I, Honghao Dai, hereby submit this original work as part of the requirements for the degree of Master of Science in Mechanical Engineering.

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Optimization of Financial Decision for Elder Care Services Using Markov Chain Modeling

A Thesis submitted to the
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ABSTRACT

This thesis presents a unified approach to integrate progressive care processes, multiple financial resources with the objective of satisfying elder’s demand for services. Under the proposed elder care framework, several modeling tasks, prediction of elder’s remaining life time, classification of elder’s preference for care services, estimation of elder’s care costs, and optimization of financial decisions are addressed systematically. An elder care financial plan is provided to maximize the elder’s care service level with optimal purchasing decisions on financial resources.

Markov Chain model is developed for predicting remaining life time. Personal physical profile is introduced in the form of covariates which are age, gender, health indicator and health history. It influences the transition probabilities from a care stage to another, so elders have their unique transition probability matrix in Markov model. Service level and pricing model aims to quantify elder’s preference of care services through classification. Three financial resources are modeled: Medicare, long-term care insurance and reverse mortgage. The optimization model selects service fill rate as objective to maximize the depth of demand that is satisfied by the benefits from financial resources. The balance of total care costs, benefits and premiums of financial resources serve as main constraint. Furthermore, a heuristic algorithm is developed to enable these models work together to search for optimal solution.

A case study is designed to show optimal decision sets. Purchasing decision set includes daily benefits, benefit period and purchasing age for long-term care insurance, purchasing age and contract rate for reverse mortgage. With the input of personal health profile, an optimal financial plan can be generated and track the wealth status (care cost, savings, benefits, and premium) of elder. Then, sensitivity analysis is carried out to investigate the uncertainty from personal health indicator. Optimal financial decisions are
recommended in each scenario. One of the result shows a contradiction with separately traditional analysis on purchasing decisions. The results also indicate the necessary of distinguish healthy condition with unhealthy condition, as well as the unhealthy severity. With these information, an optimal financial decision set is recommended based on health information and service preference of elders.
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1 Introduction

1.1 Background and Motivation

The world is aging. Every country in the world is experiencing population aging. The number and proportion of older people in the population has increased substantially in the past a few decades. According to the *World Population Ageing* report (United Nations, Department of Economic and Social Affairs, Population Division 2015), the elders aged 60 years is projected to increase by 56% within fifteen years, the global population of elders aged 60 by 2050 is expected to more than double size in 2015. Improved longevity and medicine mean that the number of eldest persons aged 80 years or over grows even faster in the group of elders, the global population of aged 80 is expected to more than triple size in 2015. The aging growth is projected to accelerate in the coming decades.

The aging population implies social transformation, including the demand for elder care and services, labor and financial markets, housing, transportation and social support. The burden of caring for elders is becoming a critical societal issue (Kinsella & He 2009). Elder care also refers to the concept “Long-Term Care (LTC)” in some contexts, which include a variety of care services that meet the demand of elders especially those with functional limitations and cognitive limitation. Functional limitations refer to physical problem related to the activities of daily livings (ADLs) like eating, bathing, dressing and incidental activities of daily livings (IADLs) which include cooking, housekeeping, transportation. Cogitative limitations refer to the restriction on these activities due to the loss in mental acuity. One study estimates that more than two-thirds of 65 years-old’s need care assistance to deal with the loss in functioning and cognition at some points during their remaining life time (Kemper *et al.* 2005). As the population ages, the need for elder care and its peripheral products will increase sharply.

Elder care is expensive and becoming more expensive. For instance, according to a survey conducted by the Genworth Inc., current national median monthly cost of Nursing Home Care (NHC) is $6844, and it’s projected to increase by 34%, up to $9198, in ten years (Genworth Financial Inc. 2016, April), while the
national median household income of 65 years and older is $3209 per month (Bernadette D. et al 2016). Paying for elder care comes from four sources, self-funding, family and friends, government aids and private financing programs.

A common assumption is that the government aids (Medicare and Medicaid) will almost pay for everything. However, the truth is the elders and their family are often shocked by what Medicare doesn’t cover (Judith 2012). A study at the Mount Sinai School of Medicine in Manhattan documents indicates out-of-pocket costs for older adults at the close of life often place a significant financial burden on individuals and families (Kelley et al. 2013). One of the key findings in this research is one-quarter of elders incurred out-of-pocket medical expenses that exceeded the total value of their assets during five-year research period. Forty-three percent of elders incurred expenses that exceeded their assets, excluding the value of their homes. Moreover, out-of-pocket cost burdens fall most heavily on those with chronic health conditions and without employer, subsidized the coverage (Crystal et al. 2000). Thus, financing elders with available funding resources to alleviate the out of pocket cost and improve care services is a critical research problem in health care area.

To introduce this proposed research problem, we need to understand the overview of care stages that elders probably go through, study the services to be provided in each stages and investigate the available financial resources in this chapter.

1.2 Elder Care Stages

Elder care continuum consists of two main forms, institutional care, and home care. Institutional care is provided within a specific living arrangement designed to meet the functional, cognitive, medical, personal, social and housing need of elders (Galik 2013). Prior to receiving these care, there are number of ways to improve the quality of life, such as joining a senior center and hiring professional caregivers, the care services are often delivered by the home health care agencies. Some early research have made effort to integrate these care stages (Grabowski et al. 2012). Pezzin et al. 1996 proposed a conceptual framework as Figure 1.1 shows that enable elders choose their care service and living arrangement under the constraints of budget and time.
Base on their conceptual model, a typical elder care progressive processes is developed as old people may be willing to stay at their own place (Aging in Place) or an Independent Living Community (ILC) as long as they can live independently. Inevitably, most elders especially who may have experienced health accident or chronic condition need to go through certain stages of institutional care including Assisted Living Facility (ALF) and Skilled Nursing Home (SNH) until the Hospice Care (HC). The detailed explanation and care services provided in each stage are discussed below.

1.2.1 Aging in Place

Aging in place is the term that an elder has the ability to live in one’s own home and regular community safely, independently, and comfortably, regardless of age, income, or ability level (The Center for Disease Control and Prevention 2016). It's a good choice for these elders who would prefer to remain in the home independently as long as possible.

From a care perspective, the Aging in Place is considered as home-based care. Various services are delivered by professional home care agencies or private caregivers include meal preparation, housekeeping, assistance with bathing, dressing, transferring and toileting, medication reminders, joyful companionship, nursing care.

In addition, some home modification need to be applied in the common household, e.g. increased lighting, grab bars, walking-in bathtubs. It will incur extra capital cost compared to independent living communities.
1.2.2 Independent Living Community

Another independent living style for elders is independent living community which often refers a retirement community that has benefit in convenient services, elder-friendly surroundings designed for senior 55 or older. It’s suitable for these elders who would wish to downsize the burden of managing a home and increase social interaction.

From a care perspective, the independent living communities are regarded as community-based care. Residents are offered with some basic services like meal preparation, housekeeping, transportation to appointment and errands. They are also allowed to employ third-party home care agencies to receive supplementary care services. Unlike institutional-based care, elders in independent living don’t require assistance for ADLs or IADLs and 24/7 skilled nursing care.

1.2.3 Assisted Living Facility

Assisted living facilities are licensed facilities that provide safe and clean living accommodations and three meals per day. It’s designed for elder who are no longer able to live on their own safely but do not require the high level of care provided in a nursing home. Elders who live in assisted living facilities may be semi-independent/ dependent on significant assistance with ADLs.

From a care perspective, the assisted living facilities are institution-based care or intermediate care facilities. According to the regulation of some state government (Utah Department of Health 2016), the personal care services (ADLs) provided in assisted living facilities include dressing, eating, grooming, bathing, toileting, ambulation, transferring, and self-administration of medications. The elders also receive general nursing care and other individualize personal and health related care services 24/7.

1.2.4 Skilled Nursing Home

Skilled nursing facility is the licensed facility that provides 24/7 licensed nursing services, eight hours of register nursing (RN) coverage. Elders in skilled nursing home are dependent and require full assistance with ADLs. They are often in the severe phases of chronic disease that requires medications by nurses.
In addition, ancillary services are always provided in these institutions. Elder could select some non-care related services like rehabilitative services, premium meal, guest parking and etc. It would charge additional fee beyond their basic service plan.

1.2.5 Hospice Care

Hospice care focus on palliation of an elder’s pain and symptoms and meet their emotional and spiritual needs. It’s usually given in your home but may also be covered in a care institution. (U.S. Centers for Medicare & Medicaid Services 2016).

Services in hospice care can be similar to their pervious care stages include hospice and palliative aide, homemaker services, skilled nursing care, physical and occupational therapy, social work services and etc. In the current public health aide system, Medicare covers almost all the cost incurred in hospice care stage.

Table 1.1 summarizes the key characters of each care stages and compares the different level of care.

