I, Mahesh Nagesh, hereby submit this original work as part of the requirements for the degree of Master of Science in Mechanical Engineering.

It is entitled:
Rotordynamic Design Analysis of a Squeeze Film Damper Test Rig

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Rotordynamic Design Analysis of a Squeeze Film Damper Test Rig

A thesis submitted to the Graduate School of the University of Cincinnati in partial fulfillment of the requirements for the degree of Master of Science

In the Department of Mechanical and Materials Engineering of the College of Engineering and Applied Science

March 2017

By

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Abstract

An analytical rotordynamic design study was conducted on a formerly operating squeeze film damper test rig comprising of an overhung rotor-bearing system; the sealed squeeze film damper test module is placed at the overhanging end. The test rig uses an eccentric rotating and hence whirling disk to provide variable parameters to test the performance of the Squeeze Film Damper. Essential parameters of the rotor-bearing system such as rotor critical speeds and sub-critical synchronous rotor response are calculated using simple Beam theory, Transfer Matrix Method (TMM) and Finite Element Method (FEM). Two approaches of FEM are used to calculate the synchronous response of rotors viz. Harmonic Response Method and Transient Time Integration Method. The critical speeds and synchronous response obtained from various methods match well. Analytical squeeze film damper relations are used to calculate approximate damping values of the damper test module. A residual unbalance response of the rotor system along with a static eccentricity provides higher rotor response and hence larger damping values than intended. A Finite Element Method study of the performance of the damped rotor-bearing system to approximate operation of the squeeze film damper test rig is provides reduced rotor orbits indicating incorrect operation of the test rig. The same is validated using previously acquired data from the test rig system.
Acknowledgements

Success is seldom a path traversed alone, and success is never achieved without expressing gratitude to all members who have made contributions throughout the journey. Foremost I would like to thank my advisor Dr. Jay Kim, for his inspiring courses and guidance throughout this thesis work. I would like to acknowledge the financial support he has arranged for me, which kept me in good times throughout my study here at the University of Cincinnati.

I am thankful to Dr. Randall Alleman for his vast range of courses in experimental modal analysis and vibrations, and for his continued support for education and learning beyond the classroom. My sincere thanks to Dr. David Thompson for his countless insights and discussions on various topics during my course of work with him. Special thanks to all the faculty members for whom I have been a Teaching Assistant. The learning experience with them has been amazing and I shall cherish them for long.

Mention must be made of my friends here in Cincinnati starting from my lab mates in SDRL, friends at my work place, roommates, teachers and trainees at Yoshisu karate dojo, members within Cincinnati community including some amazing musicians, teachers and academy award winners who never cease to inspire me, and have made my time at Cincinnati memorable.

This thesis would have never been possible without the immense support of my parents, my elder brother’s constant motivation, and care and love of family members here in the USA and back in India. I dedicate this thesis for their support and faith in my work and the word thanks can never fully acknowledge their contributions. Above all, my gratefulness and obeisance to the almighty and spiritual masters upon whom I have always entrusted my past, present and future.
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1 Introduction

1.1 Motivation and Significance

Gas turbines have played a major role in aircraft engines. There is a constant thrust to make more powerful, efficient, less noisy and safer engines that are also extremely light in weight than ones currently available. Many of these requirements are extreme engineering challenges that often conflict one another. For example, lighter engines require migration from metal alloy based components to composites and other new lightweight materials whose characteristics are not well understood and complex to predict. Vibration and related issues are also very important in design of aircraft engines. Since rotational motion creates peculiar characteristics that are not found in non-rotating systems, a thorough understanding of rotordynamics, a field that studies rotating system (turbines) vibration and associated fields, is hence essential.

Rolling element bearings are commonly applied in aircraft engines for safety reasons because fluid film journal bearings can have abrupt, unpredictable failures. However, rolling element bearings provide very little damping; therefore, additional damping is required to control high transient responses of the rotor system. A squeeze film dampers (SFD), which comprises a rolling element bearing contained within a fluid film bearing is hence used to add damping to a rotating system.

Squeeze film dampers [1, 2] consist of a rolling element bearing floating in a thin layer of fluid between the outer race of the bearing and the bearing housing. The rotational motion of the outer race of the rolling element bearing is prevented by an anti-rotation pin allowing only the small whirl motion. Thus, the outer race of the bearing undergoes a rigid body translational motion along with the whirl path. The damping effect is generated by the squeezing effect due to rotor
displacements (whirl). The optimum operations of such bearings depend on both fluid damping and associated parameters along with rotordynamic response to various load conditions. Substantial analytical [1-7] and experimental [8-12] studies have been conducted to study optimum design of SFDs.

1.2 Fundamental Concepts of Relevant Rotordynamics

Rotordynamics is a branch of dynamics dealing with mechanical components that rotate about a given axis at a high speed, and generates significant angular momentum about that axis. This rotating component is called a rotor and all parts that do not rotate are defined as a stator. Rotors are suspended through cylindrical hinges or bearings thus enabling rotational motion about its axis. Although celestial objects that rotate about an unconstrained axis and governed by angular momentum conservation principles are also termed rotors, this thesis is restricted to fixed rotors i.e. rotors that are constrained to rotate about a fixed axis by means of bearings [13-15].

1.2.1 Stationary Coordinate System

Coordinate systems used in rotordynamics may either be stationary or fixed to the rotating body, also referred as rotating coordinate system each with its own merits and limitations. An accepted convention of representing a stationary coordinate system is described as follows [13-15].

Consider Figure 1-1. A stationary coordinate system Oxyz is defined by three mutually perpendicular axes Ox, Oy, Oz intersecting at the origin O where the physical properties including center of mass is assumed to be located. The axis of rotor coincides with Oz at equilibrium; Ox, Oy, Oz in order form a right-handed coordinate system. As an accepted convention, Oz is always considered horizontal and Oy is considered vertical.
Figure 1-2 shows the coordinate system described by Figure 1-1 along the various axes. For a rotor rotating about $Oz$, the angle of rotation is given as $\theta_z$ and angular velocity is given as $\omega_z$. Unless specified alternately, non-subscript notations $\theta$ and $\omega$ generally indicate $\theta_z$ and $\omega_z$ respectively.

1.2.2 Gyroscopic Couples

Gyroscopic Couples arise in rotordynamic analysis due to conservation of angular momentum. These couples act along the axes perpendicular to axis of rotation $Oz$. If $\dot{\theta}$ and $\ddot{\theta}$ respectively
represent angular velocity and angular acceleration about an axis represented by a subscript, then the gyroscopic moments due to the various forces acting on the rotor, given as $M_x$ and $M_y$ for moments about $x$ and $y$ axis respectively is represented as follows.

$$M_x = J_t \ddot{\theta}_x + J_p \Omega \dot{\theta}_y$$  \hspace{1cm} (1.1)$$

$$M_y = J_t \ddot{\theta}_y - J_p \Omega \dot{\theta}_x$$  \hspace{1cm} (1.2)$$

Where $J_t$ and $J_p$ are the diametral and polar moment of a rigid rotor of mass $m$ undergoing whirling at $\Omega$.

1.2.3 Jeffcott Rotor

A Jeffcott Rotor undergoing a circular whirling is shown schematically in Figure 1-2. $\Omega$ is the speed of rotation about the shaft (spin) and $\omega$ is the angular velocity of the motion of center of the shaft called *whirling*. When $\Omega$ and $\omega$ are the same, the shaft motion is called synchronous whirling.

The equations of motion for synchronous whirling due to eccentricity $e$ in $x$ and $y$ directions are

$$m\ddot{x} + c\dot{x} + kx = me\Omega^2 \cos(\Omega t)$$  \hspace{1cm} (1.3)$$

$$m\ddot{y} + c\dot{y} + ky = me\Omega^2 \sin(\Omega t)$$  \hspace{1cm} (1.4)$$

Where $m$, $c$ and $k$ are the mass, damping and stiffness respectively.
1.2.4 Rigid Body Rotor

The Jeffcott is an idealistic, highly simplified approximation of rotor systems. Most turbines use large rotating bodies. Figure 1-4 shows a rigid body rotor comprising of mass \( m \), and inertias \( J_t \) and \( J_p \) with a massless shaft of stiffness \( k_{shaft} \) supported by bearings rotating about the \( Oz \) axis. This is a more meaningful representation of large rotor systems. The equations of motion in stationary coordinates for the rotor shown assuming isotropic bearings in Figure 1-4 is

\[
\begin{bmatrix}
    m & 0 & 0 & 0 \\
    0 & m & 0 & 0 \\
    0 & 0 & J_t & 0 \\
    0 & 0 & J_r & J_\theta \\
\end{bmatrix}
\begin{bmatrix}
    \ddot{x} \\
    \ddot{y} \\
    \ddot{\theta}_x \\
    \ddot{\theta}_y \\
\end{bmatrix}
\begin{bmatrix}
    c & 0 & 0 & c_c \\
    0 & c & -c_c & 0 \\
    0 & -c_c & c_\theta & \Omega J_p \\
    -c_c & 0 & -\Omega J_p & c_\theta \\
\end{bmatrix}
\begin{bmatrix}
    \dot{x} \\
    \dot{y} \\
    \dot{\theta}_x \\
    \dot{\theta}_y \\
\end{bmatrix}
\begin{bmatrix}
    k & 0 & 0 & k_c \\
    0 & k & -k_c & 0 \\
    0 & -k_c & k_\theta & 0 \\
    k_c & 0 & 0 & k_\theta \\
\end{bmatrix}
\begin{bmatrix}
    x \\
    y \\
    \theta_x \\
    \theta_y \\
\end{bmatrix} = \begin{bmatrix}
    F_x \\
    F_y \\
    0 \\
    0 \\
\end{bmatrix}
\]  
(1.5)
Where \( m \) is the mass of rigid rotor rotating at \( \omega \), supported by bearings with damping \( c \) and stiffness \( k \); \( c_c \) and \( k_c \) are the cross coupled damping and stiffness; \( c_\theta \) and \( k_\theta \) is rotational damping and stiffness; \( \Omega \) is whirl velocity. The external forces for synchronous whirling are same as that of Jeffcott Rotor.

