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I, Rumit Kumar, hereby submit this original work as part of the requirements for the degree of Master of Science in Aerospace Engineering.

It is entitled:
Position, Attitude, and Fault-Tolerant Control of Tilting-Rotor Quadcopter

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Position, Attitude, and Fault-Tolerant Control of Tilting-Rotor Quadcopter

A thesis submitted to the Graduate School of the University of Cincinnati in partial fulfillment of the requirements of the degree of

Master of Science

in the Department of Aerospace Engineering and Engineering Mechanics of the College of Engineering and Applied Sciences

by

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Committee Chair: M. Kumar, Ph.D.
The aim of this thesis is to present algorithms for autonomous control of tilt-rotor quadcopter UAV. In particular, this research work describes position, attitude and fault tolerant control in tilt-rotor quadcopter. Quadcopters are one of the most popular and reliable unmanned aerial systems because of the design simplicity, hovering capabilities and minimal operational cost. Numerous applications for quadcopters have been explored all over the world but very little work has been done to explore design enhancements and address the fault-tolerant capabilities of the quadcopters. The tilting rotor quadcopter is a structural advancement of traditional quadcopter and it provides additional actuated controls as the propeller motors are actuated for tilt which can be utilized to improve efficiency of the aerial vehicle during flight. The tilting rotor quadcopter design is accomplished by using an additional servo motor for each rotor that enables the rotor to tilt about the axis of the quadcopter arm. Tilting rotor quadcopter is a more agile version of conventional quadcopter and it is a fully actuated system. The tilt-rotor quadcopter is capable of following complex trajectories with ease. The control strategy in this work is to use the propeller tilts for position and orientation control during autonomous flight of the quadcopter. In conventional quadcopters, two propellers rotate in clockwise direction and other two propellers rotate in counter clockwise direction to cancel out the effective yawing moment of the system. The variation in rotational speeds of these four propellers is utilized for maneuvering. On the other hand, this work incorporates use of varying propeller rotational speeds along with tilting of the propellers for maneuvering during flight. The rotational motion of propellers work in sync with propeller tilts to control the position and orientation of the UAV during the flight. A PD flight controller is developed to achieve various modes of the
flight. Further, the performance of the controller and the tilt-rotor design has been compared with respect to the conventional quadcopter in the presence of wind disturbances and sensor uncertainties.

In this work, another novel feed-forward control design approach is presented for complex trajectory tracking during autonomous flight. Differential flatness based feed-forward position control is employed to enhance the performance of the UAV during complex trajectory tracking. By accounting for differential flatness based feed-forward control input parameters, a new PD controller is designed to achieve the desired performance in autonomous flight. The results for tracking complex trajectories have been presented by performing numerical simulations with and without environmental uncertainties to demonstrate robustness of the controller during flight.

The conventional quadcopters are under-actuated systems and, upon failure of one propeller, the conventional quadcopter would have a tendency of spinning about the primary axis fixed to the vehicle as an outcome of the asymmetry in resultant yawing moment in the system. In this work, control of tilt-rotor quadcopter is presented upon failure of one propeller during flight. The tilt-rotor quadcopter is capable of handling a propeller failure and hence is a fault-tolerant system. The dynamic model of tilting-rotor quadcopter with one propeller failure is derived and a controller has been designed to achieve hovering and navigation capability. The simulation results of way point navigation, complex trajectory tracking and fault-tolerance are presented.

Thesis Supervisor: Manish Kumar

Title: Associate Professor
To my loving family and friends
Saint Kabir wrote this verse to sing the glory of Teacher. He was asked, If both, Teacher and God were to appear at his door step, whose feet will he worship first? He answers, It has to be the Teacher’s feet, because without the Teacher how could he recognize God?

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Finally, I want to dedicate my graduate research thesis to my father, mother, my twin sister, my younger sister and my pet Jerry. I want to tell them that I am blessed to have them in my life and I will keep working hard to make them proud.

Rumit Kumar
Contents

Abstract iii

Acknowledgments vi

Contents viii

List of Figures xi

List of Abbreviations xiv

1 Introduction 1

1.1 Motivation 1

1.2 Objectives 2

1.3 Contributions 3

1.4 Organization of Thesis 3

2 Evolution of Quadcopters 5

2.1 Early Historical Development 5

2.2 History of Tilt-Rotor VTOL Aircraft 9

2.3 Modern Quadcopter 14

2.3.1 Conventional Quadcopter 16

2.3.2 Variable Pitch Quadcopter 19

2.3.3 Tilt-rotor Quadcopter 21

3 Mathematical Dynamic Model 26

3.1 Tilting-Rotor Quadcopters 26
4 Control Law Design

4.1 Control Design Objective ........................................... 34
4.2 Propeller RPM Control ............................................ 35
4.3 Propeller Tilt Control ............................................. 38
4.4 Complete Position and Attitude Control ......................... 39

5 Differential Flatness Based Flight Control 41

5.1 Literature Review and Flight Control Objective ................. 41
5.2 Tilting Rotor Quadcopter and Differential Flatness ............ 43
5.3 Differential Flatness Based Control ............................ 45
   5.3.1 Position Control ............................................. 46
   5.3.2 Attitude Control ............................................ 46
   5.3.3 Propeller Tilt Control ..................................... 48
   5.3.4 Control Architecture .................................... 49

6 Fault-Tolerant Tilt-Rotor Quadcopter 50

6.1 Literature Review of Fault-Tolerant Control ..................... 50
6.2 Tilt-Rotor Quadcopter and Propeller Failure .................... 53
6.3 Controller Design ................................................ 56

7 Numerical Simulations 62

7.1 Case 1a: Way Point Navigation Without Uncertainties ........ 62
7.2 Case 1b: Numerical Simulations With Uncertainties ............ 67
7.3 Case 2: Complex Trajectory Flight ............................ 72
   7.3.1 Numerical Simulations For Normal Flight ................ 72
   7.3.2 Numerical Simulations With Uncertainty Parameters .... 79
7.4 Case 3: Numerical Simulations for Fault-Tolerant Flight ........ 84
   7.4.1 Motor failure in x-axis (Pitch plane) ...................... 84
   7.4.2 Motor failure in y-axis (Roll Plane) ...................... 87
### List of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-1</td>
<td>Louis Breguet’s Gyroplane</td>
<td>6</td>
</tr>
<tr>
<td>2-2</td>
<td>Etienne Oehmichen’s Multirotor Aircraft</td>
<td>7</td>
</tr>
<tr>
<td>2-3</td>
<td>De Bothezat’s Quadcopter</td>
<td>7</td>
</tr>
<tr>
<td>2-4</td>
<td>D.H. Kaplan’s Convertawings Model A</td>
<td>8</td>
</tr>
<tr>
<td>2-5</td>
<td>Curtiss-Wright Company’s VZ-7</td>
<td>9</td>
</tr>
<tr>
<td>2-6</td>
<td>The Transcendental 1-G, Model 2, 1954</td>
<td>10</td>
</tr>
<tr>
<td>2-7</td>
<td>The Bell XV-3, 1955</td>
<td>10</td>
</tr>
<tr>
<td>2-8</td>
<td>The V-22 Osprey, 1989</td>
<td>11</td>
</tr>
<tr>
<td>2-9</td>
<td>The Curtiss-Wright X-19, 1963</td>
<td>11</td>
</tr>
<tr>
<td>2-10</td>
<td>The Bell X-22</td>
<td>12</td>
</tr>
<tr>
<td>2-11</td>
<td>The Bell Eagle Eye</td>
<td>13</td>
</tr>
<tr>
<td>2-12</td>
<td>The AgustaWestland-609</td>
<td>13</td>
</tr>
<tr>
<td>2-13</td>
<td>The AgustaWestland Project Zero</td>
<td>14</td>
</tr>
<tr>
<td>2-14</td>
<td>Bell Boeing Quad TiltRotor (QTR) Scheme</td>
<td>14</td>
</tr>
<tr>
<td>2-15</td>
<td>Modern Quadcopter Design</td>
<td>16</td>
</tr>
<tr>
<td>2-16</td>
<td>Quadcopter Free Body Diagram</td>
<td>17</td>
</tr>
<tr>
<td>2-17</td>
<td>Under-actuation in Quadcopters</td>
<td>18</td>
</tr>
<tr>
<td>2-18</td>
<td>Lateral Motion of Variable Pitch Quadcopters</td>
<td>20</td>
</tr>
<tr>
<td>2-19</td>
<td>Free Body Diagram of Tilt-Rotor Quadcopters</td>
<td>22</td>
</tr>
<tr>
<td>2-20</td>
<td>Schematic view of the Tilt-Rotor Quadcopter</td>
<td>23</td>
</tr>
<tr>
<td>2-21</td>
<td>Tilt-Rotor Quadcopter, Max Planck Institute, Germany</td>
<td>24</td>
</tr>
<tr>
<td>2-22</td>
<td>Tilt-Rotor Quadcopter, Karlsruhe Institute of Technology (KIT), Germany</td>
<td>24</td>
</tr>
<tr>
<td>3-1</td>
<td>Axes Systems for Tilt-Rotor Quadcopter</td>
<td>27</td>
</tr>
</tbody>
</table>
4-1 Control Architecture of PD-based Controller for Tilt-Rotor Quadcopters

5-1 Differential Flatness Definition

5-2 Control Architecture Differential Flatness Based PD Controller

6-1 Tilt-Rotor Quadcopters with a Failed Rotor

6-2 Flight Controller Reconfiguration Strategy

7-1 Three Dimensional Trajectory Plot (Case 1a)

7-2 Variation of Euler Angles (Case 1a)

7-3 Position Errors in Way Point Navigation (Case 1a)

7-4 Rotor Speed Variation (Case 1a)

7-5 Rotor Tilt Angle Variation (Case 1a)

7-6 Variation in Body Rates (Case 1a)

7-7 Two Dimensional Track Plot (Case 1a)

7-8 Three Dimensional Trajectory Plot (Case 1b)

7-9 Variation of Euler Angles (Case 1b)

7-10 Position Errors in Way Point Navigation (Case 1b)

7-11 Rotor Speed Variation (Case 1b)

7-12 Rotor Tilt Angle Variation (Case 1b)

7-13 Variation in Body Rates (Case 1b)

7-14 Two Dimensional Track Plot (Case 1b)

7-15 Three Dimensional Trajectory Plot

7-16 Two Dimensional Trajectory Plot

7-17 Variation of Euler Angles

7-18 Position Errors during Special Maneuver

7-19 Rotor Speed Variation

7-20 Rotor Tilt Angle Variation

7-21 Variation in Body Rates

7-22 Variation in Feed-Forward Control Parameters

7-23 Variation in Accelerations during Flight
List of Abbreviations

UAV .................. Unmanned Aerial Vehicle
ESC .................. Electronic Speed Controller
FAA .................. Federal Aviation Administration
WPN .................. Way Point Navigation
DF .................... Differential Flatness
RPV .................. Remotely Piloted Vehicle
VTOL ................. Vertical Take Off and Landing
CW .................... Clockwise
CCW .................. Counter-Clockwise
BET .................. Blade Element Theory
VRFT ................. Virtual Reference Feedback Tuning
CBT .................. Correlation based Tuning
RC .................... Remote Control
RPM .................. Revolutions per Minute
UAS .................. Unmanned Aerial System
3D ................... Three dimensional
6-DOF ................. Six Degrees of Freedom
QTR .................. Quad Tilt-Rotor
CQTR ................ Convertible Quad Tilt-Rotor
VPQTR ............... Variable Pitch Quad Tilt-Rotor
FTC .................. Fault-Tolerant Control
AFTC ................ Active Fault-Tolerant Control
PFTC ................ Passive Fault-Tolerant Control
GS-PID ............... Gain-Scheduled PID
MRAC ................ Model Reference Adaptive Control
SMC .................. Sliding Mode Control
BSC .................. Backstepping Control
MPC .................. Model Predictive Control
FTPR ................ Flatness-based Trajectory Planning/Re-planning
FDD .................. Fault Detection and Diagnosis
Chapter 1

Introduction

1.1 Motivation

Multirotor aircraft has changed the way we look at unmanned aerial systems also known as drones. The drone industry is estimated to be 85-90 billion USD over the next decade. These systems come in various designs depending on shape, size and the number of rotating propellers. They range from small micro UAVs to big size octacopters and helicopters. Quadcopters are very reliable unmanned aerial systems because of the design simplicity, hovering capabilities and minimal operational cost. They are used for numerous applications ranging from military, rescue operations and civilian use to artistic use. Aerial photography and filmmaking, drone racing are evolving as new hobbies among quadcopter enthusiasts. UAV swarming is an active area of research where group of quadcopters is deployed to accomplish complex tasks. Several companies such as Amazon and DHL are trying to use them for package delivery also. The applications of quadcopter UAV are increasing so rapidly that FAA is setting up new regulations for UAV flights in airspace.

Quadcopters are inherently unstable and rely on four propellers for their control and maneuverability. They are under-actuated systems and they experience various levels of acceleration during flight. They utilize four propellers to control all six degrees of freedom. But, complex applications such as aggressive maneuvers arise the need of a fully actuated system with more independent control inputs. This is the motivation for designing control architecture of a fully actuated quadcopter system. Further, if the quadcopters will be fully actuated it would contribute to the stability and improved efficiency during the flight. It will further enhance the
confidence of the user in the system. Researchers all over the world have explored numerous applications of quadcopters but so far very little work has been done towards addressing fault-tolerant abilities of the quadcopter UAVs. System failures are inevitable during flight of quadcopters. The failure can be attributed to faults in sensors or motors or propellers. The propeller failure is the most common in quadcopters as the propellers rotate at very high angular speeds which increases the likelihood of propeller failure if the quadcopter encounters any debris during flight. Once the propeller or motor failure happens, the quadcopter cannot continue the mission, and it either crashes or goes for fail safe emergency landing. In future, when quadcopters would find civilian applications and fly in National Airspace, tolerance to faults such as propeller failure becomes very important aspect for their safe and reliable operation. The primary motivation of this thesis is to design advanced control algorithms for autonomous position, attitude for tilt-rotor quadcopters that will allow fault-tolerance and ability to reject disturbances such as wind. It will increase the confidence of users in quadcopters and boost the UAV industry by expanding the applications.

1.2 Objectives

As discussed earlier, this research primarily focuses toward development of advanced control algorithms for tilting rotor quadcopters. This thesis will address the following aspects in this research:

* Defining the under-actuation control problem for conventional quadcopter

* Mathematical dynamic modelling of tilt-rotor quadcopter equations of motion

* Development of fully actuated control system for tilt-rotor quadcopter

* Application of differential flatness based feed-forward control for tilt-rotor quadcopter

* Introduction to fault-tolerant control in engineering systems and quadcopters

* Application of fault-tolerant control for tilt-rotor quadcopter

* Validation of concepts by numerical simulations of tilt-rotor quadcopter
1.3 Contributions

This research will contribute towards the preliminary work for development of redundancy management scheme in multirotor UAVs. Following are the main contributions of this work:

* The mathematical model of tilt-rotor quadcopter has been developed and rotor tilt functionality along with servo motor dynamics are implemented

* A PD-based flight controller is developed for a fully actuated tilt-rotor quadcopter system, the idea is to synchronize the rotor tilt with rotor angular motion for position and attitude control of the UAV

* The controller performance is enhanced by addition of differential flatness based feed-forward control parameters in the existing PD flight controller

* The numerical simulations for way-point navigation, simulations in presence of environmental and sensor uncertainties and complex trajectory tracking have been presented. The robustness of the proposed flight controller is evaluated by comparison of tilt-rotor quadcopter with a conventional quadcopter

* Mathematical simulation of one propeller failure in roll as well as pitch plane has been carried out and a new fault-tolerant flight algorithm is designed

* The list of publications resulting from the present work is provided in the appendix.