<table>
<thead>
<tr>
<th>Care Stages</th>
<th>Aging in Place</th>
<th>ILC</th>
<th>ALF</th>
<th>SNH</th>
<th>HC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Care Types</td>
<td>Home-Based Care</td>
<td>Community-Based Care</td>
<td>Assisted Living Care/ Intermediate Care</td>
<td>Skilled Nursing Care</td>
<td>Hospice Care</td>
</tr>
<tr>
<td>Elders’ Status</td>
<td>Independent</td>
<td>Independent</td>
<td>Semi-independent</td>
<td>Dependent</td>
<td>Dependent</td>
</tr>
<tr>
<td>Disability</td>
<td>Able</td>
<td>Able</td>
<td>Moderate</td>
<td>Severe</td>
<td>Profound</td>
</tr>
<tr>
<td>Service Hours</td>
<td>Intermittent or part time</td>
<td>24-hour general monitoring</td>
<td>24-hour health-related monitoring or 8-hour licensed nursing services</td>
<td>8-hour RN coverage</td>
<td>24-hour RN Coverage</td>
</tr>
</tbody>
</table>

Table 1.1 Summary of Characters for each care stage
1.3 Elder Care Services

When people grow older, physical and mental problems may expand. A large fraction of the elderly population may use a range of different care services to overcome their health issues. A variety of care services are provided in each care stages. In contrast, few researches have been done in understanding, classifying and analyzing these services. In this section, the care services are described from three perspectives: category, integration, and delivery.

The category of services can be classified by the severity of care provisions. Van Gameren & Woittiez (2005) have summarized eight service packages from long-term care data in Dutch. Five service packages for home-based care are: (1) Homemaker services (e.g. dusting, making beds); (2) Personal care services (e.g. assistance in eating, drinking, getting dressed, washing); (3) Nursing care services (e.g. injections, medicines, treatment of wound); (4) Social support services (e.g. intensive support and hospice care); (5) Semi-institutional care services (e.g. day care, night care). Three packages for institution-based care: (6) Temporary institutional care; (7) Residential care; (8) Nursing home care.

These care service packages are order of intensity, and the higher level of service packages may also include the lighter service packages. For instance, the care package “nursing care” contains the “homemaker services” and “physical care”.

In addition, some government regulations also have defined the categories for services. Utah Department of Health (2016) classified the care and services into 8 levels according to the location of services, include home health, assisted living facility type I, assisted living facility type II, small health care facility, intermediate care facilities, skilled nursing facility, hospital, and hospice. The UDH states the clear differences among these care facilities, for example, the residents in assisted living facility require services
for minimal assistance with ADLs while no more than two significant ADLs. In nursing home facility, the residents often require full assistance in ADLs. It’s obvious that both two approaches define the categories of care services by the severities.

After identifying the levels of service, the next step is to integrate them to prepare for delivery. There are two existing theories to integrate care services: (1) brokerage model; (2) consolidate direct service model (Zawadski, 2014). The definition, advantages, and disadvantages are summarized as below:

<table>
<thead>
<tr>
<th>Service Integration</th>
<th>Definition</th>
<th>Advantages</th>
<th>Disadvantages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brokerage model</td>
<td>Operational integration; several service programs are linked via case managers. The service category relies on prediction</td>
<td>“discrete fee-for-service” mode can reduce the cost of unnecessary services.</td>
<td>“prior-authorization” mode may lack of ability to meet specific requirements.</td>
</tr>
<tr>
<td>Consolidated direct service model</td>
<td>Structural integration. It directly responds to a series of needs by delivering services through their own program</td>
<td>All-inclusive” mode can match elders’ preference and willingness to receive more care services.</td>
<td>Enrollment cost needed to be a member in such program. Services may also “overfitting” elders’ demand.</td>
</tr>
</tbody>
</table>

As the development of consolidated direct service model, more and more agencies begin to consider the financial aspects in order to provide affordable service plans. A pool of funds is set up to combine diverse financial resources and payment for all services as a total package. One of the successful examples of the consolidated model is the Health Maintenance Organization (HMO). Elders who are 65+ can enroll in this organization to receive the integrated service package.

The delivery of services can be defined by three basic elements: care locations, care providers and the level of services: care locations refer to level of care, e.g. aging in place, assisted living facility and skilled nursing home; care providers can be institutions or home care services agencies; levels of services refer to the categories of services in the order of severity.
1.4 Financing Resources

1.4.1 Public Programs

A number of public programs pay for a variety of care services. However, each program has specific policies about what services are covered, how long you can receive benefits, whether you qualify for benefits, and how much you should pay in out-of-pocket costs. To accurately make a financial plan for care demand, it’s important to recognize which services may or may not covered by these public programs.

Medicare and Medicaid is the most widely used public funding option for elder care services in the United States. Medicare is eligible for people age 65 years and older. There are A (Hospital), B (Medical), C (Medical Advantage) and D (Medicare prescription drug coverage) parts in Medicare program. Medicare Part A and Part B will be automatically involved in providing benefits as long as the elders are already enrolled in Social Security or the Railroad Retirement Broad (RRB). Among the four sub-programs in Medicare, Part A is the most related to elder care including skilled nursing facility care, certified-services in home health care and hospice care (Centers for Medicare& Medicaid Service, 2017). Table 1.3 demonstrates the benefits and premiums of Medicare Part A in different elder care stages. In home care, only Medicare-approved services will be covered, for example, physical, occupational and nutrition therapies, speech-language pathology services, ambulance transportation and part-time skilled nursing care. In skilled nursing home care, Medicare only pays for some basic level of services, for example, semi-private room (two-bed room) is covered in benefits while private room (one-bed room) is not. Furthermore, Medicare doesn’t cover the whole person care, including personal care services in home-based care and all services in assisted living facilities.
### Table 1.3 Medicare Part-A Benefits and Premiums

<table>
<thead>
<tr>
<th>Stages</th>
<th>Home Care</th>
<th>Skilled Nursing Facility</th>
<th>Hospice Care</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Monthly Premium</strong></td>
<td>$0 or $441</td>
<td>(1) $0 for Medicare-approved services</td>
<td>(1) $0 for hospice care benefit period</td>
</tr>
<tr>
<td>(1) $0 for Medicare-approved services</td>
<td>(1) $0 for the first 20 days each benefit period</td>
<td>(2) Up to $5 per prescription for outpatient prescription</td>
<td></td>
</tr>
<tr>
<td>(2) 20% of the Medicare-approved amount for durable medical equipment</td>
<td>(2) $161 per day for days 21–100 each benefit period</td>
<td>(3) Full costs after day 100 in a benefit period drugs for pain and symptom management</td>
<td></td>
</tr>
<tr>
<td><strong>Beneficiary Liability</strong></td>
<td>Medicare-approved</td>
<td>100 each benefit period</td>
<td>out</td>
</tr>
<tr>
<td>(1) $0 for hospice care benefit period</td>
<td>(2) Up to $5 per prescription for outpatient prescription</td>
<td>(3) Full costs after day 100 in a benefit period drugs for pain and symptom management</td>
<td></td>
</tr>
</tbody>
</table>

As for Medicaid, it’s a funding option for long-term care which aims for low-income population. When elders’ assets and income level have dropped in to significant low levels, they may qualify for the Medicaid after assets, income and medical evaluation. The qualification processes vary from state to state and year to year, thus it’s hard to generalize and model about this processes. Medicaid maybe more helpful for those with limited financial resources and physical and mental problems that preventing them from receiving private funding sources like long-term care insurance.

We aware that our research objects are elders who have stable retirement income and available financial resources. Medicaid are not chosen in this case. Medicare Part A is the most widely used public programs in the United State and its simple qualification process bring us great modeling convenience. The limitations of Medicare coverage lead elders seeking for private funding opportunities.

1.4.2 Private Options

Most people don’t have enough money to pay for all the cost of elder care, especially the expensive services at SNF. Private payment options include long-term care insurance, reverse mortgage, life settlement, annuities, and trusts.

Long-term care insurance pays for almost all types of services in the whole care stages. The care location can be home, community and institution. The enrollment of long-term care insurance depends on many
person factors include elder’s age, health status, policy types (daily benefits, benefit periods, elimination period).

Reverse Mortgage is a type of loan based on home equality. It enables older homeowner convert their home into a source of income while still living in their house. The basic mechanism of reverse mortgage is lenders will make a monthly payment to the elders and the program costs and interests on the load don’t have to be repaid until the elders die or moving out of that house. Reverse mortgage isn’t a care-related funding source and can be saved for future expenditure.

1.5 Research Focus of Thesis
Numerous endeavors of academic researcher and industrial practitioners have made efforts on improving experience of elder care. Personality, affordability, and steadiness are the three vital goals to make care plans. Personality means the care plan should be generated based on elder’s personal health status and wealth status. Affordability means elders can pay care costs with available financial resources. Steadiness means the adaptive mechanism that can handle the potential variation caused by health status and wealth status. In this thesis, a personalized elder care planning and management has been proposed in Figure 1.2.