It must be noted that the above equation is of the form \([M]\ddot{x} + [C]\dot{x} + [K]x = [F]\), which is in the same form as a second-order differential equation describing vibratory motion of a given body. Steady state solutions can be obtained by adopting a similar approach to the one used for Jeffcott rotors.

**Figure 1-4 – Rigid Body Rotor**

### 1.2.5 Complex variables Description of Equation of Motion

A complex variable defined in the form of \( c = a + jb \) where \( j = \sqrt{-1} \) can be used to describe equations of motions of a system in planar motion by matching the real and imaginary parts respectively with \( x \) and \( y \) components. Adoption of the complex variable description serves two purposes; it reduces the total number of equations required to represent the motion and also has the important advantage that it enables tracking the direction of motion [13, 14, 16, 17]. Since
study of rotordynamics predominantly involves study of rotor orbits i.e. simultaneous tracking of \( x \) and \( y \) displacements along time, a response vector in stationary coordinates can be defined as such

\[
\mathbf{r} = x + jy
\]  

(1.6)

Applying this to equations 1.3 and 1.4, the two equations can be represented as one complex variable equation as follows.

\[
m\ddot{r} + c\dot{r} + kr = me\Omega^2 e^{j\omega t}
\]  

(1.7)

The response can be assumed to be \( \mathbf{r} = R e^{j\omega t} \). Substituting this to equation 1.7, we obtain

\[
R = \frac{me\Omega^2}{[k - \Omega^2m + j\Omega c]} = \frac{e\left(\frac{\Omega}{\omega_n}\right)^2}{1 - \left(\frac{\Omega}{\omega_n}\right)^2 + j2\zeta \left(\frac{\Omega}{\omega_n}\right)} = \frac{e\left(\frac{\Omega}{\omega_n}\right)^2}{\sqrt{1 - \left(\frac{\Omega}{\omega_n}\right)^2}^2} \left(2\zeta \left(\frac{\Omega}{\omega_n}\right)^2\right) e^{-j\phi}
\]  

(1.8)

Where phase \( \phi = \tan^{-1}\left(\frac{2\zeta \left(\frac{\Omega}{\omega_n}\right)}{1 - \left(\frac{\Omega}{\omega_n}\right)^2}\right) \)

1.2.6 Synchronous and Asynchronous Whirl

The solution in section 1.2.5 is the response to excitation by unbalance mass, which describes a shaft motion along a circular orbit the same angular velocity as the rotation. Such a motion is called synchronous whirling. Asynchronous whirling is when the shaft whirling speed is not the same as rotation speed, usually excited by friction contact force or other forcing mechanisms.
1.2.7 Critical Speed

Critical speed is defined as the rotor speed that is equal to one of the natural frequencies of the rotor.

For the Jeffcott rotor shown in Figure 1-3, the critical speed of the rotor system also called the first mode of whirling of the rotor system, is given by equating the right-hand side of equations 1.3 and 1.4 to zero. A pair of roots arise, one for forward whirl and the other for backward whirl critical frequency. A generalized expression is given below.

\[ \omega_n = \sqrt{\frac{k}{m}} \text{rad/s} \]

\[ N_1 = \frac{60}{2\pi} \sqrt{\frac{k}{m}} \text{ RPM} \]  \hspace{2cm} (1.9)

The same analogy can be applied when solving free whirling equations using complex notation as in equation 1.7, where two roots each indicating forward and backward whirl are obtained.

A similar approach to rigid body rotor in the absence of damping yields the following.

\[
\begin{bmatrix}
k - \omega^2 m & k_c \\
k_c & k_\theta + J_p \omega \Omega - \omega^2 J_t
\end{bmatrix} \begin{bmatrix}
 r_t \\
r_{\theta}
\end{bmatrix} = \begin{bmatrix}
0 \\
0
\end{bmatrix}
\]  \hspace{2cm} (1.10)

This gives the characteristics equation in 4\text{th} degree as shown in equation 1.11 which gives two pairs of roots i.e. two critical speeds each with a forward and backward whirl.

\[
\omega^4 - \Omega^2 \omega^2 \frac{J_p}{J_t} - \omega^2 \left( \frac{k}{m} + \frac{k_\theta}{J_t} \right) + \Omega \omega \frac{k J_p}{m J_t} + \frac{k k_\theta - k_c^2}{m J_t} = 0
\]  \hspace{2cm} (1.11)
In rotor systems, $k$ is the equivalent stiffness of rotor shaft and bearing system. For soft bearings, $k$ will be the parallel equivalent of both shaft and bearing stiffness values along a considered direction. For rigid bearings, $k$ will be only shaft stiffness. It must also be observed that the imbalance amplitude reaches maximum at the critical speed and hence at speeds close to critical speeds, high damping is required to reduce whirl amplitude.

### 1.2.8 Rotating Coordinate System

Many rotordynamic instabilities occur due to the effects of rotation. Additionally, various asymmetric and anisotropic rotor properties need adequate description. A robust coordinate system is required to describe the physical nature of such phenomenon. A rotating coordinate system may be used along with stationary coordinates to solve this. As shown in Figure 1-5, the rotating coordinate system $O\tilde{x}\tilde{y}\tilde{z}$ is aligned such that the coordinate system rotates along the $Oz$ axis with axis $O\tilde{z}$ coinciding with it. The stationary and rotating coordinates can be interchangeably transformed using the expression given below.

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{bmatrix} \begin{bmatrix} \tilde{x} \\ \tilde{y} \end{bmatrix} = [T] \begin{bmatrix} \tilde{x} \\ \tilde{y} \end{bmatrix}$$  \hspace{1cm} (1.12)

The rigid body rotor with only translational effects from equation 1.5 can be transformed from stationary coordinates to rotating coordinates as shown in equation 1.13.
The rotating coordinates introduces two new terms; Coriolis acceleration and spin/centripetal softening which are expressed as follows.

\[
\begin{bmatrix}
  m & 0 \\
  0 & m
\end{bmatrix}
\begin{bmatrix}
  \ddot{x} \\
  \ddot{y}
\end{bmatrix}
- 2
\begin{bmatrix}
  m & 0 \\
  0 & m
\end{bmatrix}
\begin{bmatrix}
  J_p \Omega & 0 \\
  0 & \Omega^2
\end{bmatrix}
\begin{bmatrix}
  \dot{x} \\
  \dot{y}
\end{bmatrix}
- \begin{bmatrix}
  m \Omega^2 & 0 \\
  0 & m \Omega^2
\end{bmatrix}
\begin{bmatrix}
  \ddot{x} \\
  \ddot{y}
\end{bmatrix}
+ \begin{bmatrix}
  m & 0 \\
  0 & m
\end{bmatrix}
\begin{bmatrix}
  \dot{x} \\
  \dot{y}
\end{bmatrix}
= \begin{bmatrix}
  F_x \\
  F_y
\end{bmatrix}
\] (1.13)

\[\text{Coriolis Acceleration}\]
\[
\begin{bmatrix}
  m & 0 \\
  0 & m
\end{bmatrix}
\begin{bmatrix}
  0 & J_p \Omega \\
  -J_p \Omega & 0
\end{bmatrix}
\begin{bmatrix}
  \dot{x} \\
  \dot{y}
\end{bmatrix}
\] (1.14)

\[\text{Spin or Centrifugal softening}\]
\[
\begin{bmatrix}
  m \Omega^2 & 0 \\
  0 & m \Omega^2
\end{bmatrix}
\begin{bmatrix}
  \ddot{x} \\
  \ddot{y}
\end{bmatrix}
\] (1.15)
1.3 Squeeze Film Dampers

All rotor systems require support mechanism for operation. Common types of rotor support systems are:

- Rolling element bearings
- Fluid-film bearings (sliding-element bearings)

With proper lubrication, fluid-film bearings have a very long life and also provide substantial external damping to the rotor system. However, fluid-film bearings are subject to abrupt catastrophic failure if the lubrication system fails, preventing it from being used. [7, 18].

Squeeze film dampers (SFD), as shown in Figure 1-6 can be used in systems supported by rolling element bearings. The main objective of a SFD is to add damping to the system to reduce large whirl responses when the rotor system passes through its critical speed.

A SFD consists of a thin film of oil constricted between a bearing housing and the outer race of a rolling element bearing (annulus). The outer race of the rolling element bearing is provided with a retaining (anti-rotation) pin thus restricting rotation and facilitating only whirling motion. The radial and tangential forces due to whirling in the film space contribute to stiffness and damping respectively.

Figure 1-7 shows the operational model of a squeeze film damper. The outer race of the rolling element bearing undergoes translational motion since it is constrained by the anti-rotation pin. Therefore, for the shaft whose center is at an eccentricity $e$ from the center of the stator moves along the whirl orbit as shown in Figure 1-7.
Figure 1-6 – Squeeze film damper representation

Figure 1-7 – Squeeze film damper parameters
SFDs may also be reinforced with soft springs called centering springs. These ensure constant film thickness all around the damper and provide good dynamic performance while keeping the damper centered.

Oil properties, supply and sealing mechanisms, are some important factors that have a significant role in the performance of squeeze film dampers [8, 12, 19, 20]. Some oil supply designs are center hole feed, center groove feed, end feed, etc. The oil may be sealed using piston rings or O-rings; dampers may also be unsealed. Figure 1-8 shows two different squeeze film damper configurations. Common damper designs have about 2 piston rings with 3-6 radial holes to feed oil.

