I, Sami Alshehri, hereby submit this original work as part of the requirements for the degree of Doctor of Education in Curriculum & Instruction.

It is entitled:
The Comparison of Physical/Virtual Manipulative on Fifth-Grade Students’ Understanding of Adding Fractions

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The Comparison of Physical/Virtual Manipulative on Fifth-Grade Students’ Understanding of Adding Fractions

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by

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ABSTRACT

The purpose of this quantitative study was to compare two types of manipulatives in order to see their effects upon understanding of adding fractions for three comparable groups of fifth grade students. A total of 163 students who demonstrated low mathematical performance participated in the project in order to learn the addition of fractions by using physical and virtual manipulatives for the experimental groups and the normal mathematic curriculum for the control group. The intervention occurred during a two-week time frame in six public elementary schools in Abha, Saudi Arabia where students used fraction bars for both physical and virtual manipulatives in order to build conceptual understanding of adding fractions properly. Instructions were provided to all the participants directly regarding what the participants were to do each day of the 2-week experiment.

Pre-and post-tests, an attitude survey, and a preference survey were the instruments that were used to collect data during the study. A repeated measures design with a cross over treatment was used for comparing the effects of the two modes of treatments, virtual and physical manipulatives, compared to a control group for the understanding of adding fractions for the three groups of students. Overall findings revealed that fractions performance differed significantly as a function of use of manipulative, $F(4, 320) = 506.49, p < .001, \eta^2 = .86$. Also, findings revealed that fractions performance was significantly better after students were exposed to either virtual or physical manipulatives. In addition, results indicated that fractions performance was significantly better after students were exposed to both types of manipulatives, $F(1, 161) = 1452.59, p < .001, \eta^2 = .90$. The change in the final scores indicate that using fraction bars as a manipulative tool can be helpful in teaching the concept of adding fractions because students build a better conceptual understanding of the concept of fractions.
COMPARISON OF PHYSICAL/VIRTUAL MANIPULATIVE

DEDICATION

To my wife Asma, your support throughout these many years has been resolute. Your understanding has been unwavering, and your encouragement has been infinite. Thank you for being there for me, especially in times of despair.

To my parents Musbah & Aishah, from any early age you instilled in me values that have molded and shaped my life. These values to date dedicate my aspirations, my achievements, and my resilience in persevering relentlessly through obstacles. From you I have learned to work hard and dedicate myself to accomplishing my life goals. Without such values I could not have accomplished this task. For that and more, I thank you.

To my children Yazeed, Yazan, Waleed, and Wisam, you are my joy and inspiration. Over the course of journey, as I watched you grow into young adults and childhood, your own accomplishments encouraged my efforts in realizing this goal. Thank you for your unconditional love and encouragement. You made the journey worthwhile and I love you dearly.

To my siblings, thank you for your inquiries as to my progress, your expressions of love, and your constant encouragement. I hold a deep appreciation for you love and caring. I could not have done this without all of my family’s help.
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CHAPTER 1

Introduction

It is common for students with or without disabilities to have difficulty learning and understanding fractions concepts (Flores & Kaylor, 2007; Steencken & Maher, 2002; Tourniare & Pulos, 1985). A major reason for this is because instruction is based on rules and procedural computation rather than conceptual understanding (Mills, 2011), which can lead to the student feeling that fractions are meaningless (Brown & Quinn, 2006). Mercer & Miller (1992) stated that math difficulties emerge in elementary school grades and continue as students progress through secondary school grades. Much of the confusion with fractions appears to come from different interpretations (constructs), representations (models), and coding conventions (Kilpatrick, Swafford, & Findell, 2001). In addition, a lack of fractions knowledge causes challenges for students in their future schooling (Brown & Quinn, 2006; Davis, Hunting, & Pearn, 1993; Empson, 2003; Lamon, 2007; Razak, Noordin, Alias, & Dollah, 2012; Smith, 2002) because fractions assist with the development of proportional reasoning, which is important for mathematics (Clarke, Roche, & Mitchell, 2008). For instance, a lack of understanding of fractions can cause issues with algebra, measurement, ratios, and proportion concepts (Behr & Post, 1992; Lamon, 2007; Van De Walle, 2007). Furthermore, Wu (2001) confirms that the “no matter how much algebraic thinking is introduced in the early grades . . . the failure rate in algebra will continue unless the teaching of fractions and decimals is radically revamped” (p. 11).

Learning to identify and relate with fractions in general is not the only issue. There is a particular difficulty with adding fractions. This is because students must first obtain a conceptual knowledge of fractions, such as “part-whole” and fraction interpretations, before they can move
forward to fraction operations, such as adding (Fuchs et al., 2013). In addition, students have difficulty with determining the size of a fraction. If a student understands fraction sizes, it is easier to rationalize that computations of fractions are calculated differently than the computations of whole numbers (Clarke et al., 2008). If students had a better conceptual understanding of these fraction basics, they would be able to more easily understand how to add fractions (Bruce, Chang, Flynn, Yearley, & Lakelands, 2013).

Since students need a basic conceptual understanding of fractions, many researchers suggest the use of manipulatives, physical objects used to represent mathematical ideas, to facilitate and represent fractions (Butler et al., 2003; Cramer, Post, & delMas, 2002; Reimer & Moyer, 2005). Concrete manipulatives, also called “physical” manipulatives, have been around for a long time. “Research on physical manipulatives showed that students had higher mathematics achievement and better attitudes towards mathematics when physical manipulatives were implemented into instruction” (Ozel, Ozel, & Cifuentes, 2014, p. 3). However, computers and the Internet make new methods for using manipulatives an alternative tool for teaching students fraction concepts. These virtual, technological tools can visually clarify the concepts of fractions to assist students with learning arithmetic operations of fractions (Roblyer & Doering, 2009). Thus, virtual manipulatives have become the new recommendation after many years of using physical manipulatives to improve students’ academic achievement in mathematics (Aql, 2011; Ash, 2004; Bayturan & Kesan, 2012; Pilli & Aksu, 2013; Spradlin & Ackerman, 2010; Tienken & Wilson, 2007; Traynor, 2003). These research studies have shown that there is a positive relationship between the use of software technology and students’ academic achievement improvement in mathematics.
Taking the above issues into consideration, the following is a research study that examines the effect of manipulatives on teaching fifth graders how to understand the adding of fractions. The manipulative “fraction bars” is used in both physical and virtual form. The purpose is to determine if fraction bars are a useful manipulative in teaching how to add fractions, as well as examine whether virtual or if physical fraction bars are more useful.

**Background and Problem of Statement**

**Fractions**

One of the most difficult mathematical topics for students with mathematical learning difficulties has been the study of fractions. This is because fractions do not follow the same rules which children have established and used in their study of whole numbers. According to Gallistel & Gelman (1992), when students are exposed to fractional numbers, they tend to simply read the numerator and denominator as two separate wholes (i.e. \( \frac{3}{4} \) as “three” and “four”) and fail to see them as a fraction of a whole number. These errors can occur despite using terms such as “one-half” in everyday situations outside of the classroom. Unfortunately, numerous studies have shown that students have difficulty understanding fractions (Mazzocco & Devlin, 2008; NMAP, 2008). Even before formal schooling, children conceptualize the use of fractions in everyday life (Smith, 2002). This form of conceptualization continues for students until they are well into their high school years. However, even with this early introduction, many students still have difficulty conceptualizing fractions, which can become a barrier for mathematical maturation in the future (Aksu, 1997; Behr, Harel, Post, & Lesh, 1992). As Bruce et al. (2013) explain, “We must make strides through mathematics educational research and classroom practice to ameliorate this situation” (p. 7).
While learning fractions in school, students are supposed to acquire both conceptual and procedural knowledge (NMAP, 2008). Conceptual knowledge refers to how students understand a topic and create relationships and links with previously learned information. Procedural knowledge refers to a set of rules or algorithms that students follow in order to solve a particular problem (Miller & Hudson, 2007). For example, using a number line to understand fractions, create fraction equivalences, and a perform magnitude comparison of fractions is indicative of conceptual knowledge regarding fractions. Procedural knowledge, on the other hand, involves the actual procedures of addition, subtraction, multiplication, and division of fractions. The National Mathematics Advisory Panel (NMAP) (2008) recommends that both conceptual and procedural knowledge needs to be taught because they are interdependently critical to the success of students’ understanding of fractions.

It stands to reason that teachers need to focus more on the conceptual aspect of fractions in order to assist their students. This is because a student who learns conceptually is able to incorporate and apply definitions, principles, rules and theorems, as well as compare and contrast related concepts (Hallett, Nuñes, & Bryant, 2010). Lamon (1999) hypothesizes that “as one encounters fractions . . . suddenly meanings, models, and symbols that worked when adding, subtracting, multiplying and dividing whole numbers are not as useful” (p. 22). This causes a great deal of confusion in primary school students, which continues through high school and even into adulthood (Riddle & Rodzwill, 2000). Ohlsson (1988) tries to explain this difficulty with fractions by stating, “The complicated semantics of fractions is, in part, a consequence of the composite nature of fractions. How is the meaning of 2 combined with the meaning of 3 to generate a meaning of 2/3?” (p. 53). Further, Ohlsson (1988) explains the difficulty with
understanding and retaining fractions knowledge is partially due to the “bewildering array of many related but only partially overlapping ideas that surround fractions” (p. 53).

The mathematics content in Saudi Arabia is similar to that of the United States, specifically the unit of fractions which includes equivalent fractions, comparison of fractions, operations of fractions, and so on (Alshahrany, 2015). Although the fractions unit represents approximately one-fifth of the content of the entire mathematics curriculum for fifth grade (Alshahrany, 2015; Kabli, 2013), research attests to the fact that performance of fifth grade students in learning fractions is inadequate and students struggle with basic mathematical operations of fractions. Forty-six percent of fifth grade students were not able to add two fractions with the same denominators, and 63% students were not able to add two fractions with different denominators (Ahmad, 2012). This is due primarily to limited resources, which leads to the absence of effective strategies in math classrooms (Alshahrany, 2015) and students' inabilitys to identify the common denominator and convert the mixed numbers to improper fractions (Almogerah and Al-Mohaimeed, 2013).

A small number of studies targeting conceptual issues related to understanding fractions and how elementary age students comprehend the role manipulatives play in understanding fractions have been found globally. Fuchs et al. (2013) believe students have a difficult time understanding fractions because they do not grasp a conceptual understanding of fractions; therefore, children, ages 8–11, simply revert to their old habits of solving mathematical problems. This habit includes seeing whole numbers when they attempt to add or subtract fractions and seeing fractions as a single quantity (Bogen, 2008; Newstead & Murray, 1998). The learning shift from whole numbers to fractions is a conceptual understanding shift that each student needs to master.
In order for students to learn how to add fractions, it is essential for them to have a basic understanding of fractions, such as part-whole relationships, the difference between a numerator and a denominator, and fraction equivalencies. Understanding part-whole relationships is one of the first aspects of fractions knowledge that students must understand. Many times students are able to partition correctly a whole object into pieces to represent a fraction (i.e., shade three parts of a rectangle divided into four parts to represent \( \frac{3}{4} \)); however, they still ignore the whole part (the entire rectangle) as an essential piece of information (Bruce et al., 2013). In the early stages of fraction knowledge, a conceptual understanding of part-whole relationships, where “a fraction is understood as a part of one entire object or a subset of a group of objects” is essential (Fuchs et al., 2013, p. 2). When students lack the ability to justify proper procedures or reason why a process works, in reference to fractions, the student will find it hard to relate to conceptual understanding (Bruce et al., 2013).

After understanding part-whole relationships, students must understand the difference between a numerator and a denominator and not look at them as two separate whole numbers. “The numerator represents the count and the denominator represents the unit” (Bruce et al., 2013, p. 13). Specifically, students need to understand these two aspects of a fraction have different roles because children, ages 8–11, tend to use their existing knowledge of adding whole numbers when attempting to add fractions and see the fraction as a single quantity (Bogen, 2008; Newstead & Murray, 1998). Without this understanding, students have particular difficulty with improper fractions (e.g, \( \frac{4}{3} \)). Starting in the early grades, fractions are usually represented as being part of a whole, such as dividing a pie or rectangle. However, if the student does not understand what a numerator and denominator are, they will struggle with moving from the part-whole concept to improper fractions. An over-emphasis on part-whole could inhibit the
development of a students’ understanding of other aspects of fractions, such as quotients, fair-shares, and improper fractions (Charalambous & Pitta-Pantazi, 2007; Steffe, 2002).

In addition to confusing whole numbers and fractions, students also have difficulty with determining the size of a fraction. If a student understands fraction sizes, it is easier to rationalize that computations of fractions are calculated differently than computations of whole numbers (Clarke et al., 2008). For example, if students understood fraction sizes, they would easily be able to see that 2/6 (or 1/3) is smaller than ½, thus making 1/3 an obvious incorrect answer to the equation ½ + ¼. “When transitioning from whole number thinking to working with fractions, students need to develop a strong understanding of the multiple constructs of fractions. Without this, students may not understand the possible meanings of the numerator and of the denominator, and of the distinctions between them” (Bruce et al., 2013, p. 13).

Fraction equivalency is also an area of understanding that is necessary to move on to fraction computations. Students need to fully understand that ½ and 2/4 have the same meaning and require equal partitioning (Bruce et al., 2013). The NCTM (2006, 2009a, 2009b, 2009c) notes that students should develop an understanding of fraction equivalence by the third grade. However, some researchers feel that “minimal time is allocated to understanding the general concept of equivalence” (Bruce et al., 2013, p. 14).

**Use of Manipulatives**

Early research has shown that the use of manipulatives can significantly help children in understanding fraction concepts because they can create meaningful representations of fraction computations (Bezuk & Cramer, 1989). By linking various representations to mathematical concepts, the student is able to link a concrete representation with an abstract mathematical idea (Ozel et al., 2014). In examining various research studies, Ozel et al. (2014) found that
“interaction among representation modes supports students’ conceptual understanding” (p. 2). When representations such as manipulatives are used, research has shown that students’ have better mathematics scores, as well as improved attitudes towards mathematics (Clements, 1999; Ozel et al., 2014).

Teachers who possess the ability to link their actions to physical manipulatives that can be handled and arranged to help students understand abstract mathematical ideas, and who are able to teach students this concept, liberate students to think beyond what they are just able to see (Hynes, 1986; Moch, 2001; Moyer, 2002; Moyer, Bolyard, & Spikell, 2002; Stein & Bovalino, 2001; & Terry, 1996). When using this approach, children are able to think intuitively and combine thought with visualization of images through informal language. They develop the ability to extract mentally, and think about, fractional ideas without a strong dependence on the specific procedural requirements of the mathematical problem. Instead, they look at the problem from a logical point of view. Once children have developed a conceptual knowledge-base for fraction and operation sense, they can meaningfully learn, or even create for themselves, appropriate fraction algorithms (Sharp, Garofalo, & Adams, 2002). When students develop different strategies to resolve fractions operations, they expand their knowledge of flexibility in solving fraction operations meaningfully (Huinker, 2002).

Even though concrete (or physical) manipulatives have been around for some time and have been instructionally successful, some educators believe that it is important to transition to a technological/computerized version of these manipulatives, which are called “virtual manipulatives,” to keep up with today’s technology. As explained by Goodwin (2008), “technology permeates most young children’s daily lives and social milieu [and] there is a prominence of digital media in young children’s leisure activities” (p. 105). Just as they can with
concrete manipulatives, students can rotate, flip, turn, slide, and otherwise manipulate the objects, just in a virtual manner (Moyer et al., 2002). Because this is the era of technology, the Educational Department in Saudi Arabia is ensuring that technology is used in classrooms across the country (Al-Balawi, 2010). Furthermore, there is currently a strong push for technology use and integration in fourth through twelfth grade classrooms for learning, reinforcement, and enrichment (Oyaid & Alshaya, 2015). Therefore, combining the usefulness of manipulatives with the technological advancements to make them “virtual” manipulatives provides students increased benefit because they receive instant feedback and have more tools at their disposal than they would have with limited physical manipulatives (Edwards-Johnson, Campet, Gaber, & Zuidema, 2012).

Using these virtual, technological tools to clarify visually the concepts of fractions can assist students with learning arithmetic operations of fractions (Roblyer & Doering, 2009). Virtual manipulatives allow instructional designers to present the information in a logical sequence through a computer, which allows the student to learn by reading text, observing the visual information displayed, and by receiving immediate feedback (Traynor, 2003). This model provides more tools than concrete manipulatives, such as more options within the software, instantaneous feedback, multiple representations of fractions, and a variety of ways to experiment (Goodwin, 2008; Ozel et al., 2014; Petrick, Martin, & Peacock, 2010).

Using fraction bars for both physical and virtual manipulatives can help students build a better understanding of fractions (Van De Walle, 2007) because the use of fraction bars allows students to compare fractional proportions while developing symbolic modes of flexible strategies to generate ideas for solving fraction operations. Mack (2004) stated that when students are able to establish a clear understanding of fraction size with manipulatives, then they
can progress into adding fractions by using the visual representations. The ability for students to think proportionally in solving fraction computations is vital, and fraction bars help students develop this mindset (Lesh et al., 2003; Suh et al., 2005). Fraction bars are used to notate that there are different parts to the same whole, thereby enabling students to take apart and manipulate different parts of the fraction bars and allowing them to see how different parts can be added together or compared to different fractional equations. Utilizing different sizes and colors indicates to students that different strategies can be inserted to perform operations in solving problems (Way, 2011). Fraction bars’ vital effectiveness permits students to visualize different fractional relationships. Students are able to formulate solid understanding of mixed numbers, as well as fraction equivalency. Additionally, students are able to compare, order and examine number operations with fractions. Finally, fraction bars are instrumental in assisting students in discovering how fractions can be maneuvered as part of the same whole. This concept is imperative for students to be able to solve fraction operations.

However, there are some drawbacks to virtual manipulatives versus concrete manipulatives, which has led to the need for further research in this area. For example, some research has shown that the use of concrete manipulatives is actually more effective in helping students develop conceptual knowledge (Hunt, Nipper, & Nash, 2011). In addition, some research has shown that the use of virtual manipulatives leads to a lack of teacher input and minimizes teachable moments (Edwards-Johnson et al., 2012). On the other hand, when adding virtual manipulatives as a teaching method after concrete manipulatives are used, there has been evidence that this is even more advantageous because the virtual manipulative reinforces the conceptual knowledge, as well as bridges any gaps from conceptual to procedural knowledge (Hunt et al., 2011).
Attitudes and Perceptions of Using Manipulatives

The traditional classroom environment is becoming increasingly less “traditional” with changes in technology, developments in learning theory, and cultural considerations affecting how and what students are learning each school year. Changing, also, are educator and student attitudes and perceptions toward the use of manipulatives in the classroom (Golafshani, 2013). Foundational to the discussion of using manipulatives in the classroom is that educators and researchers agree manipulatives can enhance student-teacher interaction, classroom environment, and, of greatest significance, student learning. Research speaks to how manipulatives provide greater opportunities for students and teachers to engage in the learning process together, how the classroom can be transformed into a more creative, hands-on, group-learning environment, and how students at different levels of aptitude are able to grasp various levels of instruction in one classroom setting (Ross, 2008; Olkun & Toluk, 2004; Weiss, 2006).

In the education process, teachers and students repeatedly engage in a series of instruction and learning cycles where teachers present concepts and students either process or reject those concepts. Both aspects of presenting and processing are constantly being considered by teachers in order to improve teaching and student comprehension (Mutodi & Ngirande, 2014). When teachers incorporate manipulatives into the learning process, students engage in learning by transitioning from manipulating materials to creating images from students’ perceptions of concepts to developing symbols to represent each new concept (Collins, 2011). From this reliable yet underutilized method of instruction, teachers and students together can experience learning success, which improves the interaction between the two. For instance, Moyer (2002) observed that in lessons where manipulatives were used, students appeared fascinated, active, and involved. Tymms (2001) also confirms that when students experience success in learning,
their attitudes toward learning become more positive. Further, according to Golafshani (2013), “A teacher’s affective domain will direct her or his choice of teaching strategy in the classroom; then whether the choice is appropriate or not, the teacher’s strategy will influence student learning and student attitudes about mathematics and mathematics learning” (p. 141).

From the teacher’s perspective, manipulatives enhance the student-teacher relationship by enabling students to engage with the teacher in a learning experience. Rather than relying solely on traditional instructional styles, teachers can bring visualization of concepts using hands-on activities into the learning process (Mutodi & Nhirande, 2014). In turn, Goonen and Pittman-Shetler (2012) agree when teachers add pictorial and abstract representations of concepts to their lesson plans, students are more likely to master the concepts being presented. These concrete explorations through touching, seeing, and doing, prove to have a more lasting effect in comprehension of mathematical concepts (Goonen & Pittman, 2012). When using manipulatives to teach mathematics, in particular, teachers recognize students “learning mathematics in a more enjoyable way” (Furner, Yahya, & Duffy, 2005, p. 17) as students discover connections between what they have been taught and what they are learning. McNeil and Jarvin (2007) conclude that manipulatives connect children with real-world knowledge and increase student memory and comprehension.

Educators understand the benefits of creating in the classroom environments that are conducive to learning. Indeed, much of what fosters a conducive learning environment is ensuring students are focused on the process of mastering concepts (Chouinard, Karsenti, & Roy, 2007) and by actively engaging students in their own learning process (Moyer, 2002). Using manipulatives to teach mathematics can aid in creating this type of positive learning environment in the classroom (Cockett & Kilgour, 2015). Pham (2015) reports that the hands-on aspect of
using manipulatives as well as the active seeking of mathematical theories inherent in lessons with manipulatives, can lead students in greater learning achievements. Further, when engaged with learning through manipulatives, students experience greater success with comprehension and student success, in and of itself, can contribute to fostering a positive classroom environment. That is, students’ success is a significant contributing factor to creating the type of classroom necessary to achieve success overall in mathematics instruction (Chouinard, Karsenti, & Roy, 2007).

By far, the most important factor to consider in any discussion regarding approaches to instruction is whether a student of instruction is actually learning the concepts being presented. Much research exists that speaks to how the use of manipulatives aids not only in student learning but also in students’ desire to learn and in their desire to apply what they have learned to more complex, even real-life, applications. Stein and Bovalino (2001) have observed manipulatives are a useful tool in facilitating student comprehension and developing students’ critical thinking and life application skills. Dennis (2011) confirms as well that when teachers use manipulatives in mathematical instruction, students are more flexible in their thinking, thus making them more likely to engage in problem solving while being relieved of mathematical anxiety that often hinders conceptual understanding. In addition to these benefits of using manipulatives in mathematical instruction, there is also evidence that reveals manipulatives improve advanced students’ abilities to construct deeper meanings of math concepts while also helping struggling students visualize concepts to move toward basic comprehension (Couture, 2012). According to McIntosh (2012), “It is clear that even with minimal exposure, students of all intelligence levels can benefit greatly from the use of manipulatives” (p. 6).
Nevertheless, there remain some obstacles with educators being able to maximize the benefits of using manipulatives. The most dangerous perception to which many educators cling is that manipulatives do not need teacher engagement. Educators must get over the notion that just inserting manipulatives when working with low performing students will automatically improve the student’s performance. The manipulatives must be integrated appropriately with the learning need of each particular student. That is, the manipulative idea to be taught must meet the area of improvement needed by the student.

The teacher’s guidance is essential to the student’s learning experiences (Ball, 1992; Clements & McMillen, 1996; Thompson, 1994). This fact applies to both forms of manipulatives (physical and virtual). The correct mathematical idea, matched with the appropriate manipulative, allows students to visualize concepts of the mathematical idea. The more in synch the mathematical idea and the manipulatives, the higher the level of conceptual support that will be developed by the student in his/her learning . . . the connection will be found by the student (Clements, 1999; Hiebert & Carpenter, 1992).

**Teaching Methods of Fractions**

Research has shown that teachers tend to stick to what they are comfortable with, which are “traditional” teaching methods (Takahashi, 2002; Windschitl, 2002). This results in educators trying to use manipulatives in a procedural rather than conceptual way (Schorr, Firestone, & Monfils, 2001; Stein & Bavalino, 2001). Much of the current use of manipulatives is based on a common fallacy among teachers and other educators that using manipulatives result in an automatic understanding of what educators want students to comprehend, meaning that mathematical truths can be directly “seen” through the use of concrete objects (Ball, 1992; Meira, 1998). Indeed, manipulatives are not, in and of themselves, carriers of meaning or insight.
When incorporating manipulatives into a learning environment, educators must guide students to derive intended mathematical concepts from the application. This will help students internally represent ideas and connect them with the external representations provided by the manipulative (Moyer, 2002). In other words, the specific activities used within the classroom influence the effectiveness the manipulative has on the students (Hiebert & Carpenter, 1992; Spikell, 1993).

Determining the appropriate match of the manipulative with the fitting mathematical idea should become the main objective of teachers for each individual student that needs intervention (Gibson, 1977; Takahashi, 2002; Watanabe, 2006). What is evident is that a number of teachers do not know how to choose the suitable manipulative (Ball, 1992; Moyer, 2002). Therefore, teachers need to be assisted in learning how to choose the appropriate manipulative that will be beneficial to each student’s learning. Unfortunately, no studies exist that justify the advantages or disadvantages of specific manipulatives, which could assist teachers in making these decisions. Only suggestions found in a few studies briefly mention or suggest that this approach should be used (Spikell, 1993).

Teachers should not be put into a position of assumption. Relying on surface analysis is not the answer (Goldin & Shteingold, 2001). Educators need to understand the strengths and limitations of different manipulatives so that they are able to pair the complementary mathematical idea with a manipulative that matches the student (Ball, 1993; Baroody, 1989; Kaplan, Yamamoto, & Ginsburg, 1989; NCTM, 2000). Based upon a thorough understanding, educators will be able to select whether a physical or virtual manipulative will be appropriate with the learning mathematical idea that he/she will have in mind.
Definitions of Terms

The following definitions are provided for terms that have special application to this study. These terms and definitions are extensively reviewed in Chapter 2 and discussed in Chapter 3.

- **Conceptual Understanding** – Mathematical teaching and mental constructs that focus on concepts, problem solving, and making connections (Star, 2005).

- **Constructivism** – The building of knowledge from previous knowledge structures (Sriraman & Lesh, 2007).

- **Experimental Groups** – Students learning to understand how to add fractions with the assistance of physical and virtual manipulatives of fraction bars.

- **Fraction** – “A mathematical entity that has multiple meanings and representations. It is commonly represented in the form of a/b, where a and b are integers and b is non-zero. The concept of fractions can be interpreted five ways: part-whole, measure, ratio, operator, and quotient” (Kong, 2008, p. 887). This study is focused on part-whole only.

- **Fraction Bars** – Fraction bars as a concrete manipulative are blocks that represent parts of a whole (i.e. fractions) to assist students in understanding fraction equality, inequality, addition, subtraction, division, and multiplication of fractions. Virtual manipulatives of fraction bars provide the same concepts in virtual form. Fraction bars allow for students to compare fraction sizes and determine equivalence by stacking them (if they are concrete) or sliding them over each other (if they are virtual) (Kong & Kwok, 2005).

- **Intervention** – In research studies, intervention can be defined as a manipulation of the subject or the subject’s environment (Florida State University, n.d.). For this study, the intervention is the introduction of the concrete and virtual manipulative “fraction bars.”
• **Manipulatives** – Physical (also called “concrete”) or virtual objects used by students to represent components of mathematical concepts (Moyer, 2002).

• **Physical/Concrete Manipulatives** – Objects to be handled and arranged by students and teachers that are used to convey abstract ideas or concepts by modeling or representing these ideas concretely (NCTM, 2000). Manipulatives include an array of items such as tangrams, number cubes, 3-D models, and fraction circles.

• **Virtual Manipulatives** – Applets or computer programs typically available on websites that students manipulate to better understand a mathematical concept. Virtual manipulatives are often similar to their concrete/physical counterparts (Moyer-Packenham, 2010).

**Purpose of Study**

The purpose of this research study was to determine if the manipulative “fraction bars” has an effect on fifth graders conceptual understanding of adding fractions. Since there is limited research on concrete versus virtual manipulatives, one control group and two treatment groups were compared. The control group learned the addition of fractions without manipulative tools. The first treatment group learned via the physical fraction bars manipulative and the second treatment group learned via the virtual fraction bars manipulative. Then, both treatment groups were switched to determine if there was a statistically significant difference between the groups.

**Objectives**

The main objective of this research study was to determine if the virtual and physical manipulative “Fraction Bars” would help fifth grade students form a better conceptual understanding of adding fractions. More specifically, the objectives of this study were:

1. To determine if one of the two manipulatives -physical or virtual- (Fraction Bars) is more effective for fifth graders’ conceptual understanding of the adding of fractions.
2. To determine whether or not the virtual manipulative “Fraction Bars” affects fifth graders’ conceptual understanding of the adding of fractions.

3. To determine whether or not the physical manipulative “Fraction Bars” affects fifth graders’ conceptual understanding of the adding of fractions.

4. To determine whether or not the use of the virtual and physical manipulatives “Fraction Bars” affects fifth graders’ conceptual understanding of the adding of fractions.

5. To determine whether or not the use of the physical and virtual manipulatives, "Fraction Bars", improve students' attitudes toward the understanding of adding fractions.

6. To determine whether students prefer the use of physical manipulative or virtual manipulative when learning the addition of fractions.

**Research Questions**

The research questions that have been examined in this study were:

1. Are there differences in students’ understanding of adding fractions when they are taught traditionally compared to when they are taught using physical or virtual manipulatives?

2. Does using the virtual manipulative “Fraction Bars” help fifth grade students develop a better conceptual understanding of how to add fractions?

3. Does using the physical manipulative “Fraction Bars” help fifth grade students develop a better conceptual understanding of how to add fractions?

4. What effect do the virtual and physical manipulatives "Fraction Bars" have on students’ understanding of adding fractions when use them consecutively?
5. What attitudes do students hold about the addition of fractions before and after using the physical and virtual manipulatives?

6. What type of manipulatives (physical/virtual) do students prefer when learning the addition of fractions?

**Hypotheses**

The following hypotheses have been examined:

**H1o**: There are no significant differences in fifth graders’ conceptual understanding of the adding of fractions between the control, virtual, and physical manipulatives groups.

**H1A**: There are significant differences in fifth graders’ conceptual understanding of the adding of fractions between the control, virtual, and physical manipulatives groups, thus showing one of the three groups is more effective than the others.

**H2o**: There is no significant difference in fifth graders’ conceptual understanding of the adding of fractions when using the virtual manipulative “Fraction Bars” compared to the control group.

**H2A**: There is a significant difference in fifth graders’ conceptual understanding of the adding of fractions when using the virtual manipulative “Fraction Bars” compared to the control group.

**H3o**: There is no significant difference in fifth graders’ conceptual understanding of the adding of fractions when using the physical manipulative “Fraction Bars” compared to the control group.

**H3A**: There is a significant difference in fifth graders’ conceptual understanding of the adding of fractions when using the physical manipulative “Fraction Bars” compared to the control group.
H4o: There is no significant difference in fifth graders’ conceptual understanding of the adding of fractions when using the virtual and physical manipulatives “Fraction Bars” consecutively, compared to the control group.

H4A: There is a significant difference in fifth graders’ conceptual understanding of the adding of fractions when using the virtual and physical manipulatives “Fraction Bars” consecutively, compared to the control group.

H5o: There is no significant difference in fifth graders’ attitudes toward understanding the adding of fractions when using the virtual and physical manipulatives “Fraction Bars” consecutively, compared to the control group.

H5A: There is a significant difference in fifth graders’ attitudes toward understanding the adding of fractions when using the virtual and physical manipulatives “Fraction Bars” consecutively, compared to the control group.

H6o: There are no significant differences in representation preferences between physical and virtual manipulatives.

H6A: There are significant differences in representation preferences between physical and virtual manipulatives.

**Significance of the Study**

Although there are a significant number of research studies on how to learn fractions, more in-depth research is still needed (Lamon, 2007). Because there is no research on the understanding of adding fractions by young students in Saudi Arabia, this study focused on fifth grade Saudi Arabian students to determine their understanding as they learned about the addition of fractions. The data gathered can be used to guide teachers on how to support students’ process of knowledge construction when teaching fractions concepts.
There seem to be many more in-depth studies regarding use of physical manipulatives than virtual manipulatives. Use of virtual manipulatives has recently come to the forefront of fraction studies due to the use of technology by both teachers and students. Not many published works can be found that compare virtual manipulatives to physical manipulatives, specifically for elementary education. As of now, based upon available research studies, it is almost impossible to separate the different forms of manipulatives and their effects on students. A critical factor regarding this study is that of seeking to pinpoint and distinguish the effect of virtual and physical manipulatives. The approach to accomplishing this goal hinges upon controlling the other variables such as the number of teachers, management of the school environment, instructional scripts, type of practice activities, and the time period for utilizing the manipulatives. To date, no published work of comparing physical manipulatives to virtual manipulatives of fraction bars exists. This study addresses a gap that currently exists for providing educators with appropriate studies to assist them in meeting low performing students’ needs to understand the addition of fractions.

Also, this study can guide parents, educators, and major decision makers towards developing new plans to help children learn how to conceptualize the process for solving addition of fractions using fraction bars. The large sample size of this study may provide solid data to meet elementary schools need.

Results from this study can assist teachers in determining how useful physical and virtual manipulatives are as tools to help students master the concepts of adding fractions. Additional benefits include an increased understanding of the impact that manipulatives play in teaching the concept of adding fractions. Educators will be able to see more precisely when the different types
of manipulatives should be applied, including the best ways to use technology to improving student performances.
CHAPTER 2

THEORETICAL FRAMEWORK AND REVIEW OF RELATED LITERATURE

Introduction

This chapter is comprised of two sections: a conceptual framework of social constructivism and a critical review of the empirical literature. The conceptual framework examines the theoretical basis for the hypotheses tested in this study. The critical review of the empirical literature closely examines mathematical studies that compare physical manipulatives and virtual manipulatives. This chapter also contains a review of literature on mathematics in elementary school, conceptual understanding, fractions' understanding, difficulties of fractions, fraction equivalency, addition of fractions, the integration of technology into classrooms, and intervention. In addition, this chapter explores the deficiencies in the current literature and the potential for the proposed research study to expand upon the existing literature base. The review of literature provides the empirical foundation for the hypotheses tested in this study.

It was identified by the researcher that the studies included a critical review of various empirical literature through a thorough search of the math literature. Prior to beginning the comprehensive search, the researcher established a set of criteria for identifying potential empirical studies. The inclusion criteria encompassed the need for the work to be published within the last thirty years. Also, participants in the study needed to be enrolled in grades Kindergarten to 8. Furthermore, the study must have looked at manipulative-based mathematics instruction, while also including qualitative and quantitative methods in order to make a direct comparison between the physical and virtual manipulatives. Also, included articles were written by and for practitioners who were published in the journals of the National Council of Teachers of Mathematics (NCTM). A few Internet websites have been included in the review of literature.
Following the establishment of the aforementioned criteria, the researcher started to conduct the descriptor and keyword searches of the ProQuest, ERIC, Google Scholar, Dissertation Abstracts, and PhyschInfo databases. This was followed by obtaining hard copies or electronic copies of any study appearing to meet at least some of the criteria that were set out. The researcher then took a look at the literature review section and the bibliography of each of the studies in order to determine if they possessed other potential sources that were not identified through the database searches. Several of the mathematics studies met every criteria for inclusion. Further subsections of this chapter provide a critical review of each of these studies.

**Theoretical Framework**

Constructivism assumes learning is active and students themselves construct knowledge, unlike passive methods such as lectures and textbooks (Salkind, 2008). Social constructivism assumes knowledge is formed through social interaction (Powell & Kalina, 2009). Therefore, a constructivism learning theory style favors active learning, which allows learners to build their own conceptual understanding through applying concepts, constructing their own meanings, and thinking about ideas (Ernest, 1996; Gordon, 2009). Too many students lack proficiency in mathematics as a result of traditional methods. If mathematics instruction were taught from a constructivism viewpoint rather than rote memorization, repetitive drills, and lectures, students would be encouraged to create their own understanding of the subject through social interaction and meaningful activities (Andrew, 2007).

Social constructivism theory states that the subject matter and skills developed during the lesson should be relevant to the student (Doolittle & Hicks, 2003). Personal relevancy will cause the student to become more attentive and willing to learn, thus allowing the student to become more invested in the subject matter he/she is learning (Ediger, 2000). Therefore, teachers who
utilize a social constructivist model present their students with practical real-life examples. These teachers facilitate learning in such a way that students are able to learn through self-exploration. The student is thus able to engage in hands-on experience and investigation, which facilitates a higher level of thought and cognitive connection (Kim, 2001). Physical and virtual manipulatives can aid in this process.

Social constructivism creates the ability for lesson plans to be student-centered. The importance of student-centered instruction has been demonstrated in the constructivist pedagogy (Driscoll, 1994). Additionally, if these student-centered activities are integrated, the student will be able to promote greater confidence in his/her knowledge and feel more connected to the subject that they are studying (Eggen & Kauchak, 1997). Such issues are common with mathematics, especially fractions (Behr & Post, 1992; Brown & Quinn, 2006; Davis, Hunting, & Pearn, 1993; Lamon, 2007; Verschaffel, Greer, & Torbeyns, 2006; Young-Loveridge, Taylor, Hawera, & Sharma, 2007). At this point, the teacher can then turn the classroom into a “learning community” (Eggen & Kauchak, 1997), which is supportive of the constructivist theory of learning that intends for students to be active participants in their own learning in order to help them reach new horizons of understanding and knowledge (Cobb, Yackel, & Wood, 1992; Oldfather, West, White, & Wilmarth, 1999; Ernest, 1994).

Jean Piaget, one of the most prolific psychologists in the twentieth century, proposed that learning occurs as a result of external experiences (Wood, Smith, & Grossniklaus, 2001). Ojose (2008) described how Piagetian theory has provided mathematics educators with insights into how children learn mathematical concepts during developmental stages. During the concrete operational stage of cognitive development, children begin to understand two- and three-dimensional concepts concurrently instead of linearly. Piaget’s theory implies that the use of
representations, such as manipulatives, assist with the construction of meaning (Wood et al., 2001). Therefore, using manipulatives could aid in children fully exploring and understanding mathematical concepts by demonstrating the concrete experiences of ordering and grouping (Ojose, 2008). Additionally, students operating at concrete developmental levels may also benefit from the concrete aspects of manipulatives (Uttal, Scudder, & Deloache, 1997). Manipulatives help students lay the foundation for understanding more advanced mathematical concepts through experience. Through this, the process of learning new information is simplified and allows students generate new ideas (Bruner, 1966).

Researchers and educators who endorse constructivist learning theory encourage the use of manipulatives in mathematics education in order to engage students in active, participatory learning (Uttal et al., 1997; Wood et al., 2001). Research indicates that students must learn by doing and must understand mathematics in terms of real life (Gordon, 2009; Sriraman & Lesh, 2007), which constructivism and the use of manipulatives support. This idea is established by Chung (2012) when he states, “researchers and educators who believe in constructivism claim that learners lack conceptual understanding of fractions, which results in poor performance, low interest, and anxiety in mathematics. Therefore, students should have learning experiences with visual models and hands-on activity to gain better understanding of fraction concepts as well as fraction operations” (p. 1). As computerized learning methods allow students to engage, discover, interact and explore, the theoretical framework of social constructivism supports the use of computerized learning methods (Kim, 2001).

**Literature Review**

This section provides a critical review of the empirical literature, which provides the foundation for the hypotheses tested in this study. The critical review of the empirical literature
discusses math in elementary school, how students learn math, students’ attitude toward math, and students with math difficulties (MD). This section also discusses studies that compare physical manipulatives and virtual manipulatives, conceptual understanding, fractions' understanding, difficulties of fractions, fraction equivalency, addition of fractions, and the integration of technology into classroom, as well as intervention. In addition, it draws the reader’s attention to the deficiencies in the current literature and the potential for the proposed research study to expand upon the existing literature base.

Mathematics in Elementary School

The elementary school level in both public and private schools is generally comprised of students in grades kindergarten through 5th grade, or occasionally through 6th grade. The content of instructional programs is developed from national, state, and/or local curriculum requirements, instructional guidelines and materials, and relevant national, state, and/or local assessments. However, in spite of these measures, students continue to show low achievement in mathematics. Low achievement remains an educational concern, as mathematics is often necessary for success in the professional world (Little, 2009). In recent years, many professions have required greater proficiency in mathematics and technical skills (Kloosterman, 2010). Additionally, mathematical concepts are often central to digital media, which younger generations are moving towards. Teaching mathematics in elementary school, students’ attitudes toward mathematics, and students with math learning difficulties are discussed below.

Teaching Mathematics in Elementary School

In order to positively impact student performance in mathematics, teachers must utilize teaching skills in an effective and knowledgeable way. Teachers must understand how mathematics works in order to teach effectively (Wood, 2005). Additionally, a critical need
exists for content-knowledgeable teachers at the 5th grade level, as the curriculum requirements are more demanding at this level than in the lower grades. Teachers who are proficient in practice are critical to positively impact student achievement and facilitate student mastery as they gain skills necessary for further grade levels (Slavin et al., 2009).

Teaching mathematics is heavily dependent on the environment in which the subject is taught. Students need an interactive environment conducive to learning in order for them to develop a rich understanding of mathematics (Samuelsson, 2008). Most importantly, the classroom environment should provide students with many opportunities to explain the concepts in their own words, as this strategy promotes self-directedness as well as better understanding of mathematical concepts (Wilkins & Ma, 2003). Additionally, students should be encouraged to work together toward a common goal in the classroom to remain fully engaged. Research has shown that it is beneficial for students to hear the problem-solving strategies of their peers (Tabernik & Williams, 2010). A study by Sorden (2005) suggests that increases in cognitive and affective learning were associated with positive mathematics instruction in the classroom setting.

Research has demonstrated that teachers’ attitudes towards mathematics can play a crucial role in how students perform in mathematics (Tahar et al., 2010). Teachers should strive to empower all students regardless of ability and adjust their strategies based on performance level. Research has also indicated that students may need verbal encouragement from their teachers to be successful in mathematics (Yara, 2009).

Reys and Fennell (2003) indicated that elementary school mathematics teachers must know and understand the mathematics content they teach, know how students learn mathematics, and be able to apply content-based instructional strategies that support student learning in mathematics. These standards are in accordance with the Principles and Standards for School
Mathematics, which was established by the National Council of Teachers of Mathematics (NCTM) in 2000. These standards are further supported by a study by Tabernik and Williams (2010), which explored the relationship between sustained professional development for mathematics teachers and student performance in high-achieving countries. The researchers stated, “It is not enough for teachers to develop strong pedagogical skills; they must also know their subject area well enough to understand how to teach it to students” (p. 46).

In a study by Patterson, Connolly, and Ritter (2009), the researchers demonstrated how student achievement can be enhanced using differentiated instruction, which were the traditional, lecture-heavy instructional format, collaborative groups, and the computer program. The researchers investigated how educators met the needs of students with disabilities in a 6th grade general education classroom by differentiating instruction. They observed that the educators utilized a flexible style that allowed students to shift between groups, as well as small group remediation that was designed to be responsive to their individual learning styles. Results indicated that 78% of students were on-track to achieve at grade level before the end of the school year, which increased sharply from the 28% who were on-track at the beginning of the year. The researchers concluded that this style of teaching was effective, stating, “differentiated instruction allows the teacher to meet the needs of every learner by providing students with multiple options for learning” (p.46).

Although research supports these educational techniques, the number of students performing poorly in mathematics has increased. Students who do not establish a solid foundation in mathematical concepts often carry these conceptual deficiencies into middle and high school (Nelson, 2014). Little (2009) noted that deficiencies in mathematical concepts usually emerge in elementary school “and continue as students’ progress through secondary
school, typically performing over two grade levels behind their peers” (p. 3). Little attributed this phenomenon to poor mathematical instruction that covers too wide a scope of skills and concepts, causing the students to be unable “to achieve a sufficient conceptual understanding of the core concepts that underlie operations and algorithms” (p.3). Conceptual understanding is necessary for problem solving, and students are unable to be academically successful in mathematics without these skills.

The manner in which the material is presented may also play a role in academic success. Teachers may present instructional material using the same methods every time, which fails to help students learn by other methods (Finkelstein et al., 2012). Gerretson et al. (2008) observed that elementary-level mathematics teachers often lack a deep, conceptual knowledge of the subject. As a result, they often find it difficult to teach essential mathematical concepts to young children. Additionally, many teachers lack confidence in certain areas of mathematics, or report that they dislike certain subject areas (Liu, 2011). According to Liu, many elementary teachers experience anxiety related to teaching mathematics due to a lack of confidence in the subject matter.

**Students’ Attitudes Toward Mathematics**

According to Kiamanesh (2004), the concept of attitude is one that is marked by a strong belief or particular feeling, such as approval or disagreement regarding people or a certain situation. Hannula (2012) provided a similar definition, theorizing that attitude expressed by an individual in the evaluation of an entity through cognitive, affective, and behavioral responses, incorporating like or dislike of a familiar target. According to Kiamanesh, all people have both favorable and unfavorable attitudes towards a variety of topics, including other people, politics,
and academic subjects. Research has shown that students’ attitudes are critical to the learning process (Kiamanesh, 2004). Therefore, students’ academic achievement can become associated with how strong they believe they are in a certain subject.

Many factors influence how a student achieves in mathematics. Attitude toward the subject matter has a significant impact on how students achieve in mathematics (Mohamed & Waheed, 2011), and students’ past performance related to the extent to which they enroll in more advanced mathematics courses (Ercikan, McCreith, & Lapointe, 2005; House, 2000). This phenomenon can be understood from a psychological perspective, as humans generally engage in tasks that they believe they can complete successfully, or have positively reinforcing aspects (Schunk, 1987). Parental levels of education are also related to students’ level of interest in mathematics, as well as socioeconomic status and the extent that education is emphasized in the home (Ercikan, McCreith, & Lapointe, 2005; Flores, 2007). Wilkins and Ma (2003) observed that as students proceed through their educational years, they develop less positive attitudes toward mathematics. The researchers discovered that parents, teachers, and peer groups have significant impact on how students feel about studying mathematics. The effects of these factors are discussed below.