![Figure 1.2 A Systems Approach to Personalized Elder Care Planning and Management](image-url)
Specific objectives of the proposed approach are: (1) Construct a general elder care reference model. The care stage progressing is modeled as a continue-time Markov chain. Based on initial assessment of an elder’s health status, personalized parameters of Markov model are then estimated. (2) Formula a service level and pricing approach. The elder’s preference for service is classified as level and converted to cost. The coverages from financial products are reflect on costs via this structural model. (3) Establish a service-oriented optimization model based on available financial resources. The benefits from optimal purchasing decisions of financial resources can cover more care service costs. (4) Create the adaptive mechanism to update the whole plan when changes happen. Some unexpected events may happen during care processes, e.g., a bad outcome of the health check maybe associated with increased risk of disability or death. When such events happen, the parameters of Markov model need to be updated and elder’s health status and wealth status are then reevaluated to develop new financial decisions. These will ensure that the elder enjoy high level of care service as much as possible throughout the entire care processes.

The rest of thesis is organized as follows:

Chapter 2 reviews the state-of-art modeling and optimization techniques with emphasis on elder care. Methodologies from engineering, health care, and economy are reviewed from two perspectives: health and wealth.

Chapter 3 presents the framework of proposed models, detailing the underlying parameters of Markov chain, elaborating the formation of optimization, and illustrating the modeling approaches for service level and financial resources. A heuristic algorithm is developed to collaborate above models and search for optimal financial decisions.

Chapter 4 presents a case study using heart transplant monitoring data. The impact of personal health profile on parameters of Markov chain is analyzed. Examples of optimal solutions are given and possible explanations are provided. Sensitivity analysis is carried out to explore the adaptive mechanisms with uncertainties from personal health profile.
Chapter 5 summarizes the contribution and conclusion of the research work, discusses the similarities and differences of results with previous literatures, identifies areas that need further improvement in future work.
2 State of Art Literature Review

Over the past a few decades, elder care has rapidly developed as the population aging. Financing the elder care with available funding resources requires the interdisciplinary knowledge from engineering, health care and economy. Elder care has been widely studies in these areas in order to improve the quality of elder’s remaining lifetime, enhance their ability to live independently and make better utilization of their retirement funding.

2.1 Elder Care Stage Modeling

Preliminary work in the field of engineering and health care focused primarily on technology development. Assistive technology has been proposed to meet elders’ needs for health and safety. Typical researches in this topic include fall risk detection and prevention (Rantz et al. 2015; Stone et al. 2015; Lord et al. 2003; Fortinsky et al. 2004), dementia and depression management (Galambos et al. 2013; Leontjevas et al. 2013). However, even with smart home technologies, elders still need care services best coordinated by a registered nurse. In other words, advanced technology is not one size fits all solution to replace the traditional elder care continuum and regular services.

2.1.1 Parameters in Care Processes

Several studies have been conducted on modeling and optimization of elder care continuum and services. Patrick et al. (2015) analyzed the LTC services capacity in the community-based care. They developed a Markov Decision Process simulation model to draw the connection between capacity and demand classes, elders’ preference, and services competing and transferring. Through their simulation, LTC facilities are able to maintain smooth patient’ demand flow with the acceptable wait times for elders. The results provide recommendations about capacity planning for LTC facilities. Cardoso et al. (2015) provides several strategic policies on planning location, capacity, and distribution of LTC services for institution-based care. They adopted stochastic mixed integer linear (MIL) programming model with the objective of minimizing the cost while maintaining the levels of equity. These equity levels are modeled as constraints in the
mathematic model. The uncertainty of LTC demand, along with government’s health policy options are taken into consideration. Cardoso et al. (2016) suggested medium-term planning decisions on LTC services delivery for institution. They used a multi-objective and multi-period mathematical programming model to satisfy the equity of access, socioeconomic and geographical for elders. The model inputs consist of two categories: time-dependent inputs (available budget per time period, institutional service costs per time period, and demand of institutional services per time period) and time-independent inputs (capacity of institutional service, distance of institutional service delivery, waiting time of elders and average LOS per type of institutional service). The model outputs address following concerns: the location and time to install new institutional services; extra capacity to add to each type of institutional services; and balance the equity and costs objective in the LTC service network.

In addition to demand uncertainty, there are other variables and unpredictable events affect the modeling of elder care services. Lanzarone et al. (2010) studied the changes in elder’s clinical and social conditions that impact the management of home care services. They have set up a patient stochastic model to estimate the elder’s care structure in order to support human resources planning in home-based care. Markov Chain model was developed for patient care pathway. They classified the elders by their health and social conditions after identifying the state variables. The predicted outputs in model are number of visits, length of stay (LOS) of home car service per visit, and expected workload of each care operator. The results were able to support human resource planning from medium and short term perspectives.

LOS is another significant parameter during the care processing. A study conducted by Xie et al. (2005) showed the trends and differences of (LOS) in institution-based care. They developed a continuous-time Markov Chain model to describe the movement of elders between home care and nursing home care. They estimated the parameters in Markov model e.g. transition probability by maximum likelihood approach. The model was applied to individual-level data from social service department in UK. The model shows that mixed exponential distribution with short stay mean and long stay mean could describe the pattern of LOS in nursing home well. Similarly, Xie et al (2006) have continued to study the pattern of LOS in
institution care. They have incorporated elders’ personal features into Markov model via log-linear function. They employed Kaplan-Meier to estimate the transition probability between home care and nursing home care. Their statistical results show the significant differences of gender in LOS pattern. As we can see, even the uncertainty of LOS is affected by the personal characters. Van Gameren & Woittiez (2005) used a simulation-based model to capture the transition assessment among level of service for both home-based care and institution-based care. Data were used from home care agencies, assisted living facilities and nursing care home in Netherlands as inputs for duration model. Their findings include: Age has positive effects on the initial transition assessment in home care. Living alone, receiving informal care will increase the transition probabilities. These literatures support the fact that personal health status (age, gender, and living condition) would have profound influence on their transitions parameters (LOS, probability) and model outputs (utilization, quality, and costs) during the care processes.

Research in the traditional elder care has tended to focus on institution or regional level modeling and optimization rather than individual level optimization. Elders may not get benefits directly from research outcomes. An additional problem is that most of the literatures isolated the care stages either in home-based care or institution-based care. Rare elders would stay at home or care institutions forever, linking the various care stages together will give a whole picture to manage care systematically and comprehensively.

2.1.2 Markov Model

Another key finding with much of the literature in relation to elder care is that Markov model is often used to track the changes between care stages. Actually, many attempts have already been made in health care area with the purpose of modeling the progressive processes for chronic diseases (e.g. colorectal cancer, heart disease, stroke, and diabetes) via Markov model.

The Markov model can be used to estimate care costs. Govorun et al. (2015) employed the Markov reward model to model the individual health care costs. They suggested the phase-type distribution to fit the care cost function since the health care cost naturally depend on the care stages. Their fitting results indicate, under suitable parameters, the health-dependent care costs give an approximate expectation with the net
present value of the Markov reward model with costs. They also draw the conclusion that the age significantly impacts on the net present value of care cost structure.

The hidden Markov model can model the quality of care. Mitchell et al. (2015) concerned about quality of care during the delivery. A hidden Markov model was established to model the LOS using a Coxian phase-type underlying distribution. They treated the quality of care as a hidden layer. Covariates are introduced to decide the hidden level which would affect the quality of care. Their results suggest the lower transition rates for one group of patient could be caused by long waiting time, lack of capacity, and poor care quality.

Modeling LOS in care stages is an important aspect for Markov model. Besides the papers listed above, McClean et al. (2011) studied the model of the whole care system in hospital, such as normal residence, nursing care, and death. The underlying distribution to model LOS is Coxian phase-type based on Poisson admission of patients. The Markov model is also used to build a discrete-event simulation. They developed various scenarios to reduce LOS prior to discharge to nursing home, predict the capacity needed for hospital. Castelli et al. (2007) have proposed a semi-Markov model to assess the costs with the variation from patients’ health status. Three health transition stages are defined as: alive, alive with chronic conditions, and dead. They employed Weibull distribution to model the hazard function of LOS. A regression model was developed for the cumulative cost function. Finally, the cost-effeteness analysis was carried out to compare two mechanisms for colorectal cancer study. They draw the conclusion that follow-up mechanisms and transition stages significantly influence the mean costs. Faddy & McClean (2005) applied the Markov chain model to assess the effects of covariates on geriatric patient data. The LOS in community-based care and hospital were estimated by Phase-type distribution by the maximum likelihood function. They selected the age and year of admission as the covariates. Their results show that the covariates effect on the various parameters (LOS) of Markov chain model.