Parameters affecting performance of squeeze film dampers are:

- Length of oil film \( L \)
- Radius of oil film \( R = \) Half Stator (Journal) diameter = \( D/2 \)
- Oil film thickness \( c = \) Difference between oil film radius and outer ring radius
- Film thickness at a given location \( h \)
- Absolute (Dynamic) viscosity \( \mu \)
- Whirl speed \( \Omega \) *
- Eccentricity ratio \( \epsilon = \) Ratio of eccentricity to oil film thickness \( e/c \)

* This thesis focuses on synchronous whirling; hence whirl speed \( \Omega \) is the rotation speed \( \omega \)
The Reynolds equation has been used to study of fluid effects in hydrodynamic bearings and the damping characteristics associated. The Reynolds equation is a special case of the Navier-Stokes equation where the inertia effect of fluid is ignored and one of the three dimensions is much smaller.
than the other two. Models by Crandall, El-Shafei and et al [6, 9, 11, 21-24] can be used to determine the radial and tangential forces as shown in Figure 1-10, by solving the Reynolds equation for squeeze film dampers given below.

\[ R_{e_s} = \frac{\rho c \omega^2}{\mu} \]  

\[ \frac{1}{R \partial \theta} \left( h^3 \frac{\partial p}{\partial \theta} \right) + \frac{\partial}{\partial z} \left( h^3 \frac{\partial p}{\partial z} \right) = 12 \mu \left( \frac{\partial h}{\partial t} \right) \]  

Equation 1.17 represents the Reynolds equation where \( R_{e_s} \) is defined as squeeze film Reynolds number, \( h \) is the film thickness, \( R \) is the radius of SFD, \( \mu \) is the kinematic viscosity and \( \omega \) is the angular velocity. When \( R_{e_s} \) is larger than 1, effect of inertia becomes important and the solution from Reynolds equation becomes inaccurate.
Exact solutions for equation 1.17 for circular whirling cases exist for two ideal cases; long bearing approximation and short bearing approximation. Long bearing approximation uses only the first term of the left-hand side in equation 1.17 while solving and short bearing approximation uses second term only. The former is valid for sealed dampers and latter for unsealed dampers. In addition, $L/D$ ratio is also considered when selecting long and short bearing approximations. The derivation is a two-step procedure; first calculating the pressure from the Reynolds equation and then performing an integration over the entire bearing dimension to obtain the radial and tangential damping forces.
\[ p = \text{pressure from Reynolds’ equation} \]
\[ F_r = \int_{\theta_1}^{\theta_2} \int_{z_1}^{z_2} p \cos \theta R \, d\theta \, dz \] (1.18)

\[ F_t = \int_{\theta_1}^{\theta_2} \int_{z_1}^{z_2} p \sin \theta R \, d\theta \, dz \] (1.19)

Cavitation occurs in squeeze film dampers due to a combination of factors such as oil supply pressure, whirl velocity, etc. Cavitation is when air/vapor is present in the fluid film, thus hampering damper operability and efficiency. To account for cavitation, a \( \pi \)-film solution to equation 1.17 is used and a perfect damper without cavitation requires a \( 2\pi \)-film solution to equation 1.17; the values \( \pi \) and \( 2\pi \) here imply the integration limits while solving equation 1.17.

For circular centered orbits, the radial and tangential forces obtained by solving equation 1.17 provide the damping and stiffness of squeeze film dampers as follows:

\[ F_r = -Ke \] (1.20)

\[ F_t = -Cv_t = -Ce\omega \] (1.21)

Since these derivations are based on many assumptions, a sophisticated solver such as Computational Fluid Dynamics (CFD) packages is required to calculate actual forces and force coefficients for dampers with specific geometry and other details such as feeding and sealing systems.
1.3.1 Short bearing approximation

The pressure, stiffness and damping values from a short bearing solution considering cavitation (\(\pi\)-film solution) are as follows

\[
p = -\frac{6\mu\omega\varepsilon \sin(\theta)}{c^2(1 + \varepsilon \cos(\theta))^3} \frac{L^2}{4}
\]

(1.22)

\[
C = \frac{\mu R L^3}{c^3} \frac{\pi}{2(1 - \varepsilon^2)^{3/2}}
\]

(1.23)

\[
K = \frac{\omega\mu R L^3}{c^3} \frac{2\varepsilon}{(1 - \varepsilon^2)^2}
\]

(1.24)

Where \(\varepsilon = e/c\) is eccentricity ratio; c is the clearance of SFD.

If cavitation is not considered (2\(\pi\)-film solution), \(K = 0\), \(C = \frac{2\mu R L^3}{c^3} \frac{\pi}{2(1 - \varepsilon^2)^{3/2}}\) (twice \(\pi\)-film solution)

Figure 1-12 shows variation of pressure for an uncavitated (2\(\pi\) film) short bearings squeeze film damper solution around the inner ring within the housing.

The formulae given here are force coefficients without considering inertia effects.
1.3.2 Long Bearing Approximation

The pressure, stiffness and damping values from a long bearing solution considering cavitation (π-film solution) are as follows:

\[ p = -\frac{12\mu R^2 \omega e \sin(\theta) (2 + e \cos(\theta))}{c^2(2 + e^2)(1 + e \cos(\theta))^2} \]  \hspace{1cm} \text{(1.25)}

\[ C = \frac{12\pi\mu R^3 L}{c^3(2 + e^2)\sqrt{1 - e^2}} \]  \hspace{1cm} \text{(1.26)}

\[ K = \frac{24\pi\mu R^3 L e \omega}{c^3(2 + e^2)(1 - e^2)} \]  \hspace{1cm} \text{(1.27)}
If cavitation is not considered (2π-film solution), \( K = 0 \),
\[
    C = \frac{24\pi\mu R^2 L}{c^3(2+e^2)\sqrt{1-e^2}} \text{(twice π-film solution)}.
\]

The formulae given here are force coefficients without considering inertia effects.

### 1.3.3 Experimental Determination of Squeeze Film Damper Coefficients

Squeeze film dampers have been studied for over four decades. However, some phenomenon such as cavitation, etc. associated with their actual operation is relatively less understood. Although four different methods of cavitation in squeeze film dampers have been identified and some analytical models do prevail, experimental determination of force coefficients under these circumstances is still preferred. The varying nature of different feeding systems and sealing mechanisms, combined with difficulties in quantifying exact inertia forces make it necessary for extensive experimentation of squeeze film dampers in order to obtain these force coefficients.

The first type of SFD test configuration is shown in Figure 1-13. This setup replicates the exact operation of a SFD and has both rotating and whirling components. It comprises of a shaft-bearing system immersed in a journal where the various parameters associated with operating the shaft viz. rotating speed \( \omega \), eccentricity of shaft center \( e \), and others are controllable. The shaft is attached to a rolling element bearing as shown in Figure 1-13. The inner race of the rolling element bearing has the same tangential velocity as that of the shaft at its radius. The outer race of the rolling element bearing does not undergo any rotational motion since it is constrained by an anti-rotation pin. However, due to the eccentricity of the shaft center \( e \), the shaft and the outer race of the rolling element bearing undergoes whirling i.e. rigid body motion along a curvilinear path. This outer race of the rolling element bearing setup within a journal whose housing is stationary constitutes the
SFD. Various instrumentation attached to the stationary housing provides desired parameters required for proper understanding of SFD operations. The eccentricity of shaft center is adjusted using cams and other mechanisms and trim balancing is used to ensure proper shaft balance. A formerly operable rig at the University of Cincinnati (setup by GE Aviation) is based on this test configuration [25]. However, a perfect trim balance is hard to achieve and any effects of imbalance are more profound at higher speeds.

Another configuration to test SFD operation is shown in Figure 1-14. This setup generates only whirling motion using two external shakers that provide linear motion along two mutually perpendicular with a phase difference of 90° with each other as shown in Figure 1-14. Hence, all points on the journal housing makes a circular whirl of radius of eccentricity $e$ which can be controlled either by offsetting the central stationary core or by providing an offset in terms of a static DC component to the actuator motion. This translation motion of the housing creates a relative whirl orbit of radius $e$ with respect to the stationary central core which is an equivalent representation of a SFD operation. Corollary to this setup, a SFD test configuration can be setup using a stationary journal housing with a central core that has translation only motion provided by external shakers. A SFD test rig at the Texas A&M University (setup by Pratt and Whitney Aircraft Corporation), Ecole Polytechnique de Lille (setup by SNECMA Moteurs France) and Karlsruher Institute of Technology (KIT) use this configuration for testing SFD parameters [12, 26, 27]. Both configurations of SFD test setups have merits and demerits.
Figure 1-13 – Squeeze film damper test configuration: Type 1 (Rotational and Whirling Motion)

Figure 1-14 – Squeeze film damper test configuration: Type 2 (Whirling Motion Only)
1.4 Tools for Rotordynamic Analysis

The Jeffcott rotor model and Rigid body rotor model may provide an adequate introduction to rotordynamics but more sophisticated models are needed to analyze realistic rotors.

Some of the geometries shown in Figure 1-15 may be modeled using beam deflection equations in order to determine stiffness of shafts. Such analysis may be well used during determining critical speeds and other important parameters of rotor systems [28]; these are only approximate methods.

![Figure 1-15 - Rotordynamic Analysis](image)

1.4.1 Finite Element Methods

Finite Element Method (FEM) is a discretization approach to solving complicated physical models. The complicated geometry is decomposed into a number of simple geometries which represent equivalent physical conditions of the larger bodies; larger bodies require a very large number of representative discrete elements. The discrete bodies can undergo deformations along required degrees of freedom using a derived force-deformation relationship. On a compound scale the deformation of the large body can hence be deduced, along with information such as stress, strain,
etc. Discretization may be in 1-D, 2-D or 3-D based on the complexity of the problem and computing power available. Many commercial FEM packages such as NASTRAN, ANSYS mechanical, etc. support rotordynamics [29, 30]. Genta and et al have undertaken studies comparing 1-D, 2-D and 3-D approaches to solving rotordynamics. The 3-D discretization has more advantages and superior solution qualities over other models, if the required computing power is available [31, 32].

1.4.2 Transfer Matrix Methods

Although a large number of commercial finite element software is available for rotordynamic analyses, extensive training of these commercial packages and validation of solutions is still a major issue. Simpler numerical methods are hence required to overcome these issues. Transfer Matrix Method is one such method for analyzing rotor vibrations was developed by Prohl and et al [33-36].