As previously discussed, students’ attitude toward mathematics impacts their academic performance in the subject. Research indicates that students who perform successfully in mathematics have more positive attitudes toward the subject, while those who consistently fail have a more negative attitude (Zakaria, Chin, & David, 2010). If students lack understanding of mathematics, they may lose confidence and begin to avoid the subject. This phenomenon is observed early in development, as students who struggle in their early elementary years begin to label themselves as students who are not good at mathematics. This attitude is confirmed and
thus perpetuated as mathematics increases in difficulty (Piper, 2008). Additionally, research has also shown that girls are less likely to have positive attitudes in regards to math, and also that this dislike for the subject matter continues into the higher grade levels and beyond (Hyde et al., 1990; Hannula, 2006).

Teaching styles and content knowledge of the subject matter can also impact how students feel about mathematics (Cornell, 1999; Trujillo & Hadfield, 1999). Compared to teachers in other countries, U.S teachers use techniques that are less effective in facilitating critical thinking skills and tendencies toward mathematical exploration (Trends in International Mathematics and Science Study [TIMSS], 2003). If students lack understanding of key mathematical concepts, they may have difficulty seeing interconnections of mathematics in the world around them, which may cause them to feel as though studying mathematics does not have value (Crespo, 2003). If students see mathematics as a series of irrelevant rules and formulas that must be memorized, they quickly become bored with the subject matter. The end result is that students develop negative attitudes toward mathematics.

As previously discussed, a teacher’s content knowledge can have a significant impact on confidence in teaching abilities (Cady & Reardon, 2007; Ross & Bruce, 2007). If teachers believe that they can be successful and help their students understand mathematics, they tend to set higher academic goals for themselves and their students. Confident teachers also instill in their students a persistence to achieve academic goals and confront obstacles in their education (Ross & Bruce, 2007). A student who studies mathematics with persistence and ambition is more likely to have an empowering attitude than a student who becomes discouraged.

Success in mathematics is largely dependent on the way the content is presented to the learner, as well as the way the learner interacts with the environment (Yara, 2009). Teachers with
enthusiasm, dependability, helpfulness, and strong content knowledge can inspire a more positive attitude toward mathematics. A teacher’s attitude toward the subject, the surrounding students, and the classroom environment can greatly influence a student’s preexisting disposition to mathematics (Yara, 2009). Similarly, Flores (2007) observed that the attitudes of the teachers also acted as a predictor of the student’s success in the area of math as well as their willingness to engage in the material. Overall, current research seems to suggest that learners are absorb the perspectives of teachers to create their own attitudes toward mathematics, which can have an effect on achievement outcomes (Yara, 2009).

**Students and Mathematics Difficulties (MD)**

Researchers use various definitions when describing students with difficulties learning mathematics. Some of these common terms include “mathematical disabilities,” “mathematical learning disabilities,” “dyscalculia,” and “mathematical learning difficulties” (Mazzocco, 2007). The first three terms generally describe students who have a disability and qualify to receive educational accommodations. These terms also imply an inherent disorder, as opposed to a weakness as a result of environmental influences. A disorder affects learning in multiple domains of mathematics (Gersten et al., 2007). Research estimates that approximately 6% of children have a mathematical disability (Dowker, 2005; Gersten et al., 2005). However, the concept of mathematical learning difficulty pertains to those students that have difficulties with the subject matter that are only related to a limited number of topics. According to Little (2009), approximately 7% of students struggle with basic math and problem-solving skills, causing these students to perform several grade levels below their peers. Children who achieve below the 35th percentile are often described with this term. The term implies that the individual does not
necessarily have a disability, but has low mathematical performance (Gersten et al., 2005). This was the definition used in the study.

Mathematical difficulties present themselves in various forms. Students with mathematical difficulties either differ from their peers in the degree of learning or differ in the kind of learning (Dowker, 2005). If a student differs in degree of learning, the student can learn in the same general way as their peers, but requires longer and more frequent learning sessions. A study by Staszewski (1988) supports the difference in degree theory. Students were taught methods of fast calculation, which many believe is only possible to master by individuals of higher mathematical ability. The students learned this task over a three-year period, amounting to over 300 hours of instruction. By the end of the third year, all students were able to accurately multiply five digits by two digits within 30 seconds, regardless of their mathematical ability.

Students can also differ in the number of strategies developed, as well as the amount of time required to learn concepts. A series of studies by Dowker (2005) support this phenomenon, suggesting that greater mathematical ability level leads to greater problem-solving strategies. In a study that compared the estimation abilities of college students and mathematicians, the mathematicians showed a striking difference in their methodology, and rarely used traditional algorithms. The mathematicians deeper understanding of concepts allowed more flexible problem solving (Dowker, 1992).

Research indicates that that lower achieving students differ from higher achieving students in how they process and use strategies. Sheffield (1994) compiled a list of characteristics commonly found in children with high mathematical abilities. These qualities included an ability to perceive and generalize patterns; showed an awareness and curiosity for quantitative information; the ability to reason both inductively and deductively; could transfer
learning to novel situations; and persistence with difficult problems. Research suggests that children with low mathematical abilities have less positive characteristics. Studies by Desoete, Roeyers, and Buysee’s (2001) and Lucangeli and Cornoldi’s (1997) results show that low-achieving students show inaccuracies in mathematical tasks and have difficulty evaluating their responses for correctness. Garrett, Mazzocco, and Baker (2006) found that students with mathematical learning difficulties were less effective in evaluating their solutions for accuracy.

**Summary**

Mathematics has crucial importance in the educational system, as it enhances the mathematical and critical reasoning skills of students. Positive interactions between students and teachers are essential to achieving higher levels of learning. Additionally, knowledgeable teachers who encourage positive attitudes toward mathematics and consider the student’s context support classroom achievement. However, more research is needed to evaluate effectiveness of different types of classroom organizational structures at the elementary level to identify which instructional setting will contribute the most to students' understanding of the mathematical content (Baker & Colyvan, 2011; Chang et al., 2008).

**Fractions**

The study of fractions can cause a bottleneck effect in the mathematical education of elementary and middle school students (Wu, 2005). A strong understanding of fractions is critical, as fractions are the basis for ratios, proportions, percentages, and decimals. Students who show a weak understanding of fractions may struggle with more advanced concepts such as geometry, algebra, statistics and calculus (Behr et al., 1983; Chan & Leu, 2007). Gersten and colleagues (2009) have stressed the importance of fractions, suggesting that intervention for students who struggle with mathematics in grades four through eight should center on the
development of the key concepts of rational numbers. This suggestion is aligned with the NCTM (2006) curriculum focal points. The National Mathematics Advisory Panel (2008) calls for U.S. curriculum to provide in-depth coverage of key topics related to numbers to students in kindergarten through fifth grade, as well as rational numbers from fourth through eighth grades (NCTM, 2006). This section discusses existing literature describing students’ experiences in understanding fractions, difficulties with fractions, fractions equivalence, and addition of fractions.

**Understanding of Fractions**

Failure to master fractions has significant implications. It increases the difficulty of acquiring more advanced mathematics skills and may exclude the student from many occupations (McCloskey, 2007; NMAP, 2008). One study that surveyed 1,000 Algebra I teachers from the United States found that teachers perceived fractions to be one of the greatest weaknesses of students coming into the course for the first time (Hoffer et al., 2007). Similarly, data that was collected in both the United States and the United Kingdom revealed that the degree of students’ understanding of fractions when they were in fifth grade predicted how well they would perform in math throughout high school. In a similar study, Siegler et al. (2012) found that getting other mathematical knowledge, IQ working memory, and even reading comprehension demonstrated the same results for the study. Research shows that conceptual understanding has a significant impact on gaining procedural proficiency with new knowledge (Rittle-Johnson, Siegler, & Alibi, 2001; Vukovic et al., 2014).

Understanding fractions requires knowledge of procedures for solving fraction problems, as well as considerable conceptual knowledge (Siegler and Pyke, 2013). Conceptual knowledge of fractions has several components. First, students must know that fractions are numbers that
stretch from negative infinity to positive infinity. Second, they have to know that between any two fractions are an infinite amount of other fractions. Third, students must understand that the numerator-denominator relationship determines fraction magnitudes; fraction magnitudes increase with numerator size and decrease with denominator size. Fractions can be represented as points on the number line (Kloosterman, 2010; Mazzocco & Devlin, 2008; Siegler et al., 2012).

According to recent findings, conceptual understanding of magnitudes appears to be particularly crucial to mastering fractions. Research shows that the ability to represent fraction magnitudes, such as through number line estimation, magnitude comparison, and ordering of multiple fractions, correlates significantly with knowledge of fraction arithmetic, as well as overall mathematics achievement from fifth to eighth grades (Bailey et al., 2012; Mazzocco & Devlin, 2008; Siegler, Thompson, & Schneider, 2011). Even if one were to control for fraction math competency, the relationship between fractional knowledge and a level of comprehensive success in math proves to be significant (Siegler and Pyke, 2013).

Research is currently exploring the extent to which young children can understand fractions. Watanabe (1996) conducted a study with four children who were seven years old in order to explore their conceptual understanding of fractions over a seven-week period. The four children were individually interviewed four times, then observed while they problem-solved with a partner. The children were given tasks concerning the concept “one-half.” The researcher concluded that children understand fractions in a variety of ways, and children initially understand the fraction “one-half” based on physical actions. The researchers concluded that children must conceptualize natural numbers before they have the ability to understand fractions. However, research requires different ways to evaluate children’s physical or abstract methods of
reaching mathematical understanding. Also, this investigation was a case study, meaning that classroom teachers may find it difficult to generalize to their own classrooms.

Niemi (1996) investigated understanding of fraction concepts among fifth-graders with exceptional math achievement. The researcher collected data on 540 fifth grade students who scored high on standardized tests. The students were administered two types of instruction which were employed over seven and a half days. The type of instruction was either quantitatively structured or part-whole focused. Students were administered a pretest and a posttest to determine which instruction method was most effective. The posttests showed that the group who received the quantitatively structured program on fractions in measurement contexts performed better overall than the group who received traditional part-whole instruction. Niemi’s research was the first to investigate understanding of fractions amongst high-achieving students.

Different obstacles exist in acquiring procedural knowledge of fraction arithmetic. The components of fraction arithmetic problems are complex. For example, addition and subtraction of fractions require common denominators, but multiplication and division of fractions do not. Similarly, using the arithmetic operation solely on numerators or denominators provides a means of solving multiplication problems that are related to fractions, but the same methods cannot be used in addition or subtraction problems (Siegler and Pyke, 2013). As a result, conceptual and procedural knowledge of fractions should be taught and emphasized together, because without conceptual understanding the procedures are confusing to students and difficult to remember. Research supports this idea, as both conceptual and procedural knowledge of fractions correlate significantly over a wide age range (Hecht, 1998; Hecht, Close, & Santisi, 2003; Hecht & Vagi, 2010; Siegler et al., 2011).
Several significant challenges exist in students’ acquisition of conceptual fraction knowledge. One obstacle is whole number bias, in which prior knowledge of integer rules are assumed to extend to fractions, which is often not the case (Ni & Zhou, 2005). For example, students often assume that numbers between integers do not exist. Therefore, they are unable to recognize the fractions that exist between integers. Moreover, students can have significant problems understanding the fact that multiplication can result in products that are actually smaller than the initial numbers (Vamvakoussi & Vosniadou, 2004).

**Difficulties with Fractions**

As previously discussed, one of the primary challenges in the acquisition of fractions is the whole number bias (Ni & Zhou, 2005). Children may mistakenly apply their knowledge of whole-number properties to fraction tasks (Stafylidou & Vosniadou, 2004). Other factors are also believed to cause difficulty in understanding fractions. Hasemann (1981) argued that fractions were not as apparent in everyday life as other mathematical concepts. Also, he argued that children found writing the fractions to be an incomprehensible idea and that the rules for working with fractions are far more complicated than working with whole numbers or natural numbers.

This is really problematic because whole numbers often operate on a different set of rules than fractions. Whole numbers follow a stable order of sequence. For example, 3 always come after 2 while counting, and each of the subsequent numbers has an equally larger magnitude than the one that preceded it. However, this idea is not the case when solving fraction problems and can lead to difficulty in understanding fraction magnitude. For example, students who try to apply whole number rules to fractions may fail to understand that that $1/2$ is larger than $1/3$.

Another significant difficulty observed in fraction knowledge is that students often
confuse whole number and fraction concepts when solving problems (Pitkethly & Hunting, 1996). As a result, students may attempt to solve fraction problems using whole-number concepts. For example, students may simply add numerators and denominators together in a fractional addition problem, or evaluate the magnitude of a fraction based on either the value of the numerator or denominator alone, rather than considering the whole fraction.

In a case study by Mack (1990), the researcher investigated the ways in which students' prior learning of whole numbers interfered with their understanding of fractions. The author introduced the concept of fractions to four third-graders and four fourth-graders individually using clinical interviews and co-constructing knowledge. The intervention program presented real-world problems to the students verbally, then symbolically. Results showed that students were able to discern between whole numbers and fraction numbers in their verbal responses to the real-world problems, but struggled to extend their knowledge to a symbolic representation of the same concept. Students often treated the numerator and denominator of fractional quantities as whole numbers, suggesting that whole number knowledge interfered with their learning. Mack (1990) concluded that students possessed intuitive knowledge of fractions, but were unable to represent this intuitive real-world knowledge symbolically.

Behr, Wachsmuth, Post, & Lesh (1984) observed a similar phenomenon when teaching fractions to 12 fourth-grade students. The authors administered clinical interviews throughout the experiment. Instructional techniques included establishing order and equivalence of fractional quantities with concrete manipulatives and visual representations. Results indicated that prior knowledge of whole numbers initially interfered with understanding of fractional concepts, but these incorrect whole number interpretations of fractions diminished as instruction progressed.
Most students struggle to understand that fraction parts are equally sized portions. One of the main developmental milestones in understanding fractions is recognizing that “fractional parts are equal shares or equal-sized portions of a whole or unit” (Van de Walle, 2007, p. 293). Research has demonstrated that children do not always realize that the whole must be divided into pieces of equal size (e.g., Pothier & Sawada, 1983). Reys et al. (1999) gave open-ended questions in a fifth grade classroom that had recently finished a month and a half unit on fractional concepts. When these students were asked to describe certain fractions, like 2/5, the researchers found that the children did not understand that the whole number was to be divided into pieces that were the exact same size. This misconception endured even when the children were questioned about the sections as if they were pieces of pizza. Considering that approximately one-third of the students held this misconception, it is likely that their instruction may have lacked this conceptual aspect of fractions (Reys et al., 1999).

Beginning with students’ intuitive knowledge may seem logical, but may not always be an effective technique. Pothier and Sawada (1983) discovered that presenting children with a scenario that involved cutting a birthday cake into pieces resulted in them being more likely to care that each person was given an equal number of pieces than they were about every individual getting the same amount of cake. As a result, the researchers found that informal knowledge is not always a reliable starting point. However, most other researchers and math educators have found otherwise. Research shows that a fair-sharing context may guide learning (Empson, 2002; Flores & Klein, 2005; Fosnot, 2007; Sharp et al., 2002). Fair-sharing tasks are most useful when the context is relevant to the children (Fosnot, 2007; Sharp et al., 2002), and can be used with items that are easily divided by children such as a candy bar, sandwich or confections (Van de Walle, 2007).
Additionally, in order to help students understand the importance of equal sized portions in fractions, Reys et al. (1999) suggested that teachers should take the time to give students more chances to explore fractions in the form of visual representations, such as doing projects that involve filling in parts of shapes, to engage the visual learning aspect of a child’s mind. The researchers also concluded that teachers should use such visual representations before they decide to proceed with additional fraction concepts so that the students can understand the basic concepts of fractions. Cramer et al. (2002), working in conjunction with the Rational Number Project, found that if teachers let students make a connection between a form of physical manipulatives like symbols or pictures, then students will be able to have a more beneficial and accurate rendering of fractions.

Most students have difficulty understanding fraction symbols. Research demonstrates that the symbols that are involved in fractions mathematics can be difficult for some students to conceptualize due to the fact that they are used to dealing with rational numbers (Van de Walle, 2007). Mack’s research (1995) involving children in third and fourth grade found that the students believed that the numerator in the fractions was the number of pieces that were used for the problem and that the denominator was the total number of parts that were in a whole. Mack found additional problems when attempting to help students apply the symbols they were using in fractions in the overall context of day-to-day life. The trouble was, in her estimation, that the students were using the rules that they had learned in whole numbers to support their knowledge of fractions. However, the children were able to write mixed numerals and corrected their misconceptions after Mack explained their meaning in the context of real-world situations.

One of the other misconceptions that inhibited students’ understanding was that fractions always have to be less than a whole. Mack’s research (1990) with sixth graders yielded results
that showed students had a significant amount of trouble when it came to identifying fractions that had a greater value than one whole, clinging to the aforementioned belief that fractions were always less than a whole. Still, the research noted that if the students were given a real-world example, they were typically much more able to identify the units greater than a whole correctly. Using a computer microworld, students represented various fraction units with electronic sticks of various lengths. Tzur (1999) conducted a study upon 2 fourth graders in order to see the children's construction when working with improper fractions. He found that the students would change the unit after producing non-unit fractions by reiterating a unit fraction. A unit fraction is a number that has a numerator that is one and a non-unit fraction is a fraction with a number that has a value greater than one. Students in this case believed that the non-unit number that resulted from 4/5 multiplied by 1/5 or 1/5 multiplied by 4/5 would result in 6/6, and that each subsequent part would be 1/6 (Tzur, 1999).

This misconception may be a result of the way fractions are commonly introduced in school textbooks, as well as the language of fractions. Flores and Klein (2005) reiterated that the colloquial use of “fraction” differs significantly from that of the school term of “fraction.” The concept of a fraction as it pertains to everyday use is one that means less than a singular whole. Also, the terminology that is associated with fractions, such as “improper” can make students feel as though there is something wrong with that type of fraction as it carries the connotation of something that is not correct. Therefore, the best way to explain these fractions to students would be to show them that \( \frac{3}{4} \) is not only three pieces of four, but also the total of 3 pieces of \( \frac{1}{4} \). This idea can be extended for improper fractions. For example, students could count seven pieces of size 1/4 each. Posing fair-sharing problems may also facilitate this type of learning. For example, offering to share seven brownies with four different people in the class, which is often paired
with textbook answers of answers such as 7/4 shows children that each person ends up with 1 brownie and ¾ of the remaining pieces, so each person has 7 of the ¼ pieces. These types of problems help students understand improper fractions more easily. Van de Walle et al. (2011) made the recommendation that teachers should no longer use certain terminology such as fractions that are greater than one, and instead use terminology that is more easily understood such as improper fractions to prevent the students from becoming confused. Again, it is important to begin teaching students about the concept of improper fractions when one is teaching students about fraction symbols (Van de Walle et al., 2011). Kamii and Clark (1995) also believe that mixed numbers, improper fractions, and even proper fractions need to be utilized from the beginning of the lessons on fractions so that the children will be able to use that as a basis for all future knowledge about the numbers.

Students may also have difficulty conceptualizing the relative size of unit fractions. For example, Mack (1990) found that when sixth-grade students were asked to identify the larger fraction between 1/6 and 1/8, the majority stated that 1/8 was larger. Most students’ reasoning indicated that they applied whole number concepts to fractions. However, when the same choice was presented with the context of pizza slices, students used their knowledge about whole numbers to produce an answer of 1/6. This shows that the students’ tendency to go for an inverse relationship is something that has to occur in the mind of the student and is not necessarily easily taught to them (Van de Walle, 2007). Alluding to the tenets of constructivism, Van de Walle (2007) posited that understanding cannot be “given,” and instead must be formed by the learner. Overcoming these incorrect ideas through the lens of constructivism, Van de Walle said that teachers should utilize a lesson where students have to rank fractions in terms of the least and greatest while also making them defend their ideas to the teacher and other students.
Many students are not able to estimate a sum or correctly compare the sizes of fractions, showing a need for fraction number sense in instruction (Cramer & Henry, 2002; Cramer et al., 2002; Reys et al., 1999). In order to estimate sums, students must develop an awareness of the approximate size of a fraction. One of the problematic elements of the “traditional” classroom is that the students are made to rely on rote teaching concepts like using cross multiplication to compare their fractions instead of using estimation.

Atiah (1994) examined difficulties fifth-grade students experience when learning fractions in order to determine the primary causes for the mathematical mistakes students consistently were making. The sample group consisted of 240 participants randomly chosen from eight elementary schools, including 34 mathematics teachers who completed a research. The research revealed fifth-grade students struggle most with the four operations for fractions—comparing fractions, converting fractions to decimals, and vice versa, and finding common denominators. In addition, the results pointed to weak foundational learning of fractions concepts and lack of multiple assessments as the main culprits for student difficulty in learning advanced fractions operations in fifth grade.

Teaching reference points and benchmarks can help students overcome the inability to estimate (Huinker, 1998). Examples of essential fraction benchmarks include 0, 1/2, and 1 (Van de Walle, 2007). Fosnot and Dolk (2002) found that it is important to use landmark fractions as a teaching strategy. This is a very beneficial form of teaching if the educator has intentionally applied landmarks, since it ultimately shows that the concept of fractions is one that will extend beyond the classroom.

**Fraction Equivalence**

Understanding equivalent fractions is an important building block for both fraction
addition and other mathematical topics. The Common Core State Standards (2010) recommend that fourth-grade students develop the ability to recognize two equivalent fractions, generate sets of equivalent fractions, and gain the ability to decompose fractions into unit fractions. The three main skills indicative of equivalent fraction mastery are the ability to rename fractions into their simplest form, generating sets of equivalent fractions, and determining fraction equivalence (Van de Walle, 2004).

Students may have difficulty grasping equivalent fractions (Kamii & Clark, 1995) because they may conceptually have difficulty with the idea that “a fixed quantity can have multiple names (actually an infinite number)” (Van de Walle et al., 2011, p. 310). Similarly, Huinker describes this phenomenon as difficulty understanding that “a specific amount can have many names” (p. 172), which is a critical aspect of fraction knowledge. Fosnot and Dolk (2002) theorized that children must understand two overarching ideas to fully understand equivalent fractions: “for equivalence the ratio must be kept constant” and “pieces don’t have to be congruent to be equivalent” (pp. 136-137).

In the first overarching idea, understanding a ratio implies that the student has the ability to fully understand the internal relationships of a fraction, such as the numerator and denominator, and how this form can be utilized in conjunction with other fractions. In the second overarching idea, the student must understand that congruent pieces are the same shape and size as each other. When using area models, two fractions can be equivalent as long as they are the same size, even if the pieces are not the same shape. For example, it is important to understand that if a person were to divide a sandwich into three pieces of ¼ of a sandwich, it is the same amount as if they were to have ½ of the sandwich and ¼ of the sandwich together; the overall amount stays the same while the pieces will look different. Fosnot and Dolk’s hypothesis is
supported by research. A study’s findings were such that students could explain fractions that were equivalent when they were shown in a visual representation, but the numeric notation presented them with difficulty (Jigyel & Afamasaga-Fuata’i, 2007). The researchers concluded that children must develop the understanding of a fraction in the context of a relationship between the numerator and the denominator.

Van de Walle et al. (2011) encourage teachers to approach equivalent fractions by providing students with a variety of models (area, length, and set) to help students generate different names for the same fractions. Lamon (1996) believed that using activities that rely on partitioning should be used throughout middle school grades so that students continue to develop new strategies for dealing with fractions. Moreover, this activity should not only be used for simple introductions to fractions as the visual aspect continually proves useful. Lamon also said that students were more likely to benefit from being able to make pencil and paper drawings rather than partitioning throughout the process of cutting. However, Kamii and Clark (1995) demonstrated that children’s knowledge through observation (figurative knowledge) can conflict with their knowledge of unobservable relationships (operative knowledge). As a result, children may believe that a triangle-shaped piece of a square is larger than one that is a rectangle, although they are actually the same size. Students may have trouble grasping the idea that the halves are equivalent even if the regions are shaped differently. Lamon (2002) also said that students may be able to realize fractional equivalency though unitizing, which is defined as the “process of mentally constructing different-sized chunks in terms of which to think about a given commodity” (p. 80). An advantage of this method is that it can help children reason about fractions “even before they have the physical coordination to be able to draw fractional parts accurately” (Lamon, 2002, p. 82).
When using models to partition, many students experience the significant obstacle of identifying the whole and conserving their conceptualization of the whole. In a study conducted by Kamii and Clark (1995), a group of 120 students in the fifth and sixth grade were presented with two paper rectangles that were identical, and they observed a researcher cut both of them in half, one being cut vertically and the other being cut diagonally. The students were then asked the fractional size of the pieces, and all students correctly identified the pieces as one-half. However, the students in the fifth grade only identified the pieces as being the same size in 44% of the cases, and only 51% of the sixth graders were able to see that they were the same size. Operationally, the students were aware that dividing the rectangle in half produced $\frac{1}{2}$ and that $\frac{1}{2}$ is equal to $\frac{1}{2}$, but responded to their visual interpretations that one piece was larger than the other. The same students were then presented with two additional identical rectangles. This time, the researchers folded the rectangle in fourths and cut a strip that was equivalent to one fourth of the total area. The other triangle was then cut into eight different pieces. Of the students that were in the experiment, 13% of the fifth graders that were involved in the study and 32% of the sixth graders were able to find the right answer (Kamii & Clark, 1995). The students’ struggles with these tasks show that they have trouble with the idea of maintaining the relationship of pieces to the whole. Further research indicates that students do not have the ability to visualize these parts until they are already in the fourth grade (Grobecker, 2000). However students who do not develop the concept of conservation do not think of three twelfths and one fourth as they pertain to the whole. Instead, the students do not tend to remember the relation of the three twelfths to the problem and are confused when asked to discover the equivalent fractions (Kamii & Clark, 1995).
Building sets of equivalent fractions is a critical skill in mathematics. However, students have difficulty grasping equivalent fractions when they are working to hone their understanding of fractions; they do not know that there is an infinite set of fractions that are equal to it (Ni, 2001; Smith, 2002). For many students, the first step to overcoming whole number bias is developing sets of equivalent fractions (Lamon, 1999). Students also have to overcome their lack of understanding in terms of fraction component multiplication and thinking when they are working with equivalent fractions (Kamii & Clark, 1995). Multiplicative thinking involves thinking of a fraction number as distinct groups, rather than singular objects (Ball, 1993). The transition between additive and multiplicative reasoning is difficult for many children who will initially seek to solve problems with additive principles (Chan & Leu, 2007; Kent, Arnosky, & McMonagle, 2002; Moss, 2005). When a student is evaluating a set of fractions that are equivalent, they must focus on the parts that are added instead of immediately resorting to the multiplication of the two parts of the fraction by an integer (Moss, 2005). Addition-based thinking is far different from multiplicative thinking in the sense that there is a logical progression for addition, but multiplication takes into account addition as well as multiplying (Kamii & Clark, 1995). Proficiency in equivalent fractions requires students to see both the multiplicative relationship of numerator and denominator between fractions as well as the multiplicative relationship between the numerator and denominator of a single fraction (e.g., 9/27 is equivalent to 1/3 because 9 x 3 = 27).

**Addition of Fractions**

If students do not have the precursory information in reference to equivalence and unit fractions, then they may struggle with fractional calculations. Kong (2008), Huinker (1998), Niemi (1996) and Pitkethly & Hunting (1996) all assert that learners seldom understand the
procedural knowledge associated with fractional operations such as addition, which is significantly associated with a lack of foundational understanding of many aspects of fractions. As a result, 67% of learners lacked the ability to use equivalent fractions for adding fractions with unlike denominators, and 8% of the learners grasped the idea of fraction equivalence but could not apply the concept to methods of finding equivalent fractions (Kong & Kwok, 2005).

Brown and Quinn (2006) performed a study that had 100 middle school students answer an assessment to determine their ability to work with decimals and fractions, and to test their overall understanding of computational thinking. The students used paper and pencil and they were told that they could not use calculators. Unfortunately, many of the students struggled to answer a simple algorithm. For example, when asked to add the fractions 5/12 and 3/8, 19 out of the 27 students in the class simply added the two fractions together by using the numerator and denominator. These results highlight a significant problem in mathematics learning that should be rectified, as the analysis showed that students often have many misconceptions regarding fractions and decimals.

The common mistakes of addition and subtraction of rational numbers for fifth and sixth grade students were investigated by a study of Al-Doby (1990). The research investigated the percentage of errors and types of mistake that students made. Fifty-four students participated by completing a 28-question test. The findings indicated a high percentage of errors in adding and subtracting fractions and rational numbers for students in both fifth and sixth grades. Specific types of mistakes common to most students included the following: adding and subtracting numerators and denominators, finding common denominators, changing rational numbers to improper fractions.
Ahmad’s (2012) study investigated errors fifth and sixth grade students commonly made with basic mathematical operations of both common and decimal fractions. It aimed to understand the mistakes related to fractions concepts and the addition and subtraction operations of common and decimal fractions among both boys and girls. Three hundred forty-six students participated in the study, comprised of 180 fifth-grade students and 166 sixth-grade students. The research instruments included a test for all participants and an interview with those students who could not answer the questions correctly. The results revealed approximately 46% of fifth-grade students had failed in adding fractions with the same denominators while 81% had failed in adding fractions with different denominators. The research also showed 48% of fifth-grade students had failed in subtracting fractions with the same denominators while 77% failed in subtracting fractions with different denominators. Among sixth-grade students, 91% failed in adding fractions with the same denominators while 84% failed in adding fractions with different denominators. Furthermore, 87% of sixth-graders failed to add rational numbers correctly, 82% failed in subtracting fractions with the same denominators, and 92% failed in subtracting rational numbers. However, the study found no significant differences for common errors between boys and girls or between fifth-grade and sixth-grade students.

Al-Yanbawi (1995) conducted a diagnostic study of the difficulties experienced by fifth- and sixth-grade students when they were asked to perform basic fractions operations. Participating in this study were 144 students, 72 students representing each grade level. Here again, a test was constructed that would enable researchers to gather data pertaining to their research objectives. The test consisted of 40 questions, with 10 questions covering each basic mathematical operation (addition, subtraction, multiplication, and division). The results revealed both fifth- and sixth-grade students experienced difficulty with all four of the basic operations.
However, adding and subtracting fractions was more difficult than multiplying and dividing fractions because to find the correct answer in the latter operations required more steps, such as finding the common denominators and using the Least Common Multiple (LCM) and Greatest Common Factor (GCF). In addition, the findings revealed significant differences between fifth and sixth grade students in terms of difficulties they experienced with performing basic mathematical operations on fractions. The fifth grade students struggled more than the sixth grade students did, in particular, when multiplying and dividing fractions.

One study that analyzed the skills of two classes of sixth graders resulted in the researchers finding that students have a tendency to look for patterns within the problem rather than attempt to comprehend the mathematical problem that has been posed (Lappan & Bouck, 1998). Students become confused when the symbolic configuration of the problem looks similar to a problem learned earlier, and students may use inappropriate rules based on symbols. Notably, the student errors were usually not a result of using a rule incorrectly, but using the incorrect rule for the situation. Students may also attempt to modify a rule to produce the answer that they believe looks correct. These two errors are so common that educators are able to predict the types of mistakes that students are likely to make, which demonstrate why students often make similar errors throughout their education. This research shows that students in middle school must learn to build algorithms of their own if they are to become proficient in fractions (Lappan & Bouck, 1998). Additionally, teachers may consider spending more time allowing students to construct their own algorithms, rather than teaching how to memorize a procedure (Huinker, 1998). Using a strategy such as this could allow students to comprehend the academic definitions of the operations that are involved with fractions and algorithms as a whole (Wu, 2001).
It is interesting to contemplate whether it is easier for students to learn how to manipulate fractions or if they first must grasp how to solve problems involving fractions. A study involving 155 sixth-grade students investigated performance in fractions when the subject was taught in a context of understanding the meaning, computing fractions, and solving word problems. The students were given a three-part test on fractions. Results indicated that students showed the lowest performance on the problem-solving portion of the test and showed the highest performance on the portion of the test measuring operations. This study did not investigate how the fractions were taught, but showed how the children manipulate fractions (Aksu, 1997).

When students develop a proficient understanding of fractions, they gain the ability to identify what pieces of information are relevant and which are irrelevant. Martin and Schwartz’s (2002) research into fractions used three fifth grade classes that were given the same lessons using fraction tiles and circles. The purpose of the fraction circle was to work within the confines of the area of a circle and to provide a form of visualization. However, when students use fraction tiles, it is less clear what the whole is and the students must visualize the whole for themselves. After completing the three lessons, the students solved problems using both fraction circles and fraction tiles. The results showed that both groups performed equally well on questions that used their group’s manipulative. However, the fraction tile group showed the ability to transfer their knowledge to the fraction circle problems. This group showed more accuracy when applying their knowledge to the fraction circle than the other group did when applying their knowledge to the fraction tiles. The researchers concluded that using the fraction tiles was the more effective means since it allowed students to have an encompassing focus and did not rely on the idea of the whole being built into the lesson. Additionally, a model that initially causes confusion can be beneficial for students, as the subsequent cognitive
disequilibrium can challenge students to rethink or restructure their understanding (Behr et al., 1983).

Aiming to explore how Piaget’s theory of cognitive stages might relate to students learning fractions, Almogerah and Al-Mohameed (2013) conducted an analysis among fourth, fifth, and sixth grade students of the common mistakes they experienced in fractions mathematics. Three classes from six elementary schools were chosen randomly to participate in the study. The sample size was 477 students, comprised of 163 fourth grade students, 164 fifth grade students, and 150 sixth grade students. To address the research objectives, researchers applied the following six different instruments: the test of measuring cognitive development stage for students, the test of fractions for fourth grade during the second semester, the test of fractions for fifth grade during the first semester, the test of fractions for fifth grade during the second semester, the test of fractions for sixth grade during the first semester, and the test of fractions for sixth grade during the second semester. The findings revealed fourth grade students made 20 mistakes in common, fifth grade students made 34, and sixth grade students made 20. The highest percentage of mistakes for fifth grade students was 32.9%, which was in adding and subtracting fractions. In this specific test, when students were asked to add or subtract rational numbers, most students added and subtracted only the whole numbers and not the fractions. Furthermore, the findings showed a strong positive correlation between students' errors and their cognitive stages. That is, the higher cognitive stage, the fewer mistakes were made and vice versa.

**Conceptual Understanding**

Many children in American schools show poor mathematics performance, further illustrating a need for improved mathematics instruction. Policymakers and educators have
emphasized the importance of developing a deeper conceptual understanding of mathematics through teaching problem solving (Rittle-Johnson & Alibali, 1999). Research indicates that students may gain a better learning outcome when manipulative devices and pictorial representations are used in teaching mathematics, and systematic use of pictorial representations may be particularly effective for developing conceptual understanding (Miller & Hudson, 2007). The importance of developing a conceptual understanding of both general mathematics and fractions specifically is discussed below.

**Importance of Conceptual Understanding in Mathematics**

Conceptual knowledge is critical to understanding logical relationships and interconnectedness among concepts (Hallett et al., 2010). The association of existing knowledge to new learning helps individuals develop conceptual understanding of mathematical (Arslan, 2010). Research demonstrates that when learners relate new ideas to the content that they have previously learned, they are able to make lasting connections (Miller & Hudson, 2007). In this way, conceptual knowledge becomes a cognitive network in which the relationship between the nodes are as important as is the information conveyed between them. The student who learns conceptually can incorporate and apply definitions, principles, rules, and theorems, and can compare and contrast related concepts. Research has shown that conceptual learning leads to more success than procedural learning (Arslan, 2010). For example, after conceptually learning the idea of money conversion, students could purchase items regardless of the specific coin they possessed (Miller & Hudson, 2007).

Historically, mathematics education has emphasized procedural learning over conceptual learning. However, current students must be taught with the goal of facilitating a deeper, more conceptual understanding of mathematics (Khairani & Nordin, 2011). Evidence demonstrates
that conceptual understanding plays a significant role in generation and adoption of procedures, and that children with greater conceptual understanding show more proficient procedural skill (Bryan, 2014). For example, research shows that children who have a better understanding of place value are more likely to regroup numbers successfully in manipulating multi-digit numbers (Rittle-Johnson & Alibali, 1999). Additionally, various mathematical domains such as counting, computation fractions, and so on are based on the connection between both conceptual and procedural knowledge (Rittle-Johnson & Alibali, 1999).

Success in mathematics depends upon acquiring conceptual understanding, as this type of understanding is critical for solving novel problems in a variety of settings. As a result, it is critical to develop mathematics lessons that include explicit instruction related to understanding the meaning of mathematical concepts (Miller & Hudson, 2007). Several studies investigating students’ understanding of mathematical concepts, such as equivalence in fraction and ordering fractions, demonstrated that children in all grades were more likely to understand conceptual items before the procedural items. These findings demonstrate that students learn best by understanding. From this perspective, procedural knowledge is a set of helpful tools used after learning conceptual knowledge (Hallett et al., 2010).

**Developing a Conceptual Understanding of Fractions**

Research has begun to explore the development of conceptual understanding, which enables learners to apply their knowledge flexibly and to use a variety of representations. Hecht et al. (2003) explored the fraction problem-solving skills of primary school aged children. The researchers found that students with stronger conceptual knowledge were more likely to select appropriate strategies relating to each question and review the relative success of each procedure. For example, students were more likely to show proficiency in adding fractions with different
denominators (1/4 + 1/3) if they had an appreciation of the conceptual idea of the unequal size of these fractions. The researchers posited that conceptual understanding might facilitate development of an effective mental model that provides an effective structure when considering fractional quantities.

Conceptual understanding should serve as the basis of development, and procedural understanding should only be used as a helpful tool that is applied after acquiring conceptual understanding. In a study by Byrnes and Wasik (1991), fourth- and sixth-grade students were instructed to answer questions assessing their conceptual understanding. In a second study, the students demonstrated their procedural understanding. The researchers measured conceptual understanding by the students’ ability to recognize equivalent fractions and order fractions correctly. The researchers assessed procedural understanding based on the ability to correctly add fractions with different denominators and multiply fractions. Results indicated that success in fraction addition involved knowledge that was more consistent with ordering fractions, rather than the common denominator procedure.

When students lack a sound conceptual understanding of fractions and appropriate real life application, they tend to revert to the use of half–remembered rules and algorithms. “All the evidence indicates that many children have serious misconceptions of the concept and operate fractions using incorrect rote procedures” (Orton & Frobishier, 1996, p.107). This assertion was supported by Mack (1993), who found that rote learning could significantly interfere with developing a meaningful understanding of fraction symbols. Lukhele et al. (1999) also supported this claim in their investigation of secondary aged pupils’ understanding of the addition of fractions. Their results suggest that most students’ errors result from treating the numerator and denominator separately, as well as the “urge to use familiar (even if incorrect) algorithms for
whole number arithmetic” (p. 1). The researchers found that students were likely to trust answers obtained by an established algorithm without considering whether their answers were appropriate. The researchers suggested that the children were not used to “making sense of math.”

However, some researchers believe that the concept of fractions begins as a procedural activity that guides students to produce different sized fractions with the same quantity (Charles & Nason, 2001; Gray & Tall, 2007). The process of abstraction begins when students realize that different sharing situations can result in equivalent fractions. At this point, the emphasis of the lesson changes from the process of sharing to the result, the fraction. Similarly, Kerslake (1986) found that students showed the ability to complete fraction addition problems correctly, but lacked the ability to explain the procedure. This finding demonstrates that conceptual understanding may occur after procedural knowledge.

Other studies have discovered that children may gain an understanding of a procedure before understanding the method conceptually. In a study by Peck and Jencks (1981), the researchers conducted interviews with sixth-graders to assess their knowledge of fractions. The authors discovered that only a small number of students demonstrated a deep understanding of fractions (approximately 10%) but over a third (approximately 35%) could solve fraction problems using learned procedures, in spite of their lack of conceptual understanding. Similarly, Kerslake (1986) discussed children’s methods and common errors when solving fraction problems, finding that the students could complete the problems correctly but could not explain how their method worked. One student stated, “You’re taught something, you’re never taught why” (Kerslake, 1986, p. 21).
Hallett et al. (2010) acknowledged the value of a combination of conceptual and procedural knowledge. The researchers assessed fourth and fifth graders' knowledge of fractions by studying individual differences in the use of conceptual and procedural knowledge in the students’ responses. In many cases, conceptual and procedural knowledge was found to develop in parallel. The students used either conceptual or procedural knowledge in response to particular types of questions, and did not favor one over the other as an overall approach. The researchers classified the children into five clusters indicating levels of conceptual and procedural knowledge. A key finding in this study was that students who possessed both conceptual and procedural understanding outperformed the other students. This idea was also reflected in Hecht and Vagi’s (2010) longitudinal study, which considered how a range of intrinsic and extrinsic factors affect the development of specific types of mathematical skill, including fraction computation and estimation. The researchers specifically focused on the part-whole and measurement aspects of fractions and decimals. Their results suggested that development of procedural and conceptual knowledge are influenced by each other and developed concurrently in some children.

These results are consistent with Sfard’s belief (1991) that “certain mathematical notions should be regarded as fully developed only if they can be conceived both operationally and structurally” (p. 23). Herman et al. (2004) investigated if fraction in process (1/2+1/4) and fraction as process (1/4) are interrelated according to Sfard’s theory. Their results suggested that students could represent fraction as process but only few students could produce the image for the addition of fractions even though they could find the sum of the fractions in their symbolic form. Consistent with the findings of Hallett et al. (2010), the researchers concluded that the routes of these two parts may be cognitively separate. A valid explanation of these results is that
few students can conceive this duality, and these students are more likely to succeed in developing the notion of the fraction concept.

**Using Manipulatives**

Manipulatives are visual constructions that illustrate mathematical meaning. They are commonly used in elementary school classrooms to teach mathematical concepts. As Reys (1971) discusses, manipulatives can be objects specifically designed for the purpose of representing a mathematical concept or everyday objects that are already familiar to students. Manipulatives can help students understand abstractions in a concrete way, as the learners develop an understanding of the abstract through hands-on experiences (Moyer, 2002). Reys recommended certain pedagogical methods when using manipulatives. He suggested that the objects used as manipulatives should be multipurpose and inspire motivation in the student. Additionally, he recommended that the manipulative clearly illustrate the mathematical concept at hand, allow the individual the opportunity for specific manipulations, and allow for abstraction of the mathematical concept. For many years, research has aimed to understand how manipulatives can be best used in the classroom, what types of students would benefit from them, and when they should be used. Research exploring these topics is discussed in the following section.

**Support for Manipulatives in the Classroom**

Many studies support the use of mathematical manipulatives in the classroom (Allen, 2007; Burns, 1996; Clements, 1999). According to Gardner (1991), students report that they do not understand the concept they are expected to learn because their math classes consist of instruction followed by an exam. Students do not understand why they are being taught mathematics because they do not see the relevance to their lives. Therefore, students may benefit
from learning mathematics in an illustrative and symbolic manner in order to help students match concepts to real-life situations. As such, research studies have shown that manipulatives are a useful tool in assisting students with learning conceptual mathematical ideas. Using a manipulative explicitly in a hands-on manner assists students in learning concepts more easily (Moyer, 2002).

Bruner (1966), a cognitive psychologist, posited that individuals learn by recognizing symbols and patterns. Grasping symbolic notation is therefore the first step in figuring out mathematical concepts. As children learn a concept more deeply, their understanding moves from concrete layers to abstract layers, and they are ultimately able to understand a symbol. Bruner described learning as a graduated process that “requires a continual deepening of understanding of ideas that comes from learning to use them in progressively more complex forms” (p. 13).

Hayes and Fagella (1988) stated that, “Our role, as adults, is to help each child recognize mathematics situations in their activities and encourage the children to apply their knowledge and experiences to any problems that occur” (p. 9). This idea is consistent with Bruner’s beliefs that “we must teach at the learner’s level of comprehension and continuously offer them chances of deepening their understanding” (p. 13). Children must feel a connection to the concepts that they are required to understand in order for learning to be relevant and lasting (Gardner, 1991). Manipulatives can help accomplish this goal. Research shows that when using manipulatives, students are more motivated and maintain their interest longer than with direct instruction (Heddens, 1996). Manipulatives provide students with opportunities to become actively engaged in meaningful learning experiences, thus allowing them to take ownership of their learning. After using manipulatives, students gain the ability to transfer their knowledge from concrete to
symbolic to real-life situations (Blair, 2012; Heddens, 1996).

The National Council of Teachers of Mathematics (NCTM) has strongly advocated the use of manipulatives in the classroom. Since the early 1940s, the NCTM has recommended that all students use manipulatives (Hartshorn & Boren, 1990). “Experimental education is based on the idea that active involvement enhances students’ learning. Applying this idea to mathematics is difficult, in part, because mathematics is so abstract. One way of bringing experience to bear on students’ mathematical understanding, however, is the use of manipulatives” (Bellonio, 2001, p. 1).

The NCTM (2010) recommends that teachers integrate manipulatives in to all levels of mathematics education, as these methods allow students to think algebraically and increase their conceptual understanding of mathematical ideas (Magruder, 2012). Sowell (1989) suggested that that long-term use of manipulatives is more effective in maintaining and increasing learning when compared to short-term use. As a result, manipulatives should be used consistently throughout middle school and high school. However, implementation of manipulatives in all levels of education has been limited. Moyer and Jones (2004) asserted that “it is more likely that manipulatives would increase their value in later grades, in teaching more complicated skills, as children mature and become mentally able to develop understanding of operations” (p. 5). Using manipulatives at the elementary level would allow students to bridge the gap between the procedure they are performing and the meaning it represents, ultimately increasing understanding rather than memorizing computation rules (Moyer and Jones, 2004).

