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I, Sharang Inamdar, hereby submit this original work as part of the requirements for the degree of Master of Science in Mechanical Engineering.

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Estimation of Frequency and Damping of a Rotating System using Mode Enhanced Order Tracking (MEOT) and Virtual Sensor Concept.

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Estimation of Frequency and Damping of a Rotating System using Mode Enhanced Order Tracking (MEOT) and Virtual Sensor Concept

A thesis submitted to the Graduate School of the University of Cincinnati in partial fulfillment of the requirements for the degree of Master of Science in the Department of Mechanical and Materials Engineering of the College of Engineering and Applied Science by

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**Abstract**

Currently, to obtain an understanding of the frequency and damping of the modes of a rotating system, an impact test or a shaker test is done to find the frequency at which the modes of the system exist. Then, test is conducted under operating conditions to obtain an understanding of the response of the system. A *mode enhanced order track* (MEOT) can be used to estimate the frequency and damping of various modes directly from the operating data of a rotating system but it cannot find modes which are very close together in frequency or those that are repeated. The MEOT is a method described in a PhD dissertation by Dr. Blough but has not been tested on data obtained from an actual rotating system under operating conditions. The MEOT method was evaluated utilizing data taken from a leaf blower, using a laser tachometer and seven triaxial sensors. Using MEOT, the frequency and damping of different modes was found. These were then compared to the results obtained from an impact test performed on the leaf blower. This MEOT method was also tested after using the virtual sensor concept on the data. A virtual sensor was created for each triaxial sensor. The results obtained from the impact test, the MEOT method and the MEOT method performed on the virtual sensor data were then compared and analysed.
Acknowledgments

This thesis would not have been possible without the help and support from many people. The first person I would like to thank is my thesis advisor Dr. Randall Allemang. I don’t think my thesis topic would have even occurred to me had I not taken his class, AFFT, in the fall semester of 2013. Dr. Allemang was always ready to answer any questions I had and answer them with detailed explanations. I would also like to thank Dr. Allyn Phillips for his help during my time working on this thesis. His inputs during the weekly meetings we had were of huge help.

I also want to thank Dr. Jason Blough, who I corresponded with via e-mail early on in my time working on this topic. The idea for this thesis originates from the future work section of his PhD dissertation and as a result I would not have come upon this topic had I not read his dissertation. I also want to thank Dr. David Brown, who was a co-author of the paper in which the ideas in my thesis are explained.

I would not have been able to successfully finish this thesis without the support, in all forms, provided to me by my parents and my brother. Hour long conversations with them over the weekends went a long way in helping deal with any stress that I felt during this period.

I think my time at UC, working on this thesis, would have been much harder had I not had the best of friends. Whether they were working with me in SDRL, working in other departments in UC or spread around the world living their own lives, I always had people to share my really poor sense of humour with and I cannot thank them enough for this!
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## Nomenclature

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<th><strong>Abbreviations</strong></th>
<th><strong>Description</strong></th>
</tr>
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<tbody>
<tr>
<td>CMIF</td>
<td>Complex Mode Indicator Function</td>
</tr>
<tr>
<td>DOF</td>
<td>Degree of Freedom</td>
</tr>
<tr>
<td>DSP</td>
<td>Digital Signal Processing</td>
</tr>
<tr>
<td>EVD</td>
<td>Eigen Value Decomposition</td>
</tr>
<tr>
<td>FFT</td>
<td>Fast Fourier Transform</td>
</tr>
<tr>
<td>FRF</td>
<td>Frequency Response Function</td>
</tr>
<tr>
<td>LSL</td>
<td>Least Squares Local</td>
</tr>
<tr>
<td>MEOT</td>
<td>Mode Enhanced Order Track</td>
</tr>
<tr>
<td>MPE</td>
<td>Modal Parameter Estimation</td>
</tr>
<tr>
<td>OA</td>
<td>Order track Autopower</td>
</tr>
<tr>
<td>RPM</td>
<td>Revolutions Per Minute</td>
</tr>
<tr>
<td>SDOF</td>
<td>Single Degree of Freedom</td>
</tr>
<tr>
<td>SVA</td>
<td>Singular Value Autopower</td>
</tr>
<tr>
<td>SVD</td>
<td>Singular Value Decomposition</td>
</tr>
<tr>
<td>TVDFT</td>
<td>Time Variant Discrete Fourier Transform</td>
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</table>

<table>
<thead>
<tr>
<th><strong>Symbols</strong></th>
<th><strong>Description</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta t$</td>
<td>Discrete sampling time interval</td>
</tr>
<tr>
<td>$F_{\text{samp}}$</td>
<td>Sampling frequency of the DSA</td>
</tr>
<tr>
<td>$F_{\text{nyq}}$</td>
<td>Nyquist frequency</td>
</tr>
<tr>
<td>$F_{\text{max}}$</td>
<td>Maximum frequency that can be observed in a spectrum</td>
</tr>
<tr>
<td>$O_{\text{samp}}$</td>
<td>Sampling order</td>
</tr>
<tr>
<td>$O_{\text{nyq}}$</td>
<td>Nyquist order</td>
</tr>
<tr>
<td>$O_{\text{max}}$</td>
<td>Maximum order that can be observed in a spectrum</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------------------------------------------</td>
</tr>
<tr>
<td>( \Delta o )</td>
<td>Delta Order – order resolution</td>
</tr>
<tr>
<td>( t )</td>
<td>Time variable</td>
</tr>
<tr>
<td>( T )</td>
<td>Total sample time analyzed</td>
</tr>
<tr>
<td>( N )</td>
<td>Data block size/total number of spectral lines</td>
</tr>
<tr>
<td>( \Delta f )</td>
<td>Delta frequency – frequency resolution</td>
</tr>
<tr>
<td>([A])</td>
<td>Base matrix</td>
</tr>
<tr>
<td>([U])</td>
<td>Left singular matrix</td>
</tr>
<tr>
<td>([V])</td>
<td>Right singular matrix</td>
</tr>
<tr>
<td>([\Sigma])</td>
<td>Singular value matrix</td>
</tr>
<tr>
<td>([I])</td>
<td>Identity matrix</td>
</tr>
<tr>
<td>([Q])</td>
<td>Eigen vector matrix</td>
</tr>
<tr>
<td>([A])</td>
<td>Eigen value matrix</td>
</tr>
<tr>
<td>(\sigma)</td>
<td>Singular value</td>
</tr>
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1. Introduction

1.1. Objectives.

Rotational system analysis needs to be done for many reasons. It can be used in health monitoring of machinery, diagnosis of faults, damage detection and various other applications. The information obtained from the testing these systems under operating conditions can then be used to make informed design choices, perform suitable maintenance operations or predict the failure of a part before it occurs.

The analysis of rotational systems is usually done in two steps, the first step is to analyse the static characteristics of the system, i.e. to perform either a shaker test or an impact test on the system to gathering information about the modal parameters of the system. The second step is to perform a test under dynamic conditions, i.e. while the system is running. This second step provides an understanding of the response of the system under operating conditions and takes into account system characteristics that are not found in the test to find modal parameters such as nonlinearities and force feedback issues. Using the information gathered from these steps, the system can be either designed or modified so as to have a longer life or, sometimes, damage to the internal components of the system can be detected. The problem here is, due to various factors, the characteristics of the system under dynamic conditions may vary from the characteristics of the system under static conditions. This will cause a mismatch between the results of the modal test and the operating test. Therefore, a method of obtaining all required information from the structure under dynamic conditions is required.

The first objective of this thesis is to estimate frequency and damping of the modes of a structure directly form rotating data by using mode enhanced order tracking (MEOT). The mode enhance order track can be assumed to be a single degree of freedom frequency response. This makes it possible to estimate the frequency and damping at which the mode shape being tracked by the MEOT is excited and can be done
using single degree of freedom parameter estimation techniques. The accuracy of these estimates will be tested for multiple modes of the structure.

Another objective is to integrate the virtual sensor concept with the mode enhanced order track. This will help get a MEOT curve with less noise. The major obstacle here is the spreading of phase data to the left and right singular vectors. A method to merge the phase data with the singular values of the virtual sensors is required to solve this problem. After this, the frequency and damping estimates will be compared to the estimates obtained from the virtual sensor data using the MEOT method. These estimates will also be compared to estimates obtained from a hammer test with the system not in a running condition.

1.2. Outline.

Chapter 2 gives a brief overview of the technique and procedures used in the thesis. After this, Chapter 3 deals with the procedure followed for acquiring an accurate and suitable instantaneous RPM curve, it also contains a discussion on RPM spectral maps and explains its importance in estimation of frequencies and damping. This chapter also has a discussion on the order tracks obtained, and the effects the instantaneous RPM curve can have on the order track. In Chapter 4, an overview of singular value decomposition (SVD) based techniques mentioned in Reference 19 is given followed by an in-depth discussion on their application in estimation frequency and damping. In Chapter 4, procedure used to obtain the frequency and damping values is also given Chapter 4 also deals with application of the virtual sensor concept. In Chapter 5, the order tracks, SVD plots and MEOT plots obtained from the raw data is compared to that obtained from the virtual sensor data. The frequency and damping estimates are of both the sets of data are compared to estimates acquired from an impact test. Chapter 6 presents the conclusions and gives suggestions for future works. The references follow at the end of the conclusions.
2. Theoretical Background

2.1. Spectral Maps

Spectral maps are three dimensional plots used to visualise the frequency response or auto power of a rotating machine at different RPMs. The magnitude of displacement at the frequency in question is shown on the z-axis, the frequency is represented on the x-axis and usually, the RPM is represented on the y-axis, but it can also be used to represent other variables such as temperature, time, load etc.

The spectral map is used to determine the RPM at which an order peaks, identifying the frequency at which this occurs. It is also used to investigate the RPM at which the order being analysed crosses a natural frequency. Orders themselves will have different magnitudes at different frequencies depending on the response of the structure to an input of that frequency. [7][12]

2.2. RPM Estimation

2.2.1. Tachometer Signal

For analysis of rotating systems, the most important measurement is that of the RPM. This is because the frequencies of the orders in the system are multiples of the speed of rotation (instantaneous RPM) of the system. Therefore, the accuracy of the order track is only as good as the acquired tachometer signal.