In this method, a given geometry is discretized in 1-D or 2-D and a simple force balance is performed across the element to provide the transfer matrix of the element considered. This force balance is summed across all elements to obtain the overall transfer matrix of a given geometry and on application of required boundary conditions, the response of the given system can be calculated. In addition, transfer matrices can handle complex variables; the advantages of this is discussed in earlier sections. For a disc and shaft shown in Figure 1-16 the transfer matrix by applying force balance across the element is obtained as follows.
\[ [z]_i^L = [T][z]_{i-1}^R ; \quad [T] = \text{Transfer Matrix} \] (1.28)

**Shaft**

\[
\begin{bmatrix}
  x^L \\
  \theta_y^L \\
  M_y^L \\
  V_x^L 
\end{bmatrix}_i = \begin{bmatrix}
  1 & l & \frac{l^2}{2EI} & -\frac{l^3}{6EI} \\
  0 & 1 & \frac{l}{EI} & -\frac{l^2}{2EI} \\
  0 & 0 & 1 & -l \\
  0 & 0 & 0 & 1 \\
\end{bmatrix}
\begin{bmatrix}
  x^R \\
  \theta_y^R \\
  M_y^R \\
  V_x^R 
\end{bmatrix}_{i-1}
\] (1.29)

**Disc**

\[
\begin{bmatrix}
  x^L \\
  \theta_y^L \\
  M_y^L \\
  V_x^L 
\end{bmatrix}_i = \begin{bmatrix}
  1 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 \\
  0 & J_t\Omega - J_p\omega^2 & 1 & 0 \\
  -m\omega^2 & 0 & 0 & 1 \\
\end{bmatrix}
\begin{bmatrix}
  x^R \\
  \theta_y^R \\
  M_y^R \\
  V_x^R 
\end{bmatrix}_{i-1}
\] (1.30)

**Spring – Damper**

\[
\begin{bmatrix}
  x^L \\
  \theta_y^L \\
  M_y^L \\
  V_x^L 
\end{bmatrix}_i = \begin{bmatrix}
  1 & 0 & 0 & 0 \\
  0 & 1 & 0 & 0 \\
  0 & 0 & 1 & 0 \\
  k + j\omega C & 0 & 0 & 1 \\
\end{bmatrix}
\begin{bmatrix}
  x^R \\
  \theta_y^R \\
  M_y^R \\
  V_x^R 
\end{bmatrix}_{i-1}
\] (1.31)
Complicated geometry and rotor systems can be handled well by transfer matrix methods in a simplified manner. For example, response of an overhung rotor with multiple discs can be easily calculated using this method.

Critical speeds of rotor systems in synchronous whirling can be calculated using the symbolic toolbox in MATLAB [37]. A symbolic variable in $\omega$ is used across the transfer matrices and a polynomial of very large degree is hence obtained from the overall transfer matrix when boundary conditions are applied. Roots of this polynomial will provide the critical speeds of the rotor system. MATLAB symbolic toolbox root solving capabilities may be limited due to memory and computing constraints.

For forced response in 2-D approximation of rotors, the 1-D transfer matrix can be expanded in two dimensions in a similar fashion as shown in Figure 1-1. An extended transfer matrix is used as shown in equation 1.32 to calculate the forced response.

$$
\begin{bmatrix}
z^1_L \\
z^2_L \\
1
\end{bmatrix}_1 = 
\begin{bmatrix}
T_1 & T_2 & f^1_{\text{ext}} \\
-T_2 & T_1 & f^2_{\text{ext}}
\end{bmatrix}
\begin{bmatrix}
z^1_R \\
z^2_R \\
1
\end{bmatrix}_{l-1}
$$

The previous sections contain a brief theory of rotordynamics along with some tools for analyzing complex systems. It is evident that sophisticated tools and commercial software is required to perform analysis on rotor system and/or damping systems. Additionally, such numerical methods can solve physical conditions that may and may not be feasible for experimentation. Currently, most rotordynamic systems can be solved on a structural dynamics software such as ANSYS Mechanical, NASTRAN, etc.; a CFD simulation of SFD can be carried out on ANSYS FLUENT, CFX, COMSOL, etc. [29] However, on a futuristic note, a solution of both structural and fluid
mechanics solved in a combined manner for specific geometry and physical conditions is required in order to provide accurate results.

1.5 Rotordynamics using ANSYS Mechanical

1. ANSYS Mechanical is a finite element solver capable of handling rotordynamic loads. Models may be discretized either as beam, 3-D solid or shell elements. The general second order differential equation for dynamic motion as shown in equation is interpreted in ANSYS rotordynamics as follows [29]

\[
[M]\ddot{X} + ([G] + [C])\dot{X} + ([B] + [K])U = f
\] (1.33)

Where \([M]\), \([C]\), \([K]\) are the mass, damping and stiffness matrices represented by equation 1.5, \([G]\) and \([B]\) are rotordynamic specific matrices representing the gyroscopic and rotating damping matrices respectively; these represent rotational velocity based inertia forces experienced in rotating systems. In short, \([G]\) handles the gyroscopic coupling explained in equation 1.1 and equation 1.2, and \([B]\) is used to handle centrifugal softening/stiffening explained in equation 1.13. The \([G]\) matrix is enabled in ANSYS using the CORIOLIS command; this should NOT be confused with Coriolis terms explained in the earlier sections.

Geometries may be modeled into ANSYS Mechanical using ANSYS Workbench DesignModeler or imported to ANSYS Mechanical from an external solid modeling software. Bearings are inserted as connection elements called Bearings, Springs and/or Multi Point Constraints (MPC). Connection element bearings support only linear analysis while springs are used for non-linear analysis.
Three types of analysis are supported by ANSYS rotordynamics.

- Modal Analysis – Perform modal analysis at various speeds and obtain a Campbell plot.
- Harmonic Analysis – Analyze response to synchronous (unbalance) and asynchronous loads. Other non-linear forcing is not supported.
- Transient Analysis – Time integration analysis to study response under linear and non-linear loads including unbalance, shock, startup, etc. Newmark-Beta method for time integration available.

The modal analysis and Campbell plot may be accessed directly from the ANSYS Mechanical Workbench interface. However, harmonic and transient analysis requires extensive programming using ANSYS APDL (ANSYS Parametric Design Language). The scripts may be introduced as boundary conditions in the ANSYS Mechanical workbench interface. The sample scripts are shown in the following section.

Figure 1-17 shows a sample rotor in steel ($E = 200\text{GPa}$, $\nu = 0.3$, $\rho = 7800\text{kg/m}^3$). The FEM is performed using solid elements and for a validation using TMM, the shaft is split into consecutive units of mass and shaft elements along with the disc. The modal (Campbell plot), harmonic analysis and transient analysis for the above-mentioned system are also shown below.
Figure 1-17 – Sample rotor system FEM vs TMM comparison

Figure 1-18 – Sample rotor Campbell diagram
Figure 1-19 – Sample rotor harmonic response

Figure 1-20 – Sample rotor orbit plot
1.5.1.1 ANSYS Parametric Design Language Scripts

ANSYS Mechanical – Rotordynamics Harmonic analysis

```plaintext
unb=19.68  ! Define the unbalance load, mass of rotor
ecc=7.62e-5  ! Define eccentricity of unbalance mass
f=unb*ecc  ! Net centrifugal force on rotor due to eccentric mass
disk=node(0,0,0)  ! Location of unbalance, i.e. location of disc
F,disk,fx,f  ! Force applied to disk. Real part is Fx
F,disk,fy,-f  ! Force applied to disk. Imaginary part is Fy

ALLSEL  ! Rotation i.e. w applied to all elements
CM,ALLELEM,ELEM

/SOLU
hropt,full  ! Full Harmonic analysis (Full inversion of FRF)
Kbc,1  ! Matrix type solution is performed here
SYNCHRO,,ALLELEM  ! Enable synchronous whirling
CMOMEGA,ALLELEM,,,1.0
```

ANSYS Mechanical – Rotordynamics Transient Analysis

```plaintext
! prep7  ! ********** Transient force (unbalance)
pil=acos(-1)  ! Standard definition of constant pi
spin=100*pi/30  ! Rotational velocity, converted to rad/s
tinc=5e-2  ! Time step size
totrevs=15  ! Total revolutions to carry out transient analysis
tend=2*pi*totrevs/spin  ! End time for transient
spindot=0.0  ! In case of spin-up, enter rotational acceleration
nbp=nint(tend/tinc)+1  ! Number of values for which analysis is carried out
unb=19.68  ! Define the unbalance load, mass of rotor
ecc=7.62e-5  ! Define eccentricity of unbalance mass
disk=node(0,0,0)  ! Net centrifugal force on rotor due to eccentric mass
disk=node(0,0,0)  ! Location of unbalance, i.e. location of disc

*dim,timeTab,table,nbp,,,TIME  ! Define tables for time steps, rotational velocity
*dim,spinTab,table,nbp,,,TIME  ! values, square of rotational values, Fx and Fy.
*dim,rotTab,table,nbp,,,TIME  ! Since number of steps is calculated and all values
*dim,fxTab,table,nbp,,,TIME  ! vary with time, the following lines of code are
*dim,fyTab,table,nbp,,,TIME  ! used.

*vfill,timeTab(1,0),ramp,0,tinc  ! Fill in first value of all variables in time as 0,
*vfill,spinTab(1,0),ramp,0,tinc  ! since the analysis starts at time t=0
*vfill,rotTab(1,0),ramp,0,tinc
*vfill,fxTab(1,0),ramp,0,tinc
*vfill,fyTab(1,0),ramp,0,tinc

```

```plaintext
tt=0  ! Set time increment. In this case t=0

*do,iloop,1,nbp  ! Start loop from 1 to total number of values
timeTab(1,loop,1)=tt  ! Current time step value is set
spinval=spin
spinTab(1,loop,1)=spinval
spin2=spinval**2  ! Square the spin value for centrifugal forcing
rotval=spinval*tt  ! This gives rotation i.e. angle per rotation step
```
rotTab(iloop,1)=rotval ! Enter the angle through which rotation takes place in table
sinr=sin(rotval) ! sin of angle to give x component of force for Fx
cosr=cos(rotval) ! sin of angle to give y component of force for Fy
fxTab(iloop,1)=f0*(-spin2*sinr) ! Net Fx after multiplying with component terms
fyTab(iloop,1)=f0*(spin2*cosr) ! Net Fy after multiplying with component terms
tt=tt+tinc ! Increment time to next time step *enddo 
fini ! End loop
brg1=node(0,0,0.5) ! Bearing locations. Only for user reference
brg2=node(0,0,-0.5)

ALLSEL CM,ALLELEM,ELEM
/solu
antype,transient ! Transient time integration solution
trnop,full,,,,,,,,1 ! Turn option to integration
tintp,,0.75,0.75 ! Numerical damping values for Newmark-Beta method
time,tend ! Simulation end time
deltim,tinc,
kbc,0 ! Coriolis command turns on matrix G explained earlier.
coriolis,on,,,on ! NOT TO BE CONFUSED with Coriolis forces. This is Unique
omega,,,spin

f,disk,fx,%fxTab%
f,disk,fy,%fyTab%
outres,all,all ! Apply forces in x and y direction that are stored in the tables above
! Output all results

1.5.1.2 Modeling asymmetric shafts

The above discussed FEM procedure can be applied to asymmetric shafts. However, care must be taken when modeling asymmetric shafts and how the gyroscopic matrix $[G]$ is modeled since ANSYS Mechanical has limitations executing rotation about asymmetric axes.

