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Local structure and interfacial potentials in ion solvation

by

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To friends, family, Tom Servo, and Crow T. Robot
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CONTENTS

Contents iii
List of Tables vi
List of Figures vii
Abstract ix

I Main

1 Introduction 1
  1.1 Preface 1
  1.2 Hofmeister’s puzzle: the specific ion effects 3
    1.2.1 The Hofmeister series 3
    1.2.2 The language of ion solvation: kosmotropes and chaotropes 6
    1.2.3 The law of matching water affinities 10
  1.3 Models of ion solvation 11
    1.3.1 The early models 12
    1.3.2 Modern continuum solvation approaches 15
  1.4 Ion solvation is a quantum mechanical problem 17
    1.4.1 Chemical character of ion solvation 18
    1.4.2 Temporal evolution of the solvation shell 20
    1.4.3 Nuclear quantum effects 22
    1.4.4 Ion adsorption at the air/water interface 22
    1.4.5 Additional comments on modeling non-electrostatic forces 25
  1.5 Surface potential effects on the solvation of ions 26
    1.5.1 The single-ion scale 26
    1.5.2 The air/water surface potential 29
  1.6 Ion solvation in energy storage cyclic carbonate solvents 31
  1.7 Bringing it all together 34

2 Theory 36
  2.1 Preface 36
  2.2 A primer on general electronic structure theory 36
    2.2.1 The Schrödinger equation and multielectron wave functions 36
    2.2.2 Hartree-Fock and electron correlation methods 38
    2.2.3 Density functional theory 39
      2.2.3.1 Thomas-Fermi model of the electron gas 39
2.2.3.2 Kohn-Sham density functional theory .......................... 40
2.2.4 Wannier localization ............................................. 41

2.3 Symmetry adapted perturbation theory 41
2.3.1 SAPT0 and DFT-SAPT ........................................... 42
2.3.2 SAPT2 and beyond .............................................. 43

2.4 A primer on molecular dynamics methods 44
2.4.1 Force fields and the potential energy function .............. 45
2.4.2 Periodic boundary conditions .................................. 46
2.4.3 The Ewald summation .......................................... 48
2.4.4 Equations of motion and time propagation ................. 49
2.4.5 Thermodynamic ensembles: controlling the variables ... 50

2.5 Free energy calculations using molecular dynamics 51
2.5.1 Free energy perturbation and thermodynamic integration 51
2.5.2 The potential distribution theorem ............................ 53
2.5.3 Quasichemical theory .......................................... 54
  2.5.3.1 Hard sphere conditioning .................................... 54
  2.5.3.2 Soft-core conditioning ...................................... 55

3 Toward a quantitative theory of Hofmeister effects: from quantum effects to thermodynamics 56
3.1 Preface 56
3.2 Computational methods 57
  3.2.1 Optimization and atomic partial charges .................. 57
  3.2.2 Energy decomposition using symmetry adapted perturbation theory 57
  3.2.3 The charge transfer energy .................................. 58
  3.2.4 Basis sets ...................................................... 59
3.3 Results & Discussion 59
  3.3.1 Interaction energies, atoms in molecules, and partial charges 59
  3.3.2 Energy stabilization due to charge transfer ............... 71
3.4 Conclusions 76

4 Self- and ion-induced polarization in ethylene and propylene carbonate 78
4.1 Preface 78
4.2 Computational methods 79
  4.2.1 Initial configurations ........................................ 79
  4.2.2 Dynamics and dipole moment calculation ................. 79
  4.2.3 DFT-D3 symmetry adapted perturbation theory .......... 80
4.3 Results 80
4.4 Discussion 87
4.5 Conclusions & Future Work 88
LIST OF TABLES

3.1  Optimal range parameters for regularizing potential .......................... 59
3.2  Interaction energies for ion/water clusters with $n = 1$, 2, and 3 .......... 60
3.3  Interaction energies for ion/water clusters with $n = 4$ ........................ 61
3.4  Interaction energies for ion/water clusters with $n = 5$ ........................ 62
3.5  Interaction energies for ion/water clusters with $n = 6$ ........................ 63
3.6  Atoms in molecules properties at bond critical point .......................... 66
3.7  Atoms in molecules atomic charges for ion/water dimers ....................... 68
3.8  Atoms in molecules partial charges on ions in all clusters .................... 70
3.9  Charge transfer energies for ion/water clusters with $n = 1$ and 2 .......... 72
3.10 Charge transfer energies for ion/water clusters with $n = 3$ ................... 73
3.11 Regularized SAPT energy with varying screening width ......................... 75
4.1  Gas and condensed phase dipole moments with varying basis set ............. 83
5.1  Ion pair differences in the bulk thermodynamic quantities .................... 97
5.2  Cluster data for NaF and RbI pairs ................................................. 99
6.1  Labels and parameters assigned to fictitious ions ............................. 116
A.1  Charge transfer and energies for ion/water clusters with $n = 1$, 2, and 3 135
A.2  Charge transfer and energies for ion/water clusters with $n = 4$ ............. 136
A.3  Charge transfer and energies for ion/water clusters with $n = 5$ ............. 137
A.4  Charge transfer and energies for ion/water clusters with $n = 6$ ............. 138
C.1  Quasichemical partitioning of solvation free energies in water ............... 147
C.2  Quasichemical partitioning of solvation free energies in dimethyl sulfoxide 148
C.3  Quasichemical partitioning of solvation free energies in 1,2-dichloroethane 149
LIST OF FIGURES

1.1 Collins’ volcano plot to distinguish similar and dissimilar ion pairs ......................... 12
1.2 Dynamic polarizabilities of water in gas phase and ion-water dimer geometries ........... 15
1.3 Electron density curvature in halide/water dimers versus separation ........................... 21
1.4 Schematic of the thermodynamic cycle ................................................................. 26
1.5 Illustration of interfacial potentials ........................................................................... 30

2.1 Cartoon of a cubic periodic system ........................................................................... 48

3.1 Ion/water dispersion energies and non-linear scaling ............................................... 65
3.2 Electron density difference map in $\text{F}^-(\text{H}_2\text{O})_6$ ............................................. 67
3.3 Electron density difference contours in $\text{F}^-(\text{H}_2\text{O})_2$ ............................................. 68
3.4 Charge displacement curves for ion/water dimers ....................................................... 71
3.5 Separate ion and solvent contributions to charge transfer energy ............................... 74

4.1 Snapshot of $(\text{Li}^+\text{EC}_{31})$ ..................................................................................... 81
4.2 Li$^+$ and carbonyl oxygen radial distribution functions .............................................. 84
4.3 Ethylene carbonate dipoles versus distance from ion .................................................... 85
4.4 Propylene carbonate dipoles versus distance from ion ............................................... 86

5.1 Illustration of the cluster pair approximation for the proton solvation free energy ...... 95
5.2 Half the enthalpy differences in the Na$^+\rightarrow\text{F}^-$ transition for small clusters .......... 103
5.3 Half the enthalpy differences in the Na$^+\rightarrow\text{F}^-$ transition for clusters with the ion fixed at the center of mass ................................................................. 104
5.4 Half the enthalpy differences in the Na$^+\rightarrow\text{F}^-$ transition for clusters with the ion unconstrained ................................................................. 106
5.5 Half the enthalpy differences in the Na$^+\rightarrow\text{F}^-$ transition at several levels of theory ................................................................. 107
5.6 Differences in $\frac{1}{2}h^{\text{ex}}_{X,n}$ between methods .............................................................. 108
5.7 Estimated error in the MP2-level enthalpy relative to CCSD(T) ................................. 109
5.8 Net potential with no temperature derivative ............................................................ 110
5.9 Net potential assuming experimental temperature derivative ..................................... 111

6.1 Snapshots of solvents with cavity volume excluded .................................................... 117
6.2 Inner shell (chemical) part of the solvation free energy ............................................. 119
6.3 Corrected fluctuations of the electrostatic potential at the center of an uncharged cavity .... 120
6.4 Corrected mean of the electrostatic at the center of an uncharged cavity .................... 121
6.5 Outer shell portion of the solvation free energy ......................................................... 123
6.6 Free energy differences in water .............................................................................. 124
6.7 Free energy differences in dimethyl sulfoxide ........................................................... 125
6.8 Free energy differences in 1,2-dichloroethane .................................................. 126

B.1 SAPT(KS)-D3 electrostatic and exchange energies of (LiEC₄)⁺ .................................. 140
B.2 SAPT(KS)-D3 induction and exchange-induction energies of (LiEC₄)⁺ ......................... 141
B.3 SAPT(KS)-D3 interaction energies of (LiEC₄)⁺ ....................................................... 142
B.4 SAPT(KS)-D3 electrostatic and exchange energies of (LiPC₄)⁺ .................................. 143
B.5 SAPT(KS)-D3 induction and exchange-induction energies of (LiPC₄)⁺ ......................... 144
B.6 SAPT(KS)-D3 interaction energies of (LiPC₄)⁺ ....................................................... 145
The establishment of a single-ion thermodynamic scale is essential to addressing the ubiquitous specific ion effects observed in the literature. Solvation free energies, enthalpies, and entropies when decomposed to their single-ion components include a contribution from 1) a solvophobic effect to make room for the ion and the establishment of ion/solvent interactions and 2) a distant interfacial potential (e.g., the air/water interface). Since experiment can only access the pair quantities, assumptions made to break the figures into a single-ion scale tend to clump into one of two scales. This thesis makes the case that the two scales reflect 1) (“bulk”) and 1) + 2) (“real”). Both 1) and 2) pose a significant challenge to theoretical characterization. The source of the difficulty for the bulk thermodynamic scale is in the handling of non-electrostatic forces between the ion and solvent. My results indicate that polarization plays an important role in ion/water clusters and in the modeling of energy storage solvents like ethylene and propylene carbonate which are very polarizable molecules. My work also draws attention to the strength of dispersion interactions between anions and water, as well as, the almost fluid-like nature of the excess electron’s density in anions. About 20% of the charge is diffused over waters in the first solvation shell. Fortunately these effects are found to be relatively short-ranged (1–2 solvation shells) inviting the possibility of simpler models handling longer-ranged interactions. The real scale on the other hand adds the surface potential experienced by a charge crossing the air/water interface. This is found to be \(-q0.4\) V, where \(q\) is the signed ion charge, resolving a century old problem. It is further argued that the surface potential contains two contributions a) across the air/water junction and b) across the local ion/solvent boundary. The latter of these is present even in simulations using periodic boundary conditions (PBC). My work re-interprets the findings of previous PBC simulations of large oppositely charged ions which should have equal solvation free energies on scale 1) in water, dimethyl sulfoxide, and 1,2-dichloroethane. It is found that the simulated result without any conditioning predicts large asymmetries in the free energies of the ions favoring the anion in water and 1,2-dichloroethane and the cation in dimethyl sulfoxide. Quasichemical theory is used to remove the contribution made by b) so that the result falls on thermodynamic scale 1). The asymmetries noted before are substantially reduced in both water and dimethyl sulfoxide consistent with the findings of others. Parity in these figures is not observed in 1,2-dichloroethane. Implications for the force field development community are discussed.
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INTRODUCTION

1.1 Preface

Few materials possess as great a significance as water. According to the United States Geological Survey, some 71% of Earth’s surface is covered in water[320]. Of this, about 96.5% is found in oceans, seas, and bays[320]. Adding in contributions from lakes and groundwater, some 97.5% of water on Earth (about 1.35 billion cubic kilometers)[320] is a saline solution which is mainly comprised of Na$^+$ and Cl$^-$ with smaller contributions from K$^+$, Ca$^{2+}$, Mg$^{2+}$, F$^-$, Br$^-$, I$^-$, and SO$_4^{2-}$. While total salinity of these waters varies across the globe, the average salinity of seawater is in the range of 30-35 g/L (or about 500–600 mM, assuming NaCl molar mass)[278]. However, there’s a case to be made that these simple salts are among those few essential materials.

Salts breathe life into our oceans, not just through the organisms which call them home, but also in the stratification of ocean layers based on their salinity which drives ocean circulation. This shares an intimate connection with Earth’s climate[287]. Likewise, salts play a critical role in biology: electrical signalling, regulating pH, stabilizing DNA/RNA, and catalyzing the biological function(s) of hundreds of proteins. The vital role of salts in our diet has similarly steered the course of human history by influencing the location of settlements, trade routes, and sparking human conflicts[285]. The chemical properties of salts only became of academic interest in the 1850’s[285]. Most notable among these early contributors include Arrhenius, Hofmeister, van’t Hoff, and Pfeffer. Hofmeister’s observations made note of the peculiar behavior exhibited by different salts in the precipitation of egg white proteins. It is in conjunction with Arrhenius’s theory of electrolytic dissociation that we link these observations to the unique behavior of the component positive and negative charges (ions) comprising the salt. The motion of ions powers a great many of our electronic devices allowing us to connect wherever we go with the rest of the world. For a lucky few of us their motion also powers our cars – and they provide supplemental power and warmth to our real-life Martians: Spirit, Opportunity, and Curiosity[349].

And if we want to keep up with the growing demand for energy in the developing world, batteries and other storage devices will need to evolve to make the ions flow a bit quicker, in greater numbers, and in a greater range of voltages. To do these things, we’ll need to develop quantitative and predictive models of ion solvation. This may, however, require electronic structure theory to accurately resolve. Since this is a difficult goal, the bar has been set a bit lower to the establishment of a unique, single-ion thermodynamic scale. This, it turns out, is also a difficult problem. However, once this initial goal is solved, it may be possible to translate the physics of advanced modeling into parameter sets for simpler models. The simpler models can help us rationalize ion solvation problems at scales not amenable to interrogation at the electronic structure level.
In this thesis I perform quantum chemical calculations of small alkali/water and halide/water clusters to determine the role that non-electrostatic forces such as dispersion, polarization, and charge transfer have on the first solvation shell. In anion/water clusters, these represent about 33% of the total attractive energetic contributions to the interaction energies. Polarization is also essential in the description of cation solvation, while dispersion takes a backseat. My analysis of the charge transfer in these clusters showed that about 20% of the excess anionic charge spilled out onto the surrounding waters. Charge transfer energies measured with two related methods were found to be inconclusive. These methods are not suitable to ion/water interactions and I make a recommendation of a possibly better method.

In the second chapter, \textit{ab initio} dynamics is utilized to determine that accuracy is severely compromised when using simple point charge models in simulations of Li$^+$/ethylene carbonate and Li$^+$/propylene carbonate. This is because they lack even a primitive treatment of polarization. Polarization effects are monitored through changes in the molecular dipole of solvent molecules distributed near the ion. Relative to the gas phase, the condensed phase dipole moments due to self-polarization are increased by about 30%, and around 50% in the first solvation shell of the ion. The close proximity of large dipoles pointing to a common center is believed to create a repulsive contribution that could bring previous single-ion thermodynamic data generated with point charge models into better agreement with experiment.

These initial chapters focus primarily on the interactions between ions and local solvent molecules. This is only part of the puzzle in the determination of a single-ion scale. The other part is a solvent-specific pair of interfacial potentials (one from a distant chemical interface and another from the solvent/ion interface). In a pair of studies, I determine this potential with both contributions using a novel approach to be -0.4 V for water. I argue this potential links two sets of experimental figures for the single-ion solvation properties, a hotly debated topic in the field.

In a follow-up study, I demonstrate that the potential across the solvent/ion interface is present in all molecular simulations. It has historically not been taken into account (at least not consistently) by the simulation community and represents a large ∼8 kcal/mol error in the hydration free energy of monovalent ions. I relate these findings to the purported solvation asymmetries (differences in these properties for oppositely charged ions) discussed in the literature for different models of water, dimethyl sulfoxide, and 1,2-dichloroethane. These solvents cover a wide range of dielectric constants from 10 to ∼80. Simulations of divalent ions like Mg$^{2+}$ and Ca$^{2+}$ have always been plagued by poor accuracy and it’s possible the huge errors in the free energies of these ions may be in part due to mishandling of the interfacial potential.

This thesis makes several meaningful contributions to the understanding of non-electrostatic forces in ion solvation and proposes a full resolution of a long-standing issue involving the link between two unique scales of single-ion properties derived from experiment. Several indirect experimental results support my finding, though there is no conclusive evidence at this time. The surface potential presents several challenges to the force field development community and calls for
1.2 Hofmeister’s puzzle: the specific ion effects

The story of the specific ion effects can be traced back to the systematic study of ion effects by Franz Hofmeister (b. 1850, d. 1922), Professor of Pharmacology at the University of Prague, and his students. A series titled, “Zur Lehre von der Wirkung der Salze” which translates to “About the science of the effect of salts” recounts the findings of several publications. The original document in German is available at Ref. [165] and an English translation provided by Werner Kunz et al. in Ref. [230]. A short biography of Professor Hofmeister can be found at Ref. [1].

Though the scope and importance of his work has been likened to that of Mendel to genetics[230] (and also not – Ref. [203]), Hofmeister was a proficient and interdisciplinary researcher as pharmacy tied together the fields of chemistry, physiology, and botany[1]. His work properly identified lactose as the sugar central to the condition known as lactosuria (formerly, glucosuria) common in pregnant women and newborns[1]. He published on vitamin deficiencies as well and made notable contributions to coordination chemistry and separations[1]. Altogether, Hofmeister and his students published about 400 papers[1]. And yet, this thesis concerns the lingering questions resulting from a handful of these articles. However, the challenge to address his observations, often called the Hofmeister or specific ion effects, from a theoretical perspective, has remained one of the greatest in the field of physical chemistry since.

1.2.1 The Hofmeister series

Lewith and Hofmeister’s seminal work on the specific ion effects established the relative cationic and separate anionic effects on “precipitating action, dissolving action, and lyotropic swelling of proteins along with diarrhetic action, diuretic action, water binding, osmotic pressure, and other physical chemical phenomena”[1]. By holding the cation constant, the anion effect could be isolated and ordered in a series reflecting the concentration required to, for example, precipitate the hen egg white proteins (predominantly globulins and albumins). In order of decreasing effectiveness,

\[
\text{CO}_3^{2-} > \text{SO}_4^{2-} > \text{HPO}_4^{2-} > \text{OH}^- > \text{F}^- > \text{CH}_3\text{COO}^- > \text{Cl}^- > \text{Br}^- > \text{NO}_3^- > \text{I}^- > \text{ClO}_4^- > \text{SCN}^- ,
\]

with a complementary series for cations as well,

\[
\text{C(NH}_2\text{)}_3^+ >> \text{Ca}^{2+} > \text{Mg}^{2+} > \text{NH}_4^+ > \text{Li}^+ > \text{Cs}^+ \approx \text{Rb}^+ > \text{Na}^+ > \text{K}^+ > \text{N(CH}_3\text{)}_4^+ > \text{TEA}^+ ,
\]
where TEA$^+$ is tetraethylammonium and C(NH$_2$)$_3^+$ is guanidinium. I’ve chosen the ordering presented in Ref. [276] which is most consistent with the spirit of the series Hofmeister inspired[230]. This series is presented in other forms as well, not all of which are logically consistent with the results of Hofmeister (or with each other, for that matter), see Refs. [88, 229, 263, 365, 373]. Discrepancies primarily occur within the cation series which is not as well-established as the anion one. This is because the affect on protein solubility and many other properties are dominated by the anion[152, 263]. In fact, Table 1 in Hamaguchi et al. is a wonderful example of the minimal impact cations tend to have, in this case on the denaturation of sea urchin DNA in 4 M salt solutions[152]. The cation trend also has some non-obvious placements (e.g., Cs$^+$ and Rb$^+$ between Li$^+$ and Na$^+$), with a terrific illustration of the variance in the literature presented in Ref. [276]. To explore this further, I found it instructive to revisit this data expressed in grams of salt added per 100 mL of solution and convert the grams of sulfate salts to moles of the respective cations. Using the results as published in Ref. [230], the ordering is just as published above in Eq. 1.2 with 0.1323 moles of Mg$^+$, 0.1566 moles of Li$^+$, and 0.1604 moles of Na$^+$ present. This ordering differs from that if just ordering the ions based on the mass of the salt or even moles of the whole salts required. This ambiguity greatly undermines the utility of the series and serves mainly as a source of confusion and frustration.

The trends above are also denoted as the *direct* Hofmeister series. There are a number of examples of cases where the inverse series is observed instead, called the *reverse* Hofmeister series, a few of which are summarized in Refs. [163, 307, 373]. The reverse series is also observed at pH below the isoelectric point of a protein[365]. Later I’ll discuss a less ambiguous partitioning of ionic behavior based on the ion hydration entropies.

Putting aside these details, the interesting thing about Hofmeister’s observations is that they link both salt *concentration* and *composition* to the behavior of other solutes in solution. The former is the more obvious: the solubility of various salts in water and other polar solvents is often exceedingly high, but finite all the same through the activity coefficient[66]. The compositional dependence of many of these properties is perhaps less obvious but is pervasive throughout chemical literature. Several examples of specific ion effects in both aqueous and non-aqueous media are listed below,

- activity coefficient[441],
- surface tension increments[58, 266, 317, 384],
- freezing point depression, osmotic pressure, vapor pressure[127],
- solvent viscosity[88],
- surface potential increments[266] (ion concentration changes the quantity discussed in Chapter 1.5, but these are relative shifts and so do not necessarily justify a particular value of the
air/water surface potential, but does rather significantly perturb pH measurements via glass
electrodes[57]),

- selective adsorption of ions to chemical interfaces[75, 91, 44, 204, 205, 259, 301, 338],
- isoelectric point of the air/water interface[36, 35, 64, 279, 409],
- brine rejection stratifying deep ocean waters which act as reservoirs of dissolved CO₂[377, 421]
  and lead to the formation of brinicles[98, 270],
- double-layer forces[57, 363],
- morphological and rheological properties of dipeptide-based hydrogel[362],
- clouding point temperature[452],
- kinetics of amyloid formation, especially at low ionic strength[267]
- lower critical solution temperature[453],
- CO₂ capture by nanoporous materials[378],
- bubble coalescence (see excellent study in Ref. [97] and review in Ref. [96]),
- frother stability used in foam floatation extraction particularly in separating heavy metals
  from wastewater and ore extraction[302]
- apparent surface charge of oil and air bubbles suspended in water, electrophoretic mobility (see
  recent survey in Ref. [455] with an alternative contribution discussed in Ref. [430]; ζ-potential
  is too small though, so it’s a less likely explanation)
- electrophoretic mobility of bare magnetite nanoparticles[417],
- ion channel activity[15, 81, 110, 148, 149, 147, 254, 360, 450],
- muscle contraction and neuronal signalling[340, 396],
- enzyme activity[68],
- macromolecule (i.e., protein, DNA, RNA, surfactant) stability and solubility[1, 68, 87, 88, 89,
  230, 229, 263].

An even more extensive list complete with additional references, albeit a bit dated, can be
found in Ref. [90]. In each of these examples the chemistry and concentration dependence of
these effects are difficult, if not impossible, to separate above very dilute conditions. A reasonable
cutoff is on the order of the millimolar (mM) scale. The specific behavior of salts discussed above
has profound implications in fields as diverse as soil[448], marine[205] and atmospheric[26, 64,
chemistry, climate modeling, physiology, industrial chemistry especially transport of raw ingredients[164], electrochemistry, energy storage devices[12], chemistry in mixed solvents (e.g., in liquid chromatography columns)[120], synthesis with charged intermediates, polymer self-assembly[362], and beyond.

Unsurprisingly these fields routinely deal with mM and greater concentrations of salts, motivating the need for continued efforts such as my own to develop a quantitative model of these effects. It can be difficult to know where to start however. Fortunately we can draw inspiration from other life experiences and so first work towards solving the simplest case: that of ion solvation in water[115]. The target here is the accurate determination of single-ion solvation properties and especially the solvation free energy from simulation[178]. The desired theoretical model should take care to accommodate the major contributions to the free energy change, 1) interaction part between the ion and solvent as well as 2) a change in the free energy associated with the reorganization of solvent molecules around the ion (cavity formation). The interaction part can be split up into 1a) local effects arising from the strong ≈ 1 V/Å fields around the ions[375] and 1b) distant solvation effects which may be amenable to computationally inexpensive approximations in dilute conditions, and 1c) interactions with a distant surface or chemical interface.

A focus on simulations of ion-ion interactions was common in the late 1980’s and early 1990’s and has experienced a recent resurgence in interest with publications by Duignan et al.[115], Luo et al.[258], Lund et al.[257], Fennell et al.[126], Shi and Beck[379], and others. Activities are of critical importance to these effects as well and there is no ion activity more ubiquitous to science than that of the proton H⁺. The definition of pH is only nominal as it depends explicitly on the proton activity (pH=-log αH⁺), which is not directly measurable by experiment[145, 277, 356]. This is also a big topic among researchers studying ocean acidification trends where an uncertainty of 0.003 pH units is desired to facilitate better comparison with decades worth of acidity data[73, 105, 104]. While many of the models used are predicted to struggle at moderate to high salt concentration, they are still useful for surveying the plausibility of a recent qualitative development for rationalizing the Hofmeister effects called the Law of Matching Water Affinities which I discuss below.

1.2.2 The language of ion solvation: kosmotropes and chaotropes

Though the ion solvation community has the main aim of developing a quantitative theory of the specific ion effects, it is instructive to review some of the recent attempts to discern the contributing factors to the Hofmeister series and conceptualize the differences between the ions. This exercise makes the connection between a numerical model and the single-ion solvation thermodynamic scale more apparent.

In the series presented in Eqs. 1.1 and 1.2, the ions are ranked by their ability to precipitate egg white proteins with the molar concentration of the salt required increasing from left (most effective) to right (least effective). Precipitating proteins with salt is known as “salting-out.” Salts composed of ions on the left side of the series above do this effectively and promote protein-protein interactions
which form aggregates that are less soluble in water. The anions in good precipitating salts are typically strongly hydrated[88]. Salts composed of ions on the right side of the series destabilize protein-protein interactions and initially increase their solubility (“salt-in”) and so require greater concentrations to precipitate the proteins. This property is known as chaotropicity and was initially interpreted as an entropic effect related to the “disorder” in proteins, lipids, DNA, RNA, and other macromolecules induced by the ions in solution[1, 28, 152]. Collins et al. and others argue that the cation trend should be the reverse of that displayed in Eq. 1.2 because the divalent ions interact with amide and other polar functional groups on the protein surface and enhance the solubility, see Ref. [88] and the related references therein (10, 53, 86-89). In Ref. 88 of that paper, the authors note that solution pH affects the divalent (several) results but not the monovalent ion results (Na\(^{+}\))[10]. It’s complicating factors like this which steer us from the original spirit of the Hofmeister series towards something a bit more palatable.

The work of von Klobusitzky established that the placement of anions in the Hofmeister sequence could be modified by changing the complementary cation, as in the example also summarized in Ref. [1]. Collins[90, 87, 88, 89] and Arakawa[10] also discuss a competition for waters among protein-solvating cations and anions and this interpretation is visited in Refs. [28, 263]. These findings promote a connection between the Hofmeister series and the relative ability of the ions to compete with each other and other solutes for solvating waters. This has led researchers to study the local solvation structure around the ions and ascribing the controversial tags of water ‘structure-maker’ and ‘structure-breaker’ to them. These terms apparently correlate with the ability of the solute (not necessarily ionic) to increase the order of waters around it (structure-maker, kosmotrope) or decrease their order (structure-breaker, chaotrope).

The difference between chaotrope used here and used above is the subject of much debate[28, 263], with Marcus stating the terms are more or less equivalent in this context. Much like the difficulties in deriving a consistent cation Hofmeister series above, the distinction between an ordering and disordering ion is difficult to make given that ion effects are localized within the first or second hydration shell[89, 294, 202, 203, 292, 364]. Effects on water hydrogen bonding patterning are limited to about 1 nm even for di- and trivalent ions[294, 106, 95].

The length scale of ionic forces in solution is itself a kind of debate, see Ref. [292] where long-ranged ionic forces are termed (probably jokingly) as ‘heretical’ and Ref. [80] which claims monovalent ions have a measurable effect at a range of over 20 nanometers. Keep in mind, the Bjerrum length in water (dielectric constant of 78) is about 7 Å, beyond which the Coulombic attraction is smaller than the thermal energy at room temperature. You be the judge of that.

As the tags of structure-making/breaking fell from favor, the nomenclature evolved to reflect differences in surface charge density and the relative strength of the ion/solvent to solvent/solvent interactions instead. In this context, a kosmotrope is a small ion with high surface charge density (think hard ion from hard-soft acid-base theory) which are strongly hydrated and chaotropes are weakly hydrated with low surface charge density. The ordering makes significantly more sense
(cutting it down to just the alkali-halide series) and is strongly tied to the strength of hydration forces,

$$\text{F}^- > \text{Cl}^- > \text{Br}^- > \text{I}^-,$$

(1.3)

and the cation series,

$$\text{Li}^+ > \text{Na}^+ > \text{K}^+ > \text{Rb}^+ > \text{Cs}^+.$$  

(1.4)

Though, and rather confusingly, all three versions are still in use (and sometimes interchangeably, because why not make it difficult?)\[28].

But what does it mean to be strongly hydrated? Weakly hydrated? Are there any properties that can quantify this distinction?

Collins et al.[89] relate the kosmotrope and chaotrope distinction to the sign of the Jones-Dole B coefficients taken from the Jones-Dole viscosity relation. This series quantifies the relative change in solution viscosity upon the addition of electrolytes. It is a series expansion of the form, \(\frac{\eta}{\eta_0} = 1 + A\sqrt{c} + Bc + Dc^2\), where \(\eta\) is the salt solution viscosity, \(\eta_0\) the viscosity of the pure solvent, \(c\) the salt concentration, and \(A, B, \text{and D}\) are fitted parameters. The B coefficient is typically interpreted to reflect the strength of ion/solvent interactions\[192]. The water/water interaction is assigned a value of 0. Ion/solvent interactions stronger than water/water give \(B > 0\) and those weaker \(B < 0\). The alkali/halide series match up exceptionally well with the B-coefficients. \(\text{Na}^+ (0.086)\) and \(\text{Cl}^- (-0.007)\) sit very near the 0 mark, with \(\text{Na}^+\) considered a weak/borderline kosmotrope and \(\text{Cl}^-\) considered a weak/borderline chaotrope. These data can be found in the previous Collins review in Ref. [88].

Assuming the B coefficient to track temperature and pressure behavior of the kosmo/chaotropic nature of the ion, then \(\text{Li}^+\) and \(\text{F}^-\) become less kosmotropic while all the others see increases in their B-coefficients from 0–55 °C\[192]. Oparin et al. reach similar conclusions for \(\text{LiCl, NaCl, and KCl}\) electrolytes\[300]. They also find that under extreme pressures, \(\text{Na}^+\) becomes chaotropic\[300].

In brief, the trend is inconsistent for polyatomic ions. My favorite example of this is the ammonium, tetramethylammonium (TMA), and tetraethylammonium (TEA) series where the B coefficients increase along the series (Table 5 in Ref. [192]) but these ions are actually becoming softer (increasing radius) and more weakly hydrated (TMA bulk hydration free energy \(\mu_b^{ex}\) is -48 kcal/mol and TEA \(\mu_b^{ex}\) is -41 kcal/mol)\[2]. Ammonium data is not available in this paper, but take a look at Ref. [261] to get an idea of the difference in \(\mu_b^{ex}\) between ammonium and TMA (TEA \(\mu_b^{ex}\) is set to zero in this data).

Another common metric used to distinguish the kosmotropic and chaotropic scale is the entropy of locally coordinated water molecules. An experimental quantity is discussed in Ref. [89] (Figure
5) and in more detail in Ref. [90] (Figure 3) (it’s a reproduction from an old book, which itself is a reproduction from an even older paper). The original authors measured the entropy change relative to bulk water for a water adjacent to each of the alkali/halide ions, assuming the ion solvation effect propagated no further than the first hydration shell. The quantity plotted is the bulk entropy minus the first shell entropy which is negative for all chaotropes ($K^+$, $Rb^+$, $Cs^+$, $Cl^-$, $Br^-$, and $I^-$). The interpretation here is that these ions only weakly associate with the water and liberate the bound water from the bulk. This is synonymous with the ‘structure-breaker’ term discussed previously (but importantly, the ions don’t structure waters over a great distance). Waters bound to $Li^+$ and $F^-$ are less mobile than those in the bulk. Again, the localized nature of these effects makes moot the idea of long-range structuring of water by the ions. Collins et al. also discuss the signatures of chaotrope and kosmotrope influence through proton nuclear magnetic longitudinal reorientational times[90]. Waters situated around chaotropes tend to ‘tumble’ faster than bulk waters, while those around kosmotropes ‘tumble’ slower.

The translational and rotational components of the entropy of distinguished water molecules has also been recently explored using molecular dynamics simulation[364]. Each component was reduced relative to these same quantities in bulk water, with about 80% of the effect captured in the first shell, consistent with the assumptions made above, and about 20% of the effect arising from the second hydration shell. Cations reduced both the translational and rotational entropies of first shell waters by nearly equal amounts, while monoatomic anions reduced the rotational entropy by a few multiples over the translational entropy. This is not observed with polyatomic anions possibly due to the greater number of binding sites (anions were $SO_4^{2-}$ and $ClO_4^-$. The author finds an approximately linear relationship between the reduction in the distinguished water translational entropy and surface charge density.

Finally, Beck used molecular dynamics and a free energy partitioning scheme called the local molecular field theory (with temperature derivatives) to compute spatially separable components of the solvation entropy of the alkali/halide ions[37, 38]. Spatial conditioning is applied to the electrostatic contribution, which divides into a local and far-field (distant) part. The author finds that kosmotropic ions give an ion specific, sizable, and negative $s_{ex}^{loc}$, which is the local part of the electrostatic contribution to the hydration entropy. Beck likewise found chaotropes to give positive $s_{ex}^{loc}$, with a small 0.5 cal/mol-K contribution for $Cl^-$, solidifying its reputation as a borderline chaotrope. In Figure 2 of Ref. [38], the author plots this entropic term versus the inverse of the ion size. The anion trend line is approximately linear with a large slope while the cation line (albeit of only two data points) is much flatter, implying less specificity between the ions considered. This perfectly mirrors the conclusions of Marcus and Hamaguchi et al. mentioned in a previous section[152, 263].

Three of these properties are related explicitly to solvation entropies for either the coordinated water molecule(s) or the single-ion quantity itself. The B-coefficients may also be related to the partial molar entropy of solvation as well, interestingly enough[291]. Entropic quantities also aren’t
as significantly impacted by sign-dependent surface potential effects either, which is a bonus. In my opinion, these examples serve to illustrate the connection between a quantitative model of the specific ion effects and single-ion solvation thermodynamic properties. We must pursue the latter in order to develop the former.

Additionally, authors like Evens et al.[121] have demonstrated that the simple ranking of ions based on their main effect is of limited use because 1) the one-dimensional ranking of ions gives little to no insight on the nature of ion/solvent, ion/solute (protein, DNA, etc.), or ion/ion interactions 2) the ordering depends on solution temperature, pressure, possibly on solution pH, electrolyte concentration, protein concentration, the amount of dissolved gases, begging the question what the ordering for a particular set of conditions is all that useful for, and 3) it’s rare in practical applications that only a single salt species is present in solution. Nevertheless, these series are essential. They are placeholders for a deeper understanding and spring up when we observe salt specific behavior we can’t quite wrap our heads around. That’s useful in its own right.

1.2.3 The law of matching water affinities

A kosmotropic ion is said to be strongly hydrated while a chaotropic ion is said to be only weakly hydrated. Collins linked these observations to complex Hofmeister behavior on the basis that ions of opposite charge form contact ion pairs spontaneously only when they have similar water affinities (the familiar ‘like’ seeks ‘like’ trope)[88, 89]. Unmatched ions dissolve because the larger ion screens the smaller one in strongly polar solvents[257]. The general concept is very similar to the Pearson hard soft acid base theory[313, 314, 315].

This means that kosmotrope-kosmotrope and chaotrope-chaotrope pairs are preferred to the solvent separated (i.e., dissolved) species, forming contact ion pairs which partially or entirely exclude the solvent between them. These pairs are marked as having a positive heat of solution with the individual ions having similar heats of hydration and so are not very soluble. This is best illustrated in the volcano plot of Collins et al. reproduced in Figure 1.1[89]. Crystalline salts comprised of mixed pairs of chaotrope-kosmotrope and kosmotrope-chaotrope (cation-anion) ions release heat upon being placed in solution (and so dissolve) and form the slopes of the volcano where the anion is better hydrated on the left slope and the cation better hydrated on the right slope.

This characterization has proven successful already in untangling the mystery behind these effects in many experiments which are reviewed in Refs. [88] and [89]. I mentioned this before, but this keen observation has motivated a renewed interest in simulating ion/ion interactions between matched and unmatched pairs to verify the method at least theoretically and try to understand some of the driving forces selecting the dissolved or solvent-excluded states. My colleague Yu Shi published a paper on the results he obtained applying the local molecular field theory I discussed to the free energy profiles (potential of mean force, pmf) of matched and unmatched alkali/halide salts as they approach each other[379]. This has been done before (see references mentioned above and a recent study by Baer et al.[24]). The pmf of the solvent-excluded state has a free energy minimum
where the ions are very close to each other and the solvent-separated pair have a minimum at a
greater distance which is spaced enough to allow waters to bridge the ions, see Ref. [126] for terrific
illustrations. But no one else has been able to partition the pmf into contributions made from
particular interactions. This is the subject of the rest of the opening chapter. We know there’s
specificity, we know hydration forces (ion/solvent interactions) are involved, but now we ask,

- What sort of interactions?
- How strong are they relative to one another?
- Do we need electronic structure theory to solve everything?
- Will a simpler model work far away from the ion?
- Do chemical interfaces contribute to single-ion thermodynamics?
- If so, what is the contribution?
- How can we compare our simulation results to experiment?

