area coverage task is in progress at a random location at some time.

This metric along with the visit entropy metric forms a robust measure of the area coverage and swarm dispersion [53]. Although the pop-up threat detection is not proportional to entropy, but the ability of the proposed swarm to detect these threats gives a better understanding of dispersion capabilities of this swarm. It can be measured as follow:

$$P = \frac{\sum_{i=1}^{\rho_{total}} \rho_i}{\rho_{total}}$$, where

$$\begin{cases} 
\rho_i = 1, & \text{if } i^{th} \text{ pop-up threat is detected } \psi_k \\
0, & \text{otherwise}
\end{cases}$$  \hspace{1cm} (5.28)

Here, $\rho_{total}$ are total number of threats introduced in the environment. It should be noted that each pop-up threat is introduced for a limited amount to time and only a single pop-up threat is introduced at a time.

This gives a measure of swarm mobility and quantifies its advantage to look for the areas which were otherwise unexplored in case of area monitoring task done by a stationary network.
Chapter 6

Search and Retrieval With Ant Inspired Robot Swarm

Model developed for swarm control for area coverage in chapter [3] is extended here to consider the swarm for search and retrieval task. The ant-foraging model for robot swarm can be used for search and retrieval of bulk amount of target. The equations (3.14) is modified to equation (6.1).

\[
\dot{R}_i = v_i = \left( \chi_{si} \nabla b_a(r, t) - \chi_{si} \nabla b_r(r, t) + \bar{\chi}_{si} \nabla N(r) \right) |_{R_i} + \sigma dW \tag{6.1}
\]

In this equation, first term on RHS, i.e., \( \chi_{si} \nabla b_a(r, t) \) stands for the effect of attractive pheromone deposited by the robot in the environment. The second term \( \chi_{si} \nabla b_r(r, t) \) for the repulsive pheromone and the third term \( \bar{\chi}_{si} \nabla N(r) \) for the nest potential.

For robot \( i \), the various terms mentioned in above equation are explained in equations (6.2) and (6.3)\(^\text{1}\):

\[
\chi_{si} = \begin{cases} 
1, & \text{if robot } i \text{ is in search mode} \\
0, & \text{otherwise}
\end{cases} \tag{6.2}
\]

\[
\chi_{ai} = \begin{cases} 
1, & \text{if robot } i \text{ senses attractive pheromone} \\
0, & \text{otherwise}
\end{cases} \tag{6.3}
\]

\(^{\text{1}}\text{The use of } \bar{\chi} \text{ stands for the negation of the value of } \chi.\)
The addition of two distinct pheromones in the system facilitates the search as well as return of the robot with the target to the base. The attractive pheromone $b_a$ communicates the message of available target to the other swarm agents. The repulsive pheromone $b_r$ agitates the search performance of the robots.

The equation (3.13) modified to equations (6.4) and (6.5) for adding the effect of two digital pheromones.

\[
\frac{\partial b_r(r,t)}{\partial t} = \nabla \cdot (D_{b_r} \nabla b_r(r,t)) + q_r \sum_{i=0}^{N} \chi_{ai} \bar{\chi}_ai R_i - \gamma_r b_r(r,t) \quad (6.4)
\]

\[
\frac{\partial b_a(r,t)}{\partial t} = \nabla \cdot (D_{b_a} \nabla a(r,t)) + \sum_{i=0}^{N} \bar{\chi}_{ai} q_{ai} R_i - \gamma_a b_a(r,t) \quad (6.5)
\]

In these equations, $D_{b_a}$ is diffusion coefficient for attractive pheromone, $D_{b_r}$ is diffusion coefficient for repulsive pheromone, $b_r(r,t)$ is repulsive pheromone concentration, $b_a(r,t)$ is attractive pheromone concentration, $N(r)$ is nest potential, $q_{ai}$ is attractive pheromone deposition rate of robot $i$, $q_r$ is repulsive pheromone deposition rate, $\gamma_a$ is evaporation rate of attractive pheromone and $\gamma_r$ is evaporation rate of repulsive pheromone.

The behavior of the robots while depositing the attractive pheromone is changed as per the equation (6.6).

\[
q_{ai} = \frac{1}{1 + d_{fi}} \quad (6.6)
\]

where $d_{fi} \geq 0$ is distance of robot $i$ from the target.