Historically, tachometers used tracking ratio tuners to condition the tachometer channel. This was done to achieve a clean tachometer signal, but this had the disadvantage of being unable to capture signals with
high sweep rates (or “slew” rates). To overcome this limitation, a number of techniques were developed. [13]

Vold and Leuridan developed a technique that fit multiple cubic splines to the estimated RPM data. In this technique, the time block is divided into a number of smaller blocks and then cubic splines are fit in each one of these blocks. Continuity is enforced at the end of these blocks. This, in effect, averages the blocks, removing any sudden increases and decreases in the RPM. [1][20]

There are many different types of tachometers, namely, shaft encoders, laser type tachometers, infrared light type tachometers and variable reluctance type tachometers.

### 2.2.2. RPM Estimation

To obtain an RPM estimate from the raw tachometer signal, the time between the zero crossings of tachometer pulses is measured. It is possible that no data point will exist at the zero crossing instant. This is a result of the zero crossing time instant occurring at a time that is not an integer multiple of the time between two data points (delta t). For this case, linear interpolation is done to obtain the zero crossing time instant. Therefore, a technique which can pick the zero crossing points when available and perform linear interpolation to estimate the linear crossing points when needed is used. Equation 1 is used to estimate the RPM after all the zero crossing points are found. [13][20]

\[
RPM = \frac{60}{(t_{k+1} - t_k) \times P}
\]  

(1)

Where,

\(t_{k+1}\) and \(t_k\) are the zero crossing points of two adjacent tachometer pulses.

\(P\) is the number of pulses per revolution.
For a laser tachometer, there will generally be only one pulse per revolution. Other types of tachometers can be characterised by the use of multiple pulses; for these tachometer signals, the number of pulses per revolution is an important factor. \cite{13}\cite{20}

Another method of obtaining an RPM estimate is the Hilbert transform. Here the unwrapped phase of the complex values that are obtained as the output, can be used to obtain the RPM estimate. \cite{13}\cite{20}

### 2.2.3. Original Kalman Filter Based Tachometer Analysis

Due to inaccuracies in the estimation of zero crossing times of the tachometer pulses, a smooth RPM estimate cannot be obtained. A spline fitting algorithm was developed to smooth the errors in this data by Vold and Leuridan.

This method uses the technique detailed above to estimate zero crossing times of the tachometer pulses. These zero crossing times will contain some errors due to the discrete nature of the data. The RPM estimate obtained from this data as a result, will have a large errors associated with it. After this estimate is obtained, it is then divided into many blocks of data and a least squares spline is fit to each of these blocks. To ensure a smooth spline, boundary conditions and end conditions are use. The end conditions used are the end of one spline and start of the next spline should have the same value. The boundary conditions imposed on the spline are the first derivative at the end of one spline should be equal to the first derivative at the start of the next spline. This ensures smoothness of the RPM estimate between the blocks. This RPM estimate is then used to calculate the instantaneous RPM at different times. \cite{20}

A disadvantage of this method is it cannot be used to accurately estimate the RPM for data with high sweep rates (slew rates). This is because of the stiffness of the least squares splines. \cite{20}
2.2.4 Vold – Kalman Filter Enhanced Spline Based Tachometer Analysis

This method is an improvement on the previous spline fitting methods. It was developed by Vold as a part of the Vold – Kalman filter. Two improvements were made in this method.

The first improvement was the introduction of a shaving algorithm to reduce the errors caused by missed pulses. This is done by first obtaining a regular spline fitted RPM estimate using the original Kalman filter based algorithm. This RPM estimate is referred to as the first estimate. After this, the spline fitted RPM data is compared to the initial RPM estimate. If the difference is too large, that estimated value is removed from the data set. After this is done for the whole data set, the spline fitting is done again. This procedure can be repeated multiple times until a suitable RPM estimate is reached.

The second improvement was the ability to relax the first derivative boundary condition on the splines. This helps in cases with high slew rates such as gear shifts. At the time instant the high slew rate event occurs, the first derivative boundary condition can be relaxed, thus obtaining a more accurate estimate. This also solves the problem of the primary disadvantage of the original Kalman filter based algorithm. [6][20]

2.2.5 Nth Pulse Algorithm

The $n^{th}$ Pulse Algorithm (nPA) is a method of reducing the effect of noise on the estimation of the instantaneous RPM curve. If the interval between successive pulses is small, the signal to noise ratio of the data can become a problem (high slew rates or small changes at high RPM). Skipping $n$ number of pulses in the tachometer data can cause the spacing between the pulses to be large to reduce the effect of noise on the instantaneous RPM estimate. [21]

$$RPM = \frac{60}{(t_{k+1} - t_k) \times P} \times (n + 1) \quad (2)$$
2.2.6 Smart Bayesian algorithm

The RPM of a system can also be measured directly from the acceleration data of the system using a statistical approach. Using the smart Bayesian algorithm (SBA) instantaneous RPM can be extracted by tracking the dominant order of the system. This approach can be used to track systems with high slew rates. \[^{21}\]

2.3. Order Tracking

An order is a time varying phasor that rotates with an instantaneous frequency proportional to the frequency of the rotating shaft. \[^{13}\]

\[
Order\ Frequency \propto \frac{\text{Instantaneous RPM} \times k}{60} \tag{3}
\]

Where,

\(k\) = the order being analysed.

The order can be mathematically defined as – \[^{13}\]

\[
X(t) = A(k, t)\sin(2\pi i(k/p)t + \phi_k) \tag{4}
\]

Where,

\(A(k, t)\) = Amplitude of order \(k\) at time \(t\).

\(\phi_k\) = Phase angle of order \(k\)

\(p\) = period of primary order in seconds

\(t\) = Time
k = Order being analysed

An order generally possesses frequencies that are integer multiples of the reference (shaft) frequency at any given time. This can be caused by imbalances in the system and flexibility (causing deflection) of the rotating shaft, which causes the centre of mass to shift away from the axis of rotation, causing an order 1X the reference frequency [7]. Orders that possess frequencies that are non-integer multiples of the reference frequency can also be present. The presence of such orders can be either due to the presence of gearboxes, pulleys, bearings etc. or due to defects in the system such as cracks and chipped gears. For example, defects in rolling element bearings cause an order with the frequency 0.4X to 0.6X of the reference frequency [4]. Orders possessing non-integer frequencies can also be caused by the presence of multiple sources. These orders can often be very close together in frequency, causing difficulty in measurement.

To gain a better understanding of these orders, order tracking needs to be performed

2.3.1. FFT Based Order Tracking

This method of order tracking is the oldest method and is still widely used today. It is done using a simple fast Fourier transform (FFT), which in turn is based on the Shannon sampling theorem. The Shannon sampling theorem is represented below.
\[ \Delta f = \frac{1}{T} = \frac{1}{N \times \Delta t} \]

\[ T = N \times \Delta t \]

\[ F_{Nyquist} = F_{max} = \frac{F_{Sample}}{2} \]  

(5)

\[ F_{Sample} = \frac{1}{\Delta t} \]

Where,

\( \Delta f \) = Frequency resolution

T = Observed time period

N = Number of time samples

\( \Delta t \) = Time spacing between each time sample

\( F_{Nyquist} \) = Nyquist Frequency

\( F_{max} \) = Maximum Observable Frequency

\( F_{Sample} \) = Sampling Rate

From the equations in Equation 5, it is can be observed that the Shannon sampling theorem is based on data that has a fixed \( \Delta t \) and does not take into account rotating systems with varying rotating frequencies. This leads to frequency domain results from the FFT rather than results in the order domain. This can be seen in the equation of the transform shown in Equation 6. \[^{[20]}\]
\[ a_m = \frac{1}{N} \sum_{n=1}^{N} x(n\Delta t)\cos(2\pi f_m n\Delta t) \]  
\[ b_m = \frac{1}{N} \sum_{n=1}^{N} x(n\Delta t)\sin(2\pi f_m n\Delta t) \]

Where,

\( x(n\Delta t) \) = The discrete data sample.

\( f_m \) = Frequency of the sine/cosine term.

\( a_m, b_m \) = Fourier Coefficients.

Therefore, to perform order tracking, a sliding FFT is performed on the collected time domain data. For each data block, an average RPM is calculated to estimate the frequency bin in which the order exists. There is a possibility of the orders falling between the frequency lines of the frequency domain data. For this reason, a number of spectral lines are taken around a chosen central frequency bin and summed together to obtain the frequency/phase data of the order. There are three methods which are used to choose the number of frequency bins to be summed to get a good estimate of the frequency/phase of the order in question.

The first method is called constant frequency bandwidth. In this method, a constant number of spectral lines are summed without taking the RPM or the frequency into account. The results obtained from this method generally do not match up to those obtained from more complex methods.

The second method is called constant order bandwidth. In this method, the reference or primary order is chosen and is used to determine the bandwidth. This ensures that the bandwidth adapts to the changing RPM of the system.

The third method is called constant percentage bandwidth. In this method, the number of spectral lines being summed is dependent on the frequency of the order being analysed. This method, like the constant order bandwidth method, adapts to the changing RPM or frequency of the system ensuring better results.
The weakness of these methods is that one averaged RPM value is assigned to every FFT spectrum. This assignment causes errors in the form of smearing when high slew rates are present in the data. Several other limitations of the FFT based order tracking methods are listed in References 13 and 16. Due to these limitations, these methods are generally not used in the industry.

2.3.2. Digital Resampling Based Order Tracking

In this method, uniformly sampled time data is digitally resampled to the angle domain. To do this, a reference signal is required to determine times of uniform angle intervals. Generally, a tachometer signal is used for this purpose. Once these times are found, several different interpolation algorithms are used to estimate the response channel at these points. After this an angle domain Fourier transform is performed on the resampled data set to get an estimate of the amplitude and phase of the order being analysed. To do this the time domain relationships must be formulated into angle domain relationships. This reformulation is shown below. \[13\] \[16\] \[20\]

\[
\Delta o = \frac{1}{R} = \frac{1}{N \times \Delta \theta}
\]

\[
R = N \times \Delta \theta
\]

\[O_{Nyquist} = O_{max} = \frac{o_{Sample}}{2} \quad (7)
\]

\[O_{Sample} = \frac{1}{\Delta \theta}
\]
Where,

$\Delta o$ = Order spacing

$R$ = Total number of revolutions analysed

$N$ = Number of time samples

$\Delta \theta$ = Time spacing between each time sample

$O_{Nyquist}$ = Nyquist order

$O_{max}$ = Maximum observable order

$O_{Sample}$ = Order sampling rate

Similarly, the kernel of the Fourier transform is reformulated as shown below.

$$a_m = \frac{1}{N} \sum_{n=1}^{N} x(n\Delta \theta) \cos(2\pi O_m n \Delta \theta)$$

$$b_m = \frac{1}{N} \sum_{n=1}^{N} x(n\Delta \theta) \sin(2\pi O_m n \Delta \theta)$$

Where,

$x(n\Delta \theta)$ = The discrete data sample.