1.4 Organization of Thesis

This thesis consists of eight chapters. The first chapter is the introduction. Chapter 2 is a literature review of evolution of quadcopter design and discusses various advances in quadcopter design over the years. It also provides a brief summary about the type of quadcopter platforms which are currently popular among researchers all over the world. Chapter 3 presents a brief dynamic mathematical formulation of equations of motion governing the motion of tilt-rotor quadcopter. Chapter 4 presents the control system design for the tilt-rotor quadcopter and
discusses the detailed control architecture for this UAV platform. Chapter 5 presents the differential flatness based feed-forward control approach which enhances the performance of the existing PD controller and makes the quadcopter capable of performing aggressive and tight maneuvers. Chapter 6 presents the dynamics of tilt-rotor quadcopter with a failed propeller and discusses about the fault-tolerant controller design. Chapter 7 presents the numerical simulation results to support the claims of the proposed flight controller capabilities from chapters 4, 5 and 6. Chapter 7 highlights simulation results of various flight modes, robustness studies of flight controller and fault-tolerant control. Chapter 8 presents the conclusive remarks, future works and summarizes this thesis by discussing the contributions and objectives achieved through this research work. The list of publications and intellectual property information resulting from this work is provided in the appendix.
Chapter 2

Evolution of Quadcopters

Unmanned aerial vehicle (UAV) is an aircraft with no onboard human pilot or passengers. It carries the required navigation sensors, power plant, fuel and payload for flight. UAVs include autonomous drones as well as remotely piloted vehicles (RPVs). They are different from cruise missiles as they can be recovered after flight. UAVs can be categorized as fixed wing, flapping wings, and multirotor unmanned airplanes. The fixed wing UAV requires a runway for takeoff and landing, it generates lift force by fixed wings and it is powered by a reciprocating or jet engines. There is another class of drones called ornithopters and they imitate the flapping-wing characteristics of birds. On the other hand, a multirotor UAV possesses VTOL and hovering abilities as it is lifted and propelled by rotating propellers. These propellers are powered by motors or engine and the rotational speed of these propellers is controlled by the flight control law of the multirotor aircraft. Historical study of multirotor aircraft show that initial design of multirotors were not unmanned aerial vehicles but they were big size piloted aircraft. The following sections will describe how multirotors evolved to be the most popular unmanned aerial vehicles of the present day.

2.1 Early Historical Development

The use of unmanned aerial vehicle technology was first reported in 1849 when Austria attacked the Italian city of Venice with unmanned balloons loaded with explosives. After world war 1, Elmer Sperry of the Sperry Gyroscope company developed the first RPV Hewitt Sperry Automated Airplane which was a fixed wing aircraft.
The early designs of multirotors were not UAVs or RPVs. Actually, they were piloted aircrafts. In 1907, Louis Breguet designed the first four rotor helicopter and it was the first rotary aircraft to lift off the ground [59]. This flying vehicle was called Gyroplane as shown in figure (2-1) [58]. This aircraft was not controllable and the design was improved in development of Gyroplane-II. The second gyroplane was reported to have couple of successful in 1908 [58].

![Figure 2-1: Louis Breguet’s Gyroplane](image)

In 1920s, Etienne Oehmichen designed six multirotor aircrafts and his second design had four rotors and eight propellers which were driven by a single engine. This aircraft had a steel-tube frame, with two propellers at the ends of the four arms. The angle of these blades could be varied by warping which is identical to a variable pitch configuration. The aircraft made over thousand successful flights and showed promising results in terms of stability and control by setting up a world record of flying one kilometer [25] [59]. The Etienne Oehmichen’s multirotor aircraft is shown in figure (2-2).
Figure 2-2: Etienne Oehmichen’s Multirotor Aircraft

Dr. George de Bothezat and Ivan Jerome designed another quadcopter configuration aircraft and it utilized two small propellers with variable pitch for thrust and yaw control. This aircraft was built for US army and it made nearly 100 successful flights. It was capable of reaching an altitude of about 5m whereas US army’s system requirement was 100m. Although this multicopter aircraft showed feasibility but it was underpowered and mechanically complex. Eventually, this program was scrapped. The De Bothezat’s quadcopter is shown in figure (2-3).

Figure 2-3: De Bothezat’s Quadcopter

As discussed earlier, variable propeller pitch was the primary control input in early quadcopters. Those quadcopters mainly contained a single engine in the fuselage structure to power all propellers simultaneously via belt, chain or shaft drives. These mechanical components added to the mechanical complexity and weight of the whole system and the drive components
often had brake down problems. More importantly, one propeller would usually differ from other propeller because of manufacturing errors. So, the quadcopter was never inherently stable because the propellers generated different amount of thrust force though powered by same engine. It increased pilot’s work of manually adjusting the pitch of individual propellers for a sustained flight. There were not any digital computers or sensors for autonomous control for the quadcopter and the pilot had to do everything. So, these early designs of quadcopters proved to be impractical and inefficient. But the engineering technologies evolved and in 1956 D.H. Kaplan revived the previous designs of quadcopters. He designed and piloted the Convertawings model of a quadcopter figure (2-4).

As shown in figure (2-4), the four rotors were positioned in an H configuration. In the Convertawings model, the control mechanism was extremely simplified and it employed differential change of thrust between the rotors. The cyclic control was eliminated in this design and only collective control was used. This aircraft employed two 90hp Continental power plant engines connected to the rotor drive system by multiple belt drives. It had a tricycle arrangement undercarriage with two wheels in the rear section and a nose wheel which could be steered. This aircraft is considered the first true quadcopter as it was capable of sustaining a controlled flight with a much simplified design. The Curtiss-Wright VZ-7 was another VTOL quadcopter aircraft designed by the Curtiss-Wright company for the US Army and it was also built around the same time as shown in figure (2-5) [59].
2.2 History of Tilt-Rotor VTOL Aircraft

Tilt-rotor design combines the vertical take off and hovering capability of a helicopter with the conventional fixed-wing aircraft flight. For vertical flight, the rotors are angled so the plane of rotation of rotors is horizontal and the aircraft takes off like helicopter. As the aircraft gains altitude and speed, the rotors are tilted and the plane of rotation becomes vertical. Now the wing provides the lift force, and the rotors provide propulsion [60]. This design also eliminates the blade stalling problems and the tilt-rotor can achieve higher speeds than helicopters.

The VTOL aircraft using helicopter-like rotors design was patented by George Lehberger in 1930, but he did not develop the concept. The Focke-Achgelis Fa 269 was developed by Germany during world war-II, but it never made any flights [60]. PL-16 was the first American tilt-rotor aircraft but this project was also shut down because of the lack of capital. Transcendental Aircraft Corporation worked towards development of Transcendental Model 1-G for US air force, two prototypes were developed. The first prototype flew in 1954 and it crashed after one year. The second aircraft in this series did not performed much as the US Air Force withdrew funding in favor of the Bell XV-3 project. The Transcendental 1-G was the first tilt-rotor aircraft to have flown and accomplished most of a helicopter to aircraft transition in flight [60]. The Transcendental 1-G is shown in figure (2-6) [3].
The Bell XV-3 also known as Bell 200 proved the feasibility of tilt-rotor concept and provided the necessary technical data for improvements for future tilt-rotor aircraft projects. This aircraft made nearly 110 successful transitions from helicopter to airplane mode but it was severely damaged in a wind tunnel accident. The technical data and experience gained from the Bell XV-3 program were key elements in development of the Bell XV-15, which later transformed into V-22 Osprey. Currently, the Bell Boeing V-22 Osprey is a prestigious asset to all forms of US military and it has been extensively utilized as a cargo aircraft. The Bell XV-3 and V-22 Osprey are shown in figure (2-7) and (2-8).
Tilt-wing configuration was another design variant in tilt-rotor aircraft research but it was not put into the production as the actual tilt-rotor design had better hovering efficiency over the tilt-wing design [60]. Curtiss-Wright X-19 and Bell X-22 were the first tilt-rotor quadcopter designs from 1960s. The wing and tail plane of X-19 had propellers mounted at the end. These propellers could be rotated through 90 degrees and the aircraft was capable to take off and land like a helicopter. The Curtiss-Wright X-19 propellers were powered by Avco Lycoming T55-L-5 turboshaft engines [4]. The Curtiss-Wright X-19 aircraft is shown in figure (2-9) [60], this tilt-rotor aircraft was destroyed in a crash and the program scrapped.

The Bell X-22 was developed for U.S. Navy in V/STOL research. Bell company had experience with VTOL aircraft and it utilized the experience gained from previous VTOL research
This aircraft is one of the most versatile and long-lived project in tilt-rotor aircraft research. The design of X-22 consisted of four ducted fans that could rotate together between vertical and horizontal positions for the various flight modes as shown in figure (2-10) [5].

![Figure 2-10: The Bell X-22](image)

The Bell X-22 first flew in 1966 and the aircraft succeeded in transitions between hovering and horizontal flight mode. During flight tests there were technical flaws reported in prop control. The second prototype of Bell X-22 was equipped with a variable flight control and stabilizer system from Cornell Aeronautical Laboratory, which improved flight performance [60]. The X-22 was considered as the best aircraft in V/STOL research at its time. US Navy required a maximum speed of 525 km/h but it never reached that speed but it gave many valuable lessons for future research. The ducted fan propellers concept was developed during X-22 research and it proved to be very useful for F-35B project of US military.

Presently, Bell Boeing and Augusta are working extensively towards developing modern tilt-rotor aircrafts. Bell’s TR918 Eagle Eye, Bell/Augusta BA609 or AW609 are the latest members of the tilt-rotor family. AgustaWestland is working towards a manned hybrid tilt-rotor aircraft called Project Zero. The rotors are mounted inside the wingspan [60]. Bell and Boeing are working to conceptualize a larger Quad TiltRotor (QTR) for the US Army. The QTR is a larger, four rotor version of the V-22 osprey. This aircraft would have a cargo capacity roughly
equivalent to the C-130 Hercules.

Figure 2-11: The Bell Eagle Eye

Figure 2-12: The AgustaWestland-609
2.3 Modern Quadcopter

In the previous section, historical journey through rotary aircraft research showed that early multirotor aircrafts were big size aircrafts and they required human operators inside the cockpit during flight. In the previous century, aircrafts were primarily used for transportation, military, surveillance and reconnaissance applications, e.g. SR-71 Blackbird was extensively utilized during cold war for reconnaissance purpose. Similarly, helicopters were utilized for search, rescue operations and for cargo transportation. But maintenance and operation of such big size aircrafts is very time consuming and costly.

Technological advances in the final quarter of last century was the turning point and remotely piloted vehicles gained attention of scientific community. Researchers scaled down and
modified the conventional aircraft designs into unmanned aerial vehicles. The multirotors also gained popularity because of their hovering ability and agility. Cost and ease of operation are key components for flying vehicles and multirotor aircraft emerged as a viable solution, soon it started replacing conventional UAVs in most applications. The need for aircraft with greater maneuverability and hovering ability led to a rise in number of quadcopters flying in airspace. The quadcopters are relatively simple in design yet highly reliable and maneuverable. The figure (2-15) [59] shows a typical modern day quadcopter.

In the last few decades, extensive research have been carried out on quadcopter research. Quadcopter is regarded as one of the most versatile UAV design. They have been utilized for reconnaissance, surveillance and exploration of disasters (such as fire, earthquake, flood), search and rescue operations, data collection for remote land surveying, mapping and agriculture [18] [32]. Quadcopters have been proved as a great robotics tool for university researchers. They have been utilized to test and evaluate new control theories. Many research groups have shown quadcopters performing increasingly complex aerial manoeuvres and making UAV swarms and formation flight [59]. The applications of quadcopter UAV are increasing rapidly and FAA is setting up new regulations for flying UAVs in national airspace. Several companies are trying to use them for package delivery and transporting goods from one location to other. UAV swarming is an active area of research and it has attracted many researchers all over the world [28]. Swarming and cooperative control to accomplish complex missions are more sophisticated scenarios where quadcopter UAVs can play a key role. Swarms of quadcopters can be deployed for remote sensing, land and tunnel mapping. Recently, there was a demonstration from ETH Zurich, where a quadcopters swarm constructed a temporary cable bridge, such systems can be used during rescue operations in future [11] [29]. Similarly, There was a drone display by Intel Corporation (USA) involving 100 small quadcopter drones in formation which setup a new Guinness world record for the most UAVs airborne simultaneously.
It can be seen that quadcopters hold great potential for future research and development. If the developing technologies are combined and utilized on quadcopters, they would be capable of exercising very complex autonomous flight missions which can not be achieved with conventional fixed wing vehicles. Presently many research groups are working towards exploring new design iterations for quadcopters which can improve the efficiency of quadcopters during flight. The following subsections present a brief literature review about different types of quadcopter designs used in industry.

### 2.3.1 Conventional Quadcopter

The conventional quadcopter is lifted and propelled by four rotors. The quadcopter possesses one pair of propellers rotating in clockwise (CW) direction, while the other pair rotates in counterclockwise (CCW) direction \[18\] \[8\]. Each rotor produces thrust and torque about its center of rotation whereas the drag force is produced opposite to the quadcopter’s direction of flight. If all rotors are spinning at the same angular speed the net aerodynamic torque, and angular acceleration about the yaw axis is zero. This configuration eliminates the need
for a tail rotor as in conventional helicopters. Yaw is induced by mismatching the balance in aerodynamic torques produced by four rotors. The free body diagram of a conventional quadcopter is shown in figure 2-16.

![Quadcopter Free Body Diagram](image)

Figure 2-16: Quadcopter Free Body Diagram

In conventional quadcopters, attitude and position are controlled by varying the rotational speed of each motor. This configuration balances the moments created by rotating propellers. The roll angle, roll rate and lateral position of the quadcopter are controlled by varying the rotational speeds of second and fourth propeller. Whereas the pitch angle, pitch rate and forward movement are achieved by changing the rotational speeds of first and third propeller. By increasing or decreasing the speeds of all four propellers simultaneously, the collective thrust is generated for the UAV. The on-board autopilot in quadcopters enables semi-autonomous and complete autonomous capabilities to assist the pilot during the flight. The development and testing of a flight test-bed and onboard PID flight controllers for quadcopters have been discussed in [28]. In [8], the authors have derived an accurate mathematical model and parameter identification of a conventional quadcopter by using quaternions to represent Euler angles and rates. Development of a nonlinear flight controller for the quadcopter during way point navigation flight has been presented in [17]. The nonlinear PID control structure consisted an inertia moment module along with PID control. Intelligent control of quadcopter has also been
an active area of research. Neural network based PID [10] [11], fuzzy logic based PID [52], expert system based PID are the latest advances in intelligent control of quadcopters. Neural network based PID control technique enhances the existing PID controller in order to reach better response and overcome the shortcomings of a classic PID controller. An expert system PID controller employs human operator knowledge and expertise to tune the parameters of the controller without using any mathematical model of the UAV. Fuzzy systems are termed as universal approximators because of their ability to approximate any real continuous function to a great degree of arbitrary accuracy [13]. They possess a robust behavior and have been widely utilized in several decision making engineering applications where uncertainty is present and a very precise mathematical model can not be obtained [13].

Figure 2-17: Under-actuation in Quadcopters

In previous works, different type of flight controllers have been developed for quadcopters covering all aspects of control theory. The conventional quadcopter design is inherently unstable and under-actuated system as four propellers are used to control all six degrees of freedom. Figure (2-17) shows the under-actuation phenomenon of a quadcopter UAV [35]. The conventional quadcopter UAV can only hover when roll and pitch angles are zero. Once the UAV changes its orientation angles, the forces produced by each propeller, can be resolved into horizontal and vertical components respectively. As a result, the quadcopter starts to drift either forward or sideways depending on the orientation of the UAV. Hence, a traditional quadcopter is an under-actuated system and cannot hover with specified attitude angles. Authors in [32] [35] have shown that the under-actuation problem of conventional quadcopters can be solved
by the tilt-rotor quadcopter design. Their tilt-rotor quadcopter design is the precursor to this thesis.