These are all questions I address in the following sections and in the following chapters where I
discuss my own work.

1.3 Models of ion solvation

Modeling is one of the most effective tools in scientific research. Models unify results, rationalize
observations, and grant some level of predictive power under unexplored conditions. In this section
I briefly review several important advancements in the development of numerical models of the ion
solvation problem.

As discussed above, all of these methods target analytical or numerical solutions to several target
properties of the simple ion-into-water scenario: 1) solvation free energies, enthalpies, and entropies
of individual ions at infinite dilution, 2) activities and osmotic coefficients, and 3) surface tension
increments. Each of these methods make assumptions on the nature of ion/solvent and ion/ion
interactions. Their shortcomings help us address issues with the underlying physics and so have
been instrumental in the evolution of such models over the last century.

The methods discussed here also share the treatment of solvent as a mathematical continuum in
common, meaning no ball-and-stick molecular models are required except for the ion itself. The
earliest models covered here ignore the solvent reorientational contributions to the free energy and
assume the ion/solvent interactions to be purely electrostatic. Successes and failures of these models
will be discussed.

In the second section, I’ll explore the corrections considered in modern continuum solvation
models. The corrections consider the role of several additional ion and solvent properties which
Figure 1.1: Ions can be classified as chaotropes (weakly hydrated) or kosmotropes (strongly hydrated). This plot illustrates the relationship between the standard heat of solution (at infinite dilution) of crystalline alkali/halide salts and the difference between the absolute heats of hydration of the respective ions. The degree of similarity or dissimilarity between the ions comprising a salt can be qualitatively assessed from its placement into one of four distinct regions (labelled above). Figure recreated from Ref. [89] (©Elsevier Biophysical Chemistry, reprinted with permission) in order to enhance image clarity.

are believed to impart ion specific behavior: dispersion interactions, polarization, charge transfer, cavitation energy (solvophobic effect), ion size (charge density), and surface potentials across chemical interfaces. Extra attention will be paid to the model of Duignan et al. which I believe, despite my criticisms levied against it throughout this thesis, is an exceedingly simple and elegant theory of ion solvation. The model is capable of handling ions in dilute solutions, ions moving towards the air/water interface (this is the primary area of disagreement), and ion/ion interactions.

1.3.1 The early models

Probably the most familiar of these models is the Debye-Hückel theory. Debye-Hückel theory assumes the interactions between ions to be purely electrostatic (which is rigorously true, see the Hellmann-Feynman theorem[331]). The solvent is modeled as a dielectric continuum through the
dielectric constant and the ion charge density with a Boltzmann distribution. It is only valid in the limiting case of low concentrations (<100 mM).

Debye and Hückel expressed the low concentration limit of the free energy of an ion ($\mu_i$) in this field as

$$\mu_i = k_B T \ln \Lambda_i^3 + k_B T \ln c_i - \frac{kq_i^2}{2\varepsilon}$$

(1.5)

where $\lambda$ is the de Broglie thermal wavelength, $c_i$ the concentration, and the third term is the electrostatic part of the free energy, making use of $\kappa^2 = \frac{4\pi \varepsilon}{k_B T} \sum_i q_i^2 c_i$, where $\kappa^{-1}$ is the inverse Debye length. This equation can be rewritten as

$$\mu_i = k_B T \ln \Lambda_i^3 + k_B T \ln c_i + k_B T \ln \gamma_i$$

(1.6)

where $\ln \gamma_i = -\frac{kq_i^2}{2\varepsilon k_B T}$ is the activity coefficient of the $i$th ionic species. This is not accessible to experiment, so the mean activity coefficient is measured instead, giving $\ln \gamma_{\pm} = -|q_+ q_-| \frac{k}{2\varepsilon k_B T}$. A plot of the $\ln \gamma_{\pm}$ versus $\sqrt{I}$ is linear at low concentrations, where $I = \frac{1}{2} \sum_i q_i^2 c_i$ and is the (typically) molal ionic strength. That is, the approximation of ion-ion interactions as essentially non-interacting, screened point charges is reasonably accurate for low concentrations. At higher concentrations, the limiting expressions break down and exhibit significant deviation from experiment. Application of the Debye-Hückel model to osmotic coefficients suffers similar limitations. In a later advancement, Onsager and Samaras extended the model to predict the surface tension of electrolyte solutions[299].

Aaron Klug, winner of the 1982 Nobel Prize in Chemistry, once quipped that the theory was only applicable to “slightly dirty water”[308]. This is because the model neglects ion specific properties such as the size and shape of the particle(s) and how these change with solvation (see Figure 3.2 for an example of how electron density is drawn to poles which point directly at neighboring solvent molecules). The solvent response is also neglected and is assumed to be uniform throughout space which neglects polarization. Dispersion interactions are also neglected. Debye-Hückel theory is still often used to this day in the interpretation of experimental measurements (and so these interpretations also lack these important contributions)[321, 354].

Some of these features have been introduced in a number of extended models which draw from the Debye-Hückel model, see Pitzer ion interaction model[329] and a very recent iteration of an extended Debye-Hückel theory[440, 439] which is applied to ionic liquids. The Derjaguin-Landau-Verwey-Overbeek (DLVO) model for describing the stability of colloidal suspensions has been successfully adapted to address ion solvation as well. This theory incorporates repulsive electrostatic and Lifshitz-like attractive dispersion forces. The electrostatic potential assumes a form very similar to that of the Debye-Hückel model. Ionic-dispersion can be modeled by incorporating dynamic polarizabilities from high level electronic structure calculations[309]. Overall, the DLVO model has seen success across a diverse array of fields[310] but is not predictive of Hofmeister effects even with a series of fitting parameters and the addition of terms correcting for hydration and interaction.
Evolving around the same time, the Born model of ion solvation is another approximation of the Poisson relation using the Coulomb potential [54]. The Born formula is derived from 2nd order perturbation theory [40] and assumes purely electrostatic interactions between the ion and solvent and represents the solvating molecules via a dielectric continuum. It is an expression for the single-ion solvation free energy and takes the form,

\[
\mu_b^{ex} = -\frac{N_A q^2}{8\pi\varepsilon_0 r_0} \left( 1 - \frac{1}{\varepsilon_r} \right)
\]

where \(\mu_b^{ex}\) is the bulk free energy of solvation, bulk meaning without chemical interfaces, \(q\) the ion charge, \(\varepsilon_0\) the permittivity of free space, \(r_0\) an ionic radius (must be spherical; empirically fit), and \(\varepsilon_r\) the dielectric of the solvating medium. The 1 assumes transfer from the gas phase where the dielectric constant is 1. Note that the dependence on the charge in this theory is quadratic – ions of the same size but opposite charge will have precisely the same solvation free energy. All ions are expected to be repelled from the air/water interface, just as in the Debye-Hückel theory [299]. This is, however, not the case, as I explain in Chapter 1.4.4.

The model addresses the finite size of the ions but requires that they be best modeled as an excess charge confined to a spherical cavity. The results can vary quite a bit with the selection of ionic radii. However, the theory neglects polarization of, reorganization (cavitation) of, and other interactions with the nearby solvent. The model has been built on just as Debye-Hückel theory and forms of the basis of the bulk thermodynamic scale [14, 261, 348] which is discussed in more detail in Chapter 1.5.1. These models refit the crystal radii by increasing the ionic radii [240], selecting vdW radii for the vacuum part [392], or simply extend the Born model with additional terms [348]. Additionally, some of the more advanced theoretical models condense down to the Born model under certain conditions [361]. There’s good evidence a model like this is perfect for quickly accounting for the distant ion/solvent interactions as only the electrostatic contributions are expected to remain significant beyond the first hydration shells [38, 37, 175, 380]. I even liken some of the results from my own simulations in Chapter 6.1 to the Born model.

In 1936, Onsager extended the Born model to include polarization interactions between the ion and continuum; however, the model still required a spherical cavity and required knowledge of the ion dipole and polarizability. Others issues include 1) Spherically symmetric ions (our alkali and halide friends) won’t solvate in this model. 2) The dipole and polarizability of the ion (and nearby solvent) can change with solvation [144, 274, 34, 143, 336, 454, 383]. Figure 1.2 gives an example of this.

Here the dynamic polarizability of the water molecule is plotted for a range of applied frequencies. The water geometries used include the native gas phase one and that when the water is complexed with the alkali-halides. In the case of each of the ion-induced geometries, the polarizability at a given frequency is slightly above that of the native geometry. This small change enhances the polarization and dispersion interactions. The water geometry when complexed with F− distorts to such an extent
that the polarizability increases from about $1.42 \text{ Å}^3$ to $1.5 \text{ Å}^3$, a far greater change than any of the others. This really does have a significant impact on the interactions; the polarization and dispersion energies for this complex are listed in Table 3.2. It is this kind of complex chemistry through electronic charge rearrangement that continuum models cannot reproduce. However, the effect likely becomes less pronounced with increasing cluster size. According to several studies, the dipole moment of waters in the first shell isn’t all that different from the bulk\cite{160, 161, 162, 225, 252}.

### 1.3.2 Modern continuum solvation approaches

Recall that a solvation free energy model should consider ion/solvent interactions and solvent reorganization in the form of a cavity formation energy (sometimes referred to as the solvophobic effect). Modern continuum solvent models are often built into ab initio codes to add approximate solvation effects to molecules handled with electronic structure theory. With the already high computational cost of many of these types of calculations, quality continuum solvation models are
in extreme demand. There are many of such methods out there, but the bulk of them are derived from the polarizable continuum model (PCM). This method expresses the solvation free energy in the form,

\[ \mu^{ex} = \mu^{ex}_{elst} + \mu^{ex}_{disp-repulsive} + \mu^{ex}_{cavity}. \]  

(1.8)

A derivative method called the conductor-like polarizable continuum model (CPCM) and many other electronic structure solvation models involve small rearrangements of or substitutions in the PCM equations. Despite the cavity and dispersion-repulsion terms showing up in Eq. 1.8, the PCM nor CPCM models handle dispersion or cavity parts of the free energy very well[227]. Cavities in these theories are handled as a series of overlapping spheres centered at each of the atoms in the modeled molecule(s). The PCM method is also really expensive with numerous derivatives to compute. CPCM simplifies the procedure greatly and performs very well with high dielectric solvents. The CPCM model was used in a very recent single-ion free energy scale paper[185].

Truhlar et al. have developed a general solvation model which uses the full electron density of the molecule(s) to estimate solvent accessible surface area (SASA) and atomic surface tensions[268]. These figures are related to the cavitation and dispersion-repulsion energies. Because the model actually attempts to solve for cavitation and dispersion-repulsion interactions, it is often considered the best method for calculating solvation free energies – the others essentially calculate the electrostatic part. The higher quality comes at the expense of computation time though. This model has been used to develop what was once was known as the ‘gold standard’ scale in single-ion free energies, enthalpies, and entropies[400, 214].

Other models exist that are similar to those discussed above: COSMO, COSMO-RS, SMX (X=6, 8, 12, etc.), and DESMO[236].

Duignan and coworkers have developed a method recently which also solves for the terms given in Eq. 1.8 and applied it to the determination of single-ion solvation free energies, entropies, partial molar volumes, salt activities, osmotic coefficients, ion/ion potentials of mean force upon approach (investigating Collins’ law of matching water affinities), and the free energy profile of the hydroxide and hydronium ions approaching the air/water interface[113, 112, 114, 115, 116, 111]. Electrostatics are handled at the Born level for simplicity but the authors also use the COSMO model (essentially any PCM model will work). Dispersion was handled through determination of the conventional C6, C8, and C10 coefficients from dynamic polarizability functions evaluated at the density functional level. They hope to treat dispersion using Green’s functions in the future. A simple cavity term is modeled as the product of a solvent-related constant and the excluded volume due to the ion. The volume term takes the distance to the first peak in the ion/water (oxygen-atom) radial distribution function from simulation or experiment as input. For ion-ion interactions, this volume excludes regions of overlap between the ions. When the electrostatics are handled with COSMO, an approximate polarization energy is taken into account, else it is left out and suffers the usual difficulties associated with Born solvation. For their study of hydroxide and
hydronium approaching the air/water interface a surface potential of $+0.13\,V$ was assumed\[116\]. I argue later why this assumption is very likely wrong and how their results change if using the potential I estimate in Chapter 5.1. Other small errors may be due to the use of spherical cavities in the COSMO model for non-spherical ions.

Their results highlight the special importance of dispersion through $\mu_{\text{disp-repulsive}}^{\text{ex}}$ but also in the determination of $\mu_{\text{elst}}^{\text{ex}}$ from the COSMO model. They show in Ref. [115] that the 2nd virial coefficient (inputs for both osmotic and activity coefficients – this is a common extension of the Debye-Hückel model to higher concentrations) for chaotrope-kosmotrope pairs are especially sensitive to the inclusion of electron correlation in the COSMO energy. The model overall exhibits excellent qualitative agreement with experiment and shows the benefits of hybridizing approaches.

I’ve reflected on a number of theoretical frameworks for modeling ion interactions with bulk solvent, interfaces, and other ions. These methods are of limited predictive value given their oversimplified description of solvation which lacks the granularity of explicit solvation models\[205\]. Even with extensive parameterization, they struggle to mimic the strong and ion-specific local solvation forces. However, I mentioned (admittedly, in passing) that these models are appropriate for accounting for long-range solvation effects which are almost entirely electrostatic in nature (at least in high dielectric solvents). Therefore, the pursuit is not necessarily in vain. Pairing these models with explicit handling of the first solvation shell(s) could lead to significant advances in the efficiency and accuracy of hybrid electronic structure/continuum approaches. This may also allow researchers to forego the use of periodic boundaries in simulation and simulate proper clusters free from artificial forces due to the boundary conditions. The quasichemical theory of solutions which I discuss in Chapter 2.5.3 and implement in Chapter 6.1 measures parts of the free energy as the work to solvate a cavity of considerable size. Dipoles and quadrupoles forming at this junction can interact over a significant distance and may produce spurious and unaccounted errors when interacting with its image in a neighboring cell\[351\].

Regardless, it is critical to stress that ion solvation is an inherently quantum mechanical problem due to the strong electric field around the ion and the importance of localized, mutually polarizing, non-electrostatic forces, and other as yet undisclosed contributions.

1.4 Ion solvation is a quantum mechanical problem

This section motivates the use of electronic structure theory in the characterization of ion/ion and ion/solvent interactions. Though it is also instructive to point out that “well-parameterized classical models can capture important aspects of specific ion hydration, including high resolution single-ion thermodynamic quantities”\[334\]. Indeed, the size of and timescales of simulations run today are virtually impossible to carry out at the electronic scale. So where does the need for very costly quantum chemistry come into play? Our successes in modeling ion solvation to spectroscopic and/or thermochemical accuracy can be used to train simpler models which can in turn be used to address
matters requiring many atoms or long timescales.

So what is going on in these inner shells that continuum models cannot reproduce (even in an average sense)? In short, a lot. There’s a surprisingly large number of things to discuss on this matter so I’ll break things down into a few categories (by no means an exhaustive list): 1) spectroscopic and theoretical evidences of chemical character in the closest shells, 2) dynamical effects, particularly on solvent exchange between the inner shells, 3) nuclear quantum effects, 4) ions at interfaces, and 5) some additional thoughts on atomistic modeling of non-electrostatic contributions for some perspective on how far we’ve come and the limitations of many existing force fields.

1.4.1 Chemical character of ion solvation

First, I re-establish the fact that monovalent ionic effects are limited to the first or second solvation shell. Markovich et al. measured the photoelectron spectra of sequentially solvated ion/water clusters[269]. They measured the binding energy of the valence electrons of Cl$^{-}$, Br$^{-}$, and I$^{-}$ ions with increasing cluster size, relating the change in the binding energy between subsequent cluster sizes to a stabilization energy. This figure was monitored with increasing cluster size to determine the size of the first solvation shell. The authors found this energy largely stabilized by a cluster size of $n = 6$. Interestingly, each of the spectra reflected an increase of over 3 eV in the binding energy associated with solvation. A later study by Kurahashi et al. determined these values more accurately, with all of the respective binding energies increasing[231]. These authors observed that the change in the highest occupied molecular orbital (HOMO) binding energy for the solvated water molecule decreases $\sim$1.3 eV relative to the gas phase water molecule. A redshift in other spectral features of water are observed as well and are linked in Ref. [128] to the formation of the hydrogen bonding network in bulk water. Intermediate these studies was one from Winter et al.[435]. The spectra in this paper are argued to have been superseded by those of Ref. [231], but these authors also attempted to compute the binding energies using a combination of continuum and explicit particle methods. They found that continuum methods performed well for cations because solvent reorganization could be largely neglected (the transition is from +1 to +2 charge), while for anions the continuum methods struggled (transition is from -1 to $\pm 0$ charge). In water, the dipole orientation around a neutral cavity most resembles that of a cation[370, 371, 369, 175], leading to the poor comparison. A later study embedded partially solvated anion/water clusters in the same model and achieved significantly improved agreement with experiment[107]. A separate study postulated that the neutral state of a particularly difficult case for band assignment in the spectrum of F$^{-}$/water more resembles F$^{-}$/H$_2$O$^+$[72], likely owing to the large proton affinity of F$^{-}$[243]. In a review by Seidel et al.[374], the vertical detachment energies (the binding energies assuming no change in solute or solvent structure) are reported to correlate very well with the UV charge-transfer-to-solvent (CTTS) energy, even getting the Hofmeister ordering of monovalent anions largely correct. Some of these authors recently reported on a new method for probing
solvated electron, which is the simplest form of electron transfer reaction and so is a surrogate for understanding more complex chemistry. It is also believed to play a role in radiation induced damage of DNA\cite{94}. Iodide is the most commonly used electron donor since it is relatively easy to excite and the simple nature of the neutral atom with no vibrational degrees of freedom reduces the number of variables to contend with in the measurements\cite{223}. The solvated electron is generated by exciting the iodide valence orbitals to produce an I\textsuperscript{0}-e\textsuperscript{−} state (the nature of which is poorly understood)\cite{223}. The intermediate survives a reported 1-2 picoseconds before generating the solvated electron which relaxes after about 100-500 femtoseconds\cite{223}. Spectra of this sort have been produced both experimentally and theoretically for the halide series and even the sodium anion\cite{33, 60, 131, 219, 243, 221, 245}.

But there’s growing evidence that you don’t need to excite the valence electrons of anions to see a portion of the excess charge spill out onto the solvent\cite{9, 18, 343, 399, 342, 27, 219, 243, 99, 242, 358, 281, 34, 454, 389, 5, 390, 355}. Many of these studies relate intermolecular charge transfer (also known as charge displacement) to spectral shifts in IR or Raman stretching frequencies of the solvating water(s)\cite{442}. Another pair of studies (and the experimental and theoretical references therein) discuss modeling of the vibrational spectra for water when complexed with an ion\cite{207, 339}. This too is a very localized effect and can be so strong an influence that in the case of F\textsuperscript{−} and water, the ion is often said to act as though it is “prying apart” the molecule\cite{89, 358}. In a study by Choudhuri et al., the authors monitor the vibrational frequency of the O-D stretch over the course of a simulation as the molecule diffuses away from F\textsuperscript{−}\cite{85}. There is about a 200 cm\textsuperscript{−1} blueshift in the frequency as the molecule returns to the bulk. In a follow-up studies on Br\textsuperscript{−} and I\textsuperscript{−}, the authors noted that the O-D stretching mode actually decreased as the water diffused into the bulk\cite{210, 209}. Diffusion coefficients in the first shell were also found to be greater than in the bulk, consistent with discussion above on the “tumbling” of waters around the alkali-halides measured with NMR\cite{209}. The use of heavy water minimizes the impact of nuclear quantum effects with hydrogens.

Using the quantum theory of atoms in molecules\cite{22}, I looked into this idea of F\textsuperscript{−} somehow “prying apart” the water molecule from the perspective of changes in the electron density Laplacian (curvature) as a function of ion/water separation as in Ref. [119]. In Figure 1.3, I show a previously unpublished plot of the curvature of the electron density measured at the junction between the halide anions and a bound water molecule. I monitor this quantity over a range of separations and compare between two levels of theory. In the atoms in molecules theory, a positive curvature indicates ionic character, while negative indicates covalent character. While not negative in value, it is very interesting to note that the equilibrium geometry of the F\textsuperscript{−}/H\textsubscript{2}O dimer falls on the part of the curve where the curvature becomes less positive with decreasing distance, while both Cl\textsuperscript{−} and Br\textsuperscript{−} exhibit the opposite behavior. In Ref. [119], this behavior is likened to a closed shell (electrostatic) interaction with emerging covalent character. It’s easy to rationalize that F\textsuperscript{−}
forms stronger hydrogen bonds with water than the other halides. Kim et al. point out the high
electronegativity and large proton affinity of F\(^-\) relative to the other halides\[219\]. But does the
unique location of F\(^-\) on the Laplacian curve necessarily support this covalent narrative? It’s
possible, but we mustn’t jump to conclusions. At the least I think it implies the exchange of density
between the fragments is more fluid, but then again this behavior is difficult to characterize because
there is no unique way to carve individual atoms out of a molecular system. Why is this definition
of an “atom” better than another? That’s a discussion beyond the scope of this thesis. Safe to say,
Bader’s theory does not impress some\[328\] – probably one of the most scathing reviews of anything
I’ve ever seen.

In spite of the difficulties associated with charge partitioning, partial charge transfer is believed
to *symmetrize* the first solvation shells by promoting internal ion solvation over surface solvation\[454,
389\]. Though induction is typically taken to include intramolecular polarization and intermolecular
charge transfer, high polarizability has been linked in several studies with increased solvation
asymmetry (surface binding)\[74, 100, 158, 296, 319\]. Reconciliation may be found in an explanation
that charge transfer reduces the excess charge on the ion, effectively making it harder and less
polarizable.

Rick and coworkers have also argued that the cumulative charge transfer from the bulk to the
interface leads to the apparent negative surface charge of air bubbles or oil droplets suspended
in water\[410, 390, 397\]. However, the calculated excess surface charge falls well short of those
back-converted from experimental \(\zeta\)-potentials (about 60-150x smaller). The alternative hypothesis
is OH\(^-\) adsorption at the interface which is a *highly* contentious topic I’ll address later.

The sheer number and diversity of structures reflected in many of the above studies and others
such as Refs. \[188, 215, 233, 234\] and those used in Chapter 3.1 very likely places an approximate
description of local structure beyond the reach of continuum models as well.

### 1.4.2 Temporal evolution of the solvation shell

Perhaps one of the most difficult aspects of ion solvation and possibly the best argument for molecular
resolution is the *softness* of the hydration shell conferred through the finite time of ion/solvent
interactions, diffusion, and exchange between the first and second solvation shell. The earlier results
of Heuft and Meijer suggest the first hydration shell is rather *hard* around F\(^-\) and Cl\(^-\) – in this
sense meaning sharply cut off from the rest of the solution\[160, 161\]. Though they find the solvation
shell around I\(^-\) to be rather unstructured with rapid exchange between shells\[162\]. Karmakar
et al. in their series of studies on halide ion solvation also addressed the diffusion coefficients of
ion-bound waters, the lifetime of hydrogen-bonds, and dynamics of water reorientation in the first
shell\[85, 209\]. The dynamics of first shell waters was found to fall into three distinct timescales:
short-time relaxation dynamics (~100 fs), lifetime of the X\(^+\)···H bond (F\(^-\) (~7.5 ps) > Cl\(^-\) (~3
ps) \(\approx\) Br\(^-\) (~3-4 ps) > I\(^-\) (~2-3 ps) > H\(_2\)O (~1-2 ps)\[31, 298\]). The somewhat labile nature of the
coordination shell is supported also by the work of Laage and Hynes\[232\]. A somewhat dated study
Figure 1.3: Laplacian (curvature) of the electron density in halide/water dimers measured at the bond critical point linking the ion and water. A positive value indicates that density is not shared across the point, while a negative value denotes the opposite. According to Ref. [119], the position of $F^-$ on this curve is unique in that it coincides with the emergence of covalent character in the ‘bond.’ Results are otherwise unpublished but are related to the calculations performed in Chapter 3.1.

by Koziński et al. estimated the lifetime of the hydrogen bond between water and $CN^-$ to be about 2.9 ps[224]. Interestingly these changes come with no discernible change in the dipole moments of the waters between the first and second shell[160, 161, 162, 225].

Water residence times also differ significantly across the halide series ($F^- (\sim 26 \text{ ps}) > Cl^- (\sim 20 \text{ ps}) \approx Br^- (\sim 20 \text{ ps}) > I^- (\sim 14-16 \text{ ps}))$. As shown by Heuf et al., the solvation shell of the ion when undergoing solvent exchange will take on unique characteristics not observed for low energy conformers used to parameterize continuum and simple all-atom models[160, 161, 162]. $F^-$, for example, will take on a short lived hexahydrate coordination sphere before returning one of the waters to the bulk[161]. The inherent flexibility in the coordination number can only be reasonably approximated with an all-atom model and then again only accurately with electronic structure resolution. Residence times are also critically important in understanding the dynamics of solvation around proteins[303] which can be modified by the presence of electrolytes (and hence the Hofmeister
These results also stress the importance of including dispersion effects in density functional modeling of hydrogen bonding dynamics in electrolyte solutions[29, 32]. However, some care must be taken to be sure the corrected functional doesn’t over/under structure water, as is common. Recent simulations also highlight the critical importance of dispersion in modeling the first shell of Na\(^+\) and K\(^+\) to accurately determine coordination numbers from the integral of the radial distribution function[30]. A systematic review of the challenges ion solvation pose for density functional theory can be found in Ref. [391].

1.4.3 Nuclear quantum effects

Much of the simulation work discussed above was done with heavy water, but why? As the proverb goes, “when it rains it pours.” Extending beyond the usual difficulties in describing the electronic wave function, the small mass of the proton means nuclear quantum effects play an important role in hydrogen bonded systems. Nuclear quantum effects (NQEs) are purported to increase the mobility of both H\(^+\)[272] and OH\(^-\)[405], which may be important in proton shuttling along membranes and hydrophobic surfaces[450]. NQEs are also likely responsible for a number of anomalous behaviors in liquid water and in ice[304].

Nuclear effects are generally thought to occur in hydrogen bonds through two competing modes 1) proton sharing through a stretching mode (HO···H···O···H\(_2\)) and 2) distortions in the bond due to rotation of the participating molecules[79]. As a general rule of thumb, nuclear effects weaken weak hydrogen bonding interactions and strengthen stronger ones[79, 150]. You might think this makes NQEs more important for anion solvation – but it has actually been found to be more important for Li\(^+\) than F\(^-\) due to the competing effects discussed above[431]. Suzuki et al. have also found that NQEs in small F\(^-\)/water clusters led to elongation of the hydrogen bond distance[212]. Wang et al. observed similar behavior in small Cl\(^-\)/water clusters.

These effects can be included for both quantum and classical descriptions of electrons. Including NQEs generally improves the quality of simulations across a broad range of temperatures[415]. In conventional \textit{ab initio} dynamics a 30–50 K increase in the temperature has been found to mimic NQEs and corrects the radial distribution function (rdf) of liquid water at 300 K (simulation temperature is 330–350 K) and slightly decreases the overall structuring (lower peak in rdf)[283]. However, this is a questionable approximation[426]. Additional influences of NQEs are addressed in a recent review[79].

1.4.4 Ion adsorption at the air/water interface

Ion adsorption at the air/water interface is a relatively recent discovery. For quite some time it was believed that all ions were repelled from the interface because 1) the surface tension of water increases with the addition of most inorganic salts predicting a negative surface excess by the Gibbs
adsorption isotherm and 2) the primitive dielectric models predict that ions solvated in a higher
dielectric medium are repelled from the interface due to an image charge force. The air/water
interface was believed to be ion-free and relatively inert[186, 205]. This all changed with Hu et
al.[172] who noticed that the kinetics of Cl\textsubscript{2} and Br\textsubscript{2} uptake in solutions of NaI and NaBr did not
fit the rate predicted by a bulk-phase reaction mechanism (X\textsubscript{2} + Y\textsuperscript{−} → XY + X\textsuperscript{−}). The authors
inferred significant surfactant behavior of Br\textsuperscript{−} and I\textsuperscript{−} as Y\textsuperscript{−} to fit their model to their experimental
measurements. Since then, research into the surface activities of inorganic ions and the acid/base
chemistry of the water surface has become wildly popular.

Cheng et al., Netz et al., and Ou et al. have found strong correlation between anion affinity for
the air/water interface and ion radius[82, 170, 301]. Researchers more commonly ascribe surface
propensity to polarizability[204, 205, 316, 317, 323, 324, 429]. Polarizability (or an especially large
dipole on the waters in nonpolarizable models) was found to be a critical factor in determining
whether this behavior would be seen from molecular dynamics trajectories[394]. However, the
role of polarization in polarizable force fields such as AMOEBA may be overestimated somewhat,
exaggerating the role polarization may play here[454]. There may be several competing effects also
including surface capillary waves, desolvation, cavity formation, and the surface potential across the
air/water boundary[11, 44, 346].

Surface affinity appears to also change with the counterion in solution[83, 174, 401]. The ordering
of ions in the double layer can switch when the anion is strongly kosmotropic and the depth of the
region of ion accumulation/depletion at the interface can extend to more than a nanometer into the
solution[63]. However, the region of enhanced ion density relative to the bulk is more commonly
limited to about half that distance. This is based on density profiles of the ionic species extracted
from molecular dynamics simulations and also the anion/cation ratios probed at various depths
by high-pressure photoelectron spectroscopy[205]. The density profiles and spectra also show that
the net charge density profile over the interfacial region produces a negative surface excess. The
depleted layer a few water diameters deep more than makes up for the surface active layer(s) above
it, an illustration of this is given in Ref. [205].

Hua et al. have also shown that the interface can be depolarized at sufficiently high concentrations
of surface active ions; the authors used 1.7 M perchlorate in this case[173]. Depolarizing the interface
with an applied voltage while monitoring relative surface excesses may be used to approximate the
contact/surface potential essential to the establishment of an absolute single-ion thermodynamic
scale[91].

While the surface activity of some ions is well-established, there are still some lingering issues
with quantifying the enhancement (see Refs. 253–256 in Ref. [51]) due to discrepancies between
X-ray methods, sum-frequency (SFG) and second harmonic generation (SHG) spectroscopies, and
theoretical approaches. Adsorption of hydronium or hydroxide ions at the interface remains an
extremely contentious topic of research, however. Hydroxide adsorption at the interface is suspected
because 1) the ζ-potential in neat water is negative as mentioned previously, 2) the surface tension
of a freshly formed air/water interface relaxes from 80–100 mN/m to 73 mN/m in about 1 ms (about the same time required for OH\(^{-}\) buildup due to autolysis and diffusion); competing mechanisms occur on ps (too fast) and s (too slow) timescales\[253\], 3) other anionic and/or fatty acid impurities are not observed at the air/water interface in SFG/SHG experiments\[191\], 4) the isoelectric point of the air/water interface is at pH 2.5–4\[35, 64, 253, 279\], and 5) the surface tension of water is largely pH-independent from pH 4–13. This last point does not necessarily point to one ion over the other making an appearance at the interface (as noted in Ref. [206]), but demonstrates the robust nature of the surface charge, strengthening the conclusions generated from other studies.

Theoretical studies and SFG/SHG experiments typically predict hydronium adsorption at the interface\[64, 284, 325, 404, 432\]. A notable exception is a study by Mundy et al. which found a small \(~0.6\) kcal/mol attraction of OH\(^{-}\) to the air/water interface\[286\]. The free energy profile of H\(_3\)O\(^{+}\) is more often shown to have a minimum near the interface, while the OH\(^{-}\) profile is purely repulsive. This is interpreted to reflect the loss of water contacts by OH\(^{-}\) and a favorable enthalpic contribution through exclusion of H\(_3\)O\(^{+}\) from the bulk\[404\]. However, a study by Brorsen et al. made the interesting observation that hydronium did not prefer surface solvation when the ion and water were modeled at the \textit{ab initio} level, but did when the water was handled with the TIP5P water model\[62\]. This is a 5-point classical point charge model which is known to have almost no surface potential contribution\[351\]. The surface potential is believed to play a role in ion adsorption at interfaces\[11, 116\]. The surface potential I derive in Chapter 5.1 acts at about \(\frac{1}{2}\) its full value near the interface and if included in the study of Tse et al. in Ref. \[404\] reverses their prediction. Adsorption of the OH\(^{-}\) ion is selected over H\(_3\)O\(^{+}\) by \(~4–6\) kcal/mol, similar to that discussed in the conclusions of Ref. \[253\]. This difference corresponds to a \(~10^4\) preference for OH\(^{-}\). This is consistent with the pH needed to neutralize the surface of water.

There are also several complicating factors with surface sensitive spectroscopies which are in need of resolution: 1) the signal can be convoluted by poorly understood quadrupolar or bulk solvent contributions\[75, 228\], 2) the probe depth is poorly characterized and appears to be able to change with ionic strength from about 1 nm to over 1 \(\mu\)m\[138\], 3) the signal which is thought to measure surface adsorption does not necessarily translate back to propensity\[75\], and 4) the OH\(^{-}\) may (weakly) or may not be visible to SFG/SHG\[184, 279\]. These are important concerns to address considering that SFG/SHG are also used to probe structure of the air/water, oil/water, and other non-aqueous solvent interfaces (e.g., directionality of the O-H bond: pointing into vapor phase or into bulk?)\[91, 259, 423\]. Issues with theoretical approaches are typically related to the small size of simulations and the poor performance of classical force fields for this type of work\[35\].

See Refs. \[51, 53, 186, 423\] for additional reviews on interfacial effects and surface sensitive spectroscopies and Ref. \[4\] for a recent survey of literature related to the ongoing debate about H\(_3\)O\(^{+}\)/OH\(^{-}\) surface affinity.
1.4.5 Additional comments on modeling non-electrostatic forces

Polarizable force fields are hit and miss with over-polarization being a pretty severe problem in classical models[69] in comparison with more realistic quantum models[23]. Another concern is whether the model should make use of full gas phase or reduced condensed phase ion polarizabilities[144, 34]. The popular AMOEBA force field has seen some refinement in the parameters over the years to combat these issues[241, 353, 143, 336, 438, 424] but still over-polarizes anions. This can be corrected by variation of a damping parameter employed in the self-consistent polarization calculation[332]. However the adjustment necessarily distorts the solvation structure and thermodynamic quantities without full re-parameterization. In the calculations I perform in Chapter 5.1, the $F^-$/water peak in the radial distribution function is pushed about 0.25 Å beyond that when using the default model parameters. This was a fortuitous change for us however as I discuss in that chapter.

Additionally, the AMOEBA assigned polarizability of the particles is static when I have shown previously that it can in fact change. The effect may not be as pronounced in water where the dipoles of waters in the first shell may not differ drastically from those more distant from the ion, but this is not the case in every solvent[12], also see Chapter 4.1. The model of Steve Rick and coworkers reviewed in Ref. [355] I think represents the state of the art in force field modeling. It incorporates two major improvements over AMOEBA in particular. 1) Polarization is handled using a Drude oscillator which adjusts the polarizability dynamically based on the environment and takes gas phase polarizabilities as input and 2) the model incorporates a simple and efficient description of charge transfer. Stuart and Berne used a similar model, concluding that the larger dipole moment of these water models as compared to nonpolarizable ones made for a more accurate representation of both the dielectric and dynamic properties of liquid water[394]. Fortunately, despite the deficiencies in polarizable force field models, they are often qualitatively correct and have proven exceedingly useful in addressing selective adsorption of ions at the air/water interface.

The van der Waals (vdW) potential in classical force fields, meanwhile, is a kind of mathematical Frankenstein composed of dispersion, exchange repulsion, surface potential effects, and corrections for all manner of uncategorized errors necessary to produce a better fit to a desired property. Fits of interaction parameters may be done against a dizzying array of experimental variables[289] and sometimes the parameters of cations and anions are fit to different experimental quantities, best exemplified in Ref. [171]. Nezbeda et al. have published a recent review on the difficulties of force field development especially as it pertains to the theoretical prediction of electrolyte solubility[289]. The authors (and I) believe some standardization is desperately needed. My work in Chapters 5.1 and 6.1 defines and characterizes a separate surface potential effect which also can serve to clean up the vdW potential in all force fields a bit. These papers also provide a road map for relating simulated quantities to experiment. Surface potential effects and single-ion thermodynamics are addressed in the next section.
1.5 Surface potential effects on the solvation of ions

Earlier I argued that the determination of an accurate single-ion thermodynamic scale was essential in our pursuit to finally begin to unravel the specific ion effects. What follows is a summary of the state of affairs in establishing such scales in aqueous and two energy storage related media, namely ethylene and propylene carbonate.

1.5.1 The single-ion scale

Solvation properties of neutral particles can be uniquely determined through experimental methods as long as an accurate equilibrium constant between gas and solution phase partitioning can be established. This applies also to ion pairs wherein one ion neutralizes the net charge of the other. Solubility measurements can also be used to establish solvation properties though the lattice energy of the salt must be removed to convert solution properties to solvation ones[322], see the representative thermodynamic cycle in Figure 1.4. A list of lattice energies is found in Ref. [264] on page 33.