$O_m$ = Order of the sine/cosine term.

$a_m, b_m$ = Fourier Coefficients.

The result of these reformulations is that the orders are always on the spectral lines, regardless of the RPM or frequency of the order, thus reducing the amount of leakage involved in the analyses. The main disadvantage of using this method is that it is computationally heavy. More information on this topic can be found in References 6, 16 and 20.
2.3.3. Vold - Kalman Filter Based Order Tracking

The Vold-Kalman order tracking filter is a modification and improvement on the Kalman filter done by Vold in 1997. The details of this method can be seen in Reference 15, Reference 17 and Reference 20.

The Vold-Kalman filter based order tracking method is very effective in tracking data with high slew rates and can decouple close or crossing orders. These advantages make it a very appealing method for order tracking.

The Kalman filter worked on the basis of two equations. They are, the structural equation, shown in Equation 9, describes the mathematical characteristics of the order being analysed and the data equation, shown in Equation 10, which describes the relationship between the order \( x(n) \) and the data \( y(n) \). \[^{[5][20]}\]

\[
x(nΔt) - 2 \cos(ωΔt)x((n - 1)Δt) + x((n - 2)Δt) = 0
\] (9)

Note that, \( x(nΔt) \) is the nth sample \( n^{th} \) discrete data sample and \( ω \) is the frequency of the sine wave

\[
y(n) = x(n) + η(n)
\] (10)

Also note that, \( η(n) \) is the portion of the signal containing non tracked order data or noise and is called the nuisance component.

Vold simplified the structural equation by making it into a complex first order equations.

\[
x(n + 1) - x(n) \exp(iωΔt) = ε(n)
\] (11)

In Equation 11, the exponential term represents the angle that the order rotates through in the time period \( Δt \). This gets further simplified into Equation 12.

\[
x(n + 1) + x(n) = ε(n)
\] (12)

Here, \( ε(n) \) signifies the amplitude change of the order data from one-time point to the next.
2.3.4. Time Variant Discrete Fourier Transform

A time variant discrete Fourier transform (TVDFT) is discrete Fourier transform method in which the frequency value in the kernel varies with RPM of the system. The TVDFT uses constant delta-t sampled data. In this way, it is a combination of the digital resampling based order tracking and the FFT based order tracking. The difference being, the TVDFT is much less computationally intensive than the digital resampling order tracking method. Since it uses constant delta-t sampled data, Equation 5 is valid for this method but, since the kernel is making use of a varying frequency dependant on RPM, Equation 7 is also valid.  

\[ \text{Equation 7} \]

The kernel used in the TVDFT is shown in the equations below.

\[
a_m = \frac{1}{N} \sum_{n=1}^{N} x(n\Delta\theta) \cos(2\pi \int_0^{n\Delta t} (O_m \times \Delta t \times \frac{RPM}{60}) dt) \\
b_m = \frac{1}{N} \sum_{n=1}^{N} x(n\Delta\theta) \sin(2\pi \int_0^{n\Delta t} (O_m \times \Delta t \times \frac{RPM}{60}) dt)
\]

Where, \( O_m \) is the order being analysed, \( a_m \) and \( b_m \) are the Fourier coefficients for \( O_m \) and RPM is the instantaneous RPM of the system.

This method uses a constant order bandwidth approach. The bandwidth is estimated by integrating the instantaneous RPM to get the number of revolutions the shaft has gone through at any instant in time. This is done over the required number of time points as given by Shannon’s sampling theorem, resulting in the transform being applied over varying lengths of time depending on the instantaneous RPM.

The TVDFT is generally only performed for one order at a time instead of the whole time spectrum. So an understanding of the dynamic characteristics of the system is needed while choosing the order that needs to be analysed.
To minimise leakage involved in the TVDFT, the following equations are used.

\[
\frac{1}{\Delta O} = \text{integer}
\]

\[
\frac{\text{order tracked}}{\Delta O} = \text{integer}
\]  

(14)

Errors due to lack of orthogonality, caused by high slew rates and crossing orders. These can be reduced by applying an orthogonality compensation matrix (OCM) to the TVDFT. It is used as a post processing procedure to the TVDFT. Further information on TVDFT can be found in Reference 7, Reference 13, Reference 18 and Reference 20.

2.4. Singular Value Decomposition (SVD)

The singular value decomposition (SVD) is used in many of the newer methods of analysis of rotating systems. Its use in this application was first described Reference 19 and Reference 20. Another application of the SVD in analysis of rotating systems is explained in Reference 9, this application uses singular values to get a better understanding of the data collected. The concepts put forward in both these works are used in this thesis, so a brief discussion on the SVD and these concepts is required.

The SVD is closely related to the eigenvalue decomposition (ED). The eigenvalue decomposition is in the form of

\[
A = Q\Lambda Q^T
\]

(15)

Note that, A is a symmetric matrix, \(\Lambda\) is a diagonal matrix containing the eigenvalues, \(Q\) is the eigenvector matrix. The eigenvector matrix \(Q\) is always orthogonal (\(QQ^T=I\)) since \(A\) is a symmetric matrix. Therefore, Equation 15 can be written as

\[
A = IA
\]

(16)

But, most matrices are not symmetric. Which means this factorization will become -
\[ A = U\Sigma V^T \] (17)

The formulation in Equation 17 is the SVD; here the \( \Sigma \) matrix doesn’t contain eigenvalues \( \Lambda \), and instead it contains the eigenvalues of \( AA^T \). These squared positive values are called singular values.

If an SVD is performed on order track data, the operating shapes get stored in the left singular vector. These operating shapes can be very close approximations of the actual mode shapes of the system at natural frequencies. [2][14]

2.5. Virtual Dominant Sensor Concept.

The virtual dominant sensor is a SVD based concept for observing and understanding the responses recorded by the various sensors on the structure. A square matrix containing autopower terms as diagonal terms and crosspower terms as off-diagonal terms is constructed from the frequency response vectors. An SVD is then performed on this matrix. The resulting singular value matrix contains the virtual sensor data. [9]

The order track data from the sensors can be condensed into virtual data using two approaches [9], the first approach is using response data from one location on the structure, but in the three principle directions. In Figure 1, the data from one location but for three different degrees of freedom can be seen.
In Figure 1, it can be seen that the order track in the y direction has lower magnitude between 700 RPM and 1000 RPM. This is not seen in the order tracks in the x and y directions. Similarly, there are many other features that vary in the order tracks of the three different directions. Therefore, to get a clearer idea of the overall behaviour of the structure at that location, the virtual sensor concept can be used. \[9\]

The second approach is to use the order track data from multiple locations on the structure. Through this approach a better understanding of the response of the structure can be obtained. \[9\]

Figure 2 shows the need of this approach. Many different features can be seen in the order tracks of the 1st order at locations 1 to 7 on the leaf blower. The virtual sensor order track gives a better overall understanding of the response of the structure. The data from virtual sensors can be thought of as averaged data, thus providing order tracks with less noise \[9\]. More information on the virtual sensor concept is available in Reference 9.
3. RPM Estimation, RPM Spectral Maps and Order Tracking.

3.1. RPM Estimation

For any data collected from a rotating system, the most important information is the RPM estimate. Therefore, it is worth discussing this topic in detail.

The RPM data is collected using tachometers, they can be laser tachometers, infra-red tachometers or variable reluctance tachometers. This data needs to be processed properly to obtain an accurate instantaneous RPM estimate. This is done in two steps.

3.1.1 Initial RPM Estimate.

The time instant at which the zero crossings of the raw tachometer signal pulses occurs, is used to estimate the RPM, but due to the discrete nature of the data, there may not be a data point at the zero crossing instant.
Figure 3 – Raw tachometer pulses a low speed with data points closest to the zero crossing instant.

As can be seen in Figure 3, none of the data points available fall on the zero crossing instant. The data displayed in Figure 3 is at a lower speed as compared to the data in Figure 4, which has a high speed. In this figure the data point is further away from the zero crossing point on many instances.

Figure 4 - Raw tachometer pulses at high speed with data points closest to the zero crossing instant.
To identify the time at which the zero crossing takes place for each pulse, linear interpolation is performed, as explained in Section 2.2. Linear interpolation was done using the following equation.

\[
X_{\text{zero crossing}} = \left( (0 - Y_1) \times \frac{X_2 - X_1}{Y_2 - Y_1} \right) + X_1
\]  

(18)

Where, 

\(X_{\text{zero crossing}}\) is the zero crossing point calculated through linear interpolation, 

\(Y_1\) and \(Y_2\) are the Y values of the points immediately before and after the zero crossing and 

\(X_1\) and \(X_2\) are the X values of the points immediately before and after the zero crossing.

Using these zero crossing points, an estimate of the instantaneous RPM can be calculated using Equation 1. This estimate can be very inaccurate, since there is a chance of missed or extra pulses occurring in the raw tachometer data. This can cause the tachometer estimate to be very 'noisy'.

![First Estimate of RPM from raw tachometer signal.](image)

*Figure 5 – First estimate of RPM from raw tachometer signal.*
3.1.2 Cubic Spline Fitting.

Cubic splines are used to approximate a smoother instantaneous RPM estimate. This is done, as explained in Section 2.2, by dividing the first RPM estimate into many smaller blocks. Then a cubic spline is fit into each of these block with certain boundary and end conditions.