2.3.2 Variable Pitch Quadcopter

The main difference between a conventional quadcopter and variable pitch quadcopter is the method of achieving motion control. As we know that the conventional quadcopter achieves motion control by variation of angular speed of propellers but in a variable pitch quadcopter, the primary control of translational and rotational motion is achieved by the change in blade pitch angle of different rotors in various combinations. The change of blade pitch angles causes a change in thrust force produced by the respective propeller due to variation in thrust coefficient. It should be noted that all the rotors are operated at same nominal RPM in case of a variable pitch quadcopter. The propeller’s rpm is regulated about the specified value for setting the baseline value of thrust [19].

The vertical motion is controlled by collectively changing the pitch angles for all the propellers simultaneously. Rolling and lateral flight can be achieved as explained in figure (2-18) [19]. By changing the collective pitch of two left rotors, the thrust force produced by the two rotors increases and the UAV attains a roll angle and starts moving sideways. The change in moment produced by these two rotors is still equal and the UAV does not attain any yawing motion, this maneuver results in translational motion with roll angle. Similarly, the change in collective blade angle of the two rear rotors would result in forward flight and the UAV would attain a pitch angle [19]. The yawing motion is achieved by changing the collective pitch of any two diagonal rotors. The collective pitch of the rotors rotating in the same direction is increased and the collective pitch of the other diagonal pair is decreased. The increased collective pitch results in increased lift and drag forces on respective rotors, while the other two rotors experience an identical decrease in aerodynamic forces. The rotors with increased aerodynamic forces experience an increase in torque components compared to the other two rotors. This resultant combined torque of all rotors result in yawing motion of the quadrotor. This operation does not affect the translational motion as the combined thrust of all rotors is constant [19].
The control bandwidth of a conventional quadcopters is limited by the rotational inertia of the motors [9]. As the size of the quadcopter increases, the UAV can no longer be stabilized solely by propeller RPM control. It reaches a saturation point where the torque requirements to change the angular speed of motors exceeds the capacity of motors. As a result, the conventional quadcopters flight control strategy does not work for larger vehicles [19]. The variable-pitch quadcopter design overcomes these limitations. It provides increased flight controller bandwidth and reverse thrust capabilities. Cutler et al. in [9] have compared the performance of a conventional quadcopter with a variable pitch quadcopter and they predicted the reverse thrust capabilities and quick rate of change of thrust abilities in variable pitch quadcopter design which are significant advantages over conventional quadcopter design. Gupta et al. in [19] have used blade element theory (BET) along with momentum theory to estimate the thrust and torque of rotors as a function of blade pitch angle for a variable pitch quadcopter. They have developed a nonlinear flight controller using dynamic inversion approach for stabilization and tracking. Panizza et al. in [39] have carried out black-box and grey-box identification of the attitude dynamics for a variable pitch quadrotor. Further, Panizza et al. in [40] have stated that Virtual Reference Feedback Tuning (VRFT) is a suitable method for tuning the cascade control architecture and they have successfully used VRFT to tune the cascade attitude controller of a variable-pitch quadrotor. In [21], a comparative study of VRFT and correlation based tuning (CBT) has been shown for tuning the cascade attitude controller of a variable pitch quad-
copter. In [56], the control and optimization of variable pitch quadcopter with minimum power consumption has been presented and the flight performance parameters have been evaluated. In [38], Pang has presented a design methodology for the construction of a long-endurance variable-pitch quadcopter powered by a gasoline-engine.

In recent years, the interest of quadcopter hobbyists is inclining towards use of variable pitch quadcopters and many have demonstrated the construction and flight of RC variable-pitch quadcopters. This technology is evolving as a good substitute for conventional quadcopters for large size quadcopters. It has already been concluded that the variable pitch quadrotor has higher flight controller bandwidth and it is suitable UAV platform for aggressive maneuvering.

2.3.3 Tilt-rotor Quadcopter

The tilt-rotor quadcopters are of two types based on the tilt mechanism and aerodynamic configuration. The first type can fly as conventional quadcopter as well as a fixed wing aircraft. They tilt all propellers in the same direction by the same amount and they have aerodynamic surfaces to fly like a fixed wing aircraft. The first type of tilt-rotor quadcopter is also called as convertible quad tilt-rotor (CQTR) aerial vehicle. Whereas the second type of tilt-rotor quadcopter can tilt any individual propellers independently by desired amount and they do not possess any aerodynamic surfaces such as wing, elevator etc. This research focuses on position, attitude and fault-tolerant control of the second type of tilt-rotor quadcopter configuration.

In section 2.3.1, it was stated that the conventional quadcopter is inherently unstable and under actuated system as four propellers are used to control all six degrees of freedom of the aircraft. Further, it was mentioned that a traditional quadcopter cannot hover with specified attitude angles because of under actuation phenomenon. But, over the years quadcopters have been utilized for many complex applications where they need to exercise very agile maneuvers, the UAVs experience various degrees of acceleration which arises the need of more number of independent control inputs in the quadcopter. The tilt-rotor quadcopter is one such kind of UAS where individual propeller motors are actuated to tilt about the quadcopter arm which provides four additional control inputs and thus increases the number of control inputs to eight [32] [49]. The tilting rotor quadcopter is a structural advancement in existing design of the conventional
quadcopter. It adds more complexity in design as the propellers are allowed to tilt about the axes connecting them to the main body frame by using servo motors \[32\] \[49\]. It provides advantage in terms of stability and control by converting the under actuated conventional design into an fully actuated system thus providing full control over position and orientation of the UAV. It is a suitable UAV platform for complex trajectory following applications. Figure (2-19) shows a free body diagram of a single axis tilt-rotor quadcopter.

In figure (2-19), $\theta_i$, ($i = 1, 2, 3, 4$) is the parameter for tilt angle of corresponding rotors. The planes shown with dashed lines are the original planes of rotation with zero tilt angles and the planes shown with the rigid lines are the planes after exercising tilt. The propeller thrust forces are perpendicular to these respective tilted planes. The rotor tilt manipulates the direction of total vertical force by splitting it in horizontal and vertical components which is identical to tilt of tip path plane in helicopters. The angular speed of motors is changed automatically by the on board controller to follow a desired altitude whereas the horizontal component produced by the rotor tilt helps to maneuver the quadcopter in longitudinal and lateral direction. The additional controls achieved via tilting of the rotors not only makes the quadcopter more maneuverable but it also makes it more robust to external disturbances and uncertainties.

M. Ryll et al. in \[49\] have stated the shortcomings of conventional quadcopters because of under actuation. They were the first to propose quadcopter with tilting propellers. They have derived the dynamical model of tilt-rotor quadcopter and carried out a formal controllability
analysis to devise a closed-loop controller able to asymptotically track an arbitrary desired trajectory in space. The schematic view of Ryll’s design is shown in figure (2-20) [49]. M. Ryll et al. further extended their work in [50] by illustrating the control implementation and trajectory tracking performance of a real prototype tilt-rotor quadcopter developed in their group. They reported several experimental results for different flight conditions. Figure (2-21) shows the prototype used in [50].

Figure 2-20: Schematic view of the Tilt-Rotor Quadcopter
A. Nemati et al. presented detailed equations of motion for the tilt-rotor quadcopter in [32] along with hovering modes at specified orientation angles. They developed linear PD control and non-linear controller by using lie derivatives for feedback linearization for the tilting rotor quadcopter in [32] and [33] respectively. The design and fabrication of tilting rotor quadcopter along with experimental test results were presented in [35]. G. Scholz [54] has stated that in the presence of disturbances such as a sudden wind gust the tilt-rotor quadcopter can keep performing without much change in attitude, velocity or height and the reaction is faster than a conventional quadcopter. Scholz mentions that the tilt-rotor quadcopter can land on uneven surfaces and on a moving ground vehicle. Figure (2-22) shows the UAV platform used in [54].
The boxes at each end of the quadcopter arms are the servo motors to tilt the rotors. They have used model based control approach for tilting rotor quadcopter which is based on non-linear inverse dynamics and pseudo control hedging (PCH) which increases the controller performance by driving the system to its limits and if necessary slows down the commanded dynamics. A. Oosedo et al. [36] state the concept where the UAV flies and hovers with 90° pitch angle and even can flip over when the range of the tilting motor is wide enough. Such flight conditions allow the UAV to fly easier in narrow space or provide possibility to work on vertical wall surfaces etc. Experimental results for such attitude transition for pitch angles from 0° to 90° have been presented in [36] highlighting extreme maneuverable capabilities of tilt-rotor quadcopter. They developed two switching methods for the flight control systems with respect to the attitude of UAV. The control system and switching methods were verified by experiment.

M. Elfeky et al. [12] present a tilt-rotor quadcopter design with twelve control inputs as they allowed each rotor to rotate around two axes w.r.t body frame. It increases the mechanical complexity of the structure. The concept of using combustion engine in a QTR has been discussed in [14]. Though such system would be mechanically complex but it may provide a longer endurance. Variable pitch rotors have been considered in [14]. They claim that the possibility of tilting the rotors may provide unprecedented maneuver capabilities to the UAV [14]. Such configuration can be regarded as variable pitch quad tilt-rotor (VPQTR) aircraft. In [20], design of a morphing aircraft is presented. The conventional quadcopters can fly in a horizontal configuration but their platform is capable of vertical flying mode which can be helpful for navigating in narrow spaces. The multi-copter configuration proposed in [20] uses eight rotors and the UAV can fly with four rotors in case of a rotor failure. Inverse simulation of a tilt-rotor quadcopter has been presented in [41]. They have analyzed the maneuvering abilities of the tilt-rotor quadcopter w.r.t. conventional quadcopter. The results show that tilting the rotors provides a larger control power that allows to maintain a constant rotor rate [41]. Similar works as [32] showing the solution of the under actuation phenomenon and hovering abilities of the tilt-rotor quadcopter design has been shown in [6] and [55].

The work presented here focuses mainly on the design of position, attitude, and fault-tolerant control for a tilt-rotor quadcopter. The detailed discussion follows in next chapters.
Chapter 3

Mathematical Dynamic Model

In section 2.3.1 the limitations of the under-actuated conventional quadcopter are mentioned but these limitations can be overcome by the novel tilt-rotor quadcopter configuration. In this design, the propeller motors are actuated to tilt about the quadcopter arm by individual servo motors for each rotor. It provides four additional control inputs and increases the number of control inputs to eight [32] [49]. Though it increases the mechanical complexity in design but it provides advantage in terms of stability and control. In this chapter, the mathematical dynamic model of single axis tilt-rotor quadcopter is described. Detailed equation of motion along with system parameters have been presented in following sections.

3.1 Tilting-Rotor Quadcopters

The mathematical dynamic model for a conventional quadcopter has been shown in [8] [18] [28] [31]. Unlike conventional quadcopters, in tilting-rotor quadcopter, there are four more servo motors attached to each arm that adds one degree of freedom to each of the propellers, resulting in the tilting motions about their axes [34]. In this section, the equations of motion for dynamic mathematical modelling of a tilting rotor quadcopter are presented.

The quadcopter analyzed in this thesis can be considered as a connection of 5 rigid bodies in relative motion among each other: the quadcopter body (B), and the four propellers Pᵢ, ∀ᵢ ∈ (1, 2, 3, 4) actuated for tilt [49]. The world-frame (E) is the fixed frame on earth and all the quadcopter motions can be referred with respect to this frame of reference. On the other hand, the body-fixed frame (B) is attached to the quadcopter body with origin located at center
of mass of the vehicle and the propeller \((P_i)\) are mounted at the end of each quadcopter arm. The body frame \((B)\) along with propeller frames of reference \((P_i)\) move with the quadcopter in three dimensional space of world frame. The world frame is the navigation frame for the quadcopter, figure (3-1) shows various axes systems for a tilt-rotor quadcopter. It can be seen that the individual tip path plane of propellers can be tilted about the quadcopter arm by angle \(\theta_i, \forall i \in \{1, 2, 3, 4\}\).

Figure 3-1: Axes Systems for Tilt-Rotor Quadcopter

During quadcopter flight, there are forces and moment components for each rotor which arise from the rotational motion and tilt of the rotor. These forces and moments from propeller reference frames \((P_i)\) first need to be transformed to body axes system in the body fixed frame \((B)\) of the quadcopter and then they are transformed in world frame \((E)\). Rotors 1 and 3 rotate in clockwise direction to produce a counter clockwise moment about \(x_{P_1}, x_{P_3}\) direction respectively. Similarly, rotors 2 and 4 rotate in counter clockwise sense to produce clockwise moment about \(x_{P_2}, x_{P_4}\) direction respectively.

Euler angle transformations are defined by \(\psi, \theta\) and \(\phi\) which represent yaw, pitch and roll angles respectively. The orthogonal direction transformation matrix \((R_{E/B})\) relating world frame parameters to the body fixed frame is obtained by three successive rotations. The first rotation is about \(z\)-axis, followed by another rotation about \(y\)-axis and the last rotation is about \(x\)-axis. The transformation matrix \((R_{B/E})\) relating body fixed frame parameters to the world frame is the inverse of \((R_{E/B})\), since these are orthogonal matrices the inverse will be equal to transpose.
of respective matrix. The propellers tilt by \( \theta_i, \forall i \in (1, 2, 3, 4) \) about each quadcopter arm and forces and moments produced by propellers can be transformed to body axes system by means of a single rotation \( \theta_i, \forall i \in (1, 2, 3, 4) \) for respective propeller. A detailed discussion of axes transformations is given in [37]. The three rotations can be written as follows:

\[
R_{\phi} = \begin{bmatrix}
c\psi & s\psi & 0 \\
-s\psi & c\psi & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

(3.1)

\[
R_{\theta} = \begin{bmatrix}
c\theta & 0 & -s\theta \\
0 & 1 & 0 \\
s\theta & 0 & c\theta
\end{bmatrix}
\]

(3.2)

\[
R_{\phi} = \begin{bmatrix}
1 & 0 & 0 \\
0 & c\phi & s\phi \\
0 & -s\phi & c\phi
\end{bmatrix}
\]

(3.3)

\[
R_{E/B} = R_{\phi}R_{\theta}R_{\psi}
\]

(3.4)

\[
R_{E/B} = \begin{bmatrix}
1 & 0 & 0 \\
0 & c\phi & s\phi \\
0 & -s\phi & c\phi
\end{bmatrix}
\begin{bmatrix}
c\theta & 0 & -s\theta \\
0 & 1 & 0 \\
s\theta & 0 & c\theta
\end{bmatrix}
\begin{bmatrix}
c\psi & s\psi & 0 \\
-s\psi & c\psi & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

(3.5)

\[
R_{E/B} = \begin{bmatrix}
c\theta c\psi & c\theta s\psi & -s\theta \\
\theta s\phi c\psi - s\psi c\phi & \psi s\theta s\phi + c\psi c\phi & s\phi c\theta \\
\theta s\phi c\psi + s\psi c\phi & s\psi s\theta c\phi - c\psi s\phi & c\phi c\theta
\end{bmatrix}
\]

(3.6)

\[
R_{B/E} = \begin{bmatrix}
c\theta c\psi & s\theta s\phi c\psi - s\psi c\phi & s\theta c\psi c\phi + s\psi s\phi \\
c\theta s\psi & \psi s\theta s\phi + c\psi c\phi & s\psi s\theta c\phi - c\psi s\phi \\
-s\theta & s\phi c\theta & c\phi c\theta
\end{bmatrix}
\]

(3.7)

where \( c\psi \) and \( s\psi \) denote \( \cos(\psi) \) and \( \sin(\psi) \) respectively, and similarly for \( \theta \) and \( \phi \) angles.