**How Manipulatives Can Help Students Learn Mathematical Concepts**

The use of manipulatives is supported by numerous learning theories. For instance, Piagetian theory suggests that children learn by hands-on experiences and reflecting on the
results of their physical actions (Baroody, 1989). The theories of Piaget, Bruner and Montessori are developed by the idea that students must form knowledge from concrete to abstract. Furthermore, the more experience students have with the concrete, the greater their conceptual understanding will be (McNeil & Jarvin, 2007). Fennema (1972) using hands-on manipulatives provides an example of a concrete model, while representation by commonly accepted symbols is a form of an abstract model of mathematics. Many elementary students must see concrete models to make meaningful connections to symbolic models. Boeree (1999) explained how the use of manipulatives is consistent with Piagetian theory. Piaget classified young children ages two through seven as in the preoperational stage of development, which is followed by the concrete operational stage of cognitive development that lasts until about age 11. The Piagetian categories reinforce that young children need to experience concepts concretely before being introduced to the symbolic language of mathematics (Boeree, 1999).

The incomplete internal systems of representations may be to blame for many students’ struggles in mathematics. As students’ conceptual understanding of mathematics is contingent on the power and flexibility of their internal representations (Goldin & Shteingold, 2001), it is theorized that using manipulatives can help students develop the internal representations necessary to give meaning to symbolic representations (Baroody, 1989). This idea is also supported when using virtual and interactive manipulatives. Wartloft (2011) explained that the dynamic nature of interactive virtual manipulatives encourages students to manipulate mathematical concepts through clicking a mouse or dragging and dropping objects into place. These actions move or modify the objects in a way that causes learning to occur. In the case of virtual manipulatives, students learn by noting the behavior of the manipulative and forming connections about why the change has occurred. This idea is closely tied to Piaget’s theory of
children’s developmental stages, and results in conceptual understanding facilitating learning (Wartloft, 2011).

After students learn to solve basic equations by representing them with manipulatives, they can begin to progress toward an abstract level of comprehension by transferring to symbolic representations of the problem through either drawing or providing written descriptions of their work. Therefore, when used properly, manipulatives allow students to compare representations, form new representations, and subsequently form greater connections between mathematical ideas.

**Conceptual Understanding as Symbolic Representations**

Research indicates that manipulative devices and pictorial representations are related to positive outcomes in mathematics learning. Pictorial representations are particularly useful in helping students develop conceptual knowledge of mathematics problems that may otherwise appear meaningless to students (Miller & Hudson, 2007). Suh and Moyer (2007) indicate that using symbolic representations, such as manipulatives, allows students to make meaningful connections between procedural and conceptual knowledge. Additionally, using manipulatives can assist students in making connections between various mathematical concepts. Such relational thinking is the core of conceptual understanding, and symbolic learning can significantly contribute to students’ mathematical understanding (Suh & Moyer, 2007).

Representation “involves creating, interpreting, and linking various forms of information and data displays, including those that are graphic, textual, symbolic, three-dimensional, sketched, or simulated” (NCTM, 2003, p.3). Mathematical concepts and ideas are characterized using words, symbols, illustrations, charts, and graphs (NCTM, 2003).

Research from the Rational Number Project focusing on fractional representations
supports the use of manipulatives. The findings from this project are that students who learned using manipulatives significantly outperformed students taught using a simple symbolic approach (Cramer, Post, & delMas, 2002). The Rational Number Project identified four ways that manipulatives helped students understand fractions. First, manipulatives assist students in developing mental images of fraction meaning. Second, manipulatives assist students in understanding fraction size. Third, manipulatives act as a reference when justifying their answers. Finally, manipulatives discourage students from resorting to misconceptions developed as a result of applying whole number rules to fractions.

Martin and Schwartz (2005) conducted two studies to compare the effects of using manipulatives to using simpler representations. In the first study, they compared a group of students who had used a manipulative to a group of students who were only permitted to use pictorial representations. The two groups received identical instructions, but the students in the experimental group were permitted to manipulate fraction pies and tile pieces. The students in this group were found to have significantly higher abilities in problem solving and attempted more problem-solving strategies than the other group. In the second study, the researchers compared two groups of students who both had access to manipulatives. However, one group of students was encouraged to manipulate the objects themselves, while the other group had the manipulatives prearranged for them. Results indicated that students who were able to manipulate the objects themselves were more likely to correctly solve the problems. Martin and Schwartz concluded from the results of both studies that physically manipulating the objects facilitates more effective learning than simpler representations. This method allows students to conceptualize fractions and allows them to overcome whole number bias.
When using manipulatives, Martin and Schwartz (2005) described four learning procedures commonly used by students: induction, offloading, repurposing, and physically distributed learning. Induction occurs when students utilize inductive reasoning with manipulatives to deepen their understanding (Martin & Schwartz, 2005). For example, a student may take two one-eighth-fraction pieces and lay them on top of a one-fourth-fraction piece, which may facilitate their understanding between these two fraction sizes. Offloading requires students to monitor conceptual aspects through the manipulative. This method allows students to use less of their internal memory, which expedites the learning process (Cary & Carlson, 1999; Martin & Schwartz, 2005). Repurposing allows students to change their environment, causing them to implement their learning efficiently (Martin & Schwartz, 2005). In physically distributed learning, the learning occurs in both student understanding and the action of manipulation, resulting in new ideas as a result of both the physical adaptation and the individual. For example, when determining what quantity one-fourth of eight is, the student may consider one-fourth as a part of the whole object. However, when using physically distributed learning, the student may place eight objects into four groups. In this way, the child begins to assimilate the two strategies into one idea, thereby overcoming their whole quantity interpretation of one (Martin & Schwartz, 2005).

**Making Connections Between Conceptual and Procedural Knowledge**

Research has demonstrated that manipulatives can facilitate students’ mental connections between conceptual and procedural knowledge, as manipulatives support their understanding of how manipulating images replicate and represent formal symbols. Connecting these two ideas has been shown by research to improve students’ mathematical thinking and learning (Reimer & Moyer, 2005). Manipulatives allow for creative memorization rather than traditional rote
memorization. If the brain cannot make adequate connections with the material, retention of concepts will suffer (Bellonio, 2001; DeGeorge & Santoro, 2004; Suh & Moyer, 2007). When children seek to recall information with which their brains have not connected conceptually, they may have significant difficulty retrieving it. The brain learns by recognizing patterns and searches for patterns that will decrease the cognitive load. Manipulatives allow students to develop key connections, as well as form patterns that are most consistent with their learning styles (Bellonio, 2001; DeGeorge & Santoro, 2004; Suh & Moyer, 2007).

**Research Studies on Manipulative Effectiveness**

Research has demonstrated that manipulative use is beneficial in mathematical instruction. In a meta-analysis of manipulative use in elementary school classrooms, Parham (1983) examined 64 research studies conducted between 1965 and 1979. Results of the meta-analysis indicated that the students who used manipulatives in their mathematical instruction achieved in the 85th percentile on the California Achievement Test, while students who did not use the manipulatives scored in the 50th percentile.

Although the utility of manipulatives has been demonstrated across multiple topics and various grade levels, manipulative use per se is not a guarantee for success. For example, the way in which students are taught to use the manipulative can impact their achievement. Clements and McMillen (1996) established that students who were taught to use manipulatives in a rote manner struggled to see the connection between the concrete object and the symbolic idea the object represented. Additionally, students who used manipulatives in a rote manner often failed to link the concrete image to the abstract concept when compared to their peers who did not use this method. This outcome is unsurprising, as a major goal of manipulatives is to move away
from rote memorization in traditional mathematics learning. These findings indicate that the methods in which teachers implement manipulatives is critical to student outcomes.

Many research studies have attempted to compare the efficacy of virtual and physical manipulatives. Such attempts have included various grade levels and subject matter, such as geometry, algebra, and fractions. Older studies obtained inconclusive results (Nute, 1997; Pleet, 1991), and researchers have sought to improve on these studies in recent years. The results of several meta-analyses indicate that students who interact with manipulative models generally outperform those students who do not (Parham, 1983; Sowell, 1989; Suydam & Higgins, 1977).

The following section discusses the use of manipulatives in improved mathematics abilities, improved conceptual understanding, effectiveness with fractions instruction, and combining physical and virtual manipulatives.

**General Improvement of Mathematics Abilities**

In a study by Suh and Moyer (2007), the researchers investigated whether third grade students’ mathematical abilities improved by using virtual and physical manipulatives. Although the researchers did not explicitly state where the participants were obtained, it appears as though the students were selected from two intact classrooms that may or may not have been from the same school. The researchers divided the students into two groups. The first group was taught for one week about algebraic relationships using online virtual balance scales. The other group learned the same content in the same period of time, but used commercially available physical manipulatives. The results of the statistical analyses of the quantitative data showed significant improvement between the pretests and posttests in both groups. Additionally, qualitative differences suggested that the manipulatives had different strengths. The physical manipulatives appeared to allow students to invent solution strategies as well as utilize more mental
mathematics. On the other hand, the virtual environment provided students with instant feedback, step-by-step support, and linking of the visual and symbolic models (Suh & Moyer, 2007).

However, Suh and Moyer’s experimental design showed significant flaws, particularly in the statistical analyses. For example, group differences were apparent before the experiment, but were not tested for statistical significance. Additionally, the researchers did not report any statistical comparisons between groups on the posttest measures of achievement or overall learning gains. Furthermore, although the researchers made qualitative comparisons between the groups, they failed to make quantitative comparisons between them, omitting useful information. Finally, the methods used to analyze qualitative data were not explained in detail and only moderate support for the results were given.

Steen, Brooks, and Lyon (2006) conducted a study to see if virtual and physical manipulatives improved the mathematical abilities in 31 first-graders over 13 days. The researchers randomly assigned two teachers to two treatment groups after administering two pretests to the students. One teacher taught first graders geometry concepts using virtual manipulatives, while the other taught first graders geometry concepts using physical manipulatives and corresponding worksheets. At the end of the treatment, the researchers administered two posttest measures of achievement and compared the results to the two administered pretests of achievement. Even though the data tables indicated that pre-treatment differences existed between treatment groups, no statistical tests exist to determine if these differences were significant.

When analyzing the posttests, the researchers found no statistically significant difference between the two treatment groups. However, both groups showed significant improvements in
math achievement from the time of the pretests to the time of the posttests. These results indicate that virtual manipulatives may be equally effective as physical manipulatives.

In addition to this study, Steen et al. (2006) assessed a single teacher’s opinions regarding her students’ attitudes toward using virtual manipulatives. The data were collected through qualitative measures such as journal entries. The researchers determined from the content of the journal entries that the teacher involved in the virtual manipulatives perceived this method as beneficial for saving time in the classroom. The teacher believed that saving time was directly linked to an increased amount of time-on-task and an increased number of repetitions of practicing an activity. She also believed that virtual manipulatives lent themselves to more flexibility than physical manipulatives, and that virtual manipulatives caused students to learn more “in-depth” than in previous classes she taught using physical manipulatives (Steen et al., 2006).

The study by Steen et al. (2006) provided useful results. However, the research design exhibited many gaps, such as a weak research design to collect and analyze the data. For example, when the researchers collected qualitative data, the teacher involved in the physical manipulative condition provide a journal for comparison. Quantitatively, no statistical tests existed to demonstrate how the pretreatment group differences might have impacted the results. The administration of the treatment also showed poor design, as two separate teachers instructed the two groups. The researchers also were not specific about whether the teachers had any prior relationship to the students. Prior knowledge may have also interfered with the design of the study. The high pretest scores suggest that the students had been exposed to the material beforehand, increasing the likelihood that the data became distorted as a result.

Smith (2006) also attempted to compare virtual and physical manipulatives, as well as see
if they improved the mathematical abilities of students. The study also investigated how virtual and physical manipulatives impacted time-on-task, as well as how the elementary school student participants felt about this learning style. A total of 39 fifth graders from a small, rural elementary school were randomly assigned through a stratified random sample to the treatment conditions. Gifted and regular education students were included and accounted for through random assignment. However, seven special education students could not be randomly assigned, and data were analyzed both with and without these seven students. The intervention included two units: one concerning integers, and one concerning the expansion of polynomial functions. The researchers then exchanged the conditions between units, allowing the students who had used the virtual manipulative to switch to the physical manipulative, and vice versa. Four instructional lessons comprised each unit. The researcher collected data in the form of a pretest, two posttests at the end of each unit, three interest surveys, and a questionnaire regarding the use of manipulatives. The research assistants also kept a time-on-task record in which students were monitored every two minutes (Smith, 2006).

The researchers found results similar to Steen et al. (2006). The results indicated that significant improvements existed in both groups from the pretest to the posttest, but no significant differences between the virtual and physical manipulatives groups existed. The researchers found that the students’ preferences were impacted by the order in which they were exposed to the manipulatives, but there were no major differences that existed between students’ attitudes toward virtual and physical manipulative models. Time-on-task behavioral analyses were inconclusive, as students in the first unit showed less on-task behaviors, but during the second unit, no statistically significant differences existed between the two conditions (Smith, 2006).
Smith’s (2006) study has a significantly stronger research design than the other studies discussed, but also demonstrates flaws. First, the small sample size means that the results could indicate a Type II error rather than a lack of real differences between treatment conditions. The potential benefits of the crossover design may have been negated by the randomization. Real differences may have been more difficult to observe because participants interacted with each manipulative for less time than if they had been assigned to only one treatment condition. However, the results indicated that virtual and concrete manipulatives might be equally effective when learning the mathematical concepts in the study, as both groups showed significant steps in the learning process.

**Improving Conceptual Understanding**

A study conducted by Reimer and Moyer (2005) that began as a teacher’s action research project examined the impact of virtual manipulatives on students’ conceptual and procedural understanding of fractions. Reimer, the teacher, collaborated with a researcher to improve the process and structure of the research. The participants were 19 students recruited from Reimer’s third grade class. The study was conducted over a two-week period. The first week consisted of the students learning how to use the virtual, base-10 block manipulative, and no qualitative or quantitative data was collected during this time period. In the second week, the students used the virtual manipulatives for four days, one hour per day. The students were instructed on how to use the virtual manipulative through a worksheet made by their teacher. The subject matter taught during the intervention was identical to content that had been taught earlier in the year using physical manipulatives. The researchers used previously learned content because any new learning between the pretests and the posttests could be attributed to the virtual manipulatives, rather than any other method of learning (Reimer & Moyer, 2005).
The quantitative data collected for the research consisted of pretests and posttests assessing conceptual and procedural knowledge of the mathematical subject matter taught during the intervention. The qualitative data was composed of student interviews, which were conducted by both researchers during the second week of intervention. The students were asked a series of four questions and researchers analyzed their answers using a narrative analysis (Reimer & Moyer, 2005).

The results of the study showed high scores on the pretests, which was consistent with the students’ significant previous exposure to the content taught during the intervention. These high scores can result in difficulties finding differences between pretest and posttest scores. Unsurprisingly, there were no statistically significant differences between these scores. Due to the lack of statistical significance, the results were inconclusive regarding whether the method can help students with procedural knowledge. However, when the students were interviewed, they commented on the differences between virtual manipulatives and physical manipulatives based on their experience. The students stated that the virtual manipulative was beneficial in providing quick feedback. The students also stated that they felt the virtual manipulative was easier and quicker to use than physical manipulatives (Reimer & Moyer, 2005).

As with other studies addressing this topic, Reimer and Moyer’s study showed flaws in its design. For example, the study began as an action research project by Reimer, leading to a weak research design. The study notably lacks a true control condition. Although the students showed a small increase in conceptual knowledge, the usefulness of the information is questionable without a control condition, especially since the students showed high pretest scores. The students also showed significant previous exposure to the concepts being taught during the intervention. In spite of the shortcomings of this study, the results warrant further
research in this area.

**Manipulatives for Fraction Instruction**

Research has yet to extensively explore how manipulatives can help children understand fractions. However, the limited research available indicates that manipulatives can be beneficial in teaching fractions. A study by Suh, Moyer, and Heo (2005) investigated three fifth-grade classrooms that were studying fractions to demonstrate the connection between procedural knowledge and conceptual understanding. They also investigated the use of virtual applets in this setting. Students in the three classrooms were classified as low, medium, and high achievers. All students were introduced to and taught the lesson by the same teacher. The researchers discovered during interviews with the students that they did not make the connection between the new concepts and the information they had previously learned. All three achievement groups showed discovery learning, higher levels of conjectures, connections to previous learning, and greater levels of peer interaction. Results also showed that the group of low achievers improved the most (Suh et al., 2005).

Suh (2005) conducted a study with 36 third graders using a mixed-methods approach in order to demonstrate the impact of both virtual and physical manipulatives on student achievement. Suh also investigated how the virtual and physical environments differ in representation. The author sought to avoid pitfalls experienced by past researchers who explored this topic, such as prior student knowledge, teacher effects, and inability to compare data (i.e. lack of control groups) for a stronger research design.

Suh taught both classes during the intervention to control for teacher effects. The researcher did not assign students to a condition. Instead, the author utilized a within-subjects crossover repeated measures design. All participants received both treatments and served as their
own comparison. In order to prevent the residual effects of prior knowledge, the researcher introduced two new mathematical units to the students during the intervention, fractions and algebra. The fractions unit focused on adding fractions with unlike denominators while the algebra unit focused on balancing equations. One class learned the fractions unit through use of the virtual manipulative and algebra through the physical manipulative, while the other class learned the units with the opposite combination. The virtual manipulative condition allowed students to use a free set of online manipulatives, and the students completed problems on a computer screen. The physical manipulative used by the students was commercially available and included in a task sheet. Quantitative data was collected and analyzed through pretests and posttests. The researcher collected field notes, classroom videotapes, and student interviews as qualitative data (Suh, 2005).

The analysis of the pretests and posttests indicated that all students in both environments who used a manipulative improved their knowledge of the subject matter significantly. In terms of the efficacy of the manipulative type, results indicated that students who learned fractions using a virtual manipulative did better than those who learned using a physical manipulative. However, no differences existed between the types of manipulatives when learning algebra. These results suggest that virtual manipulatives may be particularly useful when students are learning fractions. The qualitative data suggested that the virtual manipulative possessed features that help guide students through the process of learning formal algorithms for adding fractions with unlike denominators. These features assisted students with linking concepts, helping with the step-by-step process of completing problems, and providing prompt feedback (Suh, 2005).

Suh’s stronger research design and methodology avoided the weaknesses in other similar studies. The within-subjects, crossover-design allowed the researcher to use each student as
his/her own control for comparisons between the algebra unit and the fractions unit. However, the validity of the results may be compromised by the lack of control group, as the features of the virtual manipulative could be inherently different in each group because of the subject matter being taught (fractions vs. algebra). For example, step-by-step procedures were available in the fractions virtual manipulative, but not the algebra virtual manipulative. This difference could have resulted in those students’ higher performance.

Aljohani (2000) investigated the effectiveness of using pattern blocks in teaching and learning the four operations of fractions for the academic achievement of fifth and sixth grade students in Al-Madinah, a city in the western region of Saudi Arabia. Approximately 191 participants from three different schools were chosen randomly to participate in the study. Two schools included the experimental groups, using the manipulative of pattern blocks, while the third school included only the control group, using the traditional method of teaching for both fifth and sixth grade students. Two tests were constructed to measure adequately the goal of the study. The first test was for fifth grade students and included 35 questions, while the second test was for sixth grade students and included 20 questions. After using ANCOVA, the results revealed significant differences between the study groups in both fifth and sixth grades. Teaching the four operations of fractions using the manipulative of pattern blocks proved to be more effective than using the traditional teaching method for both fifth and sixth grades.

A study by Westenskow (2012) investigated 43 fifth grade students with mathematical learning difficulties to see if manipulatives could help them learn fraction equivalence. The researcher utilized three interventions over 10 sessions: virtual manipulatives, physical manipulatives, and an intervention using both virtual and physical manipulatives. Westenskow used a mixed-method approach to collect and analyze data. The researcher evaluated the effect
size at the total, cluster, and questions levels of the assessments to ascertain how the type of manipulative had impacted the results. Achievement and mastery trajectories were examined through the daily assessment data. The researcher separated the learned concepts into five clusters and three sub-clusters.

Based on the pre- and post-data analysis, all three interventions showed significant gains. Data synthesis favored the physical manipulatives in two of the clusters, virtual manipulatives in one cluster, and the combination of manipulatives in two clusters. Variations were discovered in student’s ability to overcome misconceptions in the quantitative analysis, as well as their specific strategies. Virtual manipulatives were found to be more beneficial in helping students understand only symbols, but physical manipulatives were more beneficial in helping students grasp set model representations. Additionally, students who used the virtual manipulative were more likely to show increased learning on tasks that instructed them to generate three or more equivalent fractions. The results indicate that the utility of the manipulatives is dependent on the concept, and it is critical that instructors understand the benefits of each in order to understand when to integrate the manipulative (Westenskow, 2012).

Recommendations based on Westenskow’s (2012) intervention study emphasize student understanding of concepts through manipulatives. The results support that virtual and physical manipulatives are effective in instruction. However, consistent with Suh’s (2005) study, each manipulative type has its own unique traits that can impact the learning of different concepts.

Kabli (2013) examined the effect of using overhead slide transparencies to overcome the difficulties experienced by fifth-grade students learning fractions. The study used the overhead projector to present the slides in order to facilitate the learning of fractions for children. For this study, a random sample of 129 fifth grade students were divided into experimental and control
groups. The findings showed the superiority of the experimental group students in all tests. This confirms the importance of using instructional tools within math classrooms in order to facilitate the teaching of fractions and overcome students’ difficulties with learning fractions.

**Combining Physical and Virtual Manipulatives**

The results of the study by Westenskow (2012) suggested that virtual manipulatives may be the more useful method when teaching fractions. However, the research also suggested that combining the two methods, virtual and physical, may be ultimately more effective than choosing only one method. The two studies discussed below came to similar conclusions, even when the research focus was virtual manipulatives compared to physical manipulatives, as opposed to a combination of the two.

Using two sixth grade geometry classrooms, Takahashi (2002) investigated whether computer-based virtual manipulatives were more effective and promoted student participation more than physical manipulatives. The students in both classrooms were instructed to develop a formula for determining the area of a parallelogram. One classroom used geoboards in the form of a physical manipulative, while the other classroom used geoboards in the form of a virtual manipulative. Japanese Lesson Study inspired the design of the study, where teachers and anyone else involved come together to plan, observe, and discuss outcomes (Doig & Groves, 2011). Consistent with the study’s design, two experts in mathematics teaching and learning, along with the researcher and teacher, observed the classes and then discussed the relationship between the two types of geoboards in problem-solving activities (Takahashi, 2002).

Takahashi’s study found that physical and virtual manipulatives had different affordances. For example, the virtual manipulative allowed a student to place a color inside of a shape. However, the physical manipulative allowed students to easily move the objects to create
shapes. As a result, the researchers concluded that the different manipulatives had different strengths. The physical geoboard might be more useful in helping students develop the concept of area for shapes such as a rectangle or a square. However, the virtual geoboard might be more useful in developing formulas where transforming shapes is required, as with the parallelogram. The researchers concluded that students would benefit from using both physical and virtual geoboard manipulatives in the classroom to maximize learning (Takahashi, 2002). However, further research is recommended to understand this phenomenon in other mathematical subjects.

Moyer-Packenham and Suh (2012) conducted a meta-analysis evaluating the effect of virtual manipulatives on student learning. The authors analyzed 82 effect scores obtained from 32 studies, which yielded a moderate average effect size of 0.35 when compared with other methods of instruction. When virtual manipulatives were compared to physical manipulatives as a primary teaching tool, the average effect score using 38 scores was small (0.15). When virtual manipulatives were compared to traditional classroom instruction, the average effect score using 18 scores was moderate (0.75). The researchers also conducted an analysis comparing virtual manipulatives to other instructional methods for teaching fractions, which resulted in an average effect score of 0.53 (11 effect scores). The analysis also compared 26 scores in which instruction combined virtual and concrete manipulatives. The results indicated that when virtual and physical manipulatives were used together there was a moderate effect of 0.33. Based on the results, the researchers hypothesized that virtual manipulatives are effective in teaching fractions. They also concluded that using both virtual and concrete manipulatives may be even more beneficial (Moyer-Packenham & Suh, 2012).
Summary of the Studies

In all of the discussed research studies in which the researchers made quantitative comparisons between virtual and physical manipulatives, results indicated that virtual manipulatives are at minimum as effective as physical manipulatives. Occasionally these studies demonstrated that both treatment conditions failed to make a significant improvement between the pretest and posttest, but no studies demonstrated that students in the physical condition outperformed those in the virtual condition. The virtual condition subjects in Suh’s (2005) study were found to outperform those in the physical condition, but only in the fractions unit, indicating that fractions instruction may be particularly well suited for virtual manipulatives. Further benefits of virtual manipulatives include a greater tendency toward time-efficiency than physical manipulatives, which allows for students to complete more practice during class time.

These studies also emphasize how poor research design and subsequent threats to validity can impact overall results. Significant differences can be difficult to ascertain between the virtual condition and the physical condition. It may even be difficult to ascertain significant differences between pre-test and post-test measures of achievement. This study emphasizes the benefit and importance of large sample sizes, conducting a pretest to ascertain how familiar students are with the content, and providing sufficient time for manipulative use, as well as effectively assimilating manipulatives into the subject matter. Additionally, the benefits of within-class random assignment can decrease the impact of pretreatment group differences and related issues. Finally, this study demonstrates the consequences of not controlling for other variables that could disrupt the results.
Physical Manipulatives vs. Virtual Manipulatives

Puchner et al. (2008) stated, “manipulatives are concrete tools to create an external representation of a mathematical idea” (p. 314). While physical manipulatives have existed for decades, virtual manipulatives have existed for a much smaller time period. Many virtual manipulatives are modeled based on their physical manipulative counterparts (Moyer-Packenham, 2010). Balka (1993) detailed the multifaceted benefits of manipulatives when he stated, “The use of manipulatives allows students to make the important linkages between conceptual and procedural knowledge, to recognize relationships among different areas of mathematics, to see mathematics as an integrated whole, to explore problems using physical models, and to relate procedures in an equivalent representation” (p. 22).

As previously discussed, research has engendered controversy regarding whether concrete or virtual manipulatives are more effective. Lappan and Ferrini-Mundy (1993) posited that the usefulness of manipulatives is contingent on the ability to be hand-held and literally “manipulated” by the user. As a result, most available research has promoted the effectiveness of concrete manipulatives over virtual manipulatives, as content on a computer screen cannot be physically manipulated. However, technological advances have enhanced the usefulness and quality of virtual manipulatives in recent years, causing researchers to question the relevance of historical research, stating that virtual manipulatives are more effective than physical manipulatives in today’s highly technological world. Additionally, proponents of virtual manipulatives claim that they are equally effective or superior to physical manipulatives because they can be transformed and manipulated in a similar way and they are less distracting (Bouch & Flanagan, 2009; Durmus & Karakirik, 2006; Moyer et al., 2002). As a result, more recent research should be examined to identify any logical flaws in this issue. New research more
relevant to the technology that is now available as well as the cultural changes that have followed should be conducted. The following discusses the specific differences between physical and virtual manipulatives that can affect student learning.

**Ease of Use**

One of the major arguments regarding which manipulative is superior relates to ease of use. As their names suggest, students physically handle physical manipulatives, while virtual manipulatives are utilized on a computer screen. If the characteristics of the object cause manipulation to be too difficult, the manipulative will cease to be an effective teaching tool, and may even be detrimental to student learning (Boulton-Lewis, 1998).

Virtual and physical manipulatives both exhibit advantages and disadvantages in ease of use. Haistings (2009) and Izydorczak (2003) observed that students occasionally used physical manipulatives in a “sloppy” manner, which can result in incorrect answers. However, virtual manipulatives did not exhibit this issue, and students using this type of manipulative obtained more accurate answers. Kim (1993) replicated these findings with kindergarten students. When the students used virtual manipulatives, their answers were found to be more accurate, as the students used methods that had better organization.

Numerous researchers believe that physical manipulatives are superior in terms of ease of use. These researchers state that physical manipulatives are less cumbersome to use than virtual manipulatives and allow them to complete tasks quickly (Baturo, Cooper, & Thomas, 2003; Highfield & Mulligan, 2007; Hsiao, 2001; Nute, 1997; Takahashi, 2002). On the other hand, a large amount of researchers believe that virtual manipulatives are easier to use due to the features within the virtual applets, such as cloning objects and rapid repetition of computer actions, which allow students to complete more work (Beck & Huse, 2007; Clements & Sarama, 2002;
Deliyianni et al., 2006; Izydorzak, 2003; Lamberty, 2007; Steen et al., 2006; Terry, 1996; Yuan, Lee, & Wang, 2010). It is arguable that a virtual representation can be flipped, turned, slid, and rotated on a computer in the same way as a physical object (Spicer, 2000). Additionally, research suggests that students who use virtual manipulatives are able to create a larger variety of responses (Clements & Sarama, 2007; Heal et al., 2002; Highfield & Mulligan, 2007; Moyer et al., 2005; Suh et al., 2005; Thompson, 1992).

Clements and McMillen (1996) formed an opinion that virtual manipulatives are easier to use than physical manipulatives in their study investigating students’ learning of base-ten blocks. The virtual base-ten blocks showed consistency with the students’ own mental operations about the intended learning outcome. The students had the option to break the blocks apart to form ones, or fuse them together to form tens. As the activity progressed, the intended learning outcome became clearer to the children. The virtual manipulative proved to be natural for the children and contributed to building their inference skills. The children were provided with immediate feedback, as the number on the screen changed whenever the block changed. Clements and McMillen (1996) explained that a virtual manipulative is advantageous in such a situation because concrete base-ten blocks are clumsy, and students may not grasp the connection between the activity and the material they are learning. They describe this phenomenon as the students seeing “only the trees – manipulatives of many pieces – and miss the forest – place-value ideas, [whereas] the computer blocks can be more manageable and clean” (p. 3).

**Guided Instruction and Instant Feedback**

Manipulatives show usefulness through helping with problem solving activities and in explicit guided instruction, both of which are major benefits to mathematics instruction (Martin
& Schwartz, 2005; McNeil & Jarvin, 2007). Most physical manipulatives do not have defined guidelines to direct student usage. However, virtual manipulatives often exhibit defined structures for usage, as they are designed to teach specific mathematical skills and concepts by guiding students through explicit steps (Heal, Dorward, & Cannon, 2002; Suh & Moyer, 2007).

When using virtual manipulatives, applets provide instantaneous feedback to students as part of guided instruction. Research has suggested that this instant feedback that can only be obtained in virtual manipulatives is key in student learning (Deliyianni, Michael, & Pitta-Pantazi, 2006; Highfield & Mulligan, 2007; Izydorczak, 2003; Steen, Brooks, & Lyon, 2006; Suh et al., 2005).

The automatic feedback and guided instruction found in the virtual manipulatives has a distinct advantage over concrete manipulatives, as the child requires teacher assistance when difficulty arises. The need for teacher assistance in this situation could lead to a student not continuing to try and/or not asking for assistance for various reasons (i.e. embarrassment). Even if the student is able to ask for assistance, the teacher’s time is divided among multiple students and the students may not receive the attention they need (Suh & Moyer, 2005). Additionally, instructional feedback that guides students, helps them to question the task, and assists them in finding the most appropriate plan to reach a real-life resolution is a more effective method than simply supplying an answer (Herrington, Oliver, & Reeves, 2003). However, in order for the feedback to be meaningful, it must be administered in a timely fashion. Virtual manipulatives are effective in providing feedback to students immediately upon rendering their response (Crompton, 2011). If students immediately see the outcomes of their manipulations, they form more connections between procedural and conceptual knowledge (Moyer, Bolyard, & Spikell, 2002). Research has also demonstrated that instantaneous feedback encourages students to become more experimental when developing representations, making conjectures, and testing
Built-In Constraints and Amplifications

Virtual manipulatives also help guide instruction by constraining the student to perform tasks in a step-by-step manner (Behr et al., 1983). In a study by Takahashi (2002), the researcher observed students using virtual manipulatives with this constraint, as well as students using physical manipulatives for solving geometry problems. The constraints of the virtual manipulative caused students to use more time. However, these students were more likely to look for equivalent area transformations and then use the formulas they had developed. The students who used physical manipulatives were less conceptual in their approach and focused on counting squares. When they were instructed to calculate the area of various shapes to transform them into other shapes of equal area, these students continued to focus on counting rather than applying new ideas and formulas.

Additionally, many virtual manipulatives are designed to amplify mathematical concepts (Dorward & Heal, 1999; Moyer-Packenham, Salkind, & Bolyard, 2008; Suh, 2010). Research shows that student learning is impacted by three methods of conceptual amplification: requiring specific actions, demonstrating simultaneous changes, and helping students to focus on specific aspects of the object (Moyer-Packenham & Suh, 2012). For example, Beck and Huse (2007) found that students spinning a virtual spinner were able to observe the difference between experimental and theoretical probability through observing that changes in a computer bar graph decreased, while spins increased.

Clements and Battista’s (1989) research illustrated another example of virtual manipulatives amplifying mathematical concepts. Students were instructed to draw a rectangle on paper, and they simply drew a rectangle. The task was more consistent with drawing a
picture, rather than connecting the drawing to a mathematical idea. However, when the students used the virtual manipulative Logo to draw a rectangle, the program required them to enter a series of commands and procedures to complete the drawing. The students were required to analyze the shape of a rectangle in terms of mathematical concepts in order to complete the task. For example, the commands they entered into the program required them to give opposite sides of the rectangle equal lengths, yet incorrect commands would result in the shape becoming a square or parallelogram. As a result, the students gained a greater conceptual understanding and connection between their task and the result due to the program’s amplification ability (Clements & Battista, 1989).

**Linking Representations**

As previously discussed, conceptual understanding is critical, and manipulatives are thought to facilitate this process. In order to form conceptual understanding, the manipulative should ideally illustrate the link between the object representations and the symbolic. However, only a few physical manipulatives accomplish this task (e.g. fraction tiles), as the symbolic representation is written on the manipulative. However, most virtual manipulatives have this ability, as they are specifically designed to help students make connections in linking abstract symbolic representation to visual images (Bolyard, 2006; Heal et al., 2002). Virtual manipulative applets allow students to relate changes in the object representation to changes in the symbolic representation as a result of their actions (Moyer et al., 2005).

Clements and McMillen (1996) strongly believe that virtual manipulatives are more useful for linking representations than their physical counterparts, as many students fail to understand the connections when using physical manipulatives. The virtual manipulative provides instant feedback to link the concrete and symbolic after the student makes changes on
the screen. “The computer links these two actions, and students are then able to associate the concrete and symbolic easier” (Clements & McMillen, 1996, p. 3).

However, when using virtual manipulatives, students may become locked into what Sayeski (2008) calls “search space,” in which the student only uses one method to obtain solutions and does not attempt different methods to find a solution. This phenomenon is likely to occur if the program allows students to complete procedures without adequate reflection on the connections between their actions and the mathematical concepts involved, causing the use of the manipulative to become mechanical (Martin & Schwartz, 2002; Moyer, 2002). Rather than experimenting or using conceptual thinking to comprehend their mistakes, the student simply uses the program’s tools to reset the problem, ask for a new problem, or ask for help (Izydorczak, 2003).

Distracters

A problematic aspect of virtual manipulatives that has been explored by research is the potential for cognitive overload due to student computer frustrations, especially if the student experiences mathematical learning difficulties (Highfield & Mulligan, 2007; Sorden, 2005). John Sweller’s cognitive overload theory posits that a person’s working memory is limited to five to nine items at one time, and cannot absorb new information once they have reached cognitive overload (Clark, Nguyen, & Sweller, 2006). When using virtual manipulatives, computer manipulation may occupy part of the working memory, causing less memory to be available to process concepts (Highfield & Mulligan, 2007; Sorden, 2005).

In addition to cognitive overload, virtual manipulatives may also contain cognitive distractors, and that some features of this type of manipulative limit their usefulness. For example, Highfield and Mulligan (2007) as well as Izydorczak (2003) have reported that children
find the ability to change languages, colors, and shapes a form of distraction. As a result, the children become focused on altering the features of their applet, rather than learning concepts or to completing the assigned mathematical tasks.

However, it is possible to avoid cognitive overload when using virtual manipulatives. Moyer & Reimer (2005) found that providing direct instruction on how to use the program is beneficial before focusing on the concepts. The student can spend more time learning the concepts by using the program, rather than spending large quantities of time attempting to understand the program itself.

Some researchers believe that manipulatives may actually decrease cognitive overload. Martin and Schwartz (2005) stated that manipulatives are used to offload information, causing the manipulative to hold information for the user. As a result, the user has more memory available and cognitive overload is reduced. Supporting theories for this idea include the dual coding theory, which posits that utilizing more than one mode produces an additive effect, and therefore increased memory ability (Clark et al., 2006). Therefore, the requirement of dual modes in virtual manipulatives should enhance the student’s cognitive abilities rather than hinder them (Moreno & Mayer, 1999; Suh & Moyer-Packenham, 2008). Clements and McMillen (1996) believe that physical manipulatives are more distracting than their virtual equivalents. Physical pieces have potential to become broken or lost, and students may use the physical manipulatives incorrectly or inappropriately.

**Unique Affordances**

Many research studies regarding the effectiveness of manipulatives use qualitative data to examine the method’s unique affordances. Virtual manipulatives may provide more meaningful representations of objects and concepts than physical manipulatives, despite not necessarily
adhering to the traditional definition of concrete manipulatives. Students are more likely to be active participants in the acquisition of a skill when using virtual manipulatives. The unique affordances within virtual manipulatives can help bridge the gap between students’ differing learning styles. Moving from concrete to virtual manipulatives provides visual, auditory, and kinesthetic modes of instruction, allowing the students to gain an understanding of the material more easily (Gardner, 1991).

Virtual manipulatives offer more versatility than physical manipulatives, as students have the ability to change the data representation with a simple keystroke or click of a mouse. The visual flexibility of virtual manipulatives allows the student to connect the different representations to all of the possible outcomes for a given problem, and subsequently then deduce that there is more than a single way to reach a solution to a challenge, which can be applied to real-life situations (Blair, 2012; Durmus & Karakirk, 2006; Young, 2006). Virtual manipulatives also allow students to save their work and return to it later, or review their previous work. Allowing students to review their work encourages revision of strategies through true mathematical exploration (Bellonio, 2001; Crompton, 2011).

**Students' Attitudes Toward Manipulatives**

Qualitative data was collected on student attitudes toward manipulatives throughout many studies investigating manipulatives. In interviews conducted by Haistings (2009), students demonstrated both positive attitudes toward working with virtual manipulatives and an increase in conceptual understanding. The students reported specifically that they preferred an applet that contained both symbolic and pictorial representations rather than an applet with only pictorial representations. They preferred both representations for a number of reasons, including because the problem was written for them on the screen, they did not have to keep recounting the number
of blocks, they could confirm if they set up the problem correctly, they did not have to remember large numbers, and they enjoyed seeing the numbers change. The students also indicated that they made strong symbolic-pictorial links (Haistings, 2009).

In Suh and Moyer’s (2005) study investigating fraction learning with virtual manipulatives among fifth graders, the researchers assessed how the students felt about this type of learning. The students reported that the visual representations allowed them to build on previously learned class material. They also reported that the immediate feedback gave them motivation to work through challenging fraction problems, as well as prompted them to find various solutions. Additionally, the interactive nature encouraged students to collaborate, justify how they reached the solutions to the problems, and explain why a solution was mathematically correct (Suh & Moyer, 2005).

Clements and McMillen (1996) found that students preferred to spend more time on certain problems in order to learn the objective. The students felt that by gaining a deep understanding of the task, less repetitive practice was required. The students were also able to retain more information, as they developed a true understanding of the concept and were able to apply the concept, rather than simply memorizing (Clements & McMillen, 1996).

Goracke (2009) conducted a more recent action research study exploring the attitudes of eighth graders learning with manipulatives, and generally resulted in positive attitudes from the participants. The students reported enjoying the work they did with the manipulatives, and felt that manipulatives increased their overall understanding of mathematical concepts. The students reported that their enjoyment resulted from the hands-on, active participation rather than the academic benefit, but improvement was a side effect of their feelings about the activity (Goracke, 2009).
How Manipulatives Improve Students’ Attitudes Towards Math

A widespread attitude exists among students that mathematics is difficult and boring. However, several researchers theorize that students would look forward to learning mathematics if the experience were more engaging (Burns, 1996; Heddens, 1996; Steen, Brooks, & Lyons, 2006). The fear that many students feel for mathematics can be removed when they are able to take ownership of their learning. This ownership also allows students to feel an intrinsic reward for their effort. Students can proceed to build upon these positive experiences and engage in work with more thought-provoking concepts. Students who learn in this way ultimately gain the ability to take the learned concepts and apply them to their daily lives. When children visualize a mathematical concept, they experience less confusion and feel more confident in their mathematical abilities. Increased confidence can result in a great ability to process and store important mathematical concepts instead of trying to memorize seemingly meaningless procedures (Steen et al., 2006).

Manipulatives allow students to conceptualize mathematical concepts in a novel way. They include features that cannot be replicated in textbooks, such as lively, bright colors and games that direct the students (Rhodes, 2008). Virtual manipulatives are objects that students associate with on a daily basis, and therefore useful as real-life learning tools. The frequent association of these tools causes students to feel more confident in their abilities. Manipulatives can transform a mathematical frustration to a challenge that the student enjoys pursuing (Crompton, 2011). Furthermore, students’ inherent competitiveness and desire to succeed increases their enjoyment, which results in higher levels of learning and more confidence in the subject matter being taught (Burns, 1996; Moyer et al., 2002).

Picciotto (1995) posited that a deeper level of comprehension takes place when teachers
provide a visual and kinesthetic avenue for learning. Students reported that when they were able to visualize the symbolic representation, they felt more confident in their results. When using this method with their peers, they felt more secure in their efforts to explain their procedures in mathematical terms (Reimer & Moyer, 2005; Suh & Moyer, 2005; Young, 2006).

Manipulatives can improve students’ attitudes toward mathematics, which can result in better performance. This effect is especially true for virtual manipulatives, as they are engrained in today’s society. Manipulatives are able to capture students’ attention longer than traditional methods, such as textbooks and lecture. They also challenge the students to solve difficult problems (Rhodes, 2008). In a study investigating daily virtual manipulative use among third graders, Steen, Brooks, and Lyon (2006) discovered that manipulatives increased both student motivation and enjoyment when learning fractions. Other studies have replicated this finding, as they have found that students become active learners when using manipulatives, and these tools guide students to a deep level of abstract thinking, resulting in a deeper understanding of the lesson content. Additionally, students feel a sense of ownership when they find a creative way to figure out the solutions to the problems. This ownership is further enhanced when students can relate the problems to real-life situations (Goracke, 2009; Rhodes, 2008; Wiggins, 1990).

Goracke (2009) concluded from his research that when students use manipulatives, they become more optimistic about mathematics, are more confident in their abilities, and are more likely to search for multiple solutions to challenging problems.

**Students with Math Difficulties**

In the year of 1995, Piccotto et al. theorized that manipulatives that learners can perceive mathematical concepts aid children at different levels of understanding and achievement (Piccotto et al., 1995). Additionally, they surmised that for many students, manipulatives are an
important tool. However, for other students, “manipulatives create mathematical contexts, that allow children to increase their awareness at deeper levels of understanding, which is often recognized as mechanical mastery” (p. 112). Research has depicted that students having difficulties in math, benefit from practical manipulatives, in test comparing them with their classmates, who have not used manipulatives (Guevara, 2009, (Hitchcock & Noonan, 2000; Suh & Moyer-Packenham, 2008).

Butler et al., (2003) carried out a study for 50 sixth through eighth graders that were diagnosed with mild to moderate disabilities in understanding mathematics, were placed in two groups. The two groups were given the same fraction equivalence instructions, while one group worked with physical manipulatives for three of the ten lessons. The end result of the study depicted that the group that exercised using the physical manipulatives, tallied much higher scores overall. Furthermore, this same group sub-tests scores where significantly higher as well.

Witzel, Mercer and Miller (2003) conducted a study comparing the test scores of algebra post-test. The test comprised 34 matching pairs of sixth and seventh graders who were diagnosed with a mathematical learning disorder. Students who were treated with physical manipulatives overwhelmingly outscored the students that received traditional instructions. For Fifteen fifth graders, chosen as part of a study of one-third of students recognized as requiring special services, outcome results were similar (Moch, 2001). Cass et al., (2003) tested three fourth graders, who had learning disabilities using manipulatives. Maccini and Hughes (2000) investigated six adolescents identified as having learning disabilities as well. These two studies were determined as having positive results with manipulative use and better mathematics discernment.

Research supporting the use of manipulatives for students with mathematical learning
disabilities have different results when it comes to whether manipulatives are helpful for high achievement students or low achievement students. Providing an example, some research has shown that high achieving students gain more from manipulatives than low achieving students (Moreno & Mayer, 1999; Suh, Moyer, & Heo, 2005). Moyer-Packenham and Suh (2012) conducted a study that exhibited low achieving students deriving greater benefits from manipulatives.

A Moyer-Packenham and Suh (2012) research study focused on low, average, and high level fifth graders, using implicit virtual manipulatives to study fractions. By implementing paired sample t-tests, it was discovered that all three groups demonstrated improvement. Most noticeably, the low-ability group results were significant. There is mutual agreement amongst researchers regarding these findings. Lin, Shao, Wong, Li, and Niramitranon (2011) and Hativa and Cohen (1995) tested low-achieving sixth grade students plus fourth grade students. The conclusion was that the sixth graders and the fourth graders greatly benefited from using the virtual manipulatives.

Suh, Moyer and Heo (2005) studied the effects of virtual manipulatives, by placing a total of forty-six fifth grade students into instructional groups of high, middle, and low ability. The results of the test was that high ability students received greater benefit, as they increased their level of efficiency and used their mental processes more, to determine the answers to problems they worked on. The low ability students responded orderly and exactly to the steps that were listed in the program. What was additionally observed from the students recognized as having low ability was the fact that they tended to be dependent upon relying on the visual models, to establish a framework of deciphering between pictures and symbols.

