I, William H Thatcher IV, hereby submit this original work as part of the requirements for the degree of Master of Science in Physics.

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An Inquiry of the Conventional Cavity Perturbation Method, with Direct Application to Various Liquid Samples

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An Inquiry of the Conventional Cavity Perturbation Method, with Direct Application to Various Liquid Samples

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Abstract- There are a number of measuring techniques which allow for the determining of electrical properties, such as the complex permittivity and complex permeability. Of these techniques, one in particular- the conventional cavity perturbation method, was explored. Through the utilization of this method, and a combination of standard rectangular waveguides, a discrete spectrum from approximately 2-22 GHz was established for the analysis of various liquid samples. It was noticed that for a given waveguide, as mode number increased, there was a systematic increase in permittivity values, as well as a discontinuity in permittivity values going from one waveguide to the next. As a result, several corrections are introduced, with commentary on their results.
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I. Introduction

When trying to determine the dielectric properties of a material, such as the complex permittivity and/or the complex permeability, there are two general classes of measurements usually applied: resonant and non-resonant techniques.\(^{[12]}\)

Non-resonant techniques involve sending an electromagnetic (EM) signal at a dielectric material, and analyzing the reflected or the reflected and transmitted signal, in order to obtain the complex permittivity and permeability for a band of frequencies.\(^{[12]}\) Most commonly used, are free space methods, or the transmission-line method using a waveguide or coaxial line.\(^{[1-3,11]}\)

Resonant techniques are normally divided into two subsets: resonator methods, and cavity perturbation methods. For the resonator methods, a dielectric sample is used as a resonator or as a key part of a resonant circuit, and through details of the circuits resonant characteristics, dielectric properties may be obtained.\(^{[12]}\) For cavity perturbation methods, generally there are two types. The first involves wall-perturbations of the cavity, while the more common is known as material-perturbation.\(^{[13]}\) For material- perturbations, a dielectric sample is inserted into a cavity resonator, and through the observed shift in resonant frequency and quality-factor of the cavity, the complex permittivity and permeability of the sample may be obtained.\(^{[11-13]}\) Resonant
methods are usually higher in sensitivity, however they only yield information at discrete frequencies.\cite{4-10} Henceforth, the focus shall shift to only material- perturbation of resonant cavities, referred to now as simply conventional cavity perturbation theory (c.c.p.t.), and its application to various dielectric materials.

II. Conventional Cavity Perturbation Theory

Conventional cavity perturbation measurements can be quite accurate, due to their high sensitivity, however there are many assumptions involved in the theory to be considered. A brief derivation of the complex permittivity equations using perturbation theory will be carried out below, bringing attention to some of these approximations.

To start, assume that the resonant cavity is a perfect conductor. The shift in complex angular frequency is then given as:\cite{12-14}

$$\frac{\omega_2 - \omega_1}{\omega_1} = \frac{\iiint_{V_c} (\Delta \varepsilon \varepsilon_1 \mathbf{E}_2 \cdot \mathbf{E}_1^* + \Delta \mu \mu_1 \mathbf{H}_2 \cdot \mathbf{H}_1^*) dV}{\iiint_{V_c} (\varepsilon_1 \varepsilon_2 \varepsilon_1 \mathbf{E}_2 \cdot \mathbf{E}_1^* + \mu_1 \mu_2 \mu_1 \mathbf{H}_2 \cdot \mathbf{H}_1^*) dV} \quad (1)$$

Where $V_c$ is the cavity volume, $\varepsilon$ and $\mu$ are the complex permittivity and complex permeability, while $\mathbf{E}$ and $\mathbf{H}$ are the electric and magnetic field within the cavity, respectively. The subscripts “1” and “2” denote before and after the insertion of a sample.