A transient analysis of an asymmetrical shaft shown in Figure 1-21 is modeled in FEM. Orbits plots with valid gyroscopic accountability and incorrect gyroscopic matrices are shown in Figure 1-22.
Figure I-21 – Asymmetric rotor representation

Figure I-22 – Asymmetric rotor response types
2 University of Cincinnati Squeeze Film Damper Test Rig

A now defunct squeeze film damper test rig is available at the University of Cincinnati Structural Dynamics Research Laboratory (SDRL). The rig was designed in association with General Electric (GE) Aviation as a research tool to quantify dynamics of a large engine squeeze film damper’s operation in realistic operating conditions. However, due to several operating and design issues, the rig is currently not in working condition.

A Master of Science thesis by Timothy J. Copeland is currently available, which serves as a detailed source of information regarding the design, construction and operation of the rig [25].

2.1 Rig Design and Technical Specifications

The test rig in its current state is shown in Figure 2-2. The test rig is designed to test a squeeze film damper in operation as a component, in which the parameters can be evaluated in a controlled environment. The rig provides a controlled circular orbit independent of interaction from the rotor-stator-bearing system. The intent of such a rig system is two-fold; construct a non-vibrating rotating rig to measure and understand the performance of a realistic squeeze film damper, and to conduct an experimental test program to evaluate forces and pressure developed by the oil film. The following parameters of the squeeze film damper in the rig are variable:

- Journal speed
- Oil inlet pressure and temperature
- Oil viscosity
- Eccentricity ratio
A schematic of the rig is shown in Figure 2-1. The general components of the rig setup can be separated into four sections viz. motor, coupling, spindle and damper housing. The motor is rated at 50 HP unit at 460V of peak to peak vibration 0.0009 inches (2.28 * 10^{-5}m). The motor is fixed to a concrete base reinforced with a steel mounting thereby ensuring minimum vibration in the motor. The motor is equipped with a speed controlled allowing specific speed selections with variations less than 1%. A Rexnord coupling, a disk type standard shaft coupler is used to transmit rotation from the motor to the spindle unit. The rating of the coupler is 50 HP at a maximum speed of 16,000 RPM.

The motor and coupling unit is connected to a single shaft spindle unit. The spindle is a precision balanced Setco 4308BY super precision boring/milling bearing spindle with dual bearing support at both ends. The class 7 bearings used in the spindle are oil mist lubricated and rated at 50 HP and supports speeds 2000 RPM to 10,000 RPM with fluctuations ±20 RPM and maximum dynamic loads up to 25000 lbs. The maximum runout of shaft test section centerline is 0.2 mils (5.08 * 10^{-6}m). None of the natural frequencies from any components interfere with the operating range of shaft speeds.
Figure 2-2 – Squeeze Film Damper test rig at the Univ. of Cincinnati
2.1.1 Damper Housing

The most critical component is the damper housing. A detailed explanation of the damper housing construction is shown in Figure 2-3.

The housing is made of two rings. The outer primarily houses an array of instrumentation to help determine the pressure profile accurately as shown in Figure 2-5. Details about the instrumentation are shown in Table 2-1. An annular reservoir is used to provide oil supply into the squeeze film. The housing is designed in such a way that offsets from 1 mil \((2.54 \times 10^{-5} \text{m})\) through 10 mils 
\((2.54 \times 10^{-4} \text{m})\) and land length \(L\) from 0.5 inches through 1.5 inches can be accommodated using cams. Trim balancing as shown in Figure 2-4 is used to balance offset of the central core from the shaft center.

![Figure 2-3 – Damper housing detailed description](image)

The inner ring is designed with an anti-rotation device to prevent relative rotation between the two rings, but allow whirl motions (displacements). The lubricant oil supplied is mil-1-7808 or mil-1-
23699 type at supply temperature and pressure ranging from room temperature through $300^\circ F$ ($148.8^\circ C$) and 5 psig at 20 GPM through 200 psig at 0.05 GPM respectively.

*Table 2-1 – List of instrumentation in the SFD test rig*

<table>
<thead>
<tr>
<th>Instrumentation Type</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Force Transducers</td>
<td>Damper housing supported on piezoelectric force transducers to resolve forces along mutually perpendicular directions with minimum cross-coupling between them.</td>
</tr>
<tr>
<td>Accelerometers</td>
<td>2 Nos. mounted to housing to measure vibration along their line of action</td>
</tr>
<tr>
<td>Pressure Transducer</td>
<td>24 Nos for oil film pressure measurement</td>
</tr>
<tr>
<td></td>
<td>3 Nos for oil supply pressure measurement</td>
</tr>
<tr>
<td>Thermo-couple</td>
<td>3 Nos. for oil film temperature</td>
</tr>
<tr>
<td></td>
<td>1 Nos. for oil supply temperature</td>
</tr>
<tr>
<td>Flowmeter</td>
<td>Measure oil supply quantity into damper housing</td>
</tr>
<tr>
<td>Proximity Probe</td>
<td>4 Nos. along two mutually perpendicular directions</td>
</tr>
</tbody>
</table>
Figure 2-4 – Trim Balancing in Test Ring Inner Ring

Figure 2-5 – Outer Ring Instrumentation Details
The concentric representation of sensors indicates sensors distributed almost equidistant along the $z$ axis with reference to the coordinate system explained in Chapter 1 and is not physically concentric in nature. This axial distribution of sensors is to capture the pressure profile for the type of solution as represented by Figure 2-6.

### 2.2 Experimental Determination of Damping Forces

The radial and tangential forces are obtained using SURFING, an internal program within GE. The program uses a two-step process to determine the forces from the pressure transducers. A typical axial pressure distribution is as shown in Figure 2-6.

![Figure 2-6 – Axial pressure distribution from pressure transducers](image)

In the same manner of obtaining forces from pressure in section 1.3, the double integration over axial length ($L$) and angular housing value ($R\theta$) is performed by finding the area under the $P - z$ plot shown in Figure 2-6. The same is performed across all the various pressure transducers to obtain variation of pressure area with housing angle as shown in Figure 2-7.
Since the radial and tangential forces are integral of pressure in the two mutually perpendicular components over pressure area, the values $A_1$ through $A_{10}$ can be sequentially added to perform the integration equivalent and hence the radial and tangential forces can be calculated as given in equation 1.34 and 1.35.

### 2.3 Available Experimental Data and Issues

Successful operations of the rig have been carried out. Despite two separate bearing failures, the causes of which have not been conclusively investigated, a nearly circular orbit was obtained for a spin of 3000 RPM. However, the orbit plot for 8000 RPM is highly distorted and non-circular.
Figure 2-8 – Orbit plot at 3000 RPM

Figure 2-9 – Orbit at 8000 RPM
Apart from the above two orbit plots, a variety of other data is available. A few samples are provided here. Some data have available has unit inconsistency and other issues.

Of prime importance is the run-up waterfall spectrum data shown in Figure 2-11. The presence of higher order peaks at higher speeds indicates presence of higher modes of rotor whirling such as rigid body conical modes [18].

The non-circular orbits obtained at higher speeds is a deviation from the intent of rig operation. Moreover, the stiffness and damping calculations from section 1.3 is derived based on circular orbits. Therefore, operating the rig with non-circular orbits and the damper system will lead to highly erroneous data. The experimental data for rotor operation without the damper compared against rotor operation with the damper is shown in Figure 2-15.
Figure 2-11 – Run-up frequency spectrum: 500 to 5000 RPM at 250 RPM intervals

Figure 2-12 – Sample thermo-couple data
In addition to the implications of higher orders discussed earlier, a reduction in orbit radius indicates the presence of a circular whirl orbit at the frequency nearly at the same as the rotation speed with 180° out of phase. Since the eccentricity of rig is set statically, then assuming this static eccentricity does not result in any centrifugal forces (perfect system balance) and thereby assuming that the response orbit is purely due to static eccentricity controlled and created as required. However, the test rig itself is a rotor system with its own whirling modes which are superposed with the circular orbit. A more detailed explanation is available in Chapter 3.

Although the experimental data indicate unbalance issues and other implied effects that hamper correct operating of the now defunct squeeze film damper test rig, more specific details are required to validate the claims. A finite element model to demonstrate the rig operations is hence required to study the effects of imbalance and its effects.
Figure 2-14 – Rotor operation: Intent vs Actual

Figure 2-15 – Orbit comparison at 8000 RPM
3 Finite Element Analysis Model of Test Rig

A finite element model that describes the Univ. of Cincinnati Squeeze Film Damper (SFD) test rig is built using ANSYS Mechanical. The model comprises of only key components which contribute significantly to dynamics of the system ignoring minor details. The sectional view of the spindle shaft is shown in Figure 3-1.