The group that benefited the most from using manipulatives was the high achieving
students (Moreno and Mayer, 1999). Moreno and Mayer analysis results pinpointed sixth grade students worked off of the same integer applet, for the experiments involving control groups. The exception was that the applet contained symbolic representation. There were no major difference between the groups based upon the results of the post-test score analysis. Nevertheless, when students were sub-grouped by their abilities, the comparison of experimental group versus the control group revealed that students with higher abilities imparted a result size of 1.11. The gain scores of low ability students rendered an outcome size of 0.47. More analysis, performed by Moreno and Mayer (1999) broke-out the students based upon the variables of their spatial and memory abilities. The students who possessed a high level of spatial abilities displayed an average gain score that was six times higher than students that had low spatial abilities.

Statistically, there is no noteworthy difference between comparing two groups of different memory abilities.

Despite these study differences, there are studies available that display no major statistical differences between the results of achievement levels derived by the students when virtual manipulatives are incorporated. Drickey (2000) research involved 219 sixth graders and Kim’s (1993) research entailed working with 35 kindergarten children. The overview of their research surmises that all levels of students’ abilities will be positively impacted, despite not being sure of which ability level will benefit the student the most, when manipulatives for mathematics are inclusive.

**Intervention**

In the 20th Century, the growth of information processing and the methods for understanding information used in math education became commonplace (Schoenfeld, 2004). The growth of information processing is historically linked to two international studies: The
Second International Mathematics and Science Study (SIMSS) and The Third International Mathematics and Science Study (TIMSS). The centerpiece of both reports rests upon the lack of knowledge by United States students that pertains to mathematical conception when U.S. students are compared to other students in developed nations (Frykholm, 2004).

In the year of 2000, The National Council of Teachers for Mathematics (NCTM) published a mathematics book to concentrate on the conceptual and procedural understanding of mathematics, based upon principles and standards. Additionally, a universal statement was released by the NCTM (2007), which acknowledged that all children have the capacity to learn; however, all students will not learn the same way or in the same amount of time. A second universal statement is that teachers should expect some students to experience difficulties in understanding mathematics. Henceforth, teachers will need to be willing and ready to empower students to help them overcome the pain of learning mathematics. The concept of before school and after school intervention programs providing students with resource materials and classroom activity participation was setup to be the structure framework for which to help students learn mathematics

In recent years, several studies have supplied support that the use of constructivist practices slightly shifted previous years accepted practice that student intervention fell under special education practices (Miller & Mercer, 1997). Two studies recognized to be common among students that have difficulties learning mathematics are: (1) Not being able to recall easily and (2) Possessing undeveloped strategies for solving math problems (Fletcher, Huffman, Bray, & Grupe, 1998; Geary, 1990). Research performed by Dowker (2005) concluded that there is a positive association between a student's mathematical ability and his/her recognition of what strategies to use when working to solve or resolve mathematical equations. Dowker states,
“Development consists not of the replacement of a single immature strategy or by a single more mature strategy, but of the discovery of increasingly more mature strategies, which co-exist for a long time with immature strategies, before gradually supplanting them” (p. 22). Such a finding suggests that intervention should not just be catered to procedural understanding, but imperative is the inclusion of developing flexible conceptual understanding.

Over the last decade, there has been a considerable jump in the number of students in the U.S that have been classified as having learning disabilities. The percentage rate is over 200% and of particular concern to educators is the fact that a growing number of students are falling in between regular math classes and special education remedial math classes. An even larger trend that is worrying educators is that students may fall into the gap of not receiving the help or support that they really need (VanDerHeyden, Witt & Barnett, 2006). According to (NCTM, 2007), the system of mathematics education must be proactive in ensuring that a comprehensive system of remediation is established, careful consideration of what intervention programs to invest in is prioritized, and the flexibility to accelerate the intervention process when warranted is maximized to secure students’ abilities to learn (p. 2). Dowker believes that successful intervention can take place at any time. Nevertheless, he strongly believes that math difficulties can hinder a student’s performance in other areas, which can lead to a student becoming apprehensive about math or developing a negative mental attitude about math (Dowker, 2004). Dowker’s viewpoint is that it is critical to prevent a student from developing negative beliefs and a negative attitude about math. The intention for teachers is to shift the students’ belief and focus to that of getting the students to understand the many components of mathematics, so that the student can become well versed in counting sequences, estimating answers, and resolving word problems. Concentrating the intervention process on the component that a student may be
struggling with will benefit the student in a greater capacity than spending time teaching the student about components that he/she may not be struggling with.

The chief program, created as an intervention system to help students who have already developed a negative attitude about math, is the Mathematics Recovery Program (Phillips et al., 2003). The backbone of this program is detecting any student struggles with understanding mathematics early, transitioning the student into an intervention process, thereby keeping a student from developing a failure mentality about mathematics. Statistical research has been gathered that marks an intervention process to be initiated for students who fall in the bottom 25% of their mathematics class. One-on-one instruction of the student by the teacher has resulted in a rise from below grade level to on grade or above grade level. This study has shown that students’ self-confidence increased and they were able to formulate strong strategies for solving or resolving mathematical problem. There is no conclusive evidence that suggests that the intervention is based upon the program instead of any particular intervention component (Phillips et al., 2003).

Students that typically have low performances when it comes to learning math need an intervention to take place. A study was executed by Baker, Gersten, and Lee (2002) pertaining to the study of whether or not intervention improves the performance of a student who was considered a low performer, or ‘at-risk’ performer and the results suggested that different types of intervention are what causes the students’ performance to improve. Instructions utilized throughout the intervention were systematic instructions, continued diagnosis, and continued implementation of strategies as the student’s performance improved (Baker et al., 2002).

Explicit systematic instruction is an interactive process where teachers make clear to students the strategies, and the teacher demonstrates the strategy to cover visual and/or auditory
COMPARISON OF PHYSICAL/VIRTUAL MANIPULATIVES

recognition. This concept allows students to ask questions throughout the demonstration of solving the mathematical problem. Teachers also manage the instruction process by making sure that the sequences of the math problems highlight pertinent issues, as defined by the National Mathematics Advisory Panel (NMAP), (NMAP, 2008, P. 48). In 2004, Dowker ascertained that three types of interventions should take place regarding mathematics based upon students that are under performing: (1) The type of intervention that should take place, (2) the nature of the intervention that should be scheduled, (3) and the type of mathematical difficulties that the student is encountering (the range should be established as short-term or long-term). The nature and the cause of the difficulties incurred by the student will help determine the type and the extent of the intervention. A distinguishing example of initiating intervention for student low performance is the student demonstrating difficulty grasping what to do to resolve the problem, rather than a student that takes a few seconds to formulate his/her strategy within their mind and then proceeds to resolve the mathematical problem. In 2001, Fuchs and Fuchs elaborated on the principles for the intervention of mathematical difficulties. They established that there are three levels of intervention for mathematical difficulties: (1) primary prevention – centers upon the standard design of the intervention, (2) secondary prevention - hinges upon helping the student adapt to the classroom setting and getting comfortable, (3) and finally, tertiary prevention – spotlights in-depth individualized targeted conception in getting students to understand what steps to take when they see a mathematical problem to be solved (Fuchs & Fuchs, 2001).

Teachers within the classroom have voiced their need for added intervention research opportunities when they feel it is warranted. In 2008, the NCTM, consisting of 60 mathematicians, analyzed data submitted by a group of 200 teachers who answered 350 questions that formulated a pattern of questions that needed to be identified and expanded upon.
Though the NCTM analyses of the data formed a consensus of 10 questions that were relevant to all the volunteers that completed the form, three questions resonated with the NCTM. The teachers wanted to know; (1) Which intervention worked best in teaching the students who had difficulties in understanding mathematics? (2) Will technology be used to assist students in learning? (3) What is the standard process for which students should formulate their thought development? To answer these questions, the NCTM worked rapidly to get the answers back to the teachers within the classroom. Greater detail and in-depth analysis with standards, thresholds, and baselines have now been implemented. To guide educators and teachers, training materials, intervention models, formal assessments forms and reports, and cognitive research are now available (Arbaugh, Ramirez, Knuth, Kranendonk, & Quander, 2010).

In short, intervention policies are now adaptable and flexible, to meet the demands of the changing landscape of students who arrive in school, having difficulties in understanding mathematics. Also, instructional books and research approaches have to be changed, as this process will be ever evolving. Yet constant feedback from the teachers and educators in the field is a must to ensure that the right needs of students are met and that no student is left behind due to changes that meet the majority but not the minority group of students that may be on the cuff of staying ahead of their mathematical challenges.

**Conclusion**

Understanding fractions can be a nightmare for any child. This is because the learning process for studying the concept of fractions is not the same as the process students learn in their study of whole numbers. For many children, it is their first time seeing numbers with new terms or names, and then hearing about the relationship between numbers in order to resolve a problem (Smith, 2002, Van De Walle, 2004). Not grasping just basic math, let alone fractions, can be a
frightening and daunting task for any under achieving students trying to climb up the mathematics ladder (Cahoon, Emerson, Flores, & Houchins, 2007). Lamon (1999) states that when students encounter fractions, they must justify subjective complexity within their minds.

Most research studies are fixated on the students’ ambiguities of what procedure should be taken or what logical processing should be developed such as ratio, percent, and proportional reasoning to process mathematical problems (Gay, 1997; Singh, 2000). Therefore, only a small percentage of research was found that concentrates on conceptual problems that relate to understanding fractions. The process for teaching students how to understand fractions is tied directly to instructional referents (Boulet, 1998). Boulet stated that the goal of teaching children how to learn is definitely the goal of many teachers. Nevertheless, if a student is not grasping a teacher’s lesson, he/she will resolve to memorizing formulae and the mathematical rules governing them. When a student does not understand what a teacher is teaching, memorizing formulae and struggling with why he/she is unable to understand mathematics is certainty not meaningful to the student (Boulet, 1998).

Any research that directly compares virtual and physical manipulatives is, without a doubt, limited. Studies executed by different researchers all confirm what is mutually accepted—all students who learn any mathematical content using manipulatives have higher scores than those students that did not use manipulatives (Sowell, 1989). Further research has conclusively confirmed that manipulatives impact students’ conceptual and procedural comprehension of fractions. This occurs without hindrance from a negative attitude or state of mind about being not being able to solve fractions (Cramer & Henry, 2002).

The three dimensional objects that help students relate to comprehending symbolic representations of mathematics are defined as physical manipulatives (Clements, 1999; Moyer-
Packenham, 2010). Computer software programs that emulate physical manipulatives by generating geometric planes and solid figures, in the form of Java or Flash applets, are virtual manipulatives (Terry, 1996). Spicer (2000) defines virtual manipulatives as “dynamic visual representations of concrete manipulative” (p. 4). The benefits of working with physical manipulatives are both cognitive and motivational (Clements & McMillen, 1996; Sowell, 1989). There are several advantages that have been linked to using virtual manipulatives: (1) students have the opportunity to receive feedback as to how they are doing; (2) students have unlimited access to teachers; (3) the intervention process can be easily managed; (4) the process helps to stimulate higher order thinking skills (Cannon et al., 2002; Clements & Sarama, 2002; Moyer et al., 2002; Moyer et al., 2005); and (5) and the students are able to work with different representations at the same time (Kim, 1993). The primary benefit that students receive from using physical manipulatives is that they are able to participate and consciously handle, touch, and investigate (Moch, 2001).

It is a fact that there are different levels of understanding as shown by different students within a classroom. Intervention is the process of accessing the needs of a student who is performing at a low level in understanding mathematics. Selecting the correct instructional material for a low performing student is crucial to the student’s level of success in one-on-one teaching sessions. This study was designed to test the two types of instructional manipulatives (virtual and physical), with focused attention on which manipulative works best with helping students at low performing mathematical levels meet the standard or meet the upper level of mathematical understanding of fractions.
Chapter 3: Methodology

Introduction

The purpose of this research study was to compare two different types of manipulatives (physical & virtual) in order to measure their effects on students' understanding of adding fractions in fifth grade classrooms in six public elementary schools in Abha, Saudi Arabia. The design was quasi-experimental (Campbell & Stanley, 1963) with a non-equivalent control group which is considered a suitable alternative to an experimental design when randomization is not possible (Gall, Gall & Borg, 2006). The understanding of adding fractions was measured by tests created by the researcher. The following discusses the methodology that used to test the hypotheses, including research design and procedure, participant sampling, instrumentation, data collection, and data analysis.

Research Design

The research design for this study was a non-equivalent control group design. The reasoning for this choice was that the participants could not be randomly assigned to experimental and control groups (Gall et al., 2006). The study occurred during a two-week time frame during regular school hours in public elementary schools. Students participated in the study during their regularly scheduled mathematics class sessions. All subjects, except the control group, received both treatments, which allowed each student to serve as his own comparison. This approach eliminates concerns of individual differences that might occur in between-subjects designs and maximizes statistical power. One drawback of the crossover design was the potential for distortion due to carryover, that is, residual effects from preceding treatments. To avoid any residual effects, an identical form of the test was administered three times during the project (appendices T, U, V, W, & X).
Three comparable groups were chosen that were as similar as possible to provide the fairest comparison. A training session for teaching addition of fractions by using physical and virtual manipulatives was provided to the math teachers, who had previously agreed to participate in the study. The training was completed over two days (two hours each day) and ensured that teachers had the same instructions and skills in order to eliminate validity threats due to classroom and teacher differences. Training would also ensure that both treatment conditions were administered consistently. All groups completed the Understanding of Adding Fractions pretest (appendix T & V) and Attitude Survey (appendix Y & Z).

Upon completion of the pretest, the students were divided into three groups: Group One received mathematics instruction with physical manipulatives; Group Two received mathematics instruction with virtual manipulatives; and Group Three, the control group, continued with normal mathematics instruction. After five days of instruction, all groups were given the Understanding of Adding Fractions posttest (appendix U & V). In addition, the treatment groups completed an attitudinal survey to capture their views on the use of manipulatives (appendix A1 & B1). After that, the treatment groups switched. Group One taught with virtual manipulatives and Group Two was taught via concrete manipulatives. At the completion of the study, all three groups completed the equivalent form of Understanding of Adding Fractions posttest (appendix W & X). However, the treatment groups completed the Attitude Survey and Preference Survey (appendices A1, B1, C1, & D1). The students’ posttest scores were compared across groups, controlling for pretest scores using statistical analysis. Descriptive statistics have been presented for all the major characteristics. The design is depicted in Table 1 below.
Table 1

*Experimental Conditions*

<table>
<thead>
<tr>
<th>Group</th>
<th>Fraction Pretest and Attitude Survey</th>
<th>Data Collection</th>
<th>Instructional of Adding Fractions</th>
<th>Data Collection</th>
<th>Instructional of Adding Fractions</th>
<th>Data Collection</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>Physical Manipulative</td>
<td>Fraction Posttest and Attitude Survey</td>
<td>Virtual Manipulative</td>
<td>Fraction Posttest, Attitude Survey, Preference Survey</td>
<td></td>
<td></td>
</tr>
<tr>
<td>#2</td>
<td>Virtual Manipulative</td>
<td>Fraction Posttest and Attitude Survey</td>
<td>Physical Manipulative</td>
<td>Fraction Posttest, Attitude Survey, Preference Survey</td>
<td></td>
<td></td>
</tr>
<tr>
<td>#3</td>
<td>Regular Curriculum</td>
<td>Fraction Posttest</td>
<td>Regular Curriculum</td>
<td>Fraction Posttest</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

A sequence of preparatory procedures took place prior to initiating the research. A formal request was presented to the chief of the Department of Education, who then sent back a formal letter of approval to conduct the research in the public elementary schools (appendix I). This letter was presented to the Institutional Review Board (IRB) at the University of Cincinnati along with a formal request to conduct the investigational research. Once IRB approval was received (appendix E1), participant recruitment was initiated.

**Research Procedures**

Before instruction began, 428 students and their parents have signed the consent form (appendices A, B, C, D, E, F, G, and H). Students took a 6-item pretest on the understanding of fractions and an attitudinal survey that assesses students’ level of comfort when explaining the
concept of adding fractions. Students who were not able to pass 60% of the pretest, as an achievement score, were assigned to the control and experimental groups. The total of all groups was 174 participants at the beginning; however, 11 students left the study before completion due to absence and did not complete the study. Fifth grade students who were not participating in the study had access to the same curriculum as the participants. The only difference was that they did not have their data collected and used for research purposes.

During the first week of instruction, Group One learned adding of fractions using physical manipulatives in a regular classroom setting. Group Two learned how to add fractions by using virtual manipulatives in the computer lab. Group Three learned how to add fractions by using the normal curriculum of mathematics. The teachers used similar manipulatives in both the virtual and physical manipulative sessions. For example, Group One used commercially made fractions bars during the physical manipulative sessions. Each day, the teacher modeled several activities prior to allowing students to investigate addition of fractions concepts independently (appendix L & M). Group Two used a fraction applet with dynamic images of fraction tiles on the computer. Students worked in the computer lab for 5 days using the virtual manipulatives, with a 50-minute lesson on each day. Each day in the computer lab began with an introduction to the virtual manipulative applet that would be used that day and several mathematical tasks for the students. Students were given a teacher-made task sheet that provided instructions for using the virtual manipulatives, several problems, and space to record their work (appendix N & O). These directions helped students focus on the mathematical tasks during the lessons. The teacher reviewed the instructions with the class and modeled how to use the virtual manipulatives before students worked independently on the activities. The teachers led instruction and discussions with the participants during all of the class sessions, both in the classroom and the computer lab.
There was a series of four lessons on understanding of adding fractions. On the fifth day, all groups were given the paper and pencil posttest, which they took without the use of any manipulatives (See Table 2).

*Instructional Days 1 through 5*

During the fraction unit, the teachers taught fraction concepts that included fraction equivalence and addition of fraction with like and unlike denominators. The mathematics instruction for the physical and virtual fraction treatment groups was designed to be the same as the control group, except for the manipulative environment. The consistency in the lessons was important so that there would not be extraneous variability between the three conditions. The only difference between the three conditions was the task sheet. The control group used problems written in a regular math book; the physical manipulative task sheet included problems written on a paper, whereas the virtual manipulative group viewed the problems on a computer screen. Although the virtual group had a task sheet, it was for students to record their answers and write about what they learned from the lessons.

For the experimental groups, the first lesson focused on students using the manipulative tools to find lists of equivalent fractions and constructing a rule from analyzing the patterns of equivalent fractions. The second day, students were introduced to adding fractions with like denominators. The teachers modeled addition with like denominators and then posed a problem with addition of like denominators. The third and fourth days, students were introduced to adding fractions with unlike denominators. The teachers modeled addition with like denominators and then posed a problem with addition of unlike denominators. Students in each experimental treatment were asked to use the physical or virtual manipulatives to model symbolic expressions. For example, students were given a fraction problem and had to model the problem using
fraction bars in the physical manipulative treatment. Students were asked to find ways to combine two fractions with unlike denominators by using what they had learned in the previous days when finding equivalent fractions. They were asked to practice several tasks and then write a procedure that worked for them. Before the end of class, the teachers brought the group together to discuss students’ procedures for adding fractions with like and unlike denominators. These group discussions brought closure to each lesson with guided inquiry. Some questions were: 1) Is there a pattern in the list of equivalent fractions? and 2) What rule can you make to show how you add fractions with like and unlike denominators? For the control group, students learned the same four lessons that were taught to the experimental groups, except the using of physical and virtual manipulatives, in the same period of time. These lessons included fraction equivalence, addition of fraction with like denominators, and addition of fraction with unlike denominators. At the end of the first week (day 5), participants in all three groups were given the understanding of adding fraction posttest in order to measure the effects of physical manipulative, virtual manipulative, and regular math curriculum on students’ understanding of adding fractions. Also, the attitude survey was given to groups that used physical and virtual manipulatives in order to rate students’ level of comfort when explaining the concept of adding fractions to each other and determine how physical and virtual manipulatives may have improved students’ visual understanding of adding fractions.

*Instructional Days 6 through 10*

At the beginning of the second week of the study, the experimental groups were switched so that Group One worked with the virtual manipulative for adding fractions and Group Two worked with the physical manipulatives for adding fractions. The mathematics instructions for physical and virtual fraction treatment groups were the same as the first week; the problems were
not be different in order ensure that there would not be any variability between the two conditions. The control group worked on the same curriculum except the problems that were from the activity book of mathematics. It is an additional authorized book that is provided to students and has multiple questions and problems under each lesson in order to support students understanding the mathematics concepts effectively. At the end of the second week, students in all three groups were given the equivalent understanding of adding fractions posttest in order to measure the effects of physical and virtual manipulatives on students’ understanding of adding fractions. Also, the attitude survey was given to only Group One and Two in order to rate students’ level of comfort when explaining the concept of adding one fraction to another and determine how physical and virtual manipulatives may have improved students’ visual understanding of adding fractions. In addition, students in experimental groups were given the preference survey in order to determine which manipulative environment students preferred.
Table 2.

**Instructional Sequence**

<table>
<thead>
<tr>
<th>Timeline</th>
<th>Group #1</th>
<th>Group #2</th>
<th>Group #3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-instruction</td>
<td>Pretest on Understanding of adding fractions &amp; Attitude Survey</td>
<td>Pretest on Understanding of adding fractions &amp; Attitude Survey</td>
<td>Pretest on Understanding of adding fractions &amp; Attitude Survey</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Group #1:</td>
<td>Physical Manipulatives: Fraction Bars</td>
<td>Group #2: Virtual Manipulatives: Fraction Tiles</td>
<td>Group #3: Normal Math Curriculum</td>
</tr>
<tr>
<td>Day 1</td>
<td>Equivalent Fraction</td>
<td>Equivalent Fraction</td>
<td>Equivalent Fraction</td>
</tr>
<tr>
<td>Day 2</td>
<td>Addition of Fractions</td>
<td>Addition of Fractions</td>
<td>Addition of Fractions</td>
</tr>
<tr>
<td>Day 3</td>
<td>Addition of Fractions</td>
<td>Addition of Fractions</td>
<td>Addition of Fractions</td>
</tr>
<tr>
<td>Day 4</td>
<td>Addition of Fractions</td>
<td>Addition of Fractions</td>
<td>Addition of Fractions</td>
</tr>
<tr>
<td>Day 5</td>
<td>Fractions Posttest &amp; Attitude Survey</td>
<td>Fractions Posttest &amp; Attitude Survey</td>
<td>Fractions Posttest</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Group #1:</td>
<td>Virtual Manipulatives: Fraction Tiles</td>
<td>Group #2: Physical Manipulatives: Fraction Bars</td>
<td>Group #3: Normal Math Curriculum</td>
</tr>
<tr>
<td>Day 6</td>
<td>Equivalent Fraction</td>
<td>Equivalent Fraction</td>
<td>Equivalent Fraction</td>
</tr>
<tr>
<td>Day 7</td>
<td>Addition of Fractions</td>
<td>Addition of Fractions</td>
<td>Addition of Fractions</td>
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<tr>
<td>Day 8</td>
<td>Addition of Fractions</td>
<td>Addition of Fractions</td>
<td>Addition of Fractions</td>
</tr>
<tr>
<td>Day 9</td>
<td>Addition of Fractions</td>
<td>Addition of Fractions</td>
<td>Addition of Fractions</td>
</tr>
<tr>
<td>Day 10</td>
<td>Fractions Posttest &amp; Attitude Survey &amp; Preference Survey</td>
<td>Fractions Posttest &amp; Attitude Survey</td>
<td>Fractions Posttest</td>
</tr>
</tbody>
</table>

**Internal validity.** Gall et al. (2006) listed several factors that could affect the internal validity of experiments based on the work of Campbell and Stanley (1963) and Cook and Campbell (1979). According to Gay and Airasian (2000), the non-equivalent control group design effectively controls for six of these threats to internal validity, namely history, maturation, testing,
instrumentation, differential selection, and experimental mortality. These threats are discussed below, as they relate to this research study.

**History.** The duration of the treatment was 2 weeks. The treatment time was the same as the regular math time for the school. There was no different event or climate at schools that might be a concern. An attempt was made to document any significant events at the schools that might impact the interpretation of the results. During the duration of the study, there were no interruptions aside from normal schools holidays.

**Maturation.** It is normal to expect that fifth grade students may mature to some extent during the treatment period. The study design attempted to mitigate the effects of this maturation by its short duration. Also, it was assumed that all participants experience similar levels of maturation due to environmental and physical factors, and therefore any perceived changes in student achievement due to maturation would be fairly consistent for all participants.

**Instrumentation.** The instruments that were used in this study were the Understanding of Adding Fractions tests that were constructed by the researcher to measure the students’ understanding of adding fractions. Also, the Attitude Survey was used in order to rate the level of comfort when explaining the concept of adding one fraction to another. In addition, the Preference Survey was used in order to determine which manipulative environment students preferred. A further discussion of the instruments that were used is discussed later.

**Selection.** The three groups were determined based on the accessibility of computers in the mathematics classrooms within the overall population. Each group was determined according to pre-test scores that were the closest possible match to the other groups. Controlling for pre-test score differences would decrease the likelihood of characteristic differences that could affect the dependent variable.
**Mortality.** Data results illustrated which students did not participate in both tests and qualification for inclusion in the final analysis required participation in both the pre-test and post-test, attitude survey, and preference survey. This would place a limitation on the generalizability of the results, which is discussed later in more detail.

**Selection-maturation interaction.** The schools involved in the research study were in Abha, Saudi Arabia, specifically, six public elementary schools, which have age restrictions for enrollment into kindergarten. Because of these restrictions, students in the study were within the same age range. Exceptions might include transfer students from other school systems, including other countries, as well as retained students and students who have skipped a grade. During data analysis, the ages of the participants were checked for any possible confounding factors and/or outliers.

**Experimental treatment diffusion.** Participant groups were not in close proximity during the treatment period. No students had been exposed to any of the materials or activities involved with physical/virtual manipulative of fraction bars prior to the study. The schools that were used in the study had never used the physical/virtual manipulatives in the past.

**External validity.** Bracht and Glass (1968) categorized the threats to external validity into two broad classes: (1) those threats dealing with generalizations to populations of persons corresponding to population validity and (2) those threats dealing with the environment of the experiment corresponding to ecological validity. Each of these threats to external validity that relate to this study is discussed below.

**Population validity.** Experimentally accessible populations should correspond to the target populations. The results of this study should be generalizable to those populations that are similar to the experimentally accessible population. This experiment involves fifth graders;
therefore, the generalizability should be limited to fifth grade students. Any attempt to generalize beyond the defined population may increase the likelihood of this threat to external validity. The characteristic statistics for this experimental population are given in the population section discussed later.

**Ecological validity.** Threats to ecological validity as they relate to this research study are discussed below.

*Description of the independent variable explicitly.* Bracht and Glass (1968) stated that a detailed and complete description of an experiment must be given in order for the reader to make an estimate of the generalizability of the results. Therefore, a detailed description of the independent variable is given.

*Multiple-treatment interference.* The threat of multiple-treatment interference was minimized in this study by checking whether there were order effects since each group was exposed to one sequence and the two types of manipulative were compared separately with the control group.

*Experimenter effects.* Sometimes aspects of the experimenter such as gender, race, or personal attributes can unintentionally influence the participants in a study. To minimize this threat, blind data collection procedures were used, meaning that someone other than the researcher was collecting the data. The actual person involved in collecting the data was not aware of the purpose of the study or which participants were in the treatment groups.

*Effects of experimental arrangements.* Often when participants in a study become aware that they are involved in research, their responses and/or performance can be affected due to this awareness. To minimize the effects of this threat, all of the groups were under the impression that they were using the same treatment. It would be unlikely that students would find out that
there were different treatments because the groups would be pulled from different schools.

*Interaction of measurement time and treatment effects.* This threat to external validity was considered when estimating the generalizability of the results. For the reasons stated previously, Understanding of Adding Fractions tests were used. However, this was a disadvantage when attempting to minimize for this threat because the fifth graders used an equivalent form of the test during the study.

**Confounding variables.** There were several confounding variables that were controlled for in this study. First, the amount of time spent in the treatment groups, including absences, was factored into the final results analysis. To attempt to minimize this issue, students were given extra time for days missed to complete any work with the physical/virtual manipulatives and with the regular curriculum book. Second, prior understanding of adding fractions was assessed through the use of the Understanding of Adding Fractions pre-test before the use of manipulatives began.

There were other variables to consider such as computer literacy, and socioeconomic status, but measures were taken to control for such variables. First, teachers’ computer literacy was controlled for by providing training to the teachers on how to use the virtual manipulative program. Students who were not computer literate were excluded from participation in the groups. Second, socioeconomic status (SES) may influence academic achievement and learning success; however, since this study was only conducted in a small city, SES should not be a factor.

**Description of the independent variable.** One independent variable was manipulated in this study in order to compare between two types of manipulative (virtual/physical) and see their effects in students’ conceptual understanding of the addition of fractions. More information
about this variable is presented below.

Description of the Virtual Manipulative: During the virtual manipulative treatment, students used the Internet to work on the website called the Glencoe (http://www.glencoe.com/sites/common_assets/mathematics/ebook_assets/vmf/VMF-Interface.html). During the project period, they worked specifically with the “Fraction Tiles” manipulative in grade 5. Fraction Tiles illustrates how to compare between two different fractions as equivalent fractions and also illustrates what adding fractions means when finding a common denominator and combining two fractions (See Figure 1).

![Figure 1. Virtual manipulative of fraction tiles](image)

On the Fraction Tiles manipulative, students are able to write on the screen by clicking on text tool or pen tool to explain their answer easily. Also, students can learn about the equivalence of fractions by comparing two or more different fractions. Fraction Bars can be used in two ways to illustrate equality of fractions. To first introduce equality, two bars with the same size represent two equal fractions. A second method for illustrating equality involves splitting or dividing each part of a bar into 2 equal parts by using the arrow key to click on the icon of Straight Line Tool and draw dotted lines to divide the fraction into multiple parts, as shown in
On the Fraction Tiles manipulative, students are presented with two fraction bars or tiles that have the same or different denominators. To find a common denominator, students are able to rename the two fractions and find equivalent fractions. When a common denominator has been identified, students can type the name of the equivalent fractions into the text box. If they have specified a correct equivalent, students can easily combine the fractions (See Figure 3).

*Figure 2. Fraction equivalence*

*Description of the Physical Manipulative: The concrete manipulative used in this study is*
Fraction Bars, which provides visual illustrations of mathematical operations with fractions to gain better understanding of these operations. Fraction bars are rectangular pieces that represent different parts of the same whole. They can be cut apart and manipulated to see how various parts can be added together to make the whole or to compare different fractional amounts for equivalency. A fraction bar separates the numerator and denominator of a fraction. It indicates that a division of the numerator by the denominator will be performed. It consists of several bars divided into halves, thirds, fourths, fifths, sixths, eighth, tenths, twelfths, and 1 whole (See Figure 4). These bars are a part-to-whole region model for teaching the basic concepts of fractions, equality, inequality, addition, subtraction, multiplication, division, and ratios. They are plastic coated, so marking bars with water-based pens can be washed off. Students can use the fraction bars to find equivalent fractions or to find common denominators by placing their fraction pieces over the other fraction and seeing if the lines line up evenly with their fraction pieces.

*Figure 4. Fractions Bars*
Description of Control Group Lessons: In 2005, the Department of Education in Saudi Arabia began to develop the content of K–12 mathematics curriculum in order to compete with advanced countries, such as the United States (Saeed, Abdul Hamid, & Shalhoub, 2011). This is because the deficiency of mathematics content has contributed negatively to students' abilities in their academic achievement in the advanced levels. Development of mathematics content for all K–12 grades has relied on translating the mathematics content of McGraw-Hill series that have proven its effectiveness in improving the educational results (Obaid, 2010). Hassanein and Alshehri (2013) indicated that approximately 93.7% of content of fifth grade mathematics books is compatible to NCTM standards. Fifty-nine of NCTM indicators are congruent while 4 indicators that are in geometry and probabilities topics are different.

Each student in all grade levels receives two mathematics books. The first book is the primary textbook, which teaches students mathematical concepts and provides examples to build students’ comprehension. The second book is the activity book. Considered an additional source for both teacher and student, the activity book further promotes the math concepts introduced in the textbook and provides additional activities and questions in each subject (Al-Zubi & Al-Obeidan, 2013). Both books are completely identical in content and differ only in the activities and questions. The topic of adding fractions in the normal curriculum for the control group was completely similar to the content of the treatment groups, which consisted of several components: warm-up, introduction of the topic, modeling of the lesson, guided practice, class discussion, and closure of the lesson (appendix P & Q). The only differences between the control and treatment groups were the instructional tools (physical and virtual manipulatives) and the additional activities for both groups. Below is an account of the lessons in which the control group was taught each day.
The first lesson introduced to the control groups was fractions concepts, which had been taught in fourth grade, but a thorough review of the material was necessary as a foundation for the upcoming lessons on adding fractions. The goals of this first lesson were to help students understand and recognize the equivalency of fractions, simplify the fraction by using the greatest common factor, and compare between two different fractions using the least common denominator. The lesson consisted of several colored pictures representing different fractions, which then could be used to represent the meaning of equivalent fractions, compare between two different fractions, and complete the lesson's activities with ease. As part of the modeling process, teachers had to lead a discussion of identifying equivalent fractions by asking students for their own definitions and examples. Students were able to generate the definition for equivalent fractions as fractions that have the same quantity.

To introduce students to the lesson's activities, teachers demonstrated step-by-step how to illustrate equivalent fractions. Then, students were asked to determine whether 4/6 and 8/12 are equivalent fractions and provide an explanation for their answers. Students were asked also to make as many fractions as they could that were equivalent to several selected fractions such as 1/2, 1/3, 1/4, and so on. The students were to enter their findings on their papers. Next, students learned how to use the greatest common factor in order to simplify or reduce the fraction easily. Teachers asked students to determine whether or not the fraction of 12/22 is in the simplest form. Students would infer the actual value of the fraction did not change when they simplified a fraction even though the form of fraction had changed because when the numerator and denominator were divided by the same number, students were able to write the fraction in its simplest form. Then, students were asked to answer two examples in order to ensure they had a solid understanding of how to simplify fractions.
After that, students worked to understand how to compare between two different fractions by using the mathematical symbols of >, <, or = in the comparison. Students were asked to compare between three different kinds of pies: 5/8 of cheese pie, 1/4 of apple pie, and 1/8 of egg pie. Students were able to determine the greater fraction when both had similar denominators because they could easily identify the fraction with the larger numerator was greater. However, when two fractions had different denominators, students were not able to determine which fraction had the greater value. At this point, teachers explained that a common denominator would be needed to identify the relationship between two fractions with different denominators.

To this end, students learned two methods to find common denominators between two different fractions. The first method was to multiply both denominators together and, then, multiply the numerator of the first fraction with the denominator of the second fraction, and vice versa. The second method was to use the Least Common Denominator, which enabled students to produce several equivalent fractions for each fraction by multiplying both numerator and denominator by the same factor. Students underlined similar denominators that were on both lists and chose the smallest denominators for both fractions. After using one of these methods, students were able to determine which fraction was greater by identifying the fraction with the larger numerator. Finally, at the end of lesson one, students worked on the activities and examples while teachers walked around the classroom observing students’ conversations and methods of finding common denominators in order to compare fractions correctly.

The second lesson introduced adding fractions with like denominators. The goals of this lesson were for students to be able to understand the meaning of fractions with like denominators and add two different fractions with like denominators properly. Students were taught fractions
with like denominators meant fractions that have identical denominators. Also, students learned that when adding fractions with like denominators, only the numerators were added while the denominators were to stay the same. Teachers confirmed that the common mistake among students learning to add fractions with like denominators was to add the denominators together when adding the numerators. After the concept was introduced, then students started working on the activities and answering questions while teachers went around the classroom observing students’ conversations and methods to properly add two fractions with common denominators.

The third lesson, taking two days to complete this lesson plan, introduced the concept of adding fractions with unlike denominators. The goals of this lesson were for students to identify the meaning of fractions with unlike denominators, find the common denominators for two different fractions, and add two different fractions with unlike denominators properly. Students learned that adding fractions with unlike denominators is similar to adding fractions with like denominators, with one exception: They must find a common denominator by using the Least Common Denominator before adding fractions with unlike denominators. For two days, students worked with examples and word problems that required adding improper fractions and mixed numbers. While students individually or in-group worked through the problems, teachers observed students’ conversations and their methods of adding two fractions with unlike denominators.

During the second week of this study, students in the control group used the activity book instead of the primary book. This activity book was identical in content to the primary book (Appendix R & S), except the mathematical examples and questions for each lesson differed from that of the primary book.

**Training for instruction.** The teachers for all groups who signed to participated in the
study (appendix J & K) attended training workshops before treatment began. The purpose of the training was to familiarize the teachers involved with the goals, importance of the topic, and research procedures, and not to give them very detailed “scripts” to follow. The teachers met with the researcher a total of 4 hours.

During the workshops, the teachers in both groups were introduced to the objectives and overall design of the study. It was explained that evaluation of teachers and students was not the purpose of the study; rather, the objective was to investigate the effectiveness of using the virtual and physical manipulatives to enhance students’ understanding of adding fractions concepts. In addition, the teachers participated in a 2-hour staff development workshop conducted by the researcher prior to the study, which focused first on the concrete manipulative, sequence of activities, pre-test and post-test scoring, attitude survey, performance survey, and implementation procedures. Next, teachers focused on the components of the virtual manipulatives applet as well as sequence of the activities of the virtual manipulatives. The researcher assumed responsibility for answering any questions that the teachers might have both before and during the study to ensure they felt fully comfortable with participation.

**Sampling and participants**

Participants were recruited from six public elementary schools in Abha, Saudi Arabia. In 2015, the school data indicated that there were 52 public elementary schools in Abha. Also, the August 2015 student enrollment data indicated that 18,459 students attended the public elementary schools. The demographic background of the student population for the 2015-2016 school year consisted of 96.3% Saudi students and 3.7% Non-Saudi students. The total of fifth grade students are 3,421, or approximately 18.53% of the student population. At the beginning of this study, the participants were 174 fifth grade students in 12 classes (two classes in each
school). All of the participants were boys because the system of Saudi Arabian education is based on the sex-segregation in public schools. There were 141 participants who were 11 years old while 33 participants were 10 years old. Initially, Group One, Group Two, and Group Three consisted of 58 students, of which each group included four classes. However, 11 participants left before completing the study. At the end of the study, the participants were 163 fifth grade students, of which 138 participants were 11 years old and 25 participants were 10 years old. Eight participants were Non-Saudi students while the remaining participants were Saudis. In the first week of the study, Group One (Physical Manipulative) consisted of 56 students and Group Two (Virtual Manipulative) consisted of 55 students. In the second week, both groups were switched. However, Group Three (the control group) consisted of 52 students in both weeks.

Table 3: Participants who participated in the study

<table>
<thead>
<tr>
<th>School #</th>
<th>Group One (PM)</th>
<th>Group Two (VM)</th>
<th>Group Three (Control Group)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>13</td>
<td>-</td>
<td>16</td>
</tr>
<tr>
<td>2</td>
<td>13</td>
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</tr>
<tr>
<td>6</td>
<td>17</td>
<td>17</td>
<td>-</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>58</strong></td>
<td><strong>58</strong></td>
<td><strong>58</strong></td>
</tr>
</tbody>
</table>

Table 4: Participants who completed the study

<table>
<thead>
<tr>
<th>School #</th>
<th>Group One (PM)</th>
<th>Group Two (VM)</th>
<th>Group Three (Control Group)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>-</td>
<td>14</td>
</tr>
<tr>
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</tr>
<tr>
<td>6</td>
<td>15</td>
<td>16</td>
<td>-</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>56</strong></td>
<td><strong>55</strong></td>
<td><strong>52</strong></td>
</tr>
</tbody>
</table>
Table 5: Ages of participants who participated in the study

<table>
<thead>
<tr>
<th>Ages</th>
<th>Number of Participants</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 years</td>
<td>33</td>
<td>18.97%</td>
</tr>
<tr>
<td>11 years</td>
<td>141</td>
<td>81.03%</td>
</tr>
<tr>
<td>Total</td>
<td>174</td>
<td>100%</td>
</tr>
</tbody>
</table>

Table 6: Ages of participants who completed the study

<table>
<thead>
<tr>
<th>Ages</th>
<th>Number of Participants</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 years</td>
<td>25</td>
<td>15.34%</td>
</tr>
<tr>
<td>11 years</td>
<td>138</td>
<td>84.66%</td>
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<tr>
<td>Total</td>
<td>163</td>
<td>100%</td>
</tr>
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</table>

Table 7: Demographics for the students participating in the study

<table>
<thead>
<tr>
<th>Type</th>
<th>Number of Participants</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-Saudis' Students</td>
<td>9</td>
<td>5.52%</td>
</tr>
<tr>
<td>Saudis' Students</td>
<td>154</td>
<td>94.48%</td>
</tr>
<tr>
<td>Total</td>
<td>163</td>
<td>100%</td>
</tr>
</tbody>
</table>

These fifth grade students had a regular computer lab time scheduled each week for a 50-minute period where they used word processing applications to create learning projects or used the Internet to research content. They also visited the lab to work on a computer program that taught basic skills in mathematics and language arts. However, these programs were primarily drill and practice.

The non-probability technique of convenience sampling (Gall et al., 2006) was employed because fifth-grade students were the population within the school district that studies the addition of fractions, the concept being researched. The students received regular instruction in mathematics in an elementary mathematics classroom during the 2015-2016 school year. The fifth grade classroom setting was chosen because of the high amount of time spent in direct teacher contact with students in math. Specifically, fifth grade teachers spent 50 minutes per day
in direct contact with students. The researcher selected three groups for the study that were as
similar as possible so that any differences could be attributed to the independent variable.

To obtain accurate information, enhance cooperation, and increase the number of
volunteers, students and teachers were informed that their identity would be held in confidence
in perpetuity – the names of the students from which the data would be collected would not be
disclosed. Consent and assent forms would be stored in a locked cabinet away from the data and
the data would be stored in another separate locked cabinet. The researcher is the only one who
is able to access the data.
A participant sample plan was given in Figure 5.

**Target Population**

Fifth-Grade Students

---

**Experimentally Accessible Population**

A fifth grade mathematics group using the virtual manipulative, a comparable fifth grade group using the physical manipulative, and a comparable fifth grade mathematics group using the normal curriculum.

**PM Group**

58 fifth grade students in the first group who met qualifications.

**VM Group**

58 fifth grade students in the second group who met qualifications.

**Control Group**

58 fifth grade students in the third group who met qualifications.

*Figure 5.* Participant Sample Plan. A schematic showing the steps that have been used to obtain the groups samples.

The twelve teachers who participated in the study varied in years of experience and number of years teaching at the schools. The teaching demographics for the three groups can be found in Table 8.
Table 8: Demographics for the teachers participating in the study.

<table>
<thead>
<tr>
<th>Teachers</th>
<th>Total years of teaching</th>
<th>Total years of teaching elementary</th>
<th>Total years of teaching at the study school</th>
<th>Total years of teaching 5th grade</th>
<th>Nationality</th>
</tr>
</thead>
<tbody>
<tr>
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<td>6</td>
<td>7</td>
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<td>15</td>
<td>13</td>
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<td>10</td>
<td>Saudi</td>
</tr>
<tr>
<td>3</td>
<td>24</td>
<td>24</td>
<td>4</td>
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<tr>
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<td>9</td>
<td>3</td>
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</tr>
<tr>
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<td>6</td>
<td>6</td>
<td>4</td>
<td>Saudi</td>
</tr>
<tr>
<td>6</td>
<td>21</td>
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<td>3</td>
<td>1</td>
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<tr>
<td>8</td>
<td>16</td>
<td>12</td>
<td>7</td>
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<tr>
<td>9</td>
<td>7</td>
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<td>4</td>
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<td>Saudi</td>
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<tr>
<td>10</td>
<td>15</td>
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<tr>
<td>11</td>
<td>4</td>
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<td>4</td>
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<td>Saudi</td>
</tr>
<tr>
<td>12</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>Saudi</td>
</tr>
</tbody>
</table>

**Inclusion criteria.** In order for students to be included in the sample population, they needed to meet certain inclusion criteria. First, they had to be a student at the participating public elementary schools in Abha, Saudi Arabia. Student demographics were obtained by having the children filled out forms that indicated their age. The study was looking only for students’ ages 10-11 years old, which is the “Concrete Operational Stage of Cognitive Development” as per Piaget’s stages of cognitive development. This is an age frame during which Piaget has determined that most children begin thinking logically about concrete events, yet still have difficulty understanding abstract or hypothetical concepts (Piaget, 1977). A narrow age range also helps minimize maturation bias. Another demographic consideration was whether or not the student could speak Arabic fluently. If the student needed an interpreter, he was excluded from the study. In addition, students involved in the study needed to be computer literate and not need additional help with the basic use of a computer.
**Exclusion criteria.** Students who did not complete both the pre- and post-tests, attitude surveys, and preference survey were excluded from the study in addition to students who did not have the assent and consent forms completed (appendices A, B, and C). Students who were considered gifted or who had learning disabilities were excluded from the study, as they were considered outliers and could skew the data. The study allowed for all groups to make up for absenteeism; however, if students missed 20% or more of their mathematics classes during the study period, they would be excluded from the study.

**Recruitment.** The population and the sample were chosen because students enrolled in fifth grade mathematics have a curriculum that includes fractions. The researcher recruited students by visiting each classroom to ask for student assent and to request that consent forms be sent to their parents/legal guardians. During the classroom visits, the researcher explained to students the purpose of the research study and their choice to opt out of it. The research objectives were explained verbally and in writing so that they were clearly understood. Then the assent and consent forms were distributed. The assent was read aloud to the class. Students were requested to sign the assent form if they wished to participate, and to bring the consent form home for a signature and then back to the researcher at the schools. The researcher’s contact information was made available at the schools and the researcher had the support of the principals and teachers at the schools.