Any material inserted into the cavity must be prepared such that it does not perturb the resonant frequency by more than a few percent.\cite{14} As such, depending on the dielectric constant and loss
of a given material, one may have to adjust the sample size in order to fulfill the perturbation theory requirements, or to increase resolution.[12]

If one assumes:

1. The original material in the resonant cavity is lossless.

2. The sample is homogeneous with volume much smaller than that of the cavity.

3. The EM field outside of the sample does not change.

Then equation (1) may be approximated as[12-13]:

$$\frac{\omega_2 - \omega_1}{\omega_1} \approx \frac{-\iint_{V_s} (\Delta \varepsilon E_1 \cdot E_2 + \Delta \mu H_1 \cdot H_2) dV}{\iint_{V_c} (\varepsilon_1 E_1 \cdot E_2 + \mu_1 H_1 \cdot H_2) dV}$$

(2)

$$\approx \frac{-\iint_{V_s} (\Delta \varepsilon E_1 \cdot E_2 + \Delta \mu H_1 \cdot H_2) dV}{2 \iint_{V_c} \varepsilon_1 E_1 \cdot E_2 dV}$$

(3)

Where $V_s$ is the volume of the sample contained by the cavity. It may be prudent to point out that (3), when further approximated by $E_1 \approx E_2$, $H_1 \approx H_2$, and $\omega_1 \approx \omega_2$, shows that for any increase in permittivity value, there is a decrease in resonant frequency.[12-13,17] Here, the concern is with determining the complex permittivity of various liquids. And so moving forward with the assumptions that inside of the cavity is vacuum: $\mu_1 = \mu_2 = \mu_0$, and $\varepsilon_1 = \varepsilon_0$. Equation (3) becomes[12]:

$$\frac{\omega_2 - \omega_1}{\omega_2} \approx \frac{-(\varepsilon_r - 1) \iint_{V_s} (E_1 \cdot E_2) dV}{2 \iint_{V_c} |E_1|^2 dV}$$

(4)

Where $\varepsilon_r$ is the relative complex permittivity. The relative complex permittivity is defined as: $\varepsilon_r = \varepsilon'_r - i\varepsilon''_r[15]$, with $\varepsilon'_r$ being the dielectric constant, $\varepsilon''_r$ being the dielectric loss, and $i^2 = -1$. Keeping in mind:[12,16]
\[ \omega = \omega_r + i\omega_i \quad (5) \]

\[ \omega_r = 2\pi f \quad (6) \]

\[ Q = \frac{\omega_r}{2\omega_i} \quad (7) \]

With \( Q \) being the quality-factor of the cavity. Assume that: \( \omega_{r1} \approx \omega_{r2} \), and \( \omega_i \ll \omega_r \), then\(^{[12]}\):

\[ \frac{\omega_2-\omega_1}{\omega_2} = \frac{(\omega_{r2}-\omega_{r1})+i(\omega_{i2}-\omega_{i1})}{\omega_{r2}(1+i\frac{\omega_{i2}}{\omega_{r2}})} \quad (8) \]

And if \( Q_2 \gg 1 \),

\[ \approx \left[ \frac{(f_2-f_1)}{f_2} + i \left( \frac{1}{2Q_2} - \frac{1}{2Q_1} \right) \right] \quad (9) \]

From equations (4) and (9) we have:

\[ \frac{(f_2-f_1)}{f_2} + i \left( \frac{1}{2Q_2} - \frac{1}{2Q_1} \right) = -\frac{(\varepsilon_r-1)}{2} \iint_{V_c} |E_1|E_2|dV \quad (10) \]

Equating the real and imaginary components in (10), and calculating the volume integrals, results in the familiar c.c.p.t. complex permittivity equations:\(^{[7, 17]}\)

\[ \varepsilon'_r = \frac{V_c}{2V_s} \cdot \frac{(f_1-f_2)}{f_2} + 1 \quad (11) \]

\[ \varepsilon''_r = \frac{V_c}{4V_s} \cdot \left( \frac{1}{Q_2} - \frac{1}{Q_1} \right) \quad (12) \]

III. Experimental Design

Several two-port, circular-iris coupled, rectangular waveguide cavities were utilized to determine the complex permittivity of various liquid samples. A combination of WR-284, WR-137, WR-
90, WR-62, and WR-42 waveguides (S, C, X, Ku, and K- Bands, respectively) were used to create a discrete frequency range from approximately 2-22 GHz to view the odd-mode standing wave resonances in each cavity (see Appendix A). The odd-mode resonances refer to those in which the electric field is at a maximum in the center of the cavity, while the magnetic field is at a minimum. All waveguide configurations, save the K-Band, had additional rectangular waveguide components placed in between the irises and coaxial-waveguide connectors, in order to limit evanescent wave interactions with the fields inside of the central cavity.

