I, Gil Jun Lee, hereby submit this original work as part of the requirements for the degree of Doctor of Philosophy in Mechanical Engineering.

It is entitled:
Design and Analysis of a Novel Squeak Test Apparatus Developed for Objective Rating of Squeak Propensity and Its Application

Student's name: Gil Jun Lee

This work and its defense approved by:

Committee chair: Jay Kim, Ph.D.

Committee member: YiJun Liu, Ph.D.

Committee member: Allyn Phillips, Ph.D.

Committee member: David Thompson, Ph.D.
Design and Analysis of a Novel Squeak Test Apparatus Developed for Objective Rating of Squeak Propensity and Its Application

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University of Cincinnati

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By

Gil Jun Lee
M.S., Mechanical Engineering, Sogang University, Seoul, Korea, Republic of, 2013
B.S., Mechanical Engineering, Sogang University, Seoul, Korea, Republic of, 2011

Committee Chair: Dr. Jay Kim
Members: Dr. Yijun Liu
         Dr. Allyn Phillips
         Dr. David Thompson
ABSTRACT

As noises that reach the passenger cabin are substantially reduced by recent advances in noise, vibration and harshness (NVH) engineering, the noise generated inside the passenger cabin, such as squeak and rattle (S&R), stands out to form detrimental perception of the quality of vehicles. S&R noises are unwanted, annoying noises generated by friction-induced vibration or impact between surfaces. Highly nonlinear and random nature of the underlying mechanism, S&R noises pose one of most difficult problems for automotive NVH engineers. A systematic procedure that integrates analytical model, numerical simulations and experiments is required and proposed to effectively handle S&R problems in the design stage.

A computerized procedure for automatic detection and rating of S&R noises will be highly useful for automotive engineers by enabling consistent, repeatable inspection and quantitative assessment of the S&R problems. An algorithm developed in the past was enhanced in this work, which utilizes the perceived transient loudness (PTL) that approximates the human perception of transient noises. Various signal processing techniques and psychoacoustic models, such as analytic wavelet transform and Zwicker’s loudness, were applied to enable S&R detection based on the characteristics of human perception. A new algorithm to differentiate S&R noises was developed by utilizing sound quality metrics to enhance the automatic detection and rating algorithm.

Stick-slip, mode-coupling and sprag-slip mechanisms are three well-known mechanisms leading to friction-induced instability to generate squeak noise. The mode-coupling mechanism
occurs when the system has multiple modes of nearly identical natural frequencies. A simple analytical model was developed to represent dynamics characteristics of the mode-coupling phenomenon. The equation of motion was numerically solved to study the effects of the natural frequency ratio, damping and friction coefficient on the development of self-excited instability.

A novel, unique squeak test apparatus was developed by employing a modified sprag-slip mechanism (MSSM). The mechanism is designed to make a geometrically induced instability consistently and repeatedly. A single-degree-of-freedom system which represents dynamics of the system was used to obtain insights and to refine the designs. Stability analysis of the MSSM was conducted to identify the conditions that make the system unstable in terms of main design parameters. The natural frequencies of the system were obtained from analytical, finite element method (FEM) and experimental models to confirm the design. The instability condition was also identified by the FEM model. The complex eigenvalue analysis and dynamic transient analysis were adopted to investigate the effects of the main parameters on the stability of the system in both frequency and time domain.

The test apparatus that induces squeak noises for a wide range of materials pair was designed and built. The intended use of the test apparatus is to build a material database that ranks squeak propensities of various material pairs quantitatively for NVH engineers. To demonstrate the capability of the test apparatus, several material pairs were tested. Measured motion and noises were processed to obtain frequency spectrums and time-frequency patterns. The automatic detection and rating algorithm previously developed was applied to rate squeak propensities of the material pairs.
ACKNOWLEDGEMENT

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I am very grateful for the financial support by GE Aviation and thank Mr. Tod Steen for offering the opportunity to study rotordynamics, especially squeeze film dampers. I have learned not only the squeeze film dampers in jet engines, but also how industry deals with engineering problems. Also, I would like to thank Dr. Urmila Ghia for her valuable guidance and suggestion during the squeeze film damper project with GE Aviation.

I cannot thank my family enough who always encourage and trust me during the journey of my research. I am very grateful to my parents and a brother for their endless belief, support and love. Last but not least, I really thank my wife Jihyun for her consistent support, sacrifice and love.
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LIST OF SYMBOLS

\( \mu \)  
kinetic coefficient of friction

\( \mu_s \)  
static coefficient of friction

\( m \)  
mass

\( c \)  
damping constant

\( k \)  
stiffness constant of a linear spring

\( K \)  
stiffness constant of a torsional spring

\( F_n \)  
normal force

\( F_t \)  
tangential force

\( \mathbf{J} \)  
Jacobian matrix

\( \lambda_k \)  
eigenvalue

\( \mathbf{x}_k \)  
eigenvector

\( \delta_k \)  
perturbation

\( \bar{x}_k \)  
equilibrium point

\( \psi(t) \)  
analytic wavelet

\( g(t) \)  
real symmetric window

\( \zeta \)  
center frequency

\( u \)  
time location

\( s \)  
 scale parameter
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>M</td>
<td>mass matrix</td>
</tr>
<tr>
<td>M*</td>
<td>mass matrix projected to the eigen-space</td>
</tr>
<tr>
<td>C</td>
<td>damping matrix</td>
</tr>
<tr>
<td>C*</td>
<td>damping matrix projected to the eigen-space</td>
</tr>
<tr>
<td>K</td>
<td>stiffness matrix</td>
</tr>
<tr>
<td>K*</td>
<td>stiffness matrix projected to the eigen-space</td>
</tr>
<tr>
<td>K_&lt;sub&gt;s&lt;/sub&gt;</td>
<td>symmetric, structural stiffness matrix</td>
</tr>
<tr>
<td>K_&lt;sub&gt;f&lt;/sub&gt;</td>
<td>asymmetric, friction-induced stiffness matrix</td>
</tr>
<tr>
<td>u</td>
<td>displacement vector</td>
</tr>
<tr>
<td>f_&lt;sub&gt;ex&lt;/sub&gt;</td>
<td>applied load vector</td>
</tr>
<tr>
<td>f_&lt;sub&gt;in&lt;/sub&gt;</td>
<td>internal force vector</td>
</tr>
<tr>
<td>I_&lt;sub&gt;G&lt;/sub&gt;</td>
<td>moment of inertia of a rigid bar</td>
</tr>
<tr>
<td>l</td>
<td>length of a rigid bar</td>
</tr>
<tr>
<td>( \theta_o )</td>
<td>angle of attack</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>angular acceleration</td>
</tr>
<tr>
<td>y</td>
<td>linear displacement</td>
</tr>
<tr>
<td>( \theta )</td>
<td>angular displacement</td>
</tr>
<tr>
<td>( \theta_o )</td>
<td>angular displacement when the linear spring is at free length</td>
</tr>
<tr>
<td>( \xi_o )</td>
<td>initial compression of the linear spring</td>
</tr>
<tr>
<td>F_&lt;sub&gt;A&lt;/sub&gt;</td>
<td>reaction force at a frictionless guide</td>
</tr>
<tr>
<td>E</td>
<td>modulus of elasticity</td>
</tr>
<tr>
<td>I</td>
<td>moment of inertia of a beam</td>
</tr>
<tr>
<td>( \rho )</td>
<td>density of a beam</td>
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</table>
\(A\) cross-sectional area of beam

\(\omega_n\) natural frequency

\(\beta_n\) non-dimensional frequency parameter

\(N_{\text{PTL}}\) perceived transient loudness time history

\(N_{\text{inst}}\) transient loudness time history

\(N_i\) detection threshold

\(N_{\text{det}}\) detection metric

\(N_{\text{obj}}\) objective rating

\(\omega_r\) the ratio of natural frequencies

\(\zeta_x\) damping coefficient in \(x\)-direction

\(\zeta_y\) damping coefficient in \(y\)-direction

\(S_1, S_2, S_3\) squeak sub-index

\(S\) squeak index

\(R_1, R_2, R_3\) rattle sub-index

\(R\) rattle index

\(j\) imaginary index, \(\sqrt{-1}\)
### Acronyms

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>NVH</td>
<td>noise, vibration and harshness</td>
</tr>
<tr>
<td>CAE</td>
<td>computer aided engineering</td>
</tr>
<tr>
<td>S&amp;R</td>
<td>squeak and rattle</td>
</tr>
<tr>
<td>BSR</td>
<td>buzz, squeak and rattle</td>
</tr>
<tr>
<td>MSSM</td>
<td>modified sprag-slip mechanism</td>
</tr>
<tr>
<td>FEM</td>
<td>finite element method</td>
</tr>
<tr>
<td>SPL</td>
<td>sound pressure level</td>
</tr>
<tr>
<td>DOE</td>
<td>design of experiments</td>
</tr>
<tr>
<td>FFT</td>
<td>fast Fourier transform</td>
</tr>
<tr>
<td>STFT</td>
<td>short time Fourier transform</td>
</tr>
<tr>
<td>AWT</td>
<td>analytic wavelet transform</td>
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Chapter 1 Introduction

1.1 Motivation and Objective

Due to intense research and development during recent decades, the noise and vibration problems stemming from the engine and road noises in vehicles have been reduced significantly. New materials and technology related to noise and vibration lead to a considerable reduction of the noise reaching the driver or passenger. As the overall noise level of passenger vehicles has been significantly reduced, noises generated inside the passenger cabin, such as squeak and rattle (S&R) noises, stand out and contribute to a detrimental perception of the quality of vehicles. Noise, vibration and harshness (NVH) performance of a vehicle is one of the most important factors in deciding the success or failure of the vehicle due to a growing demand for better product qualities and more comfortable rides amid intense competition between auto-manufacturers.

In addition, the number of components mounted in a passenger cabin has increased substantially, which also has increased the chance of squeak noises. Intense research effort has been made by vehicle manufacturers to minimize squeak problems by utilizing new materials and state-of-art CAE techniques. However, a systematic engineering strategy that combines an analytical work, experiments and CAE tools has yet to be developed to effectively handle the squeak problems in automobiles. The purposes of this dissertation work are summarized as follows;

1) Study an analytical model of a modified sprag-slip and mode-coupling mechanism to obtain design ideas for a squeak test apparatus.
2) Design a novel, unique squeak test apparatus that can be used as a standard material test device.

3) Refine an automatic detection and rating algorithm for squeak and rattle noises.

4) Demonstrate how to build material database for squeak propensity.

5) Develop an algorithm to distinguish between squeak and rattle noises.

6) Propose an overall, systematic procedure to effectively address squeak problems in automotive industry.
1.2 Squeak and Rattle

While they are often lumped together in reference, the squeak noise and rattle (S&R) noise are each generated by different mechanisms. Squeak refers an undesirable, annoying noise generated by a self-excited vibration induced by friction force between two surfaces in sliding contact. The elastic deformation of the contact surfaces stores energy which is released to produce audible squeak noises upon the relative motion between the surfaces. Rattle is an impact-induced noise which is generally caused by loose or overly flexible elements under forced excitation. A number of factors, such as material property, friction coefficient, relative velocity, temperature and humidity, are involved in generating S&R noises.

Due to the highly nonlinear nature of the friction-induced vibration, a squeak noise presents a very difficult challenge for NVH engineers. Stick-slip, sprag-slip and mode-coupling phenomena are three well-known mechanisms that cause friction-induced vibration instability. The stick-slip phenomenon between two surfaces in contact causes an unstable, growing motion when the kinetic coefficient of friction decreases as the sliding velocity increase, which makes the effective damping of the system negative. The sprag-slip mechanism, also known as geometrically induced instability or kinematic constraint instability, makes the system unstable when certain geometrical conditions are satisfied even if the kinetic coefficient of friction is constant. The mode-coupling mechanism, which is generally recognized as the most common mechanism that causes squeak noises, refers vibration of a system that has multiple modes of nearly identical natural frequencies. Completely eliminating all occurrences of sliding contacts in an automobile is obviously impractical. However, the areas that have high possibility of sliding contact can be predicted, and materials in such areas can be chosen based on their squeak propensity if such data is available.
1.3 Outline of the Dissertation

This dissertation thesis is comprised of nine chapters as shown in Figure 1-1. Chapter 1 provides motivations and objective of the dissertation and an overview of the S&R noises that includes the definition of the S&R. Then, the outline of this dissertation is presented.

Chapter 2 provides an extensive literature review of current state-of-the-art methodologies and techniques on published scientific articles and conference papers. It also reviews analytical studies on friction-induced instability that have been conducted so far. The past and ongoing research in this field is summarized. Based on this review, several deficiencies in the published literature are identified, which are studied in this dissertation work.

Chapter 3 presents an analytical study on the modified sprag-slip mechanism (MSSM) to obtain the design guidance of the unique squeak test apparatus developed first time in this work. In this chapter, an analytical model of the MSSM and the equations of motion of the MSSM were developed. The stability of the MSSM was investigated by several methods. Effects of the main parameters on the stability of the system were also investigated. The FEM model was developed to study the stability of the MSSM in both frequency and time domain. The instability condition identified from the analytical model was compared to the results obtained from the FEM model. Models were developed based on both analytical model (Euler-Bernoulli beam model) and the finite element method (FEM), and their results were compared.

In Chapter 4, a novel, unique squeak test apparatus developed by employing the MSSM is reported. The squeak test apparatus can generate friction-induced vibration and squeak noises consistently, making nearly the same squeak noises for a given material pair whenever the pair is tested. The shape and dimension of the test apparatus were determined by the analysis methods
presented in Chapter 3. Natural frequencies of the test apparatus were calculated and measured from the analytical model, the FEM model and experimental modal analysis, and were compared to each other. Testing of material pairs were conducted for seven different materials.

Chapter 5 summarizes the automatic detection and rating algorithm developed for S&R noises. Historically, detection and rating of S&R noises have been done subjectively by human operators, which becomes time-consuming and inconsistent process. In the automatic detection and rating algorithm, the detection threshold and objective rating of S&R noises were defined by a new physical quantity, which is perceived transient loudness. The application of the developed algorithm to various noises generated by experiments is reported to demonstrate the capability of the method. As a part of effort to further improve the S&R detection and quantification method, the relationships between the objective rating estimated by the algorithm and instantaneous and time-averaged sound pressure level (SPL) were examined.

Chapter 6 presents an analytical work of the mode-coupling effect on the instability of friction-induced vibrations. A relatively simple analytical model was developed to study mode-coupling induced vibration instability. To investigate the effect of several parameters on the stability of the system, the equations of motion of the model was developed and solved numerically. Based on the results, a design idea of a squeak test apparatus which utilizes mode-coupling mechanism was discussed. In addition, the possible application of the model to the analysis of brake squeal problems was discussed.

Chapter 7 is on the development of an algorithm to distinguish S&R noises. The algorithm was developed by utilizing a combination of three sound quality metrics, specifically sharpness, roughness and fluctuation strength. A three-dimensional space defined by the maximum values of sharpness, roughness and fluctuation strength of the noises was used to differentiate S&R noises.
The developed algorithm was applied to 86 recorded S&R noises and it was shown that the type of noises was correctly classified with approximately 90% accuracy. Also discussed is possible performance improvement and best application of the developed S&R differentiation algorithm.

Chapter 8 proposes an overall, systematic procedure to effectively handle squeak problems in automotive industry in the design stage, which utilizes material database for squeak propensity discussed in Chapter 4 and the automatic detection and rating algorithm discussed in Chapter 5. CAE analysis, experiments and signal processing are combined into a systematic engineering procedure to effectively handle squeak problems.

Chapter 9 summarized achievements that have been made so far in this dissertation work and possible future work is suggested.
Figure 1-1. Organization of the dissertation
Chapter 2 Literature Review

Vehicle noises may consist of two groups, which are the persistent type and transient type [1]. The persistent noises, such as engine, road and wind noises, occur constantly during regular and wide-ranging operation conditions. Since the persistent noise is more annoying and uncomfortable to a driver and passengers, it should be minimized or eliminated first. Due to the remarkable advances in noise, vibration and harshness (NVH) engineering, the persistent noise has been reduced significantly, and the transient type noise, such as squeak and rattle (S&R) becomes much more apparent. Thus, the transient noise should also be minimized as much as possible to improve the quality of vehicles. A market survey conducted as early as 1983 [2] reported that S&R noises is the third highest customer concern during the initial three months in service period. In addition, Li et al [3] reported that the warranty cost for S&R noises in vehicles was as high as 10 % of the total warranty cost. Because of the highly nonlinear nature of the cause of the squeak noise, which is friction-induced vibration of two interfacing surfaces, squeak noise is one of most difficult problems to address for automotive NVH engineers. Moreover, friction-induced vibration is influenced by the various factors, such as contact pressure, material properties, surface profile, temperature, humidity and friction coefficient [3-9].
2.1 Mechanisms of Friction-Induced Vibrations

There are three well-known mechanisms of friction-induced vibrations: stick-slip, sprag-slip and mode-coupling mechanism. Stick-slip phenomenon occurs as a result of the negative slope characteristic of kinetic coefficient of friction against the sliding velocity of two surfaces in contact [10-21]. Figure 2-1 (a) shows decreasing kinetic coefficient of friction with increasing sliding velocity. From an analytical model shown in Figure 2-1 (b), the equation of motion of the system can be represented as:

$$m\ddot{x} + c\dot{x} + kx = \mu N = N\left[\mu_s - a(v - \dot{x})\right]$$  \hspace{1cm} (2-1)

where $m$, $c$, $k$ and $N$ are mass, damping coefficient, spring constant and normal force, respectively. $\mu$, $\mu_s$ and $a$ are kinetic coefficient of friction, static coefficient of friction and slope between $\mu$ and sliding velocity $V$. Eq. (2-1) can be rearranged as:

$$m\ddot{x} + (c - aN)\dot{x} + kx = N(\mu_s - av)$$  \hspace{1cm} (2-2)

This characteristic of friction makes the effective damping of the system negative. Energy is fed into the system at each cycle, the amplitude of vibration becomes bigger and bigger, therefore the system becomes unstable.
Figure 2-1. (a) The negative slope characteristic of kinetic coefficient of friction and (b) an analytical model to represent stick-slip phenomenon.
Sprag-slip mechanism, so-called geometrically induced or kinematic constraint instability, occurs when a certain geometrical condition is satisfied [22, 23]. Figure 2-2 shows an analytical model to describe the sprag-slip mechanism. A rigid bar is connected to the hinge $O$, and the other end of the rigid bar contacts with angle $\theta$ to the moving surface at the bottom. From the moment equation at the hinge $O$, the normal force ($F_n$) and the friction force ($F_f$) can be represented as;

$$F_n = \frac{N}{1 - \mu \tan \theta}$$  \hspace{1cm} (2-3)$$

$$F_f = \frac{\mu N}{1 - \mu \tan \theta}$$  \hspace{1cm} (2-4)$$

where $\mu$ is the kinetic coefficient of friction.