28
In flight dynamics, we generally face problem in computation of time history of Euler angles. This computation requires knowledge of Euler angle rates $\dot{\psi}$, $\dot{\theta}$, $\dot{\phi}$. These rates are not directly measured in systems but aerial vehicles generally possess onboard rate gyro sensors which provide body rates $p, q, r$ also known as roll rate, pitch rate and yaw rate respectively. The body rates are generally utilized to compute Euler angles in world frame of reference. The equations relating the Euler angle rates with body angular rates of the quadcopter as discussed in [37] and given by equation (3.8).

$$
\begin{bmatrix}
\dot{\phi} \\
\dot{\theta} \\
\dot{\psi}
\end{bmatrix} =
\begin{bmatrix}
1 & s_\phi \frac{s_\theta}{c_\theta} & c_\phi \frac{s_\theta}{c_\theta} \\
0 & c_\phi & -s_\phi \\
0 & \frac{s_\phi}{c_\theta} & \frac{c_\phi}{c_\theta}
\end{bmatrix}
\begin{bmatrix}
p \\
q \\
r
\end{bmatrix}
$$

Equation (3.8) encounters singularity issues when the system attains a pitch angle equal to $\pi/2$ and we will not be able to represent the attitude of the system, this can be termed as a limitation of Euler angles method [37]. Quaternion based representation are more advanced methods of representing the system orientation in three dimension. Quadcopter mathematical model with quaternion representation has been discussed in [8].

The equations of motion comprise of set of force and moment equations in world frame. The free body diagram of a tilt-rotor quadcopter is shown in figure (2-19). Using the Newton-Euler method, the equations of motion in world-frame can be written as:

$$
\begin{bmatrix}
\ddot{x} \\
\ddot{y} \\
\ddot{z}
\end{bmatrix} =
\begin{bmatrix}
-F_2 s_\theta_2 - F_4 s_\theta_4 \\
-F_1 s_\theta_1 - F_3 s_\theta_3 \\
F_1 c_\theta_1 + F_2 c_\theta_2 + F_3 c_\theta_3 + F_4 c_\theta_4
\end{bmatrix}
\begin{bmatrix}
C_1 \dot{x} \\
C_2 \dot{y} \\
C_3 \dot{z} + mg
\end{bmatrix}
$$

29
\[
\begin{bmatrix}
\dot{p} \\
\dot{q} \\
\dot{r}
\end{bmatrix} =
\begin{bmatrix}
I(F_2 c \theta_2 - F_4 c \theta_4 - C'_1 \dot{\phi}) \\
+ M_2 s \theta_2 + M_4 s \theta_4 + M'_2 + M'_4 \\
I(F_3 c \theta_3 - F_1 c \theta_1 - C'_2 \dot{\theta}) \\
- M_3 s \theta_3 - M_1 s \theta_1 + M'_1 + M'_3 \\
I(-F_1 s \theta_1 + F_2 s \theta_2 + F_3 s \theta_3 - F_4 s \theta_4 - C'_3 \dot{\psi}) \\
+ M_1 c \theta_1 - M_2 c \theta_2 + M_3 c \theta_3 - M_4 c \theta_4
\end{bmatrix}
\begin{bmatrix}
p \\
q \\
r
\end{bmatrix}
- \begin{bmatrix}
p \\
q \\
r
\end{bmatrix} \times I
\begin{bmatrix}
p \\
q \\
r
\end{bmatrix}
\] (3.10)

The parameters for mathematical simulation of tilt-rotor quadcopter are as follows.

<table>
<thead>
<tr>
<th>Sr. no.</th>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Mass ((m))</td>
<td>1.08</td>
<td>kg</td>
</tr>
<tr>
<td>2.</td>
<td>Gravity ((g))</td>
<td>9.8</td>
<td>(m/s^2)</td>
</tr>
<tr>
<td>3.</td>
<td>Length ((l))</td>
<td>0.12</td>
<td>m</td>
</tr>
<tr>
<td>4.</td>
<td>(k_f)</td>
<td>2.2x10^{-4}</td>
<td>N/rpm^2</td>
</tr>
<tr>
<td>5.</td>
<td>(k_m)</td>
<td>5.4x10^{-6}</td>
<td>Nm/rpm^2</td>
</tr>
<tr>
<td>6.</td>
<td>(I_{xx})</td>
<td>0.0311</td>
<td>kgm^2</td>
</tr>
<tr>
<td>7.</td>
<td>(I_{yy})</td>
<td>0.0311</td>
<td>kgm^2</td>
</tr>
<tr>
<td>8.</td>
<td>(I_{zz})</td>
<td>0.0622</td>
<td>kgm^2</td>
</tr>
</tbody>
</table>

Table 3.1: Tilt-Rotor Quadcopter Parameters

\[
I = \begin{bmatrix}
I_{xx} & 0 & 0 \\
0 & I_{yy} & 0 \\
0 & 0 & I_{zz}
\end{bmatrix}
\] (3.11)

In figure 2-19 \(\theta_i, \forall i \in (1, 2, 3, 4)\) is the parameter representing the tilted angle of corresponding rotors and it also represents the force transformation from propeller frames of reference \(P_i, \forall i \in (1, 2, 3, 4)\) to body fixed frame of reference \(B\). The planes shown with dashed lines are
the original planes of rotation with zero tilt angles whereas the planes shown with the rigid lines are the planes after exercising tilt. The propeller thrust forces are perpendicular to these respective tilted planes. In equation 3.8, \( m \) is the total mass of quadcopter, \( g \) is the acceleration due to gravity, \( x, y \) and \( z \) are quadcopter position in world frame coordinate, \( C_1, C_2, \) and \( C_3 \) are drag coefficients which is negligible at low speed. \( F_i, \forall i \in (1, 2, 3, 4) \) are forces produced by the four rotors as given by equation (3.12).

\[
F_i = k_f \omega_i^2
\]  

where \( \omega_i, \forall i \in (1, 2, 3, 4) \) is the angular velocity of \( i^{th} \) rotor and \( k_f \) is a constant. \( M_i', \forall i \in (1, 2, 3, 4) \) are the tilting moments created by servo motors to tilt the rotors. \( I \) matrix contains terms of moment of inertia about \( x, y \) and \( z \) axes. \( C_i', \forall i \in (1, 2, 3, 4) \) are rotational drag coefficients. \( M_i, \forall i \in (1, 2, 3, 4) \) are rotor moments of respective rotors and again it is proportional to square of angular speed of rotor given by equation (3.13), where, \( k_m \) is a proportional constant.

\[
M_i = k_m \omega_i^2
\]  

The servo motors used for rotor tilt is represented mathematically by the following first order transfer function that relates the motor angular velocity (\( \omega_s \)) of motor to input voltage (\( V \)) as shown in equation (3.14).

\[
\frac{\omega_s(s)}{V(s)} = \frac{K}{\tau s + 1}
\]  

Where \( \tau \) represents the time constant of the system, and \( K \) represents the steady state gain value. The angular position of the servo motor can be obtained by integrating the motor angular velocity. The transfer function relating the angular position (\( \theta_s \)) and input voltage (\( V \)) is given by equation (3.15).

\[
\frac{\theta_s(s)}{V(s)} = \frac{K}{s(\tau s + 1)} = \frac{K}{\tau s^2 + s}
\]  

This transfer function is identical to a second order actuation system. Such systems exhibit a transient response when they are subjected to external inputs or environmental disturbances. This transient response is an important factor in system design for the proposed quadcopter that

31
affects the maneuverability and responsiveness of the quadcopter to external disturbances.

Since, the tilt-rotor quadcopter is a fully actuated system it can fly like a traditional quadcopter when the rotor tilt is not exercised and it can achieve advanced flight modes such as tilt hovering flight. There are eight control inputs available in this system comprising of four independent rotational speeds of propeller motors and four tilt inputs about the axes of quadcopter arms. By referring figure (2-19), it is assumed that \( \theta_1 = \theta_3 \) and \( \theta_2 = \theta_2 \). In [31] and [32], the tilt hovering flight modes have been discussed. It has been concluded if the quadcopter is in an equilibrium hovering state it would achieve a roll angle \( \phi \) given by \( \phi = \theta_1 / 2 \) when the pitch angle is zero, and a pitch angle \( \theta \) given by \( \theta = \theta_2 / 2 \) when the roll angle is zero.

**Theorem-I:** Considering the dynamics of tilt-rotor quadcopter, the necessary hovering speed for each rotor can be written as

\[
\omega_h = \sqrt{\frac{mg}{2k_f(c\theta_1 + c\theta_2)}}
\]

**Proof:** The force equation for z-axis for very small Euler angles can be written as per the following equation (3.16).

\[
mz = F_1c\theta_1 + F_2c\theta_2 + F_3c\theta_3 + F_4c\theta_4 - mg
\]

The rotational speed of individual motors necessary for hovering \( \omega_h \) is obtained by equating z-axis acceleration to zero, the effect of drag terms at low speeds is omitted and Euler angles are considered very small which eliminates the small sine and cosine terms.

\[
F_1c\theta_1 + F_2c\theta_2 + F_3c\theta_3 + F_4c\theta_4 - mg = 0
\]

\[
F_1c\theta_1 + F_2c\theta_2 + F_3c\theta_3 + F_4c\theta_4 = mg
\]

Substituting the expression for \( F_i \), \( \forall i \in (1, 2, 3, 4) \) as per equation equation (3.12) and \( \omega_i, \forall i \in (1, 2, 3, 4) \) will be equal to \( \omega_h \).

\[
k_f\omega_n^2c\theta_1 + k_f\omega_n^2c\theta_2 + k_f\omega_n^2c\theta_3 + k_f\omega_n^2c\theta_4 = mg
\]

\[
k_f\omega_n^2(c\theta_1 + c\theta_2 + c\theta_3 + c\theta_4) = mg
\]
This above expression can further be simplified by the assumption of $\theta_1 = \theta_3$ and $\theta_2 = \theta_4$.

$$2k_f \omega_h^2 (c\theta_1 + c\theta_2) = mg$$ (3.19)

The required expression for necessary hovering speed for the tilt-rotor quadcopter can be written as per equation (3.20).

$$\omega_h = \sqrt{\frac{mg}{2k_f(c\theta_1 + c\theta_2)}}$$ (3.20)

Substituting $\theta_1 = \theta_2 = 0^\circ$, it means the rotor tilt is not exercised and the cosine terms yield a numerical value of one and equation (3.20) reduces to equation (3.21) which is the expression for required hover speed for a conventional quadcopter.

$$\omega_h = \sqrt{\frac{mg}{4k_f}}$$ (3.21)

This chapter presented a detailed dynamic mathematical formulation of tilt-rotor quadcopter along with system parameters, it also discussed about various flight modes and derived expression for hovering flight.
Chapter 4

Control Law Design

In this chapter, the controller development strategy for tilt-rotor quadcopter is presented. This UAV system has eight control inputs consisting of rotational motion of four rotors and tilt of four rotors. In order to simplify the over actuated system, we assume the opposite propellers to each other tilt by the same angle $\theta_1 = \theta_3$ and $\theta_2 = \theta_4$. It reduces the number of control inputs to six. Thus, six control inputs are used to control 6-DOF in the system. The dynamic model of the tilting rotor quadcopter was presented in chapter (3) by equations (3.8) to (3.15).

4.1 Control Design Objective

In previous works, different types of flight controllers have been developed for quadcopters covering all aspects of control theory. So far, variable angular speed of motors have been the primary maneuvering control in quadcopter research. The work presented in this chapter focuses on syncing variable angular speed of propelling motors with tilt angle of rotors. The primary objective is to have full control over the position and orientation of the quadcopter and ensure optimum disturbance rejection to uncertainties during flight. The rotor tilt manipulates the direction of total vertical force by splitting it in horizontal and vertical components which is identical to tilt of tip path plane in helicopters. The angular speed of motors is increased or decreased by the on-board controller to follow a desired altitude whereas the horizontal component produced by the rotor tilt helps to maneuver the quadcopter in any direction. The additional controls achieved via tilting of the rotors not only makes the quadcopter more maneuverable, but it also makes it more robust to external disturbances and uncertainties. A numerical study
on disturbance rejection capability of the proposed controller will be presented in upcoming chapters by simulating uncertainties in sensor readings and wind gusts.

### 4.2 Propeller RPM Control

Here, we present the method to track a desired trajectory in three-dimensional space using rotor speeds. The position errors \((e_x, e_y, e_z)\) and velocity errors \((\dot{e}_x, \dot{e}_y, \dot{e}_z)\) along \(xyz\)-axes are computed from the comparison of position feedback and desired position values. Then, these errors are utilized by the PD controller to compute commanded accelerations. The proportional gain enhances the transient response of the system whereas the derivative gain acts as a damper to stabilize the system. The formulation is given by following relations.

\[
\begin{align*}
e_x &= x^{des} - x, & \dot{e}_x &= u^{des} - u \\
e_y &= y^{des} - y, & \dot{e}_y &= v^{des} - v \\
e_z &= z^{des} - z, & \dot{e}_z &= w^{des} - w
\end{align*}
\]  

\(x^{des}, y^{des}, z^{des}\) are the desired positions and \(x, y, z\) are the current feedback of position in \(xyz\)-direction. Similarly, \(u^{des}, v^{des}, w^{des}\) are the desired velocity and \(u, v, w\) are the feedback of velocity in \(xyz\)-direction respectively. The output from system of equations shown in (4.1) is utilized by a PD controller to generate desired acceleration change in \(xyz\)-axes. The basic structure of PD controller is given by equation (4.2).

\[
\ddot{r}_i^{des} = k_p e_i + k_d \dot{e}_i \quad \forall i \in \{x, y, z\}
\]  

The term \(\ddot{r}_i^{des}\) represents the desired accelerations in \(xyz\)-axes and \(k_p\) and \(k_d\) are the proportional and derivative gains of the proposed PD controller. The significance of controller gains have already been discussed. The force equations [3.9] shown in chapter [3] are linearized about the hovering point and the commanded accelerations obtained from the PD controller are used to compute the desired orientation angles necessary for the quadcopter to navigate in 3D. The effect of drag terms is omitted during the linearization procedure. We apply small angles
approximations for sine and cosine angles and Δ perturbation is considered in acceleration state variables and rotor speeds. The perturbation in rotor tilts are considered zero for the sake of simplicity. The delta perturbation in accelerations can be visualised as ˙¨r

\[ m \begin{bmatrix}
  \ddot{x} \\
  \ddot{y} \\
  \ddot{z}
\end{bmatrix} = R_{B/E} \begin{bmatrix}
  0 & 0 \\
  0 & 0 \\
  \Sigma F & mg
\end{bmatrix} \]  

(4.3)

By small angle approximation, \( \sin \phi \approx \phi, \sin \theta \approx \theta, \cos \phi \approx 1 \) and \( \cos \theta \approx 1 \). Once perturbations are introduced, the first and second equations from (4.4) can be solved algebraically to yield desired roll and pitch angles necessary for motion in \( xy \)-direction and the third equation is solved separately to compute the change in rotor speed necessary for motion in \( z \)-direction. \( \psi \) denotes the desired yaw angle for the quadcopter. The desired yaw angle has been considered zero for simplicity. It should be noted that \( \Sigma F = mg \) for hovering flight.

\[ m \ddot{r}_x^{des} = (c_\phi \theta^{des} + s_\phi \phi^{des})\Sigma F \]  

(4.5)

\[ m \ddot{r}_y^{des} = (s_\phi \theta^{des} - c_\phi \phi^{des})\Sigma F \]  

(4.6)

The algebraic solution of equation (4.5) and (4.6) yields the following desired expressions for roll and pitch orientation angles as shown in equation (4.7) and (4.8).
\[ \phi_{\text{des}} = \frac{\ddot{x}_{\text{des}} s_\psi - \ddot{y}_{\text{des}} c_\psi}{g} \quad (4.7) \]

\[ \theta_{\text{des}} = \frac{\ddot{x}_{\text{des}} c_\psi + \ddot{y}_{\text{des}} s_\psi}{g} \quad (4.8) \]

The third equation is solved separately to compute the change in rotor speed necessary for motion in z-direction. It is given by the expression in equation (4.9) and it can be rewritten as equation (4.10) by referring equation (3.19).

\[ m \ddot{z} = \Sigma F - mg \tag{4.9} \]

\[ m \ddot{z} = 2 k_f \omega_f^2 (c \theta_1 + c \theta_2) - mg \tag{4.10} \]