Cylindrical slots were drilled through the geometric center of the broad sides of the waveguides, allowing for a borosilicate-glass capillary containing a sample (see Appendix B) to be inserted. To maximize signal response, the orientation of the capillary was chosen such that the sample would be aligned parallel to the electric field of the EM standing wave in the cavity. An aluminum block, with a through-hole slightly larger than that of the outer diameter of the capillary, was then centered and fastened atop the waveguide, to provide stability and alignment for the capillary.

The waveguide cavities were connected to an Agilent (now Keysight) PNA Network Analyzer (Model: N522A, 10 MHz- 26.5 GHz), where the S21 and S12 scattering parameters were measured, to observe the electromagnetic standing wave resonances within the cavity.

IV. Application of Theory

Since the scope of this work is to measure the complex permittivity values, only the odd resonant modes are of interest (i.e. TE_{10n} modes, where n = 1, 3, 5…), as they refer to those in which the electric field is maximum at the position of the sample. A similar process, using the even
modes (where the magnetic field is maximum in the center of the cavity), can be used to determine a given sample’s permeability.$^{[12, 17]}$

Using the Network Analyzer, the peak frequency of a given mode, $f_r$, is measured, as well as the two half-power frequencies to the left and right of the peak (also referred to as the -3 dB points), denoted $f_1$ and $f_2$, respectively. After these are determined, the quality-factor may be calculated through the relationship:$^{[14, 16]}$

$$Q = \frac{f_r}{f_2 - f_1} \quad (13)$$

Once $f_r$ has been obtained and the quality-factor for a given mode has been calculated, with and without the insertion of a sample, equations (11) and (12), may be used, and take the form:

$$\varepsilon'_r = \frac{V_c'}{2V_s} \left( \frac{f_g - f_s}{f_s} \right) + 1 \quad (14)$$

$$\varepsilon''_r = \frac{V_c'}{4V_s} \left( \frac{1}{Q_s} - \frac{1}{Q_g} \right) \quad (15)$$

With,

$$V_c' = V_c - V_g \quad (16)$$

Where $V_g$ is the glass volume of the capillary. The subscripts “g” and “s” refer to: the cavity with the empty glass capillary, and the cavity with the sample in question, respectively.

Using the above method, the frequency dependence of the complex permittivity was measured for a variety of organic, inorganic, and biological liquid samples.
V. Results

With the ever expanding fields of biological studies, various property knowledge of common organic solvents used to suspend biological samples has become increasingly more important. Results for acetone, ethanol, and methanol may be found in Appendix D. Also in Appendix D, are results for chlorophyll suspended in acetone, and three ethanol based solutions of leutin. Both leutin and chlorophyll are essential components to a plant’s photosynthesis cycle.\[20\] Due to their fairly high dielectric constant and loss values, resonant modes beyond 14 GHz could not be resolved for the capillary size used in this work.

An inorganic substance tested was octamethyltrisiloxane (DOW Corning, OS-20), along with two 1.5 M HCl mixtures of the siloxane. Siloxanes are used commonly for electrowetting applications, as well as for insulating fluids in electronics or experiments using high frequency electric fields. Acidifying of the siloxane has proved helpful thus far in removing unnecessary oxide layers which can form while constructing and operating GaLMA (Gallium liquid metal alloy) devices.\[18\]

The complex permittivity results for a 3:1 mixture of OS-20 to HCl is shown below. The plain siloxane complex permittivity data, as well as that pertaining to a 1:1 mixture of OS-20 and HCl may be found in Appendix C.