The underlying distribution is another key character in the Markov structure of stochastic models. Common distributions are exponential distribution, Weibull distribution, Gamma distribution, Coxian distribution, and Phase-type distribution. Phase-type distribution has been used widely in areas as finance, health care
system, production system, traffic modeling, and reliability theory. Fackrell et al. (2009) gave a survey of application of Phase-type distribution in health care industry. They pointed out the Phase-type distribution has such popularity due to the algorithmic tractability and exponential naturality. In his study, nearly 62% publications in health care area use Phase-type distribution in the last four years.

Since many chronic diseases have a natural interpretation in terms of staged progression as above. Multistate models based on Markov processes is a well-established method of estimating the probability of transition between stages (Jackson, 2003). Another fact is that approximately 92% of elders have at least one chronic disease, based on a survey conducted by National Council on Aging. It’s inevitable that elders in each chronic stage need to receive corresponding level of care and a variety services. It has come to our mind that modeling the transition between care stages using data of specific chronic disease may be more precise and individualized.

2.2 Financial Decisions for Elders

There is a vast amount of literature on helping elders make financial decisions during their retirement life. Most of them studies the financial literacy, retirement planning and investment behaviors (Agarwal & Mazumder 2013; Almenberg & Säve-Söderbergh 2011; Hastings & Mitchell 2011).

Few studies have been published on linking the elderly health with financial decisions. The first systematic study in this series was carried out in 1998 by Picone et al. In their analysis of medical care, health capital, and wealth, they used dynamic Grossman household production model to capture financial behaviors of elders under uncertainty. They draw the conclusion that elders purchase the medical care and consumption with a specific pattern in order to smooth their expected utility.

Based his research, a series of work has been done by Gupta and his folks to study the optimal LTC insurance purchasing decision, retirement consumption plans and risky investment. (Gupta & Murray 2003; Gupta & Li 2013; Gupta, & Li 2007; 44.Gupta & Li 2004; Gupta & Li 2011; Gupta & Murray 2005). His approach is based on a large-scale two-branching nonlinearly constrained optimization model with purpose
of maximizing lifetime utility of wealth, consumption and LTC cost for elders. In his health evaluation model, he proposed a concept of health shock (0 or -1) to represent changes in health condition. 0 denotes normal health condition while -1 denotes the accident happened. The main weakness in their study, which also mentioned many times in his extension and future work, is that their model only capture “health shock” (-1) and “no health shock” (0) scenarios. The severity of health shock is not taken in consideration. The root cause of the limitation is that expanding the health stages requires a multi-branching model, instead of the two-way branching model, which would promote computational difficulty dramatically. In our view, if we connect Markov care stage model with financial decision optimization model, a more realistic health, and wealth transition model would be established.
3 Developed Elder Care Financing Models

3.1 Markov Chain Model

3.1.1 Absorbing Continuous-Time Markov Chain and Phase-type Distribution

Let $E$ denote a set of stages, and $\{X(t)\}$ be a stochastic process with state space $E$. $\{X(t)\}$ is a Continuous-Time Markov Chain (CTMC). This process is time-homogeneous for all $u \geq 0$,

$$\text{Prob}(X(t + u) = j | X(u) = h) = \text{Prob}(X(t) = j | X(0) = h) = P_{hj}(t) \quad (3.1)$$

The transition probabilities $P_{hj}(t)$ of a homogeneous Markov chains only depend on the difference $t$ between $u$ and $t + u$. The values $P_{hj}(t)$ defines a matrix with all transition probabilities $P$.

![Figure 3.1 Absorbing Continuous-Time Markov Chain for Elder Care Stages](image)

As the Figure 3.2 shows, the stage space $\{E_0, E_1, E_2, E_3, E_4\}$ represents for Aging in place, ILC, ALF, SNH and HC. The absorbing stage $E_4$ indicates death or discharge from care the system. Then this type of Markov chain is called *absorbing Markov chain*. We define $\{X(t)\}$ consists of the set of transient stages $X(t) = \{1, 2, ..., m\}$ and a single absorbing stage $X(t) = \{m + 1\}$.

The Markov chain model has three descriptors.

I. *A state space* $E = \{0, 1, 2, 3, 4\}$

II. *An initial state probability distribution* $\alpha = (\alpha_0, \alpha_1, 0, 0, 0)$

III. *An infinitesimal generator*
The vector $\alpha(0)$ is the initial state probabilities of CTMC. $q_{ij}$ is the instantaneous transition rate between stage $i$ and stage $j$. The random times between state transitions are exponentially distributed random variables with parameters $q_i$ for state $i$.

Let the state probabilities at time $t$ are denoted by $P_j(t) = \text{Prob} \ (X(t) = j), j \in E$. Give the properties of infinitesimal generator, $Q$ can also called transition intensity rate

$$q_{ij}(t) = \lim_{\delta t \to 0} \frac{P(X(t + \delta t) = j | X(t) = i)}{\delta t} \quad (3.2)$$

Since $q(i) > 0$, all diagonal elements of the matrix $Q$ are non-positive. All the non-diagonal elements must be non-negative. Thus, the infinitesimal generator follows that

$$\sum_{j} q_{ij} = 0$$

Assume an embedded process $\{X_r\}_{r \in N}$, with $X_0 = X(0)$, is a discrete-time Markov Chain with transition probability matrix $P$. Then it can also be expressed in terms of $Q$.

$$P(i,j) = \frac{q_{ij}}{-q_{ii}}, \text{for } j \neq i, q_{ii} \neq 0$$

The infinitesimal generator $Q$ can be written as

$$Q = \begin{pmatrix} -q_{02} - q_{14} & 0 & q_{02} & q_{03} & 0 \\ 0 & -q_{12} - q_{13} & q_{12} & q_{13} & 0 \\ 0 & 0 & -q_{23} - q_{24} & q_{23} & q_{24} \\ 0 & 0 & q_{32} & -q_{34} - q_{32} & q_{34} \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$\alpha$ is the a $1 \times 5$ vector zeros. $d = (\mu_{16}, \mu_{26}, \mu_{36}, \mu_{46}, \mu_{56})^T$, $D$ is the $4 \times 4$ matrix of transition rate between stages.
A phase-type distribution (PHD.) is defined as the distribution of the sojourn time $\tau_s$, i.e., the time to enter an absorbing state from the set of transient states. In our context, the sojourn time is the total remaining life time for elders. The distribution function of a phase-type is with the PHD parameters $\left( \alpha, D, d \right)$, when $t > 0$

$$ F(\tau_s) = 1 - e^{D^t} \mathbf{1} $$

(3.4)

Where $\mathbf{1}$ is the column of $m$-vector of 1’s. The associate probability density function (PDF) of a PH distribution at time $t$ is, when $t > 0$,

$$ f(\tau_s) = \alpha e^{D^t} d $$

(3.5)

The above absorbing CTMC process is called a phase process. It’s not difficult to find that PH distribution is an exponential-based distribution but has more parameters $\left( \alpha, D \right)$, which allows multi-state modeling.

Given the property of CTMC $\{X(t)\}$, the care process stays in state $j$ at time $t$, $X(t) = j$ is determined by an exponential distribution with parameter $-D(j, j)$, $0 \leq -D(j, j) < \infty$. $D(j, j)$ is the diagonal elements of $D$, which is defined by

$$ D(j, j) = -\left( \sum_{j \neq i} D(i, j) + d(j) \right) $$

The holding time or LOS is exponentially distributed which means that the number of occurring events follows a Poisson distribution. Therefore, the exponential LOS $\tau_j$ of each care stages $j$, has distribution function

$$ G(t) = \text{Prob}(\tau_j \leq t) = 1 - e^{-(-D(j, j))t}, t \geq 0 $$

(3.6)

3.1.2 Estimation of Transition Probabilities and Infinitesimal Generator

The transition probability matrix $P$ of CTMC is defined in (3.1). Assume an empirical transition probability matrix $\tilde{P}$ as a sampling of a time-homogenous CTMC at a time interval $[t, u]$, the $\tilde{P}$ is the transition
probability matrix of Discrete Time Markov Chain (DTMC). Through this way, a CTMC model is converted to a similar DTMC model. Let $N_{ij}(t,u)$ denotes the number of elder transiting from care stage $i$ to $j$ during the time period $[t,u]$, the likelihood function of $\tilde{P}$ would be

$$L(P) = \prod_{i \in E} \prod_{j \in E} N_{ij}^{N_{ij}(t,u)}$$

(3.7)

Then its MLE (maximum likelihood estimator) is given by

$$\hat{p}_{ij} = \frac{N_{ij}(t,u)}{N_i(t,u)}$$

(3.8)

$N_i(t,u) = \sum_{j=1}^{n} N_{ij}(t,u)$ is the total number of elders in stage $i$ during the period $(t,u)$. Therefore, the transition probability is estimated by the proportion of elders those leave original care stage in the total elder of original care stage.