From Eq. (2-4), if $\mu = \cot \theta$, $F_f$ becomes infinite; therefore, the relative motion between the rod and the surface at the bottom becomes impossible. This is called spragging condition. However, in real systems, the rod is released from the spragging condition and returns to the first state because of the flexibility. Thus, the system converges to a limit cycle with very large amplitude. Earles and Soar [24] found it from an experimental investigation based on pin-on-disc system that a certain range of angles of orientation of the pin caused sprag-slip instability.
Figure 2-2. An analytical model to represent a sprag-slip mechanism.
Mode-coupling phenomenon is unstable vibration caused by coupling of two modes of components [25-35]. Generally, a normal mode and a tangential mode are coupled. Two modes which have close or identical resonance frequencies induce more energy to the system than it dissipates. Thus, mode-coupling mechanism leads to the change of the friction force which is necessary for self-excited vibration. Qian et al [34] established the FEM model of the reciprocating sliding system. They showed that, in a certain range of the kinetic coefficient of friction, two modes have the same natural frequency and the system becomes unstable. Hoffmann et al [35] developed a minimal model to study the characteristics of the mode-coupling type instability.

Murakami et al [36] used a lump stiffness-mass model to analyze friction-induced vibration, and concluded that both stick-slip and sprag-slip contribute to an increase in friction-induced instability based on simulations and experiments. Chen et al [37] used a ball against a block reciprocating test setup. They found that the friction-induced vibration occurred under positive friction-velocity slope.
2.2 Computer Aided Engineering Applied to Squeak and Rattle Noises

Due to the dramatic development of computer science, computing cost has become very affordable in recent years. Currently, the use of computer aided engineering (CAE) tools is highly recommended and desirable since they are fast and cost-effective. Simulation-based prototyping and design of experiments (DOEs) are highly cost effective as they do not require any modification or adjustment of hardware. Therefore, the cost necessary to conduct tests has been reduced significantly in automobile industry because of the powerful CAE tools. However, squeak and rattle (S&R) analysis is one of a few problems that are very difficult to be solved only by CAE because the nonlinear nature of the problem.

Engineers and researchers started to develop and use CAE simulations to perform the S&R analysis from mid 1990’s. Hsieh et al [38] reported a rattle sensitivity analysis by using finite element method (FEM). The method they proposed was a linear FEM to identify the areas of high risk of rattle. They used a linear FEM even though the impact itself is a nonlinear phenomenon because nonlinear FEM analysis is computationally intensive and the turn-around time is too long for real applications. They conducted the rattle sensitivity analysis for a glove box of instrument panel considering several parameters, such as pre-load, excitation characteristics and structural parameters. Kulkarni et al [39] and Lin and Pitrof [40] investigated cockpit modules and steering column modal frequencies to improve S&R performance and safety. Her et al [41] developed a method to detect rattle noises by using a CAE tool.

Recently, commercially available CAE software packages with user-friendly features have been developed and used in automotive industry [42, 43]. Song et al [44] demonstrated that a FEM analysis technique could be used to simulate and predict the backlite molding squeak based on the velocity spectrum, the relative motion between the window glass and the body shell and the
threshold displacement prediction. Kuo [45] proposed a method for assessing overall S&R performance of a vehicle using body-in-prime and NVH CAE models for low frequency vibration. He reported that CAE results of S&R performance improvement were well correlated with subjective evaluations carried out on prototypes. Weber and Benhayoun [46] presented the S&R simulation to accurately represent the dynamic behavior of the interior structure of a vehicle in order to correlate the relative displacement and the actual S&R phenomenon. They concluded that the relative displacement along a gap/contact line can be evaluated and it can be related to the S&R phenomenon. Narayana [47] showed a robust method based on FEM to predict S&R problems due to speaker-borne structural vibrations, and that CAE predictions were correlated with the test results of the detection of S&R noises. Kim et al [48] defined S&R index based on the relative displacements in normal and tangential directions. They concluded that the numerical prediction of S&R noises was helpful in identifying the points with potential S&R problems. Lee and Chang [49] evaluated an attachment force at the chassis to trimmed body when the load is applied to the tire patches. Several FEM based methods to improve overall S&R performance of vehicles has been reported [50 – 53].
2.3 Experimental Works on Squeak and Rattle Problems

There have been numerous efforts to address S&R problems based on experimental approaches. Because a squeak noise is highly nonlinear and transient, and influenced by a number of parameters, such as material property, normal load, temperature and humidity and so on, it is very difficult to generate and measure the noise consistently and accurately. There exist a few types of test machines developed for the purpose of squeak measurement \([54, 55]\). Those machines control temperature and humidity, and measure material properties such as static and kinetic coefficient of friction. All squeak test machines which have been developed so far utilize stick-slip mechanism to generate squeak noises.

Hunt \(et\ al\)\ [5] developed a test system for a standard S&R test that can control and measure temperature, humidity, displacement, velocity and normal force. Their results illustrated that material compatibility test can be performed to characterize and understand material compatibility and to benchmark competitors’ materials, evaluation of coatings and material blends. Lee \(et\ al\)\ [56] developed a simple test method for S&R evaluation of a door trim by using repeated loading conditions. They used a robot arm to apply repeated load accurately, and correlated the acceleration with objective results of squeak noises. Sonie \(et\ al\)\ [57] determined threshold levels of motion as an objective evaluation of S&R performance experimentally. They tried to correlate S&R noises to the thresholds levels of motion. Juneja \(et\ al\)\ [6] developed a squeak rig to understand effects of the test parameters on the squeak behavior of materials. They tested two kinds of materials (polyurethane and PVC), and found that the squeak levels increased almost linearly as excitation frequency, interference and humidity increased. Byrd and Peterson [58] compared several different S&R test methods for large modules and subsystems. Kumar \(et\ al\)\ [59] conducted the vehicle test to improve S&R performance. They used the psychoacoustic metrics (loudness and sharpness) as
objective evaluations of S&R, and conducted jury tests. They reported that the objective and subjective evaluation of the S&R were improved after modifying the design by covering the shank with a plastic sleeve and using Teflon coating on the cross bar shank. Lee et al [4] developed an experimental analysis technique of S&R and observed the influence of the controlled parameter, such as temperature, humidity and load, on the mean loudness. They found that the averaged dynamic friction coefficient and the loudness of squeak noise showed a quadratic relationship. Yang and Rediers [60] experimentally investigated a friction-induced squeak between head restrain fabric and glass. The squeak was characterized with two bursts in each reciprocating cycle. They found that humidity is one of the biggest contributors to the squeak. According to Trapp and Pierzecki [61], variations in the contact interface stiffness can lead to a significant fluctuation in the normal load that results in stick-slip frictional instability. They suggested that thermoplastic components should be designed carefully to ensure surface properties that will not lead to rattle and unsteady sliding. Hurd [62] evaluated S&R of an instrument panel with 100,000 equivalent miles of durability cycling. Cook and Ali [63] introduced a spherical beamforming technology for S&R testing and localization of the sound source. Peterson and Sestina [64] evaluated S&R performance of vehicles by using rumble strips which are the surfaces with uniform spacing of bumps or strips.

Johnsson et al [65] designed a new test track named frequency sweep test track for S&R detection. It excited a vehicle in a range from 5 to 50 Hz. Kim [66] performed reciprocal sliding tests with ceramic balls by using a ball-on-plate apparatus that was similar to automotive rear sliding rails. He investigated the frictional characteristics of several materials. Meziane et al [33] conducted experiments of friction-induced vibrations with beam-on-beam test rig. They compared experimental results to numerical results and found that the angle of contact between beams was
an important factor to induce friction-induced instability. According to Shin and Cheong [67], the S&R noise source of the instrument panel was identified by a near-acoustic-field visualization. They defined sound evaluation metric by using Zwicker’s percentile statistical measures. Chen and Zhou [68] observed the initiation process of friction-induced vibration under reciprocating sliding conditions. They found that the friction-induced instability occurred by coupling of a normal and a tangential mode as the kinetic coefficient of friction increased. Tworzydlo et al [32] presented experimental study on friction-induced vibrations of a pin-on-disc slider. They developed an analytical model of the pin-on-disc apparatus and found that numerical prediction of the friction-induced instability was well correlated to that obtained experimentally. The coefficient of friction, mass, stiffness and dimensions have a significant influence on the stability of the system. References [69, 70] presented the S&R testing of a vehicle and material pair testing.
2.4 Stability Analysis of Nonlinear Systems

In a general vibration system, there exists nonlinearity particularly when large deformation is involved. Behaviors of inertia force, damping force, and stiffness of the system could be nonlinear. In addition, the nonlinearity can stem from geometry. Generally, a system with friction involves nonlinearity because the nature of friction itself is highly nonlinear, such as the relation between friction coefficient and sliding velocity and the direction of friction force which changes depending on the direction of relative velocity.

Nonlinear systems can be linearized at equilibrium points [71]. The general equation of motion of the nonlinear system with state variables \((x_1, x_2, \ldots, x_n)\) can be represented as:

\[
\begin{align*}
\dot{x}_1 &= f_1(x_1, x_2, \ldots, x_n) \\
\dot{x}_2 &= f_2(x_1, x_2, \ldots, x_n) \\
&\quad \vdots \\
\dot{x}_n &= f_n(x_1, x_2, \ldots, x_n)
\end{align*}
\]

The equilibrium points \((\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_n)\) are defined as:

\[
\begin{align*}
0 &= f_1(\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_n) \\
0 &= f_2(\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_n) \\
&\quad \vdots \\
0 &= f_n(\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_n)
\end{align*}
\]

By introducing perturbations \((\delta_k)\) about the equilibrium points, the state variables can be represented as:

\[
x_k = \bar{x}_k + \delta_k \quad \text{where} \quad k = 1, 2, \ldots, n
\]
By substituting Eq. (2-3) into Eq. (2-1) and applying Taylor series expansion to Eq. (2-1), Eq. (2-1) becomes;

\[ \dot{x}_1 = f_1(x_1, x_2, ..., x_n) + \sum_{k=1}^{n} \frac{\partial f_1(x_1, x_2, ..., x_n)}{\partial x_k} \delta_k + \sum_{k=1}^{n} \frac{\partial^2 f_1(x_1, x_2, ..., x_n)}{\partial x_k^2} \delta_k + ... \]

\[ \dot{x}_2 = f_2(x_1, x_2, ..., x_n) + \sum_{k=1}^{n} \frac{\partial f_2(x_1, x_2, ..., x_n)}{\partial x_k} \delta_k + \sum_{k=1}^{n} \frac{\partial^2 f_2(x_1, x_2, ..., x_n)}{\partial x_k^2} \delta_k + ... \]

\[ \vdots \]

\[ \dot{x}_n = f_n(x_1, x_2, ..., x_n) + \sum_{k=1}^{n} \frac{\partial f_n(x_1, x_2, ..., x_n)}{\partial x_k} \delta_k + \sum_{k=1}^{n} \frac{\partial^2 f_n(x_1, x_2, ..., x_n)}{\partial x_k^2} \delta_k + ... \]

(2-4)

By substituting Eq. (2-2) into Eq. (2-4) and taking only linear (the first order) terms of Eq. (2-4), Eq. (2-4) can be represented as;

\[ \dot{\delta}_1 = \sum_{k=1}^{n} \frac{\partial f_1(x_1, x_2, ..., x_n)}{\partial x_k} \delta_k \]

\[ \dot{\delta}_2 = \sum_{k=1}^{n} \frac{\partial f_2(x_1, x_2, ..., x_n)}{\partial x_k} \delta_k \]

\[ \vdots \]

\[ \dot{\delta}_n = \sum_{k=1}^{n} \frac{\partial f_n(x_1, x_2, ..., x_n)}{\partial x_k} \delta_k \]

(2-5)

Eq. (2-5) can be transformed to a matrix form as shown in Eq. (2-6).
where $\mathbf{J}$ is Jacobian matrix evaluated at the equilibrium points. Therefore, the nonlinear equation shown in Eq. (2-1) is linearized with the perturbations. Because Eq. (2-6) is the first order ordinary differential equation, the solution of Eq. (2-6) can be represented as [72]:

$$
\mathbf{\delta}(t) = \sum_{k=1}^{n} c_k e^{\lambda t} \mathbf{x}_k
$$

where $\lambda_k$, $\mathbf{x}_k$ and $c_k$ are the eigenvalues of $\mathbf{J}$, the corresponding eigenvectors and the constants determined by initial conditions respectively. Therefore, if one of the eigenvalues has a positive real part, the solution of the system grows exponentially, which means that the system becomes unstable.
2.5 Analytic Wavelet Transform

Because S&R noises are highly transient and time-localized, the conventional Fourier analysis cannot capture their inherent non-stationary characteristics. Time-frequency (T-F) analysis should be used to capture the characteristics of S&R. There are two widely used T-F analysis techniques; short time Fourier transform (STFT) and wavelet transform. In STFT, the frequency resolution decreases as the time resolution increases, making the user choose either frequency resolution or temporal resolution. Several studies [73-75] have reported that the wavelet transform is superior for characterizing highly transient noises. In the present study, the analytic wavelet transform (AWT) is used to characterize S&R noises generated from experiments.

A function is called analytic if it has zero energy in negative frequency side. An analytic function can be described by its real part. Mathematically, the analytical part of a function $f$ is represented as;

$$f_a(\omega) = \begin{cases} 2f(\omega) & \text{if } \omega \geq 0 \\ 0 & \text{if } \omega < 0 \end{cases} \quad (2-8)$$

where $f_a(\omega)$ and $f(\omega)$ are the Fourier transform of $f_a$ and $f$ respectively. For example, analytic of a $\cos \omega t$ is $e^{j\omega t}$ ($j = \sqrt{-1}$).

The AWT is a kind of wavelet transforms whose wavelet is an analytical function. AWT of $f$ can be represented as;

$$Wf(u,s) = \int_{-\infty}^{\infty} f(t)\psi_{u,s}^*(t)dt = \int_{-\infty}^{\infty} f(t)\frac{1}{\sqrt{s}}\psi^*\left(\frac{t-u}{s}\right)dt \quad (2-9)$$
where $\psi(t)$, $u$ and $s$ are the analytic wavelet, the time and scale parameters respectively. Analytic wavelet can be defined by modulation of a real symmetric window ($g(t)$) [76] as represented in Eq. (2-10).

$$\psi(t) = g(t)e^{in\tau}$$  \hspace{1cm} (2-10)

Gabor wavelet which is widely used analytic wavelet is defined by utilizing the Gaussian function for $g(t)$. Therefore, the frequency distribution of wavelet function ($\psi(t)$) has a Gaussian distribution with center frequencies ($\xi$) of $\eta/s$, which is represented as;

$$g(\omega) = \left(4\pi\sigma^2\right)^{1/4} e^{-\sigma^2\omega^2/2}$$  \hspace{1cm} (2-11)

$$\psi_s(\omega) = g(s\omega - \eta)$$  \hspace{1cm} (2-12)

The time-frequency energy of the signal in a time-frequency atom centered at scale $s$ and time location $u$ ($P_w$) is defined as;

$$P_{w\,f}(u,s) = |Wf(u,s)|^2$$  \hspace{1cm} (2-13)

References [77, 78] presented the application of AWT to highly impulsive noises. The parameters of Gabor wavelet were chosen to make AWT effectively work as 1/3rd octave band filter. The parameters were calculated as $\sigma = 1.052$ and $\eta = 7.238$. 

2.6 Finite Element Method

2.6.1 Complex Eigenvalue Analysis

The complex eigenvalue analysis (CEA) has been widely used to extract the eigenvalues of the system and investigate the stability of the system in frequency domain. In finite element method (FEM) models, the effect of friction is included in the stiffness matrix and makes it asymmetric. Thus, the eigenvalues of the FEM model with friction are generally complex. The imaginary part of the complex eigenvalue represents the damped natural frequency of the corresponding mode. The real part of the complex eigenvalue represents the effective damping of the corresponding modes, which means that the stability of the system can be determined by the real part of the complex eigenvalue because the negative damping means that the system is unstable.

The governing equation of the FEM model for CEA can be represented as;

\[ M \ddot{u} + C \dot{u} + K u = 0 \]  \hspace{1cm} (2-14)

where, \( M \), \( C \) and \( K \) are the mass, damping (which can include friction-induced damping effects and material damping) and stiffness matrices, respectively, and \( u \) is the displacement vector. If the friction is involved in the FEM model, \( K \) can be represented as follows;

\[ K = K_s + \mu K_f \]  \hspace{1cm} (2-15)

where, \( K_s \), \( K_f \) and \( \mu \) are the symmetric, structural stiffness matrix, the asymmetric, friction-induced stiffness matrix and the coefficient of friction. This asymmetric stiffness matrix results in both complex eigenvalues and complex eigenvectors.
In CEA, the complex eigenvalues are calculated by using the subspace projection method. The governing equation can be transformed to the system eigenvalue equation as shown in Eq. (2-16).

\[
\left( \lambda^2 M + \lambda C + K \right) \psi = 0
\]  
(2-16)

where, \( \lambda \) and \( \psi \) are the eigenvalue and the corresponding eigenvector, respectively.

The first step of the CEA is to solve the symmetric eigenvalue problem by ignoring the damping matrix \( C \) and the asymmetric contributions of the stiffness matrix \( K_F \). Thus, the eigenvalue \( \lambda \) becomes pure imaginary \( \lambda = j\omega \), and the eigenvalue equation shown in Eq. (2-16) becomes;

\[
\left( -\omega^2 M + K_s \right) \phi = 0
\]  
(2-17)

where, \( \phi \) is the real eigenvector of the undamped system with the symmetric stiffness matrix.