*In all plots the color scheme is as follows: S-Band- yellow, C-Band- green, X-Band – blue, Ku-Band- orange, and K-Band- brown.*
Fig. 1: Relative permittivity of 3:1 OS-20 to HCl mixture. The vertical error bars correspond to the waveguide average of constituent mode standard deviations calculated over three trials. Horizontal error bars were too small to distinguish, and as such, were discarded.

The relative permittivity appears fairly constant over the entire spectrum, hovering near or just below two, with the loss on average being approximately zero.

Further inspection, however, reveals a couple of interesting results:

1. As mode number increases for a given waveguide, an upward trend in dielectric constant and loss are observed.

2. Going from one waveguide to the next, there are slight discontinuities in permittivity values for overlapping frequencies.

Fig. 2: Dielectric loss of 3:1 OS-20 to HCl mixture. The vertical error bars correspond to the waveguide average of constituent mode standard deviations calculated from three trials of data. Horizontal error bars were too small to distinguish, and as such, were discarded.
Another result worth bringing to attention is the negative dielectric loss values determined by the S and C-Band waveguides. This is due to the fact that the cavity walls are not entirely loss-less. When observing extremely low-loss dielectric materials, a fixation of the stored cavity energy on the sample can result in a reduced electromagnetic field at the cavity walls. This reduction in field strength at the cavity walls can cause an increase in the overall quality-factor of the cavity.\(^{11-12}\) From (12), it results that if the sample-loaded cavity has a higher Q, then the overall dielectric loss will be negative.

To better understand the internal field pattern inside of the waveguide cavities, a model of the X-Band waveguide was created using COMSOL Multiphysics. A study of the TE standing modes, shows that there is no apparent change in mode structure as one goes up in frequency and that the...
sample continues to be directly in the center of the maximum strength of the electric field. There was evidence, however, that near the drill holes for the sample, non-uniform fields exist. These non-uniform fields result in discrepancies between the true complex permittivity of the sample, and that which is measured.$^{[21]}$

Fig. 5: COMSOL simulation of TE$_{103}$ mode, viewed from the side. The electric field ranges from 0-1E3 V/m, from blue to red respectively.

Fig. 6: COMSOL simulation of the TE$_{109}$ mode, viewed from the top. The electric field ranges from 0-1.4E5 V/m, from blue to red, respectively.

Fig. 7: COMSOL simulation of the TE$_{109}$ mode viewed down the length of the waveguide cavity. The electric field ranges from 0-1.4E5 V/m, from blue to red respectively. Note that near the top and bottom of the sample, there are non-uniform fields.

VI. Corrections and Discussion

The first correction attempt assumed that the length of each waveguide, as experienced by the EM wave traveling down it, was frequency dependent. A quick manipulation of the equation for allowed modes in a rectangular waveguide (17)$^{[13-14,16]}$, allows one to input the observed resonant frequency of the empty cavity, and solve for the effective length of the cavity.
\[ f_r^2 = c^2 \left( \frac{m}{2a} \right)^2 + \left( \frac{n}{2b} \right)^2 + \left( \frac{p}{2d} \right)^2 \]  

(17)

Where \( c \) is the speed of light in vacuum; \( a, b, \) and \( d \), are the width \((x)\), height \((y)\), and length \((z)\), respectively; and \( m, n, \) and \( p \) are the number of half-wavelengths in the \( x, y, \) and \( z \) directions, respectively. Since the concern here is only those modes in which the electric field is maximum in the center of the cavity, \( n = 0, m = 1 \) and \( p = 1, 3, 5 \ldots \)

Note: equation (17) assumes that there is no loss of signal to the cavity walls, which is not true in application. However, at each frequency, if one calculates the effective volume corrections, due to skin depth, the average percent difference in total volume is approximately 3.332E-6%. As such this correction is generally dismissed.

With the effective length, an effective volume for each mode could be calculated. The correction is demonstrated in Fig. 5 and Fig. 6 on the 3:1 acidic siloxane solution.