We introduce the perturbation in acceleration and rotor speed to compute change in rotor speed necessary for motion in z-direction. The above equation is modified to the following expression.

\[ m (\ddot{z} + \ddot{r}_z^{\text{des}}) = 2 k_f (\omega_f + \Delta \omega_f)^2 (c \theta_1 + c \theta_2) - mg \tag{4.11} \]

\[ m (\ddot{z} + \ddot{r}_z^{\text{des}}) = 2 k_f (\omega_f^2 + \Delta \omega_f^2 + 2 \omega_f \Delta \omega_f)(c \theta_1 + c \theta_2) - mg \tag{4.12} \]

Subtracting equation (4.10) from (4.12) and ignoring higher order terms such as \( \Delta \omega_f^2 \).

\[ m \ddot{r}_z^{\text{des}} = 4 k_f \omega_h \Delta \omega_f (c \theta_1 + c \theta_2) \tag{4.13} \]

\[ \Delta \omega_f = \frac{m \ddot{r}_z^{\text{des}}}{4 k_f \omega_h (c \theta_1 + c \theta_2)} \tag{4.14} \]

Equation (4.14) gives expression for change in rotor speed necessary for motion in z-direction for the tilt-rotor quadcopter. It should be noted that if \( \theta_1 = \theta_2 = 0 \), the above equation reduces to the expression for change in rotor speed necessary for motion in z-direction for the conventional quadcopter as shown in equation (4.15).

\[ \Delta \omega_f = \frac{m \ddot{r}_z^{\text{des}}}{8 k_f \omega_h} \tag{4.15} \]
The Euler angles errors \((e_\phi, e_\theta, e_\psi)\) and angular rate errors \((e_p, e_q, e_r)\) along \(xyz\)-axes are computed from the comparison of feedback and desired values. The errors are given by following equation (4.16).

\[
\begin{align*}
    e_\phi &= \phi^{\text{des}} - \phi, \quad e_p = p^{\text{des}} - p \\
    e_\theta &= \theta^{\text{des}} - \theta, \quad e_q = q^{\text{des}} - q \\
    e_\psi &= \psi^{\text{des}} - \psi, \quad e_r = r^{\text{des}} - r
\end{align*}
\]  

\(\phi^{\text{des}}, \theta^{\text{des}}, \psi^{\text{des}}\) are the desired Euler angles and \(\phi, \theta, \psi\) are the current feedback values of Euler angles. Similarly, \(p^{\text{des}}, q^{\text{des}}, r^{\text{des}}\) are the desired angular rates and \(p, q, r\) are the feedback of angular rates respectively. The output from system of equations shown in (4.16) is utilized by a PD controller to generate desired rotor speeds for orientation control of the quadcopter. The rotor speed signals are sent to individual electronic speed controllers of propeller motors.

\[
\begin{align*}
    \Delta \omega_\phi &= k_{p,\phi}e_\phi + k_{d,\phi}e_p \\
    \Delta \omega_\theta &= k_{p,\theta}e_\theta + k_{d,\theta}e_q \\
    \Delta \omega_\psi &= k_{p,\psi}e_\psi + k_{d,\psi}e_r
\end{align*}
\]  

\(\Delta \omega_i, i \in \{\phi, \theta, \psi\}\) represents the change in rotor speeds for orientation control.

### 4.3 Propeller Tilt Control

In the previous section, we discussed the methodology for using rotor speeds for tracking a desired trajectory. This section presents the strategy of integrating rotor tilts for position and orientation control to enhance the efficiency of the flight controller.

The position errors \((e_x, e_y, e_z)\) and velocity errors \((\dot{e}_x, \dot{e}_y, \dot{e}_z)\) along \(xyz\)-axes are given from equation (4.1). Similarly, Euler angles errors \((e_\phi, e_\theta, e_\psi)\) and angular rate errors \((e_p, e_q, e_r)\) are given by equation (4.16). A parallel feedback loop is integrated in the existing flight controller. The position and orientation angle errors are utilized by another PD controller to compute the respective tilt angles of individual rotors necessary for trajectory control. Tilt of rotors 1 and 3...
are responsible for y–position and roll angle control whereas tilt of rotors 2 and 4 are utilized for x–position and pitch angle control. The primary objective of this PD controller is to guide the quadcopter along the desired trajectory by minimizing the xy position and velocity errors to zero by maintaining the UAV in the desired orientation.

\[
\begin{align*}
\Delta \theta_T_x &= k_{p,x} e_x + k_{d,x} \dot{e}_x \\
\Delta \theta_T_y &= k_{p,y} e_y + k_{d,y} \dot{e}_y \\
\Delta \theta_T_\phi &= k_{p,\phi} e_\phi + k_{d,\phi} \dot{e}_\phi \\
\Delta \theta_T_\theta &= k_{p,\theta} e_\theta + k_{d,\theta} \dot{e}_\theta 
\end{align*}
\]

\(\Delta \theta_T_i, i \in \{x, y, \phi, \theta\}\) is the change in rotor tilt for position and orientation control as shown in equation (4.18).

### 4.4 Complete Position and Attitude Control

We have already discussed the methodology for using rotor speeds and rotor tilt for tracking a desired trajectory. This section shows the complete control architecture highlighting the syncing of rotor speeds and tilts for position and orientation control.

The vector representing complete set of tilting rotor quadcopter controls can be written as a linear combination of terms shown in the equations (3.20), (4.14), (4.17) and (4.18) as shown by the following matrix.

\[
\begin{bmatrix}
\omega^\text{des}_1 \\
\omega^\text{des}_2 \\
\omega^\text{des}_3 \\
\omega^\text{des}_4 \\
\theta_1 \\
\theta_2 \\
\theta_3 \\
\theta_4
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & -1 & 1 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\
1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\
1 & -1 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\omega_h + \Delta \omega_f \\
\Delta \omega_\phi \\
\Delta \omega_\theta \\
\Delta \omega_\psi \\
\Delta \theta_T_x \\
\Delta \theta_T_y \\
\Delta \theta_T_\phi \\
\Delta \theta_T_\theta
\end{bmatrix}
\]

\[(4.19)\]
The outputs resulting from equation (4.19) are regarded as the controller outputs. The \( \omega_i^{\text{des}}, i \in \{1, 2, 3, 4\} \) terms are rotor speed signals which are sent to individual electronic speed controllers of the propeller motors. The force and moment variation in propellers is governed by equation (3.9) and (3.10). The \( \theta_i, i \in \{1, 2, 3, 4\} \) terms are the rotor tilt angle signals sent to servo motors. The transient behavior of the servo motor is governed by the second order actuator model as shown in equation (3.15). This system represents the complete control of tilt-rotor quadcopter with the designed PD controller. The complete control architecture of the proposed flight controller is shown in figure (4-1).

![Figure 4-1: Control Architecture of PD-based Controller for Tilt-Rotor Quadcopters](image)

The flight controller proposed in this chapter covers use of all control inputs for maneuvering the UAV in three dimension. In chapter (3), it was explained that the tilt-rotor quadcopter is a structural advancement of conventional quadcopter. It is capable of functioning as a conventional quadcopter as well as an advanced flying vehicle. It should be noted that if the tilt angles become zero degrees i.e. \( \theta_1 = \theta_2 = 0 \), the proposed flight controller in this chapter automatically reduces to the flight controller of a conventional quadcopter.
Chapter 5

Differential Flatness Based Flight Control

In this chapter, differential flatness concept is used to enhance the performance of the flight controller for tilt-rotor quadcopter which was discussed in Chapter 4. The control strategy for this UAV platform comprises of differential flatness based feed-forward position control. We will discuss the flight control objectives and explain the existing theory for differential flatness in case of conventional quadcopters and develop the methodology of using this concept for control of the tilt-rotor quadcopter.

5.1 Literature Review and Flight Control Objective

The tilt-rotor quadcopter is a suitable UAV platform for complex trajectory tracking applications as it has more control inputs and it provides advantage in terms of stability and provides full control over position and orientation at the same time. UAV path planning involves finding the trajectory to the desired location whereas path following is about minimizing the position errors while following a desired trajectory [26]. The UAV should be capable of discovering a flight path and avoid collision. Radmanesh et al. have implemented Mixed Integer Linear Programming (MILP) and Grey Wolf Optimization (GWO) algorithms as heuristic solutions to UAV trajectory planning in [47] and [45]. Path following is equally important as the UAV should not have position errors while following the trajectory. In previous works, [32] and [49] provide dynamic modeling, controller design and numerical simulations for the various flight modes of tilt-rotor quadcopter. [35], [54] and [51] have shown experimental work towards the flight characteristics of the tilt-rotor quadcopter. Ryll et al. in [51] have presented a numerical
simulation where the UAV has to follow the predefined path defined by the figure ‘8’. Generally, the UAVs generate some error with respect to the desired states and show limitation in tracking the desired states during such complex maneuvers and there is scope of improvement. The UAV should follow the flight path with minimum position errors and in minimum time.

Feed-forward control has been reported as a suitable solution to minimize the position errors while following a complex trajectory. Feed-forward control can be interpreted as using nominal inputs, that would result in zero errors. Michael et al. [28] discuss feed-forward terms in the controller and conclude that feed-forward terms can be ignored for low acceleration but they play an important role for larger accelerations. The feed-forward control terms can significantly improve controller performance for complex maneuvers. Differential flatness based feed-forward control was first implemented by Ferrin et al. in [15] on a hexacopter UAV. Further, Sharma et al. in [26] have used differential flatness based flight controller on a conventional AR drone for virtual target based guidance and path following. Both [15] and [26] use differential flatness based LQR flight controller and inverse mapping to compute desired attitude angles for PID attitude controllers. But, the inverse mappings are very complicated in case of tilt-rotor quadcopter due to non-linear dynamics, and only numerical solutions are possible for the inverse mapping. Here, we use differential flatness based feed-forward position control and linearization techniques on a tilt-rotor quadcopter to achieve complete control and enhance the flight efficiency during complex maneuvers.

The primary focus of this work is on path following and the flight path considered here requires a complex and aggressive maneuver in narrow space. So, the non-linear dynamics of the UAV have been linearized as per Chapter 4 to compute the desired orientation angles for the UAV during the flight and differential flatness based PD controller has been designed for a complete control of the UAV. In this chapter, the correlation between differential flatness theory and dynamics of tilt-rotor quadcopter has been explained in detail and differential flatness theory is used to compute the feed-forward control parameters. We utilize feed-forward acceleration terms for the position controller of the UAV. These terms are used to enhance the functionality of the PD controller governing the rotational and tilting motion of the propellers. The operational bandwidth of propeller angular speed control is different from rotor tilt control.
but we have synchronized the operation of rotor tilt and rotor angular speeds to control the position and attitude of the UAV. The differential flatness based flight controller is validated in Chapter 7 for tracking a complex trajectory in three dimensions by numerical simulations.

5.2 Tilting Rotor Quadcopter and Differential Flatness

In this section, we present application of differential flatness on tilt-rotor quadcopter. The state-space form of any linearized dynamic system can be expressed by state, control, output and feed-forward matrices \((A, B, C, D)\) respectively as:

\[
\dot{x} = Ax + Bu \\
y = Cx + Du
\] (5.1)

A system is differentially flat if the state \((x)\) and control input \((u)\) of the system can be expressed as functions of flat output \((y)\) and its time-derivatives \([15][27]\).

\[
x = g_x(y, \dot{y}, \ddot{y}, \ldots) \\
u = g_u(y, \dot{y}, \ddot{y}, \ldots) \tag{5.2}
\]

\(y\) is a flat output if it is a function of state \((x)\) and control input \((u)\) and successive time derivatives of control input as shown in equation (5.3). Figure (5-1) shows the dependency correlation between state \((x)\), input \((u)\) and flat output \((y)\) and their successive time derivatives.

\[
y = h(x, u, \dot{u}, \ddot{u}, \ldots) \tag{5.3}
\]
The dynamic model of tilt-rotor quadcopter is discussed in chapter 3. The terms on the right hand side of equation (3.9) are the control input terms and gravitational acceleration. It can be seen that the control input terms are related to second derivative of the position \((x, y, z)\) of the quadcopter which satisfies the differential flatness condition from equation (5.2). Similarly, the position outputs \((x, y, z)\) are dependent on the velocity, acceleration states and control inputs in the quadcopter which satisfies the equation (5.3). Hence, the position outputs \((x, y, z)\) can be regarded as flat outputs and the tilt rotor quadcopter system can be considered as a differential flat system. The control input terms on the right hand side of equation (3.9) can be considered as the the feed-forward control terms and can be represented as \([\ddot{x}, \ddot{y}, \ddot{z}]'\). So, if the desired trajectory function is twice differentiable, then from equation (3.9), second derivative of desired trajectory function will be the feed-forward acceleration for position control. The desired trajectory function should be a continuous and at least twice differentiable function or else the magnitude of the feed-forward control terms will become zero. We have assumed that the quadcopter has to follow a tight trajectory represented in [15] and [26].

\[
\begin{align*}
    r_T^{des} &= \begin{bmatrix} x^{des} \\ y^{des} \\ z^{des} \end{bmatrix} = \begin{bmatrix} a \cos \frac{\Omega}{2} t \\ b \sin \Omega t \\ n + c \sin \Omega t \end{bmatrix} & \forall T \in \{x, y, z\}
\end{align*}
\]
Here, $\Omega$ is the angular frequency, $n$ is a constant and $a, b, c$ represent the amplitudes for the respective trigonometric functions which govern the desired trajectory in three dimension.

\[
\begin{bmatrix}
\dot{x}_{des} \\
\dot{y}_{des} \\
\dot{z}_{des}
\end{bmatrix} = \begin{bmatrix}
-a\Omega^2 \cos \frac{\Omega t}{2} \\
-b\Omega^2 \sin \Omega t \\
-c\Omega^2 \sin \Omega t
\end{bmatrix} \tag{5.5}
\]

The expression of second derivative of desired trajectory can be substituted in equation (5.5) for computing feed-forward control terms $[\ddot{u}_x \ddot{u}_y \ddot{u}_z]'$. It should be noted that we are ignoring the gravity terms here from equation (3.9) as we would be compensating for the gravitational terms while calculating hovering conditions for the quadcopter.

\[
\begin{bmatrix}
-a\Omega^2 \cos \frac{\Omega t}{2} \\
-b\Omega^2 \sin \Omega t \\
-c\Omega^2 \sin \Omega t
\end{bmatrix} = \frac{R_{B/E}}{m} \begin{bmatrix}
-F_2 s \theta_2 - F_4 s \theta_4 \\
-F_1 s \theta_1 - F_3 s \theta_3 \\
F_1 c \theta_1 + F_2 c \theta_2 + F_3 c \theta_3 + F_4 c \theta_4
\end{bmatrix} \tag{5.6}
\]

\[
\begin{bmatrix}
\ddot{u}_x \\
\ddot{u}_y \\
\ddot{u}_z
\end{bmatrix} = \begin{bmatrix}
-a\Omega^2 \cos \frac{\Omega t}{2} \\
-b\Omega^2 \sin \Omega t \\
-c\Omega^2 \sin \Omega t
\end{bmatrix} \tag{5.7}
\]

Equation (5.7) shows the required feed-forward control inputs to minimize the position errors during flight and they can be computed in real time. The desired trajectory function can be a higher degree polynomial also in $xy$z- axes unlike the functions in this case. The flight controller can be scaled easily for those applications. The flight response of UAV is mainly governed by the structural limits of the UAV platform to execute any complex maneuvers.

\section*{5.3 Differential Flatness Based Control}

In this section, the differential flatness based controller development strategy is presented for tilt-rotor quadcopter. The system has eight control inputs and for the simplicity of the design
it has been considered that the opposite propellers to each other tilt by the same angle ($\theta_1 = \theta_3$ and $\theta_2 = \theta_4$). It reduces the number of control inputs to six. The dynamic model of the tilting rotor quadcopter, described in chapter (3) has been used to design the controller.