Infinitesimal Generator is defined by (3.2). From Kolmogorov’s Forward Equation, the connection between $P$ and $Q$ is denoted by

$$\frac{dP}{dt} = P(t)Q$$

(3.9)

Israel et al. (2001) proposed an approach to solve the equation in matrix language by

$$\hat{Q} = (P - I) - \frac{(P - I)^2}{2} + \frac{(P - I)^3}{3} - \frac{(P - I)^4}{4} \ldots$$

(3.10)

Where $I$ is the $N \times N$ identity matrix. The main drawback of this estimation is that $\hat{Q}$ is unable to guarantee nonnegative off-diagonal elements, which is known as “embedded problem”. We deal with the negative value in off-diagonal elements by replacing them with the average value of other elements at the same row.

In this section, we established a CTMC model to describe the elder care processes with the PH underlying distribution. The parameters of Markov Chain model are computed by MLE estimator and Kolmogorov’s Forward Equation. The output from CTMC model will be prepared for following optimization model.
3.2 Service-Oriented Optimization Model

Table 3.1 Parameters and Description for Service-oriented Optimization Model

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_j$</td>
<td>A care stage $j, j = 0,1,2,...,m, E_j \in E$</td>
</tr>
<tr>
<td>$u_j$</td>
<td>The time unit cost at stage $j$</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Aggregate service level, $0 &lt; \eta \leq 1$</td>
</tr>
<tr>
<td>$Y_j$</td>
<td>Level of service at stage $j$</td>
</tr>
<tr>
<td>$T_{s,j}$</td>
<td>Length of stay (LOS) at $E_j$</td>
</tr>
<tr>
<td>$C_j(\Theta)$</td>
<td>Service cost to stay in $E_j$ with personal physical profile $\Theta$.</td>
</tr>
<tr>
<td>$P_t$</td>
<td>Premium rate on financial products at time $t$</td>
</tr>
<tr>
<td>$B_t$</td>
<td>Benefit rate of financial resources at time $t$</td>
</tr>
<tr>
<td>$T_p$</td>
<td>Premium payment period</td>
</tr>
<tr>
<td>$T_B$</td>
<td>Benefit period/ Benefit Multiplier</td>
</tr>
</tbody>
</table>

Our objective is to satisfy the demand of care services for elders. Case fill rate (CFR) is a common criterion to measure the depth of demand that was satisfied by inventory on hand in supply chain management. It’s the proportion of products shipped to customers on time in the total ordered products. A close analogy with care services for products is made. Our customers are elders and the services just like products in supply chain that can be delivered to them, Elders select a $Y_U$, the desired upper limit of service level, as the demand, and a service plan will be made based on the health condition and available funding sources with an actual services level $Y_j$ for each care stages. A service level fill rate (SLFR) in this care stages can be computed by $Y_j/Y_U$.

The objective function is described in broad terms as determining the set of decisions to maximize the SLFR. The service-oriented optimization model is given as:
Obj:

\[
\max_{P_t,B_t,T_B,T_P} \eta = \mathbb{E}_t \left[ \sum_{j \in E} \left( \frac{Y_j}{Y_U} \right) \right]
\]  

(3.11)

S. t.:

\[
\sum_{j \in E} C^\Theta_{T,s,j}(Y_j) + T_P \cdot P_t \leq T_B \cdot B_t
\]  

(3.12)

\[
C^\Theta_{T,s,j}(Y_j) = \int_0^{T,s,j} u_j(Y_j,t) \cdot e^{\Theta(t)} dt
\]  

(3.13)

\[
B_t = \Psi_{FR} (P_t, T_P, T_B)
\]  

(3.14)

\[0 < Y_j \leq Y_U \leq 1, j \in E\]  

(3.15)

\[u_j, B_t, T_P, T_B, P_t \geq 0\]  

(3.16)

\(C^\Theta_{T,s,j}\) is the stage-related care costs associated with \(v \times 1\) dimensional personal physical profile \(\Theta\). \(u_j(Y_j,t)\) is the average cost rate accumulation in the stage \(j\) at time \(t\) when given \(\Theta\). \(\Psi_{FR}\) is the function to denote the relationship between premium payment and benefit payment for financial resources, e.g. \(\Psi_{LTCI}\), \(\Psi_{MC}\) and \(\Psi_{RM}\).

The covariates may be time-fixed or time dependent. For example, gender is time-fixed variable while age and health indicators are time dependent explanatory variables. The effects on the infinitesimal generator are modeled by

\[e^{\Theta(t)} = q^{(0)}_{ij} \cdot e^{\omega_j^T t} \]

\(q^{(0)}_{ij}\) is the baseline of transition intensities rate. \(\omega\) is the \(v \times 1\) vector that quantify the linear effect on the physical profile indicators. The personal profile covariates can be age, gender, health indicators and health history.
In this section, a service-oriented optimization model is established with the objective of satisfying elders’ demand as much as possible. The main constraints wealth balance (3.12) is stated with the support of cost function (3.13) and financial resource limitations (3.14). A personal profile is introduced to impact the infinitesimal generator in Markov Chain as well as the time-dependent care cost and thus determine the different financial decisions.

3.3 Pricing Models for Care Services

Pricing strategies and models significantly influence the selection of care services for elders. Multiple service providers might use their own service pricing models. After reviewing the service charging systems used in care institutions and home health agencies, a care services pricing model has been established. Before introducing it, a service level model is prepared to summary and classify the detailed services.

3.3.1 Service Level Model

In Section1.3, we have reviewed the care services from the marketing and government perspectives. Our conclusion is that a common approach of classification is sorting services by severity. To determine the service level, we need to distinguish a “premium” service plan with a “basic” service plan.

In home-based or community-based care, the care services are often delivered based on the need of elders. A typical “premium” plan may include higher level and more services than a “basic” plan. In the institution-based care, it’s difficult to recognize severity of service due to the all-inclusive pricing mode, therefore, ancillary services are introduced to distinguish “premium” services from “basic” services. Figure 3.3 indicates the level of services by level of care.
In the Aging in place care stage, the entry level service is homemaker services, it primarily provides elders with assistance in IADLs, including housekeeping, laundry, meal preparation, personal hygiene, and shopping. Then, if the elders have chronic condition or health issues, home health services may introduce to provide basic health aid, like check blood pressure, heart rate, temperature, administration of prescription and other drugs, injection or surgical wound. Furthermore, the elders may receive therapies include physical, occupational, speech, and nutrition therapy which will help them recover from health incidence. If they aren’t fully independent health status, personal care services could be provided to assistant with basic ADLs. If necessary, a part-time registered nurse (RN) or licensed practical nurse (LPN) would be hired to give direct professional care or teach the caregivers.

If elders choose to live in an independent living community, most of the communities provide basic services (0.4) like homemaker services and health services. The higher level of service could be delivered by a third-part home services agency.

In care institution, an all-inclusive mode is always adopted which is considered to be basic service level (0.6). Ancillary services can be guest parking, additional meal, and private rooms.
Service level model integrates category and quantity of services, connects quantitative variables with qualitative variables, and lays foundation on establishing pricing schemes.

3.3.2 Pricing Model

Most of the home service agencies and care institutions have two ways to charge services fee: flexible pricing and all-inclusive pricing. For home care agencies, the flexible pricing approach consists of four parts: (1) one-time upfront fee, refer to as “community fee”. It’s a security deposit that can be a couple of thousand dollars to keep reservation; (2) monthly base fee. Most of the base fees include the utility, weekly housekeeping, and meals. The cost also depends on the location and size of residential unit; (3) care services fee. This fee is typically based on the number of ADLs or the severity of nursing care. Determining the amount of services requires thorough, clinical and detailed assessment; (4) non-care related service fee. The care institutions also provide ancillary services to meet the extra or specific care needed. For care institution, all-inclusive pricing approach spreads the total service costs for the entire care institutions. The advantage of this approach is that elders have a relatively stable care costs over the long term, regardless the level of services. But, the care institution always set up a cap on the level of services that they can offer under the all-inclusive price. Meanwhile, extra fees are charged by the usage of ancillary services.