![Instantaneous RPM estimate with cubic spline fitting](image)

*Figure 6 – Instantaneous RPM estimate with cubic splines used to smoothen the RPM Estimate*

As can be seen in Figure 6, this first RPM estimate also has variations in instantaneous RPM that are not physically possible due to the moment of inertia of the rotating system. In this case, the rotating system was a 6 cylinder 4 stroked diesel engine and drive train. Such a system has considerable amount of moment of inertia. Thus to make the RPM estimate more physically acceptable, the shaving algorithm is used as explained in Section 2.2.4.
With the help of the shaving algorithm a smooth instantaneous RPM estimate can be obtained as can be seen in Figure 7. The above estimate was obtained after two iterations of the shaving algorithm. In the first iteration.

3.2 RPM Spectral Map.

An RPM spectral map, as explained in Section 2.1, plots the measured RPM on the y axis as opposed to temperature, time or loading. From the RPM spectral map, we can gain an understanding of the frequencies and RPMs at which high magnitude response occurs, thus giving us an understanding of the orders which are excited in the system. The presence of natural frequencies can also be seen in these spectral maps as a vertical line which represents high acceleration magnitude at a particular frequency at all RPMs. However, this vertical line tends to experience a lot of leakage which is seen in the form of ‘smearing’ of the line into a region of high magnitude over a range of frequencies around the natural frequency.
In Figure 8, the dominant orders seen as angled lines of high amplitude data, happen to be the 12th, 24th, 32nd and 48th orders. These lines are curved due to the profile of the instantaneous RPM curve. The rectangular regions of high acceleration are the natural frequencies of the system. Due to leakage, the energy at the frequency of the peak has been spread across the frequency around the peak at every RPM resulting in this smearing effect.

From the RPM spectral map in Figure 8, it can be see that the 24th order is crossing a natural frequency, which exists at around 750Hz, when the RPM is between 1850 RPM to 1900 RPM. This will cause there to be extremely high displacement when the engine reaches this RPM. Therefore, any endeavour to reduce the magnitude of vibration in a system should start with the analysis of this order.

The spectral map in Figure 9 is created from data obtained from a leaf blower. In this set of data, the leaf blower was run by connecting it to a DC motor rather than the single cylinder engine that would normally run the leaf blower. As a result, the data is not completely realistic when compared to the actual running condition of the leaf blower. Here, it can be seen that the orders between the 6th and 7th order present. These orders have a high magnitude of acceleration between 200Hz to about 250Hz due to the presence of a natural frequency. The presence of orders between the 6th and 7th order may be because of some
damage to the components of the system. There is also a region of high magnitude in a small band between 150Hz to approximately 180Hz.

Figure 9 – RPM spectral map for data collected from leaf blower.
3.3 Order Tracking.

Order tracking was done using time variant discrete Fourier transform (TVDFT) as explained in Section 2.3.4. A large number of orders were tracked at all the available degrees of freedom (DOF). The RPM spectral map was used to choose which orders need to be analysed. This is because different orders excite different frequencies at each RPM. This is clearly illustrated in Figure 10.

![Figure 10 – Comparison of frequency ranges of various orders tracked at point 1 in the +X direction.](image)

In Figure 10, it can be seen that order 5, 6 and 7 have a higher magnitude of displacement between 200Hz to 250Hz. It is also noticed that orders 1 to 4 are not even reaching this frequency band within the RPM range used. These orders (orders 1 to 4) will cross the natural frequency at a higher RPM, but this data has not been collected. In the region between 150Hz to 180Hz, orders 4 and 5 show high magnitudes.
In Figure 11, it is observed that the 6th order has a high magnitude region between 200Hz and 250Hz. This corresponds to the RPM spectral map shown in Figure 9 where a high magnitude region is observed in the same frequency range. This high magnitude region can also be observed in the order track for the 6.5th and 7th order. Therefore, the orders that should be focussed on for the data acquired from the leaf blower are the 6th, 6.5th and 7th orders.

In Figure 10, it is observed that the magnitude of the 7th order in the region of interest (200Hz to 250Hz) is less than that of the 6th order in that same region. The 5th order has higher magnitude than the 6th or 7th orders in the region of interest, but the 5th order becomes very noisy in that area. This happens because the 5th order exists between 200Hz to 250Hz at the end of the RPM sweep. The data towards the end of all the orders was collected at a constant RPM, as seen in the highlighted area in Figure 12. This causes there to be many data points at the same RPM/frequency, causing this noisy region. The speed of the leaf blower was not taken further than this frequency for fear of causing damage to it.
As observed above, the shape of the instantaneous RPM curve is extremely important for the order track. It was also observed that having a single upward or downward RPM sweep resulted in more useable data than multiple RPM sweeps going up and down in the same data set.

Figure 13 illustrates the result of having RPM data with the characteristics mentioned. The order track ends up having more than one data point at the same RPM. If this happens at an RPM where the order in question crosses a natural frequency, it can cause problems in analysing the data.
4. Estimation of Frequency and Damping from Order Track Data.

4.1. Operating Shape Decomposition by RPM.

Operating shape decomposition is done on an RPM by RPM basis. Using this method, the same number of operating shapes can be found at certain RPMs as there are orders present in the system. This is done by performing an SVD on the order tracking data. [19][20]

In this data, the singular values ($\Sigma$) represent how well excited the operating shapes are at various RPMs. The left singular ($U$) vector contains a set of linearly independent operating shapes. These operating shapes approximate the mode shapes in the system as explained in Section 2.4. The right singular vector ($V$) contains the participation factor and the linearity between the linearly independent operating shapes and the orders.

The singular values can then be plotted with RPM. This plot is read similar to the complex mode indicator function (CMIF) plot [19]. The number of singular value curves will be equal to the number of orders present in the system. The number of singular value curves that peak at any RPM is equal to the number of linearly independent operating shapes getting excited at that RPM. The highest singular value curve represents the best excited linearly independent operating shape. If the lower singular value curves, successively peak at a certain RPM, then there are as many linearly independent operating shapes excited as the number of curves that peak at that RPM. [19][20]

Operating shape decomposition was done to identify the RPMs at which the primary singular value curve ($\sigma_1$) peaked. This was done by plotting the singular values, obtained by performing an SVD, against RPM. The exact RPM bins in which these peaks were located were also identified by plotting the singular values against the RPM bins. [19][20]
From Figure 14, the RPM bins at which the $\sigma_1$ singular value curve peaks is identified. Similarly, the RPMs at which this curve peaks can be identified in Figure 15. In both Figure 14 and 15 it can be seen that at the RPM bins, and as a result the RPMs, at which the $\sigma_1$ curve peaks, $\sigma_2$ and $\sigma_3$ singular value curves are also peaking. This indicates the existence of more than one operating shape within the one $\Delta$RPM of each other.
Five distinct peaks are present at the 644th, 666th, 698th, 713th and 719th RPM bins are found at 1834 RPM, 2009 RPM, 2375 RPM, 2594 RPM and 2677 RPM.

4.2. Operating Shape Decomposition by Frequency

In this method, a matrix is created filled with operating shapes that occur at the same frequency regardless of RPM or order. Since different mode shapes can be excited by different orders passing through the same frequency at different RPMs, it is possible to group these operating shapes together for analysis. A SVD is then performed on this operating shape matrix to find the linearly independent operating shapes. This can be seen in Figure 16.

![Comparison of frequency ranges of various order in data from the leaf blower.](image)

This method is advantageous as the number of operating shapes occurring within one frequency bin can be seen very clearly using this data. At lower frequencies, more operating shapes will be present, as more orders are present, as seen in Figure 16.

The left and right singular vectors represent the linearly independent operating shapes and participation factors the same as they did in the RPM based method; the interpretation of the frequency based method...
is the same as the order based method, the difference being, the singular values are plotted with frequency instead of RPM. The number of singular value curves peaking at any given frequency show the number of linearly independent operating shapes present at that frequency. [19][20]

### 4.3. Operating Shape Singular Vector Tracking by RPM Value

This method is used to track the operating shape based on the RPM values it is most excited at. This is done by calculating the modal assurance criterion (MAC) between the chosen operating shape and all other operating shapes present at different RPM values. Where the operating shape in question is tracked, the value of MAC will be greater than 0.9[19]. The MAC is calculated using Equation 19, [19][20]

\[
MAC = \frac{\|O_i^H O_j\|^2}{O_i^H O_i O_j^H O_j}
\] (19)

Note that,

\{O_i\} is the operating shape of the vector at the RPM, i, which is being analysed and

\{O_j\} is the operating shape at any RPM value j.

This mode tracking method can also be used to track linearly independent operating shapes, obtained from the SVD, through different RPM values and different orders. This provides an idea of the RPMs at which a mode is excited if the singular value being tracked represents a mode. This is done by replacing the operating shape, \{O_i\}, in Equation 19, with the left singular vector of the SVD (U). [19][20]
4.4. Order Track Autopower.

The order track autopower shows the amount of energy each order of the system contains at various RPMs. Different orders will have high energy at different RPMs. The order with the highest energy at a given RPM can be targeted to solve noise and vibration problems occurring at that RPM. \[^{[19]}^{[20]}\]

This method is a useful way of reducing a large number of degrees of freedom (DOFs) into a composite measurement per order. Thus, making handling large amounts of data much easier.

\[
OA_{ir} = \left( \sum_{k=1}^{n} (O_{ik})^2 \right)_{r} 
\]

(20)

Note that,

n is the number of measured degrees of freedom,

\( OA_{ir} \) is the order track autopower of order i at RPM r and

\( O_{ik} \) is the order i at degree of freedom (DOF) k at RPM r.

From the singular value plot, it is seen that the RPM range at which natural frequencies are excited are between 1800RPM to 2700RPM. The objective of using this method is to get a better idea of the orders which are dominant in this RPM range.
From Figure 17, it can be seen that the first order is dominant in the RPM range of interest (highlighted in Figure 17). However, in Section 3.3, it was seen that the first order does not reach the desired frequency within this frequency range. Therefore, it can be concluded that the first order cannot be used in this particular case. Figure 17 also shows that the Orders 5, 6 and 7 are very well excited in this RPM range. These orders also occur within the frequency range of interest at these RPMs.