The solvation free energy and related properties are a specific case of transfer free energy from
vapor into solution rather than between two solutions. More generally, transfer experiments can be conducted in mutually saturated solvents that are in contact with one another or in the pure solvent. The choice can often lead to very different results as the more hydrophilic ions tend to coextract with residual waters into the non-aqueous phase[359, 102, 101]. Electrochemical methods can be used as well[137, 344]. More recently advanced mass spectrometry setups have been used to determine sequential hydration enthalpies and based on my results from Chapter 5.1 may be able to establish the most accurate thermodynamic scale to date[428, 109]. A list of pair free energies for alkali/halide salts is shown in Table 6 of Ref. [235]. There is typically very little variation in the pair solvation properties between different measurements and these are known to high accuracy[143].

The solvation free energy for a salt is very simple,

\[ \mu_{\text{pair}}^{\text{ex}} = \mu_{\text{P},b}^{\text{ex}} + \mu_{\text{N},b}^{\text{ex}}, \quad (1.9) \]

so long as we don’t try to separate the contribution made by each ion individually. Once we try to do that, the best we can come up with is the conventional scale where everything is measured relative to the proton,

\[ \mu_{\text{N}}^{\text{ex,con}} = \mu_{\text{N}}^{\text{ex}} + \mu_{\text{H}^+}^{\text{ex}} = \mu_{\text{N},b}^{\text{ex}} + \mu_{\text{H}^+,b}^{\text{ex}} \]

(1.10)

for a negatively charged ion and

\[ \mu_{\text{P}}^{\text{ex,con}} = \mu_{\text{P}}^{\text{ex}} - \mu_{\text{H}^+}^{\text{ex}} = \mu_{\text{P},b}^{\text{ex}} - \mu_{\text{H}^+,b}^{\text{ex}} \]

(1.11)

for a positive one. This is because with the exception of mass spectrometry, no other experimental method is able to make measurements on isolated charges. Even then, these studies are limited to the determination of sequential hydration enthalpies for clusters extending just beyond the first shell. It’s more common to split bulk thermodynamic and electrochemical measurements using an extrathermodynamic assumption to solve for the proton quantities and thus all others through Eqns. 1.10 and 1.11. Some of the more cited of these are reviewed below.

Another problem in the determination of a single-ion scale is the fact that dragging charges across solution interfaces also incurs a contribution from the electrostatic potential associated with the interface. This is often called the surface potential, phase potential, or contact potential[235, 338]. Pioneering minds like Gibbs and Guggenheim long ago ruled this pursuit out concluding potential shifts experienced by single ions moving across interfaces are not thermodynamically measurable[135, 145, 146].

Nevertheless, the real electrochemical solvation free energy for a single ion is expressed as[381, 338, 123, 39],

\[ \mu_{\text{X}}^{\text{ex}} = \mu_{\text{X},b}^{\text{ex}} + q\phi_{\text{np}} \]

(1.12)
where $\mu_{X,b}^{ex}$ is the bulk hydration free energy (that includes all interactions of the ion with water except for the surface potential contribution). $\mu_{X,b}^{ex}$ is also called the *intrinsic* free energy\cite{178} and $\mu_{X}^{ex}$ the *real* free energy. Similar nomenclature is extended to the hydration enthalpy which is

$$h_{X}^{ex} = h_{X,b}^{ex} + q\phi_{np} - qT \left( \frac{\partial \phi_{np}}{\partial T} \right)_p$$

(1.13)

and the hydration entropy\cite{260} is

$$s_{X}^{ex} = s_{X,b}^{ex} - q \left( \frac{\partial \phi_{np}}{\partial T} \right)_p.$$  

(1.14)

The enthalpy and entropy add a new term which corresponds to the temperature derivative of the surface potential. The enthalpy also contains a contribution made by the surface potential itself, while the entropy does not. The surface potential contributions cancel out in Eqns. 1.9–1.11 leading to the excellent agreement among the various methods for pair and conventional free energies and the like. These figures become less agreeable when the pair quantities are divided into their respective single-ion scales, see Figure 2 in Ref. [235]. With this in mind, let’s review some of the more commonly cited extrathermodynamic assumptions.

- Marcus scale\cite{261} using the method of Halliwell and Nyburg\cite{151} for the proton enthalpy and Conway’s bulk proton solvation entropy\cite{92}. The values are -254.3 kcal/mol, -261.5 kcal/mol, and -24.0 cal/mol-K for the free energy, enthalpy, and entropy respectively.

- Born model with adjusted radii (Latimer-Pitzer-Slansky)\cite{14, 240}, giving identical results to the Marcus method.

- Assuming equal solvation entropies between $H^+$ and $OH^-$ ($s_{H^+}^{ex} = s_{OH^-}^{ex}$)\cite{367}. The proton quantities are -251.4 kcal/mol, -257.6 kcal/mol, and -20.7 cal/mol-K for the free energy, enthalpy, and entropy respectively.

- Assuming very large, ligand-screened, hydrophobic ions of opposite charge have the same solvation properties in *every* solvent (tetraphenylarsonium/tetraphenylborate, TA$^+$/TB$^-$)\cite{262}. These results are slightly shifted from those of Marcus above. The hydration enthalpy of the proton is -263.6 $\pm$ 1.7 kcal/mol. The Conway entropy is assumed again\cite{92}. The hydration free energy is then -256.4 kcal/mol.

- The extrapolation of gas phase, sequential hydration properties to the bulk via the cluster pair approximation\cite{400, 86, 407, 214}. The values here are very different from those above: -265.9 kcal/mol, -274.9 kcal/mol, and -30.0 cal/mol-K for the free energy, enthalpy, and entropy respectively. More recent re-evaluations with the method have led to a new set of recommended values: -265.3 kcal/mol, -275.3 kcal/mol, and -33.3 cal/mol-K\cite{108}. 

The last result here very clearly differs from the others, but it is most certainly not the only one in that range, more are listed in Ref. [185]. This difference extends to the other ions when these proton values are inputted into the conventional free energy scale above (again, see Figure 2 in Ref. [235]). Previous efforts have determined that all but the last of the listed approaches above excludes the charge-dependent, linear surface potential term in the free energy [16, 14, 39, 380]. I refer to these values as the “bulk” quantities from Eqns. 1.12 – 1.14. Could the surface potential in Eqns. 1.12 and 1.13 be the piece that links these scales together and explains why they appear shifted from one another? What even is this surface potential contribution? These questions are central to my work in Chapter 5.1.

1.5.2 The air/water surface potential

In reference to Figure 1.5, $\phi_{sp}$ is the total surface potential across the air/water interface and $\phi_{lp}$ is the imaginary inner interface across the solvent/solute boundary where the electrostatic potential differs from that of the bulk solvent. I’ll refer to $\phi_{lp}$ as the local potential, a recent study has also identified this potential [352]. $\phi_{sp}$ has been characterized by simple point charge models, electrochemical experiments, density functional calculations, and complex electron holography measurements with values ranging from -1 to nearly +4 V [246]. The origin of this tremendous spread was beautifully illustrated by Kathmann et al. [211]. To summarize, density functional calculations and high energy electron holographic measurements have access to interrogate the full electron density while electrochemical measurements samples from the intermolecular density which leads to a smaller observed potential jump relative to the vacuum. Contributions to this potential were found to be almost entirely due to molecular quadrupoles. A large cancellation of $\phi_{sp}$ occurs crossing the solvent/cavity interface as $\phi_{lp}$ is of similar magnitude but opposite sign. Their sum produces the net potential ($\phi_{np}$), which is reduced from $\phi_{sp}$ and $\phi_{lp}$ by about an order of magnitude [154, 211, 39, 351]. The shift in free energies due to interfacial potentials has also been observed in several studies [235, 14, 39].

Point charge models yield results similar to electrochemical measurements, though there is no such simple explanation of why that is exactly. It is interesting to note that applying a Gaussian smearing function to a point charge model has been seen to somewhat increase the measured surface potential suggesting that the distribution of charge over all space may be important to correctly modeling surface chemistry [434]. Estimates of $\phi_{np}$ from classical charge distributions or quantum estimates are very nearly identical though the magnitudes of $\phi_{sp}$ and $\phi_{lp}$ are very different and even of opposite sign [246, 39, 351, 425].

The similarity between the surface potentials measured by electrochemical probes and $\phi_{np}$ led me to refer to the net potential as an electrochemical surface potential. This is the average potential experienced by an ion crossing the air/water boundary and it shifts the free energy and enthalpy of solvation by an amount equal to $q\phi_{np}$; where $q$ is the ion charge. $\phi_{np}$, it’s been argued [11], may play a role in anion adsorption to the air/water interface [204, 205]. The basicity of surface waters with
Figure 1.5: Figure showing interfacial potential contributions across two distinct media to single-ion solvation free energy with quasichemical partitioning. The net and local potentials can be measured as the average electrostatic potential felt by the ion solvated in medium 2 (i.e., water) in a large exclusion zone with radius $r$. For these calculations, the ‘ion’ is modeled only as a spectator to define a location for the cavity. It bears no charge nor interacts with the solvent in any way other than to exclude it. If medium 1 is vacuum, the potential measured in this way is the net potential. If medium 1 is excluded and medium 2 is modeled in periodic boundaries then the local potential is measured. Neither potential is considered in a consistent (or correct) way in molecular modeling using classical force fields and represents a serious error.

$\mu_{\text{real}}^{ex} = \mu_{is}^{ex} + \mu_{pk}^{ex} + \mu_{os}^{ex}$

$= \mu_{\text{bulk}}^{ex} + q\phi_{np}$

$\phi_{np} = \phi_{sp} + \phi_{lp}$

an isoelectric point in the neighborhood of 2-4\cite{64, 279, 35}, a shift in $\mu_{H^+}^{ex}$ from the bulk of +5.46 kcal/mol, also reflects the influence of $\phi_{np}$ (predicted to be half as large at the surface)\cite{11, 116}. Surface effects could play a role in atmospheric chemistry\cite{64}. This is especially true of acid-base chemistry where the interface has been found to increase dissociation of carbonic acid relative to the bulk\cite{132, 133} while it was noted by Baer et al. that both HCl and HNO$_3$ are more likely to remain undissociated at the interface\cite{26}. In follow-up work, the authors found that HNO$_3$ was about 20% less dissociated in the interfacial layers than in bulk solution even in nearly 8 M solutions\cite{248}.

Hünenberger and Reif\cite{178} have argued +0.13 V, the average of literature values, a reasonable estimate of $\phi_{np}$, which I contest is actually a composite of estimates of various $\phi_{np}$’s and $\phi_{sp}$’s and so the average has no physical meaning. However, this figure was recently adopted into a continuum model of ion solvation\cite{115} and to addressing the surface activity of the hydronium (H$_3$O$^+$) and hydroxide (OH$^-$) ions\cite{116}. The free energies reported by Duignan et al.\cite{116} were observed to be particularly sensitive to the ion radius used with the COSMO solvation model. An initial set
of parameters produced a free energy (which includes the effect of the surface potential in the
electrostatic term, see Eqn. 8, of -167.5 k_BT for H_3O^+ and -194.9 k_BT for OH^- at 293.15 K. In a
second set, the ion radius was fit to reproduce experimental values with the proper standard state
correction[71]. The radius of H_3O^+ is decreased while OH^- is increased. The corrections adjust
the electrostatic part of the free energy and reproduce the expected -189.2 k_BT and -179.4 k_BT
hydration free energies for H_3O^+ and OH^-, respectively[71].

The authors have adjusted the ion size to make up for what they attributed to errors in computing
the ion-dependent part of the free energy but an alternative explanation is that they’ve used the
wrong surface potential. The shifts in the free energies reported by Duignan et al. are similar to
the difference between Hünenberger and Reif’s average value and the one I calculate in Chapter
5.1. The charge dependent correction is another clue that the surface potential may be involved.
But the point of this exercise is very simple: using the wrong surface potential can lead to an
incorrect interpretation of ion solvation where interfacial effects are expected to be important. See
also batteries and supercapacitors where performance is linked with the partitioning of ions between
solid electrodes and some media (can be gas, water, organic solvent, or solid)[249, 372].

1.6 Ion solvation in energy storage cyclic carbonate solvents

Non-aqueous environments provide a similar challenge, though there have been far fewer experiments
conducted than in water. The scope of this discussion will be limited to carbonate solvents used in
modern energy storage devices such as the Li-ion battery and supercapacitors, see Ref. [140] for a
wonderful review of Li-ion technology and advancements over the years.

The Li-ion battery (LIB) is found in virtually all mobile devices (e.g., cell phones, laptops,
tablets, etc.). They are also used in hybrid or all-electric vehicles and have been used to power and
warm our most intrepid explorers (i.e., the Mars rovers)[349]. The Li-ion battery cell is composed
of an anode, cathode, electrolyte, and a separator. The anode is typically a carbon-based material
like graphite, the cathode is a lithium metal oxide, and the electrolyte is a non-aqueous aprotic
solvent[420]. The separator is permeable to Li-ions and keeps the two electrodes from making
contact. Charging the cell with an externally supplied voltage (i.e., plugging in your device) pushes
Li-ions and electrons from the cathode to the anode (electrons flow through an external circuit).
These particles migrate back to the cathode when discharging.

Repeated cycling between charging and discharging leads to performance degradation over time
through a series of chemical and/or thermal pathways. These include oxidation of the electrolyte by
the cathode, reduction of the electrolyte by the anode, and thermal decomposition of the electrolyte
or electrodes[420]. However, some initial decomposition of the electrolyte is desired. This forms
an ion permeable but nonconducting film at the electrolyte/electrode interface which prevents
further decomposition of the electrolyte (called the solid/electrolyte interphase, SEI). Over time,
the oxidative and reductive stresses of discharging and recharging lead to additional degradation
products which not only affect battery performance but also introduce a number of safety concerns. These issues are borne out of the fact that carbonate solvents have very low flash points compared to their expected operating temperature. The battery also requires a regularizing mechanism to ensure the output falls between a finely specified range appropriate for the application and especially avoids overcharging (voltages greater than 4.2 V)[297, 349]. Exceeding this voltage leads to rapid decomposition of the electrolyte, breakdown of the protective film, evolution of CO\textsubscript{2} at high pressure, and thermal instability resulting in irreversible damage to the cell[420].

Suggested areas of improvement to the Li-ion battery design are delineated in a recent article by Brown et al.[63]. They are “1) finding electrodes that provide the highest energy densities, 2) developing an optimized electrolyte with the high ionic conductivity and stability at high potential/temperature and 3) the appropriate solid electrode/electrolyte interphase (SEI).” These improvements address safety concerns, capacity issues, small voltage range, relatively rapid degradation (even if the cell is never used), and lengthy recharge times which have limited the adoption of Li-ion technology in the electrification of automobiles and other transport vehicles. The greater energy demands of the future (primarily from non-OECD countries) also place a great emphasis on evolving the modern battery[326]. As does the use of storage technologies to supplement fluctuations in the output of a global energy grid increasingly reliant on renewable energy sources.

Much of the focus to date has been on improving the materials of the cathode and anode to push the chemical potentials as far apart as possible, though under the new conditions the electrolyte is likely to become the bottleneck to further improvements[180]. Computational screening has proven a useful tool in screening potential electrolytes to limit the chemical space necessary to explore further with experiment. This reduces both cost and time of development[180, 179, 372]. However, screening efforts thus far have focused on a number of simple to compute properties which makes use of continuum solvation models for the measurement of solvation free energies (a crucial factor in determining the effectiveness of an electrolyte for shuttling ions between the electrodes[443, 444]). The resulting free energies lack appropriate handling of the first solvation shell and the surface potential of the solvent – each critical to the accurate assessment of ion transport in these solvents. Additionally, it has been discussed in the literature that additives can greatly influence the cycling efficiency of the cell even at part per million concentrations, including in low temperature conditions[20, 451]. This reduces the relevance of studying bulk solvation properties in pure solvents as secondary and tertiary solvent effects are likely to significantly affect ion transport and SIE properties[20].

It is also believed that the coordination structure around the Li\textsuperscript{+} is essential to the formation and structure of the SEI[385], though this is still poorly understood even in existing carbonate-based electrolytes. Thermodynamic data is almost non-existent in these solvents as well. Despite it being generally accepted that both ethylene (EC) and propylene (PC) carbonate solvents tetrahedrally solvate the Li\textsuperscript{+}, there is no consensus in the literature on this point. Most experimental and theoretical predictions place the coordination number in the neighborhood of \( n = 4-5 \), with Raman
intensity experiments and theoretical predictions on the lower end of that range\[6, 48, 49, 55, 56, 134, 182, 183, 247, 251, 282, 290, 386, 398\] while X-ray absorption, neutron scattering, and conductivity data predict 4.5 PC molecules to each Li\(^{+}\)\[208, 222, 385\]. Smith et al. found that a classical polarizable force field underestimated (the same one used in Refs. \[55, 56\]) the coordination number relative to experiment\[385\]. Bogle et al. found the coordination number in EC to be as high as 5.69\[52\]. The largest value yet was determined by Castriota et al. who estimated a Li/EC coordination number of \(\sim 7\) in 1 M solutions of LiClO\(_4\)\[77\]. My work in Chapter 4.1 also makes an estimate of the coordination number from density functional based simulation.

A universal coordination number of 4 was assumed in the computational part of a recent study by Brown et al. which predicted the differences in binding energy between Li\(^{+}\)-1s and Cl-2p\(_{3/2}\) (in ClO\(_4^−\)) core states in ethylene carbonate, dimethyl carbonate, \((3:1)\) dimethyl carbonate to ethylene carbonate mixtures, and dimethyl sulfoxide\[63\]. A comparison with near ambient pressure photoemission with liquid jet X-ray spectroscopy established that ethylene carbonate more strongly solvates Li\(^{+}\) than the other carbonates\[63\]. This finding is consistent with a study by Yang et al. which established that \(^{13}\)C chemical shifts of EC were more strongly shifted upon complexation with Li\(^{+}\) than other carbonate solvents\[445\]. A similar conclusion was drawn for the binding of EC relative to PC\[20\]. This strong binding contrasts with the weaker binding of anions such as ClO\(_4^−\) and PF\(_6^−\) which are common constituents in commercial Li-ion batteries.

Recent efforts have also addressed whether the coordination number is shifted due to ion pairing or dissolution in the vein of the law of matching water affinities. Whether ion-pairing exists in these solvents has been the subject of much debate\[385\]. A recent pair of experimental assessments of the free energy, enthalpy, and entropy of solution of KX salts, where \(X = (F^−, Cl^−, Br^−, I^−, NO_3^−, ClO_4^−, \text{and} SCN^−)\), in EC and PC were performed by Peruzzi et al.\[321, 322\]. They make some attempt to connect their observations to the “volcano” plot I showed in Figure 1.1. The authors note that the entropies of solution reverse sign between the two solvents despite their very similar coordination structures. This can only be chalked up to an anion effect which affects the EC structure more than the PC structure due to the symmetry of the molecule\[200\]. KI was the only salt to give a negative entropy of solution in each solvent, but exhibited a terrible fit relative to the other pairs in EC which may account for this observation\[321, 322\]. Given the lattice energy, the enthalpy of solvation for the ion pair could be estimated. KI was also the only ion pair whose dissolution was exothermic in both solutions, though it was less negative in PC (though again this may reflect some error in EC). LiF, KF, KCl, and KBr dissolutions are all very endothermic and larger in EC than in PC\[200, 321, 322\]. The other pairs are also endothermic but are more positive in PC than in EC. This has led to some discussion of a primitive kosmotrope v. chaotrope scale in the solvents\[321, 322\]. Since there are fewer overall sign changes in the properties of the salts it is difficult to assign kosmo/chaotropic character to the ions. Peruzzi et al. in their EC paper established a single-ion scale for the potassium salts. K\(^+\) was found to be more strongly solvated than even F\(^−\). The “volcano” plot the authors drew from their data recovered only one slope and
had no obvious kosmotrope/chaotrope cutoff as compared to water.

Attempts by Arslanargin et al. to reproduce this single-ion scale in simulation met with mixed success[12]. Symmetry adapted perturbation theory calculations which dissect parts of the interaction energy into chemically and physically interesting pieces (a more thorough discussion is given in Chapter 2.3) predicted that polarization was essential for accurate modeling of ion/EC and ion/PC complexes (even the anions). The authors also found many-body dispersion effects to play a role comparable to that of induction for the anions which interact with the diffuse positively charged back-end of these molecules[12]. Their thermodynamic results for a standard force field fall short of the experimental results of Peruzzi et al.[321, 322]. After re-fitting the force field to reproduce an \textit{ab initio} binding energy curve of the ion/solvent dimer, their results overestimated those from the same experiments. The local potentials (-0.15 V in EC and -0.08 V in PC) were subtracted from the simulation result to establish a bulk single-ion scale in each solvent, though it is noted for pair values this distinction does not matter. Thus, while these classical models may be able to appropriately mimic the dielectric behavior of pure EC and PC[447], the models are simply too inflexible to exhibit an appropriate response when polarized by a nearby ion. In fact, the improvements in dielectric properties came at the expense of lowering the dipole of the EC and PC molecules[447]!

It was hypothesized that a proper description of polarization (not simply a universal scaling of charges to adjust the dipole) would bring the solvation free energies, enthalpies, and entropies into better agreement with experiment by improving the accuracy of the ion/solvent interaction part (enthalpy) and the solvent reorganization term (enthalpy and entropy, cancelling in free energy). The solvent reorganization term is expected to become more positive as the first solvation shell around cations points 4–5 large dipoles towards a central point, a clearly repulsive contribution.

My work in Chapter 4.1 also aims to test this hypothesis through determination of the dipole moment of molecules in the first solvation shell versus the bulk. The induced dipole depends on the polarizability through a simple relation, $\vec{\mu} = \alpha \vec{E}$, where $\vec{\mu}$ is the induced dipole, $\alpha$ the molecular polarizability (a rank 2 tensor), and $\vec{E}$ the field. The polarizability of EC is 6.8 Å$^3$ and PC is 8.7 Å$^3$. Compare this with water which is 1.42 Å$^3$. As I showed above, the polarizability is responsive to changes in molecular geometry. So they very well may be slightly larger than this around strongly polarizing cations. This work is not yet ready for publication and future studies will be discussed.

### 1.7 Bringing it all together

I have presented a lot of information in this introductory section, so how best to summarize it? Ion solvation is a fundamental problem with far-reaching consequences in biological, industrial, atmospheric, and oceanic fields. Simple electrostatic theories exist to summarize and rationalize the behavior of salts in solution, though these are far from complete and often inaccurate at relevant concentrations. This is because they neglect some of the important details I discussed were essential
to the prediction of single-ion solvation free energies: 1) ion/solvent interactions and 2) cavity formation, the reorganization of solvent around the ion. I discussed that single-ion free energies in particular were essential to developing a quantitative and predictive model of ion solvation. Modern continuum theories supplement these older models with non-electrostatic interactions such as polarization and dispersion through quantum chemistry, increasing their accuracy of contribution 1) and they make an attempt to approximate 2). While these models are most certainly improvements over the originals, the granularity of an all-atom model is required to handle the most important parts of 1) and 2) which arise from the 1st and 2nd solvation shells. Chemical interfaces also contribute to the single-ion free energies and this surface potential has two contributions, at least one of which is present in all simulations with ions. Recalling that the single-ion free energy is $\mu_{ex} = \mu_{ex}^b + q\phi_{np}$, then my thesis can be broken down of as follows:

- **Chapters 3 & 4 – Focus on interactions**
  - How can we solve $\mu_{ex}^b$ more accurately?
  - What level of theory is necessary?

- **Chapter 5 – Focus on surface potential**
  - What is the value of $q\phi_{np}$?
  - Does it relate two existing thermodynamic scales?

- **Chapter 6 – Focus on surface potential**
  - $q\phi_{np}$ is $q\phi_{sp} + q\phi_{lp}$
  - I demonstrate that $q\phi_{lp}$ contributes in periodic boundaries
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2 THEORY

2.1 Preface

This chapter introduces a number of basic concepts related to the work outlined in the later chapters. I’ll begin with a cursory overview of quantum chemistry which will provide a background for a more thorough discussion on symmetry adapted perturbation theory. Following this, I’ll introduce some basics of molecular simulation to set the stage for a more in depth examination of the computational free energy methods used in my work.

Sections 2.2.1–2.2.3 draw extensively from Refs. [118, 395]. This includes equation format and outline, but the text itself is original.

2.2 A primer on general electronic structure theory

2.2.1 The Schrödinger equation and multielectron wave functions

The principle goal of quantum chemistry is to find approximate solutions to the Schrödinger equation. Below the equation is presented in its non-relativistic, time-independent form. This form is the most commonly used in chemical literature.

\[
\hat{H}\phi = \epsilon \phi
\]  

(2.1)

In Eq. 2.1, \(\phi\) is the wave function of some system, \(\hat{H}\) the Hamiltonian operator which acts on the wave function, and \(\epsilon\) the resulting energy eigenvalues. The dimensionless Hamiltonian takes the form

\[
\hat{H} = -\sum_{i=1}^{N} \frac{1}{2} \nabla_{i}^{2} - \sum_{a=1}^{M} \frac{1}{2M_{a}} \nabla_{a}^{2} - \sum_{i=1}^{N} \sum_{a=1}^{M} \frac{Z_{a}}{r_{ia}} + \sum_{i=1}^{N} \sum_{j>i}^{N} \frac{1}{r_{ij}} + \sum_{a=1}^{M} \sum_{b>a}^{M} \frac{Z_{a}Z_{b}}{R_{ab}}.
\]  

(2.2)

Above, the first pair of terms denote the kinetic energy contributions made by the \(i\)th electron and \(a\)th nucleus through twice differentiation with respect to the particle coordinates. The middle term expresses the attraction between the \(i\)th electron and \(a\)th nucleus with \(Z_{a}\) in reference to the atomic number of the nucleus. The final two terms represent the repulsion between electrons and nuclei, respectively. Here, \(r_{ia}, r_{ij}\), and \(R_{ab}\) are distances between the relevant particles and \(M_{a}\) the ratio of the mass of nucleus \(a\) to the mass of an electron.

Quantum chemists often simplify this problem by invoking the Born-Oppenheimer approximation which uncouples electronic motions (fast) from nuclear motions (slow). In Eq. 2.2, this amounts to dropping the second term on the right hand side while the final term becomes a constant.
This reduced form is often termed the electronic Hamiltonian. Nuclear motions are handled by incorporating the electronic potential (\textit{ab initio} molecular dynamics) or as a greatly simplified approximation of the electronic potential (classical molecular dynamics). In most applications, the nuclei are handled as point charges as more computationally demanding procedures are required to include nuclear quantum effects. These effects can also be mimicked somewhat by simply increasing the temperature of a simulation. This trick is utilized in Chapter 4.2 to simulate the ethylene and propylene carbonate solvents in their liquid state.

The electronic Hamiltonian we’ve reviewed so far has only considered the coordinates of the electron in space. For a complete description of an electron, however, we need to specify its spin as well. Thus, we have $x = (r, \omega)$ such that the wave function of an N-electron system is $\phi(x_1, x_2, \ldots, x_N)$ and is now a function of both spatial and spin coordinates. The wave function of a many-electron system is simply the product of all the single-electron wave functions, where $\phi(x_1, x_2, \ldots, x_N) = \chi_1 \chi_2 \ldots \chi_N$ and $\chi_N$ is the spin orbital of the $N$th electron. We place an additional requirement on the electronic wave function that it must be antisymmetric (change sign) upon exchange of the $x = (r, \omega)$ coordinate of any two electrons. Antisymmetry forms the basis for all exchange and exchange-coupled terms in the symmetry adapted perturbation theory and postulates that no two electrons can possess the same $x = (r, \omega)$ coordinates. A convenient representation of the wave function of many-electron systems which satisfies antisymmetry is called a Slater determinant. The Slater determinant takes the form of $N \times N$-matrix for an $N$-electron system.

In conventional modern electronic structure codes each of the single-electron products discussed in the previous paragraph are represented using atom-centered orbitals. Each orbital comprises a radial and a spherical part. The solutions to the spherical part of the wave function derive from spherical harmonics, imparting shape and direction to the orbital. The radial part of the wave function generally assumes one of two forms: an exponential or a Gaussian. Though the exponential description is far more accurate, the Gaussian representation is considerably more computationally efficient thanks to the Gaussian product theorem. A linear combination of one or more Gaussian functions is used to approximate the exponential representation. The collection of atom-centered Gaussian orbitals used to represent the electronic structure of an atom is called a \textit{basis set}. Molecular systems are then modeled as a linear combination of these atomic orbitals,

$$\phi_i = \sum_\mu^n c_{\mu i} \chi_\mu. \quad (2.3)$$

Above, $\phi_i$ is the resultant $i$th molecular orbital from the sum over the $n$ atomic orbitals, $\chi_\mu$, each contributing $c_{\mu i}$ to the sum. The $n$ molecular orbitals represented as a Slater determinant make up the wave function of many-electron, molecular systems used as input to Eq. 2.1. In the next section we’ll discuss the variational principle behind the Hartree-Fock method which minimizes the total energy defined by the Hamiltonian in Eq. 2.2 by optimizing the expansion coefficients,
c_{\mu i} in Eq. 2.3 using only a single Slater determinant. The Hartree-Fock method forms the basis of many of the more accurate electron correlation methods I’ll discuss in brief as well.

2.2.2 Hartree-Fock and electron correlation methods

The Hartree-Fock approximation is a mean field theory which takes the simplifications addressed above a step further, with individual electrons modeled in the averaged field of the other electrons. The Hamiltonian becomes a summation over single-electron Fock operators which may be presented in the familiar eigenvalue form of Eq. 2.1 as,

\[ f(\chi_\alpha) = \epsilon_\alpha |\chi_\alpha\rangle. \quad (2.4) \]

The Fock operator, \( f \), is convenient to separate into single-electron and two-electron terms as shown in Eq. 2.5.

\[ \hat{H}_0 = \sum_i f_i = \sum_i (h_i + \nu_{i\text{HF}}) \]

\[ h_i = -\frac{1}{2} \nabla_i^2 - \sum_a \frac{Z_a}{r_{ia}} \quad \text{where,} \quad r_{ia} \equiv |r_i - r_{ia}^{\text{nuc}}| \]

\[ \nu_{i\text{HF}} = \sum_{i<j} (J_{ij} - K_{ij}) \]

In Eq. 2.5, \( h \) are the one-electron operators over kinetic energy and nuclear attraction contributions. \( \nu_{i\text{HF}} \) denotes the two-electron Coulomb, \( J_{ij} \), and exchange integrals, \( K_{ij} \). Solving the Hartree-Fock equations requires an iterative scheme to achieve self-consistency (i.e., the orbitals found are the eigenfunctions of the Hartree-Fock Hamiltonian).

Use of the mean field treatment of electron repulsion coupled with the single Slater determinant expansion of the wave function limits the accuracy of Hartree-Fock. van der Waals complexes where dispersion forces play a significant role in the chemistry will be particularly poorly handled with this method. To improve upon the results of the Hartree-Fock method, there are a number of post-Hartree-Fock methodologies to consider for capturing the opposite-spin part of the correlation energy, \( \epsilon_{\text{corr}} = E_{\text{exact}} - E_{\text{HF}} \). Perturbation theory approaches such as Möller-Plesset perturbation theory treat the correlation energy as a perturbation of the Hartree-Fock Hamiltonian which tends to overcorrect for the missing energy. There are empirically tuned corrections such as spin-component scaling which generally muffle the same-spin two-electron integral contribution (same-spin correlation is already handled by Hartree-Fock). Configuration interaction and coupled cluster approaches make clever use of excitation operators to add additional excited determinants to the single Hartree-Fock reference determinant. These expansions are typically truncated to double, triple, or quadruple excitations to compensate for the high computational overhead associated with these methods. The perturbation or coupled cluster approaches are generally preferred as they are both size extensive.
and size consistent while truncated configuration interaction is not. With some dependence on the computational resources at hand, coupled cluster and configuration interaction methods are typically limited to small systems with no more than \( \sim 100 \) electrons with a modest basis set. Though the use of a liberally truncated virtual orbital space extends the size of systems accessible to treatment with a coupled cluster approach. I made use of this technique in the CCSD(T)-level calculations in Chapter 5.2. By contrast, I was able to use an approximate form of Møller-Plesset perturbation theory with about 350 electrons, also in Chapter 5.2. However, for many systems such as that studied in Chapter 4.2, a cheaper method for handling electron correlation may be required.

### 2.2.3 Density functional theory

#### 2.2.3.1 Thomas-Fermi model of the electron gas

The precursor to density functional theory is the Thomas-Fermi model, which describes a uniformly distributed electron gas with electron density,

\[
\rho(\vec{r}) = \frac{N}{V} = \frac{8\pi}{3h^3} p_F^3, \tag{2.6}
\]

where \( \rho(\vec{r}) \) is the electron density, \( N \) the particle number, \( V \) the volume, \( h \) is Planck’s constant, and \( p_F \) the Fermi momentum. The maximum allowed density is 2 electrons per \( h^3 \). This theory is not suitable for describing bonding without a correction to the kinetic energy which assumes the following form,

\[
T_W [\rho(\vec{r})] = \frac{\hbar^2}{8m} \int d^3r \left( \frac{1}{\rho(\vec{r})} |\nabla \rho(\vec{r})|^2 \right). \tag{2.7}
\]

Despite the deficiencies in the theory, Eq. 2.7 highlights the fact that the properties of an electronic system can be expressed uniquely through the electron density itself. Hohenberg and Kohn built on this idea and developed an energy functional \( \langle E[\rho(\vec{r})] \rangle \) which is minimized for the ground state electron density. They also proved that while the density is not known initially, it is possible to variationally optimize it so as to minimize the functional. The solution also gives the ground state energy to within an additive constant. The energy functional is expanded below in Eq. 2.8.

\[
E [\rho(\vec{r})] = F [\rho(\vec{r})] + \int d^3r \rho(\vec{r}) \nu_{\text{ext}}(\vec{r}) \tag{2.8}
\]

The \( F[\rho(\vec{r})] \) functional is simply the kinetic and electron-electron repulsion terms from Eq. 2.2 (that’s terms 1 and 4 on the rhs of the equation). \( \nu_{\text{ext}}(\vec{r}) \) is the external potential created by the distribution of nuclei (term 3 in the Hamiltonian expression above). Recalling that under the Born-Oppenheimer approximation the kinetic energy contribution made by the nuclei is neglected and the nuclear-nuclear repulsion is constant, it is seen that the density functional theory provides an alternative to Hartree-Fock theory to approximately solve the Schrödinger equation using the electronic Hamiltonian.
Orbital-free or ‘pure’ density functional theory seeks to solve Eq. 2.8 through guessing the form of $F[\rho(\vec{r})]$ and then optimizing the electron density from a trial density (i.e., a guess) to minimize the energy functional. As with the Thomas-Fermi model, the kinetic energy functional is exceptionally difficult to characterize. The use of poor functionals produces considerable errors in the properties of molecules compared to wave function theory. Nevertheless, Hohenberg and Kohn’s theory for an inhomogeneous electron gas laid the groundwork for the workhorse of modern day quantum chemistry which is the Kohn-Sham formulation of density functional theory (KS-DFT).

### 2.2.3.2 Kohn-Sham density functional theory

The Kohn-Sham equations build on the discussion above, improving the accuracy of the results through a clever trick which models $N$-uncoupled electrons as if they were fully coupled. As a result, the Hamiltonian reduces to a summation over $N$ single-electron Hamiltonians with approximate functionals to mimic proper electron-electron repulsion and electron correlation effects. This presents a similar eigenvalue problem to that discussed in Chapter 2.2.2. The benefit to this approach is that the kinetic energy of non-interacting electrons is known exactly in an orbital representation (KS-DFT uses basis sets and produces orbitals just like Hartree-Fock) and the perturbation to fully interacting electrons is small compared to the error in ‘pure’ density functional methods. The energy functional in the Kohn-Sham approximation becomes,

$$E_{KS}[\rho] = E_{\text{kin,KS}}[\rho] + E_{\text{Coul}}[\rho] + E_{\text{ext}}[\rho] + \left(\frac{\lambda}{E_{\text{XC}}[\rho]} \right) \left( E_{\text{kin}}[\rho] - E_{\text{kin,KS}}[\rho] \right) + E_{\text{XC}}[\rho]$$

(2.9)

where $\lambda$ is the uncoupled $\rightarrow$ coupled perturbation correction in the kinetic energy which is lumped in with the approximate exchange-correlation functional, $E_{\text{XC}}[\rho]$.

The $E_{\text{XC}}[\rho]$ functional is not known exactly, just as the kinetic energy functional in Thomas-Fermi or Hohenberg-Kohn theories, but it turns out that even relatively simple forms work exceptionally well for estimating the exchange-correlation (XC) energy, see the local density approximation (LDA). The LDA method assumes the XC energy depends only on $\rho(\vec{r})$ without any dependence on the gradient of the density or the Kohn-Sham orbitals. This tends to lead to an over-estimation of the XC energy. Gradient-corrected XC functionals which include some contribution from $\nabla \rho(\vec{r})$ provide a substantial improvement over the LDA methods for molecular systems. Gradient-corrected functionals can be combined with some contribution from exact Hartree-Fock exchange to improve the results even further in so-called hybrid density functionals. Nowadays there are even double-hybrid density functionals which incorporate 2$^{\text{nd}}$ order Møller-Plesset perturbation theory to add dispersion interactions without relying on empirically designed dispersion potentials (e.g., the D, D2, D3, and damped D3BJ corrections of Grimme or the exchange-hole dipole moment method of Johnson et al.[41]).

In some cases it will be appropriate to add a further correction to condition the unphysical
exponentially decaying asymptotic behavior of the XC functionals which should go as $1/r$ at long range. These corrections differ for LDA/gradient-corrected functionals and the (double-)hybrid functionals. Hybrid functionals use range separation similar to the Ewald summation method discussed in Chapter 2.4.3 to smoothly switch to Hartree-Fock exchange. LDA and gradient-corrected functionals require a more sophisticated treatment discussed by Herbert et al.[238].