The nest potential $N(r)$ is dependent on the distance from the nest. For the purpose of the implementation, it was simulated as the equation (6.7).

\[
N(r) = \frac{1}{1 + (r - r_N)^2} \quad (6.7)
\]

where $r_N$ stands for the nest location.

Model described in this chapter facilitates the distributed control for the swarm in the task.
of search and retrieval of target. If the robot detects the target, it starts depositing the attractive pheromone which in turn is observed by the other robots who are still in search of the target. The repulsive pheromone further agitates the search process from random search to a biased random walk which results in better exploratory behavior of the system.

This model is characterized with the benefits of the distributed system as it is inspired by ant foraging behavior. Thus, the problems of scalability, complexity and robustness required by the distributed robot swarm are effectively addressed by this design.
Chapter 7

Numerical Simulations And Results

The numerical results are divided in two sections. The first section is for the results of area coverage control law. The second section is for the results of search and retrieval simulation for swarm.

The area coverage with this control law was explored with various parameters. In these results, the capability of the swarm for coverage was quantified with the metrics specified in 5.2.

The detailed programmatic implementation of the approach is provided in Algorithm 1 in appendix. The swarm robot model design was implemented for a swarm of \( N = 10 \) robots. Robot step size per unit time or velocity of robot in each case was taken as 1 unit. Pheromone constant \( \chi = 1 \) was chosen as thus a negative chemotaxis for swarm. All the robots during the simulation are deployed from the single location referred as the base. The resolution of the area taken during the study for grid based environment was 1 \( \times \) 1 sq. units. This grid of unit size was counted as visited if the point robot considered in this work passes through the grid while moving. Observation range of each swarm agent was 1 unit in all the four directions.

All the simulation results were using MATLAB R2016a on Windows 10 64-bit operating system, Intel(R) Core(TM) i5-4590 @ 3.30GHz machine.

In all the results, pheromone concentration in the neighborhood is the component which influences the decision making of swarm agents. The effects of noise with parameter variations have been studied in this system. The noise in the system not only results into exploratory characteristics of the swarm but also has a major contribution in the variation of influence of the pheromone concentration on the swarm behavior.
Figure 7-1: Typical pheromone distribution by robot swarm at the end of 500 iterations with parameter values as $D_b = 0.5$, $\gamma = 0.01$, $\sigma = 0.1$, $\eta = 0.1$ and $\beta = 1.1$

Figure 7-2: Corresponding robot positions to Figure (7-1)
Table 7.1 specifies the symbols used in the results section and their descriptions. The mean of the symbols mentioned here remains the same throughout the chapter mentioned otherwise.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_b$</td>
<td>Diffusion coefficient of pheromone</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Rate of evaporation of pheromone</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Magnitude of Noise</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Pheromone threshold value</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Index of Lévy distribution</td>
</tr>
</tbody>
</table>

Table 7.1: Various symbols used and their description

Dirichlet boundary conditions of east ($e_x$), west($e_x$), north($e_y$) and south ($e_y$) were set as zero for the Crank-Nicolson method for implementing equation (5.8) in all the simulations.

7.1 Results For Area Coverage

In this section, area coverage performance for the swarm control law from Chapter 3 are presented.

7.1.1 Area Coverage Performance

The major change introduced in this swarm design as compared to conventional ant foraging technique was addition of the adaptive nature to randomness using parameter $\eta$. Area coverage performance (ACP) results provide a major insight into the benefits of this change. Table 7.2 contains the value of all the parameters considered. All the simulations were performed for 1500 simulation time units. Number of trials taken for each case are 25 and results are averaged and displayed as error bars.
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_b$</td>
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</tr>
<tr>
<td>$\gamma$</td>
<td>$1E-5$</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.1</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.2, 0.3, 0.5, 0.7, 0.8, 0.9, 1, 1.25, 1.5, 1.75, 2, 2.5, 3, 3.5, 4, 4.5 and 5 to 20 with increment of one</td>
</tr>
<tr>
<td>$\beta$</td>
<td>1.5</td>
</tr>
</tbody>
</table>

Table 7.2: Parameters used for results in figure 7-4

Figure 7-3: Plot of area coverage performance ($ACP$) vs. Time where $D_b = 0.001$, $\gamma = 1e-5$, $\sigma = 0.1$ and $\beta = 1.5$ (NOTE: $\eta = \infty$ for Brownian Noise plot and all other parameters remain same)
With the swarm robots performing only gradient following Lévy flights, it can be observed that initially the performance is low with small $\eta$ values. As the value of $\eta$ increases, the performance improves and eventually settles down at the conventional Brownian motion based ant foraging approach plot. The details can be observed in plots of figures (7-3) and (7-4).