5.3.1 Position Control

In the on-board flight controller, the desired trajectory is given as per equation (5.4) and the current position and velocity of the UAV is used in the feedback loop. The position errors ($e_x, e_y, e_z$) and velocity errors ($\dot{e}_x, \dot{e}_y, \dot{e}_z$) are computed by relative difference of the feedback and instantaneous desired values. The feed-forward output is computed as per equation (5.7) using the differentially flat equations of tilt-rotor quadcopter dynamics. The controller output is the desired linear accelerations for the system. The basic structure of the controller output is given by the following equations [28].

\[
\begin{align*}
\ddot{r}_x^{\text{des}} & = k_p x e_x + k_d x \dot{e}_x + \ddot{u}_x \\
\ddot{r}_y^{\text{des}} & = k_p y e_y + k_d y \dot{e}_y + \ddot{u}_y \\
\ddot{r}_z^{\text{des}} & = k_p z e_z + k_d z \dot{e}_z + \ddot{u}_z
\end{align*}
\]

Here $k_p_i$, $k_d_i$, $\forall i \in (x, y, z)$ represents the proportional and derivative gain of the controller respectively. The significance of proportional and derivative gains has already been discussed in chapter [4].

5.3.2 Attitude Control

The rotational speed of motors necessary for hovering ($\omega_h$) is discussed in chapter (3) by equation (3.20) and it is given by the following expression.

\[
\omega_h = \sqrt{\frac{mg}{2k_f (\cos \theta_1 + \cos \theta_2)}}
\]

Similarly, equation (5.10) gives expression for change in rotor speed necessary for motion in $z$-direction for the tilt-rotor quadcopter. It should be noted that if $\theta_1 = \theta_2 = 0$, the equation
reduces to the expression for change in rotor speed necessary for motion in z-direction for the conventional quadcopter as shown in equation (4.15) of chapter [4]:

\[
\Delta \omega_f = \frac{m \ddot{x}_{des}}{4k_f \omega_h (c\theta_1 + c\theta_2)}
\]

(5.10)

The Euler angles errors \((e_\phi, e_\theta, e_\psi)\) and angular rate errors \((e_p, e_q, e_r)\) along xyz-axes are computed from the comparison of feedback and desired values. These errors are utilized by a PD controller to generate desired rotor speeds for orientation control of the quadcopter. The rotor speed signals are sent to individual electronic speed controllers of propeller motors.

\[
\Delta \omega_\phi = k_{p,\phi} e_\phi + k_{d,\phi} e_p
\]

\[
\Delta \omega_\theta = k_{p,\theta} e_\theta + k_{d,\theta} e_q
\]

(5.11)

\[
\Delta \omega_\psi = k_{p,\psi} e_\psi + k_{d,\psi} e_r
\]

\(\Delta \omega_i, i \in \{\phi, \theta, \psi\}\) represents the change in rotor speeds for orientation control. The vector representing propeller rotational speed tilting rotor quadcopter can be written as as shown by the following matrix.

\[
\begin{bmatrix}
\omega_{1}^{des} \\
\omega_{2}^{des} \\
\omega_{3}^{des} \\
\omega_{4}^{des}
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & -1 & 1 \\
1 & 1 & 0 & -1 \\
1 & 0 & 1 & 1 \\
1 & -1 & 0 & -1
\end{bmatrix}
\begin{bmatrix}
\omega_h + \Delta \omega_f \\
\Delta \omega_\phi \\
\Delta \omega_\theta \\
\Delta \omega_\psi
\end{bmatrix}
\]

(5.12)

The outputs resulting from equation (5.12) are regarded as the controller outputs for propeller rotational speeds. The \(\omega_i^{des}, i \in (1, 2, 3, 4)\) terms are rotor speed signals which are sent to individual electronic speed controllers of the propeller motors. The force and moment variation in propellers is governed by equation (3.9) and (3.10).
5.3.3 Propeller Tilt Control

The position errors \( (e_x, e_y, e_z) \) and velocity errors \( (\dot{e}_x, \dot{e}_y, \dot{e}_z) \) along \( xyz \)-axes are given from equation (4.1). Similarly, Euler angles errors \( (e_\phi, e_\theta, e_\psi) \) and angular rate errors \( (\dot{e}_p, \dot{e}_q, \dot{e}_r) \) are given by equation (4.16). A parallel feedback loop is integrated in the existing flight controller. The position and orientation angle errors are utilized by another PD controller to compute the respective tilt angles of individual rotors necessary for trajectory control. Similarly, the position errors \( (e_x, e_y) \) and velocity errors \( (\dot{e}_x, \dot{e}_y) \) along \( xy \)-axis are utilized by the rotor tilt PD controller to compute the tilt angles of rotors for trajectory tracking. Rotor 1 and 3 are responsible for \( y \)-position control and rotors 2 and 4 are utilized for \( x \)-position control. Since, the operational bandwidth of rotor tilt control is different from rotor speed control, we use a tuned gain for adding feed-forward terms for tilt controller and it synchronizes the operation of rotor tilt and angular speeds to control the position of the UAV. The primary objective of this PD controller is to guide the quadcopter along the desired trajectory by minimizing the \( xy \) position and velocity errors to zero by maintaining the UAV in desired orientation.

\[
\Delta \theta_{T_x} = k_{p,xy}e_x + k_{d,xy}\dot{e}_x + k_i\ddot{u}_x \\
\Delta \theta_{T_y} = k_{p,y}e_y + k_{d,y}\dot{e}_y + k_i\ddot{u}_y \\
\Delta \theta_{T_\phi} = k_{p,\phi}e_\phi + k_{d,\phi}\dot{e}_\phi \\
\Delta \theta_{T_\theta} = k_{p,\theta}e_\theta + k_{d,\theta}\dot{e}_\theta \tag{5.13}
\]

\( \Delta \theta_{Ti}, i \in \{x, y, \phi, \theta\} \) is the change in rotor tilt for position and orientation control as shown in equation (5.13). The vector representing propeller tilt control can be written as as shown by the following matrix.

\[
\begin{bmatrix}
\theta_1 \\
\theta_2 \\
\theta_3 \\
\theta_4
\end{bmatrix}
= 
\begin{bmatrix}
1 & 0 & 1 & 0 \\
0 & 1 & 0 & 1 \\
1 & 0 & 1 & 0 \\
0 & 1 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\Delta \theta_{T_x} \\
\Delta \theta_{T_y} \\
\Delta \theta_{T_\phi} \\
\Delta \theta_{T_\theta}
\end{bmatrix}
\tag{5.14}
\]
The outputs resulting from equation (5.14) are regarded as the controller outputs for rotor tilts. The \( \theta_i, i \in \{ 1, 2, 3, 4 \} \) terms are the rotor tilt angle signals sent to the servo motors. The transient behavior of the servo motor is regarded as a second order actuator model.

### 5.3.4 Control Architecture

In the previous section, we discussed all flight controller parameters for tilting rotor quadcopter including differential flatness based feed-forward parameters, PD controller for position control, rotor tilt angle control and attitude control. The complete control architecture of the proposed flight controller is shown in figure (5-2).

![Control Architecture Differential Flatness Based PD Controller](image)

Figure 5-2: Control Architecture Differential Flatness Based PD Controller

The flight controller proposed in this chapter covers the use of differential flatness for maneuvering the quadcopter in tight spaces. The tilt-rotor quadcopter is a very suitable platform for aggressive maneuvering, it is capable of following specific tight trajectories with improved stability and less cross track error. Numerical simulations for tight aggressive trajectory tracking will be presented in Chapter 7.
Chapter 6

Fault-Tolerant Tilt-Rotor Quadcopter

Quadcopters are one of the most popular configurations of unmanned aerial vehicles. They are inherently unstable and rely on four propellers for their control and maneuverability. In the past, different types of flight controllers have been explored by researchers for conventional fault-free quadcopters. If we consider the mechanical design of quadcopters, they have an increased risk of motor or propeller failure during flight. The conventional quadcopter is an under-actuated system, so it is very difficult to stabilize the UAV in case of propeller or motor failure. The main focus of this chapter is to address the fault-tolerant capabilities in quadcopters. Since the tilt-rotor quadcopter is an over-actuated form of conventional quadcopter, it is capable of handling a propeller failure and thus it is a fault-tolerant system. We focus on developing a reconfigurable control law which can take over the UAV in case of loss of one of the rotors and thus guide the tilt-rotor quadcopter complete the flight mission.

6.1 Literature Review of Fault-Tolerant Control

Propeller or motor failure is one of the most common failure in case of quadcopters. Currently, the commercial solution available to deal with propeller failure is emergency parachute which assist in emergency landing of quadcopters. The operational scenario of quadcopters require the design of controllers capable of fault detection, isolation and diagnosis. Once the failure occurs the system must be capable of maintaining the stability of the system and complete the mission without much compromise in system performance. Multicopter with six or more propellers are also popular as the vehicle is able to maintain normal flight if one of the
propellers fails. But multicopters are costly as compared to the quadcopters while applications are the same. Detailed concepts and methods in fault-tolerant control have been discussed in [7]. Blanke states that faults in automated systems can lead to undesired reactions and even damage the system as well as personnel and the environment around the system. Fault-tolerant control combines several disciplines into a common framework to achieve desired performance in a system. In general, faults occurring due to sensors or actuators can be amplified by the closed-loop control systems. The closed-loop system may even hide a fault from observation and loop failure becomes inevitable. Fault-tolerant control systems (FTCS) have intelligent software that monitors behavior of components and function blocks. Faults are isolated, and appropriate remedial actions are taken to prevent that faults develop into critical failures [7].

The FTCS are classified as passive fault-tolerant control system (PFTCS) and active fault-tolerant control system (AFTCS). In passive fault-tolerant control system (PFTCS) the control algorithm is designed to achieve a given objectives in healthy or faulty situation without changing its control law. This approach does not need any fault detection and diagnosis (FDD) or any controller reconfiguration, and it possesses limited fault-tolerant capabilities [61]. Whereas In active fault-tolerant control system (AFTCS) to preserve the ability of system to achieve the objective the control law is changed according to fault situation. The fault diagnosis and identification (FDI) block also termed as diagnosis unit consists residual generator and residual evaluation sub-units. A residual is generated by comparing the process output and the model output, if the residual differs from zero. The residual evaluation compares it to a threshold to decide and indicate fault. Based on the diagnosis result the re-configuration block has to adapt the controller in such a way that the new controller is able to cope with the faulty process [34] [61]. Fault Detection and Isolation (FDI) system for actuator faults for an hexacopter vehicle has been presented in [16]. A diagnostic Thau observer is applied to the hexacopter nonlinear model to generate residual signals. In the fault-free case, residuals are close to zero, while in case of a faulty actuator the value of residuals and fault is detected. Further, Fault isolation is realized by exploiting the mathematical model of the hexacopter. By quickly detecting the fault, the control law can be modified to satisfy the closed-loop requirements of the system and thus making it an active fault-tolerant control.
The quadcopter system becomes under-actuated and it is impossible to maintain full control of all the attitude states and all the translational states [22]. The proposed method suggests spinning the vehicle in the yaw direction, thereby maintaining flight control of a spinning vehicle. They achieve flight by robust feedback linearization which linearized the nonlinear system around an operating point where roll and pitch angles are zero but body rates are nonzero. The closed-loop linearized system is controlled through an $H_\infty$ loop shaping technique. The linear displacement is achieved through another outer control loop under the assumption of small angles approximation for the pitch and roll angles. Thus, the inner attitude control loop handles robustness considerations and the failure of a primary actuator, and the outer loop enables translational flight control. In [30], a control strategy is presented using periodic solutions for a quadcopter experiencing one, two opposite, or three complete rotor failures. The strategy employed is to define an axis, fixed with respect to the vehicle body, and have the vehicle rotate freely about this axis. By tilting this axis, and varying the total amount of thrust produced, the vehicles position can be controlled in three dimension.

Emergency landing procedure of quadcopter has been presented in [23] and [24] by using PID and Backstepping control approach respectively. The strategy is to switch off the propeller aligned on the same quadrotor axis of the failed propeller. This action converts the quadcopter configuration into a birotor aerial vehicle. The UAV becomes free to spin in yaw axis while controlling the remaining attitudes of the UAV and emergency landing procedure is exercised.

Actuator faults and fault-tolerant control (FTC) methods to accommodate such failures have been discussed in [61]. The FTC algorithms have been tested by simulation and experimental means. The developed FTC algorithms cover a wide range of flight controllers including the Gain-Scheduled PID (GS-PID), Model Reference Adaptive Control (MRAC), Sliding Mode Control (SMC), Backstepping Control (BSC), Model Predictive Control (MPC) and Flatness-based Trajectory Planning/Re-planning (FTPR). An extensive detailed study of FTC methods has been presented in [61] and they summarize the main fault-tolerant control methodologies and discuss if the proposed method requires a fault detection and diagnosis (FDD) scheme and whether they have been tested in simulation or experimentally on the quadcopters. The work done by Zhang and Chamseddine in [61] is of great research value and it is considered as the
benchmark for the future fault-tolerant control research. In the present thesis work, the tilting-rotor quadcopter platform is used to explore its fault-tolerant capabilities. It is an over-actuated form of a traditional quadcopter and it is capable of handling a propeller failure, thus making it a fault-tolerant system [34]. The dynamic model of tilting-rotor quadcopter with one propeller failure is derived and a controller is designed to achieve hovering and navigation capability. A robust, fault-tolerant control law and redundant mechanical design of the quadcopter can ensure safe handling of the quadcopter even after the propeller failure. The tilt-rotor mechanism and PD control of the quadcopter have been used to stabilize the quadcopter after the propeller failure and thus control all states of the UAV.

6.2 Tilt-Rotor Quadcopter and Propeller Failure

The dynamic model of the tilt-rotor quadcopter with all functional motors is presented in Chapter 3. When all the propellers of the tilt-rotor quadcopter are working then it yields a stable configuration as a result of symmetry of forces and moments. The equations of motion of tilt-rotor quadcopter are given by equation (3.9) and (3.10) and as shown in figure (2-19).

Assuming that the first propeller/motor fails during hovering flight of quadcopter which is located in the pitch plane. Then, the quadcopter would possess three working propellers and one failed propeller. Once the failure occurs the UAV will experience asymmetry about the yaw axis because of $M_2, M_3, M_4$ moments of working propellers while $M_1 = 0$. Another asymmetry would occur in pitch plane as $F_1 = 0$ and $F_2, F_3, F_4$ would still have some magnitude. The equations of motion can be modified by putting $F_1$ and $M_1$ equal to zero. The free body diagram of tilt-rotor quadcopter with a failed motor is shown in figure (6-1).
\[
\begin{bmatrix}
\dot{x} \\
\dot{y} \\
\dot{z}
\end{bmatrix}
= \begin{bmatrix}
-F_2 s\theta_2 - F_4 s\theta_4 \\
-F_3 s\theta_3 \\
F_2 c\theta_2 + F_3 c\theta_3 + F_4 c\theta_4
\end{bmatrix}
\begin{bmatrix}
C_1 \dot{x} \\
C_2 \dot{y} \\
C_3 \dot{z} + mg
\end{bmatrix}
\] (6.1)

\[
\begin{bmatrix}
p \\
q \\
r
\end{bmatrix}
= \begin{bmatrix}
l(F_2 c\theta_2 - F_4 c\theta_4 - C'_1 \dot{\phi}) \\
+M_2 s\theta_2 + M_4 s\theta_4 + M'_2 + M'_4 \\
l(F_3 c\theta_3 - C'_3 \dot{\psi}) \\
-M_3 s\theta_3 + M'_3
\end{bmatrix}
\begin{bmatrix}
p \\
q \\
r
\end{bmatrix}
\] (6.2)

The nomenclature of various parameters has been provided in Chapter 3. It should be noted that \( F_1 \) and \( M_1 \) terms have vanished from the equations of motion and it will result in asymmetry because of one propeller failure. The components of rotor moment \( M_2, M_3, M_4 \) would not produce a symmetrical outcome which represent unstable dynamics of quadcopter upon propeller...
failure. The available inputs to stabilize and control this system are angular speed $\omega_2, \omega_3, \omega_4$ of three working rotors and tilt angle $\theta_2, \theta_3, \theta_4$.