Confidentiality of the students’ research records was strictly maintained. Students’ names were not recorded on any information collected in the study; the pre- and post-tests used coding instead of names for analyses. The codes were distributed to the students on cards. The students wrote their names on the back of the cards and placed them on their desk with their names facing upwards. The cards were then collected without looking at the code numbers and placed into an
envelope kept by the teacher. This way, each individual code number was seen only by the student to whom it was assigned. These cards were referred to only if a student forgot his code. Having the codes on separate cards enabled the researcher to search through the names of the students (without seeing their code numbers) and pull out the card of a student who had forgotten his code.

Precautionary methods were taken to ensure that all participants were aware that participation in the study was optional. To avoid coercion, a research assistant was used to recruit participants and collect data. The research assistant assured students that participation was completely voluntary and that opting out would not have any negative effects on them (nor would opting in have positive effects, such as extra credit). In addition, participants were assured that their privacy would be protected – demographic information would only be reported at an aggregate level – and that all scores would be stored in a secure place during the study duration. Furthermore, all participants’ information would be secured in the researcher’s office for at least three years with a strong electronic password to protect participant data.

**Instrumentations.** Several sources of data were collected during the project, including the pretest and posttest of students’ understanding of adding fractions, Attitude Surveys, and Preference Survey. These sources were used to triangulate the data collected during the study.

**Understanding of adding fractions.** A review of math curriculum in Saudi Arabia indicates that fractions are typically introduced in the third grade. By the end of the fourth grade, the topics of introductory work with fractions (shading and identifying parts of figures); fractions of areas and fractions of numbers; fraction-decimal conversions; and addition and subtraction of common fractions are covered. To assess students’ understanding of adding fractions for this study, a general measure of understanding was needed for use as measurement variables.
However, since no existing test has been judged as an adequate measure of understanding of adding fractions, pretest, posttest, and equivalent posttest were constructed in accordance with previous research that used similar tests.

To create the fraction tests, items were either constructed or adapted from one of two sources: (a) tasks used in previous research studies of rational number understanding (Chorman, 2009; Hannula, 2003; Nieme, 1996); or (b) fourth and fifth grade textbooks. In addition, prior to administration, the pretest and post-tests were reviewed by the researcher, two professors of mathematics education, as well as four math teachers. The purpose of the pretest was to find out the students’ level of prior understanding of the addition of fractions. The goal of the posttest and equivalent posttest was to assess students’ conceptual understanding of adding fractions beyond traditional computation by requiring explanation and reasoning to support the procedural steps used in solving such problems. Both pretest and posttests had a total of 6 items created by the researcher. The 6 questions focused on four key components of the intervention: (1) recognize the fraction on the given picture; (2) understand equivalent fractions when given improper fractions and mixed numbers; (3) compare fractions with unlike denominators; and (4) add fractions using explanation and reasoning. The pretest items were similar to the test items from the posttest and equivalent posttest to build reliability between those tests. Also, each correct answer was worth 3 points and each incorrect answer was worth zero points. Most of the items selected by the researcher would have been successfully solved by approximately 40-60% of fifth grade students in previous studies.

In some cases, students were directed to use writing and drawing to illustrate why their solutions are correct. This was because a major curriculum goal of the NCTM (2000) is an ability to communicate mathematical understanding. “A person’s knowing of conceptual domain
is a set of abilities to understand, reason, and participate in discourse…Critical components of these sets of practices include the appreciation and use of explanatory ideas that are shared within the community and provide basic modes and goals of explanatory discourse” (Greeno, 1991, p. 176). Also, explanations may provide evidence about explicit understanding of concept procedures (Behr & Post, 1992). A copy of the pretest, posttest, and equivalent posttest is provided in Appendices G, H, and I.

**Attitude Survey.** All of the participants in experimental groups completed the attitude survey at the beginning, the middle, and the end of the study. The purpose of this survey was to determine the level of comfort that students had in explaining the addition of fractions to others before and after manipulatives treatment. The attitude survey, created by Chorman (2009), included three Likert-type scale items and one open-ended item where students were asked to write down and explain their feelings toward understanding the concept of adding fractions. The responses included (1) Yes, (2) somewhat, or (3) No. One response included (1) Comfortable, (2) Somewhat, or (3) Not Comfortable.

**Preference Survey.** This survey was created by Suh (2005) in order to find out what form of manipulatives students prefer more after having used both. There are 14 items on the preference Survey. Students have a choice of virtual manipulatives or physical manipulatives. Some examples of the statements where students were asked to choose between virtual and physical manipulatives are:

1. I can stay on task easier by using this tool.
2. I would feel comfortable working with this learning tool.
3. I can explain how to do the math better with this tool.
4. This tool helped me understand work with fraction number sentences.
Data Collection

Operational procedures. This study was introduced to study participants by the research assistant at the beginning of the study in April 2016 and was completed two weeks later. The study utilized three sources of data: the Understanding of Adding Fractions pre/posttests that were built by the researcher, Attitude Survey, and Preference Survey.

Two types of baseline data were collected: a fractions pretest and an attitude survey, both dealing with the understanding of adding fractions. Following the fractions pre-test, students were given a quick-write survey in which they were asked to rate their level of comfort in explaining the concept of adding fractions to others as well as to explain in one or more paragraphs why they felt that way.

Experimental treatment. Approximately one day after the pre-test and attitude survey, the first group received an introductory session on physical manipulatives in their regular classroom while the second group received an introductory session on virtual manipulatives in the school’s computer lab. The third group continued to use their regular math curriculum in the classroom. An email group was created for the teachers to facilitate communication between the researcher and the experimental teacher groups. The experimental phase took place for 10 days in April 2016.

Pre-test assessment. Both groups participated in the pre-test during single 50-minute sessions. The Understanding of Adding Fractions test was scored by the researcher, research assistant, and teachers who were participating in the study to help minimize experimenter effects and/or researcher bias.

Post-test assessment. At the conclusion of the first week of treatment, all groups participated in the posttest during single 50-minute sessions. Participants were verbally taught
the protocol of the test. The administration of the posttest was identical to the 6-item pretest that was given prior to the treatment period. The Understanding of Adding Fractions test was administered to all groups to measure students’ understanding of adding fractions post-treatment. In addition, at the end of the second week of the treatment, all groups participated in the equivalent posttest during single 50-minute sessions. Again, the purpose was to measure students’ understanding of adding fractions post-treatment. These tests were scored by the researcher and teachers who were participating in the study.

**Attitude Survey.** Experimental groups completed the attitude survey three times: at the beginning of the study, after the first week, and the end of the study. The attitude survey helped to see if students' confidence levels increased after the intervention or not.

**Preference Survey.** Both groups took this survey in order to find out what form of manipulatives students preferred after having used both.

**Data Analysis**

**Pretest and Posttests.** In order to look at the differences in test scores between the different treatment groups, Analysis of Variance (ANOVA) was performed on the fraction posttests from the virtual, the physical treatment groups, and the control group. Using the repeated measures design with a cross over treatment for the three student groups allowed the researcher to compare the impact of the two modes of treatments, virtual and physical manipulatives, for the understanding of adding fractions for the three groups of students.

All data was entered into SPSS 22.0, cleaned, and examined for outliers and skewness prior to analysis. Descriptive statistics, including means, standard deviations, frequencies and percentages (where appropriate) were calculated. An alpha level of .05 was selected for all statistical tests. At this level \( \alpha = .05 \), there is a 5% probability of a Type I error, which means
that there is a 5% chance of believing there is a genuine effect where there is not an effect.

Lipsey (1990, p. 38) stated, “An alpha of .05 corresponds to a .95 probability of a correct statistical conclusion when the null hypothesis is true”.

**Attitude Survey.** Numerical responses from the three-item Likert scale were evaluated by analyzing the frequencies of responses and by calculating mean rating scores.

**Preference Survey.** The Preference Survey responses were tabulated to determine which manipulative environment students preferred.

**Assumptions**

Assumptions were made about the participating students. It was assumed that the students involved were of a similar age range, since they were all fifth graders and all of them had the appropriate skill level to be included in fifth grade. It was also assumed that they were all of normal children who have mathematics difficulties as low mathematics performance since they were not chosen from advanced classes or classes for children with special needs. In addition, it was assumed that some of fifth-grade students might need an orientation to the basic skills needed to use the physical and virtual manipulatives. Every attempt was made to ensure that the two groups received the same amount of fractions study time. It was assumed that students would follow directions and only use the physical or virtual manipulatives that was assigned to them for a certain period of time. It was also expected that all students were at the same point in their fractions study, meaning that they had used the same curriculum and reached the same point in the curriculum when the intervention began. Furthermore, each student would be allowed to ask questions regarding both manipulatives (physical or virtual) and how to use them to solve the fractions computation.
The following assumptions were made regarding the teachers. It was assumed that the teachers were honest in their participation of the study and that they followed the directions that were given to administer the study. In addition, it was assumed that the teachers knowledgeable in the area of mathematics and possessed the skills necessary to teach mathematics.

The study tests were created by the researcher in order to meet the needs of the population of the study groups. Several experts were chosen to evaluate the tests: two professors of mathematics education and four mathematics teachers with ten years or more experience in the teaching field. It was assumed that the tests were written at the level of the participating students and that the students were able to read and understand the simple directions and ask the teachers for help in understanding the directions for the test. It was assumed that all the participants were able to write down their answers on the answer sheet provided with the test. After consulting with the panel of experts participating in the study, it was assumed that the length of the test was appropriate for the study. It was assumed that each participant gave his or her best effort to complete the test. It was assumed that each participant’s response confirmed their understanding of the concept of adding fractions.
CHAPTER IV

RESULTS

Introduction

The purpose of the study was to determine whether the “Fraction Bars” manipulative facilitated fifth grade students’ conceptual understanding of adding fractions. More specifically, the study sought to determine whether one of two types of manipulatives (i.e., physical vs. virtual) was more effective than the other, compared to a control group, on improving fifth graders’ conceptual understanding of adding fractions. This study was designed to answer the following questions:

1. Are there differences in students’ understanding of adding fractions when they are taught traditionally compared to when they are taught using physical or virtual manipulatives?
2. Does using the virtual manipulative “Fraction Bars” help fifth grade students develop a better conceptual understanding of how to add fractions?
3. Does using the physical manipulative “Fraction Bars” help fifth grade students develop a better conceptual understanding of how to add fractions?
4. What effect do the virtual and physical manipulatives "Fraction Bars" have on students’ understanding of adding fractions when using them consecutively?
5. What attitudes do students hold about the addition of fractions before and after using the physical and virtual manipulatives?
6. Which type of manipulatives (physical or virtual) do students prefer when learning the addition of fractions?

This study used a quantitative method of research; therefore, the information presented in this chapter includes only quantitative results. The quantitative data include the results of the
statistical and descriptive analyses of the pre- and post-tests, attitude surveys, and preference surveys.

This chapter begins with presenting the methods of analyzing data and rubrics that have been used to analyze students' explanations on the pre-test and post-test. Next, the chapter presents the descriptive statistics about the study sample and study variables. The final section of the chapter presents the findings of the hypotheses tests that were used in the study.

Methods of Analyzing Data

Participants were given a general overview on how tests would be graded prior to completing the pretests and posttests. The three criteria that were used to score the tests were as follows: providing an answer, providing an explanation for each answer, and providing illustrations or drawings as part of the explanation. These three criteria were used to encourage students to complete the tests by using their reasoning ability. Participants would receive full credit as long as their answers demonstrated the three criteria.

To analyze the pretests and posttests, the study divided findings into two components focusing on the accuracy of the answer that students provided and students' ability to provide an explanation with either words or illustrations to demonstrate their understanding of the concept. For some questions of the pretest/posttest, students were to provide an explanation for their reasoning, and there were four questions which required illustrations. For analyzing the first component, the answer of each question was marked right or wrong, such as questions #1 and #4. For the remaining questions, students had to demonstrate in their explanation the understanding of the concept in order to get a correct answer clearly. To analyze the second component, the rubric in Table 9 was used to analyze students' explanations for their reasoning for the six questions, except for questions number one and four, by using a scale of zero to three.
To explore students' visual understanding, scorers looked for diagrams or illustrations that students used as part of their explanation to help compare whether the use of manipulatives helped students improve their visual understanding of adding fractions. To determine gain of visual understanding through the use of manipulatives, students were able to use diagrams to explain common denominators using a common-sized whole for both fractions and dividing the wholes in such a way that one diagram could be superimposed on the other to subdivide the fractional bars neatly into smaller common denominators.

Table 9: *Rubric for analyzing students' explanations on both pretest and posttest*

<table>
<thead>
<tr>
<th>Rubric Score</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>- Students demonstrate no understanding of the concept.</td>
</tr>
<tr>
<td></td>
<td>- Students provided no explanation for their answer.</td>
</tr>
<tr>
<td></td>
<td>- Students provided zero illustrations for their explanation.</td>
</tr>
<tr>
<td>1</td>
<td>- Students demonstrate little understanding of the concept.</td>
</tr>
<tr>
<td></td>
<td>- Students provided little explanation for their answer.</td>
</tr>
<tr>
<td></td>
<td>- Students provided little illustrations for their explanation.</td>
</tr>
<tr>
<td>2</td>
<td>- Students demonstrate some understanding of the concept.</td>
</tr>
<tr>
<td></td>
<td>- Students provided some explanation for their answer.</td>
</tr>
<tr>
<td></td>
<td>- Students provided some illustrations for their explanation.</td>
</tr>
<tr>
<td>3</td>
<td>- Students demonstrate an excellent understanding of the concept.</td>
</tr>
<tr>
<td></td>
<td>- Students provided an excellent explanation for their answer.</td>
</tr>
<tr>
<td></td>
<td>- Students provided excellent illustrations for their explanation.</td>
</tr>
</tbody>
</table>

**Descriptive Statistics**

**Description of the Sample**

A third of the participants were exposed to Virtual Manipulatives first and Physical Manipulatives second \((n = 56)\), another third of participants were exposed to Physical Manipulatives first and Virtual manipulatives second \((n = 55)\), and the last third of participants served as the control group \((n = 52)\).
Description of Study Variables

_Fractions performance_. The findings in Table 10 reveal that the mean performance rating during the pretest was 1.68 ($SD = 1.15$). After students learned fractions using physical manipulatives, the mean score increased significantly to 10.84 ($SD = 3.77; p < .001$). After students learned fractions using virtual manipulatives, the mean score increased significantly to 10.75 ($SD = 4.22; p < .001$).

Table 10

**Descriptive Statistics for Fractions Performance ($N = 163$)**

<table>
<thead>
<tr>
<th>Test</th>
<th>Range</th>
<th>$M$</th>
<th>$SD$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pretest</td>
<td>0 to 5</td>
<td>1.68</td>
<td>1.15</td>
</tr>
<tr>
<td>Physical posttest</td>
<td>3 to 16</td>
<td>10.84</td>
<td>3.77</td>
</tr>
<tr>
<td>Virtual posttest</td>
<td>3 to 16</td>
<td>10.75</td>
<td>4.22</td>
</tr>
</tbody>
</table>

Students’ scores were compared in pretest, first posttest, and second posttest by counting the number of correct answers in order to determine whether the virtual manipulative, physical manipulative, and control group were effective in improving students’ understanding of adding fractions for students. As shown in Figures 6, 7, and 8, the correct answer of each question was counted when the achieved score was two or higher from the rubric, except for question four. Participants had to be able to demonstrate their understanding of adding fractions and provide some explanations by using words or illustrations.
Figure 6: Comparison of Pre & Post-tests Results For Group 1

Figure 7: Comparison of Pre & Post-tests Results For Group 2
The first question of the pretest was to recognize the four given fractions and write them down in their correct places. Based on the rubric, points for were distributed as follows: students received 3 points for a correct answer on three or four fractions, 2 points for a correct answer on two fractions, 1 point for a correct answer on one fraction, and 0 if there was no right answer.

The number of participants who answered correctly on four fractions in the first question and received a score of 2 or above from the rubric was 6 students in the first group, 3 students in the second group, and 4 students in the control group. The rest of participants in all groups received either one point or no point.

After the first week of the intervention, students from groups 1 and 2, in particular, improved in their answers for this question of the posttest by providing answers for most of the four fractions. Students who missed the question on the pretest showed improvement by avoiding
the mistake they made on the pretest. The number of students who received a score of 2 or above from the rubric was 52 students in the first group, 51 students in the second group, and only 10 students in the control group. After the second week of the intervention, students’ scores increased greatly for the first group. There were 54 students who answered at least 2 out of 4 items correctly and received a score of 2 or above from the rubric while 48 students who received a score of 2 and above in the second group. For the control group, students’ scores decreased in the second week. There were only 6 students who received a score of 2 or above from the rubric.

For the second question of the pretest which was to using the virtual manipulative “Fraction Bars” to help fifth grade students develop a better conceptual understanding of how to add fractions, there were only 3 students in the first group, 1 student in the second group, and 2 students in the third group who received a score of 2 from the rubric. They provided some explanation of the answer when converting the improper fraction to a mixed number. The students’ reasoning demonstrated evidence of their understanding of the concept of fraction equivalence if they explained the following: two given fractions were not equal because 8/5 is 1 3/5, which is not the same as 1 4/5. The rest of students in all groups received either a score of 0 or 1 from the rubric. When students were not able to answer the question correctly due to lack of knowledge of how to convert mixed numbers to improper fractions, they received a 0. When they provided little explanation for their answer to this question because they could not provide their reasoning after converting the fractions in order to illustrate why they are not equivalent, they received 1 point.

After the first week of the intervention, improvement in students’ understanding was noted in the posttest for experimental groups. Instead of just focusing on computation for
converting a mixed number to an improper fraction, students provided more explanation using diagrams to show the equivalent fractions properly. As a result, there were 51 students of the first group who received a score of 2 and above from the rubric while there were 49 students from the second group who also received a score of 2 and above from the rubric. For the control group, there were only 19 students who received a score of 2 or above from the rubric. After the second week of the intervention, students’ understanding continued in progress for experimental groups. Therefore, there were 53 students of the first group and 50 students of the second group who received a score of 2 or above while 13 students of the control group received a score of 2 or above from the rubric at the end of the study.

For the third question on the pretest which was to using the physical manipulative “Fraction Bars” to help fifth grade students develop a better conceptual understanding of how to add fractions, no participant in all of the three groups received a score of 2 or above. Most students received a score of 0. They had difficulty in understanding fractions as “part versus whole” when comparing the size of the denominator. Some compared denominators of 8 and 9 as whole numbers, which led them to the incorrect assumption that 9 is greater than 8, therefore 4/9 is the greater fraction. In addition, there were 37 participants from all groups who received a score of 1 because little explanation was provided for their answers. After the first week of intervention, there were 49 students from the first group, 43 students from the second group, and 15 students from the third group who received a score of 2 and above from the criteria because they demonstrated their abilities to compare the two fractions by drawing pictures to see such comparison clearly. After the second week of the study, improvement of students’ understanding of comparison of fractions continued for Group One and Two only. Fifty-one students of the first group and 52 students of the second group received a score of 2 and above in the posttest.
Students realized that when a whole is divided into 8 parts, each part would be greater than if it was divided into 9 parts. For the control group, there were only 8 students who received a score of 2 from the criteria.

For the fourth question which was to measure the effect that the virtual and physical manipulatives "Fraction Bars" have on students’ understanding of adding fractions when using them consecutively, the rubric to analyze students’ answers was different from the first three questions. Students received one point for the right answer and 0 for the wrong answer. On the pretest, 11 students of the first group, 7 students of the second group, and 6 students of the control group received 1 point while the remaining students in all three groups received a score of 0. After the first week of intervention, students’ understanding improved. There were 54 students of the first group, 53 students of the second group, and 24 students of the control group who received 1 point while the remaining students in all three groups received a score of 0. After the second week of intervention, all students provided a correct answer in the first and second groups, thus receiving a score of one. For the control group, there were only 18 students who were able to answer the question correctly, thus receiving a score of one from the rubric.

For the fifth question, students were asked to find the sum of two fractions with like and unlike denominators. On the pretest, no student in all three groups received a score of 2 or above from the rubric. Some students received a score of 1 because they provided the correct answer for the first part of the question, which required the addition of two fractions with like denominators. However, no students were able to find the correct sum for two fractions with unlike denominators. To summarize, some students added the numerator and denominator together to find the sum of the fractions while other students miscalculated during the process of finding the answer.
On the posttest, students' scores increased. Most students in the treatment groups were able to find sum of the two fractions with like or unlike denominators easily. After the first week of intervention, 37 students of the first group, 40 students of the second group, and 9 students of the third group received a score of 2 and above. After the second week of the intervention, there were 43 students of the first group, 44 students of the second group, and 11 students of the control group who received a score of 2 and above from the rubric. Those students were able to calculate and find the sum of two fractions with like and unlike denominators correctly.

For the last question, students were asked to explain and illustrate their understanding of adding fractions in order to demonstrate the actual effects of the intervention during the study. On the pretest, no student in all the three groups received a score of 2 or above due to their inability to provide correct answers for the question. A few students provided little in the way of illustrations for their explanation and received a score of 1, which is non-counted in this analysis. After the first week of intervention, students’ scores of the treatment groups increased on the posttest. Forty students of the first group, 36 students of the second group, and 8 students of the control group received a score of 2 and above from the rubric because they provided some illustrations along with their explanations. After the second week of the intervention, students’ scores on understanding again increased for the treatment groups only. There were 47 students of the first group, 46 students of the second group, and 6 students of the control group who received a score of 2 and above because they explained the process of finding the sum of two fractions as converting the denominators into equivalent fractions properly.

**Attitudes towards fractions.** Prior to students being exposed to either type of manipulative, as shown in Figure 9, students in the experimental groups were asked to rate their level of comfort when explaining fraction addition to others. The scale of comfortable, somewhat
comfortable, and not comfortable was used on the first question of the attitudinal survey to rate students’ level of comfort. Sixty-one percent indicated that they were not comfortable \( (n = 68) \), 35% stated they were somewhat comfortable \( (n = 39) \), and 4% reported they were very comfortable explaining fraction addition \( (n = 4) \).

After using the physical manipulative, 12.6% of participants indicated that they were comfortable \( (n = 14) \) while 87.4% stated they were somewhat comfortable explaining addition of fractions \( (n = 97) \). After using the virtual manipulative, 17.1% of participants indicated that they were comfortable \( (n = 19) \) while 82.9% of students stated that they were somewhat comfortable explaining fraction addition to others \( (n = 92) \).

![Rating of Students’ Level of Comfort for Groups 1 and 2](image)

*Figure 9: Students’ level of comfort explaining fraction addition for groups 1 & 2*

As shown in Table 11, students agreed somewhat that manipulatives improved their visual understanding of fractions after learning fractions using physical (69.4%) and virtual manipulatives (65.8%). They indicated that use of fractions bars was somewhat helpful when
adding fractions after learning by using physical (78.4%) and virtual manipulatives (75.7%). Most noted that they were somewhat comfortable explaining fraction addition after using physical (87.4%) and virtual manipulatives (82.9%).

Table 11

*Frequencies and Percentages for Attitudes toward Fractions after Learning the Addition of Fractions Using Physical and Virtual Manipulatives (N = 111)*

<table>
<thead>
<tr>
<th>Item</th>
<th>After Using Physical</th>
<th>After Using Virtual</th>
</tr>
</thead>
<tbody>
<tr>
<td>Manipulatives improved visual understanding of fractions</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Yes</td>
<td>34 (30.6)</td>
<td>38 (34.2)</td>
</tr>
<tr>
<td>Somewhat</td>
<td>77 (69.4)</td>
<td>73 (65.8)</td>
</tr>
<tr>
<td>Manipulatives’ fraction bars helpful when adding fractions</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Yes</td>
<td>24 (21.6)</td>
<td>27 (24.3)</td>
</tr>
<tr>
<td>Somewhat</td>
<td>87 (78.4)</td>
<td>84 (75.7)</td>
</tr>
<tr>
<td>Level of comfort after using manipulatives’ fraction bars</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Comfortable</td>
<td>14 (12.6)</td>
<td>19 (17.1)</td>
</tr>
<tr>
<td>Somewhat</td>
<td>97 (87.4)</td>
<td>92 (82.9)</td>
</tr>
</tbody>
</table>
In order to further understand students’ comfort level with fractions, the study prompted students to complete surveys with the option to describe their reasoning for their level of comfort. Prior to the intervention, students often reported their not comfortable response was due
to “not being able to explain fractions to others” or that they “don't understand fractions well.”

Students who reported being comfortable with fractions generally did not describe their reasoning, with the exception of one student who mentioned that fractions were easy because they were part of a whole. Students who reported being somewhat comfortable stated their level of comfort was due to difficulties in understanding fractions, as well as difficulty memorizing the steps involved in finding the correct answer. After the completion of the intervention, students’ responses to the same quick-write question showed an increase of confidence. Students demonstrated improvement in their quick write by moving their responses from somewhat to comfortable when they figured out how to find a common denominator for adding fractions.

Preference for type of manipulative. The preference survey was created by Suh (2005) in order to examine what type of manipulatives students preferred after using both. Based on what students thought was a more true statement, they chose either the physical manipulative or virtual manipulative after reading each statement carefully. The preference survey consisted of 14 statements, of which 12 were positive and 2 were negative. The two negative statements were included in the survey to prevent students from choosing an answer without reading each statement carefully. The findings in Figure 12 and Table 12 reveal that most of the students preferred using physical manipulatives to virtual manipulatives. However, the students indicated that virtual manipulatives were more interesting in terms of the learning process and were also easier to learn.
Table 12

_Frequencies and Percentages for Preference for Type of Manipulative (N = 111)_

<table>
<thead>
<tr>
<th>Statement</th>
<th>Virtual</th>
<th>Physical</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. In the future, I would like to use this tool more.</td>
<td>47 (42.3)</td>
<td>64 (57.7)</td>
</tr>
<tr>
<td>2. Learning with this tool is a good way to spend math time.</td>
<td>29 (26.1)</td>
<td>82 (73.9)</td>
</tr>
<tr>
<td>3. It is fun to figure out how this learning tool works.</td>
<td>48 (43.2)</td>
<td>63 (56.8)</td>
</tr>
<tr>
<td>4. Using this tool becomes boring.</td>
<td>41 (36.9)</td>
<td>70 (63.1)</td>
</tr>
<tr>
<td>5. Working with math problems using this tool is fun.</td>
<td>49 (44.1)</td>
<td>62 (55.9)</td>
</tr>
<tr>
<td>6. I wish I had more time to use these types of tools in math.</td>
<td>47 (42.3)</td>
<td>64 (57.7)</td>
</tr>
<tr>
<td>7. Learning to use this tool is interesting.</td>
<td>72 (64.9)</td>
<td>39 (35.1)</td>
</tr>
<tr>
<td>8. I can stay on task more easily by using this tool.</td>
<td>54 (48.6)</td>
<td>57 (51.4)</td>
</tr>
<tr>
<td>9. I would feel comfortable working with this learning tool.</td>
<td>54 (48.6)</td>
<td>57 (51.4)</td>
</tr>
<tr>
<td>10. This learning tool makes me feel uneasy and confused.</td>
<td>54 (48.6)</td>
<td>57 (51.4)</td>
</tr>
<tr>
<td>11. I can explain how to do math better using this tool.</td>
<td>43 (38.7)</td>
<td>68 (61.3)</td>
</tr>
<tr>
<td>12. This tool was easy to use.</td>
<td>58 (52.3)</td>
<td>53 (47.7)</td>
</tr>
<tr>
<td>13. This tool helps me understand fractions.</td>
<td>49 (44.1)</td>
<td>62 (55.9)</td>
</tr>
<tr>
<td>14. This tool helps me get the right answers.</td>
<td>53 (47.7)</td>
<td>58 (52.3)</td>
</tr>
</tbody>
</table>

![Preference for Type of Manipulative](image)

*Figure 12: Preference of Physical and Virtual Manipulatives*
Results of the Hypotheses Tests

First Hypothesis

It was hypothesized there would be significant differences in fifth graders’ conceptual understanding of the adding of fractions between the control, physical, and virtual manipulatives groups. The mixed ANOVA findings reveal that fractions performance differed significantly across the groups, $F(4, 320) = 506.49$, $p < .001$, $\eta^2 = .86$. Therefore, the first hypothesis was supported.

Table 13

Means and Standard Deviations for Fractions Performance ($N = 163$)

<table>
<thead>
<tr>
<th>Test</th>
<th>Control $M$ (SD)</th>
<th>Physical First $M$ (SD)</th>
<th>Virtual First $M$ (SD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pretest</td>
<td>1.42 (1.14)</td>
<td>1.89 (1.14)</td>
<td>1.71 (1.15)</td>
</tr>
<tr>
<td>First posttest</td>
<td>6.60 (1.38)</td>
<td>10.30 (1.03)</td>
<td>15.40 (.97)</td>
</tr>
<tr>
<td>Second posttest</td>
<td>5.60 (1.24)</td>
<td>15.64 (.84)</td>
<td>10.65 (.95)</td>
</tr>
</tbody>
</table>

Note. Fractions performance differed significantly across the groups, $F(4, 320) = 506.49$, $p < .001$, $\eta^2 = .86$.

Second Hypothesis

It was hypothesized that there would be significant differences in fifth graders’ conceptual understanding of the adding of fractions between those who did not receive any manipulatives (i.e., control) and those who used virtual manipulatives. As summarized in Table 14 and depicted in Figure 13, the increase in test scores from pretest to posttest was significantly steeper for the virtual manipulatives group than it was for the control group, $F(1, 161) = 289.66$, $p < .001$, $\eta^2 = .64$. As such, the second hypothesis was supported.
Table 14

*Mixed ANOVA Results for Fractions Performance as a Function of Manipulatives (N = 163)*

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>MS</th>
<th>F</th>
<th>η²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between subjects</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Treatment</td>
<td>1</td>
<td>791.85</td>
<td>1971.63</td>
<td>.93</td>
</tr>
<tr>
<td>Error</td>
<td>161</td>
<td>.40</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Within subjects</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Test</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pretest vs. physical</td>
<td>1</td>
<td>9293.39</td>
<td>1278.44</td>
<td>.89</td>
</tr>
<tr>
<td>Pretest vs. virtual</td>
<td>1</td>
<td>8554.16</td>
<td>1351.15</td>
<td>.89</td>
</tr>
<tr>
<td>Error</td>
<td>161</td>
<td>7.27</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Test x treatment</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pretest vs. physical x treatment</td>
<td>1</td>
<td>1213.49</td>
<td>166.93</td>
<td>.51</td>
</tr>
<tr>
<td>Pretest vs. virtual x treatment</td>
<td>1</td>
<td>1833.82</td>
<td>289.66</td>
<td>.64</td>
</tr>
<tr>
<td>Error</td>
<td>161</td>
<td>6.33</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*p < .05. ** p < .01. *** p < .001.*
**Figure 13.** Fractions pretest and virtual manipulatives posttest scores as a function of treatment

**Third Hypothesis**

Also, it was also hypothesized that there would be significant differences in fifth graders’ conceptual understanding of the adding of fractions between those who did not receive any manipulatives (i.e., control) and those who used physical manipulatives. As summarized in Table 14 and illustrated in Figure 14, the improvement from pretest to posttest was significantly steeper for the manipulatives group than it was for the control group, $F(1, 161) = 166.93, p < .001, \eta^2 = .51$. Thus, the third hypothesis was supported.
Fourth Hypothesis

It was hypothesized that there would be a significant difference in fifth graders’ conceptual understanding of the adding of fractions when using the virtual and physical manipulatives “Fraction Bars” consecutively, compared to the control group. As summarized in Tables 15 and 16 and illustrated in Figure 15, the improvement from pretest to posttest was significantly steeper for the manipulatives group than it was for the control group, $F(1, 161) = 1452.59, p < .001, \eta^2 = .90$. Accordingly, the fourth hypothesis was supported.

Table 15

Means and Standard Deviations for Fractions Performance ($N = 163$)

<table>
<thead>
<tr>
<th>Test</th>
<th>Control</th>
<th>Manipulatives</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$M$ (SD)</td>
<td>$M$ (SD)</td>
</tr>
<tr>
<td>Pretest</td>
<td>1.42 (1.14)</td>
<td>1.80 (1.14)</td>
</tr>
<tr>
<td>Second posttest</td>
<td>5.60 (1.24)</td>
<td>15.52 (.91)</td>
</tr>
</tbody>
</table>

Figure 14. Fractions pretest and physical manipulatives posttest scores as a function of treatment.
Table 16

Mixed ANOVA Results for Fractions Performance as a Function of Learning with Physical and Virtual Manipulatives (N = 163)

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>MS</th>
<th>F</th>
<th>$\eta^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between subjects</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Treatment</td>
<td>1</td>
<td>940.12</td>
<td>1501.19***</td>
<td>.90</td>
</tr>
<tr>
<td>Error</td>
<td>161</td>
<td>.63</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Within subjects</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pretest vs. posttest</td>
<td>1</td>
<td>11338.19</td>
<td>5102.09***</td>
<td>.97</td>
</tr>
<tr>
<td>Pretest vs. posttest x treatment</td>
<td>1</td>
<td>3227.98</td>
<td>1452.59***</td>
<td>.90</td>
</tr>
<tr>
<td>Error</td>
<td>161</td>
<td>2.22</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* $p < .05$. ** $p < .01$. *** $p < .001$.  

![Figure 15. Fractions pretest and second posttest scores as a function of treatment](image)
Virtual vs. physical manipulatives. The repeated measures ANOVA findings indicate that fractions performance did not differ significantly as a function of type of manipulative, $F(1, 110) = .48, p = .491, \eta^2 = .00$.

Table 17

Means and Standard Deviations for Fractions Performance across Type of Manipulative ($N = 111$)

<table>
<thead>
<tr>
<th>Posttest</th>
<th>$M (SD)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>After learning with physical manipulatives</td>
<td>12.83 (2.75)</td>
</tr>
<tr>
<td>After learning with virtual manipulatives</td>
<td>13.17 (2.66)</td>
</tr>
</tbody>
</table>

*Note.* Fractions performance did not differ significantly as a function of type of manipulative, $F(1, 110) = .48, p = .491, \eta^2 = .00$.

Order of exposure to manipulatives. As shown in Table 18, order of exposure to manipulatives moderated the relationship between improvement from pretest to posttest, $F(2, 218) = 735.69, p < .001, \eta^2 = .87$. As depicted in Figure 16, the increase in test scores from pretest to posttest (after learning with physical manipulatives) was significantly steeper for the group exposed to virtual manipulatives first, $F(1, 109) = 402.89, p < .001, \eta^2 = .79$. Similarly, the findings in Figure 17 show that the improvement from pretest to posttest (after learning with virtual manipulatives) was much steeper for the group that was exposed to the physical manipulatives first, $F(1, 109) = 294.17, p < .001, \eta^2 = .73$. These findings suggest that there was a learning effect such that fractions performance was better after the second posttest.
Table 18

*Mixed ANOVA Results for Fractions Performance as a Function of Order of Manipulative (N = 111)*

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>MS</th>
<th>F</th>
<th>η²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between subjects</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Order</td>
<td>1</td>
<td>.02</td>
<td>.04</td>
<td>.00</td>
</tr>
<tr>
<td>Error</td>
<td>109</td>
<td>.40</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Within subjects</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Test</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pretest vs. physical</td>
<td>1</td>
<td>13554.27</td>
<td>7058.87***</td>
<td>.99</td>
</tr>
<tr>
<td>Pretest vs. virtual</td>
<td>1</td>
<td>14292.41</td>
<td>6563.99***</td>
<td>.98</td>
</tr>
<tr>
<td>Error</td>
<td>109</td>
<td>1.92</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Test x treatment</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pretest vs. physical x order</td>
<td>1</td>
<td>773.62</td>
<td>402.89***</td>
<td>.79</td>
</tr>
<tr>
<td>Pretest vs. virtual x order</td>
<td>1</td>
<td>640.52</td>
<td>294.17***</td>
<td>.73</td>
</tr>
<tr>
<td>Error</td>
<td>109</td>
<td>2.18</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*p < .05. ** p < .01. *** p < .001.*
Figure 16. Fractions performance after learning with physical manipulatives as a function of order of exposure

Figure 17. Fractions performance after learning with virtual manipulatives as a function of order of exposure
**Fifth Hypothesis**

It was hypothesized that there would be a significant difference in fifth graders’ attitudes toward understanding the adding of fractions after learning with virtual manipulatives and then learning with physical manipulatives (and vice-versa). Cross-tabulation procedures were conducted to determine whether attitudes toward understanding fraction addition differed after learning with virtual manipulatives and after learning with physical manipulatives. As shown in Table 19, the percentage of yes responses to the item, “Manipulatives improved my visual understanding of fractions” was significantly higher after students used virtual manipulatives (versus physical manipulatives), $\chi^2(1) = 4.05, p = .044$.

Table 19

*Cross-tabulation Results for the Manipulatives Improve Visual Understanding of Fractions Item (N = 111)*

<table>
<thead>
<tr>
<th>After Using Virtual Manipulatives</th>
<th>Yes</th>
<th>Somewhat</th>
</tr>
</thead>
<tbody>
<tr>
<td>After Using Physical Manipulatives</td>
<td>$n$ (%)</td>
<td>$n$ (%)</td>
</tr>
<tr>
<td>Yes</td>
<td>7 (18.4)</td>
<td>27 (37.0)</td>
</tr>
<tr>
<td>Somewhat</td>
<td>31 (81.6)</td>
<td>46 (63.0)</td>
</tr>
</tbody>
</table>

*Note.* Percentage of “yes” responses was significantly higher for virtual than for physical manipulatives, $\chi^2(1) = 4.05, p = .044$.

However, as displayed in Table 20, the percentage of yes responses to the item, “Manipulatives’ fraction bars were helpful when adding fractions” was not significantly higher after students used virtual manipulatives (versus physical manipulatives), $\chi^2(1) = .98, p = .323$. 
Table 20

Cross-tabulation Results for the Manipulatives Fraction Bars Helpful Item (N = 111)

<table>
<thead>
<tr>
<th>After Using Virtual Manipulatives</th>
<th>n (%)</th>
<th>n (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>After Using Physical Manipulatives</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Yes</td>
<td>4 (14.8)</td>
<td>20 (23.8)</td>
</tr>
<tr>
<td>Somewhat</td>
<td>23 (85.2)</td>
<td>64 (76.2)</td>
</tr>
</tbody>
</table>

Note. Percentage of “yes” responses was not significantly higher for virtual than for physical manipulatives, $\chi^2(1) = .98$, $p = .323$.

Lastly, as summarized in Table 21, the percentage of comfortable responses to the item, “Level of comfort after using fraction bars” was not significantly higher after students used virtual manipulatives (versus physical manipulatives), $\chi^2(1) = 1.12$, $p = .289$.

Table 21

Cross-tabulation Results for the Level of Comfort Item (N = 111)

<table>
<thead>
<tr>
<th>After Using Virtual Manipulatives</th>
<th>n (%)</th>
<th>n (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>After Using Physical Manipulatives</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Comfortable</td>
<td>1 (5.3)</td>
<td>13 (14.1)</td>
</tr>
<tr>
<td>Somewhat</td>
<td>18 (94.7)</td>
<td>79 (85.9)</td>
</tr>
</tbody>
</table>

Note. Percentage of “comfortable” responses was not significantly higher for virtual than for physical manipulatives, $\chi^2(1) = 1.12$, $p = .289$. 
Altogether, these findings suggest that attitudes towards learning fraction addition via the manipulative, Fraction Bars, did not really differ as a function of type of manipulative. Fifth grade students found both virtual and physical manipulatives somewhat helpful in learning fraction addition; they also felt somewhat comfortable with fraction addition after using both types of manipulatives. Thus, the fifth hypothesis was not supported.

**Sixth Hypothesis**

It was hypothesized that there would be significant differences in representation preferences between physical and virtual manipulatives. To test this hypothesis, one sample binomial tests were conducted; probability of occurrence was set at 50%. The findings in Table 22 reveal that fifth grade students preferred physical manipulatives to virtual manipulatives in terms of the following: that learning math with physical manipulatives was a good way to spend math time \((p < .001)\), and that they could better explain math using physical manipulatives \((p < .023)\). Furthermore, students also preferred virtual manipulatives to physical manipulatives in terms of the following: it was less boring than physical manipulatives \((p = .002)\), and it was more interesting than physical manipulatives \((p = .002)\). However, students did not prefer one type of manipulative over the other in terms of the ten other statements. As such, the sixth hypothesis was generally not supported.
Table 22

*Percentages and One Sample Binomial Test Results for Preference for Type of Manipulative (N = 111)*

<table>
<thead>
<tr>
<th>Statement</th>
<th>Virtual (%)</th>
<th>Physical (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. In the future, I would like to use this tool more.</td>
<td>(42.3)</td>
<td>(57.7)</td>
</tr>
<tr>
<td>2. Learning with this tool is a good way to spend math time.</td>
<td>(26.1)</td>
<td>(73.9) ***</td>
</tr>
<tr>
<td>3. It is fun to figure out how this learning tool works.</td>
<td>(43.2)</td>
<td>(56.8)</td>
</tr>
<tr>
<td>4. Using this tool becomes boring.</td>
<td>(36.9)</td>
<td>(63.1) **</td>
</tr>
<tr>
<td>5. Working with math problems using this tool is fun.</td>
<td>(44.1)</td>
<td>(55.9)</td>
</tr>
<tr>
<td>6. I wish I had more time to use these types of tools in math.</td>
<td>(42.3)</td>
<td>(57.7)</td>
</tr>
<tr>
<td>7. Learning to use this tool is interesting.</td>
<td>(64.9)</td>
<td>(35.1) **</td>
</tr>
<tr>
<td>8. I can stay on task more easily by using this tool.</td>
<td>(48.6)</td>
<td>(51.4)</td>
</tr>
<tr>
<td>9. I would feel comfortable working with this learning tool.</td>
<td>(48.6)</td>
<td>(51.4)</td>
</tr>
<tr>
<td>10. This learning tool makes me feel uneasy and confused.</td>
<td>(48.6)</td>
<td>(51.4)</td>
</tr>
<tr>
<td>11. I can explain how to do math better using this tool.</td>
<td>(38.7)</td>
<td>(61.3) *</td>
</tr>
<tr>
<td>12. This tool was easy to use.</td>
<td>(52.3)</td>
<td>(47.7)</td>
</tr>
<tr>
<td>13. This tool helps me understand fractions.</td>
<td>(44.1)</td>
<td>(55.9)</td>
</tr>
<tr>
<td>14. This tool helps me get the right answers.</td>
<td>(47.7)</td>
<td>(52.3)</td>
</tr>
</tbody>
</table>

* * * * *

* p < .05. ** p < .01. *** p < .001.
Chapter V: Discussion

Introduction

The purpose of this research study was to determine whether the manipulative “fraction bars” had an effect on fifth graders conceptual understanding of adding fractions. Since only limited research exists on concrete versus virtual manipulatives, one control group and two treatment groups were compared. The control group learned the addition of fractions without manipulative tools. The first treatment group learned via the physical fraction bars manipulative and the second treatment group learned via the virtual fraction bars manipulative. Then, both treatment groups were switched to determine whether there was a statistically significant difference between the orders at which they were exposed to type of manipulative.

In line with the study’s purpose, the following hypotheses were proposed:

H1a: There would be significant differences in fifth graders’ conceptual understanding of the adding of fractions between the control, virtual, and physical manipulatives groups.

H2a: There would be a significant difference in fifth graders’ conceptual understanding of the adding of fractions when using the virtual manipulative “Fraction Bars” compared to the control group.

H3a: There would be a significant difference in fifth graders’ conceptual understanding of the adding of fractions when using the physical manipulative “Fraction Bars” compared to the control group.

H4a: There would be a significant difference in fifth graders’ conceptual understanding of the adding of fractions when using the virtual and physical manipulatives “Fraction Bars” consecutively, compared to the control group.

H5a: There would be a significant difference in fifth graders’ attitudes toward
understanding the adding of fractions after learning with virtual manipulatives and then learning with physical manipulatives (and vice-versa).

H6A: There would be significant differences in representation preferences between physical and virtual manipulatives.

In the current chapter, the findings are interpreted vis-à-vis prior literature and research studies. Following that, the implications of the study findings are discussed. Lastly, the limitations of the study and directions for future studies are presented.

**Interpretation of Findings**

*Manipulatives vs. no manipulatives.* Altogether, the findings indicate that using the Fraction Bars manipulative helped students better understand the process of fractions addition. The change from pretest to posttest for students who used virtual manipulatives was greater than the change from pretest to posttest for students in the control group. In addition, the change from pretest to posttest for students who learned fractions addition by using physical manipulatives was greater than the change for those in the control group. Lastly, the change from pretest to posttest for students who used both types of manipulatives was greater than the change for students who did not use any manipulatives (i.e., those in the control group). Note further that the effect sizes of the slope differences were relatively large; partial etas ranged from .51 to .90.

These findings echo the meta-analytic findings of Parham (1983) and more recent study findings conducted by Martin and Schwartz (2005), Suh (2005), Suh and Moyer (2007), Steen et al., (2006), and Westenkow (2012). In his meta-analysis of manipulative use in elementary school classrooms, Parham (1983) examined 64 research studies conducted between 1965 and 1979. Results of the meta-analysis indicated that the students who used manipulatives in their mathematical instruction scored in the 85th percentile on the California Achievement Test, while
students who did not use the manipulatives scored in the 50th percentile. More recently, Westenkow (2012), in his well-designed study analyzing gains at the question, cluster, and total level, demonstrated that students exposed to physical manipulatives only, virtual manipulatives only, and both physical and virtual manipulatives showed significant improvement in fractions equivalence performance.