Figure (7-3) is for illustrative purpose and shows the variation of $ACP$ with respect to time. The adaptive Lévy flight based swarm showed better performance as compared to the swarm following conventional Brownian motion. This can be visualized in terms of area coverage rate as well as the amount of area covered. In given amount of time, adaptive Lévy flight approach tends to show superior performance.

![Plot of area coverage performance (ACP) vs. pheromone threshold $\eta$](image)

**Figure 7-4**: Plot of area coverage performance ($ACP$) vs. pheromone threshold $\eta$ where

$$D_b = 0.001, \quad \gamma = 1e - 5, \quad \sigma = 0.1 \text{ and } \beta = 1.5$$

Figure (7-4) shows the variation for the ACP with respect to $\eta$. It is observed that there is significant improvement in $ACP$ as the threshold values $\eta$ increases for given simulation
conditions. However, for $\eta$ greater than 4, the area coverage performance saturates to that of Brownian motion. This happens because of the dynamic nature of pheromone. The volatility of chemotactic agent, i.e., the diffusion and evaporation, do not allow pheromone to exceed $\eta$. Since this threshold value dictates the pheromone concentration at the current robot location to perform Lévy flights, the robot is unable to make the transition from Brownian to Lévy flight for higher $\eta$ values.

$$
\text{Symbol} | \text{Description} \\
\hline
D_b & 1E - 6, 1E - 5, 1E - 4, 1E - 3, 1E - 2, 1E - 1 \\
\gamma & 1E - 5, 1E - 4, 1E - 3, 1E - 2, 1E - 1 \\
\sigma & 0.05 \\
\eta & 0.5, 0.7, 0.8, 0.9, 1, 3, \infty \\
\beta & 1.5
$$

Table 7.3: Parameters used for results in figure [7-5]

A more comprehensive analysis for variable pheromone threshold value $\eta$ is presented in figure (7-5). The parameter set for these readings can be found in 7.3 and all the simulations were performed for 1500 with results averaged over 25 trials for each parameter set. In this figure, $\eta = \infty$ stands for the case of no Lévy flights but only Gaussian noise facilitating ant foraging behavior in robots.

Figure [7-5] show the plot of $ACP$ against $D_b$ and $\gamma$ for different values of $\eta$. It can be seen that $ACP$ for the swarm has improved with the increase in $\eta$ from lower values to high values. With the trend observed for $\gamma$ and $D_b$, it is not surprising that the performance shows a decrease with higher $\gamma$. The high $\gamma$ means high pheromone destruction rate and it tends to make the swarm more random with no chemotatic agent present in the surrounding.

With lower $\gamma$ and higher $D_b$, the performance increase can be thought of as better information transfer a better information transfer in the swarm. The information transfer improves with higher diffusion coefficient values because the diffusion speed increases. Therefore, the area covered with pheromone increases per unit time with diffusion. This makes it possible for the swarm agents to better evaluate their moves based on pheromone gradient.
7.1.2 Area Coverage Integral

For the results in figure 7-6, all the simulations were performed with in the total area of 100 × 100 sq. units. Total simulation time 1000 seconds. Each simulation was carried out for 30 times and the average results are plotted. The plot along the axis for \( D_b \) and \( \gamma \) are done on log scale.

The value of 1000 time steps is chosen for simulation purpose in consideration with the well planned area coverage. If the area coverage was done with planned strategy, where each robot has the speed of 1 unit per second and it observes the area of 1 × 1 sq. units per unit time, it would have taken 1000 units for 10 robots to cover the area of 100 × 100 sq. units exactly one time.
Figure 7-6: Area coverage integral plot with noise magnitude variation.

*RED* for $\sigma = 0.1$, *GREEN* for $\sigma = 0.3$ and *BLUE* for $\sigma = 0.5$

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_b$</td>
<td>$1E - 4, 1E - 3, 1E - 2, 1E - 1, 1$</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>$1E - 4, 1E - 3, 1E - 2, 1E - 1, 1$</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>$0.1, 0.3, 0.5$</td>
</tr>
<tr>
<td>$\eta$</td>
<td>$1.5$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$1.5$</td>
</tr>
</tbody>
</table>

Table 7.4: Parameters used for results in figure 7-6

All the parameter values used in these simulations are provided in tabular form in Table 7.4. All simulations presented in this section were done at constant value of the pheromone.
threshold $\eta$.