**Theorem-II:** Considering the dynamics of tilt-rotor quadcopter upon propeller failure given by equations (6.1) and (6.2), the tilt-rotor quadcopter can be controlled in yaw and pitch plane if the third rotor is tilted by an angle $\theta_3$ such that $\theta_3 = c^{-1}[\omega_2^2/(\omega_2^2 c \theta_4 + \omega_2^2 c \theta_2)]$.

**Proof:** When propeller failure occurs, the dynamics of the quadcopter are highly non-linear. The failure of propeller-1 has been considered as shown in figure (6.1), this leads to asymmetry primarily in yaw and pitch plane of the quadcopter. It can be seen that the equations of motion shown in (6.1) and (6.2) yield an unstable configuration. We ignore the drag forces and moments generated because of rotor tilt for simplification. This assumption simplifies the force and moment equations. It is important to consider that the UAV can be stabilized during flight if and only if the angular accelerations $\dot{\rho}, \dot{q}, \dot{r}$ are zero which means either the system has constant or zero body rates. So, the equations for yawing moment and pitching moment are used to predict the required stabilizing conditions for the quadcopter and we assume that the quadcopter experience very little differential forces in roll plane.

\begin{equation}
IF_3 c \theta_3 - M_3 s \theta_3 = 0 
\tag{6.3}
\end{equation}

\begin{equation}
IF_3 s \theta_3 - M_2 c \theta_2 + M_3 c \theta_3 - M_4 c \theta_4 = 0 
\tag{6.4}
\end{equation}

The equations (6.3) and (6.4) can be solved to evaluate the desired $\theta_3$ tilt angle necessary to stabilize the quadcopter after propeller failure.

\begin{equation}
IF_3 c \theta_3 = M_3 s \theta_3 
\tag{6.5}
\end{equation}

\begin{equation}
IF_3 = \frac{M_3 s \theta_3}{c \theta_3} 
\tag{6.6}
\end{equation}

55
The value of \( lF_3 \) is substituted in equation (6.4) to yield:

\[
\frac{M_3 s\theta_3 s\theta_3}{c\theta_3} - M_2 c\theta_2 + M_3 c\theta_3 - M_4 c\theta_4 = 0 \tag{6.7}
\]

\[
M_3 (s^2\theta_3 + c^2\theta_3) = c\theta_3 (M_2 c\theta_2 + M_4 c\theta_4) \tag{6.8}
\]

Substituting \( s^2\theta_3 + c^2\theta_3 = 1 \):

\[
c\theta_3 = \frac{M_3}{(M_2 c\theta_2 + M_4 c\theta_4)} \tag{6.9}
\]

\[
\theta_3 = c^{-1} \left[ \frac{M_3}{(M_2 c\theta_2 + M_4 c\theta_4)} \right] \tag{6.10}
\]

The above expression can be re-written in terms of propeller angular speed as the moment is directly proportional to square of the angular speed of the propeller given by equation (3.13):

\[
\theta_3 = c^{-1} \left[ \frac{\omega_3^2}{(\omega_2^2 c\theta_2 + \omega_4^2 c\theta_4)} \right] \tag{6.11}
\]

This condition should hold for attaining a stable configuration after one propeller failure in the tilt-rotor quadcopter. Otherwise, the system cannot be stabilized or controlled. In fact, once the system is stabilized minor deviation in angular speeds of propellers and rotor tilt angle can be utilized to maneuver the quadcopter.

### 6.3 Controller Design

In this section, the control strategy of the tilting rotor quadcopter in a case of motor failure during the flight is presented. Two PD controllers are used due to compensate the unbalance moments created by an odd number of propellers, and also to stabilize vehicles orientation and make it functional to continue its mission without crash. The vehicle originally has eight independent inputs which includes four speed of propellers and four tilted angle of each motor about its axis. In the case of motor failure, two inputs are automatically out of equations. To compensate for the moment imbalance created due to propeller failure, the speed and tilt angles of the working motors need to be controlled individually. The the tilt mechanism needs to be
set in such a way that it compensates for the moments of the failed propeller. In this work, it has been assumed that motor one stopped working during the flight. The onboard measurement sensor on quadcopter needs to report the failure immediately. Referring to the theorems, the tilting angle of motor 2 and motor 4 should not be altered and they are controlled by the onboard autopilot of the quadcopter, but the tilt angle of motor 3 should be computed by onboard logic and the tilt angle should be set as per theorem-II immediately. The controller reconfiguration strategy is shown in figure (6-2).

**Figure 6-2: Flight Controller Reconfiguration Strategy**

**Theorem-III:** Considering the dynamics of tilt-rotor quadcopter with a failed motor, the necessary hovering speed for each functional rotor can be written as \( \omega_h = \sqrt{\frac{mg}{k_f (2c\theta_2 + c\theta_3)}} \)

**Proof:** In order to compensate for the unbalanced moment after the failure, getting back to the hovering state is necessary. Then, the orientation of the vehicle to a specific pitch or roll angle is obtained to navigate the vehicle to the desired location. The equations of motion for reduced quadcopter dynamics (i.e., with one propeller/motor failure) can be linearized about a hovering state and the required controller parameters are obtained. The force equation for \( z \)-axis for very small Euler angles can be written as:

\[
m\ddot{z} = F_2 c\theta_2 + F_3 c\theta_3 + F_4 c\theta_4 - mg
\]

The rotational speeds of individual motors necessary for hovering (\( \omega_h \)) are obtained by equating \( z \)-axis acceleration to zero, the effect of drag terms at low speeds is omitted and Euler angles
are considered very small which eliminates the small sine and cosine terms.

\[ F_2 c\theta_2 + F_3 c\theta_3 + F_4 c\theta_4 - mg = 0 \]
\[ F_2 c\theta_2 + F_3 c\theta_3 + F_4 c\theta_4 = mg \]  

(6.13)

Substituting the expression for \( F_i \), \( \forall i \in (2, 3, 4) \) as per equation equation (3.12) and \( \omega_i \), \( \forall i \in (2, 3, 4) \) will be equal to \( \omega_h \).

\[ k_f \omega_h^2 c\theta_2 + k_f \omega_h^2 c\theta_3 + k_f \omega_h^2 c\theta_4 = mg \]
\[ k_f \omega_h^2 (c\theta_2 + c\theta_3 + c\theta_4) = mg \]  

(6.14)

This above expression can further be simplified by the assumption of \( \theta_2 = \theta_4 \).

\[ k_f \omega_h^2 (2c\theta_2 + c\theta_3) = mg \]  

(6.15)

The required expression for necessary hovering speed after a propeller failure for the tilt-rotor quadcopter can be written as:

\[ \omega_h = \sqrt{\frac{mg}{k_f(2c\theta_2 + c\theta_3)}} \]  

(6.16)

The expressions for desired roll and pitch orientation angles are shown in equation (6.17) and (6.18) respectively. It is same as shown previously in Chapter 4.

\[ \phi^{des} = \frac{\ddot{r}_x^{des} s\phi - \ddot{r}_y^{des} c\phi}{g} \]  

(6.17)

\[ \theta^{des} = \frac{\ddot{r}_x^{des} c\phi + \ddot{r}_y^{des} s\phi}{g} \]  

(6.18)

The change in rotor speed necessary for motion in z-direction is computed in a similar way as discussed in Chapter 4. It is given by the expression in equation (6.19) and it can be rewritten.
as equation (6.20) by referring equation (6.15).

\[ m\ddot{z} = \Sigma F - mg \quad (6.19) \]

\[ m\ddot{z} = k_f \omega_h^2 (2c\theta_2 + c\theta_3) - mg \quad (6.20) \]

Again, we introduce the perturbation in acceleration and rotor speed to compute change in rotor speed necessary for motion in \( z \)-direction. The above equation is modified to the following expression.

\[ m(\ddot{z} + \ddot{r}_z^\text{des}) = k_f (\omega^2_h + \Delta \omega_f)^2 (2c\theta_2 + c\theta_3) - mg \quad (6.21) \]

\[ m(\ddot{z} + \ddot{r}_z^\text{des}) = k_f (\omega^2_h + \Delta \omega_f^2 + 2\omega_h\Delta \omega_f)(2c\theta_2 + c\theta_3) - mg \quad (6.22) \]

Subtracting equation (6.20) from (6.22) and ignoring higher order terms such as \( \Delta \omega_f^2 \).

\[ m\ddot{r}_z^\text{des} = 2k_f \omega_h \Delta \omega_f (2c\theta_2 + c\theta_3) \quad (6.23) \]

\[ \Delta \omega_f = \frac{m\ddot{r}_z^\text{des}}{2k_f \omega_h (2c\theta_2 + c\theta_3)} \quad (6.24) \]

Equation (6.24) gives the expression for change in rotor speed necessary for motion in \( z \)-direction for the tilt-rotor quadcopter after a propeller failure. It should be noted that if \( \theta_2 = \theta_3 = 0 \), the above equation reduces to the expression for change in rotor speed necessary for motion in \( z \)-direction for a tricopter as shown in equation (6.25).

\[ \Delta \omega_f = \frac{m\ddot{r}_z^\text{des}}{6k_f \omega_h} \quad (6.25) \]

We have already discussed the methodology for controlling rotor speeds and rotor tilt for tracking a desired trajectory in chapter (4) and (5). But when the propeller failure occurs, the dynamics of the system are changed. We have shown a scheme of linearizing the dynamics of the UAV with a failed propeller. The fault-tolerant quadcopter system possesses two flight controllers. The first controller works during the normal operation of the UAV and the second flight controller takes over once there is a propeller failure as shown in figure (6-2). A detailed
discussion of the first flight controller has already been done in Chapters 4 and 5. The second flight controller has reduced number of control inputs and a different set of proportional and derivative gains. The position errors \((e_x, e_y, e_z)\), velocity errors \((\dot{e}_x, \dot{e}_y, \dot{e}_z)\), the Euler angles errors \((e_\phi, e_\theta, e_\psi)\) and angular rate errors \((e_p, e_q, e_r)\) are computed by relative difference of the feedback and instantaneous desired values. These errors are utilized by the PD parameters of the second controller to compute the desired propeller rotational speeds and rotor tilts. It should be noted that the PD parameters of the second flight controller are different than the first flight controller.

\[
\begin{align*}
\dot{r}_i^{des} & = k'_p e_i + k'_d \dot{e}_i \quad \forall i \in \{x, y, z\} \\
\Delta \omega_\phi & = k'_p e_\phi + k'_d \dot{e}_p \\
\Delta \omega_\theta & = k'_p e_\theta + k'_d \dot{e}_q \\
\Delta \omega_\psi & = k'_p e_\psi + k'_d \dot{e}_r \\
\Delta \theta_T^x & = k'_p x_T e_x + k'_d x_T \dot{e}_x \\
\Delta \theta_T^y & = k'_p y_T e_y + k'_d y_T \dot{e}_y \\
\Delta \theta_T^\phi & = k'_p \phi_T e_\phi + k'_d \phi_T \dot{e}_p \\
\Delta \theta_T^\theta & = k'_p \theta_T e_\theta + k'_d \theta_T e_q
\end{align*}
\]

(6.26)

The following matrix represents the control allocation scheme of tilting rotor quadcopter controls after the propeller failure.
We utilize the expression obtained in equation (6.11) by the name of $\theta_c$ for reference. It can be seen that $\omega_1$ and $\theta_1$ terms are no longer present in equation (6.27) which represents the failure case. The rotor tilt and rotational motion are used for hover and $xy$-position control, yaw, roll and pitch control of the UAV. The control allocation matrix represents how the available controls of a tilt-rotor quadcopter can be utilized after the propeller failure to stabilize and control the quadcopter in three dimensions and achieve navigational capabilities. This work considers failure of only one propeller during flight and does not address the scenario if there are more than one propeller failures. The above formulation discussed the method for stabilizing and controlling the quadcopter if the propeller failure occurs in $x$-axis ($1^{st}$ or $3^{rd}$ propeller). The other propellers are equally prone to failure during flight. The similar formulation can be extended if the propeller failure occurs in $y$-axis ($2^{nd}$ or $4^{th}$ propeller) but the controller gains will be different for those cases and it also requires a complete study. The simulation results for propeller failures in $x$-axis and $y$-axis will be discussed in Chapter 7.
Chapter 7

Numerical Simulations

This chapter presents simulation studies to validate the tilt-rotor quadcopter system and provides insight information for the concepts stated from chapters (3) to (6). The numerical simulation model was developed using MATLAB R2015a. The quadcopter dynamic model is discretized and the dynamic equations are solved by the Euler’s integration method.

7.1 Case 1a: Way Point Navigation Without Uncertainties

The initial position of the UAV was set to origin and the quadcopter is commanded to follow a set of predefined way points. The desired altitude was set to five meters and desired yaw angle was set zero degree. The set of predefined way points are [6, 6; 6, 12; 10, 12; 14, 16; 18, 16; 18, 20; 20, 20] and as discussed earlier the quadcopter has to maintain the desired altitude during flight. The UAV must reach the desired height and then exercise maneuvers to the destination way points. During flight, the orientation of the vehicle should change according to reference inputs obtained by equations (4.7) and (4.8) without losing the altitude. The UAV obtains necessary hovering rotor speed and change in rotor speed for movement along z-axis as per equation (3.20) and (4.14). The UAV flight is governed by the flight controller described in chapter (4). The time response of various flight parameters during the flight has been presented in the figures (7-1) to (7-7).
Figure 7-1: Three Dimensional Trajectory Plot (Case 1a)

Figure 7-2: Variation of Euler Angles (Case 1a)
Figure 7-3: Position Errors in Way Point Navigation (Case 1a)

Figure 7-4: Rotor Speed Variation (Case 1a)
Figure 7-5: Rotor Tilt Angle Variation (Case 1a)

Figure 7-6: Variation in Body Rates (Case 1a)
The time history plots of numerical simulations without uncertainties are covered from figure (7-1) to (7-7) for a tilt-rotor quadcopter. Figure (7-1) and (7-7) show the 3-D and 2-D flight trajectory plots respectively. It should be noted that the UAV is able to track the desired trajectory without any cross track error. Figure (7-2) shows the variation of Euler angles ($\phi, \theta, \psi$) during flight which are in acceptable range and ensure stable flight characteristics. The variation of rotor tilt angles and rotor speeds along the trajectory is shown in figure (7-4) and (7-5) it can be inferred that the rotor tilts and rotor speed vary simultaneously to minimize of position errors in the $xy$ space. Figure (7-6) represents the variation of body rates ($p, q, r$) respectively and it can be seen that any oscillations in the system damp out with ease.
7.2 Case 1b: Numerical Simulations With Uncertainties

Here, a comparative simulation study is performed simultaneously on tilt-rotor quadcopter and the conventional quadcopter for performance comparison. The simulation is performed with introducing uncertainties in orientation angles by introducing normally distributed random noise in \( \theta \) and \( \phi \) angles. This disturbance causes the UAVs to drift from the desired trajectory. Such drifting response can be attributed to the presence of wind gusts or error in sensor readings of aircraft. It should be noted that the mass and moment of inertia properties of both the UAVs are considered same. The controllers of rotational speed of propellers are also identical in both UAVs. The tilt-rotor quadcopter has extra control inputs in terms of rotor tilt for position and orientation control. In the simulation, both quadcopters have to start from the origin and reach the first way point \([15, 15]\) by maintaining a height above ground of five meters. Both UAVs take off together to follow this trajectory and encounter the uncertainty during the same time \( t = 0 \) to 10 seconds. Normally distributed random noise of standard deviation 0.8 deg and mean zero is introduced in roll and pitch angles. Similarly, normal distributed noise of standard deviation 5 deg and mean zero is introduced in yaw angle. The time response during the flight has been presented in the figures (7-8) to (7-14).