2.2.4 Wannier localization

Localization procedures are commonplace in quantum chemistry for concentrating charge density within atom-centered orbitals to speed up post-Hartree-Fock methods (e.g., local MP2[244]), be used in energy decomposition analysis (e.g., the absolutely localized molecular orbitals method of Khaliullin et al.[217]), or discretize large clusters for decomposition into well-defined fragments which interact via a many-body expansion (e.g., the fragment molecular orbital method[220]). In many cases, researchers may be interested in simulating crystalline materials or solutions which make use of periodic boundary conditions discussed in Chapter 2.4.2 to remove errors attributed to finite-size effects. At the \textit{ab initio} level, periodic codes make use of plane wave basis functions instead of conventional Gaussian ones or combine the technologies in a hybrid Gaussian plane wave method, which I use in Chapter 4.2. The plane waves prevent us from simply being able to produce molecular orbitals as is commonplace in conventional electronic structure codes. This can be circumvented through the use of the Wannier localization which can condense even periodic plane wave basis functions into a set of orthogonal atom-centered orbitals, each with $-2e$ charge where $e$ is the elementary charge. I use these orbitals to calculate the dipole moment of molecules of ethylene and propylene carbonate molecules in the gas, condensed, and Li$^+$-solvating phases. These orbitals can also be used to calculate a number of other molecular or spectral properties. A more thorough discussion of the method can be found here[273].

2.3 Symmetry adapted perturbation theory

The overall presentation and equation format is modeled after the Psi4 manual[166], though the text is original.

The symmetry adapted perturbation theory (SAPT) is a technique used to partition interaction energies between fragments (i.e., between molecules but also now within molecules[306, 311]). The energies are partitioned into chemically interesting components: electrostatics, exchange-repulsion, induction, and dispersion. The induction contribution in SAPT is argued to be comprised of an intramolecular polarization component and a charge transfer component due to the exchange of fractional charge between molecules. Unlike other energy decomposition schemes however, the polarization and charge transfer components of the interaction energy are not uniquely separable. A more thorough discussion on this topic is presented in Chapter 3.2.
The SAPT Hamiltonian differs from the Hamiltonians we’ve examined previously. Instead of calculating the total energy of a molecule, we’re only after the energy a system partitioned into fragments $A$ and $B$ is lowered by due to their proximity to one another. This is handled through a perturbation theoretic expansion of the interaction energy in $W_A + W_B$ and an interaction potential $V$ as shown in Eq. 2.10.

$$\hat{H} = f_A + f_B + (W_A + W_B) + V$$ (2.10)

In Eq. 2.10, the first two terms are the Fock operators we discussed in Chapter 2.2. The next terms in parentheses, $W_A + W_B$, are correlation operators. They are often referred to as fluctuation potentials for each of the monomers because the Møller-Plesset perturbation theory can also be interpreted as a measure of the deviation of the electron-electron repulsion from the mean (i.e., Hartree-Fock). These values are always the same since it makes no sense to treat one fragment at a higher perturbation order than the other. The interaction potential, $V$, to first order decomposes into electrostatic and exchange contributions and further into induction and dispersion components at second order. The symmetry adapted part of the theory comes in the form of applying antisymmetrizers to project out contributions made by Pauli-forbidden components of the interaction energy. These operators are applied to each order in $V$ so, for example, the dispersion energy is equal to the sum of an attractive component and the coupled exchange contribution. This is a separate energy from the 1st-order exchange energy. An attractive feature of the SAPT formalism is that it is inherently free of basis set superposition error effects where monomers ‘borrow’ basis functions from neighboring atoms to approximate a more complete basis set. Below I’ll discuss some of the most common truncations of the theory.

### 2.3.1 SAPT0 and DFT-SAPT

SAPT0 is the lowest order truncation of the interaction energy. The ‘0’ associated with the name of this truncation arises from each of the $W_A$ and $W_B$ perturbation operators set to 0th order. The SAPT0 method collects terms up to 2nd order in the interaction potential, however. The perturbation orders in the expressions below are given as $(VW)$.

$$\mathcal{E}_{\text{SAPT0}} = \mathcal{E}^{(10)}_{\text{elst}} + \mathcal{E}^{(10)}_{\text{exch}} + \mathcal{E}^{(20)}_{\text{ind.resp}} + \mathcal{E}^{(20)}_{\text{exch-ind.resp}} + \mathcal{E}^{(20)}_{\text{disp}} + \mathcal{E}^{(20)}_{\text{exch-disp}} + \delta^{(2)}_{\text{HF}}$$ (2.11)

Above the ‘resp’ subscript denotes inclusion of orbital relaxation effects calculated through coupled-perturbed Hartree-Fock. Where polarization effects are expected to be large, as is the case in the ion/solvent cases explored in later chapters, the modeling of orbital response effects are necessary for accurate energies. The final term in this series can be thought of as an ad hoc correction to the induction energy at low orders in $V$. 

In principle, the Hartree-Fock interaction energy accounts for electrostatic and induction components in the SAPT0 truncation to infinite order. It is also expected that the term is dominated by charge transfer effects[422]. Taking the difference in the Hartree-Fock interaction energy and the electrostatic and induction contributions recovered from SAPT0 is seen as an approximate way to add these higher order effects in with no additional computational overhead.

Density functional theory SAPT (or DFT-SAPT/SAPT(KS) for short) replaces the need for the correlation operators in Hartree-Fock based SAPT and so is an attractive option for low cost and potentially highly accurate interaction energies. The expansion in $V$ is the same between the theories.

$$E_{\text{DFT-SAPT}} = E^{(1)}_{\text{elst}} + E^{(1)}_{\text{exch}} + E^{(2)}_{\text{ind,resp}} + E^{(2)}_{\text{exch-ind,resp}} + E^{(2)}_{\text{disp}} + E^{(2)}_{\text{exch-disp}} \quad (2.13)$$

Assuming the exact functional were known, the above expression would similarly be an exact decomposition of the interaction energy within the SAPT formalism. It is critical to note that there is no singular way to carve up the interaction energy. Additionally, the poor asymptotic behavior in the Coulomb potential of modern density functionals must be corrected in order to improve the accuracy of the method relative to the Hartree-Fock analogue. I make use of the Hartree-Fock reference SAPT scheme in Chapter 3.2 and a long-range corrected DFT-SAPT model in Chapter 4.2. Dispersion interactions are treated explicitly in the former and via an empirical model[190, 156, 238] in the latter study.

### 2.3.2 SAPT2 and beyond

In this section I’ll complete our discussion on the common truncations of the symmetry adapted perturbation theory, focusing now on the higher order terms. Higher orders of SAPT include 3rd order contributions in the interaction potential, $V$, and non-zero orders in the correlation perturbation. For the following series of truncations to the SAPT interaction energy, the higher levels include all the terms for the previous levels and include correlation effects in electrostatic and induction terms (SAPT2), dispersion (SAPT2+), higher level electrostatic and dispersion effects (SAPT2+(3)), and full 3rd order terms in the exchange and induction energies (SAPT2+3). This final truncation is typically the highest level offered by modern codes.

$$E_{\text{SAPT2}} = E_{\text{SAPT0}} + E^{(12)}_{\text{elst,resp}} + E^{(11)}_{\text{exch}} + E^{(12)}_{\text{exch}} + tE^{(22)}_{\text{exch-ind}} + \quad (2.14)$$
\[ E_{\text{SAPT}2+} = E_{\text{SAPT}2} + E_{\text{disp}}^{(21)} + E_{\text{disp}}^{(22)} \]  

(2.15)

\[ E_{\text{SAPT}2+(3)} = E_{\text{SAPT}2+} + E_{\text{elst,resp}}^{(13)} + E_{\text{disp}}^{(30)} \]  

(2.16)

\[ E_{\text{SAPT}2+3} = E_{\text{SAPT}2+(3)} + E_{\text{ind,resp}}^{(30)} + E_{\text{exch-ind}}^{(30)} + E_{\text{exch-disp}}^{(30)} + E_{\text{ind-disp}}^{(30)} + E_{\text{exch-ind-disp}}^{(30)} - \delta_{HF}^{(2)} + \delta_{HF}^{(3)} \]  

(2.17)

\[ \delta_{HF}^{(3)} = \delta_{HF}^{(2)} - \left( E_{\text{ind,resp}}^{(30)} + E_{\text{exch-ind}}^{(30)} \right) \]  

(2.18)

Dispersion terms with non-zero correlation perturbation orders can be solved with coupled-cluster instead of Møller-Plesset \( t \)-amplitudes. I make this substitution in later chapters involving SAPT2+ and higher calculations. I’m also truncating the correlated virtual orbital space by discarding natural orbitals below a defined threshold occupancy. This approximation, ideally, comes with negligible losses in accuracy.

The next sections will shift our focus from highly accurate but static descriptions of the electronic structure in small ion/solvent clusters to modeling the motions of hundreds of particles for extended lengths of time. To do this, I need to greatly simplify the physics and approximate the electronic energy with efficient van der Waals potentials such as the Lennard-Jones, Buckingham, or Halgren functions.

### 2.4 A primer on molecular dynamics methods

Molecular dynamics (MD) is a computational technique for evaluating equilibrium and transport properties of a many-body system[129]. MD simulation can be applied to a diverse range of chemical problems in the gas, condensed, or solid phase. The method is often used to compliment experimental results or make predictions of behaviors that might ultimately be confirmed in subsequent experiments. However, the accuracy of these predictions hinges on a reasonable description of the interactions between molecules through some potential function, \( U \), typically modeled as a force field. A force field applies a general formula for the interactions between atoms which become atom-specific through parameters tuned for each atom, an atom in a specific functional group, or with a particular hybridization. In Chapter 4.2 and other work not included in this thesis, I simulate with an \( ab \ initio \) potential directly. Much of the discussion below is applicable to these types of simulations as well.
The scope of computer simulations has grown considerably since its infancy, with highly parallelized and efficient codes now handling up to millions of atoms. However, this still puts us many orders of magnitude below the molar scale of Avogadro’s number of particles, \( N_A \). As such, molecular simulation in vacuo operates well below the proper limit for simulating bulk liquids. This is because the forces acting on surface molecules deviate from bulk behavior, introducing a set of finite-size artifacts into many calculated properties. To circumvent this issue, we implement periodic boundary conditions to replicate the finite system throughout space to remove the surface. Coulomb forces in periodic conditions require a unique formula which splits the interactions into real space and reciprocal space. This technique is called Ewald summation. Modern MD codes typically use a more efficient solver than the one I will present in Chapter 2.4.2. Once we have solved for the potential and forces acting on the particles, we integrate Newton’s equations of motion and solve for the new positions of all of the particles. The updated positions have all new properties compared to the previous configuration – and sometimes we wish to exert some level of control over some of those properties to facilitate comparison to experiment. In this section, we’ll explore some of the tools behind the molecular dynamics method.

The format of equations and general outline are modeled after Refs. [7, 129]. The wording is otherwise original, as is the image.

### 2.4.1 Force fields and the potential energy function

At the heart of all classical simulations is the force field. The force field is two parts: (1) the potential energy function, \( U \), and (2) the optimized parameters used to represent a specific atom in the potential. The potential is an attempt to condense the Schrödinger equation (Eq. 2.1) to a more manageable level which can trace the dynamics of thousands of atoms across millions of individual force evaluations. It is commonly expressed as a sum of a system’s bonded and non-bonded contributions, with bonds, angles, and torsions modeled as harmonic springs (or periodic functions) with the stiffness controlled via a spring constant as in Eq. 2.19.

\[
U = U^\text{bnd} + U^\text{nb} = \sum_{\text{bonds}} k_r (r - r_0)^2 + \sum_{\text{angles}} k_\theta (\theta - \theta_0)^2 + \\
\sum_{\text{torsions}} k_\phi (1 + \cos (n_\phi \theta - \theta_0)) + \\
\sum_{\text{Urey}} k_u (u - u_0)^2 + \\
\sum_{i \neq j} \frac{q_i q_j}{r_{ij}} + \sum_{i \neq j} 4 \varepsilon_{ij} \left( \frac{\sigma_{ij}}{r_{ij}} \right)^{12} - \left( \frac{\sigma_{ij}}{r_{ij}} \right)^6
\]  

(2.19)

For some force fields, the torsion potential may be expressed with a harmonic potential similar to that of the bonds, Urey-Bradley, and angle potentials. Not all force fields include the Urey-Bradley
term which is sometimes called the (1,3) interaction where the indices refer to $\angle 123$. In the periodic torsional potential, $n_{\Phi}$ is the multiplicity. The last two terms in Eq. 2.19 are the non-bonded terms, corresponding to the Coulomb and van der Waals (vdW) interactions, respectively. The vdW potential can take a number of different forms. Here, it is expressed as the Lennard-Jones potential where electron-electron repulsion is modeled as an $r^{-12}$ potential instead of with an exponential as is the case with the Buckingham potential. The $r^{-6}$ dependent term is for dispersion interactions. In compliance with requirement (2) above, each atom is designated a partial charge, Lennard-Jones well-depth $\varepsilon_i$, and radius $\sigma_i$. Mixing rules are used to generate the $\varepsilon_{ij}$ and $\sigma_{ij}$ terms specific to each atom/atom interacting pair. Another common exponent pair here is 14-7, the buffered Halgren potential which I use with the AMOEBA polarizable force field in Chapter 5.2; additional sources on this force field can be found here[241].

Partial charges ($q_i$) are fit to reproduce the electrostatic potential of the atom or molecule at the Hartree-Fock, density functional, or Møller-Plesset perturbation level of theory. $\varepsilon_i$ and $\sigma_i$ for solvents are tuned to reproduce a plethora of macroscopic, mechanical, or solvation properties against experiment for the ‘pure’ solvents. This causes issues when trying to model ion solvation in energy storage solvents which haven’t seen as much attention from the community as has water. These parameters also tend to collect a lot of artificial contributions from sources that are not fully understood (see, for example, the surface potential of water). I show in Chapter 6.3 how this practice is troublesome.

2.4.2 Periodic boundary conditions

As described above, periodic boundary conditions are often used in simulations to remove non-negligible surface effects which are present when the system is surrounded by a vacuum. A representation of a periodic cell is shown in Figure 2.1. There are no hard walls to reflect molecules back into the box. There are no longer spurious surface forces at the boundary as the mirrored exchange between the unit cell and the image cells prevents the formation of a surface.

Applying periodic boundaries helps theorists to simulate the behavior of a much larger system with fewer particles and at greatly reduced computational cost. While abundantly useful in these purposes, there are a couple drawbacks to relying on periodic cells to mimic larger systems: (a) interactions between a particle and its image due to long-ranged ($r^{-\nu}$) forces, where $\nu$ is less than the lattice dimensionality, (b) suppression of fluctuations with a wavelength longer than the lattice length, and (c) the presence of poorly understood, minor artifacts[382]. The latter points (b, c) are not so critical for the work I’ve performed in this thesis, while the former (a) is a much more pressing concern. I’ll discuss in the next section the necessary corrections to the long-ranged Coulomb potential to remove unwanted image forces. However, in the simulations I describe in Chapter 6.3, I must consider the damping of ion/dipole interactions via the solvent dielectric constant for which I have no correction. In this scenario, I want the lattice vector length to be at least as large as the Bjerrum length for a solvent, where Coulomb interactions between a particle and its periodic image
become comparable to the thermal energy, $k_B T$, with $k_B$ as Boltzmann's constant. This length scale varies with the inverse of the solvent dielectric constant, $\lambda_B \propto \frac{1}{\epsilon}$. vdW forces on the other hand are short-ranged, requiring a lattice with a size of only $\approx 6\sigma$ when using the Lennard-Jones potential.
Figure 2.1: Cartoon of a periodic cell with cubic lattice vectors of length, \( L \). Particles are colored on a spectrum from red → white → blue by distance from the center of the cell. Each arrow is coupled with the one opposite it and indicates the direction of particle exchange between the unit cell \( n(0,0,0) \) and the image cells \( n(±x,±x,0) \). Because the motions are coupled between the unit and image cells, a particle exiting \( n(0,0,0) \) and crossing into \( n(1,0,0) \) is mirrored in \( n(-1,0,0) \) as crossing into \( n(0,0,0) \) on the other side of the box.

2.4.3 The Ewald summation

Assume we have a distribution of both positive and negative point charges scattered about a periodic and cubic cell of length, \( L \), and volume, \( L^3 \). The total number of particles is \( N \) and while we generally want \( N/2 \) positive and \( N/2 \) negative charges for charge neutrality, the discussion here will make no such demands. This makes sense especially since we are simulating infinitely dilute single ions where the unit cell is expected to take the same net charge as the ion inside. We want to calculate the Coulomb contribution to the potential energy of the system, which is expressed using a direct summation in Eq. 2.20 (as in above sections, Gaussian units are assumed).

\[
U_{\text{Coulomb}} = \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{\mathbf{n} \in \mathbb{R}}' \frac{q_i q_j}{|\mathbf{r}_{ij} + \mathbf{nL}|} \tag{2.20}
\]

The double sum runs over particle indices \( i \) and \( j \) up to the total number of particles \( N \) over all cubic lattices where \( \mathbf{n} = (n_x, n_y, n_z) \). \( n_x, n_y, \) and \( n_z \) are integers which provide for the location of the lattice in 3D space, \( \mathbb{R} \). The prime indicates that the \( \mathbf{n} = 0 \) sums should omit the \( i = j \) case so a particle does not interact with itself. In principle the infinite sum of the series gives the true Coulomb energy in a lattice – keyword, infinite. To solve numerically, a finite set of lattice vectors
are used under the assumption that as the distance from the unit cell increases, the contribution to the potential will decrease as well. However, because the sum converges very slowly a large cutoff must still be used to achieve sufficient accuracy. Combine this with a hefty computational overhead of $O(N^2)$ on the direct sum and it’s easy to see why a more efficient solver is desired. Ideally, an algorithm should scale linearly in cost with system size, $O(N)$. The most widely adopted algorithm in modern MD codes is the Ewald summation $O(N^{3/2})$ or an even more efficient variant (i.e., particle mesh Ewald $O(N \log N)$).

The Ewald summation splits the slowly converging series above into two components which can each be solved faster than the direct summation method. The series in $1/r$ is usually split with the use of the error function $\text{erf}(\eta r)$ and its complimentary form $\text{erfc}(\eta r)$, where

$$\frac{1}{r} = \frac{\text{erfc}(\eta r)}{r} + \frac{\text{erf}(\eta r)}{r}.$$  \hfill (2.21)

$\eta$ is a splitting parameter controlling the length scale and balance between the real space part, $\text{erfc}(\eta r)$, and the reciprocal space part, $\text{erf}(\eta r)$. In a cubic unit cell, $\eta$ has an optimal value of $5.6/L$ measured in Å with the real space part truncated to $\sim 10$ Å and the reciprocal space part capturing the rest. The resulting formula takes the form,

$$U_{\text{Ewald}} = U_{\text{real}} + U_{\text{recip}} + U_{\text{surf}} + U_{\text{self}} + U_{\text{net}}$$

$$= \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} q_i q_j \left( \sum_{n \in \mathbb{R}} \frac{\text{erfc}[\eta (|r_{ij} + nL|)]}{|r_{ij} + nL|} + \sum_{k \in K, k \neq 0} \frac{4\pi e^{- \frac{k^2}{\eta^2}} e^{-ik \cdot r_{ij}}}{L^3} \right) + \frac{2\pi}{(2\epsilon' + 1)L^3} \left( \sum_{i=1}^{N} q_i r_i \right)^2 + \frac{\zeta}{2L} \sum_{i} q_i^2 + \frac{\pi}{12\eta^2} Q^2,$$  \hfill (2.22)

where $k = \frac{2\pi}{L} n$ is the reciprocal space lattice vector, $\epsilon'$ is the dielectric of the surrounding medium, which is taken as $\infty$ in so-called ‘tin-foil’ or conducting boundary conditions common to MD simulations and so vanishes, and $Q^2$ is the square of the net charge in the unit cell. The self-energy includes a constant $\zeta = -2.837297$ for a cubic lattice. The corrections beyond the real and reciprocal space terms require little to no computational overhead and can sometimes be pre-computed and added in as a constant at each timestep, specifically when the volume is constrained. I use the more efficient $O(N^{3/2})$ scaling algorithm in Chapter 6.3 for generating trajectories but use a single sum version of the formula as presented in Eq. 2.22 to compute the local potential I described in Chapter 1.5.

### 2.4.4 Equations of motion and time propagation

Once we have evaluated the potential energy of a configuration of particles, we calculate the force as the negative derivative of the potential with respect to the coordinate, $-\frac{\partial U}{\partial r}$. With forces in
hand, we have a variety of mathematics to choose from to update the particle positions. One of the simplest and most effective integrators is called the Verlet algorithm which is nearly universally available in modern MD codes. This method results from a Taylor series expansion in $r$ about time, $t$,

$$r(t + \delta t) \approx 2r(t) - r(t - \delta t) + \frac{f(t)}{m} \delta t^2,$$

where $\delta t$ is the length of the timestep and $\frac{f(t)}{m}$ is the acceleration on the particle for the current configuration. To compute the new location of a particle, various algorithms will require knowledge of one or more previous locations. The velocity of the particle can be computed from

$$v(t) = \frac{r(t + \delta t) - r(t - \delta t)}{2\delta t},$$

which is accurate to order $\delta t^2$. From this, the kinetic energy and the system temperature can be constructed. We’ll see in the next section that this relation is important in monitoring the stability of a simulation and/or conditioned so as to compare to an experiment under the same conditions.

### 2.4.5 Thermodynamic ensembles: controlling the variables

Thermodynamic ensembles are relations which establish a complete statistical knowledge of a system under certain conditions that are linked to macroscopic observables such as particle number, $N$, temperature, $T$, energy, $U$, or volume, $V$. The natural ensemble for molecular dynamics simulations is the microcanonical ensemble, $S(N,V,U)$, which is isolated from an external bath and the myriad parts merely exchange the available energy to maximize the entropy throughout. This ensemble, in the limit of large systems, approximates the canonical ensemble where average temperature is constant. Though in the canonical ensemble, this is accomplished through coupling to an external bath.

In both of these ensembles, the particle number and system volume are also fixed. The bath allows endo-/exothermic processes to absorb/release excess energy through artificial modification of particle velocities. The canonical ensemble is widely used in molecular dynamics as precise control of the temperature is often necessary for studies of protein folding, solvation thermodynamics, transport properties, etc.

The third most commonly used ensemble in molecular dynamics is the isobaric-isothermal ensemble where particle number and temperature remain fixed as in the canonical ensemble but the volume of the system is allowed to change to match a desired pressure. Motivations for controlling the pressure are similar to those for precise control on the system temperature. Simulations incorporate barostats which scale the unit cell size in response to the virial pressure tensor which depends on the forces on the atoms. This ensemble is commonly used to relieve strain on the solvent after adding large solutes such as proteins or other biomolecules to the unit cell and achieve a proper solvent density especially at the cell edges.
Molecular dynamics relies on the ergodic principle to generate statistical averages of the behavior of molecular systems through equivalence of averages in time to ensemble averages over all space. We must take great care with the selection of an algorithm to constrain a particular property that it does not violate this condition, such as the original Nosé-Hoover thermostat. In the limit of large system size \((N \to \infty)\) the averages in properties measured from each of the ensembles become identical. And so for the free energy work below, there is often very little difference between the averages computed in the \((NVT)\) or \((NpT)\) thermodynamic ensembles\[40\].

2.5 Free energy calculations using molecular dynamics

A knowledge of free energy and temperature derivatives is essential in advancing our understanding of a number of physical phenomena including binding and equilibrium constants, rate constants of reactions, pH, pKa, activity coefficients (solubilities), osmotic coefficients, surface tension, and partitioning behavior between phases (including phase transitions). I will first present common methods for directly calculating free energies from molecular dynamics simulations. This will be followed by a thorough derivation and discussion of the quasichemical theory used in later chapters. The quasichemical method partitions the solvation free energy through spatial conditioning. Taken together, the fragments add to the conventional solvation free energy but offer additional information not available to the other methods discussed here. The quasichemical theory is particularly well-suited for the determination of single-ion solvation free energies because it is able to uniquely separate the local and distant solvation effects.

The format of equations and general outline are modeled after Refs. \[129, 40\]. The wording is otherwise original.

2.5.1 Free energy perturbation and thermodynamic integration

Free energy perturbation is one of the oldest and most successful methods for computing free energy differences between a reference state (A) and a target state (B). Recall that in the canonical ensemble, the Helmholtz free energy has the form,

\[
F(N, V, T) = -\frac{1}{\beta} \ln Q_{NVT}.
\]  

\(\beta\) is the inverse temperature \(\frac{1}{k_B T}\) and \(Q_{NVT}\) is the canonical partition function. The partition function can be split into coordinate and momentum space contributions, leading to an excess and an ideal contribution, respectively.

\[
Q_{NVT} = \frac{1}{N!} \frac{1}{\hbar^3N} \int d\text{p} e^{-\beta X} \int d\text{q} e^{-\beta V} = Q_{NVT}^{\text{id}} Q_{NVT}^X
\]  

(2.26)
The ideal part is an analytical expression,

\[ Q_{\text{NVT}}^{\text{id}} = \frac{V^N}{N!\Lambda^3} NVT \]

where the thermal de Broglie wavelength is \( \Lambda = \sqrt{\frac{\hbar^2}{2\pi mk_B T}} \). The excess part is

\[ Q_{\text{NVT}}^{\text{ex}} = V^{-N} \int d\mathbf{r} e^{-\beta V(\mathbf{r})}. \]

The free energy difference in the canonical ensemble, is

\[ \Delta F(\text{A} \rightarrow \text{B}) = -\frac{1}{\beta} \ln \left( \frac{e^{\beta(V_B - V_A)}}{\langle \ldots \rangle_{\lambda}} \right). \]

\( V_A \) is the potential energy of the system in its reference state, and \( V_B \) is the energy of the system in the target state. The subscript indicates configurations are generated in the reference state. Angled brackets indicate ensemble averaging. The single-perturbation result is only considered accurate if states A and B are very similar with strong overlap between the energy landscape outlined by their respective partition functions. If the mass does not change with the perturbation, the ideal part vanishes.

Free energies between states with large perturbations (i.e., opposite charges, structural isomers, etc.) can be modeled as incremental changes in the chemical properties of A to match B – also known as ‘alchemical transformation.’ The intermediate states can be entirely fictitious so long as the \( i \)th and \( (i+1) \)th states are sufficiently similar. A coupling parameter \( \lambda \) is used to smoothly vary the A-state to B over \( (N-1) \) intervals. The coupling parameter can be chosen to alter a single property at a time or several.

\[ \Delta F(\text{A} \rightarrow \text{B}) = -\frac{1}{\beta} \ln \left( \frac{e^{-\beta(V_B - V_A)}}{\langle \ldots \rangle_{\lambda}} \right). \]

An alternative approach that also sees a great deal of use in the community is the thermodynamic integration method. This is similar to the alchemical transformation approach described above in that we are attempting to connect two chemically dissimilar endpoints using fictitious intermediates. However, instead of sampling the ensemble average in the difference of the energies between the \( (i+1) \)th and \( i \)th states, we sample the ensemble average of the derivative of the potential with respect to the coupling parameter and then integrate,

\[ \Delta F(\text{A} \rightarrow \text{B}) = \int_0^1 d\lambda \left( \frac{\partial V(\lambda)}{\partial \lambda} \right)_{\lambda}. \]

As with free energy perturbation, numerous independent simulations at each \( \lambda \) are required to assemble the final result. These paths can be used to compute relative ion solvation free energies (e.g., K\(^+\) to Na\(^+\) or K\(^+\) to Cl\(^-\)) and absolute ion solvation free energies where the reference state is the ion uncoupled from the solvent. The ideal part of the free energy is dropped here because it can
be determined analytically, while the excess part is both the more interesting part and far more
difficult to compute. It is desirable to devise methods which require very few calculations for this
excess part.

### 2.5.2 The potential distribution theorem

The potential distribution theorem of Widom sometimes called the particle insertion method was
developed to compute the excess chemical potential (i.e., partial molar free energy) of a solute in
pure liquids or complex mixtures. In the canonical ensemble, the excess chemical potential of a
distinguished ion (α) is

\[
\mu_{\alpha}^{ex} = -\frac{1}{\beta} \ln \left( e^{-\beta \Delta V} \right) ; \quad \text{where,} \quad \Delta V = \mathcal{V}(N + \alpha) - \mathcal{V}(N) - \mathcal{V}(\alpha).
\] (2.32)

\(\Delta V\) is the interaction energy between the ion and the solvent, with an ensemble average computed
over configurations sampled with the ion uncoupled from the solvent (as indicated by the subscript
‘0’). The quantity is more often expressed as

\[
\beta \mu_{\alpha}^{ex} = -\ln \left( e^{-\beta \Delta V} \right) = \int d\epsilon P_{\alpha}^{(0)}(\epsilon) e^{-\beta \epsilon}
\] (2.33)

with an inverse form

\[
\beta \mu_{\alpha}^{ex} = \ln \left( e^{\beta \Delta V} \right) \quad (2.34)
\]

where the sampling is now done over configurations which include the fully coupled ion/solvent
interactions. Rearranging Eq. 2.33 gives

\[
e^{-\beta \mu_{\alpha}^{ex}} = \left( e^{-\beta \Delta V} \right) = \int d\epsilon P_{\alpha}^{(0)}(\epsilon) e^{-\beta \epsilon}
\] (2.35)

where \( P_{\alpha}^{(0)}(\epsilon) = \langle \delta (\epsilon - \Delta V) \rangle_0 \) is a probability distribution of ion/solvent interaction energies
from an uncoupled trajectory. Conversely, the distribution from the fully coupled trajectory is
\( P_{\alpha}(\epsilon) = \langle \delta (\epsilon - \Delta V) \rangle \). Using the law of averages we can relate these two distributions through

\[
P_{\alpha}(\epsilon) = e^{-\beta (\epsilon - \mu_{\alpha}^{ex})} P_{\alpha}^{(0)}(\epsilon).
\] (2.36)

The crossing point of these distributions at some interaction energy \( \epsilon \) will yield the exact excess
chemical potential. If however, the distributions do not overlap, the method fails outright. The
excess chemical potential can be assessed from one of the distributions as well, though the wings of
these distributions are poorly, if ever, sampled over the course of a simulation making it difficult to
integrate accurately. The quasichemical theory I discuss next addresses this by applying spatial
conditioning to the interaction energy distributions. This removes the need to sample rare events at
the high- and low-energy tails of the distributions and yields Gaussian or nearly-Gaussian behavior.
in $P_\alpha(\varepsilon)$. The conditioning is done using hard spheres with interaction energies computed only when all solvent is beyond a certain distance, or by adding an external potential into the simulation to push the solvent away. The latter is done gradually using thermodynamic integration.

### 2.5.3 Quasichemical theory

This section borrows equation format and outline from Refs. [380, 40]. The wording is otherwise original.

#### 2.5.3.1 Hard sphere conditioning

Recall that the excess chemical potential of a monoatomic ion may be evaluated using the following expressions, derived from the potential distribution theorem,

$$
\beta \mu^\text{ex} = -\ln \left\langle e^{-\beta \Delta V} \right\rangle_0 = \ln \left\langle e^{\beta \Delta V} \right\rangle,
$$

(2.37)

where $\beta = 1/k_B T$ and $\Delta V$ is the interaction energy between ion and solvent. Brackets indicate ensemble averaging. The subscript zero implies uncoupled ion/solvent motions during sampling and fully coupled ion/solvent interactions in the second case. In both cases, full solvent/solvent interactions are in place. Inclusion of a hard-sphere cavity potential with radius, $r$, in the above expressions allows us to spatially separate contributions to the free energy,

$$
\beta \mu^\text{ex} = \ln \left\langle e^{-\beta V_{\text{HS}}(r)} \right\rangle - \ln \left\langle e^{-\beta V_{\text{HS}}(r)} \right\rangle_0 - \ln \left\langle e^{-\beta \Delta V} \right\rangle_{V_{\text{HS}}(r)} \\
= \ln x_0(r) - \ln p_0(r) - \ln \left\langle e^{-\beta \Delta V} \right\rangle_{V_{\text{HS}}(r)} \\
= \mu^\text{ex}_{\text{is}}(r) + \mu^\text{ex}_{\text{pk}}(r) + \mu^\text{ex}_{\text{os}}(r).
$$

(2.38)

Above, $x_0$ is the inner shell (is) term which is the probability of finding no solvent in the excluded volume defined by $V_{\text{HS}}$ with full ion/solvent interactions, $p_0$ is the packing (pk) term which is the probability of finding no solvent in this same cavity placed in pure solvent, and the final term is the outer shell (os) or long-range term corresponding to the residual interaction energy of the ion with solvent molecules beyond this excluded volume. A hard-sphere potential suffices for small cavities typically around 0.3 nm or less, beyond which $x_0$ and $p_0$ are generally poorly sampled in typical simulations. An external potential applied to the solvent is critical for reaching length scales far enough to see convergence in the interfacial potentials monitored as the mean-field electrostatic potential at the cavity’s core.
2.5.3.2 Soft-core conditioning

I can reach any length scale by gradually introducing a soft-core $M(r)$-potential instead (Eq. 2.39).

\[
\beta \mu_{ex}^\beta = \ln \langle e^{-\beta M(r)} \rangle - \ln \langle e^{-\beta M(r)} \rangle_0 - \ln \langle e^{-\beta \Delta V} \rangle_{M(r)}
\]

\[
= \ln \langle e^{-\beta M(r)} \rangle - \ln \langle e^{-\beta M(r)} \rangle_0 + \ln \langle e^{\beta \Delta V} \rangle_{M(r)+\Delta V}
\] (2.39)

In this framework, the packing and inner shell parts of the solvation free energy are captured by the work to dig a cavity in the bare solvent and around the ion, respectively. The inner shell portion is often referred to as the chemical part of the free energy as it’s magnitude reflects the intensity of the interactions between the ion with the retreating solvent. The thermodynamic integration procedure and $M(r)$-potential used in Chapter 6.3 are the same as in a previous study[380]. This arrangement requires two simulations for the outer shell contribution where ion/solvent interaction energy distributions are sampled from an uncoupled and a separate coupled trajectory. The cavity radius, $r$, should be tuned such that both distributions are accurately Gaussian. A mean-field approximation to $\mu_{ex}^s$ can be obtained this way[175, 176], but as Shi and Beck demonstrated especially for anions, the fluctuation terms often do not match until reaching extreme length scales which can lead to inaccuracies[380]. This behavior has quite a bit to do with the similarities in the solvation shell between a neutral particle and one bearing $\pm q$ charge (e.g., orientation of solvent dipoles). I discuss more on this later in Chapter 6.3.

To work around this issue I invoke the midpoint-rule to generate an approximation to the long-range term from a single simulation. Here the average interaction energy is sampled from a half-coupled trajectory where the ion’s charge is halved but all other interactions are unchanged. The approximation is accurate to second order in perturbation theory and has proven remarkably reliable in previous free energy work[37, 380].
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3 TOWARD A QUANTITATIVE THEORY OF HOFMEISTER EFFECTS: FROM QUANTUM EFFECTS TO THERMODYNAMICS

3.1 Preface

This chapter draws text and data from a published article,


Additional data is supplied here that will be published in a forthcoming article.

These articles concern the local solvation structure around the alkali/halide ions which is critical in developing a quantitative theory of the specific ion effects discussed throughout Chapter 1.2. This is because the solvation structure is shaped not only by electrostatic forces but also dispersion and induction contributions, which requires electronic resolution. Recent simulation work has also observed that charge transfer between anions to surrounding waters decreases overall solvation asymmetry. I sought to explore the relationship between solvation structure and the presence of these non-electrostatic forces. To do this, I carried out a series of electronic structure calculations to optimize myriad $X^\pm(H_2O)_n$ clusters, where $X$ is one of $\text{Li}^+$, $\text{Na}^+$, $\text{K}^+$, $\text{F}^-$, $\text{Cl}^-$, or $\text{Br}^-$, and $n = 1, ..., 6$. Following this, I used the symmetry adapted perturbation theory (discussed in Chapter 2.3) to extract interaction energies and components. Energies are complemented with the use of quantum theory of atoms in molecules to estimate the ion charge from gas phase to several states with a (nearly) completed first solvation shell. From this I learned that dispersion and induction forces make up approximately $\frac{1}{3}$ of the attractive contributions to the interaction energy for anion/water clusters—though, these effects tended to (nearly) saturate within the first shell regardless of ion charge. I also discovered that the symmetry adapted perturbation theory is not suitable to estimating the stabilization energy of ion/water complexes due to partial charge transfer. However, I proposed that it may be better suited to providing an upper bound on the polarization energy. Combined with the polarized orthogonally localized molecular orbital method, the polarization energy can be bounded from below too (this likewise produces upper and lower bounds to the charge transfer energy). However, assuming the relative values for a given ion and cluster size were qualitatively correct, I found that for $\text{F}^-(H_2O)_6$, the charge transfer interaction energy favored clusters where the ion was internally solvated (as opposed to surface solvated). This is consistent with the findings of others which suggest that the inclusion of partial charge transfer in simulation promotes the reduction of solvent distribution anisotropy around the ion.
This section addresses three of the questions I posed in Chapter 1.2.3,

- What sort of interactions?
- How strong are they relative to one another?
- Do we need electronic structure theory to solve everything?
- Will a simpler model work far away from the ion?

3.2 Computational methods

This chapter covers the findings outlined in two separate studies. In this case, the studies share a significant degree of overlap so a combined methods section is covered below.