Higher the value of area coverage integral, faster is the area coverage and better is the area coverage performance. ACI is highest for the least noise magnitude. As evaporation rate increases, the area coverage integral shows a decreasing trend. With diffusion, it can be observed that there is a slight increasing trend at the beginning and with high values of diffusion it again shows a decrease.

With graph in figure 7-6, it can be observed the best performance is at the the low noise magnitude. With increase in noise, it shows the performance decreases in terms of area coverage. This is because, the bias introduced due to noise accounts with larger magnitude towards the decision making of the robot. Thus, pheromone gradient component from equation (3.13) significantly reduces and the robot tends to move randomly.

There is significant decreasing trend in ACI with the increase in evaporation rate of the pheromone. This can be explained with impact of evaporation on the pheromone gradient. As the value of $\gamma$ is increased, the likelihood of the robots finding the pheromone is reduced as higher the evaporation rate results in faster destruction of the laid chemical. Because of the higher depletion rate, pheromone gradient tends to zero and the robots start moving randomly.

Diffusion, resulting in the spread of pheromone, results in information transfer among the swarm agents. If diffusion is high, the information transfer among the swarm is quick and leading to better area coverage. But if it is increased beyond some optimal value, the decrease in the performance is observed. This decrease result is result of evaporation. If higher diffusion, the influence of evaporation rate results in pheromone depletion. This makes the robots approach more towards a random walk and abandon the bias induced by pheromone in robot decision.

Further observation from figure 7-7 shows some more interesting results. It presents, the system performance monitored only with the variation of noise magnitude. A specific trend is observed in the system behavior with variation of noise. A very low value of $\sigma$ in worsening the area coverage. While very high value also does the same. Therefore, noise appears to play a critical role in swarm. There appears to be a optimum value for the noise which can be considered for this swarm. If this value of randomness is assigned to each agent, it may lead to a well organized global behavior.
These results with introduction of the adaptive Lévy flights in the ant foraging inspired robotic system are in agreement with the work presented by the case of [52]. In fact this case can be considered as the special case of this work where the Lévy noise is absent.

The behavior of the system observed is intuitive and can be generalized.

![Figure 7-7: Area coverage integral plot with noise magnitude $\sigma$ variation](image)

for $D_b = 0.01$, $\gamma = 0.1$, $\eta = 1.5$, $\beta = 1.5$

### 7.1.3 Visit Entropy

Visit entropy metric was used to compare the performance of the designed swarm in area coverage in terms of ability of the swarm agents to disperse in given area.

In results from figure (7-8) the parameters same as (7.2) are used. The results are corre-
sponding to the graph in figure (7-4).

In context of dispersion of the swarm agents measured with the entropy values, figure (7-8) suggests that with low pheromone threshold values, the entropy of the system has higher values. With increase in $\eta$, $H$ saturates to the value close to that observed for gradient following ant foraging robots with pure Brownian motion.

It is interesting to notice from figure (7-8) that saturation of entropy to robot movement based on conventional ant foraging is very fast. The visit entropy for values of $\eta$ which correspond to comparatively high values of $ACP$ in figure (7-4) show competitive entropy values with respect to the case of swarm with conventional ant foraging approach.

![Figure 7-8: Visit entropy $H$ Against Pheromone Threshold $\eta$](image)

With low values of $\eta$, frequent switching from gradient following Brownian motion to Lévy flights is observed. Thus, higher disorder is observed in the system because of Lévy flights due
to variable path lengths and this appears to be the reason for high values of entropy $H$. As $\eta$ increases, it becomes unlikely for any swarm agent to switch from gradient following Brownian motion to Lévy flights. Therefore, the swarm agents keep on moving with constant speed and step lengths which results in decreased entropy.