In Section 3.3, it was also observed that the 6.5<sup>th</sup> order is also excited in this RPM and frequency range. To determine the orders that should be used while calculating the frequency and damping of the operating shapes, the order track autopower of these orders can be plotted with the RPM bins. This should determine the exact order that should be used for each peak. Initially the existence of the 6.5<sup>th</sup> order was ignored. Later a comparison between results using the 6.5<sup>th</sup> order vs without using the 6.5<sup>th</sup> order will be done.
Figure 18 – Order Track Autopower at point 3 in the +x direction for orders 6 and 7

Figure 19 – Order Track Autopower at point 3 in the +x direction for orders 6 and 7 from RPM bin 600 to RPM bin 800

From Figures 19, the dominant order at the RPM of interest can be chosen. For the RPM bin 666, the 6\textsuperscript{th} order is dominant, for the remaining RPM bins of interest, the 7\textsuperscript{th} order is prominent. If the 6.5\textsuperscript{th} order is included, it would be the dominant order for RPM bin 698, as seen in Figure 20.
Similarly, for the peak at RPM bin 644, the 5.5th order would be the dominant order if it were taken into consideration.

**4.5. Singular Vector Autopower**

The *singular vector autopower* (SVA) is used to find the rpms at which the singular values are dominant. It is calculated across all orders and RPMs. If the singular vectors represent modes of the system, then the SVA represents the RPMs at which modes are most excited. This uses the principle of a modal filter. It is done by weighting each degree of freedom with its corresponding element in the left singular vector. It is assumed that all the participation factors are unity. \[^{19}\]^{20}\)

\[
\text{SVA}_{l,r} = (U_l)^H \{O\} \{O\}^T \{I\}_{m \times 1}
\]

Note that,\n
\text{SVA}_{l,r} \text{ is the singular vector autopower of singular vector } l \text{ at RPM } r,
\( \{U_l\}^H \) is the hermitian of the left singular vector that the autopower is being calculated with respect to and 

\( \{O\} \) is the operating shape for each order of interest at RPM \( r \).

**4.6. Mode Enhanced Order Tracking.**

The left and right singular vector of a singular value from any RPM can be used to obtain the *mode enhanced order track* (MEOT) of a singular vector being analysed. This is similar to the *enhanced Frequency Response Function* (FRF) \(^{8}\)\(^{11}\)\(^{19}\) calculated to find the frequency and damping of the mode in question in the CMIF parameter estimation method. These order tracks are estimated from orthogonal functions; therefore, the system can be considered to have linearly independent inputs. This will mean that the MEOT should look like the FRF of a single degree of freedom (SDOF) system. As a result, SDOF parameter estimation methods can be used to estimate the frequency and damping of the system.\(^{19}\)\(^{20}\)

\[
MEOT_{l,r} = \{U_l\}^H 1 \times n (\{O\} \{O\} \ldots \{O\})_{n \times m} \{V_l\}
\]  

(22)

Note that,

\( MEOT_{l,r} \) is the *mode enhanced order track* of singular vector \( l \) at RPM \( r \),

\( \{U_l\}^H \) is the Hermitian of the left singular vector that the MEOT is being calculated with respect to and

\( \{V_l\} \) is the right singular vector that the MEOT is being calculated with respect to.

The MEOT is in a form similar to a FRF of a SDOF system which peaks at a specific RPM bin. This property can be used to calculate the frequency and damping of the four peaks found in Section 4.1 at the 644\(^{th}\), 666\(^{th}\), 698\(^{th}\), 713\(^{th}\) and 719\(^{th}\) RPM bins using any SDOF parameter estimation method. Sometimes, the MEOT may peak slightly at other RPM bins, this is because the same operating shape can be excited at other RPMs by other orders.

In Equation 22, the left and right singular vectors used are those that are obtained from the SVD of the RPM bin of interest, in this case, the 666\(^{th}\) RPM bin. The singular vector matrix is used is for each RPM
bin. Figure 21 shows the MEOT of the 666\textsuperscript{th} RPM bin overlapping the $(\sigma_1)$ singular value curve as well as the MEOTs calculated with respect to RPM bins 644, 698, 713 and 719. The MEOT calculated with respect to the 666\textsuperscript{th} RPM bin will only be equal to the $(\sigma_1)$ singular value curve at the 666\textsuperscript{th} RPM bin. The same procedure is followed for the other MEOTs and the results can be interpreted similar to the MEOT calculated with respect to the 666\textsuperscript{th} RPM bin.

\textit{Figure 21 – MEOTs of RPM bins 644, 666, 698, 713 and 719.}
4.7. Obtaining Frequency and Damping using SDOF Parameter Estimation Methods.

Using the least square local SDOF parameter estimation algorithm \cite{10} \cite{19}, the frequency and damping for the selected peaks can be found. Knowing the dominant order at each RPM bin is important here as the angular velocity used in the SDOF algorithm is calculated based on this.

\[
\omega = \frac{RPM}{60} \times 2\pi \times O_r
\]  

(23)

Note that,

\(\omega\) is the angular velocity and

\(O_r\) is the dominant order at the RPM r.

As seen in Section 4.4, for the RPM bin 698, if the 6.5\textsuperscript{th} order is considered, then it is the dominant order at that RPM bin. If it is not taken into account, the 7\textsuperscript{th} order is the dominant order at that frequency. As explained above, the order taken to be the dominant order can make a lot of difference to the results obtained.

![Figure 22 – RPM Spectral map showing Magnitudes of orders 6.5 and 7 at RPM bin 698.](image-url)
The RPM corresponding to RPM bin 698 is approximately 2375 RPM. The data points marked on Figure 22 represent the 6.5th order and the 7th order at this RPM. The 6.5th order crosses this RPM at a lower frequency than the 7th order, but it has a higher magnitude while at this RPM.

The 6.5th order exists in the system as shown in Figure 9 and 20. It is also the dominant order at the 698th RPM bin as seen in Figure 18. In Figure 22 it can be seen that order 6.5 has a higher magnitude than that of order 7 at this RPM bin. Therefore, the 6.5th order will be used for determining \( \omega \) at the 698th RPM bin.

To perform the least squares local SDOF parameter estimation, points around the peak are taken and plugged into Equation 24. The dominant order plays a role in this as it is used to calculate \( \omega \). \(^{[10]}\)

\[
H_{pq}(\omega_1)\lambda_r + A_{pqr} = (j\omega_1)H_{pq}(\omega_1) \tag{24}
\]

In our current application,

\( H_{pq} \) is the MEOT at a certain RPM bin,

\( \omega_1 \) is the angular velocity obtained from Equation 20 at that RPM bin and

\( \lambda_r \) and \( A_{pqr} \) are the modal frequency and residue respectively.

The above Equation needs to be repeated for several frequencies in the vicinity of the peak. This is represented in Equation 25. \(^{[10]}\)

\[
\begin{bmatrix}
H_{pq}(\omega_1) & 1 \\
\vdots & \vdots \\
H_{pq}(\omega_p) & 1 \\
\vdots & \vdots \\
H_{pq}(\omega_s) & 1 \\
\end{bmatrix}
\begin{bmatrix}
\lambda_r \\
A_{pqr}
\end{bmatrix}
= 
\begin{bmatrix}
(j\omega_1)H_{pq}(\omega_1) \\
\vdots \\
(j\omega_p)H_{pq}(\omega_p) \\
\vdots \\
(j\omega_s)H_{pq}(\omega_s)
\end{bmatrix} \tag{25}
\]

Note that,

\( \omega_p \) and \( \omega_s \) is the angular velocity at the peak and the angular velocity at the last chosen point.

Multiple points can be chosen around the peak and included into Equation 25. This is then done for all RPM bins. Figure 23 shows the points were chosen for each of the peaks.
Using Equation 26, a FRF can be synthesized using the modal frequency and residue. This can be used to see if the points taken into consideration around the peak are the correct ones since the synthesized FRF should trace the MEOT pretty closely if these chosen points are correct. \[\text{[10]}\]
\[ H_{pq}(\omega_1) \approx \frac{A_{pqr}}{j\omega_1 - \lambda_r} \]  

Figure 24 – MEOT calculated with respect to RPM bins 644, 666, 698, 713 and 719 with respective synthesized SDOF FRF.
In Figure 24, it can be seen that all the synthesized SDOF FRFs match up to the MEOTS well with regards to the magnitude and the frequency/RPM. Therefore, it can be concluded that the points chosen before and after each of the peaks were good choices. If the MEOT and synthesized SDOF FRF is plotted against RPM bins, the synthesized SDOF FRF plot will not look like an SDOF FRF because of the profile of the instantaneous RPM curve. The RPM from instantaneous RPM bins 774 to 996 is the same as the RPM at RPM bin 725 causing the plot to look unlike a synthesized SDOF.

![Figure 24: Comparison of MEOT and synthesized SDOF FRF](image)

**Figure 24** Synthesized SDOF FRF plotted against RPM bins and Instantaneous RPM plotted against RPM bins.

### 4.8. Virtual Sensor Order Track Data.

A singular value matrix can be calculated from the cross power matrix at each order of interest. The order track data used to obtain the virtual sensor order track is the same data used for the previous parts of this thesis. For each order at any given RPM, the response would be a vector of length equal to the number of measured degrees of freedom. \(^3\\)

\[
R_{k,O} = \begin{bmatrix}
        a_1 + ib_1 \\
        a_2 + ib_2 \\
        \vdots \\
        a_m + ib_m
\end{bmatrix}_{m \times 1}
\]  \hspace{1cm} (27)

\[
C_{k,O} = R_{k,O} \times R_{k,O}^H
\]  \hspace{1cm} (28)
Note that, $R_{k,O}$ is the response of order $O$ at RPM $k$,

$m$ is the number of measured degrees of freedom and

$C_{k,O}$ is the crosspower matrix.

An SVD can then be performed on the crosspower matrix. The singular values obtained represent the magnitude data of the virtual sensor order track. The phase information is divided between the left and right singular vectors. For this thesis, the first approach of obtaining virtual sensor order track was used. As a result, 7 virtual sensor order tracks were obtained from 7 locations on the structure. Each virtual sensor order track represents an overall representation of the order track at each location in all three principal directions. This approach is explained in Section 2.5.

The phase information of the order track can be obtained from the left and right singular vectors, because of this a particular value from the right singular vector is multiplied with the left singular matrix at each RPM. The value from the right singular vector was chosen by trial and error.

The left singular vector will contain a lot of complex values. This is because of the complexity of the structure. The presence of many rubber components in the leaf blower as well as many separate parts will cause there to be non-linear damping.