Why and how the use of manipulatives led to a large and significant improvement in fractions learning was not ascertained in the current study. However, as noted in the literature review, Piagetian theory suggests that children learn by hands-on experiences and by reflecting on the results of their physical actions (Baroody, 1989). Further, Boeree (1999) explained how the use of manipulatives is consistent with Piagetian theory. Piaget posited that children, ages two through seven, fall into the preoperational stage of development, which is followed by the concrete operational stage of cognitive development lasting until about age 11. As such, young children need to experience concepts concretely before being introduced to the symbolic language of mathematics (Boeree, 1999). After students learn to solve basic equations by representing them with concrete objects like manipulatives, they can begin to progress toward an abstract level of comprehension by transferring to symbolic representations of the problem through either drawing or providing written descriptions of their work. Therefore, when used properly, manipulatives allow students to compare representations, form new representations, and subsequently learn mathematical concepts at a more abstract and symbolic level.

More specifically, the Rational Number Project identified four ways that manipulatives help students understand fractions (Cramer, Post, & delMas, 2002). First, manipulatives assist students in developing mental images of fraction meaning. Second, manipulatives assist students in understanding fraction size. Third, manipulatives act as a reference for students when
justifying their answers. Finally, manipulatives discourage students from resorting to misconceptions developed as a result of applying whole number rules to fractions.

**Physical vs. virtual manipulatives.** The findings also reveal that the type of manipulative with which students worked did not affect the degree of improved performance. That is, both virtual and physical manipulatives led to similar improvements in performance. These findings support what Moyer-Packenham and Suh (2012) documented in a meta-analysis and what Martin and Schwartz (2005), Suh (2005), Smith (2006), and Westenkow (2012) demonstrated in their studies. Moyer-Packenham and colleagues (2012) reported a small effect size (.15) for differences in performance between virtual and physical manipulatives but a large effect size (.75) for differences in performance between virtual manipulatives and traditional classroom instruction. Westenkow (2012) demonstrated that virtual manipulatives were found to be more beneficial in helping students understand symbols, but physical manipulatives were more useful in helping students grasp set model representations.

**Combining physical and virtual manipulatives.** The findings further indicate that using both manipulatives consecutively led to steeper improvement in performance, in comparison to using just one type of manipulative. Order of exposure to manipulative did not have an effect on fractions performance. This finding corroborates the study findings of Takahashi (2002) and Westenkow (2012) and the meta-analysis findings of Moyer-Packenham and colleagues (2012).

These researchers all demonstrated that combining the two methods was ultimately more effective than choosing only one method. This was because physical and virtual manipulatives had different advantages. In Takahashi’s (2002) study, for example, the physical manipulative was more useful in helping students develop the concept of area, but the virtual manipulative was more useful in helping students develop formulas when shape transformation was required. Thus,
Takahashi (2002) suggested that students would benefit from using both types of manipulatives to maximize learning. Further, Suh and Moyer (2007) documented that the manipulatives had different advantages. The physical manipulatives appeared to allow students to invent solution strategies as well as utilize more mental mathematics. On the other hand, the virtual environment provided students with instant feedback, step-by-step support, and the linking of the visual and symbolic models (Suh & Moyer, 2007). Lastly, in their meta-analysis, Moyer-Peckenham and Suh (2012) noted a moderate effect size of .33 when virtual and physical manipulatives were used together. Thus, they concluded that using both virtual and physical manipulatives was even more beneficial than using either one on its own.

**Students’ preferences for type of manipulative.** The sample of students in the current study did not exhibit any clear preference for one type of manipulative or the other. However, the majority of the students indicated that learning with physical (versus virtual) manipulatives was a good way to spend math time and that it was easier for them to explain math using physical (versus virtual) manipulatives. These findings corroborate those of Goracke (2009), who documented that students reported enjoying the work they did with the manipulatives, and felt that manipulatives increased their overall understanding of mathematical concepts. The students reported that their enjoyment resulted from the hands-on, active participation rather than the academic benefit, but that improvement was a side effect of their feelings about the activity.

Furthermore, the majority of the students noted that using physical (versus virtual) manipulatives became boring after a while and that learning to use virtual (versus physical) manipulatives was interesting. According to Gardner (1993), students are more likely to be active participants in the acquisition of a skill when using virtual manipulatives. The unique affordances within virtual manipulatives can help bridge the gap between students’ differing
learning styles. Virtual manipulatives provide visual, auditory, and kinesthetic modes of instruction, allowing the students to gain an understanding of the material more easily (Gardner, 1993).

**Theoretical Implication of Findings**

*Support for conceptual understanding of fractions operations.* The findings indirectly confirm that using manipulatives help young students develop a conceptual understanding of fractions operations. As noted in the literature review, pictorial representations are particularly useful in helping students develop conceptual knowledge of mathematics problems that may otherwise appear meaningless to students (Miller & Hudson, 2007). Suh and Moyer (2007) indicate that using symbolic representations, such as manipulatives, allows students to make meaningful connections between procedural and conceptual knowledge. Additionally, using manipulatives can assist students in making connections between various mathematical concepts. Such relational thinking is the core of conceptual understanding, and symbolic learning can significantly contribute to students’ mathematical understanding (Suh & Moyer, 2007).

Representation “involves creating, interpreting, and linking various forms of information and data displays, including those that are graphic, textual, symbolic, three-dimensional, sketched, or simulated” (NCTM, 2003, p.3). Mathematical concepts and ideas are characterized using words, symbols, illustrations, charts, and graphs (NCTM, 2003).

*Support for constructivism.* As noted above, the majority of the students indicated learning with physical (versus virtual) manipulatives was a good way to spend math time and that it was easier for them to explain math using physical (versus virtual) manipulatives. From this, it can be inferred that enjoyment resulted from the hands-on, active participation rather than the academic benefit (Goracke, 2009). These findings thus provide support for constructivism.
As opposed to more passive methods such as lectures and textbooks, constructivism assumes learning is active and students themselves construct knowledge, unlike passive methods such as lectures and textbooks (Salkind, 2008). Social constructivism assumes knowledge is formed through social interaction (Powell & Kalina, 2009). Therefore, a constructivism learning theory style favors active learning, which allows learners to build their own conceptual understanding through applying concepts, constructing their own meanings, and thinking about ideas (Ernest, 1996; Gordon, 2009). Too many students lack proficiency in mathematics as a result of traditional methods. If mathematics instruction were taught from a constructivism viewpoint rather than rote memorization, repetitive drills, and lectures, students would be encouraged to create their own understanding of the subject through social interaction and meaningful activities (Andrew, 2007). As documented in this current study, students not only enjoyed learning fractions operations via manipulatives use, but also enjoyed the interactive and meaningful activity that using manipulatives offered.

**Practical Implications of Findings**

The current study’s findings corroborate the many studies that support the use of mathematical manipulatives in the classroom (Allen, 2007; Burns, 1996; Clements, 1999). According to Gardner (1991), students reported that they do not understand the concept they are expected to learn because their math classes consist of instruction followed by an exam. Manipulatives are a useful tool in assisting students with learning conceptual mathematical ideas. Using a manipulative explicitly in a hands-on manner assists students in learning concepts more easily (Moyer, 2002). Children must feel a connection to the concepts that they are required to understand in order for learning to be relevant and lasting (Gardner, 1991). Clearly, this study and other studies show that manipulatives can help accomplish this goal. Manipulatives provide
students with opportunities to become actively engaged in meaningful learning experiences, thus allowing them to take ownership of their learning. After using manipulatives, students gain the ability to transfer their concrete knowledge to symbolic knowledge, and then finally, to real-life situations (Blair, 2012; Heddens, 1996).

As noted in an earlier chapter, the NCTM (2010) recommends that teachers integrate manipulatives into all levels of mathematics education, as these methods allow students to think algebraically and increase their conceptual understanding of mathematical ideas (Magruder, 2012). Sowell (1989) suggests that that long-term use of manipulatives is more effective in maintaining and increasing learning when compared to short-term use. As a result, manipulatives should be used consistently throughout middle school and high school.

However, implementation of manipulatives in all levels of education has been limited. Jones (2009) asserted, “It is more likely that manipulatives would increase their value in later grades, in teaching more complicated skills, as children mature and become mentally able to develop understanding of operations” (p. 5). Using manipulatives at the elementary level would allow students to bridge the gap between the procedure they are performing and the meaning it represents, thereby ultimately increasing understanding rather than memorizing computation rules (Jones, 2009).

Limitations

The current study included a control group (in addition to the virtual and physical manipulatives groups) that was measured across time to allow for a strong test of the effect of manipulatives on fractions addition. Nevertheless, the study had several limitations. One minor limitation was that the attitudes measure consisted of only three items, which did not correlate well with each other. As such, the measure was not reliable and did not appear to be valid. If
researchers want to have a more nuanced understanding of the relationship between manipulatives use and attitudes towards math, they will need to develop items that measure specific concepts and processes.

A second minor limitation is that although immediate feedback and ease of use have been posited to mediate the relationship between manipulatives use and math conceptual learning (Baturo, Cooper, & Thomas, 2003; Highfield & Mulligan, 2007; Hsiao, 2001; Nute, 1997; Takahashi, 2002), these constructs were not measured. If the mediating effects of these two variables are to be ascertained, then future studies need to directly measure these two variables.

There are two additional minor limitations that should be noted in this study. The first one is that students who participated in the study had not used the physical and virtual manipulatives of fraction bars as learning tools prior the study; therefore, the newness of these tools may have influenced students' preferences for using them. The second minor limitation is due to the small number of questions in the pretest and post-tests; however, this was because the child was expected to provide a detailed explanation of his answer, which was the main goal of the study – conceptual understanding.

A major limitation of the study is the timing of the intervention. In this study, the plan of the intervention was created to teach students from the basic to higher-level concepts of adding fractions. However, since children were not familiar with the use manipulatives as a tool of learning, more time than expected was spent in teaching students the concepts that were less difficult for them before moving on to the main concept of adding fractions. Future studies need to take into consideration that the time of the intervention should be longer to allow students to practice the concept of adding fractions and to prevent non-planned events to have any effect on the focus of the core of the research questions.
Another major limitation of the study is that it was unclear what specific process or processes in manipulatives’ use led to learning across time. Several processes have been hypothesized to explain the positive effects of manipulatives on learning. For instance, some researchers have posited that children learn best by actively manipulating objects and reflecting on the results of their physical actions (Baroody, 1989). Others hypothesize that students develop and build knowledge from concrete to abstract and that the more experience students have with the concrete, the greater their conceptual understanding will be (McNeil & Jarvin, 2007). Suh and Moyer (2007) propose that the use of symbolic representations, such as manipulatives, allows students to connect conceptual and procedural knowledge, as well as recognize the relationships among different mathematical concepts. Still others believe that the instant and timely feedback regarding performance that manipulatives provide facilitates learning (Heal, Dorward, & Cannon, 2002; Martin & Swartz, 2005; McNeil & Jarvin, 2007; Suh & Moyer, 2007). Lastly, others believe that for learning to be relevant and lasting, children have to feel a connection to the material they are required to understand (Gardner, 1991) and that is where manipulatives can help. The use of manipulatives when learning mathematics motivates and holds the interests of students far longer than direct instruction (Heddens, 1996). Manipulatives provide students with opportunities to become actively engaged in meaningful learning experiences, thus allowing them to take ownership of their learning. Then, they can make the transfers from concrete to symbolic to real-life situations (Blair, 2012; Heddens, 1996). Thus, whether it is the manipulation, the process that allows for understanding links, the feedback loops, or engagement with the task provided by manipulatives remains unclear. Future studies need to address this issue. Interventions need to be very specific and detailed, with each step equated to a specific process.
Future Research Directions

This study contributed to the existing literature of manipulative effectiveness via its use of a control group and measures of fractions addition knowledge across time (via a pretest and two posttests). However, while much is known about manipulatives, many unanswered questions still exist. The findings of this study provide several proposals for future research. It is imperative, however, that researchers seek to understand why manipulatives have such a positive impact on learning math concepts. As such, studies assessing the effects of potential mediators (such as feedback, ease of use, attitudes towards math) on the relationship between manipulative use and math performance need to be conducted.

Although a wealth of research exists as to the value of manipulatives in elementary school, further research is needed to examine the effectiveness of manipulatives among students of different ages and grades. Also, further studies are recommended to examine the effectiveness of different physical and virtual manipulatives when teaching various mathematics concepts in all grade levels and measure how to improve students' academic learning conceptually and procedurally. However, this study only used one website for virtual manipulatives. Many Internet sites are provided by multiple entities. These sites differ in their presentation and are varied in their offerings of virtual manipulatives. Future studies should include other manipulative sites to see which types of manipulatives are most effective in classroom settings.

Also, a potential area of further study related to manipulatives is determining the effectiveness of manipulatives for students with different ability levels. Further research is needed to investigate the effect on low ability, average ability, and high ability students to see if significant differences exist.
Students need to receive computer training prior to using virtual manipulatives due to its newness in the learning environment. Students' background about using computers such as years of experience, daily use, level of mouse use, and so on need to be checked at the beginning because recommendations for individual students for computer training should be based on each student’s needs. In addition, when implementing virtual manipulatives as a part of content instruction, further research is needed to investigate the use of more teacher-directed instruction in developing students’ conceptual understanding of the targeted mathematics topics in conjunction with technology-based instruction. On the other hand, research may be conducted to see how research on virtual manipulatives can help software developers and textbook publishing companies create mathematics applets that are rich in building conceptual understanding and procedural fluency. The role of technology within schools for mathematics is still very ill-defined but holds many promising leads. Further research needs to be done on integrating more technology in the teaching and learning of mathematics to ensure that our students are well prepared for the future.

The time period for future studies should be increased when using physical and virtual manipulatives. It may be beneficial to increase the treatment time to give participants more time to make connections and build their conceptual understandings and procedural skills deeply. In addition, recording tools such as video can be used as a method to record students' works while using physical and virtual manipulatives in order to provide more documentation for researchers about the problems that students can resolve properly and issues that they may face during the treatment period. Also, further research is needed to examine such topics as: the relationship between age and the effectiveness of physical and virtual manipulatives; and the effects of teacher training. Besides that, one might consider investigating how the instructional books of
physical manipulatives can guide both teachers and students properly and help them understand the mathematical concepts easily.
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APPENDICES
APPENDIX A: COVER LETTER FOR PARENTS

Dear Parents or Guardian:

My name is Sami Alshehri and I am a doctoral student at the University of Cincinnati in the United States. I am doing a research study in your child's class to see if using physical and virtual manipulatives will help children to understand the adding of fractions effectively.

Enclosed with this letter are two copies of a Parent’s Permission that describe the research in more detail. Please read the Parent Permission form carefully. If you give permission for your child to be in the study, please sign one form and return it to your child's teacher. Keep the other copy for yourself.

If you have any questions about the study, please don't hesitate to call me at 317-654-6725. Or, you may contact me via email at alshehsi@mail.uc.edu

Thank you for your time, I look forward to hearing from you.

Sincerely,

Sami Alshehri, Ed.D.
Doctoral Student
Curriculum and Instruction Department
University of Cincinnati
APPENDIX B: COVER LETTER FOR PARENTS IN ARABIC

 رسالة للآباء:

عزيزي الآباء:

اسمي سامي الشهري، أنا طالب دكتوراه في جامعة ساندي في الولايات المتحدة الأمريكية. حالياً أقوم
بإجراء دراسة بحثية في الصف الخامس ببلدنا. الدراسة تتعلق بمقارنة عملية توعين مختلفين من البدويات
(المحسوس والكاذبة) والتي تساعد الأطفال على فهم كيفية إضافة الكورس بكل سهولة.

سيتم إرفاق نسختين من نموذج الموافقة الدينية للمشاركة في الدراسة مع هذه الرسالة والتي يمكنك تقبيل
أكثر عن الدراسة التي أقوم بها في هذا الوقت. نرجو منك قراءة النموذج الخاص بموقفة الولد ونقله
في حالة إعطاء ابنك الموافقة للمشاركة في هذه الدراسة ، نرجو منك التوقيع على نموذج الموافقة وإعادته
 إلى معلم الصف الخاص بطفالك، مع الاحتفاظ بالنسخة الثانية لديك.

إذا كان لديك أي تساؤلات عن هذه الدراسة ، يرجى الاتصال بي خلال أوقات العمل على الرقم
alshehsig@mail.uc.edu أو الإرسال على الأيميل 317.204.6725 أو الإرسال على الأيميل 317.204.6725
شكرا جزيلا لوقتك الثمين.

مع خالص التحية،
سامي الشهري
طالب دكتوراه
قسم المناهج وطرق التدريس
جامعة ساندي
Parent Permission for Child’s Participation in Research
University of Cincinnati
Department: Curriculum and Instruction
Principal Investigator: Sami Alshehri
Faculty Advisor: Dr. Sally Moomaw

Title of Study: The Comparison of Physical/Virtual Manipulatives on Fifth-Grade Students’ Understanding of Adding Fractions

Introduction:
You are being asked to allow your child to take part in a research study. Please read this paper carefully and ask questions about anything that you do not understand.

Who is doing this research study?
The person in charge of this research study is Sami Alshehri who is a doctoral student at the University of Cincinnati (UC) Department of Curriculum and Instruction.

What is the purpose of this research study?
The goal of this study is to compare the effects of physical and virtual manipulatives on children to enhance understanding of adding fractions.

Who will be in this research study?
About 150-200 fifth grade children will take part in this study.

What will your child be asked to do in this research study, and how long will it take?

There are three parts of this study and it will take about two weeks to complete. Below is what your child will be asked to do in this project.

1. All children will do a test about understanding of adding fractions. This will NOT count as an exam grade. It is just to find out what they already understand. Also, they will do a quick survey to explain their comfort of adding fractions.
2. Some children will use the virtual manipulative in the computer lab. The rest of the children will use the physical manipulative in the regular class. After five days, all children are going to do a test about the understanding of adding fractions and a quick survey to explain their comfort of adding fractions.
3. The two groups will be switched. This means that children who have worked with virtual manipulatives in the first week will use the physical manipulatives and vice versa. After five days, all children will do another test about addition of fractions, attitude survey, and preference survey.

The research will take place at Public Elementary Schools in Abha.

Are there any risks to being in this research study?
Not at all.
Are there any benefits from being in this research study?
If your child uses the physical and virtual manipulatives, he or she may benefit from being in this study. There is no promise that your child will benefit from this study. The study may help students to understand the process of adding fractions easily if it shows that the research tools work well.

What will your child get because of being in this research study?
None

Does your child have choices about taking part in this research study?
Children who are not in this research will do regular activities in their classroom. They will not be treated any differently. Any child who becomes restless or wants to quit using the physical/virtual manipulatives will go back to his/her classroom.

How will your child's research information be kept confidential?
Information about your child will be kept private by the research team. Children’s identity and respective research data will remain confidential and will not be revealed. A study ID number will be assigned to each child and will be used instead of a child’s name on the study test and surveys. Data will be kept on the researcher’s locked office cabinets. Also, any research data that is saved on a computer will be password-protected. Your child’s information will be kept confidentially for five years. After that, the master list of names and study ID numbers will be destroyed by shredding. Agents of the University of Cincinnati may inspect study records for audit or quality assurance purposes.

What are your and your child’s legal rights in this research study?
Nothing in this consent form waives any legal rights your child may have. This consent form also does not release the investigator, the institution, or its agents from liability for negligence.

What if you or your child has questions about this research study?
If you or your child has any questions or concerns about this research study, you should contact the principal investigator “Sami Alshehri” at (317) 654-6725 OR, by email at alshehsi@mail.uc.edu

The UC Institutional Review Board reviews all research projects that involve human participants to be sure the rights and welfare of participants are protected.

If you have questions about your child's rights as a participant or complaints about the study, you may contact the UC IRB at (513) 558-5259. Or, you may call the UC Research Compliance Hotline at (800) 889-1547, or write to the IRB, 300 University Hall, ML 0567, 51 Goodman Drive, Cincinnati, OH 45221-0567, or email the IRB office at irb@ucmail.uc.edu.

Does your child HAVE to take part in this research study?
No one has to be in this research study. Refusing to take part will NOT cause any penalty or loss of benefits that you or your child would otherwise have. You may give your permission and then
change your mind and take your child out of this study at any time. To take your child out of the study, you should tell the principal investigator, principal of the school, or classroom teacher that your child wants to quit participating in the study.

Your child will be asked if he or she wants to take part in this research study. Even if you say yes, your child may still say no.

**Agreement:**
I have read this information and have received answers to any questions I asked. I give my permission for my child to participate in this research study. I will keep one copy of this Parent Permission form and return a signed and dated copy to my child's teacher.

**You Child's Name (please print) _____________________________**

**Your Child's Date of Birth ____________ (Month / Day / Year)**

**Parent/Legal Guardian's Signature ____________________________ Date _______**
APPENDIX D: LETTER OF INFORMED CONSENT-PARENT IN ARABIC
COMPARISON OF PHYSICAL/VIRTUAL MANIPULATIVES

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COMPARISON OF PHYSICAL/VIRTUAL MANIPULATIVES
APPENDIX E: PARTICIPANTS NEEDED FOR RESEARCH STUDY

“The Comparison of Physical/Virtual Manipulatives on Fifth-Grade Students’ Understanding of Adding Fractions”

The study is open to fifth grade students who are between 10-11 years old.

The purpose of the research study is to compare the effect of two different types of manipulative (Physical and Virtual) on understanding of adding fractions.

Participation involves using both concrete and virtual manipulatives during 2 weeks. Time commitment: The study will last approximately 10 days (50 min per day). Participants will receive study related to physical and virtual manipulatives and may see improved to understand the adding of fractions.

Participants will not be reimbursed for time of participation.

The research will be conducted at your school.

For Additional information, please contact Dr. Sally Moomaw at 513-556-4414 or email Sally at: sally.moomaw@uc.edu

Principal Investigator: Sami Alshehri, Ed.D.

University of Cincinnati, Curriculum & Instruction Department

Phone #: 317-654-6725

Email: alshehsi@mail.uc.edu
APPENDIX F: PARTICIPANTS NEEDED FOR RESEARCH STUDY IN ARABIC
APPENDIX G: LETTER OF STUDENT ASSENT

Child Assent Form for Research
(Ages 10-11 Years)
University of Cincinnati
Department: Curriculum and Instruction
Principal Investigator: Sami Alshehri
Faculty Advisor: Dr. Sally Moomaw

Title of Study: The Comparison of Physical/Virtual Manipulatives on Fifth-Grade Students’ Understanding of Adding Fractions

You are being asked to do a learning project. You may ask questions about it. You do not have to say YES. If you do not want to be in this learning project, you can say NO.

This project may help you to know how children understand the adding fractions. You will see at the end how it is easy to understand fractions and answer all questions about addition of fractions.

About 200 children will be in this study. It will take about 2 weeks only. You will be using two different tools: physical manipulatives and computer based manipulatives called virtual manipulatives. This will not be part of your grade, but you will help me learn how children understand the adding fractions.

If you have any questions, you can ask your classroom teacher or the researcher “Sami Alshehri” any time and we are so glad to answer your question.

You do not have to be in this learning project. If you decide now that you want to join the study, you can still change your mind later just by telling your parents, your teacher or me. No one will be upset with you and your grade in math will not be affected if you say no. To stop being in the learning project, you should tell your parents, your teacher or me that you want to stop being in the project.

If you want to be in this learning project, write your name and your birthday below. If you do not want to be in this learning project, leave the lines blank.

Your Name (please print) ________________________________

Your Birthday __________ (Month / Day / Year)

Your Signature _____________________________ Date _________

Signature of Person Obtaining Assent ____________________ Date __________
APPENDIX H: LETTER OF STUDENT ASSENT IN ARABIC

Source Document

Letter of Student Assent in Arabic

[Image of the letter in Arabic]
APPENDIX I: LETTER OF APPROVAL FOR RESEARCH FROM PUBLIC ELEMENTARY SCHOOLS

Study Consent Form

We are pleased to give this consent to Mr. Sami Alshehri approach learners in fifth grade to participate in the research of "The Comparison of Physical/Virtual Manipulatives on Fifth-Grade Students’ Understanding of Adding Fractions." Information of the study has been read carefully in Arabic and understands that schools' participating is voluntary and each school has a right to withdraw at any time without penalty. Also, students in fifth grade will be invited to participate and that permission will be sought from them and from their parents. Only learners who consent and whose parents consent will participate in the study and they have rights to withdraw from the study at any time without penalty. Besides that, all information obtained will be treated in strictest confidence. The learners' names will not be used and individual learners will not be identifiable in any written reports about the study. The schools will not be identifiable in any written reports about the study.

Below are the names of those schools that wish to participate, as following:
1. Alhassan Albasri elementary school.
2. Al Muhammediyah elementary school.
3. Habib Bin Zaid elementary school.
4. Akashah Bin Muhsin elementary school.
5. Abdullah Bin Masood elementary school.
6. Usaid Bin Hudair elementary school.
8. Abdurrahman Bin Auf elementary school.
9. Al Sharaf elementary school.

If you need further information about what has been written above, please do not hesitate to contact my office with your questions at (+966-17-2255081) or contact the Department of Education of Asir Region at (+966-17-2246141).

Sincerely,

Jalawi Muhammad
Chair of Department of Education
Abha, Saudi Arabia

Signature: ____________________________
Date: 25-11-2015
Title of Study: The Comparison of Physical/Virtual Manipulatives on Fifth-Grade Students’ Understanding of Adding Fractions

Dear Teacher,

You are being asked to participate in this research study. The purpose of my quasi-experimental research study is to compare the effectiveness of physical and virtual manipulative, specifically “Fraction Bars”, when it is used into math instruction in fifth grade classrooms in order to enhance the understanding of adding fractions properly. Completion of a research study is part of the doctoral degree (Ed.D) requirements. Your participation in this study will not take much of your time. By participating, you will be helping achieve some insight into this matter. If you choose to assist me, I will ask you to read and sign an inform consent agreement permission to participate in the study.

The study will take place in your class. You will monitor and document the scores of pretest/posttest data and collecting the attitude and preference surveys. This study will take approximately 2-3 weeks. Your students will be a part of either a control or experimental group for the study.

Please sign this letter here _____________________________________ and return a copy to me at your earliest convenience. I assure you that all data obtained will be confidential and you will not be identified in any manner. Please accept my gratitude and thanks for taking part in this study. If you have any questions or you would like a summary of the statistical results of the study, please call me during business hours at 317-654-6725 or email me at alshehsi@mail.uc.edu. Again, thank you very much for your help and participation in this study.

Sincerely,

Sami Alshehri
Graduate Student
University of Cincinnati
APPENDIX K: TEACHERS ASSENT FORM FOR RESEARCH IN ARABIC

Source Document

نتوجه منك التوقيع على هذه الورقة هنا _______________ وإعادة هذه النسخة إلى الباحث في أقرب وقت ممكن. نود أن نؤكد لكم أن جميع البيانات التي ستتم محررها عليها ستكون في غاية السرية ولن يتم الكشف عن هويتك بأي شكل من الأشكال. أُمَّنًى أن تقتصر أعمالك ومشاركتك في هذه الدراسة. إذا كان لديك أي استفسار عن هذه الدراسة، يمكنكم التواصل معنا على البريد الألكتروني: edu.alshehri@mail.uc

مع خالص التحية,
سامي الشهري
طالب دراسات عليا، جامعة سانستا\n
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1-855-WETRANS (1-855-938-7267)  *  support@cinchttranslaion.com
APPENDIX L: INSTRUCTIONS OF ADDING FRACTIONS FOR
PHYSICAL MANIPULATIVES

Understanding of Adding Fractions

Brief Overview:
During the five days, students will develop an understanding of adding fractions. Physical and virtual manipulatives of fraction bars will be used to facilitate the teaching of adding fractions with like and unlike denominators. For this unit, students will need the prerequisite skills of representing and comparing fractions with like denominators and equivalent fractions. They will also need to be able to place a fraction on a fraction bar labeled 1/4, 1/2, and so on.

NCTM Content Standard:
The National Council of Teachers of Mathematics (NCTM) Principles and Standards emphasizes the importance of helping children deepen their understanding of fractions to include making sense of operations on fractions, noting “teachers need to be attentive to obstacles that many students encounter as they make the transition from operations with whole numbers” (NCTM, 2000, p. 218)

Number and Operations—Fractions:
• Understand the place-value structure of the base-ten number system and be able to represent and compare whole numbers and decimals.
• Recognize equivalent representations for the same number and generate them by decomposing and composing numbers.
• Develop understanding of fractions as parts of unit wholes, as parts of a collection, as locations on number lines, and as divisions of whole numbers.
• Use models, benchmarks, and equivalent forms to judge the size of fractions.
• Recognize and generate equivalent forms of commonly used fractions, decimals, and percents.
• Developing understanding of and fluency with fraction addition.
• Apply their understandings of fractions and fraction models to represent the addition of fractions with unlike denominators as equivalent calculations with like denominators.
• Develop fluency with stand procedures for adding fractions.

Grade: 5th grade.

Duration/Length: 50 minutes per day for 5 days.
Student Outcomes:
- Compare fractions with the same numerator and different denominators to determine fraction equivalency.
- Using fraction bar models (physical/virtual) makes it easier to compare fractions.
- Create equivalent fractions using fraction bars manipulatives.
- Communicate orally and in writing their understanding of equivalent fractions.
- Identify equivalent fractions.
- Compare and order fractions between 0 and 1 on fraction bars.
- Students will recognize difference between equivalent and non-equivalent fractions.
- Use fraction bars to model addition of fractions with like and unlike denominators.
- Develop students' ability to reason flexibly with fractions.
- Enable learners to retain and apply related procedures for operating on fractions with efficiency and understanding.

Materials and Resources:
- Physical manipulative of Fraction Bars.
- Virtual manipulative of Fraction Bars.
- Reteaching Template.
- 1 Whole Grouping Squares.
- ½ Squares.
- 1/3 Squares.
- ¼ Squares.
- 1/5 Squares.
- 1/6 Squares.

Development/Procedures:

Lesson 1: Equivalent Fractions

Standards for Grades 3-5, page 144

Equivalence should be another central idea in grades 3-5. Students' ability to recognize, create, and use equivalent representations of numbers and geometric objects should expand. For example, 3/4 can be thought of as a half and a fourth, as 6/8, or as 0.75.

Teacher Facilitation:
- Tell students: "This week you will be working with fraction bars (concrete/virtual) to understand the addition of fractions. At the end of the unit, you will have a chance to answer any type of questions about the adding of fractions."
• The first day you are going to learn about the equivalent of fractions by compare two different fractions together. At the end of the lesson today, you're going to be able to recognize the difference between equivalent and non-equivalent fractions.

• Have students show 1/2 using the fraction bars.

• Ask your students to find how many 1/4 fraction bars equal 1/2.

• Have students share what they discovered, and record the picture and the fraction on chart paper.

• Ask students to find how many 1/8 fraction bars equal 1/2.

• Have students share what they discovered and the teacher posts this for the class to see.

• Tell student that Fraction Bars can be used in two ways to illustrate equality of fractions. To first introduce equality, two bars with the same shaded amount represent two equal fractions, as shown in the below examples.

• A second method for illustrating equality involves splitting or dividing each part of a bar. Each of the following yellow bars has 2 out of 3 parts shaded and represents the fraction 2/3. After splitting each part of the first bar into 2 equal parts, both the total number of parts and also the number of shaded parts are doubled. So the bar now has 6 parts and 4 shaded parts, and it represents the fraction 4/6. Similarly, each part of the second 2/3 bar can be split into 4 equal parts to show that 2/3 is equal to 8/12. These are special cases of multiplying the numerator and denominator of a fraction by the same number of obtains an equal fraction.

• As a group, students are to create 1/2 using fourths, fifths, sixths, eighths, tenths, and twelfths.

• Have students individually record their findings by illustrating three of the fractions in the math journal.

• Students can share what they discovered.

• Post this for the class to see.
Student Application:
- Have students find, a yellow bar with 1 part shaded. Ask students the following: How many parts does the split bar now have?

Split each part of the bar into 2 equal parts, by drawing dotted lines as shown in the figure.

Ask students the following: How many total parts does the split bar now have? How many shaded parts? What fraction does the bar now represent? What does this show about the fractions 1/3 and 2/6?
- Repeat the process of splitting each part of a bar into 2 equal parts to obtain equal fractions with several types of bars.
- Guide students to see that splitting each part of a bar into 2 equal parts is the same as multiplying by 2. It doubles the number of parts and doubles the number of shaded parts, but the total shaded amount remains the same.
- Direct students to work independently to see if they can make 1/3, 1/4, 1/5, 1/6 using fraction bars (concrete/virtual) for fourths, fifths, sixths, eighths, tenths and twelfths.
- Students should share what they discovered. Results can be posted in a display.
- Students will record their results as they did for 1/2.

Teacher Facilitation:
- Have the class examine the fractions on the chart paper in the 1/2 box to reinforce that they are equal to 1/2.
- Tell students that these fractions are called equivalent fractions because they are fractions that have the same value.
- Have students find equivalent fractions using fraction bars (concrete/virtual) for 1/3 following the modeled process above.
- Repeat the same process for the other fractions.

Student Application:
*Activity:* Josh and Justin are twins who are celebrating their 12th birthday. They couldn’t decide on the flavor of the frosting, so they have three flavors. Their mom ordered a cake with 1/2 chocolate, 1/4 vanilla, and 1/4 strawberry frosting.

You are the bakers. Working with a partner, use the fraction bars to create a cake with equivalent fractional parts of frosting. Students will illustrate their cakes and label the parts.
Share student cakes amongst the group.
Have students go on a “Gallery Walk” to see other cakes. (A gallery walk is when students orderly walk around the classroom and view other’s work.)
Discuss different ways the students frosted the cake. Ask the students, “What is one way to show 1⁄2 of the cake in chocolate frosting? Is there another way?
Record answers on chart paper and compare them to the previous chart.
Repeat the process for 1⁄4.

Embedded Assessment:
For concrete manipulative group, instruct the students to take a fraction bar and sit at a table with students to represent a group of fractions that are equivalent. For virtual manipulative group, there are some instructions that they have to follow (See Task Sheet of the Virtual Manipulative, Lesson #1).

Extension/Reteaching:
• Review equivalent fractions by creating fractions from concrete/virtual manipulatives.
• Repeat for fourths, eighths, and so on.
• Using the fraction bars model how 2/4 equals 1/2 by placing 2/4 on top of 1/2 to show that they are the same size. Repeat for other equivalent fractions.
• Print blank copies of the “Fraction Bars” for reteaching purposes. Four copies per student are needed.

Lesson 2: Adding Fractions with Like Denominators

Standards for Grades 3-5, page 144
The development of rational-number concepts is a major goal for grades 3-5, which should lead to informal methods for calculating with fractions. For example, a problem such as ¼ + ½ should be solved mentally with ease because students can picture ½ and ¼ or can use decomposition strategies, such as ¼ + ½ = ¼ + (¼ + ¼). (NCTM, 200, p. 35)

Pre-assessment:
• Distribute a set of fraction bars for each one to review equivalent fractions.
• Label each student table with one of the following bars: 1, 1/2, 1/3, 1/4, 1/5, and 1/6.
• Place the equivalent fraction bars on a desk near the door of the classroom.
• Instruct students as they enter to take one card and sit at the table with the fraction equivalent to their card.
• Have students discuss how they know they are at the correct table.
• Monitor the groups for participation and listen for strategies students use to find the correct table.
• After sharing the table fraction and their equivalent fraction, ask the students: How do you know the fractions are equivalent? Can you find equivalent fractions differently? Explain some strategies you use to find equivalent fractions.

Teacher Facilitation:
• Review examples from yesterday.
• Direct students to work with partner and review the definition for equivalent fractions. Provide time to share some definitions aloud.
• Tell students that our goal today is to understand how to add fractions with the same denominators. By seeing the visual concept of adding fractions with the same denominators, you will understand why the numerators are added but the denominator stays the same.
• Show students, and have them find, the yellow 1/4 and 2/4 bars.

\[
\text{1/4} \quad \text{2/4}
\]

• Have them place the shaded amounts of the bars end to end and write the addition equation. \((1/4 + 2/4 = 3/4)\)

\[
\begin{align*}
1/4 + 2/4 &= 3/4
\end{align*}
\]

• Tell students that addition can be illustrated by "putting-to-gather" or "combining" two amounts.

Student Application:
• Have students to work individually to make sure they understand the addition of fractions effectively.
• Each one has to pick two different fractions with the same denominators.
• Represent those fractions visually by using the fraction bars.
• Have them place the shaded amounts of the bars end to end and write the addition equation.
• Show your answer to the class.
• Work with your partner to answer the following question: John has two different bottles of water. The first bottle has 2/8 of water and the second bottle has 5/8 of water. He combined them together in a new bottle. How much water is in the new bottle?
• Share your answer with others.

Teacher Facilitation:

• Show students, and have them find, the blue $\frac{2}{5}$ and $\frac{3}{5}$ bars.

• Have them place the shaded amounts end to end, and write the resulting equation.

\[
\frac{2}{5} + \frac{3}{5} = \frac{5}{5} \text{ OR } 1 \text{ whole bar}
\]

• Ask students what they notice.
• This is a good time to discuss improper fractions and how to write improper fractions as whole or mixed numbers.

• Show students, and have them find, the green $\frac{4}{6}$ and $\frac{5}{6}$ bars.

• Have them place the shaded amounts end to end, and write the resulting equation.

\[
\frac{4}{6} + \frac{5}{6} = \frac{9}{6} = 1 \frac{3}{6}
\]

• Confirm the meaning of improper fractions and how to write it as whole or mixed number.

Student Application:

• Have students to work individually first.
• Ask them to choose one set of the following fractions (1/4, 3/4), (1/5, 4/5), (2/7, 5/7), (5/8, 3/8) or (2/9, 7/9).
• Add the fractions that have been chosen and represent them by using the fraction bars.
• Show your final answer to others.
• Activity: work with your partners to answer the following questions:

\[
\frac{4}{7} + \frac{5}{7} = \quad \frac{5}{5} + \frac{2}{5} =
\]

• Share your answer with the class.
Embedded Assessment:

- Have students select several pairs of bars of the same color, place their shaded amounts end to end, and write the equations. Ask them to demonstrate the results with their bars. Discuss which pairs total less than 1 whole bar, which equal 1 whole bar, and which total more than 1 whole bar. For totals of more than 1 whole bar (improper fractions) have students write their answers as whole or mixed numbers.

- For concrete manipulative group, answer the following questions (explain your answer by fractions bars):
  
  \[
  \begin{align*}
  2/6 + 1/6 & \quad 2/4 + 1/4 \\
  1/3 + 2/3 & \quad 3/5 + 2/5 \\
  3/4 + 2/4 & \quad 4/7 + 4/7
  \end{align*}
  \]

  For virtual manipulative group, there are some instructions that they have to follow (See Task Sheet of the Virtual Manipulative, Lesson #2).

Extension/Reteaching:

- Review the adding fractions with like denominators by creating fractions from concrete/virtual manipulatives.

- Repeat the meaning of improper fractions with some examples.

Lesson 3: Adding Fractions with Unlike Denominators (Using Common Denominators)

Pre-assessment:

- Distribute a set of fraction bars for each one to review addition of fraction with like denominators.

- Ask each one to ask his/her partner a question of adding fraction with like denominators.

- Have students explain their answers to each other.

- Have students discuss how they find the correct answer.

- Monitor the groups for participation and listen for strategies students use to find the correct answer.

Teacher Facilitation:

- Tell students that our goal today is to understand how to add fractions with the different denominators. By seeing the visual concept of adding fractions with unlike denominators, you will understand why the common denominators are used in order to do the computation correctly.

- Have students select the green bar for 1/2 and the yellow bar for 1/3.
• Have students place them end to end to determine if the total shaded amount is less than or greater than 1 whole bar.

• Use this example to show how adding numerator to numerator and denominator to denominator results in an unreasonable answer. (1/2 + 1/3 is not 2/5. This sum is smaller than one of the addends. Demonstrate by comparing the 1/2 and 2/5 bars.)
• Have them find the red bar with the same shaded amount as the green 1/2 bar and the red bar with the same shaded amount as the yellow 1/3 bar.
• Discuss that they have found the common denominator for 1/2 and 1/3.

• Have them write the addition equation for this process.

• Tell students the above example illustrates getting common denominators for the fractions before adding.

Student Application:
• Have students to work individually to make sure they understand the addition of fractions with unlike denominators effectively.
• Each one has to pick a card that shows two fractions with unlike denominators.
• Represent those fractions visually by using the fraction bars.
• Have students get the common denominators for the fractions before adding.
• Have students write the equation that they found.
• Show your answer to the class.
• Work with your partner to answer the following question: Sara went to the store and brought 2/3 pound of apple and 1/4 pound of orange. How many pounds did she bring?
• Share your answer with others.
Embedded Assessment:
- Have students select several pairs of bars of the different colors, place their shaded amounts end to end, illustrate getting common denominators for the fractions before adding, and write the equations. Ask them to demonstrate the results with their bars. Discuss which pairs total less than 1 whole bar, which equal 1 whole bar, and which total more than 1 whole bar. For totals of more than 1 whole bar (improper fractions) have students write their answers as whole or mixed numbers.
- For concrete manipulative group, answer three of the following questions (explain your answer by fractions bars):
  
  \[
  \begin{align*}
  \frac{2}{4} + \frac{5}{8} & \quad \frac{2}{3} + \frac{3}{6} \\
  \frac{1}{3} + \frac{4}{9} & \quad \frac{3}{5} + \frac{3}{10} \\
  \frac{3}{7} + \frac{2}{3} & \quad \frac{7}{8} + \frac{1}{2}
  \end{align*}
  \]

  For virtual manipulative group, there are some instructions that they have to follow (See Task Sheet of the Virtual Manipulative, Lesson #3).

Extension/Reteaching:
- Review the adding fractions with unlike denominators by creating fractions from concrete/virtual manipulatives.
- Explain how to get common denominators for the fractions before adding.
- Repeat the meaning of improper fractions with some examples.

Lesson 4: Adding Fractions with Unlike Denominators

Pre-assessment:
- Distribute a set of fraction bars for each one to review addition of fraction with unlike denominators for getting common denominators.
- Ask each one to ask his/her partner a question of adding fraction with unlike denominators.
- Have students explain their answers to each other.
- Have students discuss how they find the correct answer.
- Monitor the groups for participation and listen for strategies students use to find the correct answer.

Teacher Facilitation:
- Have students select the brown bar for $\frac{7}{12}$ and the yellow bar for $\frac{2}{3}$.
• Have students place them end to end to determine if the total shaded amount is less than or greater than 1 whole bar.

\[
\frac{2}{3} + \frac{7}{12} = 1 \frac{3}{12}
\]

• Tell students that addition can be illustrated by "putting-to-gather" or "combining" two amounts. In the above example, the shaded amounts of Fraction Bars are placed end-to-end. It shows that \( \frac{2}{3} + \frac{7}{12} \) is one whole bar and 3 parts out of 12.

• Have students find another way to answer the previous example by getting common denominators for both fractions.

• Have students split or divide each part of a bar \( \frac{2}{3} \) into 4 equal parts in order to get 12 parts.

• It represents the fraction \( \frac{8}{12} \) (\( \frac{2}{3} \) is equal to \( \frac{8}{12} \)).

• Tell students that we have found common denominators and we can add them and write the equation easily.

\[
\frac{7}{12} + \frac{2}{3} = \frac{7}{12} + \frac{8}{12} = \frac{15}{12} (1 \frac{3}{12})
\]

Student Application:
• Have students to work with partner.
• Ask them to answer one of the following question:
  \[
  \frac{1}{4} + \frac{7}{8} \quad \frac{3}{5} + \frac{9}{10} \quad \frac{2}{6} + \frac{10}{12} \quad \frac{3}{4} + \frac{5}{6}
  \]
• Add the fractions that have been chosen and represent them by using the fraction bars.
• Show us your final answer.
• Activity: work with your partner to answer the following questions: Sam has bought 9/12 liters of oil and then bought 5/6 liters of oil. How many liters did Sam buy? Explain your answer.
• Share your answer with others.
Embedded Assessment:

- Have students select several pairs of bars of the different colors, place their shaded amounts end to end, illustrate getting common denominators for the fractions before adding, and write the equations. Ask them to demonstrate the results with their bars. Discuss which pairs total less than 1 whole bar, which equal 1 whole bar, and which total more than 1 whole bar. For totals of more than 1 whole bar (improper fractions) have students write their answers as whole or mixed numbers.

- For concrete manipulative group, answer two of the following questions (explain your answer by fractions bars):
  
  \[
  \begin{align*}
  2/4 + 7/8 & \quad \quad 2/3 + 5/6 \\
  1/3 + 8/9 & \quad \quad 6/8 + 3/4
  \end{align*}
  \]

  For virtual manipulative group, there are some instructions that they have to follow (See Task Sheet of the Virtual Manipulative, Lesson #4).

Extension/Reteaching:

- Review the adding fractions with unlike denominators by creating fractions from concrete/virtual manipulatives.
- Explain how to get common denominators for the fractions before adding.
- Repeat the meaning of improper fractions with some examples.
Physical manipulative of Fraction Bars
Resource #3

1 Whole Grouping Cards
½ Grouping Cards
1/3 Grouping Cards
Resource #6

¼ Grouping Cards
1/5 Grouping Cards
1/6 Grouping Cards
<table>
<thead>
<tr>
<th></th>
<th>1/2</th>
<th>1/3</th>
<th>1/4</th>
<th>1/5</th>
<th>1/6</th>
<th>1/8</th>
<th>1/10</th>
<th>1/12</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Activity Answer Key

2/4 chocolate
1/4 vanilla 1/4 strawberry

4/8 chocolate
2/8 vanilla 2/8 strawberry

6/12 chocolate
3/12 vanilla 3/12 strawberry

8/16 chocolate
4/16 vanilla 4/16 strawberry
تعليمات استخدم دورة الأشرطة الفضية في دروس جميع الكسور

نظرة عامة:
خلال خمسة أسابيع متحدث التمرور على عروضهم للمعلم، جميع التلاميذ من خلال استخدم دورة "الされましたي"، أثناء شرح الكور ذات الاسم المتضاربة، ذات الاسم المتضاربة، التلاميذ في هيئة إلى تنفيذ بعض المهام الأساسية خلال تدريس هذه الوحدة. مثال الكسور، مقارنة الكسور، والكسور المشتركة. التلاميذ في هيئة اللاعبون إلى عروض استخدم أشرطة الكسور، المقاتلة مثل: 1/3 و 2/6 و 1/2 و 1/4 و 1/8.