As it is discussed in chapter 4, the bounds on the Lévy index $0 < \beta \leq 2$. These parameter was varied across the range and the visit entropy metric was used to compare the performance of the algorithm. The table 7.5 gives us the values considered for the simulation. The area and the number of agents were kept as $100 \times 100$ sq. units and 10 respectively for all the simulation considered under this section. The total simulation time was 1000 time units.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<td>$D_b$</td>
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<td>$\gamma$</td>
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<tr>
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</tbody>
</table>

Table 7.5: Parameters used for results in Figure 7-9

As per table 7.5, since the value of the noise magnitude considered is zero, Gaussian Noise was not considered. Only the randomness due to Lévy distribution is simulated in these cases. The value of $\eta$ is also taken as zero, thus, the swarm agents will follow the gradient based directions and will only take Lévy flights along these gradients.

From the properties of Lévy distribution in chapter 4 it is known that as the value of $\beta$ increases and tends to 2, the distribution is Gaussian. The value of entropy with $\beta = 2$ is for area coverage with gradient following swarm as per equation (3.14).

The visit entropy observed with the Lévy index $\beta$ is decreasing from figure (7-9). It is because of the longer steps in Lévy distribution and unevenly distributed path lengths in Lévy flights. This uneven path lengths lead to more disordered system. This disorder in the system in turn leads to uneven dispersion of the swarm leading to high values of entropy.
7.1.4 Number Of Lévy Steps

Figure 7-9: Visit entropy against Lévy index $\beta$

Figure 7-10: Number of Lévy flights taken by each swarm agent vs. pheromone threshold $\eta$

62
Figure (7-10) shows the result of average number of Lévy flights taken by each swarm agent against variation of pheromone threshold $\eta$. The plot in the figure represents the result for simulations corresponding to figures (7-4) and (7-8). As it can be observed from the figure, the count of Lévy step decreases and eventually reaches zero with increase in threshold value. The reason of the saturation of swarm with adaptive Lévy behavior to the case of swarm with only gradient following Brownian noise for figures (7-4) and (7-8) can be visualized clearly from this plot. As the threshold is increasing, the average number of Lévy flights performed by each swarm agent show a decrease.

7.1.5 Pop-up Threat Detection

Here, pop-up threat is something which the robots can detect if they see it. For pop-up threat detection 2 cases were considered in this work. In first case only the comparison of threat detection was done for the swarm mentioned in table 7.6. For second case, pop-up threats were analyzed with the introduction of faults in the sensors on the robots.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Brownian motion of robots</td>
</tr>
<tr>
<td>B</td>
<td>Lévy flights of robots</td>
</tr>
<tr>
<td>C</td>
<td>Ant foraging based swarm</td>
</tr>
<tr>
<td>D</td>
<td>Ant foraging based swarm with gradient following Lévy flights</td>
</tr>
<tr>
<td>E</td>
<td>Ant foraging based swarm with adaptive Lévy flights</td>
</tr>
</tbody>
</table>

Table 7.6: Test scenarios for pop-up threat detection

To get a good statistic in this section, the simulation time considered was increased to 3000 seconds for each case. Table 7.7 gives all the parameters considered for each case. Here, $D_b = 0$ and $\gamma = 0$ are valid only for the case of pure Brownian motion and pure Lévy flights. 30 trials for each parameter set were performed and the results are averaged.
Table 7.7: Parameters used for pop-up threat detection results

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_b$</td>
<td>0, 0.1</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0, 0.001</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.1</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0, 1.5, $\infty$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>1.5</td>
</tr>
</tbody>
</table>

Pop-up threats were created in the environment at random locations throughout the simulation time. This random location was selected from uniform random distribution. The detection range of the threat for each robot was 5 units. Threat once popped popped in the environment, was alive for the duration of 50 seconds before vanishing. During any time instance, only a single pop-up threat is available to be detected by the swarm agents.

**CASE 1: No sensor failure**

From figure (7-11), it can be observed that for pop-up threat detection, ant foraging inspired approaches showed the superior performance. The behavior for the swarms with gradient following Brownian motion (C), gradient following Lévy flights (D) and gradient following with adaptive Lévy flights (E) show competitive results. While there is decrease in performance in case of pure Brownian motion based robots and pure Lévy flights based robots. The repulsive pheromone used by these robots in case of scenarios (C), (D) and (E) plays a crucial part for the success of this technique.