![Graph](image-url)  

Fig. 8: Relative permittivity of 3:1 OS-20 to HCl mixture. Error bars were discarded for ease of viewing mode behavior.
This correction slightly levels out the upward trend for lower modes. However, for higher order modes, the behavior is still present.

The second correction aimed to calibrate the waveguide via a sample with assumed constant permittivity values. It was supposed that the 3:1 acidic siloxane mixture had constant relative permittivity and loss values over the given frequency range. An average of observed values was taken, and a point-by-point correction coefficient, \( \zeta \), was determined via the relation:

\[
\varepsilon'_r \cdot \zeta = \varepsilon'_{r,avg}
\]

Where \( \varepsilon'_r \) is the observed dielectric constant, and \( \varepsilon'_{r,avg} \) is the averaged spectrum value. A similar correction can also be made for the dielectric loss.

Fig. 7 displays the results of this correction on a sample of similar dielectric properties, the 1:1 OS-20 to HCl mixture.
While this correction certainly leveled out the observed trend when analyzing a sample of similar dielectric characteristics, it was of little consequence when acting on a significantly different dielectric substance, acetone (see Appendix E). This suggests that the simple coefficient relation proposed is incomplete, and that perhaps correction factors which are functions of the magnitude of the permittivity may be needed.

VII. Summary and Future Work

When conducting permittivity measurements over a broad spectrum, using cavity perturbation theory for rectangular waveguides, two curious results present themselves: For a given waveguide, there is an upward trend in complex permittivity values as mode number is increased. Also, as one goes from a particular waveguide to the next, there seems to be some discontinuities in permittivity values for overlapping frequencies.

Two corrections were introduced, in an attempt to resolve these issues. The first, assumed that the effective length of the central cavity was frequency dependent. Using equation (17), an observed resonant frequency of the empty cavity was used to calculate the effective length for a
given mode. This correction slightly leveled out some of the upward behavior for the lower modes. However, the trend was still noticeably present for the higher modes.

The second correction sought to calibrate the cavity to a sample of known complex permittivity values. It offered correction coefficients for each mode, and proved to be effective at reducing the trend when dealing with a sample of similar electrical characteristics. However, once this correction method was used on results from a sample of significantly different electrical properties, it proved inadequate. This suggested that correction factors, which are functions of the magnitude of the permittivity, may be what is needed.

Future endeavors involve testing various iris sizes, shapes, and thicknesses to see how the observed resonant mode behavior for a cavity changes, in hopes of finding a combination which optimizes accuracy. It is also of interest to observe the mode behaviors at slightly elevated temperatures (i.e. 5-10 °C above room temperature), to see how the upward trend in permittivity values is affected.

A final pursuit that could prove useful is to study the relationship between pressure exerted on the cavity and the resulting shift in resonant frequency. It was noticed that pressing slightly on the top of the cavity would result in the shifting of resonance by several megahertz. This could perhaps explain some of the discontinuities in overlapping frequencies observed when going from one waveguide to another.

VIII. References


IX. Appendices

A. Waveguide Information

cc – central cavity
ec – evanescent cavities
cwc – coax to waveguide connector
ic – iris composition

S-Band: 2.6- 3.95 GHz
   cc – anodized aluminum
   ec – anodized aluminum
   cwc – aluminum
   ic – aluminum

C-Band: 5.85- 8.2 GHz
   cc – copper
   ec – copper
   cwc – copper
   ic – copper

X-Band: 8.2- 12.4 GHz
   cc – silver plated copper
   ec – copper
   cwc – aluminum
   ic – copper

Ku-Band: 12.4- 18.0 GHz
cc – aluminum
c – copper
cwc – aluminum
ic – copper

K-Band: 18.0-26.5 GHz
cc – copper
e – n/a
cwc – copper
ic – copper

B. Glass Capillary Information

OD (outer diameter) = 1.2 mm
ID (inner diameter) = 0.68 mm

C. Pure and Acidified Siloxane Results

Fig. 11: Relative permittivity for pure OS-20, with effective length correction. The vertical error bars correspond to the waveguide average of constituent standard deviations calculated over three trials. Horizontal error bars were too small to distinguish, and as such, were discarded.