![Figure 3-1 – Spindle Shaft cross sectional view](image)

3.1 Finite Element Model Description

Since the effects of squeeze film damper are represented using stiffness and damping values as shown in section 1.3, they can be modeled in a similar way to modeling a bearing (spring-damper system). The shaft system is made of steel with following properties:

- Density \( \rho = 7850 \text{kg/m}^3 \)
- Young’s Modulus \( E = 200 \times 10^9 \text{N/m}^2 \)
- Poisson’s Ratio \( \nu = 0.3 \)
The spindle bearings are class 7 bearings and are in double arrangements at the two ends of the spindle. However, an exact stiffness value is not known. They are rigid bearings with high stiffness values. The original thesis records two different stiffness values for the bearings. In both cases, the first critical speed of the rig is more than 1.5 times the maximum operational speed. The setup used for the critical speed calculations in the original thesis is shown in Figure 3-3.
Table 3-1 – Model properties used in Reference Thesis

<table>
<thead>
<tr>
<th>Case</th>
<th>Bearing Stiffness ($N/m$)</th>
<th>Support Stiffness ($N/m$)</th>
<th>Net Stiffness (Overhung rotor) ($N/m$)</th>
<th>1st Critical speed (RPM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>$1.4 \times 10^8$</td>
<td>$3.6 \times 10^8$</td>
<td>$1 \times 10^8$</td>
<td>14950</td>
</tr>
<tr>
<td>Case 2</td>
<td>$6.2 \times 10^8$</td>
<td>$3.6 \times 10^8$</td>
<td>$2.28 \times 10^8$</td>
<td>28660</td>
</tr>
</tbody>
</table>

The spindle shaft model used in the original thesis can be approximated to an overhung rotor model as shown in Figure 3-4.

Case 1 given in Table 3-1 is used in the finite element model of the rig setup. Although bearings may exhibit non-linear characteristics, the finite element model assumes they are linear with stiffness values $k = 1 \times 10^8 N/m$. 

![Figure 3-4 – Rig setup approximate model](image)
Theoretical analysis can also be applied if further assumptions are made, for example assuming the shaft has no mass and the bearings are rigid as seen in the analysis of Dimentberg, et al. for overhung rotors [28]. The Transfer Matrix Method described in section 1.4 can also be used.
Figure 3-6 shows the model developed for the SFD test rig comprised of 4569 solid187 elements which are 3D 10-Node tetrahedral structures [29]. The shaft is asymmetric about the axis of rotation and the center of the disk and coordinates for disk unbalance are the same as shown in section 1.5. Asymmetry of the disk is assumed since trim balance will always have some residue imbalance in the system and perfect balancing is only an ideal case.

3.1.1 Effective Shaft Stiffness from Beam Model

Another crude approximation to determine critical speed can be made using the stiffness of the beam show in section 1.4 and combining it with the bearing stiffness in series and considering the complete mass of the system to form a spring-mass system, the natural frequency of which gives the critical speed as shown in Figure 3-7.

Although the derivation of Dimentberg, et al. for the overhung rotor case on rigid supports cannot be considered directly as stated in the previous section, beam theory can be applied to obtain an approximation for the effective stiffness of the shaft. Superposition principle can be applied to rotor model in Figure 3-8 and an overhung beam model deflecting under self-weight as shown in Figure 3-9.

The deflections for an overhung rotor model with point load at the overhung as shown in Figure 3-8 are given by equations 3.34 (deflection between supports) and 3.35 (deflection in overhang section).

Similarly, the deflections for an overhung rotor acting under its self-weight as shown in Figure 3-9 are given by equations 3.36 (deflection between supports) and 3.37 (deflection in overhang section).
\[
\Delta x = \frac{Pax}{6EI} (l^2 - x^2)
\]  
(3.34)

\[
\Delta x_1 = \frac{P_x}{6EI} (2al + 3ax_1 - x_1^2)
\]  
(3.35)

\[
\Delta x = \frac{wx}{24EI} (l^4 - 2l^2x^2 + lx^3 - 2a^2l^2 + 2a^2x^2)
\]  
(3.36)

\[
\Delta x_1 = \frac{wx_1}{24EI} (4a^2l - l^3 + 6a^2x_1 - 4ax_1^2 + x_1^3)
\]  
(3.37)

\[
m_{total} = m_{disk} + m_{shaft}
\]

Figure 3-7 – Critical Speed estimation using beam stiffness

Figure 3-8 – Dimentberg Rotor (Overhung Rotor with Point Load at Overhung End)
An estimate of the weight distribution of the shaft is available and force $P$ is the unbalance at any given speed. Figure 3-10 shows deflection of the shaft due to the two loads; $P$ is the unbalance force at 1 mil and 8000 RPM due to the eccentric disk.
From Figure 3-10, it clear that deflection due to self-weight is much lesser compared to unbalance forces at high speeds. An estimate of the shaft stiffness is hence obtained using the net deflection due to both loads and unbalance force only as shown below.

\[ k_{shaft} = \frac{P \text{ (unbalance)}}{\text{Total Deflection} \Delta} = 2.15 \times 10^8 N/m \]

### 3.2 Test Rig Critical Speeds

Figure 3-11 shows the Campbell diagram for the test rig obtained from FEM. The first critical speed is obtained at 16011 RPM. Since the maximum design requirement for the rig operation is 10000 RPM, the Campbell diagram indicates this test rig does not have any critical speed in the range of operation.

<table>
<thead>
<tr>
<th>Method</th>
<th>1\textsuperscript{st} Critical Speed (RPM)</th>
<th>Variation (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FEM</td>
<td>16011</td>
<td>(Ref.)</td>
</tr>
<tr>
<td>TMM</td>
<td>16039</td>
<td>0.174</td>
</tr>
<tr>
<td>Beam Stiffness Method</td>
<td>17700</td>
<td>10.4</td>
</tr>
<tr>
<td>Previous Thesis Calculation</td>
<td>14950</td>
<td>-6.62</td>
</tr>
</tbody>
</table>
A Transfer Matrix model for the rig setup is also used to determine the critical speeds. Since the rig shaft mass is higher than the disk mass, the continuous model described in section 1.5 is used to calculate the critical speeds using MATLAB symbolic toolbox [37]. A comparison of the critical speeds obtained from the different methods is shown in Table 3-2.

![Figure 3-11 – Test Rig Campbell Diagram](image)

### 3.3 Test Rig Unbalance Response – Without Squeeze Film Damper

A Harmonic analysis can be performed to calculate response to unbalance. Figure 3-12 shows how the unbalance force is applied to ANSYS analysis.
While the harmonic response provides symmetric response along the x and y directions indicating that the response at a given speed is a circular orbit, the same can be confirmed using a transient analysis as described in section 1.5. The responses from the transient and harmonic solutions must be the same. The transient response for 8000 RPM for the same unbalance as shown in Figure 3-13 is shown in Figure 3-15.

A transfer matrix method can be performed to obtain the response to unbalance forces. The harmonic response is performed using the transfer matrices mentioned in section 1.4 whereas the transient response to unbalance requires an extended transfer matrix as described by equation 1.48. Table 3-3 shows the comparison between the different methods.
Figure 3-13 – Harmonic Response in x and y directions. \( e = 3 \text{ mils} \)

Figure 3-14 – Phase in x and y directions \( e=3 \text{ mils} \)
Figure 3.15 – Transient response. $e = 3$ mil, $\omega = 8000$ RPM

Table 3-3 – Comparison of various methods $e = 3$ mil, $\omega = 8000$ RPM

<table>
<thead>
<tr>
<th>Method</th>
<th>Response Radius (m)</th>
<th>Variation (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Harmonic – FEM</td>
<td>$1.63 \times 10^{-5}$</td>
<td>(Ref)</td>
</tr>
<tr>
<td>Transient – FEM</td>
<td>$1.623 \times 10^{-5}$</td>
<td>0.429</td>
</tr>
<tr>
<td>Transient – TMM</td>
<td>$1.415 \times 10^{-5}$</td>
<td>13.1</td>
</tr>
</tbody>
</table>

3.3.1 Correction Terms for Test Rig Unbalance Response

The response of the SFD test rig is not just from an unbalance mass. The disk is shifted by a static eccentricity, and the effect must have been minimized by a trim-balancing, which will leave a very
small unbalanced mass and cross mass moment of inertia. Therefore, the response of this system is a combination of both static eccentricity and response to dynamic unbalance as shown in Figure 3-16.

Table 3-3 is hence modified and is given in Table 3-4. The solutions obtained after the correction applied match well. Also, to be noted from Figure 3-17 is that the responses from the rig test and modeling performed here are not the same. Since the bearing stiffness values are only an assumed value and matching them stiffness with response is not considered. The lower response from the test rig data suggests more stiff bearings in use than those considered in the modeling. The total response in Table 3-4 is calculated using x response only since the phase is 0 throughout the operating range as shown in Figure 3-14.

\[ r_{total} = eccentricity_{static} + r_{unbalance} \]  \hspace{1cm} (3.38)

Figure 3-16 – Correction applied to unbalance response
Table 3-4 – Corrected Comparison of various methods $e = 3 \text{ mil}, \omega = 8000 \text{ RPM}$

<table>
<thead>
<tr>
<th>Method</th>
<th>Total Response Radius $\left( e_{\text{static}} + r_{\text{unbalance}} \right)$</th>
<th>Variation (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Harmonic – FEM</td>
<td>$7.62 \times 10^{-5} + 1.63 \times 10^{-5} = 9.25 \times 10^{-5}$</td>
<td>(Ref)</td>
</tr>
<tr>
<td>Transient – FEM</td>
<td>$7.62 \times 10^{-5} + 1.623 \times 10^{-5} = 9.243 \times 10^{-5}$</td>
<td>0.075</td>
</tr>
<tr>
<td>Transient – TMM</td>
<td>$7.62 \times 10^{-5} + 1.415 \times 10^{-5} = 9.035 \times 10^{-5}$</td>
<td>2.324</td>
</tr>
</tbody>
</table>

---

Figure 3-17 – Original test rig data $e = 1 \text{ mil}, \omega = 8000 \text{ RPM}$
From Figure 3-18 and Figure 3-19 it is clear that the actual whirl orbit will deviate significantly from the static orbit with eccentricity and speed. Another perspective is shown in Figure 3-20. The rig is required to produce a constant whirl radius in order to measure SFD characteristics. However, from Figure 3-20 it is clear that the response increases with speed for any given eccentricity value and leads to incorrect determination of SFD characteristics.