The repulsive pheromone allows the swarm to spread across the area and makes the detection of threats possible. When the repulsive pheromone is detected, it allows the robot agents in the swarm to behave aggressively and disperse more due to its gradients. Therefore, this behavior facilitates better pop-up threat detection.
Figure 7-11: Pop-up threat detection with no sensor failures

CASE 2: Sensor failures in each swarm agent

Figure 7-12: Pop-up threat detection with sensor failures probability $p_f = 0.1$
Figure 7-13: Pop-up threat detection with sensor failures probability $p_f = 0.2$

Figure 7-14: Pop-up threat detection with sensor failures probability $p_f = 0.5$
The figures (7-14), (7-13) and (7-12) show the results obtained for the situations with sensor faults in robots for detecting the threat. The faults of different magnitudes were introduced in swarm agents for each scenario.

In terms of sensor wellness, the probability of the sensor performing successfully was \( p_s \). Thus, probability of sensor of not detecting the pop-up threat in the environment was \( p_f \) given simply by (7.1).

\[
p_f = 1 - p_s \tag{7.1}
\]

This case study gives a robust understanding about advantage of this swarm in pop-up threat detection during area coverage in unknown environment. From figures (7-14) and (7-13), it is observed that even in case of faulty sensors on the swarm agents with \( p_f = 0.1 \) and \( p_f = 0.2 \) respectively, the results are similar to that observed in figure (7-11) in case of (C), (D) and (E).

The reason for this can be explained in terms of pheromone and its dynamic nature. As the concentration of pheromone changes with time in this system because of diffusion and evaporation, there is a continuous change of its gradient as well. This leads the robots to explore the area which is already explored more than one time. This also leads to leads to visitation of same area by different robot agents.

Figure (7-12) considers more sever case of sensor failure. In this figure as well, with same system for \( p_f = 0.5 \), the correct pop-up threat detection was observed was considerably closed to that of the case of zero sensor failure in figure (7-11).

For all the pop-up threat test cases with pure random motion with Gaussian noise and Lévy flights, it was observed that the threat detection is low and less efficient, i.e. scenarios (A) and (B).
7.2 Search and Retrieval of Target

The approach was developed for search and retrieval of target in chapter 6 with the proposed swarm control law. This approach was implemented with the parameter values defined in the table 7.8. This simulation was compared with the random search approach. The simulation was ran for 2000 seconds. The number of robots included in this was increased to 50 and step size of 1 unit. The pheromone deposition rate was kept constant as $q_r = 1$ unit in case of repulsive pheromone. For attractive pheromone the deposition rate $q_a$ varied as per the equation (6.6).

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_{ba}$</td>
<td>0.1</td>
</tr>
<tr>
<td>$D_{br}$</td>
<td>0.1</td>
</tr>
<tr>
<td>$\gamma_a$</td>
<td>0.01</td>
</tr>
<tr>
<td>$\gamma_a$</td>
<td>0.01</td>
</tr>
<tr>
<td>$\gamma_r$</td>
<td>0.01</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.01</td>
</tr>
<tr>
<td>$\eta$</td>
<td>$\infty$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>1.5</td>
</tr>
</tbody>
</table>

Table 7.8: Parameters values used for simulating search and retrieval approach

For these simulations, the robot home was considered at the location $x = 25$ and $y = 25$ with dimensions as 3 units along both axes. Two targets were introduced at locations $(x = 12.5, y = 27.5)$ as Target 1 and $(x = 37.5, y = 32.5)$ as Target 2 with the width and length of 5 units each. The location of the targets were unknown to the robots. Quantity of the target at both the location is kept as 500 units.

The plots show the results of the simulations. Figures (7-15), (7-16), (7-17) and (7-18) show the evolution of the swarm self-organization along the trail between the base and the target locations during the search and retrieval operations. It can be observed from with this approaches, that swarm agents are able to find the targets and communicate the information across to other robot agents.
At the beginning of search process, the robots spread out in all the directions which can be seen from figure (7-15). As the search progresses, the robots converge to the nearest target location and self-organize along a trail towards that target. This trail can be observed.
in figure (7-16). This robot behavior is analogous to actual ant behavior observed in nature.

Even in ant colonies it is observed that ants collect the food from nearest food source first.

Figure 7-18: Typical Search and Retrieval of Target by the Swarm at time $t = 1085$ seconds

Figure 7-19: Target Quantity vs. Time

Figures (7-17) and (7-18) show the further modifications in ant behavior as Target 1 is exhausted. In figure (7-17), it is observed that robots start searching for new target. The repulsive pheromone profile gives the clear indication of this search. The swarm eventually
converges to collect Target 2. One more trail is observed along the newly located target and the robot base in figure (7-18).