معيار المحتوى للتعليم المدفوع الأمريكي في التعليم الرياضي: يوضح المعلم الوطني الأمريكي المعياري المعتمد على المواضيع المدرسية للمعهد لأكثر الأذى في إعداد التلاميذ للدراسة الكورس، بالإضافة إلى الخريطة المعلمين لمعرفة المستويات والمغزيات التي يواجهها العديد من التلاميذ عند الانتقال من تدريس الأعداد السهيلة إلى الاعداد الفردية.

الإجابة على التمرين:

1. فهم الهيكل البنائي والعصري للأساسية الفضية والعصري، والخصائص견 중심ية على مستوى تعليم وإدارة الأعداد السهيلة

2. التعرف على تسلسل مسلكية للسياق الرقم من خلال تنسيق وتركيب الأرقام.

3. اتباع الكورس كجزء من الوحدة أو أجزاء من المجموعه أو جزء من خط الأعداد أو من خلال قسمة

4. استخدام المطورات أو الأوراق أو المهمة المتعلقة للمعلمين على جميع الكسور.

5. إعداد سلاسل الكسور الفضية الأكثر شيوعا واستخدامها، والإعداد العصري والخصائص العصري.

6. تطوير الفهم الرياضي للكودري عند عملية جمع الكسور من خلال إعداد الكسور المماثلة للكسور ذات

7. تطوير الفهم الرياضي عند عملية جمع الكسور.

8. تطوير الممارسة الرياضية عندما جمع الكسور.

الصف الدراسي: الخامس الإبتدائي

المادة الأساسية للمساءلة: 30 دقيقة يومياً ورامدة خمسة أيام

بعد الانتهاء من التدريس هذه الوحدة، سيكون الطلب قادر على:

1. مقارنة الكسور ذات الاسم المتضاربة، أو ذات الاسم غير المتضاربة، أو سهيلة معرفة الكسور.

2. استخدام دورة "السلاسل الكسور" لتعلم أصول مقارنة الكسور.

3. إعداد موضوع "السلاسل الكسور" باستخدام "الدورة الكورس".

4. التواصل الفعال، توضيح مدى فهم عملية "الكودري".

5. التواصل الفعال، توضيح مدى فهم عملية "الكودري".

6. تحديد الكسور المماثلة.
1. ترتيب الدورات من الصفر وصولاً إلى شريط الدور.
2. إجراء اختبار بين الدورات المتماسكة و غير المتماسكة.
3. استخدام أداة "شريط الدور" أثناء حلقة مع الدورات ذات المتتابعون المشتركة وغير المشتركة.
4. تحويل قيمة القياس على مساحة الشريط أثناء جمع الدور.
5. تمكن الطلاب من توظيف الأدوات الرياضية أثناء جمع الدور بدل قراءة وكتابة.

الدورة الأول: الدور المشتركة

الهدف العام من الدورة: أن يصبح الطالب قادرًا على معرفة وإدراك الدور المشتركة من خلال استخدام المثلثات المتساوية المتماسكة في أداة "شريط الدور". مثلاً: 3 يمكن النظر إليها على أنها 3/5 أو 30/50.

خطوات الدورة (المتداخلة)

1- الدور المشتركة من خلال تنمية مهارات الأدوات ويكون من خلال استخدام "شريط الدور" وتحديدها بأن يكون يظهر على شريحة أي نوع من الأدوات المشتركة.

2- جمع الدور على الأدوات من طرفي الورقة في هذا الأسبوع.

3- جمع الدور على الأدوات المشتركة من خلال طورليهما كمتان من ثلاثي وورد في نهاية الأسبوع.

4- جمع الأدوات المشتركة على مساحة أخرى من الورقة المشتركة كمتان من ثلاثي وورد في نهاية الأسبوع.

5- جمع الأدوات المشتركة على أداة "شريط الدور".

6- استخدام أداة "شريط الدور" وتحديدها بأن يكون يتبعه على شريحة أي نوع من الأدوات المشتركة.

7- تمكن الطلاب من استخدام "شريط الدور" وتحديدها بأن يكون يتبعه على شريحة أي نوع من الأدوات المشتركة.

8- تمكن الطلاب من استعمال أداة "شريط الدور" وتحديدها بأن يكون يتبعه على شريحة أي نوع من الأدوات المشتركة.

9- تمكن الطلاب من استعمال أداة "شريط الدور" وتحديدها بأن يكون يتبعه على شريحة أي نوع من الأدوات المشتركة.

10- تمكن الطلاب من استعمال أداة "شريط الدور" وتحديدها بأن يكون يتبعه على شريحة أي نوع من الأدوات المشتركة.

11- تمكن الطلاب من استعمال أداة "شريط الدور" وتحديدها بأن يكون يتبعه على شريحة أي نوع من الأدوات المشتركة.

12- تمكن الطلاب من استعمال أداة "شريط الدور" وتحديدها بأن يكون يتبعه على شريحة أي نوع من الأدوات المشتركة.

13- تمكن الطلاب من استعمال أداة "شريط الدور" وتحديدها بأن يكون يتبعه على شريحة أي نوع من الأدوات المشتركة.

14- تمكن الطلاب من استعمال أداة "شريط الدور" وتحديدها بأن يكون يتبعه على شريحة أي نوع من الأدوات المشتركة.

15- تمكن الطلاب من استعمال أداة "شريط الدور" وتحديدها بأن يكون يتبعه على شريحة أي نوع من الأدوات المشتركة.

16- تمكن الطلاب من استعمال أداة "شريط الدور" وتحديدها بأن يكون يتبعه على شريحة أي نوع من الأدوات المشتركة.

17- تمكن الطلاب من استعمال أداة "شريط الدور" وتحديدها بأن يكون يتبعه على شريحة أي نوع من الأدوات المشتركة.

18- تمكن الطلاب من استعمال أداة "شريط الدور" وتحديدها بأن يكون يتبعه على شريحة أي نوع من الأدوات المشتركة.

19- تمكن الطلاب من استعمال أداة "شريط الدور" وتحديدها بأن يكون يتبعه على شريحة أي نوع من الأدوات المشتركة.

20- تمكن الطلاب من استعمال أداة "شريط الدور" وتحديدها بأن يكون يتبعه على شريحة أي نوع من الأدوات المشتركة.

21- تمكن الطلاب من استعمال أداة "شريط الدور" وتحديدها بأن يكون يتبعه على شريحة أي نوع من الأدوات المشتركة.

22- تمكن الطلاب من استعمال أداة "شريط الدور" وتحديدها بأن يكون يتبعه على شريحة أي نوع من الأدوات المشتركة.

23- تمكن الطلاب من استعمال أداة "شريط الدور" وتحديدها بأن يكون يتبعه على شريحة أي نوع من الأدوات المشتركة.

24- تمكن الطلاب من استعمال أداة "شريط الدور" وتحديدها بأن يكون يتبعه على شريحة أي نوع من الأدوات المشتركة.

25- تمكن الطلاب من استعمال أداة "شريط الدور" وتحديدها بأن يكون يتبعه على شريحة أي نوع من الأدوات المشتركة.

26- تمكن الطلاب من استعمال أداة "شريط الدور" وتحديدها بأن يكون يتبعه على شريحة أي نوع من الأدوات المشتركة.

27- تمكن الطلاب من استعمال أداة "شريط الدور" وتحديدها بأن يكون يتبعه على شريحة أي نوع من الأدوات المشتركة.

28- تمكن الطلاب من استعمال أداة "شريط الدور" وتحديدها بأن يكون يتبعه على شريحة أي نوع من الأدوات المشتركة.

29- تمكن الطلاب من استعمال أداة "شريط الدور" وتحديدها بأن يكون يتبعه على شريحة أي نوع من الأدوات المشتركة.

30- تمكن الطلاب من استعمال أداة "شريط الدور" وتحديدها بأن يكون يتبعه على شريحة أي نوع من الأدوات المشتركة.

31- تمكن الطلاب من استعمال أداة "شريط الدور" وتحديدها بأنه يتبعه على شريحة أي نوع من الأدوات المشتركة.

32- تمكن الطلاب من استعمال أداة "شريط الدور" وتحديدها بأنه يتبعه على شريحة أي نوع من الأدوات المشتركة.

33- تمكن الطلاب من استعمال أداة "شريط الدور" وتحديدها بأنه يتبعه على شريحة أي نوع من الأدوات المشتركة.

34- تمكن الطلاب من استعمال أداة "شريط الدور" وتحديدها بأنه يتبعه على شريحة أي نوع من الأدوات المشتركة.

35- تمكن الطلاب من استعمال أداة "شريط الدور" وتحديدها بأنه يتبعه على شريحة أي نوع من الأدوات المشتركة.

36- تمكن الطلاب من استعمال أداة "شريط الدور" وتحديدها بأنه يتبعه على شريحة أي نوع من الأدوات المشتركة.

37- تمكن الطلاب من استعمال أداة "شريط الدور" وتحديدها بأنه يتبعه على شريحة أي نوع من الأدوات المشتركة.

38- تمكن الطلاب من استعمال أداة "شريط الدور" وتحديدها بأنه يتبعه على شريحة أي نوع من الأدوات المشتركة.

39- تمكن الطلاب من استعمال أداة "شريط الدور" وتحديدها بأنه يتبعه على شريحة أي نوع من الأدوات المشتركة.

40- تمكن الطلاب من استعمال أداة "شريط الدور" وتحديدها بأنه يتبعه على شريحة أي نوع من الأدوات المشتركة.

41- تتمكن الطلاب من استعمال أداة "شريط الدور" وتحديدها بأنه يتبعه على شريحة أي نوع من الأدوات المشتركة.

42- تتمكن الطلاب من استعمال أداة "شريط الدور" وتحديدها بأنه يتبعه على شريحة أي نوع من الأدوات المشتركة.

43- تتمكن الطلاب من استعمال أداة "شريط الدور" وتحديدها بأنه يتبعه على شريحة أي نوع من الأدوات المشتركة.

44- تتمكن الطلاب من استعمال أداة "شريط الدور" وتحديدها بأنه يتبعه على شريحة أي نوع من الأدوات المشتركة.

45- تتمكن الطلاب من استعمال أداة "شريط الدور" وتحديدها بأنه يتبعه على شريحة أي نوع من الأدوات المشتركة.

46- تتمكن الطلاب من استعمال أداة "شريط الدور" وتحديدها بأنه يتبعه على شريحة أي نوع من الأدوات المشتركة.

47- تتمكن الطلاب من استعمال أداة "شريط الدور" وتحديدها بأنه يتبعه على شريحة أي نوع من الأدوات المشتركة.

48- تتمكن الطلاب من استعمال أداة "شريط الدور" وتحديدها بأنه يتبعه على شريحة أي نوع من الأدوات المشتركة.

49- تتمكن الطلاب من استعمال أداة "شريط الدور" وتحديدها بأنه يتبعه على شريحة أي نوع من الأدوات المشتركة.

50- تتمكن الطلاب من استعمال أداة "شريط الدور" وتحديدها بأنه يتبعه على شريحة أي نوع من الأدوات المشتركة.

ب) نموذج 3/4 = 9/12
الخطة الثانية لإعداد اختبار للكسر هو عن طريق القيام بقسم كل كسر في الفئات إلى مجموعات من الأجزاء المشابهة. كما هو موضح في الشكل التالي، وينتج عن ذلك من إعداد الكسور المشابهة من خلال فصل كميات متساوية من المجموعات. ثم يتطلب التحليل في الترتيب التالي:

1. جمع كميات المتساوية من الكسور.
2. اخبار الطلاب بإعداد الكسور المشابهة.
3. عرض المثال وإعداد الطلاب أداءً الجواب.

الخطة للأطفال:

- عاطل من التالية إعداد الكسور المشابهة من خلال استخدام الكسور المشابهة من خلال إعداد الكسور المشابهة من خلال إعداد الكسور المشابهة من خلال إعداد الكسور المشابهة من خلال إعداد الكسور المشابهة من خلال إعداد الكسور المشابهة من خلال إعداد الكسور المشابهة من خلال إعداد الكسور المشابهة من خلال إعداد الكسور المشابهة من خلال إعداد الكسور المشابهة من خلال إعداد الكسور المشابهة من خلال إعداد الكسور المشابهة من خلال إعداد الكسور المشابهة من خلال إعداد الكسور المشابهة من خلال إعداد الكسور المشابهة من خلال إعداد الكسور المشابهة من خلال إعداد الكسور المشابهة من خلال إعداد الكسور المشابهة من خلال إعداد الكسور المشابهة من خلال إعداد الكسور المشابهة من خلال إعداد الكسور المشابهة من خلال إعداد الكسور المشابهة من خلال إعداد الكسور المشابهة من خلال إعداد الكسور مشابهة من خلال إعداد الكسور مشابهة من خلال إعداد الكسور مشابهة من خلال إعداد الكسور مشابهة من خلال إعداد الكسور مشابهة من خلال إعداد الكسور مشابهة من خلال إعداد الكسور مشابهة من خلال إعداد الكسور مشابهة من خلال إعداد الكسور مشابهة من خلال إعداد الكسور مشابهة من خلال إعداد الكسور مشابهة من خلال إعداد الكسور مشابهة من خلال إعداد الكسور مشابهة من خلال إعداد الكسور مشابهة من خلال إعداد الكسور مشابهة من خلال إعداد الكسور مشابهة من خلال إعداد الكسور مشابهة من خلال إعداد الكسور مشابهة من خلال إعداد الكسور مشابهة من خلال إعداد الكسور مشابهة من خلال إعداد الكسور مشابهة من خلال إعداد الكسور مشابهة من خلال إعداد الكسور مشابهة من خلال إعداد الكسور مشابهة من خلال إعداد الكسور مشابهة من خلال إعداد الكسور مشابهة من خلال إعداد الكسور مشابهة من خلال إعداد الكسور مشابهة من خلال إعداد الكسور مشابهة من خلال إعداد الكسور مشابهة من خلال إعداد الكسور مشابهة من خلال إعداد الكسور مشابهة من خلال إعداد الكسور مشابهة من خلال إعداد الكسور مشابهة من خلال إعداد الكسور مشابهة من خلال إعداد الكسور مشابهة من خلال إعداد الكسور مشابهة من خلال إعداد الكسور مشابهة من خلال إعداد الكسور مشابهة من خلال إعداد الكسور مشابهة من خلال إعداد الكسور مشابهة من خلال إعداد الكسور مشابهة من خلال إعداد الكسور مشابهة من خلال إعداد الكسور مشابهة من خلال إعداد الكسور مشابهة من خلال إعداد الكسور مشابهة من خلال إعداد الكسور مشابهة من خلال إعداد الكسور مشابهة من خلال إعداد الكسور مشابهة من خلال إعداد الكسور مشابهة من خلال إعداد الكسور مشابهة من خلال إعداد الكسور مشابهة من خلال إعداد الكسور مشابهة من خلال إعداد الكسور مشابهة من خلال إعداد الكسور مشابهة من خلال إعداد الكسور مشابهة من خلال إعداد الكسور مشابهة من خلال إعداد الكسور مشابهة من خلال إعداد الكسور مشابهة من خلال إعداد الكسور مشابهة من خلال إعداد الكسور مشابهة من خلال إعداد الكسور مشابه
COMPARISON OF PHYSICAL/VIRTUAL MANIPULATIVES

الهدف العام: تطوير مهتمة الأهداف الرئيسية هو أحد الأهداف الرئيسية للطلاب في الصف الخامس الأساسي من أجل أن يتعلموا الطريقة عبر التعلم الفعلي جملة جميع الكسور على سبيل المثال: نسبي الرياضية 2/1 + 1/3 يمكن أن يتم حلها بسهولة بطريقة عملية وذلك لأن الطلاب يمكنهم تحويل صورة الكسور إلى فورمولات باستخدام النسبة 2/1 و 1/3. ويمكن كذلك حلها عن طريق الأسرة التحويلية التي تمثل في الفن التخطيطي:

القيمة النهائية:

- يقوم المعلم بتوزيع أداة "الشريط الكسور" على كل طالب من أجل مراجعة تقنيات الكسور المختلفة.
- يقوم المعلم بوضع أداة "الشريط الكسور" على طاولة كل طالب.
- يطلب من الطلاب وضع الأعداد من الكسور المتاحة للتكساس على طول الشريط.
- يطلب من الطلاب مشاركة الأداة والأساليب إلى الأطراف الأخرى، التي تعودها له.
- يقوم المعلم بتوجيه هذه الأداة إلى الطلاب. كيف تعرف أن هذه الكسور ستتفق؟ هل يمكن توضيح الأساليب المتاحة الخطوة الخطوة الكسور المتفقة؟

خطوات العمل:

- مراجعة بعض الأسئلة والتشريعات عن الدروس السابقة.
- توجيه الطلاب للعمل مع مراجعة تقنيات الكسور المتاحة.
- يطلب الطلاب بعد أن تعلمهم الدروس التفوق على الطريقة جمع كسور بينهما البعض، وينبغي أن تكون العملية كما يلي:

1/2 + 1/4 = 3/4

بمجرد أن يتعلمو الطريقة، يمكنهم استخدامها عند)/(العديد من الأجيال المتابعة مع بعضهم البعض متناول في

الشكل التالي:

- الكسور في الانسباق الأدوات المتشابهة، مع مساحة التحكم على أن الطالب لا يتعثر عندما تكون متشابهة.

قيمة النهائية:

- من خلال استخدام أدوات القياسية، يمكن استخدامها عند جمع الكسور المتاحة مع بعضها البعض متناول في

الشكل التالي:

1/2 + 1/4 = 3/4

بمجرد أن يتعلمو الطريقة، يمكنهم استخدامها عند)/(العديد من الأجيال المتابعة مع بعضهم البعض متناول في
COMPARISON OF PHYSICAL/VIRTUAL MANIPULATIVES
الكلمات: 

- توضح هذه الكسور بصورة من خلال استخدام شريط الكسور.
- تظهر البيانات الفعلية التي تم جمعها عند جميع الطلاب.
- يتم عرض النتائج التي توصل إليها الطلاب الآلاتية، تزويد المعلومات النتائج بين عديد محققين من إدارةوم، الذين الموجودون في الدراسة الأولى، أنواع 6/8 بينما شكل بيئة الملاحظة في الدراسة الثانية 8/6، إذا فلقد يصبح هذه النتائج من في طبيعة جديدة، كن ما تكون كمية للذين تكونت وصولاً إليها من بيئة معاهلاً وتعليم.

- يقسم المعلم بعرض الكسر النتائج أمام التلاميذ، 6/2 و 3/5.


- نسأل الطلاب: ماذا تلاحظون؟

- يشرح للطلاب كيفية فتح الكسور عبر الطبقية أو تمريرها إلى عدد كسري.


- تتناسب طرق طرقية المتغيرة ثم كيفية فتح الكسور عبر الحقيقة أو العدد الكسري.


- النتائج من فهم التلاميذ لفترة تتراوح بين الكسور عبر الحقيقة وكناكية كمية للذين.

- الطلاب من جميع أن يمكنهم بشكل محدد في النتيجة.
- يقوم المعلم بجميع البيانات مع البند والعموم مع العام من أجل التوضيح للطلاب بأن هذه الطريقة التي
- يستخدمها البعض غير صحيحة إعداداً (1/2 + 3/6 = 6/12) من خلال استخدام شريطסיס: يمكن
- الناس زيادة من الكسور (1/2, 3/6) لكي نرى أن هذا الطريقة سلبية عند جميع الناس.
- توضح الطلاب ضرورة ترتيب المكاس في عملية الجمع من خلال إيجاد كل سكان مكاسية للكلور
- المعاون في الخلاصة الرئيسية كما هو موضوع أدناه.

المكاسة مع الثلاثة عن طريق ترتيب المكاسات قبل الرقم عملية الجمع، وأن يكونهم ضرب المكاسات مع

بعضها البعض.

- يقوم المعلم بإعداد النافذة النهائي بعد ترتيب المكاسات.

- فشلت الطلاب:
- هذا التكامل ينطبق بشكل فردي من أجل حساب استيعابهم وفهم لطريقة جمع الكلور ذات الائتم المختصر
- يقوم كل طالب بتشغيل النماذج مختلفة المكوين
- يقوم كل طالب بتشغيل هذه النماذج يتضمن باستخدام شريط الكلور.
- هذا التكامل ينطبق نماذج التكامل قبل الجريمة بتقنية يمكن ربطها أن تكون المكاسات مناسبة.
- بيتكلم الثلاثة النافذة التي ترتبطها بإيجاد
- يتم نشرة التكاملائية المكاسية للمستقبل
- يتم زجاج كل مكاسة وكم سكين على هذا المكاسة الثلاثة: دفع سكين إلى متجار القوارض ونظام
- 2/3 + 3/6
- 1/3 + 4/9
- 3/7 + 2/1
- 2/4 + 5/8
- 3/5 + 3/10
- 7/10 + 1/2
- 1/2
الأنشطة الخاصة:
- مراجعة جملات جميع الكسور ذات المثلث المثلثة، مع إجراء بعض الأمثلة الإشارية بخصوص مشروح.
- توفير طريقة توضيح المثلثات قبل القيام بعملية الحساب.
- إعادة فحص ملهمة الكسور غير المعلومة مع إجراء أمثلة توضيحية على ذلك.

الدرس الرائع: جميع الكسور ذات المثلث المثلثة

التعليمي المبدئي:
- بدأ المعلم في توزيع أدوات التدوين (الكرات مثلاً) بين كل طالب من أجل مراجعة طريقة جميع الكسور ذات المثلث المثلثة.
- يقوم الطالب بتحديد الطرق المناسبة للعثور على الحل.
- يقوم كل طالب بتوضيح الطرق التي استخدمها للوصول إلى النتيجة النهائية.
- يقوم المعلم برفع الجزء الثلاثي من الطلاب والتمرين على الإرشادات الموضوعة للوصول إلى النتيجة النهائية.

خطوات العمل:
- تقوم ورقة الكسور التدريبية أثناء النشاط.

الجريئة: ورقة التصوير الناقص أمثال الأمثلة: 0.7 و 0.5
- تقوم وضع الكسور مع بعضها من خلال الأجزاء المختلفة، ثم يمكننا إذا كانت الأجزاء المختلفة أصغر من أو أكبر من الوحدة الصحيح كما في النشاط التالي:

\[
\frac{2}{3} + \frac{7}{12} = \frac{1}{3} \frac{1}{12}
\]

الجريئة: أن الأجزاء المختلفة مع بعضها يتم الوحدة الصحيح والأجزاء المتبقية نسبياً على 9،
- في النشاط الثاني، إذا كانت أجزاء أخرى لوصول إلى النتيجة النهائية من خلال إعداد نظام الطلاب للمساحة المكملة في النشاط.
- نحصل على النتيجة النهائية عند تقسيم كل جزء من أجزاء الكسر 0.7/10 إلى أربعة أجزاء متساوية من أجل الحصول على نصف الكسر.

\[
\frac{2}{3} = \frac{8}{12}
\]

مسّح الأناّم على الكسر 0.7 والتي يطلق، الكسر 0.5.
تفتيض الطلاب:

- أطلب من كل طالب أن يحول مع زملائه الإجابة على الأسئلة المتبعة في هذا الفصل.

- يقوم الطالب بنتظر واحد من الأسئلة الثانية للإجابة عليها.

- يتم تشغيل الأكواد التي تم إعدادها من خلال دارة الشريط الكسار.

- توضح خطوات واستراتيجيات الحل والوصول إلى الجواب النهائي.

استخدم الجزء الثاني، شارع 278/15 من الزاوية الاستثنائية في يوم الأحد، ثم الطريقة 15/16.

أظهر من نفس الزاوية في يوم الاثنين، كل عدد الأرقام التي تغيرها مي الطلب من الزاوية.

يتشارك الطلاب إجابته مع الأعضاء الآخرين من هناك من صفحة العمل.

التعليم:

- باستخدام شريط البائيس، أجب على سؤالين فقط من الأسئلة الثانية.

- 2/3 + 1/4 = 5/3

- 1/6 + 3/8 = 1/7

الأنشطة الخاصة:

- مراجعة حلول جميع الأكواد ذات القيم المجاورة، مع إجراء بعض الاستماع لتوضيح المفهوم.

- توضيح 방법ية توجيه الطلاب قبل التعلم عملية المشتقة.

- إعادة ترتيب مفهوم الكسر عبر الحلقات مع إجراء استماع لتوضيح المفهوم على تلك.
<table>
<thead>
<tr>
<th>1/2</th>
<th>1/3</th>
<th>1/4</th>
<th>1/8</th>
<th>1/6</th>
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</table>
APPENDIX N: TASKSHEETS FOR ADDING FRACTIONS’ INSTRUCTIONS

Fraction - Virtual Manipulative Task Sheet 1

Name: Period: Date:

Part A: Renaming

1. Go to the Glencoe at
2. On the upper-left side, Click on grade 5.
3. On the upper-left side, click on Manipulative and choose Fraction Tiles.
4. Use the arrow keys to represent the given fraction on the screen.
5. Tell students that there are two ways to rename and illustrate equality of fractions. To first introduce equality, two bars with the same size represent two equal fractions, as shown in the below screen.
6. The second method for illustrating equality involves splitting or dividing each part of a bar into 2 equal parts by using the arrow key to click on the icon of Straight Line Tool and draw dotted lines to divide the fraction into multiple parts, as shown in the below screen.

7. Use the low bar and click on Text Tool to enter the name of the equivalent fraction into the boxes. You may click on the Pen Tool to write the name of the equivalent fraction on the screen.
8. Record your equivalent fraction below. Then, try more 5 problems.

9. Click on CLEAR OBJECTS icon to get a new screen.

Work area.

8. Can you make a rule for finding equivalent fractions?

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Additional Activity:

Part B: Renaming

1. Go to the Glencoe at 

2. On the upper-left side, Click on grade 5.

3. On the upper-left side, click on Manipulative and choose Fraction Tiles.

4. Use the arrow keys to represent the given fraction on the screen.

5. There are two ways to rename and illustrate equality of fractions. To first introduce equality, two bars with the same size represent two equal fractions. A second method for illustrating equality involves splitting or dividing each part of a bar into 2 equal parts by using the arrow key to click on the icon of Straight Line Tool and draw dotted lines to divide the fraction into multiple parts.
6. Use the low bar and click on Text Tool to enter the name of the equivalent fraction into the boxes. You may click on the Pen Tool to write the name of the equivalent fraction on the screen.

7. Record your equivalent fraction below. Then, try more 5 problems.

8. Click on CLEAR OBJECTS icon to have a new screen.

Work area.

8. Can you make a rule for finding equivalent fractions?

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Fraction - Virtual Manipulative Task Sheet 2

Name:  Period:  Date:

Part A: Two Step Sum (Like Denominators)

1. Go to the Glencoe at
   

2. On the upper-left side, Click on grade 5.

3. On the upper-left side, click on Manipulative and choose Fraction Tiles.

4. Represent the following problem on the screen by using the arrow keys:

   \[ \frac{2}{8} + \frac{4}{8} \]

5. Have them place the two amounts of the bars and write the addition equation.

6. Tell students that addition can be illustrated by "putting-to-gether" or "combining" two amounts together, as shown in the below screen.
7. As shown above, you may enter the appropriate numerator and denominator values for the renamed fractions. Renamed fractions are equivalent. Can you state a rule for renaming?

The second step is to combine (or add) the renamed fractions.

8. Use the low bar and click on Text Tool to enter the name of the equivalent fraction into the boxes and write the final answer. Also, you may click on the Pen Tool to write the answer on the screen.

9. Record your answer below. Then, try more 5 problems.

10. Click on CLEAR OBJECTS icon to have a new screen.

**Keep a record of your work in the space below.**
How would you state what you did in YOUR OWN words?

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Additional Activity:

Part B: Two Step Sum (Like Denominators)

1. Go to the Glencoe at
2. On the upper-left side, Click on grade 5.
3. On the upper-left side, click on Manipulative and choose Fraction Tiles.
4. Represent the following problem on the screen by using the arrow keys:
   \[
   \frac{4}{6} + \frac{5}{6}
   \]
5. Have them place the two amounts of the bars and write the addition equation.
6. Tell students that addition can be illustrated by "putting-to-gather" or "combining" two amounts together, as shown in the below screen.
7. As shown above, you may enter the appropriate numerator and denominator values for the renamed fractions. Renamed fractions are equivalent. Can you state a rule for renaming?
   The second step is to combine (or add) the renamed fractions.
8. Use the low bar and click on Text Tool icon to enter the name of the equivalent fraction into the boxes and write the final answer. Also, you may click on the Pen Tool icon to write the answer on the screen.
9. Record your answer below. Then, try more 5 problems.
10. Click on CLEAR OBJECTS icon to have a new screen.

**Keep a record of your work in the space below.**
How would you state what you did in YOUR OWN words?

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**Fraction - Virtual Manipulative Task Sheet 3**

Name:  
Period:  
Date:  

**Part A: Two Step Sum (Unlike Denominators)**

1. Go to the Glencoe at
   

2. On the upper-left side, Click on grade 5.

3. On the upper-left side, click on Manipulative and choose Fraction Tiles.

4. Represent the following problem on the screen by using the arrow keys:

   \[
   \frac{1}{2} + \frac{1}{3}
   \]

5. Have them find the common denominator for 1/2 and 1/3. Write the appropriate numerator and denominator values for the renamed fractions. Renamed fractions are equivalent. Can you state a rule for renaming?

6. Have them write the addition equation for this process, as shown in the below screen.
7. Have students know that the above example illustrates getting common denominators for the fractions before adding.
8. Use the low bar and click on Text Tool icon to enter the name of the equivalent fraction into the boxes and write the final answer. Also, you may click on the Pen Tool icon to write the answer on the screen.
9. Record your answer below. Then, try more 5 problems.
10. Click on CLEAR OBJECTS icon to have a new screen.

Keep a record of your work in the space below.
How would you state what you did in YOUR OWN words?

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Fraction - Virtual Manipulative Task Sheet 4

Name:  Period:  Date:

TASK 1: Two Step Sum (Unlike Denominator)

1. Go to the Glencoe at

2. On the upper-left side, Click on grade 5.

3. On the upper-left side, click on Manipulative and choose Fraction Tiles.

4. Represent the following problem on the screen by using the arrow keys:

\[
\frac{7}{12} + \frac{2}{3}
\]

5. Have them find the common denominator for 7/12 and 2/3. Write the appropriate numerator and denominator values for the renamed fractions. Renamed fractions are equivalent. Can you state a rule for renaming?

6. Have them write the addition equation for this process, as shown in the below screen.
7. Have students know that the above example illustrates getting common denominators for the fractions before adding.

8. Use the low bar and click on Text Tool icon to enter the name of the equivalent fraction into the boxes and write the final answer. Also, you may click on the Pen Tool icon to write the answer on the screen.

9. Record your answer below. Then, try more 5 problems.

10. Click on CLEAR OBJECTS icon to have a new screen.

Keep a record of your work in the space below.
How would you state what you did in YOUR OWN words?

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Additional Activity:

**TASK 1: Two Step Sum (Unlike Denominator)**

2. On the upper-left side, Click on grade 5.
3. On the upper-left side, click on Manipulative and choose Fraction Tiles.
4. Represent the following problem on the screen by using the arrow keys:
   \[
   \frac{2}{5} + \frac{1}{2}
   \]
5. Have them find the common denominator for 2/5 and 1/2. Write the appropriate numerator and denominator values for the renamed fractions. Renamed fractions are equivalent. Can you state a rule for renaming?
6. Have them write the addition equation for this process, as shown in the below screen.
7. Have students know that the above example illustrates getting common denominators for the fractions before adding.
8. Use the low bar and click on Text Tool icon to enter the name of the equivalent fraction into the boxes and write the final answer. Also, you may click on the Pen Tool icon to write the answer on the screen.

9. Record your answer below. Then, try more 5 problems.

10. Click on CLEAR OBJECTS icon to have a new screen.

Keep a record of your work in the space below.

How would you state what you did in YOUR OWN words?

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APPENDIX O: TASKSHEETS FOR ADDING FRACTIONS’ INSTRUCTIONS IN ARABIC
لأستخدم الإسهم أو القارورة لحساب ودمج الكسور على الشاشة.

1. يتعلم الطلاب بأن لدينا طريقة لإعادة نسبية الكسور. ووضوح الشارع بينها الطريقة الأولى أن يستخدم الطلاب أن الكسور المشتركة لها نفس المديره كما هو مبين في الشكل التالى.

2. الطريقة الثانية لشرح عملية نسبية الكسور هي من طريق تقسيم الکسر إلى عدة أجزاء مشتركة من خلال استخدام "القيم" الموجود في شكل الشاشة، ثم وضح قيمة كل جزء كما هو مبين في الشكل التالى.

3. لا يحتاج الطلاب المبتعد الأسفل في الشاشة من أجل تسجيل إجابته وذلك من خلال استخدام القيم أو الضغط على أيقونة "تسجيل النتيجة" ثم استخدام لوجة النتائج للإجابة الناجحة الذي توصل إليه.

4. إذا الطالب في تنفيذ مشالأخرى نفس المذهب.

5. ضغط على أيقونة "توضيح الشاشة" للحصول على شاشة جديدة، كما هو مبين في الموردة.
COMPARISON OF PHYSICAL/VIRTUAL MANIPULATIVES

1. How can students use digital manipulatives effectively in this way?

2. What tool is used to create digital manipulatives?

3. Can you describe the advantages and disadvantages of using digital manipulatives compared to physical ones?
نشأة إضافي:
الجزء الثاني: جميع الأساليب ذات النظام الموجه

خطوات العمل:
1. اضغط على الزر الأزرق للدخول إلى الدراسة الإلكترونية للترانسلتات الدراسية.
2. في أعلى الزاوية السفلى اضغط على "الخيار الصافي" ثم اختر الصنف "السابق" من القائمة كما هو موجود في الصورة التالية.
3. اضغط على كلمة "الدردشة" في الجانب الأيسر ثم اختر "الترانسلتات الدراسية" من القائمة كما هو موضح في الصورة التالية.
1. كما هو مبين في الشكل أعلاه، تخبر التلاميذ أن بإمكانهم كلاهما إيجاد الكسور المكافئة للنتائج النهائية.

لاستخدم الشريط الشريطي الأول في الشريحة من أجل تسجيل إجاباته وذلك من خلال استخدام القلم أو الضبط على أيقونة "تسجيل النص" ثم استخدام موجة النص على الكتابة الناتجة التي توصل إليها.

ابدأ طلبتن في تطبيق خمس متطلبات أخرى لنص النموذج.

استخدم على أيقونة "تخطيط الناتج" للحصول على ناتج جديد، كما هو مبين في الصورة.
الكؤوس - نزلة رقم (3) للدورة الإلكترونية
التاريخ:
الموضوع: جمع الكؤوس ذات النظام المعطى
خطوات العمل:
1. اضغط على الرابط التالي للدخول إلى الدورة الإلكترونية لانترنت الكؤوس
2. في الحل الراوي المبرد اضغط على "اختيار الصف" ثم اختر الصف "الحادى" من القاعدة كما هو موضوع في الصورة الثانية.
3. اضغط على كلمة "الإجابة" في الحل الأيسر، ثم اختر "الرابط الكؤوس" من القائمة كما هو موضوع في الصورة الثانية.
COMPARISON OF PHYSICAL/VIRTUAL MANIPULATIVES

1. Using the numbers or the calculator to divide the fractions by 1/2 into the equation:

\[
\frac{1}{2} + \frac{1}{4}
\]

2. The problem is to find the common denominator between the two numbers. Then, add the fractions together.

\[
\frac{1}{2} + \frac{1}{4} = \frac{2}{4} + \frac{1}{4} = \frac{3}{4}
\]

3. As shown in the figure above, the answer is clear that the sum of the two fractions is 3/4, which is the answer to the question.
COMPARISON OF PHYSICAL/VIRTUAL MANIPULATIVES

14. Explain the process of teaching children to use the computer in this section.

15. Ask the students to explain the procedure of teaching the topic they learned in this lesson.
ال kurs - ورقة عمل رقم (2) لليوبيئة الإلكترونية	

المادة

الموضوع: جمع الكسور ذات القيمة المكافئة

خطوات العمل:

1. اضغط على الرابط التالي للدخول إلى اليوبيئة الإلكترونية لـ "الكيركس:

2. في أعلى الزاوية اليمنى اضغط على "اختيار الصف"، ثم اختار الصف "الخانات" من القائمة. كما هو موضح في الصورة التالية.

3. اضغط على كلمة "اليوبيئة" في الجانب الأيسر، ثم اختار "العقرات الكيركس" من القائمة. كما هو موضح في الصورة التالية.
1. كما هو مبين في الشكل أعلاه، نُشر القالب أن عليهم إيجاد الكسور المتكافئة لكل كسر من الكسور المعطاة في الساحة ثم جمعها والحصول على النتيجة النهائية.

2. يستخدم الشريط الأيسر في الشاشة من أجل تسجيل الإجابات، وذلك من خلال استخدام القلم أو الضبطة على أبوقة "الحل النسبي" ثم استخدام لوحة المفاتيح لنجمة النتيجة الذي توصل إليه.

3. بدأ الطالب في تطبيق خمس مسائل أخرى لنفس الطريقة.

4. ضغط على أبوقة "_trim الشاشة" للحصول على نسخة جديدة، كما هو مبين في الصورة.
1- يسجل الطالب إجاباته في الجملة المنفصلة لذلك في هذه الورقة.

11. قم بالتدريس: أشرح بطريقة تجعل الطالب دام النظر المطلق التي تعلمتها في هذا الدرس؟
تشcio إمسك

التميض مجمع الكورن ذات لحلف النظام

خطوات العمل:

1. اضغط على الرابط التالي للدخول إلى اليدوية الإلكترونية لـ "نموذج الكورن". http://www.glencoe.com/sites/common_assets/mathematics/ebook_assets/vm/ F-interface.html
2. في أعلى الزاوية العلوى أضغط على "الخيار الصف" ثم اختيار الصف "الحاسم" من القائمة كما هو موضح في الصورة التالية.
3. انضغط على كلمة "اليدوية" في الجانب الأيسر، ثم اختيار "الرباط الكورن" من القائمة كما هو موضح في الصورة التالية.
1. كما هو مبين في الشكل أعلاه، تخبر الطلاب أن عليهم إيجاد الكسور المكافئة لكل كسر من الكسور المطولة في المسألة ثم جمعها والعثور على النتيجة النهائية.

2. يستخدم الطلاب النافذة الشريط الأفقي في الشاشة من أجل تسجيل إجاباته وذلك من خلال استخدام القلم أو الضغط على أيقونة "تسجيل النص" ثم استخدام لوحة المفاتيح لإدخال النتيجة التي توصل إليها.

3. يبدأ الطالب في تطبيق نفس مسالة أخرى نفس المفهوم.

4. ضعف على أيقونة "تطبيق الشاشة" للحصول على شاشة جديدة، كما هو مبين في السورة.
COMPARISON OF PHYSICAL/VIRTUAL MANIPULATIVES

10. اعمل الطالب إجاباته في الخلاصة المخصصة لذلك في هذه الورقة.

11. كم الإجابة: أشرط حفرة جمع الكلام ذات القيم المتلفة التي تمثلها في هذا الدور؟
APPENDIX P: THE UNIT OF FRACTIONS IN THE BOOK OF MATHEMATICS FOR FIFTH GRADE IN ARABIC
<table>
<thead>
<tr>
<th>الاسم</th>
<th>العمر</th>
<th>الجنس</th>
<th>-corner</th>
<th>ملاحظات</th>
</tr>
</thead>
<tbody>
<tr>
<td>محمد</td>
<td>15</td>
<td>ذكر</td>
<td></td>
<td></td>
</tr>
<tr>
<td>تيم</td>
<td>16</td>
<td>أنثى</td>
<td></td>
<td></td>
</tr>
<tr>
<td>علي</td>
<td>14</td>
<td>ذكر</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

الملاحظات:
1. محمد قريب من العمر من تيم.
2. علي هو أصغر من جميع اللاعبين.
3. تيم ش(limit) عن مهام الفريق من قبل المدرب.

التعامل مع الأعداد:

الآن، سنقوم بتقسيم الأعداد إلى عدة كتلات. كل كتلة ترمز إلى قيمة معينة من الأعداد.

<table>
<thead>
<tr>
<th>الاسم</th>
<th>العمر</th>
<th>الجنس</th>
<th>كتل</th>
<th>ملاحظات</th>
</tr>
</thead>
<tbody>
<tr>
<td>محمد</td>
<td>15</td>
<td>ذكر</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>تيم</td>
<td>16</td>
<td>أنثى</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>علي</td>
<td>14</td>
<td>ذكر</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

ملاحظات:
1. محمد يحتاج إلى تدريب إضافي في تقسيم الأعداد.
2. تيم يستخدم تقسيم الأعداد بشكل جيد.
3. علي يعمل على تحسين مهاراته في تقسيم الأعداد.

الخاتمة:

التدريب على تقسيم الأعداد هو جزء مهم من تطوير مهارات الرياضيات. كل الأبطال يحتاجون إلى تدريب إضافي لتحسين مهاراتهم في هذا المجال.

المراجعات:

1. محمد يستفيد من التدريب في تقسيم الأعداد.
2. تيم يستمر في استخدام تقسيم الأعداد بشكل جيد.
3. علي يستمر في تحسين مهاراته في تقسيم الأعداد.

القيود:

- محمد يحتاج إلى تدريب إضافي في تقسيم الأعداد.
- تيم يستمر في استخدام تقسيم الأعداد بشكل جيد.
- علي يستمر في تحسين مهاراته في تقسيم الأعداد.

النهاية:

التدريب على تقسيم الأعداد هو جزء مهم من تطوير مهارات الرياضيات. كل الأبطال يحتاجون إلى تدريب إضافي لتحسين مهاراتهم في هذا المجال.

المراجعات:

1. محمد يستفيد من التدريب في تقسيم الأعداد.
2. تيم يستمر في استخدام تقسيم الأعداد بشكل جيد.
3. علي يستمر في تحسين مهاراته في تقسيم الأعداد.

القيود:

- محمد يحتاج إلى تدريب إضافي في تقسيم الأعداد.
- تيم يستمر في استخدام تقسيم الأعداد بشكل جيد.
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المراجعات:

1. محمد يستفيد من التدريب في تقسيم الأعداد.
2. تيم يستمر في استخدام تقسيم الأعداد بشكل جيد.
3. علي يستمر في تحسين مهاراته في تقسيم الأعداد.

القيود:

- محمد يحتاج إلى تدريب إضافي في تقسيم الأعداد.
- تيم يستمر في استخدام تقسيم الأعداد بشكل جيد.
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المراجعات:

1. محمد يستفيد من التدريس إضافي في تقسيم الأعداد.
2. تيم يستمر في استخدام تقسيم الأعداد بشكل جيد.
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1. محمد يستفيد من التدريس إضافي في تقسيم الأعداد.
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المراجعات:

1. محمد يستفيد من التدريس إضافي في تقسيم الأعداد.
2. تيم يستمر في استخدام تقسيم الأعداد بشكل جيد.
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مقارنة الكسور الاعتيادية

استعد

الله أكبر يامري على طلاب الصف الخامس أن ناظر الطلاب بحوزة عطرة الحناء، وأول الطلاب بحوزة عطرة النفاذ، وأول الطلاب بحوزة عطرة البيض.

ما نوع من العطرة تwięksده مستوى الطلاب؟

بمثابك المدارنة بين الكسور باستخدام السجائر والنمط. إذا كان للكسور المدارنة تقدير، فتحدث بين السجائر، وإذا اختفت مغامرات الكسور، فتأكد الكسور مكتوبة لها تكون مقارنتها مساوية.

الفصل المعقدة الكسورين أو أكثر هو عدد من مساعدي تقديرات تلك الكسور، استعمل العوامل المدارنة الأصغر أو المضاعفات المدارنة الأصغر للمقارنات، ولكن تقارن بين الكسور مع بعض النشاط.

مقارنة الكسور

أولاً: بين 

القطر بين 

حيث استخدام السجائر والنمط المدارنة الأصغر.

بين المشكل 

الحلحلة 1: أوجد م. الكسر المماثل، م. م: 6، م. 6 هو 20

الحلحلة 2: أوجد كسره مكافئه مقامهما 

الحلحلة 3: بنا أن 

الموير: ٥٢٥٨، فإن: 

الموئر: ١٥٢، فإن: 

الموئر: ٣٤، فإن: 

الموير: ٩٠٠٠، فإن:
في السؤال 1، السهم المشترك الأصلي لدي المعلم الكسنريين \( \frac{1}{2} \) موم. يمكن استخدام هذه النماذج لتعليم الكسنريين طرقاً مختلفة من خلال توضيح تفاعلاهم بطريقة تضحية في بعضها، لكن لا يعطي النماذج المشترك الأصلي في جميع الحالات.

مقارنة الأعداد باستخدام (3.13):

1. قارن بين \( \frac{1}{3} \) باستخدام السهم المشترك الأصلي.
   - تفسير:\( \frac{1}{3} \) هو أصغر من \( \frac{1}{2} \).

2. أوجد "م\( \frac{1}{3} \) " من المعلم.
   - تفسير:\( \frac{1}{3} \) من المعلم يساوي \( \frac{1}{3} \) من المعلم.

3. أوجد السهم المشترك الأصلي باستخدام نماذج الكسرين.
   - تفسير:\( \frac{1}{3} \) يساوي \( \frac{1}{3} \) من الكسرين في المعلم.