![Figure 26 – Complexity plot of the left singular vector at location 7 on the leaf blower before and after normalization.](image-url)
The plot on the left in Figure 26 is the complexity plot of the left singular vector before normalization. The three points represent the three DOFs available at every location on the leaf blower. The system can be normalised by using central axis rotation \cite{22}. This technique is performed on single complex modal vector at a time. So, in the context of this thesis, it is performed one operating shape (left singular vector) at a time at each RPM bin. The real and imaginary part of the complex vector are separated into two real valued vectors.\cite{3}

\[
\{\psi\} = \{\psi_{\text{real}}\} + i\{\psi_{\text{imag}}\} \tag{29}
\]

Note that,

\{\psi\} is the left singular vector.

\{\psi_{\text{real}}\} and \{\psi_{\text{imag}}\} are the real and imaginary vectors created from the real and imaginary parts of each value of the left singular vector.

After these real valued vectors are created, a matrix containing real and imaginary vectors is created as shown in Equation 30. After this a SVD is performed on this matrix as shown in Equation 31.\cite{3}

\[
\begin{bmatrix}
\{\psi_{\text{imag}}\} & \{\psi_{\text{real}}\}
\end{bmatrix}
\]

\[
[U, \Sigma, V] = \text{svd}\left(\begin{bmatrix}
\{\psi_{\text{imag}}\} & \{\psi_{\text{real}}\}
\end{bmatrix}^T\begin{bmatrix}
\{\psi_{\text{imag}}\} & \{\psi_{\text{real}}\}
\end{bmatrix}\right) \tag{31}
\]

The matrix on which the SVD is performed will be a 2 by 2 matrix irrespective of the number of DOFs available. The angle of the central axis is then found using the components of the right singular vector. This is done using an arctangent function so that the signs of the denominator and numerator are taken into account.\cite{3}

After this angle is found, it is used in a complex rotational phasor to rotate the central axis so that the left singular vector is now real valued.\cite{3}

\[
\bar{\phi} = \tan^{-1}\left(\frac{V_{2,2}}{-V_{1,2}}\right) \tag{32}
\]

\[
\{\tilde{\psi}\} = (\cos\bar{\phi} - j\sin\bar{\phi})\{\psi\} \tag{33}
\]
The plot on the right of Figure 26 shows the normalized left singular vector. It can be seen that these values are normalised. After this is done, the left singular vector is then multiplied with the values of the singular vector. This then causes the phase information to also be stored in the singular values, thus giving virtual sensor data with both magnitude and phase information.

The procedures followed for the virtual sensor data to extract frequency and damping information were the same as those followed for the data from the actual sensors.
5. Comparison of Techniques and Results.

5.1. Data Acquisition.

The data for all the results were from one data set obtained from a leaf blower. The details of the data acquisition are given in this section. Multiple data sets with different slew rates were acquired from the leaf blower for this thesis. Many of these data sets turned out to be very difficult to use. Different types of accelerometers were also used but finally one set of data, with a particular type of accelerometer was selected for further analysis.

Data from an engine and drivetrain was also available but this data was far too noisy to be useable for the thesis. Also the impact test data, required as a baseline, was not of very good quality. Therefore, this data set was rejected.

5.1.1. Hardware Used for Data Acquisition.

1. Interface – VXI Technologies, CT 310 A, 16 channels per card.
2. Data Acquisition Software – VXI Technologies DAC Express 3.9.1. and X-Modal III, UC-SDRL.
3. Accelerometers – 7 Triaxial Accelerometers used, Model number PCB356B18.
4. Tachometer – Infrared Type Tachometer, Ono Soki LG-916.
5. Data Processing Software – MATLAB.
5.1.2. Rotating System Used.

Leaf Blower –

- Model Number – 30941
- Displacement – 41cc
- The spark plug and fuel lines were disconnected to reduce back pressure on the piston.
- Single cylinder crank mechanism of the leaf blower was driven by an AC motor.
- 7 Sensors were used, 1 was placed on the cylinder head and 6 were placed on the rotor housing.
- An Infrared tachometer was used.

Howard AC motor

- Rated 155/230V and 60Hz
- Max RPM of 3450 RPM.

5.1.3. Digital Signal Processing.

Two types were performed on the leaf blower one was to capture rotating data the DSP parameters used for this test are given bellow.

- Software – VXI Technologies DAC Express 3.9.1.
- 20 seconds of data was collected for each data set.
- Sampling frequency used was 51200Hz.
- Block size used was 1024 data points.

The second test performed on the leaf blower was an impact test using an impact hammer. The DSP parameters used for this test are given bellow.

- Software – X-Modal III, UC-SDRL.
• Maximum frequency was specified to be 1600Hz

• The number of frequency lines chosen was 3200.

• 5 averages were done for each set of data.

5.2. Comparison between Standard Sensor Data and Virtual Sensor Data.

5.2.1 Order Tracks.

The order tracks for the standard sensor data have a much lower magnitude on average than the virtual sensor order tracks. This is because of the averaging caused by the application of the virtual sensor. Since the absolute magnitude is not very important, no attempt was made to normalize the virtual sensor order track.

The virtual sensor order tracks shown in Figures 1 and 2 were normalized using random values to compare the standard order tracks to the virtual sensor order track. The actual difference in magnitude is shown in Figure 27
The virtual sensor order track used is actually the primary (σ₁) values from the singular value matrix. This is because the noise in the original data will be more apparent in σ₂, σ₃ and so on. It can be seen in Figure 27 that the virtual sensor order track has far less noise in the order track than the order tracks of the separate DOFs. Also, the virtual sensor order track contains all the more dramatic features of the three order tracks such as the dips in magnitude at 423 RPM, 809 RPM and 893 RPM as well as the peak at 2360 RPM.

5.2.2. Operating Shape Decomposition.

The plot of the Singular values obtained from the SVD of the order tracks provides a good idea of the location of natural frequencies of the structure. These peaks occur at the RPM at which these operating shapes are most excited. Figure 28 shows this plot of the singular values obtained from the order tracks in the +y direction.
If these plots in Figure 28 are compared to the plots obtained from the virtual sensor order tracks in Figure 29, the virtual sensor singular value plots have far less noise than the plots from the actual data. This is especially true in the valleys of the singular value plots as shown in Figure 30. An issue noticed in the singular value plots for the virtual sensor data is the peak at RPM bin 719 in the standard sensor data is almost absent in the data from the virtual sensor. The peak at RPM bin 644 is slightly more prominent in the virtual sensor MEOT.
Figure 29 - Singular value plot of for virtual sensor data from location 1.

The magnitude of the singular values obtained from the virtual sensor data is more than that of the singular values obtained from the actual sensors. This is a direct result of the magnitude of the virtual sensor order tracks being more than the magnitude of the standard order tracks.

Figure 30 – Singular value plots for location 1.
5.2.3 Mode Enhanced Order Track.

The MEOT obtained from the virtual sensor data has less noise than the MEOT obtained from the standard order track data. This is a result of the less noisy order tracks as seen in Section 5.2.1. The peaks of the MEOT obtained from the virtual sensor data have a much higher magnitude than the peaks obtained from the standard order track data, this is because the primary singular value from the virtual sensor is associated with the information that is common to all the individual sensors. This is a form of synchronous averaging.

![Figure 31 – MEOT calculated w.r.t. RPM bin 666 using standard order track data as well as virtual sensor data.](image)

There is a huge reduction in the noisiness of the data in all RPM bins but this effect is most observable between RPM bin 200 and RPM bin 500. This reduction in the noise present in the data is also very apparent between RPM bin 750 and RPM bin 996. This is the region where the RPM was kept constant.

In the region between RPM bin 660 and RPM bin 680, where the peak exists, there is not much change in the data other than the magnitude of the peak.
The only large difference that can be seen around the peak in Figure 31 is the absence of the trough present between RPM bins 672 and 674 in the MEOT calculated from the virtual sensor data. The MEOT for the 713th RPM bin and 644th RPM bin show similar properties to the MEOT of the 666th RPM bin.

Reduction of noise around the peak is observed in the MEOT calculated with respect to RPM bin 698. This would possibly contribute to a better estimation of the frequency and damping of this peak.

In Figure 33, a more prominent peak is seen in the MEOT obtained from the virtual sensor data. In this MEOT, it can also be observed that another peak at the 713th RPM bin. This peak is not taken into account.
because its magnitude is not equal to the $\sigma_1$ value at this RPM bin while the magnitude of the peak at the 698th RPM bin is equal to the $\sigma_1$ value at this RPM bin.

Figure 34 – $\sigma_1$ Singular Value curve and MEOT calculated w.r.t. RPM bin 698 for the virtual sensor data.

The peak that is observed at the 719th RPM bin for the standard order track data is not seen in the virtual sensor data.

Figure 35 – MEOT w.r.t. RPM bin 719 for the standard order track data and MEOT w.r.t. RPM bin 720 for the virtual sensor data.
5.3. Results.

5.3.1. Impact Test Data.

To get an understanding of the natural frequencies of the structure, an Impact test was conducted. The parameters used for the impact test are listed in Section 5.1. The data from the impact test was then analysed in X-Modal III. Figure 36 is a sample FRF from the data obtained from this test. The position of the accelerometers for this test were the same as the positions used in the rotating tests.

![Frequency Response Function](image)

*Figure 36 – FRF between input 1 and output 1.*

Input 1 is an input in the x direction at location 1 on the leaf blower and output 1 is the response data in the x direction at the same location. The points of interest in this FRF are between 200 Hz to 400 Hz. This is the region in which the 6th and 7th orders of the system have a high magnitude. From the CMIF plot, a number of peaks can be seen in this frequency range.
In Figure 37, there are four peaks present in the frequency range of interest. This corresponds to the results obtained from the virtual sensor data.

Using modal parameter estimation (MPE) algorithms, the frequency and damping was found. The increase in damping due to the exponential window was accounted for. These frequency and damping values will be compared to those obtained from the MEOT.
5.3.2. Comparison of Order Track MEOT Results and Impact Test Results.

Table 1 – Comparison of frequency estimates between MEOT obtained from a standard order track and an impact test.