Fig. 12: Dielectric loss for pure OS-20, with effective length correction. The vertical error bars correspond to the waveguide average of constituent standard deviations calculated over three trials. Horizontal error bars were too small to distinguish, and as such, were discarded.
D. Acetone, Ethanol, Methanol, Chlorophyll and Lutein Results

Fig. 13: Relative permittivity for 1:1 OS-20 to HCl mixture. The vertical error bars correspond to the waveguide average of constituent standard deviations calculated over three trials. Horizontal error bars were too small to distinguish, and as such, were discarded.

Fig. 14: Dielectric loss for 1:1 OS-20 to HCl mixture. The vertical error bars correspond to the waveguide average of constituent standard deviations calculated over three trials. Horizontal error bars were too small to distinguish, and as such, were discarded.

Fig. 15: Preliminary results of the relative permittivity of acetone. Only one trial of data has been taken thus far. The error bars correspond to those of the pure OS-20 oil.
Fig. 16: Preliminary results of the dielectric loss of acetone. Only one trial of data has been taken thus far. The error bars correspond to those of the pure OS-20 oil.

Fig. 17: Results for the dielectric loss of acetone, published by Warlock Engineering\cite{19}, for comparison.

Fig. 18: Preliminary results of the relative permittivity of ethanol. Only one trial of data has been taken thus far. The error bars correspond to those of the pure OS-20 oil.

Fig. 19: Preliminary results of the dielectric loss of ethanol. Only one trial of data has been taken thus far. The error bars correspond to those of the pure OS-20 oil.
Fig. 20: Results for the dielectric loss of ethanol, published by Warlock Engineering\(^{[19]}\), for comparison.

Fig. 21: Preliminary results of the relative permittivity of methanol. Only one trial of data has been taken thus far. The error bars correspond to those of the pure OS-20 oil.

Fig. 22: Preliminary results of the dielectric loss of methanol. Only one trial of data has been taken thus far. The error bars correspond to those of the pure OS-20 oil.

Fig. 23: Preliminary results of the relative permittivity of chlorophyll, suspended in acetone (concentration unknown). Only one trial of data has been taken thus far. The error bars correspond to those of the pure OS-20 oil.
Fig. 24: Preliminary results of the dielectric loss of chlorophyll, suspended in acetone (concentration unknown). Only one trial of data has been taken thus far. The error bars correspond to those of the pure OS-20 oil.

Fig. 25: Preliminary results of the relative permittivity of lutein, suspended in ethanol (concentration unknown). Only one trial of data has been taken thus far. The error bars correspond to those of the pure OS-20 oil.

Fig. 26: Preliminary results of the dielectric loss of lutein, suspended in ethanol (concentration unknown). Only one trial of data has been taken thus far. The error bars correspond to those of the pure OS-20 oil.
Fig. 27: Preliminary results of the relative permittivity of lutein with magnetite nanoparticles, suspended in ethanol (concentration unknown). Only one trial of data has been taken thus far. The error bars correspond to those of the pure OS-20 oil.

Fig. 28: Preliminary results of the dielectric loss of lutein with magnetite nanoparticles, suspended in ethanol (concentration unknown). Only one trial of data has been taken thus far. The error bars correspond to those of the pure OS-20 oil.

Fig. 29: Preliminary results of the relative permittivity of lutein with magnetite and gold nanoparticles, suspended in ethanol (concentration unknown). Only one trial of data has been taken thus far. The error bars correspond to those of the pure OS-20 oil.

Fig. 30: Preliminary results of the dielectric loss of lutein with magnetite and gold nanoparticles, suspended in ethanol (concentration unknown). Only one trial of data has been taken thus far. The error bars correspond to those of the pure OS-20 oil.
E. Correction Coefficient Results

Fig. 31: Relative permittivity for acetone, with correction coefficient. Error bars refer to the standard deviation in relative permittivity values for the 3:1 OS-20 to HCl mixture.