3.2.1 Optimization and atomic partial charges

Small ion/water clusters were optimized to the SCS-MP2/CVTZ level of theory\cite{141} (details of the basis sets used are explained later) using the Orca 3.0.3 quantum chemistry package\cite{288, 411}. A number of favorable initial geometries were extracted from the literature for each cluster size to explore the variation in charge transfer that accompanies partial solvation\cite{27, 219, 243}. Molden-type inputs of the MP2 natural orbitals were converted to standard wave function files (WFN) using the molden2aim script available here, (http://people.smu.edu/wzou/program/). The electron density was partitioned by computing the so-called zero-flux surface about the ion within the framework of the quantum theory of atoms in molecules (AIM) pioneered by Bader\cite{22}. Calculations were carried out for each of the complexes and also the isolated fragments with the difference in the net charge values taken as an estimate of the charge exchanged upon complexation. AIM calculations were performed using the AIMAll package\cite{50, 213}. All data necessary for visualization of electron density surfaces or contours were generated using Multiwfn\cite{255} and viewed using VMD\cite{177} (surface) or the native plotting utility in Multiwfn (contours).

3.2.2 Energy decomposition using symmetry adapted perturbation theory

Interaction energies were computed at the density fit SCS-MP2 level of theory with counterpoise corrections\cite{59} and the SAPT2+3(CCD)\cite{199, 168, 169} level of theory using the open source Psi4\cite{406} software. We report the error in the exchange-induction coupled terms at 2\textsuperscript{nd}- and 3\textsuperscript{rd}-orders of SAPT but otherwise do not scale the affected terms as discussed elsewhere in the literature\cite{167, 237}. Errors in the SAPT energies relative to SCS-MP2 are in the neighborhood of 6-8% for the largest cluster sizes with SAPT almost always overshooting the counterpoise-corrected estimate.
3.2.3 The charge transfer energy

The Hellmann-Feynman theorem states that once an electron distribution has been solved, all forces within the system can be calculated purely through classical electrostatics and the Coulomb operator. Partitioning of these energies has a physical basis but is purely ‘modeling’ in many cases\[331\]. However, this modeling is imported into classical force fields which tends to make them more accurate and so it is instructive to partition energies into separable components in an attempt to translate them to simpler force field based simulation. As an interesting aside, non-variational methods (e.g., MP2) do not satisfy the Hellmann-Feynman theorem\[197\]. Nevertheless, charge transfer contributions to the interaction energy between ion/water clusters were modeled using a pair of partitioning schemes within the symmetry adapted perturbation theory (SAPT)\[199\]. We’ll refer to these methods as SM09 and reg-SAPT. The SM09 method was outlined by Stone et al. and entails taking the difference in the induction energy to arbitrary order between a dimer-centered and monomer-centered basis set calculation\[393\]. In this case, charge transfer is modeled only as a basis set superposition error artifact and so vanishes in the limit of a complete basis. Recent work by Lande et al. has shown this method to be most ‘reliable’ for moderate-sized basis sets, e.g., aug-cc-pVTZ\[422\]. To overcome the shortcomings of the SM09 method, Misquittta replaced the monomer-centered basis calculation with one in which the nuclear potential of each fragment is screened from the opposite\[280\]. Inclusion of the screening potential prevents charge transfer but allows for polarization. The sole adjustable parameter in the method is the length scale of the screening potential which is modeled as a Gaussian s-orbital with width, η. Misquittta found that η = 3.0 au (units are L−2) was sufficient for most applications involving the first and second row elements\[280\]. The parameter is tuned by minimizing the difference in the total induction energy, E^{(2)}_{\text{ind,tot}} between the dimer- and monomer-centered basis over a range of intermolecular separations, including some points below the equilibrium bond length. Tuning was only done for ion/water dimers. Anions presented a bit of a challenge in that the differences varied little with changes in the width of the screening orbital. We selected the value in the range of η = 1.0 au to 3.0 au which also minimized the change in E^{(2)}_{\text{ind,tot}} computed with the dimer-centered basis set alone. Once a range parameter is determined, the charge transfer energy is simply equal to the difference in induction energies computed with and then without the conditioning, both using a dimer-centered (or extended monomer-centered) basis representation. These energies exhibit significantly less variance with increasing basis set size. SM09 calculations were performed up to the SAPT2 level using the cc-pVTZ (+ diffuse, where appropriate and def2-TZVPP for K+)\[117, 216, 436, 433, 427, 347, 155\] with the Psi4 package\[406, 168, 169\]. Regularized SAPT was handled with the regSAPT script included in the SAPT2008 package\[65\]. The rather dated ATMOL1024 SCF interface\[366\] to the SAPT2008 package required an additional change in basis set due to small contraction and total orbital limits. All clusters were successfully modeled with (aug-)cc-pVDZ (Feller’s CVDZ) and the (aug-)pc-1 polarization consistent basis set\[193, 194, 198, 195, 196\]. However we addressed optimization of the regularization length scale with the def2-QZVPP (+ diffuse, where appropriate)
A  η
H    3.0
Li+  3.0
O    3.0
F−   2.5
Na+  3.0
Cl−  1.5
K+   1.5
Br−  1.5

Table 3.1: Optimal η parameters for each element in atomic units (η has units of L−2). These results support the conclusion of Misquitta that an η value near 3.0 au produces adequate screening for the first and second row elements.

basis[347] and these results are listed in Table 3.1. All basis sets were retrieved from the EMSL Basis Set Exchange[124, 368].

3.2.4 Basis sets

Geometry optimization and subsequent generation of WFN files for use with the AIMAll package was carried out with a triple-ζ quality basis with core-valence functions included for second row and beyond elements (aug-cc-pwCVTZ)[117, 327, 103, 337]. Diffuse functions were removed from the cation basis sets to be consistent with K+ which had to be modeled with Feller’s CVTZ[125] due to a lack of parameterization in the Dunning sets. The emphasis on density fitting algorithms in Psi4 ultimately required us to pursue the valence-only flavors of the preceding basis sets (we substituted the def2-TZVPP basis in for K+ calculations[347, 155]). For density fit procedures a Cholesky decomposed basis was constructed with a tolerance 1e-4.

3.3 Results & Discussion

3.3.1 Interaction energies, atoms in molecules, and partial charges

I begin with a survey of the SAPT2+3 (CCD) interaction energies for clusters with up to 6 attached waters, see Tables 3.2–3.5. Even a cursory look over these data confirms the presence of a great deal of ion specificity across all of the components of the interaction energies – especially in the induction and dispersion contributions for F− clusters relative to both Cl− and Br−.

All the interactions described here were found to be dominated by electrostatics with induction playing a more significant role than dispersion in each of the clusters. This is not to dismiss the role of dispersion interactions as they are often of comparable magnitude to the induction term for anionic clusters and combined they represent about \( \frac{1}{3} \) of the total attractive energy contributions. I address dispersion energies in more detail in the next paragraph. Recalling that many force fields
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Table 3.2: SAPT2+3(CCD) energy decomposition: electrostatics (Elst), induction (Ind), dispersion (Disp), exchange (Exch), and total interaction energy (E<sup>PT</sup>) in the dimer-centered aug-cc-pVTZ basis set. All energies expressed in kcal/mol. Exch Err is a value quantifying the degree of error in the single-exchange approximation which affects the accuracy of exchange-coupled higher order induction terms. A value greater than unity indicates an overestimation of the induction energy and vice versa. This figure represents a large source of error in the difference between E<sup>PT</sup> and the conventional counterpoise-corrected interaction energy, E<sup>CP</sup>.
Table 3.3: SAPT2+3(CCD) energy decomposition: electrostatics (Elst), induction (Ind), dispersion (Disp), exchange (Exch), and total interaction energy ($E^{PT}$) in the dimer-centered aug-cc-pVTZ basis set. All energies expressed in kcal/mol. Exch Err is a value quantifying the degree of error in the single-exchange approximation which affects the accuracy of exchange-coupled higher order induction terms. A value greater than unity indicates an overestimation of the induction energy and vice versa. This figure represents a large source of error in the difference between $E^{PT}$ and the conventional counterpoise-corrected interaction energy, $E^{CP}$.

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The dispersion energies captured here with high level coupled-cluster theory are expected to be of higher quality than the results of Duignan and coworkers obtained with DFT-SAPT[114]. In their work, the authors compared dispersion free energies taken from an internally developed continuum solvation model to a linear scaling of the DFT-SAPT dispersion contribution which met with mixed success. A critical feature of the data they have shown is that it is assumed the dispersion...
Table 3.4: SAPT2+3(CCD) energy decomposition: electrostatics (Elst), induction (Ind), dispersion (Disp), exchange (Exch), and total interaction energy (E^{PT}) in the dimer-centered aug-cc-pVTZ basis set. All energies expressed in kcal/mol. Exch Err is a value quantifying the degree of error in the single-exchange approximation which affects the accuracy of exchange-coupled higher order induction terms. A value greater than unity indicates an overestimation of the induction energy and vice versa. This figure represents a large source of error in the difference between E^{PT} and the conventional counterpoise-corrected interaction energy, E^{CP}.

<table>
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<tr>
<th>Cluster</th>
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Table 3.5: SAPT2+3(CCD) energy decomposition: electrostatics (Elst), induction (Ind), dispersion (Disp), exchange (Exch), and total interaction energy ($E^{PT}$) in the dimer-centered aug-cc-pVTZ basis set. All energies expressed in kcal/mol. Exch Err is a value quantifying the degree of error in the single-exchange approximation which affects the accuracy of exchange-coupled higher order induction terms. A value greater than unity indicates an overestimation of the induction energy and vice versa. This figure represents a large source of error in the difference between $E^{PT}$ and the conventional counterpoise-corrected interaction energy, $E^{CP}$.

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portion of the interaction energy increase linearly with additional waters if the initial energy were estimated with an ion/water separation taken as the first peak in condensed phase radial distribution functions. The continuum model of Duignan et al. was designed to incorporate contributions from dipoles, quadrupoles, and octupoles[114]. DFT-SAPT and lower order wave function based SAPT recover only the dipole dispersion forces (the 2\textsuperscript{nd} order formula is equivalent to the generalized Casimir-Polder integral assuming no overlap of the monomer wave functions, integrating a product of the dynamic polarizabilities of each monomer[341]). The results assuming linear scaling that the authors obtained were fortuitous, but necessarily over-estimated the dispersion energy. This is especially true for anions where a significant fraction of the interaction energy is derived from fluctuations between higher order multipoles. Linearly scaling the dimer dispersion energy measured with SAPT2+3(CCD) likewise overshot the dispersion free energy $G_{\text{disp}}$ for the $N_{\text{c}}E_{\text{disp}}$ estimate. This tells us that the various interaction components do not scale linearly with additional solvent even within just the first solvation shell. Sampling dispersion energies directly from $n = 6$ clusters, I found that I underestimated $G_{\text{disp}}$, see Figure 3.1. This is the type of behavior we should expect to see if higher order contributions are missing from the SAPT2+3(CCD) energy.

A term often left out of discussions of the SAPT interaction energy is the exchange component which is evaluated with an approximation in Psi4 that scales with the square of the overlap integral between basis functions centered on the separate monomers. This property allows for a quantitative assessment of the degree of charge overlap within the clusters and is seen to track the charge transfer values from AIM in Table 3.8 reasonably well. A charge transfer component of the SAPT induction energy can be separated from polarization using one of the two methods discussed in Ref. [393] and Ref. [312, 280]. I discuss more on this later as well.

The energetic analysis is complemented here with QTAIM descriptors highlighted in Table 3.6. As with the SAPT energies, each of the properties correlates with the strongly ionic/electrostatic nature of ion/water association (positive $\nabla^2 \rho$ at the bond critical point (\textit{bcp}) and a ratio of the magnitude of the largest attractive curvature to the repulsive curvature, $|\lambda_1| / |\lambda_3|$, less than unity). However hydrogen-bonding within the AIM theory is characterized by an electrostatic presentation of the electron density curvature but a negative total energy density which is dominated by the potential energy density. This differs from a covalent linkage which would also have a negative second derivative at the \textit{bcp}. The large amount of electron density ($\rho$) located at the $F^-(H_2O)$ dimer \textit{bcp} coupled with the negative total energy per charge is consistent with the uncharacteristically high induction and exchange contributions to the SAPT interaction energies relative to $Cl^-$ and $Br^-$. This may offer some insight into the origin of the argument for the apparent chemical character of the interaction discussed in Refs. [89, 358].

It is evident that the $F^-$ ion is much more strongly associated with the water than are the other halides likely owing to its small size and relatively large proton affinity[243]. This is also seen through the extent of charge penetration of the ion into the water “electron cloud.” I found 0.79 Bohr separation of the ion from the \textit{bcp} as compared to 1.32 and 1.43 Bohr for $Cl^-$ and $Br^-$, respectively.
Figure 3.1: Figure adapted from Duignan et al.\cite{114}. Figure highlights the non-linear scaling of interaction energy components, both $N_c E_{\text{disp}}$ estimates are too large to fit the continuum model described in the same reference. The SAPT2+3( CCD) results for explicitly modeled larger clusters I present here underestimate the dispersion energy relative to the higher order estimate of $G_{\text{disp}}$ and so is the more physically consistent result. This demonstrates that interaction energy components are not additive even within the first solvation shell. $N_c$ are experimental coordination numbers and $E_{\text{disp}}$ are dispersion energies from DFT-SAPT or SAPT2+3(CCD) depending on source.
Table 3.6: AIMAll output of several chemical indicators computed at the intermolecular bond critical point (bcp). \( r(H_2O-bcp) \) is the distance from the participating atom in the water molecule to the intermolecular bcp and is a measure of ion penetration. \( \rho \) is the value of the density and \( \nabla^2 \rho \) the Laplacian at the bcp. In the fourth column, we report the ratio of the largest negative curvature (attraction) to the curvature along the bond path (repulsion). This is typically interpreted as a measure of the local balance between potential and kinetic energies and is usually much less than unity for closed-shell, primarily electrostatic interactions. Additionally, we report the total energy density per unit charge and the ratio of the potential to kinetic energy at the critical point. Hydrogen bonds in the AIM theory display largely electrostatic character in the Laplacian but somewhat chemical character in the energy terms. It is the uncharacteristically high amount of charge and enormously negative total energy density which may fuel speculation that \( F^-/water \) interactions possess some chemical character. All values are reported in atomic units.

| Ion   | \( r(H_2O-bcp) \) | \( \rho \)  | \( \nabla^2 \rho \) | \( |\lambda_1|/\lambda_3 \) | \( H/\rho \) | \( |V|/G \) |
|-------|-------------------|------------|----------------|-----------------|------------|-----------|
| Li\(^+\) | 2.16              | 0.0367     | 0.2984         | 0.17            | 0.3229     | 0.81      |
| Na\(^+\) | 2.31              | 0.0249     | 0.1865         | 0.14            | 0.3212     | 0.79      |
| K\(^+\)  | 2.40              | 0.0213     | 0.1137         | 0.15            | 0.2101     | 0.81      |
| F\(^-\)  | 0.79              | 0.0882     | 0.1351         | 0.38            | -0.4848    | 1.56      |
| Cl\(^-\) | 1.32              | 0.0291     | 0.0589         | 0.28            | -0.1456    | 1.22      |
| Br\(^-\) | 1.43              | 0.0238     | 0.0472         | 0.26            | -0.1042    | 1.17      |

However, an AIM analysis of a series of halogen and hydrogen bonded complexes together with the reduced variational space (RVS) energy decomposition analysis (at the SCF level) revealed that the decrease in the total energy density and increasing electron density at the intermolecular bcp are principally due to a marked increase in attractive electrostatic interactions\cite{9}. In the RVS theory, charge transfer is considered a component of the chemical or covalent energy terms.

My discussion on charge transfer (or delocalization) in ion/water dimers focuses on the data presented in Table 3.7. The sign convention used here is that charge depletion is given a negative value and accumulation a positive one. The data indicated that there was a loss of charge for anions and an accumulation in the virtual orbitals of each of the cations. In the anion/water dimer, charge is also lost by the ion-bound hydrogen (H\(_b\)) and populates a \( p\)-state centered on the oxygen and also a diffuse Rydberg state bridging the oxygen and trailing hydrogen (resembling \( \frac{1}{2} \) the lowest unoccupied molecular orbital, LUMO).

These features are illustrated in the electron density difference maps of Figures 3.2 and 3.3. The oxygen amasses charge also in the cation/water dimers, drawing equal quantities of charge from the dangling hydrogens. About one half of the charge exchanged in the weakest anion/water dimers (Cl\(^-\) and Br\(^-\)) is exchanged by even the most strongly bound cation/water dimer (Li\(^+\)). These results compare well to the values obtained by Soniat et al. for Na\(^+\), K\(^+\), and Cl\(^-\)\cite{389} and we agree qualitatively with trends which conclude charge transfer increases in the order of Br\(^-\) < Cl\(^-\) < F\(^-\) originating from studies of CT in halide/water dimers with Löwdin and natural bond orbital (NBO) charges\cite{27, 219, 243, 399}. 
Figure 3.2: An electron density difference map of the $\pm 0.0022 \, e$ isosurfaces of one of the F$^-(\text{H}_2\text{O})_6$ clusters.
Table 3.7: AIM-derived charge transfer values (in me, millielectrons) for dimers computed with AIMAll. Negative values reflect charge loss, positive values reflect charge accumulation.

<table>
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<tr>
<th></th>
<th>δq(A)</th>
<th></th>
<th>δq(A)</th>
<th></th>
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<td>Na⁺</td>
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<td>O</td>
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<td>H₁</td>
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<td>H₂</td>
<td>-56.8</td>
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<td>H₂</td>
</tr>
<tr>
<td>F⁻</td>
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<td>O</td>
<td>191.3</td>
<td>Cl⁻</td>
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<tr>
<td></td>
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<td>Hₐ</td>
<td>-77.3</td>
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<tr>
<td></td>
<td>H₇</td>
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<td></td>
<td></td>
<td>H₇</td>
<td>29.3</td>
<td></td>
<td>H₇</td>
</tr>
</tbody>
</table>

Figure 3.3: An electron density difference contour map of F⁻(H₂O)₂ clusters showing regions of charge accumulation and depletion.
Moving beyond dimers into clusters, I’ll be referring to Table 3.8. For trimers, there is a notable increase in the amount of charge redistribution throughout the system which falls well short of a doubling of the result for the dimers. Competition between ion/water versus water/water is not seen until reaching a coordination number of \( n = 3 \). We also point out that the trend observed for the anion/water dimers, namely that charge transferred increases in the order \( \text{Br}^- < \text{Cl}^- < \text{F}^- \), is in danger of reversing as the effect for \( \text{F}^- \) appears to saturate rather quickly. This transition in anions is consistent with a study by Patel et al. in which they’ve found that for \( \text{F}^- / \text{water} \) clusters up to \( n = 3 \), the ion polarizability was somewhat enhanced relative to the gas phase \( \text{F}^- \) polarizability\[34\]. Beyond a cluster size of \( n = 3 \), the ion polarizability was reduced relative to the gas phase and as a consequence the observed ion-to-solvent transfer is quickly exhausted. Figure 3.2 shows that the water distorts the electron density of the ion towards poles directed at neighboring hydrogens. These polarized electron-rich clouds are simply more willing to relinquish their charge from softer ions like \( \text{Cl}^- \) and \( \text{Br}^- \) than harder ones like \( \text{F}^- \); and we note that all anions eventually see a decrease in polarizability in the condensed phase limit – a conclusion also reached by Masia\[144, 274\] and other researchers, see page 4 of Ref. \[353\] and the related discussion for more references.

In clusters with more than 3 waters we see very little in the way of change from cation/water systems, maintaining a steady 60 to 80 millielectron draw from surrounding waters with a general preference for arrangements producing a high symmetry and maximizing the number of direct ion/water contacts. The same trend we observed for dimers is preserved in the micro-solvated states with \( \text{Li}^+ \) drawing more charge than \( \text{Na}^+ \) and \( \text{K}^+ \). However, the anion trend has reversed with \( \text{Br}^- \) shedding up to about 170 millielectrons while \( \text{F}^- \) has lost around 150 millielectrons at most. These figures are similar to the results from Zhao, Rogers, and Beck where they found \( \text{Cl}^- \) to lose about 200 millielectrons\[454\]. Their calculations included the dipole and quadrupole field of waters beyond the first hydration shell suggesting there may be a small amount of additional charge displaced as we approach the bulk limit. Finally, we note that the amounts of charge exchanged between the ions and water, just as we saw for the energies, did not increase linearly with each additional water – in fact, most of these figures barely doubled with the addition of 5 more waters!

An alternative approach to quantifying the amount of charge transfer was described by Belpassi and coworkers\[43, 42\] and is particularly suited to examining charge displacement in or between linear or planar molecules. By slicing a cube file of the electron density difference upon complexation into many planes perpendicular to the bond between the ion and nearest water atom and then integrating, I can examine the amount of charge transferred from one side of the plane to the other (direction depends on direction of integration, \( -z \) to \( +z \) or \( +z \) to \( -z \)). Belpassi has argued that between two interacting molecules there will be a minimum or a maximum in the curve which is the estimate of total charge transferred. Figure 3.4 shows the charge displacement curve for all the ion/water dimers. The integration proceeds from \( -x \) to \( +x \) and so a positive value like we obtain for cations indicates charge flow from the left side to the right (ion to water) and vice versa for negative values like that obtained for anions. The hills and valleys in this curve coincide with the
Cluster | $\delta q(X^\pm)$ | Cluster | $\delta q(X^\pm)$ | Cluster | $\delta q(X^\pm)$
--- | --- | --- | --- | --- | ---
$X^\pm(H_2O)_{1-4}$ | $X^\pm(H_2O)_{4-5}$ | $X^\pm(H_2O)_{5-6}$
Li$^+$ C$_2$v | 32.4 | F$^-$ C$_1$ | -138.7 | Br$^-$ R4A1 | -163.1
Na$^+$ C$_2$v | 27.4 | F$^-$ C$_{1''}$ | -139.7 | Br$^-$ R4A | -159.4
K$^+$ C$_2$v | 21.9 | F$^-$ C$_4$ | -134.6 | Br$^-$ R43f | -157.8
F$^-$ C$_1$ | -83.4 | F$^-$ 3+1(C$_s$) | -139.2 | Br$^-$ R3AA' | -162.3
Cl$^-$ C$_1$ | -62.1 | Cl$^-$ C$_{1''}$ | -144.6 | Br$^-$ R5 | -149.4
Br$^-$ C$_1$ | -62.5 | Cl$^-$ C$_{1'''}$ | -143.0 | Br$^-$ R4f3 | -156.1
Li$^+$ D$_{2d}$ | 58.9 | Br$^-$ C$_1'$ | -150.9 | Li$^+$ D$_{2d}$ | 82.0
Na$^+$ D$_{2d}$ | 49.7 | Br$^-$ C$_{1''''}$ | -149.1 | Li$^+$ 4+2(C$_s$) | 82.5
K$^+$ D$_{2d}$ | 37.6 | Br$^-$ C$_4$ | -139.4 | Li$^+$ C$_2$ | 82.2
F$^-$ C$_2$ | -117.4 | Na$^+$ D$_{2d}$ | 77.2
Cl$^-$ C$_1$ | -100.0 | Li$^+$ C$_2$ | 74.7 | Na$^+$ 4+2(C$_s$) | 76.9
Br$^-$ C$_1$ | -101.5 | Li$^+$ 4+1(C$_2$) | 82.4 | Na$^+$ C$_2$ | 74.7
Li$^+$ D$_3$ | 74.4 | Na$^+$ 4+1(C$_2$) | 76.9 | K$^+$ D$_{2d}$ | 63.9
Li$^+$ 2+1(C$_{2v}$) | 58.6 | K$^+$ 4+1(C$_2$) | 62.6 | K$^+$ C$_2$ | 61.5
Na$^+$ D$_3$ | 65.4 | K$^+$ C$_2$ | 59.0 | F$^-$ L3L3 | -150.9
Na$^+$ 2+1(C$_{2v}$) | 50.3 | K$^+$ 4+1(C$_1$) | 57.2 | F$^-$ R3ADA | -145.7
K$^+$ D$_3$ | 51.5 | F$^-$ R3L2 | -140.5 | F$^-$ R4AA | -142.1
K$^+$ 2+1(C$_{2v}$) | 42.3 | F$^-$ R4L | -140.9 | F$^-$ Bf' | -140.0
F$^-$ C$_3$ | -128.4 | F$^-$ L3DL | -141.3 | F$^-$ R3AAL | -141.6
F$^-$ 2+1(C$_s$) | -124.0 | F$^-$ R4A | -138.1 | F$^-$ Bf' | -138.9
Cl$^-$ C$_3$ | -121.1 | F$^-$ R43f | -137.4 | Cl$^-$ R4AA | -163.8
Cl$^-$ 2+1(C$_s$) | -119.5 | F$^-$ R4L' | -141.4 | Cl$^-$ Bf' | -159.3
Br$^-$ C$_3$ | -124.2 | F$^-$ R3AA | -140.0 | Cl$^-$ Bf | -158.7
Br$^-$ 2+1(C$_s$) | -123.1 | Cl$^-$ R4A1 | -155.5 | Cl$^-$ Bd | -150.6
Li$^+$ S$_4$ | 81.2 | Cl$^-$ R43f | -150.0 | Br$^-$ R4AA | -174.2
Li$^+$ 3+1(C$_2$) | 75.8 | Cl$^-$ R3AA' | -154.5 | Br$^-$ Bf' | -168.8
Na$^+$ S$_4$ | 76.0 | Cl$^-$ R5 | -144.0 | Br$^-$ Bf | -168.2
Na$^+$ 3+1(C$_2$) | 66.6 | Cl$^-$ R3AA | -149.4 | Br$^-$ Bd | -158.2
K$^+$ S$_4$ | 61.4 | Cl$^-$ R4f3 | -149.6 | Br$^-$ Bd' | -159.3
K$^+$ 3+1(C$_2$) | 53.5

Table 3.8: Charge transfer to ($\delta q > 0$) or from ($\delta q < 0$) the ion measured in millielectrons for each complex examined. Note the increases are not linear when additional waters are considered and the $\delta q$ to cations and from F$^-$ has already nearly plateaued when considering just the first hydration shell.
Figure 3.4: Charge displacement curves following Refs. [43, 42]. The curve indicates both direction and magnitude of charge passing through a moving plane perpendicular to the ion-water bond and so offers a unique perspective of charge redistribution in the dimers. Refs. [43, 42] and citations therein argue that an unambiguous estimation of the charge transferred can be evaluated at the minimum/maximum appearing in the curve between the ion and the nearest water atom positions. The integration proceeds from left to right and so a positive value like we obtain for cations indicates charge flows from the right side to the left (water to ion) and vice versa for negative values like those obtained for anions. All ions are located at x=0. Red and white circles not at the x=0 coordinate signify oxygen and hydrogen positions, respectively. Ions are colored magenta (F\(^-\)), green (Cl\(^-\)), red (Br\(^-\)), larger red (Li\(^+\)), blue (Na\(^+\)), and magenta (K\(^+\)).

pattern of density accumulation or depletion like what is seen in Figure 3.3 and provide an insightful new perspective. Charge transfer values from this plot are as follows: 10.1 (Li\(^+\)), 10.1 (Na\(^+\)), 27.3 (K\(^+\)), 72.6 (F\(^-\)), 49.3 (Cl\(^-\)), and 46.9 (Br\(^-\)) millielectrons which reverses the cation trend from that observed with AIM-derived partial charges but preserves the trend for anions.

3.3.2 Energy stabilization due to charge transfer

Tables 3.9 and 3.10 highlight the significant enhancement in basis set independence that regularization affords. Charge transfer energies are reported for basis sets ranging from a double-\(\zeta\) through to
Table 3.9: Charge transfer energies from regularized SAPT computed (by column) with the (aug-cc-pVDZ, (aug-)pc-1, (aug-)cc-pVTZ, (aug-)pc-2, def2-TZVPP(D), and def2-QZVPP(D) basis sets.
All values expressed in kcal/mol. Some entries are left blank due to limitations with the software.

<table>
<thead>
<tr>
<th>Cluster</th>
<th>$\text{CT}^{[2]}_{\text{Reg.aDZ}}$</th>
<th>$\text{CT}^{[2]}_{\text{Reg.pc1}}$</th>
<th>$\text{CT}^{[2]}_{\text{Reg.aTZ}}$</th>
<th>$\text{CT}^{[2]}_{\text{Reg.pc2}}$</th>
<th>$\text{CT}^{[2]}_{\text{Reg.dTZ}}$</th>
<th>$\text{CT}^{[2]}_{\text{Reg.dQZ}}$</th>
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<td>1.28</td>
<td>1.58</td>
<td>1.53</td>
<td>1.59</td>
<td>1.58</td>
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<tr>
<td>Na$^+$ C$_{2v}$</td>
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<td>0.33</td>
<td>0.51</td>
<td>0.49</td>
<td>0.46</td>
<td>0.50</td>
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<tr>
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<td>0.04</td>
<td>0.06</td>
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<td>0.06</td>
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<td>-0.62</td>
<td>-0.62</td>
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<td>0.65</td>
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<td>0.14</td>
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<tr>
<td>Br$^-$ C$_1$</td>
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<td>-0.88</td>
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Table 3.9: Charge transfer energies from regularized SAPT computed (by column) with the (aug-cc-pVDZ, (aug-)pc-1, (aug-)cc-pVTZ, (aug-)pc-2, def2-TZVPP(D), and def2-QZVPP(D) basis sets. All values expressed in kcal/mol. Some entries are left blank due to limitations with the software.
Table 3.10: Charge transfer energies from regularized SAPT computed (by column) with the (aug-)cc-pVQZ, (aug-)pc-1, (aug-)cc-pVTZ, (aug-)pc-2, def2-TZVPP(D), and def2-QZVPP(D) basis sets. All values expressed in kcal/mol. Some entries are left blank due to limitations with the software.

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<td>-1.09</td>
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<td>$\text{Br}^- C_3$</td>
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which the $\delta_{\text{HF}}^{(2)}(\text{DCBS})$ term is curiously positive. The same was observed here as well[239].

Anions consistently produced more reasonable values and in the proper ordering (from least to most favorable): $\text{Br}^- < \text{Cl}^- < \text{F}^-$. Similar to what I observed for the atoms-in-molecules charge populations for the anions, the energies stagnated beyond $n > 3$ for $\text{F}^-$ – however, without falling behind either $\text{Cl}^-$ or $\text{Br}^-$. $\text{CT}_{\text{SM09}}^{(2)}$ energies hovered around twice the energies predicted by the regularized method, especially for larger clusters. This is an interesting result as the saturation in the amount of charge transfer observed particularly for $\text{F}^-$ might have been expected to lead to better agreement between the SM09 and reg-SAPT methods.

These results are also visualized in Figure 3.5. Though here, we are able to decompose the CT energies even further into donor $\rightarrow$ acceptor and donor $\leftarrow$ acceptor contributions (here, donor and acceptor refer to electrons, not protons)[199]. The $\text{CT}^{(2)}(X\leftarrow W)$ values settled near 0. kcal/mol for the entire range of cation/water clusters considered. This is to be expected however, as $\text{CT}^{(2)}(X\leftarrow W)$ represents the charge transfer energy change from the water polarizing the ion. This leaves only the water-to-ion transfer which appeared to somewhat destabilize these complexes as discussed previously. For the anion/water clusters, inductive effects due to charge transfer appeared beneficial in both directions, though with the larger portion originating from the ion $\leftarrow$ water term. As discussed previously, it is apparent that $\text{CT}^{(2)}(X\rightarrow W)$ has saturated for $\text{F}^-$ already by $n = 3$, while both $\text{Cl}^-$ and $\text{Br}^-$ appeared to not yet converge.

So why the positive charge transfer energies? Table 3.11 tells us that for the $\text{Li}^+/\text{water}$ dimer,
Figure 3.5: Ionic and solvent contributions to total charge transfer energies computed with the aug-cc-pVDZ basis set. $\text{CT}^{(2)}(X \leftarrow W)$ is the charge transfer energy change for the ion induced by the permanent multipoles of the surrounding solvent. Solvent effects due to the neighboring ion are taken as the difference in $\text{CT}^{(2)}(X \leftarrow W) + \text{CT}^{(2)}(W \leftarrow X)$ and the included ionic contribution. A solid line with points at each coordination number ($n$) traces the mean values across all structures and the shaded region is bounded by the maximum and minimum for a given cluster size. These extrema are identical to those found in Tables 2-5. The procession of the ions from top left to bottom right is as follows: Li$^+$, Na$^+$, K$^+$, F$^-$, Cl$^-$, and Br$^-$. 
Table 3.11: Illustration of the effect of regularization in SAPT induction energy components for \( \text{Li}^+ (\text{H}_2\text{O}) \) (units of kcal mol\(^{-1}\)). The exchange part of the interaction is more significantly impacted by the screening potential than is the attractive part – since CT\(^{(2)}\) is described nearly exclusively by the polarization of the water molecule, it is seen that the positive CT energy results from the omission of the stresses leading to water-to-ion charge delocalization. Here \( E^{(2)}_{\text{tot},r} \) is the total induction energy, which is divided in other columns to the attractive induction term and the repulsive exchange coupled term. \( E^{(2)}_{\text{tot},r} \) (Reg) and CT\(^{(2)}\) are similarly decomposed. The full CT energy and its components are merely the differences from the two columns preceding it. The table itself was slightly rescaled to fit the dimensions of the page.

The presence of a regularizing potential even in the limit of large \( \eta \) more aggressively screens the repulsive contributions than it does the attractive ones. This leads to the systematic overestimation of the repulsive contribution, which may make this method more promising as an upper bound of the polarization energy instead of an estimate of the charge transfer energy.

There is much interest in correlating charge transfer energies in anion/water clusters with the degree of solvation asymmetry in the first hydration shell. While it is known that the softer anions (e.g., \( \text{Cl}^- \), \( \text{Br}^- \), and \( \text{I}^- \)) tend to prefer a highly asymmetric first solvation shell, the reduction in polarizability observed in the transition from gas phase to condensed phase for each ion\([274, 34]\) (presumably from charge transfer to solvent making the ion appear ‘harder’) has been seen to decrease solvation anisotropy when modeled with static multipoles using the AMOEBA polarizable force field\([454]\) and the polarizable and fluctuating charge model of Soniat et al.\([389]\). It would be quite useful to observe that semi-/internal bound clusters are selected for with enhanced CT energies relative to the surface-bound configurations, see the discussion on this in Ref. \([243]\). Assuming the anion figures to be at least qualitatively accurate, the internal-bound configurations for \( \text{F}^- (\text{H}_2\text{O})_6 \) (\( \text{L3L3}, \text{R3AAL}, \) and \( \text{R3ADA} \)) were selected over the surface bound states by both energy partitioning schemes. Perhaps the constrained density functional theory approach of Lande et al.\([422]\) which showed greater consistency with cation/water dimers\([239]\) than the SM09 method or the absolutely localized molecular orbital approach of Khaliullin et al.\([217]\) can provide more solid answers using even larger clusters since it is based on DFT.

This is all not to say that regularized SAPT isn’t useful, rather it may be more fitting to
interpret these energies in a similar manner to the polarized orthogonal local molecular orbitals (polMO) approach of Azar et al. [21]. Where the polMO method has been shown to underestimate polarization effects, giving a lower bound to the polarization energy, reg-SAPT may reasonably be taken as an upper bound for the polarization energy and lower bound for the charge transfer energy. My discussion on Table 3.11 clearly demonstrated that the reg-SAPT method overestimated the relaxation in the induction-exchange coupled term even in the limit of a vanishing conditioning potential. This effect leads to the positive CT energies observed in both the SM09 and reg-SAPT methods. It vanished with the CT energy itself in the SM09 method, but was retained in the reg-SAPT method which greatly improved the stability of the result with increasing basis set size.

3.4 Conclusions

My efforts show that there is an appreciable degree of ion specificity in the interactions between ions and associated solvent molecules. These interactions are principally electrostatic in character. This is not to discount the role of induction (which is part polarization and part charge transfer) and dispersion which have been found by Rogers et al. [454, 357], Masia [274], Duignan et al. [292, 113, 112, 115, 114], and others [89, 389, 390, 446, 387, 388] to be of importance as well. The current study suggests up to \( \approx \frac{1}{3} \) of the attractive contributions to the SAPT interaction energy arise from these non-electrostatic forces. Despite the high level of complexity that has been found to be involved in ionic interactions, recent studies have concluded that the effect of monovalent ions on water is surprisingly localized [38, 294]. I find evidence as well, with dispersion, charge transfer, and polarization showing signs of (near) saturation just within the first solvation shell. This invites the possibility that more distant interactions can be treated with a coarse-grained or even continuum solvation model. Di- and trivalent ions present yet another significant obstacle as the ion effects on the hydrogen bonding network in water have been found to be more far-reaching [294, 106, 95].