Eckert 2009 classified the pricing models of web and information services as Figure 3.4 shows. A variable pricing model charges the elders with a fee, which depends on the actual usage of the services. The fee directly associates with the level of services execution requests. A fixed pricing model charges elders by a fixed fee, usually ensuring unlimited access to the services for a certain period of time independent of the level of services execution request. A hybrid pricing model combines the fixed pricing model with variable pricing model. Common way for services pricing are mainly fixed fee and minor variable fee following a level. Based on the pricing policies mentioned above, the variable model could be used to quantify the home-based care services (flexible pricing) and hybrid model can support the institution-based care services (all-inclusive pricing).
According to the Genworth Inc. 2015 Cost of Care Survey, the nation median rates for theses care stages can be found in Table 3.1. To simplify service pricing model, we assume the service time in the aging in place and independent living community is 44 hours per week.
### Table 3.2 Estimation of Care Service Model

<table>
<thead>
<tr>
<th>Care Stage</th>
<th>Aging in Place (Hourly)</th>
<th>IL Community (Monthly)</th>
<th>Assisted Living Care (Monthly)</th>
<th>Nursing Home Care (Monthly)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>$11</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>0.4</td>
<td>$17</td>
<td>$1840</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>0.6</td>
<td>$20</td>
<td>$3150</td>
<td>$7698</td>
<td>$6844</td>
</tr>
<tr>
<td>0.8</td>
<td>$27</td>
<td>$3620</td>
<td>$7698</td>
<td>$6844</td>
</tr>
<tr>
<td>1.0</td>
<td>$32</td>
<td>$4650</td>
<td>$8182</td>
<td>$8182</td>
</tr>
</tbody>
</table>

1 Hourly rate is converted from annual rates based on 44 hours of care per week, multiplied by 52 weeks.

3.4 Financial Resource Model

Recall in Chapter 1, the financial resources have two categories: public programs, and private options. In this section, we opted Medicare Part A as representative in public program due to the universality and stability. Long-term care insurance is modeled as care-related private financial option and reverse mortgage models is prepared for a general private funding option. Modeling technologies are tailored to suit the characters of financial resources. Three modeling skills are considered:

1. **Deductive model**, a logical structure based on a theory or policy.
2. **Inductive model**, a model arises from empirical findings generalization from them.
3. **Floating model**, an invocation of expected structure.

In this section, a guideline is built to illustrate how to model a typical financial resource. Deductive modeling approach is fit for public program under administration of federal and state government. This type of funding sources always has detailed regulation of benefits and coverages. Moreover, the policy of these programs always change yearly, the history data may not be able reflect the current situation. Inductive model is suitable for some financial aid programs with public dataset. For instance, Long-term care insurance are often provided based on personal profiles. Empirical data can tell some clues about the factors like the relationship between the premium and age, gender, and health condition. Floating model can deal with the financial resources which have specific structure. As for reverse mortgage, it’s difficult to find
individual-level datasets, but it follows some structure within the range of economic engineering, contract rate, present value, market interest rate and those variables can be introduced to support the mechanism of the structure.

3.4.1 Medicare

Medicare is modeled via deductive models. The schema of Medicare Part-A is given in Figure 3.3.

Any person who has reached age 65 and who is entitled to Social Security benefits is eligible for Medicare Part A without premium fee, which means in $B_t = \Psi_{FR}(P_t, T_P, T_B)$ the premium $P_t = 0$.

The Medicare covers some home care services in the Aging in Place and the ILC. Capital costs $C_0(\Theta)$ (walker, wheel Chair, and hospital bed) are covered by 20%. Let $Y_{j,MC}$ denote the Medicare-approved services. Previous service level model provides us the provision on distinguishing Medicare-approved services with non-Medicare-approved services. For instance, Medicare covers home health services (0.4) not home maker services (0.2), therefore, if elders select the home health services, the desired service level $Y$ is 0.4, while the Medicare-approved service level $Y_{MC}$ is 0.2. Because in the modeling of service level, we consider the high-level service package always contains the relatively low level service package. In the above example, Medicare only covers the home health services but not the home maker services, which leads to the coverage becoming 0.2.
In home-based/community-based care services, Medicare only pays for the part-time and periodic (less than 21 days) services, which means $T_{s,j} \leq 21, j = \{0, 1\}$.

$$B_j = 0.2C_j(\Theta) + u_j(Y_{j,MC})T_{j,s} \quad j = 0, 1$$

Medicare won’t cover any costs in ALF,

$$B_2 = 0$$

Medicare Part A covers many services in the SNH under certain conditions for a limited time.

When $0 < T_{s,3} \leq 20$ days,

$$B_3 = u_3(Y_{3,MC})T_{3,s}$$

When $20 < T_{s,3} \leq 100$ days,

$$B_3 = u_3(Y_{3,MC})T_{3,s} - L$$

When $T_{s,3} > 100$ days,

$$B_3 = 0$$

$L$ is the liability that should be paid by elders in each benefit period. Based on government policy, $L = $161 per day. $u_j(Y_{j,MC})$ is the unit price covered by Medicare based on the service level $Y_j$.

### 3.4.2 Long-term care insurance

Long-term care insurance is modeled via inductive model. The online data about long-term care insurance provides the relationship of premium with some impact factors like age, gender, elimination period, daily benefits and benefit periods. The structure of long-term care insurance can be found in Figure 3.6.
Our premium and benefit model for LTCI is

\[ P_t = c_1 \rho(\theta) + c_2 B_t + c_3 T_p + c_4 T_B + c_5 T_w + c_4 \]

\( \rho(\theta) \) is the polynomial of personal physical profile, e.g. \{age, gender\}. \( B_t \) denotes the daily maximum benefit. \( T_B \) is the benefit period (benefit multiplier). \( T_w \) is the elimination period that is benefit waiting period from the time care begins until the time the policy begins paying for your care. The coefficients \( c_i \) can be estimated by the dataset collected from the Genworth Financial Inc.

Regression models are used to estimate the parameters. After comparing several fitting approaches, Scheffe Cubic Model is selected. Scheffe Cubic Model is a reduced version of full cubic model, also called the special cubic.

### Table 3.3 Summary of Model Fitting

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>RSquare</td>
<td>0.9981</td>
</tr>
<tr>
<td>RSquare Adj</td>
<td>0.99802</td>
</tr>
<tr>
<td>RMSE</td>
<td>164.5169</td>
</tr>
<tr>
<td>Mean of Response</td>
<td>5844.037</td>
</tr>
<tr>
<td>Observations</td>
<td>473</td>
</tr>
<tr>
<td>Parameters</td>
<td>Mean</td>
</tr>
<tr>
<td>-----------</td>
<td>------------</td>
</tr>
<tr>
<td>G</td>
<td>-54894.40</td>
</tr>
<tr>
<td>A</td>
<td>44.86</td>
</tr>
<tr>
<td>B</td>
<td>-134.30</td>
</tr>
<tr>
<td>T</td>
<td>68906.98</td>
</tr>
<tr>
<td>G*A</td>
<td>1564.22</td>
</tr>
<tr>
<td>G* B</td>
<td>51.12</td>
</tr>
<tr>
<td>A* B</td>
<td>1.70</td>
</tr>
<tr>
<td>G* T</td>
<td>-2374.96</td>
</tr>
<tr>
<td>A* T</td>
<td>-1992.77</td>
</tr>
<tr>
<td>G<em>A</em>(G-A)</td>
<td>12.44</td>
</tr>
<tr>
<td>G* B *(G- B)</td>
<td>0.00</td>
</tr>
<tr>
<td>G<em>A</em> B</td>
<td>-0.27</td>
</tr>
<tr>
<td>A* B *(A- B)</td>
<td>0.00</td>
</tr>
<tr>
<td>G* T *(G-T)</td>
<td>-710.24</td>
</tr>
<tr>
<td>G<em>A</em> T</td>
<td>72.54</td>
</tr>
<tr>
<td>A* T *(A- T)</td>
<td>14.33</td>
</tr>
<tr>
<td>G* B* T</td>
<td>-12.99</td>
</tr>
<tr>
<td>A* B* T</td>
<td>0.72</td>
</tr>
<tr>
<td>B* T *(B- T)</td>
<td>0.00</td>
</tr>
</tbody>
</table>
Therefore, the regression model for LTCI is given by following equation. Let $B$ denote the daily benefits, $G$ is the gender, $T_B$ is benefit period and $A$ is age.

\[
P_t = -54894G - 134B + 68906T_B + 1564(G \cdot A) + 51(G \cdot B) + 2(A \cdot B) - 1993(A \cdot T_B) \\
- 32(B \cdot T_B) + 12[G \cdot A \cdot (G - A)] - 710[G \cdot T_B \cdot (G - T_B)] + 73(G \cdot A \cdot T_B) \\
+ 14[A \cdot T_B \cdot (A - T_B)] + A \cdot B \cdot T_B
\]

Figure 3.7a shows the difference of predicted premium and actual premium. From the $R^2 = 0.999$, it means the fitted regression model can illustrate 99.9% variation in the actual dataset. In Figure 3.7b, there
is no obvious pattern for this regression model, which indicates the estimating parameters are good. Figure 3.7c gives the interactions of each factor. Take the two factors age and daily benefits as example to illustrate this point. As the age increases, the low daily benefits (e.g. $B = 50$) are less sensitive to premium when compared to high daily benefits (e.g. $B = 300$). The interactions provide us a better understanding for long-term care insurance policy and lay foundation for explain optimal purchasing decision later.