In the subspace projection method, the original eigenvalue equation of the system (Eq. (2-16)) is projected onto the subspace spanned by the eigenvectors of the undamped symmetric system (Eq. (2-17)). The original \( M, C \) and \( K \) matrices are projected in the subspace of \( N \) real eigenvectors \( \phi \) as follows;

\[
M' = \begin{bmatrix} \phi_1, \phi_2, \ldots, \phi_N \end{bmatrix}^T M \begin{bmatrix} \phi_1, \phi_2, \ldots, \phi_N \end{bmatrix} 
\]  
(2-18a)

\[
C' = \begin{bmatrix} \phi_1, \phi_2, \ldots, \phi_N \end{bmatrix}^T C \begin{bmatrix} \phi_1, \phi_2, \ldots, \phi_N \end{bmatrix} 
\]  
(2-18b)

\[
K' = \begin{bmatrix} \phi_1, \phi_2, \ldots, \phi_N \end{bmatrix}^T K \begin{bmatrix} \phi_1, \phi_2, \ldots, \phi_N \end{bmatrix} 
\]  
(2-18c)

Then, the eigenvalue equation becomes;
The eigenvalue equation above is solved by the standard QZ method for generalized asymmetric eigenvalue problems. The complex eigenvalues of the projected system are the approximation of the eigenvalues of the original system shown in Eq. (2-16). The eigenvectors of the original system should be recovered by the following equation.

\[
\Psi_k = [\phi_1, \phi_2, \ldots, \phi_N] \psi_k^*
\]  

(2-20)

where, \( \psi_k \) is the approximated \( k \)-th eigenvector of the original system. Once the complex eigenvalues are extracted, the effective damping ratio (\( \zeta \)) is defined as;

\[
\zeta = -\frac{\text{Re}(\lambda)}{|\lambda|}
\]  

(2-21)

Therefore, the system becomes unstable if \( \zeta \) is negative. In other words, the unstable system has the eigenvalue whose real part is positive.

### 2.6.2 Dynamic Transient Analysis

The dynamic transient analysis (DTA) has been used to calculate the system responses in time domain by integrating the equations of motion of the FEM model. The explicit dynamic analysis of ABAQUS implements an explicit integration rule, which is central difference rule, along with the diagonal or lumped element mass matrices. In the DTA, the following FE equation of motion is solved;
\[ \textbf{M}\dot{\textbf{u}}^{(i)} = \textbf{f}_{\text{ex}}^{(i)} - \textbf{f}_{\text{in}}^{(i)} \]  

(2-22)

where, \( \textbf{M}, \textbf{u}, \textbf{f}_{\text{ex}} \) and \( \textbf{f}_{\text{in}} \) are the mass matrix, acceleration vector, applied load vector and internal force vector, respectively. The superscript \( i \) represents time increment number. The inversion of the mass matrix is used to calculate the accelerations at the beginning of the increment as follows;

\[ \dot{\textbf{u}}^{(i)} = \textbf{M}^{-1} \left( \textbf{f}_{\text{ex}}^{(i)} - \textbf{f}_{\text{in}}^{(i)} \right) \]

(2-23)

The velocity (\( \dot{\textbf{u}} \)) and displacement (\( \textbf{u} \)) vectors of the body are integrated by using the explicit central difference integration rule as follow;

\[ \dot{\textbf{u}}^{(i+0.5)} = \dot{\textbf{u}}^{(i-0.5)} + \frac{\Delta t^{(i+1)} + \Delta t^{(i)}}{2} \dot{\textbf{u}}^{(i)} \]  

(2-24)

\[ \textbf{u}^{(i+1)} = \textbf{u}^{(i)} + \Delta t^{(i+1)} \dot{\textbf{u}}^{(i+0.5)} \]  

(2-25)

where, \( \Delta t \) is a time step size, \( i+0.5 \) and \( i-0.5 \) represent mid-increment values.

The central difference integration is not self-starting operator because the value of the mean velocity (\( \dot{\textbf{u}}^{(-0.5)} \)) should be defined. Furthermore, the initial values of velocity and acceleration at \( t = 0 \) are assumed to be zero if they are not specified. The mid-increment value of the velocity (\( \dot{\textbf{u}}^{(+0.5)} \)) can be defined by the initial values of velocity (\( \dot{\textbf{u}}^{(0)} \)) and acceleration (\( \ddot{\textbf{u}}^{(0)} \)) as shown in Eq. (2-26).

\[ \dot{\textbf{u}}^{(+0.5)} = \dot{\textbf{u}}^{(0)} + \frac{\Delta t^{(i)}}{2} \ddot{\textbf{u}}^{(0)} \]  

(2-26)

Substituting Eq. (2-26) into Eq. (2-24), \( \dot{\textbf{u}}^{(-0.5)} \) can be represented as;

\[ \dot{\textbf{u}}^{(-0.5)} = \dot{\textbf{u}}^{(0)} - \frac{\Delta t^{(0)}}{2} \ddot{\textbf{u}}^{(0)} \]  

(2-27)
The central difference integration operator is conditionally stable because the explicit procedure integrates the equations by using many small time increments. The stability limit for the explicit operator with no damping can be defined in terms of the highest eigenvalue ($\omega_{\text{max}}$) of the system as shown in Eq. (2-28).

$$\Delta t \leq \frac{2}{\omega_{\text{max}}}$$ \hspace{1cm} (2-28)

In the system with damping, a small amount of damping can be added to the system in order to control high frequency oscillations. The stable time increment ($\Delta t$) with damping is represented as;

$$\Delta t \leq \frac{2}{\omega_{\text{max}}} \left( \sqrt{1+\zeta^2} - \zeta \right)$$ \hspace{1cm} (2-29)

where, $\zeta$ is the damping ratio which is the fraction of critical damping of the highest mode. It can be seen from the Eq. (2-29) that damping of the system reduces the stable time increment.
Chapter 3 Modified Sprag-Slip Mechanism

3.1 Introduction

Squeak is an annoying, undesirable noise generated by self-excited, friction-induced vibration of large amplitude. In order to design and build a unique squeak test apparatus that can generate squeak noises consistently and with good repeatability, the sprag-slip mechanism [22, 23] which is one of well-known mechanisms that lead to friction-induced instability, was modified and employed. A modified sprag-slip mechanism (MSSM) considers the compliance of the system. MSSM was designed by using a bent cantilever beam to develop the test apparatus.

In order to understand dynamics characteristics of the MSSM, an analytical model was developed, and kinematic and dynamic analysis of the analytical model were conducted. Stability of the MSSM was investigated by linearizing the equations of motion of the system. Phase portraits of the system were obtained to study dynamic characteristics of the system. The effects of major parameters, such as kinetic coefficient of friction and the angle of attack, were investigated to obtain insight that is necessary to design the test apparatus. Even though the system is stable and converges to a limit cycle, the large amplitude of the system leads to a squeak noise. Thus, the effects of the main design parameters on the size of the limit cycle were investigated in the phase plane. The stiffness of the MSSM was estimated by using Euler-Bernoulli beam theory and compared to that obtained from FE simulation.
A FEM model was developed and studied to refine the design of the test apparatus and better understand underlying mechanism of squeak noise generation. The instability condition of the system was studied by the complex eigenvalue analysis (CEA) of the model in frequency domain. It is shown that the system instability occurs by the coalescence of two modes that makes the effective damping of one of the coalesced modes negative. The instability condition identified from the CEA was confirmed by the dynamic transient analysis (DTA) in time domain. The instability condition identified from the analytical model shows good agreement with the results obtained from the FEM model.
3.2 Analytical Model to Represent a Modified Sprag-Slip Mechanism

3.2.1 Kinematics

The squeak test apparatus designed to measure squeak propensity is shown in Figure 3-1(a). The top end of the thin bent beam shown in this figure is clamped to the stationary frame. One of the test materials is attached to the bottom end of the beam that is in contact with the other material mounted on the rotating frame at the bottom. The top frame can be lowered or raised to change the normal contact force between the two materials by elastic force of the beam.

The simple, lumped parameter system shown in Figure 3-1(b) is adopted to study dynamics of the test apparatus. The model has a linear spring \( k \) and a torsional spring \( K \) attached at the top end of the rigid bar to represent the compliance of the bent beam as shown in Figure 3-1(b). This model is used to study dynamic characteristics of the squeak test apparatus shown in Figure 3-1(a). The classic sprag-slip model is a special case when the linear spring is infinitely stiff and the torsional spring is removed, that is, \( k = \infty, K = 0 \). The force equilibrium equation of the classic sprag-slip mechanism is:

\[
\frac{F_n}{F_t} = \frac{1}{1 - \mu \tan \theta_T} \tag{3-1}
\]

where, \( \mu \) is the kinetic coefficient of friction of the contact surface. From Eq. (3-1), it is seen that the mechanism becomes unstable when \( \mu = \cot \theta_T \) because the normal force has to be infinitely large at this condition. The classic sprag-slip mechanism can find only the instability condition but not the ensuing system response after the instability occurs.
Figure 3-1. (a) A squeak test apparatus and (b) the analytical model of the modified sprag-slip mechanism. ($k$: stiffness of the linear spring, $\xi_0$: initial compression, $\xi$: dynamic displacement of spring from initial compression, $l_s$: free length of the linear spring, $K$: stiffness of the torsional spring, $l$: length of a rigid bar, $\theta_a$: angle of attack, $\theta$: dynamic variation of angle, $\theta_T$: total angle, $F_n$: normal force, $F_t = \mu F_n$: friction force)
In Figure 3-1(b), $\xi_0$ and $\theta_a$ are the compression of the linear spring and the angle of the contact when the MSSM is initially set up, and $\xi$ and $\theta$ are dynamic components of the motion. $\theta_a$ is referred as the attack angle hereafter. The mechanism is a single-degree-of-freedom system because $\xi$ is a function of the total angle $\theta_T = \theta_a + \theta$. The bottom surface moves in the horizontal direction with the velocity $V$ that is much larger than the dynamic velocity of the tip; therefore, the direction of the friction force $F_T$ acting on the bar is always in the direction shown in Figure 3-1(b).

Figure 3-2 shows the acceleration polygons of the system of point $B$ shown in Figure 3-1(b) (Figure 3-2(a)) and of the center of gravity $G$ (Figure 3-2(b)). Notice that the acceleration and velocity of $B$ are constrained to be only in the horizontal direction.

From the acceleration polygon of point $B$ shown in Figure 3-2(a), it is found that;

$$\ddot{\xi} = l\dot{\theta}_T^2 \cos \theta_T + l\dot{\theta}_T \sin \theta_T$$

(3-2)

From the acceleration polygon of point $G$ shown in Figure 3-2(b), the acceleration of point $G$ in x- and y-direction can be represented as;

$$a_{Gx} = -\frac{l}{2} \ddot{\theta}_T \sin \theta_T + \frac{l}{2} \dot{\theta}_T \cos \theta_T$$

(3-3)

$$a_{Gy} = -\ddot{\xi} + \frac{l}{2} \dot{\theta}_T^2 \cos \theta_T + \frac{l}{2} \dot{\theta}_T \sin \theta_T = -\frac{l}{2} \ddot{\theta}_T \cos \theta_T - \frac{l}{2} \dot{\theta}_T \sin \theta_T$$

(3-4)
Figure 3-2. (a) acceleration polygon of point $B$ and (b) acceleration polygon of point $G$.

Figure 3-3. The relationship between the dynamic displacement ($\xi$) and angular displacement ($\theta$).
The dynamic displacement at point $A$ ($\xi$) is related to the angular displacement ($\theta$) as shown in Figure 3-3,

$$h = l \cos \theta_a = \xi + l \cos \theta_r$$

(3-5)

Therefore, the dynamic displacement of the spring $\xi$ is;

$$\xi = l (\cos \theta_a - \cos \theta_r)$$

(3-6)

### 3.2.2 Dynamics

Figure 3-4 shows the schematic of the system considered to obtain the initial conditions.

The moment equation about the top hinge point of the bar is;

$$K(\theta_a - \theta_\circ) = k \xi_\circ l (\sin \theta_a - \mu \cos \theta_a)$$

(3-7)

where, $\theta_\circ$ is the angle of the bar when the linear spring is at free length, therefore there is no contact force. The attack angle $\theta_a$ can be obtained by solving Eq. (3-7) numerically. The necessary displacement to lower the top frame to obtain $\xi_\circ$ can be found from:

$$y = \xi_\circ + l (\cos \theta_a - \cos \theta_\circ)$$

(3-8)
Figure 3-4. Schematics of the modified sprag-slip mechanism (a) when there is no force applied and (b) when the initial compression is applied to obtain the attack angle ($\theta_a$). ($k$: stiffness of the linear spring, $\xi_o$: initial compression, $K$: stiffness of the torsional spring, $l$: length of a rigid bar, $\theta_a$: the angle of the bar when the linear spring is at free length, $F_n$: normal force, $F_i = \mu F_n$: friction force)

Figure 3-5. Free body diagram of the modified sprag-slip mechanism shown in Figure 3-1.
Figure 3-5 illustrates the free body diagram of the MSSM shown in Figure 3-1(b). The force equation in \( y \)-direction as follow;

\[
\sum F_y = -k (\xi - \xi_o) - F_n = ma_{gy}
\]  
(3-9)

where, \( F_n \) and \( m \) are the normal force acting at the contact point between the rigid bar and the moving surface at the bottom and the mass of the rigid bar, respectively.

Substituting Eq. (3-4) into Eq. (3-9), \( F_n \) can be represented as

\[
F_n = -k (\xi - \xi_o) - \frac{ml}{2} (\ddot{\theta}_T \sin \theta_T + \dot{\theta}_T^2 \cos \theta_T)
\]  
(3-10)

By using Coulomb friction model with constant friction coefficient, the friction force (\( F_t \)) can be represented as Eq. (3.11).

\[
F_t = \mu F_n = -\mu \left[ k (\xi - \xi_o) + \frac{ml}{2} (\ddot{\theta}_T \sin \theta_T + \dot{\theta}_T^2 \cos \theta_T) \right]
\]  
(3.11)

where \( \mu \) is the kinetic coefficient of friction.

From the free body diagram shown in Figure 3-5, the force equation in \( x \)-direction is;

\[
\sum F_x = F_A - F_t = ma_{gx}
\]  
(3-12)

where \( F_A \) is the reaction force at the frictionless guide. By substituting Eq. (3-3) into Eq. (3-12), \( F_A \) can be obtained as follow;

\[
F_A = F_i + \frac{ml}{2} \left( \ddot{\theta}_T \cos \theta_T - \dot{\theta}_T^2 \sin \theta_T \right) = \mu F_n + \frac{ml}{2} \left( \ddot{\theta}_T \cos \theta_T - \dot{\theta}_T^2 \sin \theta_T \right)
\]  
(3-13)

The moment equation about the center of gravity of the rigid bar (\( G \)) becomes
\[ \sum M_g = -K (\theta_r - \theta_o) - \left( k (\xi - \xi_o) - F_o \right) \frac{l}{2} \sin \theta_r - (F_i + F_A) \frac{l}{2} \cos \theta_r = I_g \ddot{\theta}_r \quad (3-14) \]

By substituting Eq. (3-10), Eq. (3-11) and Eq. (3-13) into Eq. (3-14), the equation of motion of the MSSM is obtained as follow;

\[
\begin{align*}
\left[ I_G + 0.25 m I^2 (1 - \mu \sin 2 \theta_r) \right] \ddot{\theta}_r + K (\theta_r - \theta_o) \\
+ k l^2 \cos \theta_r \left[ \cos \theta_a - \cos \theta_r - (\xi_o / l) \right] (\tan \theta_r - \mu) - 0.5 \mu m I^2 \ddot{x}_r \cos^2 \theta_r = 0 \quad (3-15)
\end{align*}
\]

Eq. (3-15) can be transformed to the state space equation as shown in Eq. (3-16) by using state variables which are \( x_1 = \theta_r, \ x_2 = \dot{\theta}_r \).

\[
\begin{align*}
x_1 &= f_1 (x_1, x_2) = x_2 \\
x_2 &= f_2 (x_1, x_2) = \frac{K (\theta_o - x_1) + k l^2 \cos x_1 \left[ \cos \theta_a - \cos x_1 - (\xi_o / l) \right] (\mu - \tan x_1) + 0.5 \mu m I^2 x_2^2 \cos^2 x_1}{I_G + 0.25 m I^2 (1 - \mu \sin 2 x_1)} \quad (3-16)
\end{align*}
\]
3.3 Stiffness of the Modified Sprag-Slip Mechanism

3.3.1 Stiffness Estimation Using Euler-Bernoulli Beam Theory

Figure 3-6. (a) A bent beam of the sprag-slip mechanism with a vertical force $F$, (b) free body diagram of Beam AB and (c) free body diagram of Beam BC.

Figure 3-6 shows a bent beam of the MSSM with vertical force $F$ and free body diagrams of two beam segments of the bent beam (Beam AB and Beam BC). As shown in Figure 3-6, the deflection at C ($\delta_C$) consists of two parts; the deflection due to bending of Beam AB ($\delta_{C1}$) and the deflection due to bending of Beam BC ($\delta_{C2}$). From the free body diagram of Beam AB shown in Figure 3-13(b), the vertical force $F$ and the moment $M_B$ are applied at B. $M_B$ due to $F$ at the point C can be represented as;

$$M_B = FL_{BC} \sin \theta_a$$  \hspace{1cm} (3-17)
The rotation at \( B (\theta_B) \) can be represented as Eq. (3-18) by Euler-Bernoulli beam theory [79] and ignoring the axial deformation of a beam.

\[
\theta_B = \frac{M_{B,AB} l_{AB}}{EI} = \frac{F l_{AB} l_{BC} \sin \theta_a}{EI}
\]  

(3-18)

where \( l_{AB}, l_{BC}, E \) and \( I \) are the length of Beam \( AB \), the length of Beam \( BC \), modulus of elasticity and moment of inertia. Therefore, \( \delta_{c1} \) can be represented as;

\[
\delta_{c1} = \theta_B l_2 = \frac{F l_{AB} l_{BC}^2 \sin \theta_a}{EI}
\]

(3-19)

From the free body diagram of beam \( BC \) shown in Figure 3-6(c), \( \delta_{c2} \) can be represented as [79];

\[
\delta_{c2} = \frac{P l_{BC}^3}{3EI} = \frac{(F \sin \theta_a) l_{BC}^3}{3EI} = \frac{F l_{BC}^2 \sin \theta_a}{3EI}
\]

(3-20)

where \( P \) is the transverse load applied at \( C \). Hence, the total deflection at \( C (\delta_c) \) is;

\[
\delta_c = \delta_{c1} + \delta_{c2} = \frac{F l_{AB} l_{BC}^2 \sin \theta_a}{EI} + \frac{F l_{BC}^2 \sin \theta_a}{3EI} = \frac{F l_{BC}^2 \sin \theta_a}{3EI} \left( 3l_{AB} + l_{BC} \right)
\]

(3-21)

Only vertical component of \( \delta_c \) is needed to calculate the linear stiffness \( (k) \) of the bent beam. Thus, \( k \) can be represented as;

\[
k = \frac{F}{\delta_c \sin \theta_a} = \frac{3EI}{l_{BC}^2 \sin^2 \theta_a \left( 3l_{AB} + l_{BC} \right)}
\]

(3-22)
Figure 3-7. (a) A bent beam of the sprag-slip mechanism with a moment $M$, (b) free body diagram of Beam $AB$ and (c) free body diagram of Beam $BC$.