The convergence of the robot swarm to the nearest target location can be explained with the close observation of figure (7-15). As the swarm search starts, the attractive pheromone concentration observed from nearest target location is strongest. This leads to more positive reinforcement on the robot swarm to converge towards this source.

The trail formation along the path between target and robot base can also be explained with positive reinforcement. The trail emerges as more and more swarm agents start following attractive pheromone gradient as it starts diffusing in the environment.

![Figure 7-20: Target Quantity vs. Time plot for comparing (1) Augmented search based on proposed area coverage law (Green) and (2) Random search (Blue)](image)

To elaborate on the advantages of positive feedback in the system, one more test was carried out for the single target location at \((x = 15, y = 15)\). The performance of the algorithm was measured for the simulation time of 1500 seconds. The number of swarm agents was kept as 50. In this case one simulation was done with the pure random search and another was done.
with the augmented approach using repulsive pheromone as per the proposed area coverage law. The home location of the robots was kept as (50, 50) with the total area dimensions as 100 × 100 square units. The results comparing the target collection rate for both the approaches can be observed in figure (7-20). In terms of positive feedback to the system, the repulsive pheromone allows for faster and more efficient area coverage even in case of unknown environment. Repulsive pheromone leads to the positive feedback to the swarm agents and increases the search space of robots. Therefore, leading to early target finding.
Chapter 8

Discussion, Conclusion and Future work

8.1 Discussion and Conclusions

The parameters $\beta$, $\gamma$, $D_b$, $\sigma$ and $\eta$ play an important role in the system performance and self emergence of self-emergence observed in this complex system.

The values of pheromone threshold $\eta$ in the system decide the system behavior. For $\eta = 0$, the system will be purely gradient following with Lévy flights. All the swarm robots in the system will perform Lévy flights only. For $\eta \to 0$, all the agents will perform gradient following Brownian motion only, i.e., conventional ant foraging approach. The system will show the adaptive Lévy flight behavior for intermediate values of $\eta$.

For the values of evaporation rate $\gamma$ and diffusion coefficient $D_b$ influence the extent of self organization behavior in the system. For very high and very low values of both the parameters, the swarm tends to follow random motion and remains less organized. For intermediate values, the coordinated behavior of all the swarm agents emerges.

The magnitude of noise $\sigma$ and the Lévy index $\beta$ are responsible for the uncertainty of the system. The randomness is a key factor for emergence in the complex systems. The emergence in this case is the organized behavior of the swarm agents. The component of randomness induces freedom for individual agents of the swarm. This microscopic randomness results in the macro level organized and societal behavior on the global scale.

In fact, with the trends of the ACI with respect to the noise magnitude variation, it can be inferred that randomness is one key factor which leads to a overall organized and desired behavior in swarm. Beyond some quantity, the inclusion of this randomness leads to complete
system disorder and therefore, reduces the system performance.

This work can be seen as the generalization of the work presented in [53]. All the cases presented in [53] can be created just by changing the parameter values.

From the results of the pop-up threat detection it can be concluded that the dynamics of the system plays a major role in making it fault tolerant. Even in case of sensor failures, the system shows the capacity to detect the threat or new features in the environment.

To speak about fault tolerance in this swarm, pheromone diffusion and evaporation are crucial. Because of the depletion of pheromone, the characteristic of forgetting the information is introduced in the system. Although some area is visited by the swarm robots, with pheromone evaporation the robots forget which area was visited in the past and this leads to the possibility of some areas to be visited multiple times. This leads to alleviation of faults in the system in terms of sensor failures.

Explanation of the adaptive nature of the swarm proposed in this work leading to better area coverage can be done based on the fact of introduction of sudden excitation in the system because of the Lévy flights. The random long steps taken in some direction based on pheromone gradient leads the swarm agents to jump out of the recently visited area and increases the possibility of these agents to end up in area which was not visited before.

With implementation of one pheromone for aggressive search and other pheromone for target retrieval, this system makes it possible to expedite the search process as well as self-organize all the swarm agents to retrieve the target to the base. In an unknown area with no prior information about the possible hazards and obstacles, this system can provide a robust solution for various situations like search and rescue, area monitoring and information collection.