Figure 7-8: Three Dimensional Trajectory Plot (Case 1b)
Figure 7-9: Variation of Euler Angles (Case 1b)

Figure 7-10: Position Errors in Way Point Navigation (Case 1b)
Figure 7-11: Rotor Speed Variation (Case 1b)

Figure 7-12: Rotor Tilt Angle Variation (Case 1b)
Figure 7-13: Variation in Body Rates (Case 1b)

Figure 7-14: Two Dimensional Track Plot (Case 1b)
The flight characteristics with uncertainty in parameters are presented in figures (7-8) to (7-14). This analysis includes a performance comparison of tilt-rotor quadcopter with respect to a conventional quadcopter. As discussed earlier, mass and moment of inertia properties of both UAVs are same. The controllers of rotational speed of propellers are also identical. The tilt-rotor quadcopter has extra control inputs of rotor tilt for position and orientation control. Figure (7-9) shows the variation of Euler angles during flight, it can be inferred that the quadcopter system has encountered uncertainties from $t = 0$ to 10 seconds. Similarly, figure (7-11) shows the variation in rotors speed subjected to uncertainties. Both the UAVs start flight at the same time to reach point [15, 15] with a height of 5 meters. Figure (7-8) shows three dimensional track plot of the UAVs and figure (7-10) shows that the position errors are minimized faster in case of tilt-rotor quadcopter as compared to the conventional quadcopter. It should be noted from figure (7-14) that the tilt-rotor quadcopter is more efficient because of extra control inputs and when both UAVs are subjected to same disturbance noise the tilt-rotor quadcopter generates lesser cross track error. Both UAVs start their flight together at time $t = 0$ seconds, the tilt-rotor quadcopter has reached close to point [15, 15] much earlier (approximately 15 sec) as compared to the conventional quadcopter. The variation of rotor tilt towards disturbance cancellation can be seen in figure (7-12). It shows the advantage in terms of system performance and stability of a tilt-rotor quadcopter when compared against the conventional quadcopter system. A more accurate comparison could be carried out by determining the optimal controller parameters for both the designs of the quadcopter and then comparing the performance.
7.3 Case 2: Complex Trajectory Flight

The quadcopter dynamic model and differential flatness based controller presented in chapter (5) has been validated by the numerical simulations in this section. The vehicles initial position was set to [0.8, 0, 5] which means the quadcopter is already in flight and then the quadcopter was commanded by the pilot to follow the desired trajectory as per equation (5.4) discussed in chapter (5) as a special maneuver. The simulation has been performed to evaluate how feed-forward control terms of differential flatness based flight controller help in minimizing the position errors and track closest to the desired trajectory as compared to conventional PD flight controller of tilt-rotor quadcopter. The simulation results include a comparative study between tilt-rotor quadcopter with and without DF based PD controller w.r.t. the conventional quadcopter with and without DF based PD controller. The comparison has been presented with and without environmental or sensor uncertainties to show a real-world application.

7.3.1 Numerical Simulations For Normal Flight

As discussed earlier, the UAV starts from [0.8, 0, 5] and set to follow a tight trajectory given by equation (5.4). During flight, the orientation of the vehicle should change according to reference inputs obtained by equations (4.7) and (4.8). The time period for trigonometric functions in equation (4.8) is taken as 15 seconds. The amplitudes [a, b, c] are [0.8, 0.8, 1] meters respectively and parameter n is equal to 5 meters. It should be noted the the desired trajectory forces quadcopter to move in all three axes simultaneously and it is an inclined FIGURE-∞ pattern in three dimension. The simulation is performed for four cases i) Tilt-rotor quadcopter with differential flatness based PD controller, ii) Tilt -rotor quadcopter with conventional PD controller, iii) Conventional quadcopter with differential flatness based PD controller, iv) Conventional quadcopter with normal PD controller. When differential flatness parameters are not present then the flight controller behaves like a normal PD flight controller. Whereas, when differential flatness parameters are introduced then the required feed-forward control terms are computed and added to enhance the existing PD flight controller. The time response of various flight parameters during flight is presented from figures (7-15) to (7-23).
Figure 7-15: Three Dimensional Trajectory Plot

Figure 7-16: Two Dimensional Trajectory Plot
Figure 7-17: Variation of Euler Angles

Figure 7-18: Position Errors during Special Maneuver
Figure 7-19: Rotor Speed Variation

Figure 7-20: Rotor Tilt Angle Variation
Figure 7-21: Variation in Body Rates

Figure 7-22: Variation in Feed-Forward Control Parameters
Figures (7-15) to (7-23) cover the numerical simulation results for normal flight condition means without any sensor or environmental uncertainty. Figure (7-15) shows a three dimensional trajectory plot and figure (7-16) shows two dimensional flight trajectory of the UAV. The initial position coordinates are [0.8, 0, 5] for all cases. The UAV is able to track the desired path in all cases when it passes through the center of the desired path. But when the UAV has to turn around the tight corners, the PD flight controller in case of conventional quadcopter develops some error w.r.t. the reference trajectory. Whereas the tilt-rotor quadcopter is able to track the desired trajectory in both cases. Figure (7-17) shows the variation of Euler angles during flight. It can be seen that when the UAV starts the tight maneuver, initial transient peak is more in case of conventional quadcopter in both the cases as compared to the tilt-rotor quadcopter. During rest of the flight the Euler angles are nearly identical. Figure (7-18) represents the position error in xyz-axes during flight. It can be seen that the position error in x and y-position are less in case of tilt-rotor quadcopter when compared against the conventional quadcopter in all cases. Figure (7-20) shows variation of tilt angles during flight to minimize the position and orientation angle errors. It can be seen that the UAV is able to change the tilt angles in real time.
to minimize the errors in position and orientation. The rotor tilt demand is slightly higher in case of PD controller as compared to differential flatness based PD controller and the rotor tilts are zero for conventional quadcopter. Figure (7-22) represents the feed-forward control terms which are computed by differentially flat equations, these terms help the UAV to minimize the position errors during tight maneuvers in case of differential flatness based PD controller. The magnitude of $\ddot{u}_x, \ddot{u}_y, \ddot{u}_z$ change in real time as per equation (5.7) to minimize the position errors during flight.
7.3.2 Numerical Simulations With Uncertainty Parameters

In this case, again the UAV starts from same initial conditions $[0.8, 0, 5]$ and is set to follow a tight trajectory given by equation (5.4). The comparative study is similar to the previous case. The desired trajectory parameters have been kept same as in case of normal flight. But, in this case simulation is performed with uncertainties in orientation angles by introducing normally distributed random noise of standard deviation 0.4 deg and mean zero in in $\theta$ angle. Similarly, normally distributed noise of standard deviation 0.6 deg and mean zero in $\phi$ angle. This disturbance in Euler angles incorporates a real world scenario. The uncertainty is present throughout the simulation. The presence of such kind of noise can be attributed to sensor measurement errors, computational and sampling delays or environmental disturbances such as wind gust during flight. The flight parameters are presented in the figures.

Figure 7-24: Three Dimensional Trajectory Plot
Figure 7-25: Two Dimensional Trajectory Plot

Figure 7-26: Variation of Euler Angles

80
Figure 7-27: Position Errors during Special Maneuver

Figure 7-28: Rotor Speed Variation
Figure 7-29: Rotor Tilt Angle Variation

Figure 7-30: Variation in Accelerations During Flight
Figure (7-24) to (7-30) show the simulation results with sensor error or environmental uncertainty during flight. It can be seen that the UAV drifts from the desired course of trajectory in most cases. But, the tilt-rotor quadcopter shows more robust behavior as compared to conventional quadcopter as shown in figure (7-24) and (7-25), the tilt-rotor quadcopter tracks the desired trajectory more closely as compared to the conventional quadcopter. Figure (7-27) highlights the position errors, the conventional quadcopter experiences more position errors in xy-plane as compared to tilt-rotor quadcopter. Figure (7-29) shows the variation of rotor tilt angles during the flight in presence of uncertainty and the tilt-rotor quadcopter is able to adjust the rotor tilt angles to reject the disturbance. The tilt-rotor quadcopter has shown a very robust behavior towards the disturbances and sudden changes in accelerations. On the other hand, the conventional quadcopter experiences cross track errors from desired trajectory as it is not able to compensate for sudden jerks and uncertainty. The robust behavior of tilt-rotor quadcopter towards uncertainty and jerks can be attributed to the presence of rotor tilt functionality and differential flatness based feed-forward control further enhances the existing PD controller.
7.4 Case 3: Numerical Simulations for Fault-Tolerant Flight

Here, the numerical simulations for showing the fault-tolerant capabilities of the tilt-rotor quadcopter are presented. This work presents two different scenarios to evaluate the UAVs response in the case of motor failure: i) Motor failure in $x$-axis (Pitch Plane), ii) Motor failure in $y$-axis (Roll Plane). In both cases, the altitude was maintained and the flight controller reconfiguration was exercised as discussed in chapter (6) controller design section.

7.4.1 Motor failure in $x$-axis (Pitch plane)

In this case, the quadcopter starts from the origin to follow a way point navigation predefined trajectory. The desired altitude is set to 10 m and motor one stops working at $t = 20$ sec. It induces asymmetry of thrust in pitch plane and the dynamics become highly non-linear, the UAV becomes unstable. The fault-tolerant controller re-configuration is exercised to control the UAV with a compromise in flight performance after motor failure to complete the mission.

![Three Dimensional Trajectory Plot](image)

Figure 7-31: Three Dimensional Trajectory Plot
Figure 7-32: Variation of Euler Angles

Figure 7-33: Position Errors in Way Point Navigation
Figure 7-34: Rotor Speed Variation

Figure 7-35: Rotor Tilt Angle Variation
Figure (7-31) shows the UAV trajectory in three dimension during flight. The UAV becomes unstable after motor failure but the new flight controller takes over to navigate the UAV. Figure (7-33) shows that position errors in xyz-axes and it can be seen that the UAV is able to minimize the position errors. Figure (7-32) shows the variation of Euler angles as they become become oscillatory at t = 20 sec after the failure of motor one. The new flight controller is able to control the attitude of the UAV with a compromise in flight performance. Figure (7-34) and (7-35) represent the variation of rotor angular speeds and tilt angles respectively. At, t = 20 sec the angular speed of rotor one becomes zero and the new flight controller takes over. The variation of rotor angular speeds (ω₂, ω₃, ω₄) and rotor tilts (θ₂, θ₄) is exercised as per the new control law and tilt angle of third rotor (θ₃) is controlled by a combination of theorem discussed in chapter (6) and the new flight controller.

7.4.2 Motor failure in y-axis (Roll Plane)

In this case, the quadcopter starts from the origin to follow a way point navigation prede- fined trajectory. The desired altitude is set to 10 m and motor two stops working at t = 20 sec. It induces asymmetry of thrust in roll plane and the dynamics become highly non-linear, the UAV becomes unstable. The fault-tolerant controller re-configuration is exercised to control the UAV with a compromise in flight performance after motor failure to complete the mission.

Figure 7-36: Three Dimensional Trajectory Plot
Figure 7-37: Variation of Euler Angles

Figure 7-38: Position Errors in Way Point Navigation
Figure 7-39: Rotor Speed Variation

Figure 7-40: Rotor Tilt Angle Variation
Figure (7-36) shows the UAV trajectory in three dimension during flight. The UAV becomes unstable after motor failure but the new flight controller takes over to navigate the UAV. Figure (7-38) shows that position errors in xyz-axes and it can be seen that the UAV is able to minimize the position errors. Figure (7-37) shows the variation of Euler angles as they become oscillatory at t = 20 sec after the failure of motor two. The new flight controller is able to control the attitude of the UAV with a compromise in flight performance. Figure (7-39) and (7-40) represent the variation of rotor angular speeds and tilt angles respectively. At, t = 20 sec the angular speed of rotor two becomes zero and the new flight controller takes over. The variation of rotor angular speeds (ω₁, ω₃, ω₄) and rotor tilts (θ₁, θ₃) is exercised as per the new control law and tilt angle of third rotor (θ₄) is controlled by a combination of theorem discussed in chapter (6) and the new flight controller.

This chapter presented a proof of concepts discussed from chapter (3) to (6) by simulation studies for the tilt-rotor quadcopter. The simulation studies included validation of position, attitude and fault-tolerant flight controller for the tilt-rotor quadcopter UAV. The results were presented by means of figures along with analysis of flight parameters.
Chapter 8

Conclusion and Future work

8.1 Conclusions

This thesis presented a detailed literature review on existing capabilities of various types of quadcopter platforms. The main focus of the thesis was to study tilt-rotor quadscooters. The mathematical model of tilt-rotor quadcopter was developed and its relationship with respect to the conventional quadcopter was presented in a mathematical form. Chapter 3 described mathematical formulation of equations of motion for tilt-rotor quadcopter. Subsequently, a novel PD flight controller was presented in Chapter 4. We described the method of synchronizing rotor tilt motion with rotor angular motion in detail for position and attitude control of the quadcopter in three dimensions. The complete control architecture was presented by means of signal flow diagram. In chapter 5, we described the applications of differential flatness based feed-forward control for a tilt-rotor quadcopter. The feed-forward control method enhances the controller performance when the UAV experiences large accelerations. The addition of differential flatness based control parameters helps the UAV to follow tight trajectories and the UAV develops lesser cross track error while following tight turns. Chapter 6 covered detailed literature review of existing fault-tolerance techniques for quadcopters and presented tilt-rotor quadcopter as a fault-tolerant quadcopter system capable of handling one motor failure. The dynamic model of tilting-rotor quadcopter with one propeller failure was derived and a re-configurable flight controller was designed. A robust, fault-tolerant control law and redundant mechanical design of the quadcopter would ensure safe handling of the quadcopter even after the propeller failure. The mathematical formulation of quadcopter dynamics with a failed propeller appeared
similar to a tri-copter and it was found that the UAV can continue the mission after a motor failure but with reduced flight performance. It was shown that the quadcopter can be controlled if the rotor diagonally opposite to the failed rotor is tilted by an angle calculated by the Euler equations. The UAV employed two functional PD controllers, one for normal flight operation and the other one to control the UAV in case of a motor failure.

Chapter (7) presented numerical simulations for validation of the concepts developed throughout the thesis. The way point navigation flight where we saw that the UAV was able to follow the desired flight path with ease by using PD flight controller. The flight simulations in presence of environmental and sensor uncertainties showed that the tilt-rotor quadcopter is more resistant towards disturbance as compared to the conventional quadcopter. The UAV was able to follow an inclined figure-∞ shaped complex trajectory by employing differential flatness based feed-forward flight controller and showed good disturbance rejection capability. The simulation for one motor failure in along x-axis or y-axis was also presented and we saw that the UAV was able to complete the flight mission by utilizing a re-configurable flight controller.

8.2 Future Works

In the present thesis, the control strategy of the tilt-rotor quadcopter have been discussed. The future research will include the following works:

* Implementation of the proposed PD flight controller on the real tilt-rotor quadcopter platform and validate the flight controller performance by means of real flight experiment.

* Implementation of differential flatness based flight controller on the actual tilt-rotor UAV to validate the complex trajectory tracking concept by real flight experiment.

* Development of re-configurable flight control software to test fault-tolerant algorithms on actual tilt-rotor UAV prototype for one motor failure by actual flight experiments.

* Study of tilt-rotor quadcopter as a fault tolerant system from a systems engineering point of view which can address various faults based on the redundancy management scheme.
Chapter 9

Appendix

Publications


Intellectual Property

- Alireza Nemati, Kelly Cohen, Manish Kumar and Rumit Kumar. Fault-Tolerant Quadcopter. Invention Disclosure Has Been Filed by University of Cincinnati, February 2016.
References


[34] A. Nemati, R. Kumar, and M. Kumar. Stability and control of tilting-rotor quadcopter in case of a propeller failure.