In order to calculate the torsional stiffness ($K$), the moment ($M$) is applied at $C$ as shown in Figure 3-7. From the free body diagram of Beam $AB$ and Beam $BC$ shown in Figure 3-7(b) and (c), the rotation at $B$ due to $M$ ($\theta_B$) and the rotation at $C$ due to $M$ ($\theta_C$) can be represented as:

$$
\theta_B = \frac{Ml_{AB}}{EI} = \frac{Ml_{AB}}{EI} \quad (3-23)
$$

$$
\theta_C = \frac{Ml_{BC}}{EI} = \frac{Ml_{BC}}{EI} \quad (3-24)
$$

The total rotation at $C$ due to $M$ is the summation $\theta_B$ of $\theta_C$ and, therefore, $K$ can be represented as:

$$
K = \frac{M}{\theta_B + \theta_C} = \frac{EI}{l_{AB} + l_{BC}} \quad (3-25)
$$
3.3.2 Comparison of Stiffness obtained from Euler-Bernoulli Beam Theory and Finite Element Method

Figure 3-8. Solid model of a bent beam of the sprag-slip mechanism and its FE model.

In order to confirm the stiffness of the bent beam of the MSSM obtained from Euler-Bernoulli beam theory, it was compared to the stiffness obtained from finite element method (FEM). Figure 3-8 shows the solid model of the bent beam and its FE model. ABAQUS was used as a FE software package. Two types of element, which are 1D beam element and 3D brick (solid) element, were used in FE simulations. The top end of the bent beam is fixed, and the vertical force $F$ and moment $M$ are applied at free-end of the bent beam. The displacement at free-end with a
given load was calculated, and the stiffness was calculated by dividing the displacement by a given load. Deformed shapes obtained from FEM are shown in Figure 3-9.

Figure 3-9. Undeformed and deformed shapes of the bent beam with the moment at free end. (a) 1D beam element and (b) 3D brick element.
The stiffness obtained from Euler-Bernoulli beam theory and FEM are compared in Table 3-1. It can be seen that the stiffness obtained from FEM with 1D beam element is exactly the same as that obtained from beam theory since beam elements are defined by beam theory. The stiffness estimated by FEM with 3D brick elements is also reasonably matched to that obtained from beam theory with about 5% difference.

Table 3-1. Comparison of the stiffness obtained from Euler-Bernoulli beam theory and FEM (k: linear stiffness, K: torsional stiffness, difference to beam theory in parenthesis).

<table>
<thead>
<tr>
<th></th>
<th>Beam Theory</th>
<th>FEM – 1D Beam</th>
<th>FEM – 3D Brick</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k ) [N/m]</td>
<td>6696.5</td>
<td>6693.4 (0.05 %)</td>
<td>6298.8 (5.94 %)</td>
</tr>
<tr>
<td>( K ) [N-m/rad]</td>
<td>0.79545</td>
<td>0.79554 (0.01 %)</td>
<td>0.7728 (2.85 %)</td>
</tr>
</tbody>
</table>
3.4 Stability Analysis

3.4.1 Analytical Model

The nonlinear equation of motion of the MSSM (Eq. (3-16)) can be linearized at equilibrium points \((\bar{x}_1, \bar{x}_2)\) that are the points where both \(f_1(\bar{x}_1, \bar{x}_2)\) and \(f_2(\bar{x}_1, \bar{x}_2)\) are zero [71]. By applying a Taylor series expansion and introducing the perturbations \((\delta_1, \delta_2)\), Equation (3-16) can be linearized as follows;

\[
\begin{bmatrix}
\frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\
\frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2}
\end{bmatrix}_{\bar{x}_1, \bar{x}_2} \begin{bmatrix}
\delta_1 \\
\delta_2
\end{bmatrix} = \begin{bmatrix}
0 & 1 \\
0 & 0
\end{bmatrix} \begin{bmatrix}
\delta_1 \\
\delta_2
\end{bmatrix} \rightarrow \dot{\delta} = J\delta
\]

(3-26)

where \(J\) is Jacobian matrix evaluated at the equilibrium points.

Therefore, the nonlinear equation of the motion is converted to the ordinary differential equation (ODE) of the perturbations. The solution of Eq. (3-26) can be obtained as [72];

\[
\delta(t) = c_1 e^{\lambda_1 t} \mathbf{x}_1 + c_2 e^{\lambda_2 t} \mathbf{x}_2
\]

(3-27)

where \(\lambda_1\) and \(\lambda_2\) are the eigenvalues of \(J\), \(\mathbf{x}_1\) and \(\mathbf{x}_2\) are the corresponding eigenvectors, and \(c_1\) and \(c_2\) are the constants determined by initial conditions. If one of the eigenvalues has the positive real part, the solution grows exponentially. Hence, the stability of the system is determined by the real part of the eigenvalues of \(J\).

From Eq. (3-16), the equilibrium point of the system is \((\bar{x}_1, \bar{x}_2) = (\theta_s, 0)\), and the eigenvalues of \(J\) are;

\[
\lambda = \pm \sqrt{a_{21}}
\]

(3-28)
The system becomes unstable if \( a_{21} \) is higher than zero since one of the eigenvalues has positive real part. Therefore, the instability condition can be represented as Eq. (3-29).

\[
\begin{align*}
    a_{21} &= \left. \frac{\partial f_2}{\partial x_1} \right|_{(x_1,x_2)=(a_1,0)} \\
    &= \frac{-K + kl^2 \sin^2 \theta a (\mu \cot \theta a - 1)}{I_g + 0.25ml^2 (1 - \mu \sin 2\theta a)} > 0
\end{align*}
\] (3-29)

If \( \mu \) is less than or equal to one as in most material pairs, the denominator of Eq. (3-29) is always positive. In order to make the system unstable, the numerator of Eq. (3-29) should be positive. Thus, the instability condition of the system is;

\[
f(\mu, \theta a) = \sin^2 \theta a (\mu \cot \theta a - 1) > \frac{K}{kl^2}
\] (3-30)

In order to investigate the effect of the major parameter on the stability of the system, \( f(\mu, \theta a) \) in Eq. (3-30) is evaluated with \( 0 < \mu < 1 \) and \( 0 < \theta a < 90 \) deg. as shown in Figure 3-10. From Figure 3-10(b), the unstable regions are identified in terms of \( \mu \) and \( \theta a \) for four different values of \( K/kl^2 \) as shown in Figure 3-11. The unstable region becomes smaller as \( K/kl^2 \) increases, which means that the system becomes unstable more easily as \( K \) decreases relative to \( kl^2 \).
Figure 3-10. (a) 3D plot of the instability condition of the modified sprag-slip mechanism as a function of the kinetic coefficient of friction ($\mu$) and the angle of attack ($\theta_a$) and (b) its contour plot.
Figure 3-11. Unstable region with different values of $K/kl^2$. (a) $K/kl^2 = 0$, (b) $K/kl^2 = 0.05$, (c) $K/kl^2 = 0.1$ and (d) $K/kl^2 = 0.15$ ( — : Stable region, — : Unstable region)
In addition, in order to investigate the effect of the parameters on the stability of the system, the eigenvalues of the system were calculated with respect to four parameters, which are $\mu$, $\theta_a$, $K$ and $k$. The eigenvalues were calculated by changing one parameter while keeping the other parameters fixed. The values of the parameters used and other constants are summarized in Table 3-2.

The evolutions of the eigenvalues of the system with respect to four parameters are shown in Figure 3-12. One of the eigenvalues becomes positive real as $\mu$ increases (Figure 3-12(a)), which means that the higher value of $\mu$ makes the system unstable. This confirms the instability condition shown in Eq. (3-30). When $\theta_a$ is fixed, the higher value of $\mu$ is located in the unstable region as shown in Figure 3-11. From Figure 3-12(b), the system becomes unstable as $\theta_a$ increases, however, the system becomes stable as $\theta_a$ increases further. This can be confirmed by the unstable regions identified by Eq. (3-30). As shown in Figure 3-11(b) through (d), when $\mu$ is fixed, the system is located in the stable region with the low value of $\theta_a$. If $\theta_a$ increases, the system crosses the unstable region and returns to the stable region. Therefore, the range of $\theta_a$ should be identified with respect to other parameters to make the system unstable. The system becomes unstable as the torsional stiffness $K$ decreases or the linear stiffness $k$ increases as shown in Figure 3-12 (c) and (d). This confirms that the unstable region shown in Figure 3-11 identified by the instability condition (Eq. (3-30)) becomes bigger as the value of $K/kl^2$ increases.
Table 3-2. The values of the parameters used to calculate the eigenvalues of the system.

<table>
<thead>
<tr>
<th>Target Parameter</th>
<th>$\mu$</th>
<th>$\theta_a$</th>
<th>$K$</th>
<th>$k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$</td>
<td>$0 &lt; \mu &lt; 1$</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>$\theta_a$ [deg]</td>
<td>10</td>
<td>$0 &lt; \theta_a &lt; 90$</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>$K$ [N-m/rad]</td>
<td>1</td>
<td>1</td>
<td>$0 &lt; K &lt; 10$</td>
<td>1</td>
</tr>
<tr>
<td>$k$ [N/m]</td>
<td>$10^4$</td>
<td>$10^4$</td>
<td>$10^4$</td>
<td>$0 &lt; k &lt; 10^6$</td>
</tr>
<tr>
<td>$l$ [m]</td>
<td></td>
<td></td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>$m$ [kg]</td>
<td></td>
<td></td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>$I_G$ [kg-m$^2$]</td>
<td></td>
<td></td>
<td>$ml^2/12 = 8.33 \times 10^{-5}$</td>
<td></td>
</tr>
</tbody>
</table>
Figure 3.12. The two eigenvalues (red and blue dot) of the system with respect to four parameters. (a) Kinetic coefficient of friction $\mu$, (b) angle of attack $\theta_a$, (c) torsional stiffness $K$ and (d) linear stiffness $k$. 


In order to confirm the instability condition obtained by the linearized equation of motion and the eigenvalue analysis, the nonlinear equation of motion (Eq. (3-16)) was solved numerically by using 4th order Runge-Kutta method. Figure 3-13 shows the transient responses with three different kinetic coefficients of friction (0.1, 0.3 and 0.5) while other parameters are kept constant, which are $\theta_a = 10$ deg, $m = 0.1$ kg, $l = 0.2$ m, $I_G = ml^2/12$, $k = 10^5$ N/m, and $K = 10^2$ N-m/rad. The amplitude of the response becomes higher as $\mu$ increases. The response is bounded when $\mu = 0.3$, however it grows unbounded when $\mu = 0.5$. In general, the system becomes unstable more easily with a high coefficient of friction, which confirms the instability condition shown in Figure 3-11 and Figure 3-12.

Figure 3-14 shows the motion of the system when the angle of attack ($\theta_a$) is changed while $\mu$ is fixed at 0.2. The amplitude of the response increases as $\theta_a$ increases, then settles into a limit cycle. When $\theta_a$ is small, the response oscillates with nearly constant amplitude.
Figure 3-13. Transient responses of the system for three different kinetic coefficient of friction ($\mu$).

Figure 3-14. Transient responses of the system with three different angles of attack ($\theta_a$).
Two sets of system parameters were chosen from the instability condition shown in Eq. (3-30) to make the system unstable (Figure 3-15) and stable (Figure 3-16). The system responses grow infinitely as shown in Figure 3-15 while the system responses converge to a limit cycle as shown in Figure 3-16, which confirms the instability conditions of the system.

**Figure 3-15.** Unstable, transient responses of the system with $\theta_a = 15$ degree, $\mu = 0.5$, $K = 100$ N-m/rad, $k = 10^5$ N/m and $l = 0.2$ m. (a) angular displacement, (b) angular velocity and (c) phase portrait.
Figure 3-16. Stable, transient responses of the system with $\theta_a = 45$ degree, $\mu = 0.5$, $K = 100$ N-m/rad, $k = 10^5$ N/m and $l = 0.2$ m. (a) angular displacement, (b) angular velocity and (c) phase portrait.
The squeak test apparatus should be able to induce an unstable motion as the one seen in Figure 3-15 for a relatively wide range of system parameters lest small changes in operating condition should influence the resulting squeak noise significantly. System parameters will be always changed slightly for various reasons; for example, the friction coefficient due to temperature or humidity changes, the spring constant due to how the specimen is mounted.

To understand the robustness of the test apparatus, simulations of the system response was conducted while changing major system parameters. Four parameters, \( m, \mu, \theta_0 \) and the initial compression of the linear spring \( (\xi_0) \), were selected. Only one parameter was changed by ±10 % from the baseline case used in Figure 3-15, and the response of the system was obtained by numerically solving Eq. (3-16).

The responses with different values of \( m \) are shown in Figure 3-17. In all three cases, the responses of the system grow indefinitely, which means that the system becomes unstable in all cases. The amplitude of the responses increases more quickly as \( m \) decreases, which is expected when the inertia decreases.
Figure 3-17. The responses of the MSSM with three different values of mass \( m \). (a) Angular displacement and (b) angular velocity.

Figure 3-18 shows the responses of the system when the coefficient of friction \( \mu \) is change. The amplitude of the responses increases as \( \mu \) increases, which is again expected because the friction force is the driving force of the instability. The result shows that the system will generate a similar squeak noise even if the friction coefficient changes due to changes of humidity or temperature.

The responses of the MSSM with different values of attack angle \( \theta_a \) are shown in Figure 3-19. The amplitude of the system tends to increase as \( \theta_a \) decreases, which means that a lower value of \( \theta_a \) can make the system unstable more easily, at least around the current value of \( \theta_a = 20 \) deg. In all three cases, the response of the MSSM always grows indefinitely. This implies that the test system will work fine even if the mounting of the test specimen becomes slightly different.
Figure 3-18. The responses of the MSSM with three different values of kinetic coefficient of friction ($\mu$). (a) Angular displacement and (b) angular velocity.

Figure 3-19. The responses of the MSSM with three different values of an angle of attack ($\theta_a$). (a) Angular displacement and (b) angular velocity.
Figure 3-20 shows the responses of the MSSM with different values of $\zeta_0$ that determines the average value of the normal force ($F_n$) as shown in Eq. (3-10). The amplitude of the response becomes bigger and bigger with decreasing $\zeta_0$. In other words, the system tends to become unstable more easily when $\zeta_0$ is lower. However, all three cases induce the system instability, thus small changes in initial loading will not make significant difference in resulting squeak noises.

Simulations shown above indicate that the designed apparatus will be a quite robust squeak test machine. In actual testing, small variations of system parameters over different tests will be unavoidable, however, the result obtained from the test apparatus will not be affected significantly by those unintended variations.

![Figure 3-20](image)

**Figure 3-20.** The responses of the MSSM with three different values of initial compression ($\zeta_0$). (a) Angular displacement and (b) angular velocity.
3.4.2 FEM Model

A FEM model of the MSSM developed for the stability analysis is shown in Figure 3-21. The top end of the bent beam is fixed in x-direction, and the normal force $N$ is applied at the top end of the beam in y-direction. The rigid surface at the bottom which contacts with the bottom end of the bent beam moves in negative x-direction at the constant velocity $V$. The material property and dimensions of the FEM model are summarized in Table 3-3.

![FEM Model Diagram](image)

**Figure 3-21.** A finite element model of the MSSM. $N$, $V$ and $\theta_a$ are the applied force at the top end of the beam, the velocity of the rigid surface at the bottom and the angle of attack. $l_1$ and $l_2$ are the length of the beam segments.
Table 3-3. Material property and dimensions of the FEM model shown in Figure 3-13.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus, $E$</td>
<td>70 GPa</td>
</tr>
<tr>
<td>Poisson’s Ration, $\nu$</td>
<td>0.3</td>
</tr>
<tr>
<td>Length of the beam</td>
<td>$l_1 = 10$ mm, $l_2 = 70$ mm</td>
</tr>
<tr>
<td>Angle of attack, $\theta_a$</td>
<td>8 deg</td>
</tr>
<tr>
<td>Cross section of the beam</td>
<td>15 mm $\times$ 1 mm</td>
</tr>
</tbody>
</table>

In order to investigate the stability of the FEM model shown in Figure 3-21, the complex eigenvalue analysis (CEA) was used to extract the eigenvalues and corresponding eigenvectors of the FEM model. The governing equation and procedures of the CEA are presented in Chapter 2.6.1. The stability of the system was determined by the effective damping ratio $\zeta$ shown in Eq. (2-21), which means that, if at least one of the modes has the negative $\zeta$, the system is considered unstable.

The effect of the kinetic coefficient of friction $\mu$ on the stability of the system was investigated by changing $\mu$ while keeping the other parameters the same ($\theta_a = 8$ deg, $N = 10$ N, $V = 2$ mm/s). The eigenvalues of the system obtained from the CEA with three different values of $\mu$ (= 0.1, 0.2, 0.4) are shown in Figure 3-22. When $\mu = 0.1$, $\zeta$ of all eigenvalues are zero, which means that the system would oscillate with a finite amplitude because there is no damping effect in the system. However, when $\mu = 0.2$ or 0.4, two or three eigenvalues have the negative value of $\zeta$, which means the system becomes unstable. In addition, in order to determine the boundary of the instability, the eigenvalues were traced as $\mu$ increased as shown in Figure 3-23. It is seen that two eigenvalues corresponding to the mode 7 and 8 coalesce when $\mu = 0.1367$, and $\zeta$ of the mode 7 becomes negative as $\mu$ increases further. Also, the eigenvalues corresponding to the mode 13 and
14 coalesce when $\mu = 0.1238$, and $\zeta$ of the mode 13 becomes negative as $\mu$ increases. Therefore, the instability condition of the FEM model is $\mu > 0.1238$ when $\theta_a = 8$ deg, $N = 10$ N, $V = 2$ mm/s.