I have also conducted an in-depth analysis of small ion/water clusters in an effort to better understand what role(s) partial charge transfer between the ion and waters is expected to play. In particular, I was motivated by the recent attempts to develop fluctuating charge models which incorporate solute/solvent charge transfer and the suspected role this interaction may play in reshaping/restructuring the solvation shell around ions. This work explored the use of two approximate methods developed within the symmetry adapted perturbation theory to disentangle the charge transfer and polarization energies from the total induction energy. I found the SM09 method based on differences in the induction term computed in the dimer- and monomer-centered basis sets tended to overestimate the charge transfer energy relative to a method employing the use of regularized nuclear potentials to filter out charge transfer (which takes the place of the monomer-centered calculation). Charge transfer energies in anion/water clusters were found to be negative as expected, while cation/water clusters gave either positive or negative but very small energies. I showed that this effect was due to the regularizing potential overestimating the relaxation of the exchange-induction
term even in the limit of the full Coulomb operator. I hypothesized that the regularized SAPT method may be better suited to providing an upper bound for the polarization energy rather than an estimate of the charge transfer energy. However, assuming the relative values for a given ion to be qualitatively accurate, I observed that both partitioning schemes predicted that $F^-$ would prefer internal-bound states relative to surface binding in the $n = 6$ clusters. This behavior is consistent with the prediction of previous simulations which saw reduced solvation anisotropy when including charge transfer explicitly or mimicking it by reducing the ion polarizability. A more thorough study would be needed to correlate the two with greater certainty and we discuss alternative approaches which may, in time, help us address this question.
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4 SELF- AND ION-INDUCED POLARIZATION IN ETHYLENE AND PROPYLENE CARBONATE

4.1 Preface

Disclaimer: this chapter is subject to additional changes as it is not yet published.

In this section I explore what effect an accurate treatment of polarizability has in simulations of liquid ethylene and propylene carbonate versus the gas phase. Previous solvation thermodynamic work by Arslanargin et al. hypothesized that their results could be improved with a more physical treatment of polarization[12]. This is because in a tetrahedrally coordinated complex with either of these solvents, each of the dipoles points in towards the ion, producing an effective repulsive interaction that the classical force field failed to capture. I begin with an examination of self-polarization which is an important effect in water, for example, where the dipole moment increases from $\sim 1.85$ D in the gas phase to somewhere between 2.3 – 2.9 D. I find in both of these solvents the average dipole moment increase about 34% over the gas phase monomer sampled at the same temperature. Following this, I address the change of the dipole moment in molecules solvating a single Li$^+$. Unlike water where the dipole moment tends back towards the bulk value with increasing coordination number, the dipole moments of ethylene and propylene carbonate increase to just over 50% greater than the gas phase average. The classical model using charges that reproduce bulk properties of each of these solvents underestimates the polarization effect, while re-fitted parameters to the ion/solvent dimer lead to an overestimation of the effect. I also examine interaction energies computed with the density functional flavor of SAPT between the \textit{ab initio} and classically generated trajectories. These results indicate that Li$^+$ likely binds ethylene carbonate more tightly than propylene carbonate, though there remains no consensus on this in the literature. The coordination number is determined from the integral of the first peak in the ion/solvent radial distribution function. Comparison to the literature indicates that my predicted coordination number of 3.8 in ethylene carbonate and 3.98 in propylene carbonate are consistent with previous simulation results. However, these pretty uniformly underestimate the figure versus experimental methods. Since this is still a work in progress, some additional calculations and analysis steps are briefly discussed.

This section addresses three of the questions I posed in Chapter 1.7,

- What sort of interactions?
- How strong are they relative to one another?
- Do we need electronic structure theory to solve everything?


4.2 Computational methods

4.2.1 Initial configurations

Separate cubic boxes of 32 ethylene and propylene carbonate molecules were constructed and equilibrated in the NPT ensemble using Gromacs 4.6.7\cite{413}. The molecules were modeled with the general Amber force field (GAFF). Simulations were propagated for 4 ns with configurations saved every 10 steps to monitor solvent densities. Each simulation was performed at 310 K and 1 atm pressure. Velocity-rescaling and the Berendsen barostat were used to control temperature and pressure fluctuations, respectively. Final configurations were used as starting points for optimization at the density functional level.

Single molecule and the previously described 32-molecule cells were optimized with CP2K 2.6.1\cite{181} at the PBE/DZVP-MOLOPT-SR-GTH level\cite{136, 318, 414}. The single molecule cells were created by simply deleting all but one ethylene carbonate or propylene carbonate, while retaining the initial box dimensions. For optimization, force and displacement tolerances were set to 1e-6 au (single molecule) or 1e-4 au (32-molecules). The coordinates and wave function were retained to reduce the cost of the first dynamics step.

4.2.2 Dynamics and dipole moment calculation

Born-Oppenheimer dynamics were conducted with the PBE/DZVP-MOLOPT-SR-GTH potential energy surface. Appropriate PBE-optimized GTH pseudopotentials were used to model core electrons throughout the study. The cutoff was set to 500 Ry with a relative cutoff of 60 Ry which minimized the error relative to a calculation with a 2,000 Ry cutoff and 200 Ry relative cutoff. These values were also used for the single molecule calculations for consistency. All systems were thermally equilibrated in the NVT ensemble with the velocity rescaling thermostat using a time constant of 10 fs. To prevent freezing, temperatures of 450 K and 350 K were used for and PC, respectively. A 1.0 fs timestep was permissible by mutating hydrogens to tritium as was done in Ref. \cite{247}. Constant temperature dynamics were carried out for 12 ps to allow for thermal equilibration saving restart files every 20 steps. Four configurations were selected at regular intervals from each simulation following no less than 8 ps of equilibration. Production dynamics were performed in the NVT ensemble for 40 ps, with a change to the Nosé-Hoover chain thermostat\cite{271} with an 80 fs time constant. Positions and full restart files were written every 20 steps during the production simulations. Restart files were altered to compute Wannier centers using the 2x2 Jacobi transformation method with a tolerance of 1e-5 in thousands of separate single-point calculations. The dipole moment of individual molecules was computed as a sum over nuclear and electronic contributions given by,

\[ \mu = \mu_n + \mu_e = \sum_i Z_i \vec{r}_i + \sum_j -2e \vec{r}_j^W. \]  

(4.1)

In this expression, \( Z_i \) is the nuclear charge less any electrons replaced by the pseudopotential,
\( \vec{r}_i \), denotes the position of each nucleus, while \( \vec{r}_W^j \) is the position of the Wannier centers. Each Wannier function counts for -2e charge. To test variation in the dipole moment with basis set, energy calculations using DZVP-MOLOPT-GTH, TZVP-MOLOPT-GTH, TZV2P-MOLOPT-GTH, and TZV2PX-MOLOPT-GTH basis sets were performed on configurations saved from the double-\( \zeta \) trajectory. All calculations were performed on machines hosted by the Ohio Supercomputer Center\[295\]. A quarter of the condensed phase configurations were run with the larger basis sets; the configurations that were sampled were taken at regular intervals from the parent trajectory. All larger basis set calculations used cutoffs optimized as described above. The calculations were carried out using the TRAVIS analyzer\[61\].

A Li\(^+\) ion was introduced into the 32-molecule cells by replacing one of the carbonate molecules. A representative snapshot of the resulting cell for Li\(^+\)EC\(_{31}\) is given in Figure 4.1. An analogous cell was created for Li\(^+\)PC\(_{31}\). Dynamics were handled just as above as well but with a total charge of +1.

### 4.2.3 DFT-D3 symmetry adapted perturbation theory

Clusters of the nearest 4 molecules centered around the Li\(^+\) ion were extracted from both classical and density functional based trajectories of the slightly larger Li\(^+\)EC\(_{31}\) and Li\(^+\)PC\(_{31}\) systems. These coordinates were then used as inputs for a symmetry adapted perturbation theory (SAPT) decomposition of the ion(solvent) interaction energies at the SAPT(KS)-D3 level. D3 refers to the 3\(^{rd}\) generation dispersion potential of Herbert et al.[238]. We picked 600 configurations initially, excluding only 2 outliers from the Li\(^+\)EC\(_{31}\) DFT-generated trajectory and 8 each from the Li\(^+\)PC\(_{31}\) trajectories for which had produced nonsensical results. We have chosen to retain the the PBE functional in this analysis but slightly increased the accuracy of the basis set to jun-cc-pVDZ and resolution of the identity (RI) approximation optimized jun-cc-pVDZ-RI. Li\(^+\) uses the cc-pVDZ equivalent in both of these basis sets. Benchmarking shows SAPT(KS)-D3 and the related XSAPT method performance is significantly enhanced through the introduction of diffuse functions. The version of SAPT(KS) implemented in QChem 4.4[226, 376] takes advantage of a long-range correction added to the Coulomb potential at a range controlled via a splitting parameter, \( \omega \). We optimized the parameters as reported by Herbert et al. with the above basis sets and method[238]. A value of 8000 bohr\(^{-1}\) was required for Li\(^+\), 540 bohr\(^{-1}\) for EC, and 365 bohr\(^{-1}\) for PC.

### 4.3 Results

Table 4.1 lists the average dipole moment of the carbonate molecules from the gas and condensed phase simulations. Error bars were calculated with the block-averaging method. A single standard deviation is also included to assess the spread of width the distribution of dipoles. Gas phase dipoles for EC exhibited small fluctuations about a mean of 5.44 D. The change from H to CH\(_3\) in PC produced a small increase in the average dipole moment to 5.65 D. In the condensed phase,
Figure 4.1: Snapshot of $(\text{Li}^+ \text{EC}_{31})$. 
these dipoles rose by 34% each! The bulk EC value averaged to 7.30 D and the PC value to 7.56 D. Each of these solvents exhibited fluctuations in excess of 0.5 D with EC producing slightly larger fluctuations, possibly owing to the more mobile H group and potential steric clashes with neighboring molecules due to the larger CH$_3$ group.

Basis set dependence of these quantities is also examined and I found virtually no change in the condensed phase dipoles. There was some fluctuation in the gas phase dipoles just beyond my error estimate, but these settle back down to those predicted by the smallest basis set (which I used to generate the trajectories) for the largest basis considered.

Solvation structure around the Li$^+$ ion is shown in Figure 4.2. Both solvents give peaks in the radial distribution function at a distance of 200 pm. The coordination number resulting from integration to the first minimum is about 4 for each solvent. The actual values at the minima are 3.88 at 287.5 pm for EC and 3.98 at 277.5 pm for PC. The cutoff is very clean for PC and less so for EC where the g(r) smoothly increases through to the second, much broader shell which peaks in both solvents just shy of 800 pm.

The distribution of dipole moments of EC and PC molecules as a function of distance from the Li$^+$ are shown in Figures 4.3 and 4.4. Horizontal lines in these plots point out the gas phase and condensed phase averages from above (from the DZVP-MOLOPT-SR-GTH data). These dipoles are also computed with the short range basis. The x-axis reflects the distance between the ion and carbonyl oxygen. The density distribution of the EC molecules maxes near the ion at a dipole moment of $\sim$8.4–8.5 D and in PC at $\sim$8.7–8.8 D. Each of these figures is slightly greater than 50% larger than those measured in the gas phase and approaching 20% larger than those measured in neat solutions. The wide spread of the dipole moments in the first shell is indicative of the fairly labile nature of the solvation shell for EC especially. There’s definitely more accumulation between the first and second solvation shells for this system which corresponds to the solvent in reverse orientation with the carbonyl directed away from the ion. This appeared to occur less frequently in PC, probably because PC is modeled at a lower temperature.

SAPT(KS)-D3 results are shown in Figures B.1 through B.6. The more rigid nature of the classical force field universally produced less broad distributions in the energy terms than the density-functional configurations. The greater bond length between the ion and solvent in the GAFF model likewise produced smaller attractive energies, though the disparity is greater for PC configurations than EC. This also produced less overlap between the atoms and so gave smaller exchange and exchange-coupled energy contributions. For EC, the greatest differences were reflected in the induction energies. In PC, both 1$^{st}$ and 2$^{nd}$ order energy contributions failed to overlap significantly. In the total intermolecular energies (which include dispersion and the infinite order corrections), there is greater overlap between the classical and $ab$ initio models with EC and very little for PC. These data suggest that the force field relies on error cancellation (the smaller exchange energies) to make up for their underestimation of attractive components due to the longer bond lengths.
<table>
<thead>
<tr>
<th>System</th>
<th>(µ)</th>
<th>σ</th>
<th>(µblk)/(µgp)</th>
</tr>
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<tr>
<td><strong>DZVP-MOLOPT-SR-GTH</strong></td>
<td></td>
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<td></td>
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<tr>
<td>EC&lt;sub&gt;1, gp&lt;/sub&gt;</td>
<td>5.44 ± 0.05</td>
<td>0.28</td>
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<td>5.65 ± 0.07</td>
<td>0.29</td>
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<td>7.56 ± 0.24</td>
<td>0.52</td>
<td>1.34</td>
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<td><strong>DZVP-MOLOPT-GTH</strong></td>
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<tr>
<td>EC&lt;sub&gt;1, gp&lt;/sub&gt;</td>
<td>5.51 ± 0.05</td>
<td>0.29</td>
<td></td>
</tr>
<tr>
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<td>0.56</td>
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<tr>
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<td>5.71 ± 0.06</td>
<td>0.30</td>
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<tr>
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<td>7.55 ± 0.14</td>
<td>0.53</td>
<td>1.32</td>
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<tr>
<td><strong>TZVP-MOLOPT-GTH</strong></td>
<td></td>
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</tr>
<tr>
<td>EC&lt;sub&gt;1, gp&lt;/sub&gt;</td>
<td>5.50 ± 0.05</td>
<td>0.29</td>
<td></td>
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<tr>
<td>EC&lt;sub&gt;32, blk&lt;/sub&gt;</td>
<td>7.34 ± 0.11</td>
<td>0.57</td>
<td>1.33</td>
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<tr>
<td>PC&lt;sub&gt;1, gp&lt;/sub&gt;</td>
<td>5.70 ± 0.06</td>
<td>0.30</td>
<td></td>
</tr>
<tr>
<td>PC&lt;sub&gt;32, blk&lt;/sub&gt;</td>
<td>7.59 ± 0.15</td>
<td>0.54</td>
<td>1.33</td>
</tr>
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<td><strong>TZV2P-MOLOPT-GTH</strong></td>
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</tr>
<tr>
<td>EC&lt;sub&gt;1, gp&lt;/sub&gt;</td>
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<tr>
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<td>7.58 ± 0.15</td>
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<td>1.34</td>
</tr>
</tbody>
</table>

Table 4.1: Dipole moments (in Debye) ± standard error in the mean using the block averaging method. A single sample standard deviation is provided to get a sense of the distribution of molecular dipoles about the mean. The third column is the ratio of the bulk dipole moment to the gas phase measurement.
Figure 4.2: Li$^+$ and carbonyl oxygen radial distribution functions for ethylene carbonate (top) and propylene carbonate (bottom). The distribution function is depicted with a solid line while the number integral is shown as a dashed line. A dashed horizontal line provides an estimate of the coordination number in the first solvation shell which are both very close to $n = 4$. 
Figure 4.3: Combined density distribution of ethylene carbonate dipole moment as a function of radial distance from the ion. Colors map to raw counts delineated in the color bar. Solid, horizontal lines highlight average gas and condensed phase dipole moments from Table 4.1. Images aren’t centered because I will be adding an image to the upper right corner.
Figure 4.4: Combined density distribution of propylene carbonate dipole moment as a function of radial distance from the ion. Colors map to raw counts delineated in the color bar. Solid, horizontal lines highlight average gas and condensed phase dipole moments from Table 4.1. Image not centered, same reason.
4.4 Discussion

The dipole moments I report are a fair bit larger than those often used in literature. For pure EC and PC, 4.61 D and 4.81 D are most often cited\[12, 322\]. These are each about 85% of the values reported in Table 4.1. However, the larger dipole moments found here match those from the theoretical predictions of Hammer et al.[153], Luber[256] (including the condensed phase \(\langle \mu \rangle\) and standard deviation for EC), Masia et al.[275], Park et al.[305], Stark-effect measurements by Alonso et al.[8], and back-calculating the dipole from capacitance measurements by Chernyak[84]. Only the works of Park et al. and Chernyak made an attempt at the dipole moment of PC. Chernyak’s estimate of the EC dipole is 4.81 D but 5.36 D for PC, closer to the result I obtained[84].

Interaction energies are an interesting story as well. Averages taken of my data in Figures B.3 and B.6 are nearly identical, though the EC distribution is obviously a fair bit broader than the PC one. This reflects the increased frequency of structures where one of the solvent molecules is being exchanged with the bulk and is often nearly completely turned around. This is seen in Figures 4.3 and 4.4 as the increased density of ion-carbonyl oxygen distances around 300 pm. Given that an increased temperature in the Li\(^{+}\)/PC simulations would serve to broaden the interaction energy distribution and that the largest binding energies were observed for Li\(^{+}\)/EC despite the greater thermal stresses, it is reasonable to conclude that the binding energy between Li\(^{+}\)/EC is greater than Li\(^{+}\)/PC, consistent with some of the experiments discussed in Chapter 1.6. However, Peruzzi et al. found the solvation enthalpies of KX salts were more favorable in PC than EC. The binding energies of Park et al. support this conclusion as well[305], as does the SAPT work by Arslanargin et al.[12]. There’s a split in the thermodynamic data of anions at Cl\(^{-}\) which was determined to be better solvated in EC than PC. The similar solvent reorganization energies in this study suggest that the differences lie primarily in the strength of ion/solvent interactions. My SAPT energies using classically generated coordinates favor Li\(^{+}\)/EC over Li\(^{+}\)/PC while K\(^{+}\) was found to be better solvated in PC[12]. There may be a cutoff here as well, similar to that in water. Clearly, there is no consensus on relative binding energies of ions with the cyclic carbonates.

Radial distribution functions and the coordination number have been another point of contention in the literature. As reviewed earlier in Chapter 1.6, the coordination number is typically taken to be somewhere in the range of 4–5, though it has been determined to be as high as \(\sim 7\)[77]. The integral taken to the minimum of the first peak of my radial distribution functions in Figures 4.2 is \(\sim 4\) in each solvent. Again, the actual values were found to be 3.88 for EC and 3.98 for PC. These values are identical to those from other theoretical studies but smaller than most of the experimental measurements discussed previously. Since the methodology I used was based off of the work of Leung et al.[247], it’s excellent to see that our g(r) have very similar profiles. Both g(r)’s peak at 200 pm for the Li\(^{+}\)-O(carbonyl) distance and have an intensity of just over 25. Distributions taken from Arslanargin et al.[12] and Masia et al.[275] show over-structuring of the first solvation shell, each with intensities over 40. The location of the maximum in each of these distributions also differs
from both mine and Leung’s. The classical models tend to give too small an average bond length between Li$^+$ and EC, though one of the results from Arslanargin et al.[12] gives too great a distance (and a coordination number of 6). This model is identical to the one used in this study to prepare the initial coordinates for AIMD work. For PC, the location of the first peak was found by Smith et al.[385] to also be around 200 pm, though their simulated model is less structured than mine with an intensity similar to that of my Li$^+$/EC result. This may simply reflect temperature differences.

4.5 Conclusions & Future Work

Up to this point, I have determined that polarizability is an important consideration in the condensed phase behavior of cyclic carbonates ethylene carbonate and propylene carbonate. The dipole moments which were found to be 5.44 D and 5.65 D in the gas phase increase by $\sim34\%$ through self-polarization in the condensed phase to 7.30 D and 7.56 for EC and PC, respectively. This increase may be approximately handled in classical models which appear to reproduce the dielectric constant of the pure solvents over a broad temperature range[447]. However, when attempting to address problems of ion solvation, the dipole moments of these solvents increase even further by about 50% over the gas phase dipole moments to about $\sim8.4$–$8.5$ D and in PC at $\sim8.7$–$8.8$ D in the first solvation shell around Li$^+$. While in water there is some evidence to suggest that the dipole moments of waters coordinated to the ions more or less return to the bulk value or produce modest increases (<10%)[160, 161, 162, 225, 252], it is clear from my results that this may not be the case in solvents with larger polarizabilities. Interaction energies suggested better solvation of Li$^+$ in EC than in PC, though the extent to which EC is favored cannot be predicted from these results. The coordination structure of EC and PC around Li$^+$ compared well against other simulation results and the coordination numbers were found to be very close to 4 in each of the solvents, consistent with other theoretical calculations. These fall short of the assumed values from experimental measurements which tend closer to 5 EC or PC per Li$^+$.

Before publishing this work I have several other pieces of data to add and I may collaborate with Matthew Brown to calculate X-ray absorption spectra to predict Li$^+$-1s binding energies. The binding energy of the core state was shown by Matt Brown to be a probe of the strength of ion/solvent interactions[63]. By comparing the binding energy in a fictitious state where the gas phase optimized EC and PC molecules are superimposed on the positions of solvent molecules in Li$^+$/XC$_4$ clusters (X = E and P) versus the binding energy of the Li-core state in the fully interacting cluster, I hope to approximately determine the effect of polarization. Alternative approaches will likely be considered as well. Another possibility is to perform a many-body expansion where a distinguished EC or PC interaction energy is measured with Li$^+$ left out. I am also going to add data for PC simulated at 450 K as well. The choice of 350 K was motivated to mimic the temperature increase in EC over its melting point to simulate a liquid phase just above each solvent’s freezing temperature. It is clear that a 450 K simulation of PC would prove useful in addressing some
other concerns about the binding energy and X-ray spectra by eliminating the thermal differences. The additional data includes sampling the dipole moments from the simulations using the GAFF model, calculating interaction energies for small clusters using the GAFF interaction potential, and measuring EC and PC structural properties of molecules in the first shell and relating them to their dipole moment (to address how the structure changes to produce the larger dipole moment).
5 THERMODYNAMICS OF PROTON HYDRATION AND THE ELECTROCHEMICAL SURFACE POTENTIAL

5.1 Preface

Much of the text presented here has been adapted from the following publications,


These articles summarize the findings of my computational analysis of the cluster-pair approximation (CPA) for the determination of single-ion free energies and enthalpies from the extrapolation of cluster data to the bulk. The CPA expression of these quantities for the proton includes an interfacial potential contribution by definition. It is shown, however, that the CPA involves an extra-thermodynamic assumption that does not guarantee uniform convergence to a bulk free energy or enthalpy value with increasing cluster size. I use computational modeling to examine the size-dependence of the differences in the hydration enthalpies for the Na$^+$/$F^-$ ion pair in water clusters of size $n=1, 2, 3, ..., 200$ (not perfectly sequential, there are some breaks). A sizable shift from the accepted cluster-pair approximation value is found for the proton hydration enthalpy for cluster sizes just beyond the limits used in the CPA. The shifts arise from a combination of sequential hydration and interfacial potential effects and converges at large $n$ to a value of -0.4 V to -0.5 V depending on the temperature derivative of the interfacial potential being used. The -0.4 V result is argued to be preferable and assumes the temperature derivative to be close to 0. cal/mol-K-e. In response to concerns raised that the 2nd order Møller-Plesset (MP2) energy overestimates the dispersion energy in $F^-$/water clusters leading to the shift[157], I have included previously unpublished evaluations of the enthalpy differences using the ‘gold standard’ CCSD(T) method. Though the cluster size is severely limited, the beginnings of the shift are shown to be preserved.

This section addresses three of the questions I posed in Chapter 1.2.3,

- Do chemical interfaces contribute to single-ion thermodynamics?
- If so, what is the contribution?
- How can we compare our simulation results to experiment?

5.2 The cluster pair approximation

The cluster-pair approximation\cite{400} (CPA) is widely viewed as the most accurate approach for determining the hydration free energy of the proton\cite{71, 214} Given the hydration free energy of one ion, all other single-ion free energies can be obtained from bulk thermodynamic data\cite{92, 261}. The CPA approach utilizes ion-water cluster data and bulk conventional ion hydration free energies (referred to the proton), along with an insightful analysis, to infer the proton hydration enthalpy and free energy. Recently, the issue has been raised as to whether the CPA may involve an extra-thermodynamic assumption that could result in proton values that deviate from those predicted based on small-cluster data\cite{108, 178, 419, 418}. This deviation is predicted to result from two parts: 1) hydration effects and 2) the net potential of water $\phi_{np}$. At large $n$ it is expected that the shift be comprised of $\phi_{np}$ alone.

The excess chemical potential can be written as\cite{381}

\[
\mu^e_X = \mu^e_{X,b} + q \phi_{np} \tag{5.1}
\]

where $\mu^e_{X,b}$ is the bulk hydration free energy (that includes all interactions of the ion with water except for the net potential contribution), see discussion in Chapter 1.5. $\mu^e_{X,b}$ is also called the intrinsic free energy\cite{178} and $\mu^e_X$ the real free energy. Similar nomenclature is extended to the hydration enthalpy which is

\[
h^e_X = h^e_{X,b} + q \phi_{np} - q T \left( \frac{\partial \phi_{np}}{\partial T} \right)_p \tag{5.2}
\]

and the hydration entropy\cite{260} is

\[
s^e_X = s^e_{X,b} - q \left( \frac{\partial \phi_{np}}{\partial T} \right)_p . \tag{5.3}
\]

In these papers, standard states of 1 M concentration in both the vapor and liquid phases are used so that the free energies and entropies reflect excess quantities that do not include superfluous volume changes. The experimental value for the temperature derivative is taken as -9.9 cal/mol-K-e\cite{345}, though there is evidence this value is much too negative\cite{108, 419}.

To understand the origins of the CPA, consider the exact QCT expression for the ion excess chemical potential from Ref. \cite{15}

\[
\mu^e_X = -kT \ln[K_{X,n}^{(0)} \rho_W^n] + kT \ln p_X(n) + \mu^e_{XW_n} - n \mu^e_W \tag{5.4}
\]

where $K_{X,n}^{(0)}$ is the equilibrium constant for the formation of the $XW_n$ cluster in the gas phase, $\rho_W$ is the bulk density of water, $p_X(n)$ is the probability of observing $n$ waters complexed with the ion in bulk water, $\mu^e_{XW_n}$ is the free energy to insert the cluster into bulk water, and $\mu^e_W$ is the free energy to insert one water molecule into bulk water.
Collecting all terms but the free energy to hydrate the cluster into a single term $\mu_{X,n}^{ex}$

$$\mu_{X}^{ex} = \mu_{X,n}^{ex} + \mu_{XW,n}^{ex}$$

where

$$\mu_{X,n}^{ex} = -kT \ln K_{X,n}^{(0)} + kT \ln p_{X}(n) - n [kT \ln \rho_{W} + \mu_{W}^{ex}]$$

The first two terms on the right side of Eq. 5.6 are ion specific, while the last term involves properties of bulk water. The cluster experimental data employed in the CPA\[400\] involves the temperature dependence of $K_{X,n}^{(0)}$ for small clusters $n = 1–6$. The CPA relies on differences of free energy and enthalpy terms for positive/negative ion pairs. Then all terms in $\mu_{X,n}^{ex}$ except $K_{X,n}^{(0)}$ cancel when taking the differences (for large $n$, the term involving $p(n)$ should also cancel nearly exactly).

The terms in Eq. 5.5 correspond to a 2-step process for analyzing the hydration free energy: formation of the ion-water cluster followed by insertion of the cluster into bulk water. It is clear on physical grounds that cation/anion differences of $\mu_{XW,n}^{ex}$ approach zero as the cluster size approaches infinity, since this is the free energy to insert a large ion/water cluster into bulk water, and on average the ion is thus shielded from the surrounding bulk water.

The input data for the CPA\[400\] includes differences of bulk conventional free energies and cluster formation free energies for cation/anion pairs; the cluster free energies are available for a range of ions with $n = 1–6$ clustered waters. These data have been assembled from mass spectrometric analyses on ion/water clusters with size discrimination based on collision energy\[428\] or other means and theoretical calculations[400, 86, 407, 108, 214].

The bulk conventional free energies are defined as

$$\mu_{N}^{ex,con} = \mu_{N}^{ex} + \mu_{H_{1}}^{ex} = \mu_{N,b}^{ex} + \mu_{H_{1},b}^{ex}$$

for a negatively charged ion and

$$\mu_{P}^{ex,con} = \mu_{P}^{ex} - \mu_{H_{1}}^{ex} = \mu_{P,b}^{ex} - \mu_{H_{1},b}^{ex}$$

for a positively charged one, where the bulk free energies are $\mu_{X,b}^{ex} = \mu_{X}^{ex} - q\Phi_{NP}$. The conventional free energies can be obtained from bulk thermodynamic data, and it is apparent that the net potential does not appear explicitly in Eqns. 5.7 and 5.8 due to the cancellation between the ion and proton terms. The conventional free energy scale is set relative to the proton, just as the electrochemical scale (the standard hydrogen electrode has a potential of 0. V, when the real potential is estimated at -4.44 ± 0.02 V\[403\]).

The CPA is an analysis method that leads to an estimation of the proton hydration free energy, enthalpy, and entropy. For brevity, I’ll review only the free energy ‘path.’ First, consider the free energy difference between a cation (P) and anion (N), defined for example as
\[ \Delta \mu_{X,\text{ex,con}}^{\text{ex,con}} = \mu_{N,\text{ex,con}}^{\text{ex,con}} - \mu_{P,\text{ex,con}}^{\text{ex,con}} \]  

so the change refers to the P → N transition for a given ion pair. Combining this expression with those above in Eqns. 5.7 and 5.8 results in the following relationships

\[ \frac{1}{2} \Delta \mu_X^{\text{ex,con}} - \frac{1}{2} \Delta \mu_X^{\text{ex}} = \mu_{H^+}^{\text{ex}} \]  

and

\[ \frac{1}{2} \Delta \mu_{X,W_n}^{\text{ex,con}} - \frac{1}{2} \Delta \mu_{X,W_n}^{\text{ex}} = \mu_{H^+}^{\text{ex}}. \]  

It is also true that

\[ \frac{1}{2} \Delta \mu_X^{\text{ex,con}} - \frac{1}{2} \Delta \mu_X^{\text{ex}} + \phi_{np} = \mu_{H^+}^{\text{ex}} \]  

and

\[ \frac{1}{2} \Delta \mu_X^{\text{ex,con}} - \frac{1}{2} \Delta \mu_X^{\text{ex}} = \mu_{H^+}^{\text{ex}}. \]  

Since \( \Delta \mu_X^{\text{ex}} \) and \( \Delta \mu_{X,W_n}^{\text{ex}} \) are not available from experiment, the CPA makes an assumption to deal with these terms.

\( \Delta \mu_{X,W_n}^{\text{ex,con}}/2 \) contains no explicit net potential contribution; it is clear from Eq. 5.5, however, that \( \Delta \mu_{X,W_n}^{\text{ex,con}}/2 \) can develop an explicit net potential contribution for large \( n \) due to the cluster term (since \( \Delta \mu_X^{\text{ex,con}} = \Delta \mu_X^{\text{ex,n}} + \Delta \mu_{X,W_n}^{\text{ex,con}} \)). It is also clear from Eq. 5.11 that, in the limit of large \( n \), \( \Delta \mu_{X,W_n}^{\text{ex,con}}/2 = \mu_{H^+}^{\text{ex}}. \)

Eq. 5.10 can be rewritten as

\[ \frac{1}{2} \Delta \mu_X^{\text{ex,con}} - \frac{1}{2} \left[ \frac{\mu_N^{\text{ex}}}{c_{N,n}} + \frac{\mu_P^{\text{ex}}}{c_{P,n}} \right] = \mu_{H^+}^{\text{ex}} \]  

where \( c_{N,n} = \mu_{N,n}^{\text{ex}}/\mu_N^{\text{ex}} \) and \( c_{P,n} = \mu_{P,n}^{\text{ex}}/\mu_P^{\text{ex}} \). The central assumption of the CPA[400] is then to create a composite term for the chosen N/P pair:

\[ \bar{c}_{X,n} = \frac{\mu_{N,n}^{\text{ex}} + \mu_{P,n}^{\text{ex}}}{\mu_N^{\text{ex}} + \mu_P^{\text{ex}}} \]  

where now the quantities are experimentally available. Then

\[ \frac{1}{2} \Delta \mu_X^{\text{ex,con}} - \frac{1}{2 \bar{c}_{X,n}} \Delta \mu_X^{\text{ex}} \approx \mu_{H^+}^{\text{ex}} \]  

Since the exact relation involves

\[ \bar{c}_{X,n}(\text{exact}) = \frac{\mu_{N,n}^{\text{ex}} - \mu_{P,n}^{\text{ex}}}{\mu_N^{\text{ex}} - \mu_P^{\text{ex}}} = \frac{\Delta \mu_{X,n}^{\text{ex}}}{\Delta \mu_X^{\text{ex}}} \]
it is seen that the CPA involves an extra-thermodynamic assumption. In plain language, the assumption is that the differences in the pair solvation free energies for the small clusters should converge rapidly to the bulk limit and remain roughly constant to large $n$. As mentioned previously, this is not necessarily the case\[108, 419, 418\].

Assembling many differences for the alkali/halides and even broader ion sets, the results can be plotted as in Kelly, Cramer, and Truhlar\[214\] (Figure 1 of Ref. [214]). The $y$ axis is
\[
y(n) = \frac{1}{2} \Delta \mu_{\text{ex, con}}^{\text{XW}}
\]
and the $x$ axis is
\[
x = \frac{1}{2} \Delta \mu_{\text{X}}^{\text{ex, con}}.
\]

The original CPA paper\[400\] uses a related expression but takes from Eq. 5.16 the $y$-axis as $\mu_{H^+}^{\text{ex}}$, and the $x$-axis as $\Delta \mu_{\text{X}_n}^{\text{ex}}$. Refs. [332] and [333] use the former, but the latter is important for understanding why I only needed to consider the $\text{Na}^+/\text{F}^-$ ion pair for this study. A more exhaustive compilation of ion pairs would have produced identical results to within some error, but I’m using a trick here. The CPA figures when plotted with the latter pair of variables, produce the exact, real proton hydration free energy, enthalpy, or entropy when the difference for an ion pair at a given $n$ is exactly zero. See Figure 5.1 for reference. The differences in solvation free energies and enthalpies for the $\text{Na}^+/\text{F}^-$ pair sit near zero for $n = 1–6$, making them the ideal pair of ions to explore the shift in the difference at larger $n$. I also modify the $\text{F}^-$ ion somewhat (discussed below) and this coincidentally further idealizes the pair.

This next point draws heavily from the discussion in the caption of 5.1. Two observations from the data presented in Refs. [400] and [214] appear to support the CPA assumption. Figure 1 of Ref. [400] displays the cluster enthalpy (obtained as in Figure 5.1 for the free energy) vs $n^{-1/3}$, showing an apparent convergence towards the bulk value with increasing cluster size. Second, the slopes of the curves in Figure 1 of Ref. [400, 214, 108] are substantially reduced by $n = 6$. Ref. [400] suggests that, if there were to be any transition in the enthalpy or free energy values, it should occur for intermediate cluster sizes. This is reasonable since $\Delta \mu_{\text{X}_n}^{\text{ex}}$ approaches zero with increasing size, and thus from Eq. 5.11 the proton enthalpy or free energy should approach an asymptotic (flat) form with increasing $n$.

Then for the large $n$ limit,
\[
\Delta \mu_{\text{X}}^{\text{ex}} = \lim_{n \to \infty} \Delta \mu_{\text{X}_n}^{\text{ex}}.
\]
Figure 5.1: Illustration of the cluster pair approximation for the proton solvation free energy in the form described by Coe et al. [400]. Lines represent linear fits through all possible combinations of pairs and their differences (slopes taken directly from Ref. [400]). With increasing cluster size, the slope of the lines is reduced. In the limit of $n \to \infty$, the slope falls to zero defining $\mu_{\text{H}^+}^{\text{ex}}$. The lines also share an approximately common crossing point. This point sits very near the value of $\mu_{\text{H}^+}^{\text{ex}}$ even for $n = 1$. Remarkable, really. The ion pair $\text{Na}^+$ and $\text{F}^-$ also sit very near the crossing point for $n = 1$–6. By modeling the $\text{Na}^+$ to $\text{F}^-$ transition in $\mu_{\text{X},n}^{\text{ex}}$ or other properties as a surrogate for the crossing point, I can test my prediction that the point will drift in response to the surface potential and other potential factors as $n$ becomes larger.
physical circumstance of ions that are free to roam throughout the clusters, it serves to enhance the approach to large system behavior by ensuring that the ions are surrounded by hydration shells that more closely mimic the bulk environment. I also compare this to calculations where the ion is free to roam the cluster as it will.

To summarize, the discussion in this section located an extra-thermodynamic assumption in the original CPA approach\[400\] that allows for decoupling of salt hydration free energies, enthalpies, and entropies into single-ion contributions. The central assumption here is that the differences in these properties between microsolvated ions should rapidly converge for small \( n \), where the necessary input data is accessible to specialized mass spectrometric and theoretical analyses. Up to \( n = 6 \), these differences appear to be converging towards the bulk limit and the real proton solvation free energy, etc. However, there is evidence to suggest that at some intermediate cluster size, the differences begin to drift in part due to solvation effects and in response to a more fully formed electrochemical surface potential. In Ref. [332], the shift is measured in each of \( \Delta \mu_{X,n}^{ex} \), \( \Delta h_{X,n}^{ex} \), and \( \Delta s_{X,n}^{ex} \) by extending the cluster size out to \( n = 105 \). In Ref. [333], the enthalpy expression is rewritten to solve for \( \phi_{np} \) taking \( \frac{1}{2} \Delta h_{X,n}^{ex} \) as input for the Na\(^+\) and F\(^-\) pair. This analysis reaches out to \( n = 200 \) with direct comparison of the classical results to MP2-level quantum chemistry to \( n = 35 \), and CCSD(T) to \( n = 6 \) (triple-\( \zeta \) basis) and 8 (double-\( \zeta \) basis). The equation used to solve for the net potential is

\[
\phi_{np} = \frac{\Delta h_{X,b}^{ex}}{2} + T \left( \frac{\partial \phi_{np}}{\partial T} \right)_p - \frac{\Delta h_{X,n}^{ex}}{2}. \tag{5.21}
\]

\( T \left( \frac{\partial \phi_{np}}{\partial T} \right)_p \) is taken as -9.9 cal/mol-K\(-e\)\[345\] but I also use a value of 0 cal/mol-K\(-e\) based on the work outlined in Ref. [332]. An expression starting from the free energy exists as well

\[
\phi_{np} = \frac{(\Delta \mu_{X,b}^{ex} - \Delta \mu_{X}^{ex})}{2}. \tag{5.22}
\]

The surface potential itself is not present in the entropy, only its temperature derivative is present, refer to Eq. 5.3.