**Figure 3-18 – Harmonic Response; No Correction Applied**
Figure 3.19 – Total Response Radius; Correction Applied

Figure 3.20 – Comparison: Intended Working vs Actual Working $e = 3\ mil, \omega = 8000\ RPM$
From this data, it can be said that the actual operation of the rig deviates from the actual intent of rig operation. The actual response radius is almost 17 percent more at 8000 RPM than intended orbit radius for operation. The reduction in clearance is correspondingly the same as shown in Table 3-5.

<table>
<thead>
<tr>
<th>Case</th>
<th>Response Radius (m)</th>
<th>Stator Radius (m)</th>
<th>% Clearance reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intended Operation</td>
<td>$7.62 \times 10^{-5}$ (3 mil)</td>
<td>$2.84 \times 10^{-4}$</td>
<td>-</td>
</tr>
<tr>
<td>Correction Applied</td>
<td>$9.243 \times 10^{-5}$</td>
<td>$2.84 \times 10^{-4}$</td>
<td>16.8</td>
</tr>
</tbody>
</table>

3.4 Impact of Correction Term on Squeeze Film Damper calculations

From section 1.3, the damping and stiffness coefficients depend on eccentricity ratio $\epsilon$ defined as ratio of whirling radius to clearance $e/c$. Since the whirl radius is greater than the eccentricity due to the harmonic response, the damping and stiffness coefficients vary as well. This variation may not be significant to low eccentricity and low speed whirling, but has a great impact with increase in spin speed and static eccentricity. Hence the damper parameters determined using this rig setup will have to reflect the rotordynamic response.
4 Test Rig Operating with Squeeze Film Damper

It is assumed that Aeroshell 500 Turbine Oil is used as damper fluid for the test shown in Figure 4-1. The oil has density $\rho = 1005 \text{ kg/m}^3$ and kinematic viscosity $\nu = 27 \text{ mm}^2/\text{s}$. A constant set of damper dimensions are used to calculate the damper characteristics given in section 1.3. Details of the damper geometry are shown in Figure 4-2. The main dimensions are:

- Land length $L = 2.54 \times 10^{-2} \text{ m}$
- Radius $R = 0.08 \text{ m}$
- Clearance $c = 2.87 \times 10^{-4} \text{ m}$

![Diagram of Squeeze Film Damper Test Rig Configuration]

*Figure 4-1 – Squeeze Film Damper Test Rig Configuration*
4.1 SFD Performance for an Ideal Case

The SFD test rig will operate with a perfect circular orbit with the radius of the eccentric distance if there is no rotordynamic response of the test rig. In this case, the Reynolds equation [6, 21, 23] provides fairly accurate results for SFDs running at low speed, therefore when the Reynolds number is low. Figure 4-3 and Figure 4-4 show damping coefficient and stiffness as a function of eccentricity calculated using the analytical solutions of Reynolds equation for $2\pi$ film case (no cavitation case) and $\pi$ film case (cavitation present over half range of SFD), each for the Short Bearing Assumption (SBA) and Long Bearing Assumption (LBA).

The damping coefficient calculated from Reynolds solution for a circular orbit does not change as a function of spin speed but the stiffness varies with spin speed. It is reminded that the solution does not consider effects of fluid inertia (by using Reynolds equation) and the effect of dynamic whirling the rotor on the SFD. Nevertheless, results from analytical solutions provide a brief insight into the effects of applying a squeeze film damper.
Figure 4-3 – Damping Values: Short Bearing Approximation

Figure 4-4 – Damping Values: Long Bearing Approximation
Figure 4-5 – Stiffness Values: Long Bearing Approximation (With Cavitation)

Figure 4-6 – Stiffness Values: Short Bearing Approximation (With Cavitation)
4.2 Actual SFD Response Observed Damper Module Working

The actual response of the SFD is a combination of the static response due to eccentricity and the dynamic response of the SFD test rig. The immediate consequence of this is increase in eccentricity ratio $\varepsilon$. While the actual response of SFD is limited to $\varepsilon < 1$, it is evident that the net response across many cases results in $\varepsilon \geq 1$ as seen From Figure 3.16 and Figure 4-7. Damping values are independent of spin speed if the whirl radius is constant, but the whirl radius is a function of the speed, which is added to the static whirl radius, the damping coefficient changes as a function of speed.

![Figure 4-7 – Actual Whirl Orbit Radius and Limitations](image-url)
Figure 4.8 – Actual Damping Values: LBA $\pi$ solutions

Figure 4.9 – Actual Damping Values: LBA $2\pi$ solutions
Figure 4-10 – Actual Damping Values: SBA $2\pi$ solutions

Figure 4-11 – Actual Damping Values: SBA $\pi$ solutions
Figure 4-8 and Figure 4-11 show the regions of valid and invalid solution. Similarly, for the $\pi$ solution, the stiffness values in the invalid region are negative for Long Bearing solution but the stiffness only decreases for Short Bearing solution as indicated by the formulae for stiffness in section 1.3. This is clearly shown in these results are further evidence of deviation of rig operation from actual design intent. Data acquired by the test rig operation under these forcing conditions will produce erroneous results at higher speeds and eccentricity ratios. If more rigorous rotordynamics analysis is conducted and the results are reflected, the results may be improved to some extent but not completely.

Figure 4-12 – Actual Stiffness Values: LBA $\pi$ solutions
The intended values of damping and stiffness are compared against the actual values for 1 mil eccentric unbalance as shown in Figure 4-19 through Figure 4-22. The short bearing approximations for both $\pi$ and $2\pi$ solutions show significant increase in damping values. A similar trend is also clear in the long bearing approximation. The $\pi$ solutions have additional stiffness values. Particularly in case of the long bearing solution with $\pi$ solution, where the stiffness values are comparable to the bearing stiffness values, an increase in stiffness in the actual system in comparison to the intended operation significantly affects the analysis of the squeeze film damper characteristics.
4.3 Optimum Bearing Damping

Based on the optimum bearing damping given by Barrett L.E., et al. [38] and the relevant explanation in section 1.3, the optimum bearing damping ratio and hence the optimum damping coefficient values for the rotor system are calculated.

The bearing stiffness ratio can be calculated as $K = \frac{(k_{\text{All bearings}})}{k_{\text{shaft only}}}$. Since the two bearings each are assumed to have a stiffness $k = 1 \times 10^8 \text{N/m}$ and the shaft only stiffness is calculated from chapter 3 as $k_{\text{shaft}} = 2.15 \times 10^8 \text{N/m}$

$$K = \frac{2 \times (1 \times 10^8)}{2.15 \times 10^8} = 0.93$$

The optimum bearing damping ratio is hence $\zeta_o = \frac{1+K}{2} = 0.96$

The critical speed of the shaft on rigid supports (shaft only) is $\omega_{cr} = \sqrt{\frac{k_{\text{shaft}}}{m_{\text{total}}}} = 2677 \text{ rad/s}$

The optimum damping is hence given as $C = 2\zeta_o m\omega_{cr} = 154,198 \text{ Ns/m}$

Comparing this optimum damping coefficient value with the actual damping values given in Figure 4-8 through Figure 4-11, it is observed that damping values obtained from the Long Bearing Approximation (LBA) are much higher than the critical damping value (over 4 times larger) and the Short Bearing Approximation (SBA) is lower than or comparable to the optimum damping. Since the damper module in the UC test rig system is a sealed damper, the LBA solutions are used to study their behavior; the test rig has extremely large damping values in comparison to the optimum value.
4.4 Harmonic Response with Squeeze Film Damper

Analysis to obtain harmonic response to unbalance with application of squeeze film damper is performed in a similar manner as discussed in chapter 3. Although the previous section discusses in detail the effects of unbalance on squeeze film damper stiffness and damping values, a harmonic response with squeeze film damper is required to assure the previously made claims.

To perform harmonic analysis with squeeze film dampers, the system shown in Figure 4-1 has squeeze film damper values where the eccentricity ratio is calculated using the sum of both unbalance response and the static eccentricity as discussed in the previous section. This provides however, only the response to unbalance loads at a given eccentricity or harmonic response with squeeze film dampers. The actual response will consider both static eccentricity and harmonic response with squeeze film dampers.

Figure 4-14 through Figure 4-17 show the reduction of harmonic response only to the different eccentric loading cases for various damper configurations. The sealed squeeze film damper description provided by the long bearing approximation shows a higher reduction in harmonic. Figure 4-23 and Figure 4-24 indicates the total response i.e. sum of both harmonic response and static eccentricity for $e = 1\ mil$ for the two different solutions of long bearing approximation and short bearing approximation across the various speeds.

Table 4-2 shows a comparison of the rotor rig operation at $e = 1\ mil, \omega = 8000\ RPM$ for different operating conditions. The long bearing approximation (sealed damper solution) shows significant reduction in orbit radius. The orbit radius for long bearing case is almost equal to the intended response i.e. the harmonic response due to squeeze film damper according the two long bearing approximations is negligible and the response is due to static eccentricity alone.
Figure 4-14 – Harmonic Response Reduction: SBA $2\pi$ solution

Figure 4-15 – Harmonic Response Reduction: SBA $\pi$ solution
Figure 4-16 – Harmonic Response Reduction: LBA $2\pi$ solution

Figure 4-17 – Harmonic Response Reduction: LBA $\pi$ solution
Although the orbit value is close to intended response in the long bearing solutions, the damping and stiffness are higher than the intended response. This higher damping and stiffness causes such a reduction in orbit radius and cannot be taken as working close to intended operation since that leads to erroneous damping and stiffness calculations.