**Figure 3-22.** Eigenvalues of the FEM model of the MSSM with three different values of the kinetic coefficient of friction ($\mu$).
Figure 3-23. The evolution of the eigenvalues with respect to the kinetic coefficient of friction ($\mu$). (a) Mode 7 and 8, (b) Mode 13 and 14.
The instability condition identified from the CEA was confirmed by the dynamic transient analysis (DTA) in time domain. The DTA numerically integrates the governing equation of the FEM model. The details of the DTA are explained in Chapter 2.6.2. Figure 3-24 shows the velocity and acceleration responses at the tip of the bent beam which contacts to the rigid surface at the bottom and the frequency spectra of the responses. It is shown that, when $\mu = 0.1$, the system oscillates with very small amplitude, which means that the system is stable. However, when $\mu = 0.2$ or 0.4, the response grows to very high amplitude and recedes periodically, which means that the system becomes unstable. In addition, from the frequency spectra of the velocity and acceleration shown in Figure 3-24 (e) and (f), it can be seen that the magnitudes of the frequency contents with $\mu = 0.2$ or 0.4 are much higher than those with $\mu = 0.1$. The eigenvalues of the FEM model obtained by the CEA (Figure 3-14 and 3-15) show that the system becomes unstable when $\mu = 0.2$ or 0.4. Hence, the time history of the system obtained by the DTA confirms the instability condition identified by the CEA.

The relative velocity of the tip of the bent beam of the MSSM to the velocity of the rigid surface and the contact force between the two of the unstable case ($\mu = 0.2; N = 10 \text{ N}; V = 2 \text{ mm/s}$) are shown in Figure 3-25. From the relative velocity shown in Figure 3-25 (b), it is seen that the tip of the bent beam sticks to the rigid surface and is deformed. Then, the bent beam starts to slip from the rigid surface because the elastic force of the deformed beam exceeds the friction force, and the vibration with the large amplitude occurs. From the contact force shown in Figure 3-25 (d), the contact force is approximately 10 N, which is the same as $N$, when the tip of the beam sticks to the rigid surface. When the beam starts to slip and the large-amplitude vibration occurs, the contact force shows the oscillation between 0 N and 20 N, which means the separation and slip between the tip of the beam and the rigid surface occur alternatively.
Figure 3-24. The responses and frequency spectra of the tip of the bent beam of the MSSM obtained by the dynamic transient analysis with three different values of the kinetic coefficient of friction ($\mu$). (a) Velocity time history, (b) acceleration time history, (c) zoom-in of the velocity time history, (d) zoom-in of the acceleration time history, (e) frequency spectrum of the velocity, and (f) frequency spectrum of the acceleration.
Figure 3-25. (a) Relative velocity at the tip of the bent beam of the MSSM obtained by the dynamic transient analysis, (b) zoom-in of the relative velocity, (c) contact force between the tip of the bent beam of the MSSM and the rigid surface obtained by the dynamic transient analysis, and (d) zoom-in of the contact force. (the kinetic coefficient of friction, $\mu = 0.2$; the normal force applied at the top end of the bent beam, $N = 10$ N; the velocity of the rigid surface, $V = 2$ mm/s)
The effect of the angle of attack $\theta_a$ on the stability of the system was investigated by changing $\theta_a$ while keeping the other parameters the same ($\mu = 0.2; N = 10$ N; $V = 2$ mm/s). The eigenvalues of the system obtained from the CEA with four different values of $\theta_a (= 3, 8, 15, 45 \text{ deg})$ are shown in Figure 3-26. When $\theta_a = 15$ or 45 deg, $\zeta$ of all eigenvalues are zero, which means that the system would oscillate with a finite amplitude. However, when $\theta_a = 3$ or 8, two eigenvalues have the negative value of $\zeta$, which means the system becomes unstable. Therefore, the system becomes unstable more easily as $\theta_a$ decreases. In addition, the acceleration response of the tip of the beam was obtained by the DTA as shown in Figure 3-27 (a). It is shown that, when $\theta_a = 15$ or 45 deg, the system oscillates with very small amplitude, which means that the system is stable. However, when $\theta_a = 3$ or 8 deg, the response grows to very high amplitude and recedes periodically, which means that the system becomes unstable. From the frequency spectrum of the acceleration shown in Figure 3-27 (b), it is seen that the magnitudes of the frequency contents with $\theta_a = 3$ or 8 deg are much higher than those $\theta_a = 15$ or 45 deg. The eigenvalues of the FEM model obtained by the CEA (Figure 3-26) show that the system becomes unstable when $\theta_a = 3$ or 8 deg. Therefore, the response of the system obtained by the DTA confirms the instability condition identified by the CEA.
Figure 3-26. Eigenvalues of the FEM model of the MSSM with four different values of the angle of attack ($\theta_a$).
Figure 3-27. (a) the acceleration responses of the tip of the bent beam of the MSSM with four different values of the angle of attack ($\theta_a$), and (b) their frequency spectra.
The effect of the velocity of the bottom rigid surface of the MSSM ($V$) on the stability of the system was investigated by changing $V$ while keeping the other parameters the same ($\mu = 0.2; \theta_a = 8 \text{ deg}; N = 10 \text{ N}$). Figure 3-28 shows the velocity and acceleration responses at the tip of the bent beam, which contacts to the rigid surface at the bottom, and the frequency spectra of the responses with $V = 1, 2, 5 \text{ mm/s}$. It is shown that the maximum amplitude of the velocity and acceleration look almost the same and the system is unstable in all cases. However, the period of the growing responses decreases as $V$ increases. From the frequency spectra of the velocity and acceleration shown in Figure 3-28 (e) and (f), it is shown that the magnitude of the frequency contents increases slightly as $V$ increases. Therefore, the effect of $V$ on the stability is not as significant as that of $\mu$ or $\theta_a$. 
Figure 3-28. The responses and frequency spectra of the tip of the bent beam of the MSSM obtained by the dynamic transient analysis with three different values of the velocity of the rigid surface at the bottom (v).  
(a) Velocity time history, (b) acceleration time history, (c) zoom-in of the velocity time history, (d) zoom-in of the acceleration time history, (e) frequency spectrum of the velocity, and (f) frequency spectrum of the acceleration.
The effect of the normal force applied to the MSSM ($N$) on the stability of the system was studied. The simulations were repeated with $N = 10, 15, 20$ N while the other parameters were fixed ($\mu = 0.2; \theta_a = 8$ deg; $V = 2$ mm/s). The velocity and acceleration responses at the tip of the bent beam and the frequency spectra of the responses are shown in Figure 3-29. From the velocity and acceleration responses in time domain, it can be seen that the system is unstable in all cases because the response grows to very large amplitude and recedes periodically. The period of the growing and receding response increases as $N$ increases. This is due to the fact that the maximum deformation of the beam until the beam starts to slip becomes larger as $N$ increases. The amplitude of the response increases with increasing $N$, therefore, the MSSM generates the unstable motion and the squeak noise more easily with the large value of $N$. 
The responses and frequency spectra of the tip of the bent beam of the MSSM obtained by the dynamic transient analysis with three different values of the normal force at the top end of the beam ($N$). (a) Velocity time history, (b) acceleration time history, (c) zoom-in of the velocity time history, (d) zoom-in of the acceleration time history, (e) frequency spectrum of the velocity, and (f) frequency spectrum of the acceleration.
3.4.3 Comparison of the Instability Condition Identified by the Analytical and FEM Model

The instability conditions obtained from the analytical and FEM model are compared to understand the dynamic characteristics of the MSSM and refine the design of the squeak test apparatus. By substituting the stiffness of the bent beam (Eq. (3-22) and (3-25)) into the instability condition identified by the analytical model (Eq. (3-30)) and assuming that the length of the rigid bar of the MSSM ($l$) is the same as the total length of the bent beam ($l_1 + l_2$), the instability condition from the analytical model becomes;

$$
\mu \cot \theta_a > 1 + \frac{l_2^2 (3l_1 + l_2)}{(l_1 + l_2)^3}
$$

As shown in Table 3-3, $l_1 = 10$ mm, $l_2 = 70$ mm. Hence, Eq. (3-31) becomes;

$$
\mu \cot \theta_a > 1.957
$$

Total 25 cases were simulated by changing $\mu$ and $\theta_a$ ($\mu = 0.1, 0.2, 0.3, 0.4, 0.5; \theta_a = 3, 8, 15, 30, 45$ deg). The stability of each case was identified by the effective damping ratio $\zeta$ obtained by the CEA. The unstable region identified by the analytical model (Eq. (3-32)) is compared to the unstable cases identified by the FEM model in terms of $\mu$ and $\theta_a$ as shown in Figure 3-30. It illustrates that the stability of the MSSM obtained by the analytical model shows a good agreement to the results of the FEM model. The FEM model shows the broader range of the instability than the analytical model does. Therefore, the squeak test apparatus based on the instability condition of the MSSM obtained by the analytical model (Eq. (3-31)) is more conservative to generate unstable motion which leads to squeak noises than that obtained by the FEM model.
Figure 3-30. Comparison of the instability region identified from the analytical and FEM model of the MSSM in terms of the kinetic coefficient of friction ($\mu$) and the angle of attack ($\theta_a$).
3.5 Effects of the Main Parameters on the Amplitude of the Limit cycle of the MSSM

After the instability breaks out, the system response settles into a limit cycle because the nonlinear effect of the system takes over. The post-instability behavior of the system, specifically the amplitude of the limit cycle, is another important factor which has an influence on the squeak noise since the limit cycle with a larger amplitude will make a louder squeak noise. Effects of the main parameters in the analytical model, which are the mass of the rigid bar $m$, the kinetic coefficient of friction $\mu$, the angle of attack $\theta_a$ and the initial compression of the linear spring $\zeta_0$, were studied by solving the equation of the motion of the MSSM (Eq. (3-16)) numerically to investigate their effects on the amplitude of the limit cycle.

Figure 3-31 shows the limit cycle of the system with different value of $m$. It is shown that, as $m$ decreases, the amplitude of the angular displacement ($\theta_T$) is nearly constant, however, the amplitude of angular velocity ($\dot{\theta}_T$) increases. Hence, the inertia of the system affects the velocity of the system significantly, but not the amplitude of $\theta_T$. The amplitudes of the displacement and velocity of the limit cycles with different values of $m$ are shown in Figure 3-32. It can be seen that the amplitude of displacement is nearly constant but the amplitude of the velocity decreases as $m$ increases.

The phase portraits of the limit cycles of the system with different value of $\theta_a$ are shown in Figure 3-33. Figure 3-34 shows the amplitudes of the limit cycle as a function of $\theta_a$ for different values of $\mu$. The amplitude of the system tends to increase as $\theta_a$ increases when $\mu$ is 0.1. When $\mu$ is relatively large (0.7), the size of limit cycle increases initially as $\theta_a$ increases (from 10 deg to 25 deg), decreases as $\theta_a$ increases to 30 deg, and then starts to increase as $\theta_a$ further increases (40 deg and beyond).
Figure 3-31. Phase portraits of the limit cycles of the stable system with different mass \((m)\) of the system and friction coefficient \((\mu)\). (a) \(\mu = 0.1\) and (b) \(\mu = 0.7\).

Figure 3-32. Amplitude of the limit cycles with different values of mass \((m)\). (a) Displacement and (b) velocity.
Figure 3-33. Phase portraits of the limit cycles of the stable system with different angle of attack ($\theta_a$) of the system and friction coefficient ($\mu$). (a) $\mu = 0.1$ and (b) $\mu = 0.7$.

Figure 3-34. Amplitude of the limit cycles with different values of the angle of attack ($\theta_a$). (a) Displacement and (b) velocity.
Figure 3-35 shows the phase portraits of the limit cycles with different values of the initial compression ($\zeta_0$). As $\zeta_0$ increases from a very small value, the size of the limit cycle becomes smaller initially; then the size of the limit cycle becomes larger as $\zeta_0$ increases further beyond a certain value. The amplitudes of the responses of the limit cycles as a function of $\zeta_0$ are shown in Figure 3-36, which shows the trend. It is interesting to see that there is a value of $\zeta_0$ at which the amplitude becomes nearly zero. This observation tells that self-excited vibration of larger amplitude, i.e., squeak noises can be generated more consistently by choosing the initial compression of $\zeta_0$ properly.

Figure 3-35. Phase portraits of the limit cycles of the stable system with different initial compression ($\zeta_0$) and friction coefficient ($\mu$). (a) $\mu = 0.3$ and (b) $\mu = 0.7$. 
Figure 3-36. Amplitude of the limit cycles with different values of the initial compression ($\xi_0$). (a) Displacement and (b) velocity.
3.6 Conclusions

An analytical model of the MSSM was developed to study dynamic characteristics of the system and obtain design guidance of a squeak test apparatus. The analytical model consists of the rigid bar that contacts to the moving surface and the compliance. The acceleration of the rigid bar was obtained by kinematic analysis. The equations of motion of the analytical model were developed and solved. Stability analysis of the analytical model was conducted by using linearized equation of motion about its equilibrium points, and the instability conditions were identified by parameter study. The results illustrate that the system becomes unstable more easily with higher torsional stiffness ($K$) relative to the linear stiffness ($k$). In addition, higher kinetic coefficient of friction ($\mu$) makes the system unstable. Transient responses and phase portraits were obtained by solving the nonlinear equations of motion numerically, and studied to further understand dynamic characteristics of the system. The phase portrait converges to a limit cycle of a finite amplitude for a stable system, and grows indefinitely for an unstable system. The instability conditions can be used to design the squeak test apparatus that induces unstable, self-excited vibration between two surfaces in sliding contact.

The MSSM is modeled by a bent cantilever beam to consider the compliance and the angle of attack ($\theta_a$). The stiffness of the bent beam was obtained by using Euler-Bernoulli beam theory and FEM. The stiffness obtained from the beam theory shows good agreement with that obtained from FEM within 5% difference. Therefore, the stiffness of the bent beam in the squeak test apparatus can be estimated accurately by beam theory.

In order to confirm the instability condition identified from the analytical model, the FEM model of the MSSM was developed and studied. The CEA was employed to extract the eigenvalues of the FEM Model, and the stability of the FEM model was identified by the effective damping
ratio of each mode. In addition, the instability condition obtained by the CEA was confirmed by
the DTA in time domain. In the unstable systems, the response grew to very large amplitude and
receded periodically, while the stable system showed the oscillation with very small amplitude.
The instability conditions obtained from the analytical model showed a good agreement with the
results obtained from the FEM model, and the analytical model provides more conservative
instability condition when compared to the FEM model.

The effect of the parameters on the size of the limit cycle was also studied because the
squeak noise is the result of a large-amplitude, unstable oscillation. As the mass of the system ($m$)
increases, the amplitude of the displacement remains the same but the amplitude of the velocity
decreases, therefore the size of limit cycle becomes smaller. The size of limit cycle of the system
tends to increase as $\theta_o$ increases when $\mu$ is small. However, if $\mu$ is relatively large, there is a value
of $\theta_o$ that makes the size of the limit cycle maximum or minimum. Moreover, the size of the limit
cycle is affected by the initial compression of the system ($\zeta_0$). The size of the limit cycle becomes
smaller as the initial compression $\zeta_0$ increases, but it becomes larger after $\zeta_0$ increases beyond a
certain value, which means that there is a value of $\zeta_0$ which makes the size of the limit cycle
minimum. In actual tests, $\zeta_0$ should be carefully selected and adjusted to induce squeak noises more
effectively.
Chapter 4 Squeak Test Apparatus Based on the Modified Sprag-Slip Mechanism

4.1 Introduction

A novel, unique squeak test apparatus was designed and built in this work, which can generate squeak noises consistently and with good repeatability. The sprag-slip mechanism, one of well-known mechanisms that lead to friction induced instability, was modified and employed to design and develop the test apparatus. The sprag-slip mechanism induces a large-amplitude, unstable motion when a certain geometrical condition is satisfied. A modified version of the sprag-slip mechanism (MSSM) with a bent cantilever beam was designed and studied in Chapter 3. Based on the results in Chapter 3, the test apparatus was designed and fabricated.

In order to correlate the analytical model with the test model, natural frequencies of the system were measured and compared to the results obtained from the analytic model and FEM model. Several pairs of materials were tested to demonstrate the capability of the test apparatus. The motion and noises generated were measured during material pair tests. Frequency spectra and time-frequency patterns were obtained by using FFT and analytic wavelet transform (AWT).
4.2 Design and Fabrication

The modified sprag-slip mechanism (MSSM) was applied to the squeak test apparatus as a thin, bent cantilever beam bracket to that one of the test materials is mounted as shown in Figure 4-1(b). The other material is mounted on the flat surface at the bottom that rotates at a constant angular velocity. The upper specimen contacts with the bottom surface by the angle of attack ($\theta_a$). The top end of the cantilever beam is bolted to a fixed frame. The dimensions of the bent cantilever beam were determined by the instability conditions presented in Chapter 3 to make the test apparatus unstable as shown in Figure 4-1(a).

![Diagram of dimensions of a bent cantilever beam](image1)

![Picture of squeak test apparatus](image2)

**Figure 4-1.** (a) Dimensions of a bent cantilever beam of the squeak test apparatus and (b) the picture of squeak test apparatus
4.3 Modal Analysis

The natural frequencies of the test apparatus were obtained from an analytical model, a FEM model and an experimental modal analysis. In the analytical model, a bent beam of the test apparatus was assumed to be a straight cantilever beam. The natural frequencies of the cantilever beam [80] can be represented as Eq. (4-1).

\[ \omega_n = \beta_n^2 \sqrt{\frac{EI}{\rho A}} \]  \hspace{1cm} (4-1)

where \( E, I, \rho \) and \( A \) are the modulus of elasticity, moment of inertia, density and cross sectional area of the beam, and \( \beta_n^2 = 22.03, 62.70, 120.9, 199.9, \ldots \) for \( n = 1, 2, 3, 4, \ldots \) are non-dimensional frequency parameters.
Figure 4-2. (a) FEM model of the squeak test apparatus and (b) test setup for experimental modal analysis
A FEM model was built as shown in Figure 4-2 (a). The top end of the beam is clamped and the other end is free. The first three bending modes of the bent beam obtained from the FEM are shown in Figure 4-3. An impact test was used for experimental modal analysis as shown in Figure 4-2 (b). An accelerometer is attached at the tip of the bent beam to measure the acceleration, and the opposite side where the accelerometer was attached was hit by an impact hammer. The FRF obtained from the impact test is shown in Figure 4-4. Three peaks that represent bending modes of the beam appear in the FRF.