3.4.3 Reverse Mortgage

Reverse Mortgage is modeled via floating model which is the invocation of expected structure. Reverse mortgage is not a public aid but under regulation of government. Before modeling, there are some key assumptions: (1) The elders have only one house and prefer to live there; (2) when the elders pass away, the house is the only source for repaying the reverse mortgage balance, and (3) fixed contract interest rate independent of time. A reverse mortgage policy can be influenced by those factors: (1) the parameters of the RM contracts: principle limit factor (PLF), closing cost, and RM contract interest rate, (2) the parameters of elder’s: health status (e.g. gender, age) and wealth status (e.g. house value, care expense).

When apply for a Reverse Mortgage, elders need to pay for the closing costs $C$ (the initial mortgage insurance premium (IMIP), servicing fee, origination fee and third party fee). The initial value of the house as collateral is $V_0$. Then the constant payment (unit time cash flow) to elders until the contract is terminated, Chiang & Tsai (2016) proposed a theoretical economic model to derive the RM annual benefits,

$$B_t = \frac{1}{\rho} \cdot (\varphi V_0 - C)$$

Where $\varphi$ is the principle limit factor (PLF) and $\rho$ denotes the expected present value of the annuity based on contract rate.

$$\rho = \sum_{t=0}^{T} \left(1 + \frac{r}{12}\right)^{-12t}$$
Table 3.5 Input Parameters for RM model

<table>
<thead>
<tr>
<th>Parameter Name</th>
<th>Symbol</th>
<th>Value or Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Closing cost</td>
<td>$C$</td>
<td>$3,500</td>
</tr>
<tr>
<td>Initial house value</td>
<td>$V_0$</td>
<td>$20,000</td>
</tr>
<tr>
<td>Contract interest rate</td>
<td>$r$</td>
<td>Range from 3% to 10%</td>
</tr>
<tr>
<td>PLF</td>
<td>$\varphi$</td>
<td>U.S Department of Housing and Urban Development</td>
</tr>
</tbody>
</table>

The relationship of output variable (benefits) and input parameters (closing cost, initial house cost, contract interest rate) can be draw as Figure 3.8.

![Figure 3.8](image)

*Figure 3.8 Plot of Yearly Benefit Payment with RM start age for three typical contract interest rate*

From the figure, we know that a solid combination of older age, higher initial home value and lower contract interest rate will help garner you the most funds possible. Single financial product is not hard to find optimal purchasing decisions. However, if we turn to multiple financial resources and link the health status with wealth statutes, the issue is becoming more and more complicated. In next section, a heuristic algorithm is developed to find optimal financial decisions with multiple financial decisions in progressive health conditions.

3.5 Heuristic Algorithm

In above discuss, the integrated problem was formulated as a service-oriented optimization nonlinear-programming model, which consisted of three individual financial resource models: Medicare, long-term
care insurance, and reverse mortgage. To explain how these three models working together with service pricing model and obtain the optimal (SLFR), a heuristic algorithm was developed as following

**Function** SERVICE-ORIENTED OPTIMIZATION (desired service level, personal profile) returns optimal financial decisions

**Inputs:** desired service level, ∈ (0,1]

Personal profile, {gender, age, health indicators, health history}

0. Set SLFR = 0, c = 0, p = 0, b = 0, s = 0

1. Input Personal profile into pre-trained CTMC model to estimate expected sojourn time in each care stage.

2. Update care cost from pricing model by desired service level and remaining life time

3. Calculate actual Medicare coverage by mapping Medicare-approved services to service level model.

4. Update b and p for Medicare.

5. Search for the optimal LTCI purchasing decisions {Daily Benefits, Benefit Period, LTCI Purchasing Age} with the highest SLFR. Update b and p for LTCI.

   **While** c < b **do**

   
   \[ b_{LTCI} = c - b_{Medicare}; \]

   **End While**

6. Search for the optimal RM purchasing decisions {Contract Rate, RM purchase Age} set with the highest SLFR. Update benefits and premium for RM.

   Search Method: **Loop** through RM purchasing decision set

   For i = 1: m remaining life time m months

   **If** (b(i) > c(i))

   \[ s(i) = s(i - 1) + b(i) - c(i); \]

   \[ b(i) = c(i); \]

   **Else**

   \[ s(i) = \max(s(i - 1) - (c(i) - b(i)), 0) \]

   \[ b(i) = b(i) + \max(s(i - 1) - s(i), 0) \]

   **End if**

   \[ SFRL(i) = \frac{b(i) - p(i)}{c(i)} \]

   **End For**

**Returns** {Daily Benefits, Benefit Period, LTCI Purchasing Age, Contract Rate, RM purchase Age, SFRL}
In the heurist algorithm, the financial resources are consumed in the order of public programs, care-related private insurance, general elder financial funding, which are Medicare, Long-term care insurance, reverse mortgage. The extra money from the reverse mortgage can be saved for future use.
4 Case Studies

4.1 Data Description

The heart transplant monitoring dataset consists of health records from 622 patients. The stage at each time is a grade of cardiac allograft vasculopathy (CAV), a deterioration of the arterial walls. There are 4 stage to describe the severity of the disease. The data frame has 8 features as Table 4.1 shows.

<table>
<thead>
<tr>
<th>Name</th>
<th>Attribution</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>PTNUM</td>
<td>numeric</td>
<td>Patient identification number</td>
</tr>
<tr>
<td>age</td>
<td>numeric</td>
<td>Recipient age at examination (years)</td>
</tr>
<tr>
<td>years</td>
<td>numeric</td>
<td>Examination time (time stamp)</td>
</tr>
<tr>
<td>dage</td>
<td>numeric</td>
<td>Age of heart donor (years)</td>
</tr>
<tr>
<td>sex</td>
<td>numeric</td>
<td>sex (0=male, 1=female)</td>
</tr>
<tr>
<td>pdiag</td>
<td>factor</td>
<td>Primary diagnosis (reason for transplant) IHD=ischaemic heart disease, IDC=idiopathic dilated cardiomyopathy.</td>
</tr>
<tr>
<td>cumrej</td>
<td>numeric</td>
<td>Cumulative number of acute rejection episodes (range from 0 to 12)</td>
</tr>
<tr>
<td>statemax</td>
<td>numeric</td>
<td>Maximum observed state so far for this patient</td>
</tr>
<tr>
<td>state</td>
<td>numeric</td>
<td>mild/moderate CAV and state 3 is severe CAV. state 4 indicates death.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>firstobs (numeric) 0 = record represents an angiogram or date of death.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1 = record represents transplant (patient’s first observation)</td>
</tr>
</tbody>
</table>

Based on the available of the data set, the personal profile in the following case study is defined as \{age, gender, years, dage, pdiag, cumrej, statemax\}. After inputting these covariates to the CTMC model, we have found out only the covariates \{age, gender, cumrej, statemax\} lead to effective transition probability matrix and infinitesimal generator. An approach to interpret the covariates in personal profile is age and gender represent basic information, cumrej denotes the key health indicator at each time stamp, and statemax means the health history. In the next section, a discussion will be made to show how the changes in personal profile impact the CTMC model.
4.2 Personal Profile Changes

Transition probability matrix is the primary parameter to motivate Markov Chain model. In the description in Section 3.2, transition probability matrix can determine not only the probability of transition from one stage to another, but also the LOS in a care stage. Thus, transition probability matrix can serve as a measurement to investigate the changes in personal profile.

Age and \( \text{cumrej} \) are the two covariates that we would like to identify. Let’s assume an elder receives periodic medical examination after heart transplanting. He was found out more acute rejection episodes (the higher value of \( \text{cumrej} \)) at . The changes of \( \text{cumrej} \) would update the transition probability matrix as Figure 4.1 shows. The four columns represent four age groups: 65, 70, 75 and 80 and the four rows represent different \( \text{cumrej} \): 0, 4, 8 and 12. For example, the top-left subfigure denotes the transition probability matrix of an elder aged 65 with \( \text{cumrej} = 0 \). In each subfigure, the X-Y axis means the From-To states and each square represents probability transition from state X to state Y. ]
As we detect from above heat map, the elements on the diagonal have higher probabilities than the non-diagonal elements, which means elders tend to stay in their original state in most cases. The red block on the top-right in each subfigure shows that Stage 4 is an absorbing state. Another observation is that the higher cumrej value can facilitate the rate jumping out of original stage. For instance, the transition probability from stage 1 to stage 2 increase as cumrej increases for all the four age groups. Another key finding is that age rather than cumrej might be the domain factor that causes death. For example, the transition probability from stage 1 to absorbing stage. In addition, the higher cumrej can reduce the recovery probability like transition probability from stage 2 to stage 1.