### 4.5 Experimental and Analytical Data Comparison

Figure 4-18 shows original data from the UC SFD test rig. The LBA from the previous section show nearly all harmonic i.e. vibrational component being reduced. Thus, it can be conclusively said that in Figure 4-18, the case with damper flow represents only the orbit due to static offset of the rig. Since the radius from experimental data for damped flow corresponds to 1 mil approximately, this value is considered for further interpretation of results. For an undamped case of the test rig, the corresponding radius values and a comparison with the damped radius value is shown in Table 4-1.

The experimental data indicates 23% reduction in response radius on comparing damped and undamped data. Comparisons are made for experimental data and analytical solutions for a case $e = 1$ mil and $\omega = 8000$ RPM.

<table>
<thead>
<tr>
<th>Case</th>
<th>Orbit Radius (mil)</th>
<th>Radius Reduction (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Damper Flow</td>
<td>1.7</td>
<td>(Ref)</td>
</tr>
<tr>
<td>B,C,D – Damper Flow</td>
<td>1.3</td>
<td>23.5</td>
</tr>
</tbody>
</table>

Table 4-1 – Experimental Data: Damped vs undamped comparison
Figure 4-18 – Original test rig data $e = 1$ mil, $\omega = 8000$ RPM

Figure 4-19 and Figure 4-20 show the particular case of variation of damping coefficient for the two Reynolds solutions for both $2\pi$ and $\pi$ film cases for $e = 1$ mil. Figure 4-21 and Figure 4-22 show variation of stiffness in case of the cavitated $\pi$ film case for SBA and LBA solutions. The net variation in rotor orbit i.e. inclusive of static offset for the LBA and SBA is shown in Figure 4-23 and Figure 4-24. A summary of damped cases is available in Table 4-2, clearly indicating that when the test rig is operated with the damper module, the LBA case almost provides the static eccentricity alone while the SBA case provides no significant damping to the rotor system. The LBA system is of more relevance since the test rig damper module is inclusive of seals.
Figure 4.19 – SBA Actual vs Intended Damping $e = 1$ mil

Figure 4.20 – LBA Actual vs Intended Damping $e = 1$ mil
Figure 4-21 – LBA $\pi$ solution Actual vs Intended Stiffness $e = 1$ mil

Figure 4-22 – SBA $\pi$ solution Actual vs Intended Damping $e = 1$ mil
Figure 4-23 – Whirl Orbit Radius: Undamped vs SBA Damped Actual working $e = 1$ mil

Figure 4-24 – Whirl Orbit Radius: Undamped vs LBA Damped Actual working $e = 1$ mil
Table 4-2 – Comparison of Orbit Values at different operating conditions $e = 1 \text{ mil}, \omega = 8000 \text{ RPM}$

<table>
<thead>
<tr>
<th>Case</th>
<th>Orbit Radius (m)</th>
<th>Difference (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intended Response (Damped &amp; Undamped)</td>
<td>$2.54 \times 10^{-5}$ (1 mil)</td>
<td>-</td>
</tr>
<tr>
<td>Actual Undamped</td>
<td>$3.09 \times 10^{-5}$</td>
<td>(Ref)</td>
</tr>
<tr>
<td>Damped: SBA $2\pi$ solution</td>
<td>$3.08 \times 10^{-5}$</td>
<td>0.54</td>
</tr>
<tr>
<td>Damped: SBA $\pi$ solution</td>
<td>$3.07 \times 10^{-5}$</td>
<td>1.08</td>
</tr>
<tr>
<td>Damped: LBA $2\pi$ solution</td>
<td>$2.57 \times 10^{-5}$</td>
<td>16.9</td>
</tr>
<tr>
<td>Damped: LBA $\pi$ solution</td>
<td>$2.58 \times 10^{-5}$</td>
<td>16.52</td>
</tr>
</tbody>
</table>

Figure 4-25 - Time Integration Plot Comparison
Results show in Table 4-2 can be confirmed from the time integration plots shown in Figure 4-25, where the undamped rotor orbit is higher than the damped rotor orbit at 8000 RPM and 1 mil static eccentricity; the damped orbit almost representing intended static eccentricity orbit. This is similar to the experimental data as seen in Figure 4-18

4.6 Observations

- Since the rig setup has a sealed damper, the long bearing approximation provides the best description of the damper properties. The reduction in radius using long bearing approximation is close to 17% compared to a 1% maximum using the short bearing approximation (unsealed dampers).
- There is significantly more reduction in the orbit radius in the experimental data. This may be due to the fact that the long bearing approximation does not provide a detailed description of the sealing properties and use a correct cavitation description.
- The variation in orbit radius reduction may also be because the roller bearing considered for the spindle shaft system are assumed to have a stiffness $k = 1 \times 10^8 N/m$. If the bearing stiffness is much smaller, the first natural frequency of the system may be lower than the operating speed. In this case, the whirl response is 180° out of phase to the eccentric mass. Therefore, the dynamic and static response are subtracted and not added.
- The experimental values for rotor orbits is higher than 1 mil, close to 1.3 mil indicating some harmonic response due to unbalance and other misalignment based loads indicated by the order plots. However, the long bearing approximation case almost reduces all harmonic responses. Other advanced modeling is required to obtain better correlation between experimental and theoretical modeling.
5 Conclusion

The research is aimed at study of dynamic response characteristics of the squeeze film damper (SFD) test rig built at the University of Cincinnati in early 1990s. The test rig is no longer operational, but its design details and measurement results are still available.

To understand basic dynamics of SFD, an overview of rotordynamics, rotor-bearing interaction and damping using squeeze film dampers is studied. Various loads acting on such systems and their classification along with typical response patterns is also studied. Some experimental techniques used in parameter estimation for squeeze film dampers along with advantages and disadvantages and their impact on rotodynamic system are studied. A few techniques used in rotordynamic analysis are also described in detail.

Although the rig operated below its critical speeds, a circular centered orbit required to test squeeze film damper characteristics was not achieved at higher speeds; presence of higher order terms associated with the rig operation at higher speeds indicate misalignment issues. Additionally, it was investigated that some inherent design issues exist in the rig, which affects the correct operation of the rig. The reduction of rotor orbit radius for undamped and damped flow comparison indicates harmonic and other loads acting on the rig system.

A theoretical model of the shaft spindle system is used to study the dynamics of the SFD test rig. The rotordynamic equivalent of the system is analyzed using Transfer Matrix Methods (TMM) and a Finite Element Method (FEM) using ANSYS Mechanical. Apart from an acceptable match of critical speeds, a satisfactory comparison between the two techniques is found for general rotor systems and for the rig case, particularly in analyzing rotor orbits.
Two different approaches are used in ANSYS Mechanical to check consistency between results for orbit radius for unbalance harmonic loads viz. Harmonic analysis and transient analysis. The results from these two methods show a perfect match. The transient analysis is also included here to indicate computational capabilities of ANSYS Mechanical. Non-linear loads require transient analysis since transient analysis is a time integration (time marching) where numerically the responses are added over time to obtain the overall response. Some complicated rotor-bearing issues such as rubbing require such analysis.

Two different conditions of rotor operating conditions are studied here; rotor operation without a squeeze film damper at the damper module location of the test rig and rotor operation with squeeze film damper simulating the working of the damper module. Rotor operation without the damper is a harmonic analysis of the rig system under varying eccentricities mentioned in the design criteria across a range of rotor speeds. The unbalance responses are further used to predict the damping and stiffness for the various squeeze film damper approximate models (long bearing and short bearing approximation).

A spring-damper system is modeled at the damper module location of the test rig. The same unbalance loads across varying eccentricity values and rotor operating speeds is studied. A reduced rotor orbit is obtained. A comparison is made between the damped and undamped response. The reduction between the two cases for long bearing approximation (sealed damper description) provides a response radius reduction of about 17%. This orbit radius reduction is compared to the experimental data obtained from the test rig. The test rig data indicates about 23% reduction in orbit radius. This indicates that the test rig actual operation deviates from its intended design. The radius reduction values are close thereby indicating unbalance as the primary load on the rotor
bearing system. The presence of a higher orbit in experimental data i.e. 1.3 mils against a required 1 mil indicates presence of higher order responses that are not eliminated successfully by the rotor-bearing system. Additionally, a higher reduction in the orbit radius for long bearing approximation satisfactorily proves sealed dampers have higher damping capabilities in comparison to unsealed dampers.

Supplementary to the above discussions, the test rig configuration of the University of Cincinnati has many operational issues not due to the nature of test rig configuration, but due to rotordynamic effects of using a spindle-shaft to provide the circular centered orbit. Although the squeeze film damper properties can be tested in a controlled manner, the rotor response cannot be controlled. Provided the test rig configuration is operated without any rotordynamic interference to obtain circular centered orbits, the test rig configuration can provide accurate results for squeeze film damper operation. An alternate test rig configuration given as test rig configuration Type 2 in section 1.3 eliminates such issues. There are other challenges however, in operating test rig configuration Type 2.

### 5.1 Future Work

- The rotor material in this thesis is assumed to have no internal damping. Internal damping destabilizes rotor system. Rotor systems incorporating various such damping can be studied on similar lines as this thesis.

- Most rotor systems are more complicated with many disks and gearing systems for various purposes that are critical in predicting a correct response. A time integration of the entire rotor system inclusive of all possible non-linearity cases may be studied.
• The damping and stiffness values for the squeeze film damper are obtained based on an approximate solution to the Reynolds equation. A more realistic value can be obtained using Computational Fluid Dynamics (CFD) analysis. A combined of the Finite Element Model and a CFD of a SFD may be considered. Some commercial software such as ANSYS provide ANSYS Fluent co-simulation with ANSYS Mechanical to solve these complicated Fluid-Structure Interaction problems.
6 References


[31] Genta, G., Silvagni, M., and Qingwen, C., September 2013, "Dynamic Analysis of Rotors: Comparison between the Simplified One-Dimensional Results and those obtained through 3-D Modeling," Congresso Nazionale AIMETA, XXI.