**Figure 4-3.** Mode shapes of the first three modes obtained from FEM.
Figure 4-4. Frequency response function (FRF) of the bent cantilever beam of the test apparatus obtained from the impact test.

The natural frequencies obtained from the analytical model, FEM and experiment are compared in Table 4-1. The natural frequencies calculated from the analytical model are almost identical to those obtained from FEM. The natural frequencies measured from the experiments are approximately 15\% lower than those obtained from analytic model and FEM, as expected because the actual boundary condition is not exactly the clamped condition and the analytical and FEM models do not include the specimen attached to the tip. When the tip mass is added to the FEM model, the frequencies are obtained much closer to the measurement as shown in Table 4-1.
Because the effect of the friction force will further change the frequencies in actual motion of the beam, it is considered that the second and the third modes are mainly responsible for the generation of the squeak noise. If a stiffer beam is used, the first mode may become the main mode participating in generation of the squeak noise. Because the first mode is more sensitive to the clamping boundary condition, small changes in the clamping force can potentially change the squeak noise more significantly. On the other hand, if a too thin beam is used, the tip material will make a significant change the system characteristics, which will effectively make variations in specimen-to-specimen test conditions. Therefore, proper sizing of the beam relative to the size of the specimen needs to be studied further.

\[
\begin{array}{|c|c|c|}
\hline
\text{Natural Frequency [Hz]} & \text{Mode 1} & \text{Mode 2} & \text{Mode 3} \\
\hline
\text{Analytical Model} & 141.1 & 884.5 & 2476.6 \\
\text{FEM without a specimen} & 145.2 & 892.0 & 2471.0 \\
\text{FEM with a specimen} & 116.1 & 762.0 & 2157.9 \\
\text{Experiment} & 120.4 & 751.2 & 2063.0 \\
\hline
\end{array}
\]

Table 4-1. Natural frequencies of bending modes of the squeak test apparatus.
4.4 Material Pair Testing

Material pair testing was conducted to generate and measure the squeak noises with given material pairs. Seven materials (aluminum and six polymers) were used, therefore, total 28 pairs of materials were tested. An accelerometer was attached at the tip of the bent beam to measure acceleration as shown in Figure 4-5. The sound pressure of the generated sound was measured by a microphone located 10 cm far from the contact point between two specimens.

Figure 4-5. Experimental setup for material pair testing
The measured time histories of the acceleration, the sound pressure and their frequency spectra generated from the aluminum-polymer A pair and the aluminum-polymer B that generated the squeak noise are shown in Figure 4-6 and Figure 4-7. The acceleration and sound pressure level periodically grow to very large amplitude and recede, showing that the motion of the system becomes unstable periodically. From the frequency spectra of the acceleration and the generated sound, it can be seen that the three peaks of the acceleration and sound pressure spectra around 500 Hz, 1,700 Hz and 3,500 Hz are highly correlated with each other, which suggests that the three modes around these frequencies are participating in unstable motions. As the sound pressure level at 505 Hz is higher than that at 1,773 Hz by more than 40 dB, the single degree of freedom system model developed based on the mode corresponding to this frequency will provide quite good results qualitatively. Side robes around the first peak are considered to be due to nonlinear effects. The acceleration and sound pressure measured during the second run are very similar to those measured during the first run, which indicates the repeatability of the test apparatus. The time-frequency patterns of the generated squeak noises obtained by analytic wavelet transform (AWT) [78] are shown in Figure 4-8. It shows that broad-band, high intensity sounds were repeated with variable intervals, which is a commonly observed pattern of the squeak noises.
Figure 4-6. Measured time histories and frequency spectrums generated from the aluminum-polymer A pair that generated the squeak noise. (a) time history of the acceleration, (b) frequency spectrum of the acceleration of the 1st run, (c) time history of the sound pressure and (d) frequency spectrum of the sound pressure level of the 1st run.
Figure 4-7. Measured time histories and frequency spectrums generated from the aluminum-polymer B pair that generated the squeak noise. (a) time history of the acceleration, (b) frequency spectrum of the acceleration of the 1st run, (c) time history of the sound pressure and (d) frequency spectrum of the sound pressure level of the 1st run.
Figure 4-8. Time-frequency pattern of the generated squeak noise obtained from analytic wavelet transform. (a) aluminum-polymer A pair and (b) aluminum-polymer B pair.
The measured time histories of the acceleration, the sound pressure and their frequency spectra generated from the polymer A-polymer A pair and the polymer A-polymer B that did not generate the squeak noise are shown in Figure 4-9 and Figure 4-10. The acceleration and sound pressure level periodically grow and recede, however, the amplitude is not large enough to generate the squeak noise. Hence, the motion of the system did not become unstable. The time-frequency patterns of the sounds generated from the polymer A-polymer A pair and the polymer A-polymer B that were obtained by AWT are shown in Figure 4-11. It illustrates that SPL is higher in low frequency range and very low in all other frequency ranges, which means that the squeak noises did not occur.
Figure 4-9. Measured time histories and frequency spectrums generated from the polymer A-polymerr A pair that did not generate the squeak noise. (a) time history of the acceleration, (b) frequency spectrum of the acceleration, (c) time history of the sound pressure and (d) frequency spectrum of the sound pressure level.
Figure 4-10. Measured time histories and frequency spectrums generated from the polymer A-polymer B pair that did not generate the squeak noise. (a) time history of the acceleration, (b) frequency spectrum of the acceleration, (c) time history of the sound pressure and (d) frequency spectrum of the sound pressure level.
Figure 4-11. Time-frequency pattern of the generated sound obtained from analytic wavelet transform. (a) polymer A-polymer A pair and (b) polymer A-polymer B pair.
4.5 Conclusions

A novel, unique squeak test apparatus was developed by employing the MSSM. The test apparatus consists of a bent cantilever beam with an upper specimen and a rotating flat surface with the other specimen at the bottom. The shape and dimensions of the bent cantilever beam were determined by dynamic analysis discussed in Chapter 3 in order to induce unstable, self-excited vibration between two specimens in sliding contact and generate squeak noises with good repeatability.

Several material pairs were tested to demonstrate the capability of the test apparatus. It was seen that the measured acceleration and the generated squeak noises were well correlated in both time and frequency domains. The peaks of the frequency spectrum of the measured acceleration and natural frequencies from the modal analysis were correlated, but the former was higher than the latter because of the difference in the boundary conditions of two tests. In the modal analysis, the natural frequencies were measured with clamped-free condition, however, in the material pair testing, the other end of the test apparatus was in a sliding contact with the other material. The test and FEM analysis results indicate that the second and third modes of the clamped beam of the current design of the test apparatus are main modes that are responsible for generating squeak noises. This is desirable because the first mode is more susceptible to the boundary condition, therefore more difficult to make the test condition repeatable if the noise is generated by the first mode. The measured acceleration was well correlated with the generated squeak noise both in time and frequency domain as expected. The time-frequency patterns of the squeak noises generated by the test apparatus showed that broad-band, high intensity sound appeared with variable intervals, which is commonly observed pattern of the squeak noise.
A difficulty in squeak test is consistency of the test. For example, if a pair of test materials squeaks on-and-off manner, generating completely different noises at each time of testing, therefore, squeak propensity of the pair will not be able to be defined. The test apparatus developed in this chapter can generate squeak noises very consistently, making nearly the same squeak noises for the given pairs in all tests. In addition, the squeak noise was never generated with the material pair that did not generate squeak noises. Therefore, the test apparatus can be used to rate squeak propensity in conjunction with an automatic detection and rating algorithm of squeak and rattle noises that will be discussed in Chapter 5.
Chapter 5 Automatic Detection and Rating of Squeak and Rattle Noises

5.1 Introduction

Noises reaching the passenger cabin are substantially reduced by recent advances in noise, vibration and harshness (NVH) engineering. Because of this, the noise generated inside the passenger cabin, such as squeak and rattle (S&R), stands out and forms detrimental perception of the quality of vehicles. Market surveys conducted as early as in 1983 reported that S&R is the third most important customer concern in passenger vehicles after 3 months of ownership. [2]

This chapter focuses on automatic detection and rating of S&R noises which historically have been done by using subjective methods that are often inconsistent and time-consuming. Squeaks refer an annoying noises generated by friction-induced, self-excited vibration of the surfaces in contact. Rattles refer impact-induced noises generally caused by loose or overlay flexible elements under forced excitation. Various factors dictate S&R noise, such as material property, friction coefficient, sliding velocity, temperature and humidity. [81]

S&R noises cannot be detected only by basic physical quantities, such as sound pressure level (SPL), since they are highly nonlinear and transient, and their detection involves subjective human perception. Therefore, the characteristics of human hearing, both the temporal and spectral, should be considered. Chandrika and Kim [82] developed a computer program for automatic
detection of S&R events by considering spectral and temporal masking effects and utilizing an analytic wavelet transform (AWT) [78].

In this chapter, the applications of the developed algorithm to various noises generated by test setups to demonstrate one of effective applications of the method. The method was applied to S&R noises produced by experimental setups and rated to obtained quantitative design data of various materials. As a part of effort to further improve the S&R detection and quantification method, the relationship between the objective rating estimated by the algorithm and instantaneous and time-averaged SPL was examined. In addition, the squeak propensities of the noises, which were generated by the squeak test apparatus discussed in Chapter 4, were obtained as an application of the algorithm.
5.2 Algorithm to Automatically Detect and Rate Squeak and Rattle Noises

S&R events generally have widely different spectral characteristics as shown Figure 5-1. In a time-averaged sense, S&R noises may be embedded in the background noise which has much higher overall SPL. Therefore, the detection algorithm has to rely on a transient signal analysis technique and has to consider both temporal and spectral characteristics of the human auditory system.

![Figure 5-1. 1/3rd octave band spectra of selected squeak and rattle events.](image)
Figure 5-2 shows the algorithm of the S&R detection and rating schematically. Perceived transient loudness (PTL) is a single time series data processed from the recorded sound and devised to represent the human perception of transient noises embedded in the ambient noise. An occurrence of the S&R noise is detected if the PTL value of the S&R noise is higher than detection threshold, which is the PTL values determined by jury tests. The signal processing procedure of the algorithm for detecting and rating S&R noises consists of five steps as shown in Figure 5-2.
Figure 5-2. Procedure for obtaining perceived transient loudness time history ($N_{PTL}$).
The first step is time-frequency analysis using analytic wavelet transform (AWT). AWT is very effective to characterize highly transient noises [77, 78]. By applying AWT, 24 1/3rd-octave SPL time histories are obtained at the center frequencies in the range from 1 to 24 barks. In the second step, the 1/3rd-octave time histories obtained in the first step are converted to 24 specific loudness time histories by applying the Zwicker’s loudness model [83, 84]. This model accounts for the transmission characteristics of the outer and middle ear and the spectral processing of human auditory system.

The third step is computation of transient specific loudness time histories. Humans feel a certain sound in presence of the background noise when the auditory stimulation caused by the sound exceeds the masking effect of the background noise. Transient specific loudness time history is defined to represent this auditory stimulation exceeding the masking effect. The transient specific loudness time histories can be obtained by subtracting the contribution of the background noise from the specific loudness time histories obtained in the second step. It can be achieved by the leaky integration technique [85]. This technique filters the signal with an infinite impulse response (IIR) filter of low pass characteristics, and averages out sharp loudness time history of the background noise. The specific loudness time histories obtained in the third step are summed over the selected frequency range, 4 kHz or higher, which leads to the transient loudness time history (a single time history). The frequency range is selected based on the spectral energy distribution in typical S&R noises and background noise. The S&R noise has relatively high signal to noise ratio in high frequency bands [82]. Thus, the transient specific loudness time histories of 4 kHz or higher are summed to utilize the high signal to noise ratio.

The last step for obtained PTL is a temporal integration conducted to reflect the temporal masking effect of the human auditory system. The temporal integration from Glasberg and Moore
[86] is used to convert the transient loudness time histories into the PTL time history as shown in Eq. (5-1).

\[
\begin{align*}
\text{if } & N_{\text{inst}}(i) \geq N_{\text{inst}}(i-1) \\
\text{then } & N_{\text{PTL}}(i) = (1 - \alpha_a)N_{\text{PTL}}(i-1) + \alpha_a N_{\text{inst}}(i) \\
\text{else } & N_{\text{PTL}}(i) = (1 - \alpha_r)N_{\text{PTL}}(i-1) + \alpha_r N_{\text{inst}}(i)
\end{align*}
\] (5-1)

where, \(N_{\text{inst}}\) is the transient loudness time history, \(N_{\text{PTL}}\) is the PTL time history, \(\alpha_a\) and \(\alpha_r\) are attack and release time constants of integration respectively. Two integration constants indicate the vibration of loudness perception due to forward and backward masking effect of the ambient noise. From Glasberg and Moore [86], they used 0.045 and 0.02 for \(\alpha_a\) and \(\alpha_r\) respectively, to achieve accurate prediction of vibrations in loudness perception of short duration sounds. \(N_{\text{PTL}}\) obtained in this step represents the human perception of the S&R noises exceeding the masking effect of the background noise. Thus, the S&R noise is detected when the value of \(N_{\text{PTL}}\) exceeds the detection threshold determined from jury tests.

In order to determine the detection threshold, a percentile value of \(N_{\text{PTL}}\) distribution was used as the property of the noise. The 75th percentile value (\(n_{75}\)) of the \(N_{\text{PTL}}\) showed the best correlation with the identified detection threshold [82]. This correlation is represented as Eq. (5-2) by applying a linear regression.

\[
N_t = 1.77 \times n_{75}(N_{\text{PTL}}) + 0.0022
\] (5-2)

The S&R noise is detected when \(N_{\text{PTL}}\) of the noise is higher than the detection threshold \((N_t)\). In other words, the S&R noise occurs when the difference between \(N_{\text{PTL}}\) and \(N_t\) is positive. Thus, the detection metric \((N_{\text{det}})\) is defined as;

\[
N_{\text{det}} = N_{\text{PTL}} - N_t
\] (5-3)

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After the detection, a modified PTL ($N'_{PTL}$) is used for a quantitative rating of the detected event, which is defined as a sum of all frequency components above 200 Hz [82], and works better in estimating the subjective loudness of the S&R noise. From the linear regression of the sound set of jury tests, the objective rating of S&R noises ($N_{obj}$) is represented as;

$$N_{obj} = 2.9N'_{PTL} + 1.33$$ (5-4)

The detailed information about jury tests for identifying $N_i$ and $N_{obj}$ can be found in Chandrika and Kim [82].

Figure 5-3 shows an application of the algorithm for detection and rating of S&R noises from a typical cabin noises pre-recorded. Three different squeak and rattle noises were embedded in the cabin noise. When the detection metric ($N_{det}$) is above the zero line, the S&R event is detected. The objective rating ($N_{obj}$) estimated by the algorithm is shown in Figure 5-3(b).
Figure 5-3. (a) Detection metric curve of the cabin noise with multiple S&R noises and (b) objective rating of the S&R noises.
5.3 Experiments to Produce Squeak and Rattle Noises

Various S&R noises were produced and recorded from experiments. Figure 5-4 shows the experimental setup for producing S&R noises that are designed to test any combination of materials. Since squeak is friction-induced noise, a pair of materials was rubbed against with a set level of the normal force. While one material was fixed, the other material slid back and forth by an exciter (Figure 5-4(a)). To produce the rattle noises, a pair of the materials was impacted. One material was moved in vertical direction by an exciter and impacted with the other material fixed (Figure 5-4(b)). The sound pressure of the S&R noises were recorded by a microphone located at 20 cm far from the contact spot of the material pair. The S&R detection and rating algorithm was applied to the set of noises generated by the experimental setup.
Figure 5-4. Experimental setup for producing (a) squeak noises and (b) rattle noises.
5.4 Comparison of Sound Pressure Level to Objective Rating of Squeak and Rattle Noises

Quantitative rating of the S&R noises relied on the empirical equation developed by a limited set of jury testing; therefore, has a potential room to improve. As one quick assessment of the algorithm, the relationship between the SPL, a more common metric, and objective rating of the S&R events were calculated. The total SPL and the maximum SPL were used for this purpose.

A-weighting was applied to SPL to reflect human perception of sounds. The total SPL was obtained in frequency domain by applying FFT. One second time block, which is sufficiently long to capture the S&R noises, was used for each average, and approximately 50 % overlap processing was used. The procedure for obtained the total SPL is below:

1) FFT was applied to each time block to obtained a linear spectrum in frequency domain
2) Magnitude were converted to SPL (dB, un-weighted).
3) Un-weighted SPL was converted into A-weighted SPL (dBA).
4) Step 1 through 3 were repeated for all time blocks.
5) SPL of all time blocks were summed and averaged in frequency domain.
6) Total SPL was calculated by Eq. (5-5)

\[
\text{SPL}_{\text{tot}} = 10 \log_{10} \left( \sum_{i} 10^{\text{SPL}_{i}/10} \right)
\]  

The maximum SPL was obtained in time-frequency domain by applying AWT. From AWT, the un-weighted SPL of the S&R noises was calculated in time-frequency domain. Then, A-weighting was applied to each frequency spectrum at each time step to convert dB into dBA. After
applying A-weighting, the maximum SPL was identified. The objective rating of the S&R noise was obtained by the S&R detection and rating algorithm.

The detection/rating algorithm was applied to 21 test noises recorded from the experimental setup (3 squeak noises and 18 rattle noises). One example of the detection metric curve and objective rating of the squeak noises produced by the experiments are shown in Figure 5-5. Time-frequency pattern obtained by AWT, which was used to identify the maximum SPL of S&R noises, is shown in Figure 5-6.
Figure 5-5. (a) Detection metric curve of the squeak noise generated from the experiment and (b) its objective rating.

Figure 5-6. Time-frequency pattern of the SPL obtained by AWT. (a) Rattle between polymers and (b) squeak between materials of steel.
The relationship between the total SPL and objective rating (Figure 5-7(a)) and between the maximum SPL and objective rating (Figure 5-7(b)) are shown in Figure 5-7 with a line obtained by a linear regression. The correlation value between total SPL and the maximum objective rating was 0.711, and that between the maximum SPL and the maximum objective rating was 0.922. This indicated that the instantaneous level of the noise is more responsible to the S&R perception.
Figure 5-7. Comparison of maximum objective rating to SPL (dBA) and its linear regression. (a) The relationship between the total SPL and the maximum objective rating with correlation value of 0.711 and (b) The relationship between the maximum SPL and the maximum objective rating with correlation value of 0.922.
5.5 Conclusions

The automatic detection/rating algorithm of S&R noises, which is developed in the previous study [82], was refined and applied to various squeak and rattle noises obtained from the experiments. This algorithm utilizes various signal processing techniques suitable for highly transient noises, such as AWT, temporal integration and Zwicker’s loudness model, to obtain the PTL time history which approximates the human perception of the S&R noises.