Computationally the differences can be evaluated as in Eqns. 5.23–5.25

\[
\Delta \mu_{X}^{ex} = \Delta \mu_{X,b}^{ex} - 2\phi_{np} \tag{5.23}
\]

\[
\Delta h_{X}^{ex} = \Delta h_{X,b}^{ex} - 2\phi_{np} + 2T \left( \frac{\partial \phi_{np}}{\partial T} \right)_p \tag{5.24}
\]

\[
\Delta s_{X}^{ex} = \Delta s_{X,b}^{ex} + 2 \left( \frac{\partial \phi_{np}}{\partial T} \right)_p \tag{5.25}
\]

The bulk differences in these equations come from Marcus\[261\] and some relevant pairs are given in Table 5.1.
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<th>K⁺</th>
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Table 5.1: Marcus bulk free energy, enthalpy, and entropy difference data.[261]

5.3 Computational methods

The work in Ref. [333] builds on that done in Ref. [332], so I’ll focus principally on this second study.

The molecular dynamics package Tinker[335] was used to perform simulations of ion-water clusters with the polarizable AMOeba force field[336]; the sodium and fluoride ions were studied. Cluster sizes were estimated assuming a density of 0.997 g cm⁻³ for a cluster of \(n\) waters. A half-harmonic bounding potential was applied a further 2.0 Å beyond the computed size to reflect any evaporating waters back into the cluster. The default AMOeba bonded and nonbonded parameters were assigned to sodium and all waters. In the case of fluoride, the Thole damping parameter was reduced from 0.39 to 0.2, having the effect of bringing the ion induced dipole distribution into better agreement with quantum mechanical calculations performed previously[144, 454, 23]. Simulations were also performed with the default values for comparison.

The simulations were carried out at a temperature of 300 K for 6 ns with all nonbonded cutoffs removed. In one set of calculations each ion was free to explore the extent of the droplet. A second set of calculations freezes the ion at the droplet center of mass with the aim of accelerating convergence of the solvation behavior to the bulk limit. For subsequent quantum chemical analysis,
500 configurations from each case were extracted from the trajectory.

Dynamics using the B3LYP-D3/6-31++G(d,p)[142] potential energy surface as implemented in the QChem 4.0.1 package[376, 226] was performed for clusters of \( n = 1 \)–6 waters about a sodium or fluoride ion. Four simulations were performed for 36 ps under constant energy conditions for each cluster/ion pair. The initial velocities were randomly generated from a Boltzmann kinetic energy distribution; the same initial configuration was employed for each run and 12 ps of the dynamics was discarded to allow for deviation of the cluster state due to the differing initial velocities. No external potential was used for these simulations. From each of these trajectories, 250 configurations were extracted and energies were computed at the B3LYP-D3/6-31++G(d,p) and RI-MP2/aug-cc-pVDZ/aug-cc-pVDZ-RI levels of theory.

Extending the analysis of Ref. [332] to the quantum chemical level requires extensive computational resources. Determination of \( \phi_{np} \) through enthalpy differences as in Equation 5.21 is particularly direct, however, requiring only calculation of cluster energies and the energy of the ion with the dimer-centered basis set to account for basis set superposition errors. For larger clusters, these calculations become more difficult as the differences being sought become very small compared to the magnitude of the energies involved. For the current study, all energies were computed using the Psi4 quantum chemistry software (beta version 5)[406]. These were done at the RI-MP2/aug-cc-pVDZ/aug-cc-pVDZ-RI level of theory considering nearest noble gas frozen core approximation and neglecting zero point contributions.

As mentioned in the previous section, the MP2-level calculations came under fire because MP2 is known to overestimate dispersion interactions[402]. This is the reason I used spin-component scaled MP2[141] in Chapter 3.2. By rescaling the same-spin and opposite-spin components of the MP2 energy, I can more accurately dial in the dispersion energy compared to higher levels of theory. However, for the purposes of addressing these concerns, I have chosen to use the ‘gold standard’ quantum chemistry method of CCSD(T). To that end, I also include unpublished calculations performed on a subset of the initial 500 configurations for each Na/water and F/water cluster size. Some 30 configurations were chosen so as to reproduce the mean and standard deviation of the larger population to within 0.05 kcal/mol for a particular cluster size using the existing RI-MP2/aug-cc-pVDZ data. I use Cholesky decomposition instead of the conventional resolution-of-the-identity approximation to speed up the calculation of the repulsion integrals. The Cholesky decomposition tolerance was set to 5e-6 for CCSD(T)/aug-cc-pVDZ and CCSD(T)/jun-cc-pVTZ calculations. This however is quite prohibitive in probing CCSD(T) energies for larger clusters, so I was only able to perform the calculations on clusters with up to \( n = 6 \) waters (jun-cc-pVTZ basis) and \( n = 8 \) waters (aug-cc-pVDZ basis).
Table 5.2: Cluster data from AMOEBA simulations and experiment\cite{400,108} All values are in kcal/mol except for the $\Delta s_{X,n}^{ex}$ results, which are in cal/mol-K. The net potential values are computed using Eq. 5.22. The proton hydration free energy, enthalpy, and entropy derived using the above data are -264.7 kcal/mol, -271.9 kcal/mol, and -24.0 cal/mol-K, respectively. The corresponding values from Ref. \cite{400} are -265.9 kcal/mol, -274.9 kcal/mol, and -30.0 cal/mol-K.

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<th>$\Delta h_{X,n}^{ex}$</th>
<th>$\Delta s_{X,n}^{ex}$</th>
<th>$\phi_{np}$</th>
<th>$\partial \phi_{np} / \partial T$</th>
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5.4 Results

5.4.1 Drift in $\mu_{X,n}^{ex}$, $h_{X,n}^{ex}$, and $s_{X,n}^{ex}$ as $n \to \infty$

Refer to Table 5.2 for a discussion of the results from Ref. \cite{332} beginning with the Na$^+/F^-$ pair in the $n = 5$ water cluster. The results for the cation to anion free energy and enthalpy changes agree with experiment within a range that is of the same magnitude as the difference between the listed experimental values. Throughout the calculations, the expected errors in the calculated free energy differences are estimated to be in the range 0.5 to 1.0 kcal/mol, while the enthalpy difference errors are small for the $n = 5$ cluster (0.1 kcal/mol) and larger for the $n = 25$ and $n = 105$ cases (1 to 1.5 kcal/mol). The small-cluster results suggest that the AMOEBA model (with the reduced polarization parameter) represents the local ion-water interactions with decent accuracy for the Na$^+/F^-$ pair.

For the Na$^+/F^-$ pair in the $n = 25$ water cluster, there is a shift in the free energy difference (-2.6 kcal/mol) and a larger shift in the enthalpy difference (-8.0 kcal/mol). The computed entropy difference is seen to be large and negative, exhibiting a compensating effect between enthalpy and entropy. The entropy change implies that, for the $F^-$ ion, the second hydration shell has more induced order due to interactions with the ion and first shell waters relative to the Na$^+$ ion.

Approaching the bulk limit with a cluster size of $n = 105$, free energy and enthalpy differences do not change significantly from the $n = 25$ results. This suggests that, already with a second hydration shell, bulk-like behavior is emerging for the cation/anion difference (for this kosmotropic pair). The agreement of the $n = 25$ cluster results with the larger-cluster limit will be exploited in quantum mechanical calculations of the same quantities in the next section.
The computed temperature derivative of the net potential (from Eq. 5.25), is -2.5 cal/mol-K-e for the n = 105 cluster. This value has the same sign but is reduced in magnitude by roughly a factor of 4 from the value reported by Randles[345] and [178] based on a range of experiments. It’s unknown whether this discrepancy is due to the experimental values being inaccurate (discussion on this possibility in Ref. [178]), the curvature of the small clusters used here changing the derivative, or the AMOeba model simply not doing a good job reproducing this property. Fair to say, it might be all of the above, but it is suspected that the experimentally derived derivative is too negative[108]. I elaborate on this a bit more in the next section as well.

The derived net potential (at 300 K) from this data is -10.1 kcal/mol-e or -0.44 V. It is interesting that I observe a negative net potential along with the negative temperature derivative. If anything, at higher temperatures the derivative should become more positive because the cavity/water boundary behaves much like a hydrophobic particle which has a positive hydration entropy.

While examining the Marcus bulk values[261] for alkali halide ion pairs (Table 5.1), it was observed that the chaotropic Rb+/I− ion pair displays near-zero values for the differences of the bulk (Marcus) free energies and enthalpies; this contrasts with the Na+/F− case that exhibits a near-zero value for the small cluster free energy difference in the CPA[400]. In Eqns. 5.21 and 5.24, this suggests that the enthalpy change is nearly entirely a net potential effect, or conversely, the net potential comes primarily from the large-cluster enthalpy change (since the term involving the temperature derivative of the net potential is only of magnitude in the range ~0 to -3 cal/mol, based on the results reported here and experimental estimates[345]). Simulations were also performed of the Rb+/I− pair. Clusters of sizes n = 5, 25, 45, 105, and 242, since this ion pair displays slower convergence with increasing cluster size.

The Rb+/I− pair yields several interesting results. First, the computed free energy difference for the n = 5 cluster is 2.3 kcal/mol smaller than the experimental value, while the computed enthalpy difference is 6.1 kcal/mol smaller than the experimental value (sugesting deficiency in the AMOeba ion-water interactions for this ion pair). Second, the enthalpy differences for the n = 25 and n = 45 clusters and the n = 105 cluster differ substantially, suggesting convergence is not reached until larger cluster sizes for the chaotropic pair. Third, the free energy difference for the n = 105 cluster is quite close to the enthalpy difference, indicating a small entropy difference. The net potential obtained for this very different ion pair is consistent with that for the Na+/F− pair (negative and of substantial magnitude), but differing by 2.3 kcal/mol-e. Applying an ad hoc correction of 2.3 kcal/mol due to the deviation from experiment for the n = 5 cluster, the net potential becomes -9.0 kcal/mol-e, in agreement with the result using the NaF pair. Fourth, the n = 242 free energy difference and the resulting net potential confirm that the results are relatively well converged by n = 105. Compare this with the kosmotropic pair where these properties were seemingly converged by n = 25.

Using Eq. 5.10 (and the corresponding enthalpy equation), the predicted values for the proton hydration quantities taken as averages between the kosmotropic and corrected chaotropic pairs...
are: $\mu_{H^+}^{\text{ex}} - 264.7$ kcal/mol, $h_{H^+}^{\text{ex}} - 271.9$ kcal/mol, and $s_{H^+}^{\text{ex}} - 24.0$ cal/mol-K. The net potential is estimated (also as an average of the two measurements) as -10.4 kcal/mol-e or -0.45 V. The 6.1 kcal/mol correction to the enthalpy for the RbI pair is probably too large as the resulting entropy prediction is -28.3 cal/mol-K. It is expected that with the small but still negative prediction of the temperature derivative discussed above, the average would be more positive than the Marcus value of -24 cal/mol-K [261].

The shifts in the free energy and enthalpy are due to 1) sequential hydration effects and 2) the net potential effect. These effects work in opposite directions, so if the shifts were due entirely to a net potential effect, free energy and enthalpy shifts in the opposite direction would be observed for the Na$^+$F/− pair. This suggests a significant contribution from sequential hydration effects in which the first shell strongly interacts with the second shell (with a corresponding large negative value for the entropy for the F$^-$ ion). This effect is apparently large enough to overcome the shift in the opposite direction due to the net potential. For the Rb$^+/I^-$ pair, on the other hand, the net potential effect on the enthalpy difference is largely isolated due to the small value of the bulk enthalpy difference in Eq. 5.24. The net potential is very small for clusters of $n = 5$ (on the order of about 10% as large). For RbI, the shift is due to the emergence of the remaining 90% of the potential as the free energy differences due to the ion specific hydration effects cancel. The Rb$^+/I^-$ results provide a further indication of the stronger hydration of anions relative to cations (for a given ion size), reflected in the nearly equal bulk hydration free energies but smaller radius of the Rb$^+$ ion compared with the I$^-$ ion.

Given the issues with the RbI pair, the value of $\Delta h_{X,n}^{\text{ex}}$ is adapted from Ref. [400] which is 15.7 kcal/mol for $n = 5$. The slope of this line is $\Delta h_{X}^{\text{ex}}/2\Delta h_{X,n}^{\text{ex}}$ and is equal to 0.64. Rearranging to solve for $\Delta h_{X}^{\text{ex}}$, the CPA estimate is 20.1 kcal/mol (which is reduced from the 21.6 kcal/mol from the simulation results). This difference gives a new net potential of -10.1 kcal/mol-e with the temperature derivative set to zero and -13 kcal/mol-e when set to -9.9 cal/mol-K-e. Either way, it is clear that the net potential is negative and is of large magnitude. The -13 kcal/mol-e potential with a -9.9 cal/mol-K-e temperature derivative being larger than that predicted by Ashbaugh et al. [14] also suggests that the temperature dependence of the net potential may be smaller than is currently accepted [345, 178].

5.4.2 Size dependence of ion-pair enthalpy differences

For my analysis of the size dependence of ion-pair enthalpy differences, I start by examining the small-cluster size range ($n = 1$–6 water molecules) for the NaF ion pair. With the modified Thole parameter, Figure 5.2 shows excellent agreement of the computed enthalpy difference with the experimental values listed in Ref. [108] (except for a deviation for the $n = 1$ case). Using the default Thole parameter results in significant deviation from experiment for the $n = 2$–6 clusters. The small cluster data tabulated in Ref. [108] includes a wider range of experimental studies than in the original CPA paper [400].
Examination of radial distribution functions (RDFs, not shown) for the F$^-$ ion/water(oxygen) pair shows that reduction of the Thole parameter leads to an increase in the average ion-water oxygen distance of 0.25 Å. Related to this structural observation, Ref. [212] presents fundamental simulations of $n = 1–3$ F$^-$/water clusters. The electrons are treated at the MP2 level, while the nuclear motions include quantum effects through path integral simulations. The results show a significant broadening of the RDFs for the $n = 3$ cluster with a substantial tail developing at large distances (and some increased density at small distances) due to nuclear quantum effects. (Note that the ion/hydrogen and ion/oxygen RDFs are mislabeled in Ref. [212].)

This suggests that the reduced Thole parameter has two effects: 1) production of a more realistic polarization state of the F$^-$ ion and 2) a larger bond length that fortuitously mimics the inclusion of nuclear quantum effects; both then lead to better agreement with experiment for the enthalpy differences. It is interesting that, in the results of Ref. [212], a large distance tail is not observed for the $n = 1$ case; in fact, when nuclear quantum effects are included, the distributions display some contraction to smaller ion-water distances. This may explain the deviation of the $n = 1$ modified AMOEBA results in Figure 5.2 from experiment, and the better agreement of the default AMOEBA model. The results in Ref. [212] also suggest that water-water interactions are important, even for the small $n = 2–6$ clusters.

To further explore the role of electronic quantum effects in the small clusters, I computed enthalpy differences for several other cases. First, I generated configurations with both the modified and default Thole parameter simulations, and computed the energies at the MP2 level. The MP2-computed enthalpy differences (using the modified Thole parameter for the sampling) accurately match the modified AMOEBA results. When the default Thole parameter is used to generate the configurations, the MP2 results lie between the default AMOEBA results and experiment. Finally, configurations were also generated with B3LYP-D3/DFT simulations. When the energies are computed at the same level of theory, it is apparent the quantum model yields results more negative than experiment (except for the $n = 5$ case). It is interesting that, when energies are computed at the MP2 level (with DFT configurations), larger deviations from experiment are observed. Nuclear quantum effects were not included in the DFT simulations, and based on the discussion above these would likely move the computed values upward toward the experimental results. It is important to note the AMOEBA model was parametrized based on MP2 level calculations (with no inclusion of nuclear quantum effects)[336]. From the above results, I can conclude that the modified AMOEBA model for the F$^-$ ion yields quite accurate results for the energetics of the cation to anion transition in the small clusters.

Next I'll explore the size dependence of the cation-anion enthalpy difference over a large range of cluster sizes (up to $n = 200$). As in Ref. [332], I first constrain each ion to the cluster center of mass for clusters of $n \geq 5$. Figure 5.3 displays the enthalpy difference as a function of cluster size using the modified AMOEBA model and MP2 calculations. First, it is clear that a shift to more negative values develops as the cluster size is increased beyond $n = 6$. The shift appears to
Figure 5.2: Half the difference in the enthalpy for the Na\(^+\) to F\(^-\) transition in the \(n = 1\)–6 clusters. All simulations allowed free motion of the ions in the clusters. The plot labeled “Tissandier” (black solid squares) is for experimental data from Ref. [400], and the plot labeled “Williams” (black solid circles) is for experimental data from Ref. [108] obtained from a larger range of experiments. “m. AMOEBA” (black open squares) refers to results from the modified AMOEBA model, while for “m. RI-MP2” the energies were computed at the MP2 level (with modified AMOEBA sampling). “d. AMOEBA” (red open squares) indicates the default AMOEBA model, and “d. RI-MP2” (red open circles) is for energies computed at the MP2 level (with default AMOEBA sampling). “DFT(QM/QM)” (blue open squares) labels the DFT simulation results, and “RI-MP2(QM/QM)” (blue open circles) indicates DFT sampling with energy calculations at the MP2 level. The cluster size is displayed as \(n^{-1/3}\).
Figure 5.3: Half the difference in the enthalpy for the Na\(^{+}\) to F\(^{-}\) transition in larger clusters (up to \(n = 200\)). The energies were computed from modified (reduced Thole factor) AMOEBA trajectories block-averaging every 100 steps for the case of the ion bound to the cluster center of mass (CoM). The results are compared out to \(n = 35\) with a sample of configurations modeled at the RI-MP2/aug-cc-pVDZ/aug-cc-pVDZ-RI level of theory. The cluster size is given as \(n^{-1/3}\). Error bars for AMOEBA averages are smaller than the size of the symbols.

stabilize (with some oscillation) at a value of roughly \(-4\) kcal/mol; already by the \(n = 10\) cluster size, the majority of the shift has occurred. This is just beyond the scope of the myriad CPA studies\[400, 86, 407, 108, 214\] but still within the range of both experimental and theoretical used to generate the smaller cluster data\[428\].

In Ref. [332] there was some discussion of the difficulties in obtaining converged enthalpy differences for the larger cluster sizes (involving the difference of two large numbers and possible partial evaporation events). However, the previous free energy results (which are less affected by the difficulties in the enthalpy calculations) fully support the notion that the net potential stabilizes for clusters in the size range considered here.

There is remarkable agreement of the MP2 results out to \(n = 35\) with the classical results. Since the configurations were generated with the modified AMOEBA model, perhaps this is not so surprising; it does show that, for a given set of configurations, the model accurately mimics the
energetics obtained from the realistic quantum electron distribution. The configurations are for the ions fully coupled to the nearby waters, and do not rely on a particular cavity definition.

The agreement of the modified AMOEBA results with experiments on small clusters discussed above then suggests this model is accurately representing the energetics over the full size range. It seems highly unlikely that errors in half the enthalpy difference of magnitude 10 kcal/mol occur (pushing the asymptotic values of the curves in Figures 5.3 and 5.4 downward by ≈ 10 kcal/mol); such a shift magnitude would be necessary to produce a net potential close to zero.

For the case in which the ions are free to move about the entire cluster, similar results are obtained (Figure 5.4). The enthalpy differences first stabilize at a slightly higher energy relative to Figure 5.3, but then relax to the asymptotic value of −4 kcal/mol. Again, the quantum results are very close to the AMOEBA results. The results show that locating the ion at the cluster center (aimed at more rapid convergence to the bulk limit) has only a small effect on the computed results for this kosmotropic ion pair.

Considering the extension of these differences to the ‘gold standard’ CCSD(T) method, I found similar agreement to that which I established earlier between AMOEBA and the MP2 method, see Figure 5.5. Out to a size of \( n = 3 \), the quantum results fall more negative than the AMOEBA result at an intermediate energy to that predicted by Tissandier et al. and Williams et al. in Refs. [400] and [108], respectively. Interestingly, there seems to be better overlap between the CCSD(T)/jun-cc-pVTZ and MP2 trends than when using CCSD(T)/aug-cc-pVDZ implying a fortuitous result at the lower level of theory which is of similar quality to the much more prohibitive triple-\( \zeta \) coupled-cluster method. Most importantly however, each of the trends tracks the drift in the Na\(^+\)/F\(^-\) pair enthalpy differences from near zero.

A comparison between the AMOEBA and MP2 energies to the higher levels of theory revealed that the bulk of the figures fall within ± 0.5 kcal/mol of the higher level estimate, see Figure 5.6. The AMOEBA model tended to produce more positive energies than CCSD(T) except beyond \( n = 3 \) for CCSD(T)/jun-cc-pVTZ, while MP2 was slightly more negative than CCSD(T)/jun-cc-pVTZ and more positive than CCSD(T)/aug-cc-pVDZ. Differences between the AMOEBA and quantum methods tended to exhibit non-linear fluctuations while those between MP2 and CCSD(T) were approximately linear over the range considered. Overall, however, I find remarkable agreement between the methods.

Projecting the MP2 to CCSD(T) errors to the bulk limit, assuming the \( n^{-\frac{1}{3}} \) dependence remains linear, it is clear that the errors in the reported enthalpy differences aren’t likely to exceed 1 kcal/mol, see Figure 5.7. Granted, the trend approaches an asymptotic limit of about -4 kcal/mol at large \( n \). However, even the AMOEBA to MP2 differences start off relatively linearly before oscillating in the range of ±0.5 kcal/mol from zero. It’s likely given that AMOEBA was parameterized against MP2 that the MP2 to CCSD(T) errors will also start to oscillate, making the linear projection a sort of worst case scenario. Errors assuming the linear and oscillatory trends are discussed below.

Using Equation 5.21, I display in Figure 5.8 the size dependence of the derived net potential. As
Figure 5.4: Half the difference in the enthalpy for the Na\(^+\) to F\(^-\) transition in larger clusters (up to \(n = 200\)). The energies were computed from modified (reduced Thole factor) AMOEBA trajectories block-averaging every 100 steps for the case of the ion free to move throughout the cluster. The results are compared out to \(n = 35\) with a sample of configurations modeled at the RI-MP2/aug-cc-pVDZ/aug-cc-pVDZ-RI level of theory. The cluster size is given as \(n^{-1/3}\). Error bars for AMOEBA averages are smaller than the size of the symbols.

Discussed above, I assume a temperature derivative of the net potential of zero. It is clear from the plot that the net potential approaches an asymptotic value of close to \(-9.4\) kcal/mol-e discussed in Refs. [39] and [332]. Compare this with the result assuming the -9.9 cal/mol-K-e temperature derivative in Figure 5.9. Given my discussion of the errors previously, it is instructive to note that the nearest alternative estimate of the electrochemical surface potential is +3 kcal/mol-e (+0.13 V). The linear estimate produces a 25% error in my estimate of \(\phi_{np}\) with a lower limit of \(\approx -6.8\) kcal/mol-e. Taking the oscillations of the AMOEBA to MP2 trend as an estimate of the error instead, I expect a more realistic uncertainty of \(-9.4 \pm 1.1(25)\) kcal/mol-e which is half the estimate of the worst case scenario spelled out above. In volts that’s -0.41 \pm 0.05 V, though I always round down to -0.4 V.

The final result presented here is the size dependence of the net potential assuming the experimental temperature derivative to be reasonable[345] as is asserted in Ref. [178]. Interestingly this
Figure 5.5: Half the enthalpy differences in the transition of Na → F, expressed in kcal/mol across varying theoretical treatments for clusters with up to $n = 35$.

shifts the asymptote of the curve to around -11.6 kcal/mol-e which is the average of the charge dependent shifts in Ref. [14]. However, as has been noted by others[108, 332], this temperature derivative drives the proton solvation entropy much too negative. The CPA predicted proton solvation entropy is on the order of -30.3 to -33.5 cal/mol-K[400, 108] while bulk thermodynamic measurements predict a bulk solvation entropy of -24.7 cal/mol-K[92] using 0.878 mV/K for the temperature dependence of the hydrogen electrode (from Ref. [93]). This result is identical to that reported by Marcus[261]. Adding the negative surface potential temperature derivative should push this quantity less negative to around -15 cal/mol-K. The corrected results from Ref. [419] require a larger and negative temperature derivative and give a proton hydration entropy of only -5.3 cal/mol-K. This value cannot be recommended when the derivative is suspected of being very small. Note, these entropies reflect the 1 M ideal gas and 1 M solution standard state which adds a factor of 1900 cal/mol / 300 K to the literature values reported here. The -1900 cal/mol is the free energy standard state correction and the entropy correction is the negative of the free energy correction divide the temperature.
Figure 5.6: Comparison of $(1/2)\Delta h^\text{ex}_{X,n}$ for the Na$^+$ to F$^-$ transition, expressed in kcal/mol between the various methods, 1) AMOEBA minus MP2/DZ, 2) AMOEBA minus CCSD(T)/DZ, 3) AMOEBA minus CCSD(T)/TZ, 4) MP2/DZ minus CCSD(T)/DZ, and 5) MP2/DZ minus CCSD(T)/TZ. A is shorthand for AMOEBA, M for MP2, CDZ for CCSD(T)/DZ, and CTZ for CCSD(T)/TZ. DZ and TZ refer to aug-cc-pVDZ and jun-cc-pVTZ basis sets, respectively.

5.5 Discussion

In conjunction with the results from Chapter 5.4.1, my efforts suggest a proton hydration free energy of -264.7 kcal/mol, an enthalpy of -271.9 kcal/mol, an entropy of -24 cal/mol-K, an electrochemical surface potential of water of -0.4 V, and a temperature derivative of the potential that is close to zero at room temperature. These were revised in Ref. [333] to a free energy of -264.2 kcal/mol, an enthalpy of -271.6 kcal/mol, an entropy of -24.5 cal/mol-K presented in the paper but not touched on here. In general, the values reported in Ref. [333] should supersede those presented in Ref. [332].

The above values differ in several respects from previous discussions and ongoing work. Hünenberger and Reif [178] present an exhaustive compilation of proton hydration data, and based on an average of 98 values, recommend -264.8 kcal/mol for the free energy, -272.2 kcal/mol for the enthalpy, and -25 cal/mol-K for the entropy (labelled as intrinsic or bulk values). While these results appear to agree well with the real values listed above, they are obtained as an average over data
Figure 5.7: Differences in $\frac{1}{2}\Delta h_{\text{ex}}^{\text{X,n}}$ (MP2 minus CCSD(T)) in the transition of Na → F, expressed in kcal/mol. Here M is shorthand for MP2, CDZ for CCSD(T)/aug-cc-pVDZ, and CTZ for CCSD(T)/jun-cc-pVTZ. The solid black lines are linear fits through the data and provide an estimate of the error in the enthalpy shift to large cluster size.

that includes both bulk and real quantities. The averaging then blurs the distinction drawn here between bulk and real quantities and its likely importance in determining the surface potential. The recommended real values in Ref. [178] are -261.7 kcal/mol (free energy), -266.2 kcal/mol (enthalpy), and -15.2 cal/mol-K, respectively, showing a large deviation from the results obtained here.

Ref. [178] also recommends an electrochemical surface potential value of +0.13 V and a temperature derivative of -9.9 cal/mol-e (-0.42 mV/K-e). The temperature derivative is similar to the result previously discussed by Randles[345]; from that temperature derivative, and an extrapolation to the critical point of water, Randles obtained a value for the electrochemical surface potential close to the +0.13 V value. Ref. [178] discusses several possible concerns related to the previous experimental analysis of the temperature derivative. Also, it’s worth mentioning that a smooth extrapolation from room temperature and atmospheric pressure to the critical point is not likely a justified route for obtaining the net potential.

In a recent theoretical study aimed at examining the role of dispersion in ion solvation,[112] the
surface potential value recommended by Hünenberger and Reif[178] was used, and it was argued that the CPA values such as those in Refs.  [400] and [108] are bulk (or intrinsic) values that include no surface potential contribution. Previous simulations of large clusters, however, have been shown to include a surface potential contribution[39, 332]. The agreement of the computed (large cluster) results with the modified CPA analysis presented here (that is free of extra-thermodynamic assumptions) provides clear evidence that the CPA-derived values correspond to the real quantities and not the bulk quantities. My results for the net potential rely on the assumption that the Marcus values are the bulk quantities, however. The main evidence in favor of this interpretation is the agreement between the Marcus[261] and LPS[14] scales (after adjusting for the -11.6 kcal/mol potential).

Finally, recent simulation studies of the electrostatic potential at the center of neutral cavities.
Figure 5.9: The electrochemical surface potential as a function of cluster size up to \( n = 200 \) (using Equation 5.21). The temperature derivative of the net potential was assumed to be -9.9 cal/mol-K-e, as taken from Randles[345]. The plots are for the cases in which the ions was either restricted to the cluster center of mass or free to move throughout the cluster. The F− Thole damping parameter has been reduced from the default 0.39 value to 0.2 in generating the trajectories. The cluster size is given as \( n^{-1/3} \). Error bars are comparable to or smaller than the size of the points (approximately ± 0.10 kcal mol\(^{-1}\) or less). The dashed horizontal line is the -11.6 kcal/mol-e value taken as an average of sign dependent deviation in the LPS model of Ref. [14].

in water implies a value close to zero for the net potential[25, 351]. In this work, the cavity was created by providing a repulsive force on the water oxygens. A previous study[380] showed, however, that the potential can change significantly if repulsions are included (in classical models) on the water hydrogens. An interesting alternative would be to strictly enforce the cavity by excluding all electron density.

### 5.6 Conclusions

The present chapter thus provides a contrasting view of proton hydration and the water surface potential. I have approached the problem from an alternative thermodynamic direction that does not
rely on direct electrostatic calculation of the net potential but rather infers its value by comparing ion enthalpy differences in large clusters with tabulated bulk values. Due to the relatively rapid convergence with cluster size of the enthalpy differences for the NaF kosmotropic pair (in the range $n = 10–20$), it is hoped that experimental results on clusters in this size range may provide a test of the theoretical proposals outlined here.

Additionally, this work has independently derived a single-ion thermodynamic scale when combined with the conventional (relative to $H^+$) thermodynamic scale. This scale defines real hydration properties which differ from the bulk values by inclusion of the net potential I reviewed previously. This scale differs somewhat from the CPA scale of Tissandier et al.[400] and a separate recently derived scale of Vlcek et al.[419] but is similar to that described by Zhan et al.[449]. I would recommend that additional work to resolve the issues with the temperature derivative be performed. These calculations are planned using the Tinker package[335] and the iAMOEBA (improved AMOeba) model which performs well for a number of properties over a broad temperature and pressure range. A model equation of state will be useful in directing targeted simulations with greater accuracy with \textit{ab initio} molecular dynamics.
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6  REVISITING THE TA+/TB- HYPOTHESIS: INTERFACIAL POTENTIALS IN PERIODIC BOUNDARIES

6.1  Preface

Disclaimer: this chapter is subject to additional changes as it is not yet published.

The free energy change for transferring an ion across chemical interfaces includes a surface potential contribution. This potential has two parts: 1) the actual surface or contact potential between two immiscible phases and 2) a solvent-specific local potential around the ion. In periodic boundary calculations without an explicit vapor or vacuum region, the second potential still contributes to the solvation free energy and enthalpy. Simulation results therefore fall on an unphysical, intermediate thermodynamic scale which needs to be adjusted to remove 2) or add 1) in order to properly facilitate comparison with one of the two major branches of single-ion quantities. Using ionic parameters which have not been optimized to fit one of these scales and which approach the size limit of the tetraphenylarsonium and tetraphenylborate ions, I illustrate that the intermediate free energy scale is the one which predicts the $\approx 20 \text{ kcal/mol}$ hydration asymmetry in water[370, 371, 369]. These large ions are otherwise assumed to have the same solvation free energy due to ample charge screening effects[262]. The bulk or *intrinsic* free energy scale (removing 2) above) predicts a 2-4 kcal/mol deviation from this ideal behavior. I have also made predictions of the deviation in dimethyl sulfoxide and 1,2-dichloroethane, covering a broad spectrum of dielectric constants (10 to 80). I find dimethyl sulfoxide satisfies this condition as well while 1,2-dichloroethane violates it thanks to a large, anion favoring asymmetry. My results in water and dimethyl sulfoxide compare favorably against another theoretical assessment which also falls on the bulk free energy scale[76]. This study builds on the emerging narrative that surface effects can differ substantially from the bulk and also has important implications in the force field development community.

Single-ion solvation free energies were calculated using the quasichemical theory to spatially resolve the free energy, dividing the solvation process into 3 distinct contributions: 1) a packing term related to cavity formation in the pure solvent, 2) an inner shell term related to the ion specific work necessary to dig the same size cavity around the ion, and 3) long-range solvation effects not captured in the inner shell term. In Chapter 1.2.1 I said,

“The desired theoretical model should take care to accommodate the major contributions to the free energy change, 1) interaction part between the ion and solvent as well as 2) a change in the free energy associated with the reorganization of solvent molecules around the ion (cavity formation). The interaction part can be split up into 1a) local effects arising from the strong $\approx 1 \text{ V/Å}$ fields around the ions[375] and 1b) distant solvation effects which may be amenable to computationally
inexpensive approximations in dilute conditions, and 1c) interactions with a distant surface or chemical interface.”

It is seen that the quasichemical theory addresses each of these contributions, though it is not immediately obvious where the interfacial potentials come into play. Consider that the outer shell contribution can be split into vdw and electrostatic components via a cumulant expansion as illustrated below,

\[
\beta \mu_{ex}^{os} = \mu_{ex,vdW}^{os} + \mu_{ex,elst}^{os}
\]

\[
= \mu_{ex,vdW}^{os} + q \langle \phi_{lp} \rangle_{M(\lambda)} - \frac{\beta q^2}{2} \left( \frac{\phi_{lp}^2}{M(\lambda)} - \frac{\xi}{\beta L} \right) + \frac{\beta^2 q^3}{6} \langle \phi_{lp}^3 \rangle_{M(\lambda)} + \ldots
\]

(6.1)

where \( M(\lambda) \) refers to sampling configurations with an external cavitation potential included as discussed in Chapter 2.5.3.2. A cumulant expansion of the electrostatic part of the outer shell free energy generates a mean-field, quadratic, cubic, and higher order terms (though I truncate this series to third order). The quadratic term depends on the square of the ion charge and fluctuations in the electrostatic potential felt by the ion, with the other part here representing a self-energy correction. This form also resembles the Born solvation model discussed in Chapter 1.3.1 and has been shown to be accurate for the description of long-range interactions[38, 37, 175, 380]. The first and third order terms exhibit a sign dependence and are summed together to give the local potential.

The free energy is measured using a mid-point approximation discussed below and the local potential by accumulating statistics of the electrostatic potential at the center of a large, uncharged cavity over the course of a simulation. There are several deficiencies associated with this method, but it is likely accurate for cavities of \( \geq 0.9 \) nm radius where Ashbaugh shows the potential converges in water[14]. This distance is assumed to be a reasonable length scale in the other solvents as well. At any rate, differences in \( \mu_{ex,elst}^{os} \) across the range of ions considered are nearly zero for each of the solvents. The result is certainly less ambiguous in the non-aqueous solvents given the fact that hydrogens in these molecules have non-zero Lennard-Jones parameters in the force field used (OPLS/AA).

This section addresses three of the questions I posed in Chapter 1.2.3,

- Do chemical interfaces contribute to single-ion thermodynamics?
- If so, what is the contribution?
- How can we compare our simulation results to experiment?
6.2 Solvation free energies from the TA$^+/TB^-$ assumption

The TA$^+/TB^-$ hypothesis and its variants (e.g., replacing TA$^+$ with tetraphenylphosphonium) represents one of the most widely used extrathermodynamic assumptions used to tease apart individual ion hydration free energies, enthalpies, entropies, partial molar heat capacities, equivalent conductivities, and Dole-Jones B-coefficients[262]. Marcus speaks highly of the method, remarking that it has found support over the years and is generally regarded as the “least objectionable” of the myriad extrathermodynamic assumptions[265]. In a field rife with complex thermodynamic cycles and often suspect assumptions, TA$^+/TB^-$ is also without doubt one of the simplest models relying only on the disappearance of ion-specificity when incorporating a pair of oppositely charged ions into a large molecular framework to isolate it from the solvent. The hypothesis is not entirely without issue, however, as the molecular ion pair exhibit a small inequality in their vdW radius leading to a 1.2% error in electrostatic interactions and 2.2% in the solvation energy of the neutral particle[218, 265].

The method provides a means to easily construct a universal thermodynamic scale in all solvents as the peripheral ligands always mask the identity of the charge. Solubility or electrochemical measurements are used to measure free energies of transfer from water, W, to another solvent, S, with an accuracy argued to be on the order of ± 2 kJ/mol[265]. Measurements of the W→S transfer may be taken in mutually saturated solvents that are in contact with one another or in the pure solvent. The choice can often lead to very different results as the more hydrophilic ions tend to coextract with residual waters into the non-aqueous phase[359, 102, 101]. A universal thermodynamic scale is essential to linking theoretical measurements to their experimental counterparts, providing a means to accurately predict the pH of mixed electrolyte solutions used in liquid chromatography[120], measure absolute electrochemical potentials and convert to non-aqueous solvents (in both theory and experiment)[187], and in the development of force fields for molecular simulation.