<table>
<thead>
<tr>
<th>RPM bin</th>
<th>MEOT (Hz)</th>
<th>X-Modal (Hz)</th>
<th>Percentage Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>644</td>
<td>168.1413</td>
<td>164.6201</td>
<td>2.138985458</td>
</tr>
<tr>
<td>666</td>
<td>200.9454</td>
<td>207.6498</td>
<td>3.228705253</td>
</tr>
<tr>
<td>698</td>
<td>257.3778</td>
<td>249.7797</td>
<td>3.04192054</td>
</tr>
<tr>
<td>713</td>
<td>302.6949</td>
<td>293.3494</td>
<td>3.185791415</td>
</tr>
<tr>
<td>719</td>
<td>311.1453</td>
<td>NA</td>
<td>NA</td>
</tr>
</tbody>
</table>

The frequency estimates obtained from the MEOT of the standard order track data seems to mostly estimate higher than the frequency estimates obtained from the impact test. The only exception is the peak at RPM bin 666 and the peak at the RPM bin 719. The peak at the RPM bin 719 does not exist in the impact test data as seen in Figure 37. At RPM bin 666 the MEOT underestimates the frequency of the peak. This could be explained by non-linearity in the structure. The leaf blower is a very complex structure, with a number of parts.

Table 2 – Comparison of Damping ratio estimates between MEOT obtained from a standard order track and an impact test.

<table>
<thead>
<tr>
<th>RPM bin</th>
<th>MEOT (%)</th>
<th>X-Modal (%)</th>
<th>Percentage Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>644</td>
<td>0.0565</td>
<td>4.7044</td>
<td>98.79</td>
</tr>
<tr>
<td>666</td>
<td>0.0267</td>
<td>2.2820</td>
<td>98.83</td>
</tr>
<tr>
<td>698</td>
<td>0.0345</td>
<td>2.3800</td>
<td>98.55</td>
</tr>
<tr>
<td>713</td>
<td>0.0124</td>
<td>1.1654</td>
<td>98.94</td>
</tr>
<tr>
<td>719</td>
<td>0.0229</td>
<td>NA</td>
<td>NA</td>
</tr>
</tbody>
</table>

As can be seen in Table 2, the damping estimates obtained from the MEOT are very inaccurate and should not be considered. These inaccuracies can be because of the complexity of the structure. This complexity
would lead to the existence of non-linear damping. Bad estimation of damping is commonly seen in operating data.

5.3.3. Comparison of the Virtual Sensor Order Track MEOT Results and Impact Test Results.

Table 3 – Comparison of frequency estimates between MEOT obtained from the virtual sensor order track and an impact test

<table>
<thead>
<tr>
<th>RPM bin</th>
<th>MEOT(Hz)</th>
<th>X-Modal(Hz)</th>
<th>Percentage Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>644</td>
<td>168.1804</td>
<td>164.6201</td>
<td>2.162737114</td>
</tr>
<tr>
<td>666</td>
<td>200.886</td>
<td>207.6498</td>
<td>3.257311107</td>
</tr>
<tr>
<td>698</td>
<td>257.2597</td>
<td>249.7797</td>
<td>2.994638876</td>
</tr>
<tr>
<td>713</td>
<td>302.5388</td>
<td>293.3494</td>
<td>3.13257842</td>
</tr>
<tr>
<td>719</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
</tr>
</tbody>
</table>

The frequency estimates obtained using the virtual sensor data are marginally better than the standard sensor estimates. The virtual sensor estimates also seem to mostly overestimate the frequency except in the case of the peak at the 666th RPM bin. The virtual sensor data does match up to the impact test data better in that it does not find a peak at the 719th RPM bin.

Table 4 – Comparison of damping estimates between MEOT obtained from the virtual sensor order track and an impact test.

<table>
<thead>
<tr>
<th>RPM bin</th>
<th>MEOT (%)</th>
<th>X-Modal (%)</th>
<th>Percentage Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>644</td>
<td>0.0793</td>
<td>4.7044</td>
<td>98.31</td>
</tr>
<tr>
<td>666</td>
<td>0.0223</td>
<td>2.2820</td>
<td>99.02</td>
</tr>
<tr>
<td>698</td>
<td>0.0106</td>
<td>2.3800</td>
<td>99.55</td>
</tr>
<tr>
<td>713</td>
<td>0.0240</td>
<td>1.1654</td>
<td>97.94</td>
</tr>
<tr>
<td>719</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
</tr>
</tbody>
</table>

The virtual sensor estimate of damping, similar to the standard sensor estimate, is too inaccurate to be considered.
6. Conclusion and Future Recommendations.

6.1 Conclusion.

Instantaneous RPM estimation is extremely important while conducting tests on any rotational systems. The details of how to obtain an accurate instantaneous RPM estimate are mentioned in this thesis. Also, it is important to have the correct instantaneous RPM profile which ensures no overlapping of the order track when plotted against the instantaneous RPM.

The orders were tracked using the TVDFT. It is important to have an understanding of the dominant orders at various RPMs. This can be done with the help of RPM spectral maps and order track autopower plots. A singular value plot can be used to find the RPMs at which the singular value curve peaks. These peaks signify RPMs at which a specific operating shape is highly excited.

Knowledge of the RPM at which the peaks are located is needed while calculating the MEOT because it is calculated with respect to specific RPM bins. The RPM bins chosen for this calculation are those which contain peaks. To estimate the frequency and damping, it is important to know the dominant order at each of these RPM bins since this information is needed to calculate $\omega$. The least square local SDOF parameter estimation algorithm was used to estimate these parameters.

The singular value decomposition technique was used on the crosspower matrix at each order and RPM, this resulted in a set of virtual data. The crosspower matrix contained response data from one location but three DOFs. The resulting data set is called the virtual sensor order track data. This data was then processed in the same way the standard order track data was processed. It was observed that the virtual sensor order track data contained much less noise than the standard order track data,

An impact test was conducted to find the natural frequencies of the structure. It was found that the frequencies of the peaks estimated from the two sets of data and the impact test matched up well while
the estimated damping ratio did not match at all. The virtual sensor estimates were marginally better than
the standard sensor estimates. The standard order track data showed the presence of a peak which was
missing in both the virtual sensor data and the impact test data.

6.2. Recommendations for Future Work.

A more robust method is required for choosing the order which will be used to calculate $\omega$ for parameter
estimation. As explained in Section 4.7, wrongly choosing the order that is dominant at the required RPM
bin can cause large errors in estimation of frequencies and damping. The goal here would be to come up
with a procedure for choosing the order rather than the method used in this thesis.
In Section 4.8 it was mentioned that the right singular value that the left singular vector was multiplied
with was chosen through trial and error. While looking at SVD based modal parameter estimation
techniques, the right singular value chose is the value from one of the driving points in the system. In the
case of rotational data, there is no driving point available. Finding a method of choosing the right singular
value used can be another topic of study. This can lead to a more in-depth study of the phase data in the
MEOT.
Another topic which needs further study is the scaling of the virtual sensor order track. As discussed in
Section 5.2.1 and in Reference 9, the virtual sensor order track has a much higher magnitude than the
standard order tracks. The details about choosing the right scaling factor and how it affects the estimation
of frequency and damping should be studied further.
As seen in Section 5.3.2 and 5.3.3, the damping estimates obtained are extremely inaccurate. A better
method to estimate damping of the structure is required.
References


Appendices.

Appendix A – Matlab Scripts.

RPM Estimation from Raw Tachometer Data and Spline Fitting

clear all; close all; clear;

data=readDX('Dx2015_0513_1613.sdf');

tachsig=data.ydata(11,:);

bsize=2048;
olap=0.25;
fsamp=51200;
freq= linspace(0,fsamp/2,bsize/2);
deltaT=1/fsamp;
t=0:deltaT:(length(tachsig)*deltaT-deltaT);
mTach1=(max(tachsig)+min(tachsig))/2;
tachsig1=tachsig-mTach1;
Nn=floor(((length(tachsig)-bsize)/((olap)*bsize))+1);
w=hann(bsize);
w=w.';

%%% RPM Estimation
jj=0;
for ii=1:length(tachsig1)-1
    if sign(tachsig1(ii))==1 && sign(tachsig1(ii+1))==-1
        jj=jj+1;
        tn(jj)=(0-tachsig1(ii))*(t(ii+1)-t(ii))/(tachsig1(ii+1)-tachsig1(ii))+t(ii);
    end
end

for ii=1:length(tn)-2
    RPMest1(ii)=60/(tn(ii+1)-tn(ii));
tTach1(ii)=(tn(ii+1)+tn(ii))/2;
end

figure()
plot(tTach1,RPMest1);
xlabel('Time(s)');
ylabel('RPM');
title('Initial Tach Estimate');
grid;

%%% Spline Fitting

spl=2;
RPMest2=RPMest1;
tTach2=tTach1;

while spl==2
  clearvars RPMS;
  iteration=2;
  while iteration==2
    seg=30;
    N1=floor(length(tTach1)/seg);
    clearvars RPMs;
    time2=linspace(tTach2(1),tTach2(end),seg);
    RPMs=spline(tTach2,RPMest2,time2);
    figure()
    plot(time2,RPMs);
    xlabel('Time(s)');
ylabel('RPM');
title('Spline Fitted Tach Estimate');
grid;
    iteration=input('Initiate Shaving Algorithm Iteration: 1.No 2.Yes - ');
    if iteration==2
      RPMs=spline(time2,RPMs,tTach2);
      limit=input('Enter Max Allowable Difference - ');
      redRPM=RPMest2(abs(RPMest2-RPMs)<limit);
      redtTach=tTach2(find(abs(RPMest2-RPMs)<limit));
      RPMest2=redRPM;
      tTach2=redtTach;
    end
  end
figure()
plot(tTach2,RPMest2)
xlabel('Time(s)');
ylabel('RPM');
title('Spline Fitted Tach Estimate');
grid;
end
RPMs=spline(tTach2,RPMest2,time2);
RPMS = spline(time2, RPMs, 1);
RPMS(RPMS < 0) = 0;

clearvars RPMest2;
RPMest2 = RPMest1;
Tach2 = Tach1;
spl = input('1 - Accept RPM Estimate 2 - Re-do RPM Estimation')

counter = 0;
for ii = 1:bsize*olap:length(RPMS)-bsize
    counter = counter + 1;
    instRPM(counter) = mean(RPMS(ii+1:ii+bsize));
tRPM(counter) = mean(t(ii+1:ii+bsize));
end