The relationship between the objective rating of the S&R noises estimated by the algorithm and the SPL was investigated. Two types of the SPLs were calculated; the total SPL calculated from frequency domain and the instantaneous SPL calculated by applying AWT in time-frequency domain. The results illustrate that the maximum SPL has much better correlation than the total SPL with the objective rating of the S&R noises, which indicates that the human perception of the S&R noise is influenced mainly by instantaneous changes of the sound.

One of the best use of the developed algorithm is to build database of materials for designers. By using the squeak test apparatus that generates squeak noises consistently and with good repeatability, such as the test apparatus developed in Chapter 4, the squeak noises can be generated and recorded with given material pairs. The developed algorithm can be applied to test various pairs of materials for their squeak propensities quantitatively in a repeatable manner. Such a database will be very helpful for engineers to select materials in places where squeak occurrence is highly possible.
Chapter 6 Analytical Study of the Mode-Coupling Effect on the Friction-Induced Instability

6.1 Introduction

Squeak is a loud noise generated by a self-excited vibration due to friction. Because of the highly nonlinear nature of the friction-induced vibration, squeak noise presents a very difficult challenge for NVH engineers. Stick-slip, sprag-slip and mode-coupling phenomena are three well-known mechanisms that cause friction-induced vibration instability. The stick-slip phenomenon between two surfaces in contact causes an unstable, growing motion when the kinetic coefficient of friction decreases as the sliding velocity increases, which makes the effective damping of the system negative [13, 19]. The sprag-slip mechanism, also known as geometrically induced instability or kinematic constraint instability, makes the system unstable at certain geometrical conditions even if the friction coefficient is constant [2, 23]. The mode-coupling mechanism, that is generally recognized as the most common mechanism that causes squeak noises, refers vibrations of a system that has multiple modes of nearly identical resonance frequencies [87, 88].

In this chapter, a relatively simple analytical model which accounts for the mode-coupling mechanism was developed and investigated. The equation of motion of the analytical model was solved numerically to study effects of main design parameters, such as the frequency ratio and damping, on the stability of the system. Implication of the results in the design of a squeak test apparatus and an application to brake squeal are discussed.
6.2 Analytical Model

The analytical model studied is shown in Figure 6-1. With two masses, two degrees-of-freedom, the model presents the simplest system that can be used to study the mode-coupling effect. Mode-coupling occurs indirectly because the motion in \( x \)-direction (in-plane) is independent to the motion in \( y \)-direction (out-of-plane). The in-plane motion, however, is influenced by the out-of-plane motion because the normal force that affects the friction force varies due to the out-of-plane motion. The friction coefficient is assumed to be constant. Initial compression (\( \delta \)) is applied in the vertical spring to ensure the friction surfaces in contact. It is assumed that the system in motion in the horizontal direction, that is composed of \( m_1, k_1 \) and \( c_1 \), moves in \( x \)-direction with a constant velocity \( V \) of large magnitude. This makes that the friction force is always in the negative \( x \)-direction, thus the direction of the friction force is unchanged during the motion. Therefore, the mass oscillates in the moving reference frame that moves in \( x \)-direction with constant velocity \( V \).
Figure 6-1. (a) Schematic of the analytical model to study a mode-coupling instability in friction-induced vibration and (b) the free body diagram.
From the free body diagram shown in Figure 6-1(b), the equations of motion of the model
in x- and y-direction are obtained as;

\[
m_1 \ddot{x} + c_1 \dot{x} - \mu c_2 \dot{y} + k_1 x + \mu k_2 y = c_1 V + k_1 \dot{V} - \mu k_2 \delta
\]  

(6-1)

\[
m_2 \ddot{y} + c_2 \dot{y} + k_2 y = k_2 \delta
\]  

(6-2)

where \( k_1 \) and \( k_2 \) are the spring constant, \( c_1 \) and \( c_2 \) are the damping constant, \( \mu \) is the kinetic coefficient of friction, and \( V \) is the velocity of the reference frame. Eq. (6-1) and Eq. (6-2) can be represents as a matrix form as follows;

\[
\begin{bmatrix}
m_1 & 0 \\
0 & m_2
\end{bmatrix}
\begin{bmatrix}
\ddot{x} \\
\ddot{y}
\end{bmatrix} +
\begin{bmatrix}
c_1 & -\mu c_2 \\
c_2 & 0
\end{bmatrix}
\begin{bmatrix}
\dot{x} \\
\dot{y}
\end{bmatrix} +
\begin{bmatrix}
k_1 & -\mu k_2 \\
0 & k_2
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix} =
\begin{bmatrix}
c_1 V + k_1 \dot{V} - \mu k_2 \delta \\
k_2 \delta
\end{bmatrix}
\]

(6-3)

\[
\rightarrow \mathbf{M} \ddot{\mathbf{x}} + \mathbf{C} \dot{\mathbf{x}} + \mathbf{K} \mathbf{x} = \mathbf{F}(t)
\]

Eq. (6-3) can be transformed into a form of the state space equations as follows;

\[
\dot{\mathbf{y}}(t) = \mathbf{A} \mathbf{y}(t) + \mathbf{f}(t), \quad \mathbf{y}(t) = \begin{bmatrix}
\mathbf{x}(t) \\
\dot{\mathbf{x}}(t)
\end{bmatrix}
\]

where \( \mathbf{A} = \begin{bmatrix}
0 & \mathbf{I} \\
-M^{-1} \mathbf{K} & -M^{-1} \mathbf{C}
\end{bmatrix} \) and \( \mathbf{f}(t) = \begin{bmatrix}
0 \\
M^{-1} \mathbf{F}(t)
\end{bmatrix} \)

(6-4)

where \( \mathbf{I} \) is an identity matrix. Eq. (6-4) is numerically integrated by using the 4th order Runge-Kutta method to study the behavior of the system.
6.3 Parameter Study

Four parameters were chosen to examine their effects on the stability of the system. The natural frequency ratio \((\omega_r)\) is defined as;

\[
\omega_r = \frac{\omega_{rx}^2}{\omega_{ry}^2} = \frac{k_1}{m_1} / \frac{k_2}{m_2}
\]

\(\omega_r\) is adjusted by changing \(k_2\) while keeping \(k_1\) constant as 10 N/m. In order to study the role of damping, two damping parameters are selected; the damping ratio of the in-plane motion \((\zeta_x)\) shown in Eq. (6-6) and the damping ratio of the out-of-plane motion \((\zeta_y)\) shown in Eq. (6-7).

\[
\zeta_x = \frac{c_1}{2\sqrt{m_1k_1}} \quad (6-6)
\]

\[
\zeta_y = \frac{c_2}{2\sqrt{m_2k_2}} \quad (6-7)
\]

In addition, the kinetic coefficient of friction \((\mu)\) is chosen as a major parameter. All other parameters are kept constant; \(m_1 = m_2 = 0.1\) kg, \(\delta = 0.1\) m, \(V = 10\) m/s, \(x_0 = y_0 = 0\), \(\dot{x}_0 = \dot{y}_0 = 1\) m/s, \(\mu = 0\).

The transient responses of the system without damping \((C = 0)\), that has no loss mechanism other than friction, with different \(\omega_r\) are shown in Figure 6-2. If the natural frequencies of two modes are significantly different, the responses stay bounded with a nearly constant amplitude (Figure 6-2(a)). If the natural frequencies of two modes are only slightly different, the in-plane response (x-direction) shows beating phenomenon while the out-of-plane motion (y-direction) oscillates with constant amplitude (Figure 6-2(b)). If the natural frequency of the in-plane motion
is exactly matched to the natural frequency of the out-of-plane motion, the in-plane response indefinitely grows, which indicates occurrence of the instability (Figure 6-3(c)).

Figure 6-2. Transient responses of the undamped system with different ratios of natural frequencies ($\omega_r$). (a) $\omega_r = 0.2$, (b) $\omega_r = 0.9$ and (c) $\omega_r = 1.0$. 
Figure 6-3 shows the response of the in-plane motion of the undamped system with three different coefficients of friction. The amplitude of the in-plane motion becomes bigger as the coefficient of friction increases for all natural frequency ratio. Therefore, the instability occurs more easily if the coefficient of friction higher.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fric.png}
\caption{Transient responses of the undamped system with different kinetic coefficient of friction ($\mu$). (a) $\omega_r = 0.2$, (b) $\omega_r = 0.9$ and (c) $\omega_r = 1.0$.}
\end{figure}
When damping exists in the system in addition to the friction, the in-plane motion does not increase unbounded even if two natural frequencies are the same ($\omega_r = 1.0$). Figure 6-4(a) shows the response of the in-plane motion when damping exists only in the motion in $x$-direction with three different magnitude of damping. The in-plane motion for all cases starts to grow, then converges to a limit cycle of different amplitude. As expected, the amplitude of the limit cycle increases as the damping decreases, which can become significantly large, and will possibly generate squeak noises. Figure 6-4(b) shows the response of in-plane motion of the system when damping exists also in the $y$-direction motion. If the damping in both $x$- and $y$-direction is applied, the amplitude of the response of the in-plane motion grows into a certain amplitude then its envelope decays, settling into a bounded amplitude.
Figure 6-4. Transient responses of in-plane motion of the damped system with different damping ratio in x-direction ($\zeta_x$) and $\omega_r = 1.0$. (a) $\zeta_y = 0$ and (b) $\zeta_y = 0.02$. 
Figure 6-5 shows the motions of the system in both x- and y-direction with the damping in only y-direction. The out-of-plane motion (Figure 6-5(b)) decays exponentially if there is damping in y-direction \((\zeta_y \neq 0)\). Since the change in the friction force due to the out-of-plane motion is the mechanism that feed energy into the motion of the in-plane vibration, the response of the in-plane motion eventually stops growing, then oscillates with a constant amplitude because there is no damping in x-direction.
Figure 6-5. Transient responses of in-plane motion of the damped system with different damping ratio in $y$-direction ($\zeta_y$) and $\omega_r = 1.0$. (a) in-plane motion and (b) out-of-plane motion.
6.4 Conclusions

A relatively simple analytical model was developed to represent mode-coupling type instability. The basic parameter study was conducted to understand the effects of stiffness, damping and kinetic coefficient of friction on the stability of the system. The results illustrate that the ratio of natural frequencies between in-plane and out-of-plane motion is the most important parameter that makes the system unstable. The system becomes unstable more easily if the friction coefficient is higher. This is due to the fact that the friction force is the driving mechanism that injects a net positive energy per cycle into the system. Any damping in $x$- and $y$-direction makes the system response more stable or bounded as expected.

If the mode-coupling mechanism is employed to design a squeak test apparatus that generates unstable, large amplitude friction-induced vibration motion for a pair of materials for most cases consistently, the test apparatus should be designed by carefully adjusting stiffness and inertia of the apparatus to match the natural frequencies of two modes as closely as possible. Also, the system damping should be minimized. Such a test apparatus will generate large friction-induced vibrations consistently, therefore, enable to test the squeak propensity of given material pairs.

The analytical model developed can explain underlying physics of squeak as well as brake squeal. Brake squeal refers a sustained vibration of brake system components during a braking action resulting in an undesirable, annoying noise which is audible to passengers or passers-by. In general, brakes that squeal do not squeal in every braking action, which means that the occurrence of brake squeal is intermittent or perhaps even random [10]. Brake squeal is also generated by the friction-induced instability of the brake system. From the analytical model shown in Figure 6-1(a), the disc of the brake system can be modeled as the mass-spring-damper at the bottom which moves
at high constant velocity in $x$-direction. The caliper of the brake system can be modeled as the mass-spring-damper which moves in $y$-direction. The disc of the brake system has a huge number of modes, and their natural frequencies are subject to change by various factors, such as humidity, temperature and wear [89, 90]. If one of the natural frequencies of the disc is changed and close to that of the caliper, the large amplitude vibration occurs, therefore, brake squeal occurs.
Chapter 7 Algorithm to Automatically Detect and Distinguish
Squeak and Rattle noises

7.1 Introduction

As the overall noise level of passenger cars has been significantly reduced by recent advances in noise, vibration and harshness (NVH) engineering, the squeak and rattle (S&R) noises generated inside the passenger cabin stand out and contribute to a detrimental perception of the quality of vehicles. Market surveys conducted as early as 1983 reported the S&R as the third most important customer concern in passenger vehicles after three months of ownership [2].

While they are often lumped together in reference, the squeak noise and the rattle noise are each generated by different mechanisms. Squeaks are a noise generated by friction-induced vibrations between interfacing surfaces. The elastic deformation of the contact surfaces stores energy which is released and produces audible squeak noises upon the relative motion between the surfaces. Rattles are impact-induced noises generally caused by loose or overly flexible elements under forced excitation. A number of factors, such as material property, friction coefficient, relative velocity, temperature and humidity, are involved in S&R noises [5].

Historically, S&Rs have been detected and rated by using subjective methods; therefore, the detection and rating of S&R noises often become an inconsistent and time-consuming process. There have been numerous efforts to develop a method to automate the detection and rating of S&Rs [82, 51, 91]; however, no work has been reported the algorithm which distinguishes S&R
Noises after detection. Most automatic detection algorithms of S&R noises utilize the fact that both noises are highly transient and momentarily become discernably louder than the background noise. Therefore, it is difficult to automatically distinguish squeak and rattle noises. This chapter reports a special algorithm that has developed to reliably distinguish the squeak and rattle noises.

A computer algorithm to automatically detect and rate S&R noises [82, 92] was previously developed. The method developed in this chapter can be used in conjunction with the detection/rating algorithm in Chapter 5. At first, the detection algorithm identifies the existence of the S&R noise, then the algorithm developed in this chapter can be used to identify whether the noise is a squeak or rattle. This will help automotive engineers to choose an appropriate approach to solve the problem.
7.2 Sound Quality Metrics

Three sound quality metrics were employed to build the algorithm, specifically sharpness, roughness and fluctuation strength, which are briefly explained and defined below.

Sharpness describes the tone color of a sound in terms of its pleasantness or aggressiveness. It depends on the weighted centroid of the specific loudness ($N'$) content. Thus, sharpness is proportional to spectral center of gravity. Sharpness is defined as Eq. (7-1). [83]

$$ S = 0.11 \int_{0}^{24\text{Bark}} \frac{N'g(z)dz}{\int_{0}^{24\text{Bark}} N'dz} $$  \hspace{1cm} (7-1)

where $S$ denotes the sharpness in acum, and $g(z)$ denotes the weighting function with respect to the critical band rate ($z$). The integral in numerator means the first moment of specific loudness. One acum is referenced to a band of noise centered at 1 kHz with 60 dB. A higher value of sharpness means higher energy in high frequency bands.

Roughness is a sensation caused by rapid temporal variation of sounds, or by amplitude- or frequency-modulated tones. Roughness is represented as [83];

$$ R = 0.3 \frac{f_{\text{mod}}}{\text{kHz}} \int_{0}^{24\text{Bark}} \frac{\Delta L_E(z)dz}{\text{dB/Bark}} $$ \hspace{1cm} (7-2)

where $R$ is roughness in asper, $f_{\text{mod}}$ is the modulation frequency, and $\Delta L_E$ is the range of excitation level within an auditory filter. 1 asper defined as a 1 kHz tone at 60 dB with 100 % amplitude modulation at 70 Hz. Roughness increases as modulation depth of the temporal masking pattern of sounds increases. In addition, roughness is related to fast modulations.
On the contrary to roughness, fluctuation strength represents human sensitivity of relatively slow modulations. The unit of fluctuation strength is vacil, and 1 vacil is referenced to a 1 kHz, 60 dB tone with 4 Hz 100% amplitude modulation. Fluctuation strength can be expressed as [83];

\[
F = \frac{0.008 \int_{0}^{24 \text{Bark}} (\Delta L / \text{dB Bark}) dz}{(f_{\text{mod}}/4 \text{Hz}) + (4 \text{Hz}/f_{\text{mod}})}
\]  

(7-3)

where \( \Delta L \) is the masking depth which is the difference between the maxima and the minima in the temporal masking pattern.
7.3 Experiments to Generate Squeak and Rattle Noises

Various squeak and rattle noises used in this work were generated and recorded from experiments. Figure 5-4 shows the experimental setups that were used to produce S&R noises. These setups were used with numerous combinations of two different materials along with 86 test noise signals. In order to produce squeak noise, a pair of materials was rubbed against each other. Figure 5-4(a) is the setup used to generate squeak noises. A thin cantilever beam with one material applied on its surface was moved back and forth by a shaker while rubbing the other material on the fixed thin beam. Figure 5-4(b) shows the setup used to generate rattle noises. A rectangular shaped piece of material mounted on the shaker was in a reciprocal motion in the vertical direction in order to hit the other material applied on the fixed beam. The sound pressure due to S&R noises was recorded by a microphone located 20 cm away from the contact point. The sound quality metrics of the S&R noises were calculated using the recorded sound pressure time histories.
7.4 Defining Squeak and Rattle Regions

As a preliminary analysis, the maximum values of sound quality metrics of S&R noises obtained from the experiments were calculated and compared. Figure 7-1 shows the distribution of the sound quality metrics of S&R noises using box plots. The means (± standard deviation) of the sharpness of the squeak noises and the rattle noises are 1.77 (± 1.15) acum and 1.56 (± 0.34) acum, respectively. The means of the roughness of the squeak noises and the rattle noises are 0.46 (± 0.34) asper and, 13.55 (± 12.51) asper, respectively. The means of the fluctuation strength are 0.53 (± 0.55) vacil for the squeak noises and 2.69 (± 3.93) vacil for the rattle noises. As shown in Figure 7-1, the range of the distribution of the sound quality metrics is very broad even though the mean values of the sound quality metrics are different between squeak and rattle noises respectively. Therefore, it is clear that squeak noises and rattle noises cannot be distinguished by using only one sound quality metric.
Figure 7-1. Box plots of maximum sound quality metrics of squeak and rattle noises produced by experiments. (a) sharpness, (b) roughness, and (c) fluctuation strength.
The respective squeak and rattle regions in a space defined in terms of the three sound quality metrics were constructed. A total of 86 noise recordings were used, where each signal had been previously identified as a squeak or a rattle based on the respective test setup. Figure 7-2 shows the three-dimensional space of the sound quality metrics which includes the sharpness-roughness plane, sharpness-fluctuation strength plane, and fluctuation strength-roughness. Each data point represents the set of maximum values of sound quality metrics of the S&R noises generated from the experiments. For example, each point in Figure 7-2(a) represents the maximum sharpness value and the maximum roughness value of the corresponding noise. Based on the plotted sound quality metrics, the squeak region and rattle region were each grouped in an elliptical area which best characterized the data.
Figure 7-2. Squeak region and rattle region in planes of the sound quality metrics. (a) sharpness-roughness plane, (b) sharpness-fluctuation strength plane, and (c) fluctuation strength-roughness plane.
7.5 Algorithm to Distinguish between Squeak and Rattle Noises

Figure 7-3 shows the three-step procedure of the algorithm developed to distinguish squeak noises and rattle noises. As stated in the previous section, the squeak region and rattle region in three planes of the sound quality metrics were pre-defined using the data obtained from the experiments. Without previous knowledge of the characteristics of a given noise signal, the maximum values of sound quality metrics are calculated. The type of noise is subsequently identified based on its squeak index and rattle index.