There is however evidence of sign specific effects on water structure around the ions which suggest anions are more strongly hydrated than cations. Several of these evidences are visited by Grossfield et al. (see the bottom left column on page 2)[143]. These observations are primarily spectroscopic in nature. I also find that approaching the TA$^+/TB^-$ limit, anions are better solvated in water and 1,2-DCE and cations in DMSO. However, the asymmetry changes after modifying the simulation result to exclude the interfacial potential and reflects any true sign specificity in the solvation free energy.

6.3 Computational methods

All calculations in this Chapter were performed in double-precision with Gromacs 4.6.7[413] installed locally on the Oakley and Ruby clusters at the Ohio Supercomputer Center[295]. The OPLS-AA[201] force field was used for DMSO and 1,2-DCE. SPC/E[45] parameters were used for water. Bonds and angles involving hydrogen atoms were constrained with the LINCS algorithm[159] in all simulations.
Ions parameters are listed in Table 6.1 and range from 0.3 nm to 0.55 nm in radius, the same size used in the Schurhammer and Wipff studies[370, 371, 369]. Dispersion parameters were chosen to surround the Schurhammer and Wipff value of 0.42 kJ/mol with the smaller value matching that of I\(^-\) in the Horinek and Netz force field[171]. The larger value is just an order of magnitude greater.

The Good-Hope[139] and Berthelot[47] combining rules were used to generate the pair \(\sigma_{ij}\) and \(\varepsilon_{ij}\) parameters, respectively. The ion/cavity/solvent or cavity/solvent systems were equilibrated in the NPT ensemble for 400 ps at 1.0 bar and 298.15 K using the Berendsen barostat[46] and V-rescale algorithms[67] for pressure and temperature coupling. Real-space and vdW interactions were truncated at 1.2 nm, with a 0.16 nm grid spacing and 6\(^{th}\)-order spline for reciprocal space interactions. I used an Ewald convergence parameter of \(\eta = 5.6/L\) where \(L\) is the box length after NPT equilibration. Energies were sampled every 20 frames from 2.0 ns of a 2.2 ns production run in the NVT ensemble, also held at 298.15 K with the V-rescale algorithm[67]. For quantities obtained through thermodynamic integration, 21 unique \(\lambda\) values were used. Each simulation, including at each \(\lambda\), was performed 3 separate times with a timestep of 2.0 fs. Statistical errors were evaluated with the block-averaging method; errors were propagated through numerical integration as well. The solvation environment around an ion with the fully grown cavity in each solvent is illustrated in Figure 6.1. The cavity had a 0.9 nm radius and did not change with ion size. This leads to decreasing inner shell contributions with increasing ion size and increasing outer shell contributions through the vdW potential.

The solvents modeled here cover a broad range of dielectric constants from 10 to 80. For additional validation of the methods used here I also computed dielectric constant for each solvent. I performed 400 ps of equilibration of the pure solvent without cavities as above followed by 10 ns constant volume dynamics at 298.15 K. The first 500 ps of each trajectory was discarded as additional equilibration. The SPC/E dielectric constant was found to be 72 (71), in DMSO it was...
Figure 6.1: Visualization of solvation structure at the boundary (white surface) of the 0.90 nm radius cavity used to compute solvation free energies. The ion is at the center of this void, and is shown in purple. The solvents from left to right are as follows: water, dimethyl sulfoxide, and 1,2-dichloroethane.

45 (42), and in 1,2-DCE it was 12 (13). Values in parentheses were taken from Reddy et al.[350] and Caleman et al.[70]. I also performed this study with acetonitrile but the OPLS/AA force field produced a dielectric of 20 which is about half that of the experimental value and so performed very poorly[70].

6.4 Results

6.4.1 Packing and inner shell terms

I begin with some discussion of the cavity-related terms in each solvent: the packing and inner shell contributions. The packing free energy is independent of the ion so I can very simply list the values for each solvent. For water this came out to $82.77 \pm 0.63$ kcal/mol which compares favorably with the result from Shi and Beck who produced a fit to a scaled-particle theory model[380]. This value reflects the work required to clear a void of only the water oxygens since the hydrogens do not participate in the vdW potential. However, for both DMSO and 1,2-DCE, the hydrogens in these molecules have been assigned non-zero $\sigma$ and $\varepsilon$ parameters. The cavity potential I used made no distinction between this atom or that based on mass and so acted the same on all particles provided the force field recommended non-zero vdW parameters. A slightly larger value of $83.50 \pm 0.91$ kcal/mol was obtained for DMSO, while a much smaller result of $49.45 \pm 0.56$ kcal/mol was found for 1,2-DCE. It’s interesting to note that the cavity formation free energy in DMSO remains so large in spite of the pair of methyl groups the solvent is forced to accommodate. Solvent/solvent contacts in 1,2-DCE however do appear to be relatively weak compared to both water and DMSO. Consequently, the solvent is less tightly packed and so offers substantially less resistance to the insertion of large cavities and likewise to large, hydrophobic molecules and ions. You can get a sense
of this visually from Figure 6.1 as well.

A connection between the quasichemical packing contribution and the surface tension of the solvent was developed in Ref. [380]. The surface tension for OPLS 1,2-DCE is 23.2 mN/m (expt 31.86 mN/m) while OPLS DMSO matches the experimental figure of 42.92 mN/m remarkably well (calc 42.4 mN/m)[70]. Like 1,2-DCE, the surface tension of the SPC/E model (63.6 mN/m) is somewhat smaller than the experimental value (71.73 mN/m)[416]. Based on the model of Shi and Beck, I should still expect to see the packing contribution of water to be larger than DMSO, despite the smaller than expected SPC/E water surface tension. However, my finding is consistent with a scaled particle theory estimate from Abraham et al.[3]. Assuming the cavity term in Ref. [3] is positive too, then \( \text{pack(DMSO)} - \text{pack(water)} > 0 \) means it costs more energy to put the cavity (here the cavity has a diameter of 0.84 nm) into DMSO. Their model appears to take into account the size of the solvent particles, which may account for the unexpected result.

I continue the discussion focusing on Figure 6.2. For all ion \( \sigma \) and \( \varepsilon \) considered, the chemical part of the free energy showed that the anion was better solvated in water and 1,2-DCE than the cation and generally followed the opposite trend in DMSO due to the hard S=O moiety. The gap between the \( \mu_{ex}^{\text{is}} \) of complementary ions pairs (oppositely charged pairs with all other properties equal) diminishes to nearly zero for the largest radii in DMSO and water. This appeared to not be the case for ion solvation in 1,2-DCE where the charge-specific differences in smaller ion pairs remained essentially constant even when I simulated \( \text{TA}^+/\text{TB}^- \)-sized particles. These results shed an initial favorable light on the validity of the \( \text{TA}^+/\text{TB}^- \) assumption in water and in DMSO as the large ions share nearly identical free energies in the ion specific region. A small amount of ion specificity is expected to survive based on the finite-size corrected fluctuation contributions in Figure 6.3 which still show preference for one ion over the other, even using ions of \( \text{TA}^+/\text{TB}^- \) size. This hints that the validity of the \( \text{TA}^+/\text{TB}^- \) assumption may rely on cancellation of lingering ion specificity by the \( \phi_{lp} \).

### 6.4.2 Outer shell and local potential terms

I begin this section with some discussion of the motivations for electing to use the mid-point approximation to the outer shell free energy. Normally, I would sample the fully coupled interaction energies from the final \( \lambda \) state in both the packing (uncoupled) and inner shell (coupled) trajectories. However, as pointed out by Shi and Beck[380], there needs to be continuity in the solvation environment between that sampled with \( \pm q \) and a neutral but otherwise identical particle. Monitoring the mean-field part of the electrostatic potential at the center of a cavity bearing integer and fractional charges in the range of \(-q\) to \(q\) is a particularly useful way for interrogating the solvation shell for charge-specific asymmetries, see Figure 6.4. As noted by Wipff et al., the water dipole orientation around the neutral cavity more resembles the solvation environment around a positively charged ion[370]. Unsurprisingly, I found the mean-field part of the electrostatic potential for the neutral particle fell on the same line drawn through the points with positive charge. The same observation
Figure 6.2: Chemical or inner shell part of the solvation free energy for the ions in water (blues), dimethyl sulfoxide (reds), and 1,2-dichloroethane (greens). The darker shade corresponds to ions with $\varepsilon = 1.60$ kJ/mol and the lighter shade for $\varepsilon = 0.16$ kJ/mol. The cations are presented here with open circles at the sampled radii linked together with a solid line; anions at the sampled radii are represented as diamonds and are joined together with dashed lines. Error bars are omitted as they are about the same size as the symbols and are reported in the tables in Appendix C.

was made from the TA$^+/TB^-$-sized ion data set as well. Lines through the negatively and positively charged ion data intersect at -0.22e and -0.17e for the smaller and larger ion sizes, respectively. The deviation between these lines is reduced with the larger particles, but not enough to allay concerns over errors introduced by sampling energies from dissimilar solvation environments. I’ve found likewise in DMSO as well, where the lines intersect at -0.16e and -0.11e. In contrast, the solvation environment around the neutral cavity in 1,2-DCE slightly favored the anion and produced intersections at 0.22e and 0.03e. An additional point, the potential at the center of the neutral cavity can be taken as a crude estimate of $\phi_{lp}$, with some caveats discussed below.

The mid-point approximation for $\mu_{ex}$ provides a straightforward decomposition pathway to separate vdW and electrostatic sources as in Equation 6.1. Assuming any differences in the vdW energy between the ions to be small, the top plot in Figure 6.5 depicts the results from the cumulant expansion of the electrostatic contributions to $\mu_{ex}$. Across the range of ion radii covered here and in all solvents, this plot shows a very nearly constant gap between the complementary sets of ions. I
Figure 6.3: Finite-size corrected fluctuations of the electrostatic potential at the center of the Horinek and Netz Cl\(^-\)-ion (top) and a TA\(^+\)/TB\(^-\)-ion (bottom). Figures are reported as solid lines in blue for water, red for DMSO, and green for 1,2-DCE.
Figure 6.4: Finite-size corrected means of the electrostatic potential at the center of the Horinek and Netz Cl\(^{-}\)-ion (top) and a TA\(^{+}\)/TB\(^{-}\)-ion (bottom). Colors correspond to solvents: blue for water, red for DMSO, and green for 1,2-DCE. A dashed line is used to show the linear fit through the negatively charged points and a solid line for the fit through the neutral and positively charged data points.
argue this gap is due to the presence of a $q\phi_{lp}$ factor for each ion. As the bottom plot in Figure 6.5 illustrates, the removal of this factor shifts the free energy differences into near perfect overlap with some residual due to non-Gaussian behavior in the vdW contributions for the organic solvents. The electrostatic contributions are within 0.5 kcal/mol of each other across 3 separate trajectories with small differences in the variance and higher order terms. A slightly larger cavity may be needed for DMSO and 1,2-DCE to remove the remaining small, but not insignificant non-Gaussian behavior.

The local potential in each solvent [in (mean + third cumulant); total ± error format] was determined to be (8.76 + 0.20); 8.95 ± 0.14 kcal/mol-e, (6.71 + -0.27); 6.44 ± 0.24 kcal/mol-e, and (-1.42 + 0.16); -1.26 ± 0.20 kcal/mol-e in water, DMSO, and 1,2-DCE respectively. These potentials were sampled from configurations generated with an empty 0.9 nm radius cavity buried in each solvent. The potential makes no distinction between light and heavy atoms so long as they are described with non-zero terms in the conventional LJ function, else those atoms are ignored for consistency with the force field. This method was shown by Ashbaugh to produce well-converged results at the same length scale I used here[13]. Simply using the neutral point in the fluctuating electrostatic potential calculations discussed previously can, in some cases, net you a fortuitously close result, but it displays significant size variance and may even predict the wrong sign for solvents with smaller $\phi_{lp}$. This is a consequence of using standard LJ interactions in the simulation which operate on different length scales for each atom type, so smaller elements tend to settle a bit closer to the ion than they would with my modified WCA potential, for example. The local charge balance at the cavity is skewed in favor of the charge of these smaller elements, usually hydrogens. The positive charge on these atoms introduces an artificial shift driving $\phi_{lp}$ more positive, or negative as the case may be. A similar shift occurs when applying the cavity-growth potential to hydrogens in the SPC/E or other single vdW-site water models and actually yields a $\phi_{np}$ around -0.4 V, though this is almost assuredly coincidental[39].

6.4.3 Free energy differences

I now have all the pieces needed to assemble the total solvation free energies and examine how the differences between the ions transition with increasing size. These values are reported in Appendix C in a tabular format complete with error bars for both the intermediate and bulk free energy scales. The differences between the single-ion solvation free energies listed in those tables are depicted in Figures 6.6, 6.7, and 6.8. Where appropriate I have included figures taken from Wipff et al.[370] and Pliego et al.[76] to highlight not only my agreement with these additional studies but to showcase that their respective results fall on very different thermodynamic scales and lead to very different conclusions. Removal of $q\phi_{lp}$ from $\mu_{os}^{ex}$ extinguishes the artificial stabilization of anions relative to their cation analogue in both SPC/E water and OPLS/AA DMSO that produced the ~20 kcal/mol disparity in TB$^-$ versus TA$^+$ solvation free energies reported by Schurhammer and Wipff. Withdrawing $q\phi_{lp}$ from $\mu_{os}^{ex}$ in 1,2-DCE created a slightly greater rift in the ion solvation free energies with a deviation from ideal behavior of about 6-10 kcal/mol in favor of the anion. This
Figure 6.5: Full outer shell (top) and outer shell less the local potential (bottom) part of the solvation free energy. Color and labeling scheme are the same as described in Figure 6.2.
Figure 6.6: Free energy differences in complimentary ion pairs in water. Open circles represent intrinsic free energy points while open diamonds are on the bulk free energy scale. Blue coloring corresponds to ions with $\varepsilon = 0.16$ kJ/mol while green marks $\varepsilon = 1.60$ kJ/mol differences. Where appropriate, comparisons to the Wipff and Pliego results are reported.

range is similar to that of acetonitrile as discussed by Pliego et al.\[76\].

6.5 Discussion

Qualitatively, the result that anions are more strongly hydrated than cations falls in step with a number of other studies\[370, 76, 353, 89, 87\]. Assuming the $F^-$ and $K^+$ ions to be of nearly identical size (or perhaps more conservatively, between $Na^+$ and $K^+$)\[437\], Rose et al. also concluded that the anion interacted more favorably with 1,2-DCE than the cation\[359\]. And finally, in agreement with the somewhat dated conductometric studies of Exner et al.\[122\] and more recent simulation results of Pliego et al. for intermediate\[330\] and bulk free energies\[76\], I found the cation produced a larger $\mu^\text{ex}$ part than its complementary anion. A more quantitative assessment of the accuracy of my results was a bit more difficult to come by, but the extensive work of Abraham et al. (Table 8)\[2\] allowed me to get an approximate sense of where my bulk free energy results fell for the more physically motivated ion parameter sets (cations with $\varepsilon = 0.16$ kJ/mol and anions with $\varepsilon = 1.60$ kJ/mol).
kJ/mol). For example, I properly recovered that the transfer free energy from water to 1,2-DCE was unfavorable for smaller ions, up to the size of tetraethylammonium (TEA\(^+\)) which has an ionic radius in the neighborhood of 0.3 to 0.4 nm\(^{\text{[19]}}\). Given the trend reported in their article and second harmonic generation spectroscopic investigation of the water/1,2-DCE interface\(^{\text{[91]}}\), I know that larger ions eventually become better suited to the oil over water. Likewise, the cation solvation free energies in DMSO were found to be greater than that in water which was also observed by Pliego et al.\(^{\text{[330]}}\). For anions, the smallest one gave a positive transfer free from water to DMSO consistent with these other authors but beyond that reverses. Within the limitations of the force field, these results compare rather favorably against other studies, but it is important to note the parameters used here were not designed to handle ion solvation problems. They neglect or oversimplify difficult to characterize polarization and partial charge transfer contributions\(^{\text{[334]}}\), which can be difficult to overcome even when fitting some of the parameters to high level electronic structure calculations\(^{\text{[12]}}\). Above all though, this exercise is meant to be illustrative of a broader concern and even the poorest force fields are susceptible to these interfacial potential issues.
Figure 6.8: Free energy differences in complimentary ion pairs in 1,2-DCE. Open circles represent intrinsic free energy points while open diamonds are on the bulk free energy scale. Blue coloring corresponds to ions with $\varepsilon = 0.16$ kJ/mol while green marks $\varepsilon = 1.60$ kJ/mol differences.

In Table 4 of the Wipff manuscript for the spherical $S^+$ and $S^-$, the authors show that the electrostatic part of the interaction energy favors the anion by 23 kcal/mol[371]. This difference is also reflected in Table 5, where they report the electrostatic potentials at the center of both charged and uncharged solutes[371]. The mean electrostatic potential at the center of the uncharged $S^0$ particle with a radius of 0.55 nm was found to be 7.4 kcal/mol with TIP3P water. This figure is similar to that reported by Ashbaugh at the same distance in SPC/E water[13]. This is $\phi_{lp}$, albeit with too small a cavity. The authors do not remove this contribution from the figures reported in Table 2[371]. With a positive $\phi_{lp}$, the -21.0 kcal/mol $\Delta\mu^{ex}$ is increased by +14.8 kcal/mol to give a difference of -6.2 kcal/mol which compares favorably against my own figure.

As a point of comparison, the $\phi_{lp}$ reported for TIP5P is much smaller, on the order of about 1 kcal/mol-e[371]. TIP5P water has been observed by Mundy et al.[351] and Ichiye et al.[78] to exhibit rather unusual behavior in relation to the ongoing dialogue about the conventional surface potential, $\phi_{sp}$, and the electrochemical surface potential, $\phi_{np}$. Recently, Pollard and Beck calculated the latter, $\phi_{np}$, felt by a charge in water to be -0.4 V[332, 333]. Water models such as TIP3P and SPC/E underestimate this potential somewhat, but the potential from TIP5P is very nearly
zero. This is because the quadrupole and octupole moments of the TIP5P water model compare poorly against the experimental and \textit{ab initio} predicted values\cite{293}. Though an exact relationship between the quadrupole moment (and higher order moments) is unknown, $|\phi_{sp}|$ has been found to be proportional to the quadrupole moment of the water model for the basic point charge models (i.e., TIPXP, SPC, SPC/E)\cite{78}. Note, that the SSDQO model which does not follow the trend in the Ichiye et al. paper also incorporates oxygen-centered point dipoles, quadrupoles, and octupoles.

With respect to the TA$^+/TB^-$ hypothesis, I found a 2.13 kcal/mol deviation from cation/anion solvation parity in SPC/E water. The error bars include 1.28 kcal/mol prediction of Pliego et al.\cite{76} who combined gas phase electronic structure calculations at the MP2 and MP4 levels of theory with continuum solvation using the SMD model. Their results found a similarly small deviation from the TA$^+/TB^-$ scale in DMSO of 1.8 kcal/mol which fell just outside the error bars of my result. A key tenet of the TA$^+/TB^-$ hypothesis is that such ions give the same solvation free energy in \textit{every} solvent. Calculations in acetonitrile, however, revealed a nearly 10 kcal/mol shift from TA$^+/TB^-$ behavior. I observed a comparable disparity in the solvation free energies on the bulk/Marcus scale in 1,2-dichloroethane spurred by the solvation asymmetry in the inner shell term favoring the anion and a negative $\phi_{lp}$ which further separated $\Delta \mu_{ex}(X^+ \rightarrow X^-)$.

6.6 Conclusions

I have presented a case that simulations of ionic species even in periodic boundary conditions with no vapor region include a potential centered about the ion. The potential is the product of broken multipolar symmetry relative to the bulk. It also impacts the solvation free energy, solvation enthalpy, and related properties such as pKa of charged species. The local potential is solvent dependent but applies in increments of $\pm q$, where $q$ is the ion charge. Difficulties in the development of proper force fields for divalent cations like Mg$^{2+}$ and Ca$^{2+}$ may in part owe some of their trouble to the mishandling of this potential which would impart a $\sim18$ kcal/mol error in $\mu_{ex}$ and $h_{ex}$ (assuming $\frac{\partial \phi_{lp}}{\partial T} \approx 0$, V, same physical origin as $\phi_{sp}[130]$) in SPC/E water. Also of note, the TA$^+/TB^-$ scale agrees with other surface potential free (bulk scale) methods such as quasichemical theory\cite{16, 17, 332}, the Latimer-Pitzer-Slansky model with re-fit ionic radii\cite{14}, and a second set of values predicted by Marcus with a model similar to that of Born\cite{261}.

I’ve also re-examined the tetraphenylarsonium/tetraphenylborate assumption which postulates that oppositely charged ions in a large, shielding ligand network have the same solvation free energy in every solvent. Previous work had shown the anion of this pair to be on the order of $\sim20$ kcal/mol more strongly hydrated than the cation. I showed this difference reflected the influence of a factor of $q\phi_{lp}$ for each ion and that these ions possessed nearly identical free energies in the absence of the potential. I also managed to extend the analysis to dimethyl sulfoxide (DMSO) and 1,2-dichloroethane (1,2-DCE) using the OPLS/AA force field. It was found that DMSO behaved similarly to water, where removing the potential allowed me to compare favorably against a recent
continuum model investigation, while 1,2-DCE violated this assumption even after removing $\phi_{lp}$. I concluded that the assumption was possibly valid in water and DMSO despite the model deficiencies thanks to my excellent comparison to MP4 level quantum chemistry calculations by Pliego and coworkers. Finally, it is my opinion that these interfacial potentials have profound and far reaching implications in the world of force field optimization. However, I discuss in detail a road map for how to go about comparing existing force field results to experiment.

Leela: “Five thousand feet!”
Farnsworth: “Dear Lord! That’s over one hundred and fifty atmospheres of pressure.”
Fry: “How many atmospheres can the ship withstand?”
Farnsworth: “Well, it’s a space ship. So I’d say anywhere between zero and one.”

— Futurama
Intentionally left blank
Conclusions

This thesis has highlighted the significance of ion solvation across the biological, chemical, and physical spectrum of science. I began the discussion with a review of Hofmeister’s observations noting the varying precipitating capacity of different salts on egg white proteins. From this, I developed the narrative that explaining these effects from a modeling perspective was simply too difficult and the conclusions of different experiments often conflicted with one another. This in turn motivated study of the simplest case, where building a model for more complex systems might be more approachable. The target of these simpler studies is the determination of a single-ion solvation free energy scale from which other properties such as activities, osmotic coefficients, surface tensions increments, and surface potential increments might also be established. The challenge in addressing this scale was found to be two-fold: 1) it is unclear what level of theory is appropriate for handling ion solvation problems and 2) experimental determination of the bulk values of these properties is impossible without the use of a hopefully reasonable assumption. As regards the first challenge, I reviewed an extensive body of literature which found non-electrostatic contributions evaluated at the electronic structure level and the granularity of all-atom models to be necessary to achieve spectroscopic/thermochemical accuracy. For the second, I made the case that the generally binary split of experimental single-ion scales was linked by the surface potential of water and that such potentials would be present for other systems as well.

In Chapters 3.1 and 4.1 I discussed my own contributions addressing the first of these challenges. I used quantum chemistry calculations and an energy partitioning scheme on small alkali/water and halide/water clusters to divvy up the interaction energy into chemically informative contributions. These fragments included non-electrostatic pieces like polarization, charge transfer, and dispersion. I discovered that in anion/water clusters, these contributions accounted for about 33% of the total attractive part of the interaction energy. The rest was due to electrostatics. While dispersion was found to be appreciably less important in cation/water clusters, polarization played a significant role as well. The electronic charge assigned to individual atoms can be partitioned as well. The charge on the ion was monitored with increasing hydration number. With \( n = 6 \) waters attached to the ion, some 20% of the excess charge of anions was shared with neighboring waters. A visual of this was provided as well, outlining poles in the electron density of the anion directed at each of the waters. It was found, in agreement with previous studies, that the non-electrostatic interactions saturated or nearly saturated within the first solvation shell. This implies that simpler electrostatic models of ion solvation are suitable for handling longer-ranged aspects of ion solvation, greatly reducing the computational cost of calculating accurate properties.

The focus on local interactions was extended in Chapter 4.1 to the non-aqueous solvents ethylene and propylene carbonate which are used in Li-ion batteries and supercapacitors. Ion solvation is especially important in the continued evolution of energy storage materials with solvation free
energies an indicator of an electrolyte’s ion transport properties\cite{180}. Very little free energy work has been done with these solvents despite their presence in our everyday lives through cell phones, laptops, and many other devices. Previous simulation results from our group suggested classical force fields lacked the flexibility to properly mimic polarization, with a standard model underestimating the effect and a corrected model overestimating it. \textit{Ab initio} molecular dynamics simulations were performed for the ethylene and propylene carbonate solvents. I compared the average dipole moment of these solvents in the gas phase, liquid phase, and solvating a single \( \text{Li}^{+} \) ion. The induced dipole moment of a molecule in a field is related to the polarizability of the molecule. Relative to the gas phase, I found the dipole moment of each of these solvents increased 34\% over the gas phase average when simulating the liquid phase and slightly over 50\% for molecules in the first solvation shell of the ion. This is odd behavior compared to water where it’s been found that the water dipole moment near ions is nearly the same as in the bulk. This study is not yet complete, but I have already learned that an explicit treatment of polarizability in these solvents is essential for modeling the local solvation structure near ions. I hope to also determine the strength of the induced dipole-induced dipole repulsion interaction between the first shell molecules and relate it back to our initial study of the thermodynamic data.

The last pair of chapters of my thesis dealt with the surface potential of water and the implications of my interpretation of this potential on force field development and molecular simulation in general. In Chapter 5.1, I used a novel approach involving the sequential hydration energies of \( \text{Na}^{+}/\text{water} \) and \( \text{F}^{-}/\text{water} \) clusters. This builds off an existing approach called the cluster pair approximation but makes no extrathermodynamic assumption to extrapolate from small cluster data to the bulk. I evaluated the energy differences to \( n = 200 \), well past the bulk limit of \( n = 105 \), using a polarizable force field model. For clusters up to \( n = 35 \), I was able to compare the results with MP2 level quantum chemistry. These results showed a shift in the sequential hydration enthalpy not observed by the cluster pair approximation, which assumed the small differences in these values for small clusters remained small in the limit of \( n \to \infty \). The shift reflected preferential anion solvation with the addition of the second hydration shell. The shift is supported by even higher level calculations at the CCSD(T) level of theory. The deviation from the reportedly \textit{ideal} behavior of this ion pair settled around -8 kcal/mol. A simple rearrangement in the expression for the solvation enthalpy allowed me to solve for the electrochemical surface potential of water at -0.4 V. Several indirect evidences support this figure over the recommended literature value of +0.13 V which is an average of values ranging from -0.5 V to +3–4 V in the literature. I argued this average is a compilation of two different surface potential contributions (\( \phi_{np} \) and \( \phi_{sp} \)) and so is nonphysical. My work also supports a small surface potential temperature derivative which is supported by an overlooked paper by Gabdoulline et al.\cite{130}.

Recalling that \( \phi_{np} = \phi_{sp} + \phi_{lp} \), in the last chapter of this thesis I show that a local interfacial potential \( \phi_{lp} \) contributes to ion solvation properties even in periodic boundary conditions. This potential depends on the solvent and force field used to simulate it. I illustrated its influence by
calculating the solvation free energy of large spherical ions approaching the size of the tetraphenyl arsonium and tetraphenyl borate ions. The large size and screening ligands are believed to mask the sign of the charge at the core, giving these ions equal solvation free energies in *all* solvents. Previous studies had shown water to significantly favor the anion by approximately 20 kcal/mol. I used quasichemical theory to partition the solvation free energy into an ion nonspecific packing contribution, ion specific inner shell or chemical contribution, and a long ranged term. The long ranged term contains contributions from the vdW potential and if conditioned properly an accurately Gaussian electrostatic part. I showed that the mean-field term in a cumulant expansion of this energy, which is associated with the local potential, was the source of the disparity between the anion and cation solvation free energies. By removing it, I found that the tetraphenyl arsonium and tetraphenyl borate ions shared nearly identical solvation properties in SPC/E modeled water and OPLS/AA modeled dimethyl sulfoxide. However, an anion favoring asymmetry persisted in the low dielectric 1,2-dichloroethane solvent also modeled with OPLS/AA. This result has profound implications in the force field development community as it brings to light the source of a glaring problem which has led to a great amount of inconsistency in the fitting of parameters. My work almost demands that we as a community attempt some level of standardization as to how we work with these potentials, how changing parameters to match this experiment or that (which can differ by the presence of these potentials) affects the solvation structure, and how best to relate simulation to experiment so that the symbiotic relationship developed between the approaches can be maintained.

*It’s a dangerous business, Sodium, crossing the air/water interface. You step into the solvent, and if you don’t know the real surface potential, there’s no knowing where you might be swept off to.*

— #LOTRYourResearch

High-five, you did it!

*Indeed so. Most indeedly.*

— Philip J. Fry, Futurama

In triumph, we sing!
Oh baby, Rowsdower saves us and saves all the world!

— Tom Servo, Mystery Science Theater 3000
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Part II

Appendices
A  CHARGE TRANSFER ENERGIES FOR $X^\pm(\text{H}_2\text{O})_n$

This supplementary section contains tables of all the charge transfer energy data using both the SM09 and conditioned nuclear potential methods that was referenced throughout the main body of the text. The amount of charge transfer between the ions and surrounding waters are expressed in millielectrons (me) and all energies are expressed in kcal/mol. A detailed breakdown of the relevant terms is described in each table’s caption. The methods used to compute each of these values were also discussed in the main text. These tables have been moved here because they require a different formatting to accommodate them properly.
Table A.1: Clusters are identified as in the main text by point group or structure-related code. Atoms in molecules estimated charge transfer in millielectrons to (+)/from (-) the ion is shown in the second column. The remaining columns represent energy quantities and are all given in units of kcal/mol. These charge transfer energies are breakdowns of the SM09 method first and the regularized estimate second. The SM09 energies are decomposed into 2nd-order CT energies at the Hartree-Fock level, followed by the correlation correction by itself, and finally the sum. The infinite-order HF correction to the 2nd-order induction energy is listed separately. Regularized CT energies are given using the aug-cc-pVDZ and aug-pc1 basis sets and can be compared to the $CT_{SM09}^{(2)}$ energy directly.
### Table A.2: Clusters are identified as in the main text by point group or structure-related code. Atoms in molecules estimated charge transfer in millielectrons to (+)/from (-) the ion is shown in the second column. The remaining columns represent energy quantities and are all given in units of kcal/mol. These charge transfer energies are breakdowns of the SM09 method first and the regularized estimate second. The SM09 energies are decomposed into 2nd-order CT energies at the Hartree-Fock level, followed by the correlation correction by itself, and finally the sum. The infinite-order HF correction to the 2nd-order induction energy is listed separately. Regularized CT energies are given using the aug-cc-pVDZ and aug-pc1 basis sets and can be compared to the $\text{CT}_{\text{SM09}}^{(2)}$ energy directly.

<table>
<thead>
<tr>
<th>Cluster</th>
<th>$\delta q(X^\pm)$</th>
<th>$\text{CT}_{\text{SM09}}^{(2)}$</th>
<th>$\text{CT}<em>{\text{SM09}}^{\epsilon</em>{\text{corr}}}$</th>
<th>$\text{CT}<em>{\text{SM09}}^{(2)+\epsilon</em>{\text{corr}}}$</th>
<th>$\delta_{\text{HF}}^{(2)}(\text{DCBS})$</th>
<th>$\text{CT}_{\text{Reg},\text{aDZ}}^{(2)}$</th>
<th>$\text{CT}_{\text{Reg},\text{pc1}}^{(2)}$</th>
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<td>-8.96</td>
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Table A.3: Clusters are identified as in the main text by point group or structure-related code. Atoms in molecules estimated charge transfer in millielectrons to (+)/from (-) the ion is shown in the second column. The remaining columns represent energy quantities and are all given in units of kcal/mol. These charge transfer energies are breakdowns of the SM09 method first and the regularized estimate second. The SM09 energies are decomposed into 2nd-order CT energies at the Hartree-Fock level, followed by the correlation correction by itself, and finally the sum. The infinite-order HF correction to the 2nd-order induction energy is listed separately. Regularized CT energies are given using the aug-cc-pVDZ and aug-pc1 basis sets and can be compared to the CT_{SM09} energy directly.

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<th>Cluster</th>
<th>δq(X\pm)</th>
<th>CT_{SM09}^{(2)}</th>
<th>CT_{SM09}^{e_{corr}}</th>
<th>CT_{SM09}^{(2)+e_{corr}}</th>
<th>δ_{HF}^{(DCBS)}</th>
<th>CT_{Reg,aDZ}^{(2)}</th>
<th>CT_{Reg,pcl}^{(2)}</th>
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Table A.4: Clusters are identified as in the main text by point group or structure-related code. Atoms in molecules estimated charge transfer in millielectrons to (+)/from (-) the ion is shown in the second column. The remaining columns represent energy quantities and are all given in units of kcal/mol. These charge transfer energies are breakdowns of the SM09 method first and the regularized estimate second. The SM09 energies are decomposed into 2nd-order CT energies at the Hartree-Fock level, followed by the correlation correction by itself, and finally the sum. The infinite-order HF correction to the 2nd-order induction energy is listed separately. Regularized CT energies are given using the aug-cc-pVDZ and aug-pc1 basis sets and can be compared to the CT$_{\text{SM09}}^{[2]}$ energy directly.
This supplementary section contains histograms of the distribution of interaction energies taken from SAPT(KS)-D3 calculations that was referenced in the main body of the text. All the energies are expressed in kcal/mol. These graphs were relocated to minimize disruptions in the main text.
Figure B.1: SAPT(KS)-D3 electrostatic (top) and exchange (bottom) energies of (LiEC₄)⁺. All energies in kcal/mol.
Figure B.2: SAPT(KS)-D3 induction (top) and exchange-induction (bottom) energies of (LiEC₄)⁺. All energies in kcal/mol.
Figure B.3: SAPT(KS)-D3 interaction energies of (LiEC$_4$)$_+^+$ with dispersion and $\delta_{\text{HF}}^{(2)}$ included. All energies in kcal/mol.
Figure B.4: SAPT(KS)-D3 electrostatic (top) and exchange (bottom) energies of (LiPC_4)^+. All energies in kcal/mol.
Figure B.5: SAPT(KS)-D3 induction (top) and exchange-induction (bottom) energies of (LiPC$_4$)$^+$. All energies in kcal/mol.
Figure B.6: SAPT(KS)-D3 interaction energies of (LiPC$_4$)$_+^+$ with dispersion and $\delta^{(2)}_{HF}$ included. All energies in kcal/mol.
Document presents tables of all the single-ion free energy components (and errors) and totals on both the intrinsic and bulk free energy scales for water, DMSO, and 1,2-DCE. All units are kcal/mol. Packing and local potential with errors are listed in the caption for each table. The mean-field and third order contributions to the local potential are also reported. These data were used to generate several of the figures in the main text.
Table C.1: Single-ion free energies of all ions in SPC/E water using the QCT method. Ion labels are as they were given in the main text. By column, we report inner shell, outer shell, intrinsic, and bulk free energy quantities all in kcal/mol. The intrinsic free energy is the resultant sum of the inner and outer shell figures and also includes the packing term evaluated as 82.77 ± 0.63 kcal/mol. Bulk free energy quantities differ only by inclusion of a \(-q\phi_{lp}\) term for the local potential, where q is the ion charge and \(\phi_{lp}\) was determined to be 8.95 ± 0.14 kcal/mol-e at the center of a 0.9 nm radius cavity. \(\phi_{lp}\) is constructed from the sum of the first and third cumulants which are 8.76 kcal/mol-e and 0.20 kcal/mol-e, respectively.

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Table C.2: Single-ion free energies of all ions in OPLS/AA dimethyl sulfoxide using the QCT method. Ion labels are as they were given in the main text. By column, we report inner shell, outer shell, intrinsic, and bulk free energy quantities all in kcal/mol. The intrinsic free energy is the resultant sum of the inner and outer shell figures and also includes the packing term evaluated as 83.50 ± 0.91 kcal/mol. Bulk free energy quantities differ only by inclusion of a -$q\phi_{lp}$ term for the local potential, where $q$ is the ion charge and $\phi_{lp}$ was determined to be 6.44 ± 0.24 kcal/mol-e at the center of a 0.9 nm radius cavity. $\phi_{lp}$ is constructed from the sum of the first and third cumulants which are 6.71 kcal/mol-e and -0.27 kcal/mol-e, respectively.

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Table C.3: Single-ion free energies of all ions in OPLS-AA 1,2-dichloroethane using the QCT method. Ion labels are as they were given in the main text. By column, we report inner shell, outer shell, intrinsic, and bulk free energy quantities in kcal/mol. The intrinsic free energy is the resultant sum of the inner and outer shell figures and also includes the packing term evaluated as 49.45 ± 0.56 kcal/mol. Bulk free energy quantities differ only by inclusion of a -$q\phi_{lp}$ term for the local potential, where $q$ is the ion charge and $\phi_{lp}$ was determined to be -1.26 ± 0.20 kcal/mol-e at the center of a 0.9 nm radius cavity. $\phi_{lp}$ is constructed from the sum of the first and third cumulants which are -1.42 kcal/mol-e and 0.16 kcal/mol-e, respectively.
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Part III

References
# List of References


[366] V. Saunders and M. Guest. ATMOL 3 program system. *Computational Science Group, SERC, Daresbury Laboratory*.