4. أوجد السهم المشترك الأصلي باستخدام نماذج الكسرين.
   - تفسير:\( \frac{1}{3} \) يساوي \( \frac{1}{3} \) من الكسرين في المعلم.

5. أوجد السهم المشترك الأصلي باستخدام نماذج الكسرين.
   - تفسير:\( \frac{1}{3} \) يساوي \( \frac{1}{3} \) من الكسرين في المعلم.

6. أوجد السهم المشترك الأصلي باستخدام نماذج الكسرين.
   - تفسير:\( \frac{1}{3} \) يساوي \( \frac{1}{3} \) من الكسرين في المعلم.

7. أوجد السهم المشترك الأصلي باستخدام نماذج الكسرين.
   - تفسير:\( \frac{1}{3} \) يساوي \( \frac{1}{3} \) من الكسرين في المعلم.

8. أوجد السهم المشترك الأصلي باستخدام نماذج الكسرين.
   - تفسير:\( \frac{1}{3} \) يساوي \( \frac{1}{3} \) من الكسرين في المعلم.

إذاً، النتائج أُعيدت استنادًا إلى النماذج الأصلية.
قارن بين كلّ كسريين معاً باستعمال العمليات الحسابية أو استخدام النماذج أو أعمدة الخطابين الأصفر:

ارجع إلى قاعدة النماذج.

الجöz: قارن بين المغذيين في كلّ منهما معاً باستعمال العمليات الحسابية أو أعمدة الخطابين الأصفر و أعمدة الخطابين الأحمر.  

بسم الله الرحمن الرحيم

التدريب: قارن بين كلّ كسريين معاً باستعمال عمليات الأعداد، أو الأعمدة الأحمر والأصفر:

الجöz: قارن بين المغذيين في كلّ منهما معاً باستعمال عمليات الأعداد، أو أعمدة الخطابين الأصفر و أعمدة الخطابين الأحمر.  

لمحة: قارن بين المغذيين في كلّ منهما معاً باستعمال العمليات الحسابية أو أعمدة الخطابين الأصفر و أعمدة الخطابين الأحمر و أعمدة الخطابين الأصفر.

التدريب: قارن بين كسريين معاً باستعمال عمليات الأعداد أو أعمدة الخطابين الأصفر و أعمدة الخطابين الأحمر و أعمدة الخطابين الأصفر.
جمع الكسور المتشابهة

يمكنك استخدام طريقة جمع الكسور لجمع وطرح كسور العددات للدالة، والكسر أبلى إذا كانت عليه نفس الكسور متشابهة. حسبًا الكسرًا 

استخدام ثلاث أعظم لكميات لا يمكن تمتلك الكسر 

الخضوعة 1: أصل سرودفة للكرس 

الخضوعة 2: أصل سرودفة للكرس 

الخضوعة 3: أصح 

أصل أصل الفنجر أبلى استعمالها من مساحة الكسر 

1/4
COMPARISON OF PHYSICAL/VIRTUAL MANIPULATIVES

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استكشاف

جمع الكسور غير المتشاربة

باعتبار فائدة الكسور المتشاربة من الكسور التي لها المتشار، فشلت انا الكسور التي تكون فيها متشار، فسوز الكسور غير المتشاربة.

<table>
<thead>
<tr>
<th>الكسور المتشاربة</th>
<th>الكسور غير المتشاربة</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{3}$</td>
</tr>
<tr>
<td>$\frac{1}{4}$</td>
<td>$\frac{1}{5}$</td>
</tr>
</tbody>
</table>

ويمكن استخدام نماذج الكسور لجمع الكسور غير المتشاربة.

خطة العمل

1. استخدم نماذج الكسور من الحلقات لإعداد صنع قطع كبيرة، إذا كان طول عدد القطع معقداً، طول القطع الأخر لا Entre، نما الطول الكلي للمزوحيين؟

الخطوة 1: أصل النمودج لكل كسر، وضع النمودج على جنب

الخطوة 2: أرفع نموذج إلى الطول النمودج أعلاه، وضع أسفل

الخطوة 3: جمع

لا حاجة لاستخدام خصائص أخرى لنمودج الكسور.

إذا $\frac{1}{3} = \frac{1}{4} \times \frac{4}{4}$.

التقاطع النمودج النهائي الموثوق بالنسبة.$\frac{4}{4}$.
COMPARISON OF PHYSICAL/VIRTUAL MANIPULATIVES

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جمع الكسور غير المشابهة

استعد
المستوى 3-9

شغب الكسور غير المشابهة

جمع الكسور غير المشابهة

جمع الكسور غير المشابهة

جمع الكسور غير المشابهة
Appendix Q: The Unit of Fractions in the Book of Mathematics for Fifth Grade in English

**Equivalent Fractions**

**Objective:** Students should be able to write equivalent fractions to other fractions.

**Warm-up:** Joseph divided his garden into three parts. The width of his garden was 9 meters, and he decided to put a part that its width was 3 meters to plant tomatoes. Was his decision true?

Equivalent fractions are the fractions that have equal ratios. The fractions $\frac{1}{3}$ and $\frac{3}{9}$ are represented the same part of one whole. Thus, they are equivalent fractions. Therefore, Joseph’s decision was true because when we multiply 3 with the numerator and denominator of $\frac{1}{3}$, we would have the fraction of $\frac{3}{9}$.

$\frac{1}{3} \times 3\times 3 = \frac{3\times 3}{3} = \frac{3}{9}$

Just remember that the fraction of $\frac{3}{9}$ is an equivalent form of numerator 1 and multiply in number 1 does not change the value of fraction. Thus, to find equivalent fractions for any fraction, you multiply the fraction in an equivalent form of number 1 such as $\frac{2}{2}, \frac{3}{3}, \frac{4}{4}$, and so on.

**Example:** Find two equivalent fractions for $\frac{1}{4}$.

You have to multiply $\frac{1}{4}$ in equivalent forms of number 1 such as $\frac{2}{2}$ and $\frac{3}{3}$.

$\frac{1}{4} = \frac{2\times 2}{2\times 2} = \frac{2}{8}$

$\frac{1}{4} = \frac{3\times 3}{3\times 3} = \frac{3}{12}$

Thus, $\frac{2}{8}$ and $\frac{3}{12}$ are equivalent fractions for $\frac{1}{4}$.
Example from real life: Hazel measured the length of her pillow and found it is 3/5 meter. Find two equivalent fractions for the length of her pillow.

First, multiply 3/5 in equivalent terms of number 1 such as 2/2 and 3/3.

\[ \frac{3}{5} \times \frac{2}{2} = \frac{3 \times 2}{5 \times 2} = \frac{6}{10} \]
\[ \frac{3}{5} \times \frac{3}{3} = \frac{3 \times 3}{5 \times 3} = \frac{9}{15} \]

Thus, 6/10 and 9/15 are equivalent fractions for the length of her pillow.

Example: Find the appropriate number instead of X symbol that makes both fractions 2/7 = 3/X to be equivalent.

\[ \frac{2}{7} = \frac{3}{X} \]
\[ 2 \times X = 3 \times 7 \]

Looking for the number that when you multiply it with 7, you get 21.
\[ 2 \times 7 = 2 \times 3 = 6 \times 3 \]

The missing number is 6. Thus, \( \frac{2}{7} = \frac{3}{6} \)

Question #1: Find two equivalent fractions for the below fractions and explain your answer by drawing pictures.

\[ \frac{1}{5} \quad \frac{3}{4} \quad \frac{6}{10} \]

\[ \frac{1}{5} \quad \frac{3}{8} \quad \frac{5}{6} \]

Question #2: Find the appropriate number instead of X symbol in order to make both fractions become equivalent.

\[ \frac{1}{2} = \frac{X}{4} \quad \frac{2}{3} = \frac{10}{X} \quad \frac{4}{18} = \frac{12}{X} \]

Question #3: Explain how to find an equivalent fraction for \( \frac{4}{9} \)?
Simplify Fractions

Objective: Students should be able to write a fraction in its simplest form.

Warm-up: The length of chosen box is 12 cm while the width of the box is 22 cm. This means that the length of chosen box equals 12/22 of the width. Is the fraction of 12/22 in its lowest form?

The fraction comes in its simplest or lowest form when the Greatest Common Divisor (GCD) for numerator and denominator is number 1 and the simplest form of fraction is the same from different equivalent fractions.

Example: Based on the warm-up example, what is the fraction that represents the length of chosen box to its width? Write down the fraction in its lowest form.

First step: Determine the GCD for numerator and denominator.

GCD of 12 is 1, 2, 3, 4, 6, 12

GCD of 22 is 1, 2, 11, 22

The GCD for 12 and 22 is number 2.

Second step: the numerator and denominator are divided by the GCD. Remember, when you divide the numerator and denominator by the same number, it equals to dividing by number 1.

Thus, the form of fraction is changed while a value of fraction is still the same.

\[
\frac{12}{22} = \frac{12 \div 2}{22 \div 2} = \frac{6}{11} \quad \text{(GCD for 6 and 11 is 1)}
\]

Thus: 12/22 = 6/11
Example: Write $\frac{18}{30}$ in its lowest term.

First method: divide the numerator and denominator by the same number.

$18 \div 18 = \frac{2}{30} = \frac{2}{9} \div 3$

$9 \div 15 = \frac{9}{3} \div 15 = \frac{3}{5}$

The numbers 7 and 3 don't have any common factors instead of number 1. Thus, we will stop dividing by any number here.

Second method: divide by GCD.

GCD of 18: 1, 2, 3, 6, 9, 18
GCD of 30: 1, 2, 3, 5, 10, 15, 30

The GCD for 18 and 30 is 6.

$18 \div 18 = \frac{6}{30} = 6 \div 5$

Notice: the simplest form of $\frac{18}{30}$ is $\frac{3}{5}$ whether we use the first method or the second one.

\[ \frac{18}{30} \quad \frac{3}{5} \]

Question 1: Write each fraction in its simplest or lowest term. If the fraction is in its simplest form, just write that, "Fraction is in its simplest term".

\[ \frac{4}{6} \quad \frac{2}{12} \quad \frac{4}{24} \quad \frac{8}{16} \]

Question 2: Reehim bought 24 cakes and 10 of them were with chocolate flavor. What is the fraction that represents chocolate cakes? Write the fraction in its simplest term.

Question 3: Write 8.8 as a fraction and then write it in simplest term.

Question 4: Explain in two sentences how you simplify fractions in its lowest form.
Comparing Fractions

Objective: Students are able to compare between fractions by using common denominators.

Warm-up: A survey presented that

\[
\begin{array}{c|c|c|c|c|c|c|c|c|c|c}
& \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\
\end{array}
\]

\[
\begin{array}{c|c|c|c|c|c|c|c|c|c|c}
\text{of fifth grade students liked cheese pie,} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\
\text{of students liked apple pie, and} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\
\text{of students liked pepper pie.} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\
\end{array}
\]

Which kind of pies did most students prefer?

Students can compare between fractions by drawing or pattern. If the fractions have the same denominators, you just compare the numerators. However, if the denominators are different, you have to find equivalent fractions for getting equal denominators.

The similar denominators of two or more fractions are called common denominators. The Least Common Denominator (LCD) is the least common multiple of the denominators of a set of fractions that is used to compare between them.

Example: Compare between 3/5 and 1/2 by using pattern and LCD.

By using pattern:

\[
\begin{array}{c|c|c|c|c|c|c|c|c|c|c}
\end{array}
\]

You can see that 3/5 > 1/2

By using LCD:

First, you have to find the LCD for both denominators 5 and 2 and it is 10.

Second, you have to find equivalent fractions that their denominators are 10.

\[
\begin{array}{c|c|c|c|c|c|c|c|c|c|c}
\end{array}
\]

Then, because 6 > 5, thus, \(6/10 > 5/10\)

Therefore, \(3/5 > 1/2\)
Notice: In the previous example, the LCD of the denominators of both fractions \( \frac{3}{7} \) and \( \frac{4}{7} \) is \( 7 \).

This could be found by multiplying both denominators together in order to get a common denominator for both fractions. However, this way does not always give you the LCD.

**Example:** Compare between \( \frac{5}{6} \) and \( \frac{7}{9} \) by using the LCD.

First, you have to find the LCD for the fractions. The LCD for 6 and 9 is 18. Note, the product of multiplying 6 and 9 is 54. This is the common denominator, but it is not the LCD.

Second, you have to find equivalent fractions that have denominators as 18.

\[
\frac{5}{6} = \frac{15}{18} \quad \quad \quad \frac{7}{9} = \frac{14}{18}
\]

Third, since 15 > 14, then, \( \frac{15}{18} > \frac{14}{18} \)

Thus, \( \frac{5}{6} > \frac{7}{9} \)

**Example:** Royal made two out of three saves that his team did while Sanders made 5 out of six saves that his team did. Which one of them has scored high percentage of goals for his team?

The picture shows that:

\[
\begin{array}{ccccccc}
& & & & & & \\
2/3 & 5/6 & 1/6 & 1/6 & 1/6 & 1/6 & 1/6 \\
\end{array}
\]

By using the LCD:

First, you have to find the LCD for the fractions. The LCD for 3 and 6 is 6.

Second, you have to find equivalent fractions that have denominators as 6.

\[
\frac{2}{3} = \frac{4}{6} \quad \quad \quad \frac{5}{6} = \frac{5}{6}
\]

Third, since 4 < 5, then, \( \frac{4}{6} < \frac{5}{6} \)

Thus, \( \frac{2}{3} < \frac{5}{6} \)

This means that Sanders has scored the high percentage of goals.
Question #1: Compare between each two fractions by using the pattern or LCD.

\[
\begin{align*}
\frac{1}{2} & \text{ and } \frac{1}{3} \\
\frac{1}{4} & \text{ and } \frac{3}{8} \\
\frac{1}{6} & \text{ and } \frac{1}{9} \\
\frac{1}{8} & \text{ and } \frac{1}{16}
\end{align*}
\]

Question #2: Compare between fractions by using (<, >, =)

\[
\begin{align*}
\frac{1}{2} & \text{ and } \frac{1}{3} \\
\frac{1}{4} & \text{ and } \frac{1}{8} \\
\frac{1}{8} & \text{ and } \frac{1}{12}
\end{align*}
\]

Question #3: If you want to make a cake, you need \( \frac{1}{3} \) cup of sugar and \( \frac{2}{3} \) cup of flour. Which one is more?

Question #4: Explain the relation between the Least Common Multiple and Least Common Denominator?

Additional activities:

Question #1: Compare between each two fractions by using the pattern or LCD.

\[
\begin{align*}
\frac{2}{3} & \text{ and } \frac{3}{4} \\
\frac{3}{5} & \text{ and } \frac{5}{11} \\
\frac{1}{5} & \text{ and } \frac{5}{12}
\end{align*}
\]

Question #2: Compare between fractions by using (<, >, =).

\[
\begin{align*}
\frac{2}{9} & \text{ and } \frac{3}{10} \\
\frac{3}{6} & \text{ and } \frac{4}{12} \\
\frac{4}{9} & \text{ and } \frac{5}{16}
\end{align*}
\]
Adding Fractions with Like Denominators

Objective: Students are able to add two fractions that have the same denominators.

Warm-up: Every one can see patterns to add fractions that have the same denominators. It is called fraction with like denominators. For instance, 1/5 and 3/5 are fractions with like denominators because both have the same denominator that is 5.

Activity: Manita cut an apple to several slices. We ate 3/5 of the apple and gave her some 1/5 of the apple. How many did both eat?

First, make a pattern of 3/5 by using three parts of 1/5 in order to represent 3/5.

Second, make a pattern of 1/5 by adding one part of 1/5 to the previous pattern.

Third, add both parts and get the result.

Because 3/5 + 1/5 equals 4/5, then, Manita and her friend ate 4/5 of apple together.

To add fractions with like denominators, you just add the numerators together and keep the denominator the same.
Example: This table explains how much Sami read everyday. What is the fraction that represents the days of Saturday and Monday altogether?

<table>
<thead>
<tr>
<th>Day</th>
<th>Fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Saturday</td>
<td>1/2</td>
</tr>
<tr>
<td>Sunday</td>
<td>1/3</td>
</tr>
<tr>
<td>Monday</td>
<td>1/6</td>
</tr>
<tr>
<td>Tuesday</td>
<td>2/3</td>
</tr>
</tbody>
</table>

First, add 1/2 and 1/3

\[ \frac{1}{2} + \frac{1}{3} = \frac{4}{6} \]

Thus, simplify the fraction 4/6

\[ 4 = \frac{2}{3} = 3 \frac{1}{3} \]

Thus, Sami read 2 1/3 in Saturday and Monday altogether.

Example: Find the sum of 2/5 + 4/5 and then explain your answer by using the pattern.

\[ 2/5 + 4/5 = 2/10 = 2/6 \]

(add numerators only and keep the denominators the same)

\[ = 6/15 \]

\[ = 1/5 \]

Thus, 2/5 + 4/5 = 1 1/5
Question #1: Find the sum of two fractions in its simplest form and then explain your answer by using the patterns.

\[
\begin{align*}
\frac{1}{7} + \frac{3}{7} &= 2/7 \\
\frac{1}{8} + \frac{3}{4} &= 5/8 \\
\frac{3}{5} + \frac{5}{8} &= 23/40 \\
\end{align*}
\]

Question #2: Salith has painted \( \frac{5}{12} \) of the garden fence and Mohammad has painted \( \frac{4}{12} \) of the same fence. What is the fraction that represents the part that has been painted?

Question #3: In two sentences, explain how you answered the previous question.

Additional activities:

Question #1: Find the sum of two fractions in its simplest form and then explain your answer by using the patterns.

\[
\begin{align*}
\frac{4}{7} + \frac{2}{7} &= 6/7 \\
\frac{2}{8} + \frac{2}{8} &= 1 \\
\frac{3}{4} + \frac{1}{4} &= 1 \\
\frac{2}{5} + \frac{4}{5} &= 2 \\
\end{align*}
\]

Question #2: What is the total of two-fifths and one-fifth?

Question #3: What is the total of six-ninths and three-ninths?
Adding Fractions with Unlike Denominators

Objective: Students use the patterns to add two fractions with different denominators.

Warm-up: You have learned that fractions with like denominators are the fractions that have the same denominators. However, fractions with different denominators are called fractions with unlike denominators.

<table>
<thead>
<tr>
<th>Different Fractions</th>
<th>Similar Fractions</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/2, 1/3</td>
<td>3/4, 4/6</td>
</tr>
</tbody>
</table>

We can use patterns to add two fractions with unlike denominators.

Activity:
Carpenter has used two boards of wood to build a kennel. If the length of one board is 1/2 meter and the other one is 1/3 meter, what is the total length for both?

First step: Make a pattern for each fraction and put them next to each other.

Second step: Make a new pattern that is comparable to the length of the two patterns and put it below them.

Third step: Adding: 1/2 + 1/3 = 5/6
Thus, the length of both boards are 5/6 meter.
Example: Sarah bought $\frac{3}{4}$ kg of grapes and $\frac{5}{8}$ kg of cherry. What is the total weight of grapes and cherry together?

First step: make a pattern for each fraction and put them next to each other:

\[
\begin{array}{cccc}
\frac{1}{8} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\
\frac{1}{8} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\
\end{array}
\]

Second step: make a new pattern that is comparable to the length of the two patterns and put it below them:

\[
\begin{array}{cccc}
\frac{1}{8} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\
\frac{1}{8} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\
\end{array}
\]

Third step: adding:

\[
\frac{3}{4} + \frac{5}{8} = \frac{11}{8} = 1 \frac{3}{8}
\]

Thus, the total weight of grapes and cherry is $1 \frac{3}{8}$ kg.

Question: Use the patterns to find the sum of below fractions:

\[
\begin{array}{ccc}
\frac{2}{3} + \frac{1}{6} & \frac{3}{4} + \frac{1}{3} & \frac{3}{8} + \frac{1}{4} \\
\frac{1}{2} + \frac{3}{8} & \frac{1}{3} + \frac{3}{8} & \frac{3}{8} + \frac{1}{4} \\
\frac{5}{2} + \frac{1}{4} & \frac{3}{4} + \frac{3}{8} & \frac{4}{7} + \frac{2}{3}
\end{array}
\]
Adding Fractions with Unlike Denominators

Objective: Students use the pattern to add two fractions with different denominators.

Warm-up: Sally spent \( \frac{1}{3} \) hour to write an article about integrity and \( \frac{1}{4} \) hour to review her article. How much time did Sally spend to write and review her article?

Before adding two fractions with unlike denominators, we have to rewrite one or both fractions until they get on similar denominators.

Steps of adding two fractions with unlike denominators:
- Rewrite the fractions by using the LCD for them.
- Add the fractions as what you do when adding fractions with the same denominator.
- Simplify the result fraction in its simplest form.

Example: Answer the question that has mentioned in the warm-up.

First step: write the equation:

\[
\frac{1}{3} + \frac{1}{4}
\]

Second step: rewrite both fractions by using the LCD for both of them.

\[
\frac{1}{3} = \frac{4}{12} \\
\frac{1}{4} = \frac{3}{12}
\]

Third step: adding

\[
\frac{1}{3} + \frac{1}{4} = \frac{4}{12} + \frac{3}{12} = \frac{7}{12}
\]

Thus, Sally spent \( \frac{7}{12} \) hour to write and review her article.
Example: Nadia spent $\frac{1}{6}$ of her time in reading and $\frac{5}{12}$ of her time in cleaning her house. What is the fraction that represents the total time in which she spent in both reading and cleaning?

First step: write the equation

$$\frac{1}{6} + \frac{5}{12}$$

Second step: rewrite both fractions by using the LCD for both of them.

$$\frac{1}{6} = \frac{1 \times 2}{12} = \frac{2}{12}$$

$$\frac{5}{12}$$

Third step: adding

$$\frac{2}{12} + \frac{5}{12} = \frac{7}{12}$$

Thus, Nadia spent $\frac{7}{12}$ of her time in reading and cleaning her house.

**Question #1**: Find the sum of each one in its simplest form

$$\frac{1}{4} + \frac{1}{8}$$

$$\frac{2}{9} + \frac{1}{9}$$

$$\frac{2}{3} + \frac{2}{3}$$

$$\frac{5}{12} + \frac{5}{12}$$

$$\frac{5}{8} + \frac{3}{8}$$

**Question #2**: The farmer got $\frac{3}{8}$ of the wheat crop in Wednesday and got $\frac{1}{3}$ of the crop on Thursday. What is the fraction that represents the total of harvest?

**Question #3**: Explain the steps of adding $\frac{5}{12}$ and $\frac{5}{6}$ together. What is the total?
APPENDIX R: THE UNIT OF FRACTIONS IN THE ACTIVITY MATHEMATICS BOOK
FOR FIFTH GRADE IN ARABIC
مقارنة التكسور

ضع الإشارات المناسبة، لذا تكون جمعًا صحيحًا في كلّ مربع يأتي:

\[ \frac{1}{4} + \frac{1}{4} = \frac{1}{2} \]
\[ \frac{1}{6} + \frac{1}{6} = \frac{1}{3} \]

وقد قررت أن تجعل الإطار الأصغر في الفئة عند تحليل الإجراءات تتبعها بعناية بعض على المحاصلة ما طول الإطار الذي تستخدمه في الوسط؟

دروسية الفصل الدراسى

الكتب الأول محتاجات مصنوعة لكل مجموعة أعدام مشترك:

<table>
<thead>
<tr>
<th>العدد الأول</th>
<th>العدد الثاني</th>
<th>العدد الثالث</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

لوجد د.م.آ، الكلّ مجموعه أعدام مشتركاً يأتي:

<table>
<thead>
<tr>
<th>العدد الأول</th>
<th>العدد الثاني</th>
<th>العدد الثالث</th>
</tr>
</thead>
<tbody>
<tr>
<td>32</td>
<td>18</td>
<td>5</td>
</tr>
<tr>
<td>24</td>
<td>18</td>
<td>6</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>8</td>
</tr>
</tbody>
</table>

النقطة 18: التفسير والملاحظات
الفصل 9: جمع الكسور ومخرجها
جمع الكسور المتشابهة
أوجد نتائج الجمع في أبسط صوره:

\[ \frac{1}{4} + \frac{1}{4} = \frac{2}{4} \]

\[ \frac{1}{3} + \frac{1}{3} = \frac{2}{3} \]

\[ \frac{1}{5} + \frac{1}{5} = \frac{2}{5} \]

\[ \frac{2}{6} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2} \]

\[ \frac{3}{8} + \frac{1}{8} = \frac{4}{8} = \frac{1}{2} \]

\[ \frac{1}{10} + \frac{3}{10} = \frac{4}{10} = \frac{2}{5} \]

أحسب بالروتين لجمع الكسور:

\[ \frac{1}{4} + \frac{1}{2} = \frac{1}{4} + \frac{2}{4} = \frac{3}{4} \]

ما المسافة التي قطعتها الولدان لذا؟

كتبت الكنش في أبسط صوره:

\[ \frac{2}{6} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2} \]

\[ \frac{1}{3} + \frac{1}{3} = \frac{2}{3} \]

\[ \frac{3}{10} + \frac{2}{10} = \frac{5}{10} = \frac{1}{2} \]

\[ \frac{4}{8} + \frac{1}{8} = \frac{5}{8} \]

\[ \frac{3}{6} + \frac{1}{6} = \frac{4}{6} = \frac{2}{3} \]

\[ \frac{4}{10} + \frac{2}{10} = \frac{6}{10} = \frac{3}{5} \]

جمع الإشارة المناسبة: »» لتكون نتائج صحية في البداية!"
### جمع الكسور غير المتشابهة

| 
| --- |
| $\frac{1}{2} + \frac{1}{3}$ |
| $\frac{1}{4} + \frac{1}{5}$ |
| $\frac{2}{3} + \frac{1}{6}$ |
| $\frac{3}{4} + \frac{1}{8}$ |
| $\frac{1}{2} + \frac{1}{4}$ |
| $\frac{1}{3} + \frac{1}{6}$ |
| $\frac{2}{3} + \frac{1}{9}$ |
| $\frac{3}{4} + \frac{1}{12}$ |

### إخراج الجواب المتعمّق في أ 쉽ّة شورع

| 
| --- |
| $\frac{1}{2} - \frac{1}{3}$ |
| $\frac{1}{4} - \frac{1}{5}$ |
| $\frac{2}{3} - \frac{1}{6}$ |
| $\frac{3}{4} - \frac{1}{8}$ |
| $\frac{1}{2} - \frac{1}{4}$ |
| $\frac{1}{3} - \frac{1}{6}$ |
| $\frac{2}{3} - \frac{1}{9}$ |
| $\frac{3}{4} - \frac{1}{12}$ |

### خلّ الاشعة المبكرة

احتياج كرهمة إلى $\frac{1}{2}$ ساعةً لكي تنهي راجب الرياضياتي، واتبً $\frac{1}{3}$ ساعةً لكي تنهي راجب الرياضياتي. كم يوماً الوقت الذي قضاه الكريمة في خلّ راجب الرياضياتي على الوقت الذي قضاه في خلّ راجب الرياضياتي؟ حاول إجابتك في أ 쉽ّة شورع.

الفصل: 9، جمع الكسور وطرحها.
### Equivalent Fractions

**Question #1:** Find two equivalent fractions for each fraction below:

<table>
<thead>
<tr>
<th>Fraction</th>
<th>Equivalent Fraction 1</th>
<th>Equivalent Fraction 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{2}{4} )</td>
<td>( \frac{6}{12} )</td>
<td>( \frac{5}{10} )</td>
</tr>
<tr>
<td>( \frac{3}{12} )</td>
<td>( \frac{11}{33} )</td>
<td>( \frac{4}{8} )</td>
</tr>
<tr>
<td>( \frac{8}{28} )</td>
<td>( \frac{3}{9} )</td>
<td>( \frac{12}{16} )</td>
</tr>
<tr>
<td>( \frac{6}{24} )</td>
<td>( \frac{12}{20} )</td>
<td>( \frac{8}{16} )</td>
</tr>
</tbody>
</table>

**Question #2:** Find the right number instead of \( x \) so that the two fractions are equivalent:

<table>
<thead>
<tr>
<th>Fraction</th>
<th>Equivalent Fraction 1</th>
<th>Equivalent Fraction 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{4}{5} = \frac{x}{13} )</td>
<td>( \frac{14}{16} = \frac{x}{8} )</td>
<td>( \frac{3}{5} = \frac{12}{20} )</td>
</tr>
<tr>
<td>( \frac{2}{6} = \frac{x}{3} )</td>
<td>( \frac{x}{18} = \frac{3}{4} )</td>
<td>( \frac{3}{4} = \frac{6}{x} )</td>
</tr>
<tr>
<td>( \frac{14}{42} = \frac{x}{3} )</td>
<td>( \frac{3}{x} = \frac{15}{27} )</td>
<td>( \frac{9}{36} = \frac{6}{x} )</td>
</tr>
</tbody>
</table>

**Question #3:** Write “YES” if the two given fractions are equivalent OR write “NO” if they are not equivalent:

<table>
<thead>
<tr>
<th>Fraction 1</th>
<th>Fraction 2</th>
<th>Equivalent</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{3}{4} = \frac{6}{8} )</td>
<td>( \frac{3}{8} = \frac{7}{16} )</td>
<td>( \frac{5}{9} = \frac{15}{27} )</td>
</tr>
<tr>
<td>( \frac{2}{3} = \frac{4}{5} )</td>
<td>( \frac{3}{7} = \frac{13}{21} )</td>
<td>( \frac{10}{12} = \frac{7}{14} )</td>
</tr>
</tbody>
</table>

**Review the previous lesson:**

Determine whether the given numbers below are prime numbers or not:

<table>
<thead>
<tr>
<th>Number</th>
<th>Prime or Not</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>Prime</td>
</tr>
<tr>
<td>33</td>
<td>Prime</td>
</tr>
<tr>
<td>47</td>
<td>Prime</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Number</th>
<th>Prime or Not</th>
</tr>
</thead>
<tbody>
<tr>
<td>26</td>
<td>Prime</td>
</tr>
<tr>
<td>28</td>
<td>Prime</td>
</tr>
</tbody>
</table>
Simplify Fractions

Question #1: Find the right number instead of (x) in order to simplify the fraction to its simplest or lowest term.

\[ \frac{6}{12} = \frac{3}{x} \]

\[ \frac{10}{20} = \frac{x}{7} \]

\[ \frac{25}{50} = \frac{4}{x} \]

Question #2: Write each fraction in its simplest or lowest term. If the fraction is in its simplest term, just write that, "Fraction is in its simplest term".

\[ \frac{1/4}{1/4} \]

\[ \frac{12}{16} \]

\[ \frac{4}{13} \]

\[ \frac{15}{9} \]

\[ \frac{19}{22} \]

\[ \frac{210}{210} \]

Review the previous lesson:
Find two equivalent fractions for each below fraction:

\[ \frac{3/4}{1/5} \]

\[ \frac{6/8}{2/5} \]

\[ 1/2 \]
Comparing Fractions

Question #1: Use the proper symbol (<, >, =) below:

\[
\frac{1}{3} \quad \frac{1}{5} \\
\frac{7}{8} \quad \frac{7}{9} \\
\frac{9}{16} \quad \frac{9}{20} \\
\frac{3}{4} \quad \frac{7}{8}
\]

Question #2: Hana wants to put pictures in three different frames in which their length are \( \frac{1}{2} \) cm, \( \frac{1}{3} \) cm, and \( \frac{5}{6} \) cm. She decides to put the smallest frame in the middle when hang the frames on the wall next to each other. What is the length of the frame that will be in the middle?

Review the previous lesson

Write two equivalent fractions for the following fractions:

\[
\frac{24}{26} \\
\frac{16}{18} \\
\frac{17}{19} \\
\frac{24}{24}
\]
Adding Fractions with Like Denominators

Question #1: Find the sum of two fractions in its simplest form:

\[ \frac{2}{5} + \frac{3}{5} = \quad \frac{5}{9} + \frac{1}{9} = \]
\[ \frac{6}{8} + \frac{5}{8} = \quad \frac{3}{4} + \frac{2}{4} = \]
\[ \frac{9}{9} + \frac{1}{9} = \quad \frac{7}{8} + \frac{2}{8} = \]
\[ \frac{1}{2} + \frac{2}{2} = \quad \frac{4}{5} + \frac{3}{5} = \]
\[ \frac{12}{15} + \frac{3}{15} = \quad \frac{4}{7} + \frac{1}{7} = \]

Question #2: Jasmine ate \( \frac{3}{8} \) of a pie while Mona ate \( \frac{2}{8} \) of the same pie. How much did they eat from the pie? Write down the fraction in its simplest form.

Question #3: Salem walked \( \frac{4}{15} \) Kilometers while Khalid walked \( \frac{3}{15} \) Kilometers. How much was the distance that they walked together? Write down the fraction in its simplest form.

Review the previous lesson

Use the proper symbol (\( > \), \( < \), or \( = \)) for the following questions below:

\[ \frac{1}{4} \quad \frac{5}{8} \]
\[ \frac{1}{2} \quad \frac{5}{9} \]
\[ 3 \quad \frac{5}{9} \]
\[ \frac{2}{3} \quad \frac{4}{9} \]
\[ \frac{3}{5} \quad \frac{2}{7} \]
\[ \frac{7}{12} \quad \frac{4}{3} \]
Adding Fractions with Unlike Denominators

Question #1: Find the sum of two fractions in its simplest form:

\[
\begin{array}{c}
\frac{2}{3} + \frac{3}{5} = \_
\\
\frac{2}{7} + \frac{3}{14} = \_
\\
\frac{5}{12} + \frac{1}{4} = \_
\\
\frac{5}{8} + \frac{3}{8} = \_
\\
\frac{2}{9} + \frac{5}{9} = \_
\\
\frac{5}{10} + \frac{4}{10} = \_
\\
\frac{3}{5} + \frac{7}{10} = \_
\\
\frac{2}{3} + \frac{6}{14} = \_
\\
\frac{5}{4} + \frac{6}{13} = \_
\\
\frac{2}{3} + \frac{2}{9} = \_
\\
\frac{5}{7} + \frac{3}{7} = \_
\\
\frac{5}{6} + \frac{2}{3} = \_
\\
\frac{3}{10} + \frac{2}{3} = \_
\\
\frac{3}{10} + \frac{3}{4} = \_
\\
\end{array}
\]

Review the previous lesson:

1. Find the sum of two fractions in its simplest form:

\[
\begin{array}{c}
\frac{1}{7} + \frac{3}{7} = \_
\\
\frac{1}{6} + \frac{2}{6} = \_
\\
\end{array}
\]

2. Kartina spent 2/3 hour to finish her math assignment and 4/5 hour to finish her science assignment. How much time did she spend to finish both assignments? Write your answer in its simplest form.
APPENDIX T: PRETEST (A) FOR UNDERSTANDING OF ADDING FRACTIONS

Pretest (A)

Understanding of Adding Fractions
Understanding of Adding Fractions

1) For each picture below, write a fraction to show what part is gray:

   a. ________  b. ________  c. ________  d. ________

2) Determine whether the given fractions below are equivalent fractions. Explain your reasoning.

   \[
   \frac{8}{5} \quad \text{and} \quad 1\frac{4}{5}
   \]

3) Laura is having difficulties comparing \( \frac{5}{8} \) and \( \frac{4}{9} \) to determine the greater fraction of the two. Can you help her determine the greater fraction? Be sure to provide reasoning for your comparison.
4) Choose the correct answer below to show what part of this rectangle is gray:

![Rectangle with shaded parts]

- a. $\frac{2}{3} + \frac{2}{6}$
- b. $\frac{1}{9} + \frac{3}{9}$
- c. $5 + \frac{1}{9}$
- d. $5 \frac{4}{9}$

5) Add the two fractions:

- a. $\frac{4}{9} + \frac{2}{9}$
- b. $\frac{1}{5} + \frac{3}{6}$
6) a. Draw a picture to show whether this is true or not. Explain your reasoning.
\[
\frac{1}{2} + \frac{1}{6} = \frac{4}{6}
\]

b. Make a drawing to illustrate the sum of the two fractions below. Explain your reasoning.
\[
\frac{2}{5} + \frac{1}{3}
\]
APPENDIX U: POSTTEST (A) FOR UNDERSTANDING OF ADDING FRACTIONS

Posttest (A)

Understanding of Adding Fractions
Understanding of Adding Fractions

Code ____________________________
Age ______________________________
School ____________________________

1) For each picture below, write a fraction to show what part is gray:

![Fractions Diagram](image)

a. _______  b. _______  c. _______  d. _______

2) Determine whether the given fractions below are equivalent fractions. Explain your reasoning.

\( \frac{8}{5} \) and \( 1\frac{4}{5} \)

3) Laura is having difficulties comparing \( \frac{5}{8} \) and \( \frac{4}{9} \) to determine the greater fraction of the two. Can you help her determine the greater fraction? Be sure to provide reasoning for your comparison.
4) Choose the correct answer below to show what part of this rectangle is gray:

a. $\frac{2}{3} + \frac{2}{6}$  

b. $\frac{1}{9} + \frac{3}{9}$  

c. $5 + \frac{1}{9}$  

d. $5 \frac{4}{9}$

5) Add the two fractions:

a. $\frac{4}{9} + \frac{2}{9}$

b. $\frac{1}{5} + \frac{3}{6}$
6) a. Draw a picture to show whether this is true or not. **Explain your reasoning.**

\[
\frac{1}{2} + \frac{1}{6} = \frac{4}{6}
\]

b. Make a **drawing** to illustrate the sum of the two fractions below. **Explain your reasoning.**

\[
\frac{2}{5} + \frac{1}{3}
\]
APPENDIX V: POSTTEST (A) FOR UNDERSTANDING OF ADDING FRACTIONS IN ARABIC
لا) باستخدام الرسم التوضيحي: هل هذه العملية صحيحة أم لا (شرح إجابتك من فضلك)

أ) 

ب) باستخدام الرسم أو إحدى القواعد المعروفة (شرح إجابتك من فضلك)
APPENDIX W: POSTTEST (B) FOR UNDERSTANDING OF ADDING FRACTIONS

Posttest (B)

Understanding of Adding Fractions
Understanding of Adding Fractions

Code ______________________________
Age ______________________________
School _____________________________

1) For each picture below, write a fraction to show what part is gray:

![Fraction Pictures]

a. _______  b. _______  c. _______  d. _______

2) Determine whether the given fractions below are equivalent fractions. Explain your reasoning.
   \[
   \frac{7}{4} \quad \text{and} \quad \frac{8}{9}
   \]

3) John is having difficulties comparing \( \frac{2}{3} \) and \( \frac{4}{5} \) to determine the greater fraction of the two. Can you help her determine the greater fraction? Be sure to provide reasoning for your comparison.
4) Choose the correct answer below to show what part of this circle is gray:

![Circle with parts shaded]

a. \(\frac{2}{10} + \frac{2}{10}\)

b. \(\frac{1}{4} + \frac{3}{6}\)

c. \(6 + \frac{4}{10}\)

d. \(3 \frac{4}{10}\)

5) Add the two fractions:

a. \(\frac{3}{8} + \frac{5}{8}\)

b. \(\frac{2}{4} + \frac{2}{6}\)
6) a. Draw a picture to show whether this is true or not. Explain your reasoning.

\[
\frac{1}{3} + \frac{1}{6} = \frac{3}{6}
\]

b. Make a drawing to illustrate the sum of the two fractions below. Explain your reasoning.

\[
\frac{2}{4} + \frac{2}{3}
\]
APPENDIX X: POSTTEST (B) FOR UNDERSTANDING OF ADDING FRACTIONS
IN ARABIC
APPENDIX Y: ATTITUDE SURVEY PRIOR TO THE PRETEST

Code:…………………………

Date:…………………………

Attitude Survey on Pretest

Quick Write

1. Using your pretest, rate your level of comfort when explaining the concept of adding fractions to others by circling on that best describes you.

   1  2  3
   Not Comfortable  Somewhat Comfortable  Very Comfortable

2. In one or more paragraph, describe and explain why you feel that way.
APPENDIX Z: ATTITUDE SURVEY PRIOR TO THE PRETEST IN ARABIC

1) ما هو شعورك عندما تقوم بشرح عملية جمع الكسور للأخرين؟

2) تكتب سبب اختيارك للإجابة السابقة؟
APPENDIX A1: ATTITUDE SURVEY AFTER THE POSTTEST

Code:…………………………

Date:…………………………

Attitude Survey-Posttest
Quick Write

Read the question and choose the best answer.

1. Using the physical and virtual manipulatives “fraction bars”, did it improve your visual understanding of fractions?
   A. Yes                        B. Somewhat                        C. No

2. Was it helpful to use the physical and virtual manipulatives “fraction bars” when adding fractions?
   A. Yes                        B. Somewhat                        C. No

3. After using the physical and virtual manipulatives “fraction bars”, rate your level of comfort when explaining the concept of adding fractions to others.
   A. Comfortable               B. Somewhat Comfortable           C. Not Comfortable

4. In one or more paragraph, describe and explain your feelings toward understanding the concept of adding fractions.
APPENDIX B1: ATTITUDE SURVEY AFTER THE POSTTEST IN ARABIC
APPENDIX C1: PREFERENCES SURVEY

Preference Survey

Code: ........................................
School’s Name: ..............................

Read the statements and circle the tool that is more true of each statement.

<table>
<thead>
<tr>
<th>Statement</th>
<th>Virtual</th>
<th>Physical</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. In the future, I would like to use this tool more.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. Learning with this tool is a good way to spend math time.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. It is fun to figure out how this learning tool works.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. Using this tool becomes boring.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. Working with math problems using this tool is fun like solving a puzzle.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6. I wish I had more time to use these types of tools in math.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7. Learning using this tool is interesting.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8. I can stay on task easier by using this tool.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9. I would feel comfortable working with this learning tool.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10. This learning tool makes me feel uneasy and confused.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11. I can explain how to do math better using this tool.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12. This tool was easy to use.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>13. This tool helps me understand work with fractions.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>14. This tool helps me get the right answers.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>المتفرج</td>
<td>المسمار</td>
<td>الموسوعة</td>
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<td>---------</td>
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<td>---------</td>
</tr>
<tr>
<td>1. أحب أن استخدم هذه الأداة بشكل أكبر في المستقبل</td>
<td>7. تعتبر هذه الأداة رائعة جداً للممارسة الخاصة بالرياضيات</td>
<td></td>
</tr>
<tr>
<td>2. أعلم أن تعلم كيف استعمل هذه الأداة</td>
<td>8. ذكر الأداة التي عملت في استخدام هذه الأداة على نطاق واسع من خلال الأدوات الرياضية</td>
<td></td>
</tr>
<tr>
<td>3. استعمل هذه الأداة في عادةً</td>
<td>9. أعد في طريقة هذا الأداة في ممارسة الرياضيات</td>
<td></td>
</tr>
<tr>
<td>4. أشعر بالسعادة عند استخدام هذه الأداة من خلال الأدوات الرياضية</td>
<td>10. أعيد في طريقة هذا الأداة في ممارسة الرياضيات</td>
<td></td>
</tr>
<tr>
<td>5. استعمل هذه الأداة في عادةً</td>
<td>11. أشعر بالسعادة عند استخدام هذه الأداة</td>
<td></td>
</tr>
<tr>
<td>6. أشعر بالسعادة عند استخدام هذه الأداة من خلال الأدوات الرياضية</td>
<td>12. أعيد في طريقة هذا الأداة في ممارسة الرياضيات</td>
<td></td>
</tr>
<tr>
<td>7. تعتبر هذه الأداة رائعة جداً للممارسة الخاصة بالرياضيات</td>
<td>13. هذه الأداة سهلة الاستخدام</td>
<td></td>
</tr>
<tr>
<td>8. ذكر الأداة التي عملت في استخدام هذه الأداة على نطاق واسع من خلال الأدوات الرياضية</td>
<td>14. هذه الأداة سهلة الاستخدام</td>
<td></td>
</tr>
<tr>
<td>9. أعيد في طريقة هذا الأداة في ممارسة الرياضيات</td>
<td>15. هذه الأداة سهلة الاستخدام في هدف تعلم الأدوات</td>
<td></td>
</tr>
<tr>
<td>10. أشعر بالسعادة عند استخدام هذه الأداة من خلال الأدوات الرياضية</td>
<td>16. هذه الأداة سهلة الاستخدام في هدف تعلم الأدوات</td>
<td></td>
</tr>
<tr>
<td>11. أشعر بالسعادة عند استخدام هذه الأداة من خلال الأدوات الرياضية</td>
<td>17. هذه الأداة سهلة الاستخدام في هدف تعلم الأدوات</td>
<td></td>
</tr>
<tr>
<td>12. أعيد في طريقة هذا الأداة في ممارسة الرياضيات</td>
<td>18. هذه الأداة سهلة الاستخدام في هدف تعلم الأدوات</td>
<td></td>
</tr>
</tbody>
</table>
Institutional Review Board - Federalwide Assurance #00003152
University of Cincinnati

Date: 3/24/2016

From: UC IRB

To: Principal Investigator: Sami Alshehri
CECH Academic Affairs

Study ID: 2014-0126

Re: Study Title: The Comparison of Physical/Virtual Manipulatives on Fifth-Grade Students’ Understanding of Adding Fractions

The above referenced protocol and all applicable additional documentation provided to the IRB were reviewed and RE-APPROVED using an EXPEDITED review procedure set forth in 45 CFR 46.110(b)(1), Category(ies)(see below) on 3/24/2016.

Please note the following requirements:

Consent Requirements: Per 45 CFR 46.116 (21 CFR 50.20) the IRB has determined that informed consent must be obtained from all adult participants and that this consent must be documented by signature on the IRB approval consent form.

Parental Permission Requirements:
Per 45 CFR 46.408 the IRB has determined that at least _1_ parent(s) (or guardian) must give permission for the inclusion of a child in this research and that permission must be documented by signature on the IRB approved parental permission form.

Per 45 CFR 46.408 the IRB has determined that documented assent must also be obtained from all child participants between _10_ and _11_ years of age.

This study will be due for continuing review at least 30 days before 3/23/2017.