![Figure 7-3. Procedure to distinguish squeak and rattle noises.](image-url)
**Step 1: Calculation of the Maximum Values of the Sound Quality Metrics**

In step 1, the maximum values of the three sound quality metrics of an unknown noise are calculated. The recorded time series of the unknown noise is converted to three time series of sound quality metrics (sharpness, roughness, and fluctuation strength) by Eq. (1) through Eq. (3). Calculated over the recorded time interval, the maximum values of the metrics are used.

**Step 2: Plotting the calculated maximum values in the planes of the sound quality metrics**

The maximum values of the sound quality metrics calculated in Step 1 are plotted in the planes of the sound quality metrics (sharpness-roughness plane, sharpness-fluctuation strength plane, and fluctuation strength-roughness plane), which are defined in the previous section as shown in Figure 7-2.

**Step 3: Calculation of squeak index and rattle index**

Squeak index (S) and rattle index (R) are defined to distinguish squeak noises and rattle noises. Prior to the calculation of S and R, squeak sub-indices (S₁, S₂, and S₃) and rattle sub-indices (R₁, R₂, and R₃) are calculated. Figure 7-4 illustrates how S₁ and R₁ are calculated in the sharpness-roughness plane. P represents the point corresponding to the maximum sharpness and the roughness values of the noise whose type is unknown, Cₛ and Cᵣ represent the center points of the squeak region (or ellipse) and the center of the rattle region (or ellipse), respectively. S₁ and R₁ are defined as
where \( l_S \) and \( d_S \) are the distance between \( C_S \) and \( P \), and the distance between \( C_S \) and a point of intersection between the boundary of the squeak region and \( l_S \). \( l_R \) and \( d_R \) are the distance between \( C_R \) and \( P \), and the distance between \( C_R \) and a point of intersection between the boundary of the squeak region and \( l_R \), respectively. \( S_1 \) is higher than one if the unknown noise \( P \) is located inside the squeak region, or lower than one if \( P \) is located outside the region. Therefore, higher \( S_1 \) value means that \( P \) is located closer to the center of the squeak region. The same discussion can be made for \( R_1 \). That is, \( R_1 \) becomes higher if \( P \) is located closer to the center of the rattle region. Applying the same algorithm to all three planes (sharpness-roughness, sharpness-fluctuation strength, fluctuation strength-roughness), \( S_2 \) and \( R_2 \) are calculated in the sharpness-fluctuation strength plane, and \( S_3 \) and \( R_3 \) are calculated in the fluctuation strength-roughness plane. Once all sub-indices are calculated, the overall squeak index and the overall rattle index of the noise, \( S \) and \( R \), can be represented as Eq. (7-5).

\[
S = S_1 \cdot S_2 \cdot S_3 \quad \text{and} \quad R = R_1 \cdot R_2 \cdot R_3 \quad (7-5)
\]

The algorithm identifies whether the given noise is squeak or rattle by the \( S \) and \( R \) numbers. That is, if \( S \) is bigger than \( R \), the unknown noise is classified as squeak noise and vice versa.
Figure 7-4. Calculation of squeak sub-index ($S_1$) and rattle sub-index ($R_1$) in the sharpness-roughness plane. $P$ represents the maximum sharpness and roughness of the unknown sound. $C_S$, $l_S$, and $d_S$ are the center of squeak region, the distance between $C_S$ and $P$, and the distance between $C_S$ and a point of intersection between the boundary of squeak region and $l_S$, respectively. $C_R$, $l_R$, and $d_R$ are the center of squeak region, the distance between $C_R$ and $P$, and the distance between $C_R$ and a point of intersection between the boundary of rattle region and $l_R$.

Squeak sub-index: $S_1 = \frac{d_S}{l_S}$

Rattle sub-index: $R_1 = \frac{d_S}{l_R}$
7.6 Test of the Algorithm Using the Recorded Squeak and Rattle Noises

A total of 41 squeak noises and 45 rattle noises obtained from the experiments were used to test the performance of the algorithm developed in this work. The algorithm was applied to these 86 acoustic signals whose types are known. The results were summarized in Table 7-1. All squeak noises were identified as the squeak noise by the algorithm, and 36 of 45 rattle noises were identified as the rattle noise. The overall accuracy of the algorithm was 89.5 %.

Table 7-1. Classified squeak and rattle events by the algorithm to distinguish squeak and rattle noises.

<table>
<thead>
<tr>
<th></th>
<th># of specimens</th>
<th># of specimens classified as squeak</th>
<th># of specimens classified as rattle</th>
<th>True</th>
<th>False</th>
</tr>
</thead>
<tbody>
<tr>
<td>Squeak</td>
<td>41</td>
<td>41</td>
<td>0</td>
<td>100 %</td>
<td>0 %</td>
</tr>
<tr>
<td>Rattle</td>
<td>45</td>
<td>9</td>
<td>36</td>
<td>80 %</td>
<td>20 %</td>
</tr>
<tr>
<td>Total</td>
<td>86</td>
<td>50</td>
<td>36</td>
<td>89.5 %</td>
<td>10.5 %</td>
</tr>
</tbody>
</table>
7.7 Conclusions

A new algorithm was developed to distinguish between the squeak noise and the rattle noise. The method utilizes three sound quality metrics: sharpness, roughness, and fluctuation strength. Based on noise recordings, the squeak and rattle regions were each determined along in three planes (sharpness-roughness plane, roughness-fluctuation strength plane, fluctuation strength-sharpness plane). Acoustic signals were subsequently identified as either a squeak or a rattle, based on the distance from the corresponding data point to each region. The algorithm was able to distinguish S&R noises within nearly 90% accuracy.

Because the causes of the squeak noise and rattle noise are very different, knowing the type of the noise can be very helpful for engineers. The developed algorithm may be used in conjunction with a S&R detection algorithm such as the one developed in Chapter 5. The steps listed below are suggested as an example:

1. Run a S&R test of a vehicle and record the noise.

2. Run the S&R detection algorithm to detect the occurrence of S&Rs.

3. Run the algorithm developed in this chapter to assess whether each detected noise is a squeak or rattle.

4. Develop and apply a proper method to reduce the noise.

One limitation of this work is that the developed algorithm to distinguish squeak and rattle noises used only the noises generated in the test bench. More tests with S&R noises recorded in real car tests will further validate the performance of the algorithm.
Chapter 8 Engineering Procedure to Minimize Squeak Problems of Automobiles in the Design Stage

8.1 Introduction

Most squeak problems in automobiles are caused by structural deficiencies and use of incompatible material pairs. Typically, squeak noises in automobiles have been handled by a find-and-fix type method in the prototype test, not in the design stage, which is very inefficient and costly because it often requires a new prototype and possible solutions are severely constrained because the solution cannot alter the vehicle design significantly. Therefore, a systematic, overall procedure to minimize squeak problems in the design stage is proposed in this chapter, which will reduce the overall cost dramatically.

While it is desirable to address all squeak problems in the early design stage, it is impractical to try to eliminate all sliding contacts completely in a vehicle that has the very huge number of structural parts and is subjected to a wide range of dynamic loading. A good structural design can minimize the occurrence of sliding contacts, but cannot eliminate them completely. In addition, a squeak noise will occur more likely if the material pair which has a high squeak propensity is used. Hence, the regions of high possibility of sliding contact can be identified first based on numerical simulations or experiments in the design stage, then material pairs can be selected based on the squeak propensity in such regions. A material database of squeak propensity will be necessary to enable such an approach.
8.2 Material Database for Squeak Propensities

The ultimate objective of the squeak test apparatus developed in Chapter 3 and 4 is to build a material database that NHV engineers can utilize in the early design stage. In order to define material database for squeak propensities, the test apparatus should generate a squeak noise consistently. It is confirmed that the squeak test apparatus based on the modified sprag-slip mechanism (MSSM) can generate squeak noises very consistently, making nearly the same squeak noises for a given pair of materials in all tests.

Squeak noises generated by the apparatus can be assessed by the automatic detection and rating algorithm for squeak and rattle noises discussed in Chapter 5 to rate squeak propensities of the sounds quantitatively. For the squeak noise detected, the algorithm calculates the objective rating of the squeak noise. Figure 8-1 shows the objective rating curves of squeak noises generated from three different material pairs. Based on the rating of the generated noises shown in Figure 8-1, squeak propensities of the three pairs are rated as 5.3, 13.5 and 9.2, which are taken as the average of the peaks in the rating curves shown in Figure 8-1. An example of the material database of squeak propensities of 28 material pairs from 7 materials (aluminum and 6 polymers) is shown in Table 8-1.
Table 8-1. Material Database for Squeak Propensity.

<table>
<thead>
<tr>
<th>Material</th>
<th>Aluminum</th>
<th>Polymer A</th>
<th>Polymer B</th>
<th>Polymer C</th>
<th>Polymer D</th>
<th>Polymer E</th>
<th>Polymer F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>2.2</td>
<td>5.3</td>
<td>13.5</td>
<td>9.2</td>
<td>4.5</td>
<td>5.2</td>
<td>11.2</td>
</tr>
<tr>
<td>Polymer A</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Polymer B</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1.8</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Polymer C</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1.9</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Polymer D</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Polymer E</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1.7</td>
<td>-</td>
</tr>
<tr>
<td>Polymer F</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

- Squeak Detected
- Squeak Not Detected
Figure 8-1. Objective rating of the generated squeak noises. (a) aluminum-polymer A pair, (b) aluminum-polymer-A pair and (c) aluminum-polymer C pair.
8.3 Roles of Experiments and Computer Aided Engineering

A major purpose of experiments in squeak problems of automobile has been to verify the structural design of a vehicle in a prototype test. The part of vehicle or a full vehicle is tested by shakers or four-poster in a laboratory or on-road test in a proving ground or a public road. Johnsson et al. [65] developed a new test track for automotive squeak and rattle detection, which is named the frequency sweep test track. Shin and Cheong [67] reported the method to localize the noise source of buzz, squeak and rattle in an instrument panel. Hurd [62] reported a squeak and rattle evaluation in accelerated laboratory durability test.

In addition, the experiment should be used to obtain the load conditions that will be used as inputs or boundary conditions in CAE simulation because accurate, realistic boundary conditions of FEM simulation lead to more accurate, valuable results. Therefore, in a squeak design of automobiles, the on-road test of a vehicle is necessary to obtain the load conditions that cover the operation range in that squeak problems should not appear.

The role of CAE has become much more important due to the recent advances in computer science. In automobile industry, a part of a vehicle or even a full vehicle has been simulated, however, the appropriate use of CAE is much more important than just running the simulations. Although a number of works [38, 47, 48, 93] have been done and reported to predict and prevent squeak problems using CAE, their load conditions could not describe the actual operation of vehicles. Therefore, in order to obtain accurate results from CAE, the load conditions obtained from the experiments should be applied in numerical simulations.
8.4 Conclusions

A systematic, overall procedure to minimize squeak problems in the design stage shown in Figure 8-2 will reduce the overall cost dramatically. The engineering procedure based on the developed techniques, which are the squeak test apparatus and the algorithm to automatically detect and rate squeak noises, is proposed as follows;

1) Test various material pairs by utilizing the squeak test apparatus.
2) Rate their squeak propensities and build material database by using automatic detection/rating algorithm.
3) Obtain the realistic load conditions that cover the operation range in that squeak problems should not appear.
4) Run CAE simulations of a component or a system to identify the areas with high probability of the squeak occurrence.
5) Modify the design to avoid sliding contacts as much as possible.
6) Select material pairs of low squeak propensity in the areas identified for high risk of squeak.
Figure 8-2. Flow chart of the engineering procedure to minimize squeak problems in automobiles.
Chapter 9 Conclusions and Future Work

9.1 Conclusions

One of the most difficult challenges in quantitatively measuring squeak propensity of the pair of materials is to generate squeak noise consistently and in a repeatable manner. If a squeak test machine produces a squeak noise in on-and-off manner and generates totally different squeak noises with the same pair of materials, the objective rating of the squeak propensity of the generated noises is impossible and meaningless. Since the squeak noise is generated by friction-induced, self-excited vibration and affected by various factors, such as material properties, temperature and humidity, the squeak test apparatus should induce the unstable motion between two materials in a robust manner. In this dissertation, the squeak test apparatus based on the modified sprag-slip mechanism (MSSM) was designed, built and studied to consistently generate the unstable motion, therefore the squeak noise. In addition, the algorithm for automatic detection and rating of squeak and rattle (S&R) noises and the algorithm to automatically detect and distinguish between S&R noises were developed in this work. Furthermore, an overall, systematic engineering procedure to minimize squeak problems in automobiles was proposed as the applications of the developed squeak test apparatus and algorithms. A list of the conclusions can be summarized as follows:

1) An extensive literature survey of current state-of-art methodologies and techniques on published scientific articles and conference papers, analytical and numerical studies on
friction-induced instability and S&R noises are conducted in Chapter 2, which enables a broad and detailed understanding of the main research objects. Several deficiencies were found from the published literature, which have been improved through this dissertation work.

2) The squeak test apparatus based on the MSSM was developed. The stability analysis of the MSSM was conducted by the analytical and FEM model. The effects of the main parameters on the stability of the system were investigated. The system becomes unstable more easily with higher torsional stiffness relative to the linear stiffness and the higher kinetic coefficient of friction. The size of the limit cycle of the system was also investigated because a large-amplitude oscillation can result in the squeak noise even in the stable system. There exists a value of the angle of attack which makes the size of the limit cycle maximum or minimum when the friction coefficient is relatively large. In addition, the initial compression of the test apparatus should be carefully adjusted to induce squeak noises more effectively because there is a value of the initial compression that makes the size of the limit cycle minimum.

3) The instability condition identified from the analytical model was confirmed by the FEM model of the MSSM. The complex eigenvalue analysis (CEA) was adopted to extract the eigenvalues of the FEM model and determined the stability of the model based on the effective damping of each mode. Moreover, the instability condition identified by the CEA was confirmed by the dynamic transient analysis (DTA) in time domain. The instability condition obtained from the analytical model showed a good agreement with the results obtained from the FEM model.
4) In order to demonstrate the capability of the squeak test apparatus developed, several pairs of materials were tested. The results showed that the measured acceleration and the generated squeak noise were well correlated in both time and frequency domains. In addition, it was seen that the second and third modes of the clamped beam of the current design of the test apparatus are main modes which are responsible for generating squeak noises. From the time-frequency patterns of the squeak noises generated by the test apparatus, the broad-band, high-intensity sound appeared with variable intervals, which is a commonly observed pattern of a squeak noise.

5) The relatively simple analytical model was developed to represent mode-coupling type instability, which is the one of well-known mechanisms that lead to friction-induced instability. The basic parameter study was conducted and showed that the ratio of the natural frequencies of in-plane and out-of-plane motion is the most important parameter that makes the system unstable. If the mode-coupling mechanism is adopted to design squeak test apparatus which generates unstable, large-amplitude motion consistently, the test apparatus should be designed by carefully adjusting stiffness and inertia of the apparatus to match the natural frequencies of two modes as closely as possible.

6) The automatic detection and rating algorithm of S&R noises was developed, refined and applied to various S&R noises obtained from the experiments. The algorithm utilized various signal processing techniques, which are suitable for highly transient noises, to obtain the perceived transient loudness (PTL) time history that approximates the human perception of the S&R noises.

7) A new algorithm to distinguish between the squeak and rattle noises was developed by utilizing the sound quality metrics. The recorded sounds were identified as either a squeak
or a rattle, based on the distance from the corresponding data point to each region. The algorithm could differentiate S&R noises within nearly 90% accuracy.

The ultimate objective of this dissertation is to develop the squeak test apparatus that generates squeak noises consistently and repeatedly and to establish the systematic engineering procedure to minimize squeak problems in automobiles at the design stage, which utilizes the developed test apparatus and algorithms. Squeak noises generated by the squeak test apparatus developed can be assessed by the automatic detection and rating algorithm for S&R noises. For the squeak noises detected, squeak propensity of the material pair is defined as the average of the peaks in the objective rating curve calculated by the algorithm. Therefore, the material database of the squeak propensity of given material pairs can be built and highly useful for automotive NVH engineers.

The overall, systematic procedure to minimize squeak problems in the design stage, that integrates analytical work, CAE simulations and experiment, is proposed as follows: (1) Test various pairs of materials by using the squeak test apparatus developed in this work; (2) Rate their squeak propensities and build material database by using automatic detection/rating algorithm; (3) Obtain the realistic load conditions that cover the operation range in that squeak problems should not appear; (4) Run CAE simulations of a component or a system to identify the areas with high risk of the squeak occurrence; (5) Modify the design to avoid sliding contacts as much as possible; and (6) Select material pairs of low squeak propensity from the material database in the areas identified for high risk of squeak.
9.2 Recommendations for Future Work

Some of the work continued out of this dissertation work are proposed as follows:

(1) The structural design of the squeak test apparatus can be improved to generate squeak noises more effectively. For instance, the angle of attack between two specimens can be adjusted instead of changing the whole bent beam. The initial compression applied to the specimens can be adjusted to set the normal force precisely.

(2) The material database of squeak propensity can be expanded. As shown in Chapter 4 and Chapter 7, seven kinds of materials (aluminum and six polymers) that provides 28 material pairs were tested, and their squeak propensities were summarized as a material database. The other kinds of materials, such as leather, which are commonly used in a seat, door or instrumental panel of a vehicle, can be selected to conduct additional material pair tests.
BIBLIOGRAPHY