All these changes in transition probability can be pass through entire Markov Chain and update infinitesimal generator and length of stay. For instance, the total length of stay of \{age=65, gender=0, cumrej=0\} in each stage are : 5.79 years for Stage1, 1.35 years for Stage 2, 1.01 years for Stages 3 and
Inf for Stage 4. While the total length of stay of \(\{\text{age}=65, \text{gender}=0, \text{cumrej}=12\}\) in each stage are: 1.644 years for Stage 1, 1.39 years for Stage 2, 0.98 years for Stages 3 and Inf for Stage 4. The results also verify our observation in transition probability that the higher cumrej value can facilitate the rate escaping from original stages.

4.3 Optimization of Individual-level Financial Decisions

In this case, the personal profile is selected to be \(\{\text{age}=65, \text{gender}=\text{male}, \text{cumrej}=0, \text{statemax}=0\}\). Figure 4.2 demonstrates the performance all the feasible decision sets in the heuristic algorithm. The optimal solutions which have the highest SLFR is marked with red dots. It can be observed that the performances of these feasible decision sets are as high as 0.99 or as low as 0.11. The optimal solution sets are \{Daily Benefits, Benefit Period, LTCI Purchasing Age, RM Contract rate, RM Purchasing Age\} = \{150, 5, 69, 3%, 65\} or \{250, 5, 70, 3%, 65\} or \{200, 4, 70, 3%, 65\}.

![Figure 4.2 Performance of Feasible Decision Sets](image)

One of the advantages of the model is that we can track the elder’s status monthly. Figure 4.3 illustrates the care cost and benefits of financial products during the remaining lifetime. The elder will experience three progressive care stages and care cost in each stage is represented as solid red line. The care costs is
calculated based on the desired service level $Y_U$. The elder may have different requests for home care services so that the care cost is not constant during the first care stage. The Medicare benefits (blue dash line) are able to cover most of costs for home care services but it depends on the level of service. Since Medicare won’t pay for any costs in assisted living facility and cover fees in skilled nursing home for a limited time period with liability, our optimal decisions tell us 69 years old is the best time to purchase the long-term care insurance (solid green line).

![Figure 4.3 Example of Optimal Decision Set: Care Cost and Benefits](image)

The optimal solution recommends elder to buy the reverse mortgage as early as possible. It conflicts with the conclusion in Section 3.5.3: older purchasing age can help elders garner more funding back. The reasons can be explained by Figure 4.4. Reverse mortgage can convert to personal savings. In this case, the low month cash saved for a long time can deal with the out-of-pocket expenditure better than the high month cash for short time with high utilization of house value. After about 90 months, the elder can save about $30K from reverse mortgage and it’s served as self-funding to pay the high cost in skilled nursing home.
4.4 Sensitivity Analysis

In the last section, we discuss the optimal financial decision sets. It’s a personalized optimal solution for elders with specific personal profile. In our previous model, we distinguish elders via different personal profile \{age, gender, cumrej, statemax\}. The personal profile is going to impact transition probabilities in the Markov Chain model, thus change the LOS and care cost in each care stage. In this section, a technique is used to determine how covariates affect the dependent variables in financial decision set.

Sensitivity analysis studies the uncertainty in the output of a system is impacted by the different sources of uncertainty in its input. The follow analysis can answer the question: if elder’s personal profile updates, what kind of adaptive reaction should financial decisions make to deal with this type of uncertainty? The covariate \textit{cumrej} is selected as source of uncertainty for health indicator.
Figure 4.5 Impact of cumrej on Optimal LTCI Daily Benefits

As Figure 4.5 (a) shows, the higher cumrej doesn’t lead to the lower SLFR as expected, because the high cumrej causes short remaining life time thus less care costs. The higher daily benefits don’t mean the higher coverage, an optimal LTCI daily benefits can be found in the Figure 4.3(b), the higher cumrej require higher LTCI daily benefits exclude cumrej = 0. It’s probably caused by the factors that an elder with zero cumrej may live longer than those who appear cumrej. It’s a tradeoff between long service time at low care stage and short service time at a high care stage.
Figure 4.6 Impact of cumrej on Optimal LTCI Benefits Period

The trend of SLFR with LTCI benefit period is easy to capture in Figure 4.6, the higher LTCI benefit period tends to have lower SLFR. The optimal LTCI benefits period is consistent to be 3 years which is probably caused by the character of LOS in each care stage for this group of elders. The elders with this chronic disease probably die before receiving the full coverage of LTCI benefits period.

Figure 4.7 Impact of cumrej on Optimal LTCI Purchasing Age

Generally, the older LTCI purchasing age leads to the higher SLFR due to the high care cost at the late stages of chronic disease. The pattern of optimal LTCI purchasing age shows a similar but reverse curve with the LTCI daily benefits. If elder has no cumrej, he or she stays much longer in care stages, which may lead to the later optimal purchasing age compared to the 1 and 2 cumrej.
The pattern of influence of cumrej on optimal reverse mortgage purchasing age (Figure 4.8) has the same extraordinary trend on cumrej = 0. It tells us the fact that if 65-year old people have an incidence in cumrej indicator, like transition from 0 to 0 +, they’d better postpone revere mortgage purchasing age.

Another parameter of determining reverse mortgage program is the contract rate. As the Figure 4.7 indicates, it’s difficult to tell the absolute relationship between optimal reverse mortgage contract rate and cumrej which might be caused by the insignificant effects from contract rate.
Figure 4.9 Impact of cumrej on Optimal RM Contract Rate

In addition to the sensitivity analysis on health condition, gender can be another key factor that impacting the financial decision making.

Figure 4.10 Impact of Gender on the Optimal Financial Decisions

As Figure 4.10 indicates, some of elements in financial decision set show similar pattern both for male and female such as the purchasing age and contract rate in reverse mortgage. Because the gender has little effect on the inner characters of reverse mortgage. For long-term care insurance, the different patterns for male and female are caused by the premium of the long-term care insurance and length of stay in each care stage.

5 Conclusion and Discussion

The most striking outcome to emerge from the model is that we associate multi-states health processes with financial decisions under the uncertainty from personal health profile. We believe our continuous-time Markov Chain model can describe the care stages for most of the chronic condition with impacts
from covariates. Our case study investigates the individual-level organ transplant data with four selected covariates: age, gender, health indicator \((\text{cumrej})\), and health history \((\text{statemax})\). The analysis on one of the key health indicators \(\text{cumrej}\) shows the covariates in personal profile influence the transition probability matrix and thus impact the infinitesimal generator and LOS in Markov chain model. The approaches to model financial resources provide a guideline to build up other financial resources with similar characters. Medicare represents for a group of federal and state public care aid funding. Long-term care insurance stands for a variety of private care-related insurance programs with support of data. Reverse Mortgage is one of the retirement income sources that follows economic structure. Our service level model and pricing model serve as a bridge to connect the health conditions with financial decisions, or in other words, associate demand with money. The parameters in service level model and pricing model are quite flexible that can be updated with specific value to obtain more accurate estimation. All these models lay a solid foundation to solve the service-oriented optimization model.

Our results show a clear advantage for elder to optimize their financial products purchasing decision. Traditional opinion may mislead elders to buy the longer benefits period, higher daily benefits and earlier purchasing age for long term care. However, according to our optimal results for elders, it really depends on the personal profile. Furthermore, the interaction coming from other financial products also affect purchasing decisions. Our separate analysis on reverse mortgage indicates that the older purchasing age can utilize the benefits better. In contrast, the service-driven optimal decisions suggest the earlier enrollment can save the funding to cover more care services. Our optimization recommends the better financial decisions should rely on the personal health condition and adjust by the changes of health status.

It is interesting to note that the particularity of healthy elders with zero \(\text{cumrej}\) distinguished from unhealthy elders with more than zero \(\text{cumrej}\). It’s similar with the concepts of “health shock” in Gupta’s work as we mentioned above. A healthy elder (no health shock) shows different patterns in long-term care purchasing age, daily benefits, and reverse mortgage purchasing age, which support the previous findings
in his literatures to some degree. Our modeling technique extended the severity of health shock to multiple health shocks via updating the covariates in personal profile.

The future work on the thesis can be improved from three perspectives. Firstly, the better data guarantee the better results. If we can acquire data with more personal features, e.g. heart rate, blood pressure, and income level, the Markov model can be updated with more covariates and then more precision estimation. Machine learning algorithms (e.g. classification) can be introduced when deal with the large amount of data before imputing to the multi-state model. Secondly, the financial resources can be modeled more completed. There are many other financial products that associated with elder care. In the end, the more advanced algorithms should be developed. Current heuristic algorithm only search the optimal solutions through the blind trial-and-error method. On the other hand, unlike machines, the status of people especially older people experiences a variety of uncertainties, the robustness should be considered in financial decisions.
Reference


Routledge.