**RPM Spectral Maps.**

response = zeros(21, 512000);
response(1,:) = data.ydata(1,:);
response(2,:) = data.ydata(2,:);
response(3,:) = data.ydata(3,:);
response(4,:) = data.ydata(4,:);
response(5,:) = data.ydata(5,:);
response(6,:) = data.ydata(6,:);
response(7,:) = data.ydata(8,:);
response(8,:) = data.ydata(9,:);
response(9,:) = data.ydata(10,:);
response(10,:) = data.ydata(12,:);
response(11,:) = data.ydata(13,:);
response(12,:) = data.ydata(14,:);
response(13,:) = data.ydata(7,:);
response(14,:) = data.ydata(15,:);
response(15,:) = data.ydata(16,:);
response(16,:) = data.ydata(17,:);
response(17,:) = data.ydata(18,:);
response(18,:) = data.ydata(19,:);
response(19,:) = data.ydata(20,:);
response(20,:) = data.ydata(21,:);
response(21,:) = data.ydata(22,:);

freqresp = zeros(Nn, 21, bsize);
instRPM1 = zeros(Nn, 1);
counter = 0;
w = hann(bsize);
w = w';
for ii = 1:bsize*olap:length(RPMS)-bsize
    responseblock = zeros(21, bsize);
    responseblock = response(:, ii+1:ii+bsize);
    f_response = zeros(21, bsize);
    for jj = 1:21

Order Tracking using TVDFT with OCM.

```matlab
f_response(jj,:)=fft(responseblock(jj,:).*w);

end

f_response=(f_response/bsize)*2;
counter=counter+1;
freqresp(counter, :) = f_response;
instRPM1(counter) = mean(RPMS(ii+1:ii+bsize));

end
C=colormap;
for ii=1:30
    C(ii,:)=[1 1 1];
end;
kk=input('Enter location number');
figure();
waterfall(freq,instRPM1,log(squeeze(abs(freqresp(:,kk,1:end/2))));
xlabel('Frequency(Hz)');
ylabel('RPM');
Zlabel('Magnitude(volts)');
title('RPM Spectral Map');
axis([0,2100,1300,20000,-5,3]);
view(0,90);
```

```matlab
%% TVDFT
r=1;
z=floor(length(RPMS)/length(instRPM));
oo=input('Enter maximum order to track - '); 
for o=1:oo*2
for ii=1:length(instRPM)
    Nsize=floor(r*60/instRPM(ii)/deltaT);
    Responseblock=response(:,(ii-1)*z+1:(ii-1)*z+Nsize);
    instrpmblock=RPMS((ii-1)*z+1:(ii-1)*z+Nsize);
    cosvec=zeros(1,Nsize);
    sinvec=zeros(1,Nsize);
    for jj=1:Nsize
        cosvec(jj)=cos(2*pi*(o*0.5)*deltaT/60*sum(instrpmblock(1:jj)));
        sinvec(jj)=sin(2*pi*(o*0.5)*deltaT/60*sum(instrpmblock(1:jj)));
    end
```
A=[cosvec;sinvec];
B=A*Responseblock;'
B=B./length(Responseblock);
coeff(:,o,ii)=B(1,:)+1i*B(2,:);
clear A B cosvec sinvec Nsize instrpmblock timeblock;

end
end

%% OCM

term1=zeros(21,15,length(instRPM));
term2=zeros(21,15,length(instRPM));

for kk=1:length(instRPM)
    for ii=1:15
        Coeffi=squeeze(coeff(:,ii,kk));

        for jj=1:15
            
            Coeffj=squeeze(coeff(:,jj,kk));
            term1=exp(2*pi*(Coeffi(:)*instRPM(kk).*deltaT/60));
            term2=exp(2*pi*(Coeffj(:)*instRPM(kk).*deltaT/60));
            term3=term1.*term2;
            term4=sum(term3);
            OCM(ii,jj,kk)=term4/length(Responseblock);
        end
    end
end

end

for ii=1:21
    for kk=1:length(instRPM)
        invOCM=inv(squeeze(OCM(:,:,kk)));
        Coeff(ii,:,kk)=(invOCM(:,:))*squeeze(coeff(ii,:,kk)).';

    end
end

%% Plotting Order Tracks

K=input('Enter order track to display - ');
O=input('Enter response location - ');

plot(instRPM,abs(squeeze(Coeff(O*2,K,:))));
xlabel('RPM');
ylabel('Magnitude');
title('Order Track');
grid;

SVD, Order Track Autopower, MEOT and LSL SDOF Parameter Estimation Algorithm

counter=0;
for ii=1:3:21
    counter=counter+1;
    coeff(counter,:,:)=squeeze(Coeff(ii,:,:));
end

%% Singular Values
for ii=1:length(instRPM)
    [u1 s1 v1]=svd(coeff(:,:,ii));
    S(:,ii)=diag(s1);
    U1(:,:,ii)=u1;
    V1(:,:,ii)=v1;
end

%% OA
for jj=1:length(instRPM)
    OA(:,:,jj)=squeeze(coeff(:,:,jj)).^2;
end

%% MEOT
y=zeros(25,1);
y(:,1)=1;
I=input('Enter RPM bin of peak to be analysed');
for ii=1:length(instRPM)
    MEOT(ii)=(U1(:,1,I)'*(Coeff(:,:,ii))*(V1(:,1,I)));
end

xx=1:length(instRPM);

%% Synthesis of SDOF
O=input('Enter dominant order');
w=(instRPM./60).*(O).*2.*pi;
Hlsl=[meot(I-1) 1;meot(I) 1;meot(I+1) 1];
WHlsl=[j*w(I-1)*meot(I-1);j*w(I)*meot(I);j*w(I+1)*meot(I+1)];
Hlsli=pinv(Hlsl);
parameters=Hlsli*WHlsl;
for ii=1:length(instRPM)
    HH(ii)=(parameters(2)/((1j*w(ii))-parameters(1)))+\(conj(parameters(2))/((j*w(ii))-conj(parameters(1))));
end

Crat=(real(parameters(1)))/(sqrt(imag(parameters(1))^2+(real(parameters(1))^2)));
Crat=Crat*100;

F1=imag(parameters(1))/(2*pi);

%% Plots
semilogy(instRPM,S(1,:));
xlabel('RPM');
ylabel('Singular Values');
title('Plot of Sigma 1 Singular Values');
grid;

semilogy(xx,S(1,:));
xlabel('RPM bin');
ylabel('Singular Values');
title('Plot of Sigma 1 Singular Values');
grid;

semilogy(instRPM,abs(MEOT),instRPM,S(1,:));
xlabel('RPM');
ylabel('Singular Values');
title('Plot of Sigma 1 Singular Values and MEOT');
grid;

semilogy(xx,abs(MEOT),xx,S(1,:));
xlabel('RPM bin');
ylabel('Singular Values');
title('Plot of Sigma 1 Singular Values and MEOT');
grid;

semilogy(instRPM,abs(MEOT),instRPM,abs(HH));
xlabel('RPM/Frequency(Hz)');
ylabel('Singular Values/Magnitude');
title('Plot of MEOT and synthesized SDOF FRF');
grid;

semilogy(xx,abs(MEOT),xx,abs(HH));
xlabel('RPM bin/Frequency(Hz)');
ylabel('Singular Values/Magnitude(Volts)');
title('Plot of MEOT and Synthesized SDOF FRF');
grid;

plot(xx,abs(squeeze(OA(1:7,3,:))));
xlabel('RPM bin');
ylabel('Autopower(Volts^2)');
title('Order Track Auto Power');
gird;
Virtual Sensor Order Tracks.

l(1,:)=[1;3];
l(2,:)=[4;6];
l(3,:)=[7;9];
l(4,:)=[10;12];
l(5,:)=[13;15];
l(6,:)=[16;18];
l(7,:)=[19;21];

%% Virtual Sensor

for ii=1:length(instRPM)
    for jj=1:25
        for kk=1:7
            or=Coeff(l(kk,:),jj,ii);
            orh=or';
            AA=zeros(length(or),length(or));
            AA=or*orh;
            [u,s,v]=svd(AA);
            Uex(:,:,ii)=u;
            UU=u*v(2,1);
            for ll=1:3
                UI=[imag(UU(:,ll)) real(UU(:,ll))];
                UUI=UI.'*UI;
                [uui ssi vvi]=svd(UUI);
                theta=atan(vvi(2,2)/(-vvi(1,2)));
                urot(:,ll)=(cos(theta)-(j)*sin(theta))*UU(:,ll);
            end
            S(kk,jj,ii)=max(max(urot(:,1)*diag(s).'));
            Urot(:,:,ii)=urot;
        end
    end
end

%% plots

plot(Uex);
xlabel('Real Values');
ylabel('Imaginary values');
title('Complexity Plot Before Normalization');
grid;

plot(Urot);
xlabel('Real Values');
ylabel('Imaginary values');
title('Complexity Plot After Normalization');
grid;

semilogy(instRPM, abs(squeeze(S(O,K,:))));
xlabel('RPM');
ylabel('Magnitude');
title('Virtual Sensor Order Track');
grid;
Appendix B – Rotating System and Experimental Setup.

Leaf Blower –

![Experimental Setup](image)

*Figure 38 - Experimental Setup*

![Infrared Tachometer](image)

*Figure 39 - Infrared Tachometer*
Figure 40 - Accelerometer Positions

Figure 41 – Variac
Appendix C – Plots.

Sample RPM Spectral Maps.

Figure 42 - RPM Spectral Map From Location 1-4 in the Y Direction.
Figure 43 - RPM Spectral Map From Location 5-7 in the Y Direction.
Sample Order Tracks (Standard and Virtual).

Figure 44 - Order track of order 6 at location 3 in the Y direction

Figure 45 - Order track of order 7 at location 3 in the Y direction
Figure 46 - Order track of order 5.5 at location 3 in the Y direction

Figure 47 - Order track of order 6.5 at location 3 in the Y direction
Figure 48 - Virtual Sensor Order track of order 6 at location 3

Figure 49 - Virtual Sensor Order track of order 7 at location 3
Figure 50 - Virtual Sensor Order track of order 5.5 at location 3

Figure 51 - Virtual Sensor Order track of order 6.5 at location 3