### 8.2 Future Works

It is possible to implement more changes in the swarm design and improve it. Some of the possible work in this direction is given below:

1. Inclusion of the dynamics of the actual robots in the system.
2. Hardware implementation of this swarm with the simulated pheromone.
3. Replacing the pheromone with some communication beacons to reduce computational loads.

4. Using this approach in the recent idea of *Self-Driving Cars* will be a great idea. The control law proposed here is simple and robust which can be applied to self organization of these automated cars.
References


[54] David W Sims, Emily J Southall, Nicolas E Humphries, Graeme C Hays, Corey JA Bradshaw, Jonathan W Pitchford, Alex James, Mohammed Z Ahmed, Andrew S Brierley,


Appendices
Pseudocode for Algorithm Implementation

This pseudocode summarizes the implementation of the swarm control law design for area coverage. For this implementation, a fixed area was considered without any obstacles in it.

Algorithm 1 gives the details of implementation of proposed approach with $N$ number of robots. Here, the value of $\gamma$, $v$ and $q$ are kept constant throughout the process.

The symbols used in this pseudocode are explained in the following table:

<table>
<thead>
<tr>
<th>Parameter/Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_o$</td>
<td>Total simulation time</td>
</tr>
<tr>
<td>$dt$</td>
<td>Time step</td>
</tr>
<tr>
<td>$P_r$</td>
<td>Robot position</td>
</tr>
<tr>
<td>$P_l$</td>
<td>Lévy destination</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Measurement accuracy</td>
</tr>
<tr>
<td>$q$</td>
<td>Unit pheromone deposition rate</td>
</tr>
<tr>
<td>$v$</td>
<td>Robot speed or robot step length per unit time</td>
</tr>
<tr>
<td>$T_c$</td>
<td>Current simulation time</td>
</tr>
<tr>
<td>$N_r$</td>
<td>Robot under consideration</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Repulsive pheromone constant</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Pheromone threshold value</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Pheromone evaporated per unit time</td>
</tr>
<tr>
<td>LevyFlag</td>
<td>Flag to keep track if robot is performing Lévy flight</td>
</tr>
<tr>
<td>$dist(P_1, P_2)$</td>
<td>Distance between points $P_1$ and $P_2$</td>
</tr>
</tbody>
</table>

Table 1: Symbols Mentioned in Pseudocode
**Result:** Robots emulating Ant foraging with Lévy flights

**Data:** \( \text{Levyflag} = F; T_o; T_c = 0; N; N_r = 0; P_r; q; \eta; b_r; \gamma; v; P_l \)

**Algorithm 1: Pseudocode for swarm control law for area coverage**

```
while \( T_c \leq T_o \) do
    while \( N_r \leq N \) do
        if Robot is outside the area under consideration then
            Move within the area;
        else
            if LevyFlag = T then
                Move towards Lévy destination \( P_l \);
            else
                if \( b_r > \eta \) then
                    LevyFlag = T;
                    Choose Lévy destination \( P_l \) with equations (5.13) and (5.14);
                    Move towards \( P_l \);
                else
                    LevyFlag = F;
                    Choose next destination with equation (5.12) and move towards it;
                end
            end
        end
        if LevyFlag = T AND dist \( (P_l, P_r) \) < \( \delta \) then
            Reached the Lévy destination \( P_l \);
            LevyFlag = F;
        end
    end
    Pheromone Deposition \( q \) at \( P_r \);
    \( N_r = N_r + 1; \)
end
Evaporation as per equation (5.11) with rate of \( \gamma \);
Pheromone Diffusion as per equation (5.5);
\( T_c = T_c + dt \)
MATLAB Code for Lévy Random Number Generation

MATLAB code is provided below which was used to generate random numbers from Lévy distribution. This algorithm for random number generation was taken from [71].

```matlab
function z = levy(n,beta)
    % This function implements Levy’s flight.
    % Input parameters
    % n --> Number of steps
    % beta --> Power law index  % Note: beta < 2
    % Output
    % z --> 'n' Levy steps
    num = gamma(1+beta)*sin(pi*beta/2); % used for Numerator
    den = gamma((1+beta)/2)*beta*2^((beta-1)/2); % used for Denominator
    sigma_u = (num/den)^(1/beta); % Standard deviation
    u = random('Normal',0,sigma_u^2,n,1);
    v = random('Normal',0,1,n,1);
    z = (u./(abs(v).^(1/beta)));
end
```
List of Publications and Presentations
